AD-754 356

A PROPOSED MODEL FOR A PARTICULAR NAVAL PATROL SYSTEM

Guerrero Eustaquio Guzman

Naval Postgraduate School Monterey, California

September 1972



National Technical Information Service U. S. DEPARTMENT OF COMMERCE 5285 Port Royal Road, Springfield Va. 22151

NAVAL POSTGRADUATE SCHOOL Monterey, California





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by

Guerrero Eustaquio Guzman

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A Proposed Model for A

Particular Naval Patrol System

by

Guerrero Eustaquio Guzman Lieutenant (junior grade), Philippine Navy B.S., Philippine Military Academy, 1965

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL September 1972

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I. INTRODUCTION

The primary purpose of this paper is to develop a preliminary model of a naval patrol system to be used in a particular geographic region. The naval patrol's mission is to prevent the penetration of waters by smuggling craft. Topefully the model will in part provide the following:

1) A workable managerial tool to assist in the analysis of patrol operations.

2) An analytical tool to use in developing patrol operations strategy.

3) A basis for further mathematical modelling of the system.

There is a need for managerial tools with which to assess whether or not the means being employed to combat smuggling activity are economically sound.

There is a rich collection of intelligence information and operational data which could be used with a patrol model for the development of patrol operation strategy. At present, there are no means with which to utilize these data to develop strategy, outside of some ad hoc subjective ones.

As a basis for further mathematical modelling, it is hoped that this model will provide a first step in the difficult task of modelling such an operation.

The purpose of this paper is not to give an immediate solution to the problem which motivated it.

Time is not considered to be a relevant variable in this model of the system.

II. DESCRIPTION OF THE SCENARIO

For years, Country A has been plagued by unscrupulous investors who find that smuggling contraband goods into the country is a lucrative business, so much so that local products have been undermined and legitimate businesses discouraged by unequal competition and the government of Country A has been deprived of revenues amounting to millions of dollars per annum.

It is known that the sources for these contraband goods are the specific ports in Country B which are marked S1, S2, ..., S5 in figure 1. These goods are readily available to the smugglers at these points and they are free to come and go in the territory of Country B. It is a welcomed business for Country B and hence tolerated by its government.

It is not specifically known where these goods are landed in Country A, however these goods find markets in the general areas marked M1, M2, M3, and M4.

The means being used by the smugglers to transport these goods from the source to the destination are small motorized boats. The only navigational equipment these boats have are magnetic compasses. Thus, once land marks are outside visual range these crafts navigate by dead reckoning techniques. The departure points in the territory of Country B and the specific routes taken in transiting the region marked W in the figure are not known. However, from its departure point in the territory

of Country B, a smuggling craft can be assumed to follow a straight line course. In proceeding toward the intended market area, the smuggling craft takes one of the approaches marked by arrows A1, A2, ..., A5 in the figure.

The Coast Guard Force of Country A has the task of apprehending this smuggling craft in order to stop this illicit activity. To carry out this task, the Coast Guard Force employs patrol craft to detect and intercept the inuggling craft. The detection systems of the patrol craft are radar and visual means.

There are several ways of deploying the patrol craft. An effective deployment would be to station them just off the harbors where the sources are located. Unfortunately this is not possible since there is at present no agreement between the two countries for mutual cooperation. Hence, the patrol operation must be confined within the territorial waters of Country A.

An alternative to this is to deploy them off landing points, however, this is not practical because of the number of different landing points used by the smugglers, even if intelligence information were to pin-point the exact locations of the various landing points.

Another alternative is to deploy them along the approaches to the market areas, however, this is not advisable for the following reasons:

1) At this point in their transit the tactical movement of the smuggling craft is unpredictable.

2) Identification becomes quite difficult, since smuggling craft can easily intermix and blend with local fishing boats which compound the identification problem.

Therefore, the region for deployment considered in this model is the area marked W which is between the territorial boundaries of the two countries. Here the possible courses for the smuggling craft are more predictable and the tracks are basically straight lines. Also, the number of local boats that ply this area is quite small and the problem of identification is practically nil.

There is no problem of intercepting a smuggling craft once it is detected. Usually a mere signal from a patrol craft is enough to cause a smuggling craft to surrender. In extreme cases, warning shots are required to stop an attempt to escape. There are no reported tactical evasive maneuvers, such as zigzagging, taken by the smuggling craft.

