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**PREDICTED MULTILINE CONTINUOUS WAVE
POWER OUTPUT FROM A PERFECTLY STIRRED
REACTOR CHEMICAL LASER**

I. T. Osgerby

ARO, Inc.

February 1973

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FOREWORD

The work reported herein was conducted at the Arnold Engineering Development Center (AEDC) under sponsorship of the Air Force Systems Command (AFSC), under Program Element 65802F.

The results of the research presented were obtained by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC, AFSC, Arnold Air Force Station, Tennessee. The research was conducted from March 1 through June 30, 1972, under ARO Project No. RW5204, and the manuscript was submitted for publication on October 31, 1972.

This technical report has been reviewed and is approved.

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ABSTRACT

The results of a previous study of the concept of a stirred reactor laser with lasing restricted to the zeroth fundamental band are compared with the predicted power output of a multilevel laser for the hydrogen-fluorine-helium (H_2-F_2-He) system. Lasing is assumed to occur on all fundamental bands for which the calculated maximum gain equals the cavity threshold value. Power output is significantly increased over the zeroth band calculations, and the effect of cavity pressure is shown to be less pronounced because of the cascade enhancement from higher bands. Vibrational de-excitation caused by collisional processes appears to limit this kind of chemical system to low cavity pressures. The effect of cavity losses was evaluated with three assumed values for the cavity loss coefficient. These values were estimated as a small fraction (0.1) of the average small signal gain coefficient at different cavity residence times.

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NOMENCLATURE

A_V^{v+1}	Einstein spontaneous emission coefficient
a	Mirror absorption loss coefficient
c	Vacuum speed of light
$F_V(J)$	Constant in rotational energy expression
g	Gain coefficient
g_0	Small signal gain coefficient
g_v	Degeneracy
h	Planck's constant
I	Intensity
J	Rotational quantum number
K^V	Degree of inhomogeneous broadening
k	Boltzmann's constant
L	Length of active medium in cavity
\dot{m}	Mass flow rate into reactor
N_A	Avagadro's number

n	Number of vibrational levels
n_0	Refractive index
P	Cavity pressure
P_R	Power output
P_{R_u}	Useful power output
Q_R^v	Rotational partition function for vibrational level v
q_v	Stimulated emission function
r, r_1, r_2, r_m	Mirror reflection coefficient
S_v	Integrated line intensity
T	Translational temperature
t_c	Cavity residence time
V	Volume of reactor
v	Vibrational quantum number
W	Molecular weight
w	Energy
w_s	Saturation parameter
z	Distance along cavity
α_v, α'_v	Coefficients in analytical model
β_L	Loss coefficient
γ_v	Mole/mass ratio of vibration level v
$\dot{\gamma}_{v_c}$	Chemical production rate of vibrational level v
$\dot{\gamma}_{v_s}$	Stimulated emission/absorption production rate of vibrational level v
ν	Frequency
ν_0	Frequency at line center
ξ_ν	Local value of gain
ρ	Gas density
ρ_ν	Photon flux density
$\bar{\rho}_\nu$	Stimulated emission rate constant

$\tau \gamma$ Gain
 ω Wave number

SUPERSCRIPTS

D Denotes Doppler effect
o Denotes reactor feed quantity
V Denotes Voigt effect
 \pm Direction indicator
- Average quantity

SUBSCRIPTS

D Denotes Doppler effect
opt Optimum
v, o, 1, etc. Denotes vibrational levels
 ν Denotes frequency

SECTION I INTRODUCTION

The concept of a continuous wave (cw) stirred reactor chemical laser was presented in Ref. 1 for a hydrogen/fluorine/helium (H_2-F_2-He) system. Calculations were carried out with stimulated emission/absorption transitions restricted to the zeroth fundamental vibrational band. An optical gain coefficient was calculated for the rotational line having maximum gain, and lasing was assumed to occur at the center of the Doppler-broadened line. The single fundamental band restriction was used so that one could specify a radiation flux density for parametric calculations without having to specify detailed optical cavity boundary conditions and losses. The results correspond to a laser cavity with a mirror at one end and a grating at the other to select the zeroth fundamental band. In this report, all fundamental bands (up to six) with sufficient optical gain were allowed to lase. A cavity gain threshold equation was used to determine the radiation flux densities and which bands were capable of lasing. The predicted power output reported in Ref. 1 indicated a dramatic decrease as the cavity pressure level was increased. As cavity pressure increases, collisional relaxation rates increase, thus reducing the optical gain coefficient. Allowing all fundamental bands to lase should reduce this effect since the decrease in gain on higher bands is offset by an equivalent increase in gain on lower bands. This cascade enhancement is particularly prevalent during lasing where the reduction in population of an upper vibrational level appears as an increase in population of the next lower level.

The purposes of this research were (1) to apply the AEDC expertise in propulsion technology and reaction kinetics to the stirred reactor concept in order to determine its potential as a chemical laser device and (2) to become familiar with chemical laser devices which could conceivably be tested in AEDC facilities.

A simplified analysis of the effect of cavity losses is included to determine the effect on power output and to determine whether an optimum mirror transmission coefficient exists as predicted for other chemical laser models (Ref. 2).

SECTION II THEORETICAL MODEL

2.1 CHEMICAL AND RADIATIVE INTERCHANGE EQUATIONS

In accordance with Ref. 1, the vibration levels $v = 0, 1, \dots, n$ of the radiating molecule are considered as an individual chemical

species denoted by γ_v . The equations which describe the chemical and radiative interchanges (assuming only fundamental band transitions are allowed) in a perfectly stirred reactor are

$$\gamma_v^0 - \gamma_v = -t_c \dot{\gamma}_{v_c} - t_c \dot{\gamma}_{v_s} \quad \text{and} \quad v = 0, n \quad (2.1)$$

where γ_v^0 is the concentration (moles/gm) in the reactor feed, γ_v is the concentration in the reactor, $\dot{\gamma}_{v_c}$ is the rate of production of vibrational level v from chemical reaction, $\dot{\gamma}_{v_s}$ is the rate of production attributable to stimulated emission/absorption transitions (spontaneous radiation assumed to be negligible), and t_c is the average residence time of particles in the reactor. The quantity $\dot{\gamma}_{v_c}$ is evaluated conventionally in terms of a forward-rate constant and a backward-rate constant (obtained from the forward-rate constant and an equilibrium constant). The quantity $\dot{\gamma}_{v_s}$ is evaluated as the net production rate of vibrational level v as follows:

$$\dot{\gamma}_{v_s} = -\bar{\rho}_{\nu_{v-1}} (\gamma_v - q_{v-1} \gamma_{v-1}) + \bar{\rho}_{\nu_v} (\gamma_{v+1} - q_v \gamma_v) \quad (2.2)$$

$$\bar{\rho}_{\nu_v} = \rho_{\nu_v} \left\{ \frac{1}{8\pi h \omega_v^3} A_v^{v+1} g_v(J_v-1) \exp \left[-F_{v+1}(J_v-1) \frac{hc}{kT} \right] / Q_R^{v+1} \right\} \quad (2.3)$$

$$q_v = \frac{Q_R^{v+1}}{Q_R^v} \exp \left\{ \left[F_{v+1}(J_v-1) - F_v(J_v) \right] \frac{hc}{kT} \right\} \quad (2.4)$$

The quantity ρ_{ν_v} is the radiation flux density, q_v is the minimum population ratio for lasing to occur, A_v^{v+1} is the Einstein coefficient for spontaneous transitions, $g_v(J_v-1)$ is the degeneracy of the transition, J_v denotes the rotational line having maximum gain, $F_v(J_v)hc/k$ is a characteristic rotational temperature, Q_R^v is the rotational partition function for vibrational level, v , ω_v is the wave number of the transition, h is Planck's constant, k is Boltzmann's constant, c is the vacuum speed of light, and T is the rotational temperature (assumed equal to the translational temperature).

2.2 CAVITY THRESHOLD EQUATION AND RADIATION FLUX DENSITIES

The problem in solving Eqs. (2.1) is that of specifying the radiation flux densities, ρ_{ν_v} . In Ref. 1, only ρ_{ν_0} was considered, with

radiation transitions between vibrational levels $v = 1$ and $v = 0$ allowed. Values of ρ_{ν_0} were assumed and parametric calculations carried out for various pressure levels and residence times, t_c . The radiation flux densities may be calculated implicitly through some algebraic manipulation of Eqs. (2.1) and the introduction of the laser (gain) threshold equation (Ref. 2).

$$2Lg_\nu = 2L\beta_L - \ln r_1 r_2 \quad (2.5)$$

where L is the active cavity length, g_ν is the gain coefficient, β_L is a cavity loss coefficient, and r_1 and r_2 are the reflection coefficients of the cavity mirrors.

Substituting Eq. (2.2) in Eqs. (2.1) and noting that $\bar{\rho}_{\nu-1}$ is zero, one obtains

$$\dot{\gamma}_0 = - \sum_{\nu=1}^n \dot{\gamma}_\nu \quad (2.6)$$

and

$$\gamma_0^o - \gamma_0 = - \sum_{\nu=1}^n (\gamma_\nu^o - \gamma_\nu) - t_c \sum_{\nu=0}^n \dot{\gamma}_\nu \quad (2.7)$$

There is thus one equation instead of n , and an additional $n-1$ equations are required for the remaining $n-1$ unknown γ_ν . Equation (2.5) together with the spectral line properties for each transition is used to provide the additional equations.

The gain coefficient g_ν is related to the spectral line through the integrated line intensity \hat{S}_ν as

$$g_\nu = K^V g_\nu^D = -K^V \sqrt{\frac{\ln 2}{\pi}} \frac{S_\nu}{\Delta\nu_D} \quad (2.8)$$

The coefficient K^V is evaluated in Ref. 1 as a Voigt broadening coefficient in order to account for combined collision/Doppler broadening processes, g_ν^D is the pure Doppler-broadened gain coefficient, and $\Delta\nu_D$ is the Doppler half-width at half maximum intensity for the transition at frequency ν .

From the definition of S_v it can be determined that

$$-S_v = h\omega_v \rho N_A a'_v (\gamma_{v+1} - q_v \gamma_v) \quad (2.9)$$

where

$$a'_v = \frac{1}{8\pi h\omega_v^3} A_v^{v+1} g_v(J_v-1) \exp \left[-F_{v+1}(J_v-1) \frac{hc}{kT} \right] \quad (2.10)$$

From Eqs. (2.5), (2.8), and (2.9) one can derive

$$\gamma_{v+1} = q_v \gamma_v + a_v \quad \text{and} \quad v = 0, n-1 \quad (2.11)$$

where

$$a_v = g_v \frac{1}{K^v h N_A} \sqrt{\frac{2\pi k}{W}} \frac{\sqrt{T}}{\rho} \frac{1}{a'_v} \quad (2.12)$$

Here, N_A is Avagadro's number, ρ is the mixture density, and W is the molecular weight of the radiating molecule.

Equations (2.11) thus enable γ_v , $v = 1, n$ to be evaluated in terms of γ_0 . Nonradiating species are assigned an equation similar to Eq. (2.1) (without the radiation production term), except for those evaluated with atom conservation equations. The system of equations is solved by the iterative technique described in Refs. 1 and 3 after a loss coefficient is defined. The mixture density and temperature are obtained from the equation of state and an energy balance which includes the radiated power and radiative losses.

