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THE COMBAT USE AND COMBAT EFFECTIVENESS OF FIGHTER-INTERCEPTORS

by

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ANNOTATION

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The book is intended for specialists in Air Force units, AA aviation, for students at aviation schools, and also for workers in the aviation industry.

INTRODUCTION

The combat use of fighter-interceptors involves various branches of science and engineering. For successful use of armament, pilots study the design of vehicles and armament, flight dynamics, the theory of combat efficiency, air tactics, and other disciplines. As a survey of the open foreign literature (some of which is given at the end of the book) shows, general principles of operation of aircraft and rockets have been described in greatest detail up to the present time. Much less study has been devoted to quantitative methods of evaluating the combat use of aviation; these are given individually in various sources.

The dynamics of fighter maneuvering in aerial combat have been examined thoroughly in the familiar book by V. A. Bulinskiy [1]. However, in this book he does not examine the next stage in the interception of an aerial target – guiding the rocket to the target – or questions of the effectiveness of interception. General assumptions and methods for calculating the combat effectiveness of weapons are described in [3, 4, 8], while questions of the combat effectiveness of airborne vehicles are described in [3, 14, 15]. In the literature [9-21] there are described principles of control, rocket flight kinematics and dynamics, and methods of guiding vehicles to aerial targets. Many questions of the combat use of fighter-interceptors are presented in the form of articles in the open Soviet [22-24] and particularly the foreign [25-28] press, although most of this material is of a descriptive nature and quantitative relationships are reduced to simple calculation formulas. Thus, although the combat use of fighter-interceptors has been the object of study in many sources, nonetheless at the present time there are no books from which one could get a sufficiently complete picture of the use of analytical methods when studying the combat use of fighter-interceptors.

This book will attempt to at least partially fill this gap.

In this book we have generalized, to some extent, the mathematical methods and procedures so that they can be used to solve problems of the combat use and organization of combat operations, for quantitative determination of combat capabilities and the combat effectiveness of fighter-interceptors under various conditions.

The concept of combat capabilities includes all properties which define the ability of the interceptor to fulfill various combat missions. Depending on the conditions of combat use, the fighter-interceptor can have various missions:

— to intercept a target on a given line or in a given belt;

— to intercept a target from a state of "airfield readiness" or flying alert patrols in a zone remote from the airfield at a given distance;

— to destroy the maximum possible number of targets;

— to destroy the leading target with maximum probability;

— to enter into combat with the target within the minimum possible length of time; etc.

If there are definite combat operations, the following types of questions always arise: in what area of space, how rapidly in time, and with what quality can these missions be fulfilled?

The spatial capabilities in carrying out combat missions are quantitatively characterized by the areas of combat employment (interception line, altitudes and speeds of the targets to be intercepted, maneuvering altitudes and speeds of the interceptor in aerial combat), by areas of possible attacks, and by areas of possible rocket firings. The time capabilities in carrying out combat operations are quantitatively characterized by the combat-readiness factors: the time for going from one state of readiness to another, the time for takeoff on command from the command point, and the time for reaching the given line. The quality and degree or level with which the interceptor carries out the functions or missions for which it is intended are characterized by its combat effectiveness. For a quantitative evaluation of the effectiveness in carrying out combat operations we use various criteria, the numerical combat-effectiveness factors.

The selection and basis for a specific combat-effectivess factor is a logic problem, and is determined entirely by the combat mission at hand or by the aim of the conducted investigation.

A clear classification of combat-effectivness factors was first given by Academician A. N. Kolmogorov, who examined two limiting cases of combat operations [37]. The first case was characterized by the fact that it was necessary to solve a quite definite problem or attain a quite definite result (e.g., hit a target, destroy all targets in a raid). In this case the result can either be achieved or not achieved. Success is evaluated by a "yes-no" scheme. Therefore, the effectiveness of a combat operation against the enemy in this case is evaluated by the probability of carrying out the combat operation: the probability of hitting the target, the probability of destroying all targets in the raid. The second case is characterized by the fact that quantitatively a specific problem is not posed, but all operations are directed toward inflicting maximum possible damage on the enemy according to the principle "the more the better" (e.g., destroy the maximum possible number of targets, maximum possible damage prevention). In this case the combat effectiveness against the enemy is evaluated by the mathematical expectation of damage inflicted on the enemy: the mathematical expectation of the number of targets destroyed, the mathematical expectation of preventing damage.

For a fighter-interceptor we can give the following typical examples of combat conditions, the combat mission arising from them, and their corresponding combat-effectiveness factors:

Combat conditions	Combat problem	Effectiveness factor
Single aerial target penetrates AA system	Shoot down target	Probability of downing target
Group of aerial targets penetrates AA system	Shoot down maximum number of targets	Mathematical expectation of the number of downed targets
	Shoot down all targets	Probability of downing all targets
	Shoot down at least a given number of targets	Probability of downing at least a given number of targets
A raid in the form of a random number of aerial targets penetrates the AA line (area) defended by an n-channel system	Mathematical expectation of the number of downed targets on the given line (area)	Mathematical expectation of the number of downed targets on the given line (area)
		Probability that all n channels are occupied and the next target penetrates the line (area) unintercepted

The effectiveness of interceptor group operations is influenced not only by the effectiveness and tactical-technical characteristics of single interceptors but also by the quality of the organization and control of combat operations and also tactical ways and means for solving the posed problems. It is also possible to indirectly increase the effectiveness of combat operations by decreasing the expenditures of forces and equipment for carrying out combat operations. Thus it becomes necessary to investigate ways of optimizing combat operations, seeking methods for assuring a given level of effectiveness at minimum cost.

When the commander selects a solution, it is usually a matter of rational utilization of the equipment available to him.

Identical combat problems can be solved, in terms of effectiveness, the sphere of combat utilization, and time, in identical ways by using various interceptors, differing in their tactical-technical characteristics and their economic factors. Therefore it becomes possible to introduce, for estimating the combat operations of interceptors, the criterion of **specific effectiveness**, which is equal to the ratio of the total output (the combat effectiveness when carrying out the stated mission) to the total expenditures to accomplish it. Such a complex effectiveness criterion characterizes the maximum possible damage inflicted on an enemy with consideration of restrictive conditions, e.g., minimum forces and equipment used to complete the mission. Therefore, an evaluation of combat effectiveness should always be associated with an estimate of the cost or expenditure of forces and equipment to complete a combat mission.

The basis of the combat use of fighter-interceptors, as we know, is to intercept an aerial target. By interception of an aerial target we mean the method of operation of a fighter-interceptor to destroy the enemy's means of an air strike, consisting of establishing contact with the target and subsequently destroying it. The interception process can arbitrarily be divided into two basic steps:

1) surface guidance, i.e., guiding an interceptor to an aerial target using ground equipment situated so as to assure detection and attack of the target;

2) target attack (aerial combat), consisting of independent approach of the interceptor to the target after detection or lock-on by airborne radar, assuming a position favorable for rocket firing, firing the rockets, assuring control of the rockets enroute to the target, triggering of the proximity fuse, and withdrawal of the interceptor from the attack.

The interceptor's success in destroying the enemy's air attack equipment is characterized by the combat effectiveness of interception, which is determined primarily by the tactical-technical characteristics of the aircraft, the airborne radar, the air-air rockets, and the accuracy of the surface guidance system.

A basic quantitative criterion of the combat effectiveness of interception of a single interceptor in operation against a single aerial target is the probability of interception, which is determined by the probability of successful surface guidance, the probability that a rocket will destroy a target, and the reliability of the interceptor.

With group combat operations of fighter-interceptors against group targets, the criteria for combat effectiveness are the mathematical expectation of the number of downed targets, the probability of downing all targets in a raid, and the probability of downing at least a given number of targets. These factors depend on the balance of forces (the number of interceptors and targets), the effectiveness of a single operation, the quality of combat control, the organization and guaranteeing of combat operations, the serviceability of the aircraft fleet, combat readiness, etc.

On the basis of what has been said, problems in the study of the combat use of fighter-interceptors can be arbitrarily divided into the following areas: surface guidance, interception lines, interceptor homing, target attack, air-air rocket homing, the combat effectiveness of a single interceptor, the combat effectiveness of interceptor group combat operations, combat readiness, and interceptor reliability. The book is divided into chapters based on this breakdown. To make the book more useful for direct practical application, it is presented in the form of the posing of individual problems and discussion of their solutions.

The solutions are presented, as a rule, first in general form and then numerical data are substituted. This makes it possible to use the derived formulas directly in tactical and operational calculations. The solution of complex problems involving unwieldy calulations is given in the form of graphs and nomograms, many of which are valid for the entire range of examined characteristics. Such solutions include:

– dependences between interceptor and target characteristics, which determine the areas of possible target attacks;

— dependences among interceptor, target, and rocket characteristics, serving to construct the zones of possible firings;

— dependences which determine the range (areas) of combat use of the interceptor;

— a nomogram connecting the basic tactical-technical characteristics of the interceptor;

— dependences of the probability of surface guidance on random and permissible guidance errors, and dependences of guidance errors on the tactical-technical characteristics of the aircraft, the armament system, and the accuracy of the radar field of surface guidance;

— dependences of the mathematical expectation of the number of downed targets on the quantity of targets, the designated duty of the interceptors, the probability of downing a single target, the probability of an interceptor's being destroyed, the quality of target distribution;

— dependences of the throughput of a multichannel surface guidance system on the number of guidance channels, the guidance time, the intensity of the target raid, the width of the target intercept area;

— dependences of the coefficient of combat readiness of a group of interceptors on the reliability, the recoverability of aviation equipment, and the number of service personnel.

A brief description of the problems and results of their solutions will be found at the beginning of each chapter.

I would like to thank at this time V. G. Nevzorova, Candidate of Technical Sciences, who critiqued the manuscript for this book.

I realize that my attempt to present basic analytical methods for studying the combat use of fighter-interceptors in a book of limited size has its drawbacks. I would therefore appreciate all comments.

