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INVESTIGATION OF STABILITY AND CONTROL  
CHARACTERISTICS OF AC130 LINEAR MODELS

Robert G. Lorenz

Air Force Institute of Technology  
Wright-Patterson Air Force Base, Ohio

March 1972

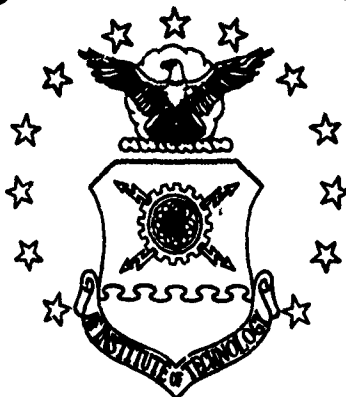
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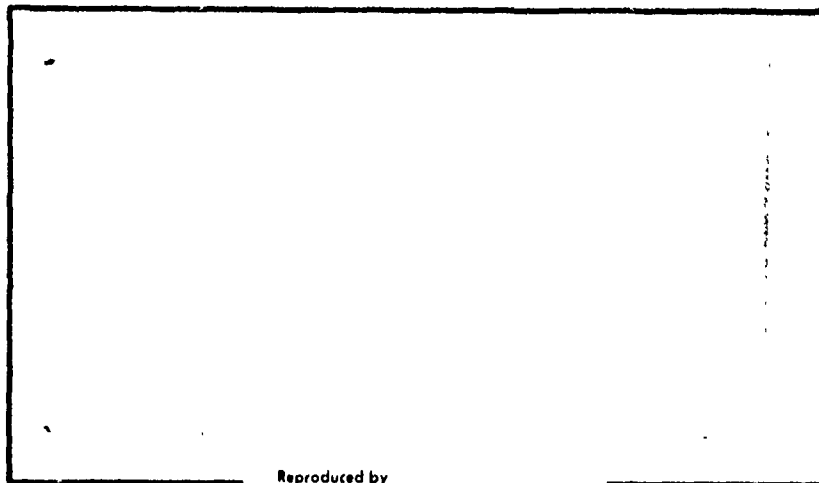
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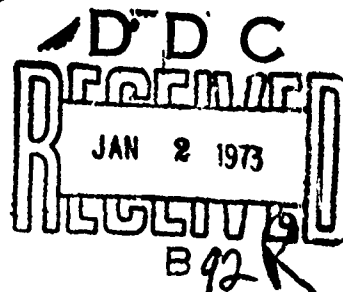
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Unclassified

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

|                                                                                                                                |                                                    |
|--------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------|
| 1. ORIGINATING ACTIVITY (Corporate author)<br>AIR FORCE INSTITUTE OF TECHNOLOGY (AFIT-TH)<br>Wright-Patterson AFB, Ohio, 45433 | 2a. REPORT SECURITY CLASSIFICATION<br>Unclassified |
| 2b. GROUP                                                                                                                      |                                                    |

|                                                                                                     |
|-----------------------------------------------------------------------------------------------------|
| 3. REPORT TITLE<br>Investigation of Stability and Control Characteristics of AC130<br>Linear Models |
|-----------------------------------------------------------------------------------------------------|

|                                                                          |
|--------------------------------------------------------------------------|
| 4. DESCRIPTIVE NOTES (Type of report and inclusive dates)<br>AFIT Thesis |
|--------------------------------------------------------------------------|

|                                                                                           |
|-------------------------------------------------------------------------------------------|
| 5. AUTHOR(S) (First name, middle initial, last name)<br>Robert C. Lorenz<br>Captain, USAF |
|-------------------------------------------------------------------------------------------|

|                              |                              |                       |
|------------------------------|------------------------------|-----------------------|
| 6. REPORT DATE<br>March 1972 | 7a. TOTAL NO. OF PAGES<br>92 | 7b. NO. OF REFS<br>11 |
|------------------------------|------------------------------|-----------------------|

|                           |                                                                             |
|---------------------------|-----------------------------------------------------------------------------|
| 8a. CONTRACT OR GRANT NO. | 9a. ORIGINATOR'S REPORT NUMBER(S)<br>GAM/AE/72-5                            |
| b. PROJECT NO.            | 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) |
| c.                        |                                                                             |
| d.                        |                                                                             |

|                                                                                   |
|-----------------------------------------------------------------------------------|
| 10. DISTRIBUTION STATEMENT<br>Approved for public release, distribution unlimited |
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|--------------------------------------------------------------------------------------------------------|-----------------------------------------------|
| Approved for public release; LAW AFR 1902-17<br>Jerry C. Hix, Captain, USAF<br>Director of Information | 11. SPONSORING MILITARY ACTIVITY<br>AFFDL/FGL |
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| 12. ABSTRACT<br>Mathematical models of the AC130A and AC130E aircraft are proposed. The models are developed from linearized equations and are referred to trim conditions of level turning flight.<br><br>The proposed AC130E model is compared to an existing model to ascertain whether any significant differences exist between the two. A qualitative comparison is conducted by investigating each model's response to control deflections.<br><br>The proposed AC130A model is used to predict general trends and probable values for stability derivatives and selected mode parameters over an extensive flight envelope.<br><br>The proposed AC130E model exhibited increased phugoid damping, and its dutch roll oscillations and divergent modes were generally weaker than those of the existing model. Short period characteristics were identical.<br><br>The data compiled to estimate trends and values of stability parameters for the AC130A aircraft produced no unusual results. |
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~~Unclassified~~

Security Classification

| 14. KEY WORDS                                                                                               | LINK A |    | LINK B |    | LINK C |    |
|-------------------------------------------------------------------------------------------------------------|--------|----|--------|----|--------|----|
|                                                                                                             | ROLE   | WT | ROLE   | WT | ROLE   | WT |
| AC130A/E<br>Stability and Control<br>Linear Mathematical Models of Aircraft<br>Digital Computer Simulations |        |    |        |    |        |    |

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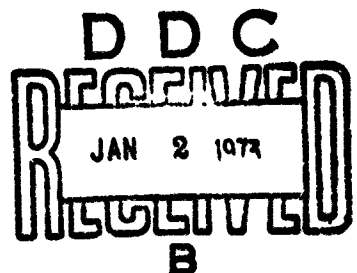
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INVESTIGATION OF STABILITY  
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AC130 LINEAR MODELS

THESIS

GAM/AE/72-5

Robert G. Lorenz  
Capt USAF



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INVESTIGATION OF STABILITY  
AND CONTROL CHARACTERISTICS OF  
AC130 LINEAR MODELS

THESIS

Presented to the Faculty of the School of Engineering  
of the Air Force Institute of Technology  
Air University  
in Partial Fulfillment of the  
Requirement for the Degree of  
Master of Science

by

Robert G. Lorenz, B.S.  
Captain                      USAF

Graduate Aerospace-Mechanical Engineering

March 1972

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Preface

This study was undertaken for the Control Systems Development Branch, Air Force Flight Dynamics Laboratory (Flight Control Division). The first objective of the effort was to ascertain whether the existing linear mathematical model of the AC130 was accurate. I thought it best to begin with the basic equations of motion and develop a parallel model, the characteristics of which could be compared to those of the existing model.

Using the model developed in this study, the second purpose was to compile data on selected stability and control parameters for the AC130A aircraft to predict general trends and values for these parameters throughout an extensive flight envelope.

I wish to express my gratitude to the following personnel: to Lt. Col. James Thompson, for sponsoring this study; to Capt. Kenneth Bassett, for supporting the project and guiding my efforts to produce useful results; to Capt. Len Kruczynski, for supplying the basic computer routines used in the computations; and to my wife, for her unending patience with the author.

Robert G. Lorenz

Contents

|                                                                    |      |
|--------------------------------------------------------------------|------|
| Preface. . . . .                                                   | ii   |
| List of Figures. . . . .                                           | iv   |
| List of Tables . . . . .                                           | vi   |
| Notation . . . . .                                                 | viii |
| Abstract . . . . .                                                 | xii  |
| I. Introduction. . . . .                                           | 1    |
| Background . . . . .                                               | 1    |
| Purpose and Scope. . . . .                                         | 2    |
| II. Governing Equations . . . . .                                  | 4    |
| III. Development of Model. . . . .                                 | 11   |
| Equations. . . . .                                                 | 11   |
| Initial (Trim) Conditions. . . . .                                 | 14   |
| Stability Derivatives. . . . .                                     | 16   |
| Model Characteristics. . . . .                                     | 22   |
| A and E Model Differences. . . . .                                 | 23   |
| IV. Results . . . . .                                              | 24   |
| Comparison of AC130E Models. . . . .                               | 24   |
| Mode Parameters and Stability<br>Derivatives, AC130A. . . . .      | 28   |
| V. Conclusions and Recommendations . . . . .                       | 30   |
| Bibliography . . . . .                                             | 32   |
| Appendix A: Basic Data. . . . .                                    | 33   |
| Appendix B: C130E Model Response to Control<br>Deflection. . . . . | 36   |
| Appendix C: Stability Parameters for the AC130A . .                | 43   |
| Appendix D: Computer Programs . . . . .                            | 58   |
| Vita . . . . .                                                     | 76   |

List of Figures

| <u>Figure</u> |                                                                                                  | <u>Page</u> |
|---------------|--------------------------------------------------------------------------------------------------|-------------|
| 1             | Body Axis System. . . . .                                                                        | 35          |
| 2             | Aircraft Response to Control Surface<br>Deflection. . . . .                                      | 37          |
| 3             | Aircraft Response to Control Surface<br>Deflection. . . . .                                      | 38          |
| 4             | Aircraft Response to Control Surface<br>Deflection. . . . .                                      | 39          |
| 5             | Aircraft Response to Control Surface<br>Deflection. . . . .                                      | 40          |
| 6             | Aircraft Response to Control Surface<br>Deflection. . . . .                                      | 41          |
| 7             | Aircraft Response to Control Surface<br>Deflection. . . . .                                      | 42          |
| 8             | Phugoid Parameters. . . . .                                                                      | 44          |
| 9             | Short Period Parameters . . . . .                                                                | 45          |
| 10            | Dutch Roll Parameters . . . . .                                                                  | 46          |
| 11            | Spiral and Rolling Modes Parameters . . .                                                        | 47          |
| 12            | Stability Derivative Variation<br>( $C_{x_u}$ , $C_{z_u}$ , $C_{m_u}$ ) . . . . .                | 48          |
| 13            | Stability Derivative Variation<br>( $C_{x_\alpha}$ , $C_{z_\alpha}$ , $C_{m_\alpha}$ ) . . . . . | 49          |
| 14            | Stability Derivative Variation<br>( $C_{x_d}$ , $C_{z_d}$ ) . . . . .                            | 50          |
| 15            | Stability Derivative Variation<br>( $C_{x_q}$ , $C_{z_q}$ ) . . . . .                            | 51          |
| 16            | Stability Derivative Variation<br>( $C_{m_q}$ , $C_{m_d}$ ) . . . . .                            | 52          |

|    |                                                                                                                             |    |
|----|-----------------------------------------------------------------------------------------------------------------------------|----|
| 17 | Stability Derivative Variation<br>( $C_{x_{\delta e}}$ , $C_{z_{\delta e}}$ , $C_{m_{\delta e}}$ ) . . . . .                | 53 |
| 18 | Stability Derivative Variation<br>( $C_{y_{\beta}}$ , $C_{l_{\beta}}$ , $C_{n_{\beta}}$ ) . . . . .                         | 54 |
| 19 | Stability Derivative Variation<br>( $C_{y_r}$ , $C_{l_r}$ , $C_{n_r}$ ) . . . . .                                           | 55 |
| 20 | Stability Derivative Variation<br>( $C_{l_{\delta a}}$ , $C_{n_{\delta a}}$ , $C_{n_{\delta r}}$ ) . . . . .                | 56 |
| 21 | Stability Derivative Variation<br>( $C_{y_p}$ , $C_{l_p}$ , $C_{n_p}$ , $C_{y_{\delta r}}$ , $C_{l_{\delta r}}$ ) . . . . . | 57 |

List of Tables

| <u>Table</u> |                                           | <u>Page</u> |
|--------------|-------------------------------------------|-------------|
| I            | Stability Derivatives . . . . .           | 17          |
| II           | Model Parameters - AC130E . . . . .       | 25          |
| III          | Mode Parameters - AC130E Models . . . . . | 26          |
| IV           | Perturbations of Proposed AC130E Model. . | 27          |

Notation

| <u>Symbol</u>   | <u>Description</u>                                       | <u>Units</u>              |
|-----------------|----------------------------------------------------------|---------------------------|
| A               | Altitude                                                 | ft                        |
| a               | Speed of sound                                           | ft/sec                    |
| A.C.            | Aerodynamic center, % MAC                                |                           |
| AR              | Aspect ratio                                             |                           |
| b               | Span                                                     | ft                        |
| C.G.            | Center of gravity, % MAC                                 |                           |
| c               | Section chord                                            | ft                        |
| $\bar{c}$       | Mean aerodynamic chord (MAC)                             | ft                        |
| D               | Drag                                                     | lbf                       |
| d               | Diameter of propeller                                    | ft                        |
| F               | Flap position in %                                       |                           |
| g               | Acceleration due to gravity                              | ft/sec <sup>2</sup>       |
| $h_x, h_y, h_z$ | Scalar components of angular momentum of spinning rotors | slug ft <sup>2</sup> /sec |
| I               | Total moment of inertia of propeller                     | slug ft <sup>2</sup>      |
| $I_x$           | Moment of inertia                                        | slug ft <sup>2</sup>      |
| $I_y$           | Moment of inertia                                        | slug ft <sup>2</sup>      |
| $I_z$           | Moment of inertia                                        | slug ft <sup>2</sup>      |
| $I_{xz}$        | Product of inertia                                       | slug ft <sup>2</sup>      |
| $I_{xy}$        | Product of inertia                                       | slug ft <sup>2</sup>      |
| $I_{yz}$        | Product of inertia                                       | slug ft <sup>2</sup>      |
| i               | Angle of incidence                                       | deg                       |
| K               | Induced drag coefficient, $1/\pi eAR$                    |                           |
| L               | Lift                                                     | lbf                       |

|                             |                                                                                                                            |                 |
|-----------------------------|----------------------------------------------------------------------------------------------------------------------------|-----------------|
| $L$                         | Rolling moment about mass center                                                                                           | lbf ft          |
| $l_t$                       | Longitudinal distance between center of gravity and of aerodynamic center horizontal tail (assumed to be at $.25\bar{c}$ ) | ft              |
| $l_f$                       | Longitudinal distance between center of gravity and of aerodynamic center vertical tail (assumed to be at $.25\bar{c}$ )   | ft              |
| $M$                         | Mach number                                                                                                                | ft              |
| MAC                         | Mean aerodynamic chord, $\bar{c}$                                                                                          | ft              |
| $m$                         | Pitching moment about mass center                                                                                          | lbf ft          |
| $m$                         | Mass                                                                                                                       | slug            |
| $n$                         | Yawing moment about mass center                                                                                            | lbf ft          |
| $P, Q, R$                   | Scalar components of angular velocity                                                                                      | rad/sec         |
| $p, q, r$                   | Perturbations of $P, Q, R$                                                                                                 | rad/sec         |
| $\hat{p}, \hat{q}, \hat{r}$ | Nondimensionalized perturbations, $p\bar{b}/2U_o, q\bar{c}/2U_o, r\bar{b}/2U_o$                                            |                 |
| $R$                         | Radius of turn                                                                                                             | ft              |
| $S$                         | Total wing area                                                                                                            | ft <sup>2</sup> |
| $S_t$                       | Total area of horizontal tail                                                                                              | ft <sup>2</sup> |
| TAS                         | True air speed                                                                                                             | ft/sec          |
| $T$                         | Thrust                                                                                                                     | lbf             |
| $T_c$                       | Specific thrust, $T(\text{eng})/\rho U^2 d^2$ , where $d$ is propeller diameter                                            |                 |
| $U, V, W$                   | Scalar components of $V_c$                                                                                                 | ft/sec          |
| $u, v, w$                   | Perturbation of $U, V, W$                                                                                                  | ft/sec          |
| $\hat{u}, \hat{v}, \hat{w}$ | Nondimensionalized perturbations, $u/U_o, v/U_o, w/U_o$                                                                    |                 |
| $V_c$                       | Velocity of mass center                                                                                                    | ft/sec          |
| $X, Y, Z$                   | Scalar components of aerodynamic force acting on mass center                                                               | lbf             |

|                                        |                                                       |                      |
|----------------------------------------|-------------------------------------------------------|----------------------|
| $x, y, z$                              | Displacements along axes                              | ft                   |
| $\alpha$                               | Angle of attack (see Fig. 1)                          | rad                  |
| $\alpha_w$                             | Angle of attack of wing, $\alpha + i_w$               | rad                  |
| $\beta$                                | Sideslip angle (see Fig. 1)                           | rad                  |
| $\delta_a, \delta_e, \delta_r$         | Perturbation angles of ailerons, elevator, and rudder | rad                  |
| $\rho$                                 | Air density                                           | slug/ft <sup>3</sup> |
| $\theta, \phi, \psi$                   | Euler angles                                          | rad                  |
| $\delta\theta, \delta\phi, \delta\psi$ | Perturbations of $\theta, \phi, \psi$                 | rad                  |
| $\Omega$                               | Propeller RPM                                         | rad/sec              |
| $\xi$                                  | Damping ratio                                         |                      |

#### Subscripts

|     |                                                                          |
|-----|--------------------------------------------------------------------------|
| $a$ | Aileron                                                                  |
| $e$ | Elevator                                                                 |
| $f$ | Vertical tail                                                            |
| $i$ | Inertial                                                                 |
| $o$ | Denotes evaluation at trim condition<br>(except for $C_{L_o}, C_{d_o}$ ) |
| $r$ | Rudder                                                                   |
| $t$ | Horizontal tail                                                          |
| $T$ | Component due to thrust                                                  |
| $w$ | Wing                                                                     |

DescriptionNondimensional Coefficients

|                    |                                            |                        |                                                             |
|--------------------|--------------------------------------------|------------------------|-------------------------------------------------------------|
| $C_D$              | $(D/1/2\rho S V_c^2)_0$                    | $C_{m_\alpha}$         | $(\partial C_m / \partial \alpha)_0$                        |
| $C_{D_0}$          | $(C_D \text{ at } C_L=0)$                  | $C_{m_{\dot{\alpha}}}$ | $[\partial C_m / \partial (\dot{\alpha} \bar{c} / 2U_0)]_0$ |
| $C_{D_\alpha}$     | $(\partial C_D / \partial \alpha)_0$       | $C_{m_{\delta e}}$     | $(\partial C_m / \partial \delta e)_0$                      |
| $C_L$              | $(L/1/2\rho S V_c^2)_0$                    | $C_n$                  | $(n/1/2\rho S b V_c^2)_0$                                   |
| $C_{L_0}$          | $(C_L \text{ at } \alpha=0)$               | $C_{n_p}$              | $(\partial C_n / \partial \dot{p})_0$                       |
| $C_{L_\alpha}$     | $(\partial C_L / \partial \alpha)_0$       | $C_{n_r}$              | $(\partial C_n / \partial \dot{r})_0$                       |
| $C_{l_1}$          | $(\ell/1/2\rho S b V_c^2)_0$               | $C_{n_\beta}$          | $(\partial C_n / \partial \beta)_0$                         |
| $C_{l_p}$          | $(\partial C_{l_1} / \partial \dot{p})_0$  | $C_{n_{\delta a}}$     | $(\partial C_n / \partial \delta a)_0$                      |
| $C_{l_r}$          | $(\partial C_{l_1} / \partial \dot{r})_0$  | $C_{n_{\delta r}}$     | $(\partial C_n / \partial \delta r)_0$                      |
| $C_{l_\beta}$      | $(\partial C_{l_1} / \partial \beta)_0$    | $C_T$                  | $(T/1/2\rho S V_c^2)_0$                                     |
| $C_{l_{\delta a}}$ | $(\partial C_{l_1} / \partial \delta a)_0$ | $C_x$                  | $(X/1/2\rho S V_c^2)_0$                                     |
| $C_{l_{\delta r}}$ | $(\partial C_{l_1} / \partial \delta r)_0$ | $C_{x_q}$              | $(\partial C_x / \partial \dot{q})_0$                       |
| $C_m$              | $(M/1/2\rho S c V_c^2)_0$                  | $C_{x_u}$              | $(\partial C_x / \partial u)_0$                             |
| $C_{m_q}$          | $(\partial C_m / \partial \dot{q})_0$      | $C_{x_\alpha}$         | $(\partial C_x / \partial \alpha)_0$                        |
| $C_{m_u}$          | $(\partial C_m / \partial u)_0$            | $C_{x_{\dot{\alpha}}}$ | $[\partial C_x / \partial (\dot{\alpha} \bar{c} / 2U_0)]_0$ |

|                    |                                                       |
|--------------------|-------------------------------------------------------|
| $C_{x\delta e}$    | $(\partial C_x / \partial \delta e)_o$                |
| $C_y$              | $(Y/1/2\rho S V_c^2)_o$                               |
| $C_{y_p}$          | $(\partial C_y / \partial \hat{p})_o$                 |
| $C_{y_r}$          | $(\partial C_y / \partial \hat{r})_o$                 |
| $C_{y_\beta}$      | $(\partial C_y / \partial \beta)_o$                   |
| $C_{y_{\delta a}}$ | $(\partial C_y / \partial \delta a)_o$                |
| $C_{y_{\delta r}}$ | $(\partial C_y / \partial \delta r)_o$                |
| $C_z$              | $(Z/1/2\rho S V_c^2)_o$                               |
| $C_{z_q}$          | $(\partial C_z / \partial \hat{q})_o$                 |
| $C_{z_u}$          | $(\partial C_z / \partial \hat{u})_o$                 |
| $C_{z_\alpha}$     | $(\partial C_z / \partial \alpha)_o$                  |
| $C_{z_{\delta c}}$ | $[\partial C_z / \partial (\delta \bar{c} / 2U_o)]_o$ |
| $C_{z_{\delta e}}$ | $(\partial C_z / \partial \delta e)_o$                |

INVESTIGATION OF STABILITY  
AND CONTROL CHARACTERISTICS OF  
AC130 LINEAR MODELS

I. Introduction

Background

The weapons and fire control system of the AC130 gunship possesses a capability for extremely high accuracy, if the aircraft platform can be rigidly controlled within crucial tolerances. A novel automatic control system, termed the Sight Line Autopilot (SLAP) by its innovators, is presently under development. The system will be capable of maintaining the aircraft in its firing orbit within stringent limits, allowing the pilot to devote his attention to firing the weapons.

The replacement of the human pilot by this control system is conditional on that system's capability to anticipate accurately the aircraft's perturbed response to a disturbance from trim. A comprehensive mathematical model is, therefore, a necessity. In determining such a model, two sets of parameters are needed to describe adequately the dynamic idiosyncrasies of the airframe: stability derivatives and transfer functions. Stability derivatives, introduced into the perturbation equations, relate the changes in the

aerodynamic forces and moments with changes in the aircraft's state vectors when the aircraft is disturbed from trim. The state vectors in this study were: angle of attack, forward airspeed, sideslip angle, yaw angle, pitch angle, bank angle, and angular velocities. Transfer functions, determined primarily by the stability derivatives, are input/output ratios used within the autopilot to program the required changes back to the trim condition.

#### Purpose and Scope

The objectives of this study were twofold. First, more comprehensive, linear models of both the AC130A and AC130E aircraft were developed. The dynamic characteristics of the proposed AC130E model were then compared with those of an existing model. Second, data were generated for the AC130A throughout an extensive flight envelope to establish general trends and probable values of stability derivatives and selected stability and control parameters. This data will be used to compare model response with flight test results on the AC130A aircraft. Aeroelastic effects were not included in the model as no test data were available and no reliable technique has been developed for estimating these phenomena. Gust inputs and quantitative determination of transfer functions were not treated in the study as the author feels that, until a valid model has been confirmed, such efforts would not produce useful results. Because of the large number of calculations involved, the iterative

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procedures, the dynamic response modeling, and the plotting of data were all programmed in Extended Fortran language for the CDC 6600 digital computer and the Calcomp package.

