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AN ANALYSIS OF PICKETT'S SOLUTION TO WESTERGAARD'S EQUATION FOR RIGID PAVEMENTS

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Champaign, Illinois

January 1973

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13. ABSTRACT

In this report, influence coefficients used by the Corps of Engineers for the development of design curves for rigid airfield pavements have been re-computed and extended to permit an expanded and more accurate analysis of large military aircraft such as the C-5A. Calculations were performed using a digital computer in conjunction with a more refined numerical integration technique. Results are compared with the original influence coefficients, and the impact of the more accurate extended table of influence coefficients is evaluated by employing the new and old influence coefficients to calculate edge stress resulting from several representative aircraft gear loads.

It is also demonstrated that further accuracy can be obtained by using a non-linear interpolation scheme derived from a bivarient quadratic regression analysis in place of a linear interpolation procedure to extract intermediate values from the table of influence coefficients. Finally, other areas of research which may possibly lead to improvements in the Corps design procedure for rigid airfield pavements are suggested.

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AN ANALYSIS OF PICKETT'S SOLUTION TO WESTERGAARD'S EQUATION FOR RIGID PAVEMENTS

1 INTRODUCTION

Background. The Corps of Engineers method[1] of determining minimum thickness requirements for concrete pavements relies in part on Pickett's procedure[2] for determining maximum stress at the edge of a concrete slab. Pickett presented influence coefficients for stresses in two forms. Table 1 presents in tabular form influence coefficients for stress. These influence coefficients are also available in graphic form as shown in Figure 1. For many years the graphic form was the predominant tool for manual determination of stresses in concrete pavements.

More recently, however, Pickett's method of determining stresses in concrete pavements has been programmed for computer by inputting Pickett's tabular values and using linear interpolation to calculate intermediate values. A typical example is the computer program H51 developed by Kreger.[3]

Objective. To facilitate the application of Pickett's work to determine stresses in concrete pavements, it was necessary to determine the effect of ignoring all aircraft loads, typical of large complex landing gear systems, which fall outside the limits of the original table. Thus, only those loads acting within a transverse and longitudinal distance of twice the radius of relative stiffness from the point at which stress is required were taken into consideration. The impact of this limitation was determined by extending the original table to include loads at distances equal to four times the radius of relative stiffness in both the transverse and longitudinal directions. At the same time, a more rigorous interpolation scheme was compared to the

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[1] TM 5-824-3/AFM 88-6, "Rigid Pavements for Airfields Other Than Army," Chapter 3, (Departments of the Army and Air Force, December 1970).

[2] G. Pickett, M. E. Raville, W. C. Janes, and F. J. McCormick, "Deflections, Moments, and Reactive Pressures for Concrete Pavements," Bulletin No. 65, Engineering Experiment Station (Kansas State College, October 1951).

[3] W. C. Kreger, Computerized Aircraft Ground Flotation Analysis - Edge-Loaded Rigid Pavements, Research Report ERR-FW-572 (Fort Worth Division, General Dynamics, January 1967).

Table 1

Moment at the Edge of a Concrete Slab Resting on a Liquid Subgrade

NUMBER OF BLOCKS (N) PER 0.01 ℓ^2 OF AREA FOR THE REGION INDICATED; $M = \frac{q\ \ell^2\ N}{10,000}$; $\ell = (\frac{D}{K})^{1/4}$; $\nu = 0.15$

b/ℓ*: 2.0, 1.8, 1.6, 1.4, 1.2, 1.0, 0.8, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1, Pavement Edge 0.0

Old Value →
Corrected Value →
% Difference →
(In critical area only)