CHAPTER 1

SURFACE GUIDANCE. INTERCEPTION LINE. TARGET APPROACH

The bases for solving problems on the surface guidance of a fighter-interceptor to an aerial target are the analytical relationships between the parameters of motion of the interceptor and the target; these determine the guidance method selected. Therefore, at the outset let us examine the familiar methods of surface guidance and give the basic kinematic dependences between the parameters of motion of the interceptor and the target during the guidance process (Problem 1.1).

We then have a group of problems (Problems 1.2-1.8) involving the calculation of the removal of the interception lines. Solution of these problems allows us to determine how the distance of the interception lines changes:

– with a change in speeds of the interceptor and the target, the recognition range, and the passive time;

– depending on the direction of attack of the target;

– depending on the surface guidance method used;

– with a change in target flight relative to the airfield from which the interceptor takes off;

– depending on the fuel supply and consumption, the aerodynamic flight characteristics, and the flight regimes.

Another group of problems (Problems 1.9-1.23) is devoted to the process of interceptor homing, i.e., interceptor approach to an aerial target from the moment of detection (or lock-on) by the airborne radar to the moment of rocket firing. Solution of these problems allows us to determine:

– the interceptor maneuvering required to arrive at the rocket-firing point;

– the characteristics of the interceptor and airborne radar required to assure realization of the required target-approach trajectories;

– satisfaction of the necessary rocket-firing conditions;

– the zones of possible attacks as a function of the conditions of combat use;

– the interceptor target-approach trajectories;

– the trajectories of the interceptor as it withdraws from the attack.

The areas of possible attacks form the basis for analyzing the possibility of establishing interceptor contact with the target. When estimating the combat capabilities of interceptors and studying rational methods for combat use it is important to be able to determine the proposed position of the interceptor relative to the target when transferring from surface-guidance to homing conditions. To assure successful target interception, the interceptor at the moment of transfer from surface guidance to homing should be within the limits of the zone of possible attacks, whose dimensions are determined by the tactical-technical characteristics of the aircraft, the airborne radar, the weapons system, and the target flight characteristics. The zone of possible attacks is a region in space from which homing of the interceptor with subsequent successful rocket firing is possible. Analysis of the zones of possible attacks and the zones of possible firings allows us to present optimum methods for attacking an aerial target.

The possibility of interceptor flight along a homing trajectory is determined by the relationship between the angular velocity required for this flight $\omega_{п.потр}$ and that available $\omega_{п.расп}$. By $\omega_{п.расп}$ we mean the available angular velocity of a 360° steady turn of the interceptor. If $\omega_{п.расп} \geq \omega_{п.потр}$, flight along an attack curve is possible. Otherwise, the interceptor goes into a flight trajectory with constant angular velocity $\omega_{п.расп}$, which causes guidance errors. Therefore, to give the initial attack conditions which assure error-free interceptor homing during the entire attack it becomes necessary to determine those zones around the target in which an interceptor will not be able to perform its required flight. These zones are called zones with $\omega_{п.расп} \leq \omega_{п.потр}$, i.e., zones for which the available angular velocity of the interceptor is not greater than that required for the given homing method. The dimensions of these zones are determined by the combat use conditions (speeds and relative bearings of the interceptor and the target) and depend on the interceptor homing method. In the problems of this chapter we examine two interceptor homing methods: "pursuit" and "pursuit with a lead."

PROBLEM 1.1. To derive the basic kinematic equations which characterize the familiar guidance methods: "constant-bearing approach," "interception," "direct approach," "pursuit," "line-of-sight," and "maneuvering."

Solution. To obtain the desired equations let us examine the kinematics of motion of the target and the interceptor.

1. The method of "constant-bearing approach" (Fig. 1.1). With guidance by this method the interceptor flies in a straight line to the target-intercept point, and the angular velocity of the sight line is zero:

$$\frac{d\epsilon}{dt}=0. \quad (1)$$

As can be seen from Fig. 1.1, the method is described by the equation

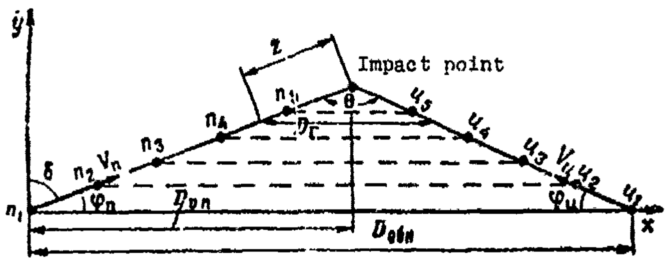


Fig. 1.1.

$$\sin\varphi_n = \frac{V_u}{V_n}\sin\varphi_u, \qquad (2)$$

where V_n and V_u are the speeds and φ_n and φ_u are the relative bearings of the interceptor and the target.

The interceptor flight time to the point of impact with the target

$$t_n = \frac{D}{V_n\cos\varphi_n + V_u\cos\varphi_u}. \qquad (3)$$

2. The "intercept" method (Fig. 1.2). This method differs from the "constant-bearing approach" method in that the finite range to the target Δl is given as a function of the characteristics of the interceptor's armament system. Analytically, the method is described by the following equations:

$$\sin\varphi_n = \frac{V_u t_n \sin\varphi_u}{V_n t_n + \Delta l}; \qquad (4)$$

$$t_n = \frac{D - \Delta l\cos\varphi_n}{V_n\cos\varphi_n + V_u\cos\varphi_u}. \qquad (5)$$

3. The method of "direct approach" (Fig. 1.3). In this method the interceptor flies directly to the lead point of impact of the

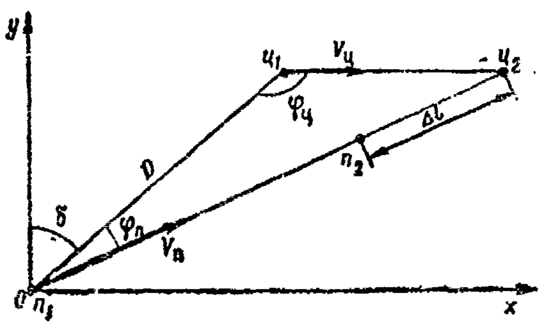


Fig. 1.2.

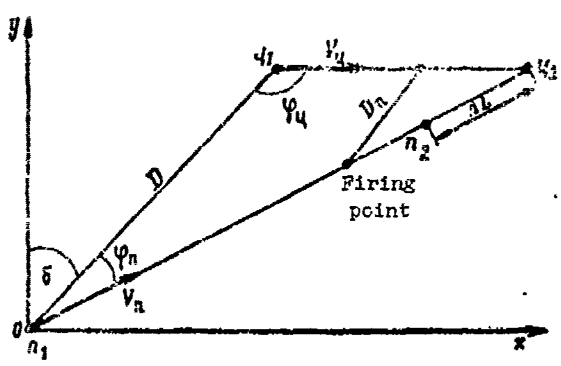


Fig. 1.3.

rocket with the target. The analytical dependences of the method are obvious from Fig. 1.3:

$$\sin\varphi_{п} = \frac{V_{ц} t_{п} \sin\varphi_{ц}}{V_{г} t_{п} + \Delta L}. \tag{6}$$

The interceptor flight time

$$t_{п} = \frac{D - \Delta L \cos\varphi_{п}}{V_{п}\cos\varphi_{п} + V_{ц}\cos\varphi_{ц}}, \tag{7}$$

where

$$\Delta L = (V_{р.\,ср} + V_{п})\, t_{р}; \tag{8}$$

$$t_{р} = \frac{D_{п}}{V_{п}\cos\varphi_{п} + V_{ц}\cos\varphi_{ц} + V_{р.\,ср}\cos\varphi_{п}}, \tag{9}$$

where t_p is the flight time of the rocket to its impact with the target; ΔL is the distance "interceptor-target" at the moment the rocket impacts with the target; V_p is the absolute velocity of the rocket

$$V_{р} = V_{р.\,ср} + V_{п}; \tag{10}$$

$V_{p.cp}$ is the natural velocity of the rocket, averaged over the time of controlled flight.

4. The "pursuit" method (Fig. 1.4). This method is characterized by the fact that the interceptor's velocity vector at each moment of time is directed toward the target. As can be seen from Fig. 1 4, the interceptor heading

$$Q = \operatorname{arctg}\frac{x_{ц} - x_{п}}{y_{ц} - y_{п}}. \tag{11}$$

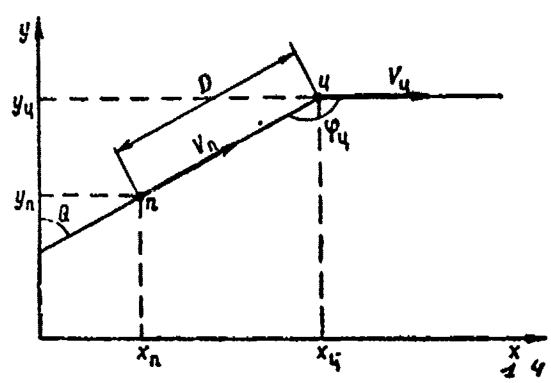


Fig. 1.4.

Thus, to determine the interceptor heading we need not know the motion parameters of the target; we need only know its coordinates. This is an advantage of the method. A disadvantage of the method is that target attack is possible only from the aft hemisphere which, with

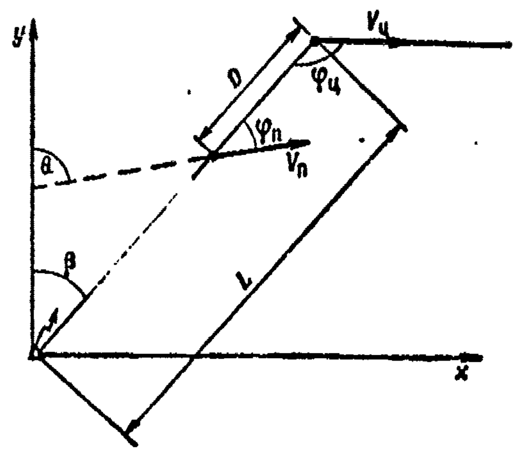


Fig. 1.5.

a low approach velocity, leads to a considerable loss in the interception line.

