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ESTIMATING THE DEPTH DEPENDENCE OF
OCEAN HORIZONTAL DIFFUSION COEFFICIENTS

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Technical Report

by

I. E. Alber

1972 December
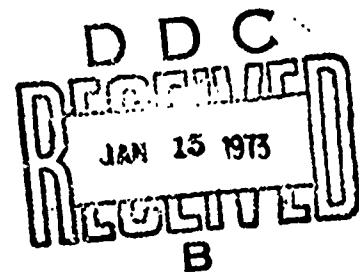
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13 ABSTRACT

A simple empirical estimate is presented for predicting the variation with depth of a scalar concentration field in the far wake of a submarine operating beneath the ocean surface. By examining near surface and intermediate depth diffusion and turbulent dissipation data, it is shown that the horizontal diffusion coefficient decreases by a factor of from 2.2 to 2.5 in the depth regime between 10 and 300 meters. Semi-empirical equations for the strength of the concentration field from instantaneous and continuous sources are developed.

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KEY WORDS	ROLE	WT	ROLE	WT	ROLE	WT
Empirical correlations						
Stratification						
Scalar concentration						
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Diffusion						
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II

ESTIMATING THE DEPTH DEPENDENCE OF
OCEAN HORIZONTAL DIFFUSION COEFFICIENTS

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IV

SUMMARY

The objective of this study is to develop a simple empirical estimate for predicting the variation with depth of a scalar concentration field in the wake emitted by a submarine operating beneath the ocean surface. This study treats only that portion of the wake field for which the initial effects of sub generated turbulence have diminished, and for which the effects of ocean turbulence and scalar field size are the primary factors affecting the process of scalar diffusion.

After the wake is 1 to 2 hours old, the subsequent wake diffusion is dominated by horizontal transport processes, vertical diffusion being inhibited by the effects of ocean stratification. The horizontal diffusion coefficient, K, has been shown by Richardson and Batchelor to be proportional to the 4/3 power of the diffusion scale, ℓ, $K = a\ell^{4/3}$. The constant of proportionality 'a' is evaluated from near surface ocean diffusion data in Section 2 of this study. This parameter is also evaluated at a depth of 300 meters from the one available piece of ocean diffusion experimental data in Section 3. It is shown that there is an apparent decrease in 'a' (or the diffusion coefficient K) of approximately 2.5 when one compares the near surface and intermediate depth (300m) data.

It is shown in Section 4 that this depth dependence of 'a' can be estimated by application of the inertial subrange relation for the diffusivity, $K = A\varepsilon^{1/3}\ell^{4/3}$, where ε is the rate of turbulence dissipation, and A is a universal constant. From the available ocean diffusion data and from the variation of ε with depth, A is shown in Section 4 to be nearly invariant with depth. The depth dependence of ε indicates a factor of 2.2 decrease in the diffusion coefficient, K, between the surface and 300 meters.

A correlation of the data of Webster for the rate of dissipation, ε, with depth shows that ε decreases as the depth to the -.7 power. This variation is nearly identical to that of the Brunt-Väisälä frequency, N, below the thermocline. Hence a means is provided for estimating the decrease of ε with depth in different regions of the ocean.

Finally in Section 5, it is shown that peak scalar concentrations from a sublike source will be larger at 300m depth by approximately a factor of 3 than from a source moving near the surface.

TABLE OF CONTENTS

1. INTRODUCTION

The rate of diffusion of a passive scalar, emitted by a submarine into the ocean, is determined by ocean turbulence and the size of the scalar field once the initial effects of sub-generated turbulence have diminished. This initial sub-dominated period corresponds to a diffusion time of less than 1 to 2 hours. As noted by Ko and Alber[1], the subsequent wake diffusion is dominated by horizontal transport processes, vertical diffusion being inhibited by the effects of ocean stratification. The horizontal diffusion coefficient, K, has been shown by Richardson and Batchelor (on a theoretical basis) to be proportional to the 4/3 power of the scale of diffusion ℓ

$$K = a\ell^{4/3} \tag{1a}$$

'a' is a constant with units of $[cm^{2/3}]/sec$, with ℓ measured in cm. If the scale of the diffusing substance falls within a portion of the ocean turbulence spectra characterized by an inertial subrange (with dissipation rate ϵ), then the diffusivity coefficient, K, and its constant, 'a', can be related to the dissipation rate, ϵ, by the relation

$$K = A\epsilon^{1/3}\ell^{4/3} \qquad A\epsilon^{1/3} = a \tag{1b}$$

The parameter, A, appearing in equation 1b is assumed to be a universal constant, independent of ϵ.