3.13 2.974	3.0 2.918	3.0 2.807	2.9 2.648	2.7 2.445	2.4 2.206	1.92 1.799	1.33 1.209	0.74 0.622	0.19 0.087	-0.43 - .367	-0.6 - .724	-0.92 - .982
4.05 3.960	4.0 3.880	3.84 3.723	3.59 3.497	3.22 3.212	2.97 2.980	2.31 2.319	1.5 1.524	0.7 0.751	-0.07 .0634	-0.6 - .505	-0.9 - .941	-1.25 -1.248
5.3 5.241	5.3 5.125	5.0 4.899	4.7 4.575	4.12 4.171	3.69 3.705	2.96 2.932	1.6 1.863	0.7 0.856	0.15 - .014	-0.8 - .712	-1.2 -1.232	-1.7 -1.586
6.99 6.913	6.75 6.740	6.51 6.408	5.99 5.937	5.4 5.355	4.75 4.696	3.4 3.628	2.0 2.199	0.85 0.903	-0.23 - .176	-1.0 -1.012	-1.66 -1.613	-2.26 -2.007
9.15 9.118	8.90 8.855	8.39 8.350	7.67 7.646	5.8 6.795	5.9 5.852	4.3 4.373	2.48 2.480	0.89 0.845	-0.47 - .459	-1.3 -1.430	-2.0 -2.102	-2.5 -2.522
12.14 12.083 0.5	11.71 11.657 0.5	10.87 10.857 0.1	9.84 9.769	8.26 8.495	7.1 7.132	5.0 5.090	2.62 2.626	0.5 0.619	-0.83 - .904	-2.0 -1.991	-2.8 -2.711	-3.5 -3.136
16.27 16.191 0.5	15.51 15.447 0.4	14.19 14.091 0.7	12.36 12.332 0.2	10.42 10.381 0.4	8.3 8.406	5.5 5.630	2.53 7.525	0.13 0.160	-1.67 -1.545	-2.80 -2.708	-3.5 -3.443	-4.03 -3.848
20.64 20.452 0.9	19.45 19.222 1.2	17.29 17.081 1.2	14.5 14.473 0.2	11.94 11.765 1.5	9.2 9.186	5.81 5.783	2.08 2.213	-0.6 - .369	-2.3 -2.163	-3.4 -3.348	-4.2 -4.069	-4.7 -4.442
24.1 24.170 0.3	22.5 22.340 0.7	19.5 19.326 0.9	15.8 15.891 0.6	12.8 12.543 2.0	9.60 9.515	5.83 5.702	1.96 1.872	-0.8 - .813	-2.7 -2.638	-3.8 -3.919	-4.6 -4.520	-5.0 -4.862
28.81 28.967 0.5	26.08 26.045 0.1	21.74 21.658 0.4	17.20 17.137 0.4	13.19 13.076 0.9	9.72 9.611	5.6 5.447	1.54 1.423	-1.46 -1.325	-3.2 -3.158	-4.4 -4.322	-5.0 -4.994	-5.4 -5.301
35.12 35.473 1.0	30.50 30.314 0.6	23.7 23.783 0.3	17.8 17.981 1.0	13.36 13.245 1.9	9.7 9.428	5.2 5.015	1.0 .875	-1.8 -1.898	-3.8 -3.718	-4.9 -4.854	-5.5 -5.488	-5.9 -5.755
43.86 44.963 2.5	35.5 34.571 2.7	25.0 25.135 0.5	18.1 18.193 0.5	13.13 12.992 1.1	9.4 8.967	4.72 4.427	0.3 .243	-2.6 -2.521	-4.4 -4.313	-5.3 -5.409	-6.1 -6.000	-6.3 -6.223
56.42 59.605 5.3	40.66 36.833 10.4	25.18 25.229 0.2	18.0 17.757 1.4	12.5 12.381 1.0	8.6 8.288	3.95 3.718	-0.6 - .456	-3.56 -3.187	-5.2 -4.936	-6.0 -5.986	-6.5 -6.528	-6.8 -6.702

a/ℓ*: 0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0

*NOTE: Symbol 'a' denotes the distance from point 0 parallel to the edge and 'b' denotes the distance from point 0 perpendicular to the edge.

