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THE COHERENCE OF SIGNALS, NOISE AND REVERBERATION IN THE SEA

R. J. Urick

Naval Ordnance Laboratory White Oak, Maryland

10 November 1972



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THE COHERENCE OF SIGNALS, NOISE AND REVERBERATION IN THE SEA

This is the text of an invited paper presented at a recent meeting of the Acoustical Society of America. It will be of interest to those interested in the second-order statistics of underwater sound and its effects on the gain of receiving arrays.

> ROBERT WILLIAMSON II Captain, USN Commander

Z.I. SLAWSKY

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# THE COHERENCE OF SIGNALS, NOISE AND REVERBERATION IN THE SEA\*

## INTRODUCTION

1. Over the past ten years, we, at the Naval Ordnance Laboratory, have been interested in the coherence of sounds of various kinds in the sea. During this period of time, a succession of field trips has been made and the results have been published in a number of different papers and reports (References 1 - 9).

2. The present report is a synopsis of this prior work aimed at comparing the ccherence of different signals and noises in the sea with one another. In what follows, we will first describe in a tutorial way what is meant by coherence and how it is measured. After that, we will see some of the results of field measurements where coherence is expressed in terms of a normalized hydrophone separation. Finally, we will consider briefly how such information can be used in the design of arrays for maximum improvement of signal to noise ratio.

## DEFINITIONS

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3. To start with, we should make it clear what we mean by the term "coherence". Coherence means the degree of similarity of the output of two hydrophones spaced a distance apart in a sound field. It can be expressed quantitatively by a <u>cross-correlation coefficient</u>,  $\rho$ , defined as the time-averaged normalized product of the two outputs.  $\rho$  is a number ranging between + 1 and - 1. When a variable time delay is inserted in one of the two channels,  $\rho$  becomes a function of time delay called a time-delay correlogram.

4. Different kinds of correlation coefficients can be defined, depending upon what is done to the signals before they are correlated or multiplied - together. Figure 1 gives definitions of two of these kinds of correlation. The correlation coefficient  $\rho$  is a function of averaging time T and delay time  $\tau$  and is equal to the normalized time average of the product of  $s_1(t)$  and  $s_2(t+\tau)$ . The bracketed quantity in the denominator is a normalizing factor equal to the rootmean-square product of the two signals. The "true" correlation coefficient  $\rho_T$  involves doing nothing to the signals before multiplying except bandpass filtering. Another coefficient results when the signals are hard-clipped or limited before multiplying. When

\* Invited paper presented at the 84th Meeting of the Acoustical Society of America, 28 November - 1 December 1972.

TRUE CORRELATION COEFFICIENT (MULTIPLIER CORPELATOR)

$$\rho(T,\tau) = \frac{\frac{1}{T} \int_0^T s_1(t) \cdot s_2(t+\tau) dt}{\left[ \right]}$$

$$\left[ \right] = \left[ \frac{1}{T} \int_0^T s_1^2(t) dt \cdot \frac{1}{T} \int_0^T s_2^2(t) dt \right]^{\frac{1}{2}}$$

CLIPPED CORRELATION COEFFICIENT (CLIPPER CORRELATOR)

$$\rho(T,\tau) = \frac{\frac{1}{T} \int_0^T \operatorname{sgn} s_1(t) \cdot \operatorname{sgn} s_2(t+\tau) dt}{\left[\right]}$$

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FIG. 1 DEFINITIONS OF TRUE (MULTIPLIER) AND CLIPPED CORRELATION COEFFICIENT

this is done, all amplitude information is removed by clipping, and the device is called a clipper correlator. Its output is the <u>clipped</u> <u>correlation coefficient</u>,  $\rho_c$ , defined as indicated at the bottom of the figure. Conceivably, many other kinds of correlation could be done, depending on what property of the signals is desired to be compared. For example, an <u>envelope correlator</u> is one wherein the signals are rectified and low-pass filtered before correlating; here carrier phase is removed and only the relatively slow amplitude changes in the envelope are preserved.

5. Figure 2 is a generalized block diagram that illustrates how the processes indicated by the definitions of the quantities are implemented to produce, on a display, the time delay correlogram  $\rho(\tau)$ .

