AD-751 546

INTERACTIONS OF STRESS WAVES WITH CRACKS IN ROCK MEDIA

Paul J. Blatz

Shock Hydrodynamics

Prepared for:

Advanced Research Projects Agency
Bureau of Mines

September 1972

3230-09

AD751546

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SEMI-ANNUAL REPORT

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Sponsored by

Advanced Research Projects Agency
ARPA Order No. 1597, Amend. 3
Program Code 2F10

Prepared by

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Sherman Oaks, California 91403

31

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) SHOCK HYDRODYNAMICS A Division of the Whittaker Corporation 15010 Ventura Blvd., Sherman Oaks, California 91403	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP N/A

3. REPORT TITLE

INTERACTIONS OF STRESS WAVES WITH CRACKS IN ROCK MEDIA

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

SEMI-ANNUAL REPORT - 1 March - 1 September 1972

5. AUTHOR(S) (First name, middle initial, last name)

Paul J. Blatz

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1972	28	6

8a. CONTRACT OR GRANT NO. H0220025 b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 3230-09 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	ARPA

13. ABSTRACT

A number of model problems involving the elastic deformation of a body perforated by a pore or crack have been solved. For each of these problems effective elastic moduli are defined as a function of the flaw content. These effective elastic parameters are introduced into the Griffith criterion and flaw growth is followed. A cubic viscosity has been introduced into the SHEP Program. The purpose of this fictive term is to provide better control over the oscillation introduced in an algorithm based on a difference scheme and also based on out-of-plane motion. By contrast, quadratic viscosity which is ordinarily used does not control out-of-plane motion.

DD FORM 1 NOV 65 1473

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Randomly Faulted Media						

ib

3230-09

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This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by the Bureau of Mines under Contract No. H0220025.

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Effective Date of Contract:	10 February 1972
Contract Expiration Date:	10 April 1973
Amount of Contract:	$50,315

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1. INTRODUCTION

This report summarizes the work conducted under Contract No. H0220025 during the period 1 March - 1 September 1972. The program is generally concerned with an understanding of how stress waves interact with discrete faults, randomly distributed faults and tunnels in rock media. The study comprises both theoretical analyses and two-dimensional computer calculations of selected interaction problems. The work carried out during this period was primarily concerned with the characterization of randomly faulted media and the development of an algorithm for controlling crack propagation in the SHEP code.

2. TECHNICAL PROGRESS: RANDOMLY FAULTED MEDIA

2.1 POROSITY

In trying to understand the effects of strong stress waves on rock media, e.g., hard granite, it is first necessary to assign a length scale within which damage occurs. This length scale depends, of course, on the energy associated with the excavation process or damage process caused by a high explosive burst. There are two categories of bursts, those which are used for excavation (a few tons of high explosive), and those which are designed to damage underground structures or tunnels (several kilotons or more). It is generally agreed[1] that the scaled elastic radius (the radius of a spherical region within which plastic yield and significant fracture damage may occur) surrounding a burst center is of the order of 400 $ft/kt^{1/3}$. This means that the elastic radius associated with a megaton burst is of the order of 4000 ft. On the other hand the elastic radius associated with an excavation burst may be as little as 40 ft.

From tests such as PILEDRIVER and HARD HAT which were conducted at Nevada Test Site 15 in hard granite, it is known[2] that fractures, faults or blocks are distributed with spacings significantly less than 1 ft. Thus a sphere of radius, as little as 40 ft. will contain a large number of flaws. In such a sphere or a larger one, one is led to the view that the fracture area is so complicated and so diversified that one may profitably approach the problem of defining the flaw structure on a stochastic basis, and assume that the flaw area is randomly distributed. This idea has been subsumed by other investigators[3].

With this thought in mind, we undertake to characterize a randomly faulted medium by elastic moduli which depend on the fracture area per unit volume. There arise immediately a number of model problems that one can use as a guide to this characterization. The simplest one and the first of several we shall discuss is the hydrostatic tension of a porous medium. If one makes the first order approximation that a unit cell of a randomly porous medium is a spherical cell, then it is easy to show that the effective bulk modulus of such a cell is given by

$$K_{eff} = \frac{1 - \nu_p}{\frac{1}{K} + \frac{3\nu_p}{4G}} \tag{1}$$

where ν_p is the volume fraction of pores, and

K, G are the elastic moduli of the continuum

It is also easy to relate ν_p to the surface area per unit volume α, namely

$$\nu_p = \left(\frac{b\alpha}{3}\right)^{2/3} \tag{2}$$

where b is a length scale which has to do with porosity or flaw size. We suspect that interaction among the adjacent flaw will result in a not dissimilar relation where:

$$\nu_p = \left(\frac{b\alpha}{3}\right)^{n} \tag{3}$$

and where n and b must be determined experimentally.

Expression (1) is based on linear elastic theory. Because the applied stress becomes concentrated at the pore surface, it is of interest to see under what conditions(s) (1) breaks down. We approach the question by analyzing the effect of geometric nonlinearity.

Assume that a spherical shell of outer radius, b, inner radius, a, is subjected to isotropic tension, S, at the outer surface, i.e.

$$\bar{\sigma}_r \Big|_{r=b} = + S \tag{4}$$

We desire to determine the amount of dilatation. The bulk material will actually dilate very slightly, while the pore will dilate more or less highly depending upon the tension. Let us assume that the material properties are given by:

$$W = \frac{G}{2} (\Sigma \lambda_i^2 - 3) + G f(J) \tag{5}$$

where	W	is the strain energy function
	λ_i	is the stretch ratio in the i^{th} principal direction
	J	is the volume ratio, $J = \pi \lambda_i$ (6)
and	G	is the shear modulus of the material.

The Piola stress is given by:

$$\sigma_i = \frac{\partial W}{\partial \lambda_i} = G [\lambda_i + \frac{J f'(J)}{\lambda_i}] \tag{7}$$

The Cauchy stress is given by:

$$\bar{\sigma}_i = \frac{\lambda_i \sigma_i}{J} = G [\frac{\lambda_i^2}{J} + f'(J)] \tag{8}$$

Let the deformed coordinate be related to the undeformed coordinate by

$$\bar{r} = \bar{r}(r) \tag{9}$$

Then

$$\lambda_r = \frac{d\bar{r}}{dr} \tag{10}$$

$$\lambda_\theta = \lambda_\phi = \frac{\bar{r}}{r} \tag{11}$$

Thus $$J = \frac{\bar{r}^2}{r^2}\frac{d\bar{r}}{dr} \tag{12}$$

We can also write

$$\lambda_r = \frac{J}{\lambda^2} \tag{13}$$

and $$\lambda_\theta = \lambda_\phi = \lambda \tag{14}$$

Thus $$\frac{\sigma_r}{G} = \frac{J}{\lambda^2} + \lambda^2 f'(J) \tag{15}$$

$$\frac{\sigma_\phi}{G} = \frac{\sigma_\theta}{G} = \lambda + \frac{J f'(J)}{\lambda} \tag{16}$$

In writing (5), we have included in the first term on the right hand side all the geometrical nonlinearity associated with shear. The second term has been left arbitrary, but does represent the effect of dilatation. Since J will nowhere throughout the spherical shell be significantly different from unity, it matters little what function f(J) we choose so long as it reduces to Hooke's law in the limit of infinitesimal strain. When we make this identification, we find that:

$$f(1) = 0 \tag{17}$$

$$f'(1) = -1 \tag{18}$$

$$f''(1) = \frac{1}{1-2\nu} = \frac{K}{G} + \frac{1}{3} \equiv C \tag{19}$$

Apart from these mild restrictions, f may have any form whatsoever. We are going to choose f to make the ensuing analysis convenient.

