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INFORMATION MATERIALS ADAPTIVE SYSTEMS

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13. ABSTRACT

Includes the realization of adaptive control algorithms, elements of adaptive control systems, self-adjusting control systems for an aeroelastic aircraft, adaptive control of flight vehicle roll, flight vehicle control during atmospheric reentry, parametrically invariant control systems, statistical estimation in monitoring and control problems, choosing the transfer function of a standard model of a self-adjusting control system, the equations of motion of bodies of variable mass, the synthesis of nonsearching self-adjusting control systems, the development of a criterion for comparing adaptive control systems, the structure of a digital self-adjusting system with a model, the synthesis of a self-adjusting autopilot, and the special features of fixed-adjustment control systems.
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Scientific Council for the Comprehensive
Study of the Problem
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INFORMATION MATERIALS
ADAPTIVE SYSTEMS

Proceedings of the Anniversary Scientific
and Engineering Conference, dedicated to the
100th anniversary of V. I. Lenin's birth
Edited by Academician B. N. Petrov

MOSCOW

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INTRODUCTION

The anniversary conference, celebrating the 100th anniversary of V. I. Lenin's birth, devoted to the topic "Adaptive Systems" took place at the Moscow Aviation Institute on March 26 - 27, 1970. The conference was organized by the Chair of "Aircraft Automatic Control Systems" at the Institute.

The conference attracted a large number of specialists in the area of adaptive systems. It was attended by professors, graduate students, engineers associated with the Laboratory Chair, and representatives of a number of scientific research organizations.

The present edition contains lectures and communications that were delivered at the conference. They deal with various aspects of the cybernetics of aircraft.

Zh. S. Ageyev, B. V. Viktorov, B. V. Kirsanov, N. A. Shokolo took part in the preparation and the editing of the present volume.

INTRODUCTORY GREETING

Academician B. N. Petrov

Comrades!

Our country celebrates a momentous anniversary. One hundred years have passed from the birth of V. I. Lenin, the founder of our party, the creator of the world's first socialist nation.

V. I. Lenin made an enormous contribution to the development of Marxism, and to the development of social and natural sciences. A clear illustration of Lenin's deep foresight is included in his famous statement: "The electron is also inexhaustible like the atom, nature is infinite."*

Lenin built the foundation for the scientific management of government and social development. He devoted a great deal of attention to scientific and technological progress, and to the scientific organization of work. He emphasized that the proletariat should take advantage of the best elements in what has been created during the centuries of the development of man's civilization: " . . . Everything that has been conquered by science, technology, all the improvements, the entire knowledge of specialists, all this should serve the united proletariat."**

*V. I. Lenin, Complete works, Vol. 38, p. 26.

**V. I. Lenin, Complete works, Vol. 18, p. 275.

In an outline of the plan of scientific and technological development, written by Lenin in April of 1918, he placed a whole series of the most important tasks before the members of the Academy of Sciences.

Lenin's GOELRO*plan was born in a difficult period for the young Soviet state. It was, however, not only the first state economic plan designed for a number of years, but also the first state plan of scientific research. The plan was full of optimism. It was based on a deep belief in the primary role of electrification as a basis of technological progress and development of the national economy.

Our nation, guided by the Communist Party, has been following Lenin's path for over 50 years already. The first socialist state created a powerful industry, leading technology, and advanced agriculture. Soviet science has flourished as never before, and in a number of important fields it has taken the leading role in the world. The Soviet people made a great contribution to the progress of modern science and technology. Our nation showed man the way to the universe, and the way to a peaceful use of atomic energy. Our country is a leading power in aviation.

The development of aerospace technology in recent years has introduced a whole series of completely new problems in the theory and technology of aircraft control. The traditional methods of automatic flight and attitude control of aircraft are no longer sufficient to satisfy the increasing demands on control systems. In order to solve the control problems arising in that area, it is necessary to use ideas and methods that one can call "cybernetic" in the full sense of the word. The development of microelectronics and computers, the creation of control devices enables us to realize effective control algorithms and to create automatic control systems capable of controlling both an individual aircraft and their complicated combinations.

The wide range of flight conditions, the great range of effectiveness of control surfaces, and the increasingly complicated tasks facing aircraft, gave rise to the emergence of a new class of control systems,

*Translators' Note: This designates State Commission for Electrification of Russia.

namely adaptive systems. The latter are capable of providing the required control quality under changing flight conditions, with the parameters of the controlled objects varying within a wide range.

Such systems, as we know, include self-adjusting automatic pilots, systems of variable structure, parametrically invariant systems, and others. The theory of terminal control has progressed a great deal. The foundations of the theory of a new class of systems, systems of coordinate-parametric control, are being created. Control theory makes effective use of the methods of game theory and statistical decision making, operations research, theory of mass servicing, methods of linear and nonlinear programming, and many other theories included in modern theory of large systems.

The theory of optimal control has developed to a great extent. The idea of optimization is the fundamental and decisive concept in the design of aircraft and power plant control systems.

This conference was arranged in order to provide a comprehensive view of some aspects of the work being done in the area of the theory and methods of design of adaptive control systems, and to outline the fundamental objectives in further development of this important class of modern aircraft control systems.

It is my pleasure now to open this anniversary conference of the Chair of Automatic Control Systems associated with the Moscow S. Ordzhonikidze Aviation Institute, devoted to the theory and methods of adaptive control, and dedicated to the 100th anniversary of V. I. Lenin's birth.

IMPLEMENTATION OF ADAPTIVE CONTROL ALGORITHMS
BY MEANS OF ON-BOARD DIGITAL COMPUTERS

Alekseyev, K. B.; Teryayev, Ye. D.;
and Ukolov, I. S.

The solution of present-day control problems requires the use of digital computers in control systems. At the present time the problem can be solved if digital computers are used.

The use of computers in control systems assumes particular importance and timeliness in connection with the technical execution of adaptive (self-adjusting) control systems. In this area one is faced with the specific problems of studying the dynamics of automatic control systems that include computers, the problem of determining the basic characteristics of on-board digital computers, and their interaction with the dynamic characteristics of aircraft.

In the present report, we discuss certain problems that arise with adaptive control algorithms in automatic control systems.

An adaptive control algorithm will be defined as a changing program of operations that takes into account the actual state of the surrounding medium, the parameters of the input and output coordinates of the object, and optimizes in a certain sense the value of a given functional of the quality of the control system.

Similarly, an adaptive control system will be defined as a system that realizes an adaptive algorithm in the sense described above.

On-board digital computers offer a great deal of promise as far as their application in aircraft control systems is concerned. The effectiveness of using on-board computers in control systems depends on whether it will be possible to use radically new methods of solving the control problems, methods that would result in better performance parameters for a control system as a whole.

The use of on-board digital computers is based on the following considerations:

a. Great logic and computational capabilities, permitting the solution of diverse and complicated problems with practically unlimited accuracy;

b. Stability of operation within a wide range of working conditions;

c. High speed, permitting the simultaneous solution of several control problems;

d. High level of adaptation and self-control of the control systems.

On-board digital computers offer great possibilities to the designers of control systems when they are involved in designing complicated adaptive control algorithms. The reliability of control systems is always improved both as a result of a programmed correction of errors in the operation of certain reserved elements of the control systems, and by controlling the apparatus directly during its operation.

In the design of control systems involving on-board digital computers, we are faced with the following technical tasks:

1. Analysis of the structure of control systems for the purpose of finding those versions that possess highest reliability and are least costly;

2. Determination of rational requirements on the on-board computers;

3. Organization of efficient parallel programming of individual problems solved with the aid of an on-board computer;

4. Execution of the on-board computer programs in real time, while satisfying the requirement that different levels of reliability be achieved when solving problems of various degrees of importance (i.e., various degrees of influence on flight safety), and while satisfying the time requirements of various computer users;

5. Taking advantage of the multichannel structure of a control system and an on-board computer in order to use programming to increase the reliability of the most important control algorithms;

6. Debugging computer programs using general-purpose digital computers and simulation programs;

7. Debugging computer programs using an analog-digital (hybrid) unit.

An on-board digital computer used as a component of aircraft control systems must perform a large number of operations needed to solve the following functional problems:

1. Navigation and control of aircraft in various stages of flight;

2. Collection and processing of information about the surrounding medium;

3. Solution of auxiliary problems; control of the operating conditions of individual units, selection of optimal flight conditions, etc.

The basic feature of control systems and control based on the use of on-board computers is the possibility of flexible, adaptive and automatic changes to the control program and the possibility of making decisions based on an assessment of the flight situation.

On-board digital computers may be classified according to different basic characteristics which assign them to a given class.

One possible classification is based on the location and purpose of the on-board digital computer within the control system:

a. Command computers, performing strict control according to a preset program, and independently of the course of the controlled process;

b. Compensating computers, used in loops of strict load control;

c. Information computers, where the information about the state of a system is processed, received by man, and finally used to control the system;

d. Control computers, used in a closed control loop. This form of application is basic and most promising. Their location in the control system and their tasks may be quite diverse.

The use of on-board digital computers in control systems poses particularly great demands on such parameters of a computing system as weight, volume, reliability, required power, range of operating temperatures, radiation resistance, transmission of vibrations, etc. In addition, the on-board computers must satisfy a number of specific demands. A computerized control system should: 1) operate reliably without preventive maintenance and servicing during the entire period of effective service, i.e., during the flight of an aircraft. This means that an on-board computer, consisting of thousands of components, must have an acceptable service life (in the case of an on-board computer which cannot self-repair, it is equal to the time interval up to the first failure), 2) the range of problems solved by the computer will usually remain the same during the entire service life of the machine, 3) input and output operations must be carried out in real time, and the time intervals must be strictly specified. An on-board computer must be capable of being used for a wide range of problems, since it may become necessary to change the scale of the telemetry data, change the calculations needed to prepare the initial data, etc.

When synthesizing a computerized control system, the following parameters are given:

1. Program running time;
2. Speed, defined as the time needed to perform a short operation;
3. System of commands;
4. Bit size of numbers;
5. Storage capacity.

Program running time is determined by the maximum permissible length of the elementary time interval for the most dynamic control algorithm realized using the on-board computer.

Programs for on-board computers have a complicated structure and consist of subprograms that perform individual control tasks, and of standard subprograms that perform typical algorithms used in control and processing of numerical information; a subprogram which adjusts the special and the standard subprograms in accordance with data (obtained from the outside) on the working conditions, the time, and external control signals, and an initializing subprogram that reads the initial codes into the operative memory.

The type of on-board digital computer is determined by:

1. Complexity of algorithms;
2. Speed;
3. Accuracy required in algorithms;
4. Allowable weight and size;
5. Ease of adjustment, so that other problems can be solved;
6. Freedom from interference;
7. Possibility of controlling the control system.

The demands placed on on-board computers, as well as on their structure and the structure of the input and output units, depend on the specific problem and algorithm.

Depending on the type and purpose of an aircraft, an on-board computer may perform the following tasks:

1. Data processing;
2. Computation of controlled quantities;
3. Generation of control signals;
4. Control functions.

The tasks in this group, in particular the navigational algorithms, require a higher accuracy of calculations. In this case, the computer word length should be no less than 20 - 24 bits, and the speed — at least 5 - 8 thousand operations per second.

Control of the on-board equipment by means of a computer consists of the algorithms of automatic control and those for predicting defects that can occur while the basic problems are being solved. In order to solve the control problems, one needs increased capacity of the fixed and operative computer memories.

Certain characteristics of foreign on-board third-generation computers are given in the table on the following page.

Normally, an on-board computer should guarantee the realization of a large number of algorithms, and thus it is important to first consider those algorithms that constitute the largest computational load (n):

On-Board Computer	Purpose	Speed T^{op}/Sec	Memory (in words)	Word Length	Reliability (in hours)	Weight (in kg)	Volume (in dm^3)	Power (in W)
N	Navigation, guidance, and control	50	IS* - 4096	24	10,000	16.8	28.3	90
387	(X-15)		PS** - 32768					
	Flight control (F-111)	100 - 200	8,192 - 32,768	16	2,500	22.5	22	--
F_{opR}- S***	Data processing	200 - 400	16,384 - 131,072	16.32 64	--	34	52	--

*IS designates internal storage

**PS designates permanent storage

***S probably designates scanner

$$\eta = \frac{A}{A + L} \qquad (1)$$

where A and L are the numbers of simple arithmetical and logical operations.

Those algorithms include primarily the algorithms used in data processing and the computation of controlled quantities.

The structure of the on-board computer should be maximally adapted to the composition of the on-board systems, the problems being solved, and should be optimal in the sense defined in the design criterion. It is clear that it is necessary to have a certain number of possible structural versions.

Unfortunately, at the present time we do not know of an established procedure for solving this problem, even though there are some approaches that permit us to formalize the problem.

The increasing demands as to the minimization of energy losses incurred in orienting flight vehicles on trajectories with a long flight time (on the order of one year) force us to seek new, more economical methods of controlling the motion of the vehicle about its center of mass. One of these methods involves extensive control of the attitude, and can be realized by using a computerized control system. The method consists of selecting the moments applied to the vehicle relative to the axes attached to it in such a way that the vehicle is set in motion about the axis of equivalent rotation (the Euler axis).

As compared with existing methods, the method of extensive control enables us, other conditions being equal, to solve the basic problems of orientation with much smaller energy losses (~ by a factor of 2) and much more rapidly (~ by a factor of 2 - 3). However, extensive control involves a relatively large number of computations, which assumes that an on-board computer is used. It is characteristic that the use of an on-board computer in systems of extensive control is a necessary condition for their technical implementation. Moreover, the advisability of the use of an on-board computer is justified by significant energy gains and higher speed, which permits us to lengthen the active life of a vehicle, and improve its tactical and engineering characteristics without additional expenditures.

Let us consider the principal problems involved in the construction of computerized systems of extensive control, focussing our attention on the physical content of the problem and the computer functions resulting from it.

The formulation of the control problem is as follows. Suppose we are given the initial ($t = 0$) and the final ($t = T$) angular attitudes of a vehicle, determined by means of modified Euler angles Ψ, ν and γ, subject to a given set of constraints.

We must determine a program of the time dependence on the bounded control moments M_1, M_2, M_3 applied to the vehicle relative to the axes attached to it, which are such that the vehicle is taken from its initial attitude to the final one during:

1. a minimum time T_m,
2. a given time T with minimum energy losses.

We assume that the vehicle is an ideal rigid body, and the effect of external perturbations on the angular motion of the vehicle in the course of control is negligibly small.

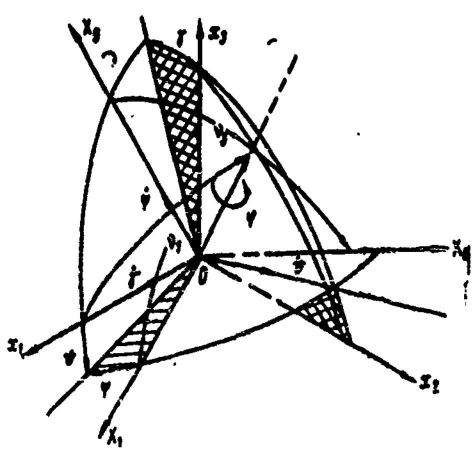


Figure 1-2.

Without giving detailed calculations, we shall only state that the problem of extensive control has an exact analytic solution. This fact is of great importance in constructing the computer functions.

Let us analyze the sequence of operations performed by the on-board computer. The geometry of the problem is shown in Figure 2. We shall give the computational operations necessary to determine the resulting angle of turn ϕ, direction of the Euler axis $l_\phi = [\nu_1, \nu_2, \nu_3]$ (in the system Ox_1, x_2, x_3 and OX_1, X_2, X_3), and the sign of the angle of turn, sgn ϕ. Using the initial measured values of φ, ϑ and γ, we determine the matrix of direction cosines $\alpha = \|\alpha_y = \|$; i, $\alpha' = 1, 2, 3$, which we then use to calculate

$$\varphi = \arccos \frac{1}{2}(\alpha_{11} + \alpha_{22} + \alpha_{33} - 1)$$

$$\nu_l = \frac{\alpha_{l+2,l+1} - \alpha_{l+1,l+2}}{2 \sin \varphi}$$

$$\operatorname{sgn} \varphi = \operatorname{sgn} \left\| \begin{matrix} a_1 & b_1 & \nu_1 \\ a_2 & b_2 & \nu_2 \\ a_3 & b_3 & \nu^4 \end{matrix} \right\|_{b = \alpha a}^{a \neq l} \frac{\varphi}{} . \qquad (2)$$

The last operation sgn ϕ involves the largest amount of labor.

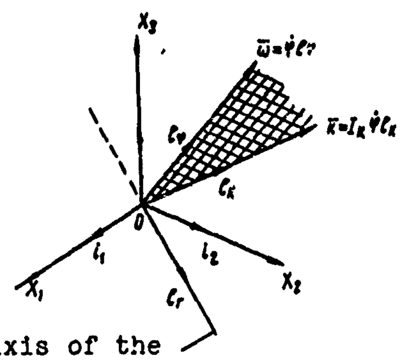


Figure 3.

Knowledge of the direction of the Euler axis gives us the direction of the kinetic moment vector and the axis of the gyroscopic moment (Figure 3).

The directions of these vectors can be found from

$$l_k = \sum_{s=1}^{3} \frac{I_s \nu_s}{I_k} i_s \tag{3}$$

If

$$I_x = \left\| \begin{matrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{matrix} \right\| \tag{4}$$

then

$$I_k = \sqrt{\sum_{s=1}^{3} I_s^2 \nu_s^2} \tag{5}$$

where

$$I_\theta = \sqrt{\sum_{s=1}^{3} (I_{s+2} - I_{s+1})^2 \nu_{s+1}^2 \nu_{s+2}^2}$$

Here i_s are the unit vectors of the attached coordinate system. We note that the unit vector of the Euler axis is $l_\varphi = \sum_{s=1}^{3} \nu_s i_s$.

Knowledge of the orientation of l_ϕ, l_k, and l_r enables us to specify the direction of $l_m = \sum \beta_m i_s$ of the axis of the resultant moment vector $\overline{M}$, and write implicit expressions (in terms of the angle 0) for its direction cosines (Figure 4).

$$\begin{gathered} \beta_n = (\widetilde{a}_n + b_n \operatorname{tg} \Theta) \cos \Theta \\ a_n = \frac{I_n \nu_n}{I_k}; \quad |\widetilde{a}_n| = a_n \\ \widetilde{b}_n = \frac{(I_{n+2} - I_{n+1})}{I_\theta} \nu_{i+1} \nu_{i+2} \quad |\widetilde{b}_n| = b_n \\ \beta_s^* = |\beta_n|_{max} \end{gathered} \tag{6}$$

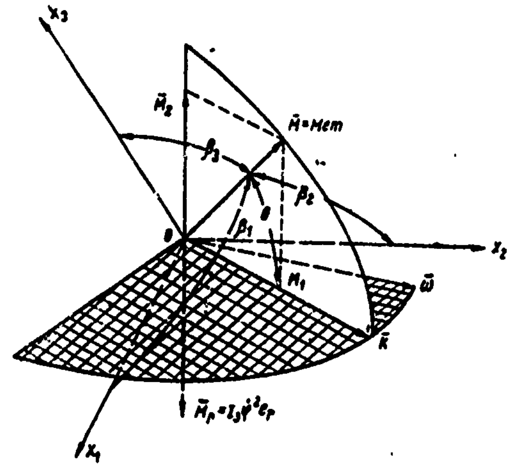


Figure 4.

Denoting the maximum allowable value of the moment applied to the vehicle along an attached axis by M_{ms} and the maximum value of a direction cosine by β_s, we find that the maximum length of the vector M is equal to

$$|\overline{M}|_{max} = \frac{M_{ms}}{\beta_s^{*}} \tag{7}$$

Then the equations of motion of the vehicle can be written using the condition that the dynamic reaction of the vehicle to the applied control moment be compensated: The equations of motion are:

$$\ddot{\varphi} = \frac{M}{I_k}\cos\Theta = \frac{M_{ms}}{I_k\,|a_s \pm b_s \operatorname{tg}\Theta|} \tag{8}$$

$$\dot{\varphi}^2 = \frac{M}{I_s}\sin\Theta = \frac{M_{ms}}{I_s}\,\frac{\operatorname{tg}\Theta}{|a_s \pm b_s \operatorname{tg}\Theta|} \tag{9}$$

The subsequent computational procedure used to synthesize the system is given without explanations.

Synthesis of the System

$$\text{a) } I = \int_0^T dt = \min \tag{10}$$

$$t_1 = \frac{1}{n}\operatorname{arc\,th}\sqrt{\operatorname{th}\frac{I_k a_s n^2 \varphi}{M_{ms}}} \text{ for 1)} \tag{11}$$

or $$t_1 = \frac{1}{n} \operatorname{arc\,th} \sqrt{\operatorname{th} \frac{I_k a_s n^2 \varphi}{M_{ms}}} \tag{12}$$

$$T_m = \frac{1}{n}\left[\operatorname{arc\,th}\sqrt{\operatorname{th}\frac{I_k a_s n^2 \varphi}{M_{ms}}} + \operatorname{arc\,th}\sqrt{\operatorname{th}\frac{I_k a_s n^2 \varphi}{M_{ms}}}\right] \text{for 2)} \tag{13}$$

For $0 \leq t \leq t_1$ and 1)

$$M_s = M_{ms} \frac{\beta_s}{\beta_s^\circ} \tag{14}$$

$$M_{s+l} = M_{ms} \frac{\tilde{a}_{s+l} + \tilde{b}_{s+l} \frac{a_s}{b_s} \cdot \operatorname{sh} nt}{a_s \operatorname{ch}^2 nt} \tag{15}$$

and 2)

$$M_s = M_{ms} = \frac{\beta_s}{\beta_s^\circ} \tag{16}$$

$$M_{s+l} = M_{ms} \frac{\tilde{a}_{s+l} + \tilde{b}_{s+l} \frac{a_s}{b_s} \sin^2 nt}{a_s \cos^2 nt} \tag{17}$$

For $t_1 \leq t \leq T$ and 2)

$$M_s = M_{ms} \frac{\beta_s}{\beta_s^\circ} \tag{18}$$

$$M_{s+l} = M_{ms} \frac{\{\tilde{a}_{s+l} + b_{s+l} \frac{a_s}{b_s} \sin^2[x(t - t_1)]\}}{a_s \cos^2[x - n(t - t_1)]} \tag{19}$$

and 2) $$M_s = M_{ms} \frac{\beta_s}{\beta_s^\circ} \tag{20}$$

$$M_{s+l} = M_{ms} \frac{\{\tilde{a}_{s+l} + b_{s+l} \operatorname{sh}^2[x - n(t - t_1)]\}}{a_s \operatorname{ch}^2[x - n(t - t_1)]} \tag{21}$$

$$l = 1,2 \quad x = \operatorname{arc\,tg\,th} nt_1 \tag{22}$$

$$\text{b) } I = \int_0^T \sum_{n=1}^{3} K_n |M_n| \, dt = \min \tag{23}$$

$$t_1 = \frac{T}{4}\left[1 - \sqrt{1 - \frac{8 I_k a_k \varphi}{T^2 M_{ms}}}\right] \tag{24}$$

$$t_2 = \frac{T}{2}\left[1 + \sqrt{1 - \frac{8 I_k a_k \varphi}{T^2 M_{ms}}}\right] \tag{25}$$

For $t_1 \leq t \leq t_1 + t_2$

$$M = \frac{M_{ms}}{b_s} \operatorname{th}^2 n t_1 \tag{26}$$

$$M_i = \frac{M_{ms}}{b_s} \operatorname{th}^2 n t_1 \cdot b_i \quad i = 1,2,3 \tag{27}$$

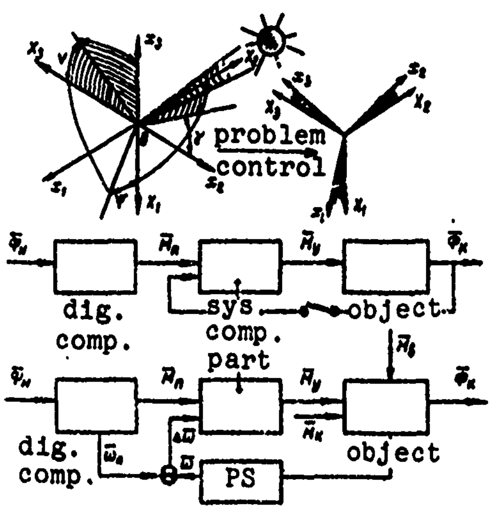
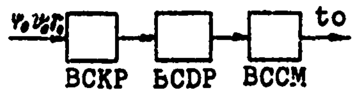


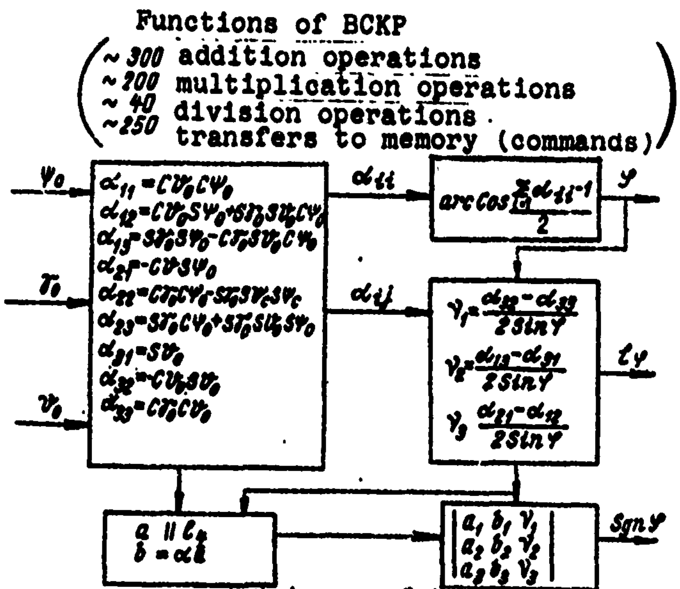
Figure 5.

The existence of the analytic solution of the problem involving the control of a spatial turn maneuver makes it possible to use an on-board computer as a command computing unit (Figure 5). Such a use of the computer excludes any feedback (rigid program). However, this does not mean that feedback coupling will occur in the controlling part of the system and when eliminating small angular deviations.



BCKP - block computing kinematic parameters of motion
BCDP - block computing the dynamic parameters of motion
BCCM - block computing the programmed values of the control moments

Figure 6. Structure of command on-board digital computer.



	+	×	÷	no. of transfers to memory
$Sin\,\psi_0(\vartheta_0\gamma_0)$	84	54	12	120
$Cos\,\psi_0(\vartheta_0\gamma_0)$	87	54	12	120
α_{ij}	4	16	–	–
$arc\,Cos\frac{\sum \alpha_{ii}-1}{2}$	33	28	5	43
ℓ_ψ	31	21	7	47
$Sgn\,\psi$	12	21	–	–

Figure 7.

When performing a turn by ϕ = 180° the effect of the external disturbances may result in an inadmissibly large error in the final attitude of the vehicle. Therefore, it becomes necessary to introduce a correction, which can be effected by maintaining a programmed angular velocity of the vehicle (Figure 6). Then the computer functions are somewhat widened, but the purpose of the computer remains the same.

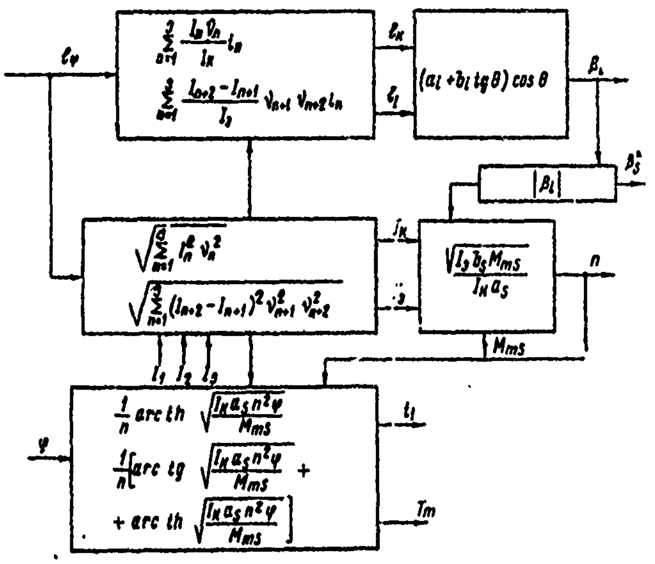


Figure 8. BCDP function.

As for its structure, an on-board computer containing arithmetic and logic units may be subdivided into three blocks which are designed to compute:

1. Kinematic parameters;
2. Dynamic parameters;
3. Programmed values of controlled moments.

A general characteristic of one of these blocks is shown in Figure 7. We can see that extensive control using simple computing devices is impossible. The functions performed by the other blocks are shown in Figures 8 and 9.

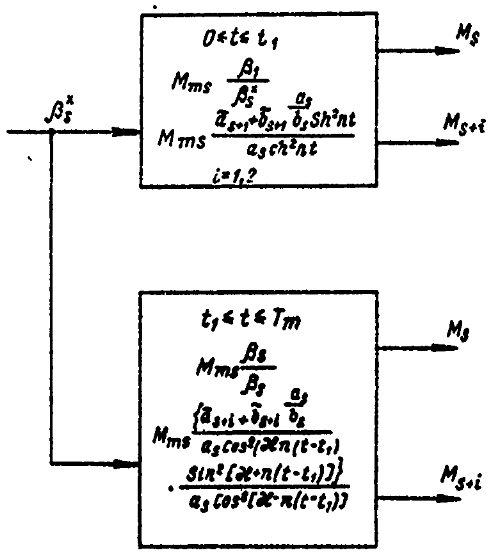


Figure 9. Functions of BCCM.

An important question arises in the control of the attitude with adaptation. This type of control occurs when the moments of inertia of a vehicle either change significantly or are to be determined in the course of the flight. A study of this problem shows that in this case it is also advisable to retain the above-mentioned function of the on-board computer that ensures optimal control. However, there appears a new computer function which involves the problem of identifying the object. It is characteristic that this function is performed by the computer in a closed control loop.

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ELEMENTS OF ADAPTIVE AUTOMATIC CONTROL SYSTEMS

Braslavskiy, D. A. and Yakubovich, A. M.

Adaptive control systems are constructed using both general- and special-purpose elements. The general-purpose elements, used to construct both the adaptive and linear control systems, are in the form of linear converters. They include data sensors (angle-angular velocity, altitude sensors, etc.) and amplifying converters (operation amplifiers, power amplifiers, differentiators, integrators, etc.). The special-purpose elements are mainly used in adaptive control systems. They include the following basic elements:

- test pulse generators,
- narrow-band filters, which select signals in a given narrow frequency band,
- multipliers,
- integro-multipliers,
- functional multipliers,
- quorum elements.

The basic requirements placed on the above elements are given in the table.

Pulse generators and narrow-band filters can be constructed using electronic amplifiers and threshold elements in combination with RC networks. Pulse generators and narrow-band filters having a higher

No.	Element	Basic Requirements
1	Pulse Generator	Frequency range 2 - 30 Hz Frequency stability 2 - 5% Amplitude stability 5 - 10%
2	Narrow-band filter	Resonance frequency 2 - 30 Hz Frequency stability 2 - 5% Amplitude stability 5 - 10%
3	Multipliers	Null level accuracy 0.2 - 0.5% Multiplication accuracy 3 - 5% 3 dB level frequency band 100 - 200 Hz
4	Integro-multiplier	Integration time constant 5 - 50 sec Null level accuracy of input 0.2 - 0.5% Multiplication accuracy 15 - 20%
5	Functional multiplier	Null level accuracy 0.2 - 0.5% Accuracy of functional conversion 5 - 10% 3 dB level frequency band 100 - 200 Hz
6	Quorum element	Accuracy of conversion 1 - 2% Threshold accuracy 5 - 10%

stability of characteristics (1 - 3% as to frequency, 3 - 6% as to amplitude) are constructed using electromechanical devices with mechanical moving systems.

The multipliers are constructed using various techniques which involve both direct and indirect transformation of signals. It should be noted that the multipliers for closed adaptive systems can be constructed more simply than others, since the requirements on the accuracy of multiplication can be much lower. Integro-multipliers which perform the operation $z(t) = y(t) \cdot \int x(t)dt$, where $y(t)$, $x(t)$ are input coordinates, are used in systems used with a standard model. The integro-multipliers may be constructed according to various principles. As an example we can mention an electrochemical integro-multiplier which is called a ministor. Low temperature stability is the ministor's basic drawback. We shall consider two important and promising elements in more detail: the functional multiplier and the quorum element.

Functional Multipliers

The functional multipliers form the product of the input coordinates $y_1 \ldots y_N$ entering through N channels and of the functions $f_1(x) \ldots f_N(x)$ of the coordinate x, so that the output coordinates of the channels, $z_1 \ldots z_N$, are equal to

$$z_1 = y_1 \cdot f_1(x), \ldots, z_N = y_N \cdot f_N(x) \quad (1)$$

The most common functions used in automatic control systems include algebraic and transcendental functions of the forms x^α, $e^{-\alpha x}$, ln x, $1 - x^\alpha$, $(1 - x)^\alpha$, where α is an integral or fractional positive number.

One of the most promising versions of the functional multiplier is the discrete dynamic version [1, 2] with analog input and output, and pulse signals used in all intermediate transformations.