6. Of these various kinds of correlation, the clipped correlation is the most important and useful in sonar. One advantage of clipped correlation is that carefully matched hydrophones are not required, since the outputs are hard-clipped at the outset. Another is that the output of a clipper correlator averages to zero, without the troublesome d.c. output of a true multiplier. A third is that clipper correlation is readily adaptable to digital beam-forming and processing. Finally, the price to be paid for clipping, in terms of reduction of processing and array gains, is small and tolerable. For these reasons, clipper correlation is the one used most often in modern sonars.

In terms of the properties of the sound field in which the two 7. separated hydrophones are placed, the clipped coefficient  $\rho_{C}$  is a measure of the phase distortion, or the wave-front crenulation, of the sound field. What is meant by this is illustrated in Figure 3. Here we see a series of crenulated distorted wave-fronts impinging from the left upon two hydrophones labelled A and B. These wavefronts may be imagined to separate the portions of the sound wave of positive and negative phases. After clipping, the outputs of A and B become the square topped signals shown below, where time is imagined to run from right to left as the impinging wave travels from left to right. Now, it is easy to show that the clipped coefficient equals the fraction of the time that the outputs of A and B have like signs, minus the fraction of the time that they have un-like signs. At the bottom of Figure 3 there is shown a little calculation for the wavefronts we have drawn, using the symbols M and N for the lengths of time that the outputs are of similar or dissimilar polarities. The result of the calculation is a value of  $\rho_{\rm C}$  equal to 0.36. In an actual clipper correlator this process is of course executed electronically.

### COHERENCE MEASUREMENTS

8. With this much tutorial matter disposed of, we may now have a look at some actual clipped time-delay correlograms. In Figure 4 we have a series of correlograms that were taken sequentially over an interval of 0.9 seconds during the return of an explosive shot from the sea bottom. The receivers were a pair of hydrophones 7-1/2 ft. apart in water 5800 ft. deep; the shot depth was 800 ft. and the hydrophone depth, 300 ft. In the figure, the sequence of correlograms begins at يتي الانتقاصا الوالي أراضيمه

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FIG. 3 CLIPPED CORRELATION COEFFICIENT & AS RELATED TO CRENULATIONS OF THE WAVEFRONT.





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the upper left, at a time just before 2.0 seconds after the explosion, and continues on to 2.9 seconds. Now, if we take a close look at this series of wiggles, we will see agroup of four correlograms at a time near 2.0 seconds that have peaks occurring to the left of zero time de-These represent the bottom reflection arriving from below. lay. Shortly thereafter, at about 2.1 seconds, there is another series of correlogram; having peaks to the right of zero time delay; these represent the bottom-surface arrival coming in from above. Later on, at 2.3 seconds, there is a series with peaks again to the left of zero that represents the surface-bottom arrival. Finally, after a confused interval, another series of correlograms, representing oblique back scattering from the ocean floor, begins to emerge. If the sequence were to be continued beyond 2.9 seconds, these correlogram peaks on the bottom back-scattering would be seen to approach zero time delay as, with increasing time, the back scattering comes in more and horizor tally.

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Figure 5 illustrates some more correlograms, this time of the low 9. frequency ambient noise background received by pairs of vertically separated hydrophones. The hydrophones were those of the Trident Vertical Array located near the bottom in 14,400 feet of water south of Bermuda. In the figure, the top correlogram in each of the three octave bands is a calibration correlogram made on random electronic noise fed to all recorder inputs in parallel, giving the amplitude corresponding to unit correlation coefficient as well as the location of the zero point on the time delay scale. In perusing the ambient noise correlograms shown below, we may observe two features of interest. First, when we scan the correlograms from top to bottom, we see that the coherence decreases with increasing spacing at the same frequency. Second, when we scan from left to right, we see that the coherence decreases with frequency at the same spacing. In addition, we note that the correlograms always remain centered at zero time delay, in keeping with the theory that low frequency noise in the deep sea originates in noise sources located at long ranges, and so arrives in horizontal directions corresponding to the zero time delay.