Elastostatic equilibrium is guaranteed by the statement:

$$\frac{r}{2}\frac{d\sigma_r}{dr} + \sigma_r - \sigma_\theta = 0 \tag{20}$$

We also have

$$\bar{r} = \lambda r \tag{21}$$

so that:

$$\frac{d\bar{r}}{dr} = \lambda_r = \frac{J}{\lambda^2} = \lambda + r\frac{d\lambda}{dr} \tag{22}$$

and

$$r\frac{d}{dr} = \left(\frac{J}{\lambda^2} - \lambda\right)\frac{d}{d\lambda} \tag{23}$$

Equation (22) will be used to determine r as a function of λ, or vice versa, when J is known, as a function of λ. Using (23), we can rewrite (20) in the form:

$$\left(\frac{J}{\lambda^2} - \lambda\right)\frac{d\sigma_r}{d\lambda} + 2\,(\sigma_r - \sigma_\theta) = 0 \tag{24}$$

After introducing (15) and (16) and simplifying, one obtains

$$\frac{dJ}{d\lambda} = -\frac{2\,(\lambda^3 - J)}{\lambda\,(1 + \lambda^4 f'')} \tag{25}$$

Equation (24) is immediately separable if we set

$$f'' = \frac{C}{J^{4/3}} \tag{26}$$

Now let

$$J = w^3\lambda^3 \tag{27}$$

so that

$$d\ell n\,\lambda = -\frac{(C + w^4)\,dw}{w\,[C + \frac{2}{3}\,w + \frac{1}{3}\,w^4]} \tag{28}$$

Now observe:

$$\text{as } r \to b,\ \lambda \to 1 + \epsilon,\ J \to 1 + O(\epsilon^2),\ w \to 1 - \epsilon \tag{29}$$
$$\text{as } r \to a,\ \lambda \to \frac{1}{\epsilon},\ J = 1 + O(\epsilon^2),\ w \to \epsilon \tag{30}$$

Thus, at either limit, the governing equation behaves like:

$$d\ell n\lambda \approx -d\ell nw \tag{31}$$

Thus, to an excellent approximation:

$$\lambda w = C_1 \tag{32}$$

and

$$J = C_1^3 \tag{33}$$

Insertion of this result into (23) yields:

$$\lambda = \left(C_1^3 + \frac{C_2^3}{r^3}\right)^{1/3} \tag{34}$$

The radial component of the Cauchy stress is given by:

$$\frac{\bar{\sigma}_r}{G} = \frac{J}{\lambda^4} + 3C - 1 - \frac{3C}{J^{1/3}} \tag{35}$$

which, with (33) and (34) yields:

$$\frac{\bar{\sigma}_r}{G} = C_1^3 \left(C_1^3 + \frac{C_2^3}{r^3}\right)^{-4/3} + 3C - 1 - \frac{3C}{C_1} \tag{36}$$

We now set:

$$\bar{\sigma}_r = S, \text{ at } r = b \tag{37}$$

$$\bar{\sigma}_r = 0, \text{ at } r = a \tag{38}$$

We can solve (37) and (38) to an excellent approximation by first setting

$$C_1^3 = 1 + \theta \tag{39}$$

and then linearizing both expressions in θ. This is permitted only because the materials of interest have Poisson's ratio $\nu = 1/4$, or $C = 2$. In the case of a rubberlike material, this approximation breaks down because C is more like 1000.

If we define

$$\lambda_b^3 - 1 \equiv \delta \tag{40}$$

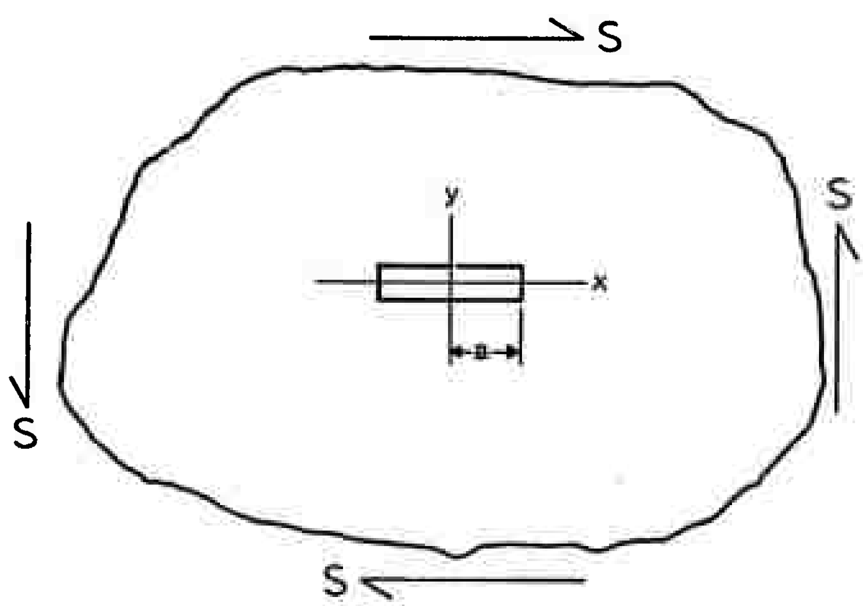
then we arrive at:

$$\frac{\delta}{S} \equiv B = \frac{\frac{1}{K} + \frac{3\nu_P}{4G}}{1 - \nu_P} \tag{41}$$

which is identical with (1). Thus geometrical nonlinearity has no overriding effect even though strain at the pore may be very large and even though the pore may buckle in compression.

2.2 PLANE CRACK

In the previous section 2.1, we considered the effect of flaw volume (porosity) upon bulk modulus. In this section we shall show how to analyze the stress field around a plane crack in an infinite plane space subjected to simple shear at infinity, cf sketch:



Within the framework of linear theory, the problem can be solved by the Kolosov-Muskhelishvili technique of complex variables. It is first convenient to map the Cartesian frame into an elliptic frame by the transformation:

$$z = \frac{a}{2}\left(\zeta + \frac{1}{\zeta}\right) \tag{42}$$

where $$\zeta = \rho\sigma \tag{43}$$

and $$\sigma = e^{i\vartheta} \tag{44}$$

Far away from the crack, the mapping approaches:

$$z \rightarrow \frac{a}{2}\rho e^{i\vartheta} \tag{45}$$

which is polar in nature. For this problem we have the BC, as $\rho \rightarrow \infty$,

$$\sigma_x = \sigma_y = 0, \ \tau_{xy} = S \tag{46}$$

These stress components may be converted by tensor transformation into the corresponding polar components, and we obtain

$$\text{as } \rho \rightarrow \infty, \ \sigma_\rho - i\tau_{\rho\vartheta} \rightarrow -iS\sigma^2 \tag{47}$$

$$\text{as } \rho \rightarrow 0, \ \sigma_\rho - i\tau_{\rho\vartheta} \rightarrow 0 \tag{48}$$

In terms of the two complex potentials ϕ and ψ, the cartesian displacements and elliptic stresses are given by:

$$2G(u + iv) = (3-4\nu)\varphi - \frac{z}{\bar{z}_{\bar{\zeta}}}\bar{\varphi}_{\bar{\zeta}} - \bar{\psi} \tag{49}$$

$$\sigma_\rho + \sigma_\vartheta = 2\left(\frac{\phi_\zeta}{z_\zeta} + \frac{\bar{\varphi}_{\bar{\zeta}}}{\bar{z}_{\bar{\zeta}}}\right) \tag{50}$$

and

$$\sigma_\rho - i\tau_{\rho\vartheta} = \frac{\phi_\zeta}{z_\zeta} + \frac{\bar{\phi}_{\bar{\zeta}}}{\bar{z}_{\bar{\zeta}}} - \frac{\sigma^2}{\bar{z}_{\bar{\zeta}}}\left[\bar{z}\left(\frac{\phi_{\zeta\zeta}}{z_\zeta} - \frac{\phi_\zeta z_{\zeta\zeta}}{z_\zeta^2}\right) + \psi_\zeta\right] \tag{51}$$

