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ORGANIZATION OF ERROR DETECTION AND COR-
RECTION IN NONPOSITIONAL SYSTEMS IN COM-
PUTERS

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Foreign Technology Division
Wright-Patterson Air Force Base, Ohio

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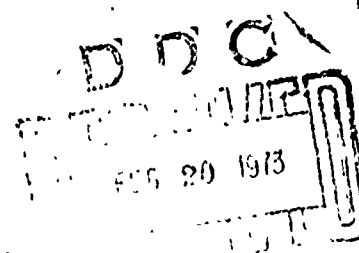
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IN COMPUTERS

by

I. Ya. Akushkiy and D. I. Yuditskiy



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Correcting codes in a system of residual classes with one test base are studied. It is suggested that the method of zeroing of a number be used for testing, allowing the number of the interval in which the number is located to be determined unambiguously. If the result of zeroing is other than zero, this indicates that there is an error present. The result of zeroing allows possible values of errors to be determined for each base. Due to determination not only of the location but also of the value of possible errors, an average of three operations is sufficient to localize an error. An example of correction of a single error by a correcting code with one test base is presented.
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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З з	<i>З з</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere.
 When written as ѣ in Russian, transliterate as yě or ě.
 The use of diacritical marks is preferred, but such marks
 may be omitted when expediency dictates.

ORGANIZATION OF ERROR DETECTION AND
CORRECTION IN NONPOSITIONAL SYSTEMS
IN COMPUTERS

I. Ya. Akushskiy and D. I. Yuditskiy

A rather prodigious arsenal of methods for detecting and correcting errors during information transfer has been developed at the present time for positional notation systems (binary, ternary, decimal, etc.). The basic idea inherent to all methods is the introduction of redundant information represented in the form of any generalized characteristic of the entire number which is taken with respect to one or several moduli.

In other words, a positional number is accompanied by a certain complementary integral characteristic having a pronounced nonpositional nature and specially organized for the purpose of detecting and correcting errors which can arise when storing and transmitting a number.

Since the representation of the number itself is assumed positional, and redundant information has a nonpositional nature, the complication of the additional equipment in attempting to control the correctness of operation execution in the arithmetic unit and the tremendous complexity of the equipment for correcting a detected error is naturally noticeable.

The situation is somewhat different in nonpositional notation systems, in particular in a system of residue classes. At the basis of such systems lies a certain set of relatively prime numbers called the bases [radices] of the system. Let us designate them by p_1, p_2, \dots, p_n . Number A is represented in the form of residues with respect to these bases, namely

$$A = (a_1, a_2, \dots, a_n).$$

where

$$a_i \equiv A \pmod{p_i}$$

$$i = 1, 2, \dots, n$$

Here digits a_1, a_2, \dots, a_n due to the very representation of the number are values independent of each other, as a result of which a number of arithmetic operations (addition, subtraction, multiplication, formal division, etc.) can be carried out digitwise, i.e., disregarding the connection between the digits of the number. The range of the number representation in such a system is defined as

$$\mathcal{P} = \prod_{i=1}^n p_i.$$

Let us now introduce redundancy into the number representation in the form of an additional relatively prime base p_{n+1} with the remaining bases of the systems. The number range will now be defined as

$$P = \prod_{i=1}^{n+1} p_i = p_{n+1} \mathcal{P},$$

i.e., the introduced redundancy increases the numerical range by p_{n+1} times.

Number A will now be represented in the form of residues with respect to all the bases of the system in the form

$$A = (a_1, a_2, \dots, a_n, a_{n+1}).$$

and as a result of the accepted representation of the number all its digits, including α_{n+1} , are independent. And this means that the digit α_{n+1} of a number with respect to additional base p_{n+1} subsequently called, the check base, will participate in the execution of the arithmetic operations equally with the remaining digits of the number. In other words, if as a result of the additional base it is possible to check the appearance of an error, then this check will also be effective in analyzing the result of an arithmetic operation without additional equipment or program operations.