5. The "line-of-sight" method (Fig. 1.5). This method is characterized by the fact that in the guidance process the interceptor, target, and ground guidance station are on one line. The interceptor azimuth is equal to that of the target:

$$\beta_п = \beta_ц = \beta. \qquad (12)$$

To satisfy this condition it is necessary that the angular velocities of the interceptor and target be equal to each other:

$$\omega_п = \omega_ц. \qquad (13)$$

Since

$$\omega_п = \frac{V_п \sin \varphi_п}{L - D}; \qquad (14)$$

$$\omega_ц = \frac{V_ц \sin \varphi_ц}{L}, \qquad (15)$$

equating the right sides of Eqs. (14) and (15) we obtain the following expression for determining the relative bearing of the interceptor $\varphi_п$:

$$\varphi_п = \arcsin \left[\frac{V_ц}{V_п} \left(\frac{L - D}{D} \right) \sin \varphi_ц \right]. \qquad (16)$$

Having determined $\varphi_п$, let us find the interceptor heading:

$$Q = \beta + \varphi_п = \beta + \arcsin \left[\frac{V_ц}{V_п} \left(\frac{L - D}{D} \right) \sin \varphi_ц \right]. \qquad (17)$$

All of the above methods of surface guidance bring the interceptor into an arbitrary position relative to the target.

The limited capabilities of the interceptor's armament system make it necessary to lead the interceptor, at the end of surface guidance, into a quite specific zone from which the airborne radar detects and locks on to the target and the rocket is fired. The "maneuvering" method of surface guidance satisfies this requirement.

6. The "maneuvering" method (Fig. 1.6). The target and the interceptor are in one horizontal plane, their velocities are constant and equal to $V_{ц}$ and $V_{п}$, respectively. Points O and A in Fig. 1.6 are the positions of the target and interceptor at the moment of time at which the interceptor begins to be homed onto the target. Point C is the proposed intercept point.

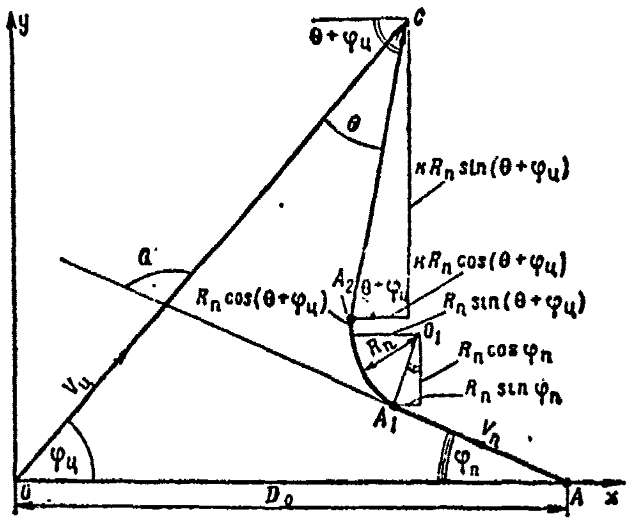


Fig. 1.6.

Let us introduce the following designations: D_0 is the "interceptor-target" range at the start of surface guidance; θ is the given intercept angle; $R_{п}$ is the turn radius of the interceptor for exiting to the line of motion of the target at the given angle θ; $A_2C = l$ is the segment of given length after the turn, intended for correcting surface-guidance errors; $AA_1 = L$ is that segment of the interceptor trajectory prior to turning; $\varphi_{п}$ is the relative bearing of the interceptor at the start of surface guidance; $\varphi_{ц}$ is the relative bearing of the target at the start of surface guidance; $D_{ц}$ is the path covered by the target from the start of surface guidance to the intercept point; t is the balanced guidance time.

Since we assume that intercept point C exists, the figure in Fig. 1.6 is a closed polygon, which can be represented in the form of a vector polygon. Projecting the vector polygon $OAA_1O_1A_2CO$ onto axis x and then onto line OC we obtain two basic equations for the "maneuvering" guidance method:

$$V_{\text{ц}} t \cos\varphi_{\text{ц}} = L\cos\varphi_{\text{п}} + sR_{\text{п}}[\sin(\varphi_{\text{ц}}+\theta) - \sin\varphi_{\text{п}}] + l\cos(\varphi_{\text{п}}+\theta) - D_0; \qquad (18)$$

$$V_{\text{п}} t = D_0\cos\varphi_{\text{ц}} - L\cos(\varphi_{\text{ц}}+\varphi_{\text{п}}) + l\cos\theta + sR_{\text{п}}[\sin(\varphi_{\text{ц}}+\varphi_{\text{п}}) - \sin\theta]. \qquad (19)$$

The path traversed by the interceptor in the balanced guidance time can be expressed as follows:

$$V_{\text{п}} t = L + s(Q-\theta)R_{\text{п}} + l, \qquad (20)$$

where

$$s \equiv \operatorname{sign}(Q-\theta) = \operatorname{sign}(180^\circ - \varphi_{\text{ц}} - \varphi_{\text{п}} - \theta) \qquad (21)$$

characterizes the sign of the expression $Q - \theta$ (signature): when $Q > \theta$, in place of s in Formulas (18) and (19) we place the "+" sign; when $Q < \theta$, we place the "-" sign.

Let us substitute into (18) and (19) the expression, obtained from (20), for the phase of the interceptor trajectory before the turn:

$$L = V_{\text{п}} t - s(Q-\theta)R_{\text{п}} - l, \qquad (22)$$

the velocity ratio

$$p = \frac{V_{\text{ц}}}{V_{\text{п}}} \qquad (23)$$

and the path of the target up to the intercept point. We then get

$$D_{\text{ц}} = \frac{sR_{\text{п}}[\sin(\varphi_{\text{ц}}+\varphi_{\text{п}}) - \sin\theta + (180^\circ - \varphi_{\text{ц}} - \varphi_{\text{п}} - \theta)\cos(\varphi_{\text{ц}}+\varphi_{\text{п}})] + l[\cos\theta + \cos(\varphi_{\text{ц}}+\varphi_{\text{п}})] + D_0\cos\varphi_{\text{ц}}}{1 + p\cos(\varphi_{\text{ц}}+\varphi_{\text{п}})}; \qquad (24)$$

$$D_{\text{ц}} = \frac{sR_{\text{п}}[\sin(\varphi_{\text{ц}}+\theta) - \sin\varphi_{\text{п}} - (180^\circ - \varphi_{\text{ц}} - \varphi_{\text{п}} - \theta)\cos\varphi_{\text{п}}] - l[\cos(\varphi_{\text{ц}}+\theta) + \cos\varphi_{\text{п}}] - D_0}{-(\cos\varphi_{\text{ц}} + p\cos\varphi_{\text{п}})}. \qquad (25)$$

Equations (24) and (25) connect eight variables

$$D_{\text{ц}},\ D_0,\ \varphi_{\text{ц}},\ R_{\text{п}},\ l,\ \theta,\ \varphi_{\text{п}},\ p,$$

six of which can be taken as independent variables; then System (24) and (25) can be solved unambiguously for the two remaining unknowns. For example, if we know the mutual position and velocities of the interceptor and target (D_0, $\varphi_ц$, p) and we are given the radius of turn of the interceptor $R_п$, the required relative bearing of the interceptor $\varphi_п$, and the interception line $D_ц$, System (24) and (25) allows us to determine the intercept angle θ and the length of the last phase of the interceptor trajectory l for which target interception is possible.

If we equate the right sides of (24) and (25), the relative bearing of the interceptor and, consequently, the interceptor heading can be determined from the known or given values $D_ц$, D_0, $\varphi_ц$, $R_п$, l, θ, and p.

PROBLEM 1.2. To determine the withdrawal of the line of interception of an aerial target moving on a straight line in the direction of the airfield at which the interceptors are based with velocity $V_ц$ = 900 km/h, if the warning range D_0 = 1000 km, the interceptor velocity $V_п$ = 1000 km/h, and it takes off in the so-called passive time $t_{пасс}$ = 2 min after the first target fix. The target attacks strictly from the forward hemisphere. How does withdrawal of the target-intercept line change if, under these same conditions, the velocities of the interceptor and the target increase by a factor of 2 and if $t_{пасс}$ doubles?

Solution. The condition of time balance to the moment the interceptor meets the target gives us the following relationship:

$$D_0 = V_ц (t_{пасс} + t) + V_п t, \tag{1}$$

where t is the interceptor flight time until it meets the target.

Withdrawal of the intercept line from the interceptor takeoff field is equal to

$$D_{р.п} = V_п t. \tag{2}$$

Let us find t from (1) and insert it into (2). Then

$$D_{p.n} = \frac{D_0 - V_u t_{nacc}}{1 + \frac{V_u}{V_n}} = 510 \text{ km}. \qquad (3)$$

With a doubling of the velocities, $D_{p.n}$ = 495 km. If t_{nacc} doubles, $D_{p.n}$ = 463 km.

Thus, with constant warning range and constant passive time, withdrawal of the target intercept line is determined basically not by the absolute values of the interceptor and target velocities but by their ratio.

PROBLEM 1.3. For the conditions of Problem 1.2, determine how close the target intercept line approaches the takeoff field if the interceptor attacks the target from the aft hemisphere, making a 180° turn with radius R_n = 20 km. Compare two cases:

1) the interceptor, after pulling out of the turn, is precisely at the firing range D_n = 20 km;

2) after the turn the range to the target is 30 km.

Solution. Withdrawal of the intercept line is defined by the formula

$$D_{p.n} = V_n t - D_n, \qquad (1)$$

where t is the interceptor flight time to the start of the turn; this is found from the relationship obtained from the time balance:

$$D_0 + D_n = V_u (t_{nacc} + t + t_{180°}) + V_n t, \qquad (2)$$

where $t_{180°}$ is the time for the interceptor to turn 180°. Thus,

$$D_{p.n} = \frac{D_0 + D_n - V_u (t_{nacc} + t_{180°})}{1 + \frac{V_u}{V_n}} - D_n; \qquad (3)$$

$$t_{180°} = \frac{\pi R_n}{V_n} = \frac{\pi 20}{16{,}7} = 3{,}76 \text{ min}; \qquad (4)$$

$$D_{p.n} = 472 \text{ km}.$$

For the second case, the line moves toward the takeoff field by the distance covered by the target as the interceptor pursues it up to the rocket-firing range. Since the approach speed is 1.67 km/min, the pursuit time is 6 min and the target covers a distance of 90 km. Consequently, $D_{p.п} = 362$ km.