We now expand ϕ and ψ in power series in ζ which are holomorphic for $\zeta \to \infty$; we then equate powers of σ, and are led to the results:

$$\phi = \frac{aiS}{2} \frac{1}{\zeta} \tag{52}$$

$$\psi = \frac{aiS}{2} \left(\zeta - \frac{1}{\zeta} + \frac{2\zeta}{\zeta^2 - 1}\right) \tag{53}$$

After transforming back to Cartesian coordinates we obtain:

$$\tau_{xy} = \frac{(\zeta^2+1)(\zeta^4-2\zeta^2+1)}{2(\zeta^2-1)^3} + \frac{\zeta^3 \, (\bar{\zeta}+\frac{1}{\bar{\zeta}})}{(\zeta^2-1)^3} + \frac{(\bar{\zeta}^2+1)(\bar{\zeta}^4-2\bar{\zeta}^2+1)}{2(\bar{\zeta}^2-1)^3}$$

$$+ \frac{\bar{\zeta}^3 \, (\zeta+\frac{1}{\zeta})}{(\bar{\zeta}^2-1)^3} \tag{54}$$

What we desire now is an average value of τ_{xy} within some unit cell of outer radius R, inner radius 1. If one defines

$$\frac{G_{eff}}{G} = \frac{\langle\tau_{xy}\rangle}{S} \tag{55}$$

where
$$\langle\tau_{xy}\rangle = \frac{\int_1^R \oint \tau_{xy} \, \rho d\rho d\vartheta}{\pi(R^2-1)} \tag{56}$$

one arrives at the <u>incorrect</u> result: $\langle\tau_{xy}\rangle = S$ (57)

The reason that $\langle\tau_{xy}\rangle$ turns out to be independent of R under this definition of the average is that there is a stress singularity at $[\rho = 1; \vartheta = 0, \pi]$. This stress singularity (or stress concentration) affects the reduced stress at $\vartheta = \frac{\pi}{2}$, in such a way that $\langle\tau_{xy}\rangle$ is a constant. In order to define $\langle\tau_{xy}\rangle$ correctly, one has to treat the body with a crack in it as a "black box" and use the phenomenological definition of shear modulus, namely:

$$G_{eff} = \frac{\tau_{xy}}{\frac{u}{y} + \frac{v}{x}} \tag{58}$$

with $$\langle G \rangle = \frac{\int_1^R \oint G_{eff}\, \rho\, d\rho\, d\vartheta}{\pi (R^2 - 1)} \tag{59}$$

This integral can in principle be done but the work is prohibitive. In the interest of expediency, we turned our attention to another problem which can be done in a more straight forward fashion with equally useful results.

2.3 EFFECTIVE POISSON'S RATIO AND YOUNG'S MODULUS

Consider now an elastic space perforated by a sphere of radius a subjected to simple tension at $z = \pm \infty$. In the absence of the hole, the solution in Cartesian coordinate is, of course:

$$\sigma_x = \sigma_y = 0, \ \sigma_z = S \tag{60}$$

$$u = -\nu \frac{S}{E} x \tag{61}$$

$$v = -\nu \frac{S}{E} y \tag{62}$$

$$w = \frac{S}{E} z \tag{63}$$

In spherical coordinates, the solution $(r \to \infty)$ takes the form,

$$\sigma_r = S \, cs^2 \theta \tag{64}$$

$$\sigma_\theta = S \, sn^2 \theta \tag{65}$$

$$\tau_{r\theta} = S \, sn\theta \, cs\theta \tag{66}$$

$$\sigma_\theta = 0 \tag{67}$$

$$\tau_{r\theta} = - S \, sn\theta \, cs\theta \tag{68}$$

$$u = \frac{S}{r} r \, [cs^2\theta - \nu sn^2\theta] \tag{69}$$

$$v = - \frac{Sr}{2G} sn\theta \, cs\theta \tag{70}$$

$$w = 0 \tag{71}$$

At the hole, we have: $\sigma_r = \tau_{r\theta} = 0$ (72)

We present here only the pertinent details; the solution is straight forward. We expand u and v in spherical harmonics, and write:

$$u = \sum_{n=0}^{\infty} a_n P_n (cs\,\theta) \tag{73}$$

$$v = \sum^{\infty} b_n P'_n (cs\,\theta) \tag{74}$$

where the prime denotes differentiation re $\underline{r}$, and a_n and b_n depend on $\underline{r}$. We use the method of equating coefficients of P_n, and after much calculation, obtain:

$$\frac{\sigma_z}{S} = [1 + \frac{4-5\nu}{2(7-5\nu)} (\frac{a}{r})^3 + \frac{9}{2(7-5\nu)} (\frac{a}{r})^5] + \frac{cs^2\theta}{7-5\nu} [\frac{33+15\nu}{2} (\frac{a}{r})^3 - 45 (\frac{a}{r})^5]$$

$$+ \frac{15\, cs^4\theta}{2(7-5\nu)} [7 (\frac{a}{r})^5 - 5 (\frac{a}{r})^3] \tag{75}$$

$$\frac{E\tilde{w}}{Sz} = [1 + \frac{(4-15\nu)(1+\nu)}{2(7-5\nu)} (\frac{a}{r})^3 + \frac{9(1+\nu)}{2(7-5\nu)} (\frac{a}{r})^5]$$

$$+ \frac{15(1+\nu)\, cs^2\theta}{2(7-5\nu)} [(\frac{a}{r})^3 - (\frac{a}{r})^5] \tag{76}$$

where $\tilde{w}$ is the Cartesian displacement component.

The effective Young's modulus is defined by:

$$\frac{E_{eff}}{E} = \frac{z\sigma_z}{\tilde{w}E} = \frac{A + B\, cs^2\theta + C\, cs^4\theta}{(D + E_o\, cs^2\theta)} \tag{77}$$

where $\{A, B, C, D, E_o\}$ are defined above in (75) and (76).

The average tensile modulus is then given by:

$$\frac{<E>}{E} = \int_0^{\frac{\pi}{2}} \frac{A + B\, cs^2\theta + C\, cs^4\theta}{(D + E_o\, cs^2\theta)} \Big|_{r=b} sn\theta d\theta = \{\frac{C}{3E_o} + \frac{BE_o - CD}{E_o^2}$$

$$+ [\frac{A}{\sqrt{DE_o}} - \frac{B}{E_o}\sqrt{\frac{D}{E_o}} + \frac{CD}{E_o^2}\sqrt{\frac{D}{E_o}}] \arctan\sqrt{\frac{E_o}{D}}\}_{r=b} \qquad (78)$$

$$\simeq \{\frac{A}{D}(1 - \frac{E_o}{3D} + \frac{E_o^2}{5D^2}) + \frac{B}{D}(\frac{1}{3} - \frac{E_o}{5D}) + \frac{C}{5D}\}_{r=b} \qquad (79)$$

Figure 1 shows how $\frac{<E>}{E}$ behaves as a function of $(\frac{a}{b})^3$. In plotting $\frac{<E>}{E}$, it was sufficient to use the approximate representation given by (79), which is valid for $0 \le (\frac{a}{b})^3 \le 1$. In the limit of small $(\frac{a}{b})^3$, (81) behaves like:

$$\frac{<E>}{E} \simeq 1 - \frac{3(1+\nu)(3-5\nu)}{2(7-5\nu)} (\frac{a}{b})^3 \qquad (80)$$

which means that the initial downward slope of the two lines shown in Figure 1 for ($\nu = \frac{1}{2}, \frac{1}{3}$) and respectively $\frac{1}{4}, \frac{1}{2}$. Equation (80) can now be used in a fracture analysis as long as $(\frac{a}{b})^3$ is small with respect to unity in a sense that can be defined. We prefer however, to represent $\frac{<E>}{E}$ empirically by an expression that fits over the whole range, $0 \le (\frac{a}{b})^3 \le 1$. Such an expression is of the form:

$$\frac{<E>}{E} \simeq 1 - \frac{13 - 2\nu - 15\nu^2}{27 - 12\nu - 15\nu^2} (\frac{a}{b})^3 \qquad (81)$$

Equation (81) assumes that the two curves in Figure 1 are roughly linear.

