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BALLISTIC IMPACT OF SINGLE AND SPACED
PLATES

William P. Dunn, et al

Watervliet Arsenal
Watervliet, New York

December 1972

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BALLISTIC IMPACT OF SINGLE AND SPACED PLATES



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W.P. DUNN

Y. K. HUANG

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III

BALLISTIC IMPACT OF SINGLE AND SPACED PLATES

ABSTRACT

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Cross-Reference Data

Plate Impact
Ballistic Impact
Hypervelocity
Impact
Hypervelocity
Cratering
Terminal Ballistics
Ballistic Limit
Residual Velocity
Shatter Velocity
Spray Angle
Shock Waves
Nonlinear
Mechanics
Nonlinear
Mathematics
Armor Piercing Ammunition
Armor Plate Protection
Metal Plates

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INTRODUCTION

This is the final report which summarizes some results of my investigation for the research task of the fiscal year 1972. The investigation has been concerned with the penetration dynamics of plate impacts in three ballistic categories.

This research should be of value for consideration of weapons design and armor protection from kinetic energy threats.

A survey of the literature [1-4] shows that many approaches have been taken to provide solutions which characterize the ballistic impact with dominant features in one way or the other. But there exists no single approach which has offered a convincingly satisfactory solution. For this reason and others, we seek to consider the impact of plate from the viewpoint of nonlinear mechanics. Such an approach is especially suitable for the analytical treatment of terminal ballistics which is often empirical (and confidential). It is not surprising that remarkable simplification and idealization have to be made in order to arrive at an analytical description of the problems considered. As mentioned we will consider normal impacts and their ballistic effects only. A few modes of plate penetration

1. Effects of Impact and Explosion, Summary Technical Report of NDRC, Division 2, Vol. 1, Chapters 6, 7, and p. 279 (Washington, D. C., 1946).
2. W. Herrmann and A. H. Jones, Survey of Hypervelocity Impact Information, A.S.R.L. Report No. 99-1 (Massachusetts Institute of Technology, 1961).
3. W. Goldsmith, Impact, pp. 240-248 (Edward Arnold, London, 1960).
4. N. Cristescu, Dynamic Plasticity, Chapter 6 (North-Holland, Amsterdam, 1967).

and perforation can be formulated as one-dimensional nonlinear problems. This method we have exploited not only for less mathematical effort but also for more physical support by scaling. Therefore our analytical results are derived from concepts such as nonlinear resistance and shock fragmentation.

For the categories of impact to be considered, there are a large number of parameters which affect the processes and results of these impacts. Here we refer only to the more important factors such as the projectile velocity, mass (size, shape, density), tensile strength, plate thickness, density, hardness, and shock-compression properties. Upon impact, the projectile and plate interact with or without perforation/shatter. Plate perforation may occur as a result of plug formation, hole enlargement, bulging petal, spallation, or fragmentation. At the same time severe shock loading and unloading may cause large deformation or complete disintegration of the projectile. Let us summarize these situations as depicted [1] in Figure 1.

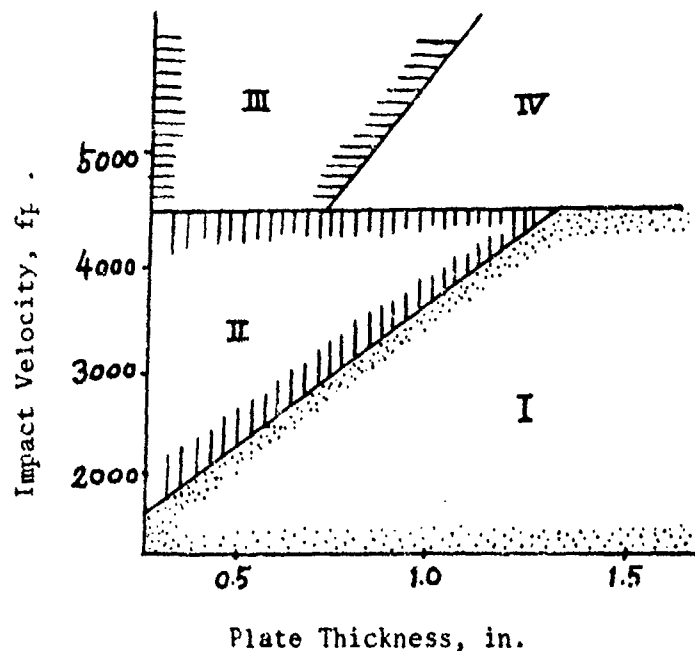


FIGURE 1 - BALLISTIC IMPACT OF PLATE

There we note three lines of demarcation: (a) the ballistic limit without shatter, separating impact categories I and II; (b) the ballistic limit with shatter, separating categories III and IV; and (c) the transition velocity dividing the no shatters and the shatters.