PROBLEM 1.4. A target flying in a straight line can be intercepted by an interceptor by two methods: constant-bearing approach and pursuit. We must determine the difference in withdrawal of the intercept lines if the mutual positions of the interceptor and the target at the moment of the start of approach are characterized by the following parameters: "interceptor-target" range $D_0 = 100$ km, interceptor speed $V_п = 1000$ km/h, target speed $V_ц = 900$ km/h, target relative bearing $\varphi_ц = 90°$. The relative bearing of the interceptor during constant-bearing approach $\varphi_п = 20°$, while with pursuit $\varphi_п = 0°$.

Solution. The difference in withdrawal of the intercept lines can be estimated from the difference in distances traversed by the target when it is intercepted by the two methods given above.

For the constant-bearing-approach method (Fig. 1.1), according to the law of sines we have

$$\frac{\sin \theta}{\sin \varphi_п} = \frac{D_0}{D_ц}, \qquad (1)$$

where $D_ц$ is the distance covered by the target from the start of approach to the moment of encounter with the interceptor; θ is the aspect angle.

Thus, when the target is intercepted by the constant-bearing method we have

$$D_ц = \frac{D_0 \sin \varphi_п}{\sin \theta} = \frac{100 \sin 20°}{\sin 70°} = 36{,}4 \text{ km}. \qquad (2)$$

When the pursuit method is used the target covers the distance

$$D_ц = V_ц t_{пог}, \qquad (3)$$

where $t_{пог}$ is the interceptor flight time along the pursuit curve,

$$t_{пог} = \frac{D_0\left(\frac{V_п}{V_ц} - \cos\varphi_ц\right)}{V_ц\left(\frac{V_п^2}{V_ц^2} - 1\right)}. \qquad (4)$$

Consequently,

$$D_ц = D_0 \frac{\frac{V_п}{V_ц} - \cos\varphi_ц}{\left(\frac{V_п}{V_ц}\right)^2 - 1} = \frac{100\,(1.11 - \cos 90°)}{1.21 - 1} = 465 \text{ km}.$$

Solution of the analyzed problem shows graphically how sharply the withdrawal of the intercept line decreases with the "pursuit" guidance method.

PROBLEM 1.5. An interceptor is directed toward a target flying at a constant heading over an airfield with x = 250 km. The intercept line is set at a distance $D_{р.п}$ = 100 km from the airfield line. The interceptor is guided by the direct-intercept method (with constand heading). The armament system allows an attack from the forward hemisphere at a target relative bearing $\varphi_ц \leq 40°$. Determine whether the target can be intercepted on the given line if the interceptor takes off at the first fix of the target by the surface radar whose detection range $D_{обн}$ = 500 km. The target and interceptor speeds are constant; $V_ц$ = 1300 km/h, $V_п$ = 1500 km/h. The interceptor guidance time (the time from takeoff to the intercept line) $t_н$ = 15 min. Determine the maximum possible flight x for which target interception is possible (Fig. 1.7).

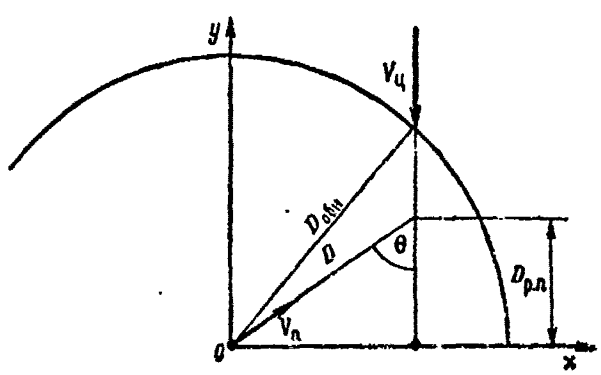


Fig. 1.7.

Solution. According to the stipulations of the problem the aspect angle θ should satisfy the relationship

$$\theta < \arccos\left(\frac{D_{р.п}}{V_п t_н}\right), \qquad (1)$$

where $D_{р.п}$ is the distance of the intercept line from the airfield line.

Since

$$D_{р.н} = \sqrt{D_{обн}^2 - x^2} - V_ц t_н, \qquad (2)$$

then

$$0 < \arccos\left(\frac{\sqrt{D_{обн}^2 - x^2} - V_ц t_н}{V_п t_н}\right). \qquad (3)$$

When the conditions of the problem are adhered to, the aspect angle θ, with x = 250 km, is, according to Formula (3), 77.2°.

Thus, the conditions for target intercept are not met with respect to angle θ. Here the distance of the intercept line from the airfield line is 107 km. In order that angle θ be no more than 40° it is necessary to increase $D_{обн}$ (e.g., increase the warning range by using radar data from neighboring radars). From the inequality

$$\cos 40° \geq \frac{\sqrt{D_{обн}^2 - 250^2} - 325}{375}$$

we find that with

$$D_{обн} \geq 550 \text{ km}, \quad \theta < 40°.$$

According to (3), target interception with the maximum possible pass can be achieved with θ = 90°. Then

$$\frac{\sqrt{D_{обн}^2 - x^2} - V_ц t_н}{V_п t_н} = 0$$

and we find x from the relationship

$$\sqrt{D_{обн}^2 - x^2} = V_ц t_н.$$

If $D_{обн}$ = 550 km, then x = 445 km with $V_ц t_н$ = 325 km.

PROBLEM 1.6. The interceptor is in the forward hemisphere of the target. We must determine the maximum distance of the intercept line from the standpoint of fuel, if the fuel supply $G_т$ = 10 tons, while the intercept flight profile consists of segments (Fig. 1.8) characterized by corresponding lengths in the horizontal plane and by per-kilometer C_k or total fuel consumptions (Table 1.1).

Table 1.1

Route leg	Leg length	Fuel consumption
Climb and accelerate to $V_{п.макс}$	$l_{н.р} = 40$ km	$G_{н.р} = 1000$ kg
Horizontal flight at $V_{п.макс}$	$l_{г.п}$	$C_{к.г.п} = 7$ kg/km
Correct errors of surface guidance, and attack	$l_{н} = 60$ km	$G_{н} = 500$ kg
180° turn	$l_{р}$	$G_{р} = 100$ kg
Return to airfield under cruise conditions	$l_{в}$	$C_{к.в} = 2$ kg/km
Descent and airfield maneuver	$l_{сн} = 50$ km	$G_{сн} = 200$ kg

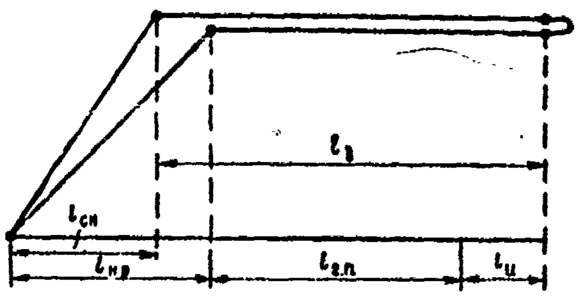


Fig. 1.8.

During the calculation it is necessary to consider the emergency fuel supply $G_{т.а} = 800$ kg.

Solution. The fuel distance of the intercept line is determined by selection of the horizontal flight leg $l_{г.п}$ and the return leg $l_{в}$. Let us designate the difference of these legs by

$$l = l_{в} - l_{г.п}. \qquad (1)$$

According to Fig. 1.8 we have

$$l_{сн} + l_{в} = l_{н.р} + l_{н} + l_{г.п}, \qquad (2)$$

while with consideration of (1)

$$l_{в} = l_{н} + l_{г.п} + l_{н.р} - l_{сн} = l + l_{г.п}, \qquad (3)$$

from which

$$l = l_{н.р} + l_{н} - l_{сн}. \qquad (4)$$

The total fuel balance

$$G_{т} = G_{н.р} + G_{г.п} + G_{н} + G_{р} + G_{в} + G_{сн} + G_{т.а}. \qquad (5)$$

The fuel expended on legs $l_{г.п}$ and $l_{в}$ is, respectively,

$$G_{\text{г.п}} = C_{\text{к.г.п}} l_{\text{г.п}}; \quad G_{\text{в}} = C_{\text{к.в}} l_{\text{в}}. \tag{6}$$

The total fuel expended on these two legs, considering (3), is

$$G_{\text{г.п}} + G_{\text{в}} = C_{\text{к.г.п}} l_{\text{г.п}} + C_{\text{к.в}} (l + l_{\text{г.п}}), \tag{7}$$

from which the length of the horizontal path at $V_{\text{п.макс}}$ is

$$l_{\text{г.п}} = \frac{G_{\text{г.п}} + G_{\text{в}} - C_{\text{к.в}} l}{C_{\text{к.г.п}} + C_{\text{к.в}}}. \tag{8}$$

Having determined the value of $l_{\text{г.п}}$, let us find the maximum fuel distance of the intercept line:

$$l_{\text{макс}} = l_{\text{н.р}} + l_{\text{г.п}} + l_{\text{в}}. \tag{9}$$

Substituting the numerical values for our problem into (4), (5), (8), and (9), we get

$$l = 50\ \text{km};\ G_{\text{г.п}} + G_{\text{в}} = 7400\ \text{kg};\ l_{\text{г.п}} = 609\ \text{km};\ l_{\text{макс}} = 709\ \text{km}.$$

Thus, tne maximum fuel distance of the intercept line is 709 km.

Similarly, let us determine $l_{\text{макс}}$ when attacking the target from the aft hemisphere. In this case, as a rule, after completing the turn there is still one short leg of horizontal flight at $V_{\text{п.макс}}$.

The fuel distance of the intercept line during cruising flight is defined by the formula

$$l = \frac{3{,}6 K V_{\text{п}}}{C} \ln \frac{G_{\text{н}}}{G_{\text{к}}}, \tag{10}$$

where $G_{\text{н}}$ and $G_{\text{к}}$ are the weights of the aircraft at the beginning and end of the cruise leg, K is the lift-to-drag [L/D] ratio, C is the specific fuel consumption [s.f.c].