A functional multiplier (Figures 1 and 2) consists of a control channel and N channels. x is the input to the control channel, and y_i is the input to the ith channel. The unit consists of the following subunits: linear dynamic network, comparator, pulse keys, delay lines, and memory capacitors. Let us analyze the operation of the unit. Suppose that in the control tract in a linear dynamic network the process $y(t) = u_0 e^{\frac{-t}{T}}$ occurs involving a discharge of the capacitor C_1 charged by the standard voltage $u_0 > x$. At a time T_n when the equality $u_0 e^{\frac{-T_n}{T}} = x$ is satisfied, the comparator sends a fixing pulse. This pulse goes through the delay line and forms the tact pulse. The tact pulse at a time τ closes the pulse switch through which C_1 is then charged by the standard voltage U_0. The process is then repeated. The interval between the tact pulse and the fixing pulse is

$$T_n = -T \ln \frac{x}{u_0} \quad (2)$$

Simultaneously with the charging of C_1 by the tact pulses through the pulse switches, the capacitors C_i in the linear dynamic networks LN_{2i} become charged (or discharged). The amplitude to which the capacitor in the i^{th} channel is charged is proportional to y_i. The transient processes $w_i(t) = y_i \cdot u_i(t)$ in the channels are periodic. Their start coincides with the start of the process $u_0 \cdot e^{\frac{-t}{T}}$ in the control channel. At a time T_n the fixing pulses connect the output capacitors C_n to $LN_{21} \ldots LN_{2N}$, through the switches $PS_1, \ldots, LS_N$, storing the signal from the output of LN_{21} in the i^{th} channel equal to

$$z_i = w_i(T_u) = y_i \cdot u_i(T_u) = y_i \cdot u_i\left(-T \ln \frac{x}{u_0}\right) \tag{3}$$

where

$$u_i\left(-T \ln \frac{x}{x_0}\right) = f_i(x) \tag{4}$$

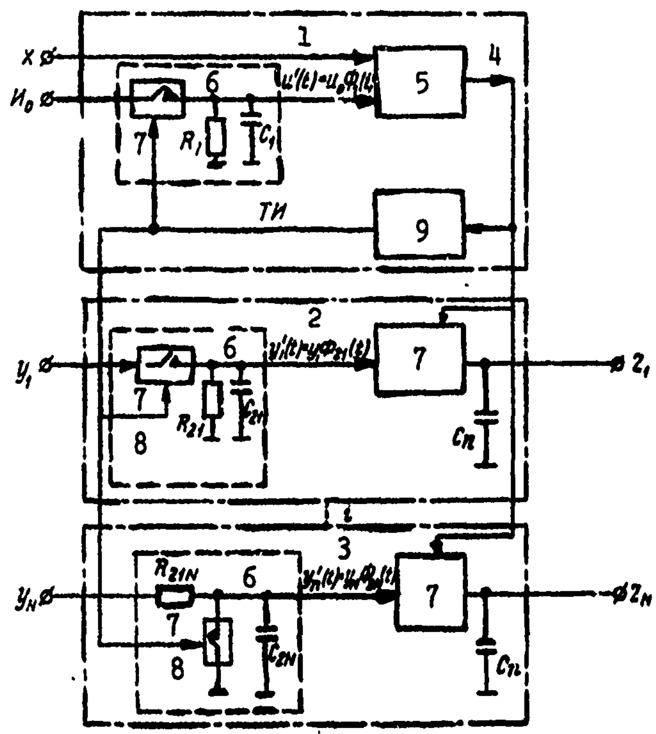
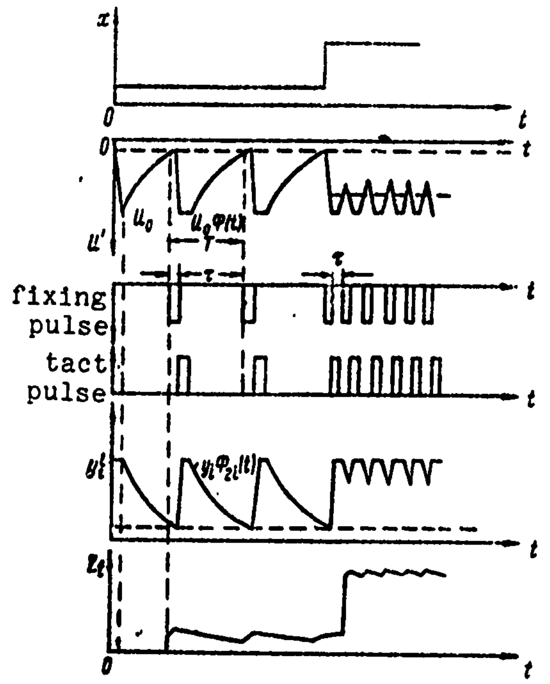


Figure 1.

1. control channel; 2. channel 1; 3. channel N;
4. fixing pulse; 5. comparator; 6. LDN*;
7. pulse switch; 8. tact pulse; 9. delay line.

Using various transient processes $u_i(t)$ in the channels, we obtain various $f_i(x)$. For example, if $u_i'(t) = e^{\frac{-t}{T}}$, then $f_i(x) = x$, and $z_i = y_i(x)$, for $u_i(t) = 1 - e^{-\frac{t}{T}}$, $f_i(x) = 1 - \frac{x}{u_0}$, $z_i = y_i\left(1 - \frac{x}{u_0}\right)$.

*LDN designates linear dynamic network



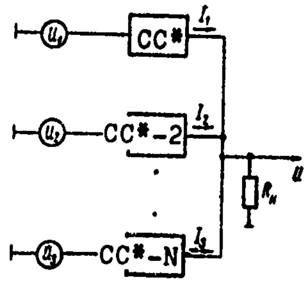


Figure 2.

Figure 3.
*CC designates current clipper.

Quorum Element

The quorum element [3] is an analog logic unit with N inputs $y_1 \ldots y_N$ (where N is odd), and one output y equals

$$y \approx \frac{y_1 + y_2 + \ldots + y_m}{m}, \tag{5}$$

for $|y - y_i| < \Delta_i$ $i = 1 \ldots m$ $|y - y_j| > \Delta_j$; $j = m+1, \ldots N$, , where $m > \frac{N}{2}$, Δ_i and Δ_j are threshold values. In the special case when $\Delta_i = \Delta_j = 0$, ,

$$y = \mathrm{med}(y_1 \ldots y_N),$$

i.e., is a median of the signals $y_1 \ldots y_N$. According to [4], Equation (5) represents an algorithm which is close to being optimal for adaptive systems. A diagram of the quorum element for N electric signals is given in Figure 3.

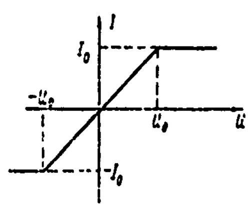


Figure 4.

The basic unit of the quorum element is in the form of a current clipper. The volt-ampere characteristic of the current clipper is shown in Figure 4. Let us analyze the operation of the device for N = 3, assuming for simplicity that the load resistance is $R_L = \infty$. Three modes of operation are possible: averaging, blocking the circuit of one of the signals, and generation of a median.

If the input voltages u_1, u_2, u_3 are such that

$$\left| u_i - \frac{u_1+u_2+u_3}{3} \right| < u_0, \quad i = 1, 2, 3,$$

then the balancing currents in the circuits, I_1, I_2, I_3, are smaller than the limiting current I_0. In this case the current clippers are linear resistances, and

$$u = \frac{u_1+u_2+u_3}{3}.$$

Let us consider the mode in which circuit 1 is blocked. Let $u_1 > u_2 > u_3$, and

$$|u_1 - u| > u_0 \qquad (6)$$

$$|u_2 - u| < u_0; \; |u_3 - u| < u_0. \qquad (7)$$

When the inequality (6) is satisfied, the current in the circuit 1 attains the maximum value I_0. This current splits into identical currents in circuits 2 and 3, equal to $\frac{I_0}{2}$. In addition to $\frac{I_0}{2}$, the channels 2 and 3 carry an additional current due to the fact that u_2 is different from u_3. If the condition (7) is satisfied, then the resultant currents in the circuits 2 and 3 will not reach the maximum value I_0. The dynamic resistance of the nonlinear current clipper CC_1 is practically infinite, and the operating currents in the elements CC_2, CC_3 remain in the region of small linear resistances. The circuit 1 is blocked. The device averages the signals u_2 and u_3, i.e.,

$$u \approx \frac{u_2+u_3}{2}.$$

The mode in which a median is generated will occur for $|u_1 - u_2| > u_0$, $|u_2 - u_1| > u_0$; $u_3 - u_1| > u_0$. Suppose that $u_1 > u_2 > u_3$. Then the balancing current I_0 is shunted through the circuits 1 and 3, and $I_1 = -I_3$. In the second circuit there is no current. Since CC_1 and CC_3 are saturated, the dynamic resistance in those circuits will be close to infinity. Since the resistances in the circuit 2 are small, $u = u_2$.

It will be noted that the quorum element has been constructed according to a generalized structural diagram given in [3]. [3] discusses circuits of quorum elements for the case when input signals are in the form of displacements and pressures.

In conclusion, we note that a quorum element is a new basic element of adaptive automatic control systems that enables one to solve several independent problems, including:

- realization of the above-mentioned majorizing function,
- construction of adaptive systems of variable structure,
- construction of low-frequency filters with small phase distortions (quorum filters).

Fundamental Problems and Prospects for the Further Development of Elements of Automatic Control Systems

The principal trends in the work in progress are:

1. The use of new physical principles enabling one to simplify elements and to increase their reliability. In particular, the use of optical-electronic and electrochemical converters in the construction of multipliers and integro-multipliers seems to offer promise.

2. Unit miniaturization of the elements of automatic control systems.

The miniaturization has two aspects:

a. A reduction of the weight of the on-board equipment in order to improve the performance parameters of flight vehicles;
b. An increase of the capacity in order to apply the method of functional redundancy to a greater extent.

The miniaturization of purely electronic elements is based on the use of integral monocrystal and hybrid-film microelectronic circuits.

The miniaturization of the electromechanical elements is in some cases based on the use of new technological processes without changing the construction of the element itself. In other cases, one has to develop new designs of the elements, and sometimes even switch to new physical principles.

3. Transition from analog control systems to discrete ones with the maximum use of universal or specialized computers.

As digital computers become more sophisticated, and as their speed and reliability increase while the size becomes smaller, it seems that analog systems will be replaced by discrete ones. With a transition to digital computers, the nomenclature of the converting elements used in automatic control systems will be greatly reduced, but in all cases we shall have to retain the data sensors and the I/O units.

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CERTAIN PROBLEMS OF THE ADAPTIVE BANK CONTROL OF A FLIGHT VEHICLE

Ageyev, Zh. C.; Viktorov, B. V. and Ukolov, I. S.

The bank control of the effective aerodynamic quality of a flight vehicle as we know presupposes a flight with a constant value of the equilibrium angle of attack which is achieved by proper centering. The control of the effective aerodynamic lift-to-drag ratio is achieved by turning the vehicle about the air velocity vector by a given bank angle. In an emergency situation it is advisable to change the bank angle of the vehicle with an angular velocity on the order of 15 1/sec.

However, a number of studies (for example, [1]), have shown that it is advisable to control flights by simultaneously changing the angle of attack and the bank angle. In such a system, a change in the angle of attack substantially modifies the effectiveness of the bank regulator. In addition, there are several well-known dynamic problems of controlled lateral motion, for example, due to the fact that the air velocity vector and the vector of the controlling moment of bank nozzles do not coincide. These circumstances may necessitate the introduction of additional (including also adaptive) structural couplings in the control system.

This article is devoted to certain aspects of this problem.

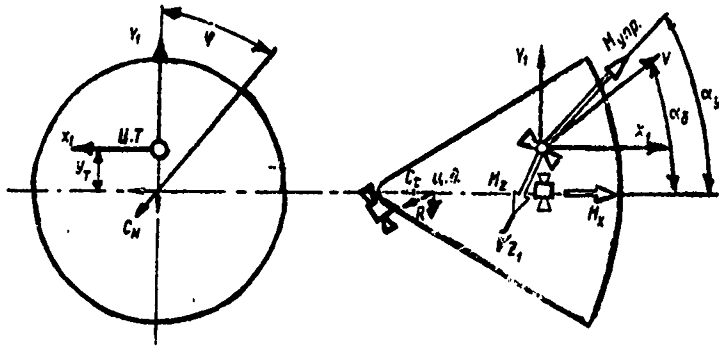

Figure 1.

Figure 1 is a diagram of an aircraft. The aerodynamics of flow over an axisymmetric body includes the peculiarity that in any motion of the aircraft relative to its center of mass, due to the symmetry of the field of aerodynamic forces, an instantaneous plane of aerodynamic symmetry appears. This plane includes the linear velocity vector $\overline{V}$, the geometric axis of symmetry, and the total aerodynamic force vector $\overline{R}$. It should be noted that the control of the attitude of the aerodynamic symmetry plane is the basic objective of controlling the aerodynamic lift-drag ratio of aircraft, by varying the bank angle.

The flight trajectory in the atmosphere of vehicles of this type is characterized by very small angular velocities of the linear velocity vector $\overline{V}$. Therefore, in the study of the angular motion of the vehicle it is convenient to use the semi-velocity system XYZ as the reference system for measurement.

Let us determine the attitude of a vehicle (or in other words, the attitude of the attached system $X_1Y_1Z_1$) relative to the following semi-velocity system of the Euler angles κ, α and ϕ (Figure 2). For this combination of angles, we clearly have the following kinematic relations

$$\dot{\varkappa} = \frac{1}{\sin\alpha}(\omega_{z_1}\sin\varphi - \omega_{y_1}\cos\varphi); \quad (1)$$

$$\dot{\varphi} = \omega_{x_1} - \operatorname{ctg}\alpha\,(\omega_{z_1}\sin\varphi - \omega_{y_1}\cos\varphi); \quad (2)$$

$$\dot{\alpha} = \omega_{z_1}\cos\varphi + \omega_{y_1}\sin\varphi. \quad (3)$$

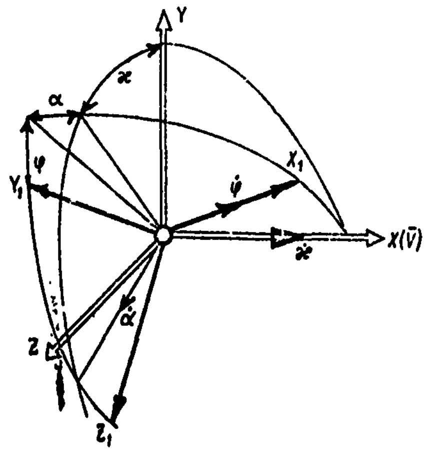

Figure 2.

In writing the dynamic equations of angular motion, we make the following assumptions:

- the inertia ellipsoid of the vehicle is sufficiently close to a sphere, i.e., it is permissible to consider the centrifugal moments of inertia and the differences of the axial moments of inertia to be of first order of smallness in the axial ones;

- the characteristics of the servo engines do not have any delay and are linear; their firing does not disturb the dynamics of flow over the vehicle;

- the direction of the linear air velocity vector and the magnitude of the ram pressure do not change in the process of a command turn;

- the vehicle is symmetric in weight and aerodynamics;

- the hypothesis of stationarity is valid;

- specific damping of the vehicle is negligibly small.

In accordance with these assumptions and the location of the servo units (see Figure 2) the approximate dynamic equations for the vehicle become

$$I_{x_1}\dot{\omega}_{x_1} = -C_n(\alpha)\cdot\overline{Y}_T\cdot qSe\sin\varphi + M_{ynp_x} + M_x \qquad (4)$$

$$I_{y_1}\dot{\omega}_{y_1} = C_n(\alpha)\,[C_d(\alpha) - \overline{X}_T]\,qSe\sin\varphi + M_{ynp_y} \qquad (5)$$

$$I_{z_1}\dot{\omega}_{z_1} = \{C_n(\alpha)\,[C_d(\alpha) - \overline{X}_T]\cos\varphi - C_\tau(\alpha)\,\overline{Y}_T\}\,qSe + M_{z_1} \qquad (6)$$

where

I_{x_1}, I_{y_1}, I_{z_1} are the moments of inertia of the fuselage in the $X_1Y_1Z_1$ system;

C_n (α) is a coefficient of the normal component of the aerodynamic force;

C_τ (α) is a coefficient of the tangential component of the aerodynamic force;

$\overline{X}_T$, $\overline{Y}_T$ are the dimensionless coordinates of the center of mass;

C_d is the dimensionless coordinate of the center of pressure;

q is the ram pressure;

l is the length of the vehicle;

S is the area of the midsection;

M_{c_x}, M_{c_y}, M_x, M_z are the components of the control moments in the $X_1Y_1Z_1$ system axis.

Equations (1 - 6) describe the total angular motion of the vehicle. An analysis of this system of equations shows that a statically stable attitude of the vehicle is characterized by $\alpha = -\alpha_{bal}$ and $\phi = 0$ for any x. This means that the bank motion of the vehicle for $\alpha = -\alpha_{bal}$ and $\phi = 0$ is similar to the rotation of a rigid body in a vacuum

$$\ddot{x} = \frac{M_{ynp}}{\sin\alpha_0 I_{y_1}} \qquad (7)$$

From the condition of ideal turn (performing obvious transformations of (1 - 6) for $\alpha = -\alpha_{bal}$ and $\phi = 0$) we can determine the magnitude of the angle of placement of jet engines needed for bank control

$$\mathrm{tg}\,\alpha_{\text{уст}} = \frac{I_{y_1}}{I_{x_1}}\cdot \mathrm{tg}\,\alpha_{\text{б}}. \tag{8}$$

The structure of the bank control unit in the simplest case should contain the self-equalizing and angular bank velocity signals, i.e.,

$$M_{\text{упр}} = k_1(\varkappa_{\text{к}} - \varkappa) - k_2\dot{\varkappa}. \tag{9}$$

However, a realization of the signal x is difficult in practice. Instead of x one normally uses a signal from the PS oriented properly relative to the fuselage. In this case, the instrument signal is

$$\omega_{\text{приб}} = \dot{\varphi}\cos\alpha_{\text{уст}} + \dot{\varkappa}\cos(\alpha_{\text{уст}} - \alpha_{\text{б}}). \tag{10}$$

The dynamic properties of the channels α and ϕ in accordance with the ideas of bank turn require the introduction of active damping

$$M_z = -k_4\omega_{z_1} \tag{11}$$

and

$$M_x = -k_3\dot{\varphi}. \tag{12}$$

It should be noted that for $\phi = 0$ from (3) it follows that

$$\omega_{z_1} = \dot{\alpha}. \tag{13}$$

The information about the rate of change of the angle ϕ may to a first approximation be obtained by differentiating the ratio of lateral loads

$$\mathrm{tg}\,\varphi = \frac{n_{z_1}}{n_{y_1}} \approx \varphi, \tag{14}$$

or the ratio of the angular readings of a pendulum suspended at the center of gravity at the axis of rotation, the collinear geometric axis of symmetry of the vehicle.

It should be noted that the effectiveness of damping in the ϕ channel (the location of the servo units is shown in Figure 2) does not depend on the value of the angle of attack.

A study of the controlled angular motion of aircraft [system of equations (1 - 6)] shows that the latter can be decomposed into "longitudinal," described by Equations (3) and (6), and "lateral," described by Equations (1 - 2) and (4 - 5).

Let us consider the lateral motion of aircraft. For this purpose, let us represent the motion in the form of a system of differential equations solved for the higher derivatives $\ddot{x}$ and $\ddot{\phi}$,*

$$\ddot{x} = A\varphi + B\{k_1(x_k - x) - k_2[\dot{\varphi}\cos\alpha_{ycr} + \dot{x}\cos(\alpha_{ycr} - \alpha_6)]\}, \tag{15}$$

$$\ddot{\varphi} = -\omega^2\varphi + C\{k_1(x_k - x) - k_2[\dot{\varphi}\cos\alpha_{ycr} + \dot{x}\cos(\alpha_{ycr} - \alpha_6)]\} - \frac{k_8}{I_{x_1}}\dot{\varphi}, \tag{16}$$

where

$$A = \frac{C_n(\alpha)[C_d(\alpha) - \bar{X}_T]qSe}{I_{y_1}\cdot\sin\alpha_6};$$

$$B = \frac{\sin\alpha_{ycr}}{I_{y_1}\cdot\sin\alpha_6};$$

$$C = \frac{\cos\alpha_{ycr}}{I_{x_1}} - \frac{\sin\alpha_{ycr}\cdot\mathrm{ctg}\,\alpha_6}{I_{y_1}};$$

$$\omega^2 = \frac{C_n(\alpha)\bar{Y}_T qSc}{I_{x_1}} + \frac{C_n(\alpha)\,\mathrm{ctg}\,\alpha_6[C_d(\alpha) - \bar{X}_T]qSe}{I_{y_1}}.$$

When condition (8) is satisfied, the ϕ motion does not depend on the bank motion since

$$C = \left(\frac{\cos\alpha_{ycr}}{I_{x_1}} - \frac{\mathrm{ctg}\,\alpha_6\cdot\sin\alpha_{ycr}}{I_{y_1}}\right) = 0.$$

Therefore when α_{bal} is constant, it is necessary in practice to make sure that the vector $\bar{M}_c$ follows $\bar{V}$, so that Equation (8) can be satisfied. Then, as we know, the effectiveness of the bank regulator remains constant.

Otherwise the motion of the channels becomes coupled in both directions. The character of the motion becomes very complicated, and the easily visualized physical considerations used in the construction of the structure of regulators and a selection of their parameters become invalid.

In addition, the peculiarities of the partial motions of the system, including the well-known effect of the bank regulator on the dynamics of the ϕ channel for $M_x = 0$, can be clearly explained by the

*System (1 - 6) is solved for $\ddot{x}$ and $\ddot{\phi}$ under the assumption that $\alpha = -\alpha_{bal}$, $\alpha \approx 0$, ϕ is small (i.e., $\sin\phi \approx \phi$ and $\cos\phi \approx 1$), and M_x, M_z, M_c are determined from (9), (11), and (12).

the-ry of differential equations with small parameters which multiply the derivatives [4]. We shall limit ourselves to the explanation of this effect, setting for simplicity $\dot{\varphi}=z$.

Considering the relatively high frequency of the eigen motion with respect to ϕ, it is permissible to assume that ω is "large." Therefore, $\mu=\frac{1}{\omega}$ is below always considered a small parameter. Let us represent Equation (16) in the form of the system: $\dot{\phi} = z$,

$$\mu\dot{z}=-\frac{1}{\mu}\varphi+\mu C\{-k_1\varkappa-k_2[\Delta\cos(\alpha_{ycr}-\alpha_0)+z\cos\alpha_{ycr}]\}, \tag{17}$$

where $\dot{\varkappa}=\Delta$.

System (17) contains a small parameter μ which multiplies the derivative of z. In addition, the system also contains a small parameter on the right-hand side of the second equation, where it enters in an irregular fashion: at $\mu = 0$ the right-hand side is discontinuous. Following the procedure used in [3], the substitution of the variables $\frac{1}{\mu}\varphi=u$ is performed, and then System (17) becomes $\mu\dot{u}=z$,

$$\mu\dot{z}=-u+\mu C\{-k_1\varkappa-k_2[\Delta\cos(\alpha_{ycr}-\alpha_0)+z\cos\alpha_{ycr}]\}. \tag{18}$$

In turn it is advisable to write Equation (15) in normal form. Thus $\dot{\chi} = \Delta$

$$\dot{\Delta}=A\mu u+B\{-k_1\varkappa-k_2[\Delta\cos(\alpha_{ycr}-\alpha_0)+z\cos\alpha_{ycr}]\}. \tag{19}$$

Let us consider the dynamics of the coupled system (18 - 19), consisting of two "fast" (18) and two "slow) (19) equations. In this we shall confine our attention to just the "fast" partial motion, generated by System (18), considering the fact that it is coupled with (19).

Let us introduce a new independent variable ("fast" time) $\tau = t/\mu$ and make the substitution in (18) and (19). We get $\frac{du}{d\tau}=z$,

$$\frac{dz}{d\tau}=-u+\mu C\{-k_1\varkappa-k_2[\Delta\cos(\alpha_{ycr}-\alpha_{0\text{зл}})+z\cdot\cos\alpha_{ycl}]\} \tag{18'}$$

and $\frac{d\varkappa}{d\tau}=\mu\Delta$,

$$\frac{d\Delta}{d\tau}-A\mu^2u+B\mu\{-k_1\varkappa-k_2[\Delta\cos(\alpha_{ycr}-\alpha_{0\text{зл}})+z\cdot\cos\alpha_{ycl}]\}. \tag{19'}$$

The report [3] gives and justifies a procedure for an approximate analysis of the "fast" phase of the solution of systems such as (18′ - 19′). With certain improvements the "slow" system (19′) is replaced by another one which can be obtained from (19′) by performing the following iterations:

1. "Slow" variables x and Δ on the right-hand side of (19′) are set equal to their initial values (here — zeroes), i.e., (19′) is replaced by $\frac{d\varkappa_1}{d\tau}=0$,

$$\frac{d\Delta_1}{d\tau}=A\mu^2 u-B\mu k_2\cdot\cos\alpha_{\text{уст}}\cdot z. \tag{20}$$

Therefore, taking into account the null initial conditions and the first equation in (18′), $\varkappa_1(\tau)=0$ and $\Delta_1(\tau)=A\mu^2\int\limits_0^\tau u(\xi)\,d\xi-B\mu k_2\cos\alpha_{\text{уст}}\cdot u(\tau)$.

2. "Slow" variables z and Δ on the right-hand side of (19′) are set equal to $\varkappa_1(\tau)$ and $\Delta_1(\tau)$, i.e., System (19′) is replaced by

$$\frac{d\varkappa_2}{d\tau}=A\mu^3\int\limits_0^\tau u(\xi)\,d\xi-B\mu^2 k_2\cos\alpha_{\text{уст}}\,u(\tau),$$

$$\begin{aligned}\frac{d\Delta_2}{d\tau}=A\mu^2 u-B\mu\cdot k_2\{\cos(\alpha_{\text{уст}}-\alpha_0)\cdot[A\mu^2\int\limits_0^\tau u(\xi)\,d\xi-\\ -B\mu k_2\cos\alpha_{\text{уст}}\,u(\tau)]\}-B\mu\cdot\cos\alpha_{\text{уст}}\cdot z.\end{aligned} \tag{21}$$

In essence we use here Picard's method of successive approximations, but only in application to the "slow" system.

Let us stop after, for example, the second iteration and neglect the integral components in (21), since they are proportional to μ^3. Then, System (19′) is replaced by

$$\begin{aligned}\frac{d\varkappa}{d\tau}&=-Bk_2\cos\alpha_{\text{уст}}\cdot\mu^2\cdot u.\\ \frac{d\Delta}{d\tau}&=A\mu^2 u+B^2k_2^2\cos(\alpha_{\text{уст}}-\alpha_{\text{бал}})\cos\alpha_{\text{уст}}\cdot\mu^2 u-\\ &\quad-Bk_2\cos\alpha_{\text{уст}}\cdot\mu\cdot z\end{aligned} \tag{22}$$

The system of equations composed of (18′) and (22) is structurally equivalent to the scheme shown in Figure 3: it describes the dynamics of an oscillating circuit enclosed by a series of feedback couplings.

With null initial conditions, the integral feedbacks reduce to a single feedback with a resultant amplification coefficient. It is important to note that the amplification coefficients for all feedback couplings are small. In this case the effect of each feedback coupling on the dynamics of the oscillating loop can be taken into account independently of one another (this follows from the theory of small parameters).

The effect of the feedback couplings I and II (Figure 3) is obvious: depending on the sign of C they respectively damp the oscillating link and increase its frequency (or conversely). Let us consider the effect of the resultant integral feedback coupling. The characteristic polynomial (upon disconnecting the loops I and II) in terms of the time τ becomes

$$(p^2+1)\,p+Cf, \tag{23}$$

where Cf is the resultant amplification coefficient of the integral couplings (small quantity).

For $f = 0$ one of the oscillating roots of the polynomial (23) p $p_1 = s$. For $f \neq 0$ the same root is $p_1 = s + \Delta p_1$. Below, f and Δp_1 are assumed to be small. Substitution of the new value of p_1 into (23) and a retention of the terms of the first order of smallness yield $\Delta p_1 = fC/2$. Thus, the oscillating root of the system moves to the left or to the right depending on the sign of C, but its imaginary part remains the same *). Therefore, from the point of view of the oscillatory partial motion, the integrating feedback coupling with the amplification coefficient "Cf" is equivalent co a differentiating feedback coupling with the amplification coefficient "-Cf."

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*) With the aid of the Rouche theorem [5], one can prove the admissibility of the calculations as the first approximation in i.

The resulting effect (damping or anti-damping) of the circuit is determined by the mutual relationship of the "weight" coefficients of the feedbacks II or III.

In particular, for the apparatus shown in Figure 2, the resulting effect for $\alpha_{pl} > \alpha_{bal}$ will be, as we know, in the form of damping (and conversely).

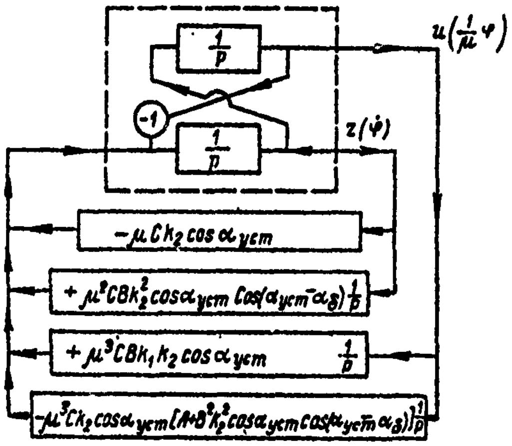

Figure 3.

It is clearly advisable (in order to retain the working capacity of the system in case of a malfunction of the damper in the "ϕ" channel) to make sure that in a steady-state motion $\alpha_{pl} - \alpha_{bal}$ is positive and equal to several degrees (up to 10°). The latter should be taken into account in the design of the tracking system used to place $\bar{M}_{pl}$ in the same direction as $\bar{V}$ for the adaptive channel of a system for controlling the lateral motion.

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CONTROL OF THE FLIGHT OF SPACECRAFT DURING ATMOSPHERIC RE-ENTRY BY MEANS OF ON-BOARD COMPUTERS

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Formulation of the Problem

During atmospheric re-entry at parabolic velocity and a flight range on the order of 7,000 - 10,000 km, the flight of a re-entering vehicle must be organized in such a way that its trajectory outside the atmosphere [1] is as shown in Figure 1. The altitude of 100 km is taken as the upper boundary of the atmosphere.

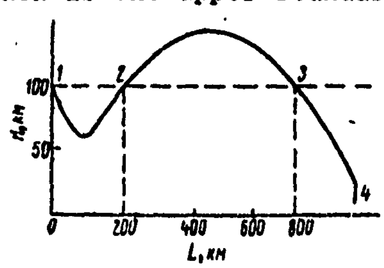

Figure 1.

The most important segment, from the point of view of maximum accelerations and its effect on the longitudinal range, is that of the first immersion. Therefore, below the control system operation will be considered only within that segment, and the accuracy of the system will be determined with respect to the total range of the first immersion and the elliptical path outside the atmosphere.

The system of the equations of motion for a re-entry vehicle in the longitudinal plane, written in the velocity coordinate system, has the following form

$$\begin{cases} \dfrac{dV}{dt} = -n_X \cdot g - g\sin\theta \\ \dfrac{d\theta}{dt} = \left(\dfrac{V}{R+H} - \dfrac{g}{V}\right)\cos\theta + \dfrac{1}{2}\rho V \dfrac{SC_X}{m} \cdot K_{эф} \\ \dfrac{dH}{dt} = V\sin\theta \\ \dfrac{dL}{dt} = V\cos\theta \end{cases} \qquad (1)$$

$$\rho = \rho_0 e^{-\beta H}, \; g = g_0\left(\frac{R}{R+H}\right)^2, \; n_X = \frac{1}{2}\rho V^2 \frac{C_X S}{mg}$$

The basic task of the control system considered in the present paper consists of bringing the re-entry vehicle to the point with a given value of the longitudinal range $L_{lз}$, irrespective of the large scatter of the initial conditions and the existing disturbances. The motion is controlled by changing the effective aerodynamic lift-to-drag ratio of the vehicle

$$K_{эф} = \frac{C_Y}{C_X}\cos\gamma \qquad (2)$$

where γ is the bank angle.

One of the possible methods of determining the free fall portion of the trajectory consists of successive refinement of the lift-to-drag ratio of the vehicle by predicting the longitudinal flight range using an on-board computer to solve System (1). Thus, for example, if we take the simplest flight program in which the lift-to-drag ratio is constant on the entire trajectory, then the iteration process used to calculate the required lift-to-drag ratio of the vehicle in the $(i + 1)^{th}$ step is done using the following relation

$$K_{эф,i+1} = K_{эф,i} - \frac{L_i - L_{з}}{\dfrac{L_i - L_{i-1}}{K_{эф,i} - K_{эф,i-1}}} \qquad (3)$$

Another method for computing the program of lift-to-drag ratio variation is proposed in [1].

When the motion is predicted by integrating System (1), it is necessary to somehow determine the actual value of the atmospheric density $\rho(H)$ and the correction

$$\xi(t) = \frac{\rho \text{ actual}}{\rho \text{ nominal}} = \frac{\rho_a}{\rho_n} \tag{4}$$

In addition, due to the strong dependence of the exit (from the atmosphere) parameters on the variation of the density ρ_a (H, L), it is necessary to extrapolate $\xi(t)$ and somehow predict its behavior over the entire remaining portion of the first immersion.

The limited possibilities of modern on-board computers require a very thorough study of the methods of solving the boundary problem, and a thorough analysis of the equations used to predict the flight with a view toward their simplification.

In order to lessen the demands on the storage capacity and speed of the on-board computer, it is advisable to construct a control system that combines the principles of flight prediction with the feedback principle. In this case during the time interval needed by the computer to calculate a new programmed motion, the object is controlled not by an open system, but instead by a feedback principle with the advantage that the motion becomes stabilized even in the presence of various perturbations. This enables us to eliminate the unpleasant situation when the parameters of motion at the end of the interval needed to calculate the control command differ significantly from the calculated values due to perturbations and using an open system. This results in an instability of the control process and in a sharp change of the controlling action in the consecutive control intervals.

Selection of Equations Describing the Process of Spacecraft Re-entry

In order to select the mathematical model which will then be used by the on-board computer to predict the re-entry process, it is desirable to simplify the equations describing the dynamics of spacecraft descent, or if possible, to use approximate analytic solutions of the equations of motion. The wide range of variation of the initial conditions and the required range, the large values of the perturbations, and the strong dependence of the elliptical range on the parameters of motion at the point of departure from the atmosphere complicate the application of approximate analytic solutions.

Let us replace the system of differential equations (1) with a simpler system (5), using the flight velocity V as the independent variable, replacing the equations for the variation of altitude with the equation for the variation of the acceleration, and assuming $g = g_{av} = \text{const}$ [2].

$$\begin{cases} \dfrac{d\theta}{dV} = \dfrac{-\left[\dfrac{V}{(R+H)g} - \dfrac{1}{V}\right]\cos\theta - \dfrac{n_x}{V}K_{s\phi}}{(n_x + \sin\theta)} \\ \dfrac{dn_x}{dV} = \dfrac{\beta n_x V \sin\theta}{g(n_x + \sin\theta)} + \dfrac{2n_x}{V} \\ \dfrac{dL}{dV} = -\dfrac{R}{R+H} \cdot \dfrac{V\cos\theta}{g(n_x + \sin\theta)} \end{cases} \tag{5}$$

The transition from System (1) to System (5) has been made using the following relation

$$n_x = \frac{1}{2}\rho_0 e^{-\beta H} V^2 \frac{C_x S}{mg} = \frac{1}{2}\rho_H V^2 \frac{C_x S}{mg}$$

$$\frac{dn_x}{dt} = 2\frac{n_x}{V} g(n_x + \sin\theta) - n_x \beta V \sin\theta, \tag{6}$$

Let us consider the effect of various variations of the density $\rho = \rho_n \cdot \xi$ on Systems (1) and (5).