Let us now examine one of the possible methods of checking a number for correctness in the presence of a check base. First of all let us introduce the following stipulation. Let us proceed from the fact that in the execution of a given program by a computer all operands and the results of any arithmetic operations are correct numbers, i.e., numbers lying in the range $(0, \mathcal{P})$ — in the initially determined number range.

Moreover, let us note that in the numerical sequence

$$A_s = (z_1, z_2, \dots, z_n, s);$$

$$s = 0, 1, 2, \dots, p_{n+1} - 1$$

only one number A_s^0 with one value $s = s^*$ is located in the range $(0, \mathcal{P})$, i.e., is correct:

$$A_{s^*}^0 = (z_1, z_2, \dots, z_n, s^*).$$

Any other value $s \neq s^*$ shifts the number from range $(0, \mathcal{P})$ into range $(\mathcal{P}, \mathcal{P}^2)$. The examined fact is also valid for any digit α_1 with respect to base p_1 .

This means that in considering the accepted stipulation the arithmetic unit operates only with the correct numbers of form A_S^0 and any error with respect to any base changes it to an incorrect number.

Here, by the word error or single error is understood the misrepresentation of any one digit of a number with respect to any base. The magnitude of the misrepresentation or the duration of its effect does not play a role, i.e., by the word error is understood both the random short duration failure of a digit and the total failure of a computer circuit with respect to a given base.

Thus, the check for the presence of an error is reduced to the determination of the correctness of an individual operand or the result of an arithmetic operation.

The so-called method of number zeroing can be proposed as one of the possible check methods. The essence of this method consists in the following. Let us be given a number of the following form

$$B := (0, 0, \dots, 0, a_i, a_{i+1}, \dots, a_n, a_{n+1}),$$

having the first $i - 1$ digits zero.

Let us select the smallest possible number of the form

$$M_i := (0, 0, \dots, 0, a_i, \beta_{i+1}, \dots, \beta_n, \beta_{n+1}).$$

It is evident that the number M_i is one of the numbers of the sequence

$$0, \gamma, 2\gamma, \dots, (p_i - 1)\gamma,$$

where

$$\gamma = \prod_{j=1}^{i-1} p_j.$$

Let us find the difference between numbers B and M_i :

$$C := B - M_i = (0, 0, \dots, 0, c_{i-1}, \dots, c_{n+1}).$$

As we see, in the obtained difference the i first digits are already zero. We carried out one zeroing step, as a result of which one more digit of the number became zero. Let us now return to the initial number

$$A = (a_1, a_2, \dots, a_n, a_{n+1})$$

and let us carry out n zeroing steps, beginning with digit a_1 and ending with digit a_n .

Then, as a result, we will obtain a number of the form

$$D = (0, 0, \dots, 0, d_{n+1}),$$

having the first n digits zero.

Let us now examine, what the magnitude of the greatest possible sum $\sum_{i=1}^n M_i$, being subtracted from number A is:

$$\left(\sum_{i=1}^n M_i \right)_{\max} = (p_1 - 1) + (p_2 - 1) p_1 \dots \dots$$

$$\dots \dots (p_i - 1) \prod_{j=1}^{i-1} p_j + \dots + (p_n - 1) \frac{P}{p_n} = P - 1.$$

That is, even in the extreme case the grand total of the numbers being subtracted is less than the magnitude of range P . In other words, the zeroing process reduces to the following. If the entire range $(0, P)$ is broken down into p_{n+1} intervals, and the incorrect number is situated in the interval

$$(jP, (j+1)P),$$

then as a result of each zeroing step, not going out of the interval with the number $j + 1$, we transform the initial number into a number situated closer to the left margin of the interval in question. As a result of the entire zeroing process we obtain a number lying at the left end of the interval.

Thus, the zeroing reduces to the determination of the interval, in which the number is situated.

Actually, let us examine number D

$$D = (0, 0, \dots, 0, d_{n+1}) = (d_{n+1}m_{n+1}) \pmod{p_{n+1}} \mathcal{P},$$

where m_{n+1} - the constant of a base system, and, taking into account that the number D lies in the same interval $(j\mathcal{P}, (j+1)\mathcal{P})$, as the initial number A, we obtain:

$$j = (d_{n+1}m_{n+1}) \pmod{p_{n+1}}.$$

Selecting a base system in such a way that condition $m_{n+1} = 1$ is fulfilled, we will somewhat simplify the obtained expression:

$$j = d_{n+1}.$$

That is, digit d_{n+1} obtained as the result of number zeroing, uniquely defines the number of the interval, in which the initial number was situated.