PROBLEM 1.7. Warning of a raid by aerial targets allows realization of the maximum possible fuel distance of the intercept lines. Target interception takes place in the horizontal plane and with a constant speed of the fighter-interceptor. Let us show how the distance of the intercept lines changes for the F-4C "Phantom" fighter as a function of horizontal-flight altitude and speed, if we know the aerodynamic characteristics (Fig. 1.9), the fuel supply for horizontal flight G_T, the s.f.c. as a function of the flight conditions and the degree of throttling of the engine (Figs. 1.10 and 1.11), the average flight weight G_{cp} = 17,600 kg, and the wing area S = 49.2 m^2.

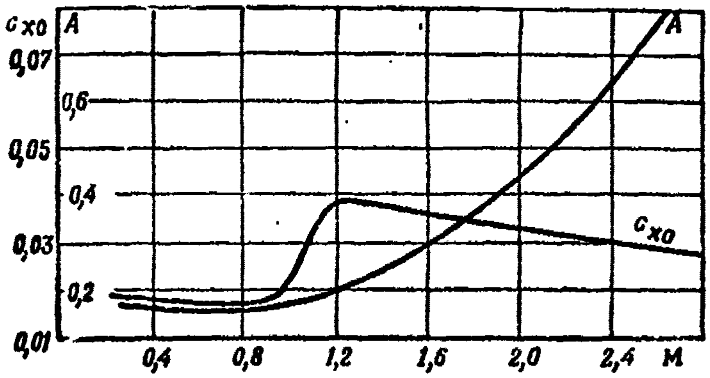


Fig. 1.9.

Solution. The distance of the intercept lines with a constant supply of fuel intended for level flight is unambiguously determined by the per-kilometer fuel consumption, which in turn depends on the flight mode and the degree of throttling of the engine. The optimum mode, assuring maximum flight range, occurs with minimum per-kilometer fuel consumption. Turbojet aircraft, as a rule, have two flight modes that are optimum from the standpoint of fuel consumption — subsonic and supersonic. To determine which of these modes is best we must consider the thrust-

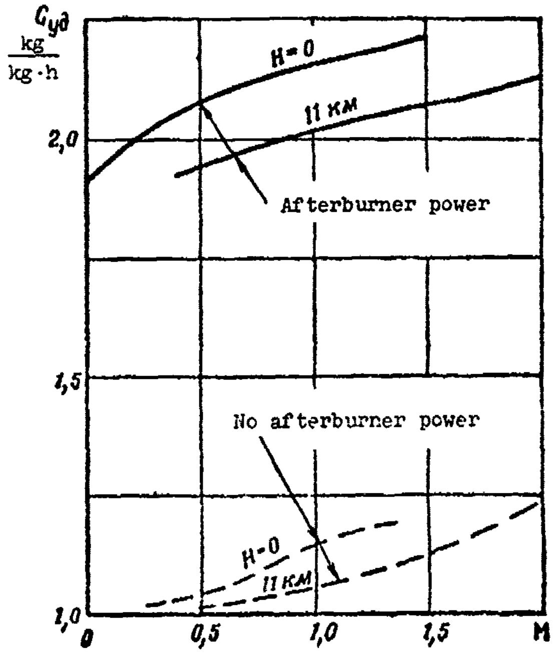


Fig. 1.10.

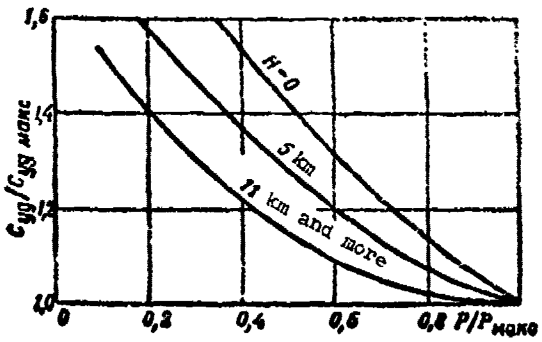


Fig. 1.11

to-weight ratio, the fuel load ratio, and the L/D ratio in these modes. For comparative estimates of the distance of the intercept lines it is convenient to introduce into the examination a value which is the reciprocal of the per-kilometer fuel consumption $C_к$ – the distance in kilometers covered by the interceptor with the expenditure of one kilogram of fuel:

$$\bar{L}_{г.п} = 1/C_к. \quad (1)$$

Let us express the per-kilometer fuel consumption $C_к$ in terms of the hourly fuel consumption and the true air speed of the interceptor:

$$C_к = \frac{C_ч}{V_п}. \quad (2)$$

Since

$$C_ч = PC_{уд}, \quad (3)$$

where P is thrust and $C_{уд}$ is s.f.c. (kg of fuel/kg of thrust·hour), then

$$\bar{L}_{г.п} = \frac{V_п}{PC_{уд}} = \frac{aM}{QC_{уд}} = \frac{aMK}{GC_{уд}}. \quad (4)$$

Here $C_{уд}$ is the s.f.c. by a throttled engine during level flight at constant speed when the condition of equality of drag Q and thrust P is satisfied:

$$Q = P = \frac{G}{K}; \quad (5)$$

G is the weight of the aircraft; K is the L/D ratio; a is the speed of sound.

To take into account the influence of the aerodynamic characteristics and the velocity head, let us determine the value of Q.

Considering the parabolic dependence for the aircraft drag polar (Fig. 1.9)

$$c_x = c_{x0} + Ac_y^2 \quad (6)$$

and the conditions of the equilibrium of forces for level flight

$$Y = G = KQ \tag{7}$$

we get

$$Q = (c_{x0} + Ac_G^2)\, qS. \tag{8}$$

Now let us calculate the dependence of the relative fuel consumption (r.f.c.) on the degree of throttling of the engine (Fig. 1.11). Substituting (7) and (8) into (4) we get a formula for the interpolated relative level-flight range:

$$\overline{L}_{г.п} = \frac{aM}{\left(c_{x0} + AC_G^2\right) qSC_{уд.\,макс}\left(\frac{C_{уд}}{C_{уд.\,макс}}\right)}\left[\frac{km}{kg}\right]. \tag{9}$$

The results of calculation using this formula are conveniently represented in the form of the graph H = f(M) for fixed values of $\overline{L}_{г.п}$. We then obtain a combination of flight modes M, H for which the interceptor flight range at constant speed and altitude has identical value with the fuel supply given for level flight.

The calculation algorithm for these graphs is as follows:

– we are given the Mach number M and the altitude H of level flight;

– from Fig. 1.9 we determine the aerodynamic coefficients c_{x0} and A for the given M;

– we calculate the value of the velocity head

$$q = \frac{\rho V_n^2}{2}, \tag{10}$$

where ρ is air density (found from ISA [International Standard Atmosphere] tables);

– we calculate the weight coefficient

$$c_G = \frac{G}{qS}; \tag{11}$$

– from Figs. 1.10 and 1.11 we determine the values of $C_{уд}$ and

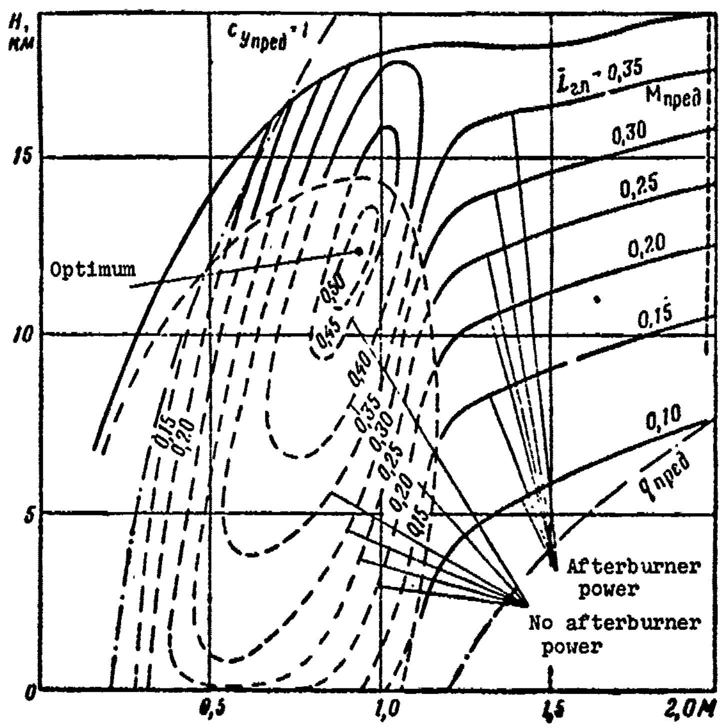


Fig. 1.12.

$C_{уд}/C_{уд.макс}$;
– from Formula (9) we find the relative level-flight range.

Since the graph H = = f(M) for $\bar{L}_{г.п}$ = const was obtained for a certain set of whole numbers of $\bar{L}_{г.п}$ (e.g., 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50), this sequence of calculations is repeated by the iteration method until the given whole number of $\bar{L}_{г.п}$ is obtained.

The results of the calculation for the given conditions of the problem are shown in Fig. 1.12.

PROBLEM 1.8. To determine the width of possible action against aerial targets in the direction of the airfield, if the target speed is in the range $V_ц$ = 900-1800 km/h, the target-detection range by ground radar D = 300 km, the speed of departure of the interceptor from the airfield is within the limits $V_п$ = 1450 ± 150 km/h, the time from the moment of target detection to interceptor takeoff $t_{пасс}$ = 2-4 min, and the minimum permissible line of target action with respect to the object is at distance $D_{мин}$ = 100 km from the airfield. Let us assume that the ground radar is located at the airfield.

