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AN ANALYTICAL STUDY OF HYPERSONIC INLETS IN FREE-MOLECULE FLOW

Max Kinslow and M. R. Busby

ARO, Inc.

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NOTICES

AN ANALYTICAL STUDY OF HYPERSONIC INLETS IN FREE-MOLECULE FLOW

Max Kinslow and M. R. Busby
ARO, Inc.

FOREWORD

The research reported herein was conducted at the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Station, Tennessee, under Program Element 65802F.

The results of the research presented were obtained by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC. The research was conducted from August 1971 to June 1972 under ARO Project No. BE2256. The manuscript was submitted for publication on December 15, 1972.

This technical report has been reviewed and is approved.

ROBERT O. DIETZ
Director of Technology

ABSTRACT

This is a report of an analytical study of free-molecule, hypervelocity air inlet configurations which might be used to supply gas for ion engines aboard satellites. The basic equations for the inlet are developed based upon gas-surface interactions. These equations are numerically solved for various gas-surface interaction parameters. Based upon these inlets, ion rocket performance is determined for various efficiencies.

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SECTION I
INTRODUCTION

This report presents an analytical study of free-molecule hypervelocity molecular inlets for use in conjunction with an ion rocket aboard an artificial satellite. For altitudes above approximately 150 km, where the mean free path is greater than 40 m, the flow may be considered free-molecule for satellites with a characteristic dimension on the order of a few meters. Taking into consideration the effects of the interaction between an atmospheric molecule and the inlet surface, an analytic expression is obtained for the optimum inlet shape.

SECTION II
THEORETICAL ANALYSIS

The optimum inlet shape for maximum molecule collection would be one which would "focus" the molecules to a point. In a monodirectional free-molecular flow field, the optimum geometrical form of a perfectly (i. e., specular) inlet is a paraboloid, since all molecules upon reflection are directed toward the focus. However, in reality there are no specular surfaces, and thus the angle of reflection of an incident molecule depends upon the physics of the gas-surface interaction. An example of the interaction of an argon molecular beam whose incident energy is typical of orbital flight (12 eV) and a silver surface is shown in Fig. 1, Appendix I (Ref. 1). The spatial distribution is quite lobular and supraspecular. For high-speed ratios, S_w (e.g., if the vehicle has a typical velocity of 8 km/sec and surface temperature of 300°K, then $S_w \approx 20$), the relation between the local zenith angles of the incident molecular flux, θ_i, and that of the maximum reflected molecular flux, θ_m, is given by Ref. 2, as follows:

$$\theta_m = \tan^{-1}\left[\frac{\sqrt{1-\alpha_t}}{\sqrt{1-\alpha_n}}\tan\theta_i\right] \quad , \qquad S_w >> 1 \tag{1}$$

where α_t and α_n = tangential and normal energy accommodation coefficients. Using Fig. 2, the following geometrical relations may be derived:

$$\tan\theta_i = dx/dr \tag{2}$$

$$\tan\left[\theta_i + \theta_m - (\pi/2)\right] = \frac{x}{r} \tag{3}$$

and

$$\tan\left[\theta_i + \theta_m - (\pi/2)\right] = \frac{\tan\theta_i - \cot\theta_m}{1 + \tan\theta_i \cot\theta_m} = \frac{x}{r}$$

From Eq. (1),

$$\tan\theta_m = z\tan\theta_i, \text{ where } z = \frac{\sqrt{1-a_t}}{\sqrt{1-a_n}}$$

or

$$\cot\theta_m = \frac{1}{z\tan\theta_i}$$

Thus

$$\frac{\tan\theta_i - \dfrac{1}{z\tan\theta_i}}{1 + 1/z} = \frac{x}{r}$$

or

$$z\tan^2\theta_i - 1 = (1+z)\frac{x}{r}\tan\theta_i$$

Substituting $\tan\theta_i = \dfrac{dx}{dr}$ yields

$$z\left(\frac{dx}{dr}\right)^2 - (1+z)\frac{x}{r}\frac{dr}{dx} - 1 = 0$$

or

$$\left(\frac{dr}{dx}\right)^2 + (1+z)\frac{x}{r}\frac{dr}{dx} - z = 0 \tag{4}$$

Equation 4 is a first-order, second-degree differential equation which can be solved analytically by placing the equation in parametric form. The solution of an equation of this form, i.e.,

$$r(r')^2 + axr' - br = 0$$

is given by

$$x^{-2(a+b)} = \text{const}\ t^{2a}(t^2 + b)^{-2(a+b)}(t^2 + a + b)^{a + 2b}$$

$$(t^2 + b)\,r = -\,axt$$

For the case at hand, $a = 1 + z, b = -z$, and $a + b = 1$.

Thus, substituting, the solution to Eq. (4) becomes

$$x^{-2} = \text{const}\ t^{2(1+z)}(t^2 - z)^{-2}(t^2 + 1)^{1 - z}$$

$$(t^2 - z)\,r = -(1 + z)\,x\,t$$

Now, at $x = 0$, $r = r_o$, and thus the constant may be evaluated as

$$\text{const} = r_o^{-2}\, z^{-z}(z + 1)^{z-1}(1 + z)^2$$

Rearranging terms, one arrives at the parametric solution to Eq. (4),

$$x = \frac{z - t^2}{z + 1}\,\frac{r}{t}$$

$$\frac{r}{r_o} = \left(\frac{z + 1}{t^2 + 1}\right)^{\frac{1-z}{2}} \frac{z^{z/2}}{t^z}$$

For the case of $z = 1$ (i.e., a specular surface), the equations reduce to the following forms:

$$x = \frac{1}{2}(1 - t^2)\,\frac{r}{t}$$

and

$$\frac{r}{r_o} = \frac{1}{t}$$

Then

$$\frac{x}{r_o} = \frac{1}{2}\left[\left(\frac{r}{r_o}\right)^2 - 1\right]$$

which is simply the equation of a paraboloid symmetrical about the x-axis.

The parametric equations were numerically solved with the inlet shapes for various z's presented in Fig. 3. Notice that for a fixed value of x, the area, πr^2, increases as z increases. Thus, the inlet would be a more efficient molecule collector if it were constructed from a material with a large z-value for atmospheric gases. It has been shown in Ref. 2, based upon the theory of reciprocity of gas-surface interaction, that z can be a function only of the surface material and the incident gas and not of incident velocity or direction. Typical values of z determined from a theoretical analysis (Ref. 2) using experimental data are 1.23 for argon reflected from silver (Ref. 3) and 1.71 for platinum reflecting argon (Ref. 4).