## II. Governing Equations

The equations used in the mathematical model were derived from Etkin (Ref 6), and include the gyroscopic coupling terms. Assuming the airframe to be a rigid body and the earth axes to be fixed in space, the complete set of equations is:

### Equations of Motion

$$X + X_T - mg \sin \theta = m(\dot{U} + QW - RV) \quad (1)$$

$$Z + Z_T + mg \cos \theta \cos \phi = m(\dot{W} + PV - QU) \quad (2)$$

$$\begin{aligned} M + M_T = & \dot{Q}I_y + PR(I_x - I_z) + (QR + \dot{P})I_{xy} + (PQ - \dot{R})I_{yz} \\ & + (P^2 - R^2)I_{xz} + h_x R - h_z P \end{aligned} \quad (3)$$

$$Y + Y_T + mg \cos \theta \sin \phi = m(\dot{V} + RU - PW) \quad (4)$$

$$\begin{aligned} L + L_T = & \dot{P}I_x + QR(I_z - I_y) - (PQ - \dot{R})I_{xz} + (PR - \dot{Q})I_{xy} \\ & + (R^2 - Q^2)I_{yz} - h_y R + h_z Q \end{aligned} \quad (5)$$

$$\begin{aligned} N + N_T = & \dot{R}I_z + PQ(I_y - I_x) + (QR - \dot{P})I_{xz} + (PR - \dot{Q})I_{yz} \\ & + (Q^2 - P^2)I_{xy} - h_x Q + h_y P \end{aligned} \quad (6)$$

## Euler Relationships

$$P = \dot{\theta} - \dot{\psi} \sin\theta \quad (7)$$

$$Q = \dot{\theta} \cos\phi + \dot{\psi} \cos\theta \sin\phi \quad (8)$$

$$R = -\dot{\theta} \sin\phi + \dot{\psi} \cos\theta \cos\phi \quad (9)$$

$$\dot{\theta} = Q \cos\phi - R \sin\phi \quad (10)$$

$$\dot{\phi} = (Q \sin\phi + R \cos\phi) \tan\theta + P \quad (11)$$

$$\dot{\psi} = (Q \sin\phi + R \cos\phi) \sec\theta \quad (12)$$

## Inertial Velocity

$$\dot{z}_i = -U \sin\theta + V \cos\theta \sin\phi + W \cos\theta \cos\phi \quad (13)$$

To simplify these equations further, the following assumptions and restrictions were made:

1. The only significant rotors contributing to gyroscopic coupling were the propellers.  $\rightarrow h_y = h_z = 0$ ,  $h_x = \Omega I$ .
2. The xz plane was a plane of symmetry.  $\rightarrow I_{xy} = I_{yz} = 0$ ,  $Y_T = X_T = N_T = 0$ .
3. The thrust force was parallel to the longitudinal axis.  $\rightarrow Z_T = 0$ .
4. The angles of attack and sideslip were not allowed to exceed ten degrees. The following linear approximations were then used:  $U \dot{=} V_c$ ,  $\alpha \dot{=} W/U$ ,  $\beta \dot{=} V/U$ ,  $\dot{W} \dot{=} U\dot{\alpha} + \dot{U}\alpha$ ,  $\dot{V} \dot{=} U\dot{\beta} + \dot{U}\beta$ .

Employing these assumptions, the equations became:

$$K + X_T - mgsin\theta = m(\dot{U} + U\dot{\alpha} - U\dot{\beta}) \quad (1a)$$

$$Z + mgcos\theta cos\phi = m(\dot{U}\alpha + U\dot{\alpha} + U\dot{\beta} - U\dot{\phi}) \quad (2a)$$

$$\dot{m} + m_T = \dot{Q}I_y + PR(I_x - I_z) + (P^2 - R^2)I_{xz} + \Omega IR \quad (3a)$$

$$Y + mgcos\theta sin\phi = m(U\dot{\beta} + U\dot{\phi} + UR - U\dot{\alpha}) \quad (4a)$$

$$\dot{\mathcal{L}} = \dot{P}I_x + QR(I_z - I_y) - (PQ + \dot{R})I_{xz} \quad (5a)$$

$$\dot{\mathcal{H}} = \dot{R}I_z + PQ(I_y - I_x) + (QR - \dot{P})I_{xz} - \Omega IQ \quad (6a)$$

Equations (7) through (12) were unchanged.

$$\dot{z}_1 = U(-sin\theta + \beta cos\theta sin\phi + \alpha cos\theta cos\phi). \quad (13a)$$

Solving the equations for time-dependent parameters, the following set of equations referred to the trim conditions were obtained:

$$2m\dot{U}_0 = \rho_0 S U_0^2 C_x + 2T_0 - 2mgsin\theta_0 + 2mU_0(R_0\beta_0 - Q_0\alpha_0) \quad (14)$$

$$2mU_0\dot{\alpha}_0 = \rho_0 S U_0^2 (C_z - \alpha_0 C_x) - 2\alpha_0 T_0 + 2mg(cos\theta_0 cos\phi_0 + \alpha_0 sin\theta_0) \\ + 2mU_0 Q_0 (1 + \alpha_0^2) - 2mU_0 \beta_0 (P_0 + R_0 \alpha_0) \quad (15)$$

$$2I_y\dot{Q}_0 = \rho_0 S U_0^2 \bar{C}_m + 2P_0 R_0 (I_z - I_x) + 2I_{xz} (R_0^2 - P_0^2) + 2I_y \Omega IR_0 \quad (16)$$

$$\dot{\theta}_0 = Q_0 cos\phi_0 - R_0 sin\phi_0 \quad (17)$$

$$2m\dot{U}_O \dot{\beta}_O = \rho_O S U_O^2 (C_y - \beta_O C_x) - 2\beta_O T_O + 2mg(\cos\theta_O \sin\phi_O + \beta_O \sin\theta_O) \\ - 2mU_O R_O (1 + \beta_O^2) + 2mU_O \alpha_O (P_O + Q_O \beta_O) \quad (18)$$

$$2(I_x I_z - I_{xz}^2) \dot{P}_O = \rho_O S U_O^2 b (I_z C_1 + I_{xz} C_n) + 2Q_O R_O (I_y I_z - I_z^2 + I_{xz}^2) \\ + 2P_O Q_O I_{xz} (I_z + I_x - I_y) + 2Q_O I_{xz} \Omega I \quad (19)$$

$$2(I_x I_z - I_{xz}^2) \dot{R}_O = \rho_O S U_O^2 b (I_x C_n + I_{xz} C_1) + 2Q_O P_O (I_x^2 - I_x I_y + I_{xz}^2) \\ + 2R_O Q_O I_{xz} (I_y - I_z - I_x) + 2Q_O I_x \Omega I \quad (20)$$

$$\dot{\phi}_O = P_O + (Q_O \sin\phi_O + R_O \cos\phi_O) \tan\theta_O \quad (21)$$

$$\dot{\psi}_O = (Q_O \sin\phi_O + R_O \cos\phi_O) \sec\theta_O \quad (22)$$

$$\dot{z}_{i_O} = U_O (-\sin\theta_O + \beta_O \cos\theta_O \sin\phi_O + \alpha_O \cos\theta_O \cos\phi_O) \quad (23)$$

Thrust appears explicitly in equations (14), (15), and (18), while in equation (16) this term is absorbed into the coefficient  $C_m$ . This arrangement was to facilitate an iteration process used to obtain the initial (trim) conditions.

Perturbations about the trim condition were then introduced. These disturbances were considered small compared to the steady-state values of the state vectors. The sines and cosines of the perturbation angles were approximated by the angles themselves and one (1), respectively. The products and squares of the perturbation quantities were considered negligible in comparison to the perturbations themselves. The following notation is used in this report:  $U = U_O + u$ ,  $\phi = \phi_O + \phi$ , etc, where the subscript denotes the

state variable at its trim condition and the lower case character denotes a perturbation. Assuming that inertia terms ( $I_x$ , etc) and atmospheric variables (density, speed of sound, etc) remain essentially constant for the small time span under consideration, the following perturbation equations were obtained (direct thrust effects here were included in the aerodynamic forces):

$$\begin{aligned} \Delta \dot{X}/m = & \dot{u} + (\alpha_o Q_o + R_o \beta_o)u + U_o Q_o \alpha + U_o \alpha_o q + g \theta \cos \theta_o \\ & - U_o R_o \beta - U_o \beta_o r \end{aligned} \quad (24)$$

$$\begin{aligned} \Delta \dot{Z}/m = & \alpha_o \dot{u} + U_o \dot{\alpha} + (\dot{\alpha}_o - Q_o)u + \dot{U}_o \alpha - U_o q + g \theta \sin \theta_o \cos \phi_o \\ & + P_o U_o \beta + U_o \beta_o p + g \phi \cos \theta_o \sin \phi_o \end{aligned} \quad (25)$$

$$\Delta \dot{M} = I_y \dot{q} + (I_x R_o - I_z R_o + 2I_{xz} P_o)p + (I_x P_o - I_z P_o - 2I_{xz} R_o + \Omega I)r \quad (26)$$

$$\Delta \dot{\theta} = q \cos \theta_o - r \sin \phi_o - \phi \dot{\psi}_o \cos \theta_o \quad (27)$$

$$\begin{aligned} \Delta \dot{Y}/m = & \beta_o \dot{u} + U_o \dot{\beta} + (R_o - P_o \alpha_o + \dot{\beta}_o)u - P_o U_o \alpha + g \theta \sin \theta_o \sin \phi_o \\ & + \dot{U}_o \beta - \alpha_o U_o p + U_o r - g \phi \cos \theta_o \cos \phi_o \end{aligned} \quad (28)$$

$$\Delta \dot{\mathcal{L}} = I_x \dot{p} - I_{xz} \dot{r} + (I_z R_o - I_y R_o - I_{xz} P_o)q + Q_o (I_z - I_y)r - I_{xz} Q_o p \quad (29)$$

$$\begin{aligned} \Delta \dot{n} = & I_z \dot{r} - I_{xz} \dot{p} + (I_y P_o - I_x P_o + I_{xz} R_o - \Omega I)q \\ & + Q_o (I_y - I_x)p + I_{xz} Q_o r \end{aligned} \quad (30)$$

$$\begin{aligned} \Delta \dot{\phi} = & q \tan \theta_o \sin \phi_o + (\dot{\phi}_o \tan \theta_o - P_o \tan \theta_o + Q_o \sin \phi_o + R_o \cos \phi_o)\theta \\ & + p + (\tan \theta_o \cos \phi_o)r + (\dot{\theta}_o \tan \theta_o)\phi \end{aligned} \quad (31)$$

$$\dot{\Delta\psi} = (q \sin \phi_0 + \theta Q_0 \tan \theta_0 \sin \phi_0 + \theta R_0 \tan \theta_0 \cos \phi_0 + r \cos \phi_0) \sec \theta_0 + \dot{\phi} \theta_0 \sec \theta_0 \quad (32)$$

Equations (24) through (32) were then expanded by linearizing the aerodynamic forces and moments and expressing them in terms of the non-dimensional stability derivatives. The following assumptions were applied to the linearization procedure:

1. Cross derivatives ( $C_{x_y}$ ,  $C_{n_u}$ , etc) were considered negligible (Ref 6:123-4).
2. The thrust coefficient was independent of angle of attack (Ref 6:147).
3. The change in side force due to aileron deflection ( $C_{y_{\delta a}}$ ) was negligible (Ref 3:88).
4. The airflow around the aircraft was assumed to be quasi-steady. Unsteady flow effects were considered negligible and all derivatives with respect to accelerations were omitted with the exception of those due to  $\dot{\alpha}$ . The  $\dot{\alpha}$  derivatives were retained to account for the effects of downwash on the horizontal tail (Ref 9:4-52; Ref 4:370).

The linearized aerodynamic forces and moments, adapted from Ref 6, were used in the following form:

$$\Delta X = \frac{1}{2} \rho_0 S U_0^2 (2C_{x_0} + C_{x_u}) u + \frac{1}{2} \rho_0 S U_0^2 (C_{x_\alpha} \alpha + \frac{\bar{c}}{2U_0} C_{x_q} \dot{q} + C_{x_{\delta e}} \delta e)$$

$$\Delta Z = \frac{1}{2} \rho_0 S U_0^2 (2C_{z_0} + C_{z_u}) u + \frac{1}{2} \rho_0 S U_0^2 (C_{z_\alpha} \alpha + \frac{\bar{c}}{2U_0} C_{z_\alpha} \dot{\alpha} + \frac{\bar{c}}{2U_0} C_{z_q} \dot{q} + C_{z_{\delta e}} \delta e)$$

$$\Delta m = \frac{1}{2} \rho_o SU_o \bar{c} (2C_m + C_{m_u}) u + \frac{1}{2} \rho_o SU_o^2 \bar{c} (C_{m_\alpha} \alpha + \frac{\bar{c}}{2U_o} C_{m_\alpha} \dot{\alpha} + \frac{\bar{c}}{2U_o} C_{m_q} q + C_{m_{\delta e}} \delta e)$$

$$\Delta Y = \frac{1}{2} \rho_o SU_o (\dot{C}_y) u + \frac{1}{2} \rho_o SU_o^2 (C_{y_\beta} \beta + \frac{b}{2U_o} C_{y_p} p + \frac{b}{2U_o} C_{y_r} r + C_{y_{\delta r}} \delta r)$$

$$\Delta \dot{x} = \frac{1}{2} \rho_o SU_o b (2C_1) u + \frac{1}{2} \rho_o SU_o^2 b (C_{1_\beta} \beta + \frac{\bar{c}}{2U_o} C_{1_p} p + \frac{\bar{c}}{2U_o} C_{1_r} r + C_{1_{\delta a}} \delta a + C_{1_{\delta r}} \delta r)$$

$$\Delta \dot{n} = \frac{1}{2} \rho_o SU_o b (2C_n) u + \frac{1}{2} \rho_o SU_o^2 b (C_{n_\beta} \beta + \frac{\bar{c}}{2U_o} C_{n_p} p + \frac{\bar{c}}{2U_o} C_{n_r} r + C_{n_{\delta a}} \delta a + C_{n_{\delta r}} \delta r)$$

### III. Development of Model

#### Equations

The linearized forces and moments were substituted into the perturbation equations (24) through (32) and the resulting set of equations solved for the time derivatives of the state variables. The steady-state (trim) condition of level, turning flight was simulated by requiring the following restrictions:

1. All state vectors were constants except  $\psi_0$ , which was cyclic, and  $\beta_0$ , which was zero.

2. Accelerations of state variables were zero.

Applying these conditions to the perturbation equations, the following set, referred to body axes, resulted:

$$\begin{aligned} \dot{u} = & \left[ \frac{\rho_0 S U_0}{2m} (2C_{x_u} + C_{x_u}) - \alpha_0 Q_0 \right] u + \left[ \frac{\rho_0 S U_0^2}{2m} C_{x_u} - U_0 Q_0 \right] \alpha \\ & + \left[ \frac{\rho_0 S U_0 \bar{c}}{4m} C_{x_{\dot{\alpha}}} \right] \dot{\alpha} + \left[ \frac{\rho_0 S U_0 \bar{c}}{4m} C_{x_q} - \alpha_0 U_0 \right] q \\ & - [g \cos \theta_0] \theta + [U_0 R_0] \beta + \left[ \frac{\rho_0 S U_0^2}{2m} C_{x_{\delta e}} \right] \delta e \end{aligned} \quad (33)$$

$$\begin{aligned}
\left[1 - \frac{\rho_o S \bar{c}}{4m} C_{z_\alpha}\right] \dot{\alpha} &= \left[\frac{\rho_o S}{2m} (2C_z + C_{z_u}) + \frac{Q_o}{U_o}\right] u - \left[\frac{\alpha_o}{U_o}\right] \dot{u} + \left[\frac{\rho_o S U_o}{2m} C_{z_\alpha}\right] \alpha \\
&+ \left[1 + \frac{\rho_o S \bar{c}}{4m} C_{z_q}\right] q - \left[\frac{g}{U_o} \sin \theta_o \cos \phi_o\right] \theta \\
&- [P_o] \beta - \left[\frac{g}{U_o} \cos \theta_o \sin \phi_o\right] \phi + \left[\frac{\rho_o S U_o}{2m} C_{z_{\delta e}}\right] \delta e \quad (34)
\end{aligned}$$

$$\begin{aligned}
I_y \dot{q} &= \left[\frac{\rho_o S U_o \bar{c}}{2} (2C_m - C_{m_u})\right] u + \left[\frac{\rho_o S U_o^2 \bar{c}}{2} C_{m_\alpha}\right] \alpha + \left[\frac{\rho_o S U_o \bar{c}^2}{4} C_{m_\alpha}\right] \dot{\alpha} \\
&+ \left[\frac{\rho_o S U_o \bar{c}^2}{4} C_{m_q}\right] q + (I_z R_o - I_x R_o - 2I_{xz} P_o) p \\
&+ \left[\frac{\rho_o S U_o^2 \bar{c}}{2} C_{m_{\delta e}}\right] \delta e + (I_z P_o - I_x P_o + 2I_{xz} R_o - \Omega I) r \quad (35)
\end{aligned}$$

$$\dot{\theta} = (\cos \phi_o) q - (\sin \phi_o) r - (\dot{\psi}_o \cos \theta_o) \phi \quad (36)$$

$$\begin{aligned}
\dot{\beta} &= \left[\frac{\rho_o S}{m} C_y + \frac{\alpha_o P_o - R_o}{U_o}\right] u + (P_o) \alpha - \left[\frac{g}{U_o} \sin \theta_o \sin \phi_o\right] \theta \\
&+ \left[\frac{\rho_o S U_o}{2m} C_{y_\beta}\right] \beta + \left[\frac{\rho_o S b}{4m} C_{y_p} + \alpha_o\right] p + \left[\frac{\rho_o S b}{4m} C_{y_r} - 1\right] r \\
&+ \left[\frac{g}{U_o} \cos \theta_o \cos \phi_o\right] \phi + \left[\frac{\rho_o S U_o}{2m} C_{y_{\delta r}}\right] \delta r \quad (37)
\end{aligned}$$

$$\begin{aligned}
\left[\frac{I_x - I_{xz}}{I_{xz}} - \frac{I_{xz}}{I_z}\right] \dot{p} &= \rho_o S U_o b \left[\frac{C_l}{I_{xz}} + \frac{C_n}{I_z}\right] u + \frac{\rho_o S U_o^2 b}{2} \left[\frac{C_{l_\beta}}{I_{xz}} + \frac{C_{n_\beta}}{I_z}\right] \beta \\
&+ \left[\left(\frac{I_x - I_y}{I_z} + 1\right) P_o + \left(\frac{I_y - I_z}{I_{xz}} - \frac{I_{xz}}{I_z}\right) R_o + \frac{I}{I_z}\right] q \\
&+ \left[\frac{\rho_o S U_o b^2}{4} \left(\frac{C_{l_p}}{I_{xz}} + \frac{C_{n_p}}{I_z}\right) + Q_o \left(1 + \frac{I_x - I_y}{I_z}\right)\right] p
\end{aligned}$$

$$\begin{aligned}
& + \left[ \frac{\rho_o S U_o^2 b^2}{4} \left( \frac{C_{l_r}}{I_{xz}} + \frac{C_{n_r}}{I_z} \right) + Q_o \left( \frac{I_y - I_z}{I_{xz}} - \frac{I_{xz}}{I_z} \right) \right] r \\
& + \frac{\rho_o S U_o^2 b^2}{2} \left[ \frac{C_{l_{\delta a}}}{I_{xz}} + \frac{C_{n_{\delta a}}}{I_z} \right] \delta a + \frac{\rho_o S U_o^2 b^2}{2} \left[ \frac{C_{l_{\delta r}}}{I_{xz}} + \frac{C_{n_{\delta r}}}{I_z} \right] \delta r
\end{aligned} \quad (38)$$

$$\begin{aligned}
\left[ \frac{I_z}{I_{xz}} - \frac{I_{xz}}{I_x} \right] \dot{r} &= \rho_o S U_o b \left[ \frac{C_n}{I_{xz}} + \frac{C_l}{I_x} \right] u + \frac{\rho_o S U_o^2 b^2}{2} \left[ \frac{C_{n\beta}}{I_{xz}} + \frac{C_{l\beta}}{I_x} \right] \beta \\
& + \left[ \left( \frac{I_y - I_z}{I_x} - 1 \right) R_o + \left( \frac{I_x - I_y}{I_{xz}} + \frac{I_{xz}}{I_x} \right) P_o + \frac{I}{I_{xz}} \right] q \\
& + \left[ \frac{\rho_o S U_o^2 b^2}{4} \left( \frac{C_{n_p}}{I_{xz}} + \frac{C_{n_p}}{I_x} \right) + Q_o \left( \frac{I_x - I_y}{I_{xz}} + \frac{I_{xz}}{I_x} \right) \right] p \\
& + \left[ \frac{\rho_o S U_o^2 b^2}{4} \left( \frac{C_{n_r}}{I_{xz}} + \frac{C_{l_r}}{I_x} \right) + Q_o \left( \frac{I_y - I_z}{I_x} - 1 \right) \right] r \\
& + \frac{\rho_o S U_o^2 b^2}{2} \left[ \frac{C_{n_{\delta a}}}{I_{xz}} + \frac{C_{l_{\delta a}}}{I_x} \right] \delta a + \frac{\rho_o S U_o^2 b^2}{2} \left[ \frac{C_{n_{\delta r}}}{I_{xz}} + \frac{C_{l_{\delta r}}}{I_x} \right] \delta r
\end{aligned} \quad (39)$$

$$\begin{aligned}
\dot{\phi} &= (\tan \theta_o \sin \phi_o) \dot{q} + (\dot{\psi}_o \cos \theta_o - P_o \tan \theta_o) \theta \\
& + (\tan \theta_o \cos \phi_o) \dot{r} + p
\end{aligned} \quad (40)$$

$$\dot{\psi} = \left( \frac{\sin \phi_o}{\cos \theta_o} \right) \dot{q} + (\dot{\psi}_o \tan \theta_o) \theta + \left( \frac{\cos \phi_o}{\cos \theta_o} \right) \dot{r} \quad (41)$$

As can be seen, the numerical evaluation of the coefficient terms in these equations requires determination of the trim conditions and estimation of the application stability derivatives.

Initial (Trim) Conditions

The trim equations, (14) through (23), were used to compute initial conditions through iteration on  $\alpha_o$ ,  $T_o$ ,  $C_l$ ,  $C_m$ , and  $C_n$ . The iteration procedure was developed by personnel in the Department of Astronautics and Computer Science, U.S. Air Force Academy, and was expanded slightly for use in this study. The basis of the iteration was the requirement for constant state vectors at the trim condition. A listing of the computer program can be found in Appendix B.

Applying the restrictions listed on page 11 for level, steady, turning flight, the following equations were obtained:

$$\dot{U}_o = \frac{\rho_o S U_o^2}{2m} C_x + \frac{T_o}{m} - g \sin \theta_o - U_o Q_o \alpha_o \quad (42)$$

$$\begin{aligned} \dot{\alpha}_o = & \frac{\rho_o S U_o}{2m} (C_z - \alpha_o C_x) - \frac{\alpha_o T_o}{m U_o} + \frac{g}{U_o} (\cos \theta_o \cos \phi_o + \alpha_o \sin \theta_o) \\ & + Q_o (1 + \alpha_o^2) \end{aligned} \quad (43)$$

$$P_o = -\dot{\psi}_o \sin \theta_o \quad (44)$$

$$Q_o = \dot{\psi}_o \cos \theta_o \sin \phi_o \quad (45)$$

$$R_o = \dot{\psi}_o \cos \theta_o \cos \phi_o \quad (46)$$

$$\alpha_o = \tan \theta_o / \cos \phi_o \quad (47)$$

From Ref 8:203, the following relationships were obtained for the reference flight condition:

$$R = \frac{v_c^2}{g \tan \phi_0} \quad (48)$$

$$\dot{\psi}_0 = \frac{v_c}{R} \quad (49)$$

Using data from Ref 1, approximate values of  $\alpha_0$  and  $T_0$  were computed to begin the iterative process.

Iteration was first performed on equations (42) and (43). It was assumed that, for small disturbances,

$$\begin{bmatrix} \Delta \dot{u} \\ \Delta \dot{\alpha} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{u}}{\partial \alpha} & \frac{\partial \dot{u}}{\partial T} \\ \frac{\partial \dot{\alpha}}{\partial \alpha} & \frac{\partial \dot{\alpha}}{\partial T} \end{bmatrix} \begin{bmatrix} \Delta T \\ \Delta \alpha \end{bmatrix} \quad (50)$$

$$\begin{bmatrix} \Delta \dot{u} \\ \Delta \dot{\alpha} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{u}}{\partial \alpha} & \frac{\partial \dot{u}}{\partial T} \\ \frac{\partial \dot{\alpha}}{\partial \alpha} & \frac{\partial \dot{\alpha}}{\partial T} \end{bmatrix} \begin{bmatrix} \Delta T \\ \Delta \alpha \end{bmatrix} \quad (51)$$

The following equations were then solved for  $\Delta \alpha$  and  $\Delta T$  by Cramer's Rule.

$$\begin{bmatrix} \Delta \dot{u} \\ \Delta \dot{\alpha} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{u}}{\partial \alpha} & \frac{\partial \dot{u}}{\partial T} \\ \frac{\partial \dot{\alpha}}{\partial \alpha} & \frac{\partial \dot{\alpha}}{\partial T} \end{bmatrix} \begin{bmatrix} \Delta \alpha \\ \Delta T \end{bmatrix}$$

where  $\Delta \dot{u} = \dot{u}_0 - \dot{u}$  computed and  $\Delta \dot{\alpha} = \dot{\alpha}_0 - \dot{\alpha}$  computed. Final values for  $\alpha_0$  and  $T_0$  were obtained by setting  $\dot{u}_0 = \dot{\alpha}_0 = 0$  and noting that requiring  $\Delta \alpha = \Delta T = 0$  will result in  $\dot{u}$  computed  $= \dot{\alpha}$  computed  $= 0$ .