Solution. For a clear representation, let us represent the solution graphically (Fig. 1.13). In the coordinate system "range D (km) – time t (min)" let us draw, from point O, straight lines

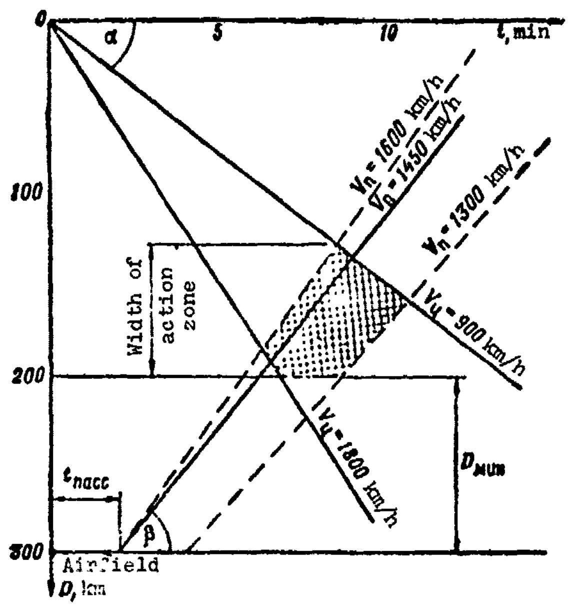


Fig. 1.13.

corresponding to target speeds $V_ц$ = 900 and 1800 km/h. The slope of these lines is calculated numerically from the relationship

$$\alpha = \operatorname{arctg}\left(\frac{\Delta D}{\Delta t}\right) = \operatorname{arctg}(V_ц). \quad (1)$$

Similarly, from points with coordinates D = 300 km (airfield) and $t_{пасс}$ = 2-4 min, at angle β we draw lines corresponding to $V_п$ = 1450 ± 150 km/h. Then at a given distance from the airfield $D_{мин}$ we draw a line for the minimum permissible line of target action against an object. Obviously, the shaded area in Fig. 1.13 characterizes the desired width of the action zone as a function of $V_ц$, $V_п$, D, $D_{мин}$, and $t_{пасс}$. In our example the width of this zone does not exceed 73 km, while target interception with $V_ц$ = 1800 km/h is possible only within a band of ~14 km, if $V_п$ = 1600 km/h and $t_{пасс}$ = 2 min.

Graphic solution of such problems is particularly simple and clear when the target and interceptor velocities change with time, and then the corresponding slopes calculated from Formula (1) are constructed from segments of the flight profiles where $V_ц$ and $V_п$ are constant or can assume equal mean values.

PROBLEM 1.9. The interceptor is guided to the target by the method of constant-bearing approach. The interceptor speed is constant, $V_п$ = 1800 km/h, the maximum value of the angle of view of the airborne radar $\varphi_п$ = 120°, the maximum interceptor aspect angle θ = 40° (θ is the angle between vectors $V_п$ and $V_ц$ at the moment of rocket firing). The interceptor flies with these parameters up to the moment of target lock-on by the homing head. The angle for automatic tracking of the target by the head is also $\varphi_п$ = 120°. We are to determine the limiting velocity $V_ц$ of the target to be intercepted. What should the target-detection range by the airborne radar $D_{обн}$ be if the target lock-on distance by the homing head $D_г$ = 20 km, if the time required by the pilot to operate the airborne radar from the moment of target detection to its lock-on by the head t = 60 s?

Solution. On the basis of the law of sines (Fig. 1.1) we have

$$V_ц = V_п \frac{\sin \varphi_п}{\sin (180° - \varphi_п - \theta)}. \qquad (1)$$

Substituting the given values of $\varphi_п$, θ, and $V_п$, we obtained the desired limiting speed of the intercepted target:

$$V_ц = 1800 \frac{\sin 120°}{\sin 20°} = 4560 \text{ km/h}.$$

Then from the condition of proportionality we have

$$\frac{D_{обн}}{D_г} = \frac{V_п t + z}{z}, \qquad (2)$$

from which

$$D_{обн} = D_г \frac{V_п t + z}{z}.$$

Segment

$$z = \frac{D_г \sin (180° - \theta - \varphi_п)}{\sin \theta}. \qquad (3)$$

Substituting Expression (3) into Formula (2) we find that the required target-detection range by the airborne radar

$$D_{обн} = D_г + \frac{V_п t \sin \theta}{\sin (180° - \theta - \varphi_п)}. \qquad (4)$$

For the given numerical values of our problem we get $D_{обн}$ = 76 km.

Thus, this problem allows us to draw an important conclusion: to intercept a target whose speed exceeds that of the interceptor, with the method of constant-bearing approach the airborne radar should have a long detection range and should allow measurement of large azimuth angles, i.e., have high values of the limiting angles of automatic target tracking.

PROBLEM 1.10. An interceptor at a speed $V_п$ = 2000 km/h attacks, from the aft hemisphere, an aerial target flying at a speed $V_ц$ = = 1600 km/h. At the moment of target acquisition the mutual position of the interceptor and target is described by target relative bearing $\varphi_ц$ = 140° and interceptor relative bearing $\varphi_п$ = 20°. The target acquisition range $D_{обн}$ = 40 km. From the moment of acquisition, in time t = 1 min the pilot identifies the target, locks on to it, and then fires the rocket. The method of interceptor guidance is direct interception; the rocket is homed by the pursuit method. For certain target destruction the firing range should be no more than $D_п$ = 35 km, while the angular firing error should not exceed 15°. We must verify if these rocket firing conditions are satisfied.

Solution. The kinematic relationships given in Fig. 1.14 allow us to obtain calculation formulas for the current range D and the current relative bearing of the interceptor $\varphi_п$:

$$D=\sqrt{A^2+B^2}, \qquad (1)$$

$$\varphi_п=\pm\left(180^\circ-\varphi_{п0}-\varphi_{ц0}-\operatorname{arctg}\frac{|A|}{|B|}\right), \qquad (2)$$

where

$$A=D_{обн}\sin(180^\circ-\varphi_{ц0})-V_п t\cos(\varphi_{п0}+\varphi_{ц0}-90^\circ); \qquad (3)$$

$$B=D_{обн}\cos(180^\circ-\varphi_{ц0})+V_ц t-V_п t\cos(180^\circ-\varphi_{п0}-\varphi_{ц0}). \qquad (4)$$

Here $\varphi_п > 0$ when the lead point is to the left of vector $V_п$; $\varphi_п < 0$ when it is to the right.

Substituting the numerical values of the problem for the firing moment we get:

A = 14.3 km, B = 26.1 km, D = 29.8 km.

Thus the distance "interceptor-target" 60 seconds after detection is 29.8 km, which is less than the maximum distance for permitted firing $D_п$ = 35 km. Consequently, firing conditions are satisfied as far as range is concerned. Now let us find the interceptor relative bearing $\varphi_п$ which, at the moment of firing, should not exceed the permissible angular error in rocket firing. For our data we have $\varphi_п = -8°45'$, i.e., the firing conditions are satisfied as far as the angle is concerned.

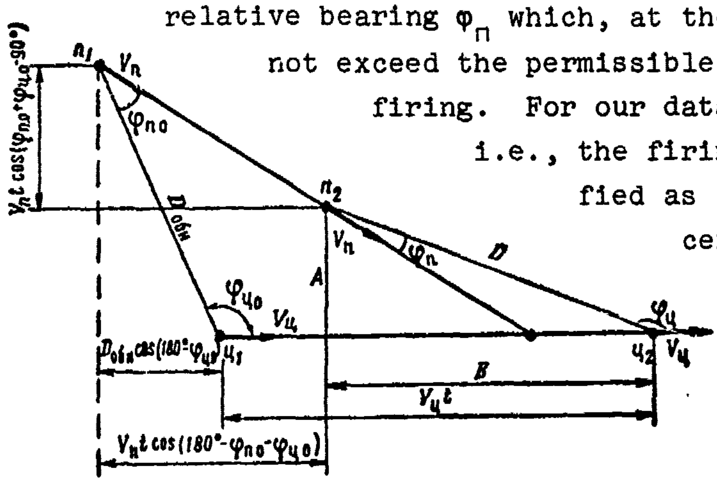


Fig. 1.14.

PROBLEM 1.11. The interceptor attacks a high-speed aerial target whose speed considerably exceeds that of the interceptor. The attack is carried out such that at the moment of target acquisition the interceptor is in a turn, somewhat ahead of the target; as the target overtakes the interceptor the interceptor pilot carries out all required recognition and lock-on operations. Here t. .ghting angle should not exceed the limiting angle of automatic target tracking by the airborne radar. When, after overtaking the interceptor, the target is ahead of the interceptor and the rocket-firing conditions relative to range and azimuth are satisfied, the pilot fires the rocket.

We must verify whether firing conditions are satisfied one minute after target acquisition, if:

- acquisition range $D_{обн} = 35$ km;
- interceptor relative bearing $\varphi_{п0} = 45°$;
- target relative bearing $\varphi_{ц0} = 125°$;
- target speed $V_ц = 2000$ km/h;
- interceptor speed $V_п = 1600$ km/h;
- interceptor turn radius $R_п = 30$ km;
- maximum range of permissible fire $D_п = 15$ km;
- maximum angle of automatic target tracking $\varphi_п = 100°$.

We are to determine how many minutes after target acquisition the range "interceptor-target" will be minimum and what the target sighting angle will be in this case.

Solution. The problem can be solved graphically by constructing kinematic lines of flight for the target and the interceptor. However, the solution accuracy in this case is low. To increase it we must derive formulas for the current values of the distance "interceptor-target" and the target sighting angle. For this, the radius of turn of the interceptor $R_п$ and the range "interceptor-target" at the moment of detection $D_{обн}$ are projected onto the target flight line and onto the perpendicular to this line. The trigonometric ratios in Fig. 1.15 allow us to obtain the following calculation formulas:

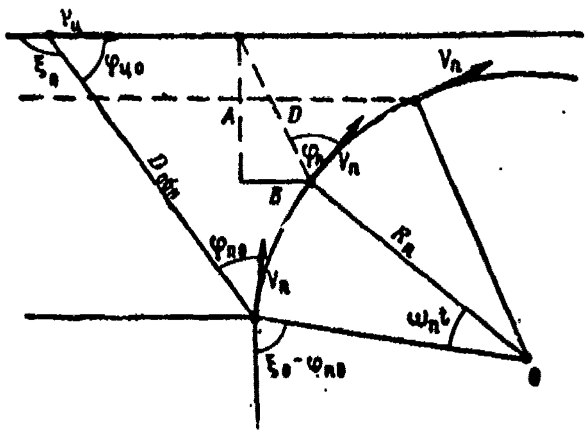


Fig. 1.15.

