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LARGE DEFLECTIONS OF PLATES WITH NON-LINEAR ELASTICITY AND HOLLOW SHELLS WITH HINGE-SUPPORTED EDGES

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Foreign Technology Division
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А а	*А а*	A, a	Р р	*Р р*	R, r
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В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

All figures, graphs, tables, equations, etc. merged into this translation were extracted from the best quality copy available.

LARGE DEFLECTIONS OF PLATES WITH NONLINEAR ELASTICITY AND HOLLOW SHELLS WITH HINGE-SUPPORTED EDGES

M. S. Kornishin, N. N. Stolyarov,
and N. I. Dedov

In this work, method of finite differences is used to solve problems dealing with large deflections of rectangular plates in plan with nonlinear elasticity and hollow shells with hinge-supported edges, which are under the effect of external normal pressure uniformly distributed along the entire surface or along the central rectangular area. The curvature parameter values at which a crack in a cylindrical panel occurs are established. Critical loads and their corresponding deflections have been found for panels with different curvature parameters.

1. Principal dependences. Let us consider a hollow shell which is rectangular in plan with sides 2a, 2b and thickness h. We will place the origin of the coordinates in the center of the shell and direct axes $\bar{x}$, $\bar{y}$ parallel to the sides of the plan.

We introduce the following designations: $\bar{\varphi}$ - stress function; $\bar{u}$, $\bar{v}$, $\bar{w}$, - corresponding displacements of a point of the middle surface along axes $\bar{x}$, $\bar{y}$, $\bar{z}$; k_1, k_2 - shell curvatures;

written here in total derivatives:

$$\frac{d^2}{dz^2}\left[EI_k \frac{d^2 f_k(z)}{dz^2}\right] + p^2 m_k \sigma_k \varphi_k(z) - p^2 m_k f_k(z) = 0, \tag{1_1}$$

$$-\frac{d}{dz}\left[GI_k \frac{d\varphi_k(z)}{dz}\right] + p^2 m_k \sigma_k f_k(z) - p^2 I_m \varphi_k(z) = 0, \tag{1_2}$$

$$\frac{d^2}{dx^2}\left[EI_\phi \frac{d^2 f_\phi(x)}{dx^2}\right] - p^2 m_\phi f_\phi(x) = 0. \tag{1_3}$$

The conditions of articulation (z = 0; x = 0):

1° $f_k(0) = f_\phi(0)$,

2° $\frac{df_k(0)}{dz} = 0$ — condition of symmetry of vibration shapes,

3° $-\varphi_k(0) = \frac{df_\phi(0)}{dx}$,

4° $\frac{d}{dx}\left[EI_\phi \frac{d^2 f_\phi(0)}{dx^2}\right] = -2\frac{d}{dz}\left[EI_k \frac{d^2 f_k(0)}{dz^2}\right]$,

5° $EI_\phi \frac{d^2 f_\phi(0)}{dx^2} = 2\, GI_{kp} \frac{d\varphi_k(0)}{dz}$.

Boundary equations when $z = l_k$:

6° $\frac{d}{dz}\left[EI_k \frac{d^2 f_k}{dz^2}\right] = 0$ - absence of shearing force,

7° $EI_k \frac{d^2 f_k}{dz^2} = 0$ absence of bending moment,

8° $GI_{kp} \frac{d\varphi_k}{dz} = 0$ — absence of torque.

Boundary conditions when $x = l_\phi$:

Assuming that $\bar{\sigma}_{33} = 0$, from relationships (1) we obtain

$$\bar{\sigma}_{11} = \frac{E\gamma(\psi^2)}{(1+\mu)(1-\nu)}(\varepsilon_{11} + \nu\varepsilon_{22}), \quad (\overline{1,2}), \quad \bar{\tau}_{12} = \frac{E\gamma(\psi^2)}{2(1+\mu)}\gamma_{12}, \tag{2}$$

$$\nu = \frac{3\varkappa - [illegible]}{2(3\varkappa + g\gamma)}, \quad g = \frac{3 [illegible]}{[illegible]}.$$

Here and subsequently 1, 2 - index transposition symbol.