Thus we may consider the various impacts under these headings:

- I. No Perforation, No Shatter
- II. Perforation, No Shatter
- III. Perforation, Shatter
- IV. No Perforation, Shatter.

In what follows, we shall examine most of the cases except for category I and petaling.

ARMOR PERFORATION WITH PLUG

Let us consider impact category II. A simple model can be constructed for it on the basis of nonlinear resistance $R = R(\dot{x}, x)$, where x is the penetration depth and $\dot{x} = dx/dt$ is the rate of penetration with t denoting the time. It is heuristic to consider $R = -Kx^n$, K and n being appropriate constants. Thus the equation of motion of the projectile may be written as

$$M\ddot{x} + Kx^n = 0 \quad (1)$$

where M denotes the projectile mass and \ddot{x} its deceleration. From equation (1) the ballistic impact appears to be a special kind of nonlinear vibration. But we will not exploit equation (1) in that

sense for the projectile's terminal trajectory might not be the same as the solution of equation (1). With certain shape factors and appropriate energy balance accounted by constants K and n, equation 1 can satisfy the initial and boundary conditions: (a) $t = 0$, $x = 0$, $\dot{x} = V =$ striking velocity; and (b) $t = T =$ penetration time, $x = P =$ total penetration depth, $\dot{x} = 0$. Inserting these values into the first integral of equation (1), we get the initial energy and momentum as given by

$$\frac{1}{2} MV^2 = KP^{n+1}/(n+1) \quad (2)$$

$$MV = [2MK/(n+1)]^{\frac{1}{2}} P^{\frac{1}{2}} (n+1) \quad (3)$$

Let P be the plate thickness. Then V stands for the ballistic limit (V_*). From Equation (2) we should have $n = 2$ for perforation by energy absorption [2] in a volume proportional to P^3 . Such proportionality is justifiable for plate thickness of about 1 caliber; otherwise a certain shape factor should be considered. From equation (3) a momentum-absorption theory [2] requires $n = 5$, which implies also the alternate form of equation (1) as

$$M\ddot{x} + k \dot{x} x^2 = 0 \quad (4)$$

for a different kind (k) of nonlinear resistance (viz. rate sensitive).

From the similarity point of view, equation (1) can also fit the classical formulations of Robins, Fairbairn, DeMarre, and Krupp

2. W. Herrmann and A. H. Jones, Survey of Hypervelocity Impact Information, A.S.R.L. Report No. 99-1 (Massachusetts Institute of Technology, 1961).

for the ballistic limit of armor plate [2]. Thus we may put $n = 0$ and $K^\infty d^2$ in equation (1) or (2) to derive the Robins formula, σ being the flow stress of plate and d being the projectile caliber. For Fairbairn we should have $n = 1$ and $K^\infty \tau d$, τ being the shear strength of plate; for DeMarre, $n = 3/10$ and $K^\infty d^{3/2}$; and for Krupp, $n = 1/3$ and $K^\infty d^{5/3}$. All these different values of n should be attributed to the empirical nature of the pertinent formulas. As far as terminal ballistics and nonlinear mechanics are concerned, our formulation of equations (1) and (4) is analytically sound. Slightly modified experimental correlations are up to individual investigators.

If the projectile has sufficient kinetic energy to pass through the armor plate, then its residual velocity is given [5] by

$$V_r = M(M + m)^{-1} (V^2 - v_*^2)^{\frac{1}{2}} \quad (5)$$

or

$$V_r = [M(M + m)^{-1} (V^2 - v_*^2)]^{\frac{1}{2}} \quad (6)$$

where m is the mass of the displaced plug. In equation (5) V should be identified as the flight velocity of the projectile, which differs from the striking velocity due to effects of friction and rotation (rifling). Equation (6) neglects such difference. Obviously, there is no difference between equations (5) and (6) for heavy projectile or thin plate. Ideally, the secondary impact of a double-plate system would occur with almost the same amount of momentum as the initial.

2. W. Herrmann and A. H. Jones, Survey of Hypervelocity Impact Information, A.S.R.L. Report No. 99-1 (Massachusetts Institute of Technology, 1961).

5. See Appendix.

But the impact energy is less. Threat from such impact could be indentation, spalling, bulging, or petaling. For a safe design of spaced plates, V_T should be such that the kinetic energy of the secondary impact be small.