These numerical results for the inlet shape will now be applied to the calculation of ion rocket performance. The net thrust, $\mathfrak{J}$, of the vehicle may be estimated for various ion engine efficiencies and inlet shapes by considering the momentum equation,

$$\mathfrak{J} = (M_2 V_2) - (M_1 V_1)$$

where V_1 is the entrance velocity, typically 8 km/sec (2.6 x 10^4 fps), V_2 is the exit ion velocity, M_1 is the mass entering inlet, and M_2 is the accelerated exiting mass. The momentum of the exiting unaccelerated particles is neglected. The ion engine efficiency, η, will be defined as that fraction of captured gas that is ionized and accelerated to velocity V_2, i.e.,

$$\eta = \frac{M_2}{M_1}$$

Therefore, the expression for the thrust becomes

$$\mathfrak{J} = M_1(\eta V_2 - V_1) = \rho_1 A_1 V_1(\eta V_2 - V_1)$$

where ρ_1 is the ambient density and A_1 is the inlet entrance area. Thus, a thrust coefficient may be defined as

$$C_{T,A_1} = \frac{\mathfrak{J}}{\rho_1 A_1 V_1^2} = \left(\eta \frac{V_2}{V_1} - 1\right) \tag{5}$$

The variation of C_{T,A_1} with η is presented in Fig. 4 for V_2 = 120 km/sec (4.2 x 10^5 fps). Also, one may define a thrust coefficient based on A_o, the inlet exit area, as follows:

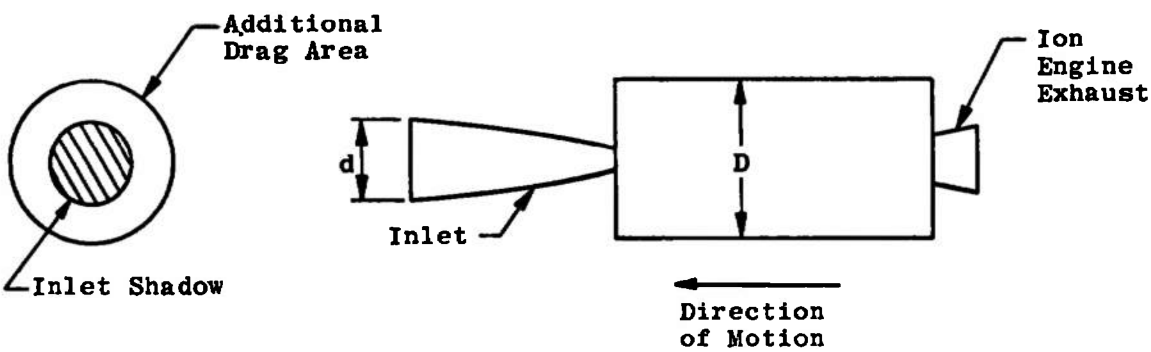
$$C_{T,A_o} = \frac{\mathcal{T}}{\rho_1 A_o V_1^2} = \left(\frac{r_1}{r_o}\right)^2 \left(\eta \frac{V_2}{V_1} - 1\right)$$

where r_1 is the inlet entrance radius and r_o is the inlet exit radius. Figure 5 presents the variation of C_{T,A_o} with z for various efficiencies, η, at an x/r_o value of 3.0 for the same values of V_2 and V_1 as given above, with r_1/r_o being determined from Fig. 3.

The fundamental purpose of an ion engine on an orbiting vehicle is to overcome atmospheric drag and provide orbit control. In order to offset drag of the inlet, it is necessary that the net thrust be zero, and from Eq. (5) it can be seen that this condition is achieved for η = 6.7 percent for the values of V_1 and V_2 previously assumed.

If the total drag of a satellite is caused by components other than the inlet, then a higher ion engine efficiency is required. In order to illustrate the results for the case of a satellite with drag other than inlet drag, the following simple example will be considered.

Assume a satellite configuration as shown below:



Let ΔD be the drag in excess of the inlet drag. The expression for ΔD is

$$\Delta D = \pi(D^2 - d^2)\rho_1 V_1^2 C_D/8 \tag{6}$$

where C_D is the drag coefficient of the satellite minus the inlet.

For the ion engine to overcome the atmospheric drag of the satellite, it is necessary that

$$\mathcal{T} = \Delta D \tag{7}$$

From Eq. (5) and the definition of A_1 it can be seen that

$$\mathcal{T} = C_{T,A_1} \ \rho_1 V_1^2 \ \pi d^2/4 \tag{8}$$

Substituting Eqs. (8) and (6) into Eq. (7) yields

$$(D^2 - d^2) C_D = 2C_{T,A_1}$$

Notice that the effect of the static density and orbital velocity has vanished. Using an estimated value for C_D of 2.11 and solving for D/d gives

$$D/d = \sqrt{0.946 \, C_{T,A_1} + 1} \tag{9}$$

Thus for the previously stated values of V_1 and V_2 and using Fig. 4, Eq. (9) becomes

$$D/d = 3.77\sqrt{\eta} \tag{10}$$

Now, if the ion rocket engine efficiency is assumed to be approximately 30 percent ($\eta = 0.3$), then Eq. (10) yields D/d = 2.06 for zero net force on the satellite.

SECTION III
CONCLUDING REMARKS

In conclusion it can be stated that, based upon Fig. 3, a material that has a larger value of $\sqrt{1 - \alpha_t} / \sqrt{1 - \alpha_\eta}$ when reflecting atmospheric gases (i.e., with high normal and low tangential accommodation) permits shorter inlets for the same inlet area. A shorter inlet is advantageous for several reasons. First, it requires less material and therefore less weight for its construction. Second, the flight path of molecules between the encounter with the inlet surface and entering into the ion engine is shorter, which will permit operations at lower altitudes before intermolecular scattering decreases efficiency. Third, a shorter inlet will increase the solid angle subtended by the inlet exit as seen from a point on the inlet surface and thus decrease "spillage" from the ion engine inlet due to the finite size of the scattering lobe (see (Fig. 1).