If it can be shown that A is indeed nearly a universal constant, and if an independent means is available for determining the variation of ϵ with depth in the ocean, then it is possible to obtain an estimate of the variation of the diffusion coefficient, K, with depth. By estimating the depth dependence of K, one can then determine the change in the concentration field, S(t), of a passive scalar emitted by a source at various depths in the ocean. It is with the objective in mind of predicting the variation of a scalar concentration field with ocean depth, for which this study was initiated.

In Section 2 below, near surface ocean diffusion data (Reference 2) are examined to determine the best estimates of the overall diffusivity parameter, 'a'. This parameter is also evaluated at a depth of 300m from the one available piece of ocean diffusion experimental data (Reference 3) in Section 3. From these observations and data on the variation of the turbulence dissipation with depth, the universal constant, A, is determined in Section 4 and shown to be nearly independent of depth. The Conclusions of Section 5 recapitulates the important empirical and theoretical formulas necessary to calculate the variation of a scalar field concentration with depth.

2. NEAR SURFACE DIFFUSION COEFFICIENT 'a'

From the data correlation of Okubo[2] [Figure 1], the 'effective' variance σ_{rc}^2 of an instantaneously released dye patch in near surface waters, grows with time to the 2.34 power, i.e.,

$$\sigma_{rc}^2 = .01t^{2.34} \tag{2}$$

where $\sigma_{rc}^2 \equiv \int_0^\infty r_e^2 S(t,r_e)2\pi r_e dr_e / \int_0^\infty S2\pi r_e dr_e = 2\sigma_x\sigma_y$

r_e = radius of circle with area equal to that within an isoconcentration contour of value S

The depths of the dye patches analyzed by Okubo varied from 2 to 60m beneath the surface of the ocean.

From simple diffusion theory, the horizontal diffusivity coefficient, K, of a circular two-dimensional patch, with variance σ_{rc}^2, is given by