SECTION III LASER POWER OUTPUT WITH CAVITY LOSSES

The intensity in a laser cavity with mirrors at $z = 0$ and $z = L$ is related to the gain coefficient as follows (losses are included later):

$$\frac{dI_v^+}{dz} = g_v I_v^+ \quad \text{and} \quad \frac{dI_v^-}{dz} = g_v I_v^- \quad (3.1)$$

In a perfectly stirred reactor, the gain coefficient g_v is independent of position z ; thus, Eq. (3.1) can be integrated to give

$$I_{\nu}^{+}(z) = e^{\xi_{\nu}} I_{\nu}^{+}(1) \quad \text{and} \quad I_{\nu}^{-}(z) = I_{\nu}^{-}(2) e^{\tau_{\nu} - \xi_{\nu}} \quad (3.2)$$

where

$$\xi_{\nu} = g_{\nu} z \quad \text{and} \quad \tau_{\nu} = g_{\nu} L$$

Under lasing conditions, the intensity has a very narrow spike at ν_0 ; thus, Eqs. (3.2) can be integrated over the narrow frequency bandwidth for the spike to give

$$I^{+}(z) = e^{\xi_{\nu_0}} I^{+}(1) \quad \text{and} \quad I^{-}(z) = e^{\tau_{\nu_0} - \xi_{\nu_0}} I^{-}(2) \quad (3.3)$$

At the mirror boundaries,

$$I^{+}(1) = r_1 I^{-}(1) \quad \text{and} \quad I^{-}(2) = r_2 I^{+}(2) \quad (3.4)$$

$$\therefore 1 = r_1 r_2 e^{2\tau_{\nu_0}} \quad (3.5)$$

$$\frac{I^{+}(1)}{I^{-}(2)} = \sqrt{\frac{r_1}{r_2}} \quad (3.6)$$

Equation (3.5) is the cavity threshold equation describing the radiative steady-state condition that says gain equals loss. In other words, the increase in intensity for two passes through the medium is balanced by the output or losses at the two mirrors.

The intensities at the boundaries can be determined by calculating an average intensity \bar{I} . Thus,

$$\bar{I}^{\pm} = \frac{1}{L} \int_0^L I^{\pm}(z) dz = \frac{1}{\tau_{\nu_0}} \int_0^{\tau_{\nu_0}} I^{\pm}(\xi_{\nu_0}) d\xi_{\nu_0} \quad (3.7)$$

$$\bar{I}^{+} = \frac{e^{\tau_{\nu_0}} - 1}{\tau_{\nu_0}} I^{+}(1), \quad \text{and} \quad \bar{I}^{-} = \frac{e^{\tau_{\nu_0}} - 1}{\tau_{\nu_0}} I^{-}(2) \quad (3.8)$$

The average intensity is given by

$$\bar{I} = \bar{I}^+ + \bar{I}^- = \frac{e^{r\nu_0} - 1}{r\nu_0} [I^+(1) + \Gamma(2)] \quad (3.9)$$

3.1 OUTPUT POWER

The total output power is given by the product of the energy per photon, the number of photons per unit volume to leave the cavity, and the cavity volume, as follows:

$$P_R = \sum_v hc\omega_v \rho_v N_A \rho_{v-1} (\gamma_v - q_{v-1} \gamma_{v-1}) V \quad (3.10)$$

The output power is also given by the flux crossing the mirror surfaces, as shown by

$$P_R = [(1 - r_1)I^-(1) + (1 - r_2)I^+(2)] \frac{V}{L} \quad (3.11)$$

It may be assumed that

$$r_1 = r_2 = r_m$$

$$\therefore P_R = \frac{1 - r_m}{r_m} [I^+(1) + I^-(2)] \frac{V}{L} \quad (3.12)$$

$$= \frac{2(1 - r_m)}{r_m} I^+(1) \frac{V}{L} \quad (3.13)$$

$$= \frac{2(1 - r_m)}{r_m} \frac{e^{r\nu_0}}{e^{r\nu_0} - 1} \bar{I}^+ \frac{V}{L} \quad (3.14)$$

$$= \frac{1 - r_m}{r_m} \frac{e^{r\nu_0}}{e^{r\nu_0} - 1} \bar{I} \frac{V}{L} \quad (3.15)$$

$$= r\nu_0 \bar{I} \frac{V}{L} \quad (3.16)$$

Equation (3.16) relates the total power output to the average cavity intensity

3.2 CAVITY POWER OUTPUT WITH LOSSES

It is necessary to add $\beta_L I_\nu^\pm$ to the right-hand side of the radiative transfer equations (3.1); thus, the τ_ν are replaced by $\tau_\nu - \beta_L L$, and Eq. (3.5) becomes

$$\tau_\nu = -\ell n r + \beta_L L \quad (3.17)$$

The useful power output is readily evaluated as

$$P_{R_u} = (\tau_\nu - \beta_L L) \bar{I} \frac{V}{L} = \left(1 - \frac{\beta_L}{g_\nu}\right) P_R \quad (3.18)$$

The quantity τ_ν is the total output of which $\beta_L L$ is lost to absorption, scattering, and diffraction losses.

Inspection of Eq. (3.18) indicates that a maximum useful power output must occur at some appropriate ratio of β_L/g_ν since power output is zero if $I = 0$ and also if $g_\nu = \beta_L$ (corresponding to $r = 1$ in Eq. (3.17)). Since P_R/\dot{m} is a function of β_L and g_ν , it is not possible to evaluate the maximum P_{R_u}/\dot{m} analytically from Eq. (3.18). Under certain circumstances an analytical solution for laser power output can be obtained (Refs. 4 through 9). The analyses (based on gain saturation formulas given in Ref. 5) predict that an optimum output coupling exists for maximum power output when cavity losses are included in the analysis. This is in contrast to the no-loss case in which the radiation flux density in the cavity can be increased to infinity as shown in Ref. 1. Two simple formulas can be derived for the optimum output coupling for the following limiting cases:

1. Homogeneous (pure collision) broadening

$$-\left(\frac{\ell n r}{2L}\right)_{opt} = \sqrt{g_o \beta_L} - \beta_L \quad (3.19)$$

2. Inhomogeneous (pure Doppler) broadening

$$g_o^2 = \left[\beta_L - \left(\frac{\ell n r}{2L}\right)_{opt} \right]^3 / \left[\beta_L + \left(\frac{\ell n r}{2L}\right)_{opt} \right] \quad (3.20)$$

It is not immediately apparent what saturation formula applies to the chemical laser described in this report. Equations (3.19) and (3.20) are limiting cases derived from an analysis of a helium/neon (He-Ne) laser (Ref. 10). For this laser, production terms corresponding to the right-hand side of Eqs. (2.1) are steady-state values and, hence, are equal to zero. Given these conditions, one can find a simple solution for the lasing species concentration by algebraic manipulation.

3.3 MIRROR ABSORPTION LOSSES

The previous development provided a means for estimating the internal cavity losses. Mirror surface losses are accounted for by reducing the value of r to its actual value and including a mirror absorption coefficient, a . The actual value of the transmissivity of each mirror is then given by $1-r-a$.

Equations (3.4) through (3.6) become

$$I^+(1) = rI^-(1) \text{ and } I^-(2) = rI^+(2) \quad (3.21)$$

$$I = re^{\tau\nu} \quad (3.22)$$

and

$$I^+(1) = I^-(2) \quad (3.23)$$

Equation (3.11) becomes

$$P_R = [(1-r-a)I^-(1) + (1-r-a)I^+(2)] \frac{V}{L} \quad (3.24)$$

$$= (1-r-a)2I^-(1) \frac{V}{L} \quad (3.25)$$

$$= \frac{(1-r-a)}{r} 2I^+(1) \frac{V}{L} \quad (3.26)$$

$$= \frac{(1-r)}{r} 2I^+(1) \left(\frac{1-r-a}{1-r} \right) \frac{V}{L} \quad (3.27)$$

Thus, with mirror absorption losses an extra correction, $\left(\frac{1-r-a}{1-r} \right)$, is

introduced. This correction factor is very useful if closed-cavity experiments are conducted. If $1 = r + a$, the cavity power is lost into the mirror, which may be water-cooled for measurement of the flux.

The value of β_L to be used cannot be readily evaluated theoretically. In a perfectly stirred reactor laser, the losses would be primarily scattering and diffraction losses attributable to the local fluctuations generated by the intense, fine-scale turbulence. In a well-stirred reactor, in which the mixing is not entirely complete, some absorption would occur also, since all the eddies would not have undergone exactly the same time history in the reactor. Large absorption losses would occur if the eddies were large; i. e., mixing occurs on a macroscale and not on a molecular scale. In this latter case, the ensemble of eddies is considered as a collection of individual stirred reactors which remain segregated. The residence time distribution in a stirred reactor is given by $\exp(-t/\bar{t}_c)/\bar{t}_c$ where \bar{t}_c is a mean residence time in the reactor. The output concentration in the segregated reactor is given by

$$\gamma_v = \frac{1}{\bar{t}_c} \int_0^{\bar{t}_c} \gamma_v(t) \exp(-t/\bar{t}_c) dt$$

where $\gamma_v(t)$ is obtained from the plug flow form of Eq. (2.1):

$$\frac{d\gamma_v}{dt} = \dot{\gamma}_{v_c} + \dot{\gamma}_{v_s}$$

Segregation of fluid elements in the reactor is possible if there is laminar or transitional flow in the reactor. This possibility may be very real at cavity pressures much below 0.1 atm. To the author's knowledge, experimental data from reactors at such low pressures has not been obtained.

An analysis of the effects of incomplete molecular scale mixing at atmospheric pressure is given in Ref. 11 for anthracite flames in a swirl combustor.

Current theoretical models of (cw) chemical lasers do not provide a means of calculating β_L ; thus, a simplified method of accounting for these loss mechanisms is used. Obviously the value of β_L must be less than the small signal gain and for this reason, values of $\beta_L = 0.001$, 0.0025, and 0.005 were chosen to take account of significant absorption losses. It will be seen in the next section that these values correspond

to one tenth of the average small signal gain coefficient at different values of residence time, t_c .

SECTION IV RESULTS AND DISCUSSION

Results of calculations are presented for a H_2-F_2-He system in the molar ratios 2:1:10. A preburner temperature is assumed such that the temperature of the fluorine molecules and fluorine atoms in the reactor feed is $600^\circ K$ at the cavity pressure. Thus, as the cavity pressure is increased, the equilibrium fluorine atom concentration decreases. Hydrogen and helium are assumed to be at $300^\circ K$ in the reactor feed. The fixed preburner temperature is considered a reasonable limit for the integrity of combustor wall materials. Refractive index fluctuations in the subsonic cavity flow are considered to be negligible as discussed in Ref. 1. Some results from Ref. 1 are included to illustrate the enhancement caused by the cascade effect of depopulating all fundamental bands capable of lasing. Results with and without cavity losses are also shown, and only one axial mode at each line center is considered.