The expression for Poisson's ratio is obtained in the following way. The Cartesian displacement is given by:

$$\frac{\tilde{u}}{x} = \frac{\tilde{v}}{y} = \frac{u + v\, ctn\,\theta}{r} \qquad (82)$$

$$\frac{\tilde{w}}{z} = \frac{u - vtn\,\theta}{r} \qquad (83)$$

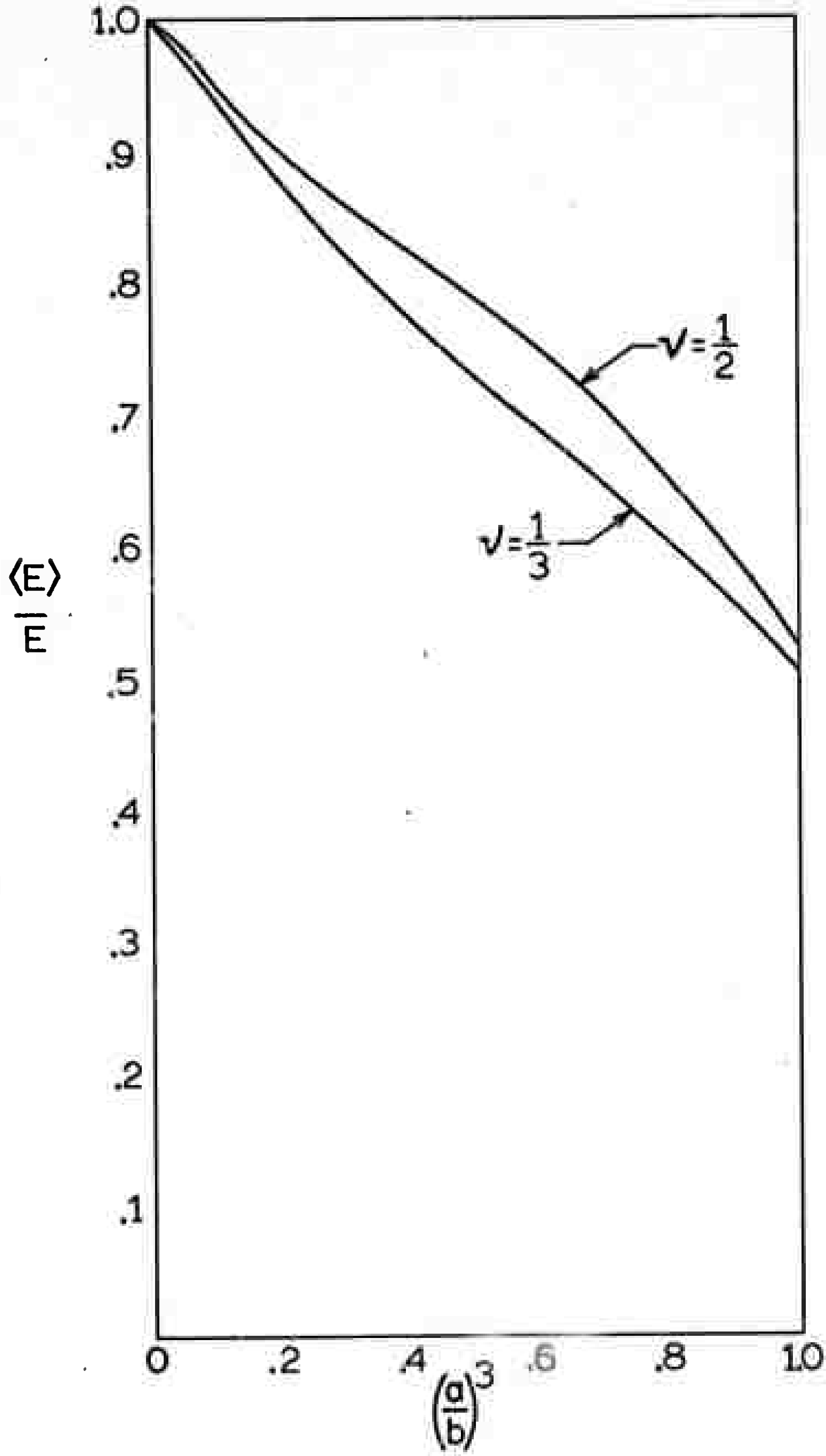


Figure 1. Effective Modulus at a Function of Pore Fraction.

so that
$$-\nu_{eff} = \frac{\tilde{u}_x}{\tilde{w}_z} = \frac{u + v\,ctn\,\theta}{u - v\tan\theta} \tag{84}$$

where u and v are given by:

$$\frac{Gu}{aS} = [\frac{1}{12}\left(\frac{a}{r}\right)^2 + \frac{1-2\nu}{6+6\nu}\frac{r}{a}] + P_2\,(cs\theta)\,[\frac{-3}{2\,(7-5\nu)}\left(\frac{a}{r}\right)^4$$
$$+ \frac{5\,(5-4\nu)}{6\,(7-5\nu)}\left(\frac{a}{r}\right)^2 + \frac{1}{3}\frac{r}{a}] \tag{85}$$

$$\frac{Gv}{aS} = P_2'\,[\frac{1}{2\,(7-5\nu)}\left(\frac{a}{r}\right)^4 + \frac{5\,(1-2\nu)}{6\,(7-5\nu)}\left(\frac{a}{r}\right)^2 + \frac{1}{6}\left(\frac{r}{a}\right)] \tag{86}$$

Substitution into (84) yields after averaging at $r = b$:

$$<\nu> = \frac{A}{\sqrt{BC}} \arctan\sqrt{\frac{C}{B}} - 1 \tag{87}$$

where

$$A = \frac{3}{2\,(7-5\nu)} + \left(\frac{a}{b}\right)^5 + \frac{5\,(1-2\nu)}{2\,(7-5\nu)}\left(\frac{a}{b}\right)^3 + \frac{1}{2} \tag{88}$$

$$B = \frac{1}{2+2\nu} + \frac{4-15\nu}{4\,(7-5\nu)}\left(\frac{a}{b}\right)^3 + \frac{9}{4\,(7-5\nu)}\left(\frac{a}{b}\right)^5 \tag{89}$$

$$C = \frac{15}{4\,(7-5\nu)}\left[\left(\frac{a}{b}\right)^3 - \left(\frac{a}{b}\right)^5\right] \tag{90}$$

Figure 2 shows how (87) behaves as a function of $\left(\frac{a}{b}\right)^3$. In plotting (87) it was sufficient to use the approximation:

$$<\nu> = \frac{A}{B}\left(1 - \frac{1}{3}\frac{C}{B} + \frac{1}{5}\frac{C^2}{B^2}\right) - 1 \tag{91}$$

In the limit of small $\left(\frac{a}{b}\right)^3$, (91) behaves like:

$$<\nu> \simeq \nu - \frac{(1+\nu)\,(14\nu^2 - 1)}{2\,(7-5\nu)}\left(\frac{a}{b}\right)^3 \tag{92}$$