$$K = \frac{1}{4}\frac{d\sigma_{rc}^2}{dt} \tag{3}$$

Hence the diffusivity K based on Okubo's correlation is

$$K = .0063t^{1.34} \tag{4}$$

To calculate K in terms of a characteristic scale, ℓ, it is assumed that

$$\ell = 2\sqrt{3}\sigma_x = \sqrt{6}\sigma_{rc} \equiv b\sigma_{rc}$$

Substituting for time, t, in terms of ℓ

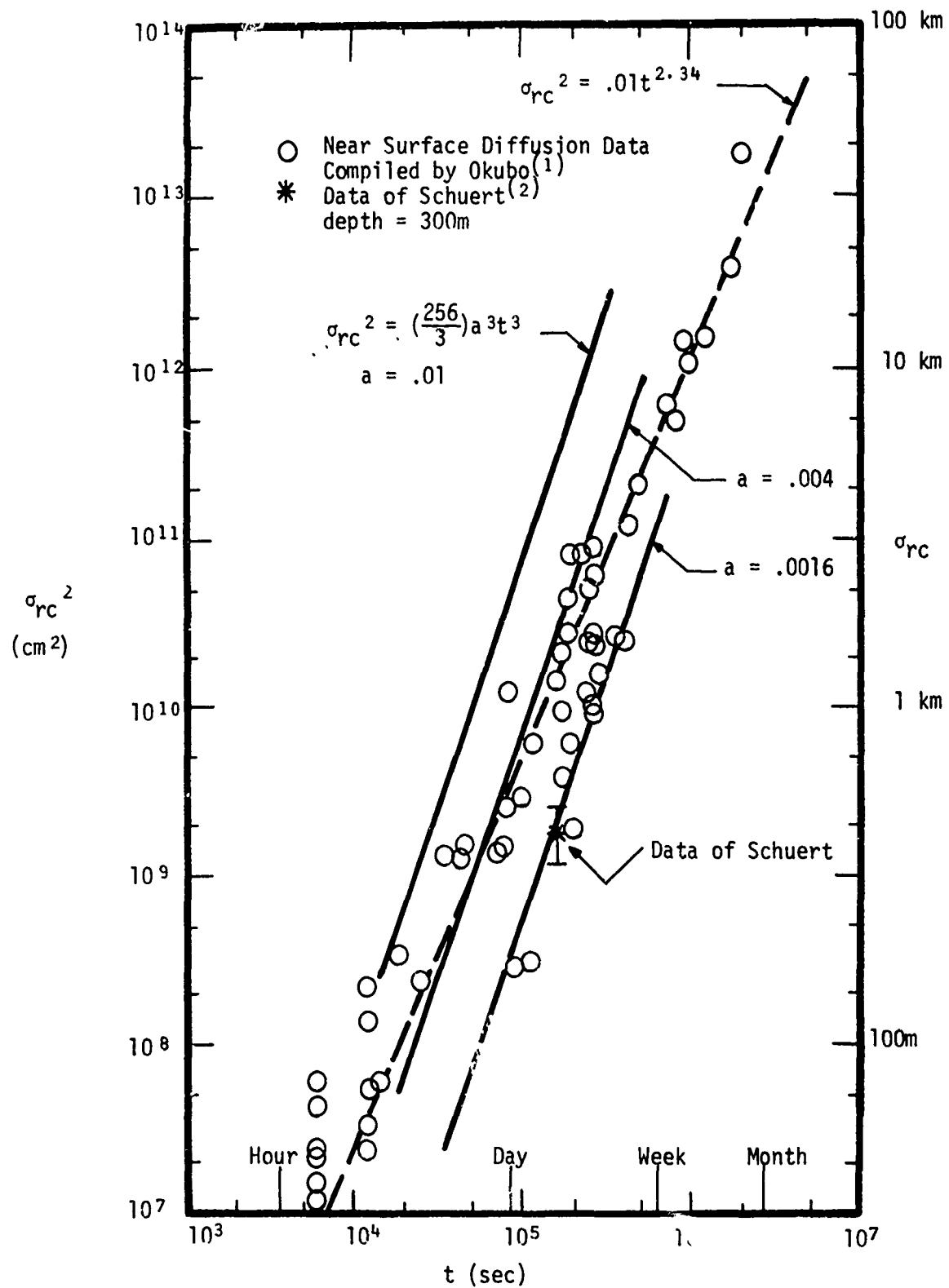$$\sigma_{rc} = (2.45)^{-1}\ell = .104t^{1.17}$$

or

$$t = (\ell/.255)^{1/1.17}$$

Figure 1. Variance, σ_{rc}^2 vs. Diffusion Time Data Compiled by Okubo[2]

4

Hence $K = .0063t^{1.34} = .0063(\ell/.255)^{1.34/1.17}$ or

$$K = .03\ell^{1.15} \qquad (5)$$

If it is assumed that the 4/3 law predicts an eddy diffusivity which is coincident with the Okubo correlation for K [equation 5] for a dye patch which is 1 Km wide, i.e., $\ell_{match} = 10^5$cm, then

$$K_{4/3} = a\ell^{4/3} = K_{Okubo} = .03\ell^{1.15}$$

$$(6)$$

$$a = \frac{.03}{\ell_{match}^{.18}} = \underline{.00375}$$

If $\ell_{match} = 10^4$cm, a = .0058.

Hence, the effective constant, 'a', for a 4/3 law diffusivity, in the depth range 60m < h < 2m, is

$$a_{\substack{near \\ surface}} \approx .004 \; \frac{cm^{2/3}}{sec} \qquad (7)$$

3. INTERMEDIATE DEPTH DIFFUSION COEFFICIENT 'a'

The only available data on turbulent transport in stable intermediate waters is the dye diffusion measurements of Schuert[3] made at a depth of 300 meters off Oahu, Hawaii.