III. THE MATHEMATICAL MODEL

The problem is to find the optimum patrol areas for a given number of patrol craft in a defined region where the objective is to maximize the number of apprehensions of smuggling craft. This can be expressed mathematically by defining the following events which are conditioned on a patrol craft having undertaken a smuggling operation:

 $\boldsymbol{\xi}_i \equiv \begin{cases} \text{Apprehension of the smuggling craft} \\ \text{by the ith patrol craft} \end{cases}$

 $\mathbf{R}_{i} \equiv \begin{cases} The smuggling craft is present in the ith patrol area \end{cases}$

 $D_{i} \equiv \begin{cases} The smuggling craft is detected in \\ the ith patrol area by the ith patrol craft \end{cases}$

In this problem it is assumed that

$$\mathcal{E}_{i} = \left\{ \mathcal{R}_{i} \cap \mathcal{D}_{i} \right\}$$

and that once a smuggling craft is apprehended, it cannot escape, that is,

$$\{\xi_i \cap \xi_j\} = \emptyset$$
 for all $i \neq j$.

The problem's objective is to maximize the probability of the event

$$\xi = \bigcup_{i=1}^{N} \xi_{i}$$

by the choice of patrol areas where N is the total number of patrol areas, that is the number of patrol craft.

It was assumed that detection by a patrol craft could not take place out of its patrol area.

In probabilistic terms, the objective function is

MAX
$$P[\xi] = \sum_{i=1}^{N} P[D_i|R_i] P[R_i]$$

by the choice of patrol areas for a given N. In this expression, P $\begin{bmatrix} D_i & R_i \end{bmatrix}$ is the conditional probability of detecting the target in the ith area given the target is present in the ith area.

It was also assumed that given a target is present in a patrol area, the probability of detecting the target is unity, that is in effect a definite range law of detection. This is not unreasonable if limited patrol areas are considered. This can be expressed mathematically as $P\left[D_i \mid R_i\right] = 1$ and, hence, $P\left[\boldsymbol{\xi}\right] = \sum_{i=1}^{N} P\left[R_i\right]$.

The decision "ariable then is the choice of patrol areas in the region W for the N patrol craft and the problem is reduced to the problem of choosing the patrol areas with the highest probability of having the smuggling craft present during its transit from the set of all possible patrol areas in the region.

The problem of how to assign the probability of a target being present in each of the possible patrol areas in the region W will now be considered. However, before doing this it is convenient to define the following terms:

OBSERVER	 a naval craft conducting a patrol operation
	designed to prevent transit through the
	playing area by specified targets.
TARGET	 a smuggling craft transporting contraband
•	goods passing through the playing area.
PLAYING AREA	- the region of concern through which the
	targets transit and where patrol deployment
	is intended. This is the region W.
SOURCE	- a specified port or place where the contra-
	band goods are procured by the smugglers.
DESTINATION	- any of the general areas where contraband
	goods find a market.
DEPARTURE	- the position from which a target enters into
POINT	the playing area.
UNIT PATROL	- the size of the area an observer is capable
AREA	of covering in order to assure a specified
	level of detection.

APPROACH POINT - the position from which a target leaves the playing area.

The system is schematically depicted in figure 2. The following are the elements of the model:

- 1) Source
- 2) Departure Points

- 3) Playing Area
- 4) Approach Points
- 5) Destination
- 6) Geographic Configuration

Geographic configuration is the element of the model that limits the variables associated with the other elements. For instance, the approach points to a market area are dictated by the obstructing islands, their relative positions, ..., etc.; thus geographic configuration limits the range of values that variables take.

Time variables such as wind velocity, visibility, availability of a patrol craft, position of the target at any point in time, ..., etc., have been ignored in the development of this preliminary model.

The problem of assigning the probability $P[R_i]$ of a target being present in the ith patrol area is a modification of a problem which could be called the hitch-hiker problem. Figure 3 illustrates a hypothetical case. In figure 3a, there are two roads from town A to town B. A hitchhiker desires to go from town A to town B and he is indifferent as to which road he takes. If he has a choice as to where to station himself anywhere below the dotted line, then obviously he would choose to station himself at the intersection. Not because it is closer to town B, but because he has a better chance of catching a ride at this point since he can signal cars moving along both roads.