The expressions for stresses and moments

$$\bar{T}_{ii} = \int_{-0.5h}^{0.5h} \bar{\sigma}_{ii}\, d\bar{z}, \quad (i=1,2), \quad \bar{T}_{12} = \int_{-0.5h}^{0.5h} \bar{\tau}_{12}\, d\bar{z},$$

$$\bar{M}_{ii} = \int_{-0.5h}^{0.5h} \bar{\sigma}_{ii}\, \bar{z}\, d\bar{z}, \quad (i=1,2), \quad \bar{M}_{12} = \int_{-0.5h}^{0.5h} \bar{\tau}_{12}\, \bar{z}\, d\bar{z}$$

after certain transformations with the use of relationships (2) will assume the form

$$\bar{T}_{11} = \frac{Eh}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2) + \Delta\bar{T}_{11}, \quad (\overline{1,2}), \quad \bar{T}_{12} = \frac{Eh}{2(1+\mu)}\gamma_{12} + \Delta\bar{T}_{12}, \tag{3}$$

$$\bar{M}_{11} = \frac{Eh^3}{12(1-\mu^2)}(\varkappa_{11} + \mu\varkappa_{22}) + \Delta\bar{M}_{11}, \quad (\overline{1,2}), \quad \bar{M}_{12} = \frac{Eh^3}{12(1+\mu)}\chi + \Delta\bar{M}_{12},$$

where

$$\Delta\bar{T}_{11} = \frac{Eh}{1+\mu}(P\varepsilon_1 + Q\varepsilon_2) + \frac{Eh^2}{1+\mu}(L\varkappa_{11} + M\varkappa_{22}), \quad (\overline{1,2}),$$

$$\Delta\bar{T}_{12} = \frac{Eh}{2(1+\mu)}R\gamma_{12} + \frac{Eh^2}{1+\mu}N\chi, \tag{4}$$

$$\Delta\bar{M}_{11} = \frac{Eh^2}{1+\mu}(L\varepsilon_1 + M\varepsilon_2) + \frac{Eh^3}{1+\mu}(X\varkappa_{11} + Y\varkappa_{22}), \quad (\overline{1,2}),$$

$$\Delta\bar{M}_{12} = \frac{Eh^2}{2(1+\mu)}N\gamma_{12} + \frac{Eh^3}{1+\mu}Z\chi.$$

Coefficients P, Q, R, ..., X, Y, Z represent integrals [Translator's note: equation (5) is not indicated - assumed to be one of the following]

$$P = \frac{1}{h}\int_{-0.5h}^{0.5h} F_1\, d\bar{z}, \quad Q = \frac{1}{h}\int_{-0.5h}^{0.5h} F_2\, d\bar{z}, \quad R = -\frac{1}{h}\int_{-0.5h}^{0.5h} (A\psi^2)^{\alpha}\, d\bar{z},$$

$$L = \frac{1}{h^2}\int_{-0.5h}^{0.5h} F_1 \bar{z}\, d\bar{z}, \quad M = \frac{1}{h^2}\int_{-0.5h}^{0.5h} F_2 \bar{z}\, d\bar{z}, \quad Z = -\frac{1}{h^3}\int_{-0.5h}^{0.5h} (A\psi^2)^{\alpha} \bar{z}^2\, d\bar{z},$$

$$X = \frac{1}{h^3}\int_{-0.5h}^{0.5h} F_1 \bar{z}^2\, d\bar{z}, \quad Y = \frac{1}{h^3}\int_{-0.5h}^{0.5h} F_2 \bar{z}^2\, d\bar{z}, \quad N = -\frac{1}{h^2}\int_{-0.5h}^{0.5h} (A\psi^2)^{\alpha} \bar{z}\, d\bar{z},$$

where

$$F_1 = \frac{\nu + \gamma(1-\mu) - 1}{(1-\mu)(1-\nu)}, \qquad F_2 = \frac{\gamma\nu(1-\mu) - \mu(1-\nu)}{(1-\mu)(1-\nu)}$$