The predicted power output when emission/absorption is restricted to $v = 1 \rightleftharpoons v = 0$ is shown in Figs. 1 through 3, Appendix. The power output is plotted as a function of mass flow rate per unit volume, \dot{m}/V . The average residence time of particles in the reactor is simply $t_c = \rho V/\dot{m}$, or, since ρ is approximately constant for a given cavity pressure, $t_c \propto V/\dot{m}$. The maximum power output is readily obtained by setting $\alpha_v = 0$ in Eq. (2.12); i. e., the radiation flux density is sufficiently high to force γ_1 to the minimum value for lasing: $\gamma_1 = q_0 \gamma_0$ ($\rho_v \rightarrow \infty$ as $g_0 \rightarrow 0$). The power output for $\beta_L = 0$ with all fundamental bands allowed to lase (providing the threshold equation (2.5) is satisfied) is shown in Figs. 4 through 6. The cascade enhancement significantly increases the output over single-band calculations shown in Figs. 1 through 3. The reduction in peak power output as pressure increases is not so dramatic in Fig. 4 as it is in Fig. 2. The peak is readily explained by a consideration of the combined actions of chemical pumping, collisional deactivation, and stimulated radiation depopulation processes. At very short times t_c (large \dot{m}/V), chemical pumping is attempting to build up the population while radiation depopulation reduces it. As t_c increases, population levels increase and collisional deactivation becomes significant. Collisional deactivation results in a higher

mixture temperature, which shifts the maximum gain line to higher J values, in addition to the reduction in population ratios of adjacent vibrational levels. This "loss" is in contrast to the carbon dioxide (CO_2) gas-dynamic laser behavior where vibrational levels of the ν_1 and ν_2 modes are preferentially reduced by rapid collisional deactivation to maintain adequate gain between the ν_3 and ν_1 modes. As the pressure is increased, collisional deactivation becomes significant earlier because of increased population levels, and the peak power shifts to shorter and shorter times (chemical pumping rates also increase because of increased pressures). The very short residence times at pressures greater than 0.025 atm probably preclude higher pressure operation because of the inability to mix reactants fast enough to satisfy the reactor mixing criterion--mixing time less than residence time.

The effect of introducing cavity losses is shown in Figs 7 and 8. The power output is significantly reduced. Values of the loss coefficient were chosen from the small signal gain calculations shown in Figs. 9 and 10 (corresponding population ratios are shown in Fig. 11). Values of $\beta_L = 0.1 g_0$ with $g_0 = 0.05, 0.025,$ and 0.01 were selected as representative. A constant average value of g_0 was used for convenience and should provide a conservative estimate of the useful power output of a well-stirred reactor. The values of β_L used result in a significant reduction in predicted useful power output with an optimum transmission coefficient $r \approx 0.8$. The results shown in Fig. 8 correspond to a predominantly Doppler-broadened gain coefficient; however, the value of $r = 0.8$ would lead to a negative value of g_0 if Eq. (3.2) were applied, since $\beta_L < -\ln r/2L$; hence, this equation does not apply as expected. The effect of mirror absorption losses is shown in Fig. 12 for $a = 0.01$ and for $\beta_L = 0$.

4.1 CAVITY RESIDENCE TIME AND MIXING

The cavity residence times for maximum power operation are quite short ($t_c \approx 100 \mu\text{sec}$ for pressures of 20 Torr or less); however, if there is adequate turbulence at these low pressure levels, the time required to mix the reactants may still be sufficiently less than this to satisfy the stirred reactor assumption. Experiments with stirred reactors have been carried out at pressures as low as 0.1 atm with residence times on the order of 40 to 60 μsec (Ref. 12).

Supersonic flow lasers reported in the literature are known to be diffusion limited. Supersonic velocities are required (1) to provide sufficiently high particle transit times and finite beam lengths in the flow direction, and (2) to provide some diffuser pressure recovery to reduce exhaust gas pumping requirements. The basic stirred reactor assumption is that the process is chemical-kinetic limited. The mixing time must, therefore, be short compared with reactor residence times. It has been well established (Refs. 13 and 14) that the mixing rates in turbulent shear flows are determined by the conversion of mean velocity (U) to the fluctuating velocity (u'). That is, the mixing time, t_m , is proportional to the ratio of these velocities:

$$t_m \propto \frac{u'}{U}$$

- a. Incompressible jet into still air, $\left(\frac{u'}{U}\right)_{\max} \leq 0.2$
- b. Incompressible jet into coflowing stream, $\left(\frac{u'}{U}\right)_{\max} < 0.2$
- c. Supersonic jet into coflowing stream, $\left(\frac{u'}{U}\right)_{\max} \leq 0.05$
- d. Stirred reactor, $\left(\frac{u'}{U}\right)_{\max} \geq 1$

The above numbers have been established in experiments at pressures of 0.1 atmospheres or higher. No information exists in the literature for pressures lower than these. The pressure levels of interest for chemical lasers are significantly less than this - on the order of a few Torr in supersonic flow lasers. Since the Reynolds number is inversely proportional to pressure, it is expected that $(u'/U)_{\max}$ will be even lower than those given above. In fact, it is possible that the supersonic flow lasers have transitional or laminar mixing rates.

The mixing rate in a stirred reactor is usually a factor of 20 to 50 times greater than that for the coflowing, supersonic, diffusion flame. In addition, the time constant for the degree of chemical conversion in a stirred reactor can be significantly longer than that in a premixed flame (plug flow reactor) as shown in Ref. 3.

For these reasons, it is hoped that chemical-kinetically controlled processes in a stirred reactor laser are possible. Only experiments can determine if there is adequate turbulence to satisfy the basic stirred reactor assumption.

4.2 NUMERICAL PROBLEMS

Some difficulties were experienced in obtaining solutions at long residence times because of the method used to eliminate lasing between adjacent lines since Eq. (2.5) was no longer satisfied. The current version of the program assumes lasing is terminated on successively higher vibrational levels as the cascade effect fills up lower levels. In fact, the gain is not distributed sequentially as shown in Figs. 8 and 9 even for no power output. This is, of course, caused by the nonuniform pumping of the chemical reactions (Ref. 15).

SECTION V CONCLUSIONS

A method of predicting the power output from vibrationally excited molecules in a stirred reactor laser has been developed and the following conclusions have been reached:

1. Radiation flux densities are determined by the requirement that the gain between vibrational levels satisfy the threshold equation for a resonance cavity.
2. The power output for multilevel lasing is significantly greater than that for the simple two-level laser analyzed previously.
3. The effect of cavity pressure level is less pronounced for the multilevel than for the two-level laser; however, the H_2-F_2-He system appears to be limited to low pressure levels because of the influence of collisional deactivation processes.
4. Losses reduce the power output significantly, with an optimum mirror reflection coefficient $r = 0.8$ for the chemical laser analyzed in this report.
5. Cavity residence times are quite short ($t_c = 100 \mu\text{sec}$ for pressures of 20 Torr or less), but if there is adequate turbulence for mixing on a molecular scale,

the mixing time may still be sufficiently less than this to satisfy the stirred reactor assumption.

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**APPENDIX
ILLUSTRATIONS**

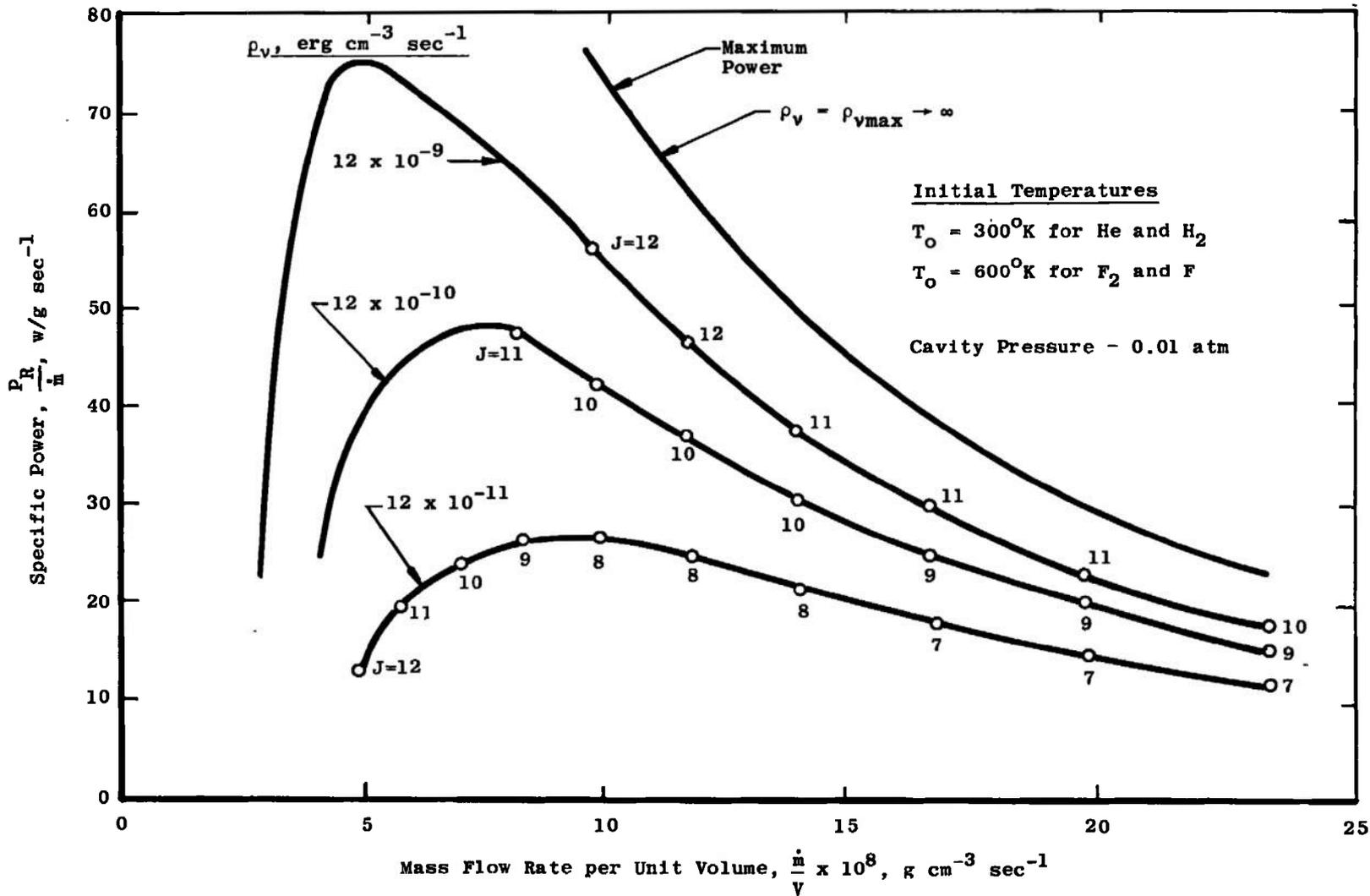


Fig. 1 Specific Power versus Mass Flow Rate per Unit Volume: Effect of Radiation Flux Density at Constant Pressure and Enthalpy