For $\xi = 1$ we obtain the standard type of variation of the actual density $\rho_a = \rho_n$. Both Systems (1) and (5) are equivalent in this case. For $\xi = 0.5 = \text{const}$ or $\xi = 1.5 = \text{const}$ Equations (6) imply that perturbations of this type in no way affect the accuracy of System (5). In general for any $\xi = \text{const}$ it is only necessary to state correctly the initial conditions for System (5), since when the initial conditions on n_x are given exactly, ξ is automatically taken into account.

For perturbations of the type $\xi = e^{k(\frac{H_{current}}{100})}$, using Equation (6) we obtain

$$\frac{dn_X}{dV} = 2\frac{n_X}{V} + \frac{n_X(\beta \pm k) V \sin\theta}{g(n_X + \sin\theta)} \tag{7}$$

For k = ± 0.5 · 10^{-5} $[\frac{1}{M}]$ the relative variation of the density at the altitude H = 100 km is

$$\xi = e^{\pm 0,5} = \begin{cases} 1,65 \\ 0,6 \end{cases}; \text{and for } k = \pm 1 \ \xi = e^{\pm 1} = \begin{cases} 2,7 \\ 0,4 \end{cases}$$

i.e, the density changes more than by a factor of two in either direction from the nominal value. When the basic system (1) is used with such perturbations, the right-hand sides of System (1) change by factors 1.5 - 2.7, respectively. When System (5) is used with analogous perturbations, only one of the terms in the second equation changes by $\frac{k}{\beta} = \frac{(0,5-1).10^{-5}}{14 \cdot 10^{-5}} \approx 3,5 - 7\%$.

When the density perturbation changes according to the law assumed in [1]

$$\xi = \left\{1 + e^{\beta H}\left[A + D\sin\left(\frac{2\pi}{H_0}H + \varphi_H\right)\right]\right\}^c \tag{8}$$

the coincidence of the solutions of Systems (1) and (5) without the use of information about the character of the perturbations is somewhat disturbed. Figure 2 shows the plot of one of the coordinates (altitude) corresponding to the exact solution of System (1), assuming that we know the character of the density perturbation (solid lines), and the corresponding solution of System (5) with an additional third equation from System (1) (dotted lines), obtained without using any information about the character of perturbations. In Equation (8) we set c = ±1; ϕ_n = $\pm\frac{\pi}{2}$.

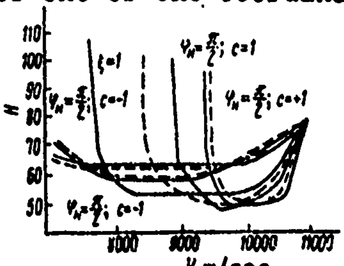

Figure 2.

Thus the use of System (5) instead of System (1) permits us in certain cases to avoid direct measurement of the flight altitude and the atmospheric density, and use only the information about the acceleration, which considerably simplifies the necessary calculations.

It should be noted that recently other simplified forms were proposed for the description of the center of mass of the spacecraft [L].

The Solution of the Boundary Value Problem

The problem of bringing a spacecraft to a given point is a boundary value problem whose solution involves great computational difficulties. In addition, in a number of cases various additional requirements are imposed on the selected control functions which complicates the solution of such problems even more.

As we know, when solving optimization problems in accordance with the classical methods of variational calculus or the method based on the Pontryagin maximum principle, it becomes necessary to solve a system of differential equations whose order is twice that of the system describing the physical process, and includes two boundary conditions.

When solving such problems numerically, one needs to supplement the incomplete boundary conditions at the initial point, then to numerically integrate the differential equations up to the final point, to see to what extent the boundary conditions at the final point thus obtained correspond to the given conditions, and finally to improve the inaccurate initial conditions. This process must be repeated until all final conditions are satisfied.

These methods are of little use if one wants to obtain numerical solutions in real time using an on-board computer.

Since the problem must be solved within a short period of time, it is desirable to use the most rapidly converging procedures. For this purpose, we use a method which is analogous to the procedure of the λ-matrix control [5, 6]. This permits us to improve the convergence of the iterative process and thus reduce the number of iterations, since when integrating the system in the forward direction one uses the information about the current deviations of the phase coordinates of the actual motion of the object from the intermediate reference trajectory.

If System (5) is written in a general form

$$\frac{dx}{dV} = f[x(V), K(V), V], \tag{9}$$

where x is the state vector of the system, then the equations of motion in variations with respect to the reference trajectory have the form

$$\frac{d\Delta x}{dV} = B(V)\Delta x + c(V)\cdot\Delta K. \tag{10}$$

Adding the conjugate system of equations to System (10)

$$\frac{d\lambda}{dV} = -B'(V)\cdot\lambda, \tag{11}$$

one can obtain the fundamental Bliss formula [3, 4], and take into account the change in the final point

$$(\lambda'\cdot\Delta x)_{V_k+\Delta V} = \int\limits_{V_k}^{V_0}(\lambda'\cdot c)(\Delta K)dV + (\lambda'\Delta x)_{V_0} \tag{12}$$

(the prime symbolizes the operation of transposition). We impose on the scalar product $(\lambda'\cdot\Delta x)$ the condition

$$(\lambda'\cdot\Delta X)_{V_k+\Delta V} = \Delta L_{13}. \tag{13}$$

Taking this condition into account, the initial conditions for System (11) are determined in the following way

$$\lambda(V_k) = \left[\frac{\partial L_{13}}{\partial x} - \frac{dL_{13}}{dV}\cdot\left(\frac{dx^*}{dV}\right)^{-1}\cdot\frac{\partial x^*}{\partial x}\right]_{V_k}, \tag{14}$$

where x^* is the phase coordinate defining the endpoint of the control interval, $\frac{\partial L_{13}}{\partial x}$ and $\frac{\partial x^*}{\partial x}$ are partial derivatives with respect to the corresponding coordinates, $\frac{dL_{13}}{dV}$ and $\frac{dx}{dV}$ are total derivatives determined by the corresponding equations of System (5).

In connection with the fact that the lift-to-drag ratio at our disposal is small, it appears advisable to choose the control functions such that the required change in the range ΔL_{13} will be achieved by minimizing the change in the control program (in the mean-square sense).

Thus we arrive at a variational problem involving a conditional extremum. We are required to determine the law of variation of the control action (ΔK) such that the following expression will be minimized

$$\int\limits_{V_0}^{V_k}(\Delta K)^2 dV \tag{15}$$

subject to the constraints (12) and (13). This type of problem involving a conditional extremum subject to constraints in an integral form is called isoperimetric.

An isoperimetric problem can be reduced to the Lagrange problem, i.e., to the minimization of $\int_{V_0}^{V_k} (\Delta K)^2 dV$ (15)

subject to the condition $\psi'_V (\Delta L_{13})'_V = (\lambda' \cdot c) \Delta K$. In order to minimize the expression in (15), we construct the intermediate function

$$H = (\Delta K)^2 + \mu(V)[\psi'_V - (\lambda' \cdot c)\Delta K] \quad (16)$$

and solve the corresponding Euler equation

$$\begin{cases} \frac{\partial H}{\partial (\Delta K)} - \frac{d}{dV} \cdot \frac{dH}{\partial (\Delta K'_V)} = 0 \\ \frac{\partial H}{\partial \psi} - \frac{d}{dV} \cdot \frac{\partial H}{\partial (\psi'_V)} = 0 \end{cases} \quad (17)$$

From Equations (17), we obtain

$$\begin{aligned} 2(\Delta K) - \mu(V)(\lambda' \cdot c) &= 0 \\ 0 - \frac{d}{dV}\mu(V) &= 0 \end{aligned} \quad (18)$$

The second equation in (18) implies that μ does not depend on V, i.e., μ = const, and from the first we obtain

$$\Delta K = \frac{1}{2}\mu(\lambda' \cdot c) = \mu_1(\lambda' \cdot c) \quad (19)$$

where μ_1 is a constant to be determined.

Using (19), (12), and (13), we obtain

$$dL_{13} = \int_{V_0}^{V_k} \mu_1 (\lambda' \cdot c)^2 dV + (\lambda' \cdot \Delta x)_{V_0} \quad (20)$$

whence

$$\mu_1 = \frac{dL_{13} - (\lambda' \cdot \Delta x)_{V_0}}{\int_{V_0}^{V_k} (\lambda' \cdot c)^2 dV} \quad (21)$$

The obtained expression (19) for the variation of the control (ΔK) has a definite physical meaning. The variation of the functional (20), when (ΔK) is selected in accordance with relation (19), always has the same sign as the sign of the variation of control ΔK.

Equation (19) gives the direction of the gradient, i.e., the direction of the maximum effect of (ΔK) on ΔL_{13}.

We introduce a function $\tilde{\lambda}(V)$ by means of the equation

$$\frac{d\tilde{\lambda}}{dV} = -(\lambda' \cdot c)^2 \tag{22}$$

for which the boundary condition is $\lambda_{V_k} = 0$.

Using (19), (21), and (22), we finally obtain

$$\Delta K(V) = -\Lambda_1(V)\left[\Delta L_{13} + (\lambda' \cdot \Delta x)_{V_0}\right] \tag{23}$$

where

$$\Lambda_1(V) = \frac{(\lambda' \cdot c)}{\lambda}$$

Equation (23) determines the control law for a closed system, which takes into account the effect of the input ΔL_{13} and the deviations of the actual motion from the reference trajectory.

The difference between this method of solving the boundary-value problem and the methods discussed in [1], is that in solving the boundary-value problem we use the information about the deviation of the actual motion from the nominal motion, and in forming the control ΔK we only use a single integration of the fundamental nonlinear system (5). Instead of a second integration of the nonlinear system describing the dynamics of a descending vehicle, we integrate the associated linear system (11) and Equation (22) in order to determine new control coefficients.

A block diagram of the control system realizing this control algorithm is shown in Figure 3.

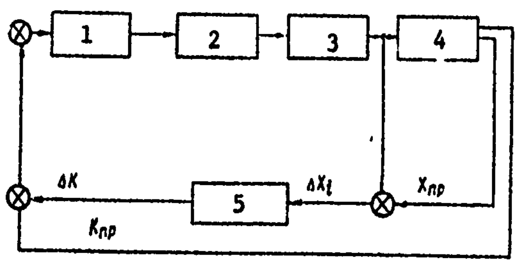

Figure 3: 1 - object; 2 - data sensors; 3 - information conversion block; 4 - prediction block; 5 - automatic pilot.

The information about the parameters of motion, obtained by means of the data sensors, is fed through the data conversion block to the parameters of motion prediction block of the spacecraft and to the automatic pilot which determine the command value of the effective aerodynamic lift-to-drag ratio.

Conclusion

The system discussed here which involves an on-board computer connected into the control loop can be applied to various objects subject to large perturbing forces and a large area of scatter of the initial conditions, where the presently known systems using a fixed nominal trajectory do not meet our requirements. In addition, such control systems may be used in such important cases as when, during the operation of the system, it is necessary to change the trajectory of motion in a direction unknown beforehand — for example, when guiding a spacecraft toward a moving point. Similar control systems can also be applied in spacecraft returning from other planets. A characteristic feature of interplanetary flights is the fact that spacecraft re-enter the atmosphere at a very large speed which is much larger than the parabolic speed. This places high requirements on the accuracy of the necessary spacecraft maneuvers in the atmosphere.

At superparabolic speeds, the operation of the control system subject to large density perturbations is greatly improved if the control system includes an on-board digital computer that can calculate in real time the maneuver required when perturbations are present.

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PARAMETRICALLY INVARIANT AUTOMATIC CONTROL SYSTEMS WITH A LINEAR PHYSICALLY REALIZABLE REGULATOR

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and Lipatov, A. V.

When synthesizing adaptive control systems, we are usually faced with the problem of maintaining a system's performance indicators constant or changing them in a desired direction with a change in the parameters of the object.

There are papers available in which the solution of the problem is attempted by using systems of variable structure, self-adjusting systems of various types, and systems with an infinitely large amplification coefficient (for example, 1, 2, 3,).

In this paper we show that it is possible to construct linear physically realizable regulators with a rigid structure and constant parameters in order to control objects whose parameters change within a wide range. It is shown that such regulators are capable of keeping the performance indicators of a dynamic system constant within any finite range of the parameters of the controlled object.

It is assumed that the object is controlled by means of some linear actuator. Suppose that the object is described by a differential equation of m^{th} order

$$x^{(m)} + \sum_{i=0}^{m-1} a_i(t)\, x^{(i)} = a_0(t) K_{ob}(t)\, \varphi, \qquad (1)$$

and that the actuator is described by a differential equation of the $(n - m)^{th}$ order

$$\varphi^{(n-m)} + \sum_{i=0}^{n-m-1} b_i \varphi^{(i)} = b_0 u, \qquad (2)$$

where x is the controlled coordinate;

ϕ is the actuator output;

u is the control;

$\alpha_i(t)$ are the variable parameters of the object, varying within finite limits;

$K_{ob}(t)$ is the variable transfer coefficient of the object

$$0 < K_{ob\,min} \leqslant K_{ob}(t) \leqslant K_{ob\,max};$$

b_i are constant parameters of the actuator.

The problem is to control the object in such a way that the performance indicators of the system will remain constant while the parameters of the object are changing.

The control must be effected using the input coordinate of the system and the signals generated by physically realizable correctors that transform the output coordinate of the system and a number of measurable intermediate coordinates of the system.

In the case of complete observability, i.e., when it is possible to use n - 1 derivatives of the output coordinate of the system, one can obtain the required dynamic properties within any range of the object parameters [1, 3, 4].

Under the actual conditions, it is not possible to obtain and use "pure" derivatives, particularly those of high order. Therefore, it is of interest to consider the possibilities inherent in linear systems with physically realizable correctors. Physically realizable correctors will be defined as correctors for which the order of the numerator of the transfer function does not exceed the order of the denominator.

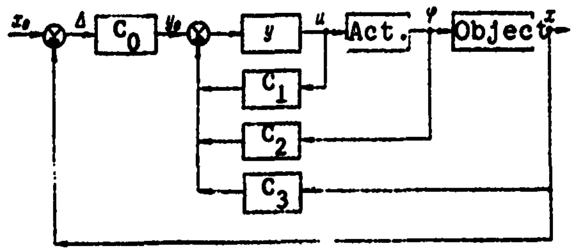

Figure 1.

Figure 1 is a diagram of the control system for which the controlled object and the actuator are described by Equations (1) and (2), and the physically realizable correctors C_0, C_1, C_2, C_3 are described by the differential equations

$$y_0^{(r)} + \sum_{i=0}^{r-1} c_i y_0^{(i)} = c_0 \Delta; \tag{3}$$

$$y_1^{(r)} + \sum_{i=0}^{r-1} c_i y_1^{(i)} = \sum_{i=0}^{r-1} \mu_i u^{(i)}; \tag{4}$$

$$y_2^{(r)} + \sum_{i=0}^{r-1} c_i y_2^{(i)} = \sum_{i=0}^{r} \lambda_i \varphi^{(i)}; \tag{5}$$

$$y_3^{(r)} + \sum_{i=0}^{r-1} c_i y_3^{(i)} = \sum_{i=0}^{r} d_i x^{(i)}. \tag{6}$$

The corrector C_0, described by Equation (3), is introduced to eliminate the derivatives on the right-hand side of the differential equation for the closed system [5]. The motion of the closed system is then described by the equation

$$F_{n+r}(p)\, x = c_0 K K_{\text{об}} a_0 b_0 x_0, \tag{7}$$

where $F_{n+r}(p)$ is a differential operator of the form

$$\begin{aligned} F_{n+r}(p) &= p^{n+r} + \sum_{j=0}^{n+r-1} A_j p^j = \\ &= p^{n+r} + \sum_{j=0}^{n+r-1} \sum_{l=\max[0,\, j-r]}^{\min[j,\, n]} \sum_{i=\max[0,\, l-n+m]}^{\min[l,\, m]} a_i b_{l-i} (c_{j-l} + K\mu_{j-l})\, p^j + \\ &+ \sum_{j=0}^{m+r} \sum_{l=\max[0,\, j-r]}^{\min[j,\, m]} K b_0 a_l \lambda_{j-l} p^j + \sum_{j=0}^{r} K K_{\text{об}} a_0 b_0 d_j p^j + K K_{\text{об}} a_0 b_0 c_0 \end{aligned} \tag{8}$$

Depending on the degree j of the operator p, the form in which the selected parameters $c_l + K\mu_l,\ \lambda_l,\ d_l$ enter into the coefficients A_j of the characteristic polynomial (8) changes. Three forms of the coefficients A_j of the characteristic polynomial are possible

a) $m+r+1 < j \leqslant n+r$

$$A_j = \sum_{i=\max\{0,\ j-r\}}^{\min\{j,\ n\}} \sum_{l=\max\{0,\ i-n+m\}}^{\min\{i,\ m\}} a_l b_{i-l}(c_{j-i} + K\mu_{j-i}) \tag{9}$$

b) $r+1 < j \leqslant m+r$

$$A_j = \sum_{i=\max\{0,\ j-r\}}^{\min\{j,\ n\}} \sum_{l=\max\{0,\ i-n+m\}}^{\min\{i,\ m\}} a_l b_{i-l}(c_{j-i} + K\mu_{j-i}) + \sum_{l=\max\{0,\ j-r\}}^{\min\{j,\ m\}} K b_0 a_l \lambda_{j-l} \tag{10}$$

c) $0 < j \leqslant r$

$$A_j = \sum_{i=\max\{0,\ j-r\}}^{\min\{j,\ n\}} \sum_{l=\max\{0,\ i-n+m\}}^{\min\{i,\ m\}} a_l b_{i-l}(c_{j-i} + K\mu_{j-i}) + \sum_{l=\max\{0,\ j-r\}}^{\min\{j,\ m\}} K b_0 a_l \lambda_{j-l} + KK_{oc} a_0 b_0 d_j. \tag{11}$$

Let us find the relationship between the smallest necessary order of the correctors and the orders of the equations describing the object and the actuator. In order to uniquely construct the coefficients A_j of the characteristic polynomial (8), it is necessary that each coefficient of the characteristic polynomial, starting with the coefficient A_{n+r-1}, contain successively a new parameter out of the set of the selected parameters c_l, λ_l, d_l. The number of the arbitrarily selected parameters c_l is equal to r. The number of the arbitrarily selected parameters λ_l is equal to (r+1), and the number of the parameters d_l is equal to (r+1). Since the parameter λ_l enters the coefficient A_{m+r} first, the value of r must be chosen such that the parameters c_l will form the coefficients of the characteristic polynomial from A_{n+r-1} to A_{m+r+1} inclusive, i.e.,

$$r = (n+r-1) - (m+r+1) + 1 \tag{12}$$

or

$$r_1 = n - m - 1.$$

Since the parameter d_l enters the coefficient A_r first, the r parameters c_l together with the (r+1) parameters λ_l must form the coefficients of the characteristic polynomial from A_{n+r-1} to A_{r+1} inclusive, i.e.,

$$r+(r+1)=(n+r-1)-(r+1)+1$$

or

$$r_2=\frac{n-2}{2}. \qquad (13)$$

We obtain two possible values of r: r_1 and r_2. The larger is taken as the required value of r. If the number is a fraction, which occurs when n is odd, then the value of r is rounded off to the nearest higher integer

$$r=\max\left[n-m-1,\ \frac{n-2}{2}\right]. \qquad (14)$$

We shall give an example of how to select the order of the correctors. Suppose that the order of the equation of the object is m = 2, and that the actuator has the order n - m = 3, (n = 5).

Then from (12)

$$r_1=3-1=2,$$

and from (13)

$$r_2=\frac{5-2}{2}=1{,}5$$

and $r=\max[r_1,\ r_2]=2.$

If the order r of the corrector is selected properly, then it becomes possible to obtain any desired performance for any possible mode of operation of the system described by Equation (7). However, due to the variation of the parameters of the object — in particular the component $K_{ob}a_0$ in the case of other operating modes — it is possible for the dynamic properties of the system to deteriorate all the way to a loss of stability.

In order to obtain a system which is operational in the entire range of the admissible operating conditions, we suggest the following approach to the formulation of the variable characteristic polynomial (8). Each of the coefficients A_j of this polynomial contains both constant terms (due to the fact that $a_m = 1$), and variable terms. By properly selecting the value of K, we can achieve a situation in which the constant terms will be larger by an order of magnitude than the sum of the variable terms in each coefficient A_j for $j > r$. Since the terms containing the component $K_{ob}a_0$ usually vary the most, it is advisable to write the characteristic polynomial (8) in an approximate form reflecting the variability of this component.

$$F_{n+r}(p) = p^{n+r} + A_{n+r-1}p^{n+r-1} + \ldots + A_{r+1}p^{r+1} + \\ + (A'_r + A''_r\gamma)p^r + \ldots + (A'_0 + A''_0\gamma) \tag{15}$$

where
$$\gamma = \frac{K_{ob}a_0}{(K_{ob}a_0)_{min}}$$

and changes from 1 to
$$\gamma_{max} = \frac{(K_{ob}a_0)_{max}}{(K_{ob}a_0)_{min}}$$

A'_i, A''_i are constants.

From the structure of the characteristic polynomial (15), we can see that a change in γ may result in a displacement of its roots to the right half-plane of the root plane even assuming that the system has a sufficient stability margin for any γ. The effect of γ on the roots of the characteristic polynomial can be controlled to suit our purposes by writing the polynomial (15) in the form of a product of two polynomials: one of order $n - 1$ and the other of order $r + 1$:

$$F_{n+r}(p) = F_{n-1}(p)F_{r+1}(p) = (p^{n-1} + B_{n-2}p^{n-2} + \cdots + B_0)\times \\ \times(p^{r+1} + C_rp^r + \cdots + C_0) \tag{16}$$

If the natural frequency of the polynomial $F_{n-1}(p)$ is greater than the natural frequency of the polynomial $F_{r+1}(p)$ by approximately one order, i.e.,

$$\sqrt[n-1]{B_0} \gg \sqrt[r+1]{C_0}, \tag{17}$$

then the polynomials $F_{n-1}(p)$ and $F_{r+1}(p)$ may be approximately expressed in terms of the coefficients of the polynomial (15) in the following way [6]:

$$F_{n-1}(p) \approx p^{n-1} + A_{n+r-1}p^{n-2} + \cdots + A_{r+2}p + A_{r+1}; \tag{18}$$

$$F_{r+1}(p) \approx \frac{1}{A_{r+1}}[A_{r+1}p^{r+1} + (A'_r + A''_r\gamma)p^r + \cdots + (A'_0 + \gamma A''_0)]. \tag{19}$$

If the condition (17) is satisfied for all γ (and this can always be achieved by properly choosing $c_i + K_{\mu i}$, λ_i, d_i for any finite range of the object parameters and any finite speed), then the performance indicators of the system are completely determined by the polynomial (19), and a_l can be selected such that the inequality $A'_i \ll A''_i$ is satisfied, and then the polynomial (19) becomes

$$F_{r+1}(p) \approx \frac{1}{A_{r+1}}[A_{r+1}p^{r+1} + A''_r\gamma p^r + \cdots + A''_0\gamma] \tag{20}$$

One can always achieve a situation in which for $\gamma = 1$ one of the roots of the polynomial (20) is so far from the imaginary axis that it will have little influence on the dynamic properties of the system. According to [6], the value of this root α is approximately $\alpha \approx -\frac{A''_r\gamma}{A_{r+1}}$, and as γ increases the root will affect the dynamic properties of the system even less. However, the remaining roots of the polynomial (20), determining the performance indicators of the system, can be obtained from the polynomial $F_r(p)$ with constant coefficients

$$F_r(p) \approx \frac{1}{A_{r+1}}[A''_r p^r + A''_{r-1}p^{r-1} + \cdots + A''_0] \tag{21}$$

If the absolute value of the increasing root α of the polynomial (20) approaches the average absolute value of the roots of the polynomial $F_{n-1}(p)$, then the group of roots with the largest modulus will be determined by a polynomial of degree n with a free term dependent on γ. As γ increases, the system may become unstable (for $n > 2$) as a result of an increase of the free term in the polynomial.

Thus, on one hand, the variable root α must be greater than the roots of the polynomial $F_r(p)$ which mainly determines the performance indicators of the system, and on the other hand, it must not approach the roots of the polynomial $F_{n-1}(p)$, i.e., thus for all γ we must have the inequalities

$$\sqrt[r]{\frac{A''_0}{A''_r}} \ll \frac{A''_r\gamma}{A_{r+1}} \ll \sqrt[n-1]{A_{r+1}} \tag{22}$$

The inequalities in (22) are a sufficient condition (but not necessary!) for the stability and constancy of the performance indicators of the system for any finite range of the parameters of the controlled object.

In practice, a similar result can be obtained with less restricting inequalities. It is not necessary to require that for any γ the right-hand inequality in (22) be satisfied. We may assume that for large values of γ the root α reach the region of the roots with the largest modulus, corresponding to the polynomial (18). As γ increases further a pair of complex conjugate roots appears of the polynomial of degree n with a free term dependent on γ, and the pair moves toward the imaginary axis of the root plane. The absolute value of this pair of roots remains quite large, and as a result these roots have hardly any effect on the performance indicators of the system, even if they move all the way to the imaginary axis in the root plane. The character of motion of the roots as γ changes is shown in Figures 2 and 3.

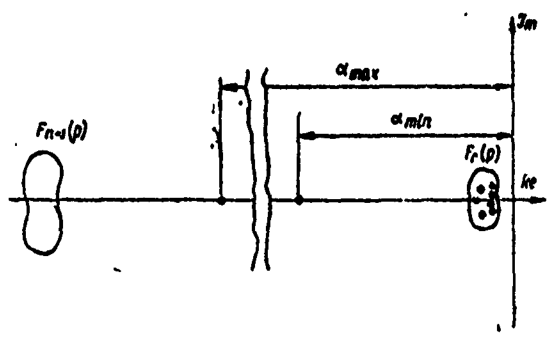

Figure 2.

The polynomial $F_{r+1}(p)$ (20) can be constructed in such a way that — as γ increases — not just one but two, three, or more roots become larger. The condition for the separation of the absolute values of the roots (17) of the polynomials ($F_{n-1}(p)$ and $F_{r+1}(p)$ then changes. For example, in the case when two roots of the polynomial $F_{r+1}(p)$ increase with an increase in γ, we write

$$\sqrt[r-1]{\frac{A_0^{*}}{A_{r-1}^{*}}} \ll \sqrt{\frac{A_{r-1}^{*}\gamma}{A_{r+1}}} \ll \sqrt[n-1]{A_{r+1}}, \tag{23}$$

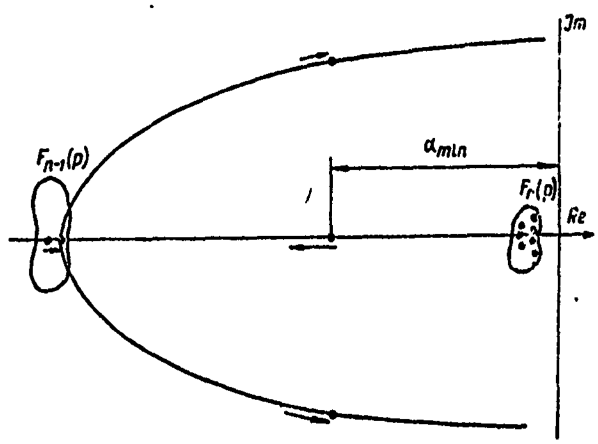

Figure 3.

which is a less restricting condition than Condition (22).

Finally, if the polynomial $F_{r+1}(p)$ is constructed in such a way that as γ increases so do all of its $(r + 1)$ roots, then the performance indicators of the system do not remain constant, but instead vary within a certain range.

Thus linear systems with a finite amplification coefficient with physically realizable correctors can in principle satisfy the requirements on the dynamic properties of a system for a finite range of the parameters of the controlled object. Moreover, the structure and the parameters of the regulator remain constant.

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APPLICATION OF "APPROXIMATE" STABILITY CRITERIA
TO THE SYNTHESIS OF ADAPTIVE SYSTEMS

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When synthesizing adaptive systems, we usually deal with a set of systems with "frozen" coefficients. Usually it is required that the entire set of systems as a whole or some of its subsets satisfy certain performance criteria. In the analysis and synthesis of systems, one has to know the relationships between the performance on one hand, and the variable parameters of the object or parameters of the regulator (constant or variable), on the other. In particular, it may become necessary to establish such relations between the varying parameters of the object and the varied (or constant) parameters of the regulator such that the performance remains constant or changes according to a certain law.

In this paper, we attempt to supply the means for establishing this type of relationship for one of the partial performance criteria of a system — namely, stability. The application of existing stability criteria to establish such relationships is possible, but in practice this is impossible for systems of high order.

There is a need for approximate but simpler stability conditions which could eliminate this drawback. The approximate stability conditions will be defined as either only necessary conditions for stability or only sufficient conditions. Geometrically this corresponds to replacement of the stability region in a k-dimensional space of "k" parameters that are of interest to us, and for which the equation of its boundary is too complicated, with another region inscribed into the exact stability region, or described around it, for which the equation of the boundary has a simpler form.

One of such "approximate" simple stability conditions involves the necessary condition requiring that all the coefficients of the characteristic polynomial of a system be positive. This condition also specifies a certain region in parameter space, but it differs too much from the exact stability region for systems of order higher than two. This makes it impossible to use this condition by itself.

Let us formulate two theorems about the necessary conditions of stability.

Theorem 1. In order that the polynomial

$$F(z) = a_0 z^n + a_1 z^{n-1} + \ldots + a_n;\ a_i > 0 \tag{1}$$

have all roots lying in the left half-plane, it is necessary that the following inequalities be satisfied

$$\frac{a_i a_{i+1}}{a_{i-1} a_{i+2}} > 1;\ i = 1, 2, \ldots, n-2. \tag{2}$$

Theorem 2. In order that the polynomial (1) have all roots lying in the left half-plane, it is necessary that the following inequalities be satisfied

$$\frac{a_i^2}{a_{i-1} a_{i+1}} > C_i;\ i = 2, 3, \ldots, n-2, \tag{3}$$

where

$$C_i = \frac{\left(n - i + \frac{(-1)^{n+i} + 3}{2}\right)\left(i + \frac{(-1)^i + 3}{2}\right)}{\left(n - i + \frac{(-1)^{n+i} - 1}{2}\right)\left(i + \frac{(-1)^i - 1}{2}\right)}. \tag{4}$$

The values of C_i for each n and i can be calculated and listed in a table.

Both conditions involve a certain parameter λ_i, defined as

$$\lambda_i = \frac{a_i a_{i+1}}{a_{i-1} a_{i+2}}. \tag{5}$$

In terms of this parameter conditions (2) and (3) can be rewritten as:

$$\lambda_i > 1;\ i = 1, 2, \ldots, n-2 \tag{6}$$

and

$$\lambda_i \lambda_{i+1} > C_i;\ i = 2, 3, \ldots, n-2. \tag{7}$$

Each characteristic polynomial of order "n" is characterized by n - 2 parameters λ. The number of the parameters, λ_i, is smaller by one than the number of the coefficients determining the characteristic polynomial in a normalized form. Nevertheless, it can be shown that the stability of the system describing this polynomial depends only on the parameters λ_i.

Suppose that we have a polynomial of degree n with a given system of parameters λ_i. Since for a complete description of the polynomial in this case we need two or more parameters, they will be assigned arbitrary values. Let us write the polynomial in the form

$$F_n(z) = z^n + Az^{n-1} + Bz^{n-2} + \frac{AB}{\lambda_1} z^{n-3} + \frac{B^2}{\lambda_1\lambda_2} z^{n-4} + \\ + \frac{AB^2}{\lambda_1^2\lambda_2\lambda_3} z^{n-5} + \frac{B^3}{\lambda_1^3\lambda_2^2\lambda_3\lambda_4} z^{n-6} + \ldots \tag{8}$$

Here A and B are arbitrary parameters which define a set of polynomials having the same system of parameters λ_i. The only conditions on A and B are that they must be greater than zero. We know that, if the initial polynomial has all roots in the left half-plane, then they remain there upon the substitution $z = z'k$ (by the scaling theorem), and when all even or odd coefficients are multiplied (or divided) by the same positive number, the roots also remain in the left half-plane (by Mikhaylov's stability test). Therefore, without changing the character of stability of the polynomial in (8), we make the following transformations. We make the substitution $z = z'\sqrt{B}$. Then we divide the even coefficients of the polynomial thus obtained by $(\sqrt{B})^n$, and the odd ones by $A(\sqrt{B})^{n-1}$. We obtain the polynomial (dropping the primes):

$$F_n(z) = z^n + z^{n-1} + z^{n-2} + \frac{1}{\lambda_1} z^{n-3} + \frac{1}{\lambda_1\lambda_2} z^{n-4} + \\ + \frac{1}{\lambda_1^2\lambda_2\lambda_3} z^{n-5} + \frac{1}{\lambda_1^3\lambda_2^2\lambda_3\lambda_4} z^{n-6} + \dots \tag{9}$$

Thus, it is clear that any polynomial in the set of possible polynomials having the same system of parameters λ_i can be reduced to a polynomial whose coefficients depend on λ_i alone by means of transformations that do not affect its stability. Since the stability of a system described by this polynomial depends only on the parameters λ_i, the stability of a system describing the original polynomial also depends on the parameters λ_i.

The system of parameters λ_i completely describes the stability of the system but hardly characterizes its quality. For example, the two characteristic polynomials:

$$\begin{aligned} F_{51}(z) &= z^5+5z^4+10z^3+10z^2+5z+1 \\ F_{52}(z) &= z^5+50z^4+10z^3+100z^2+5z+10 \end{aligned} \tag{10}$$

are described by the same system of parameters λ_i, and are factored as follows:

$$\begin{aligned} F_{51}(z) &= (z+1)^5 \\ F_{52}(z) &= (z+49{,}84)(z^2+0{,}047z+0{,}105)(z^2+0{,}11z+1{,}89) \end{aligned} \tag{11}$$

The character of roots from the point of view of oscillations is different, but from the point of view of stability, these two polynomials are equivalent. The systems described by these polynomials will also have the same stability margin in any of the parameters entering the coefficients of the characteristic polynomial in identical fashion. We shall assume that any parameter enters the coefficients of polynomials in identical fashion if it produces identical fractions in identical coefficients. Thus, the free term in both equations can be increased until instability occurs the same number of times. This example shows that it is incorrect to judge the stability margin of a system on the basis of the oscillatory character of the roots of its characteristic polynomial.

Since the system of parameters λ_i determines completely the stability of a system, it is natural to construct the regions of stability of the system in the (n - 2)-dimensional space of the parameters. Then the necessary conditions for stability (2) and (3) will also yield a certain region which comprises the exact region of stability and may serve as an "approximate" region of stability.