Having expressed $\bar{T}_{11}$, $\bar{T}_{22}$, $\bar{T}_{12}$ in terms of the stress function $\bar{\varphi}$,

$$\bar{T}_{11} = \bar{\varphi}_{yy}, \qquad \bar{T}_{22} = \bar{\varphi}_{xx}, \qquad \bar{T}_{12} = -\bar{\varphi}_{xy},$$

from relationships (3) we obtain

$$\begin{aligned}
\varepsilon_1 &= \frac{1}{Eh}(\bar{\varphi}_{yy} - \mu\bar{\varphi}_{xx}) - \frac{1}{Eh}(\Delta\bar{T}_{11} - \mu\Delta\bar{T}_{22}),\\
\varepsilon_2 &= \frac{1}{Eh}(\bar{\varphi}_{xx} - \mu\bar{\varphi}_{yy}) - \frac{1}{Eh}(\Delta\bar{T}_{22} - \mu\Delta\bar{T}_{11}),\\
\bar{\gamma}_{12} &= -\frac{2(1+\mu)}{Eh}\bar{\varphi}_{xy} - \frac{2(1+\mu)}{Eh}\Delta\bar{T}_{12}.
\end{aligned} \tag{6}$$

Using the well known nonlinear equations of equilibrium and the condition of compatibility of deformations of the theory of hollow shells, and introducing dimensionless variables

$$x=\frac{\bar{x}}{a},\quad y=\frac{\bar{y}}{b},\quad \lambda=\frac{b}{a},\quad w=\frac{\bar{w}}{h},\quad K_1=\frac{4a^2}{R_1 h},\quad K_2=\frac{4b^2}{R_2 h},$$

$$\Phi=\frac{\bar{\Phi}}{Eh^3},\quad p=\frac{16\bar{p}b^4}{Eh^4},\quad z=\frac{2\bar{z}}{h},\quad T_{11}=\frac{\bar{T}_{11}b^2}{Eh^3},\quad T_{22}=\frac{\bar{T}_{22}b^2}{Eh^3},$$

$$T_{12}=\frac{\bar{T}_{12}b^2}{Eh^3},\quad M_{11}=\frac{\bar{M}_{11}b^2}{Eh^4},\quad M_{22}=\frac{\bar{M}_{22}b^2}{Eh^4},\quad M_{12}=\frac{\bar{M}_{12}b^2}{Eh^4},$$

we will obtain a system of two nonlinear differential equations

$$\lambda^4\Phi_{xxxx}+2\lambda^2\Phi_{xxyy}+\Phi_{yyyy}+0{,}25K_1\lambda^2 w_{yy}+0{,}25K_2\lambda^2 w_{xx}=$$
$$=\lambda^2 w_{xy}^2-\lambda^2 w_{xx}w_{yy}+\Delta T_{11,yy}-\mu\lambda^2\Delta T_{11,xx}-\mu\Delta T_{22,yy}+$$
$$+\lambda^2\Delta T_{22,xx}-2(1+\mu)\lambda\Delta T_{12,xy}, \tag{7}$$
$$\frac{1}{12(1-\mu^2)}\left(\lambda^4 w_{xxxx}+2\lambda^2 w_{xxyy}+w_{yyyy}\right)-0{,}25K_1\lambda^2\Phi_{yy}-$$
$$-0{,}25K_2\lambda^2\Phi_{xx}=p/16+\lambda^2 w_{xx}\Phi_{yy}+\lambda^2 w_{yy}\Phi_{xx}-2\lambda^2 w_{xy}\Phi_{xy}+$$
$$+\lambda^2\Delta M_{11,xx}+2\lambda\Delta M_{12,xy}+\Delta M_{22,yy}.$$

Presented below are solution results of the geometrically and physically nonlinear problems of bending a square plate and a square cylindrical panel using external normal pressure with boundary conditions of hinged support, which can be presented as:

with $x=\pm 1$

$$\Phi_{yy}=\Phi_{xy}=0,\qquad w=M_{11}=0;$$

with $y=\pm 1$

$$\Phi_{xx}=\Phi_{xy}=0,\qquad w=M_{22}=0. \tag{8}$$

Conditions (8) indicate the equality to zero on the contour of normal and tangential stresses, and also the deflection and moment.