Values were thus obtained for  $\alpha_o$ ,  $\theta_o$ ,  $P_o$ ,  $Q_o$ , and  $R_o$ .  $C_x$  and  $C_z$  were determined by summing forces along the respective axes, giving

$$C_x = C_T + C_L \sin \alpha_o - C_D \cos \alpha_o \quad (52)$$

$$C_z = -C_L \cos \alpha_o - C_D \sin \alpha_o \quad (53)$$

Trim conditions were then impressed upon equations (16), (18), (19), and (20) and these equations solved for  $C_m$ ,  $C_y$ ,  $C_l$ , and  $C_n$ . A second iterative procedure was performed using equations (16), (19), and (20). The matrix which was used to solve for final values of  $C_l$ ,  $C_m$ , and  $C_n$  was

$$\begin{bmatrix} \dot{\Delta p} \\ \dot{\Delta q} \\ \dot{\Delta r} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{p}}{\partial C_l} & \frac{\partial \dot{p}}{\partial C_m} & \frac{\partial \dot{p}}{\partial C_n} \\ \frac{\partial \dot{q}}{\partial C_l} & \frac{\partial \dot{q}}{\partial C_m} & \frac{\partial \dot{q}}{\partial C_n} \\ \frac{\partial \dot{r}}{\partial C_l} & \frac{\partial \dot{r}}{\partial C_m} & \frac{\partial \dot{r}}{\partial C_n} \end{bmatrix} \begin{bmatrix} \Delta C_l \\ \Delta C_m \\ \Delta C_n \end{bmatrix}$$

where  $\dot{\Delta p} = \dot{P}_o - \dot{p}$  computed,  $\dot{\Delta q} = \dot{Q}_o - \dot{q}$  computed, and  $\dot{\Delta r} = \dot{R}_o - \dot{r}$  computed.

### Stability Derivatives

Formulas used in estimating stability derivatives are presented in Table I.

Table 1. Stability Derivatives

| Formula                | Source       | Restrictions                                                                                            |
|------------------------|--------------|---------------------------------------------------------------------------------------------------------|
| $C_{x_u}$              | Ref 6:148-52 | $C_{D_M} = n$ , $T(U_o + u) = \text{const}$ ,<br>Neglect Aeroelastic Effects                            |
| $C_{z_u}$              | Ref 1:35     | $C_{L_u} = \frac{-\alpha C_{L_\alpha}}{1 - M^2}$ , $C_{D_u} = -\alpha C_{D_\alpha}$<br>Body Axis System |
| $C_{m_u}$              | Ref 6:151-2  | Neglect Aeroelastic Effects<br>$-0.1 \leq C_m \leq 0.1$                                                 |
|                        | Ref 1:39, 41 | Stability Axis System                                                                                   |
| $C_{x_\alpha}$         | Ref 1:39, 41 | Body Axis System                                                                                        |
| $C_{z_\alpha}$         |              |                                                                                                         |
| $C_{m_\alpha}$         | Ref 8:392    | Stability Axis System,<br>$T_c \leq 0.2$ , $-0.1 \leq C_m \leq 0.1$                                     |
| $C_{x_{\dot{\alpha}}}$ | Ref 6:158-65 | 10% Added for Wing-Body Contribution                                                                    |
| $C_{z_{\dot{\alpha}}}$ | Ref 4:369    |                                                                                                         |
| $C_{m_{\dot{\alpha}}}$ | Ref 3:54, 55 | Stability Axis System                                                                                   |

Table 1. (continued)

| $C_{x_q}$          | Negligible                                                                                                 | Ref 4:376      |                           |
|--------------------|------------------------------------------------------------------------------------------------------------|----------------|---------------------------|
| $C_{z_q}$          | $-2.2 \frac{l_t}{\bar{c}} \eta_t$                                                                          | Ref 6:153-55   | Stability Axis System     |
| $C_{m_q}$          | $-2.2 \left(\frac{l_t}{\bar{c}}\right)^2 \eta_t$                                                           | Ref 3:54,55    |                           |
| $C_{x_{\delta e}}$ | -.080                                                                                                      | Ref 3:40       | $\delta_e \pm 10$ degrees |
| $C_{z_{\delta e}}$ | $-\frac{\bar{c}}{l_t} \eta_t C_{m_{\delta e}}$                                                             | Ref 3:44       | Stability Axis System     |
| $C_{m_{\delta e}}$ | -1.644                                                                                                     | Ref 1:68-9     |                           |
| $C_{y_\beta}$      | $-.888 + .345F - (1.72 - 2.91F)T_c$<br>for $M \geq .225$ , $C_{y_\beta} = C_{y_\beta} (1 + .15(M - .225))$ | Ref 1:78,79,81 |                           |
| $C_{l_\beta}$      | $-.0544 + .0115F + (-.0014 + .0029F)\alpha_o, \alpha_o$ in degrees                                         | Ref 1:94-7     | $\beta \pm 8$ degrees     |
| $C_{n_\beta}$      | $.0785 + .0344F - .0859T_c$<br>for $M \geq .225$ , $C_{n_\beta} = C_{n_\beta} (1 + .2(M - .225))$          | Ref 1:84-7     | Stability Axis System     |

Table 1. (continued)

|                |                                                                                                                                                                                                          |                                      |
|----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------|
| $C_{y\beta t}$ | $-.645 + .0650F + (1.47 - 2.81F)T_C - (1.75 - 2.03F)T_C^2$<br>for $M > .225$ , $C_{y\beta t} = C_{y\beta t} (1 + .2(M - .225))$ Ref 3:94, 119                                                            | -4 degrees $\leq T_C \leq 8$ degrees |
| $C_{l\beta t}$ | $-.0638 - .0034F - (.023 + .090F)T_C - (.090 + .500F)T_C^2$<br>Ref 3:120                                                                                                                                 | $T_C \leq 0.2$                       |
| $C_{n\beta t}$ | $.218 - .0343F + (.745F - .430)T_C$<br>for $M > .225$ , $C_{n\beta t} = C_{n\beta t} (1 + .4(M - .225))$ Ref 1:105                                                                                       | Stability Axis System                |
| $C_{yp}$       | $-.0167C_{L_w} - 2\alpha_o \left(\frac{l_f}{b}\right)C_{y\beta t}$ Ref 5:7.4.2.1                                                                                                                         |                                      |
| $C_{lp}$       | $-.530$ $M < .225$<br>$-.530 - .114(M - .225)$ , $M \geq .225$ Ref 1:99                                                                                                                                  | Stability Axis System                |
| $C_{np}$       | $-.0015 - .09C_{L_w} - 2\left(\frac{l_f}{b}\right)\alpha_o C_{n\beta t}$ Ref 5:7.4.2.3                                                                                                                   |                                      |
| $C_{yr}$       | $2 C_{n\beta t}$ Ref 5:7.4.3.1                                                                                                                                                                           |                                      |
| $C_{lr}$       | $.29 \left[ 1 + \frac{10.09M^2}{20.18(1-M^2) + 4\sqrt{1-M^2}} \right] C_{L_w} + .0518F$<br>$- C_{l\beta} - .0167C_{y_r} + 2 \left[ \frac{C_{l\beta t} C_{n\beta t}}{C_{y\beta t}} \right]$ Ref 5.7.4.3.2 | Stability Axis System                |

Table 1. (continued)

| $C_{n_r} = .02C_{L_w}^2 - .3C_{D_{O_w}} + 2\left[\frac{C_{n_{\beta_t}}}{C_{y_{\beta_t}}}\right]$ |                                                                                                                              | Ref 5:7.4.3.3                                                                                                      | Stability Axis System                                        |
|--------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------|
| $C_{y_{\delta_a}}$                                                                               | negligible                                                                                                                   | Ref 3:88                                                                                                           |                                                              |
| $C_{l_{\delta_a}}$                                                                               | $-.0602 - .0170F + .0290(C_L - .8)$<br>$-.0544 - .0172F + (.0140 + .03F)(C_L - 1.0)$<br>$-.0516 - .0114F + .0290(C_L - 1.2)$ | $, 0.8 \leq C_L < 1.0$<br>$, 1.0 \leq C_L < 1.2$<br>$, 1.2 \leq C_L < 1.6$                                         | $\delta_a$ (total) $\pm 15$ degrees<br>Stability Axis System |
| $C_{n_{\delta_a}}$                                                                               | $.00332 + .00115\alpha_o$<br>$.00556 + .00072(\alpha_o - 2)$                                                                 | $, 0 \text{ degrees} \leq \alpha_o < 2 \text{ degrees}$<br>$, 2 \text{ degrees} \leq \alpha_o < 8 \text{ degrees}$ | Ref 1:105-6                                                  |
| $C_{y_{\delta_r}}$                                                                               | 0.264                                                                                                                        | Ref 1:108                                                                                                          |                                                              |
| $C_{l_{\delta_r}}$                                                                               | $-.0264 - .0015\alpha_o, \alpha_o$ in degrees                                                                                | Ref 1:88-90                                                                                                        | $\delta_r \pm 10$ degrees<br>Stability Axis System           |
| $C_{n_{\delta_r}}$                                                                               | -.0917                                                                                                                       |                                                                                                                    |                                                              |

Because the model was to be referred to the body axes, the following transformations (Ref 10:57-8) were then applied to those stability derivatives which were originally referenced to the stability axes.

$$\begin{aligned}
 C_{m_{u_B}} &= C_{m_u} - \alpha_o C_{m_\alpha} & C_{l_{\beta_B}} &= C_l - \alpha_o C_n \\
 C_{m_{\alpha_B}} &= C_{m_\alpha} + \alpha_o C_{m_u} & C_{n_{\beta_B}} &= C_n + \alpha_o C_l \\
 C_{x_{q_B}} &= C_{x_q} - \alpha_o C_{z_q} & C_{l_{p_B}} &= C_{l_p} - \alpha_o (C_{l_r} + C_{n_p}) + \alpha_o^2 C_{n_r} \\
 C_{z_{q_B}} &= C_{z_q} + \alpha_o C_{x_q} & C_{n_{p_B}} &= C_{n_p} + \alpha_o (C_{l_p} - C_{n_r}) - \alpha_o^2 C_{l_r} \\
 C_{x_{\dot{\alpha}_B}} &= C_{x_{\dot{\alpha}}} - \alpha_o C_{z_{\dot{\alpha}}} & C_{l_{r_B}} &= C_{l_r} + \alpha_o (C_{l_p} - C_{n_r}) - \alpha_o^2 C_{n_p} \\
 C_{z_{\dot{\alpha}_B}} &= C_{z_{\dot{\alpha}}} + \alpha_o C_{x_{\dot{\alpha}}} & C_{n_{r_B}} &= C_{n_r} + \alpha_o (C_{l_r} + C_{n_p}) + \alpha_o^2 C_{l_p} \\
 C_{x_{\delta e_B}} &= C_{x_{\delta e}} - \alpha_o C_{z_{\delta e}} & C_{l_{\delta a_B}} &= C_{l_{\delta a}} - \alpha_o C_{n_{\delta a}} \\
 C_{z_{\delta e_B}} &= C_{z_{\delta e}} + \alpha_o C_{x_{\delta e}} & C_{n_{\delta a_B}} &= C_{n_{\delta a}} + \alpha_o C_{l_{\delta a}} \\
 C_{y_{p_B}} &= C_{y_p} - \alpha_o C_{y_r} & C_{l_{\delta r_B}} &= C_{l_{\delta r}} - \alpha_o C_{n_{\delta r}} \\
 C_{y_{r_B}} &= C_{y_r} + \alpha_o C_{y_p} & C_{n_{\delta r_B}} &= C_{n_{\delta r}} + \alpha_o C_{l_{\delta r}}
 \end{aligned}$$

All other derivatives were either unchanged or were initially defined in the body axis system.

Model Characteristics

After initial conditions and stability derivatives had been evaluated, equations (33) through (41) were arranged in matrix form

$$\dot{X} = A X$$

where  $\dot{X}$  was the 9x1 matrix of time derivatives ( $\dot{u}$ ,  $\dot{\alpha}$ ,  $\dot{q}$ ,  $\dot{\theta}$ ,  $\dot{\beta}$ ,  $\dot{p}$ ,  $\dot{r}$ ,  $\dot{\phi}$ ,  $\dot{\psi}$ ),  $X$  was the 12x1 matrix of state vectors and control terms ( $u$ ,  $\alpha$ ,  $q$ ,  $\theta$ ,  $\beta$ ,  $p$ ,  $r$ ,  $\phi$ ,  $\psi$ ,  $\delta a$ ,  $\delta e$ ,  $\delta r$ ), and  $A$  was the 9x12 coefficient matrix determined by trim conditions and the stability derivatives.

Mode parameters were estimated by factoring the coefficient matrix  $A$ , using an eigenvalue program from Ref 11. The procedure yielded one zero root, two real roots, and three complex pairs. Factoring the complex roots gave the natural (undamped) frequencies and damping ratios for the Phugoid, Short Period, and Dutch Roll modes. The real roots determined the time constants for the Rolling and Spiral modes.

The perturbation equations were then integrated using a fourth-order Runge-Kutta routine. The subroutines were arranged so that the perturbed response of the state vectors could be estimated from a single control deflection or any combination of aileron, elevator, and rudder deflections. The computer programs used to determine the above characteristics are listed in Appendix D.

C130 A and E Model Differences

Only two parameters differ significantly between the two aircraft, both due to the fact that different propellers are used. The propeller moment of inertia ( $I$ ) and the specific thrust ( $T_c$ ) for each aircraft are listed in Appendix A.

#### IV. Results

##### Comparison of AC130E Models

Response to a one-second pulse of 0.1 radian was investigated for each of the primary control surfaces. Perturbations produced by the model developed in this study were compared to those produced by an existing model. The trim condition was turning flight at a constant altitude of 10,500 feet, at 28 degrees of bank, at a gross weight of 110,000 lb, and with C.G. at 25% MAC. Model parameters are listed in Tables II and III, and results of the computer simulations are presented in Appendix C.

A modified sensitivity analysis was performed by equating pairs of parameters which differed considerably and noting any subsequent change in response. Table IV contains the results of this analysis.

Compared to the existing model, the proposed model showed an increase in damping of phugoid oscillations. The primary dutch roll oscillations of the proposed model were generally weaker, but the  $\beta$  and  $r$  response to aileron deflection showed a marked decrease in dutch roll damping. Pronounced dutch roll was evident in several of the cross coupled perturbations of the proposed model, but the magnitudes of these oscillations were quite small. The divergent mode tended to be weaker in the proposed model. The short period oscillations of both models were identical.

Table II. Model Parameters - AC130E

| Proposed   |                   | Present  | Proposed           |          | Present  | Proposed           |         | Present |
|------------|-------------------|----------|--------------------|----------|----------|--------------------|---------|---------|
| I          | 175.4             | 0.000    | $C_m$              | -0.00216 | -0.00002 | $C_{m\delta e}$    | -1.644  | -1.634  |
| $I_x$      | $1.4 \times 10^6$ | same     | $C_n$              | -0.00012 | -0.00000 | $C_{y\beta}$       | -0.747  | -0.745  |
| $I_y$      | $9.7 \times 10^5$ | same     | $C_{x_u}$          | -0.261   | -0.145   | $C_{l\beta}$       | -0.0464 | -0.0461 |
| $I_z$      | $2.2 \times 10^6$ | same     | $C_{z_u}$          | 0.154    | -0.0757  | $C_{n\beta}$       | 0.0870  | 0.0819  |
| $I_{xz}$   | $6.1 \times 10^4$ | same     | $C_{m_u}$          | 0.0354   | -0.0167  | $C_{y_p}$          | -0.134  | 0.000   |
| $\alpha_o$ | 0.0307            | 0.0276   | $C_{x_\alpha}$     | 0.752    | 0.760    | $C_{l_p}$          | -0.542  | -0.539  |
| $\theta_o$ | 0.0271            | 0.0244   | $C_{z_\alpha}$     | -7.544   | -7.343   | $C_{n_p}$          | -0.1630 | -0.0391 |
| $\phi_o$   | -0.489            | same     | $C_{m_\alpha}$     | -1.739   | -1.600   | $C_{y_r}$          | 0.386   | 0.404   |
| $\psi_o$   | -0.0578           | same     | $C_{x_\delta}$     | 0.0982   | 0.0000   | $C_{l_r}$          | 0.323   | 0.263   |
| $P_o$      | 0.00156           | 0.00141  | $C_{z_\delta}$     | -3.203   | -3.619   | $C_{n_r}$          | -0.174  | -0.147  |
| $Q_o$      | 0.0271            | same     | $C_{m_\delta}$     | -10.263  | -11.530  | $C_{y_{\delta a}}$ | 0.000   | 0.000   |
| $R_o$      | -0.0510           | same     | $C_{x_q}$          | 0.240    | 0.000    | $C_{l_{\delta a}}$ | -0.0648 | -0.0637 |
| $C_x$      | 0.0289            | 0.0259   | $C_{z_q}$          | -7.822   | -7.870   | $C_{n_{\delta a}}$ | 0.0034  | 0.0000  |
| $C_y$      | -0.00037          | -0.00015 | $C_{m_q}$          | -25.067  | -25.100  | $C_{y_{\delta r}}$ | 0.263   | 0.266   |
| $C_z$      | -0.943            | same     | $C_{x_{\delta e}}$ | -0.0625  | 0.0000   | $C_{l_{\delta r}}$ | 0.0265  | 0.0252  |
| $C_l$      | -0.0001           | same     | $C_{z_{\delta e}}$ | -0.572   | -1.125   | $C_{n_{\delta r}}$ | -0.0910 | -0.0944 |

Table III  
Mode Parameters - AC130E Models

| <u>Parameter</u>         | <u>Present Model</u> | <u>Proposed Model</u> |
|--------------------------|----------------------|-----------------------|
| <b>Phugoid Mode</b>      |                      |                       |
| $w_n$ (rad/sec)          | 0.146                | 0.144                 |
|                          | 0.001                | 0.043                 |
| <b>Short Period Mode</b> |                      |                       |
| $w_n$ (rad/sec)          | 1.984                | 2.055                 |
|                          | 0.636                | 0.606                 |
| <b>Dutch Roll Mode</b>   |                      |                       |
| $w_n$ (rad/sec)          | 0.866                | 0.968                 |
|                          | 0.215                | 0.253                 |
| <b>Rolling Mode</b>      |                      |                       |
| Time Constant (sec)      | 0.663                | 0.686                 |
| <b>Spiral Mode</b>       |                      |                       |
| Time Constant (sec)      | 69.31                | 66.09                 |

Table IV. Perturbations of Proposed AC130E Model

| Response                                                                                                                                | Comparison with Existing Model | Reason                               |
|-----------------------------------------------------------------------------------------------------------------------------------------|--------------------------------|--------------------------------------|
| $\frac{\alpha}{\delta a}$                                                                                                               | Less dutch roll damping        | I                                    |
| $\frac{\theta}{\delta a}$                                                                                                               | Less dutch roll damping        | $C_{n_p}$                            |
| $\frac{u}{\delta a}, \frac{\alpha}{\delta a}, \frac{q}{\delta a}, \frac{\theta}{\delta a}$                                              | More phugoid damping           | $C_{x_u}, C_{m_u}, C_{n_p}$          |
| $\frac{\beta}{\delta a}, \frac{r}{\delta a}, \frac{\phi}{\delta a}, \frac{\psi}{\delta a}$                                              | Less dutch roll damping        | $C_{n_p}$                            |
| $\frac{u}{\delta e}, \frac{\theta}{\delta e}$                                                                                           | More phugoid damping           | $C_{x_u}, C_{n_p}$                   |
| $\frac{\beta}{\delta e}, \frac{p}{\delta e}, \frac{r}{\delta e}$                                                                        | Less dutch roll damping        | I                                    |
| $\frac{\beta}{\delta e}, \frac{r}{\delta e}$                                                                                            | More phugoid damping           | $C_{x_u}, C_{l_r}, C_{n_r}, C_{n_p}$ |
| $\frac{\phi}{\delta e}, \frac{\psi}{\delta e}$                                                                                          | More divergence                | $C_{l_r}$                            |
| $\frac{\alpha}{\delta r}, \frac{q}{\delta r}$                                                                                           | Less dutch roll damping        | I                                    |
| $\frac{u}{\delta r}, \frac{\alpha}{\delta r}, \frac{q}{\delta r}, \frac{\theta}{\delta r}$                                              | More phugoid damping           | $C_{m_u}, C_{n_p}$                   |
| $\frac{\theta}{\delta r}, \frac{\beta}{\delta r}, \frac{p}{\delta r}, \frac{r}{\delta r}, \frac{\phi}{\delta r}, \frac{\psi}{\delta r}$ | More dutch roll damping        | $C_{n_p}$                            |
| $\frac{\phi}{\delta r}, \frac{\psi}{\delta r}$                                                                                          | Less divergence                | $C_{m_u}, C_{l_r}, C_{n_p}$          |

Mode Parameters and Stability Derivatives, AC130A

Selected mode parameters and stability derivatives were calculated for the AC130A aircraft in a level, left-hand turn at 30 degrees of bank, altitudes from 6000 feet to 15,000 feet in increments of 1000 feet, gross weights of 100,000 lb, 110,000 lb, and 120,000 lb, and C.G. locations of 15% MAC, 25% MAC (nominal), and 30% MAC. Results are presented graphically in Appendix D. From these data, probable ranges of the selected quantities were estimated to be

## Phugoid Mode

$w_n$  (rad/sec)-----0.115 to 0.145

$\xi$ -----0.025 to 0.050

## Short Period Mode

$w_n$  (rad/sec)-----2.200 to 2.700

$\xi$ -----0.550 to 0.650

## Dutch Roll Mode

$w_n$  (rad/sec)-----0.850 to 1.250

$\xi$ -----0.180 to 0.300

## Rolling Mode

time constant (sec)-----0.400 to 0.900

## Spiral Mode

time constant (sec)-----50.00 to 70.00

$C_{x_u}$ ----- -0.260 to -0.190

$C_{z_u}$ ----- -0.150 to 0.150

$C_{m_u}$ ----- 0.040 to 0.040

$C_{x_\alpha}$ ----- 0.350 to 0.770

|                        |         |            |         |
|------------------------|---------|------------|---------|
| $C_{z_{\alpha}}$       | -7.700  | to         | -7.500  |
| $C_{m_{\alpha}}$       | -1.780  | to         | -1.730  |
| $C_{x_{\alpha}}$       | -0.040  | to         | 0.100   |
| $C_{z_{\dot{\alpha}}}$ | -3.100  | to         | -2.850  |
| $C_{m_{\dot{\alpha}}}$ | -9.900  | to         | -9.200  |
| $C_{x_q}$              | -0.080  | to         | 0.250   |
| $C_{z_q}$              | -7.700  | to         | -7.400  |
| $C_{m_q}$              | -25.000 | to         | -23.500 |
| $C_{x_{\delta e}}$     | -0.090  | to         | -0.060  |
| $C_{z_{\delta e}}$     | -0.564  | to         | -0.540  |
| $C_{m_{\delta e}}$     | -1.644  | (constant) |         |
| $C_{y_{\beta}}$        | -0.749  | to         | -0.740  |
| $C_{l_{\beta}}$        | -0.050  | to         | -0.045  |
| $C_{n_{\beta}}$        | 0.088   | to         | 0.093   |
| $C_{y_p}$              | -0.134  | (constant) |         |
| $C_{l_p}$              | -0.550  | to         | -0.535  |
| $C_{n_p}$              | -0.170  | to         | -0.110  |
| $C_{y_r}$              | 0.388   | to         | 0.396   |
| $C_{l_r}$              | 0.396   | to         | 0.420   |
| $C_{n_r}$              | -0.173  | to         | -0.155  |
| $C_{l_{\delta a}}$     | -0.075  | to         | -0.065  |
| $C_{n_{\delta a}}$     | 0.00328 | to         | 0.00344 |
| $C_{y_{\delta r}}$     | 0.264   | (constant) |         |
| $C_{l_{\delta r}}$     | 0.0264  | (constant) |         |
| $C_{n_{\delta r}}$     | -0.0920 | to         | -0.0910 |

## V. Conclusions and Recommendations

The only dramatic differences between the two AC130E models occurred in cross-coupled responses which were at least one order of magnitude less than their respective primary perturbations. It is concluded that the existing model is reasonably accurate, based on available data; however, the qualitative comparison provided the following suggestions:

1. Significant cross-coupling occurs in the  $\frac{\theta}{\delta a}, \frac{\theta}{\delta r}, \frac{\phi}{\delta e}$ , and  $\frac{\psi}{\delta e}$  responses, but these are unaffected by the gyroscopic coupling due to propellers. On this basis, the gyroscopic coupling is negligible; however, if other cross-coupling is to be considered, these effects should be included, as the phenomenon substantially increases the dutch roll in several of the smaller cross-coupled reactions.

2. The u-derivatives should be more rigidly defined and the contribution to  $C_{x_u}$  and  $C_{z_u}$  due to  $\partial \alpha / \partial u$  in the body axis system, as well as the quantitative change in  $C_{m_u}$  due to axis transformation from stability to body axes, should be taken into account. These effects were not included in the existing model.

3. Flight test verification is needed to determine values for the rotary derivatives. None of the values used in either model (except  $C_{l_p}$ ) can be analytically justified as all were obtained from empirical formulae.