Now suppose another road has been added as in figure 3b and each road has a different degree of usage, for example, suppose 70% of the

time a car going from town A to town B takes road "a," 20% of the time the car takes road "b," and 10% of the time the car takes road "c." Among the three roads he naturally prefers road "a" since it offers the greatest expected number of cars from which to obtain transportation in a given time. If he moves down the road to one of the two intersections "d" or "e," he will increase his probability of obtaining a ride assuming the probability of obtaining a ride from a car is the same for all cars. Between these two points, "e" should be preferred, since starting at some time the ratio of the number of cars passing point "e" to the number passing point "d" should approach 9 to 8 as time increases. Now, the above percentages can be taken as probabilities, if no information about departure rates along the three roads is available. That is, given a car leaves town A for town B, the probability that the car goes on road "a" is .7 and the probability the car passes intersection "e" is the probability that the car goes on road "a" or road "b" which is .9 since the events are mutually exclusive. This is the basis for initially assigning probabilties in the mathematical model to target tracks and their intersections. These probabilities and the condition that an apprehended target cannot escape provide the basis for determining the probabilities $P[R_i]$.

Assigning probabilities to the event that given a target is to go from a source toward a destination, the target will go on a particular track in the playing area should be aided by the kind of prior information which is provided by intelligence evaluations. A possible way of determining the probabilities is by means of statistical inference using information

gathered in the region such as the location of past apprehensions, the areas where patrols have been conducted, and intelligence informations on unapprehended cases. However, to do this, the information available should provide a sample large enough to make valid statistical inferences.

The following events relate to the problem of assigning probabilities to target tracks:

$$M_{i} \equiv \begin{cases} \text{Destination i is chosen} \\ S_{i} \equiv \\ \text{Source i is chosen} \\ \end{bmatrix}$$
$$B_{i} \equiv \begin{cases} \text{Departure point i is chosen} \\ \end{bmatrix}$$
$$A_{i} \equiv \\ \text{Approach point i is chosen} \\ \end{bmatrix}$$
$$T_{ij} \equiv \begin{cases} \text{The track between departure point i} \\ \text{and approach point j is chosen} \end{cases}$$

and

In assigning a probability to the event a particular source is chosen, it could be assumed, for example, that this choice is determined only by the availability of goods and then relate this probability to the probability of the event a particular source has the goods which could be assigned by use of prior information.

It might be assumed that the factor which motivates a smuggler while planning a smuggling trip is the promise or prospect of profit and that the destination chosen will be the one at which the smuggled goods could be sold at a maximum profit. It is assumed that this choice will influence the choice of the approach point. Thus, for instance, if the intended destination were M3 in figure 1, M6 would appear to be a likely choice for the approach point. However, the approach point chosen might be A1, although this event would appear to be an unlikely one.

The departure point could be assumed to depend on the destination as well as the source. When a departure point and an approach point have been chosen, the straight line joining the two is the track of the target. For each departure point and approach point pair, a unique track is determined, that is $T_{ij} = \{B_i \cap A_j\}$.

Once an approach point and a departure point have been chosen and hence a track determined, the target is, in effect, advanced along the track from the departure point until the destination is reached or a patrol area is encountered.

In this model, when a target reaches a patrol area it is apprehended and thus eliminated and, hence, there is no target traffic past a patrol area on tracks which enter the patrol area.

IV. CONCLUSION

In order to obtain a solution to the problem of determining the areas with the highest probabilities of having a target present, it is necessary to determine the prior probabilities of the various events in the model in order to specify the probability that a particular track will be chosen. An initial location for the patrol areas can be made with this information and then various modifications tried. Since only intersection points or tracks of high probability need be considered, the number of possible patrol areas is limited and for a limited number of patrol craft and target tracks a solution could possibly be obtained by visual inspection.

As an illustration of the use of the model, suppose there are two patrol craft, one source, three departure points, two approach points, and one destination. This is illustrated schedmatically in figure 4 where track intersections are labelled I_1 , I_2 , and I_3 . Also, suppose the following probability distributions were specified:

Р	[B _i]	.2	.5	.3
	i	1	2	3
Р	[A _i]	.3	.7	
	i	1	2	

Now, in addition, suppose B_i and A_j are independent, then $P[T_{ij}] = P[B_i \cap A_j] = P[B_i] P[A_j]$ and, hence,

P [T _{ij}]	.06	.14	.15	.35	.09	.21	
(i,j)	(1, 1)	(1,2)	(2,1)	(2,2)	(3, 1)	(3,2)	

These probabilities are shown adjacent to the corresponding tracks in figure 4. The probability $P[I_j]$ that a target which leaves S for M will cross an intersection I_j when the patrol system is in operation is the sum of the probabilities $P[T_{ij}]$ of those tracks which comprise it which do not cross a patrol area prior to the intersection. In this case it can be seen by inspection that, since there are only two patrol craft, if the patrol areas chosen are the intersection I_1 and I_3 , then $P[R_1] = P[I_1] = .29$, $P[R_2] = P[I_3] = .44$, and $P[\xi] = .73$, and $P[\xi]$ is a maximum.