From the point of view of the regulator synthesis, it would be more useful to have some simple sufficient conditions for stability that would guarantee the stability of a system. It turns out that such conditions may be obtained also by using the parameters λ_i.

We can prove the following theorem about the sufficient conditions for the stability of a system having the characteristic polynomial (1).

Theorem 3. If all roots of all possible polynomials of fifth degree, formed out of six consecutive coefficients of the polynomial (1), lie in the left half-plane, then all the roots of the polynomial (1) also lie in the left half-plane.

Thus, one can obtain a stable system of n^{th} order by achieving the stability of n - 4 systems of fifth degree, which is much simpler to do. The stability conditions for a system of fifth degree, expressed in terms of the parameters λ, have the form

$$\begin{gathered}\lambda_1 > 1 \\ \lambda_1^2\lambda_2\lambda_3^2 + 2\lambda_1\lambda_3 + \lambda_1\lambda_2\lambda_3 - \lambda_1\lambda_2\lambda_3^2 - \lambda_1^2\lambda_2\lambda_3 - \lambda_1^2\lambda_3^2 - 1 > 0\end{gathered} \tag{12}$$

As we can see from Equation (12), these conditions are also quite complicated which will hinder their use in practice when investigating the stability of systems of n^{th} degree. But this expression can be considerably simplified. We can use the "approximate" sufficient conditions instead of the exact, necessary and sufficient, conditions for the stability of a system of fifth degree. By investigating the second inequality in (12) graphically or analytically, one can show that it is valid if all $\lambda_i > 2.144 \ldots$ ($i = 1, 2, 3$) or if all $\frac{1}{\lambda_i} + \frac{1}{\lambda_{i+1}} < 0.89 \ldots$ ($i = 1, 2$). Using this and Theorem 3, we can formulate the following two theorems about the sufficient conditions for the stability of linear stationary systems.

Theorem 4. If the polynomial in (1) satisfies

$$\lambda_i > 2{,}144 \ldots, \quad i = 1, 2, \ldots, n-2, \tag{13}$$

then all its roots lie in the left half-plane.

Theorem 5. If the polynomial in (1) satisfies

$$\frac{1}{\lambda_i} + \frac{1}{\lambda_{i+1}} < 0{,}89 \ldots, \quad i = 1, 2, \ldots, n-3, \tag{14}$$

then all its roots lie in the left half-plane.

Conditions (13) and (14) determine a certain region in the space of the parameters λ_i, which is inscribed into the exact stability region, determined using the exact necessary and sufficient conditions for stability. This region may also be used as an approximate region of stability.

Let us consider several examples showing how "approximate" conditions for stability are used.

Example 1. Suppose we are given the characteristic equation of a system

$$z^6 + 12z^5 + 47z^4 + 108z^3 + 122z^2 + K_2 z + K_1 = 0 \quad (15)$$

We are required to find the "approximate" regions of stability in the space of the parameters K_1 and K_2.

In this case, we have selected a simple polynomial, so that for comparison we can obtain the "exact" boundary of stability using the method of D-decomposition.

The necessary conditions for stability of (2) and (3) and the requirement that the coefficients of the polynomial be positive impose on the parameters K_1 and K_2 the following conditions:

$$\begin{aligned} 0 < K_1 &< 105 \\ K_2 &< 242 \\ K_2 &> 0{,}89 K_1 \end{aligned} \quad (16)$$

If even one of these conditions is not satisfied, the system becomes unstable. On the other hand, if all the conditions are satisfied, this still does not guarantee stability.

The sufficient conditions for stability (13) and the requirement that the coefficients be positive impose on the parameters K_1 and K_2 the following conditions:

$$\begin{aligned} 0 < K_1 &< 0{,}52 K_2 \\ K_2 &< 130 \end{aligned} \quad (17)$$

If these two conditions are satisfied, the stability of a system is guaranteed.

And, finally, the sufficient conditions in (14) yield

$$\begin{gathered} 0 < K_1 < K_2 - 0{,}004K_2^2 \\ K_2 < 169 \end{gathered} \tag{18}$$

If both of these conditions are satisfied, independently of the validity of the inequalities in (17), the system is also guaranteed to be stable.

The inequalities (16) - (18) determine certain regions in the space of the parameters K_1 and K_2. In the same space we can also construct the exact boundary of the region of stability by using the method of D-decomposition. Three regions, constructed using the sufficient, necessary, and exact conditions for stability, respectively, are shown in Figure 1. The sufficient conditions for stability determine a region, lying completely inside the exact region of stability, and the boundary of the region obtained using the necessary conditions for stability, encompasses the exact region of stability.

Example 2. In the preceding example, the approximate tests for stability were used to solve a problem that can also be solved using the exact methods (D-decomposition). Let us make the problem a little more complicated. Suppose we are given the following characteristic control function

$$z^6 + 12z^5 + 47z^4 + 108z^3 + 122z^2 + \tilde{K}K_2z + \tilde{K}K_1 = 0, \tag{19}$$

where $\tilde{K}$ is a variable parameter of the object, and K_1 and K_2 are the unknown parameters of the regulator. The method of D-composition is difficult to use here, since first of all the problem is three-dimensional which makes it hard to visualize the solution. Secondly, it does not reduce to a system of three linear equations with three unknowns. However, the use of the "approximate" conditions for

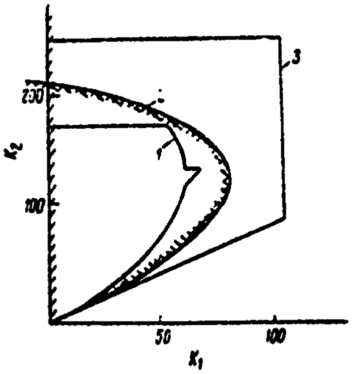


Figure 1. Exact and "approximate" regions of stability.

1 — region of stability obtained using the sufficient conditions for stability (17) and (18); 2 — exact boundary of stability, obtained using the method of D-decomposition; 3 — boundary of stability obtained using the necessary conditions for stability (16).

stability gives simple inequalities that do not require geometrical interpretation. Thus, the use of the necessary conditions for stability (2) and (3) yields:

$$\begin{gathered} 0<K_1<\frac{105}{\tilde{K}} \\ K_2<\frac{242}{\tilde{K}} \\ K_2>0{,}89K_1 \end{gathered} \qquad (20)$$

The sufficient stability conditions (13) yield

$$\begin{gathered} 0<K_1<0{,}52K_2 \\ K_2<\frac{130}{\tilde{K}} \end{gathered} \qquad (21)$$

Finally, the sufficient conditions (14) yield

$$\begin{gathered} 0<K_1<K_2-0{,}004K_2^2\tilde{K} \\ K_2<\frac{100}{\tilde{K}} \end{gathered} \qquad (22)$$

For any given range of change in the parameter $\tilde{K}$, we may find the values of the parameters K_1 and K_2 necessary for stability.

Thus, it may be seen that the "approximate" stability criteria may successfully replace the "precise" stability criteria for high order systems. The use of the "approximate" stability criteria yields simple relationships between the system parameters, which makes it possible to synthesize the regulator simultaneously for a certain set of values for individual parameters of the system. The use of "approximate" stability criteria greatly increases the productivity of the engineer-designer.

CERTAIN PROBLEMS OF THE CONSTRUCTION OF PARAMETRICALLY INVARIANT AUTOMATIC CONTROL SYSTEMS

N. I. Sokolov

The control of nonstationary objects with their parameters changing within wide limits is in principle possible with the aid of adaptive automatic control systems and systems equivalent to them.

Here adaptive regulators will be defined as regulators in which certain (adjustable) parameters are functions of the corresponding variable parameters of a nonstationary controlled object. Consequently, the operation of such a regulator requires, in principle, that information be given about the variable parameters of the controlled object. Technically the variation of the regulator parameters is achieved with the aid of parametric couplings. A parametric coupling is a necessary (but not sufficient) trait of an adaptive system. The basic problem in the construction of adaptive automatic control systems is to organize the information about the variation of the parameters of the nonstationary controlled object.

In adaptive control systems with open loops for the adjustment of the regulator parameters, the information about the change of the parameters of the nonstationary object is obtained directly by measuring the external conditions that result in the nonstationarity of the object, or in certain cases by using a specially programmed unit.

In both cases the accuracy with which the problem is solved is small.

Adaptive control systems with closed loops for the adjustment of the regulator parameters offer much greater possibilities. The information about the changes in the parameters of a nonstationary object is obtained from the signals circulating in a closed system. However, a signal, circulating in a system, has no natural characteristics that would lead in a technically simple fashion to information about changes in the parameters of an object.

On the other hand, we cannot exclude the theoretical possibility of building such characteristics into a signal by selecting structures and parameters of the basic contour of a control system.

The best known methods of extracting information about changes in the parameters of an object are the correlation methods with the method of the energy balance as one of their forms. These methods have become widespread in solving engineering problems. However, the energy balance method is applicable only in the case when the parameters of an object change very little during the transition process. The technique in which the effect of changing parameters (parameter) is translated into a high-frequency signal by properly selecting the structure and parameters of the basic circuit can theoretically be used to extract information about changes in a certain parameter of an object. This is the so-called indirect method of determining parameters of an object.

The equivalent adaptive regulators will be defined as those regulators in which the information about the variable parameters of an object is not used and is not needed to provide a given type of control.

The usual dynamic and static properties of the equivalent adaptive regulators are achieved by introducing proper coordinate links. It is also theoretically possible to use parametric links in such regulators. However, the parametric linkage, in contrast to adaptive systems, changes the parameters of a regulator depending on the instantaneous value of certain phase coordinates of the control system.

The equivalent adaptive control systems may be subdivided in two classes:

1) parametrically invariant systems are characterized by the fact that, when the parameters of a control object change, the output coordinates of the performance indicators of the system remain the same or change in a desired direction within given limits;

2) ultra-coarse systems, characterized by the fact that — as the parameters of an object vary — the stability of a control system is preserved, and the performance indicators are not subject to control.

These classes in turn may be subdivided into groups that differ from one another in design, and naturally, in their capacity.

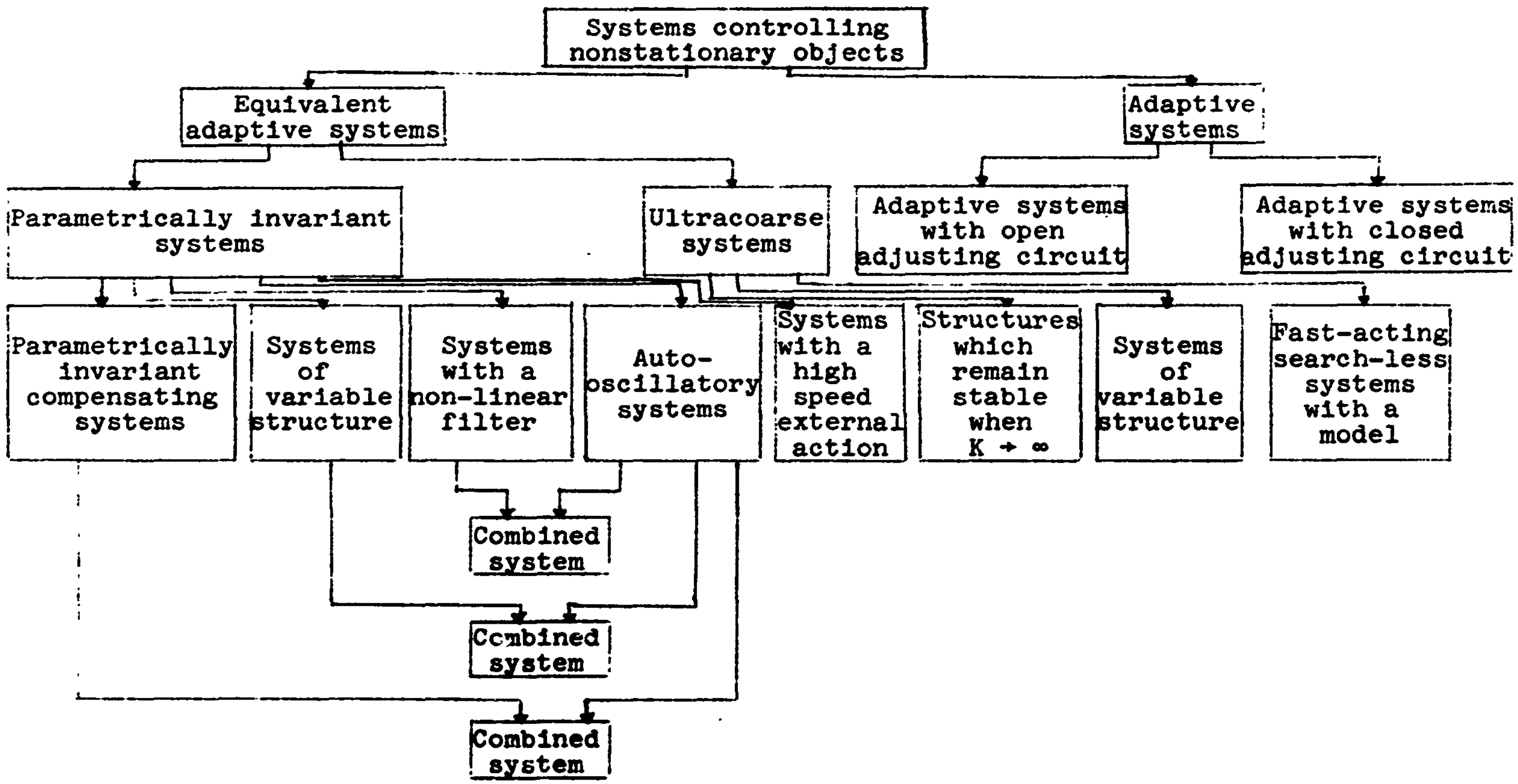
Figure 1 shows the classification table.

The definitions given here and the classification table do not pretend to be complete, have purely utilitarian objectives, and are used in the further discussion of the material.

Let us briefly discuss the basic differences and capabilities of the structures in question.

The parametrically invariant control systems can be subdivided into the following groups:

a) parametrically invariant compensating systems are linear within a limited range. In order to assure stability and independence of the performance indicators when the parameters of an object undergo a change, one introduces inversely parallel correcting devices which comprise the object and certain elements of the regulator, and are in the form of physically realizable filters. In order to compensate for the lag introduced by physically realizable filters, a low frequency filter—whose poles are equal to the poles of the inversely



Systems controlling nonstationary objects
Equivalent adaptive systems
Adaptive systems
Parametrically invariant systems
Ultracoarse systems
Adaptive systems with open adjusting circuit
Adaptive systems with closed adjusting circuit
Parametrically invariant compensating systems
Systems of variable structure
Systems with a non-linear filter
Auto-oscillatory systems
Systems with a high speed external action
Structures which remain stable when K → ∞
Systems of variable structure
Fast-acting search-less systems with a model
Combined system
Combined system
Combined system

Figure 1

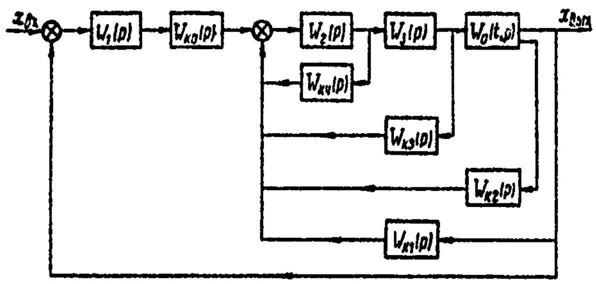


Figure 2.

parallel correcting devices — is introduced in the direct circuit between the measuring element and the adder.

The structural diagram of the parametrically invariant compensating systems in their general form is given in Figure 2, where $W_1(p)$ is the measuring element; $W_2(p)$ is the amplifier; $W_3(p)$ is the drive, $W_0(t, p)$ is the control object; and $W_{ki}(p)$ are the correcting devices.

The poles of the transfer function $W_{k0}(p)$ are identical with the poles of the functions $W_{k_4}(p)$, $W_{k_3}(p)$, $W_{k_2}(p)$, $W_{k_1}(p)$ (or at least identical with the smallest ones among them).

A differential equation for parametrically invariant compensating systems can in general be written as

$$\sum_{j=0}^{n+m} A_j \frac{d^j x_{вых}}{dt^j} + \sum_{j=0}^{n+m-1} C_j(t) \frac{d^j x_{вых}}{dt^j} + \sum_{j=0}^{m} K_p K(t) b_j \frac{d^j x_{вых}}{dt^j} = K_p K(t) x_{вх}, \qquad (1)$$

where n is the order of the differential equation for an uncorrected system, m is the order of the differential equation describing the processes in the correcting devices, $C_j(t)$ are the coefficients which contain the variable parameters of the object and their derivatives, K(t) is the product of the amplification factor and the natural frequency of the nonstationary object. The introduction of a compensating filter minimizes the coefficients multiplying the derivatives in the equation. By the same token, assuming there is interference and nonlinearities of the saturation type, it keeps the control system operational when the parameters of the object vary within a wide range.

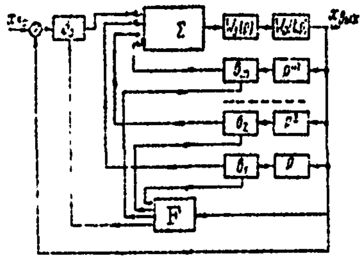


Figure 3.

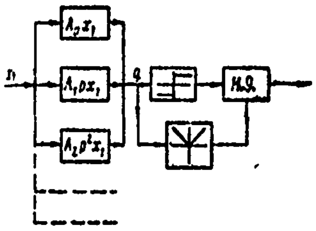


Figure 4.

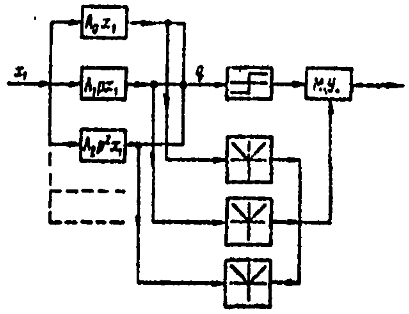


Figure 5.

b) Systems of variable structure with internal feedbacks linking the control object and certain elements of the regulator, technically based on the ψ-cells involving sufficiently small lags in the correcting devices. Also theoretically assure parametric invariance when the parameters of an object vary within a certain range.

A diagram of a system of variable structure is shown in Figure 3, where F is the unit in which the switching functions are formed, b_j is the coefficient of action with respect to the derivatives.

Taking into account the real lag involved in the formation of the derivative signals, the differential equation for a system of variable structure, shown in Figure 3, may be written as

$$\sum_{j=0}^{n+m} A_j \frac{d^j x_{вых}}{dt^j} + \sum_{j=0}^{n+m-1} C_j(t) \frac{d^j x_{вых}}{dt^j} + \sum_{j=0}^{m} K_p K(t) b_j \times$$
$$\times \left(x_{вых} \frac{dx_{вых}}{dt} \dots \frac{d^m x_{вых}}{dt^m}\right) \frac{d^j x_{вых}}{dt^j} = \sum_{j=0}^{m} K_p K(t) B_j \frac{d^j x_{вх}}{dt^j}, \qquad (2)$$

where $b_j = \begin{cases} b_{j1} \\ b_{j2} \end{cases}$, Since here we deal with systems of variable structure which are characterized by sliding regimes, when a sliding regime does occur, the above equation may be replaced with a linear one, where the coefficients are equal to the averages $b_j = \frac{b_{j1} + b_{j2}}{2}$.

The linear equation thus obtained will describe the motion in a hyperplane. Fast movements relative to the hyperplane are not represented by this equation. However, since the frequency of a sliding regime in systems of variable structure of this type is much higher than the band of the working frequencies of the system, these fast oscillations will have practically no effect on the output coordinate. Consequently, the systems of variable structure with physically realizable correcting devices may in this case be investigated using linear equations.

The occurrence of an uncompensated lag, introduced by physically realizable correcting devices, interferes with the minimization of the coefficients of the differential equation for the system. Therefore, when there is interference and a system involves additional nonlinearities, its capacity is less than that of a parametrically invariant compensating system. In addition, the nonlinear control part (the control part might be said to have a source of internal interference) also results in a higher sensitivity to interference than in the case of parametrically invariant compensating systems.

A system of variable structure may serve as an example of a parametrically invariant system widely utilizing parametric links in the phase coordinates due to which the system acquires certain nonlinear properties. In general, the presence of parametric links does not necessarily result in nonlinear properties, which can be directly seen in Figures 4 and 5. Figure 5 shows an equivalent model of the control portion of a system of variable structure [2]. A system of variable structure is characterized by the parametric links in the coordinates x, x, x..., resulting in a change of the "bank" of relays.

Figure 4 shows the control portion which is fairly close to the control portion of a system of variable structure, with the exception that here the modulus is taken of the total signal "q." The parametric linkage does not disturb the linear properties of the system. Thus, in the case of Figure 4, we have a technical realization of a linear system composed of nonlinear elements.

c) Systems involving a special nonlinear filter. These systems are conceptually similar to systems of variable structure. They differ from the latter in that a signal entering a relay element is formed by one operator, whereas a signal entering modulus-type nonlinear elements is formed by another operator. In the case of a single frequency input signal, one can form any amplitude and phase frequency characteristics (in the first harmonic). However, in the case of a mixed signal, in view of the invalidity of the superposition principle, these characteristics do not hold. If in such a system high frequency sliding regimes occur, then processes in the system, just as in a system of variable structure, are approximately described by linear equations. The capabilities of such systems in the case of a nonstationary automatic control are similar to those of systems of variable structure.

d) Auto-oscillatory control systems (with a passive self-adjustment). These systems perform harmonic linearization.

For a certain structure of the regulator with respect to slow actions, the system will operate as a linear and stationary system, independently of the variable parameters of the control object. The amplitude of the auto-oscillations will vary depending on the changes in the parameters of the object.

The amplitude of the auto-oscillations must for technical reasons lie within a certain range, which considerably limits the capabilities of such systems.

The systems in question are also more sensitive to noise than linear systems.

e) Nonlinear systems with high-frequency external signals. Such systems perform linearization at the expense of external high-frequency signals. The system has quasilinear properties with respect to slowly varying signals. The capabilities of such systems approximate those of the auto-oscillatory systems.

We have in mind the following systems:

a) parametrically invariant compensating systems;

b) systems of variable structure;

c) systems with special nonlinear filters.

We can formally construct a combined system of the form linear-variable structure auto-oscillatory. However, this type of system is of no technical interest, since all problems are solved more completely by a combined system: parametrically invariant compensating-auto-oscillatory (optimality at large deviations and stability, as well as given values of performance indicators at small deviations).

To illustrate the parametrically invariant systems and ultra-coarse control systems, we shall use an example from [3] (Kokhenburger's system). The equation of the control object will be

$$\frac{x_{вых}}{x_p} = \frac{K_0(t)\,5000}{(p+5)^2(p+200)}, \text{ where } 0{,}1 \leqslant K(t) \leqslant 10 \tag{4}$$

The regulator elements were assumed to be inertia-free $W_1(p) = K_1$, $W_2(p) = K_2\, W_3(p)$; for simplicity we assume that $K_1 = 1$, $K_3 = 1$.

The control system to be designed must satisfy the following technical requirements:

a) static error no greater than 5%;

b) maximum regulation time no greater than $t_p = 0.28$ sec (the process is considered to be over when the dynamic error is less than 5%);

c) over-regulation is no greater than 15%.

A Parametrically Invariant Compensating System

1) Excluding the static error, we include an integral link in the adder (See Figure 3.).

Then $W_{k0}(p) = W_{k10}(p)\frac{1}{p}$.

In a general form $W_{k1}(p) = \frac{A_{k1}(p)}{B_{k1}(p)}$; $W_{k2}(p) = \frac{A_{k2}(p)}{B_{k1}(p)}$; $W_{k0}(p) = \frac{A_{k0}}{B_{k0}(p)}$.

The transfer function for the system will be

$$K(p) = \frac{K_2 5000 K_0(t)}{(p+5)^2(p+200)B_{k1}(p)p + K_2 5000 K_0(t) + K_1 A_2(p)p(p+5)^2(p+200) + K_2 5000 K_0(t) A_{k1}(p)p} \qquad (5)$$

In case of correcting devices of the type

$$W_{k1}(p) = \frac{0{,}383p^3 + 29p^2 + 3850p + 78\,000}{(p+150)(p+80)(p+40)} - \qquad (6)$$

(includes the object, power drive, amplifier),

$$W_{k2}(p) = \frac{0{,}135p^2 + 9{,}84p + 252}{(p+150)(p+80)(p+40)} \qquad (7)$$

(includes amplifier),

$$W_{k0}(p) = W_{k10}(p)\frac{1}{p} = \frac{480\,000}{(p+150)(p+80)(p+40)p} \qquad (8)$$

(The last correcting device was put between the measuring element and the adder) and K_2 = 2140, all technical requirements are satisfied.

Let us consider the operational ability of the system in the presence of interference and saturation-type nonlinearities in the drive.

Suppose that the drive becomes saturated by an input signal x_{dr} = 16 V. Consider x_{dr} with noise of the form 0.001 sin 2000 t_{in}.

a) The noise $x_{in.n}$ = 0.001 sin 2000 t_{in} acts on the input of the system. The object has a sufficient inertia and does not pass a signal of frequency ω = 2000. Therefore, at that frequency the system might be said to be open with respect to the principal feedback, and we can write approximately

$$\frac{x_{np}}{x_{Bx.n.}} \approx \frac{80\,000}{(p+150)(p+80)(p+40)p} \cdot \frac{2140}{1+\frac{(0,135p^2+0,84p+252)\,2140}{(p+150)(p+80)(p+40)}} = \frac{480\,000\cdot 2140}{(p^3+560p^2+23000p+1020000)\,p} \tag{9}$$

$$|x_{np}| \approx 0{,}001 \cdot \frac{480\,000\cdot 2140}{8250\cdot 10^6\cdot 2000} = 0{,}0000000615 \text{ V} \tag{10}$$

The noise at the input to the drive will be greatly weakened.

b) The noise $x_{out.n}$ = 0.001 sin 2000 t enters the input of the correcting device $W_{c.d.1}$ (p).

$$\frac{x_{np}}{x_{Bux.n.}} \approx \frac{0{,}383p^3+29p^2+3850p+78\,000}{(p+150)(p+80)(p+40)} \cdot \frac{2140}{1+\frac{(0,135p^2+0,84p+252)\,2140}{(p+150)(p+80)(p+40)}} = \frac{(0{,}383p^3+29p^2+3850p+78\,000)\,2140}{p^3+560p^2+23\,000p+1020000} \tag{11}$$

$$|x_{np}| \approx 0{,}001\cdot 0{,}368\cdot 2140 = 0{,}785 \text{ V} \tag{12}$$

The system will be operational with this type of noise.

2) Correcting devices may be chosen to be simpler.

For example, the above dynamic requirements are satisfied for

$$W_{k1}(p) = \frac{0{,}054p+5{,}38}{p+45};\; W_{k2}(p) = \frac{0{,}0204}{p+5};\; W_{k0}(p) = \frac{45}{(p+45)\,p} \tag{13}$$

The operational capacity of the system in the presence of noise is

a) $$\frac{x_{\text{пр}}}{x_{\text{Вх.п.}}} \approx \frac{45 \cdot 36\,500\,(p+5)}{p(p+45)(p+800)} \quad |x_{\text{пр}}| \approx 0{,}001 \cdot 0{,}38 = 0{,}00038 \text{ V} \tag{14}$$

b) $$\frac{x_{\text{пр}}}{x_{\text{Вых.п.}}} \approx \frac{(1900p+196\,000)(p+5)}{(p+45)(p+800)} \quad |x_{\text{пр}}| \approx 0{,}001 \cdot 1{,}77 \cdot 10^3 = 1{,}77 \text{ V} \tag{15}$$

In the second version, the scheme is also operational, but its resistance to noise is lower than in the first version.

Systems of variable structure and the system with nonlinear filters without compensating filters will have a lower resistance to interference than the above structure.

(Control Systems with Passive Self-Adjustment. Auto-Oscillatory Control Systems)

Auto-oscillatory control systems are also parametrically invariant but their operational regimes differ from the preceding ones.

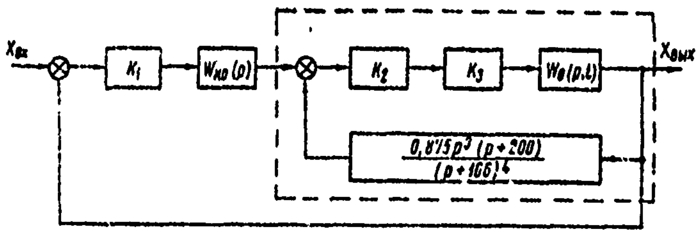


Figure 6.

Let us consider Kokhenburger's solution (Figure 6).

The equation of the oscillatory loop for K_2 = 300,000 will be

$$K_k(p) = \frac{5000K_2K_0\,(p^4+664p^3+165\,110p^2+18{,}28\cdot 10^6 p+760\cdot 10^6)}{(p^6+674p^5+171777p^4+132\cdot 10^6 p^3+947\cdot 10^6 p^2+8160\cdot 10^6 p+19050\cdot 10^6)(p+200)} \tag{16}$$

Under those conditions, the loop becomes unstable. Due to the saturation of the amplifier (or the drive) the system will be auto-oscillatory. The expression in the brackets can be factored into a product of two polynomials

$$(p^3+667p^2+167\,046p+130{,}8\cdot 10^6)(p^3+7{,}2p^2+61p+145) \qquad (17)$$

The first polynomial has roots with positive real parts. The instability will occur at the frequency $\omega = 510$. If, just as before, the saturation occurs at 16 V, then the amplitude of the auto-oscillations of the object will be less than the amplitude of the noise $x_{out.dr} = 0.001$ V. The system will not be operational. One can increase the amplitude of the auto-oscillations at the output from the object by introducing an amplifier after the nonlinear element, and a divider before the nonlinear element. However, the result will be that the amplitude of the auto-oscillations at the input to the object will be very high, which is not permissible under real conditions.

Ultra-Coarse Control Systems. A Structure that Remains Stable for $K \to \infty$

The system has $t_p = 0.28$ sec for $K_0 = 0.1$; for $K_0 = 10$ the system has $t_p = 09.02$ sec. The over-regulation for $K_0 = 0.1$ is 15%.

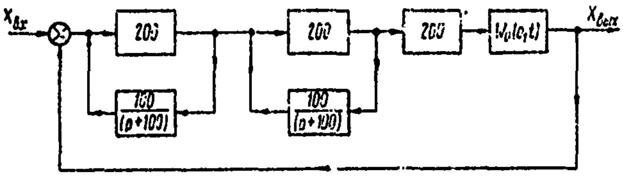


Figure 7.

The operational ability in the presence of noise.

Here the effect of the noise $x_{in.n}$ and $x_{out.n}$ is equivalent and similar to what was used earlier

$$\frac{x_{np}}{x_{bx.n.}} \approx \frac{200^3(p+100)^2}{(p+20100)^2}; \qquad (18)$$

$$|x_{np}| = 0{,}001\cdot 80\,000 \approx 80 \text{ V} \qquad (19)$$

At 16 V the drive will become saturated due to noise, For this type of noise, vibrational linearization will occur, the effective amplification coefficient of the drive will be reduced several times, t_p will increase, and in this case the process will become more oscillatory.

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CERTAIN PROBLEMS OF STATISTICAL ESTIMATION IN CONTROL THEORY

A. S. Golubkov

The desire to satisfy the increasing requirements placed on technical systems leads in a majority of cases to the use of optimal design methods in the construction of control systems.

When one solves applied problems in that area, there always arises the problem of obtaining reliable information that would be sufficient for the construction of optimal systems. Attempts to solve this problem also encounter certain specific theoretical difficulties, particularly when the problem is statistically formulated. As a rule, problems of that type cannot be tackled by a single theory. They require the use of probability theory, mathematical statistics, and theory of optimal processes.

The results obtained in those areas indicate that to construct high-quality systems it is absolutely imperative that measurements be made. If this is impossible, the values of the parameters characterizing the state of the processes investigated must be calculated with high accuracy and with no substantial delay. The occurrence of various random disturbances and measurement errors, and changes in the parameters and characteristics within a wide range in a way impossible to predict, force us to use the statistical approach in the evaluation of processes. As always, the estimates are a result of certain operations on the measurement data, permitting us to estimate the state of the process.

The state of a process can, as we know, be described by a stochastic model to any degree of accuracy. Moreover, the correspondence between the model and the real process occurs only if the model is statistically sufficient. Considering only one class of processes — namely, the class of controlled Markov processes — we shall assume below that their space-time behavior is described by the trajectory x(t) satisfying a model written in the form of a stochastic differential equation

$$\dot{x} = F(x,t) + \varphi(x,t)\omega(t) \tag{1}$$

where $\omega(t)$ is a random process of the type of white noise with the parameters $E\{\omega(t)\}$ and $E\{\omega(t)\omega(\tau)\}$ (E is a symbol denoting the averaging operation).

Information about the state of the coordinate x(t) can be obtained by measuring a certain y(t) which is related to the original x by equations of the type

$$y(t) = H(x,t) \tag{2}$$

or

$$\dot{y}(t) = H(x,t,\omega) \tag{3}$$

and is given as a solution of the stochastic differential equation

$$\dot{z}(t) = A(z,y,t) + B(y,t)v(t) \tag{4}$$

where v(t) is a random process of the type of white noise with the parameters:

$$E\{v(t)\} = 0 \text{ and } E\{v(t)v(\tau)\}$$

This type of description allows us to take into consideration the peculiarity of the measuring process which includes the dynamic properties and the additive noise of measurements.

Assuming for simplicity that x, y, and z are scalar processes and H(x, t) = x, the equation of measurements (4) becomes

$$\dot{z}(t) = A(x, z, t) + B(x, t)v(t) \quad (5)$$

According to the terminology accepted in theory of controlled random processes [1], the set {x(t); z(t)}, where x(t) is the unobserved coordinate, and z(t) is the observable, came to be called partially observable, and the procedures based on the use of the second coordinate z(t) alone came to be called the procedures based on incomplete data.

In our case, the set {x(t); z(t)} satisfying (1) and (5), represents a two-dimensional Markov process in which the first coordinates x(t), z(t), being fixed, form the so-called conditional Markov process.

The problem as applied to a conditional Markov process belongs to the class of problems of mathematical statistics, and reduces to the question of how one can best estimate the component x(t) on the basis of measurements of z(t). The technical solution of the problem involves processing large amounts of data, and acceptable solutions can be obtained only if one extracts "sufficient statistics" based on the set of the measurement data $\gamma(z)$, that are the central question in those problems [2].