Further study is necessary to provide information in the following areas:

1. Further computer simulation using different combinations of control deflections.
2. Possible changes in stability parameters to account for aeroelastic phenomena and the effects thereof on model response.

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## Appendix A

Basic Data

(from Ref 1, 2, and 3)

|           | <u>Wing</u> | <u>Horizontal<br/>Tail</u> | <u>Vertical<br/>Tail</u> | <u>Units</u>    |
|-----------|-------------|----------------------------|--------------------------|-----------------|
| Area      | 1745.50     | 545.00                     | 300.00                   | ft <sup>2</sup> |
| Span      | 132.60      | 52.70                      | 23.10                    | ft              |
| MAC       | 13.71       | 11.18                      | 14.82                    | ft              |
| AR        | 10.09       | 5.02                       | 1.78                     |                 |
| TR        | 0.52        | 0.37                       | 0.30                     |                 |
| Dihedral  | 1.50        | 0.00                       | N/A                      | degrees         |
| Incidence | 1.50        | -1.75                      | N/A                      | degrees         |
| Twist     | -3.00       | 0.00                       | N/A                      | degrees         |

Fuselage

|                        |                   |                 |
|------------------------|-------------------|-----------------|
| Max Length             | 97.74             | ft              |
| Frontal Area           | 180.00            | ft <sup>2</sup> |
| Horizontal Tail Length | 47.33-C.G.(13.71) | ft              |
| Vertical Tail Length   | 46.03-C.G.(13.71) | ft              |

Propeller

RPM 106.80 rad/sec

|          |         |       |    |
|----------|---------|-------|----|
| Diameter | A Model | 16.00 | ft |
|          | E Model | 13.50 | ft |

|   |         |        |                      |
|---|---------|--------|----------------------|
| I | A Model | 139.00 | slug/ft <sup>2</sup> |
|   | E Model | 175.40 | slug/ft <sup>2</sup> |

$$T_c = 0.375 - 0.01A - (0.00338 - 0.0009A)\Delta V \quad (a)$$

$$T_c = 0.275 - 0.008(A-10) - [0.00248 - 0.00007(A-10)]\Delta V \quad (b)$$

where (a) is valid for  $A \leq 10000$  ft, (b) is valid for

$10000 \text{ ft} < A \leq 20000 \text{ ft}$ , and  $V = \text{TAS}-120$  (knots).

$$C_T = \frac{8d^2}{S} T_c \rightarrow \begin{array}{ll} C_T = 1.032 T_c & \text{A Model} \\ C_T = 0.836 T_c & \text{E Model} \end{array}$$

$$\begin{aligned} C_{L_o} &= 0.25 + 0.12T_c + 0.90F \\ C_L &= C_{L_o} + \alpha C_{L_\alpha}, \quad C_{L_\alpha} = 6.30 + 0.435T_c + 2.00F, \quad M \leq 0.225 \\ C_{L_\alpha} &= C_{L_\alpha(0.225)} [1 + 0.33 (M-0.225)], \quad M > 0.225 \end{aligned}$$

$$C_D = [C_{D_o} + K(C_L^2 - 0.20)](1 + 0.18FC_T), \quad \begin{array}{ll} C_{D_o} &= 0.030 + 0.048F \\ K &= 0.044 - 0.030F \end{array}$$

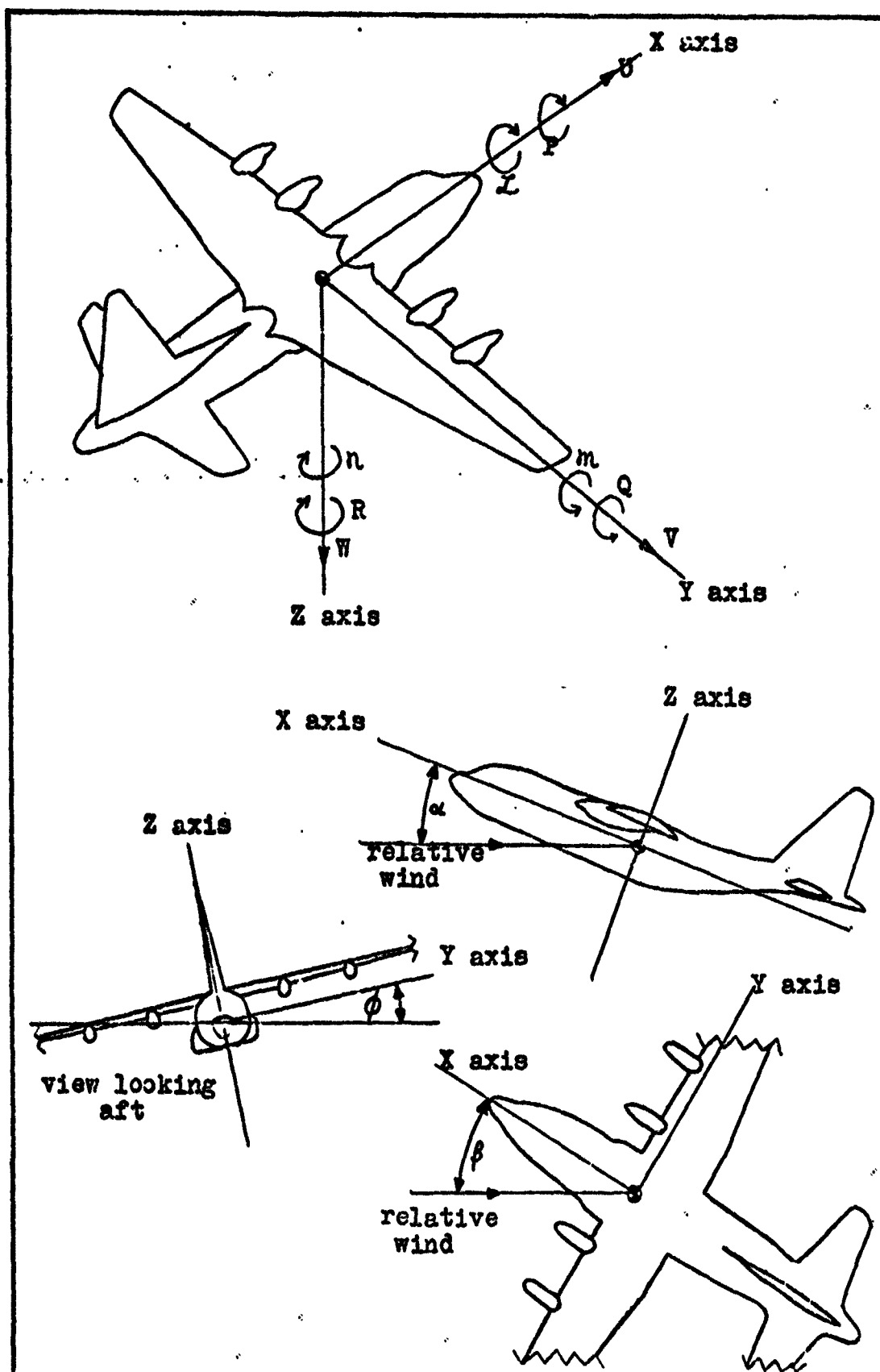


Figure 1. Body Axis System

Appendix B

C130E Model Response to Control Deflection

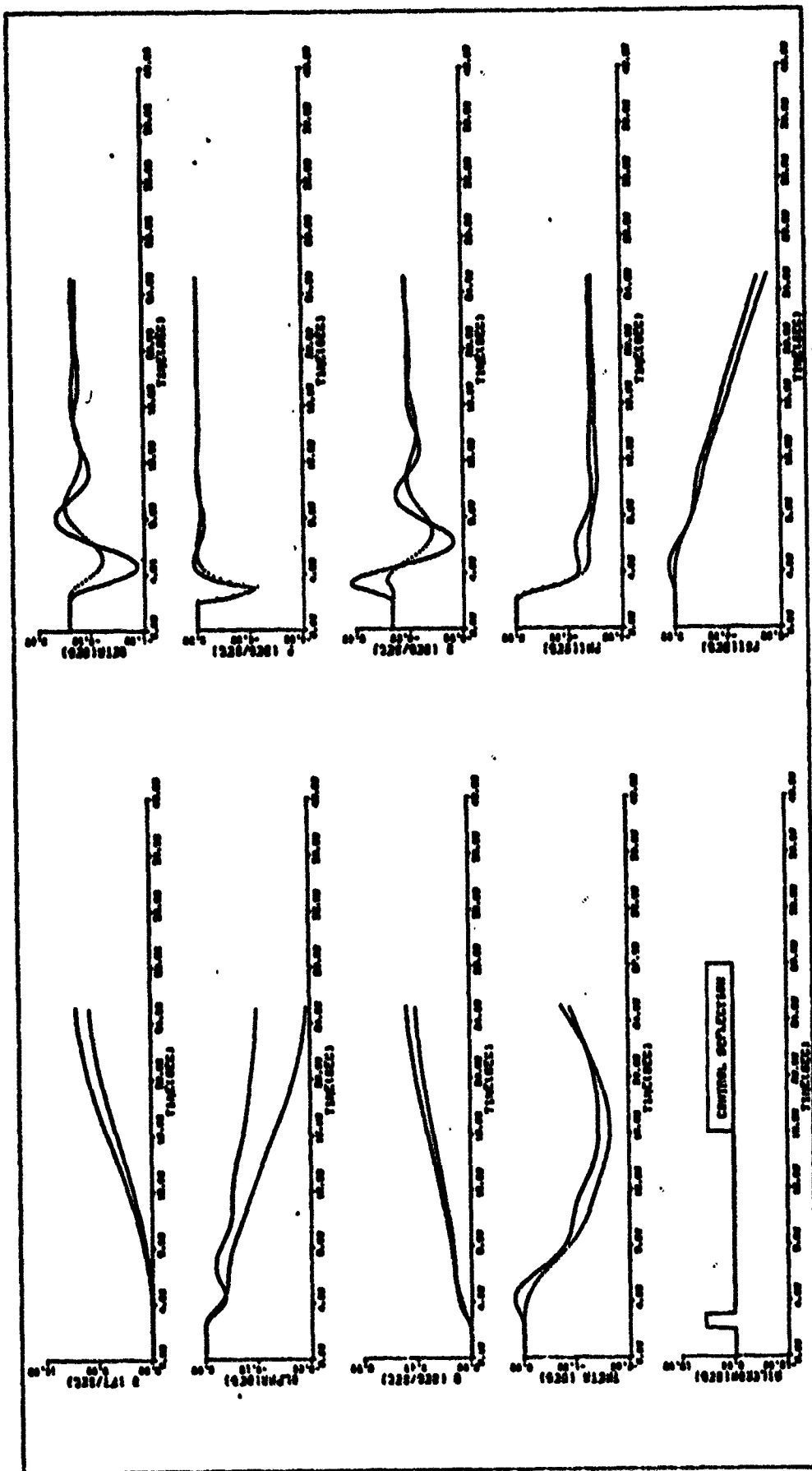


FIG. 2 - CURRENT VERSION TO PREVIOUS VERSION SECTION  
 PREVIOUS VERSION .....  
 CURRENT VERSION .....

Reproduced from  
 best available copy.

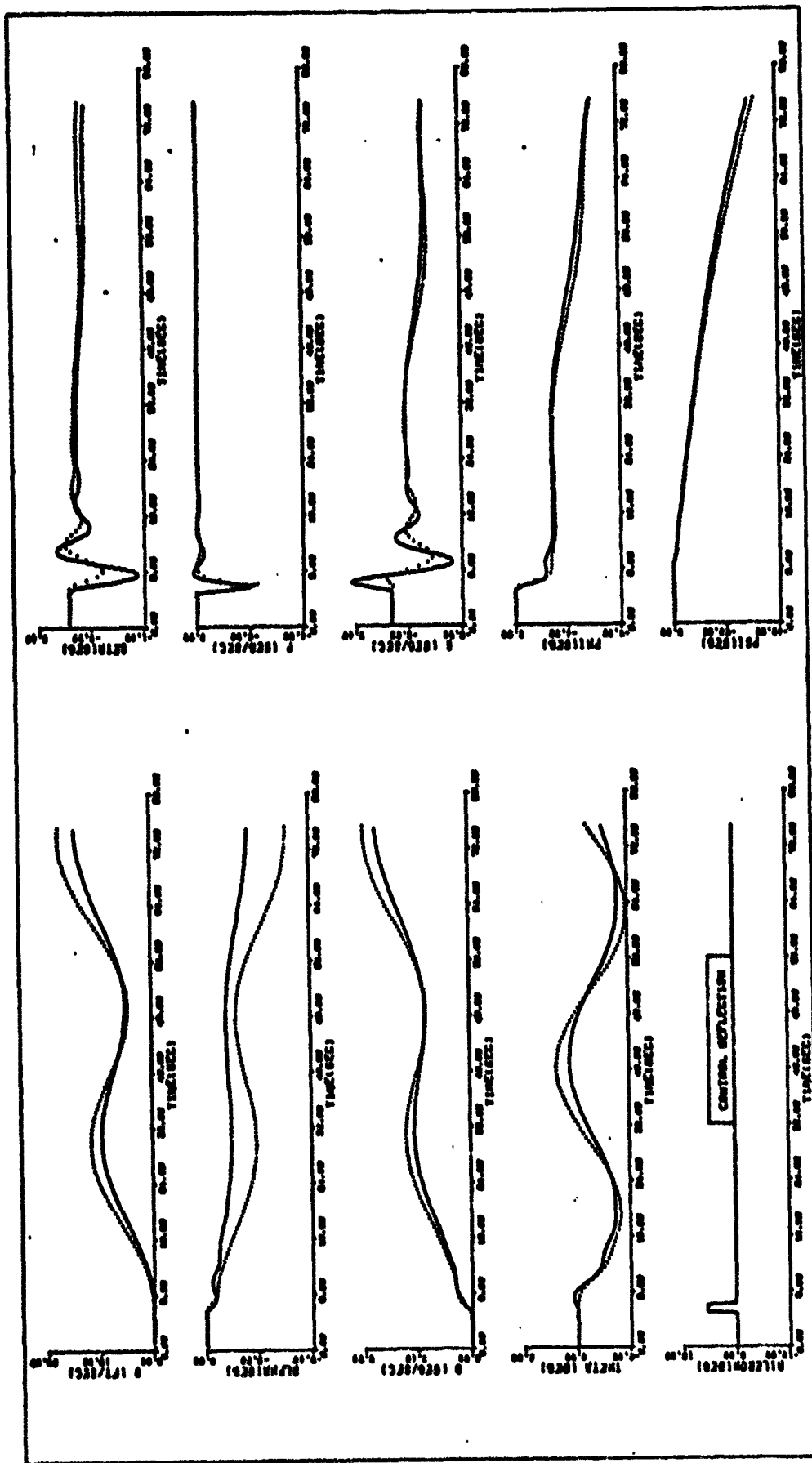


FIG. 3 - SURFACE DEFLECTION TO CONTROL SURFACE DEFLECTION  
 PERFECT MODEL .....  
 CONTROL SURFACE DEFLECTION

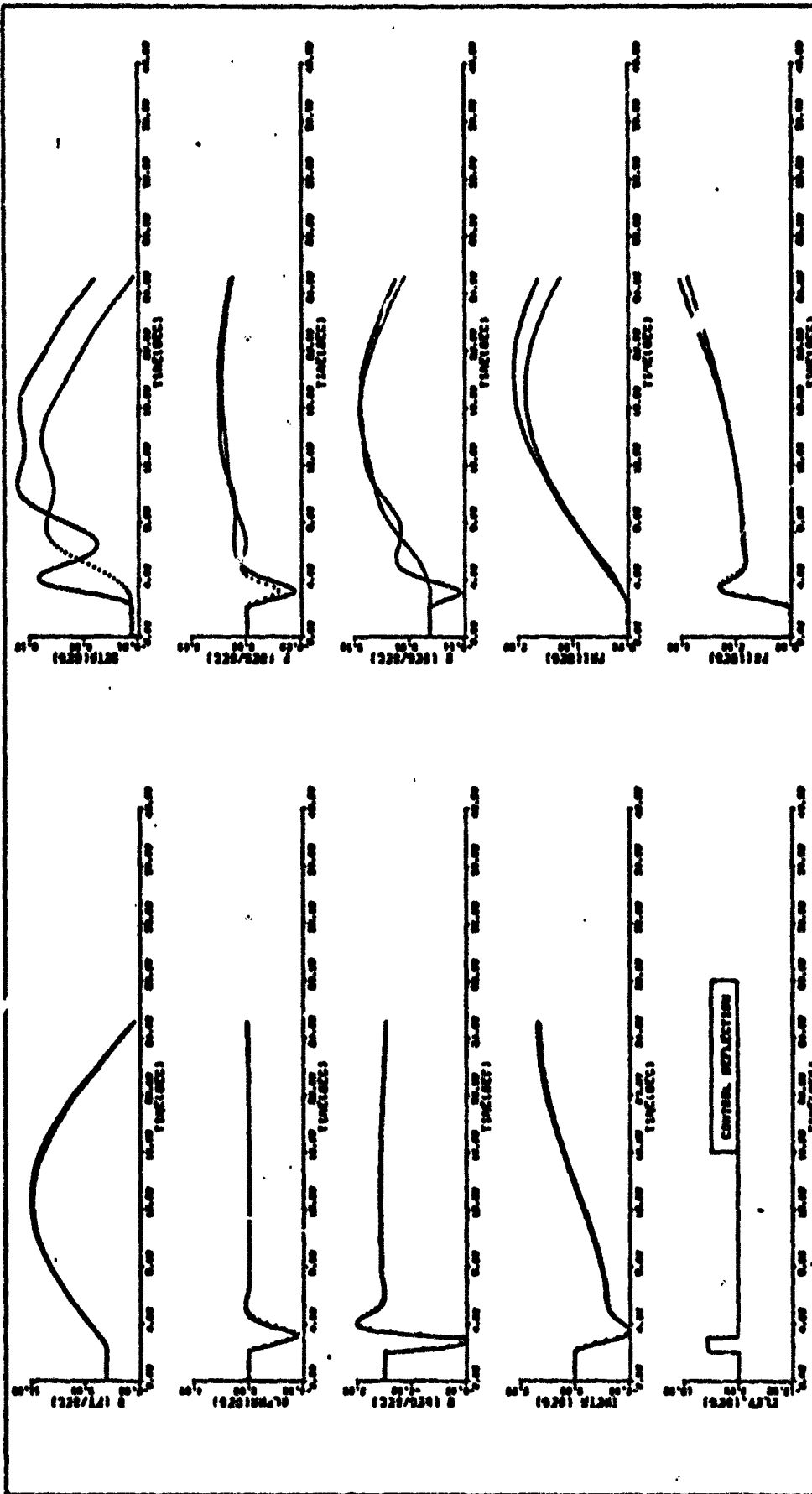


FIG. 4 - AIRCRAFT RESPONSE TO CONTROL SURFACE DEFLECTION  
 ..... FREQUENCY AND LOAD FACTOR FEEDBACK  
 ———— CONTROL SURFACE DEFLECTION

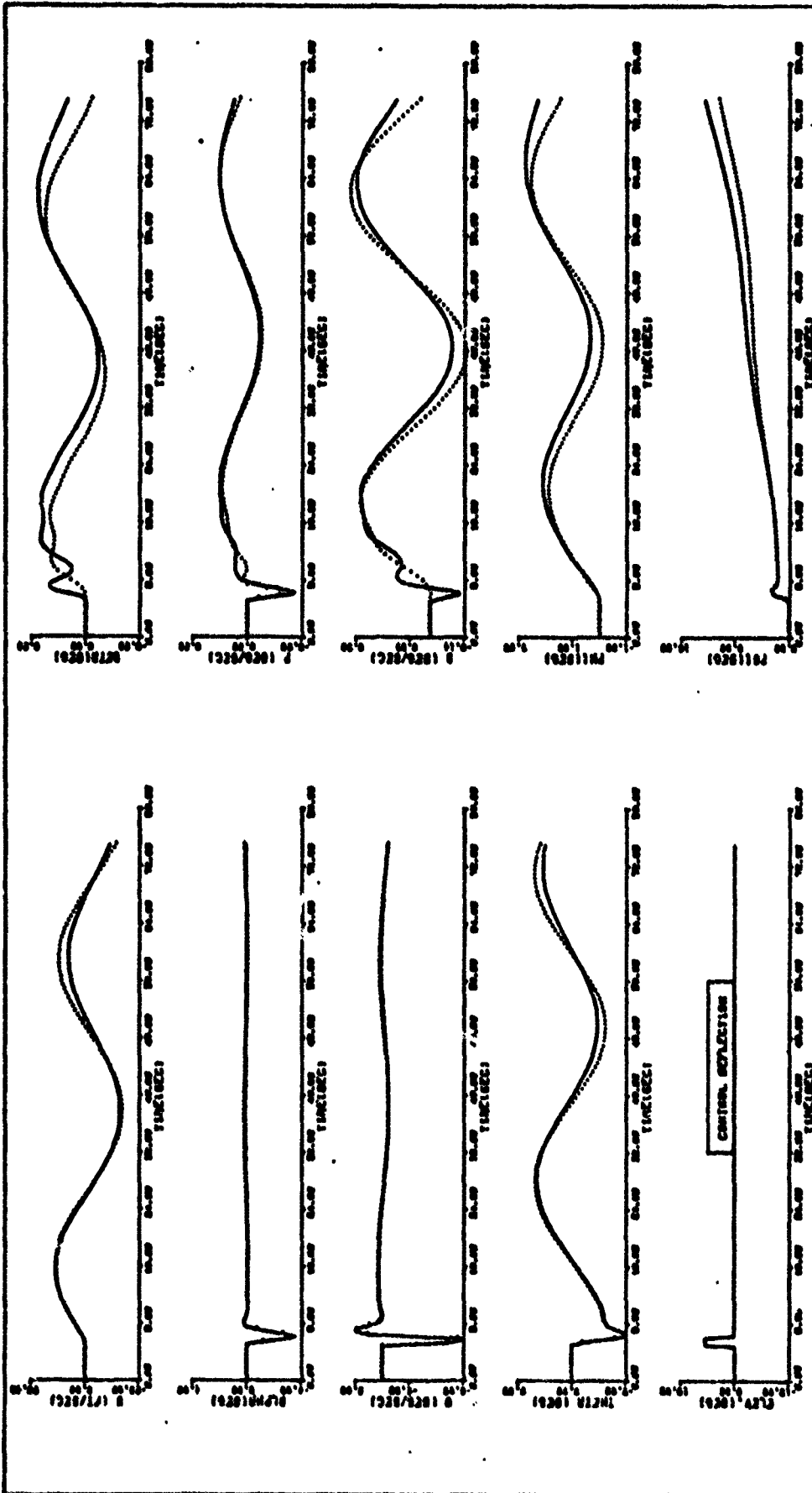


FIG. 5. AIRCRAFT RESPONSE TO CONTROL SURFACE DEFLECTION  
 PRESENT MODEL .....  
 LOAD MODEL .....  
 PROPOSED MODEL .....