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APPENDIX
ILLUSTRATIONS

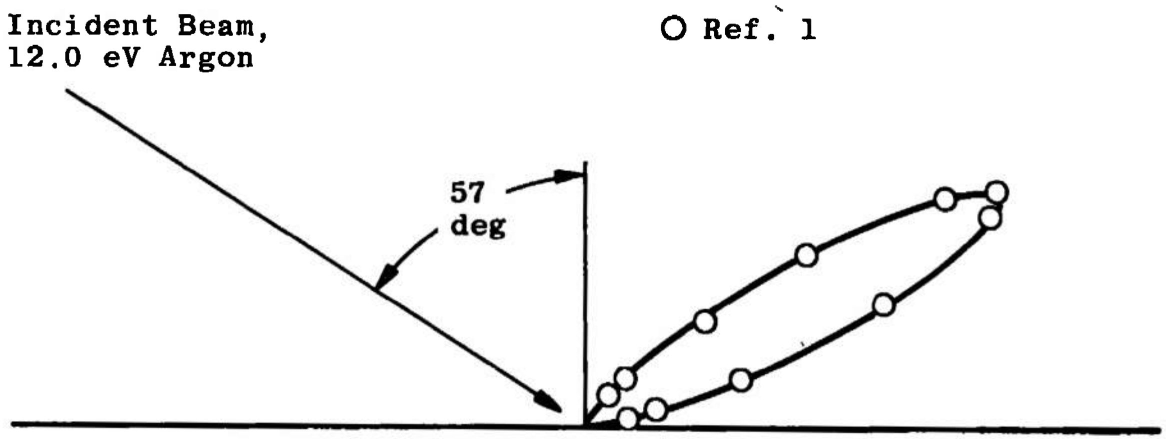


Fig. 1 Spatial Distribution of a High-Energy Argon