The 4/3 law diffusivity constant $'a'_{300m}$ can be determined from a set of data if the measured variance σ_{rc}^2 is known as a function of time and can be fitted to the theoretical relation,

$$\sigma_{rc}^2 = \left(\frac{4}{3}\right)^3 a^3 b^4 t^3 \tag{8}$$

obtained by integrating

$$K = ab^{4/3} \sigma_{rc}^{4/3} = \frac{1}{4} \frac{d\sigma_{rc}^2}{dt} \tag{9}$$

The variance σ_{rc}^2 of the dye pool at a given time, t, can be determined from a semi-log plot of dye concentration, S, vs. the square of the effective radius, r_e, by fitting the data to the Gaussian relation

$$S(t,r_e) = S(t,0)\exp(-r_e^2/\sigma_{rc}^2) \tag{10}$$

where $S(t,0) = \dfrac{\widetilde{M}}{\pi\sigma_{rc}^2}$

$\widetilde{M}$ = total mass per unit depth of mixed layer

Such a plot is presented in Figure 2 from Schuert's data at a time 46.5 hours after the initial dye release.[†] From Figure 2 the variance σ_{rc}^2 is found to be

$$\sigma_{rc}^2 = .17 km^2 = 1.7 \times 10^9 cm^2$$
$$@ \ t = 1.68 \times 10^5 \ sec \tag{11}$$

[†]Unfortunately, Schuert was able to obtain a complete concentration map at this one time only. At other periods the data was insufficient to close the isoconcentration lines.
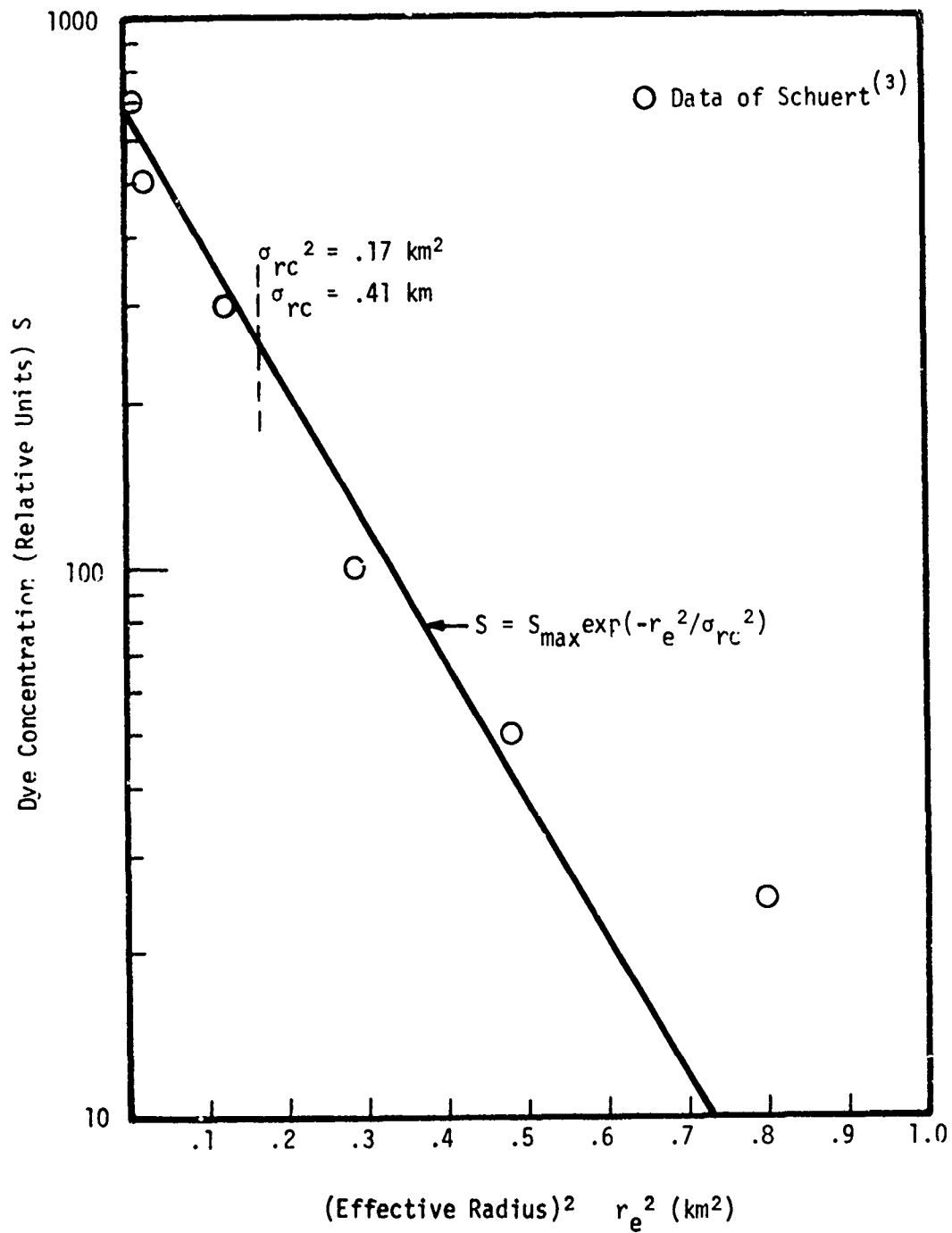
Figure 2. Fit of Schuert's Diffusion Data to
Gaussian Distribution