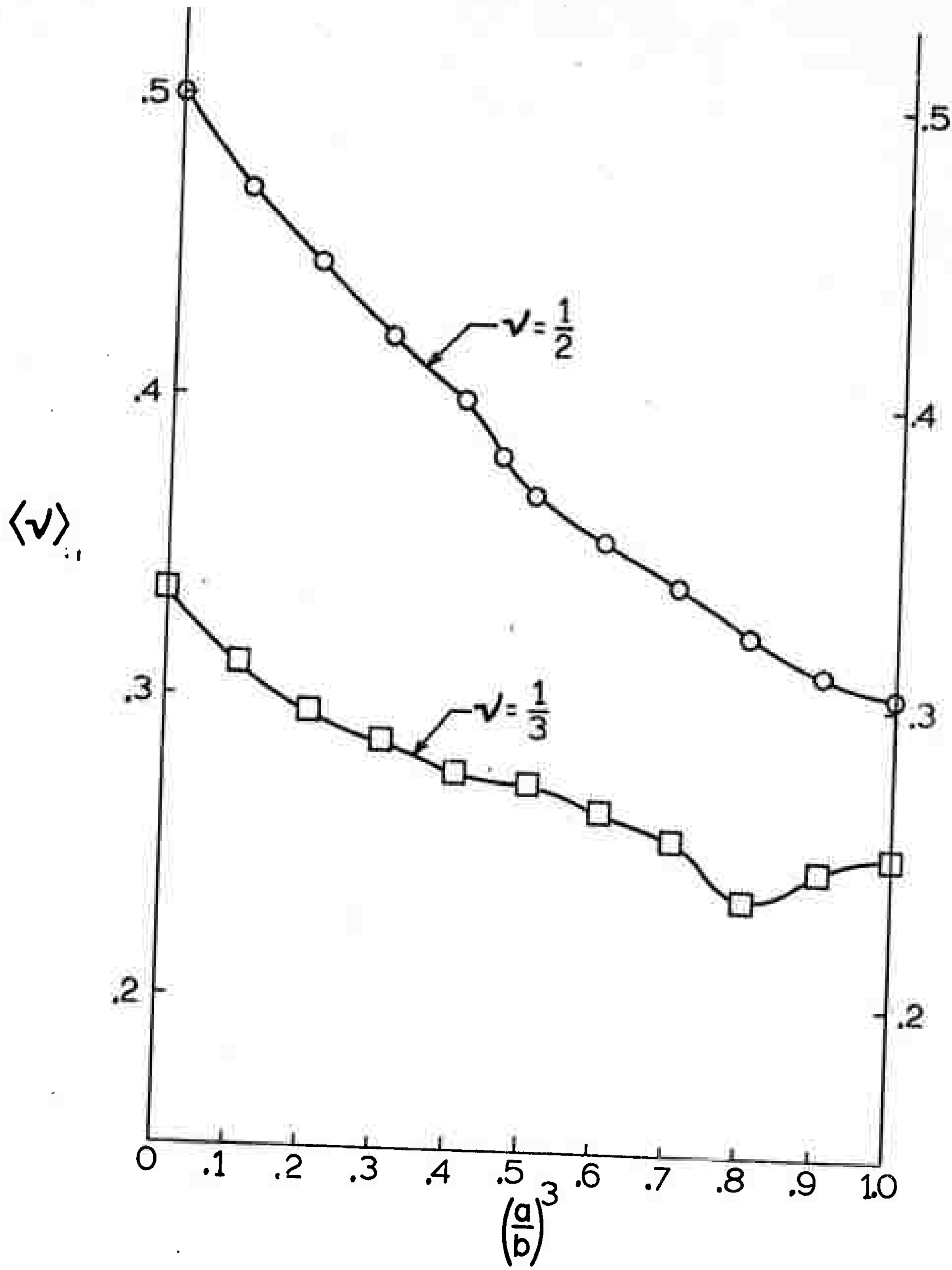


Figure 2. Effective Value of Poisson's Ratio vs Pore Fraction.

which means that the initial downward slope of the lines in Figure 2 (for $\nu = \frac{1}{2}, \frac{1}{3}$) are respectively $\frac{3}{8}, \frac{1}{4}$. Over the whole range (91) can be better represented by an empirical function of the form:

$$\nu \simeq \nu - k \left(\frac{a}{b}\right)^3 + l \left(\frac{a}{b}\right)^6 \tag{93}$$

where k is the linear coefficient in (92), and l is adjusted so that, at $(\frac{a}{b})^3 = 1$,

$$<\nu> = \frac{1 + 4\nu - 5\nu^2}{9 - 4\nu - 5\nu^2} \tag{94}$$

which is the value obtained from (94).

The results of the preceeding analysis can be summarized by saying that, for a porous material, an effective Young's modulus and an effective Poisson's ratio can be defined by an expression of the form:

$$\frac{<E>}{E} = 1 - k_o \nu_p \tag{95}$$

and

$$<\nu> = \nu - k_1 \nu_p + k_2 \nu_p^2 \tag{96}$$

where, if the holes interact, ν_p is given by:

$$\nu_p = \left(\frac{b\alpha}{3}\right)^n \tag{97}$$

with b and n empirical. Also the parameters $\{k_o\ k_1\ k_2\}$ depend on ν as given above.

Before using $<E>$ and $<\nu>$ to determine the fracture behavior of a randomly flawed material, it is prudent to point out that the representations (95) and (96) are, strictly speaking, only valid in a simple tensile field. It would be necessary to determine these same parameters for two or three other types of loading and to strike some sort of an average behavior. In general, however the algebra involved is cumersome and tedious and, at the best, we must think of k_o, k_1, and k_2 as semi-empirical parameters anyway. Thus we prefer to eschew the analysis of other model flaw problems and will go ahead on the basis that, roughly speaking, effective Young's Modulus decreases linearly with flaw content, and effective Poisson's ratio decreases parabolically with flaw content.

2.4 INCORPORATION OF EFFECTIVE MODULI IN THE GRIFFITH CRITERION

Again, we start slowly and look at a model problem, one which was first treated by Max Williams[4], and which is the first problem considered in this section, namely, the hydrostatic tension of a sphere. Imagine now that as the sphere is stretched radially by a monotonically increasing stress at its outer boundary $(r = b)$, that the inner boundary $(r = a)$ starts to ablate (fracture) when some critical stress concentration is reached. In this analysis, we ignore inertia and do a quasi-static analysis (the algebraic details are suppressed).

In the case of linear theory, it is easy to show that, as the stress is increased monotonically from zero, a critical stress is reached which depends on the initial flaw size, and which is determined by the Griffith criterion to have the form:

$$S = \left[1 - \left(\frac{a}{b}\right)^3\right] \sqrt{\frac{8E\Gamma}{9a_o(1-\nu)}} \tag{98}$$

where a_o is the critical hole size and Γ is the specific surface energy.