Molecular Beam Reflected from a Silver Surface

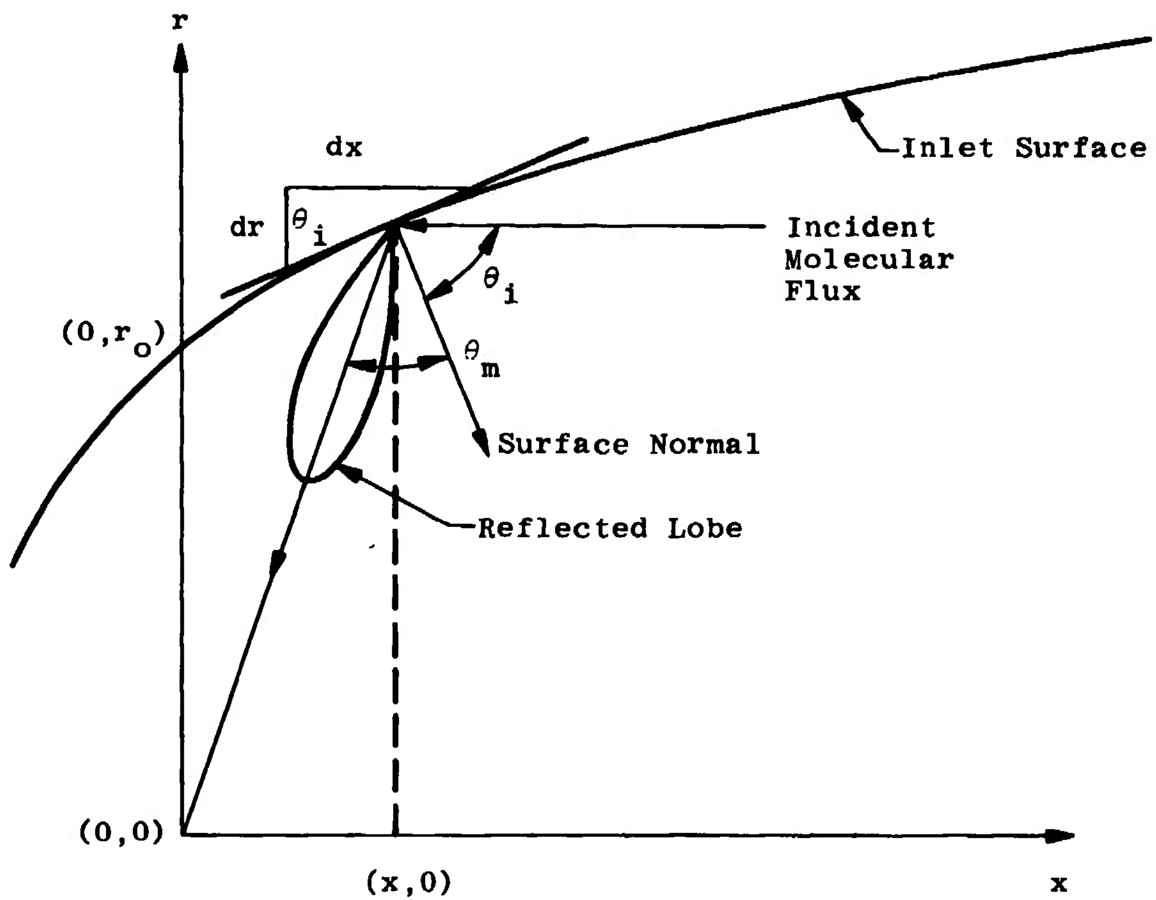
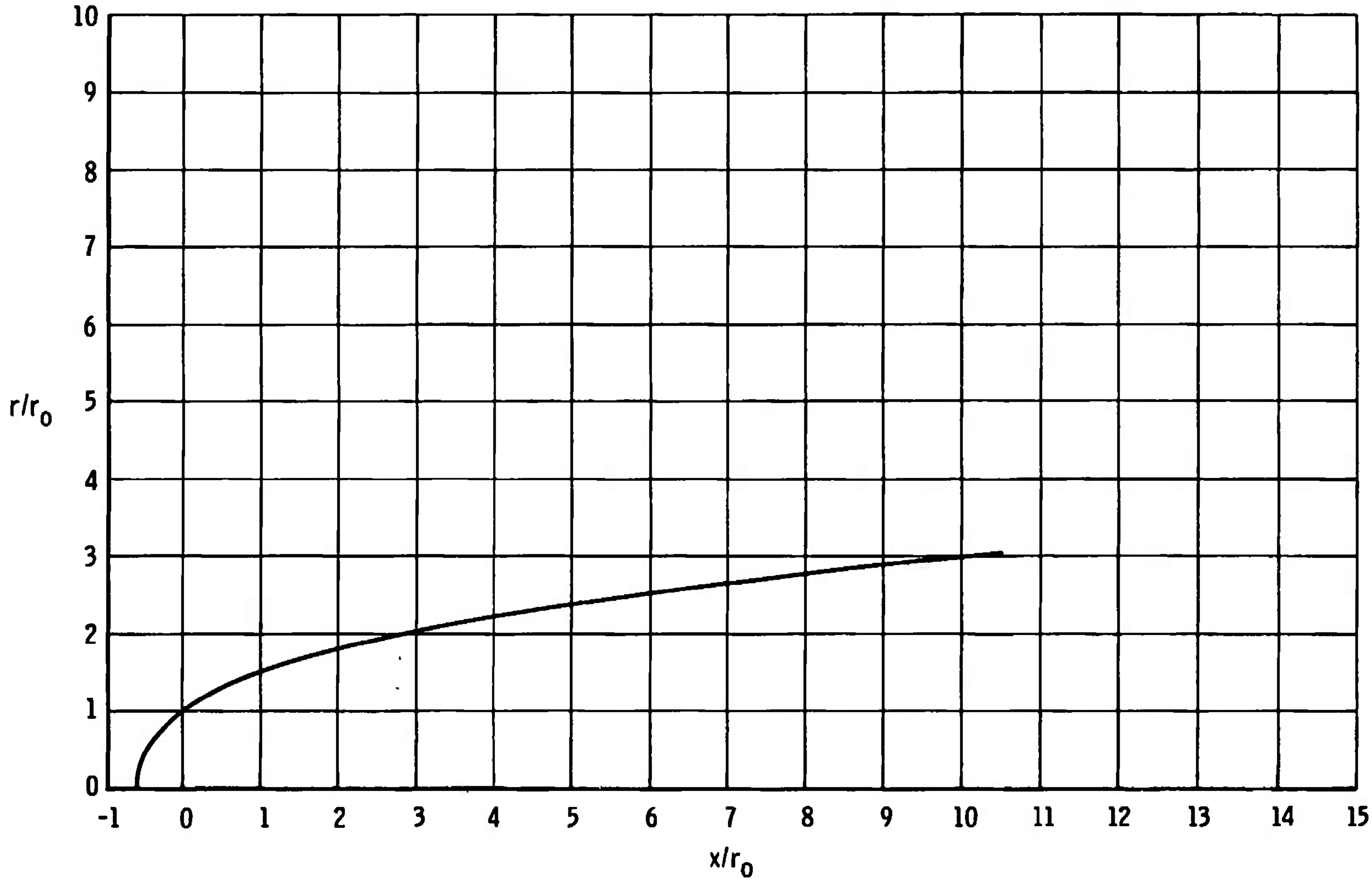
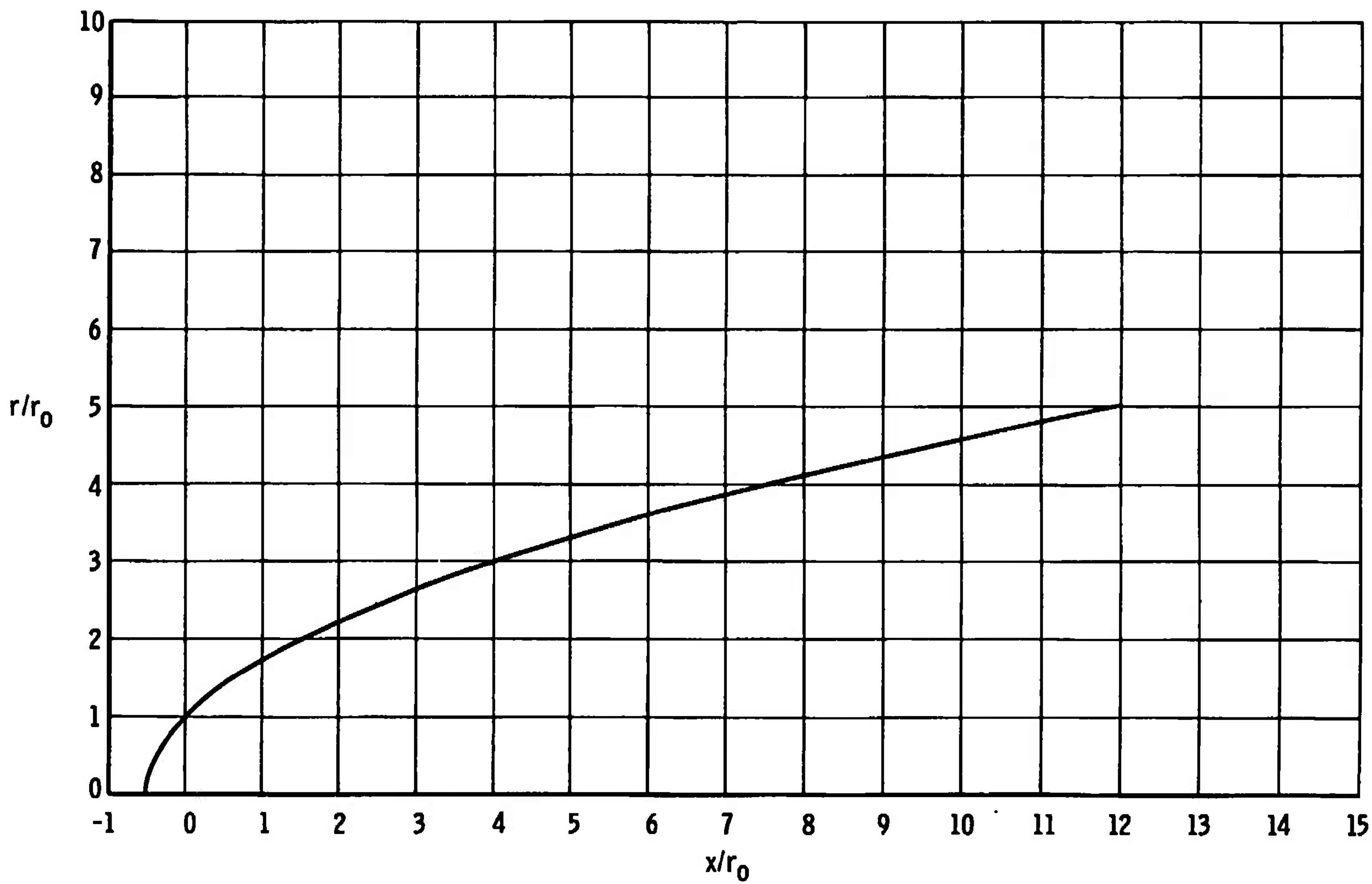