One of the principal difficulties in this approach is the volume of measurement data, which increases as time goes on, and whose processing in real time and in full is difficult using the computers available today.

In a majority of problems of sequential analysis, where the present estimate problem also belongs, the a posteriori probabilities turn out to be sufficient statistics.

Referring to [3], we can state that the use of the Markov properties of the set {x(t); z(t)} enables us to considerably reduce the volume of data required to obtain sufficient statistics. In the case of a finite observation interval, the a posteriori probability is a Markov sufficient statistic if the following conditions are satisfied:

1) unobserved process x(t) is a Markov process;

2) observed process consists of a sequence of conditionally independent random variables;

3) optimality test is additive;

4) preceding measurements and estimates do not impose any restrictions on the following estimates.

If the conditions (3) [3] and (4) [4] are satisfied, we can consider the synthesis of optimal estimates as an extremal problem of mathematical statistics in relation to a conditional Markov process subject to conditions (1) [1] and (5) [5].

The problem will consist of making decisions, $\dot{x}(t)=\gamma(z(\tau))$ $\dot{x}\in X$, constructed out of known data $z(\tau)$, $z\in Z$, $0 \leq \tau \leq T$, which must be optimal in the sense that they must give the best estimate of the parameter $x(t)$, $x\in X$, $t \leq T$.

When the Bayesian approach is used, such an estimate (here we have in mind a point estimate) must minimize the mean risk [4]

$$R(\dot{x})=\int l_0(z,x,\gamma(z))\,p(x,z)\,dx\,dz \tag{6}$$

where l_0 [z, x, γ (z)] is a loss function.

To satisfy (6), it is sufficient to minimize the conditional risk

$$r(\dot{x}/z)=\int l_0(z,x,\dot{x})\,p(x/z)\,dx \tag{7}$$

at each point individually.

The conditions (1) and (5) on the set $\{x(t); z(t)\}$ require that one consider additional conditions in the extremal problem, in particular, that one introduce a new loss function $L(x, z, x)$

$$L(x, z, \hat{x}) = l_0(x, z, \hat{x}) + \sum_{l=1}^{n-1} \lambda_l l_l(x, z, \hat{x}) \tag{8}$$

and minimize the mean risk $R(x)$ under the condition that

$$\int l_l(z, x, \hat{x})\, p(x, z)\, dx\, dz = c_l \tag{9}$$

In view of (8) and (9), we assume that the estimate has the form $x = \gamma(z, \lambda)$, where the parameter $\lambda_l = \lambda_l(c_l)$ is determined by

$$\int l_l(x, z, \gamma(z, \lambda_l))\, p(x, z)\, dx\, dz = c_l \tag{10}$$

If System (10) has a solution, then the estimate $x = \gamma(z, \lambda(c))$ is optimal for the loss function (8) and is the desired solution of the extremal problem. Next, assuming that the measurement interval is bounded $0 = \leq \tau \leq T < \infty$, and the process in which we obtain and process the data is sequential in time, the introduction of time in the problem leads to the following relations for the loss function L and the estimate x

$$L(x, z, \hat{x}, T) = \int_0^T L(x(t), z(t), \hat{x}(t))\, dt + L^T(x_T, z_T, \hat{x}_T) \tag{11}$$

$$\hat{x}(T) = \gamma_T(z(\tau)) \quad 0 \leqslant \tau \leqslant T \tag{12}$$

respectively. The optimal solution, satisfying (11) and (12) for fixed T, can be found from the minimum condition

$$r_T(\hat{x}(T)/z(\tau)) = \int L(x(\tau), z(\tau), \hat{x}(\tau))\, p(x(\tau)/z(\tau))\, dx \tag{13}$$

The conditional risk (13) depends on $z(\tau)$ $0 \leq \tau \leq T$ through the a posteriori distribution $p(x(t)/z(\tau))$ $0 \leq \tau \leq T$, which contains the entire necessary information about the structure of the optimal estimates.

Since $\{x(t); z(t)\}$ is a conditional Markov process, $p(x(t)/z(\tau))$ may be found for all $\tau \leq T$ by applying the theory of conditional Markov processes [5]. The evolution of the a posteriori probability $p(x/z)$ is obtained as a solution of the stochastic differential equation, and for the model (1) and (5) it is characterized by the solution of an infinite-dimensional system of stochastic differential equations

$$\begin{aligned} \dot{p}_{x/z}(t) &= p_{x/z}(t) \frac{[A(z,t) - \lambda(z,x)]}{B^2(x,t)} [\dot{z}(t) - \hat{\lambda}(z,x,t)] + \\ &+ \left\{ -\frac{\partial}{\partial x} [F_x(t) p_{x/z}(t)] + \frac{1}{2} \frac{\partial^2}{dx^2} [\varphi_x^2(x,t) p_{x/z}(t)] \right\} \\ \hat{A}(z,x,t) &= \int A(x,z,t) p_{x/z}(t) \, dx \end{aligned} \tag{14}$$

Suppose that $p^*_{x/z}(t)$ is a solution of (14). Then we shall assume that the estimate of $x(t)$ will have the form $x(z, t) = \gamma^*(p^*_{x/z}(t)$.

We know that in the case when the probability density $p_{x/z}(t)$ is unimodal and symmetric in its expectation, and the loss function is symmetric, i.e., $L(x, x) = L(x, x)$, then the conditional expectation is the best estimate. Therefore, the optimal estimate will always be chosen as

$$\hat{x} = \gamma(z) = \int x p(x/z) \, dx = \frac{\int x p(x) p(z/x) \, dx}{\int p(x) p(z(x) \, dx} \tag{15}$$

Such an estimate being asymptotically unbiased represents an efficient estimate in the sense that the deviation is minimized [2, 4].

The above expression for the a posteriori probability clearly shows that the process of finding estimates requires a solution of parabolic partial differential equations, which is very difficult.

Since Equation (14) is infinite-dimensional, we are actually not interested in it as much as in the fact that from it one can obtain equations of certain characteristics of the process $p_{x/z}(t)$, for example moments, characteristic functions, and semi-invariants.

However, even with this simplification, the equations for the first two moments can be obtained and solved only for an exponential family of Markov process distributions, and then only in the case when it is possible to establish a one-to-one correspondence among the sets X, X and Z.

As an example, let us consider a Markov process described by the equation

$$\dot{x} = c + a\omega(t) \tag{1E}$$

$x_0 = x(0)$ is a Gaussian random variable with the parameters $E\{x\}$ and $E\{x^2\}$. $\omega(t)$ is a random process of the white noise type with the parameters $E\{\omega\} = 0$; $E\{\omega^2\}$.

The process x(t) is observed in the presence of white noise by a system of measurements described by the equation

$$\dot{z} = x(t) + bv(t) \tag{2E}$$

v(t) is a random process of the white noise type with the parameters $E\{v\} = 0$ and $E\{v^2\}$.

The processes $\omega(t)$ and v(t) are statistically independent. On the basis of the measurements $z(\tau)$, $0 \leq \tau \leq T$, we are required to obtain optimal estimates of the coordinates x(t), which are optimal in the sense that they minimize

$$L(x, \hat{x}) = \|x(t) - \hat{x}(t)\|^2 \tag{2Ea}$$

For this purpose, we have to determine the first and second moment of the a posteriori density $p_{x/z}(x, t)$.

The first moment

$$\hat{x}(t) = \int x p_{x/z}(t)\, dx \tag{2Eb}$$

is an optimal estimate, and the second

$$D(t) = \int (x(t) - \hat{x}(t))^2 p_{x/z}(t)\, dx \tag{2Ec}$$

is its minimum deviation.

The a posteriori probability according to (14) satisfies, in the case of Example 1E and 2E, the stochastic differential equation

$$\frac{dp_{x/z}(t)}{dt} = \left(-c\frac{\partial p_{x/z}(t)}{\partial x} + \frac{a^2 \partial^2 p_{x/z}(t)}{\partial x^2}\right) + \frac{p_{x/z}(t)}{b^2}(x - \hat{x}(t)) \times (\dot{z}(t) - \hat{x}(t)) \tag{3E}$$

with the initial conditions $p_{x/z}(x, 0/0) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\hat{x}_0)^2}{2\sigma^2}\right\}$.

The first term on the right-hand side of (3E) is the right-hand side of the direct Kolmogorov equation, and the second is a result of measurements.

Equation (3E) for $p_{x/z}(t)$ yields an equation for x(t) and D(t)

$$\dot{\hat{x}}(t) = c + D(t)(\dot{z}(t) - \hat{x}(t)) \tag{4E}$$

$$\dot{D}(t) = a^2 - \frac{D^2(t)}{b^2} \tag{5E}$$

The solution of the problem is thus given by a system of nonlinear differential equations whose analytic solution in closed form may be obtained only in certain particular cases. Nevertheless, analog and digital computers can be used without too great difficulty in conjunction with an algorithm for the processing of data. Using digital computers the operations on the data are in the form of recursive computational procedures, using at each step only the current data, which means that the required estimates can be obtained with a delay equal to the machine time of computation for one measurement.

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AN ANALYSIS OF A TYPICAL STRUCTURE OF AN AUTOMATIC CONTROL SYSTEM AND A METHOD OF SELECTING THE TRANSFER FUNCTION FOR A STANDARD MODEL OF A SELF-ADJUSTING SYSTEM

N. A. Shokalo

In designing a self-adjusting control system, one has to analyze the original control system to discover the possibilities of satisfying the T. T. Z.* with respect to the performance indicators of the transfer process, which may be in the form of:

1. Maximum over-regulation of the bank angle (γ) no greater than +15% of γ_{giv}.
2. The length of the transfer process no greater than 2 - 3 sec with a deviation of the bank angle from its steady value by no more than ±5%.
3. Maximum over-regulation of the bank rate (ω_x) no greater than +50%.

Methods of the theory of sensitivity can be used to solve this problem for a system of any order, but the solution is hindered by the necessity of choosing a reference transfer function for the control system [1, 3].

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* Translator's Note: Expansion unknown.

The purpose of this paper is to select a reference transfer function for a control system. For this, it is necessary to establish a relation between the performance indicators for the transfer process with a distribution of the poles and zeros, and the coefficients of the transfer function for a typical control system of fifth degree in the denominator and third degree in the numerator.

$$W_{3c} = \frac{A_2 p^3 + A_3' p^2 + A_4' p + A_5'}{p^5 + A_1 p^4 + A_2 p^3 + A_3 p^2 + A_4 p + A_5} \quad (1)$$

To solve the problem, we have to vary simultaneously from 5 to 8 coefficients of the transfer function. The second problem to be solved consists of selecting the optimal coefficients of the control system's regulator and the reference model such that certain quality tests will be minimized.

Mutual Relationship among the Coefficients of the Transfer Function on the Boundary of the Region of Transfer Processes of Given Quality

The transfer function is studied using the method of mathematical models. The problem to be solved can be simplified by fixing coefficients in the denominator A_1, A_2, A_3, which contain only the time constants of the object and the director, or by fixing the coefficients A_4 and A_5, containing the typical regulator and control surface efficiency coefficients. The coefficients are varied successively from the average value in both directions, and the variation is limited by tolerances in the given quality characteristics. Experiments serve to establish the range of the coefficients of a normalized and unnormalized tranfer function of the system, and thus we could construct the grids of the regions of transfer processes of given quality in the coordinates of any two denominator coefficients. The effect of each numerator coefficient on the dimensions of the regions was determined, and the restrictions on the coefficients were selected.

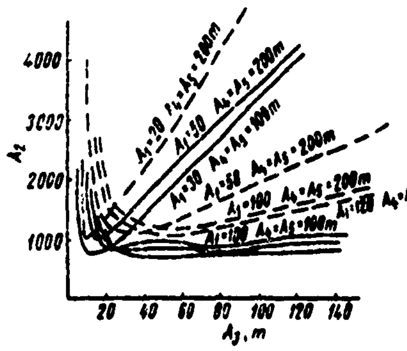

Figure 1.

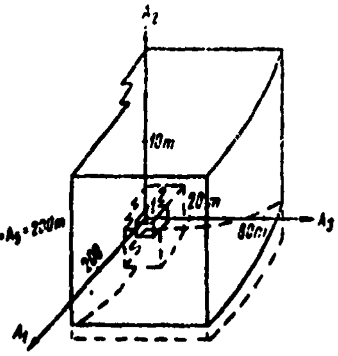

Figure 2.

The relationships thus obtained permit us to claim that:

1. The transfer processes satisfy a given performance level for a wide range of the first three coefficients A_1, A_2, A_3 (Figures 1 and 2). For comparison, we plotted the volumes "1" and "2," which define the maximum possible variations of the coefficients A_1, A_2, A_3 depending on the time constant of a real director.

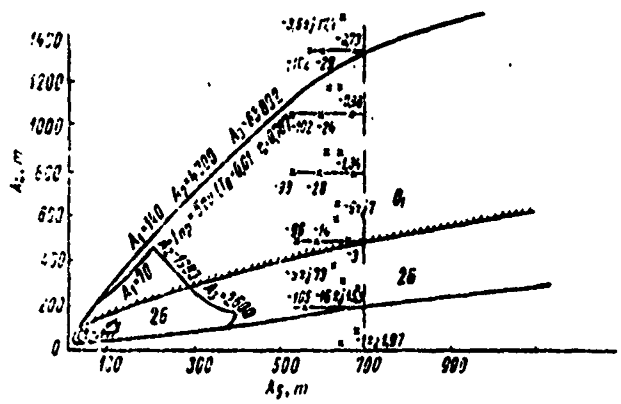

Figure 3.

2. The "lower" boundary of the region with the two last coefficients in the denominator A_4 and A_5 being fixed is determined by the high frequency oscillation. In order to assure a given level of performance, the coefficients A_1, A_2, A_3 must be selected such that the boundary of the high frequency instability will shift beyond the area determined by the performance indicators, since for values close to those on the boundary the region of performance shifts toward the origin and has small dimensions as compared with the stability region.

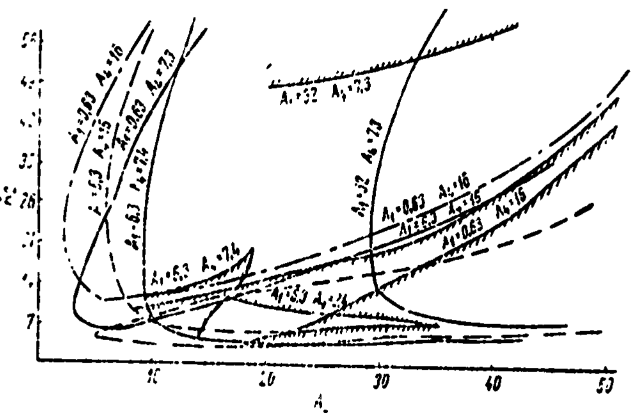

Figure 4.

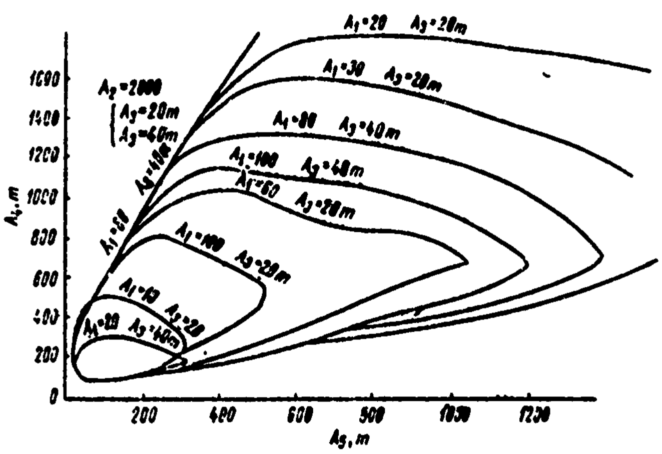

Figure 5.

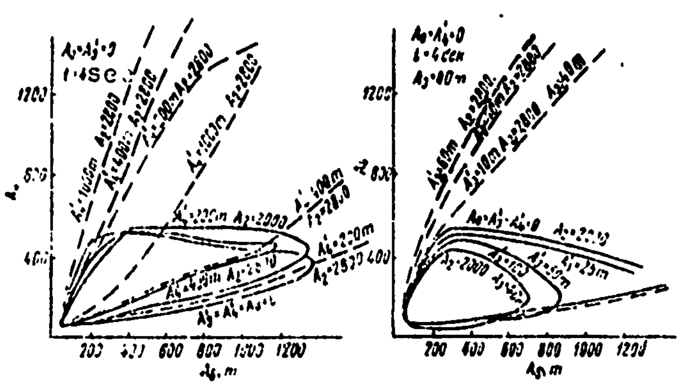

Figure 6a, b.

3. The region of transfer processes of given performance may be widened in two ways:

a) by increasing all three coefficients A_1, A_2, A_3 simultaneously (Figure 3);

b) by fixing the first coefficient A_1 and simultaneously increasing the coefficients A_2, A_3 (Figure 5).

4. It is found that an increase of the coefficient A_3 in the coordinates A_4, A_5 widens the region of transfer processes of given performance only in the direction of the coefficient A_5, and a correction to increase the coefficient A_2 is necessary (Figure 6).

5. When analyzing the functional relationship along the boundary of the high-frequency oscillation between the coefficients A_2 - A_4 and A_3 - A_5, we find a relationship which is close to linear. By expressing this relationship as a formula, one can reduce the number of the independently varied coefficients of the equation. The modeling scheme is shown in Figure 7.

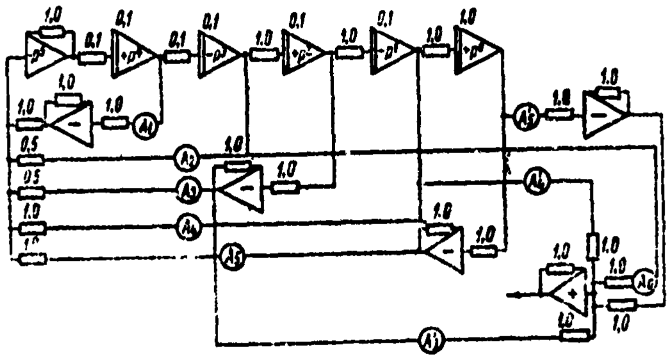

Figure 7.

An Investigation of the Distribution of the Poles of the Transfer Function Inside the Region of Given Performance

On the grids of the selected regions of transfer processes of given performance, we have plotted the regions where the roots are located and constructed the boundary curves that separate them (Figures 3 and 4). In constructing the regions with the roots, we have used a procedure described in [2]. The classes of the roots were determined as proposed by A. A. Fel'dbaum: 2A is a class with one real root and two pairs of complex roots, the nearest complex roots with respect to the imaginary axis. 2B is a class with the real root nearest to the imaginary axis and two pairs of complex roots. O_1 is a class with three real roots and one pair of complex roots, as well as the real root nearest to the imaginary axis; O is a class in which all five roots are real.

The method is essentially as follows. If the characteristic equation of a closed system has the form

$$\sum_{k=0}^{n} A_k p^{n-k} = 0, \text{ where } A_0 = 1 \qquad (2)$$

then each set of coefficients A_k (k = 1, 2 ... n) is associated with a single class of distribution of roots of Equation (2). We know from higher algebra that the necessary and sufficient condition for Equation (2) to have r identical roots is that the polynomial in (2) and its

$(r - 1)^{th}$ derivative vanish simultaneously when the multiple root is substituted in them. To simplify the problem, we set r = 2. Thus, when the multiple root is substituted, both the polynomial in (2) and its first derivative should vanish. Thus, we have two equations

$$\sum_{k=0}^{n} A_k \mu_0^{n-k} = 0; \quad \sum_{k=0}^{n} A_k (n-k) \mu_0^{n-k-1} = 0, \qquad (3)$$

where $A_0 = 1$.

Letting μ_0 take on values from 0 to $-\infty$ we obtain various sets of the values of coefficients satisfying System (3) when μ_0 is substituted in the latter. Each of the sets is, in the hyperspace of parameters, a figurative point lying on the boundary separating the subregions with a different combination of roots. The boundary of the subregion with a different combination of roots is constructed in the plane of any two parameters, where the remaining parameters are fixed, i.e., one intersects the hyperspace of parameters by a plane of two parameters A_l and A_m. One derives an expression for the boundary curve [4] separating the subregions with a different combination of roots in the canonical form, and its behavior is considered by studying the derivative of the boundary curve. The shading of the boundary curve from $\mu_0 = 0$ to $\mu_0 = -\infty$ is done from the left if $(m - l)$ is even, and from the right if $(m - l)$ is odd in case the "m" coefficient lies on the axis of ordinates.

The distribution of the zones of roots in the coordinates of any two normalized coefficients permits us to establish that the coefficients A_2, A_3, A_4 have the greatest effect on the redistribution of the zones of the curve inside the selected region. It is found that in the coordinates A_4, A_5, which may first of all include the parameters of the regulator, a wide performance region can be obtained in two ways:

1. If the director has a large time constant and the values of A_1, A_2, A_3 are close to the boundary of the high-frequency oscillation, then fixing the coefficient A_1 one can correct the coefficients A_2

and A_3 in such a way that the boundary of the high-frequency oscillation will move beyond the region of transient processes of given performance. In this case, the values of the coefficients A_2 and A_3 should be equal to $A_2 = 2000$, $A_3 = 30,000$. The region of the transient processes is here filled with roots of the 2B type (with the real root nearest to the imaginary axis and two pairs of complex conjugate roots). The region of the roots O_1 is small and is close to the coordinate origin (Figure 6).

2. A wide region of transient processes of given performance can be obtained by simultaneously increasing the values of three coefficients A_1, A_2, A_3, i.e., by bringing their values closer to the values corresponding to the drive constant $T_{dr} = 0.03$ sec and the booster constant $T_b = 0.015$ sec. In this case the entire region of given performance, with the exception of a narrow lower strip, is filled with roots of type O_1 (where the real root turns out to be nearest to the imaginary axis, followed by a pair of complex conjugate roots, and by two real roots). It is important to know that by varying the values of A_1, A_2, A_3 one can superpose the regions of the distribution of O_1 roots on the region of transient processes of given performance (Figure 5). Thus, having the grids of transient processes of a given performance, and given the aerodynamics and parameters of the real director of an aircraft, one can determine in each case the values of the nominal coefficients of the transfer function, the nominal distribution of roots, and thus the nominal transfer function of the fundamental system, which may be used as the transfer function of the reference model of a self-adjusting system.

Tests of the Optimization of the Coefficients of a Control System Regulator and a Reference Model

The optimization of the coefficients of the control system regulators and the reference model of self-adjusting systems is accomplished by calculating the partial derivatives of the phase coordinates in terms of the parameters of the regulator in the two basic regimes of the control system: damping (pilot's manual control) and stabilization. Partial derivatives can be best calculated by using the method of the sensitivity function models which may be linear or nonlinear. The advantage of the method is that the extremal values of the coefficients are obtained fast, and it is not necessary to vary those parameters by trial and error. The regulator coefficients are optimized in the presence of two actions: control γ_{giv}, X_f and wind (f_w). As the optimization test, we propose a square integral test determining the minimum loop error in the stabilized regime

$$I=\int_0^T E_i^2(t,\ \mu,\ i)\,dt, \text{ where } E_i=\gamma_{зз}-\gamma \tag{4}$$

μ is the regulator coefficient in the damping loop, i is the regulator coefficient in the stabilization contour. The gradients will be

$$\nabla_\mu I=2\int_0^T E_i(t,\ \mu)\frac{\partial E_i}{\partial\mu}(t,\ \mu)\,dt \tag{5}$$

$$\nabla_i I=2\int_0^T E_i(t,\ \mu)\frac{\partial E_i}{\partial i}(t,\ i)\,dt \tag{6}$$

The extremal values of the coefficients μ^* and i^* for equal values of integral estimates (Figure 8) correspond to the values of μ, lying in the selected region of transient processes of given quality, and the values of i^* lie beyond the performance region — for example, in the middle of the stability region. With wind disturbances (of the same spectrum as the control ones) the optimal values of μ^*, i^* lie at the upper boundary of the stability region (Figure 12). Thus, the test $I=\int_0^T E_i^2 dt$ determines the minimum of the stabilization error. For a flight in a turbulent atmosphere, this test may result in considerable overloads acting on the pilot and large values of vane deflections. For aircraft, the damping loop may usually be considered to be autonomous by using its integral performance test to optimize its coefficient μ. This is advisable if one wants to adjust the coefficient μ independently of the adjustment of the stabilization loop coefficient i.

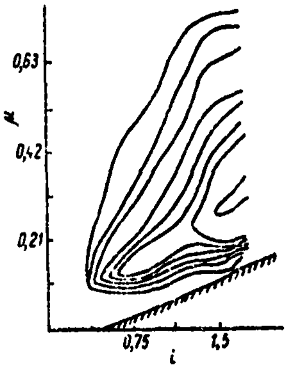

Figure 8.

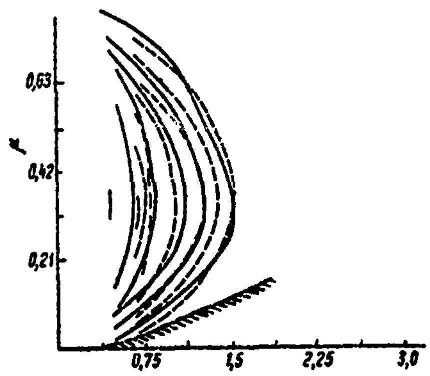

Figure 9.

In this case, we can take the following integral tests:

$$I = \int_0^T E_\mu^2 dt \text{ where } E_\mu = X_{лет} - 3 \tag{7}$$

$$I = \int_0^T \delta_{б\,ст}^2 dt \quad I = \int_0^T (\dot{\omega}_x)^2 dt$$

The phase coordinate E_μ characterizes the loop error for manual control, the second phase coordinate δ_b characterizes the deflection of vanes, and is of practical interest since it can be measured when forming the integral estimate. The third phase coordinate ω_x characterizes the minimum overloads acting on the pilot. In the case of a sufficiently fast director (f_{dr} - 4 - 5 Hz), these estimates turn out to be close (Figures 9, 11). The values of μ^* and i^* for the tests being minimized lie within the region of transient processes of given performance. In addition, the value of μ^* is inversely proportional to the effectiveness of the vanes as the flight regimes change.

The advantages of the practical test $I = \int_0^T \delta_b^2\, dt$ are as follows:

1. Independence of the adjustment of coefficients μ and i.

2. Sufficient stability margins relative to μ and i, corresponding to the desires of the pilot (in the linear regime),

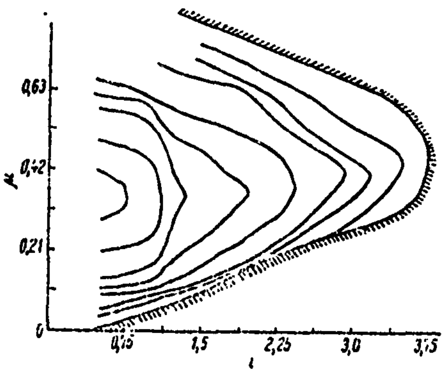

Figure 10.

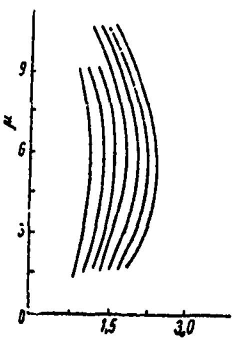

Figure 11.

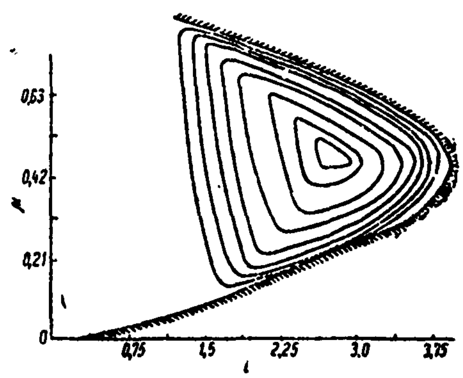

Figure 12.

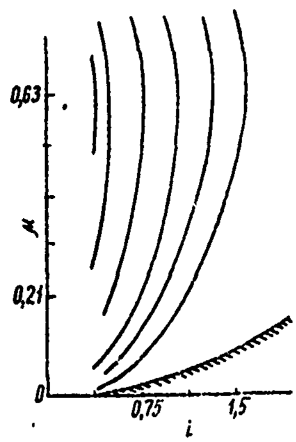

Figure 13.

3. Constancy of the stability margins as the regimes of flight change for both the control (γ_{giv}, X_f) and wind disturbances (of the same spectrum). (Figures 9, 10, and 12).

4. The possibility of realizing a fairly simple self-adjusting scheme (when adjusting the coefficients μ and i by different tests).

5. The average deflection of vanes is minimized, which is implied by the test itself.

The shortcomings of the test include a lower accuracy of regulation in the stabilized regime, and a higher necessary value of the coefficient μ in the nonlinear regime (Figure 13).

Thus, from the point of view of satisfying the requirements on the processes of given performance, of minimizing the loads acting on the pilot, and the deflections of the vanes, we see that the efficient test involves optimizing the coefficients of the automatic pilot

$$I=\int_0^T E^2\mu\,dt \quad \text{or} \quad I=\int_0^T \delta^2_{\text{бует}}\,dt \tag{8}$$

In the stabilized regime during bombing, the test $I=\int_0^T E_i^2 dt$ is efficient or (in the case of wind disturbances or nonlinear control systems) its modified version is

$$I=\int_0^T (E_i+\tau\dot{E}_i)^2\,dt. \tag{9}$$

To achieve independent adjustment of the coefficient of the damping loop regulator, it is advisable to use different performance tests in the stabilized regime for the adjustment of the coefficients μ and i.

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AN APPROXIMATE PROCEDURE FOR DESIGNING A SEARCHLESS SELF-ADJUSTING SYSTEM SENSITIVE TO THE PROPER FREQUENCIES OF THE BASIC CONTROL LOOP

B. V. Viktorov, G. N. Izmaylov, B. V. Kirsanov, V. A. Pokhvalenskiy

The analysis and synthesis of complicated control systems is more often than not extremely laborious or completely intractable analytically. For this reason, it is of great practical importance to develop approximate procedures based on permissible simplifications.

One of the possible ways of investigating nonperiodic processes in complex systems is the method of separation of composite motion. It is based on the theory of differential equations with a small parameter multiplying the derivatives. Its application enables us to lower the order of the original system of equations and estimate the accuracy of the approximate solution [5].

The system under investigation is shown in Figure 1. This system, known in the literature as the Marx system [1], was studied in quite a few papers [2,3,4], and is used at the present time in aircraft.

In this paper, we do not investigate the dynamics of the system, its stability, or the performance of the adjustment processes. We shall only be interested in the possible ways of obtaining the analytic relationship

$$\mu_{\text{ст}} = \mu(T_{\omega}, \xi_{\omega}, T_{\omega}, T_{\text{п}}, \xi_{\text{п}}, T_{\text{в}}, \xi_{\text{в}}, T_{\text{н}}, \xi_{\text{н}}, T) \qquad (1)$$

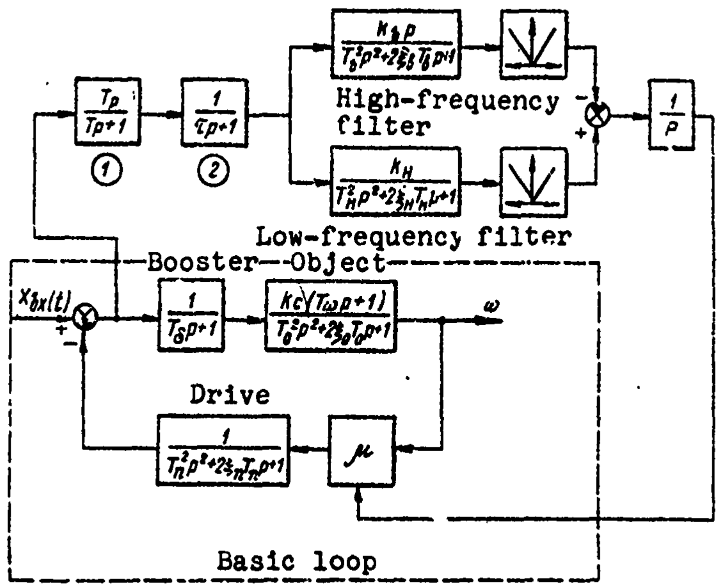

Figure 1.

where μ_{adj} is the adjusted value of the amplification coefficient, T_o, T_{dr}, T_{up}, T_{low}, T are the time constants of the object, drive, upper filter, lower filter, and forming filter, respectively; ξ_o, ξ_{dr}, ξ_{up}, ξ_{low} are the damping coefficients of the object, drive, lower filter, and upper filter, respectively.

To solve the problem, we propose to make a number of simplifications that reduce to the following:

1. As shown by simulation, stand, and flight tests of this type of system, wind disturbances have hardly any effect on the adjustment processes and on the steady-state values of the coefficient μ (with the possible exception of the takeoff and landing regimes). Therefore, the pilot's signal in the course of piloting was taken as the test signal.

2. The disturbance used as the input into the control system was taken in the form of a single function. Stand tests showed the correctness of this simplification. The error in the adjusted value

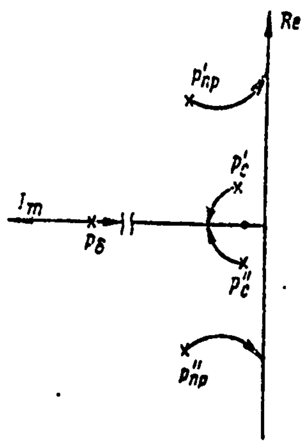

Figure 2.

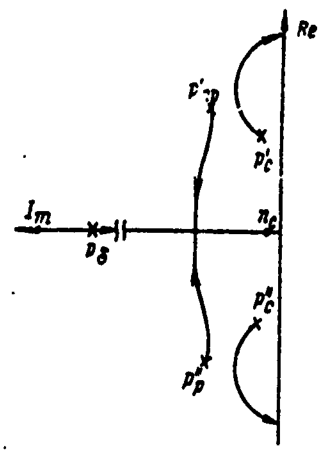

Figure 3.

of the coefficient under various actions (also including manual piloting) did not exceed 10%.

3. The dynamics of damping loops of present-day aircraft is such that one can distinguish two qualitatively different types of flight regimes:

a) regimes with a fairly large scatter of the roots of the drive and the aircraft. The root hodograph, illustrating this type of regimes, is shown in Figure 2. In this case, the control law assumed here is satisfied with sufficient accuracy;

b) regimes with a small scatter of the roots of the drive and the aircraft. In this case, the control law $\delta = \mu\omega$ might be said to change to $\delta \cong \mu\alpha$. Here δ is the angle of deflection of the control surfaces of the object, ω is the angular velocity, α is the angle of attack. This type of regimes is illustrated by the root hodograph in Figure 3.