– the current range "interceptor-target"

$$D = \sqrt{A^2 + B^2}; \qquad (1)$$

– the current target sighting angle

$$\varphi_п = 90^\circ - (\xi_0 - \varphi_{п0}) + \omega_п t + \operatorname{arctg}\frac{|B|}{|A|}. \qquad (2)$$

Formula (2) is valid for the case when the sighting line is to the right of perpendicular line A. When the sighting line is to the left of A,

$$\varphi_ц = 90^\circ - (\xi_0 - \varphi_{п0}) + \omega_п t - \operatorname{arctg}\frac{|B|}{|A|}. \qquad (3)$$

Here

$$\xi_0 = 180^\circ - \varphi_{ц0};$$

$$A = D_{обн} \sin\varphi_{ц0} + R_п[\cos(\xi_0 - \varphi_{п0}) - \cos(\xi_0 - \varphi_{п0} - \omega_п t)]; \qquad (4)$$

$$B = V_ц t - D_{обн}\cos\varphi_{ц0} - R_п[\sin(\xi_0 - \varphi_{п0}) - \sin(\xi_0 - \varphi_{п0} - \omega_п t)]; \qquad (5)$$

$\omega_п$ is the angular velocity of the interceptor turn;

$$\omega_п = \frac{V_п}{R_п}$$

in radians per second.

In time t = 1 min the angle of turn for the interceptor, expressed in degrees, is

$$\omega_п t = \frac{57.3 V_п t}{R_п} \approx 51°.$$

Substituting the numerical values into (4) and (5), we get A = 7.63 km, B = 1.67 km.

The distance "interceptor-target" 60 seconds after target acquisition is, according to (1), D = 7.8 km, i.e., the range conditions for firing are satisfied.

The target sighting angle 60 seconds after detection is, according to Formula (2), $\varphi_п = 73°20'$, i.e., $\varphi_п < \varphi_{макс} = 100°$, and the azimuth firing conditions are also satisfied. To determine the moment of time for which the distance "interceptor-target" will be minimum we must differentiate function D with respect to t, set the derivative equal to zero, and solve this equation.

The minimum of function D = f(t) is at t = 90 s.

The desired sighting angle $\varphi_п$ according to (3) is 68.5°, which is less than the maximum angle for automatic target tracking by the airborne radar.

PROBLEM 1.12. At the moment of detection of a high-speed aerial target the position of the interceptor relative to the target is characterized by acquisition range $D_{обн}$ = 23 km, target relative bearing $\varphi_{ц0}$ = 83°, and interceptor relative bearing $\varphi_{п0}$ = 76°. From the moment of detection the target, in time t_1 = 30 s, carries out avoiding action with radius $R_ц$ = 15 km toward the interceptor, and then continues its flight at a constant heading. Here the interceptor continues to make its turn with radius $R_п$ = 20 km. We are to determine the distance "interceptor-target" and the target sighting angle 80 seconds after acquisition, if the interceptor speed $V_п$ = 660 km/h while the target speed $V_ц$ = 1650 km/h. We are to see whether the rocket-firing conditions are satisfied in this case if the maximum range of permitted fire should not exceed $D_{п.макс}$ = 10 km and the maximum permissible target sighting angle during firing $\varphi_п$ should be no more than 30°.

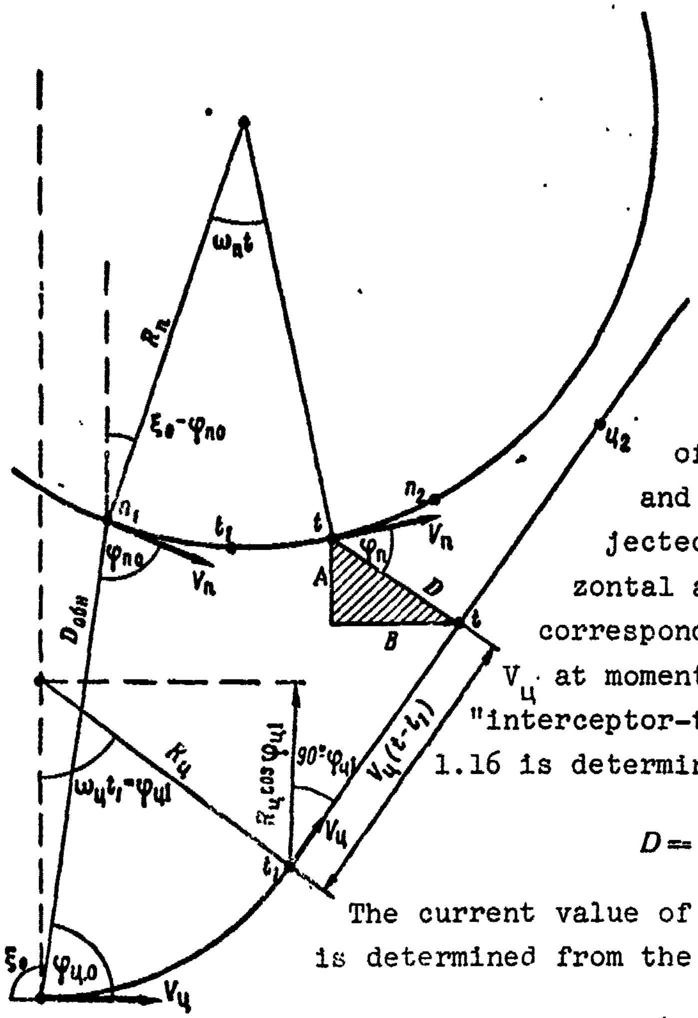


Fig. 1.16.

Solution. The calculation formulas, by analogy with Problem 1.11, are obtained from the kinematic relationships shown in Fig. 1.16. The distance "interceptor-target" at the moment of detection $D_{обн}$ and the radii of turn of the interceptor and the target $R_п$, $R_ц$ are projected onto the vertical and horizontal axes. The horizontal axis corresponds to the direction of vector $V_ц$ at moment t = 0. The current range "interceptor-target" according to Fig. 1.16 is determined from the formula

$$D = \sqrt{A^2 + B^2}. \qquad (1)$$

The current value of the target sighting angle is determined from the formula

$$\varphi_п = -\xi_0 + \varphi_{п0} + \omega_п t + \operatorname{arctg} \frac{|A|}{|B|}, \qquad (2)$$

where

$$t > t_1 = \frac{\varphi_{ц1}}{\omega_ц}. \qquad (3)$$

Here

$$\xi_0 = 180° - \varphi_{ц0};$$

$\varphi_{ц1}$ is the angle by which, during maneuvering time t_1, the target veers from its previous heading;

$$A = D_{обн} \cos(\xi_0 - 90°) + R_п [\cos(\xi_0 - \varphi_{п0}) - \cos(\xi_0 - \varphi_{п0} - \omega_п t)] - R_ц (1 - \cos \omega_ц t_1) - V_ц (t - t_1) \sin \varphi_{ц1}; \quad (4)$$

$$B = D_{обн} \cos(180° - \xi_0) + R_ц \sin \varphi_{ц1} + V_ц (t - t_1) \cos \varphi_{ц1} - R_п [\sin(\xi_0 - \varphi_{п0}) - \sin(\xi_0 - \varphi_{п0} - \omega_п t)]. \quad (5)$$

The angular turning speed of the target

$$\omega_ц = \frac{57{,}3 V_ц}{R_ц} \ °/s. \quad (6)$$

The angular turning speed of the interceptor

$$\omega_п = \frac{57{,}3 V_п}{R_п} \ °/s. \quad (7)$$

Let us substitute numerical data for the problem. Then $\omega_ц = 1.75$ °/s while $\omega_п = 0.526$ °/s.

With $t = 80$ с $\omega_п t = 42{,}08°$, $\omega_ц t = 140°$, $\xi_0 = 97°$, $\varphi_{ц1} = 52{,}5°$,

$$A = -1{,}13 \text{ km}, B = 8{,}9 \text{ km}, D = 8{,}98 \text{ km}, \varphi_п = 14°.$$

Since D and $\varphi_п$ are less than the given values, the firing conditions are satisfied.

PROBLEM 1.13. The interceptor is guided to an unmaneuvering target from the aft hemisphere at an aspect angle 0/4 by the method of direct interception (direct approach). The mutual positions at the start of surface guidance are described by the following parameters: distance "interceptor-target" $D_{обн} = 150$ km, target relative bearing $\varphi_{ц0} = 100°$, interceptor relative bearing $\varphi_{п0} = 50°$. The interceptor and target speeds are constant and equal to the following: $V_п = 2200$ km/h, $V_ц = 2000$ km/h. Determine how long it will take for the interceptor to reach the firing distance $D_п = 20$ km.

Solution. As we know, unlike the method of constant-bearing approach, with direct interception the fighter is guided not to the point of target encounter but to the rocket-firing point. Therefore we have

$$\frac{V_{п}t + D_{п}}{\sin \varphi_{ц0}} = \frac{V_{п}t}{\sin \varphi_{п0}}, \qquad (1)$$

from which

$$t = \frac{D_{п} \sin \varphi_{п0}}{V_{ц} \sin \varphi_{ц0} - V_{п} \sin \varphi_{п0}} = 3{,}2 \text{ min}. \qquad (2)$$

Thus the interceptor arrives at the firing distance in 3.2 minutes.

PROBLEM 1.14. The mutual positions of a linearly moving interceptor and target at the start of the approach are described by the range $D_{обн}$ = 500 km, interceptor relative bearing $\varphi_{п0}$ = 45°, and target relative bearing $\varphi_{ц0}$ = 90°. To arrive at the attack curve the interceptor must decrease its relative bearing to $\varphi_{п}$ = 5° by turning toward the target (Fig. 1.14). Determine the time and the distance from the target at which the interceptor should begin its turn. $V_{п}$ = 30 km/min, $V_{ц}$ = 25 km/min.