Fig. 2 Coordinate System and Inlet Geometry

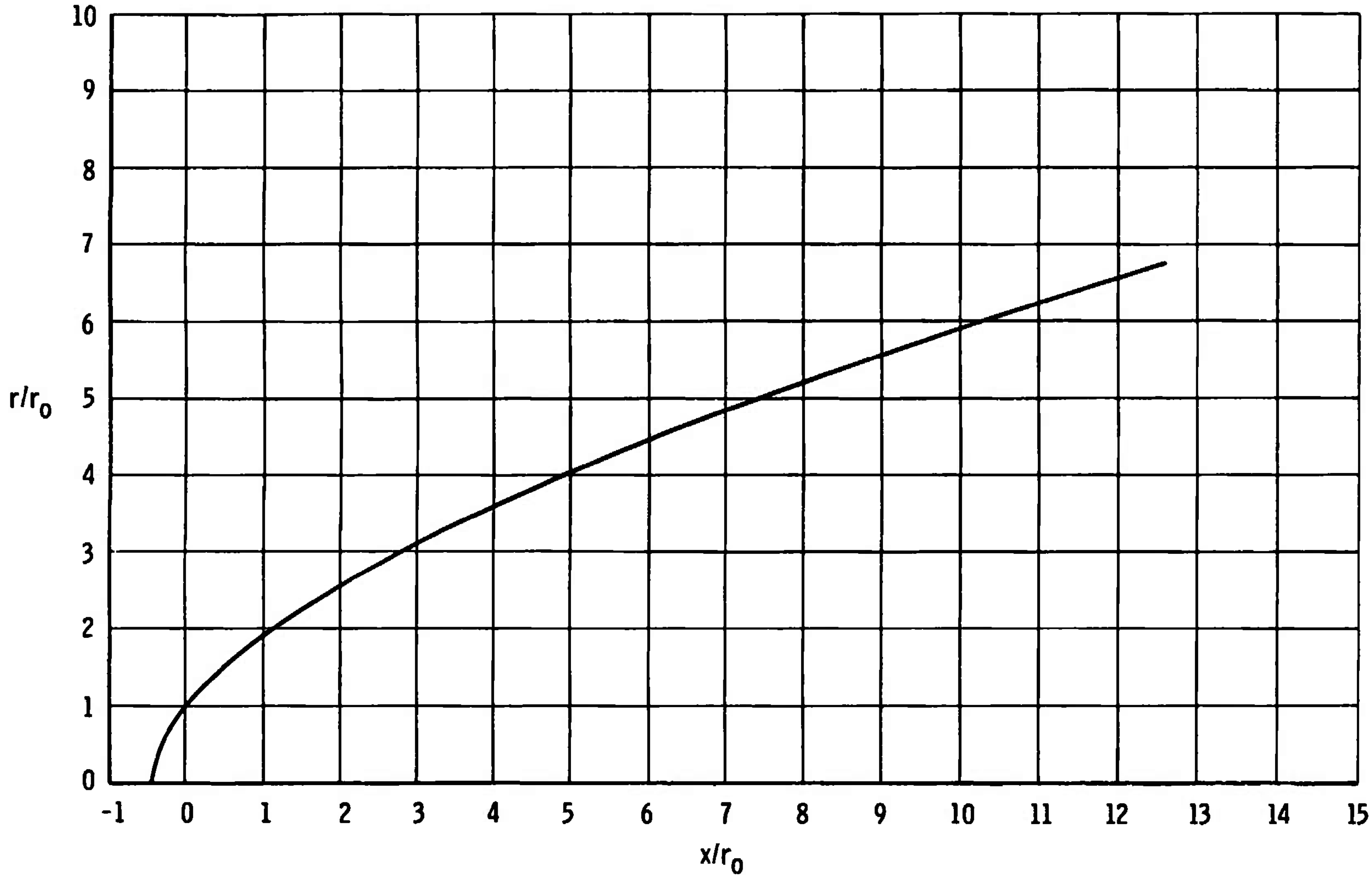


a. z = 0.50

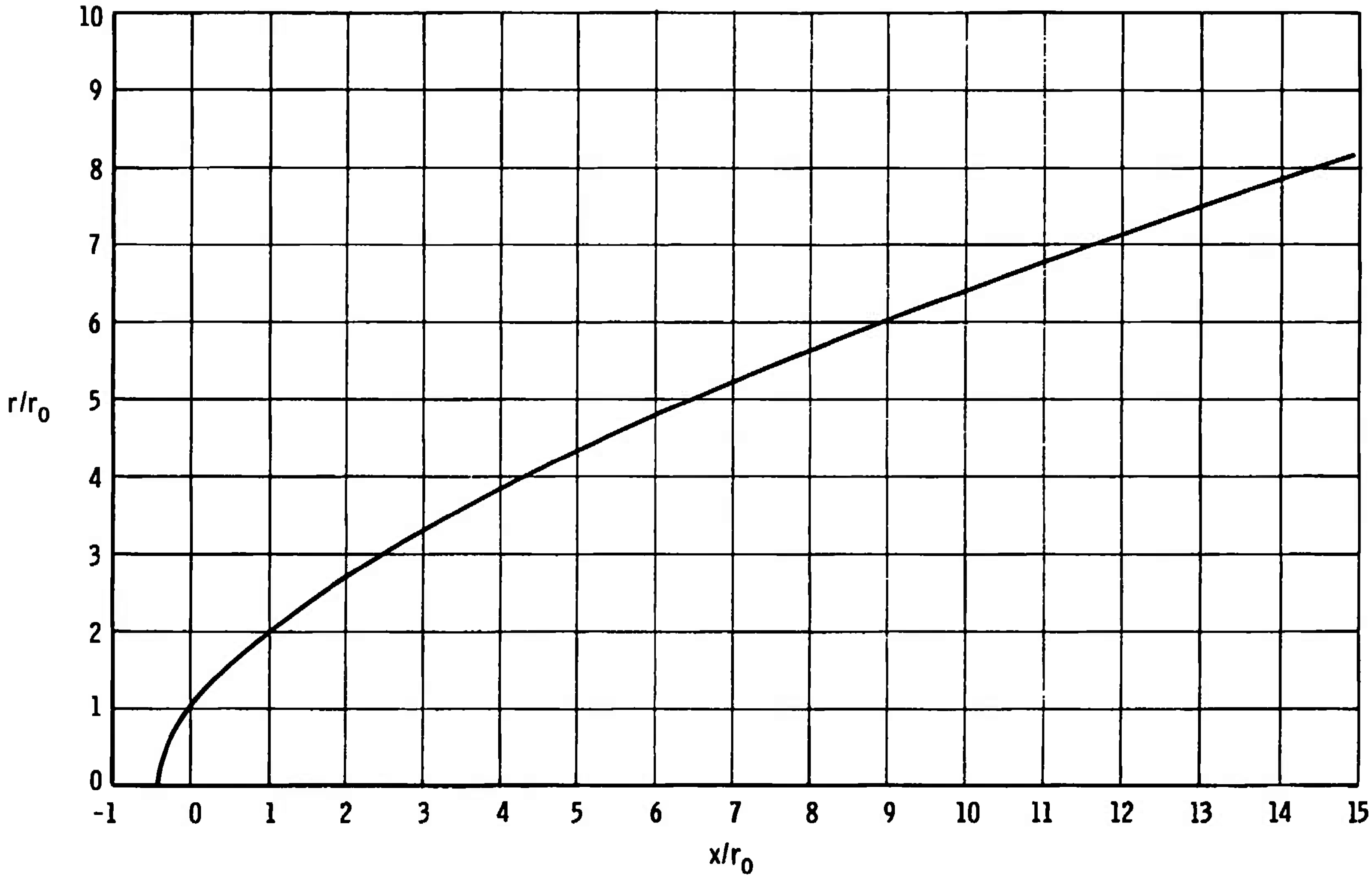
Fig. 3 Optimized Hypersonic Inlet Shapes for Various Values of z