$M = \frac{q l^2 N}{10{,}000}$

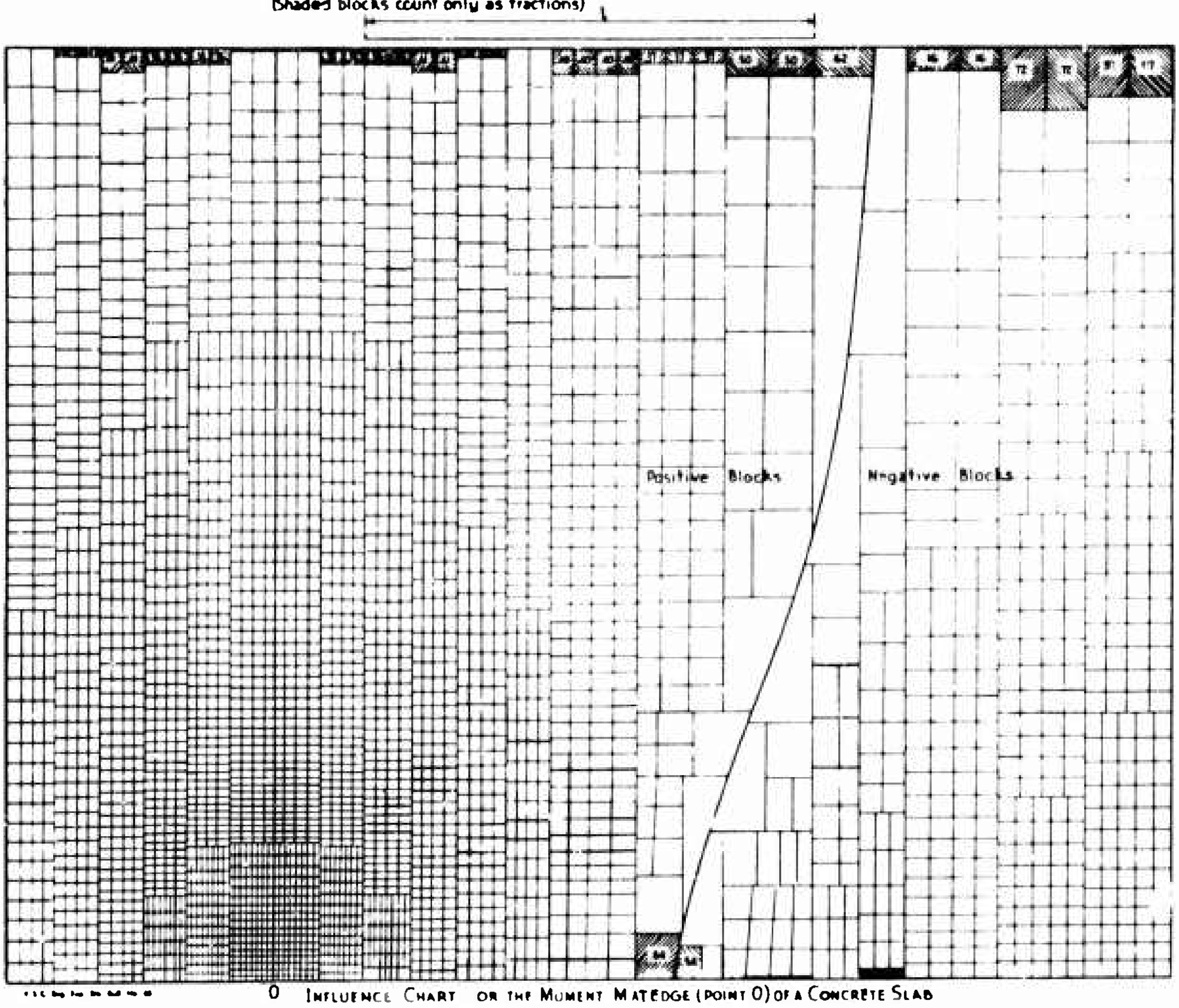
q = Intensity of loading

N = Number of positive blocks minus number of negative blocks (Shaded blocks count only as fractions)