PLATE PERFORATION WITH SPRAY

This is our Category III. See Figure 1. For a given projectile there exists a shatter velocity depending upon shock-compression properties and the impedance match, at and above which it will be disintegrated due to the sudden release of strong shock pressure around its free surface. Strong shock compression of solids requires hypervelocity impact at the level of 1 km/sec and higher [6]. We shall give expression for evaluating the shatter velocity after we consider the shock waves of the explosive impact.

Upon impact at hypervelocity, the two colliding bodies are compressed by two strong shock waves (S_1, S_2) which can be described as follows:

$$\rho_{10} (W_1 + V) = \rho_1 (W_1 + v - u_1) \quad (7)$$

$$S_1 \quad p_1 = \rho_{10} u_1 (W_1 + V) \quad (8)$$

$$E_1 = \frac{1}{2} p_1 (\rho_{10}^{-1} - \rho_1^{-1}) \quad (9)$$

6. Y. K. Huang and N. Davids, J. Franklin Inst. 276, 39 (1963).

$$\rho_{20} W_2 = \rho_2 (W_2 - u_2) \quad (10)$$

$$S_2 \quad p_2 = \rho_{20} u_2 W_2 \quad (11)$$

$$E_2 \approx \frac{1}{2} p_2 (\rho_{20}^{-1} - \rho_2^{-1}) \quad (12)$$

where V is the impact velocity, W = shock velocity, u = particle velocity behind the shock front, ρ = density, p = shock pressure, E = specific internal energy, and subscripts 0, 1 and 2 refer to the initial value, striker and plate, respectively. It should be noted that only u_1 is a velocity relative to a reference frame moving with V . The interface between the two colliding bodies furnishes the boundary conditions:

$$p_2 = p_1 \quad (13)$$

$$u_2 = V - u_1. \quad (14)$$

Thus far we have set up only eight equations for ten unknowns, namely, p_1 , ρ_1 , E_1 , W_1 , u_1 , p_2 , ρ_2 , E_2 , W_2 , and u_2 . Since the strongly compressed material will expand immediately as spray, we may approximate the shocked and expanded states by a polytropic gas with carefully chosen index r (in the limit $r \rightarrow 3$). So two additional equations are given by:

$$E_1 = p_1 / \rho_1 (r_1 - 1) \quad (15)$$

$$E_2 = p_2 / \rho_2 (r_2 - 1). \quad (16)$$

Now we can solve equations (7)-(16) simultaneously for the ten unknowns whose explicit results are given by:

$$\rho_1 = \rho_{10} (r_1 + 1)/(r_1 - 1) \quad (17)$$

$$u_1 = \xi V/(\xi + 1) \text{ with}$$

$$\xi = [\rho_{20}(r_2 + 1)/\rho_{10}(r_1 + 1)]^{\frac{1}{2}} \quad (18)$$

$$W_1 = \frac{V}{2} \left[\frac{\xi(r_1 - 1) - 2}{\xi + 1} \right] \quad (19)$$

$$P_1 = \frac{1}{2} \rho_{10}(r_1 + 1)(\xi V)^2/(\xi + 1)^2 \quad (20)$$

$$E_1 = \frac{1}{2} (\xi V)^2/(\xi + 1)^2 \quad (21)$$

$$\rho_2 = \rho_{20}(r_2 + 1)/(r_2 - 1) \quad (22)$$

$$u_2 = V/(\xi + 1) \quad (23)$$

$$W_2 = \frac{1}{2} V(r_2 + 1)/(\xi + 1) \quad (24)$$

$$P_2 = \frac{1}{2} \rho_{20}(r_2 + 1)V^2/(\xi + 1)^2 \quad (25)$$

$$E_2 = \frac{1}{2} V^2/(\xi + 1)^2 \quad (26)$$

Further results can be sought. Equations (15) and (16) imply $dp/d\rho = a^2 = rp/\rho$, a being the sound velocity. From this and

equations (17), (20), (22), and (25) we get

$$a_1 = \xi(\xi + 1)^{-1} [r_1(r_1 - 1)/2]^{1/2} v \quad (27)$$

$$M_{s1} = (W_1 + V)/a_1 = (r_1 + 1) [2r_1(r_1 - 1)]^{-1/2} \quad (28)$$

$$a_2 = (\xi + 1)^{-1} [r_2(r_2 - 1)/2]^{1/2} v \quad (29)$$

$$M_{s2} = W_2/a_2 = (r_2 + 1) [2r_2(r_2 - 1)]^{-1/2} \quad (30)$$

with M_s denoting the shock Mach number. Using the Riemann invariant [7], we also get the velocity component of lateral expansion:

$$u_\ell = 2a/(r - 1) \quad (31)$$

the spray angle of particles is given by $\phi = \tan^{-1} (u_\ell/u_2)$.