Regimes of this type do not satisfy the basic requirement on the control system — namely, that the drive should act faster than the control object.

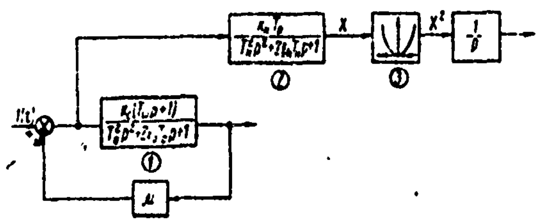

Figure 4.

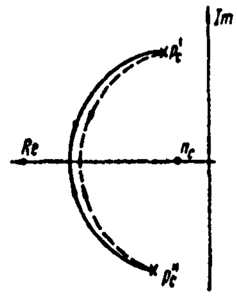

Figure 5.

Recently there has been a tendency to increase the speed of operation of the drives. In view of this fact, we shall consider only the "slow" regimes or "fast" ones in which filters or special control laws are used to obtain differences in motion.

The procedure we propose is based on extracting from the general transfer function (of fifth degree) two component second-degree functions which reflect the characteristic types of motion: aircraft ("slow" and drive ("fast").

Strictly speaking, three types of motion occur in the loop under consideration. In the order of speed of action, we have the aircraft, drive, and booster motions.

The fastest motion (booster motion) is not taken into account when considering the aircraft motion. When considering the drive motion, the booster motion may be taken into account in the form of several terms in the expansion of the transfer function.

The zero-order approximation is used as the transfer function generating the "slow" component of composite motion. Structure 1 in Figure 4 illustrates how this approximation is obtained. In principle, one could also take the first-order approximation in which the drive is represented by the expansion (which does not increase the order of the system)

$$W_n(p) \approx 1 - 2\xi_n T_n p \tag{2}$$

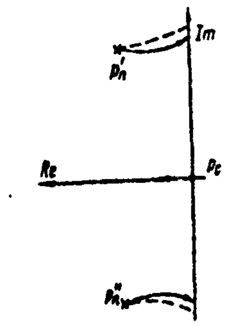

Figure 6.

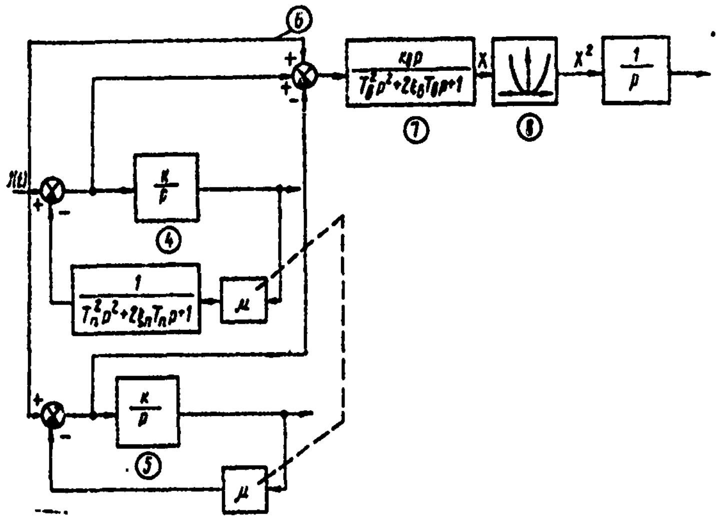

Figure 7.

The root hodograph of aircraft roots with real and inertialess drives is shown in Figure 5. The transfer function, generating the "slow" motion, is written as

$$W_{\text{м}}(p) = \frac{T_0^2 p^2 + 2\xi_0 T_0 p + 1}{T_0^2 p^2 + (2\xi_0 T_0 + k_c T \cdot \mu) p + 1 + k_c \mu} \qquad (3)$$

The "fast" component of the composite motion is conveniently taken in the form of the first-order approximation. Then the drive (the unit generating this type of motion) is closed through the object taken in the form of an integrating unit. The root hodographs of the drive roots in real and idealized schemes, confirming this approximation, are shown in Figure 6.

The formation of the "drive" transfer function is shown in Figure 7. Loop 4 does the "generation," and loop 5 extracts the desired motion.

Analytically this has the following form. We assume that the transfer function of loop 4

$$W_4(p) = \frac{p\left(T_n^2 p^2 + 2\xi_n T_n p + 1\right)}{p\left(T_n^2 p^2 + 2\xi_n T_n p + 1\right) + k\mu} \tag{4}$$

consists of a sum of the desired W_b and the transfer function of loop 5, which is the image of the "slow" motion

$$W_5(p) = \frac{p}{p + k\mu} \tag{5}$$

The denominator of the unknown transfer function is determined by the expansion of the function

$$\frac{p\left(T_n^2 p^2 + 2\xi_n T_n p + 1\right) + \mu k}{p + \mu k} = T_n^2 p^2 + \left(2\xi_n T_n - T_n^2 \mu\right) p + \\ + 1 - \mu\left(2\xi_n T_n - \mu T_n^2\right) + \sum_{i=1}^{\infty} a_i p^{-i} \tag{6}$$

in a Laurent series. Equation (6) can be obtained by dividing the denominator of the transfer function $W_4(p)$ by the denominator of $W_5(p)$.

Neglecting the remainder in (6), we obtain an approximate expression for the denominator of the unknown transfer function. An analysis of the transient error processes for the system in Figure 1 shows that, taking into account the transfer function for the "slow" component, $W_b(p)$ should have the form

$$\frac{k_1 p}{T_n^2 p^2 + \left(2\xi_n T_n - T_n^2 \mu\right) p + 1 - \mu\left(2\xi_n T_n - \mu T_n^2\right)} \tag{7}$$

where $k_1 \approx 2\xi_{dr} T_{dr}$ is obtained using the method of indeterminate coefficients and the condition that the original $W_4(p)$ is approximately equal to the sum of the approximate transfer functions in (5) and (7).

Thus, the reaction of the original system has been decomposed into two superimposed components, given by the transfer functions in (3) and (7).

4. Next, we assume that both components of the composite motion are sufficiently different in speeds of action. This assumption enables us to direct each one of them to the "lower" and "upper" filters, respectively of the spectral analyzer.

The forming filters are considered as follows:

The value of the root corresponding to Loop 1 in Figure 1 is chosen so that its modulus lies between the roots of the drive and the aircraft, and the value of the root corresponding to Loop 2 is much greater than the modulus of the drive root.

In view of the above, in the case of a "slow" signal, the forming filters 1, 2 in Figure 1 may be accounted for by the differential loop Tp, 2 in Figure 4. In fact, a "slow" signal is a sum of a stepwise component and a component that increases from 0 [see Equation (3)]. The stepwise signal is separated at the input to the low-frequency filter 2 in Figure 4 in the form of a short (as compared with the time of the transient process in the low-frequency filter) pulse, whose effect on filter 2 will be practically identical with the effect of a delta function (since the areas of the delta function and the real pulse are identical and equal to T). However, the "flat" part of the signal, in view of its small slope, will practically be differentiated by Loop 1 (Figure 1) and passed without distortions by Loop 2.

The "fast" signals, due to its high "speed of action," will be passed by either filter with hardly any distortion.

The above is reflected in the structural schemes in Figures 4 and 7, showing two separate channels along which the input signal proceeds.

In addition, to account for the effect of the stepwise component of the "slow" signal [see Equation (3)] on the high-frequency filter 7 in Figure 7, we introduced an additional single connection 6.

5. To make an analytic calculation of the possible scheme, the moduli were replaced by quadratures 3 in Figure 4, and 8 in Figure 7.

A simulation of the complete Figure 1 and the separated Figure 4 and 7 showed a good agreement between the results with respect to the adjusted value of the coefficient μ_{adj}.

An error in the value of μ_{adj} was no greater than 10% in various regimes.

It is important that, when the convolution integral is applied (in a complex domain), the order of the original integrand in the exact scheme (Figure 1) is nine. When the procedure proposed here is used, the order is lowered to four.

6. The subsequent design of the system can be made by finding the quadratic estimate at the output from each of the channels. This permits us to obtain an analytic expression which relates the basic parameters of the system.

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AN INVESTIGATION OF ADAPTIVE CONTROL SYSTEMS FOR RANDOM ACTIONS

M. F. Rosin and V. I. Ul'yanov

At the present time, adaptive control systems are studied statistically by using the interpolation method [3], the method of equivalent perturbations [4], and the method of coefficients [1]. Among the methods in which the output signal is represented by polynomians, the interpolation method has the highest accuracy. In this method, the interpolation points are taken to coincide with the roots of the orthogonal polynomials, with the weights equal to the coefficient distribution densities in the canonical expansion of the input signal to the control system.

The accuracy of the probabilistic characteristics obtained by the interpolation method is checked by comparing the results for various degrees q of the approximating polynomial. The entire computational procedure is repeated again for a new value of q, since the optimal interpolation points change. In contrast with the interpolation method, the method of coefficients uses the results of previous simulations. This lowers substantially the amount of work involved in the calculation of the probabilistic characteristics of the adaptive control system. For example, for $q = 4$ and $\rho = 7$ in the interpolation method and the method of coefficients, one has to carry out 2187 and 113 simulations of the control system, respectively.

However, due to the nonoptimal selection of the approximating points, the method of coefficients is inferior in accuracy to the interpolation method. One can increase the accuracy of the coefficient method by taking into account different effects of the coefficients in the canonical expansion of the input signal, and by combining the coefficient method with the interpolation method.

Determination of the Probabilisitic Characteristics of a Control System by Taking into Account Various Effects that the Coefficients in the Canonical Expansion of the Input Signal Have on the Output Signal

The selection of the degree q in each random coefficient in the coefficient method is achieved by improving the accuracy of the probabilistic characteristics of the control system, and does not require any additional simulations of the control system. Let us select Chebyshev-type points in such a way that their number will be sufficient to determine the coefficients in the expansion of the output signal in a Maclaurin series. These points satisfy the requirements of the coefficient method: they lie within the ranges of the real values of the random coefficients in the canonical expansion of the input signal, and the output signals for them are not very small.

Let us consider the procedure in which the variance of the output signal from an adaptive control system is calculated taking into account different effects of the random expansion coefficients.

We have

$$D_y(t)=M\{\Psi(t, X_1, X_2, \ldots, X_n)\} \quad (1)$$

Here

$$\Psi(t, X_1, X_2, \ldots, X_n)=[Y(t, X_1, X_2, \ldots, X_n)-m_y(t)]^2 \quad (2)$$

$X_1, X_2, \ldots, X_\rho$ are the random coefficients of the canonical expansion of the input signal with the variances

$$M[X_1^2]=\sigma_1^2,\ M[X_2^2]=\sigma_2^2,\ \ldots,\ M[X_\rho^2]=\sigma_\rho^2. \tag{3}$$

The expansion coefficients will be assumed to be distributed normally. The degrees $q_1, q_2, \ldots, q_\rho$ of the expansion coefficients $X_1, X_2, \ldots, X_\rho$ in the polynomial approximating the function $\Psi(t, X_1, X_2, \ldots, X_\rho)$ are unknown. To a first approximation, we assume

$$q_1=q_2=\ldots=q_\rho=2. \tag{4}$$

According to the coefficient method, we find the first approximation to the variance of the output signal from the control system

$$D_y^0(t)=\Psi(t,0,0,\ldots,0)+\sum_{k=1}^{\rho}\frac{1}{2}[\Delta_k(t,x_k+\Delta_k(t-x_k)]\frac{\sigma_k^2}{x_k^2} \tag{5}$$

Here

$$\Psi(t,0,0,\ldots,0)=\Psi(t,x_1,x_2,\ldots x_\rho)=A_0(t)$$
$$\text{for}\quad x_1=x_2=\ldots=x_\rho=0 \tag{6}$$

$$\Delta_k(t,x_k)=\Psi_k(t,x_k)-\Psi(t,0,0,\ldots,0)$$
$$\Psi_k(t,x_k)=\Psi(t,x_1,x_2,\ldots,x_\rho) \tag{7}$$

$$\text{for}\quad x_1=x_2=\ldots=x_{k-1}=x_{k+1}=\ldots=x_\rho\equiv0 \text{ and } x_k\neq0 \tag{8}$$

Chebyshev-type points are chosen as the concrete values of the random coefficients [3].

Process No.	Values of $X_1, X_2, \ldots, X_\rho$			
	x_1	x_2	......	x_ρ
1	$+1{,}732\sigma_1$	0	......	0
2	$-1{,}732\sigma_1$	0	......	0
3	0	$+1{,}732\sigma_2$	......	0
4	0	$-1{,}732\sigma_2$	......	0
......	......	......	......	......
$2\rho-1$	0	0		$1{,}732\sigma_\rho$
2ρ	0	0		$-1{,}732\sigma_\rho$
$2\rho+1$	0	0	0	2

We increase the degree q in the expansion of the function $\Psi(t, x_1, \ldots, x_\nu, \ldots, x_\rho)$ in a Maclaurin series in only one ("large") random coefficient x_ν, so that the highest degree of the approximating polynomial of the function $\Psi(t, x_1, x_2, \ldots, x_\nu, \ldots, x_\rho)$ does not exceed four. The algorithm in which the random coefficients are divided into "large" and "small" is given in Section 2.

$$\Psi(t, x_1, \ldots, x_\nu, \ldots x_\rho) = A_0(t, x_\nu) + \sum_{\substack{k_1=1\\ k_1\neq\nu}}^{\rho} A_{k_1}(t, x_\nu)\cdot x_{k_1} + \frac{1}{2}\sum_{\substack{k_1=1\\ k_1\neq\nu}}^{\rho}\sum_{\substack{k_2=1\\ k_2\neq\nu}}^{\rho} A_{k_1k_2}(t, x_\nu)\cdot x_{k_1}\cdot x_{k_2} \tag{9}$$

$$A_0(t, x_\nu) = A_0(t) + A_0^{x_\nu}(t)\cdot x_\nu + \frac{1}{2}A_0^{x_\nu^2}(t)\cdot x_\nu^2 + \frac{1}{3!}A_0^{x_\nu^3}(t)\,x_\nu^3 + \frac{1}{4!}A_0^{x_\nu^4}(t)\,x_\nu^4 \tag{10}$$

$$A_{k_1}(t, x_\nu) = A_{k_1}(t) + A_{k_1}^{x_\nu}(t)\cdot x_\nu + \frac{1}{2}A_{k_1}^{x_\nu^2}(t)\,x_\nu^2 + \frac{1}{3!}A_{k_1}^{x_\nu^3}(t)\,x_\nu^3 \tag{11}$$

$$A_{k_1k_2}(t, x_\nu) = A_{k_1k_2}(t) + A_{k_1k_2}^{x_\nu}(t)\,x_\nu + \frac{1}{2}A_{k_1k_2}^{x_\nu^2}(t)\,x_\nu^2 \tag{12}$$

Substituting Equations (10), (11), (12) in Equation (9), we have

$$\begin{gathered}\Psi(t, x_1, \dots x_\nu, \dots x_\rho) = A_0(t) + A_0^{x_\nu}(t)\, x_\nu + \frac{1}{2} A_0^{x_\nu^2}(t)\, x_\nu^2 + \\ + \frac{1}{3!} A_0^{x_\nu^3}(t)\, x_\nu^3 + \frac{1}{4!} A_0^{x_\nu^4}(t)\, x_\nu^4 + \sum_{\substack{k_1=1 \\ k_1 \neq \nu}}^{\rho} A_{k_1}(t) + A_{k_1}^{x_\nu}(t)\, x_\nu + \\ + \frac{1}{2} A_{k_1}^{x_\nu^2}(t)\, x_\nu^2 + \frac{1}{3!} A_{k_1}^{x_\nu^3}(t)\, x_\nu^3]\, x_{k_1} + \frac{1}{2} \sum_{\substack{k_1=1 \\ k_1 \neq \nu}}^{\rho} \sum_{\substack{k_2=1 \\ k_2 \neq \nu}} [A_{k_1 k_2}(t) + \\ + A_{k_1 k_2}^{x_\nu}(t)\, x_\nu + \frac{1}{2} A_{k_1 k_2}^{x_\nu^2}(t)\, x_\nu^2]\, x_{k_1 k_2}\end{gathered} \tag{13}$$

Applying the expectation operator to Equation (13), we obtain

$$\begin{gathered}D_y^1(t) = D_y^0(t) + \Delta D_y^{x_\nu}(t) = A_0(t) + \frac{1}{2} A_0^{x_\nu^2}(t) \cdot M\{x_\nu^2\} + \frac{1}{4!} A_0^{x_\nu^4}(t) \cdot M\{x_\nu^4\} + \\ + \frac{1}{2} A_{k_1 k_1}^{x_\nu^2}(t)\, M\{x_\nu^2\} + \frac{1}{2} \sum_{\substack{k_1=1 \\ k_1 \neq \nu}}^{\rho} ([A_{k_1 k_1}(t) + \frac{1}{2} A_{k_1 k_1}^{x_\nu^2}(t)\, M\{x_\nu^2\}]\, M\{x_{k_1}^2\}\end{gathered} \tag{14}$$

The coefficients $H_{k_l k_1}(t)$ are determined from the known formulas by using the results of the previous simulations of the control system. The coefficients $A_{k_1 k_1}^{x_\nu^2}(t)$ are calculated using the formulas

$$\begin{gathered}A_{k_1 k_1}^{x_\nu^2}(t) = \frac{4!}{6 x_\nu^2 \cdot x_{k_1}^2} \Big\{\frac{1}{4} [\Delta_{\nu, k_1}(t, x_\nu, x_{k_1}) + \Delta_{\nu, k_1}(t, -x_\nu - x_{k_1}) + \\ + \Delta_{\nu, k_1}(t, -x_\nu - x_{k_1}) + \Delta_{\nu, k_1}(t, x_\nu - x_{k_1})] - \frac{1}{2} [\Delta_\nu(t, x_\nu) + \\ + \Delta_\nu(t, -x_\nu)] - \frac{1}{2} [\Delta_{k_1}(t, x_{k_1}) + \Delta_{k_1}(t, -x_{k_1})]\end{gathered} \tag{15}$$

The number of simulations of the control system for one "large" random coefficient as compared with the first approximation to the variance $D_y^0(t)$ is increased by

$$K_{\text{доп}}^1 = 4(\rho - 1). \tag{16}$$

Since the Chebyshev-type optimal points were chosen as the approximation points, some of the terms in the variance (14) can be calculated using the interpolation method

$$M\{\Psi(t,0,0,\ldots,x_\nu,\ldots,0)=A_0(t)+\frac{1}{2}A_0^{x_\nu^2}(t)M(x_\nu^2)+$$
$$+\frac{1}{4!}A_0^{x_\nu^2}(t)M(x_\nu^4)=\Psi(t,0,\ldots,0)\rho_0+\Psi(t,0,\ldots,1{,}732\sigma_\nu,\ldots,0)\rho_1+ \tag{17}$$
$$+\Psi(t,0,\ldots,-1{,}732\sigma_\nu,\ldots,0)\rho_2$$

Here ρ_0, ρ_1, ρ_2 are the Christoffel numbers used in the normal distribution of the random coefficient with the following values [3]

$$\rho_0=0{,}666;\ \rho_1=0{,}166;\ \rho_2=0{,}166. \tag{18}$$

A partial application of the interpolation method permits us to reduce the number of simulations of the control system by $2\cdot K_l$; K_l is the number of "large" coefficients in the expansion. If the increase in the variance of the control system $\Delta D_y^{x_\nu}(t)$, caused by an increase of the degree q_ν relative to the "large" random coefficient x_ν, does not exceed $K\cdot D_y^0(t)$, then in the case of the random coefficient x_ν, we limit ourselves to the obtained value of q_ν.

The coefficient K characterizes the accuracy to which the variance is determined. It can be, for example, taken as equal to K = 0.05. If $\Delta D_y^{x_\nu}(t) \geq KD_y^0(t)$, then one should consider — in the expansion of the output signal — terms of a higher degree q_ν in the random coefficient x_ν. Upon finding $\Delta D_y^{x_\nu}(t)$, we proceed to consider two "large" random variables x_ν and x_μ. For this case, in the formula for the variance of the output signal

$$D_y^2(t)=D_y^0(t)+\Delta D_y^{x_\nu}(t)+\Delta D_y^{x_\mu}(t) \tag{19}$$

we add the terms

$$\frac{1}{4!}A_0^{x_\mu^4}(t)M(x_\mu^4),\ \frac{1}{4}A_{k_l k_l}^{x_\mu^2}(t)M(x_\mu^2)M(x_{k_l}^2)$$

whose coefficients are determined using the formulas similar to Equations (7) and (8).

The number of additional simulations of the control system in order to calculate the variance $D_y^2(t)$ as compared with the variance $D_y^1(t)$ is increased by

$$K^{2}_{\text{доп}} = 4(\rho - 2) \tag{20}$$

The increase in the variance $\Delta D_y^x\mu(t)$ due to the random coefficient X_μ is compared with the value of $K \cdot D_y^0(t)$. If $\Delta D_y^x\mu(t) \leq KD_y^0(t)$, we limit ourselves to the value $q_\mu = 4$; otherwise the degree y_μ in the random coefficient X_μ is increased to six.

The increases $\Delta D_y(t)$ and degrees q needed to take into account the successive random coefficients are calculated in the same order. For $\rho = 5$, $\rho = 10$ and $q = 4$, we give below the necessary number of simulations of the control system.

Number of simulations using the coefficient method considering different effects of the expansion coefficients											Coefficient method without considering different effects of the expansion coefficients
Kl	1	2	3	4	5	6	7	8	9	10	
$\rho=5$	27	39	47	51	51						61
$\rho=10$	57	89	117	141	161	177	189	197	201	201	221

Thus, the number of simulations of the control system, taking into account different effects of the random coefficients in the canonical expansion of the input signal, is much lower than the number of simulations as compared with the usual coefficient method; or, for the same number of simulations, the accuracy of the coefficient method taking into account different effects of the coefficients is higher than the accuracy of the ordinary coefficient method.

2. The algorithm for dividing the random coefficients into "large" and "small."

Let us consider the algorithm in which two coefficients in the canonical expansion X_1 and X_2, normally distributed, are divided into "large" and "small."

Using the coefficient method for Chebyshev-type points

x_1	x_2
0,000	0,000
$+1{,}732\,\sigma_1$	$+1{,}732\,\sigma_2$
$-1{,}732\,\sigma_1$	$-1{,}732\,\sigma_2$

we find the components of the first approximation to the variance of the output signal.

$$M\{\Psi^0(t, x_1, 0)\} = \Psi(t, 0, 0) + \frac{1}{2} A_{x_1^2}(t) \cdot M\{x_1^2\} \tag{21}$$

$$M\{\Psi^0(t, 0, x_2)\} = \Psi(t, 0, 0) + \frac{1}{2} A_{x_2^2}(t) \cdot M\{x_2^2\}. \tag{22}$$

Using the same results of a simulation of the control system, we use the interpolation method to find the expectation of the functions $\psi^1(t, x_1, 0)$, $\psi^1(t, 0, x_2)$ for $q_1 = q_2 = 4$

$$\begin{aligned} M\{\Psi^1(t, x_1, 0)\} &= \Psi(t, 0, 0) + \frac{1}{2} A^1_{x_1^2} M\{x_1^2\} + \frac{1}{4!} A^1_{x_1^4}(t) M\{x_1^4\} = \\ &= \Psi(t, 0, 0)\rho_0 + \Psi(t; 1{,}732\,\sigma_1, 0)\rho_1 + \Psi(t, -1{,}732\,\sigma_1, 0)\rho_2 \end{aligned} \tag{23}$$

$$\begin{aligned} M\{\Psi^1(t, 0, x_2)\} &= \Psi(t, 0, 0) + \frac{1}{2} A^1_{x_2^2}(t) M\{x_2^2\} + \frac{1}{4!} A^1_{x_2^4}(t) M\{x_2^4\} = \\ &= \Psi(t, 0, 0)\rho_0 + \Psi(t, 0, 1{,}732\,\sigma_2)\rho_1 + \Psi(t, 0, -1{,}732\,\sigma_2)\rho_2 \end{aligned} \tag{24}$$

We calculate the differences

$$M\{\Psi^1(t, x_1, 0)\} - M\{\Psi^0(t, x_1, 0)\} = a_{x_1} \tag{25}$$

$$M\{\Psi^1(t, 0, x_2)\} - M\{\Psi^0(t, 0, x_2)\} = a_{x_2} \tag{26}$$

The coefficients a_{X_1}, a_{X_2} characterize the effect of the fourth-degree terms

$$\frac{1}{4!} A^1_{x_1^4}(t) M\{x_1^4\}, \quad \frac{1}{4!} A^1_{x_2^4}(t) M\{x_2^4\}$$

on the value of the variance of the output signal from the control system.

The random coefficient with the largest value of a_x is taken as the "large" random coefficient when going from the first approximation $D_y^0(t)$ to $D_y^1(t)$. The transition from "small" to "large" random variables is done in the descending order of a_{x_ν}.

The transition from $q_\nu = 4$ and $q_\nu = 6$ is done similarly.

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EQUATIONS OF MOTION OF A SYSTEM OF BODIES OF VARIABLE MASS AS A CONTROL OBJECT

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A majority of objects in aircraft, missile, and space technology are mechanical systems that can be represented, if one neglects the elasticity of their construction, as ensembles of a number of ideally rigid bodies of variable mass whose relative motion and the variation of mass are assumed to be known.*) Such objects include, for example, spacecraft with cosmonauts or other movable mechanical masses moving in them, aircraft of variable geometry, and a number of other objects.

Due to the shifting of the centers of mass of the component bodies, the gyroscopic linkage between rotating bodies, the shifting of the axes of inertia, the change of the moments of inertia of the ensemble in its entirety, etc., the dynamics of the above mechanical system differ substantially from the dynamics of an isolated rigid body of variable mass. Numerically this difference is greater if the mass of the moving bodies is greater compared to the mass of the main body, and if the movements of the masses are faster.

*The term has the following meaning: the relative kinematics of bodies and the character of mass variation of each one of them are known functions of time or, in a more complicated case, are the phase coordinates of another more complicated dynamic system.

It is important to note that, for the particular case of bodies of constant mass, the equations of this type (only for rotational motion) were obtained in [1].

The mechanical system considered here has of course six degrees of freedom. Therefore, it is necessary and sufficient to set up six independent dynamic equations of first order, with the variables in the form of the components of the linear velocity vector of an arbitrary instantaneous center of mass and the components of the angular velocity relative to that center.

In the present paper, the center of reduction is the instantaneous center of inertia of the system. The scalar equations for translational and rotational motion are written relative to the instantaneous central axes.

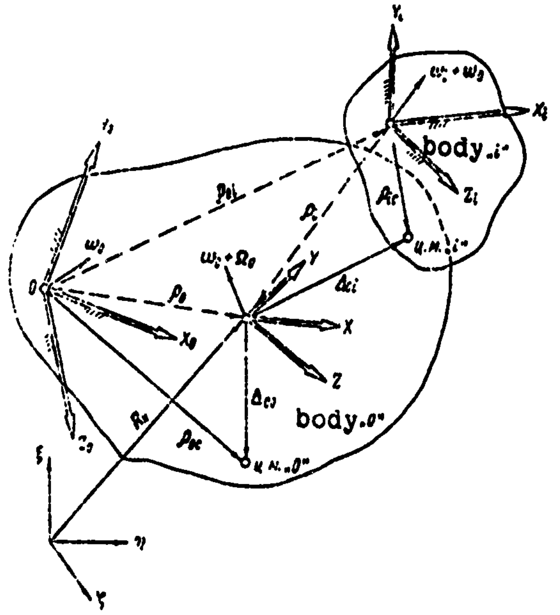
The hypothesis of near-action [2] is used as the initial assumption, as is done in a majority of papers on the mechanics of variable masses.

The Coordinate System and Kinematic Relations

Let us consider the case when the system consists of the main body "0" and a number of movable bodies "1" (Figure 1). We shall use the following coordinate systems:

- $\zeta\eta\xi$ — inertial;
- $X_0Y_0Z_0$ — attached to the main body "0";
- $X_1Y_1Z_1$ — attached to body "1";
- XYZ — movable system whose origin coincides with the instantaneous center of inertia.

Since the relative motions of bodies and the change in their mass characteristics are considered to be given, the following quantities are also assumed to be given: ρ_0 which is the radius vector giving the position of the origin of the XYZ system relative to $X_0Y_0Z_0$; the Euler angles giving the relative orientation of the XYZ and $X_0Y_0Z_0$



axes; ω_i is the angular velocity vector of the system $X_iY_iZ_i$ relative to $X_0Y_0Z_0$; ρ_{0i} is the radius vector giving the origin of the system $X_iY_iZ_i$ ($i = 1, 2, \ldots, n$) relative to $X_0Y_0Z_0$; the Euler angles giving the relative orientation of the $X_iY_iZ_i$ and $X_0Y_0Z_0$ axes; Ω_0 is the angular velocity vector of the system XYZ relative to $X_0Y_0Z_C$*; Ω_i is the angular velocity vector of the system XYZ relative to $X_iY_iZ_i$; moreover

$$\Omega_0 = \omega_i + \Omega_i \tag{1.1}$$

Figure 1.

are the Euler angles coordinating the orientation of $X_iY_iZ_i$ and $X_0Y_0Z_0$. Of course, a knowledge of the orientation of XYZ and $X_0Y_0Z_0$, $X_iY_iZ_i$ and XYZ uniquely determines the orientation of $X_0Y_0Z_0$ and $X_iY_iZ_i$.

ρ_i is the radius vector giving the location of the origin of XYZ relative to $X_iY_iZ_i$ ($i = 1, 2 \ldots n$); Δ_{0c} is the radius vector giving the position of the instantaneous center of inertia "0" relative to XYZ; Δ_{ci} is the radius vector giving the position of the instantaneous center of inertia of a body "i" relative to XYZ; ρ_{0c} is the radius vector giving the position of the instantaneous center of inertia of the body "0" relative to $X_0Y_0Z_0$; ρ_{ic} is the radius vector giving the position of the instantaneous center of inertia of a body "i" relative to $X_iY_iZ_i$; m_0 and m_i are the instantaneous masses of the body "0" and "i," where by assumption

$$m_0\Delta_{c0} + \sum_{i=1}^{n} m_i\Delta_{ci} = 0 \tag{1.2}$$

*Translator's note: Incorrectly given as "Y" in foreign text.

and

$$m = m_0 + \sum_{i=1}^{n} m_i; \qquad (1.3)$$

m_{j0} and m_{ji} are instantaneous masses of "current points j" forming the bodies "0" and "i," respectively, where

$$\sum_j m_{j0} = m_0; \quad \sum_j m_{ji} = m_i; \quad \sum_{i=0}^{n}\sum_j m_{ji} = m. \qquad (1.4)$$

The above points are referred to XYZ using the radius vectors Δ_{j0} and Δ_{ji}, respectively, where

$$\Delta_{ji} = x_{ji}l + y_{ji}d + z_{ji}n \; (i = 1 \ldots n) \qquad (1.5)$$

I_{XXi}, I_{YYi}, I_{ZZi}, I_{XYi}, I_{YZi}, I_{ZXi} are the instantaneous moments of inertia of a body "i" in the central axes XYZ. I_{XX}, I_{YY}, I_{ZZ}, I_{XY}, I_{YZ}, I_{ZX} are the instantaneous moments of inertia of the ensemble relative to the central axes XYZ.

q_0 is the local derivative of ρ_0 in the $X_0Y_0Z_0$ system, and it characterizes the velocity of the origin of XYZ relative to the boundaries of the body "0,", q_i is the local derivative of ρ_i in the $X_iY_iZ_i$ system, and it characterizes the velocity of the origin of XYZ relative to the boundaries of a body "i"; $\overset{*}{q}_0$ is the local derivative of q_0 in the $X_0Y_0Z_0$ system; $\overset{*}{q}_i$ is the local derivative of q_i in the $X_iY_iZ_i$ system; q_{0i} is the local derivative of ρ_{0i} in the $X_0Y_0Z_0$ system; q_{0i} is the local derivative of q_{0i} in the XYZ system; R_0 is the radius vector giving the origin of the XYZ system relative to ζηξ; V = d/dtR_0 is the derivative of R_0 in the ζηξ system; d/dt V is the derivative of V in the ζηξ system; l, d, n are the unit vectors along the XYZ axes. From the vector triangle in Figure 1, we can write

$$\rho_0 = \rho_{0i} + \rho_i \qquad (1.6)$$

Then the local derivative of ρ_0 in the $X_0Y_0Z_0$ system is

$$q_0 = q_{0i} + q_i + \omega_i \times \rho_i \qquad (1.7)$$

and

$$\dot{q}_0 = \dot{q}_{0i} + \overset{*}{\omega}_i \times \rho_i + \omega_i \times (\omega_i \times \rho_i) + 2\omega_i \times q_i + \dot{q}_i, \qquad (1.8)$$

where $\overset{*}{\omega}_i$ is the local derivative of ω_i in the $X_0Y_0Z_0$ system equa' ' the local derivative of ω_i in the $X_iY_iZ_i$ system. Clearly,

$$d/dt V = \dot{V} + (\omega_0 + \Omega) \times V \qquad (1.9)$$

and

$$V = V_X l + V_Y d + V_Z n, \qquad (1.10)$$

where V is the local derivative of V in the XYZ system.

The total angular velocity of a body "i" and its angular acceleration are equal

$$\omega_{in} = \omega_0 + \omega_i \qquad (1.11)$$

$$d/dt\, \omega_{in} = d/dt\, \omega_0 + d/dt\, \omega_i, \qquad (1.12)$$

respectively, where ω_0 is the angular velocity vector of the $X_0Y_0Z_0$ system;

$$d/dt\, \omega_i = \dot{\omega}_i + \omega_0 \times \omega_i. \qquad (1.13)$$

Furthermore,

$$d/dt\, \omega_0 = \dot{\omega}_0 + (\omega_0 + \Omega_0) \times \omega_0, \qquad (1.14)$$

where $\dot{\omega}_0$ is the local derivative of ω_0 in the XYZ system. Finally

$$d/dt\,\omega_0 = \dot{\omega}_0 + \Omega_0 \times \omega_0 \tag{1.15}$$

It will be noted that

$$\omega_0 = \dot{\omega}_{0X} l + \dot{\omega}_{0Y} d + \dot{\omega}_{0Z} n, \tag{1.16}$$

where $\omega_0 X$, $\omega_0 Y$, $\omega_0 Z$ are the projections of ω_0 on the XYZ axes. By analogy, we can also write

$$d/dt\,\omega_{in}^{\cdot} = \dot{\omega}_0 + \Omega_0 \times \omega_0 + \dot{\omega}_i + (\omega_0 + \Omega_i) \times \omega_i, \tag{1.17}$$

where $\dot{\omega}_1$ is the local derivative of ω_1 in the XYZ system.