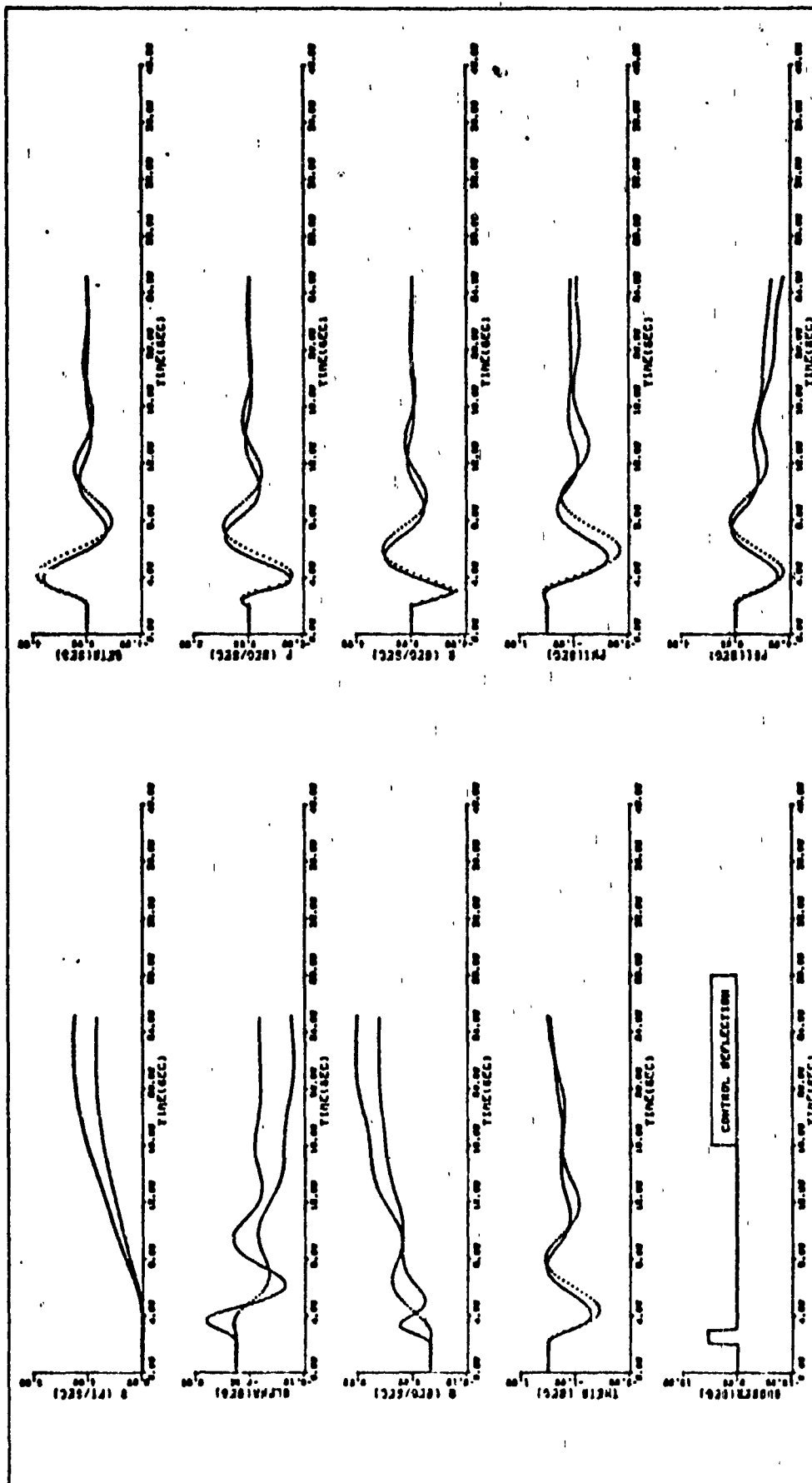


FIG. 6 - REFLECTED REFERENCE TO CONTROL SURFACE REFLECTION  
 LEARNED REFLECTED MODEL  
 CONTROL REFLECTION

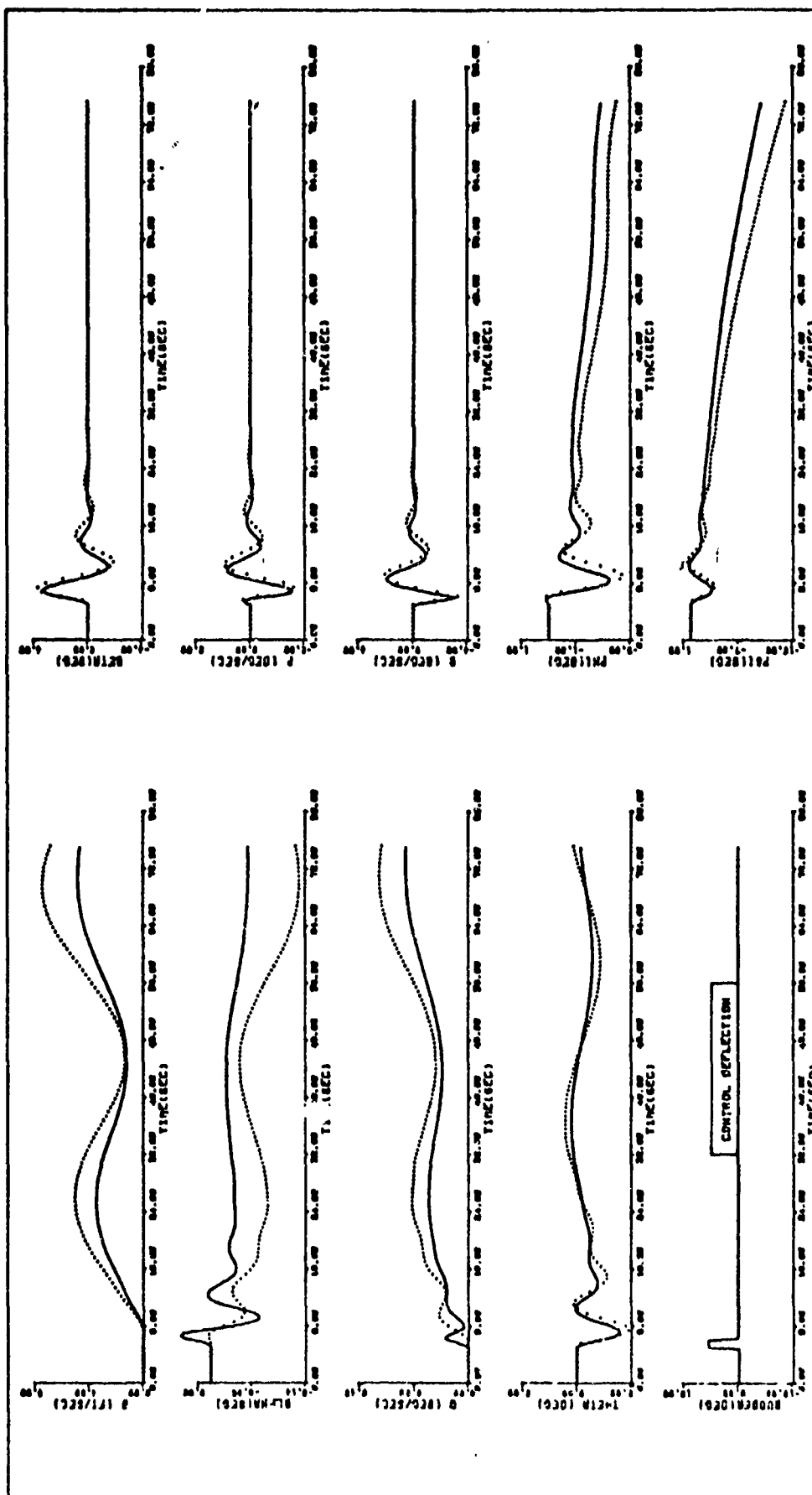


FIG. 7 - AIRCRAFT RESPONSE TO CONTROL SURFACE DEFLECTION  
 LEGEND: PRESENT MODEL .....  
 REFERENCE MODEL .....  
 REFERENCE MODEL .....

Appendix C

Stability Parameters for the AC130A Aircraft

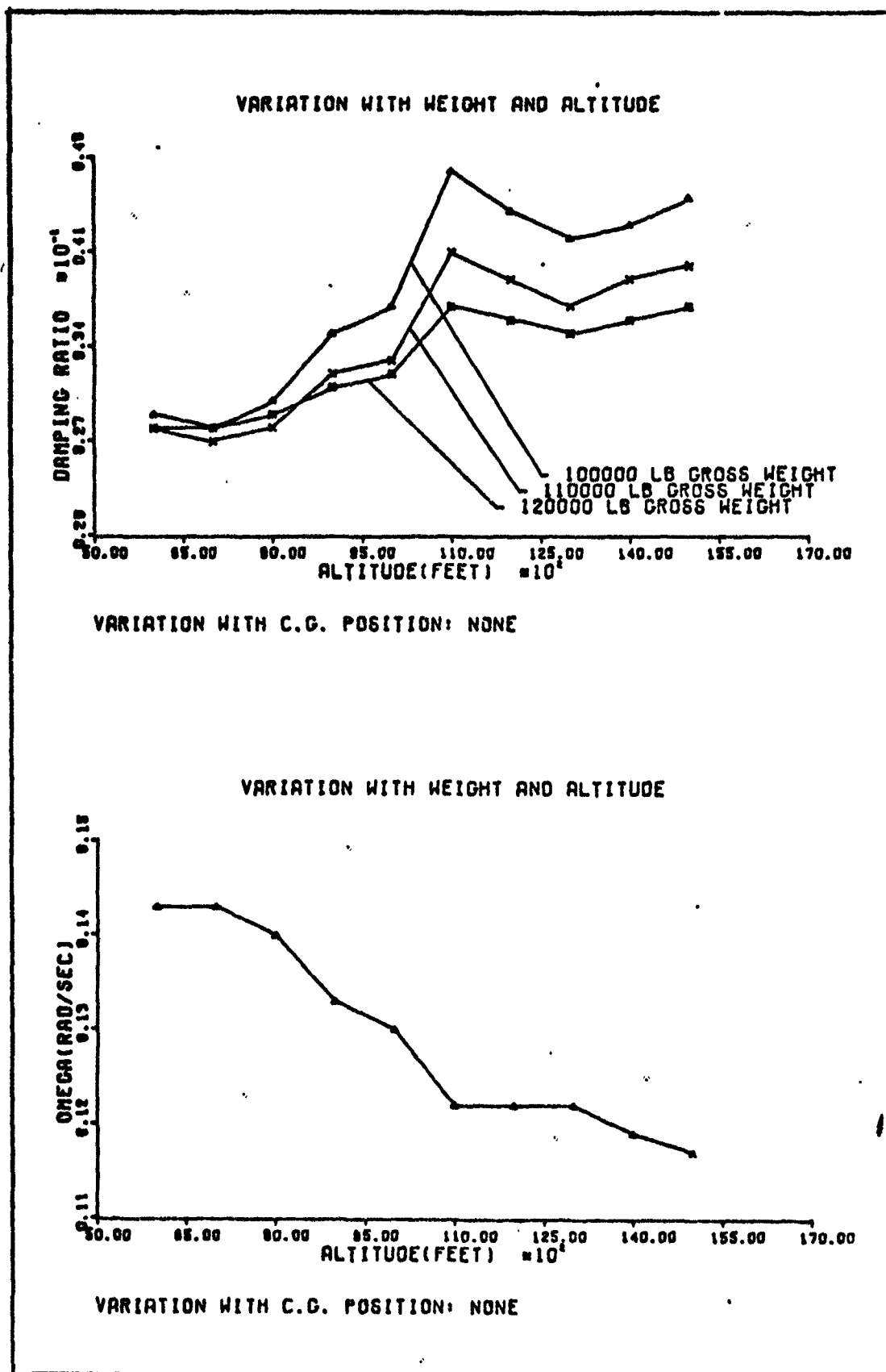


Figure 8. Phugoid Parameters

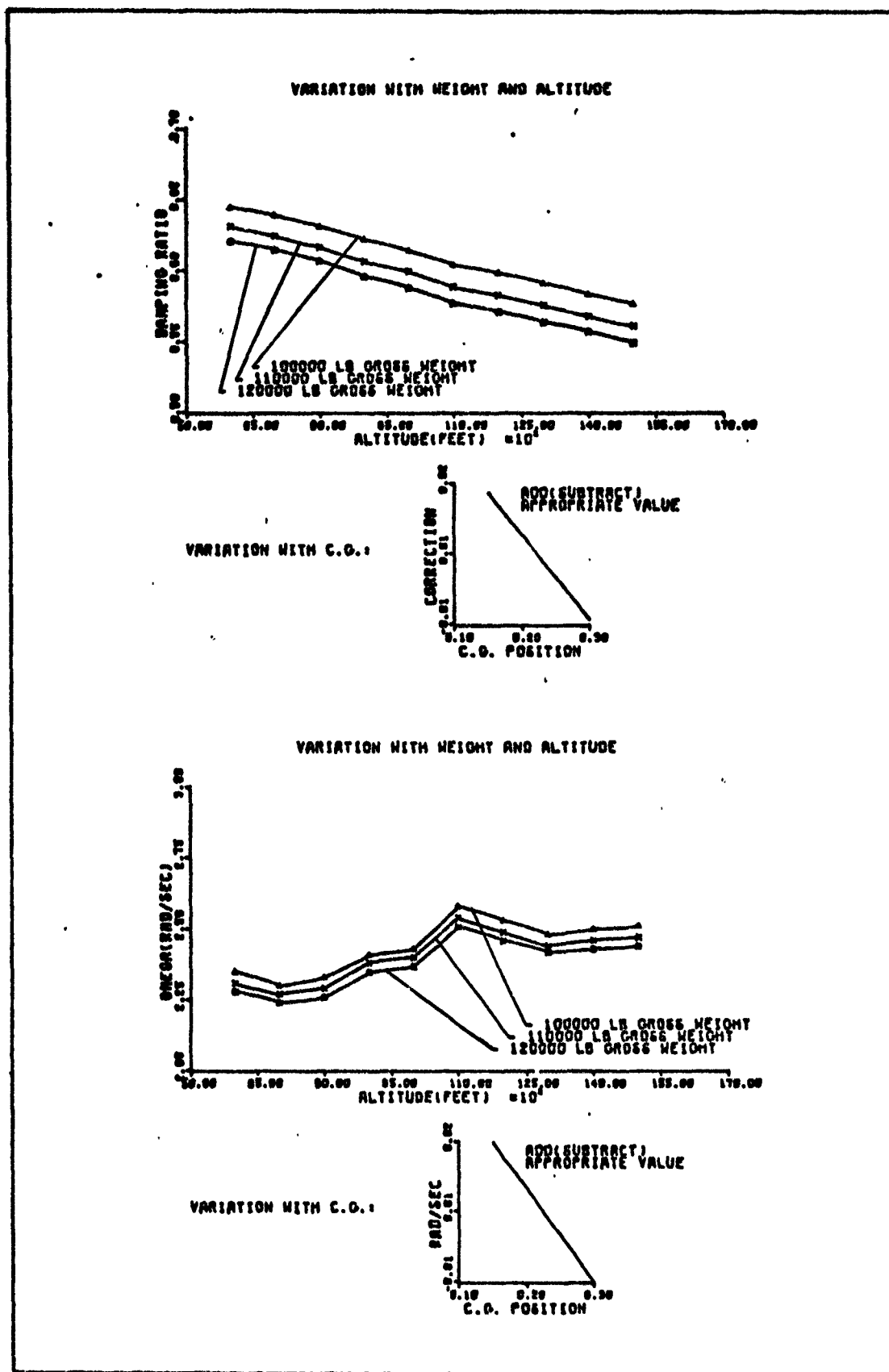


Figure 9. Short Period Parameters

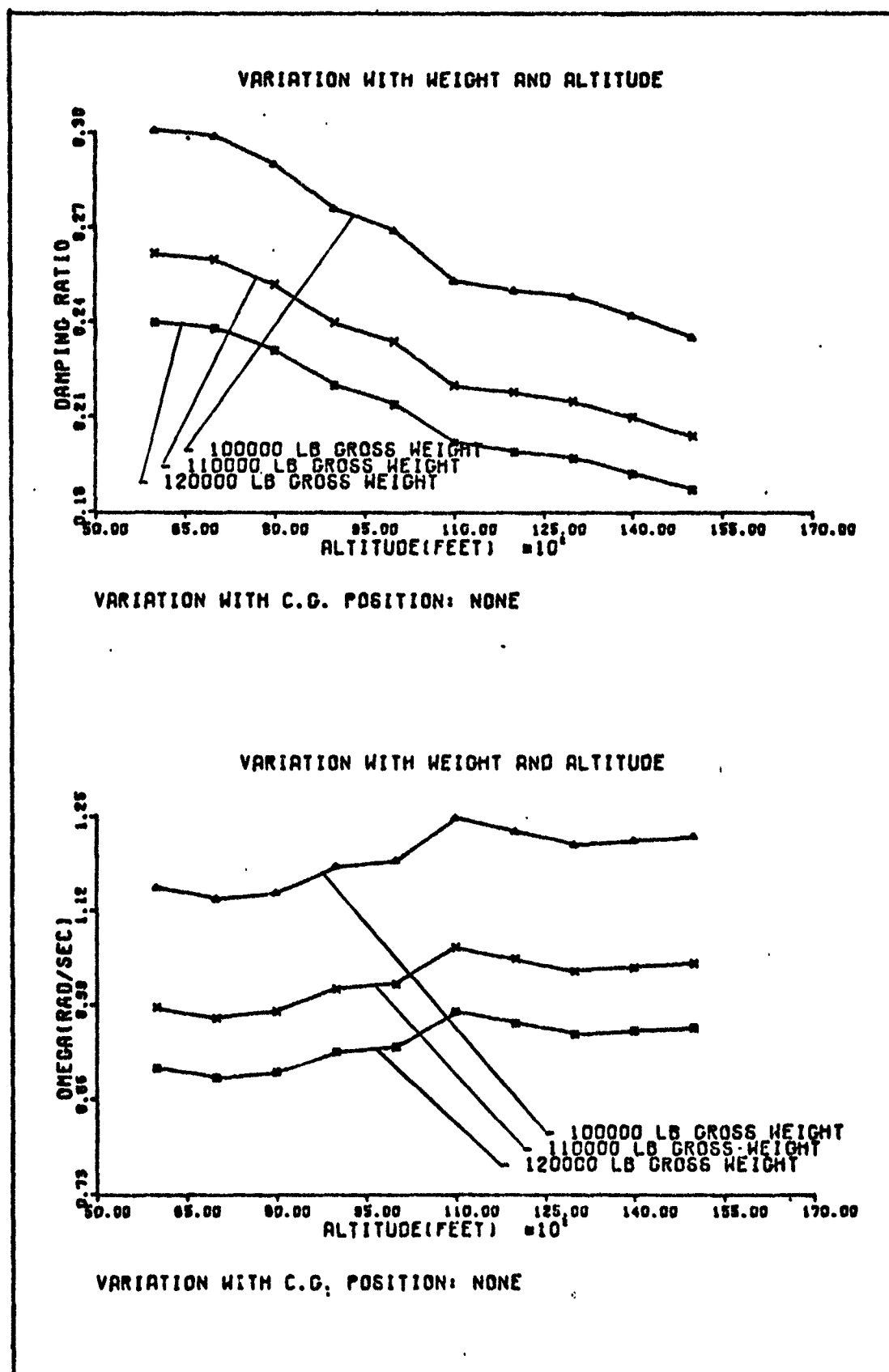


Figure 10. Dutch Roll Parameters

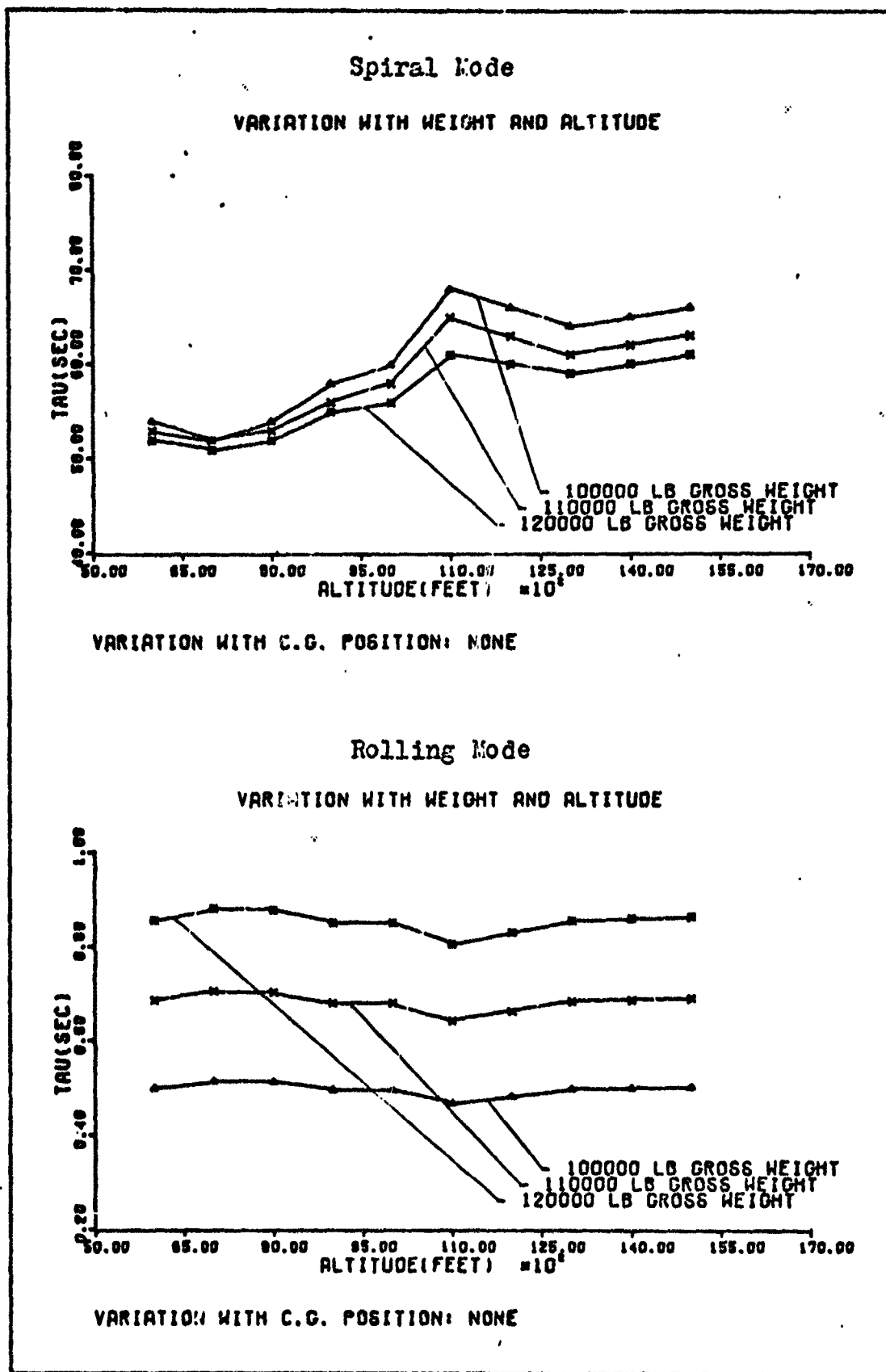


Figure 11. Spiral and Rolling Modes Parameters

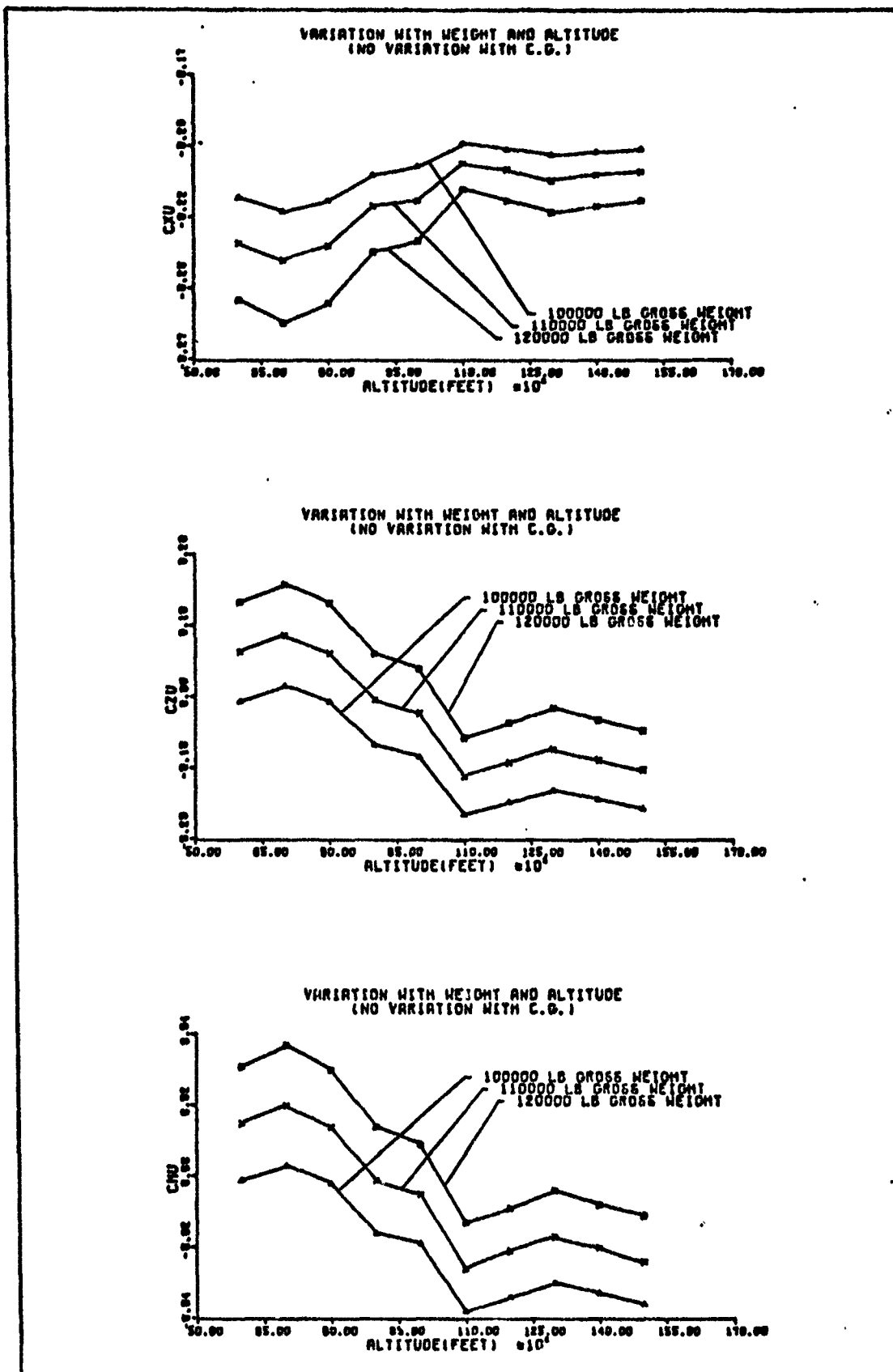


Figure 12. Stability Derivative Variation ( $C_{xu}, C_z, C_{mu}$ )