7

This data point is plotted in Figure 1, for comparison with the near surface data compiled by Okubo. One notes that Schuert's one data point falls below most of the near surface data. The value of the constant 'a' required to fit a 4/3 law theoretical curve of variance vs. time (equation 8) through Schuert's single data point, is

$$a \approx 1.6 \times 10^{-3} \frac{cm^{2/3}}{sec}$$ †

@ 300m
depth

(12)

---

†Note that the value of a is not to be confused with the value of α reported by Schuert of 6.6×10^{-3}. The constant α appears in the 1962 Okubo theory[4], where $K = 9/8\alpha^2 r^{2/3} t$ and $S = (const/t^3) exp[-r^{4/3}/\alpha^2 t^2]$.

4. EVALUATION OF THE UNIVERSAL CONSTANT A

In order to calculate the assumed universal constant 'A' appearing in the 4/3 law diffusivity formula, (equation 1) and to determine if depth variations can be adequately predicted by this relation, data on the variation of the rate of energy dissipation, ϵ, with depth are required.

Values of the rate of turbulent dissipation, ϵ, have been inferred from measurements of the inertial subrange spectrum, $E(k)$, for ocean currents using the relation

$$E(k) = (1.4)\epsilon^{2/3}k^{-5/3} \tag{13}$$

The constant 1.4 is taken from isotropic turbulence data.

Measurements of ocean velocity spectrum have been made by Grant, Stewart and Moillet (1962)[5], Stewart and Grant (1962)[6], Grant, Moillet and Vogel (1963)[7], and Webster (1969)[8]. The most recent and most extensive measurements of ocean spectra are those of Webster[8], who made deep sea current measurements in the Atlantic ocean from moored buoys using current meters. The depth of the water at the measurement site (site D) is 2600 meters.

A plot of the measured dissipation rate, ϵ, (in units of ergs/cm³/sec) is shown in Figure 3 as a function of depth. The corresponding Brunt-Väisälä frequency is shown in Figure 4.

A best fit of Webster's dissipation data is given by the straight line (in log-log coordinates)

$$\boxed{\epsilon = 4.4 \times 10^{-2} Z^{-0.7}} \ \text{cm}^2/\text{sec}^3 \tag{14}$$