Taking into account the stipulation concerning the correctness of the operands and the results of the arithmetic operations, we confirm the following.

If the initial number is correct, then the result of zeroing can only be a zero number, i.e.,

$$d_{n+1} = 0.$$

If the initial number was incorrect, then the result of zeroing is different from zero and

$$d_{n+1} \neq 0.$$

Thus, the zeroing method is a method of checking a number for the presence of a single error in any digit of the number. The word any is emphasized here, since we have in mind a digit of a check base (taking into account the equal correctness of the introduction of this digit with respect to the other digits of the number).

Let us now move on to an investigation of the possibility of correcting a number, if some error Δa has occurred with respect to the base p_1 . In this case the number A will be misrepresented and will have the form

$$\bar{A} = (a_1, a_2, \dots, a_{i-1}, \bar{a}_i, a_{i+1}, \dots)$$

where

$$a_i = a_i + \Delta a_i, \quad \Delta a_i = 1, 2, \dots, p_i - 1,$$

instead of the form

$$A = (a_1, a_2, \dots, a_{i-1}, a_i, a_{i+1}, \dots, a_{n+1}).$$

Let us carry out the zeroing process for all digits of numbers \bar{A} and A , with the exception of the digit with the base p_1 :

$$\bar{D}_1 = (0, 0, \dots, 0, \bar{d}_i, 0, \dots, 0)$$

$$D_1 = (0, 0, \dots, 0, d_i, 0, \dots, 0)$$

Since zeroing is carried out by subtractions, the magnitude of the error cannot change and the following relationship is corrected

$$\bar{d}_i = d_i + \Delta d_i.$$

The conclusion of the zeroing process should be carried out with constants of the form

$$\frac{c_i^x}{p_i}; \frac{2c_i^x}{p_i}; \dots; (p_i - 1) \frac{c_i^x}{p_i}.$$

where

$$\frac{c_i^x}{p_i} = (0, 0, \dots, 0, c_i, 0, \dots, 0).$$

Let us select the pair λ_1 and λ_1' - of such whole non-negative numbers, so that the following condition is fulfilled

$$\lambda_1' = \lambda_1 c_i \pmod{p_i},$$

$$\lambda_1 = \lambda_1' c_i \pmod{p_i}.$$

Multiplying the numbers λ_1' , λ_1 by $\frac{c_i^x}{p_i}$ and subtracting respectively from \bar{D}_1 and D_1 , we will obtain:

$$\bar{d}_{n+1} = \lambda_1' c_{n+1} = d_{n+1} \pmod{p_{n+1}},$$

$$\lambda_1 c_{n+1} = 0 \pmod{p_{n+1}}.$$

whence

$$\lambda'_i - \lambda_i = \frac{p_{i+1} - d_{n+1}}{c_{n+1}} \pmod{p_{n+1}}.$$

Here division is examined as digit-by-digit division with respect to modulus p_{n+1} . From the rule for the introduction of values λ_1 and λ'_1 we will obtain

$$(\lambda'_i - \lambda_i) c_i = (\beta_i - \alpha_i) \pmod{p_i} \cdot \Delta x_i \pmod{p_i}.$$

Now the value $\Delta \alpha_1$ can be uniquely defined with the following formula

$$\Delta x_i = \left(\frac{p_{n+1} - d_{n+1}}{c_{n+1}} \right) \pmod{p_{n+1}} c_i \pmod{p_i}.$$

Thus, in our case, if the location of the error is known, then the value of the result of the zeroing of the initial number d_{n+1} uniquely defines, by which magnitude $\Delta \alpha_1$ the faulty digit should be corrected.