2. Method for solving nonlinear problems using the electronic digital computer and the calculation results. To resolve the problem, we will use a method of finite differences. Having selected a rectangular net 10 × 10, we substitute the initial system of differential equations (7) with a system of difference equations, approximating the biharmonic operator and all the derivatives with the symmetric difference equations with an error on the order of $O(\bar{h}^2)$ where $\bar{h}$ - net spacing. The derivatives of function w in boundary conditions we approximate with an error on the order of $O(\bar{h}^4)$.

In accordance with the boundary conditions the contour values of functions $w_K = 0$, $\varphi_K = 0$, and the values of functions w_{K+1}, φ_{K+1}, beyond the contour, are expressed in terms of the intracontour values according to formulas [2]:

with $x = \pm 1$

$$w_{K+1} = -\frac{6}{11} w_{K-1} - \frac{4}{11} w_{K-2} + \frac{1}{11} w_{K-3} + \frac{5{,}75}{11} (1-\mu^2) \lambda^{-2} \Delta M_{11},$$

$$\varphi_{K+1} = 3\varphi_{K-1} - 0{,}5\varphi_{K-2};$$

with $y = \pm 1$

$$w_{K+1} = -\frac{6}{11} w_{K-1} - \frac{4}{11} w_{K-2} + \frac{1}{11} w_{K-3} + \frac{5{,}75}{11} (1-\mu^2) \Delta M_{22},$$

$$\varphi_{K+1} = 3\varphi_{K-1} - 0{,}5\varphi_{K-2}.$$

Due to the symmetry of deformational state of the shell relative to axes x and y the number of difference equations is reduced to 50. The obtained system of 50 nonlinear algebraic equations was solved using a method of total iteration, similarly to those described in monograph [2].

Part of the calculation results with $\mu = 0.3$, $A = 0,$ $18\cdot10^6$ for $k(\varepsilon_i)=1$, $\gamma = 1-\bar{\gamma}A_\gamma^2$ is presented in the table and on figures. The calculations were carried out for the following loads: load of constant intensity P_1 distributed throughout the surface $s_1 = 4ab$; with intensity P_2 - along the cylindrical area $s_2 = 4ab$; with intensity P_3 along area $s_3 = 0.36ab$; with intensity P_4 along area $s_4 = 0.04ab$. We used the following designations in the figures:

$$p_i = \frac{q_i a^2 b^2}{E h^4}, \qquad P_i = p_i \frac{S_i}{ab}, \quad i = 1,2,3,4; \quad w_0 = \frac{w}{h},$$

where p_i - load intensity parameter, P_i - parameter of total load, w_0 - deflection parameter at the center.

r_i	32	40	48	56	64	72	80
			$\gamma = 1-\bar{\gamma}A_\gamma^2$				
p_1'	48.5	69.2	97.4	133.1	176.1	225.3	[illegible]
w_0'	1.75	1.75	2	2	2.25	2.25	[illegible]
p_1''	48.3	56.4	60.2	61.7	58.2	54.0	[illegible]
w_0''	2.5	3.75	4.75	5.5	6.5	7.5	8.
			$\gamma = 1$				
p_1'	–	79.8	115.3	162.2	222.2	296.9	–
w_0'	–	2.0	2.0	2.0	2.25	2.25	–
p_1''	–	68.6	75.9	79.1	76.5	73.7	–
w_0''	–	3.5	4.5	5.75	6.5	7.5	–
$\varepsilon_1'\%$	–	13	16	18	21	24	–
$\varepsilon_1''\%$	–	18	21	22	26	27	–