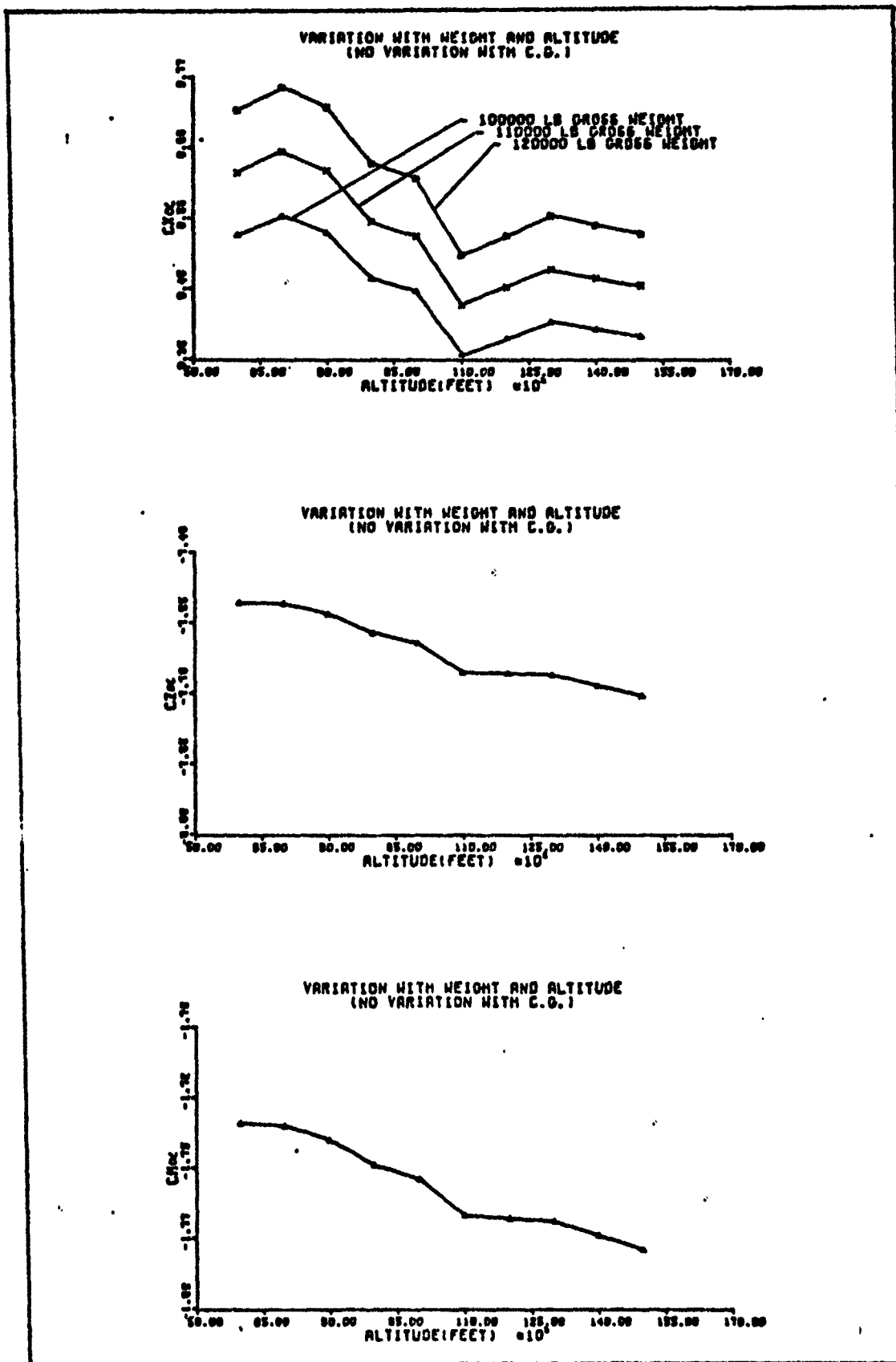


Figure 13. Stability Derivative Variation ( $C_{x\alpha}$ ,  $C_{z\alpha}$ ,  $C_{m\alpha}$ )

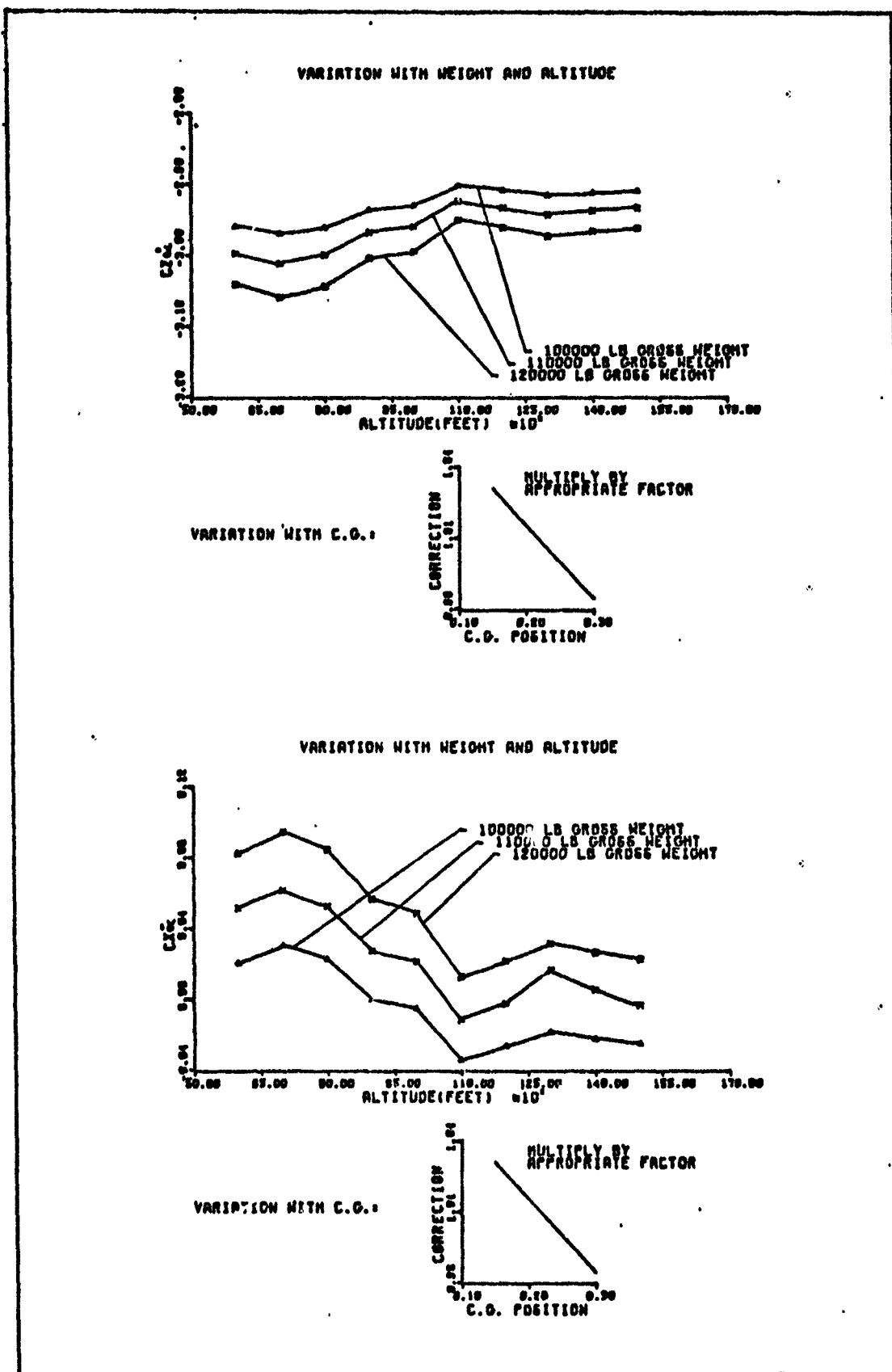


Figure 14. Stability Derivative Variation ( $C_{x\alpha}$ ,  $C_{z\alpha}$ )

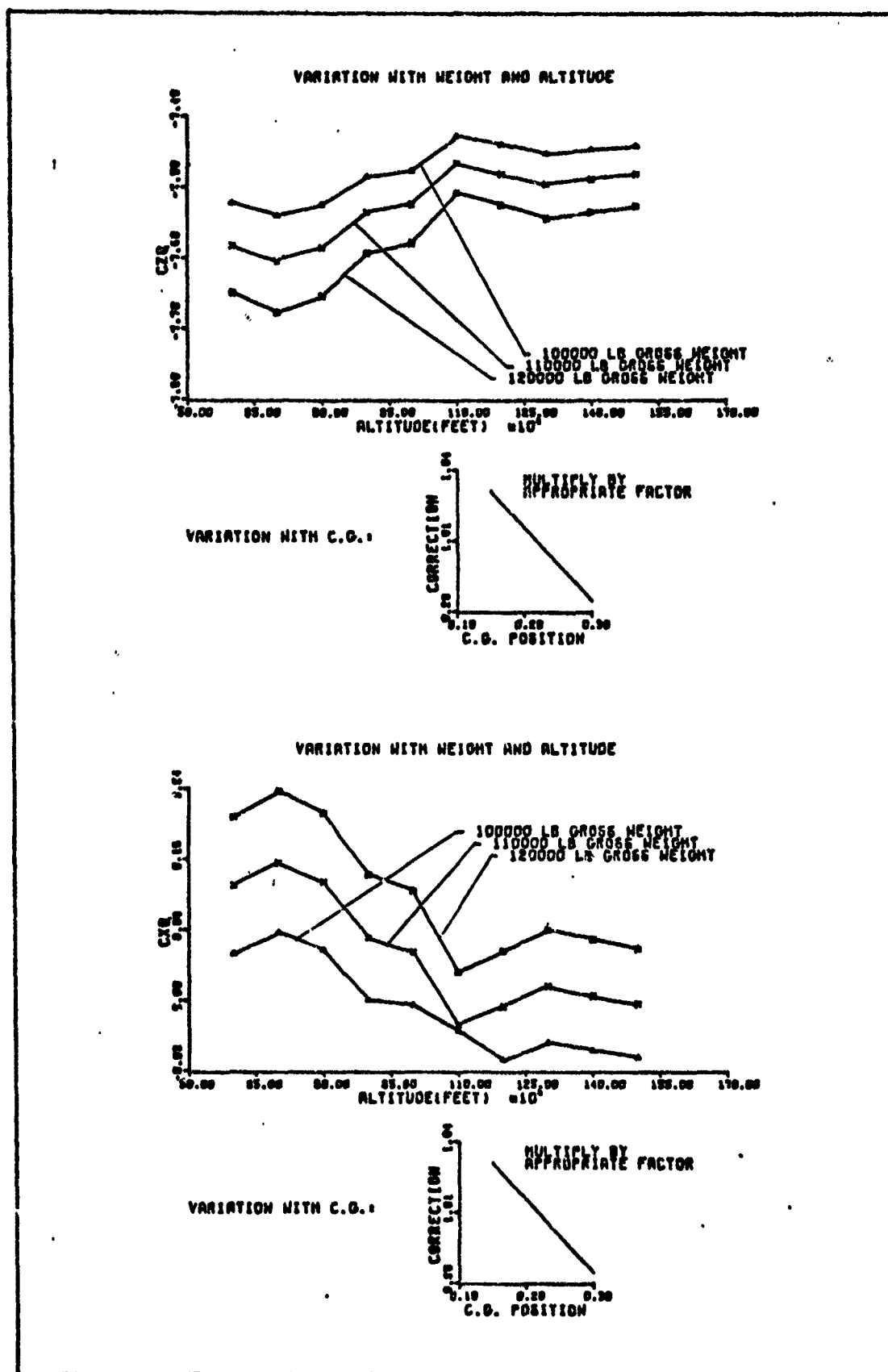


Figure 15. Stability Derivative Variation ( $C_{xq}, C_{zq}$ )

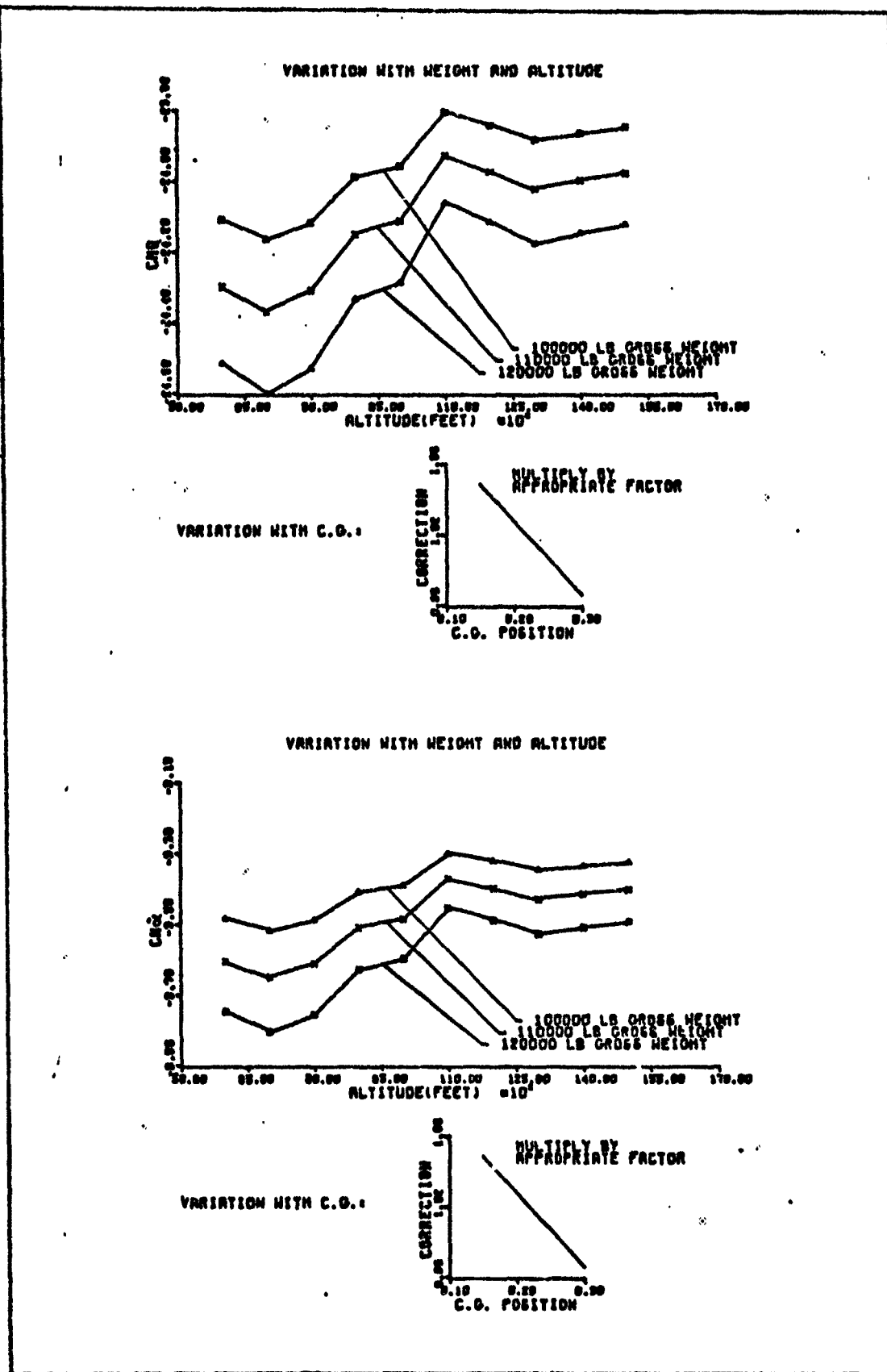


Figure 16. Stability Derivative Variation ( $C_{mq}$ ,  $C_{m\alpha}$ )



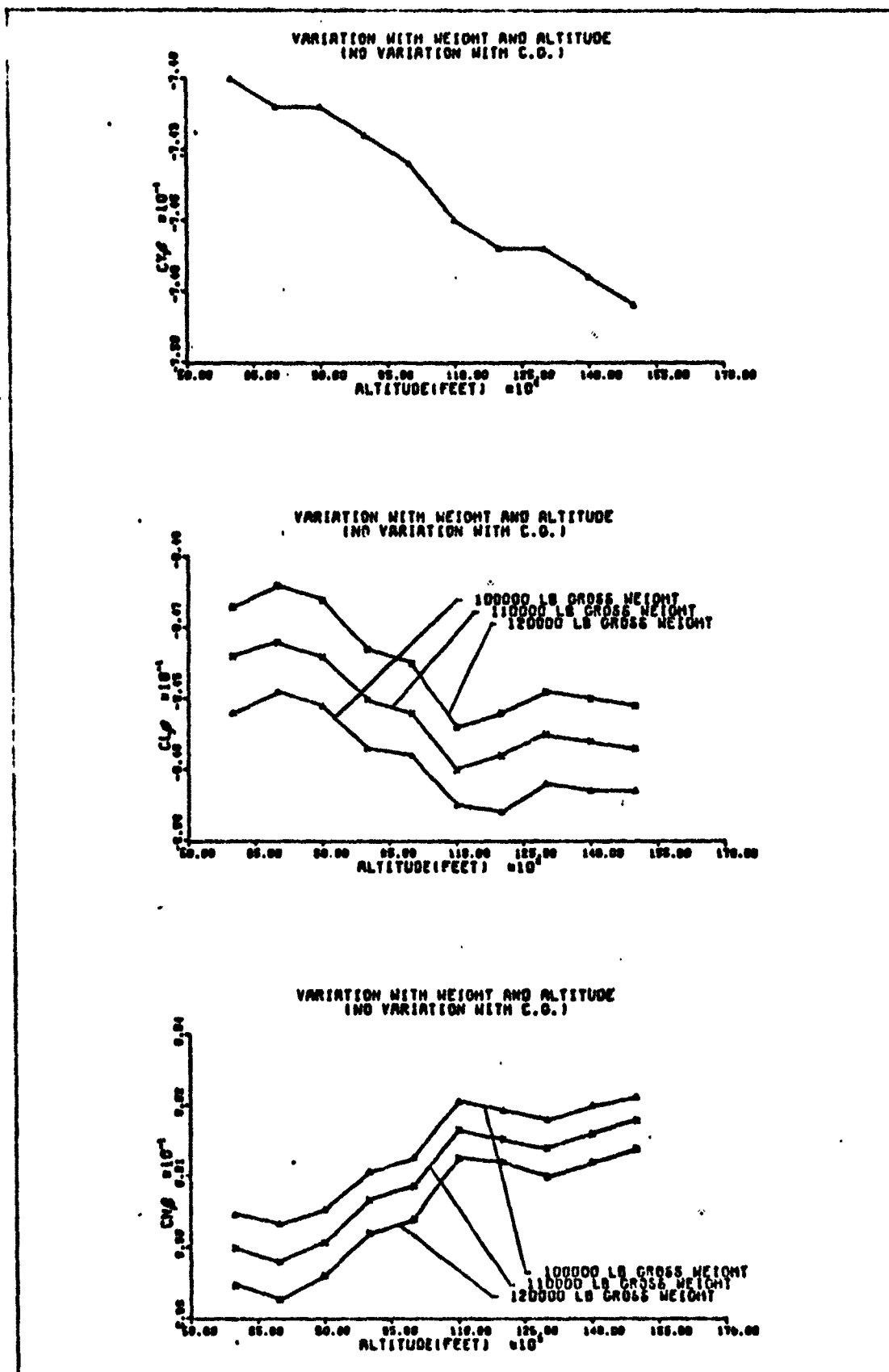


Figure 13. Stability Derivative Variation ( $C_{y\beta}, C_{l\beta}, C_{n\beta}$ )

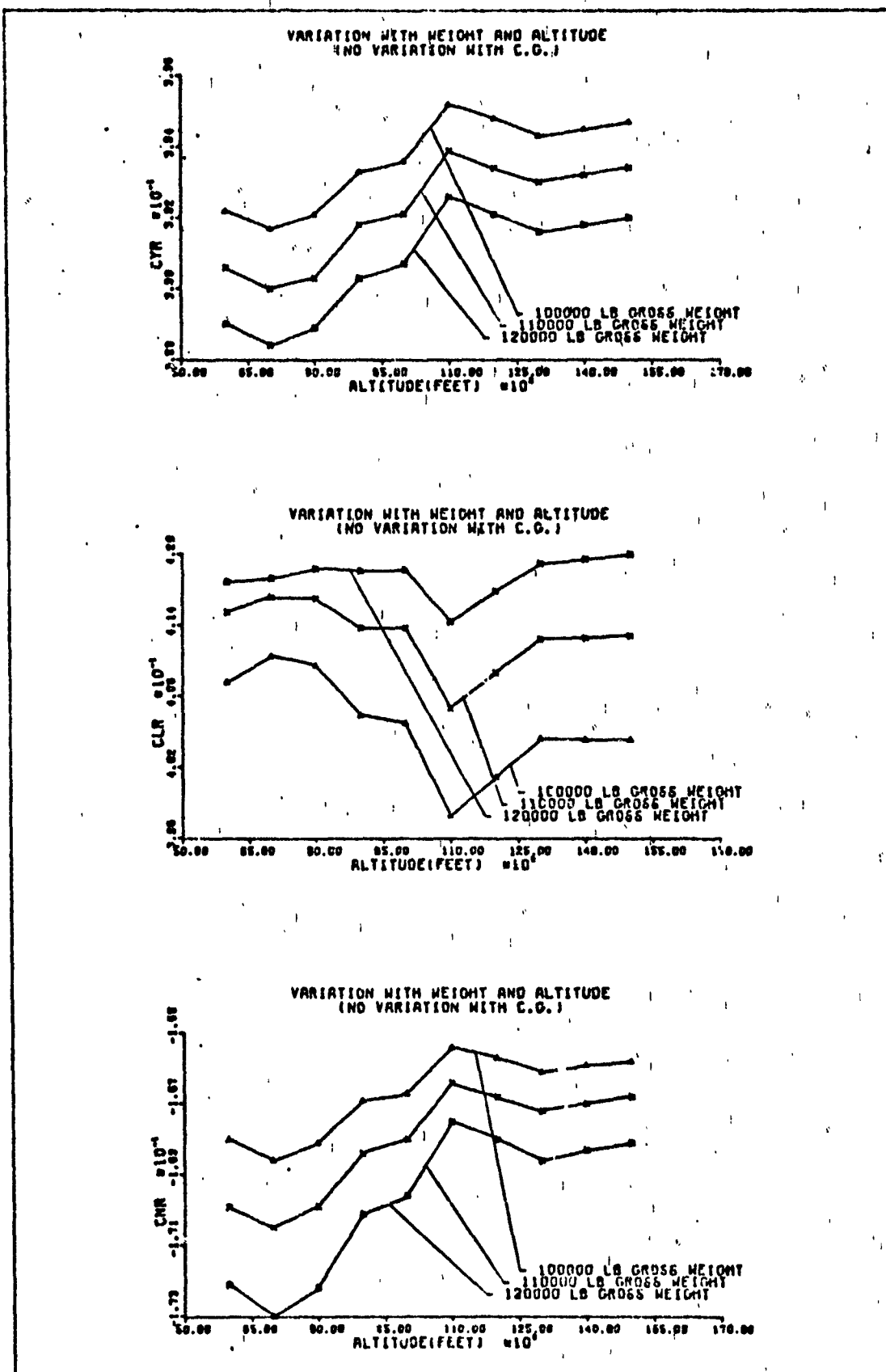


Figure 19. Stability Derivative Variation ( $C_{yR}$ ,  $C_{lR}$ ,  $C_{nR}$ )