Therefore, we have

$$\tan \phi_1 = \frac{u_{\ell 1}}{u_2} = \xi \left(\frac{2r_1}{r_1 - 1} \right)^{1/2} \quad (32)$$

$$\tan \phi_2 = \frac{u_{\ell 2}}{u_2} = \left(\frac{2r_2}{r_2 - 1} \right)^{1/2} \quad (33)$$

From a^2 or E we can also estimate the shock temperature. In this connection, the impact spray may be thought of as a result of vaporization (rather than spallation).

The foregoing is an analytical description for the perforation of plate by a small projectile shattered as a result of shock loading

7. R. Courant and K. O. Fredricks, Supersonic Flow and Shock Waves pp. 87, 90 (Wiley, New York, 1948).

and unloading. It is of interest to consider three limiting values of the index r . As $r \rightarrow 0$, equations (18), (20), (23), and (25) reduce to those equations of shaped-charge jet penetration [1,2,8]. This evidence seems to indicate the generality of shock-wave theory on the one hand and the complementarity of hydrodynamics on the other. Since two different material models are involved in the two theories, we would not seek the meaning of equations (15), (16), (17), and (22) with $r = 0$. A similar case with $r = -1$ has been considered [7,9] for isentropic flow in gas dynamics. Equations (28) and (30) impose another limit upon r . The shock waves are said to be hypersonic with $M_s \rightarrow \infty$ as $r \rightarrow 1$. Let $r = 3$ and $\xi \approx 1$. From equations (32) and (33) we get $\phi = 60^\circ$ which turns out to be qualitatively descriptive of the spray. Moreover, the pertinent p , u , and ρ/ρ_0 are comparable with those for the products of detonation [10] (TNT) based on $r = 3$. Thus we consider $r = 3$ appropriate for both dense gases.

In order to evaluate the shatter velocity V_s , we must consider some fundamental properties of shock waves in solids. It is far reaching to begin with the relation [11]:

$$W = A + Bu \tag{34}$$

where A and B are experimental constants with many physical interpretations and implications. According to Rodean [12], $A^2/2B^2$ is approximately equal to the heat of sublimation of the solid. From equation (18) the absolute particle velocity is given by $V - u_1 = V/(\xi + 1)$ and the associated kinetic energy is $V^2/2(\xi + 1)^2$. When the latter is equal to the heat of sublimation, the shocked material will expand

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12. H. C. Rodean, J. Chem. Phys. 49, 4117 (1968).

infinitely upon unloading. Therefore we establish the relation

$$V_s = (1 + \xi) A/B \quad (35)$$

where A and B belong to the projectile material, with $\xi = (\rho_{20}/\rho_{10})^{\frac{1}{2}}$.

Current criterion for the design of spaced plates is as follows. At velocities where the projectile is broken up, there is an optimum shield thickness for which the total penetration is a minimum. A semi-empirical demonstration of such calculations is given in Reference [13]. At lower velocities with no breakup of the projectile, the shield plate offers less protection for potential threat.

HYPERVELOCITY CRATERING

When a target plate is too thick to permit perforation, a crater may form therein as a result of annihilation of the projectile by strong shock compression. Category IV of Figure 1 is comprised of impacts like this. In the literature there exist a large number of papers concerning this problem. See Reference [1] and Proceedings of Symposia on Hypervelocity Impact (1958, 1960, 1961, 1963, 1964). In this report we consider this problem again with nonlinearities in one dimension.

Let us denote the nonlinear resistance by

$$R(v,x) = \alpha f(x)v^\gamma + \beta g(x)v^2$$

where v is the velocity $\dot{x} = dx/dt$, f(x) and g(x) are two arbitrary functions, and α , β , and γ are all constants. Expressing \dot{x} as $v dv/dx$

1. Effects of Impact and Explosion, Summary Technical Report of NDRC, Division 2, Vol. 1, Chapters 6, 7, and p. 279 (Washington, D. C., 1946).
13. D. J. Stone, "Designing for Meteoroid Impact," ASME Paper 67-DE-45 (March 1967).

with some constant equivalent mass M , the equation of motion is given by

$$Mv \, dv/dx = -\alpha f(x)v^\gamma - \beta g(x)v^2$$

which may be re-arranged as

$$\frac{dv}{dx} + \frac{\beta}{M} g(x)v = -\frac{\alpha}{M} f(x)v^{\gamma-1}. \quad (36)$$

Now equation (36) is of the Bernoulli type. Let $y = v^{2-\gamma}$. Then equation (36) can be linearized and integrated as

$$\begin{aligned} y \exp \left[\int \beta(2-\gamma)g(x)dx/M \right] \\ = \int (\alpha/M)(\gamma-2)f(x) \exp \left[\int \beta(2-\gamma)g(x)dx/M \right] dx \end{aligned} \quad (37)$$

It is clear that we have been motivated to formulate the idealized impact by the well-known Bernoulli's nonlinear differential equation. As a special case, we have the classical Poncelet resistance [1] with $f(x) = \text{constant}$, $g(x) = \text{constant}$, and $\gamma = 0$.