Somewhat more complex is the situation, when the location of the error is unknown to us, i.e., we do not know, with respect to which base it arose. Then, it is necessary to solve the reverse problem, namely, we must examine all the possible errors for all possible bases and resolve the question of whether the given error for a given base can lead to the fact that the incorrect number obtained as a result of the appearance of error will fall in the interval with the number $j + 1$. The incorrect number \bar{A} and the correct number A are connected by the equality

$$\bar{A} - A = (0, 0, \dots, 0, \Delta x_i, \dots, 0),$$

i.e., the number of the interval, where the incorrect number falls, will be defined as

$$j \dots i = \left[\frac{\Delta x_i m_i \frac{p}{p_i}}{\frac{p}{p_{n+1}}} \right] \cdot \left[\frac{\Delta x_i m_i \frac{p}{p_i}}{c_i} \right] \pmod{p_{n+1}}.$$

Here, the square brackets designate entire side of the expression included in them. The term δ which can take the value 0 or 1, indicates a certain ambiguity in the number of the interval due

to the dependence of the location of the incorrect number on the magnitude of the initial number.

Thus, the number of the interval and the magnitude of the assumed error are connected by the relationship:

$$j = \left[\frac{\Delta a_i m_i p_{n+1}}{p_i} \right] (\text{mod } p_{n+1}) \div \delta.$$

Now, we have the possibility to consider all possible values of the errors for each of the bases and to isolate those of them, whose presence could transform the initial correct number into an incorrect number, located in the range

$$(j\delta, (j+1)\delta).$$

and to compile a table corresponding to the interval number, or, more simply, to the magnitude of the result of the zeroing of the values of the magnitudes of the possible errors for different bases. Then, upon detecting value d_{n+1} from this table we can select the appropriate alternative set of possible errors.

It has been proven in general form that with the presence of two check bases p_{n+1} and p_{n+2} a similar alternative set is contracted to one possible error value for only one possible base, i.e., a one-to-one correspondence of values d_{n+1} and Δa is ensured. But the presence of two check bases naturally requires a larger quantity of equipment, than of one base, and it is considerably more interesting to attempt to solve the problem with only one check base. It turns out that on the basis of the accepted stipulation by enlisting the dynamics of the process, we are in a position to substantially narrow the magnitude of the alternative set and within limit to uniquely solve the posed problem. And, namely, if as a result of any operation the following set of bases is obtained

$$\mathcal{B}_1 = (p_{i1}, p_{i2}, \dots, p_{ik}),$$

by which an error could arise which shifts the result to a given interval with number j , then, assuming that in the computational

process the error cannot move to another base (the digits of the number are not connected with each other), we, obtaining another set of the bases:

$$\mathcal{A}_2 = (p_{j1}, p_{j2}, \dots, p_{jm}),$$

confirm that the error can only exist at the intersection of regions \mathcal{A}_1 and \mathcal{A}_2 .

The modeling of certain problems shows that total contraction is ensured as a result of the carrying out, on the average, of three operations. Acceleration of the contraction is ensured by also considering, besides the location of the error, its magnitude.

Let us illustrate what has been said by an example. Let us take the following system of bases

$$p_1 = 2; p_2 = 3; p_3 = 5; p_4 = 7; p_5 = 11.$$

Here, base $p_5 = 11$ is a check base. The operating range of such a system is

$$\mathcal{P} = 2 \cdot 3 \cdot 5 \cdot 7 = 210,$$

and the total range is

$$p = 11\mathcal{P} = 2310.$$

The minimum zeroing numbers will be defined in the following manner:

$$\begin{aligned} M_1^{(1)} &= (1, 1, 1, 1, 1); \\ M_2^{(1)} &= (0, 1, 4, 4, 4); \\ M_2^{(2)} &= (0, 2, 2, 2, 2); \\ M_3^{(1)} &= (0, 0, 1, 6, 6); \\ M_3^{(2)} &= (0, 0, 2, 5, 1); \\ M_3^{(3)} &= (0, 0, 3, 4, 7); \\ M_3^{(4)} &= (0, 0, 4, 3, 2); \\ M_4^{(1)} &= (0, 0, 1, 1, 0); \\ M_4^{(2)} &= (0, 0, 0, 2, 8); \\ M_4^{(3)} &= (0, 0, 0, 3, 7); \\ M_4^{(4)} &= (0, 0, 0, 4, 5); \\ M_4^{(5)} &= (0, 0, 0, 5, 4); \\ M_4^{(6)} &= (0, 0, 0, 6, 2). \end{aligned}$$

Let us examine the distribution of the incorrect numbers with respect to the intervals of the numerical range depending on the magnitude of the error.