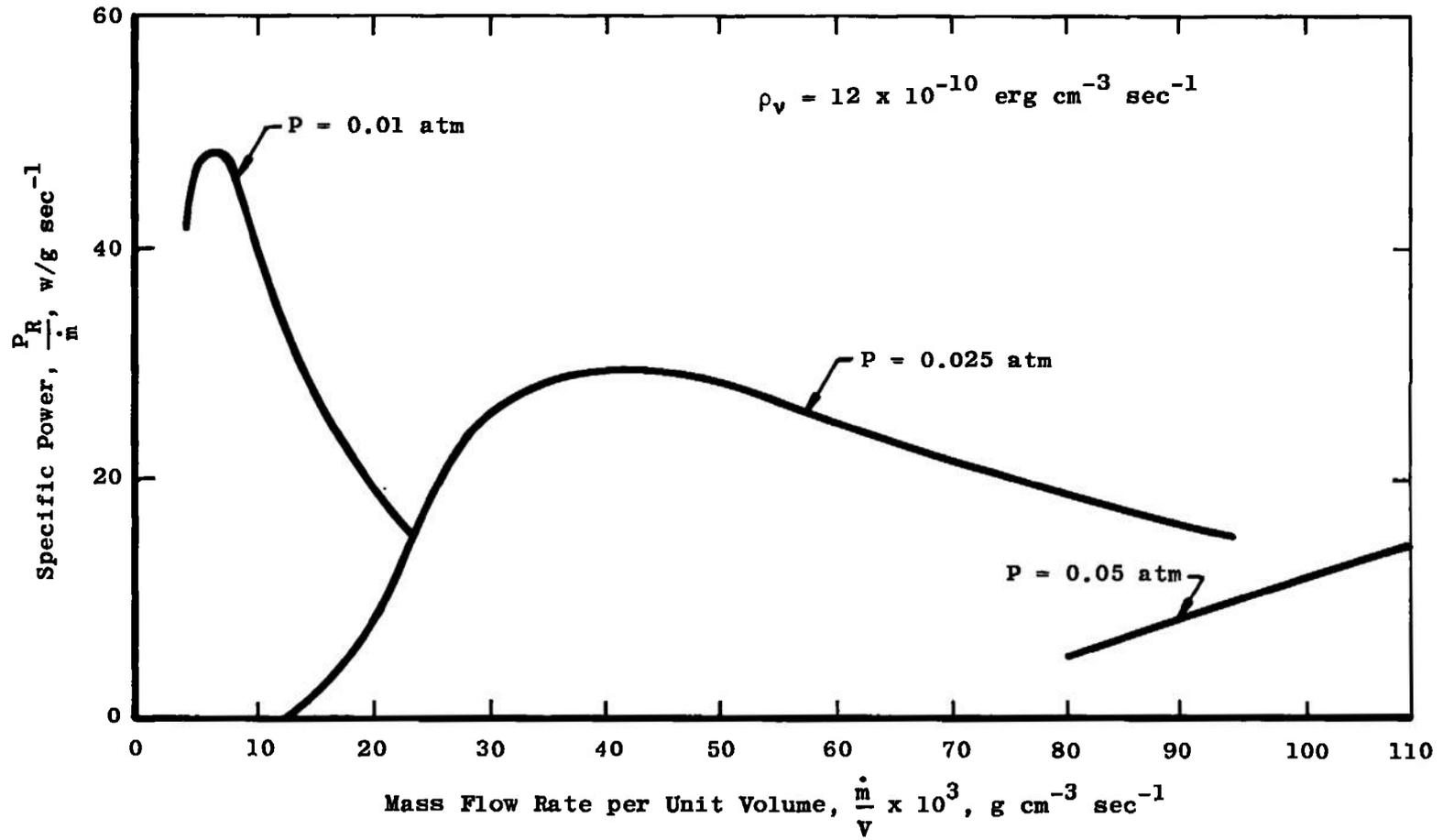


Fig. 2 Specific Power versus Mass Flow Rate per Unit Volume: Effect of Pressure at Constant Enthalpy and Radiation Flux Density

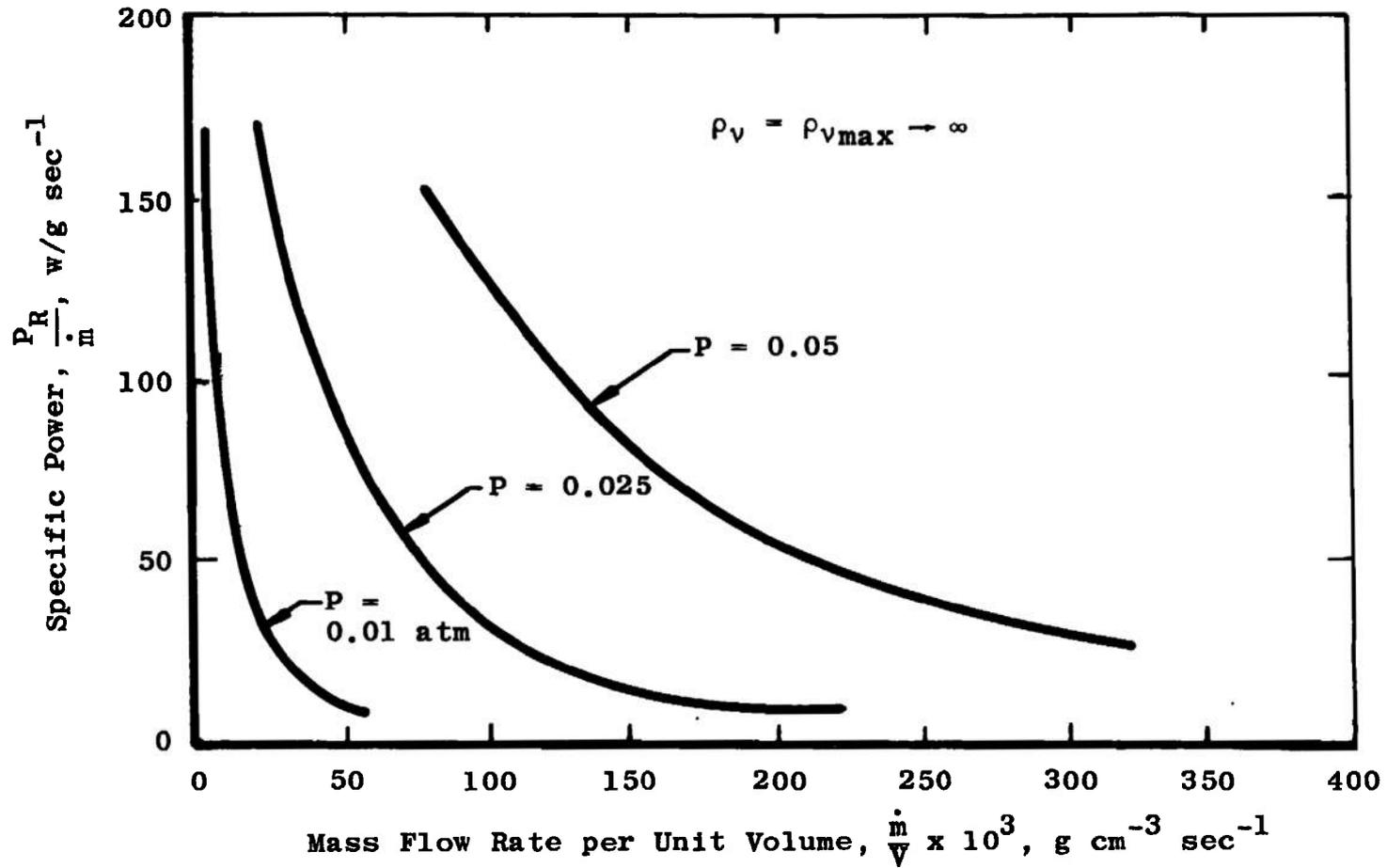


Fig. 3 Maximum Specific Power versus Mass Flow Rate per Unit Volume: Effect of Pressure at Constant Enthalpy

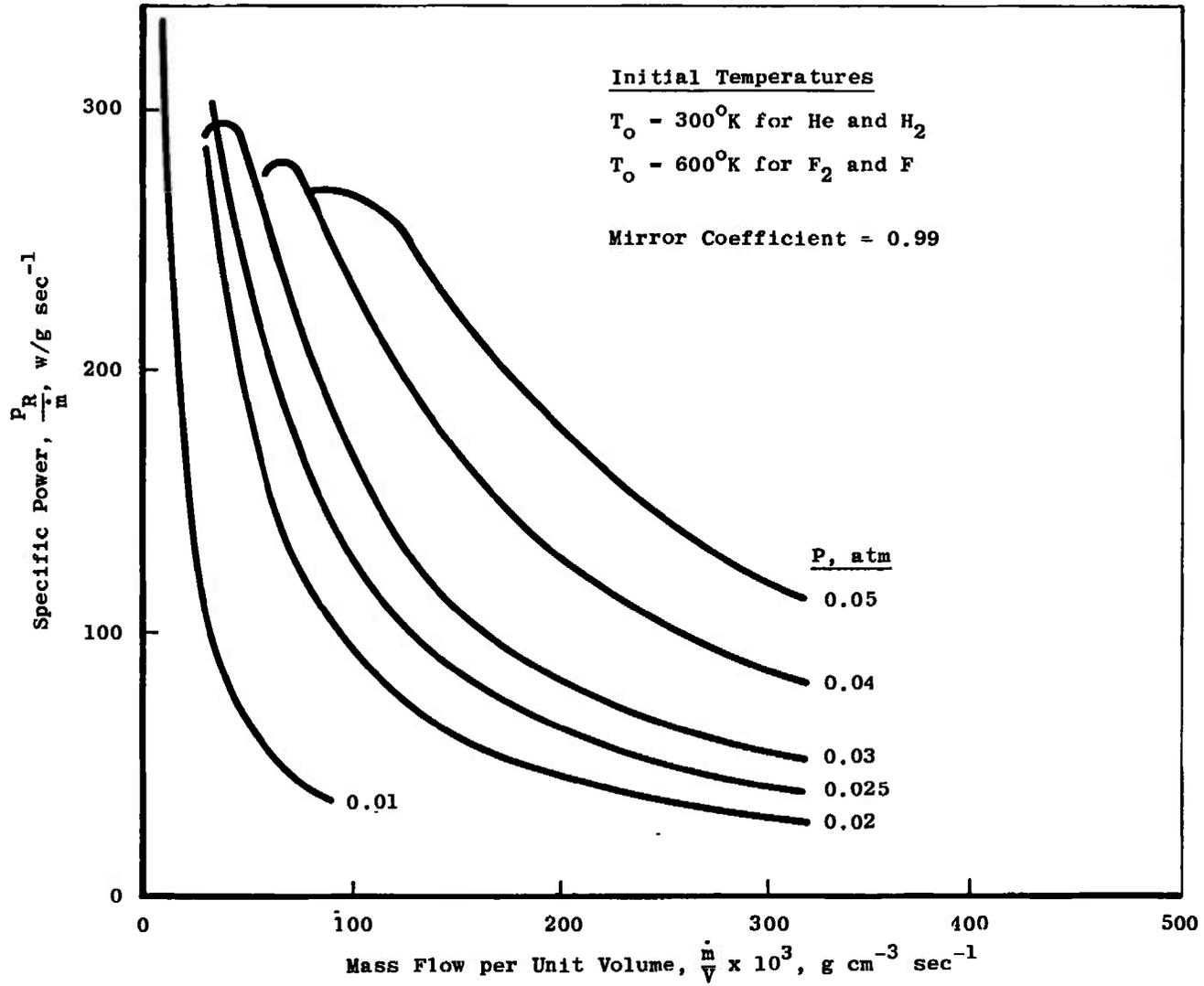


Fig. 4 Specific Power versus Mass Flow Rate per Unit Volume: Effect of Pressure at Constant (Low) Gain