In the theory of nonlinear differential equations, Riccati's equation is also well-known. It turns out also reasonable to assume a nonlinear resistance in the form:

$$R(v,x) = \lambda v^3 - \mu v x^\theta$$

where λ , μ and θ are all constants. Now the equation of motion becomes

$$\frac{dv}{dx} + \frac{\lambda}{M} v^2 = \frac{\mu}{M} x^\theta \quad (38)$$

which is the Riccati equation. Let $C_1 = \lambda/M$, $C_2 = \mu/M$, and $y = \exp(C_1 \int v dx)$. Then equation (38) reduces to

1. Effects of Impact and Explosion, Summary Technical Report of NDRC, Division 2, Vol. 1. Chapters 6, 7, and p. 279 (Washington, D. C., 1946).

$$\frac{d^2y}{dx^2} - C_1 C_2 x^\theta y = 0$$

which is the Bessel equation with solution in closed form:

$$y = \frac{1}{x^2} [B_1 I_\nu(hx^q) + B_2 I_{-\nu}(hx^q)]. \quad (39)$$

In equation (39) we have used:

$$\nu = (\theta + 2)^{-1}$$

$$h = 2i\nu(C_1 C_2)^{\frac{1}{2}}, \quad i = (-1)^{\frac{1}{2}}$$

$$q = \frac{\theta}{2} + 1$$

$$I_\nu(z) = e^{\frac{-1}{2}\nu\pi i} J_\nu(iz).$$

From equation (39) we get

$$\begin{aligned} v &= C_1^{-1} d \ln y/dx \\ &= (2C_1 x)^{-1} [1 + 2 h q x^q (B_1 I'_\nu + B_2 I'_{-\nu}) \\ &\quad (B_1 I_\nu + B_2 I_{-\nu})^{-1}]. \end{aligned} \quad (40)$$

For given initial and boundary values, we can establish appropriate penetration formulas from equation (40).

CONCLUSION

This investigation has attempted to provide a basic understanding of plate impacts from consideration of both physical and mathematical nonlinearities. Our physical concepts are nonlinear resistances and shock compression of solids. Our mathematical formalism is classically motivated. See equations (1), (4), (36), and (38). Within this framework of nonlinear mechanics and mathematics, we have examined ballistic effects of plate impact in three categories. Analytical evaluation has been given of the ballistic limit, residual velocity, shatter velocity, and spray angle.

This research is conceptually oriented for physical reality and mathematical simplicity as much as possible. Its methods and results can be extended to more general problems of high-speed impact.

APPENDIX

Following Recht and Ipson [14], we may write

$$MV = (M + m)v_i \quad (a)$$

$$\frac{1}{2} MV^2 - \frac{1}{2} (M + m)v_i^2 = \frac{1}{2} (M + m)^{-1} m MV^2 = \Delta E \quad (b)$$

with ΔE denoting the energy loss due to momentum redistribution. Let ΔW = work done for shearing the plug m from the plate. Then energy balance gives

$$\frac{1}{2} MV^2 = \Delta E + \Delta W + \frac{1}{2} (M + m)V_r^2 \quad (c)$$

From equations (b) and (c), we get

$$\Delta W_* = \frac{1}{2} (M + m)^{-1} M^2 V_*^2 \quad (d)$$

for $V_r = 0$ and $V = V_*$ = the ballistic limit. Assuming $\Delta W = \Delta W_*$ and substituting equations (b) and (d) into equation (c), we get

$$V_r = M(M + m)^{-1} (V^2 - V_*^2)^{\frac{1}{2}}$$

which is the sought equation (5).

If we write with $\Delta E = 0$

$$MV = (M + m)V_r$$

$$\frac{1}{2} MV^2 = \Delta W + \frac{1}{2} (M + m)V_r^2$$

then we obtain

$$\Delta W = \Delta W_* = \frac{1}{2} MV_*^2$$

and

$$V_r = [M(M + m)^{-1} (V^2 - V_*^2)]^{\frac{1}{2}}$$

which is the sought equation (6).

14. R. F. Recht and T. W. Ipson, J. Appl. Mech. 30E, 384 (1963).

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