Base $p_1 = 2$. The number with an error $\Delta\alpha_1 = 1$ falls into the interval with the number

$$d_s = \left[\frac{11}{2} \right] (\text{mod } 11) + \delta = 5 + \delta,$$

i.e., the error shifts the number into the fifth or sixth interval.

Base $p_2 = 3$. The error $\Delta\alpha_2 = 1$ shifts the number into the interval

$$d_s = \left[\frac{2 \cdot 11}{3} \right] (\text{mod } 11) + \delta = 7 + \delta,$$

i.e., into the 7th or 8th intervals. The error $\Delta\alpha_2 = 2$ shifts the number into

$$d_s = \left[\frac{4 \cdot 11}{3} \right] (\text{mod } 11) + \delta = 3 + \delta$$

- the third or fourth intervals.

Base $p_3 = 5$. The number with an error $\Delta\alpha_3 = 1$ falls into the interval

$$d_s = \left[\frac{3 \cdot 11}{5} \right] (\text{mod } 11) + \delta = 6 + \delta.$$

With an error $\Delta\alpha_3 = 2$ we will obtain

$$d_s = \left[\frac{2 \cdot 3 \cdot 11}{5} \right] (\text{mod } 11) + \delta = 2 + \delta.$$

With $\Delta\alpha_3 = 3$ we will obtain

$$d_s = \left[\frac{3 \cdot 3 \cdot 11}{5} \right] (\text{mod } 11) + \delta = 8 + \delta.$$

With $\Delta\alpha_3 = 4$ we will have

$$d_s = \left[\frac{4 \cdot 3 \cdot 11}{5} \right] (\text{mod } 11) + \delta = 4 + \delta.$$

Base $p_4 = 7$. The number with an error $\Delta\alpha_4 = 1$ falls into the interval

$$d_s = \left[\frac{11}{7} \right] (\text{mod } 11) + \delta = 1 + \delta.$$

Error $\Delta\alpha_4 = 2$ shifts the number into the interval

$$d_5 = \left[\frac{2 \cdot 11}{7} \right] (\text{mod } 11) + 8 = 3 + 8.$$

$\Delta\alpha_4 = 3$ shifts the number into the interval

$$d_5 = \left[\frac{3 \cdot 11}{7} \right] (\text{mod } 11) + 8 = 4 + 8.$$

With $\Delta\alpha_4 = 4$ we will obtain

$$d_5 = \left[\frac{4 \cdot 11}{7} \right] (\text{mod } 11) + 8 = 6 + 8.$$

With $\Delta\alpha_4 = 5$ we will have

$$d_5 = \left[\frac{5 \cdot 11}{7} \right] (\text{mod } 11) + 8 = 7 + 8.$$

Error $\Delta\alpha_4 = 6$ will shift the number into the interval

$$d_5 = \left[\frac{6 \cdot 11}{7} \right] (\text{mod } 11) + 8 = 9 + 8.$$

The obtained results make it possible to compile a table, in which the values of d_5 are compared with the possible errors: $\Delta\alpha_1$. It is doubtful that an error with respect to a check base can also correspond to value d_5 . Designating it by Δ_k , we will have value $\Delta_k = d_5$ as a possible error.