$l = \left(\frac{D}{k}\right)^{\frac{1}{4}}$

D = Flexural rigidity of pavement

k = Rigidity of subgrade (density of liquid)



Influence Chart of the Moment M at Edge (point 0) of a Concrete Slab
Due to a Load in the Vicinity of the Edge
(Subgrade assumed to be a dense liquid Poisson's ratio for pavement = 0.15)

(from K.S.C. Eng. Exp. Sta. Bull. No. 65)

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Figure 1

original linear interpolation method in order to justify or eliminate the use of linear interpolation. The objective of this report is to present preliminary findings obtained from this study.

Approach. Pickett's tabular values (shown in Table 1) were developed prior to 1950; therefore, all calculations were performed by applying numerical integration techniques without the use of computers. In order to reduce the possibility of error, a similar numerical integration technique was used herein to extend the tables, except that calculations were done by computer instead of desk calculators. To further guarantee accuracy, more stringent step sizes and convergence criteria were applied and Simpson's rule instead of the Trapazoidal rule was used for the integration process.

Once a reliable numerical integration scheme had been developed, the table was extended to accept "a" and "b" values for offset equal to four times the radius of relative stiffness. The stress analysis program[4] was then modified to accept the new tabular values. Six aircraft loadings were arbitrarily chosen; three separate analyses were performed on each loading. The first analysis was performed using Kreger's H51 computer program with Pickett's stress coefficients. The second solution used H51 with the new tabular values extended to include ratios of a/1 and b/1 equal to 4.0. Finally, to test the acceptability of using linear interpolation to obtain intermediate values in the table, the original computer program was modified to incorporate a higher order interpolation method developed by McDowell.*

2 RESULTS

When Pickett's integral was first evaluated by computer, several discrepancies appeared between the newly generated data and the original tabular values. Further refinement of the numerical integration scheme eventually led to a set of corrected values which are shown in Table 1 along with Pickett's original values. In the critical section of the table, all errors in Pickett's values are less than 11 percent and average only 1.2 percent. The term "critical area" was arbitrarily chosen to include only those tabular values which specify a block count of 10 or more in order to focus attention on that section of the table where accuracy is most essential.

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[4] W. C. Kreger, Computerized Aircraft Ground Flotation Analysis - Edge-Loaded Rigid Pavements, Research Report ERR-FW-572 (Forth Worth Division, General Dynamics, January 1967).

* Correspondence with Dr. E. L. McDowell of CERL, July 1972.

Table 2 presents stress comparisons for the six aircraft loadings. Use of the original influence coefficients and a linear interpolation procedure produces discrepancies of from 1 to 4 percent; when the new tabular values are used with linear interpolation, discrepancies of as much as 3 percent are obtained.

While discrepancies of less than 4 percent may not cause catastrophic failures, it should be understood that the chosen aircraft loadings are not intended to represent the most critical loading conditions. However, since the critical discrepancies in Pickett's table do not exceed 11 percent, it can be safely assumed that use of linear interpolation and Pickett's tabular values has produced errors of less than 15 percent in all cases.

3 CONCLUSIONS AND RECOMMENDATIONS

While use of the extended tables and the higher order interpolation procedure changed stresses by less than 5 percent, use of the two improvements is recommended, especially since their use causes no significant penalty in terms of increased requirements for computer size or calculation time.