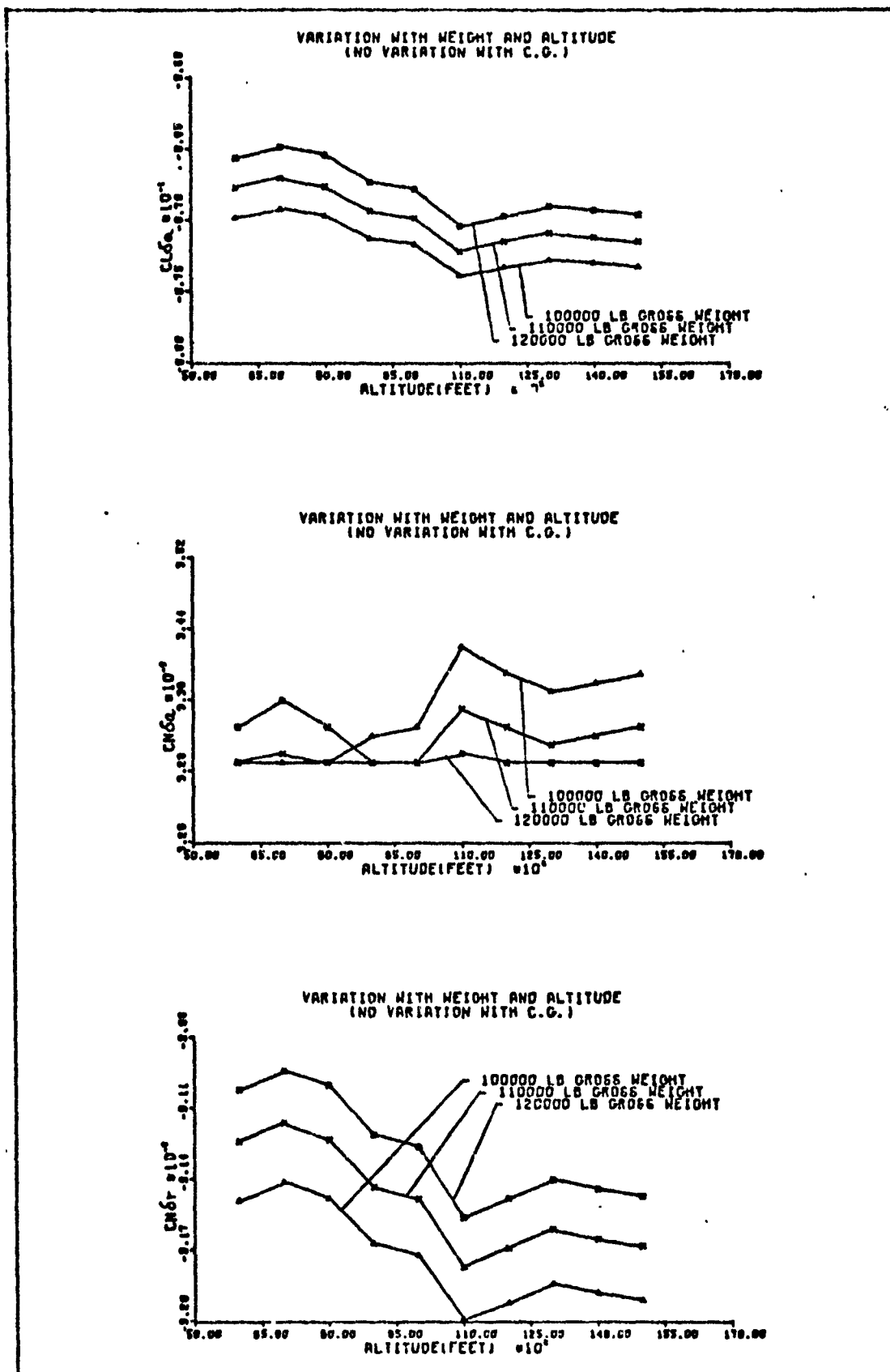
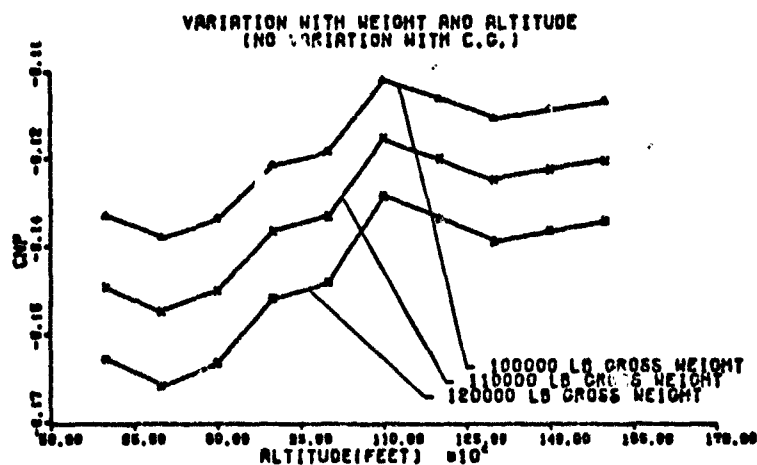
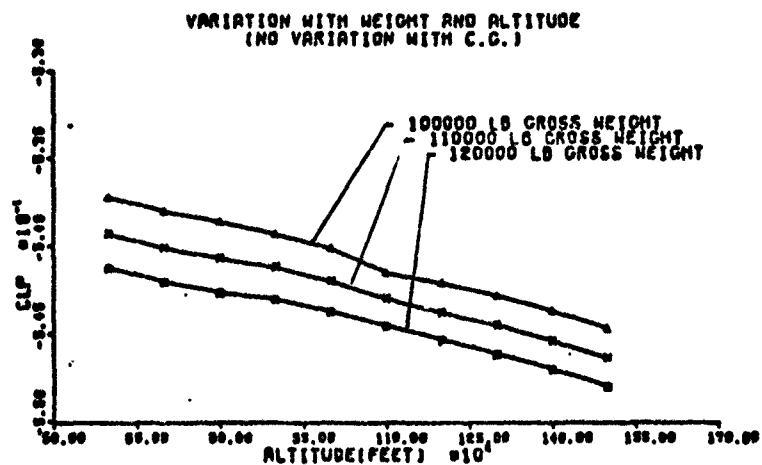


Figure 20. Stability Derivative Variation ( $C_{l\delta_a}, C_{n\delta_a}, C_{n\delta_r}$ )



$CYP = -0.134$  AND WAS CONSTANT FOR ALL FLIGHT CONDITIONS

$CY_{\delta r} = 0.264$  AND WAS CONSTANT FOR ALL FLIGHT CONDITIONS

$CL_{\delta r} = 0.0264$  AND WAS CONSTANT FOR ALL FLIGHT CONDITIONS

Figure 21. Stability Derivative Variation  $\begin{pmatrix} C_{lp}, C_{np}, C_{yp} \\ C_{y_{\delta r}}, C_{l_{\delta r}} \end{pmatrix}$

Appendix D

Computer Programs

```

PROGRAM COMPUTES INITIAL CONDITIONS, MODE PARAMETERS
AND PERTURBED RESPONSE; PRINT STATEMENTS NOT INCLUDED
ESTIMATE 65K MEMORY AND 400 SECONDS
THIS PROGRAM IS SET UP FOR THE 2 MODEL
IF A MODEL IS USED, CHANGE FOLLOWING TERMS...
TC=T/(K1*1.000), CT=1.072*TC, I=139.0
  DIMENSION X1(255), X2(255), Y3(255), X4(255), X5(255),
  *Y6(255), X7(255), X8(255), X9(255), X11(255),
  *X11(255), X12(255)
  DIMENSION T1(255), Y(12)
  COMMON AA(9,12), Y(255,12)
  DIMENSION WN(9), ZETA(9), PD(9), THALF(9), APFAL(2)
  REAL K, K1, K2, K3, K4, K5, K6, K7, K8, K9, K10, K11, K12, K13, K14,
  *K15, K16, K17, K18, K20, K21, K22, K23, K24, K25, K26, K27, K28, K29
  REAL I, IXX, IYY, IZZ, IXZ, M, M1, LT, LF
INPUT PARAMETERS WHICH ARE CONSTANT
  READ 200, G, S, P, CHAP, FI, RPM, AP
INPUT PARAMETERS WHICH VARY WITH FLIGHT CONDITION
  READ 100, U, ALTO, PHI, FLAPS, W, TEMP
WHERE U IS TAS IN KNOTS, ALTO IS PRESS ALTITUDE
IN FEET, PHI IS BANK ANGLE IN DEGREES, W IS GROSS
HEIGHT IN LB, AND TEMP IS FREE STREAM AIR TEMP
  READ 105, IXX, IYY, IZZ, IXZ, I, CG
COMPUTE PARAMETERS NEEDED TO DETERMINE AERODYNAMIC DATA
  OPM=1020.0*FI/30.0
  PHI=PHI/57.29578
  M=W/G
  PHO=.02376*(1.0-.000068825*ALTO)**4.255
  F=FLAPS/100.0
  DELV=U-120.00
  U=U*6081.0/3600.0
  VSOUND=49.1*SQRT(TEMP)
  M1=U/VSOUND
  ALTO=ALTO/1000.0
  CD=.11*(.028+.052*F)
  K=.52-.020*F
  CLMINTG=.25+.3*F
COMPUTE CONSTANTS
  K1=RHO*S*U**2/2.0
  K2=K1/M
  K3=PHO*S*U/(2.0*M)
  K4=K1*P
INITIAL VALUES OF A AND T TO BEGIN ITERATION
  IF (PHI.EQ.(.0)) GO TO 46
  R1=U**2/(G*TAN(PHI))
  PSI0CT=U/R1
  GO TO 47
46  PSI0CT=.0
47  CONTINUE
  IF (ALTO.GT.10.0) GO TO 21
  TC=.375-.01*ALTO-(.0033P-.00009*ALTO)*DELV
  GO TO 22
21  TC=.275-.002*(ALTO-10.0)
  *-(.00249-.00006*(ALTO-10.0))*DELV

```

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```

22 CONTINUE
CT=J*.836*TC
T=CT*K1
CLL=V/(K1*CCS(PHI))
CLLJ=.25+.12*TC+.9*F
IF (M1.LT..225) GO TO 3
CLLA=(6.3+.435*TC+2.*F)*(1.+*.333(M1-.225))
GO TO 4
3 CLLA=6.3+.435*TC+2.*F
4 A=(CLL-CLLJ)/CLLA
ITERATION ON ALPHA AND THRUST USING GRAMERS RULE
DO 10 J=1,5
IF (J.LT.2) GO TO 14
CLLJ=.25+.12*TC+.9*F
IF (M1.LT..225) GO TO 15
CLLA=(6.3+.435*TC+2.*F)*(1.+*.333(M1-.225))
GO TO 14
15 CLLA=6.3+.435*TC+2.*F
14 CLL=CLLJ+CLLA*A
CD=CD0+K*(CLL-CLMIN0)**2+.005
CT=T/K1
CD=CD*(1.+*.18*F*CT)
CDA=.2*K*CLLA*(CLL-CLMIN0)
CX=CLL*SIN(A)-CD*COS(A)
CXA=(CLL-CDA)*COS(A)+(CD+CLLA)*SIN(A)
CZ=-CLL*COS(A)-CD*SIN(A)
CZA=(CLL-CDA)*SIN(A)-(CD+CLLA)*COS(A)
TH=ATAN(TAN(A)*CCS(PHI)*
P=-PSIDOT*SIN(TH)
Q=PSIDOT*CCS(TH)*SIN(PHI)
R=PSIDOT*CCS(TH)*COS(PHI)
UDOT=K2*CX+T/M-G*SIN(TH)-U*Q*A
ADOT=K3*(CZ-A*CX)-(A*T)/(M*U)+Q*(1.+A**2)
*(G/U)*(COS(TH)*COS(PHI)+A*SIN(TH))
DTHDA=COS(PHI)/(1.-SIN(PHI)*SIN(A)**2)
DPDA=-PSIDOT*COS(TH)*DTHDA
DQDA=-PSIDOT*SIN(PHI)*SIN(TH)*DTHDA
DRDA=-PSIDOT*CCS(PHI)*SIN(TH)*DTHDA
DUDDA=K2*CXA-G*CCS(TH)*DTHDA-U*(Q+A*DQDA)
DUDDT=1./M
DADDA=K3*(CZA-CX-A*CXA)-T/(M*U)+(G/U)*(SIN(TH)
+A*CCS(TH)*DTHDA-SIN(TH)*COS(PHI)*DTHDA)+2.*A*G
+DQDA*A**2
DADDT=-A/(M*U)
DET=DQDDA*DADDT-DUDDT*DADDA
DELA=(DUDDT*ADDT-DADDT*UDOT)/DET
DELT=(DADDA*UDOT-DUDDA*ADDT)/DET
A=A+DELA
T=T+DELT
TC=T/(K1*.836)
10 CONTINUE
TRANSFORM INERTIA TERMS TO BODY AXIS
ALPHA=A*.57.2357A
IXZ=IXZ*(1.-ALPHA/4.35)

```

```

IXXS=IXX
IZZS=IZZ
IXZS=IXZ
IXX=IZZS*SIN(A)**2+IXXS*COS(A)**2+2.*IXZS*SIN(A)*COS(A)
IZZ=IXYS*SIN(A)**2+IZZS*COS(A)**2-2.*IXZS*SIN(A)*COS(A)
IXZ=IXZS*(COS(A)**2-SIN(A)**2)+(IZZS-IXXS)*SIN(A)*COS(A)
COMPUTE CONSTANTS
K5=IXZ/IZZ
K6=IYY-IZZ+(IXZ**2/IZZ)
K7=IXZ*(1.0+(IXX-IYY)/IZZ)
K8=K5*I*PPM*4.0
K9=K1*CBAR/IYY
K10=(IZZ-IXX)/IYY
K11=IXZ/IYY
K12=-4.0*I*FPM/IYY
K13=IXZ/IXX
K14=IXZ*((IYY-IZZ)/IXX-1.0)
K15=IXX-IYY+(IXZ**2/IXX)
K16=I*PPM*4.0
K17=IXX-(IXZ**2/IZZ)
K18=IZZ-(IXZ**2/IXX)
K20=K5*K14+K6
K21=K5*K15+K7
K22=K5*K16+K8
K23=K4*(1.0-K5*K13)
K24=K6*K17+K14
K25=K7*K13+K15
K26=K8*K13+K16
K27=K11/K9
K28=-K10/K9
K29=-K12/K9
TC=T/(K1*.935)
CLL0=.25+.12*TC+.005*F
IF (M1.LT.(.225)) GO TO 15
CLL4=(6.3+.475*TC+2.0*F)*(1.0+.333*(M1-.225))
GO TO 17
16 CLL4=6.3+.435*TC+2.0*F
17 CLL=CLL0+CLL4*A
CD=CD0+K*(CLL-CLMIN0G)**2+.105
CT=T/K1
CD=CD*(1.+.18*F*CT)
CDA=2.*K*CLL4*(CLL-CLMIN0G)
TH=ATAN(TAN(A)*COS(PHI))
P=-PSIDOT*SIN(TH)
Q=PSIDOT*COS(TH)*SIN(PHI)
R=PSIDOT*COS(TH)*COS(PHI)
CY=(P-Q*A-(G/U)*COS(TH)*SIN(PHI))/K7
CL=(K2.00*P+K21*Q**2+K22*U)/K23
CM=K27*(Q**2-P**2)+K28*P+K29*U
CN=(K24*Q**2+K25*P**2+K26*U)/K23
ITERATION ON CM,CL,CN USING CRAMER'S RULE
DO 44 JJ=1,17
CDOT=(.4*(CL+K3*CM)+.5*(CDA+K7*P**2+K9*U))/K17
CDOT=K5*CD+K10*P**2+K11*(P**2-P**2)+K12*P

```

$PDOT = (Y4 * (C1 + K13 * CL) + K14 * S * R + K15 * P * C + Y15 * C) / Y18$   
 $DPDDCN = 1.0$   
 $DPDDCL = K4 / K17$   
 $CPDDCN = K4 * K5 / K18$   
 $CDDDCN = K9$   
 $DDDDCL = 1.0$   
 $CDDDCN = 1.0$   
 $DRDDCN = 1.0$   
 $DRDDCL = K4 * K13 / K19$   
 $DRDDCN = K4 / K19$   
 $DET1 = DCDCCN * (DPDDCL * CPDDCN - DRDDCL * CDCCCN)$   
 $DELCN = CDOT * (DPDDCN * DPDDCL - DPDDCL * DRDDCN) / DET1$   
 $DELCN = DCDCCN * (PDOT * DPDDCN - PDOT * DPDDCL) / DET1$   
 $DELCN = DCDCCN * (RCOT * DRDDCL - PDOT * DRDDCN) / DET1$   
 $CM = CM + DELCN$   
 $CL = CL + DELCL$   
 $CN = CN + DELCN$

44 CONTINUE  
 MISCELLANEOUS PARAMETERS

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$AWR = ALPHA + 1.53$   
 $CALL ETA (CLL, TC, F, ETATAIL)$   
 $DEPDA = .300 + .14 * F + (.92 - .2 * F) * TC - (1.0 - .2 * F) * TC ** 2$   
 $CLATAIL = 3.209 * ETATAIL$   
 $CLAW = (.114 + .052 * F)$   
 $CLW = .26 + 1.82 * F + CLAW * AWR$   
 $CDLW = .0352 + .0044 * F$   
 $IF (TC > .1) GO TO 23$   
 $DCMDCLL = -.244 + .35 * TC + (.022 - .72 * TC) * F$   
 $GO TO 24$   
 23  $DCMDCLL = -.209 + .09 * (TC - .1) - (.05 + .16 * (TC - .1)) * F$   
 24 CONTINUE  
 $CLAW = CLAW * 57.29578$   
 $BETA1 = SQRT(1.0 - M1 ** 2)$   
 $XX1 = (AP * M1 ** 2) / (2.0 * (AP * BETA1 + 2.0) * BETA1)$   
 $LT = 47.37 - CG * CPAR$   
 $LF = LT - 1.30$

COMPUTE DERIVATIVES REFERENCED TO STABILITY AXES

$CTU = -3.0 * CT$   
 $CXU = CTU * COS(A)$   
 $CZU = -(CLL * M1 ** 2) / (1.0 - M1 ** 2) - CTU * SIN(A)$   
 $CMU = DCMDCLL * (CLL * M1 ** 2 / (1.0 - M1 ** 2))$   
 $CXA = CLL - CDA$   
 $CZA = -CLLA - CC$   
 $CMA = DCMDCLL * CLLA$   
 $CX0 = 0.0$   
 $CZ0 = -2.2 * ETATAIL * (LT / CPAR)$   
 $CM0 = -2.2 * ETATAIL * (LT / CPAR) ** 2$   
 $CXA0 = 0.0$   
 $CZA0 = -2.0 * ETATAIL * DEPDA * (LT / CPAR)$   
 $CMA0 = -2.0 * ETATAIL * DEPDA * (LT / CPAR) ** 2$   
 $CMDE = -.0287 * 57.29578$   
 $CZDE = (CPAR / LT) * ETATAIL * CMDE$   
 $CXDE = -.080$   
 $CYB = -(0.155 - .006 * F + (.030 - .0508 * F) * TC) * 57.29578$

```

CLB=(-.00005-.0002*F-(.00025+.00015*F)*ALPHA)*57.29578
CNR=(.00137+.0005*F-.0015*TC)*57.29578
*F (TC-.05-.2) GO TO 5
CYRT=(-.045+.065*F+(1.474-2.315*F)*TC-(1.75-2.03*F)*TC**2
CLRT=(-.0638-.0034*F-(.023+.09*F)*TC+(.09+95*F)*TC**2
GO TO 6
5 CYBT=-.400-.688*F
CLBT=-.0648-.011*F+(.009-.026*F)*(TC-.20)
6 CNRT=(.0039-.0005*F+(.013*F-.0075)*TC)*57.29578
CYBR=.0046*57.29578
CLBR=(.00046-(.00021/8.0)*ALPHA)*57.29578
CNBR=-.0016*57.29578
CYBA=0.0
IF (CLL.GT.1.0) GO TO 11
CLDA=-.0602-.017*F+.029*(CLL-.80)
GO TO 13
11 IF (CLL.GT.1.20) GO TO 12
CLDA=-.0544-.0172*F+(.0140+.030*F)*(CLL-1.00)
GO TO 13
12 CLDA=-.0516-.0114*F+.029*(CLL-1.20)
13 CONTINUE
IF (ALPHA.GT.2.0) GO TO 1
CNDA=(.000053+.00002*ALPHA)*57.29578
GO TO 2
1 CNDA=(.001097+.0000125*(ALPHA-2.0))*57.29578
2 CYP=-.0167*CLAW-2.*A*CYRT*LF/R
CLP=-.0300
8 CNP=-.09*CLW-.0015-2.*A*CNBT*LF/R
CYR=2.*A*CNRT
CLR=.20*CLW*(1.+XX1)+.0518*F-CLB-CYR*Z*CCS(A)**2/R
*+2.0*CNBT*CLBT/CYRT
CNR=-.02*CLW**2-.3*CLW+2.0*CNBT**2/CYRT
CORRECTION FOR MACH EFFECTS
IF (M1.LE..225) GO TO 470
CYB=CYP*(1.+.15*(M1-.225))
CNB=CNP*(1.+.2*(M1-.225))
CYBT=CYRT*(1.+.2*(M1-.225))
CNBT=CNRT*(1.+.4*(M1-.225))
CLP=CLF-.13*(M1-.225)
470 CONTINUE
TRANSFORM DERIVATIVES TO BODY AXES
CX=CT+A*CLL-CD
CZ=- (CLL+A*CD)
CXA3=CLL-CDA+A*(CD+CLLA)
CZA3=A*(CLL-CDA)-CD-CLLA
CMA3=CMA+A*CMU
CXU3=CXU+CZA*A**2-A*(CXA+CZU)
CZU3=CZU-CXA*A**2+A*(CXU-CZA)
CMU3=CMU-A*CMA
CXQ3=CXQ-A*C7Q
CZQ3=CZQ+A*CXQ
CMQ3=CMQ
CXAD3=CXAD-A*C7AD
CZAD3=CZAD+A*CXAD

```