Value	Possible errors
0	The number is correct, there are no errors:
1	$\Delta x_1 = 1; \Delta_2 = 1.$
2	$\Delta x_2 = 2; \Delta x_4 = 1; \Delta_k = 2.$
3	$\Delta x_2 = 2; \Delta x_3 = 2; \Delta x_4 = 2; \Delta_k = 3.$
4	$\Delta x_2 = 2; \Delta x_3 = 4; \Delta x_4 = 2; \Delta x_1 = 3; \Delta_k = 4.$
5	$\Delta x_1 = 1; \Delta x_3 = 4; \Delta x_4 = 3; \Delta_k = 5.$
6	$\Delta x_1 = 1; \Delta x_3 = 1; \Delta x_4 = 4; \Delta_k = 6.$
7	$\Delta x_2 = 1; \Delta x_3 = 1; \Delta x_4 = 4; \Delta x_1 = 5; \Delta_k = 7.$
8	$\Delta x_2 = 1; \Delta x_3 = 3; \Delta x_4 = 5; \Delta_k = 8.$
9	$\Delta x_3 = 3; \Delta x_4 = 6; \Delta_k = 9.$
10	$\Delta x_4 = 6; \Delta_k = 10.$

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Let the incorrect number A' be given:

$$A' = (1, 1, 4, 1, 1)$$

to which in the process of resolving the problem correct number B is added:

$$B = (0, 2, 0, 4)$$

Let us attempt to determine the location and the magnitude of the error in number A' . Let us carry out zeroing of number A' .

$$\begin{array}{r} (1, 1, 4, 1, 1) \\ - (1, 1, 1, 1, 1) \\ \hline (0, 0, 3, 0, 0) \\ \\ (0, 0, 3, 0, 0) \\ - (0, 0, 3, 4, 7) \\ \hline (0, 0, 0, 3, 4) \\ \\ (0, 0, 0, 3, 4) \\ - (0, 0, 0, 3, 7) \\ \hline (0, 0, 0, 0, 8) \end{array}$$

Since $d_5 = 8$, then from the given table let us determine the following alternative set:

- error $\Delta\alpha_2 = 1$ in a digit with respect to base p_2 ;
- error $\Delta\alpha_3 = 3$ in a digit with respect to base p_3 ;
- error $\Delta\alpha_4 = 5$ in a digit with respect to base p_4 ;
- error $\Delta_k = 8$ in a digit with respect to the check base.

Let us further carry out the operation of adding number A' to number B :

$$A' + B = (1, 1, 4, 1, 1) + (0, 2, 0, 4, 2) = (1, 1, 5, 5, 3)$$

Since, according to our assumption, the arithmetic unit operates with correct numbers and the result of the operation is correct and since it is assumed that during the time of the execution of the operating a second error cannot appear, obviously, in the obtained sum the error has the same value, as in number A .

An error cannot move from a digit with respect to one base to a digit with respect to another base.

Let us zero obtained sum:

$$\begin{array}{r}
 -(1, 0, 3, 5, 3) \\
 (1, 1, 1, 1, 1) \\
 \hline
 (0, 2, 3, 4, 2) \\
 (0, 2, 3, 4, 2) \\
 -(0, 2, 2, 2, 2) \\
 \hline
 (0, 0, 1, 2, 0) \\
 (0, 0, 1, 2, 0) \\
 -(0, 0, 1, 6, 6) \\
 \hline
 (0, 0, 0, 3, 5) \\
 -(0, 0, 0, 3, 7) \\
 \hline
 (0, 0, 0, 0, 0)
 \end{array}$$

For the sum $d_5 = 9$ is obtained.

Now the following alternative set of errors is possible:

- error $\Delta\alpha_3 = 3$ for base p_3
- error $\Delta\alpha_4 = 6$ for base p_4
- error $\Delta_k = 9$ for the check base.

Since neither the magnitude, nor the location of the error changes, it is possible to assert that error $\Delta\alpha_3 = 3$ exists for base p_3 .

Hence the desired corrected number A can be represented in the following manner

$$A = A' - (0, 0, 3, 0, 0) = (1, 1, 1, 1, 1).$$

The examined example shows that if during the operation of adding the correct number to the incorrect number the magnitude of d_5 changes, then the location of the error and its value in a number of cases are uniquely defined.

The examined example demonstrates the fact that an increase in the informativeness of the alternative set made it possible in one step not only to determine the location of the error, but also to indicate its magnitude, i.e., to completely solve the check problem - finding the location of incorrectness and removing the error.