The Force Equation

In accordance with Equation (22) from [3] and the notation used here, the vector equation of the translational motion of a body of variable mass, written for the body "0," has the form

$$\begin{aligned} m_0 d/dt V = F_0 + R_0 + m_0 [&- d/dt\,\omega_0 \times \Delta_{c0} - (\omega_0 \Delta_{c0})\,\omega_0 + \\ &+ \omega_0^2 \Delta_{c0} + \dot{q}_0 + 2\omega_0 \times {}'q_0], \end{aligned} \tag{2.1}$$

where F_0 is the resultant of all the external forces acting on the body "0." This may include forces which are internal relative to the ensemble. R_0 is the resultant of all the external (with respect to the body "0") reaction forces. It may also include forces that are internal relative to the ensemble.

The structure of the expression in (2.1) remains the same for a body "1." One must only change the indices and write out the total angular acceleration and velocity vectors in more detail.

In view of (1.13), (1.14), and (1.15) after a number of transformations, we obtain

$$\begin{aligned} m_l d/dt\,V = F_l + R_l + m_l[-d/dt\,\omega_0 \times \Delta_{cl} - (\omega_0 \cdot \Delta_{cl})\omega_0 + \omega_0^2 \Delta_{cl}] + \\ + m_l[-\dot{\omega}_l \times \Delta_{cl} - (\omega_l \Delta_{cl})\omega_l + \omega_l^2 \Delta_{cl}] + 2m_l(\omega_0 \omega_l)\Delta_{cl} - \\ - 2m_l(\omega_0 \cdot \Delta_{cl})\omega_l + m_l(\dot{q}_l + 2\omega_0 \times q_l + 2\omega_l \times q_l). \end{aligned} \tag{2.2}$$

Adding Equations (2.1) and (2.2), we obtain in view of (1.2) and (1.3)

$$\begin{aligned} md/dtV = F + R + \sum_{l=1}^{n} m_l[-\dot{\omega}_l \times \Delta_{cl} - (\omega_l \Delta_{cl})\omega_l + \omega_l^2 \Delta_{cl}] + \\ + 2\sum_{l=1}^{n} m_l(\omega_0 \cdot \omega_l)\Delta_{cl} - 2\sum_{l=1}^{n} m_l(\omega_0 \cdot \Delta_{cl}) + \\ + \left(m_0 \dot{q}_0 + \sum_{l=1}^{n} m_l \dot{q}_l\right) + 2\omega_0 \times \left(m_c q_0 + \sum_{l=1}^{n} m_l q_l\right) + \\ + 2\sum_{l=1}^{n} m_l(\omega_l \times q_l), \end{aligned} \tag{2.3}$$

where

$$F = F_0 + \sum_{l=1}^{n} F_l \quad \text{and} \quad R = R_0 + \sum_{l=1}^{n} R_l.$$

It should be noted that F and R do not now contain any components internal to the ensemble.

Equation (2.3) is a vector equation determining the motion of the instantaneous center of inertia of the system. If another point is taken as the center of reduction, the structure of the right-hand side of (2.3) will turn out to be much more complicated.

Let us project the vector Equation (2.3) onto the axes of the moving system XYZ. Here in order to simplify the structure of the scalar equations, we shall represent only the principle terms from (2.3) in the projection onto the XYZ axes. They characterize the dynamics of the translational motion of an instantaneously "fixed" system. The remaining terms in an expanded form will be written separately, outside of the equations.

Thus, denoting the additional terms in (2.3) by a vector Q_1 and taking Equations (1.9) and (1.10) into account, we find

$$\begin{aligned} m[\dot{V}_X + (\omega_{0Y} + \Omega_{0Y})V_Z - (\omega_{0Z} + \Omega_{0Z})V_Y] &= F_X + R_X + Q_{1X}, \\ m[\dot{V}_Y + (\omega_{0Z} + \Omega_{0Z})V_X - (\omega_{0X} + \Omega_{0X})V_Z] &= F_Y + R_Y + Q_{1Y}, \\ m[\dot{V}_Z + (\omega_{0X} + \Omega_{0X})V_Y - (\omega_{0Y} + \Omega_{0Y})V_X] &= F_Z + R_Z + Q_{1Z} \end{aligned} \tag{2.4}$$

where ω_{0X}, ω_{0Y}, ω_{0Z}, Ω_{0X}, Ω_{0Y}, Ω_{0Z} are the components of ω_0 and Ω_0 in the XYZ system; F_X, F_Y, F_Z, R_X, R_Y, R_Z, Q_{1X}, Q_{1Y}, Q_{1Z} are the components of F, R, and Q_1, respectively, in the XYZ system.

Let us write the expanded scalar expressions for the vector Q_1 in the form of separate groups of terms.

The first term is

$$\sum_{i=1}^{n} m_i[-\overset{*}{\omega}_i \times \Delta_{ci} - (\omega_i \cdot \Delta_{ci})\,\omega_i + \omega_i^2 \Delta_{ci}] \tag{2.5}$$

In view of

$$\overset{*}{\omega}_i = \dot{\omega}_i + \Omega_0 \times \omega_i, \tag{2.6}$$

Equation (2.6), considering (2.1), becomes

$$\overset{*}{\omega}_i = \dot{\omega}_i + \Omega_i \times \omega_i \tag{2.7}$$

Substituting (2.7) and (2.5), and considering that

$$\Delta_{cl} = x_{cl} l + y_{cl} d + z_{cl} \cdot n,$$

we get

$$\begin{aligned}
&\sum_{l=1}^{n} m_l [(-\dot{\omega}_{lY} - \omega_{lX} \cdot \Omega_{lZ} + \omega_{lZ} \cdot \Omega_{lX} - \omega_{lX}\omega_{lZ}) z_{cl} + \\
&+ (\dot{\omega}_{lZ} - \omega_{lX} \cdot \Omega_{lY} + \omega_{lY} \cdot \Omega_{lX} - \omega_{lX} \cdot \omega_{lY}) y_{cl} + (\omega_{lY}^2 + \omega_{lZ}^2) x_{cl}] l + \\
&+ \sum_{l=1}^{n} m_l [(-\dot{\omega}_{lZ} - \omega_{lY} \cdot \Omega_{lX} + \omega_{lX} \cdot \Omega_{lY} - \omega_{lY} \cdot \omega_{lX}) x_{cl} + \\
&+ (\dot{\omega}_{lX} - \omega_{lY} \cdot \Omega_{lZ} + \omega_{lZ} \Omega_{lY} - \omega_{lY} \cdot \omega_{lZ}) z_{cl} + (\omega_{lZ}^2 + \omega_{lX}^2) y_{cl}] d + \\
&+ \sum_{l=1}^{n} m_l [(-\dot{\omega}_{lX} - \omega_{lZ} \cdot \Omega_{lY} + \omega_{lY} \cdot \Omega_{lZ} - \omega_{lZ} \cdot \omega_{lY}) y_{cl} + \\
&+ (\dot{\omega}_{lY} - \omega_{lZ} \cdot \Omega_{lX} + \omega_{lX} \cdot \Omega_{lZ} - \omega_{lZ} \cdot \omega_{lX}) x_{cl} + (\omega_{lX}^2 + \omega_{lY}^2) z_{cl}] n.
\end{aligned} \tag{2.8}$$

The second term is

$$\begin{aligned}
&2 \sum_{l=1}^{n} m_l (\omega_0 \cdot \omega_l) \Delta_{cl} - 2 \sum_{l=1}^{n} m_l (\omega_0 \cdot \Delta_{cl}) \omega_l = \\
&= 2 \sum_{l=1}^{n} m_l [(\omega_{0Y} \cdot \omega_{lY} + \omega_{0Z} \cdot \omega_{lZ}) x_{cl} - \omega_{0Y} \cdot \omega_{lX} y_{cl} - \omega_{0Z} \cdot \omega_{lX} z_{cl}] l + \\
&+ 2 \sum_{l=1}^{n} m_l [(\omega_{0X} \cdot \omega_{lX} + \omega_{0Z} \cdot \omega_{lZ}) y_{cl} - \omega_{0Z} \cdot \omega_{lY} z_{cl} - \omega_{0X} \cdot \omega_{lY} \cdot x_{cl}] \mathrm{d} + \\
&+ 2 \sum_{l=1}^{n} m_l [(\omega_{0Y} \cdot \omega_{lY} + \omega_{0X} \cdot \omega_{lX}) z_{cl} - \omega_{0X} \cdot \omega_{lZ} \cdot x_{cl} - \\
&- \omega_{0Y} \cdot \omega_{lZ} \cdot y_{cl}] \mathrm{n}.
\end{aligned} \tag{2.9}$$

The third term is

$$m_0 \dot{q}_0 + \sum_{l=1}^{n} m_l \dot{q}_l.$$

In view of (1.1) and the scalar expressions for the local derivatives of q_0 and q_l in XYZ

$$\begin{aligned}
\dot{q}_0 &= \dot{q}_{0X} l + \dot{q}_{0Y} d + \dot{q}_{0Z} n, \\
\dot{q}_l &= \dot{q}_{lX} l + \dot{q}_{lY} d + \dot{q}_{lZ} n,
\end{aligned}$$

we obtain the following scalar expression for the third term

$$\begin{aligned}&\{m_0(\dot{q}_{0X}+q_{0Z}\cdot\Omega_{0Y}-q_{0Y}\cdot\Omega_{0Z})+\\&+\sum_{i=1}^{n}m_i\,[\dot{q}_{iX}+(\Omega_{0Y}-\omega_{iY})\,q_{iZ}-(\Omega_{0Z}-\omega_{iZ})\,q_{iY}]\}\,l+\\&+\{m_0\,(\dot{q}_{0Y}+q_{0X}\cdot\Omega_{0Z}-q_{0Z}\cdot\Omega_{0X})+\\&+\sum_{i=1}^{n}m_i\,[\dot{q}_{iY}+(\Omega_{0Z}-\omega_{iZ})\,q_{iX}-(\Omega_{0X}-\omega_{iX})\,q_{iZ}]\}d+\\&+\{m_0(\dot{q}_{0Z}+q_{0Y}\cdot\Omega_{0X}-q_{0X}\cdot\Omega_{0Y})+\\&+\sum_{i=1}^{n}m_i\,[\dot{q}_{iZ}+(\Omega_{0X}-\omega_{iX})\,q_{iY}-(\Omega_{0Y}-\omega_{iY})\,q_{iX}]\}\,n.\end{aligned}\qquad(2.10)$$

The fourth term is

$$\begin{aligned}&2\omega_0\times\left(m_0q_0+\sum_{i=1}^{n}m_iq_i\right)=2\omega_0\times\sum_{i=0}^{n}m_iq_i=\\&=2\sum_{i=0}^{n}m_i(\omega_{0Y}\cdot q_{iZ}-\omega_{0Z}\cdot q_{iY})\,l+2\sum_{i=0}^{n}m_i(\omega_{0Z}\cdot q_{iX}-\omega_{0X}\cdot q_{iZ})\,d+\\&+2\sum_{i=0}^{n}m_i(\omega_{0X}\cdot q_{iY}-\omega_{0Y}\cdot q_{iX})\,n.\end{aligned}\qquad(2.11)$$

The fifth term is

$$\begin{aligned}&2\sum_{i=1}^{n}m_i(\omega_i\times q_i)=2\sum_{i=1}^{n}m_i(\omega_{iY}\cdot q_{iZ}-\omega_{iZ}\cdot q_{iY})\,l+\\&+2\sum_{i=1}^{n}m_i\,(\omega_{iZ}\cdot q_{iX}-\omega_{iX}\cdot q_{iZ})\,d+\\&+2\sum_{i=1}^{n}m_i(\omega_{iX}\cdot q_{iY}-\omega_{iY}\cdot q_{iX})\,n.\end{aligned}\qquad(2.12)$$

Using Equations (2.8), (2.9), (2.10), (2.11) and (2.12), one can make a numerical estimate of each additional term in Equation (2.3). In addition, these expressions can be combined with the corresponding equations in (2.4) in order to obtain a more exact solution for V_X, V_Y, V_Z.

The Moment Equation

According to Equation (42) in [3] and the notation used here, the vector equation of the rotational motion of a body of variable mass, set up for the body "0" relative to the origin of XYZ, has the form

$$\sum_j m_{j0}\Delta_{j0} \times [d/dt\omega_0 \times \Delta_{j0} + (\omega_0\Delta_{j0})\omega_0 - \omega_0^2\Delta_{j0}] = M_{F_0} + M_{R_0} + \\ + m_0[d/dtV \times \Delta_{c0} + \Delta_{c0} \times \dot{q}_0 + 2\Delta_{c0} \times (\omega_0 \times q_0)], \tag{3.1}$$

where M_{F_0} is the moment of all external (relative to "0") forces; M_{R_0} is the moment of all external (relative to "0") reaction forces. M_{F_0} and M_{R_0} may include components which are internal relative to the ensemble.

The structure of (3.1) remains the same for a body "i." One must merely change the indices and write the total angular acceleration and velocity vectors in more detail [see (1.11), (1.12), and (1.13)]. After a number of transformations, the torque equation for a body "i" will become

$$\sum_j m_{ji}\Delta_{ji} \times [d/dt\omega_0 \times \Delta_{ji} + (\omega_0\Delta_{ji})\omega_0 - \omega_0^2\Delta_{ji}] = M_{F_i} + M_{R_i} + \\ + m_i[d/dtV \times \Delta_{ci} + \Delta_{ci} \times \dot{q}_i + 2\Delta_{ci} \times (\omega_0 \times q_i)] + 2m_i\Delta_{ci} \times \\ \times (\omega_i \times q_i) - 2\sum_j m_{ji}(\omega_0\Delta_{ji})(\Delta_{ji} \times \omega_i) - \\ - \sum_j m_{ji}\Delta_{ji} \times [\dot{\omega}_i \times \Delta_{ji} + (\omega_i\Delta_{ji})\omega_i - \omega_i^2\Delta_{ji}]. \tag{3.2}$$

Adding Equations (3.1) and (3.2), we obtain in view of (1.2)

$$\sum m_{ji}\Delta_{ji} \times [d/dt\omega_0 \times \Delta_{ji} + (\omega_0\Delta_{ji})\omega_0 - \omega_0^2\Delta_{ji}] = M_F + M_R + \\ + \sum_{i=0}^{n} m_i\Delta_{ci} \times \dot{q}_i + 2\sum_{i=0}^{n} m_i\Delta_{ci} \times (\omega_0 \times q_i) + 2\sum_{i=1}^{n} m_i\Delta_{ci} \times (\omega_i \times q_i) - \\ - 2\sum_{i=1}^{n}\sum_j m_{ji}(\omega_0\Delta_{ji})(\Delta_{ji} \times \omega_i) - \sum_{i=1}^{n}\sum_j m_{ji}\Delta_{ji} \times [\dot{\omega}_i \times \Delta_{ji} + \\ + (\omega_i\Delta_{ji})\omega_i - \omega_i^2\Delta_{ji}], \tag{3.3}$$

where $\sum = \sum_{i=0}^{n} \sum_{j}$, $M_F = \sum_{i=0}^{n} M_{F_i}$ and $M_R = \sum_{i=0}^{n} M_{R_i}$. The vectors M_F and M_R do not contain components internal to the ensemble.

Equation (3.3) is a vector relation determining the rotation of a mechanical system about the center of reduction, in this case the instantaneous center of inertia. Just as in the case of Equation (2.3), the structure of the right-hand side of Equation (3.3) will be more complicated if another point is taken as the center of reduction. In particular, it is precisely for this reason that Equation (3.3) does not contain parameters of the translational motion.

Thus, if M_F and M_R are functions of time and parameters of rotational motion alone, Equation (3.3) may be integrated independently of (2.3). Thus, the total system of equations, composed of Equations (2.3) and (3.3) and determining the total motion of the mechanical system, in view of the remarks with respect to the functional dependence of F, R, M_F and M_R, is connected in only one direction. It should be noted that Equations (2.3) and (3.3) are not total time derivatives of the momentum and angular momentum vectors, respectively. This is the peculiarity of the mechanics of bodies of variable mass.

Let us project the vector Equation (3.3) onto the axes of the moving system XYZ. In so doing, we shall give the components in the XYZ system of only the basic terms in (3.3). They characterize the dynamics of the angular motion of an instantaneously "fixed" system.

$$
\begin{aligned}
&I_{XX}(\dot\omega_{0X} + \Omega_{0Y}\omega_{0Z} - \Omega_{0Z}\omega_{0Y}) - (I_{YY} - I_{ZZ})\omega_{0Y}\omega_{0Z} - I_{XY}(\dot\omega_{0Y} - \\
&- \omega_{0X}\omega_{0Z} + \Omega_{0Z}\omega_{0X} - \Omega_{0X}\omega_{0Z}) - I_{XZ}(\dot\omega_{0Z} + \omega_{0X}\omega_{0Y} + \Omega_{0X}\omega_{0Y} - \\
&- \Omega_{0Y}\omega_{0X}) + I_{YZ}(\omega_{0Z}^2 - \omega_{0Y}^2) = M_{F_X} + M_{R_X} + Q_{X}, \\
&I_{YY}(\dot\omega_{0Y} + \Omega_{0Z}\omega_{0X} - \Omega_{0X}\omega_{0Z}) - (I_{ZZ} - I_{XX})\omega_{0Z}\omega_{0X} - \\
&- I_{YZ}(\dot\omega_{0Z} - \omega_{0Y}\omega_{0X} + \Omega_{0X}\omega_{0Y} - \Omega_{0Y}\omega_{0X}) - I_{YX}(\dot\omega_{0X} + \omega_{0Y}\omega_{0Z} + \\
&+ \Omega_{0Y}\omega_{0Z} - \Omega_{0Z}\omega_{0Y}) + I_{ZX}(\omega_{0X}^2 - \omega_{0Z}^2) = M_{F_Y} + M_{R_Y} + Q_{Y}, \\
&I_{ZZ}(\dot\omega_{0Z} + \Omega_{0X}\omega_{0Y} - \Omega_{0Y}\omega_{0X}) - (I_{XX} - I_{YY})\omega_{0X}\omega_{0Y} - \\
&- I_{ZX}(\dot\omega_{0X} - \omega_{0Z}\omega_{0Y} + \Omega_{0Y}\omega_{0Z} - \Omega_{0Z}\omega_{0Y}) - I_{ZY}(\dot\omega_{0Y} + \omega_{0Z}\omega_{0X} + \\
&+ \Omega_{0Z}\omega_{0X} - \Omega_{0X}\omega_{0Z}) + I_{XY}(\omega_{0Y}^2 - \omega_{0X}^2) = M_{F_Z} + M_{R_Z} + Q_{Z},
\end{aligned}
\tag{3.4}
$$

where M_{F_X}, M_{F_Y}, M_{F_Z}, M_{R_X}, M_{R_Y}, M_{R_Z}, Q_{2X}, Q_{2Y}, Q_{2Z} are the components of the vectors M_F, M_R and Q_2 in the XYZ system.

The vector Q_2, similarly as Q_1 in(2.4), includes the additional components, and expresses the specific features of the mechanical system under consideration.

Let us write the scalar expressions for the vector Q_2 in the form of individual groups of terms. The first term is

$$\begin{aligned}
&\sum_{l=0}^{n} m_l \Delta_{cl} \times \overset{*}{q}_l = \sum_{l=0}^{n} m_l \Delta_{cl} \times (\dot{q}_l + \Omega_l \times q_l) = \\
&= \sum_{l=0}^{n} m_l [(\dot{q}_{lZ} + \Omega_{lX} q_{lY} - \Omega_{lY} q_{lX}) y_{cl} - (\dot{q}_{lY} + \Omega_{lZ} q_{lX} - \Omega_{lX} q_{lZ}) z_{cl}] l + \\
&+ \sum_{l=0}^{n} m_l [(\dot{q}_{lX} + \Omega_{lY} q_{lZ} - \Omega_{lZ} q_{lY}) z_{cl} - (\dot{q}_{lZ} + \Omega_{lX} q_{lY} - \Omega_{lY} q_{lX}) x_{cl}] d + \\
&+ \sum_{l=0}^{n} m_l [(\dot{q}_{lY} + \Omega_{lZ} q_{lX} - \Omega_{lX} q_{lZ}) x_{cl} - (\dot{q}_{lX} + \Omega_{lY} q_{lZ} - \Omega_{lZ} q_{lY}) y_{cl}] n.
\end{aligned} \tag{3.5}$$

In (3.5), it is possible to make a change of variables with the help of Equations (1.1) and (1.7) - (1.8). The second term is

$$\begin{aligned}
&2 \sum_{l=0}^{n} m_l \Delta_{cl} \times (\omega_0 \times q_l) = \\
&= 2 \sum_{l=0}^{n} m_l [(\omega_{0X} q_{lY} - \omega_{0Y} q_{lX}) y_{cl} + (\omega_{0X} q_{lZ} - \omega_{0Z} q_{lX}) z_{cl}] l + \\
&+ 2 \sum_{l=0}^{n} m_l [(\omega_{0Y} q_{lZ} - \omega_{0Z} q_{lY}) z_{cl} + (\omega_{0Y} q_{lX} - \omega_{0X} q_{lY}) x_{cl}] d + \\
&+ 2 \sum_{l=0}^{n} m_l [(\omega_{0Z} q_{lX} - \omega_{0X} q_{lZ}) x_{cl} + (\omega_{0Z} q_{lY} - \omega_{0Y} q_{lZ}) y_{cl}] n.
\end{aligned} \tag{3.6}$$

If it is necessary, one can make a change of variables also in (3.6).

The third term is

$$2\sum_{i=1}^{n} m_i \Delta_{ci} \times (\omega_i \times q_i).$$

By analogy with (3.6), we obtain

$$\begin{aligned} &2\sum_{i=1}^{n} m_i\left[(\omega_{iX}q_{iY}-\omega_{iY}q_{iX})\,y_{ci}+(\omega_{iX}q_{iZ}-\omega_{iZ}q_{iX})\,z_{ci}\right] l+\\ &+2\sum_{i=1}^{n} m_i\left[(\omega_{iY}q_{iZ}-\omega_{iZ}q_{iY})\,z_{ci}+(\omega_{iY}q_{iX}-\omega_{iX}q_{iY})\,x_{ci}\right] d+\\ &+2\sum_{i=1}^{n} m_i\left[(\omega_{iZ}q_{iX}-\omega_{iX}q_{iZ})\,x_{ci}+(\omega_{iZ}q_{iY}-\omega_{iY}q_{iZ})\,y_{ci}\right] n. \end{aligned} \tag{3.7}$$

The fourth term is

$$\begin{aligned} &-2\sum_{i=1}^{n}\sum_{j} m_{ji}(\omega_0\Delta_{ij})(\Delta_{ji}\times\omega_i) = -2\sum_{i=1}^{n}\left[I_{XYi}\omega_{0X}\omega_{iZ}+I_{YZi}(\omega_{0Z}\omega_{iZ}-\right.\\ &-\omega_{0Y}\omega_{iY})-I_{ZXi}\omega_{0X}\omega_{iY}+0,5(I_{ZZi}+I_{XXi}-I_{YYi})\,\omega_{0Y}\omega_{iZ}-\\ &\left.-0,5(I_{XXi}+I_{YYi}-I_{ZZi})\,\omega_{0Z}\omega_{iY}\right] l-2\sum_{i=1}^{n}\left[I_{YZi}\omega_{0Y}\omega_{iX}+I_{ZXi}\times\right.\\ &\times(\omega_{0X}\omega_{iX}-\omega_{0Z}\omega_{iZ})-I_{XYi}\omega_{0Y}\omega_{iZ}+0,5(I_{XXi}+I_{YYi}-I_{ZZi})\,\omega_{0Z}\omega_{iX}-\\ &\left.-0,5(I_{YYi}+I_{ZZi}-I_{XXi})\,\omega_{0X}\omega_{iZ}\right] d-2\sum_{i=1}^{n}\left[I_{ZXi}\omega_{0Z}\omega_{iY}+I_{XYi}\times\right.\\ &\times(\omega_{0Y}\omega_{iY}-\omega_{0X}\omega_{iX})-I_{YZi}\omega_{0Z}\omega_{iX}+0,5(I_{YYi}+I_{ZZi}-I_{XXi})\,\omega_{0X}\omega_{iY}-\\ &\left.-0,5(I_{ZZi}+I_{XXi}-I_{YYi})\,\omega_{0Y}\omega_{iX}\right] n. \end{aligned} \tag{3.8}$$

The fifth term is

$$\begin{aligned} &-\sum_{i=1}^{n}\sum_{j} m_{ji}\Delta_{ji}\times\left[\overset{*}{\omega}_i\times\Delta_{ji}+(\omega_i\Delta_{ji})\,\omega_i-\omega_i^2\Delta_{ji}\right]=\\ &=-\sum_{i=1}^{n}\left[I_{XXi}(\dot{\omega}_{iX}+\Omega_{iY}\omega_{iZ}-\Omega_{iZ}\omega_{iY})-(I_{YYi}-I_{ZZi})\,\omega_{iY}\omega_{iZ}-\right.\\ &-I_{XYi}(\dot{\omega}_{iY}-\omega_{iX}\omega_{iZ}+\Omega_{iZ}\omega_{iX}-\Omega_{iX}\omega_{iZ})+I_{YZi}(\omega_{iZ}^2-\omega_{iY}^2)-\\ &\left.-I_{ZXi}(\dot{\omega}_{iZ}+\omega_{iX}\omega_{iY}+\Omega_{iX}\omega_{iY}-\Omega_{iY}\omega_{iX})\right] l-\sum_{i=1}^{n}\left[I_{YYi}(\dot{\omega}_{iY}+\Omega_{iZ}\omega_{iX}-\right.\\ &-\Omega_{iX}\omega_{iZ})-(I_{ZZi}-I_{XXi})\,\omega_{iZ}\omega_{iX}-I_{YZi}(\dot{\omega}_{iZ}-\omega_{iY}\omega_{iX}+\Omega_{iX}\omega_{iY}- \end{aligned} \tag{3.9}$$

$$
\begin{aligned}
&-\Omega_{iY}\omega_{iX}) + I_{ZXi}(\omega_{iX}^2 - \omega_{iZ}^2) - I_{XYi}(\dot{\omega}_{iX} + \omega_{iY}\omega_{iZ} + \Omega_{iY}\omega_{iZ} - \Omega_{iZ}\omega_{iY})]\mathrm{d} - \\
&- \sum_{i=1}^{n} [I_{ZZi}(\dot{\omega}_{iZ} + \Omega_{iX}\omega_{iY} - \Omega_{iY}\omega_{iX}) - (I_{XXi} - I_{YYi})\omega_{iX}\omega_{iY} - I_{ZXi} \times \\
&\times (\dot{\omega}_{iX} - \omega_{iZ}\omega_{iY} + \Omega_{iY}\omega_{iZ} - \Omega_{iZ}\omega_{iY}) + I_{XYi}(\omega_{iY}^2 - \omega_{iX}^2) - \\
&- I_{YZi}(\dot{\omega}_{iY} + \omega_{iZ}\omega_{iX} + \Omega_{iZ}\omega_{iX} - \Omega_{iX}\omega_{iZ})]\,\mathrm{n}.
\end{aligned}
$$

By summing the terms (1) - (5) [see Equations (3.5) - (3.9)] using the same unit vectors l, d, n, and by including them in (3.4), we obtain a system of three differential scalar equations determining the rotational motion of the system under consideration.

Using Equations (3.5) - (3.9), we can make a numerical estimate of each additional term in Equation (3.4). In order to obtain a more accurate solution for ω_{eX}, ω_{eY} and ω_{eZ}, the above expressions should be complemented by the expressions in (3.4).

If the scalar equations are set up with respect to the principal central axes of inertia, then of course the structure of the equations will become simplified. The transition from the components of the vector ω_e in one coordinate system to another can be made using the matrices of direction cosines.

<u>Supplement:</u> The orientation of the XYZ system relative to the stationary $\zeta\eta\xi$ system is given by the vector R_s which determines the origin of XYZ relative to $\zeta\eta\xi$, and by the Euler angles ψ, ϑ, and γ.

The relationship between variables V_X, V_Y, V_Z and the components of V in the $\zeta\eta\xi$ system is the usual one: it is formed by means of the direction cosines expressed in terms of the angles ψ, υ, γ, and yields three equations. Since the projections of the total angular velocity vector of the frame XYZ are $(\omega_e + \Omega_e)_X$, $(\omega_e + \Omega_e)_Y$, $(\omega_e + \Omega_e)_Z$, respectively, the variables ψ, ϑ and γ must be expressed in terms of these projections using well-known relations. This shows the difference between the structure of the given kinematic angular Euler equations and the analogous Euler equations used in the mechanics of bodies of constant mass: in the latter the projects are equal to ω_{eX}, ω_{eY}, ω_{eZ}.

It will be noted that by recalculating the projections ω_{0X}, ω_{0Y}, ω_{0Z} in the $X_0Y_0Z_0$ system (attached to the body "0"), one can use the ordinary kinematic angular equations. In this case, one of course assumes that the angles ϑ, ψ and γ specify the orientation of the X $X_0Y_0Z_0$ axes (and not XYZ). Any of these approaches will determine the last three deficient equations.

In obtaining Equations (2.3), (3.3), and their scalar analogs, the vector quantities ρ_0, ρ_1, Δ_{0C}, Δ_{1C}, q_0, q_1, $\overset{*}{q}_0$, $\overset{*}{q}_1$, etc. were used as parameters. However, ρ_{0C}, ρ_{01}, ω_1, ρ_{1C}, and their derivatives are of course the starting vector quantities which are determined by the specific relative kinematics of the bodies, distribution of mass in them, rate of change of the mass, and other design factors. Similar remarks can be made also with respect to the moments of inertia of the bodies. It is more natural to consider the latter as given functions of time relative to the $X_0Y_0Z_0$ system (for "0") and $X_1Y_1Z_1$ system (for bodies "1"), but not relative to XYZ. These quantities can be easily converted using well-known relations.

Conclusions

As a result of the present study carried out in a fairly complete form (considering the dynamics of the variability of mass characteristics and configurations), we obtained a mathematical model, in particular, of a flight vehicle as a control system which can be used in the synthesis and development of control systems.

We have shown the fundamental difference between the dynamics of systems of variable geometry and mass and the dynamics of bodies of constant mass, and this must be taken into account when designing a number of objects.

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ON THE PROBLEM OF SYNTHESIZING SEARCHLESS SELF-ADJUSTING CONTROL SYSTEMS FOR AIRCRAFT

V. D. Yeliseyev

It is advisable to design self-adjusting control systems by minimizing a certain statistical performance index. The performance index for aircraft should usually include the error determining the accuracy of the system, and the coordinates characterizing the energy losses and the accelerations acting on the craft.

The control system may contain the following adaptive blocks: blocks of identification, blocks of self-adjustment, and blocks of learning. After the system is used many times, the blocks of learning make it possible to establish and use the average functional relationships between the adjusted coefficients and the readings of the flight regime sensors.

In practical applications, the control system may be considerably simplified. Thus, the learning block may be absent, and the functional relationship between the adjustable coefficients and the regime sensor readings may be fixed in the design stage. To reduce the level of complexity, the identification blocks may also be removed. However, the self-adjusting blocks can be realized quite easily if one uses the gradient method in conjunction with simplified models of sensitivity [1].

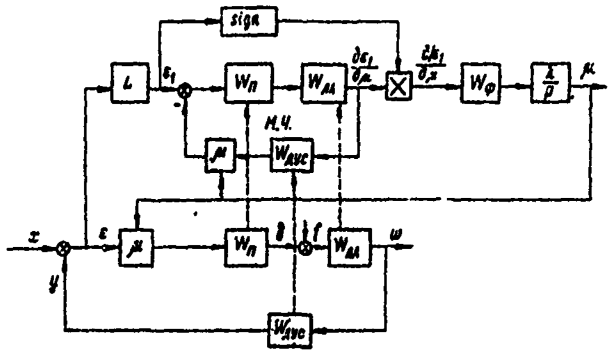

Figure 1. A scheme for optimizing the error produced by the angular velocity circuit for the aircraft fuselage.

W_d — drive operator with a rigid feedback; W_a — aircraft operator; W_{ang} — operator of the angular velocity sensor; L — operator of transforming filter; W_f — operator of the correcting filter; f — wind disturbance resulting in a deflection of a control surface; dotted lines — links necessary in designing a nonlinear sensitivity model.

Thus, in the case of a single-loop system, in a number of cases the sensitivity model may be taken to have the form of a stationary filter maintaining an energy balance between the high and low error signal frequencies, which is proper for an optimal system. The possibility of using a stationary filter is related to the stability of the system's pass band or adjusted values of the regulator coefficients.

A study of a single-loop self-adjusting system, using a stationary filter as a sensitivity model, shows that the adjusted value of the amplification coefficient depends less on the spectral composition of the input signals than in the case of an optimal system.

We shall give a simplified procedure for designing a single-loop self-adjusting system with one adjustable coefficient assuming that the basic loop operates quasilinearly. First, we find the structure of the optimization scheme (Figure 1) to calculate the current value of the gradient $\frac{\partial I}{\partial \mu}$ of the performance index I, taken, for example, in the form of the absolute value of the transformed error signal

$$\frac{\partial I}{\partial \mu}=\frac{\partial \varepsilon_1}{\partial \mu}\,\mathrm{sign}\,\varepsilon_1,$$

where $I = \varepsilon_1$; μ is the adjusted parameter; $\varepsilon_1 = L(p)\varepsilon$; $L(p)$ is the transfer function (operator) of the transforming filter; $\varepsilon = x - y$ is the error signal, where x is the input, and y is the output from the system; $\frac{\partial \varepsilon_1}{\partial \mu}$ is the sensitivity function of the signal ε_1 to the variation of the parameter μ, obtained using the sensitivity model.

For a given type of random or determined signals, the optimal values of the adjustable parameter μ are calculated in all flight regimes using the adjustment law $\frac{d\mu}{dt} = -\lambda \frac{\partial I}{\partial \mu}$, where λ is a sufficiently small parameter.

By properly choosing the transforming filter, we can eliminate the low-frequency components (if the system is non-astatic), change the optimal value of the coefficient μ (stability margin), and reduce the effect of noise.

A part of the optimization scheme consisting of the transforming filter, sensitivity model, and the multiplication unit may be called the spectral analyzer, since the signal produced by it depends on the ratio of the power of high and low frequencies.

When constructing a real self-adjusting system, the nonstationary model of sensitivity may be in many cases "frozen" at an adjusted value of μ in a certain average regime of flight, i.e., it may be taken in the form of a stationary linear filter. If the accuracy of adjustment is insufficient in certain regimes that differ greatly from the chosen selected regime, it is possible to use nonstationary filters by introducing a readjustment of the filter parameters in the function μ (for example, in the sensitivity model, one can change the analog of the coefficient μ and simultaneously the analog of the control surface efficiency coefficient which is proportional to it). Thus, the structure of a self-adjusting system is similar to the structure of the optimization scheme (Figure 2).