At this point the linear theory predicts that, in order for a_o to increase, S must be decreased; and conversely, as S is increased, a_o decreases. Thus (98) possibly holds only at the point of initiation of flaw growth, and predicts, as is known from studies on glass, that the critical stress depends inversely on the square root of the hole size. In order to find how the hole grows as S increases indefinitely, one must do a geometrically exact analysis. In order to gain insight into how the hole grows, we present here an exact geometrically non-linear analysis for an incompressible neo-Hookean material. Following the same approach as used above in Section 2.1, it is easy to show that the Griffith Criterion predicts that:

$$\frac{4\Gamma}{Gb} = \nu_p^{1/3} \left[2 \left(1 + \frac{\lambda_b^3 - 1}{\nu_p}\right)^{2/3} + \left(1 + \frac{\lambda_b^3 - 1}{\nu_p}\right)^{-4/3} - 3\right] \tag{99}$$

and

$$\frac{S}{G} = \frac{1}{2} \left[\frac{1}{\lambda_b^4} - \left(1 + \frac{\lambda_b^3 - 1}{\nu_p}\right)^{-4/3}\right] + 2 \left[\frac{1}{\lambda_b} - \left(1 + \frac{\lambda_b^3 - 1}{\nu_p}\right)^{-1/3}\right] \tag{100}$$

where

$$\lambda_b^3 = 1 + 3\epsilon_b \tag{101}$$

In the limit of small strain, i.e., as $\frac{3\epsilon_b}{\nu_p} \rightarrow 0$ we have:

$$\epsilon_b = \sqrt{\frac{\Gamma}{Eb}}\, \nu_p^{5/6} \tag{102}$$

$$\frac{S}{G} = 8 \sqrt{\frac{\Gamma}{Eb}}\, \frac{1-\nu_p}{\nu_p^{1/6}} \tag{103}$$

which relations are valid for:

$$\nu_p >> \left(\frac{3\Gamma}{Gb}\right)^3 \tag{104}$$

In the limit of large strain, i.e., as $3\frac{\epsilon_b}{\nu_p} \rightarrow \infty$ we have:

$$\epsilon_b = \frac{1}{3} \left(\frac{2\Gamma}{Gb}\right)^{3/2} \sqrt{\nu_p} \tag{105}$$

$$\frac{S}{G} = \frac{5}{2} - 2\left(\frac{Gb}{2\Gamma}\right)^{3/2} \sqrt{\nu_p} \tag{106}$$

Which relations are valid for:

$$\nu_p << \left(\frac{2\Gamma}{Gb}\right)^3 \tag{107}$$

Thus, in summary, for large strain, ν_p increases quadratically with strain, and for small strain, the rate of increase drops to the $(6/5)^{th}$ power. This behavior is shown in Figures 3 and 4 where ν_p is plotted versus log ϵ_b for various values of $(4\Gamma/Gb)$. In Figure 5 we present a crossplot of log ϵ $(\nu_p = 1)$ versus by $(4\Gamma/Gb)$. In the small strain limi , (103) compares favorably with (98) within a factor of two, whereas behavior beyond 20% strain is just not forthcoming from the linear theory. Whether or not one uses the nonlinear theory depends on the value of (Γ/Gb).

In general, for weakly cohesive materials, linear theory is adequate.

Let us now redo this problem in the framework of linear theory, and replace the cavitated shell by a solid shell of radius b with an effective bulk modulus given by (1).

A simple calculation shows that:

$$S = \frac{1-\nu_p}{\nu_p^{1/6}} \sqrt{\frac{8\Gamma E}{9b(1-\nu)}} \tag{108}$$

Thus a Griffith calculation based on effective elastic constants rather than flaw geometry leads to a solution of the same form as that obtained in the "flaw" calculation, except that the length scale a_o is replaced by b. In effect, this means it is equivalent to filling up the void with material of the correct effective modulus. Thus the randomly flawed theory leads to reasonable results.

It is our intention to examine in similar fashion the problem of simple tension of the space with a hole replaced by "effective" material. If this checks out, we will then program the relations (95) and (96) into the SHEP code, and will then run several wave problems using the Griffith criterion for materials with simple geometry and moduli which depend on flaw fracture. It is then our intention to define a critical α or ν_p at which sufficient damage is assumed to have occurred so that the fractured material is now ready for excavation by shoveling processes. Thus we have here a simple damage theory, a precursor to excavation.

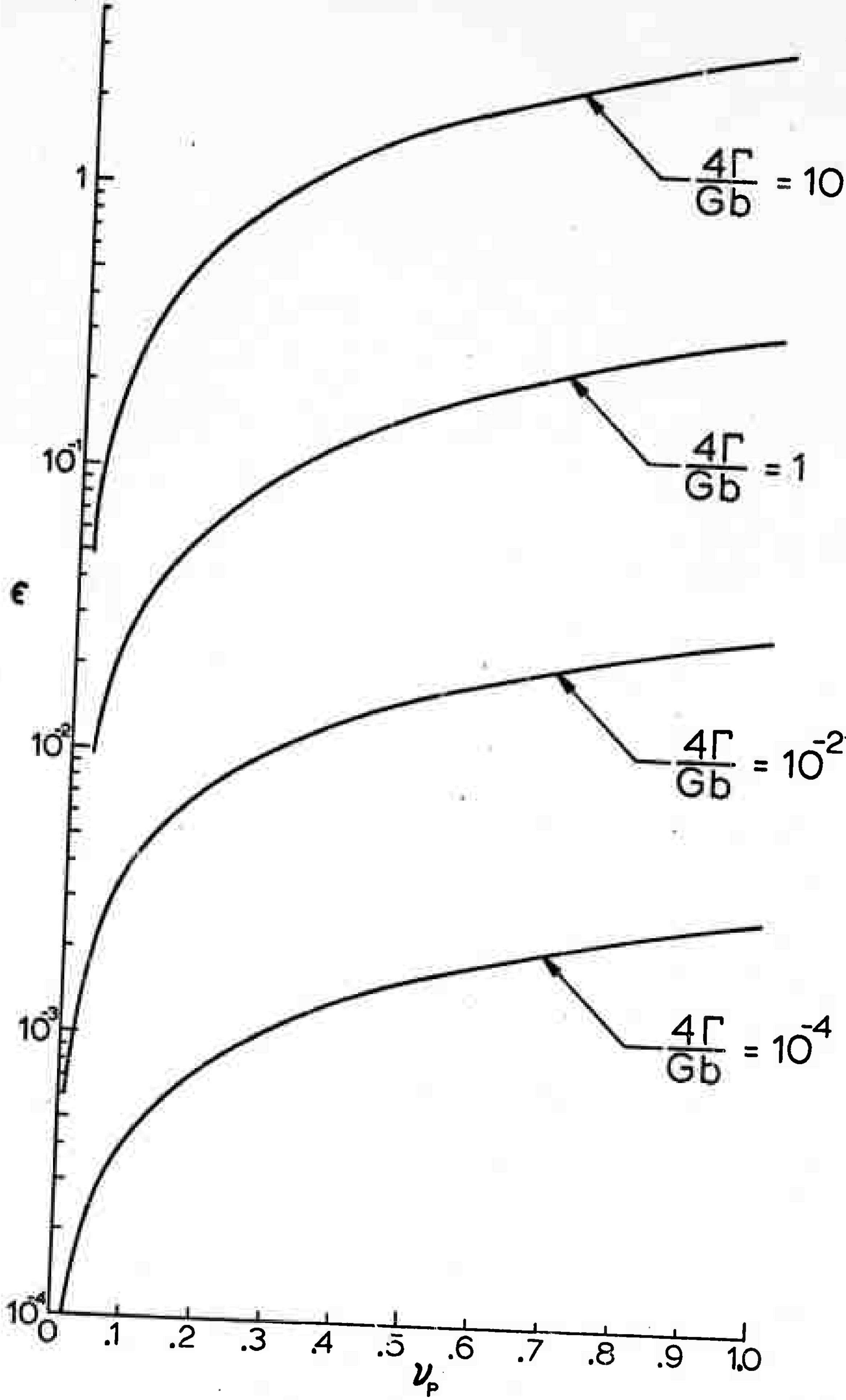


Figure 3. Griffith Strain vs Flaw Content.