$$Z = \text{depth (cm)} \qquad \rho = 1 \ \text{gm/cm}^3$$

Note that below 50 - 100 meters, the Brunt-Väisälä frequency

$$N = \left(\frac{g}{\rho}\frac{\partial \rho}{\partial z}\right)^{1/2} \sim Z^{-0.7}$$
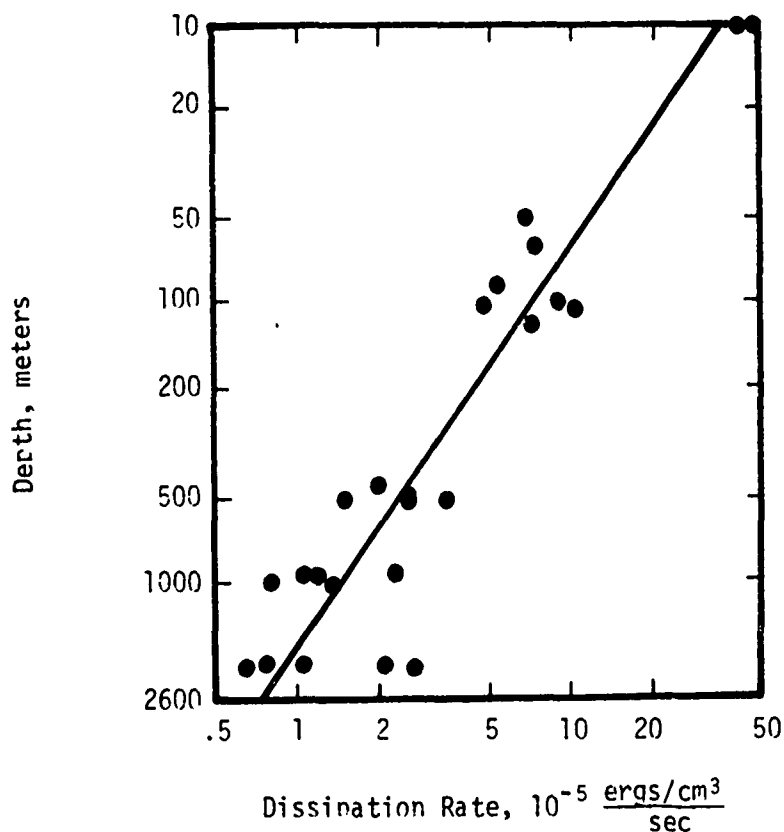
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Figure 3. The Variation of the Rate of Dissipation of
Energy with Depth. The plot is log-log and the
straight line has a slope of $z^{-0.7}$. Webster (1969)
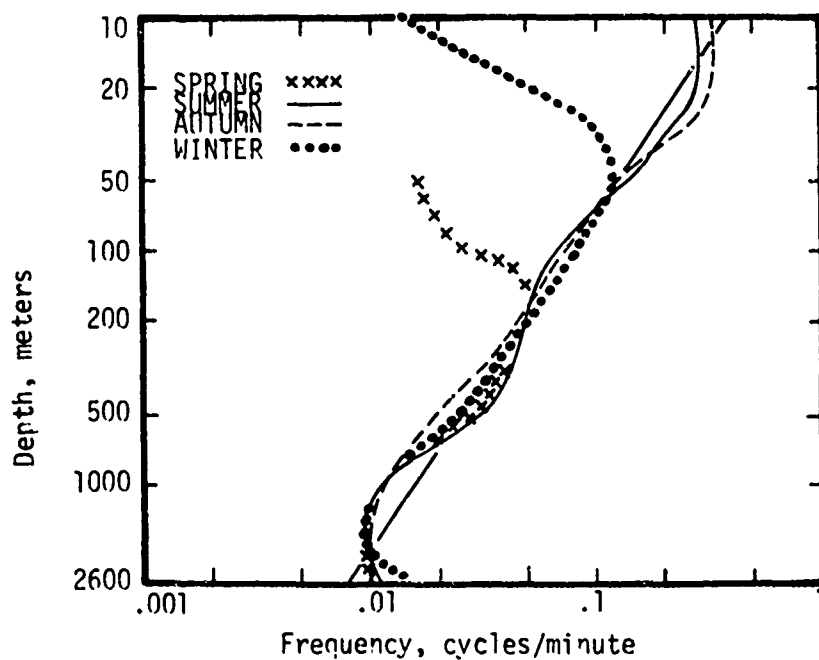

Figure 4. Profiles of Log Brunt-Vaisala Frequency at Four
Seasons vs. Log Depth at Site D. A straight line
having a slope proportional to $z^{-0.7}$ has been
drawn. Webster (1969)

10

It thus appears that the variation of ϵ with depth below the thermocline is proportional to N

$$\boxed{\epsilon \sim N} \tag{15}$$

Using equation 14, one can estimate the near surface dissipation rate (at $Z \simeq 10m$) and the intermediate depth dissipation rate, corresponding to Schuert's measurements at 300 meters (see Table I below). Using the values of the diffusion constants, 'a' for near surface and intermediate depth diffusion, the universal constant A is then calculated by the relation $A = a/\epsilon^{1/3}$ (see Table I).

Table I

$Z(m)$	ϵ cm^2/sec^3	a $\dfrac{\text{cm}^{2/3}}{\text{sec}}$	A
10m	3.53×10^{-4}	4×10^{-3}	.0565
300m	3.20×10^{-5}	1.6×10^{-3}	.0505

$$K = a\ell^{4/3} = A\epsilon^{1/3}\ell^{4/3}$$
$$\ell = \sqrt{6}\sigma_{rc}$$

The interesting result found from combining the data from several different sources is that the value of the universal constant A is nearly constant with depth, within a range of 10 - 15%.