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10. We have obtained time delay clipped correlograms such as those we have just seen for various kinds of backgrounds in deep and shallow water. From these correlograms the peak clipped coefficient  $\rho_{\rm C}$  was read off and plotted in Figure 6 in order to obtain a synoptic view of the observed values. In the figure, the abcissa is the ratio of the vertical separation distance of the two hydrophones correlated together, divided by the mid-band wave-length. Each plotted point is a peak reading of a single correlogram using a short integration time - hence the scatter of the points. Included are various kinds of reverberation in deep water, reverberation in shallow water and ambient noise near the deep sea bed. Because the data points can never fall below correlogram noise, they level off at a value that depends on bandwidth and integration time. The measured values lie, as they should, between the theoretical curves for a plane wave ( $\rho_c = 1.0$ ) and for isotropic noise. In between these limits are drawn theoretical curves for noise distributed uniformly within vertical angles of

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EXAMPLES OF TIME DELAY CORRELOGRAMS OF AMBLENT NOISE IN THREE OCTAVE BANDS BETWEEN VERTICALLY SPACED HYDROPHONES NEAR THE DEEP SEA BED.

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 $\pm 1^{\circ}$ ,  $\pm 10^{\circ}$  and  $\pm 30^{\circ}$ . These equivalent vertical angles may be called "coherence and les", since they are the angles in the vertical within which all the sound appears to be uniformly distributed, with nothing arriving beyond. The most coherent kind of noise is the return from the bottom in deep water just after it arrives at a shallow hydrophone; this kind of background appears to have a coherence angle of 2°. Somewhat less coherent is reverberation in shallow water having a coherence angle of about 5°. Ambient noise near the ocean floor appears to have a coherence is the volume reverberation in the deep sea in the absence of a deep scattering layer.

11. A corresponding plot for CW transmitted signals at frequencies of 1120 and 1500 Hz is shown in Figure 7. The 1120 Hz transmissions were received on a vertical string of hydrophones located inside and outside of the first convergence zone at a distance of about 33 miles from the source. The signals inside the zone are more coherent than those outside of the zone (i.e. at a range shorter or longer than the range to the zone) because they are received via an essentially single transmission path instead of paths involving bottom and surface reflections. On the other hand, the 1500 Hz signals transmitted over a 24 mile path extending out to  $d/\lambda_0 = 100$  were once reflected from the sea surface. Here the degradation of coherence is due to a single bottom-bounce multipath that remains nearly equal in intensity to the primary path not involving the bottom. As a result, the coefficient remains at a constant low value out to large separation distances.

### ARRAY GAIN

12. Data of this kind, which depend on the medium in which sonars must operate, are essential to the design of arrays having a maximum of array gain in that medium. Array gain (AG) is simply related to the cross-correlation coefficients of the signal and the noise, taken between all elements of the array in pairs. For an n<sub>z</sub>element array it can be shown (10) that

$$AG = 10 \log \frac{\sum_{j=1}^{n} \sum_{i=1}^{n} s_i s_j(\rho_s) ij}{\sum_{j=1}^{n} \sum_{i=1}^{n} s_i s_j(\rho_n) ij}$$

where <sup>S</sup>i, <sup>S</sup>j are the amplitude shading coefficients of the i-th an j-th elements and  $(\rho_s)_{ij}$  and  $(\rho_n)_{ij}$  are the correlation coefficients for signal and noise, respectively, between the i-th and j-th array elements. AG is thus defined as 10 times the log of the ratio of the sums of the elements of the coherence matrix for the signal to the corresponding sum of matrix elements for the noise. It involves summing the  $\rho$ 's for all the separations of pairs of elements, as read off from curves such as those just seen, at a time delay corresponding to that used in steering the array. In a design problem, such sums would be obtained for an initial trial array by means of a computer program,





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and then the array would be repeatedly modified, using the designer's intuition, to obtain a greater array gain. In the data just presented only vertically separated hydrophones were used and the results are applicable only to vertical arrays.