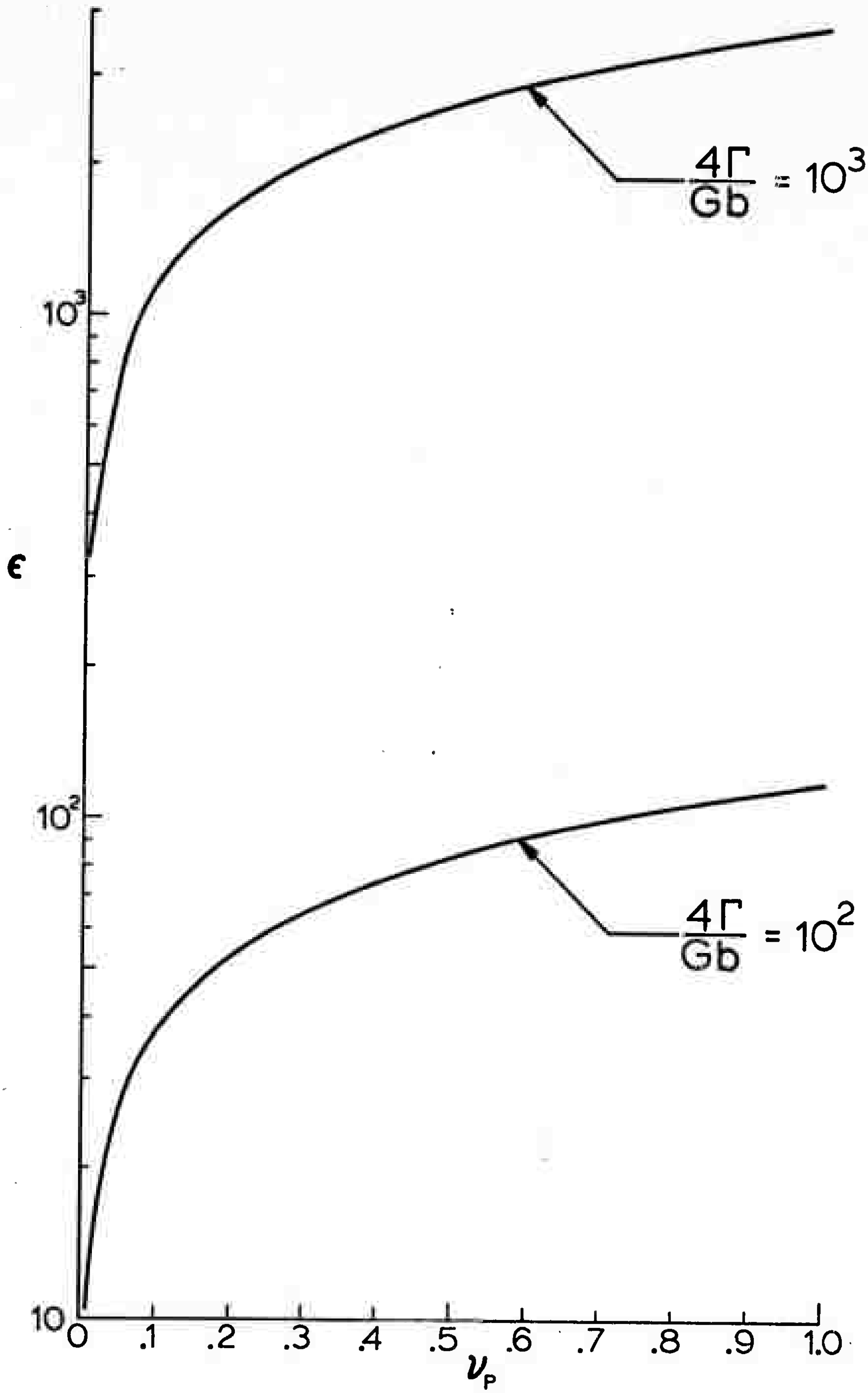


Figure 4. Griffith Strain vs Flaw Content.

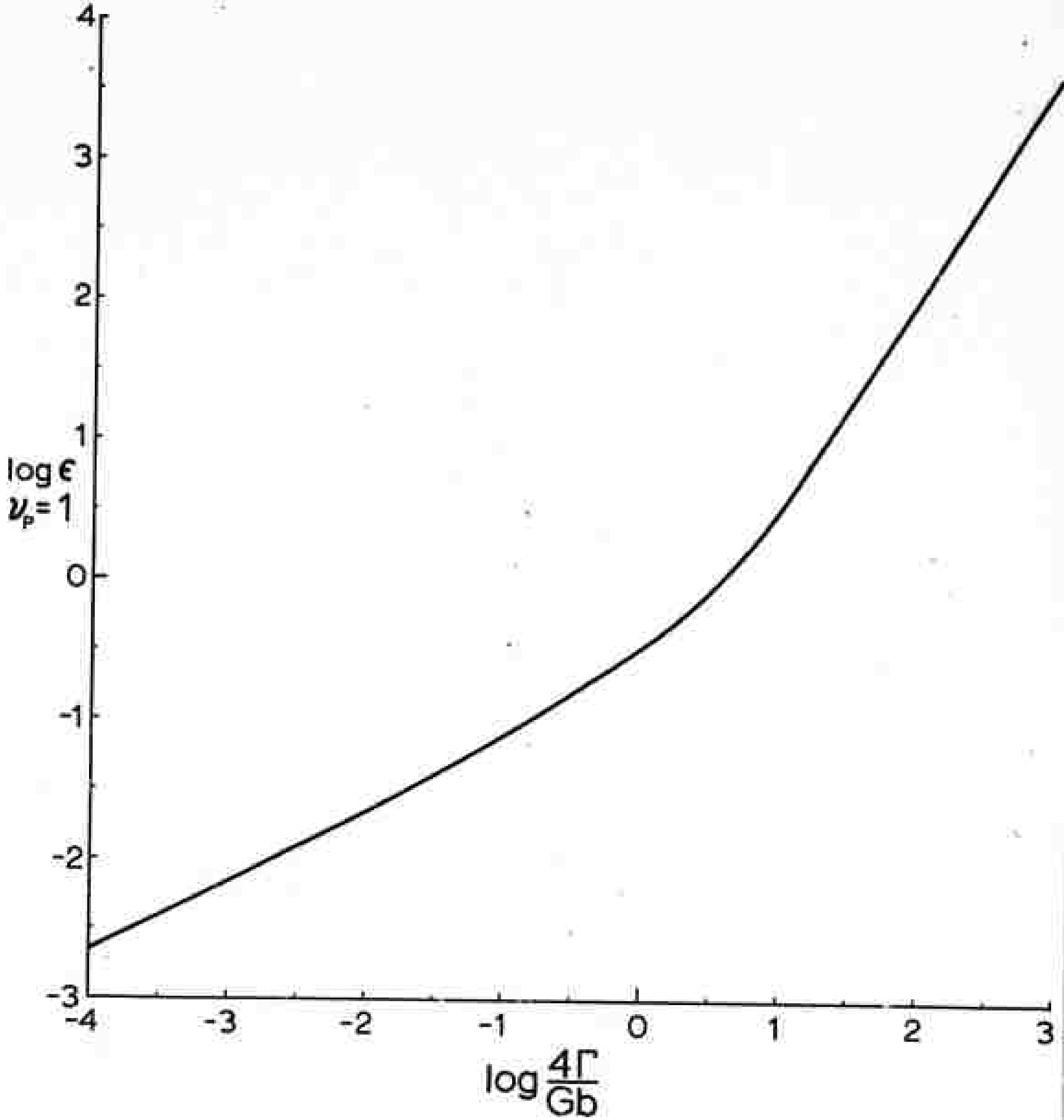


Figure 5. Griffith Strain vs Surface Energy.