The table shows the obtained, with the consideration of $(\gamma = 1-\bar{\gamma}A_\gamma^2)$ and without the consideration of $(\gamma = 1)$ of physical nonlinearity, parameter values for upper (p_1') and lower (p_1'') critical loads and their corresponding deflection parameters in center $(w_0^{\text{в}}, w_0^{\text{н}})$ for a square cylindrical panel uniformly loaded along the entire surface in the plan with different curvature parameters K_2 and with ratio $h/b = 0{,}005$. Given here also are the values expressed in per cent for relations

$$\varepsilon_1 = \frac{p^{\text{в}}_{1,\gamma=1} - p^{\text{в}}_{1,\gamma\neq 1}}{p^{\text{в}}_{1,\gamma=1}} \cdot 100\% \quad . \quad \varepsilon_2 = \frac{p^{\text{н}}_{1,\gamma=1} - p^{\text{н}}_{1,\gamma\neq 1}}{p^{\text{н}}_{1,\gamma=1}} \cdot 100\%$$

It is evident from the table that physical nonlinearity lowers the critical loads significantly. With an increase in parameter K_2 the effect of physical nonlinearity noticeably increases, moreover, its effect on $p_1^{\text{н}}$ is somewhat greater than on $p_1^{\text{в}}$. At the same time, for the critical states the corresponding deflection parameters $w_{0,\gamma=1}$ and $w_{0,\gamma\neq 1}$ are very close or coincide.

For a twice nonlinear case $(\gamma \neq 1)$, Fig. 1 shows relationships $p_1(w_0^{*})$ for a number of values K_2 and $h/b = 0{,}005$, and Fig. 2 shows relationships $p_n(w_0^{*})$ for a square panel with $K_2 = 40$ and $h/b = 0{,}002$ with loading areas which differ with respect to size.
Here p_n - parameter of the total load per area.

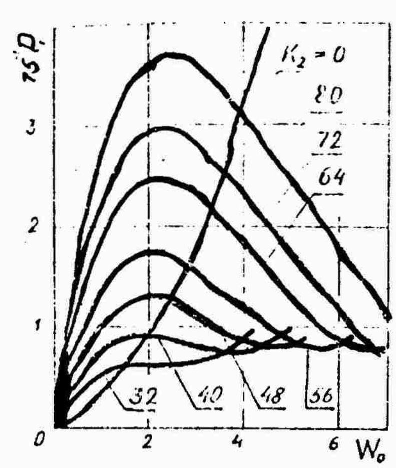

Fig. 1.

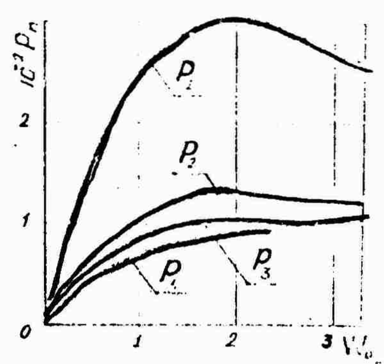

Fig. 2.

obvious algebraic transformations we get:

$$\{(\vec{\alpha}_1\cdot\dot{\alpha}_*)-(\dot{\beta}_1\cdot\vec{\alpha}_k)\}\cdot f_k(0)+\{(\vec{\alpha}_1\cdot\vec{\alpha}_*)-(\vec{\gamma}_1\,\dot{\alpha}_k)\}\,\varphi_k(0)=$$
$$=-p^2(\dot{\gamma}_1\,A_2\,\dot{f}_k)+p^2(\dot{\beta}_1\,A_1\,\dot{f}_k)+p^2(\vec{\gamma}_1\cdot B_2\,\ddot{\varphi}_k)+$$
$$+p^2(\dot{\beta}_1\,B_1\,\dot{\varphi}_k)-p^2(\vec{\alpha}_1\,C_2\,\dot{f}_*). \qquad (26)$$