This result is not a direct proof of the validity of the diffusion law

$$K = A\epsilon^{1/3}\ell^{4/3}$$

since the dissipation rate and diffusion coefficients were not measured at the same location. Further, the hot film data of Grant, Moillet and Vogel[7], taken from a submarine in 700m water indicates a much higher near surface dissipation rate

$$\epsilon_{\text{Grant, et. al}} \approx 10^{-2} \text{ to } 10^{-3} \text{cm}^2/\text{sec}^3$$
$$Z = 15 - 27 \text{ meters}$$

11

which would yield a value of A nearly 3 times that given in Table 1. Whether this data is a local phenomena related to measurement technique and near surface flow behavior awaits further clarification.

The only data for ϵ extending from the surface to at least 300 meters in depth is that of Webster. His data shows a factor of 11 decrease in ϵ with depth which corresponds to a decrease in the diffusivity factor, 'a' ($\sim\epsilon^{1/3}$) of 2.22. The decrease of a with depth based on the Okubo and Schuert data is

$$a_{surface}/a_{300m} \approx 2.5 \tag{16}$$

12

5. CONCLUSIONS

The effect of depth on the apparent diffusivity, K, of a passive scalar can be ascertained by estimating the variation of the rate of turbulent dissipation, ϵ, with depth and by applying the relation

$$K = A\epsilon^{1/3}\ell^{4/3}$$

where $\ell = \sqrt{6}\sigma_{rc}$

$A \approx .05 - .055$

$a = A\epsilon^{1/3}$

$b = \sqrt{6}$

The data of Webster indicates for site D in the Atlantic, that

$$\epsilon = 4.4 \times 10^{-2} Z^{-0.7} cm^2/sec^3$$

$$Z = \text{depth in cm}$$

For extrapolation to othe areas of the ocean, a linear relation between ϵ and N might be used below the thermocline

$$\frac{\epsilon_Z}{\epsilon_{Z=50m}} = \frac{N}{N_{Z=50m}} \qquad N = \left(\frac{g}{\rho}\frac{\partial\rho}{\partial z}\right)^{1/2}$$

The data of Schuert and Webster indicates a decrease in apparent diffusivity at 300m depth of

$$\frac{K_{300m}}{K_{surface}} \approx \left[\frac{\epsilon_{300m}}{\epsilon_{10m}}\right]^{1/3} \approx 1/2.2 \text{ or } 1/2.5$$

The variance of a dye or scalar patch varies with ϵ and time as

$$\sigma_{rc}^2 = \left(\frac{4}{3}\right)^3 A\epsilon b^4 \epsilon t^3 \text{ if } K = A\epsilon^{1/3}\ell^{4/3}$$

13

The peak concentration, S, of a circular dye patch decreases as the reciprocal of the variance

$$S_{max} \approx \frac{M/D}{\pi \sigma_{rc}^2} \; gm/cm^3$$

or

$$S_{max} \approx \frac{M/D}{\pi \left(\frac{4}{3}\right)^3 A^3 b^4 \epsilon t^3}$$

For diffusion from a line source or from a continuous source moving at a fixed speed

$$S_{max} \simeq Q^{**}/\sqrt{\pi \sigma_{rc}^2} = Q^{**}/[(4/3 \; A)^{3/2} b^2 \epsilon^{1/2} t^{3/2}]$$

Thus the peak concentration for a given time after release will decrease inversely with ϵ for a point source and $\epsilon^{1/2}$ for a line source. Hence, peak concentrations will decrease much slower with time at depths than near the surface, and hence will be significantly larger than near the surface;

$$\frac{S_{Z=300m}}{S_{Z=10m}} = \frac{\epsilon_{10m}}{\epsilon_{300m}} \sim 10 \qquad \text{point source} \\ \text{instantaneous release}$$

$$\frac{S_{Z=300m}}{S_{Z=10m}} = \left(\frac{\epsilon_{10m}}{\epsilon_{300m}}\right)^{1/2} \sim 3 \qquad \text{line source or} \\ \text{continuous (sublike)} \\ \text{source at fixed speed}$$

14

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