```

CMAJP=CMAJ
CYDE=CXDE-A*CXDF
CZDE=CZDE+A*CXDF
CMDE=CMAJF
CYDE=CYP
CLRB=CLB-A*CNB
CNRB=CNB+A*CLB
CYPB=CYP-A*CYR
CLPB=CLP-A*(CLB+CNB)+CNP*A**2
CNPB=CNB-A*(CNB-CLP)-CLP*A**2
CYRB=CYP+A*CYR
CLRB=CLB-A*(CNP-CLP)-CNP*A**2
CNRB=CNB+A*(CLB+CNB)+CLP*A**2
CYDA=CYDA
CYDR=CYDR
CLDA=CLDA-A*CNDA
CLDR=CLDR-A*CNDR
CNDA=CNDA+A*CLDA
CNDR=CNDR+A*CLDR
COMPUTE COEFFICIENT MATRIX
U0=U
A0=A
Q0=Q
TH0=TH
P0=P
R0=R
PHI1=PHI
AA1=PHC*S*U0/(2.*M)
AA2=AA1*U0
AA3=AA1/U0
AA4=PHC*S*CPAR/(4.*M)
AA5=PHC*S*U0*CPAR/(2.*IYY)
AA6=AA5*U0
AA7=AA5*CPAR/2.
AA8=PHC*S*R/(4.*M)
AA9=PHC*S*U0*B
AA10=AA9*U0/2.
AA11=AA9*B/4.
AA12=1.-AA4*CZAD
AA13=IXX/IX7-IXZ/IZ7
AA14=IZ7/IXZ-IX7/IXX
AA15=PHC*S*CPAR*U0*CXADP/(4.*M)
AA16=AA15/AA12
AA17=1.+AA16*AC/U0
AA18=PHC*S*U0*CPAR/(4.*M)
AA19=1.-AA4*(CZADB-AC*CXADR)
AA(1,6)=AA(1,7)=AA(1,8)=AA(1,9)=AA(1,10)=AA(1,12)=0.0
AA(2,6)=AA(2,7)=AA(2,9)=AA(2,10)=AA(2,12)=0.0
AA(3,9)=AA(3,10)=AA(3,12)=0.0
AA(4,1)=AA(4,2)=AA(4,4)=AA(4,5)=AA(4,6)=0.0
AA(4,9)=AA(4,10)=AA(4,11)=AA(4,12)=0.0
AA(5,9)=AA(5,10)=AA(5,11)=0.0
AA(6,2)=AA(6,4)=AA(6,8)=AA(6,9)=AA(6,11)=0.0
AA(7,2)=AA(7,4)=AA(7,8)=AA(7,9)=AA(7,11)=0.0

```

$AA(3,1)=AA(3,2)=AA(3,3)=AA(3,4)=0.$   
 $AA(3,9)=AA(3,10)=AA(3,11)=AA(3,12)=0.$   
 $AA(4,1)=AA(4,2)=AA(4,3)=AA(4,4)=AA(4,5)=AA(4,6)=AA(4,7)=AA(4,8)=AA(4,9)=AA(4,10)=AA(4,11)=AA(4,12)=0.$   
 $AA(1,1)=(AA1*(2.*CX+CYUR)-A1*Q1$   
 $+AA16*(AA7*(2.*C7+C7UB)+C7/UL))/AA17$   
 $AA(1,2)=(AA2*CYAB-U1*Q1+AA16*AA1*C7AB)/AA17$   
 $AA(1,3)=(-A1*U1+AA15*(CYCB+CXAD*(1.+AA4*C7CB)/AA12))$   
 $*/AA17$   
 $AA(1,4)=(-G*COS(TH1)-AA16*(G*SIN(TH1)*COS(PHI1)/UL))$   
 $*/AA17$   
 $AA(1,5)=(U1*R1-AA16*P1)/AA17$   
 $AA(1,8)=AA16*G*COS(TH1)*SIN(PHI1)/(U1*AA17)$   
 $AA(2,1)=(AA3*(2.*C7+C7UB-A1*(2.*CX+CXUR)$   
 $+Q1*(1.+AA4*Q1)/U1)/AA19$   
 $AA(2,2)=(AA1*(C2*B-A1*CYAB)+A1*Q1)/AA19$   
 $AA(2,3)=(1.+A1*Q1+AA4*(C2B-A1*CXCB))/AA19$   
 $AA(2,4)=((G/U1)*(A1*COS(TH1)-SIN(TH1)*COS(PHI1)))/AA19$   
 $AA(2,5)=(-P1-A1*P1)/AA19$   
 $AA(2,8)=(-(G/U1)*COS(TH1)*SIN(PHI1))/AA19$   
 $AA(3,1)=AA5*(2.*CM+CMUR)+AA(2,1)*AA7*CMADP$   
 $AA(3,2)=AA5*CMAP+AA(2,2)*AA7*CMADP$   
 $AA(3,3)=AA7*(CMCT+AA(2,3)*CMADP)$   
 $AA(3,4)=AA(2,4)*AA7*CMADP$   
 $AA(3,5)=AA(2,5)*AA7*CMADP$   
 $AA(3,6)=((IZ7-IXX)*P1-2.*IXZ*P1)/IYY$   
 $AA(3,7)=((IZ7-IXX)*P1+2.*IXZ*P1-RPM*I*4.)/IYY$   
 $AA(3,8)=AA(2,8)*AA7*CMADP$   
 $AA(4,7)=COS(PHI1)$   
 $AA(4,8)=-SIN(PHI1)$   
 $AA(4,9)=-PSTDOT*COS(TH1)$   
 $AA(5,1)=RHC*S*CY/M-P1/UL+A1*P1/UL$   
 $AA(5,2)=P1$   
 $AA(5,4)=-G*SIN(TH1)*SIN(PHI1)/U1$   
 $AA(5,5)=AA1*CYRB$   
 $AA(5,6)=AA9*CYRB+A1$   
 $AA(5,7)=AA8*CYRB-1.$   
 $AA(5,8)=+G*COS(TH1)*COS(PHI1)/U1$   
 $AA(6,1)=AA9*(CL/IX7+CN/I77)/AA13$   
 $AA(6,3)=(((IXX-IYY)/IZZ+1.)*P1$   
 $+((IYY-IZZ)/IXZ-IXZ/I77)*P1+RPM*I*4./IZZ)/AA13$   
 $AA(6,5)=AA11*(CLRB/IXZ+CNRB/IZZ)/AA13$   
 $AA(6,6)=(AA11*(CLPB/IX7+CNPB/IZZ)$   
 $+Q1*(1.+(IXX-IYY)/IZZ))/AA13$   
 $AA(6,7)=(AA11*(CLRB/IXZ+CNRB/IZZ)$   
 $+Q1*((IYY-IZZ)/IXZ-IXZ/IZZ))/AA13$   
 $AA(7,1)=AA9*(CN/IXZ+CL/IXX)/AA14$   
 $AA(7,3)=(((IXX-IYY)/IX7+IXZ/IXX)*P1$   
 $+((IYY-IZZ)/IXX-1.)*P1+RPM*I*4./IXZ)/AA14$   
 $AA(7,5)=AA11*(CNPB/IXZ+CLPB/IXX)/AA14$   
 $AA(7,6)=(AA11*(CNPB/IXZ+CLPB/IXX)$   
 $+Q1*(IX7/IXX+(IXX-IYY)/IXZ))/AA14$   
 $AA(7,7)=(AA11*(CNRB/IXZ+CLRB/IXX)$   
 $+Q1*((IYY-IZZ)/IXZ-1.))/AA14$

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```

AA(9,3)=TAN(TH)*SIN(PHI)
AA(9,4)=PSICOT/COS(TH)-PD*TAN(TH)
AA(9,6)=1.0
AA(9,7)=TAN(TH)*COS(PHI)
AA(9,3)=SIN(PHI)/COS(TH)
AA(9,4)=PSICOT*TAN(TH)
AA(9,7)=COS(PHI)/COS(TH)
AA(1,11)=(AA2*CXDEB+AA16*AA1*CZDEB)/AA17
AA(2,11)=(AA1*(CZDEB-A1*CXDEB))/AA19
AA(3,11)=AA6*CMDEB+AA7*CMADP*AA(2,11)
AA(5,10)=0.0
AA(5,12)=AA1*CYDPR
AA(6,10)=((CLOAB/IXZ+CNDAB/IZ7)*AA10)/AA13
AA(6,12)=((CLOAB/IXZ+CNDAB/IZ7)*AA10)/AA13
AA(7,10)=((CNDAP/IXZ+CLOAB/IXX)*AA10)/AA14
AA(7,12)=((CNDAP/IXZ+CLOAB/IXX)*AA10)/AA14
CALL MEGVAL (9,AA,WN,ZETA,PD,THALF,AREAL)
WHERE WN IS NATURAL FREQUENCY,ZETA IS DAMPING
RATIO,PD IS PERIOD,THALF IS TIME TO DAMP TO HALF
AMPLITUDE AND AREAL IS MATRIX OF TIME
CONSTANTS FOR ROLLING AND SPIRAL MODES
INTEGRATION BY RUNGE KUTTA
THIS ARRANGEMENT RECORDS EVERY 10TH VALUE
SET INITIAL VALUES OF PERTURBATIONS TO ZERO
DO 454 KJ=1,12
454 X(1,KJ)=0.0
T1(1)=0.0
T=0.
VALUE OF DELT USED WOULD BE SPECIFIED HERE
II=1
DO 314 J=1,12
314 Y(J)=X(1,J)
318 CONTINUE
DO 316 J=1,12
316 CALL RUNGE1 (T,MFCOUNT,DELT,Y)
II=II+1
DO 317 J=1,12
317 X(II,J)=Y(J)
T1(II)=T
IF (II.LE.250) GO TO 318
JJJ=II
CONVERT RADIAN MEASURE TO DEGREES
DO 99 JJ=1,JJJ
III=JJ
X1(III)=X(JJ,1)
X2(III)=Y(JJ,2)*57.29578
X3(III)=X(JJ,3)*57.29578
X4(III)=X(JJ,4)*57.29578
X5(III)=Y(JJ,5)*57.29578
X6(III)=X(JJ,6)*57.29578
X7(III)=X(JJ,7)*57.29578
X8(III)=Y(JJ,8)*57.29578
X9(III)=Y(JJ,9)*57.29578
X10(III)=X(JJ,10)*57.29578

```

```

X11(III)=X(JJ,11)*57.29578
X12(III)=X(JJ,12)*57.29578
99 CONTINUE
SUBROUTINES
TAIL EFFICIENCY
SUBROUTINE ETA (CLL,TC,F,ETATAIL)
IF (TC.GT.0.10) GO TO 2
IF (CLL.GT.1.20) GO TO 1
ETATAIL=1.0+(1.25-.5*F+(1.25-.5*F)*(CLL-1.80))*TC
GO TO 4
1 ETATAIL=1.0+(1.75-.7*F+(.50+1.5*F)*(CLL-1.20))*(TC-0.10)
GO TO 4
2 IF (CLL.GT.1.20) GO TO 3
ETATAIL=1.125-.05*F+(.125-.05*F)*(CLL-0.80)
X+(.93-1.16*F+(.375+.5*F)*(CLL-0.80))*TC
GO TO 4
3 ETATAIL=1.175-.07*F+(.1625+.125*F)*(CLL-1.20)
X+(1.08-.46*F+(.3125+.875*F)*(CLL-1.20))*(TC-0.10)
4 CONTINUE
RETURN
END
RUNGE KUTTA
SUBROUTINE RUNGE1 (T,MECOUNT,DT,X)
DIMENSION P1(9),P2(9),P3(9),X(12),XC(12),XT(12),
*XP(12)
TQ=T
T=T+DT
HT=DT
TT=T
DO 5 I=1,12
XC(I)=X(I)
5 XT(I)=X(I)
ASSIGN 6 TO K
GO TO 20
6 DO 7 I=1,9
7 XP(I)=X(I)
HT=0.5*DT
XT(10)=X(10)
XT(11)=X(11)
XT(12)=X(12)
ASSIGN 9 TO K
GO TO 20
9 DO 10 I=1,12
10 XT(I)=X(I)
TT=T+HT
ASSIGN 11 TO K
20 CALL FDOT (TT,MECOUNT,XT,P0)
DO 21 I=1,9
21 X(I)=XT(I)+.5*HT*P0(I)
CALL FDOT (TT+.5*HT,MECOUNT,X,P1)
DO 22 I=1,9
22 X(I)=XT(I)+HT*(.207116781*P0(I)+.292493219*P1(I))
CALL FDOT (TT+.5*HT,MECOUNT,X,P2)

```

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```

      DO 23 I=1,9
23  Y(I)=X(I)+HT*(.707106781*(P2(I)-P1(I))+P2(I))
      CALL FOOT (IT+HT,RECOUNI,X,P2)
      DO 24 I=1,9
24  X(I)=YT(I)+HT*(P2(I)+.505736+38*P1(I)+3.41421356*P2(I)
      +P3(I))/6.0
      GO TO K,(6,9,11)
11  DO 12 I=1,9
12  X(I)=X(I)+(X(I)-XP(I))/15.0
      RETURN
      END

```

#### DERIVATIVES FOR RUNGE KUTTA

```

SUBROUTINE FOOT (T,RECOUNI,X,XDOT)
  DIMENSION XDOT(9),X(12)
  COMMON AA(9,12)
  CONTROL DEFLECTION WOULD BE SPECIFIED HERE
  X(11) IS ATLETON,X(11) IS ELEVATOR,X(12) IS RUDDER
  EXAMPLE: FOR ONE SECOND PULSE...

```

```

      IF (T.LT.5.0.OR.T.GE.6.0) GO TO 26
      X(11)=0.1
      GO TO 27
20  IF (T.LT.5.0.OR.T.GE.6.0) GO TO 26
      X(11)=0.1
      GO TO 27
21  IF (T.LT.5.0.OR.T.GE.6.0) GO TO 26
      Y(12)=0.1
      GO TO 27
25  X(11)=X(11)+X(12)=0.1
27  CONTINUE
      DO 1 J=1,9
      XDOT(J)=0.0
      DO 3 J=1,9
      DO 2 K=1,12
2  XDOT(J)=XDOT(J)+AA(J,K)*X(K)
3  CONTINUE
      RETURN
      END

```

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#### SUBROUTINES TO DETERMINE EIGENVALUES

```

SUBROUTINE MEGVAL (NP,A,WN,ZETA,PD,THALF,AREAL)
  DIMENSION A(9,9),PSR(9),RSI(9),ZETA(9),WN(9),PD(9)
  DIMENSION AREAL(2)
  DIMENSION THALF(9)
  DOUBLE PRECISION DA(9,9)
  IF (NP.GT.1) GO TO 180
  PSR(1)=A(1,1)
  RSI(1)=0.0
  GO TO 200
180 DO 190 I=1,NP
  DO 190 J=1,NP
190  DA(I,J)=A(I,J)
  CALL CHAPD (NP,9,DA,PSR,RSI)
200 DO 300 I=1,NP
  PD(I)=0.0
  WN(I)=0.0

```

```

    THALF(I)=J.0
300 ZETA(I)=J.0
    I=1
310 IF (I.GT.NP) GO TO 320
    IF (PSI(I).NE.0) GO TO 720
    I=I+1
    GO TO 310
320 WN(I)=SQRT(PSI(I)**2+RSI(I)**2)
    ZETA(I)=-RSI(I)/WN(I)
    PD(I)=6.28318530718/(WN(I)*SQRT(1.-ZETA(I)**2))
    THALF(I)=.693/(WN(I)*ZETA(I))
    I=I+2
    GO TO 310
330 CONTINUE
    K=1
    DO 340 I=1,NP
    IF (PSI(I).EQ.0.0.AND.PSR(I).NE.0.0) 341,740
341 AREAL(K)=1./ABS(PSR(I))
    K=K+1
342 CONTINUE
    IF (AREAL(2).LT.AREAL(1)) 342,343
342 S1=AREAL(1)
    AREAL(1)=AREAL(2)
    AREAL(2)=S1
343 CONTINUE
    DO 30 I=1,8
    K=I+1
    DO 31 J=K,0
    IF (WN(I).GE.WN(J)) GO TO 31
    R1=WN(I)
    R2=ZETA(I)
    R3=PD(I)
    R4=THALF(I)
    WN(I)=WN(J)
    ZETA(I)=ZETA(J)
    PD(I)=PD(J)
    THALF(I)=THALF(J)
    WN(J)=R1
    ZETA(J)=R2
    PD(J)=R3
    THALF(J)=R4
31 CONTINUE
30 CONTINUE
    RETURN
    END

```

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SUBROUTINE CHARD (N,NVAR,A,CPIT,FR,RI)  
THIS SUBROUTINE COMPUTES THE EIGENVALUES OF A REAL MATRIX,  
SYMMETRIC OR NONSYMMETRIC. THE INPUT MATRIX IS TRANSFORMED BY  
SIMILARITY TRANSFORMATIONS INTO ONE OF THE FROBENIUS FORMS  
WHERE ROW 1 CONTAINS ALL BUT THE LEADING COEFF OF THE  
CHARACTERISTIC EQN. THE LEADING COEFF IS 1. ACCURACY IS IN-  
CREASED BY MAXIMIZING DIVISOR BY INTERCHANGING ROWS AND  
COLUMNS. THE ROOTS ARE SOLVED USING A D-ALEMBERT LEMMA  
WHERE ALL VALUES IN A ROW TO THE LEFT OF THE DIAG ARE LESS

THAN INPUT CRITERION. PROGRAM SUBDIVIDES PROBLEM USING PF TO  
 OPERATE ON 2 OR MORE LOWER POLYNOMIALS WHERE :  
 A IS NVAR BY NVAR DOUBLE PREC MATRIX, N IS ORDER OF MATRIX,  
 RR,PI ARE STORAGE ARRAYS, CRIT IS DIVISOR CRITERIA (NORMAL 0).  
 CHARD USES POLYPF,LEMPRT,POLYCV AND COEFER

```

    DIMENSION A(NVAR,NVAR),RR(1),RI(1)
    DOUBLE PRECISION COEF(11)
    DIMENSION XX(1),YY(10)
    DOUBLE PRECISION SUM,DIV,RCN(10),COL(10)
    DOUBLE PRECISION A,X,YDP
  C   MATRIX NORMALIZATION FOR A SPECIAL CLASS OF PROBLEMS
  C   IF N GE 20 DIVIDE ALL MATRIX ELEMENTS BY 10.0
    IF (N.LT.20) GO TO 3100
    DO 3050 I = 1,N
    DO 3050 J = 1,N
 3050 A(I,J) = A(I,J)/10.0
 3100 CONTINUE
    JACK=0
    M=N
    NR=1
 1   L=M
 2   K=L-1
    RIG=CRIT
    JJ=1
  C   FIND LARGEST ROW ELEMENT TO LEFT OF DIAGONAL
    DO 10 J=1,K
    YDP = A(L,J)
    AA = DABS(YDP)
    IF (AA.LT.SIC) GO TO 10
    RIG = AA
    JJ=J
 10  CONTINUE
  C   IF ALL ELEMENTS LEFT OF DIAGONAL ARE LF CRITERIA GO TO
  C   COMPUTE EIGENVALUES OF REDUCED MATRIX
    IF (JJ.EQ.0) GO TO 70
  C   SHIFT ROWS AND COLS IF NECESSARY
    IF (JJ.EQ.K) GO TO 40
    DO 20 J=1,M
    X = A(JJ,J)
    A(JJ,J) = A(K,J)
 20  A(K,J)=X
    DO 30 I=1,L
    X= A(I,JJ)
    A(I,JJ)=A(I,K)
 30  A(I,K)=X
 40  CONTINUE
  C   MAKE SIMILARITY TRANSFORMATION ON MATRIX
    DIV=A(L,K)
  C   ROW IN EFFECT IS THE LEFT OR INVERSE SIMILARITY MATRIX
  C   COL IN EFFECT IS THE RIGHT SIMILARITY MATRIX
    DO 42 J=1,M
    ROW(J)=A(L,J)
 42  COL(J)=-ROW(J)/DIV
    COL(K)=1.0/DIV

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```

C      (PCW+I) * A WHERE ROW IS KTH ROW, I THE IDENTITY MATRIX
      DO 50 J=1,M
      SUM=0.000
      DO 45 I=1,M
45     SUM=SUM+A(I,J)*PCW(I)
50     A(K,J)=SUM
C      (COL+I) * A WHERE ROW IS KTH ROW, I THE IDENTITY MATRIX
C      FIRST K ROWS LESS KTH COL.
      DO 60 I=1,K
      DO 60 J=1,M
      IF (J.EQ.K) GO TO 60
      A(I,J)=A(I,J)+A(I,K)*COL(J)
60     CONTINUE
C      LTH ROW
      DO 65 J=1,M
65     A(L,J)=0.000
C      KTH COL
      A(L,K)=1.000
      DO 68 I=1,K
68     A(I,K)=A(I,K)*COL(K)
      L=L-1
      IF (L.EQ.1) GO TO 70
      GO TO 2
C      SET UP TO COMPUTE ROOTS OF REDUCED OR FULL MATRIX
70     CONTINUE
      IF (L.EQ.M) GO TO 200
      COEF(1) = 1.0
      J=1
      DO 80 I=L,M
      J=J+1
      COEF(J)=-A(L,I)
80     CONTINUE
C      J BECOMES DEGREE OF POLYNOMIAL
      J=J-1
      CALL POLYRF(COEF,J,XY,YY,IERR)
      IF (IERR.NE.0) PRINT 1080, IERR
1080  FORMAT(1H0,10X,13HPOLRF IERR =,I5)
C      STORE J ROOTS
      DO 90 I=1,J
      NR=NR+1
      RR(NR)=XX(I)
90     RI(NR)=YY(I)
      IF (NR.GE.N) GO TO 500
      M=N-NR
      IF (M.EQ.1) GO TO 220
      GO TO 1
C      ONE EIGENVALUE IS A DIAGONAL ELEMENT
200    NR=NR+1
      RR(NR)=A(L,L)
      RI(NR)=0.0
210    IF(NR.LT.N) GO TO 500
      IF(L.EQ.2) GO TO 220
      M=N-NR
      GO TO 1

```

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```

220 NR=NR+1
    RR(NR)=A(1,1)
    RI(NR)=J.0
    GO TO 210
C   IF N GE 20 MULT. ALL ROOTS BY 10.0
500 IF (A.LT.20) RETURN
    DO 3150 I = 1,N
    PR(I) = 10.0*RR(I)
3150 RI(I) = 10.0*RI(I)
    RETURN
    END
    SUPROUTINE LEMPRT
C   THIS ROUTINE SYSTEMATICALLY FINDS A ROOT OF A POLYNOMIAL
C   USING A SIMPLE GAGING SCHEME BASED ON D-ALFEMBERTS LEMMA
    COMMON /COEFER/PP(11),M,X,Y,AC,RP,RI,IERRR
    DIMENSION NFLAG(5),U(5),V(5),P(5)
    DOUBLE PRECISION PP
    EQUIVALENCE (P,P1),(P(2),P2),(P(3),P3),(P(4),P4),
    * (P(5),P5)
    L = 1
    RR = 0.0
    RI = J.0
    SIGN = 1.0
    IFLAG = 0
    JFLAG = 0
    KFLAG = 0
    DEL = 0.5
    DDEL = 0.0
    DO 5 I = 1,5
5   NFLAG(I)=0
10  IF (IFLAG.LT.5) GO TO 25
    IF (JFLAG.GT.0) GO TO 25
20  IFLAG = 0
    IF (KFLAG.LT.3) GO TO 22
    RR = RP + SIGN/10.0
    RI = RI + SIGN/13.0
    SIGN = -2.0*SIGN
21  CONTINUE
    IF (ABS(RI).LE.1.0) GO TO 22
    SIGN = SIGN/97.0
    RP = SIGN/3.0
    RI = -SIGN
    GO TO 21
22  KFLAG = KFLAG + 1
    DEL = DDEL*DEL
    DDEL = DDEL + 1.3
24  NFLAG(L) = 0
    GO TO 30
25  IFLAG = IFLAG + 1
30  CONTINUE
    DO 40 I = 1,5
    IF (NFLAG(I).NE.0) GO TO 38
    X = RR
    Y = RI

```

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```

      IF (I.EQ.1) GO TO 35
      IF (I.EQ.2) X = X + DEL
      IF (I.EQ.3) X = X - DEL
      IF (I.EQ.4) Y = Y + DEL
      IF (I.EQ.5) Y = Y - DEL
35  U(I) = X
      V(I) = Y
      CALL POLYEV
      P(I) = AP
38  NFLAG(I) = 0
40  CONTINUE
      IF (JFLAG.GT.27) GO TO 60
      DO 45 I = 1,5
      IF (P(I).GT. 1.0E-17) GO TO 48
45  CONTINUE
      GO TO 60
48  DIF1 = AMAX1(P1,P2,P3,P4,P5)
      DIF2 = AMIN1(P1,P2,P3,P4,P5)
      DIF = DIF1 - DIF2
      IF ((DIF.GE.1.0).AND.(P1.LT.1.0)) GO TO 55
      IF (P1.EQ.0.0) GO TO 60
      DIF = DIF/P1
      IF (DIF.LT.0.001) GO TO 20
55  CONTINUE
60  CONTINUE
      DO 70 J = 1,5
      I = J
      IF (P(J).EQ.0.0) GO TO 100
70  CONTINUE
      DIF2 = AMIN1(P2,P3,P4,P5)
      IF (P1.GT.DIF2) GO TO 80
      IF (DEL.LT.10.E-30) RETURN
      DEL = 0.5*DEL
      XX = PR + DEL
      YY = RI + DEL
      IF ((XX.EQ.PR).AND.(YY.EQ.RI)) RETURN
      IF ((XX.EQ.PR).AND.(PI.EQ.0.0)) RETURN
      IF ((RP.EQ.0.0).AND.(RI.EQ.YY)) RETURN
      IF (JFLAG.GT.100) GO TO 220
      JFLAG = JFLAG + 1
      NFLAG(1) = 1
      GO TO 30
80  AMINY = P2
      N = 2
      DO 95 I=3,5
      IF (P(I).GT.AMINY) GO TO 85
      N = I
      AMINY = P(I)
85  CONTINUE
      L = 3
      IF (N.EQ.3) L = 2
      IF (N.EQ.4) L = 5
      IF (N.EQ.5) L = 4
      NFLAG(1) = 1

```

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```

NFLAG(L) = 1
U(L) = U(1)
U(1) = U(N)
V(L) = V(1)
P(L) = P1
V(1) = V(N)
P1 = P(N)
RP = U(1)
RI = V(1)
GO TO 13
100 RR = U(I)
PI = V(I)
RETURN
220 IERRR = 2
RETURN
END
SUBROUTINE POLYEV
COMMON /COEFFR/PR(11),M,X,Y,AP,RR,RI
DOUBLE PRECISION PR,U,V,US
U = PR(1)
V = 5.000
DO 20 I = 2,M
US = U
U = X*U - Y*V + PR(I)
20 V = X*V + Y*US
AP = DABS(U) + DABS(V)
RETURN
END
SUBROUTINE POLYRF(P,M,X,Y,IERR)
COMMON /COEFFR/PR(11),M,A,E,AP,RP,RI,IERRR
DOUBLE PRECISION P(1)
DIMENSION X(1),Y(1)
DOUBLE PRECISION PR,D,X2,XY
IF ((N.LT.1).OR.(N.GT.30)) GO TO 200
IERR = 0
IERRR = 0
J = 1
M = N+1
DO 10 I = 1,M
PR(I) = P(I)
10 CONTINUE
IF (N.EQ.1) GO TO 100
15 CONTINUE
CALL LEMRT
IF (RI.EQ.0.0) GO TO 40
IF (RR.EQ.0.0) GO TO 16
TEST = RI/RR
IF (ABS(TEST).LT.0.000001) GO TO 40
16 CONTINUE
X(J) = RR
Y(J) = RI
IF (IERRR.NE.0) GO TO 220
IF (J.EQ.N) RETURN
J = J + 1

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```

      X(J) = RP
      Y(J) = -RI
      IF(J.EQ.N) RETURN
      J = J + 1
      M = M - 2
1     X2 = 2.0*RP
      YY = -(PR*PR + RI*RI)
      DO 20 I = 2,M
      PR(I) = PR(I) + X2*PP(I-1)
      PR(I+1) = PR(I+1) + YY*PR(I-1)
20    CONTINUE
      IF (M.EQ.2) GO TO 100
      GO TO 15
40    CONTINUE
      X(J) = RP
      Y(J) = 0.0
      IF (IERRP.NE.0) GO TO 220
      IF (J.EQ.N) RETURN
      J = J + 1
      M = M - 1
      DO 50 I = 2,M
50    PR(I) = PR(I) + PR*PR(I-1)
      IF (M.EQ.2) GO TO 100
      GO TO 15
100   D = PR(1)
      X(J) = -PP(2)/D
      Y(J) = 0.0
      RETURN
200   PRINT      12(7, N
1200  FORMAT(1H0,10X, 3HN =,I4,21HOUTSIDE LIMITS POLYRF)
      IERR=1
      RETURN
220   IERR = IERRR
      RETURN
      END
DATA CARD FOR CONSTANT TERMS
32.174 1745.5 132.6 13.71 3.1416 1020.0 10.09
END

```

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Vita

Robert G. Lorenz was born on 25 March, 1942, in Miami, Florida. He graduated from Miami Jackson High School in June, 1959, and in that same year, entered the United States Air Force Academy at Colorado Springs, Colorado. He graduated from the Academy in June, 1963, being awarded a degree of Bachelor of Science and a commission in the United States Air Force. He spent the next six years in rated duties, amassing 3000 flying hours in the Lockheed C130A Hercules. His last tour of duty prior to entering the Air Force Institute of Technology in November, 1970, was as a pilot in the AC130 Gunship.

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