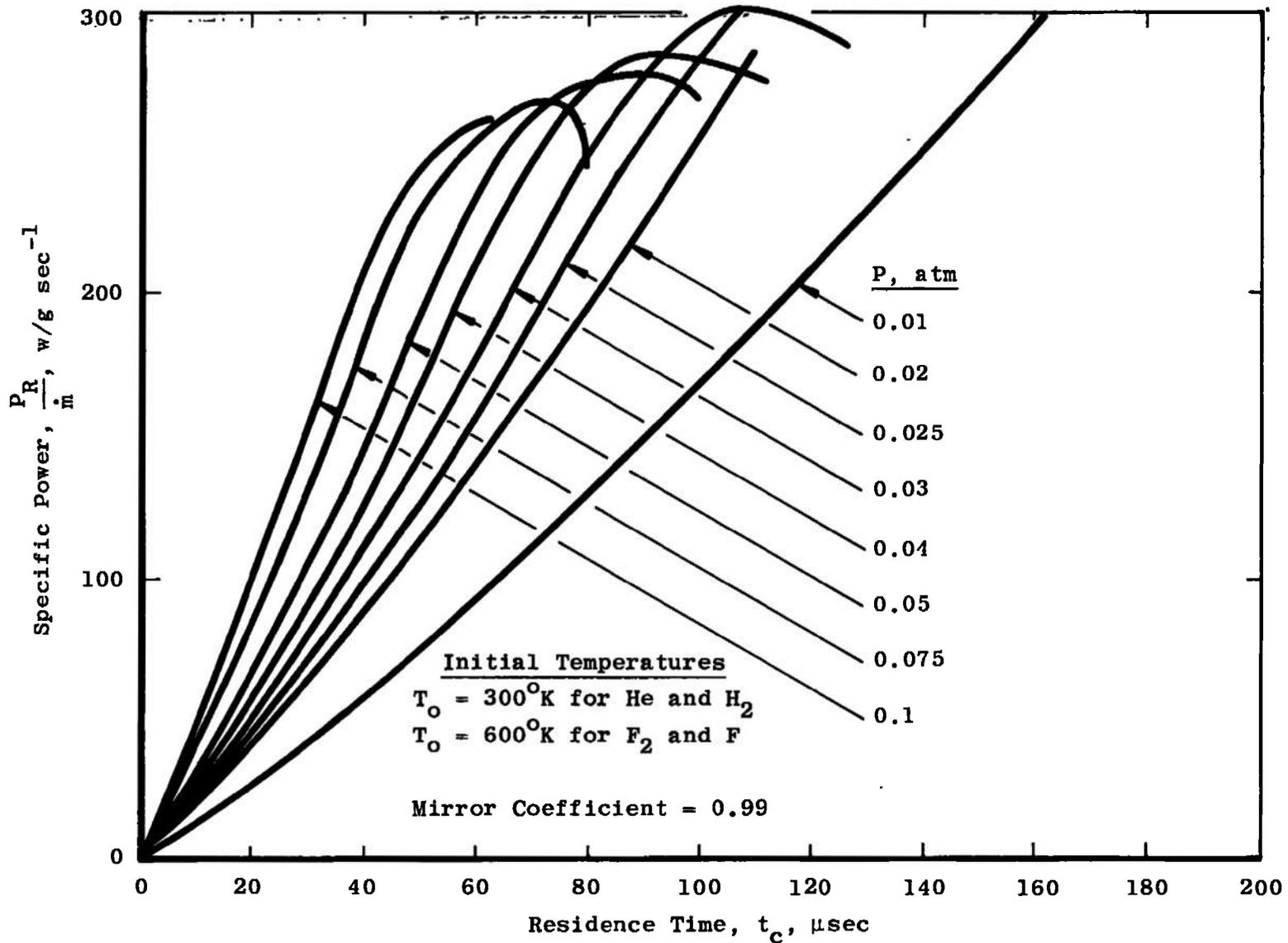


Fig. 5 Specific Power versus Residence Time: Effect of Pressure at Constant (Low) Gain

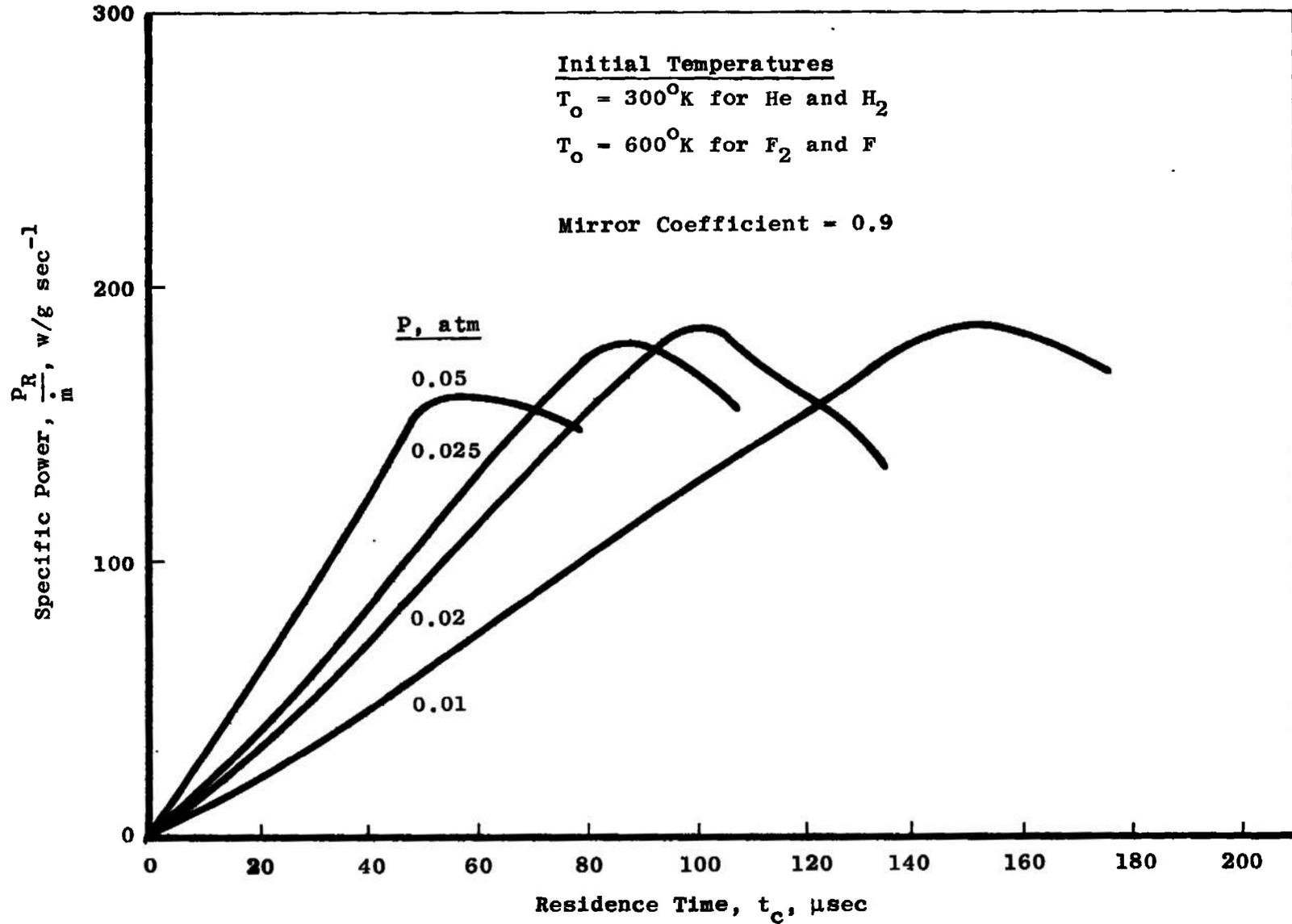
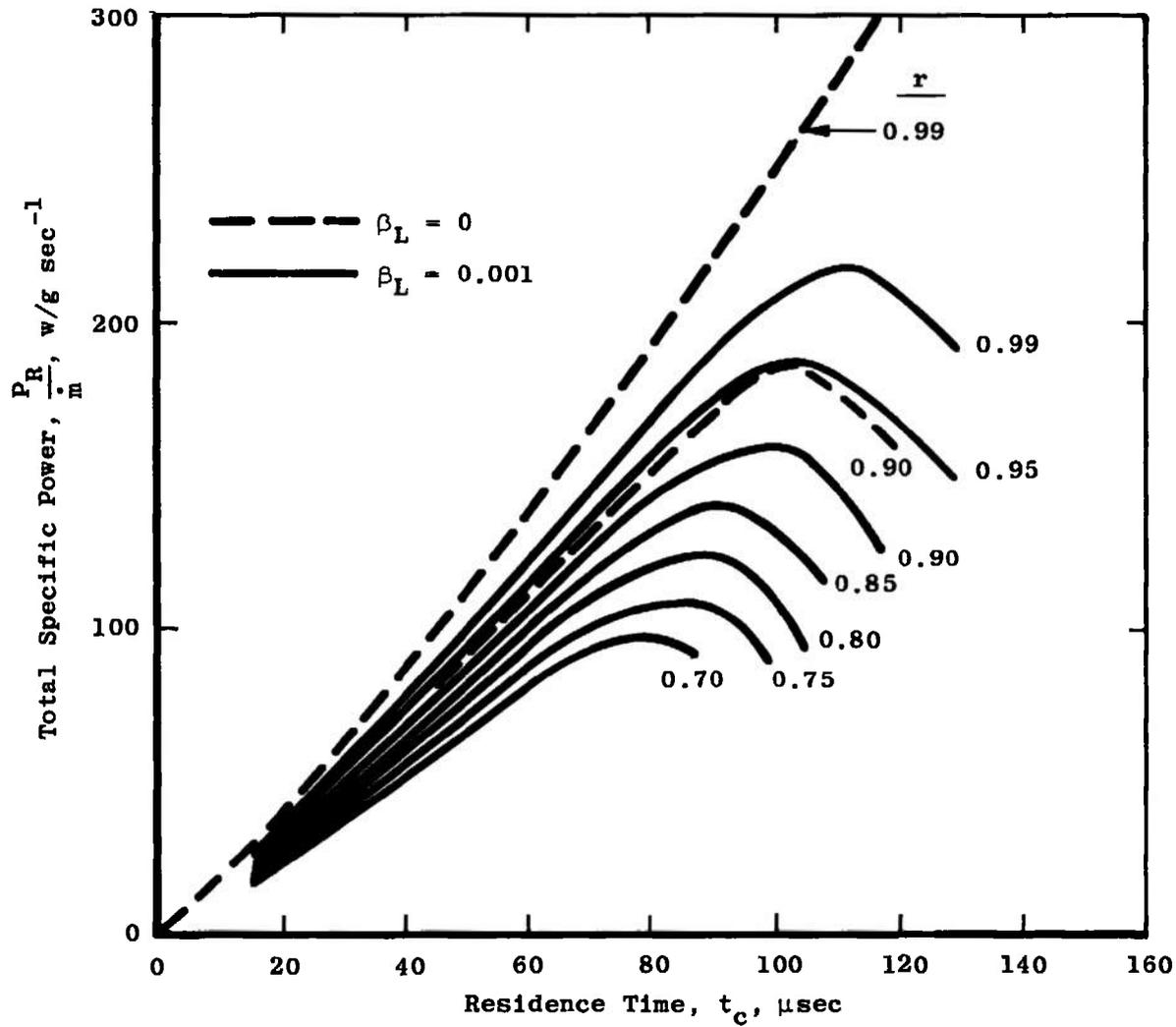
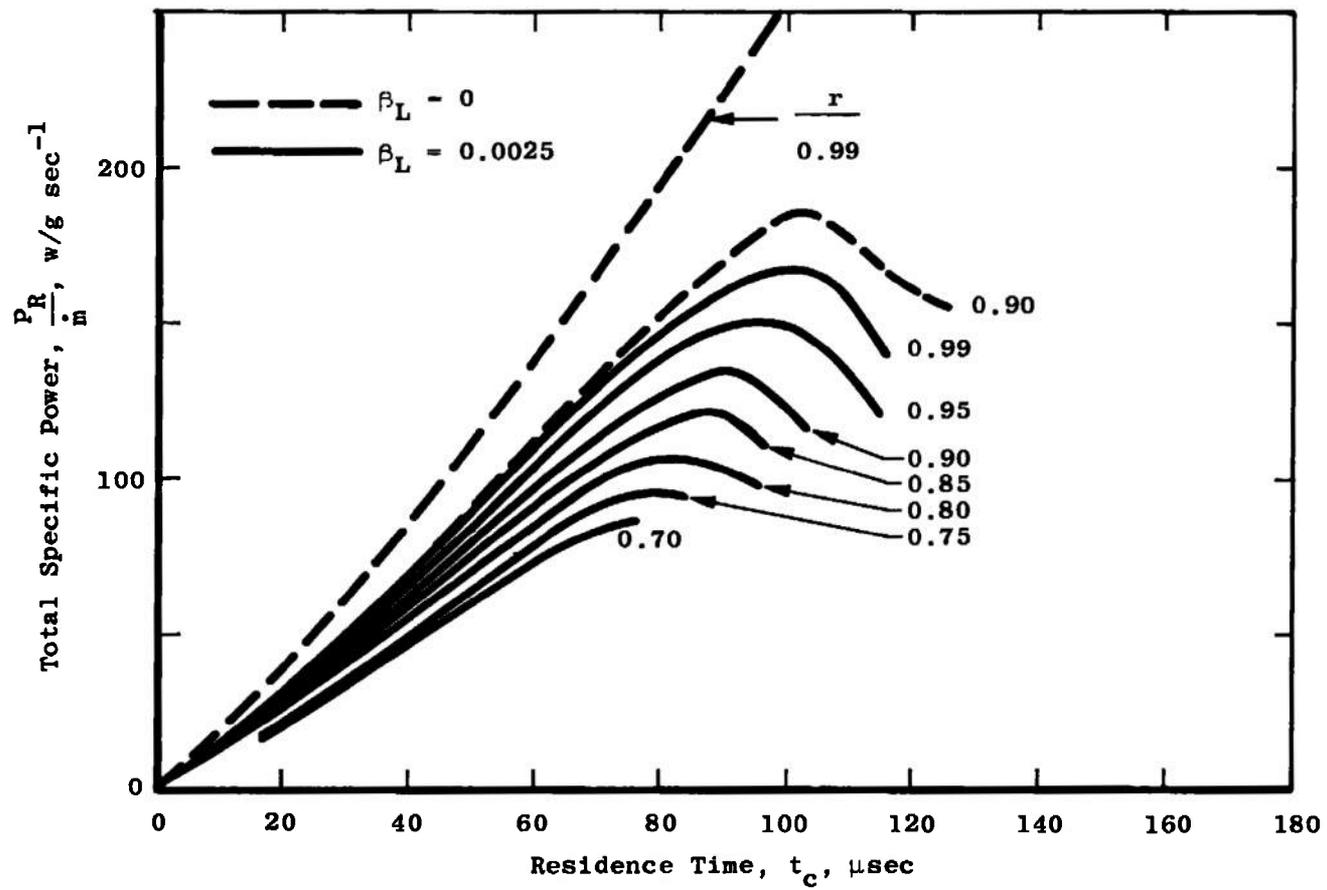