The present theory has so far been developed for an isotropic elastic continuum. In the forthcoming period, we propose to examine both an orthotropic elastic material as well as an elasto-plastic material.

3. TECHNICAL PROGRESS: DISCRETELY FAULTED MEDIA

In the work completed on Contract No. H0210019, it was shown that a computer solution obtained by SHEP provided excellent agreement with the same features of the analytical solution derived by Achenbach and later extended by Blatz.[5] One feature of the computer solution is not, however, desirable. Since the SHEP code is a difference equation representation of the continuum equation, it is naturally resonant and "coughs back" both oscillations in the wave profile as well as spreading of the wave profile. This behavior is shown in Figure 6 where the computer solution is compared with the theoretical solution for the velocity profile from the crack tip to the bow wave front. Such a numerical behavior makes it impossible for us to compute precisely enough the integrals in the algorithm for crack propagation. An attempt to smooth out the curve is being made with the aid of cubic viscosity, an extension of the form introduced by Von Neumann and Richtmeyer.[6]

If we decompose the stress tensor into elastic and viscous components, we find:

$$\vec{\vec{t}} = \vec{\vec{t}}_E + \vec{\vec{t}}_D \tag{109}$$

where $\vec{\vec{t}}_D$ is the dissipative stress. This component generally can be written as:

$$\vec{\vec{t}}_D = \alpha_1 \vec{\vec{d}} + \alpha_2 \vec{\vec{d}} \cdot \vec{\vec{d}} \tag{110}$$

where $\vec{\vec{d}}$ is the strain rate tensor, and:

$$\alpha_k = \alpha_k \left(I_d, II_d, III_d\right), \; k = 1, 2 \tag{111}$$

with $\{I_d, II_d, III_d\}$ the three invarients of the strain rate tensor. Von Neumann and Richtmeyer took:

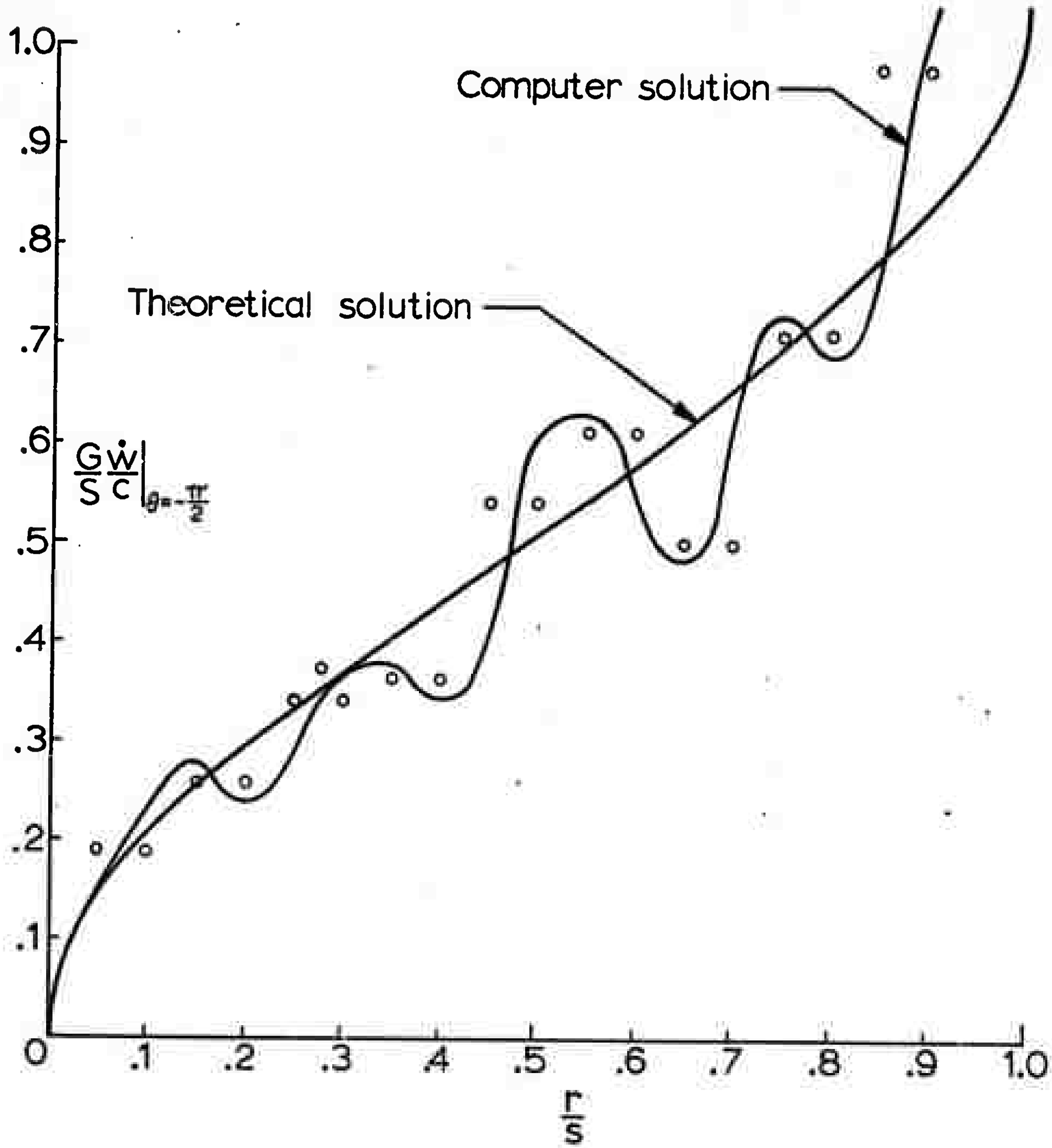


Figure 6. Computer Oscillations for Out of Plane Motion.

$$\vec{\vec{t}}_D = 2\nu_o \rho \, (1-\tau I_d) \, \vec{\vec{d}} \tag{112}$$

where ν_o is a kinematic viscosity and τ is a volumetric relaxation time.

We take
$$\vec{\vec{t}}_D = 2\nu_o \, \rho \, (1-\beta II_d) \, \vec{\vec{d}} \tag{113}$$

where β is a higher order rate constant.

Equation (112) involves quadratic and linear terms, (113) involves cubic and linear terms. Equantion (113) provices a twofold advantage over (112). First it can be used for incompressible or isochoric deformation (such as our model antiplane shear problem). Equation (112) can only be used for compressible media. Equation (113) contributes to both the inplane and out-of-plane terms. In (112) the quadratic term contributes only to the inplane terms. Thus (113) fills the bill for our problem.

The coefficients ν_o and β are given by:

$$\nu_o = C_1 \frac{(\Delta x)^2}{\Delta t} \tag{114}$$

$$\beta = \frac{C_2}{(\Delta t)^2} \tag{115}$$

Equations (113), (114), and (115) have been programmed and an optimization study of the constants C_1 and C_2 is underway.

REFERENCES

1. W. R. Perret, Free Field Ground Motion in Granite, Sandia Laboratory, POR-4001 (1968).

2. I. Borg, Survey of PILE DRIVER Results, Lawrence Radiation Laboratory, UCRL-50865 (1970).

3. R. G. Hoagland, et al., Influence of Microstructure on Fracture Propagation in Rock, Batteile, Columbus Laboratories, Final Report, Contract H0210006, January, 1972.

4. M. L. Williams, Int. J. Fract. Mech. (1968).

5. P. J. Blatz and M. H. Wagner, Quarterly Management Report, Nov. 1971 to U. S. Bureau of Mines, Prepared by Shock Hydrodynamics.

6. J. Von Neumann and R. D. Richtmeyer, J. App. Phys. 21, 232 (50).