13. Short of a precise computation, a quick estimate of array gain may be obtained in the following way. Let us postulate, for a particular signal or background, that a "coherence distance" exists, within which the signal or background is perfectly coherent ( $\rho$ =1) and beyond which it is perfectly incoherent ( $\rho$ =0). The coherence distance may be readily found by integrating a curve of  $\rho$  against separation, and equating areas, as shown in the upper part of Figure 8. Then, with these values and for a trial array a count is made of the number of elements, including each element taken with itself, that lie within the coherence distance for the signal and for the background. If we denote these numbers by n<sub>s</sub> and n<sub>n</sub>, the array gain becomes simply

$$AG = 10 \log \frac{n_s}{n_n}$$

At the bottom of Figure 8 is an example, showing a circular array of 24 elements in a signal and noise field characterized by the coherence distances  $D_s$  and  $D_n$ ; these must not depend on direction, within the limits of the array. Then, in this example, the number of pairs of array elements lying within  $D_s$  is 24 x 19 and the number within  $D_n$  is 24 x 7. The array gain is then 10 log 19/7 or 4.3 db.

14. Such a calculation presupposes that the array is "steered" in the direction of the signal so as to maximize the correlation between elements of the value  $\rho_s$ . At the same time, steering will often decorrelate the background below the peak correlation coefficient  $\rho_n$ In a many-element array,  $\rho_n$  may be taken to average out to zero for all array pairs, so that  $n_r$  in the denominator reduces to the number of array elements. And the second

15. We have seen that coherence of noise and reverberation falls off with vertical separation in a way consistent with the idea that sound arrives at the hydrophones within a certain angular range in the vertical plane. While this model may serve to explain the observed decorrelation of noise and reverberation, We might ask, what about signals transmitted from a single distant source? What causes transmitted signals to decorrelate?

16. Two answers to this question may be surmised. One possible cause is the irregular velocity structure of the sea, which acts to cause focussing, de-focussing and scattering of sound, and thereby produces an irregular crenulation of the wavefronts of a plane wave. However, if we use as a guide what little is known quantitatively about the thermal structure of the sea along with the theory of Chernov (11), we find results that do not seem consistent with the observed coherence data. Accordingly, it does not appear likely that the inhomogeneous structure of the sea is by itself the explanation we seek.



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Array Gain = 10 log  $\frac{19}{7}$  = 4.3 db.

# FIG. 8 SIMPLIFIED ARRAY GAIN CALCULATION.

17. A more likely possibility is multipath interference. At long ranges the sound from a distant source reaches the receiver via a variety of different paths involving, in the vertical plane, different numbers of bounces from the sea surface and bottom. If the array is steered toward one of these signals by introducing suitable time lags between the array elements, the multipath contributions act as noise and so reduce the achievable array gain.

18. It can be shown that, if the array gain of a linear, additive, steerable n-element array is defined as

$$AG = 10 \ \log \frac{(S/N)_{array}}{(S/N)_{one \ element}}$$

then, when interference is present, the array gain becomes

$$AG = 10 \log \frac{nI_s + I_i}{I_s + I_i}$$

where  $I_s$  is the intensity of the signal (assumed to be perfectly coherent) to which the array is steered, and I. is the intensity of all the multipaths, such as multiple bottom bounces, that are uncorrelated among the array elements because of the steering that had been done in the direction of the signal. In most cases it will happen that  $nI_s > I_i$  and the array gain then becomes more simply

$$AG = 10 \log n \left[ \frac{I_s}{I_s + I_i} \right]$$

From this it is clear that in the presence of interference the gain of the array is reduced below its value 10 log n applying for a single coherent signal in incoherent noise. The reduction factor is the ratio of the signal intensity in the steering direction to the intensity of the total sound field. It follows that not all of a complex sound field is useful; the multipaths act as noise to the array. In the sonar equations, if a value of transmission loss based on measurements of the total sound field is used, then the <u>degraded</u> value of array gain, not the value 10 log n, must be used to find the S/N ratio at the array output. 白田子

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### CONCLUSION

19. It is apparent, from what we have just seen, that the subject of coherence of sound in the sea is still in its infancy. Little really definite is known about it, and observations based on field data, such as those of Figures 6 and 7 are, to be charitable, inadequate. Yet the subject is basic for the design of arrays to operate in the real ocean. Ignorance of the coherence characteristics of the real world, or even a failure to recognize that the sea is not the ideal medium with unidirectional signals and isotopic noise is one reason (there are others) why arrays do not always work as well

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as they should when they are taken out to sea. Much more needs to be done on both on its theoretical and observational aspects before it can be said that the coherence of sound in the sea, and how to mitigate and exploit its effects, is at all well understood.

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