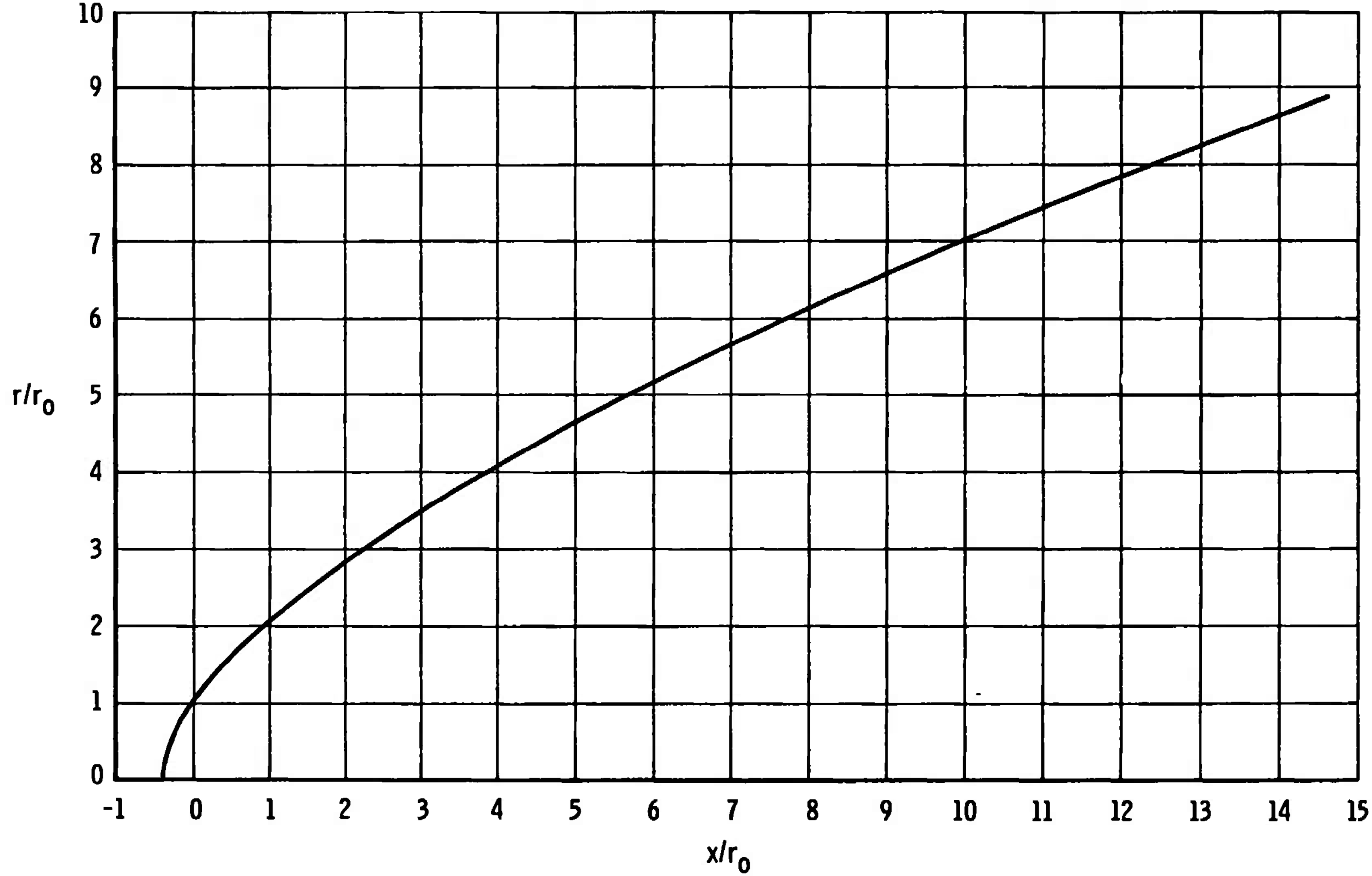
b. z = 1.00
Fig. 3 Continued



c. z = 1.50
Fig. 3 Continued

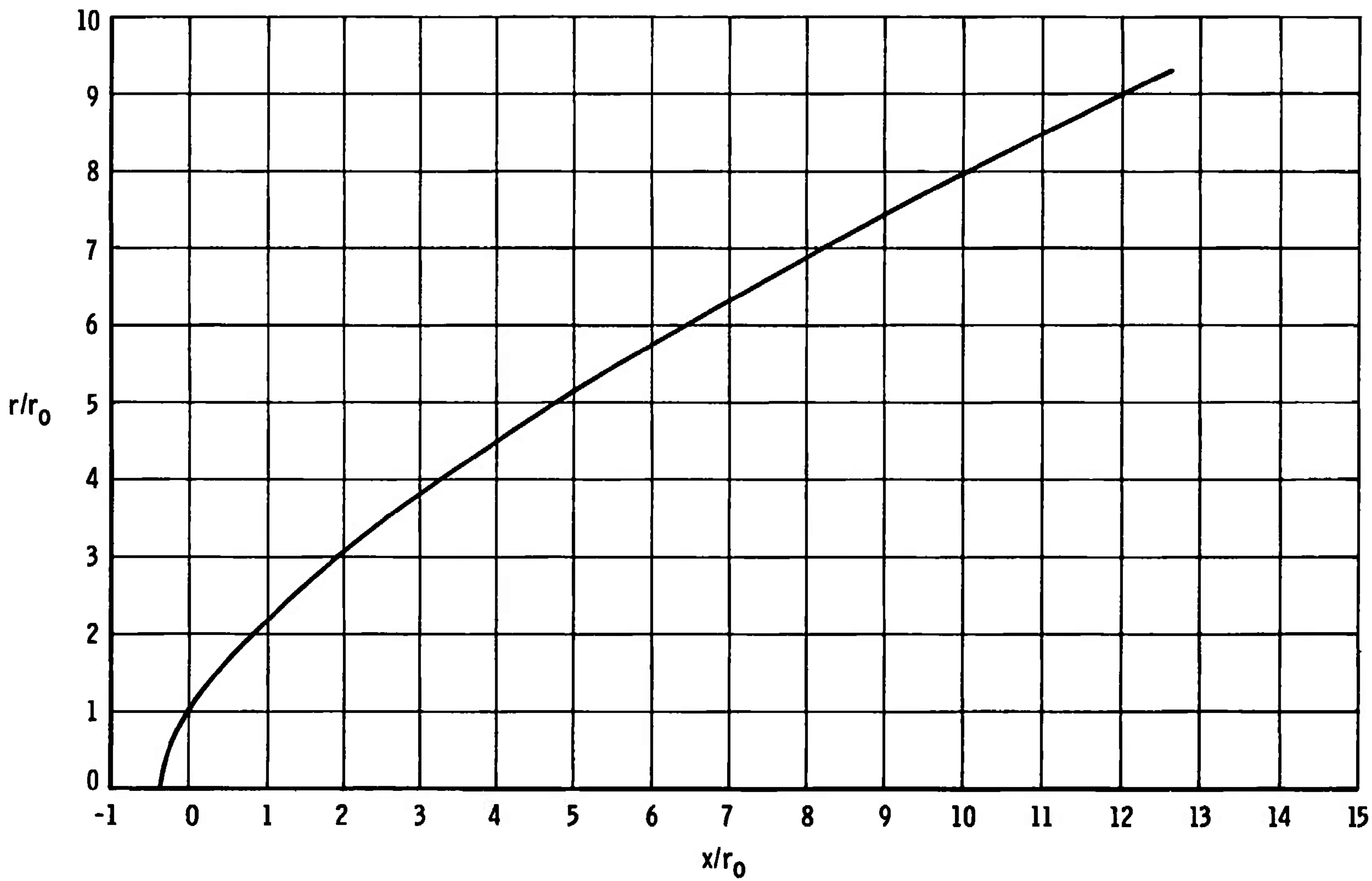


d. $z = 1.71$
Fig. 3 Continued

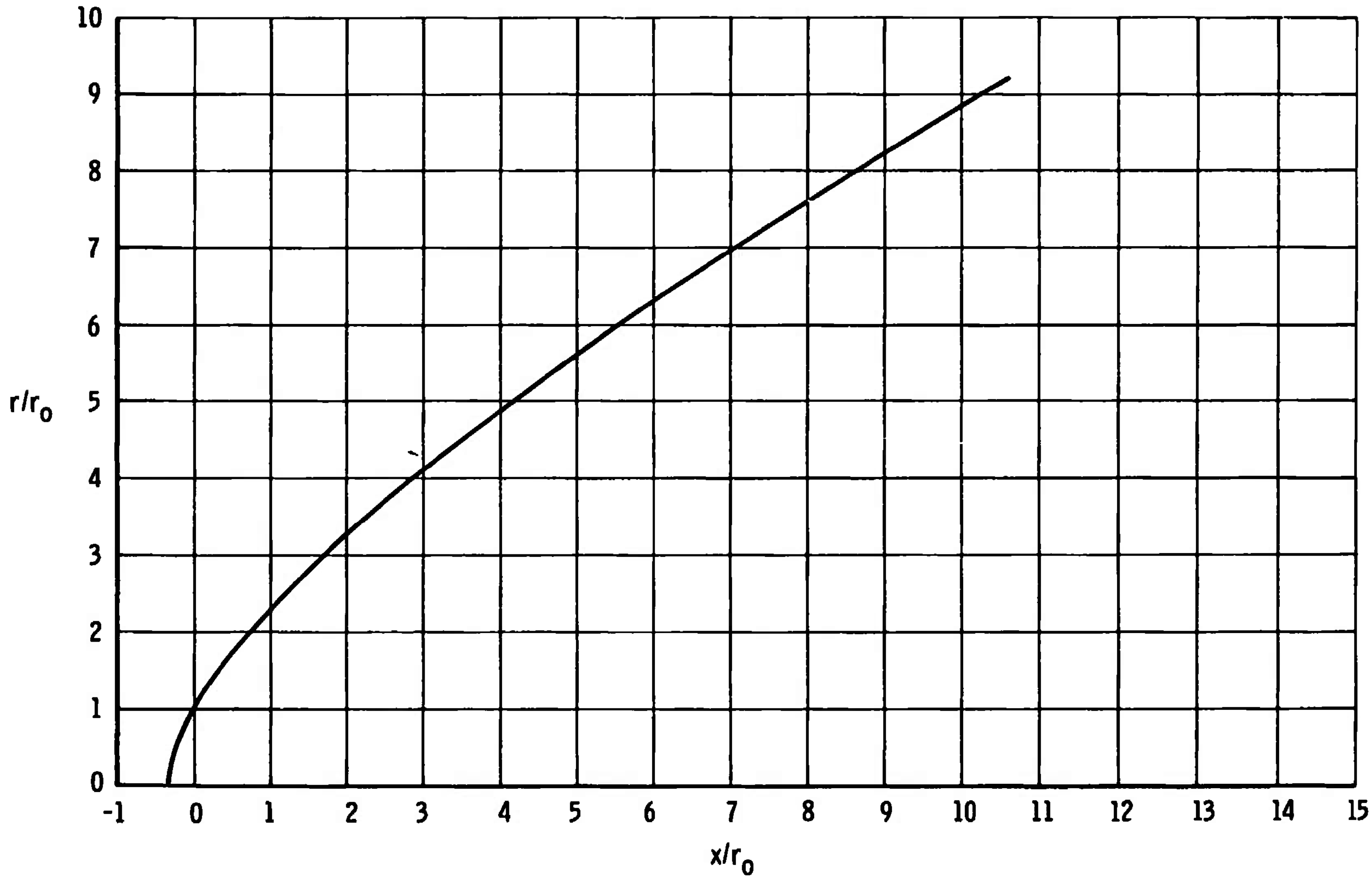


e. $z = 2.00$