If we introduce still more designations to certain equations

$$l_{11}=(\vec{\alpha}_1\cdot\dot{\alpha}_*)-(\dot{\beta}_1\cdot\vec{\alpha}_k),$$
$$l_{12}=(\vec{\alpha}_1\cdot\vec{\alpha}_*)-(\ddot{\gamma}_1\cdot\dot{\alpha}_k),$$
$$(\vec{a}_1\cdot\vec{f}_k)=(\dot{\gamma}_1\cdot A_2\cdot\vec{f}_k)+(\dot{\beta}_1\cdot A_1\cdot\ddot{f}_k),\qquad (27)$$
$$(\vec{b}_1\cdot\dot{\varphi}_k)=(\ddot{\gamma}_1\cdot B_2\cdot\dot{\varphi}_k)+(\vec{\beta}_1\cdot B_1\cdot\ddot{\varphi}_k),$$
$$(\vec{C}_1\cdot\vec{f}_*)=-(\vec{\alpha}_1\cdot C_2\cdot\vec{f}_*),$$

then equation (26) is rewritten in the form of

$$l_{11}\cdot f_k(0)+l_{12}\cdot\varphi_k(0)=p^2(\vec{a}_1\cdot\vec{f}_k)+p^2(\vec{b}\cdot\vec{\varphi}_k)+p^2(\ddot{C}_1\cdot\dot{f}_*). \qquad (28)$$

By performing algebraic calculations for equation (23_2) and the second trio of vectors $\vec{\beta}_2$, $\vec{\gamma}_2$, $\vec{\alpha}_2$, we get equation

$$l_{21}\cdot f_k(0)+l_{22}\cdot\varphi_k(0)=p^2(\ddot{a}_2\cdot\vec{f}_k)+p^2(\vec{b}_2\cdot\ddot{\varphi}_k)+p^2(\vec{C}_2\cdot\vec{f}_*). \qquad (29)$$

Here the numbers l_{21}, l_{22}, and vectors $\vec{a}_2$, $\vec{b}_2$, $\vec{C}_3$ are determined by the same formulas as numbers l_{11}, l_{12} and vectors $\vec{a}_1$, $\vec{b}_1$, $\vec{C}_1$, by, of course substituting index 1 in these formulas for the indicated quantities and index 2 for vectors $\vec{\alpha}$, $\vec{\beta}$, $\vec{\gamma}$.

From Fig. 2 it is evident that the localization of the load on a smaller area leads to a considerable increase in maximum deflection and to a decrease in the value of the upper critical load. Thus, with a load distributed along the central area S_2 equalling one fourth of the total panel area (curve p_2) deflections w_0 in the precritical state having increased almost three times, while cracking load p_n^b has decreased by approximately two times, as compared to the corresponding values for a panel uniformly loaded along the entire surface (curve p_1). With a load of p_3 the crack virtually disappears.

Figure 3 shows relationships $p_1(w_0)$ with various ratios h/b for a square panel ($\lambda = 1$, $k_2 = 40$) uniformly loaded along the entire surface. It is evident that with an increase of this ratio the effect of physical nonlinearity on deflections and critical loads increases significantly.

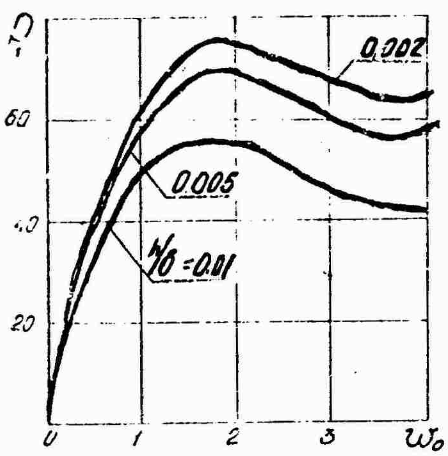

Fig. 3.

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