Fig. 6 Specific Power versus Residence Time: Effect of Pressure at Constant (Medium) Gain

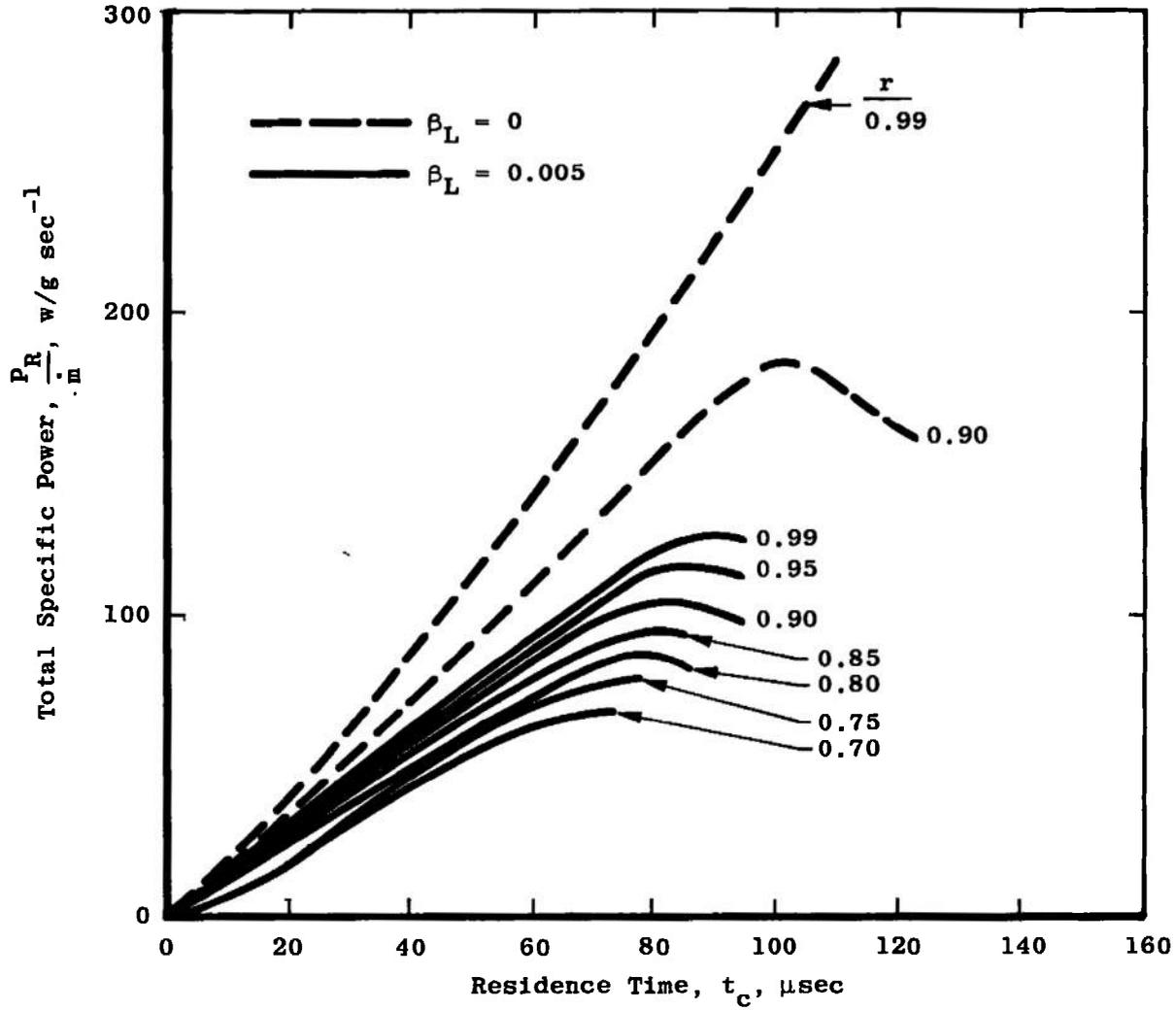


a. $\beta_L = 0.001$

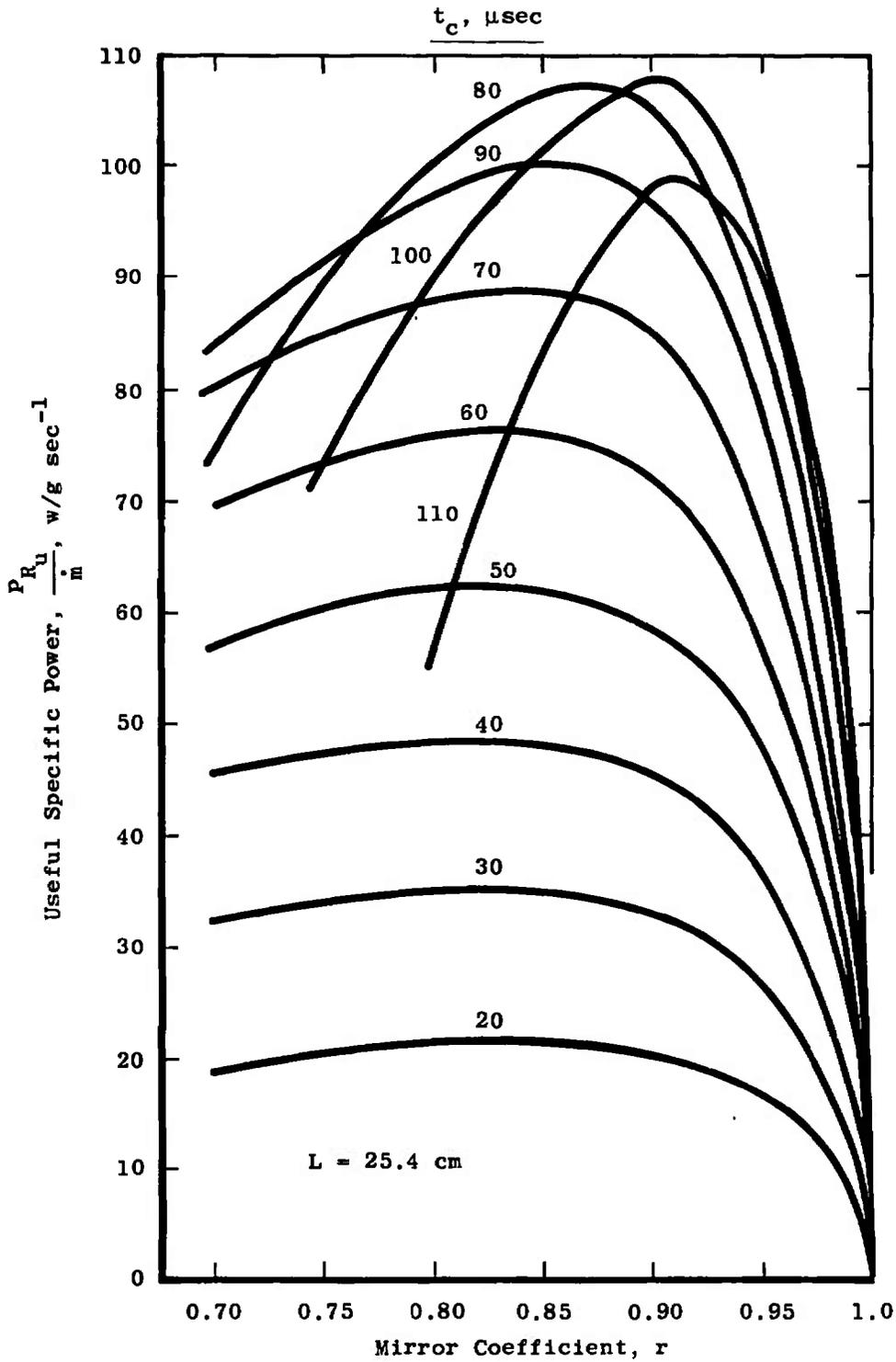
Fig. 7 Specific Power versus Residence Time: Effect of Mirror Reflection Coefficient at a Cavity Pressure of 0.02 atm