Fig. 3 Continued



f. z = 2.50
Fig. 3 Continued



g. z = 3.00
Fig. 3 Concluded

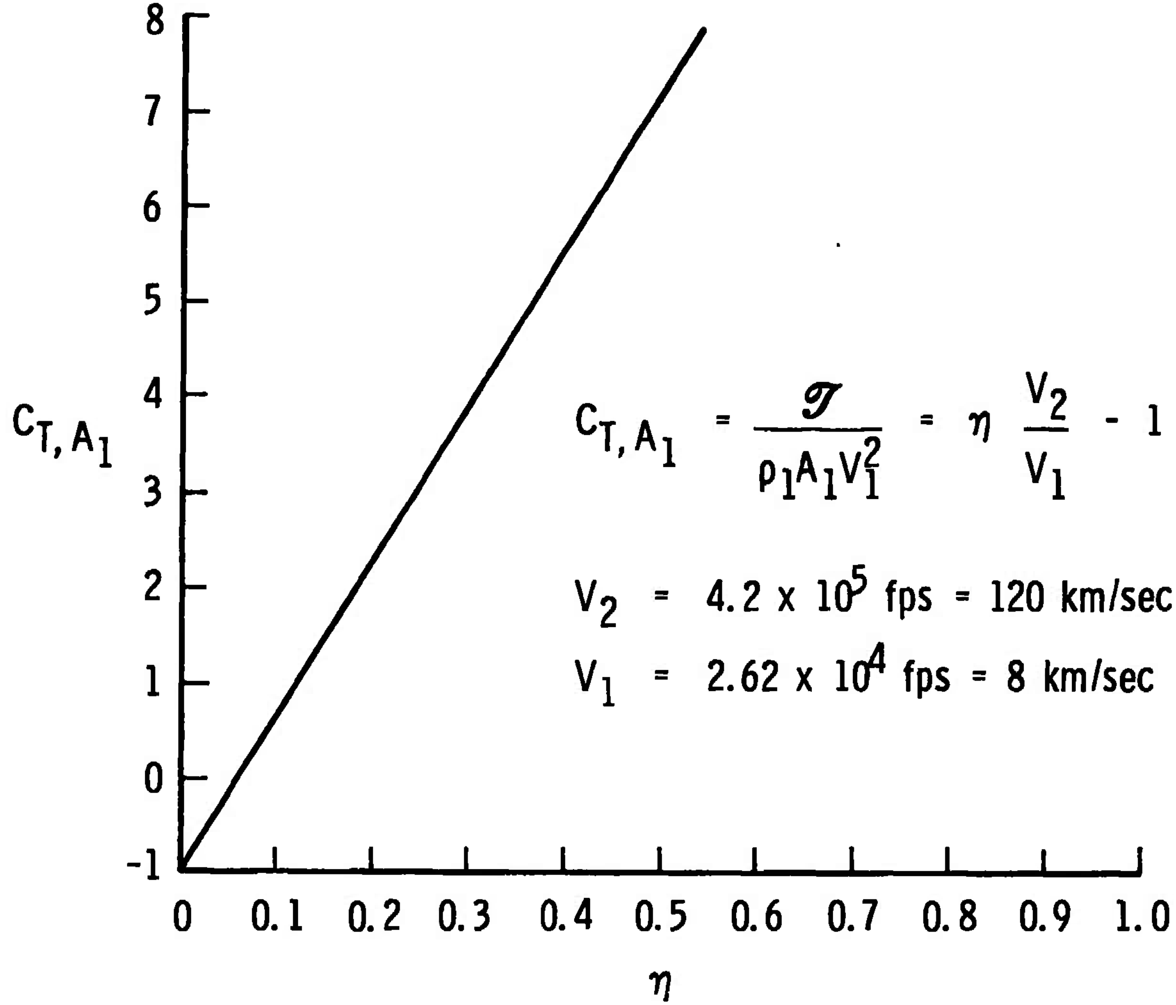


Fig. 4 Thrust Coefficient versus Ion Rocket Efficiency

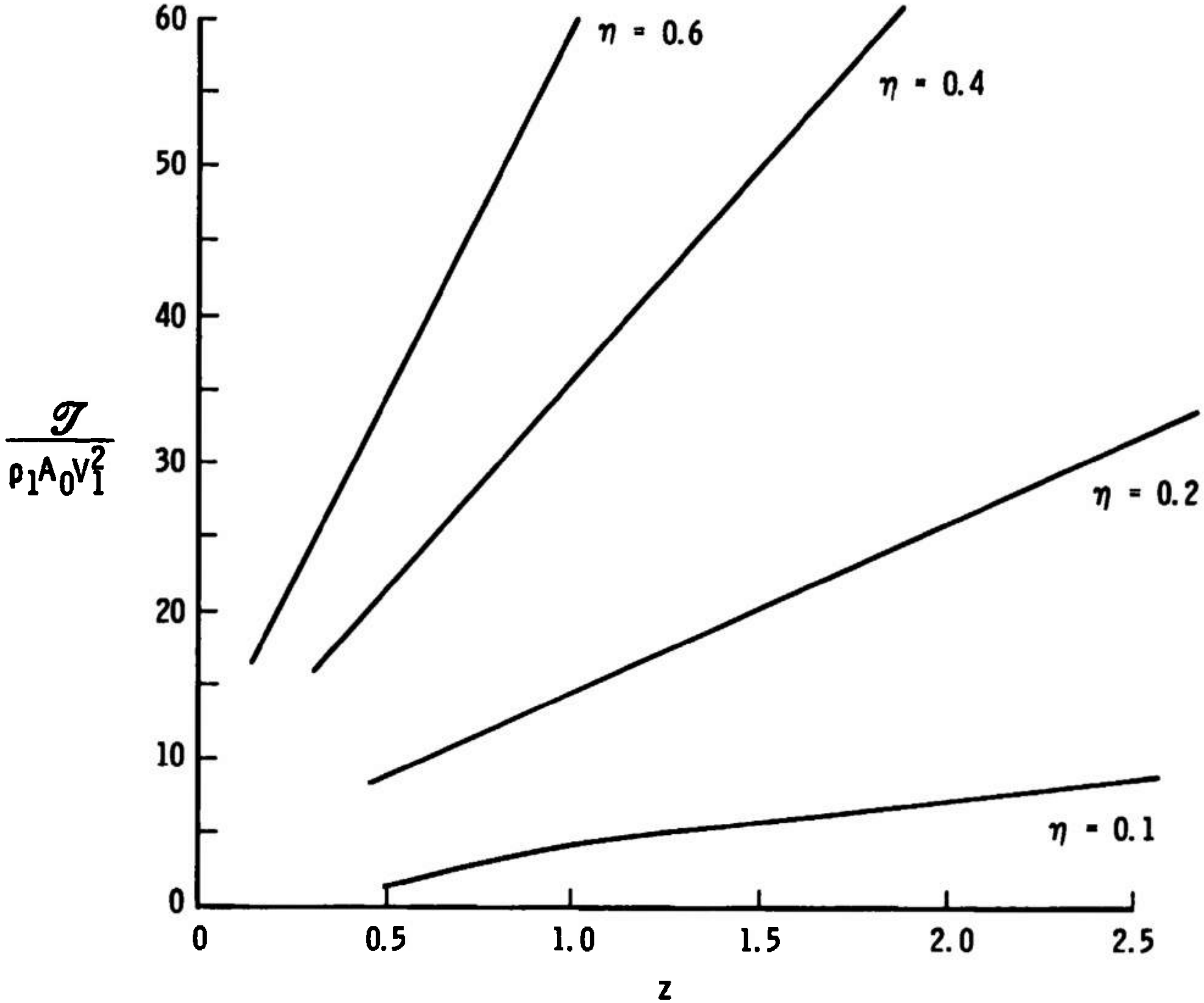


Fig. 5 Variation of Thrust Coefficient with z for Various Ion Rocket Efficiencies

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free molecular flow						
artificial satellites						

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