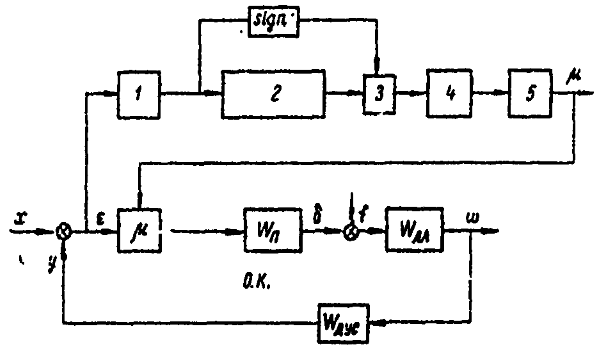

Figure 2. A scheme of a self-adjusting system controlling the angular velocity of an aircraft.

1 — transforming filter; 2 — a model filter of an optimal basic loop; 3 — multiplication unit; 4 — filter-amplifier; 5 — integrating unit.

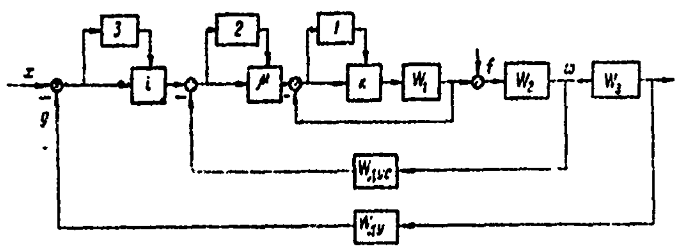

Figure 3. An example of a multi-loop self-adjusting system involving an adjustment of the coefficient of each loop.

W_1 — operator of the open drive depending, for example, on the hinge moment of the control surface; W_2 — angular velocity operator of the aircraft; W_3 — operator which transforms the angular velocity into either an angular deflection or a linear acceleration; W_{ang} — operator of the angle or linear acceleration sensor; 1, 2, 3 — self-adjustment blocks.

After selecting the parameters of the analyzer of the self-adjusting system, one must assure the desired dynamics of the self-adjusting circuit, in particular, one must select a coefficient λ which can be adjusted depending on μ and on the average value of the signal modulus ε_1.

It will be noted that, to simplify the system, one may sometimes use the control surface deflection signal, for example, instead of the error signal.

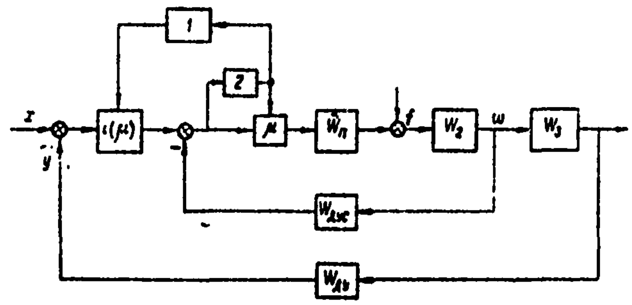

Figure 4. An example of a two-loop self-adjusting system with the adjustment of the damping loop coefficient and a functional readjustment of the outer aircraft control loop coefficient.

1 — nonlinear converter; 2 — inner loop self-adjustment block.

This procedure may be extended to the case of multi-loop control systems for which it is advisable to use a separate self-adjusting circuit for each loop (Figure 3). Here the self-adjusting circuit of each loop uses the error of its loop and contains a simplified model of the sensitivity of this loop. Calculations of a two-loop system controlling the angular motions of an aircraft show that the adjusted value of the internal circuit coefficient is in many cases practically independent of the value of the outer circuit coefficient. In addition, often the outer circuit coefficient may be readjusted depending on the inner circuit coefficient, which requires only one self-adjusting circuit (Figure 4).

Thus, the design of a searchless self-adjusting system may be based on the property of optimal systems which maintain the energy balance of the frequency components in the error signal and can be carried out using the gradient method with simplified sensitivity models.

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A CRITERION FOR COMPARING ADAPTIVE CONTROL SYSTEMS

V. I. Kozlov and N. I. Savkin

Recently there have appeared many different schemes of adaptive control systems, which sooner or later have to be compared.

As we know, a comparison of systems reveals the advantages of some and shortcomings of others. The work done to eliminate those shortcomings may bring the systems being compared to the same level. For example, historically the systems with constant adjustments of the regulator ceased to satisfy the technical requirements, and were replaced by parametrically invariant adaptive systems. Due to the method for designing high-accuracy structures, developed by Professor Meyerov, M. V., and the rational physically realizable structures, developed by Professor Sokolov, N. I., systems with constant adjustments have acquired "coarse" properties with respect to the change in the parameters of an object. In this connection, it would be interesting to compare the above adaptive systems with constant adjustments with the self-adjusting systems.

Unfortunately, the experience accumulated thus far in the area of the appraisal of adaptive control systems is grossly inadequate, due to the complexity of the systems themselves and the existing criteria for their comparison.

SKB-3 MAY* made a comparison of such systems when designing an adaptive stabilization network for controlling the aircraft bank channel:

$$W_{ЛА}(p)=\frac{\gamma(p)}{\delta(p)}=\frac{K}{p(Tp+1)}. \quad (1)$$

Here K = const (2) and the time constant is T = 0.1 - 1.5 sec, depending on the regime. The desirable low-frequency transfer function of the system corresponded in all regimes to the aperiodic block $\frac{1}{T_c p+1}$ with the time constant T_c = 0.15 sec.

The system was designed in the constant-adjustment and self-adjustment versions using the frequency characteristics of the elements. Simultaneously, an attempt was made to find a criterion for comparing adaptive systems that would be simultaneously fairly simple and objective. During the design, a comparison was made of: a system with constant adjustment and a relay auto-oscillatory self-adjusting system; a relay system operating in the forced oscillatory regime, and a system on the boundary of stability.

The designed adaptive systems in various versions satisfied the requirements placed on them. However, it turned out that the information about the frequency characteristics of an object (aircraft, drive, and sensors), which is necessary in designing the systems, varies from system to system, and is as follows:

0 - 100 Hz — for systems with constant adjustment;

0 - 30 Hz — for a relay self-adjusting system in a forced oscillatory regime;

0 - 25 Hz — for a relay auto-oscillatory self-adjusting system;

0 - 15 Hz — for a self-adjusting system on the boundary of stability.

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*Translator's note: This designates the Special Design office-3 of the Moscow Aviation Institute.

The slightly larger amount of information needed by a self-adjusting system in the forced regime, as compared with a relay auto-oscillatory system, is due to the fact that the frequency of forced oscillations ω_b of the first system is chosen to be larger than the frequency of the auto-oscillations ω_a of the second system. This is how one achieves the greater stability under interference in the relay system.

These results lead us to conclude that the amount of information about an object that is needed for its design may be utilized as a criterion in comparing adaptive control systems. This criterion will enable us to find systems whose design (design of correcting blocks) requires information in the frequency range in which the specific features of an object begin to exert influence. We are talking about features that were not taken into account by the differential equations describing the elements of the system.

For example, at high frequencies the bending oscillations of an aircraft become important, and the transfer functions for the object as an "ideally rigid body" cease to be valid.

At high frequencies also, the transfer functions of the system drive may turn out to be invalid due to additional lags introduced by parameters of the drive that were not considered before.

Finally, there are frequency bands in which the frequency characteristics of the drive — obtained for example by means of experiments — may no longer be utilized in the design of the correcting devices using the ordinary methods, since the real drive may exhibit nonlinear behavior. Of course, a system — for which the design of the correcting devices was made in the frequency range for which one has no reliable information about the frequency characteristics of the drive — may in practice turn out to be very sensitive to the neglected small parameters of the drive.

Thus, we propose a criterion for comparing adaptive control systems in the form of the frequency range in which the information about an object, necessary for the design of correcting devices, is concentrated.

This criterion permits us in the design stage to exclude those systems which require information about the control object in the frequency region in which that information is practically inaccessible.

The criterion enables us to see if it is necessary to consider certain small parameters of an object, and gives limits within which the effect of those parameters must be conside. . .

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DESIGN OF THE STRUCTURE OF A DIGITAL SELF-ADJUSTING SYSTEM WITH A MODEL FOR AN AIRCRAFT AND ITS IMPLEMENTATION IN AN ON-BOARD DIGITAL COMPUTER

N. D. Litvinov

Design of the Structure of the Control Program for the On-Board Computer

Let us consider a control system consisting of a continuous part (composite object) and an on-board digital computer.

The continuous part will include an interpolator, usually of zero order, an amplifier with the actuator (control surface machine), and the control object which has varying parameters.

It will be assumed that the control object is linear and has a given structure.

Digital sensors will be used as th` sensors of the data cn the coordinates of the object and the control signals.

The control signals involve specific and random inputs.

The continuous part or the composite object may be described by a linear difference equation with time-dependent coefficients.

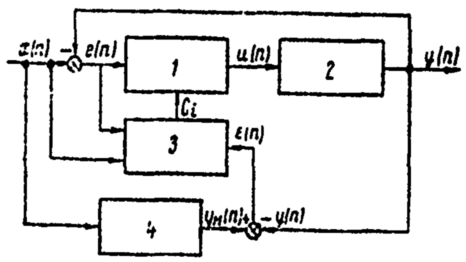

Figure 1. A block diagram of the control program.

1 — control program; 2 — composite control object; 3 — optimization program; 4 — program representing the model of the desired closed system.

We shall consider quasi-stationary processes for the cases when the parameters (coefficients of the equation) of the composite object change continuously and slowly or stepwise, and then remain constant.

It is required to design a searchless adaptive control program for the on-board digital computer as applied to the composite linear nonstationary control object, that would result in the desired dynamic characteristics of the closed system.

We shall represent the control program in the form of a control program — generating the control signal in the on-board computer, and the optimization program — which optimizes the closed system by minimizing the index used to evaluate the performance of the entire system by adjusting the coefficients of the control program.

The control program may be represented in the form of a block diagram (Figure 1).

Let us determine the structure of the control program.

The equation of the control object may be written as

$$y(n+k)+\sum_{i=0}^{k-1} a_i(n)\, y(n+i) - \sum_{j=0}^{k-1} b_j(n)\, u(n+j), \qquad (1)$$

where y_n is the output from the control object, u_n is the control signal generated by the on-board computer.

For the basic regime of operation of the control object, we can assume that the control coefficients $a_i(n)$ and $b_i(n)$ are constant and given.

Let us determine the structure of the control program using a variational method [1]. For this purpose, we obtain a difference equation of the control program from the conditions:

a) stability of the closed system;

b) astatism of first order relative to external inputs, i.e., exact generation of a step input in a steady state motion;

c) optimality of the system in the sense that the quadratic estimate of the transient error process is minimized.

For this purpose, we write Equation (1) for the error in the closed system, and take the first difference of both sides of the equation. Setting $x(n) = x_0 1(n)$, we obtain the following equation:

$$\sum_{s=0}^{k+1} g_s e(n+s) = -\sum_{j=0}^{k-1} b_j u(n+j), \tag{2}$$

where

$$g_s = a_{s-1} - a_s, \ (a_k = 1, \ a_{k+1} = a_{-1} = 0) \ (s = 1, 2, \ldots, k+1) \tag{3}$$

$$\Delta u(n) = u_1(n), \tag{4}$$

in which the initial conditions of Equation (1) are replaced with certain new conditions corresponding to the action of the perturbation

$$\varphi(n) = \sum_{i=0}^{k} a_i x_0 \sigma(n+1+i), \tag{5}$$

where

$$\sigma(n+\Upsilon) = \Delta 1(n+\Upsilon). \tag{6}$$

We represent Equation (2) in the form of an equivalent system of first-order difference equations in generalized coordinates in terms of the error with certain new initial conditions

$$e_{\mu}(0)=e_{\mu 0}\ (\mu=0,1,\ldots,k+1). \tag{7}$$

Now we can formulate the variational problem stating that it is necessary to determine the structure of the control program for which the closed system, described by the system of first-order difference equations, moves from point 7 to the origin

$$e_{\mu}(\infty)=0 \tag{8}$$

while minimizing the functional

$$F(u_1)=\sum_{n=0}^{\infty}\left(\sum_{\mu=1}^{k+1}\alpha_{\mu}e_{\mu}^{2}(n)+\beta u_1^{2}(n)\right),\ (\alpha_{\mu}\beta=\text{const}), \tag{9}$$

which represents a generalized quadratic estimate of the error transient process.

The problem is variational and involves a conditional extremum. However, it can be easily reduced to finding an unconditional extremum, which — when solved employing the usual methods — will lead us to an optimal control law in the form

$$u_1(n)=\sum_{\mu=1}^{k+1}\nu_{\mu}e_{\mu}(n), \tag{10}$$

where ν_{μ} are fully determined coefficients expressed in terms of the coefficients of the equation for the composite control object in the coefficients α_{μ} and β, which in turn are determined using the method of the standard difference equations [1], starting with a given performance index of the transient error process in the closed system.

Substituting in (10) the expressions for $e_{\mu}(n)$ and $u_1(n)$ in terms of e(n) and u(n), we obtain a difference equation which will determine the structure of the control program. The structure is stable, since it was determined from the condition in (8).

The coefficients in the difference equation of the control program will be adjusted by the gradient method with the aid of the optimization program.

In order to estimate the approximation of the desired process, given by the model, as compared with the real process, we introduce a performance index of the adjustment in the form of the functional

$$I = P(E)\cdot\varphi(\varepsilon), \tag{11}$$

where P(E) is a certain difference operator, E is the forward shift operator, $\phi(\varepsilon)$ is the differentiated function of the quadratic form in the deviation $\varepsilon(n)$.

For the gradient method of adjustment, the variation of the coefficients of the control program is given by

$$c_i(n+1) = c_i(n) + \lambda_i \frac{\partial I(c)}{\partial c_i}, \tag{12}$$

where i specifies the adjusted parameter, λ_i are coefficients which generally depend on the deviation of the position of extremum and gradient components [2].

It will be assumed that the coefficients λ_i have been selected in such a way that a steady adjustment process is obtained.

We obtain the following expression for the gradient for the case of the above block diagram (Figure 2)*

$$\frac{dI(c)}{dc_i} = P(E)\frac{d\varphi(\varepsilon)}{d\varepsilon}\Phi(E,\alpha,c)\,[1-\Phi(E,\alpha,c)]\cdot\frac{1}{W_n(E,c)}\cdot\frac{dW_n(E,c)}{dc_i}\cdot x(n), \tag{13}$$

*Translator's note: Foreign text has no Figure 2.

where $\Phi(E, \alpha, c)$ is the operator of transformation of the closed system, dependent on the adjusted coefficients of the control program, c_i, and the time-dependent coefficients of the control object α, $W_n(E, c)$ is the operator of transformation for the control program, $W_{ob}(E, \alpha)$ is the operator of transformation for the composite object.

Since for the deviations $\varepsilon(n)$ equal to zero we have $\Phi(E, \alpha, c) = \Phi(E)$, where $\Phi_M(E)$ is the operator of transformation for the model, the algorithm for adjusting the coefficients c_i of the control program may be written as

$$c_i(n+1) = c_i(n) - \lambda_i P(E) \frac{d\varphi(\varepsilon)}{d\varepsilon} \Phi_M(E) [1 - \Phi_M(E)] \frac{1}{W_n(E, c)} \times \\ \times \frac{dW_n(E, c)}{dc_i} x(n). \tag{14}$$

The algorithm in (14) represents an optimization program, and is described by a nonlinear difference equation.

To form the algorithm in (14), we can use the intermediate coordinates of the closed system. For this, we use the relation

$$[1 - \Phi(E, \alpha, c)] x(n) = e(n), \; \frac{e(n)}{u(n)} = \frac{1}{W_n(E, c)}. \tag{15}$$

Thus, the control program, the optimization program, which adjusts coefficients of the control program, and the program with a model of the desired closed system will altogether constitute a searchless adaptive control program for the on-board digital computer.

A Method of Implementing the Control Program

In those cases where a composite control object is described by a high-order difference equation, the structure of the control program is also of high order. From the equation of the optimization program, we can see that its order will be determined by the orders of the equations of the model and the control program. Of course, for a given computer speed the elementary time interval of the control program will be very high, which will affect the accuracy and quality of control. On the other hand, for a given elementary time interval of the control program, we need an on-board digital computer of very high speed in order to realize the program, and this is usually impossible to achieve in practice. Therefore, other ways of realizing the control program are necessary.

As one of the possible ways of realizing the control program, one can use the following method.

Due to the fact that one considers quasi-stationary processes in which the coefficients of the equation for a composite control object change continuously and slowly or stepwise, and then remain constant, by analogy with continuous self-adjusting systems one can assume that the rate of the process in a real closed system is higher than the rate of the process of adjustment of the control program coefficients, which will be determined by the length of the range in which the parameters of the composite control object stay constant. Therefore, one can organize the operation of the closed loop which includes the control program and the composite control object, using the length of the elementary time interval selected from the conditions determined by the requirements on the accuracy of control, by the dynamic properties of the composite control object, and the frequency properties of external inputs. The operation of the optimization program and the program of the model of the desirable closed system may be organized for the length of the elementary time interval determined by the length of the interval in which the parameters of the composite control object stay constant. Of course, we have to impose the condition that time intervals be multiples of the elementary time interval thus obtained, due to the discreteness in the deviation $\varepsilon(n)$.

When the operation of the control program is as described above, one will meet the requirements for accuracy of control, and will obtain lower requirements on the speed of an on-board digital computer.

Conclusion

In designing a searchless adaptive control program for an on-board digital computer as applied to a linear nonstationary object, the design must be such that the requirements on accuracy and quality of control will be met. The above method of realizing the control program for multiples of the elementary time interval enables us to meet the requirements for accuracy and quality of control in a wide range of the control object parameters.

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A DESIGN OF A SELF-ADJUSTING AUTOMATIC PILOT

A. A. Kotenko and V. A. Serdyukov

The behavior of the rigid fuselage of an object may be approximately described by the equation

$$\ddot{\nu}_0 + a_{01}\dot{\nu}_0 + a_{00}\nu_0 = a_{02}\delta \qquad (1)$$

The blocks characterizing the elastic oscillations of the fuselage are represented by equations of the type

$$\ddot{\nu}_i + a_{i1}\dot{\nu}_i + a_{i0}\nu_i = a_{i2}\delta; \; i = 1, 2, \ldots, n. \qquad (2)$$

where

$$\nu = \sum_{i=0}^{n} \nu_i. \qquad (3)$$

We assume that the dynamics of the control organ is described by the equation

$$\dot{\delta} + b\delta = \delta_c. \qquad (4)$$

In Equations (1), (2), (3), (4) we used the following notation: ν is the pitch angle of the elastic object; ν_0 is a component of the angle ν characterizing the pitch deflection of the rigid fuselage of the object; ν_i ($i = 1, 2, \ldots, n$) are components of the angle ν characterizing the elastic oscillations of the object; a_{ij} ($i = 1, 1, \ldots, n$, $j = 0, 1, 2$) are time-dependent coefficients; δ is the output of the control organ; δ_c is the input of the control organ; b is a constant coefficient.

It is assumed that: 1) during flight the dynamic characteristics of the elastic object undergo changes (coefficients a_{ij} change arbitrarily with time, but slowly enough so that within a given time interval one can take a_{ij} = const, which is the condition of quasi-stationarity); 2) measuring apparatus has zero inertia; 3) the coordinates ν_i, $i = 0, 1, \ldots, n$ may be measured.

It is required to design a self-adjusting autopilot that will compensate for the effect of the changing parameters of a rigid object and the elastic oscillations of the fuselage.

The control law is as follows

$$\delta_c = (D+b)\left[k_2 g - \sum_{j=0}^{n} (k_{j1} D + k_{j0}) \nu_j\right] \tag{5}$$

Here $D = \frac{d}{dt}$, g is the control signal, k_{j0}, k_{j1}, k_2 are adjustable coefficients.

We substitute (5) in (1) and (2), taking (3) and (4) into account, and obtain

$$\ddot{\nu}_i + a_{i1}\dot{\nu}_i + a_{i0}\nu_i = k_2 a_{i2} g - \sum_{j=0}^{n} a_{i2} k_{j1} \dot{\nu}_j - \sum_{j=0}^{n} a_{i2} k_{j0} \nu_j \quad (i = 0, 1, \ldots, n.) \tag{6}$$

We approximate the desired dynamic characteristics of the basic loop of the system "object-autopilot" by a standard filter (model) of the form

$$\ddot{v}_M + \bar{a}_1 \dot{v}_M + \bar{a}_0 v_M = \bar{a}_2 g \tag{7}$$

Here a_i (i = 0, 1, 2) are constant coefficients of the model.

We set

$$\varepsilon = v - v_M = \sum_{i=0}^{n} v_i - v_M \tag{8}$$

We shall sum Equations (6) and subtract (7) from this sum. In view of (8), we obtain

$$\ddot{\varepsilon} + \bar{a}_1 \dot{\varepsilon} + \bar{a}_0 \varepsilon = \left(\sum_{i=0}^{n} k_2 a_{i2} - \bar{a}_2 \right) g - \sum_{i=0}^{n} \sum_{j=0}^{1} (a_{ij} - \bar{a}_j + n a_{i2} k_{ij}) v_i^{(j)} \tag{9}$$

We assume that the coefficients of the control law (5) are adjusted by the algorithm

$$\begin{aligned} \dot{K}_{ij} &= \psi_{ij}, \quad i = 0, 1, \ldots, n; \quad j = 0, 1. \\ \dot{K}_2 &= \psi_2. \end{aligned} \tag{10}$$

We represented (9) and (10) in matrix form

$$\begin{aligned} \dot{\varepsilon} &= A\varepsilon - B, \\ \dot{k} &= \psi, \end{aligned} \tag{11}$$

where

$$A = \left\| \begin{matrix} 0 & 1 \\ -\bar{a}_0 & -\bar{a}_1 \end{matrix} \right\|; \quad B = \left\| \begin{matrix} 0 \\ \left(\sum_{i=0}^{n} k_2 a_{i2} - \bar{a}_2 \right) g - \sum_{j=0}^{1} \sum_{i=0}^{n} (a_{ij} - \bar{a}_j + n a_{i2} k_{ij}) v_i^{(j)} \end{matrix} \right\|$$

$$\varepsilon = \left\| \begin{matrix} \varepsilon \\ \dot{\varepsilon} \end{matrix} \right\|; \quad k = \left\| \begin{matrix} k_{00} \\ k_{10} \\ \vdots \\ k_{n0} \\ k_{01} \\ \vdots \\ k_{n1} \\ k_2 \end{matrix} \right\|; \quad \psi = \left\| \begin{matrix} \psi_{00} \\ \psi_{10} \\ \vdots \\ \psi_{n0} \\ \psi_{01} \\ \vdots \\ \psi_{n1} \\ \psi_2 \end{matrix} \right\|. \tag{12}$$

The adaptation algorithm (10) will be formed using the condition of the existence of the Lyapunov function for System (11). The Lyapunov function is assumed to have the form

$$V = \varkappa \varepsilon' P \varepsilon + \lambda \sum_{j=0}^{1} \sum_{i=0}^{n} (a_{ij} - \bar{a}_j + n a_{i2} k_{ij})^2 + \lambda_2 \sum_{i=0}^{n} (a_{i2} k_2 - \bar{a}_2)^2 \tag{13}$$

Here κ, λ_2 and λ are certain coefficients, and P is a square positive symmetric 2 x 2 matrix; ε' is a transposed matrix.

In view of Equations (11) and the condition that the coefficients a_{ij} ($i = 0, 1, \ldots, n$; $j = 0, 1, 2$) be quasi-stationary, the derivative of (13) will be

$$\dot{V} = \varkappa\varepsilon' M\varepsilon + 2\varkappa(P_{21}\varepsilon + P_{22}\dot{\varepsilon})\left[\left(\sum_{l=0}^{n} a_{l2}k_2 - \bar{a}_2\right) g - \right.$$
$$\left. - \sum_{j=0}^{1}\sum_{l=0}^{n}(a_{lj} - \bar{a}_j + na_{l2}k_{lj})v_l^{(j)}\right] + 2\lambda_2\left(\sum_{l=0}^{n} a_{12}k_2 - \bar{a}_2\right)\left(\sum_{l=0}^{n} a_{l2}\right)\Psi_2 + \tag{14}$$
$$+ 2\lambda\sum_{j=0}^{1}\sum_{l=0}^{n}(a_{lj} - \bar{a}_j + n_{l2}k_{lj})\left(\sum_{l=0}^{n} na_{l2}\right)\Psi_{lj} \leqslant 0.$$

Here M is a 2 x 2 negative matrix. The elements of the matrix M which are specified arbitrarily are related to the elements of the matrix P in (13) by the equation

$$A'P + PA = M. \tag{15}$$

If the real parts of the roots of the characteristic equation

$$|A - \lambda E| = 0 \tag{16}$$

are negative, then according to [2] to the definite negative form $\varepsilon' M\varepsilon$ in (14) there always corresponds a definite positive form $\varepsilon' P\varepsilon$ in (13). Consequently, in order for the adaptation process in the "object-autopilot" system to be stable, it is sufficient that the following conditions be satisfied

$$\varkappa(P_{21}\varepsilon + P_{22}\dot{\varepsilon})\left[\left(\sum_{l=0}^{n} a_{l2}k_2 - \bar{a}_2\right) g - \sum_{j=0}^{1}\sum_{l=0}^{n}(a_{lj} - \bar{a}_j + na_{l2}k_{lj})v_l^{(j)}\right] +$$
$$+ 2\lambda_2\sum_{l=0}^{n}(a_{l2}k_2 - \bar{a}_2)\left(\sum_{l=0}^{n} a_{l2}\right)\Psi_2 + 2\lambda\sum_{j=0}^{1}\sum_{l=0}^{n}(a_{lj} - \bar{a}_j + na_{l2}k_{lj}) \times \tag{17}$$
$$\times\left(\sum_{l=0}^{n} na_{l2}\right)\Psi_{lj} < 0$$

Since changes in the deviations of the coefficients are independent

$$(a_{lj} - \bar{a}_j - na_{l2}k_{lj}),\ \left(\sum_{l=0}^{n} a_{l2}k_2 - \bar{a}_2\right),\ (l = 0, 1, \ldots n;\ \ j = 0, 1)$$

we have the following sufficient conditions for the inequality in (17) to be satisfied

$$\varkappa (P_{21}\varepsilon + P_{22}\dot{\varepsilon}) g = -\lambda_2 \left(\sum_{i=0}^{n} a_{i2} \right) \Psi_2,$$
$$\varkappa (P_{21}\varepsilon + P_{22}\dot{\varepsilon}) v_i^{(j)} = \lambda \left(\sum_{0}^{n} n a_{i2} \right) \Psi_{ij}, \qquad (18)$$
$$i=0,\ 1, \ldots, n; \quad j=0,\ 1.$$

We take

$$\lambda_2 = \frac{1}{\sum_{i=0}^{n} a_{i2}}$$

$$\lambda = \frac{1}{\sum_{i=0}^{n} n a_{i2}}.$$

Then the algorithms for the adjustment of the coefficients of the control law (5) become

$$\dot{K}_2 = -\varkappa (P_{21}\varepsilon + P_{22}\dot{\varepsilon}) g,$$
$$\dot{K}_{ij} = \varkappa (P_{21}\varepsilon + P_{22}\dot{\varepsilon}) v_i^{(j)},\ i=0,\ 1, \ldots, n; \quad j=0,\ 1. \qquad (20)$$

In (20) the factor κ may be arbitrary (including equal to infinity). In this case with the coordinate K_{ij} or K_2 restricted in absolute value, we obtain a relay switching of the coefficients of the control law

$$K_2 = -B \operatorname{sgn} (P_{21}\varepsilon + P_{22}\dot{\varepsilon}) \operatorname{sgn} g$$
$$K_{ij} = B \operatorname{sgn} (P_{21}\varepsilon + P_{22}\dot{\varepsilon}) \operatorname{sgn} v_i^{(j)} \quad i=0,\ 1, \ldots, n;\ j=0,\ 1. \qquad (21)$$

According to the design of the algorithms (20) and (21), the self-adjusting loops for the system "object-autopilot," designed according to (20) or (21), assure the stability of motion of the output relative to the output of the standard model, and consequently, assure the invariance to within ε of the dynamic characteristics of the system "object-autopilot" with respect to the changing parameters of the object and the elastic oscillations of the fuselage.

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ADAPTATION OF SYSTEMS WITH CONSTANT ADJUSTMENTS

M. I. Savkin

We know of adaptive systems in which a change in the parameters of the control object is compensated for by a corresponding change in the adjustment of the regulator. If a system contains a nonstationary object and it includes special correcting devices with constant parameters, one can reduce the range of its performance indices. As a result, such systems with constant adjustment acquire properties that are analogous to adaptation.

This will be shown using an example of a simple system. Here, we assume that conditions are satisfied so that the system may be regarded as quasi-stationary.

Suppose we are given the transfer function of the object $W_0(p)=\frac{K_0^{I,II}}{T_0^{I,II}p+1}$, the amplification coefficient $K_0^{I,II}$, and the time constant $T_0^{I,II}$ that satisfy the following inequalities in the design regimes (I, II)

$$K_0^{I,II}\gg 1;\ K_0^{II}\gg K_0^{I};\ T_0^{I}\gg T_0^{II}.$$

Suppose that we must select the structure and parameters of a sequential correcting device $W_K(p)$, such that the cutoff frequency $\omega^{I,\ II}$ of the system $\Phi^{I,\ II}(p)$ closed by a single negative feedback, will be greater than or equal to a certain desired value ω_d, where

$$\frac{K_0^{I}}{T_0^{I}} < \omega_{ж} < \frac{K_0^{II}}{T_0^{II}}.$$

If $W_K(p) = K$ is taken such that $\frac{K \cdot K_0^{I}}{T_0^{I}} = \omega_d$, then we obtain

$$\Phi^{I,II}(p) \approx \frac{1}{T^{I,II}p+1}, \tag{1}$$

where

$$\omega^{I} = \frac{1}{T^{I}} = \omega_{ж},$$

$$\omega^{II} = \frac{1}{T^{II}} = \frac{KK_0^{II}}{T_0^{II}} \gg \omega_{ж}.$$

In this case

$$\frac{\omega^{II}}{\omega^{I}} = \frac{K_0^{II}}{K_0^{I}} \cdot \frac{T_0^{I}}{T_0^{II}}. \tag{2}$$

Here the performance indices of system (1) will differ greatly depending on the design regime.

However, if $W_K(p) = \frac{K(T_2p+1)}{T_1p+1}$; $\frac{K \cdot K_0^{I}}{T_0^{I}} = T_1 = \omega_{ж}$;

$$\frac{1}{T_2} = \alpha \frac{1}{T_1}, \quad \alpha > 1,$$

then

$$\omega^{I} = \omega_{ж}, \quad \tilde{\omega}^{II} = \frac{K \cdot K_0^{II}}{T_0^{II}} \cdot \frac{-}{\alpha},$$

and

$$\frac{\tilde{\omega}^{II}}{\omega^{I}} = \frac{K_0^{II}}{T_0^{II}} \cdot \frac{T_0^{I}}{K_0^{II}} \cdot \frac{1}{\alpha}. \tag{3}$$

By properly selecting the value of α (for example, based on the permissible stability margin), one can bring the cutoff frequencies ω^{I} and ω^{II} closer by a factor of α, and thus achieve the same effect in the range of the performance indices of the system. This effect may be amplified if one takes a "closing" correcting device

$$W_K(p) = \frac{(T_2^2p^2 + 2\xi_2T_2p + 1)^m}{(T_1^2p^2 + 2\xi_1T_1p + 1)^m}. \tag{4}$$

The example considered above is illustrated in Figure 1. In conclusion, we note that, if at the input to the system one places a low-frequency filter with constant parameters

$$W_\phi(p) = \frac{1}{T^2p^2 + \xi Tp + 1}, \tag{5}$$

where $T \geq (2 - 5)\omega_d^{-1}$, then the system thus obtained will have approximately constant dynamic characteristics. It is important to note (see the preceding article) that such a system with "closing" correcting devices requires an identification of the object $W_0(p)$ in a smaller frequency range than a system with fixed adjustments without such devices.

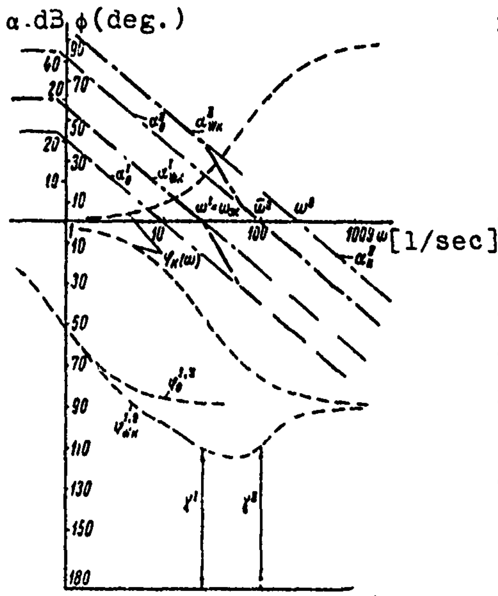

Figure 1. Logarithmic frequency response characteristic of the system.

Thus we have considered a way of designing a single-loop system with constant adjustments that possesses a property similar to adaptation. This method can be applied to stabilize the dynamic properties of multi-loop systems with constant adjustments. In this case, the "closing" correcting devices must be put in each loop of the system that includes given elements with variable parameters.

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<u>SYMBOL LIST</u>

УПР	c	control
Ц Ц	n	network
БАЛ	bal	balance
УСТ	pl	placement
ПРИБ	ins	instrument
Б	l	lateral
ЭФ	eff	effective
ОБ	ob	object
ВЫХ	out	output
ВХ	in	input
Р	p	control
ПР	dr	drive
П	n	noise
В	in	in
КУ	cd	correcting device
ЗАД	giv	given
Б	b	booster
ЛЕТ	f	flight
ВЕТ	w	wind
БУСТ	b	booster
СТ	adj	adjust
О	o	object
В	up	upper filter
Н	low	lower filter
М	s	slow
Б·ч	b	undefined
ДОП	add	additional
Б	l	large
Ц·М	c.m.	center of mass
Н	o	origin
Н	s	stationary
ДУС	ang	angular velocity sensor
ЛА	a	aircraft
Ф	f	filter
ДУ	s	sensor
ОБ	ob	object
Ж	d	desired

ДЕЙСТ	a	actual
Н	n	nominal
СР	av	average
ТЕК	current	current
Э	e	Euler