b. $\beta_L = 0.0025$
 Fig. 7 Continued

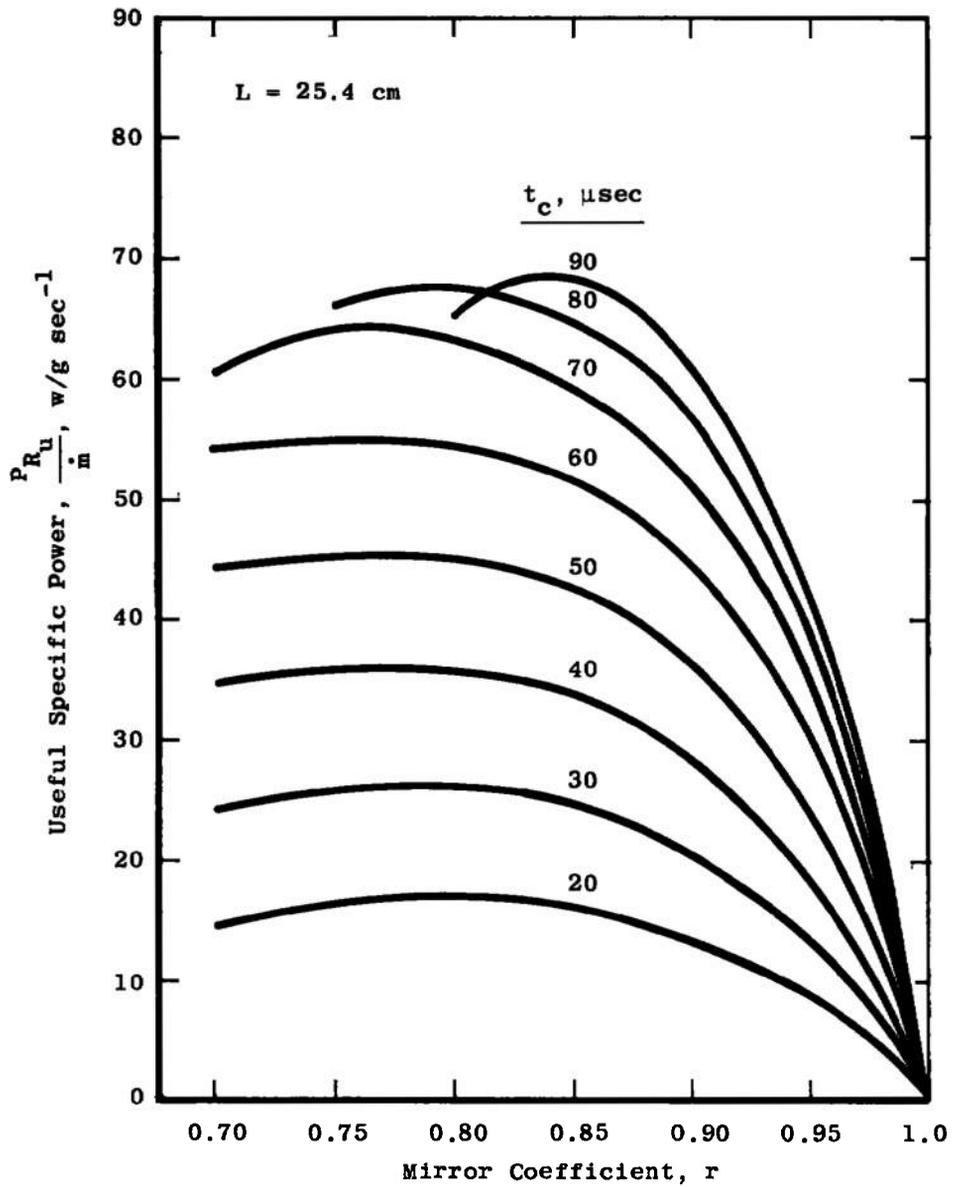


c. $\beta_L = 0.005$
 Fig. 7 Concluded

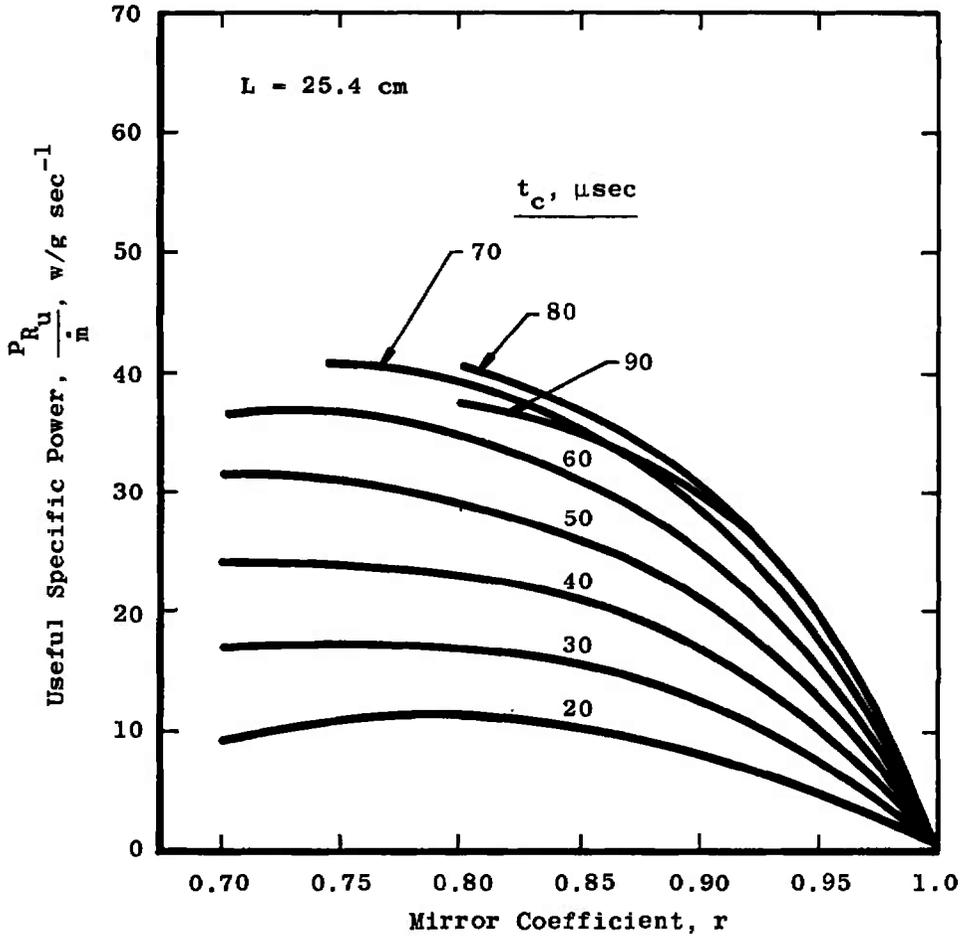


a. $\beta_L = 0.001$

Fig. 8 Useful Specific Power versus Mirror Reflection Coefficient at a Cavity Pressure of 0.02 atm