Based on information supplied by the heavy-load design curves for a twin-twin wheel gear of the B52, it can be demonstrated that a 2.5 percent increase in load at the edge of a 24-inch concrete slab would require an additional 1/2-inch of pavement, whereas a similar 1/2-inch increase in thickness will allow up to a 4 percent increase in load in a 15-inch pavement. Turning to the light-load design curves for a single wheel gear load, a 1/2-inch increase in thickness will increase the load capacity of a 12-inch slab by 7 percent and the load capacity of a 7-inch slab by 11 percent. Therefore, it is seen that a 3 percent error will probably affect the design of a heavily-loaded thick pavement, whereas a lightly-loaded thin pavement is much less sensitive to errors in the design procedure.

Regardless of whether or not these suggested modifications to Pickett's approach have an impact on pavement design, this study brings out the fact that there are areas of possible improvement in the Corps procedure for rigid pavement analyses as the result of the development of computers and analytic procedures. Therefore, one begins to wonder if there may not be other improvements that could be made, especially since there appears to be a trend towards eliminating the use of design curves altogether and going to an entirely computerized design process.

Other areas to be considered for improvement are:

1. Pickett's influence coefficients were generated using a value of 0.15 for Poisson's ratio. Currently, Poisson's ratio for

Table 2

Comparison of Stresses Showing Effect of Extended Tables and Parabolic Interpolation

LOAD TYPE	PAVEMENT THICKNESS (INCHES)	SUBGRADE MODULUS P/CU IN	S1	S2	$\frac{S1-S2}{S1}$ (%)	S3	$\frac{S1-S3}{S1}$ (%)
Single-Wheel Light Load	10	100	720.7	705.7	2.1	693.5	3.8
Twin Wheel Medium Load	16	100	704.3	699.0	0.8	694.8	1.3
Twin-Twin Wheel Heavy Load	23	100	617.2	599.3	2.9	592.3	4.0
C-5A	12	100	704.5	700.4	0.6	687.2*	2.5
C-141A	15	100	688.7	681.2	1.1	680.9	1.1
KC135	13	100	709.5	696.7	1.8	685.3	3.4

S1 = Stress (psi) from the extended table using parabolic interpolation.
S2 = Stress (psi) from the extended table using linear interpolation.
S3 = Stress (psi) from the Pickett table using linear interpolation.
*Some wheel loads partly off chart.

concrete is assumed to be 0.20. This has led to the current Corps practice[5] of using a value of 0.15 to develop a plot of radius of relative stiffness versus block count and then changing to a value of 0.2 when performing the remaining calculations needed to complete a set of design curves for a given aircraft.

2. How valid is the assumption that an edge-loaded pavement can be considered to act as a semi-infinite plate? Behrmann[6] found from small-scale tests that the semi-infinite assumption is valid for a single-wheel loading only as long as the minimum slab dimension is greater than three times the radius of relative stiffness. This indicates that Pickett's analysis will produce questionable results as the pavement thickness approaches 24 inches. It logically follows that the semi-infinite assumption is further invalidated by the fact that new aircraft, such as the C-5 and the 747, incorporate landing gears which cover much larger areas of pavement than do the older twin-tandem gears. If this is the case, an alternate method of analysis which more closely models the true boundary conditions should be adopted.

An increase in design reliability might be achieved through a total re-evaluation of Corps pavement design procedures. Special attention should be given to locating inaccuracies that occur as a result of assumptions and approximations. Such a study would then permit one to determine the extent to which inaccuracies accumulate during the design of a typical airfield pavement. In view of the major changes that have occurred in aircraft design since 1950, it appears quite likely that many assumptions and approximations may no longer be valid.

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[5] R. L. Hutchinson, Basis for Rigid Pavement Design for Military Airfields, Misc. Report No. 5-7 (Ohio River Division Laboratories, CE, May 1966).

[6] R. M. Behrmann, Small-Scale Model Study to Determine Minimum Horizontal Dimensions for 'Infinite Slab Behavior, Technical Report No. 4-32 AD 603-345, (Ohio River Division Laboratories, July 1964).

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TM 5-824-3/AFM 88-6, "Rigid Pavements for Airfields Other Than Army," Chapter 3, (Departments of the Army and the Air Force, December 1970).