b. $\beta_L = 0.0025$
 Fig. 8 Continued



$c. \beta_L = 0.005$
Fig. 8 Concluded

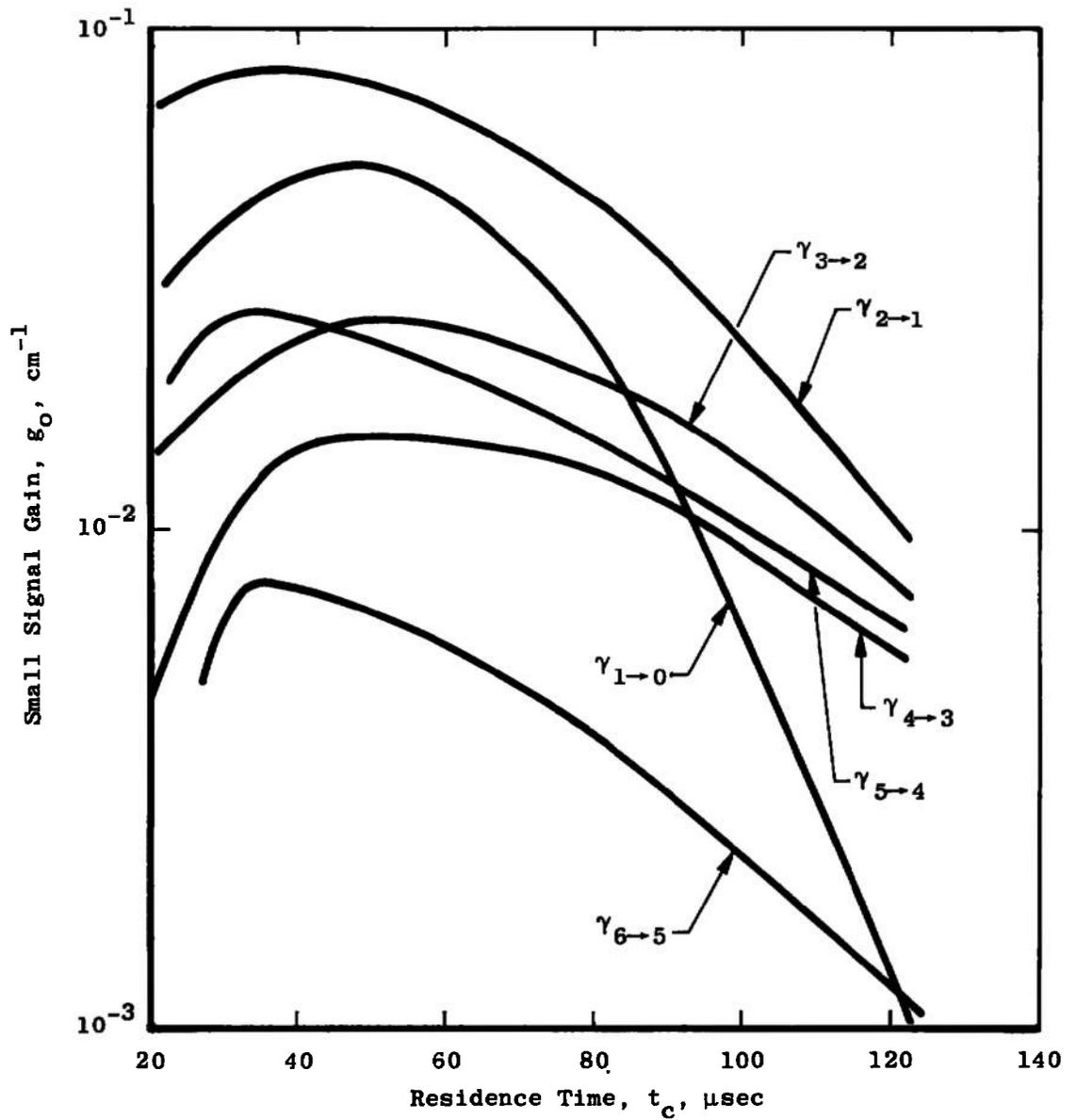


Fig. 9 Small Signal Gain Coefficient versus Cavity Residence Time

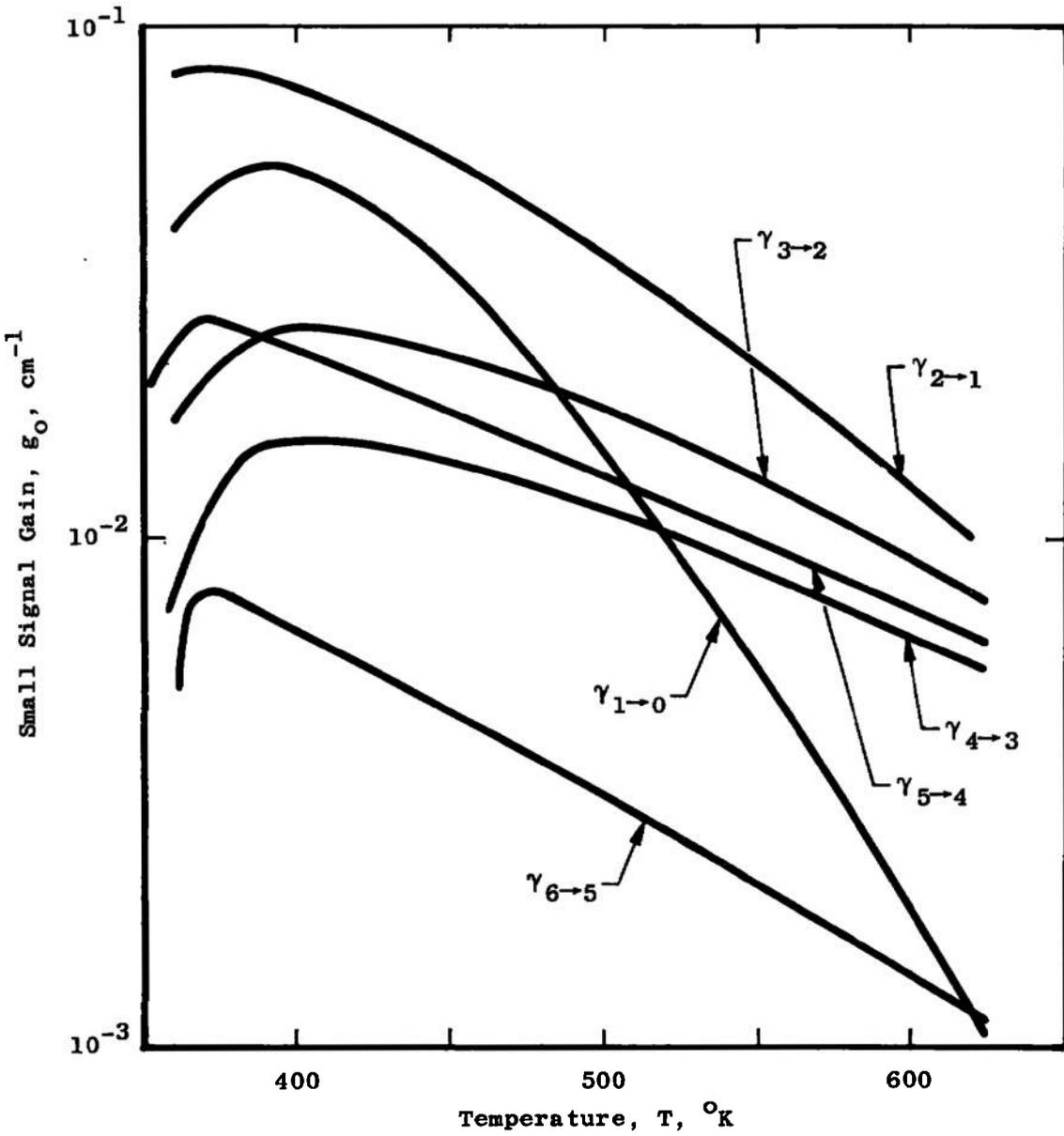


Fig. 10 Small Signal Gain Coefficient versus Cavity Temperature

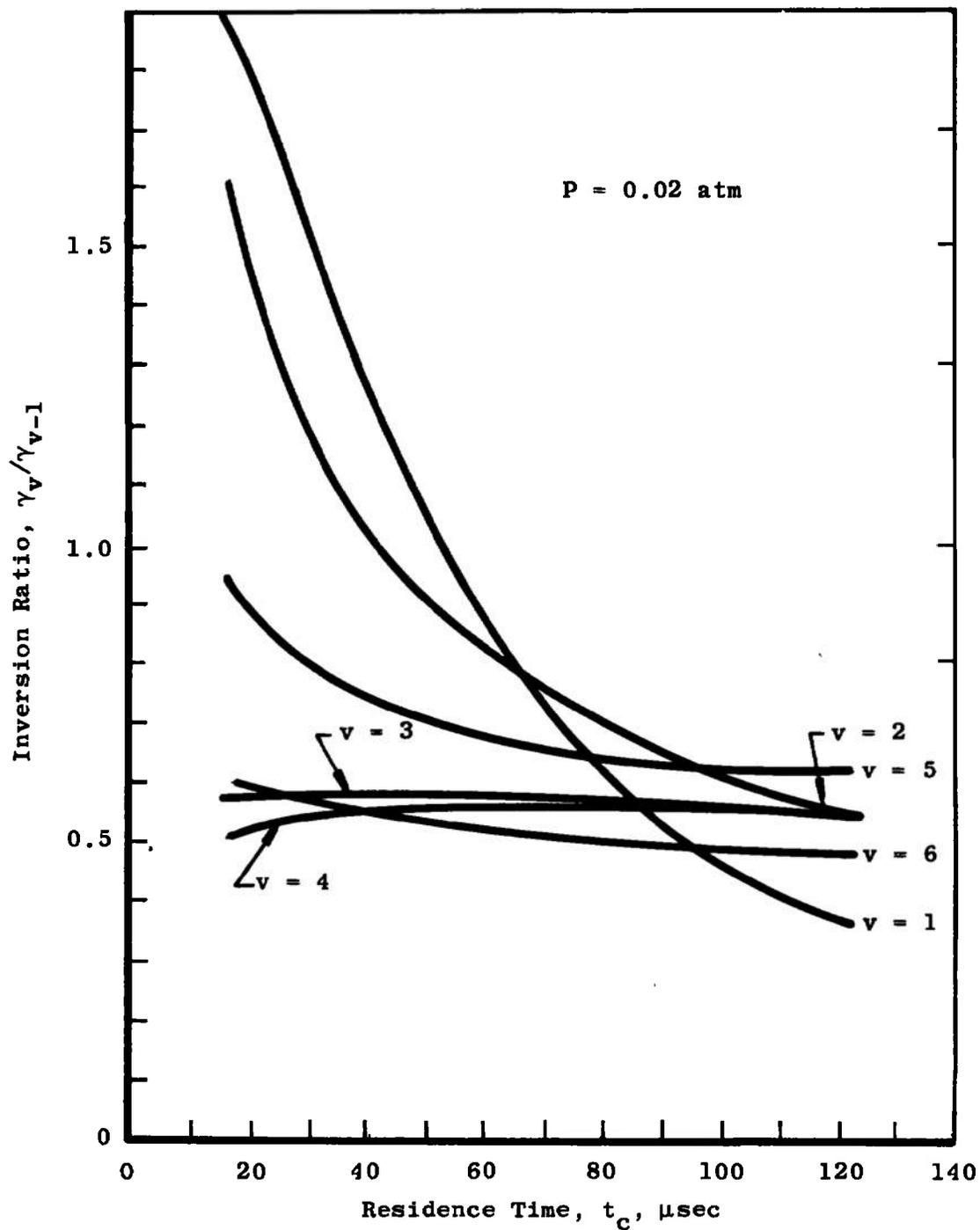


Fig. 11 Inversion Ratio versus Residence Time at Zero Power

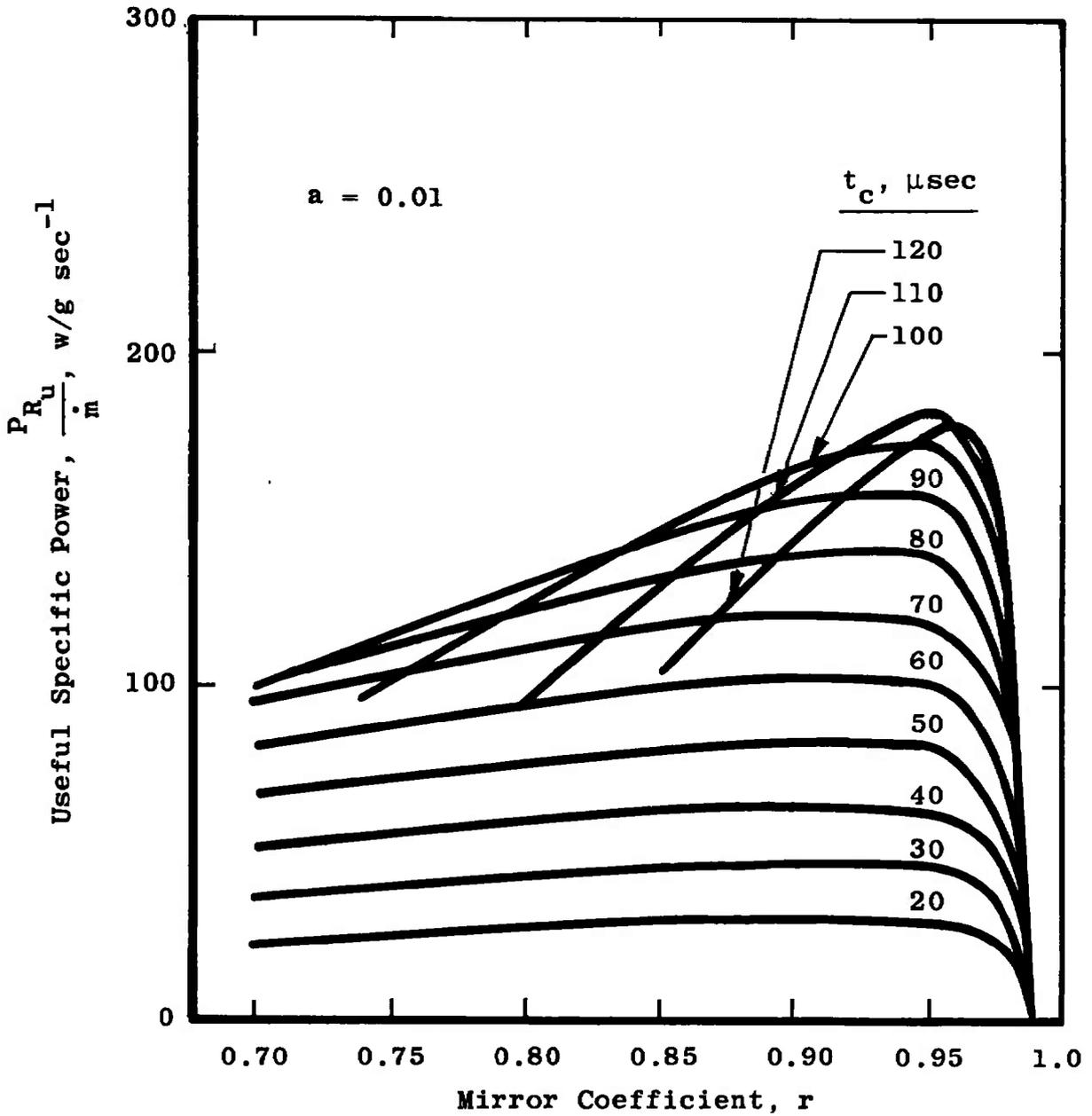


Fig. 12 Effect of Mirror Absorption Losses on Useful Specific Power

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13 ABSTRACT The results of a previous study of the concept of a stirred reactor laser with lasing restricted to the zeroth fundamental band are compared with the predicted power output of a multilevel laser for the hydrogen-fluorine-helium (H ₂ -F ₂ -He) system. Lasing is assumed to occur on all fundamental bands for which the calculated maximum gain equals the cavity threshold value. Power output is significantly increased over the zeroth band calculations, and the effect of cavity pressure is shown to be less pronounced because of the cascade enhancement from higher bands. Vibrational de-excitation caused by collisional processes appears to limit this kind of chemical system to low cavity pressures. The effect of cavity losses was evaluated with three assumed values for the cavity loss coefficient. These values were estimated as a small fraction (0.1) of the average small signal gain coefficient at different cavity residence times.			

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