Solution. The projection of the desired distance D onto the axis coinciding with the initial distance $D_{обн}$ is

$$D \cos(\varphi_{ц0} - \varphi_{п}) = D_{обн} - (V_{ц} \cos \varphi_{ц0} + V_{п} \cos \varphi_{п0})\, t. \qquad (1)$$

The projection of D onto the perpendicular to $D_{обн}$ is

$$D \sin(\varphi_{п0} - \varphi_{п}) = (V_{ц} \sin \varphi_{ц0} - V_{п} \sin \varphi_{п0})\, t. \qquad (2)$$

Dividing (2) by (1) we get

$$\operatorname{tg}(\varphi_{п0} - \varphi_{п}) = \frac{(V_{ц} \sin \varphi_{ц0} - V_{п} \sin \varphi_{п0})\, t}{D_{обн} - (V_{ц} \cos \varphi_{ц0} + V_{п} \cos \varphi_{п0})\, t}. \qquad (3)$$

Substituting into (3) the numerical values of our problem, we find from the equation

$$\operatorname{tg}(45° - 5°) \frac{(25 \sin 90° - 30 \sin 45°)\, t}{500 - (25 \cos 90° + 30 \cos 45°)\, t};$$

$$t \approx 1{,}8 \text{ min}.$$

Thus the turn must be started 1.8 min from the moment of the start of the approach. Here, according to (2), the distance "interceptor-target" is

$$D = 107 \text{ km}.$$

PROBLEM 1.15. An interceptor, speed $V_п$ = 2000 km/h, attacks a target, speed $V_ц$ = 1700 km/h, from the aft hemisphere at an aspect angle 0/4. After firing a heat-seeking rocket the interceptor withdraws from the attack with maximum permissible g-load. The firing distance $D_п$ = 30 km. The rocket flight time to impact with the target $t_р$ = 60 s. Determine the bank angle required to withdraw from the attack if the turn is made in one plane at constant speed, while the minimum permissible distance of the interceptor from the explosion point, for safety purposes, $D_{мин}$ = 10 km.

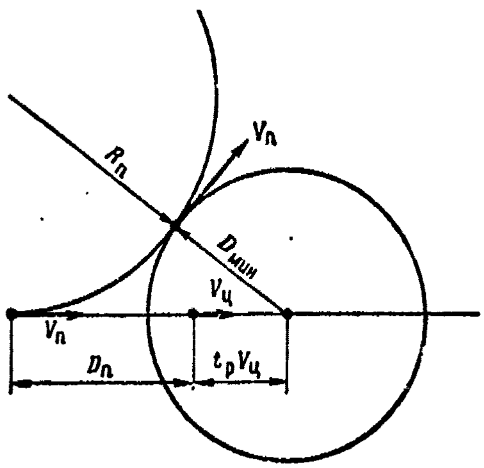


Fig. 1.17.

Solution. The kinematic diagram of withdrawal from the attack (Fig. 1.17 allows us to express the minimum possible radius of banked turn $R_п$, which assures safe departure of the interceptor relative to the explosion point:

$$R_п^2 = (R_п + D_{мин})^2 - (D_п + t_р V_ц)^2, \quad (1)$$

from which

$$R_п = \frac{(D_п + t_р V_ц)^2 - D_{мин}^2}{2D_{мин}}. \quad (2)$$

Substituting the numerical values we get $R_п$ = 12.85 km. This radius corresponds to bank angle γ, which is defined by the familiar relationship

$$\text{tg}\,\gamma = \frac{V_п^2}{gR_п} = 2{,}46; \quad (3)$$
$$\gamma = 67°50'.$$

PROBLEM 1.16. Show how the value of the region of possible attacks of an aerial target depends on the available load factor n_y of the interceptor, and on the speeds of the interceptor and target.

Solution. The centripetal force which distorts the trajectory of the interceptor in the horizontal plane is directed along the normal to this trajectory, and is numerically equal to

$$F_{\text{ц}} = ma_{\text{п}} = mV_{\text{п}}\omega_{\text{п}}, \tag{1}$$

where m is the mass, $a_{\text{п}}$ the acceleration, and $\omega_{\text{п}}$ the angular velocity of the interceptor.

On the other hand,

$$F_{\text{ц}} = \sqrt{Y^2 - G^2} = mg\sqrt{n_y^2 - 1}, \tag{2}$$

where Y is the lift, G is the weight of the interceptor, and g is the acceleration of gravity.

From (1), considering (2), we get a formula for calculating the available angular velocity of an attacking interceptor:

$$\omega_{\text{п. расп}} = \frac{F_{\text{ц}}}{mV_{\text{п}}} = \frac{g\sqrt{n_y^2 - 1}}{V_{\text{п}}}. \tag{3}$$

The required angular velocity for the interceptor for flight along the attack curve (pursuit curve) in the horizontal plane is

$$\omega_{\text{п. потр}} = \frac{V_{\text{ц}} \sin \varphi_{\text{ц}}}{D}. \tag{4}$$

Equating the required and available interceptor angular velocities, we find an equation for the boundary of the region of possible attacks in the horizontal plane:

$$D = \frac{V_{\text{п}} V_{\text{ц}} \sin \varphi_{\text{ц}}}{g\sqrt{n_{y\,\text{расп}}^2 - 1}}. \tag{5}$$

According to (5), the boundary for the region of possible attacks is a circle which is tangent to the target velocity vector, with a change in target relative bearing $\varphi_{\text{п}}$ from 0 to 180°, while with a change from 180 to 360° we get a second circle, symmetric to the first.

The equation for the boundary of the region of possible attacks in the vertical plane is obtained analogously. When the attack curve lies in the vertical plane, the available angular velocity of the interceptor

$$\omega_{\text{п. расп}} = \frac{g(n_y - \cos\varphi_{\text{ц}})}{V_{\text{п}}}. \tag{6}$$

The angular velocity required for the interceptor to fly along the attack curve in the vertical plane is

$$\omega_{\text{п. потр}} = \frac{V_{\text{ц}} \sin\varphi_{\text{ц}}}{D}. \tag{7}$$

Equating the available and required angular velocities of the interceptor, we find a formula for determining the boundary of the region of possible attacks of the interceptor in the vertical plane:

$$D = \pm \frac{V_{\text{п}} V_{\text{ц}} \sin\varphi_{\text{ц}}}{g(n_y + \cos\varphi_{\text{ц}})}. \tag{8}$$

Obviously, Eq. (8) is also a pair of circles, but asymmetric relative to the target velocity vector. In Formula (8) we must use the "+" when the interceptor attacks from above, and "-" when it attacks from below.

PROBLEM 1.17. Determine the conditions which define the position of the point of tangency of the attack curve with the boundary of the region of possible attacks. Show on what the position of the attack curve depends relative to the boundary of the region of possible attacks.

Solution. The attack curve is defined by the equation

$$\frac{d\varphi_{\text{ц}}}{dt} = \frac{V_{\text{ц}} \sin\varphi_{\text{ц}}}{D} - \omega_{\text{ц}} = \omega_{\text{д}} - \omega_{\text{ц}}. \tag{1}$$

The closing speed

$$\frac{dD}{dt} = -V_{\text{п}} - V_{\text{ц}} \cos\varphi_{\text{ц}}. \tag{2}$$

Dividing (1) by (2) we find the dependence between increments of D and $\varphi_{\text{ц}}$ along the attack curve:

$$\frac{d\varphi_{\text{ц}}}{dD} = -\frac{\omega_{\text{п}} - \omega_{\text{ц}}}{V_{\text{п}} + V_{\text{ц}} \cos \varphi_{\text{ц}}}. \tag{3}$$

On the other hand, we have

$$\frac{\partial D}{\partial \varphi_{\text{ц}}} = \frac{V_{\text{ц}} \cos \varphi_{\text{ц}}}{\omega_{\text{п. расп}}}, \tag{4}$$

which is obtained by differentiating the equation

$$D = \frac{V_{\text{ц}} \sin \varphi_{\text{ц}}}{\omega_{\text{п. расп}}}. \tag{5}$$

At the point of tangency of the attack curve with the boundary of the region of possible attacks the following condition should be fulfilled:

$$\frac{d\varphi_{\text{ц}}}{dD} = \frac{\partial \varphi_{\text{ц}}}{\partial D}, \tag{6}$$

i.e.,

$$-\frac{\omega_{\text{п}} - \omega_{\text{ц}}}{V_{\text{п}} + V_{\text{ц}} \cos \varphi_{\text{ц}}} = \frac{\omega_{\text{п}}}{V_{\text{ц}} \cos \varphi_{\text{ц}}}. \tag{7}$$

Solving (7) we find that for the point of tangency the following condition should be fulfilled:

$$\cos \varphi_{\text{цА}} = \frac{p}{\frac{\omega_{\text{ц}}}{\omega_{\text{п}}} - 2}, \tag{8}$$

where

$$p = \frac{V_{\text{п}}}{V_{\text{ц}}}.$$

With $\omega_{\text{ц}} = 0$, when the target flies in a straight line, we have

$$\cos \varphi_{\text{ц.А}} = -0{,}5p. \tag{9}$$

From (9) we can draw the following important practical conclusions: 1) the point of tangency of the attack curve with the boundary of the region of possible attacks can be only in the aft hemisphere of the target; 2) with p > 2 there is generally no tangency of the attack curve, i.e., all attack curves begin within the regions of possible attacks and terminate at its boundaries.

The attack curve tangent to the boundary of the region of possible attacks is called the **limiting attack curve**. The region of possible attacks is defined by the boundary of this region. The boundaries of the regions of possible attacks when the interceptor flies along the pursuit curve are defined unambiguously by the available limiting load factors. From the load factors we calculate the corresponding limiting angular velocities.

For the horizontal plane

$$\omega_{\text{п. пред}}=\frac{g\sqrt{n_{y\,\text{пред}}^{2}-1}}{V_{\text{п}}}. \qquad (10)$$

For the vertical plane:

– with attack from above

$$\omega_{\text{п. пред}}=\frac{g\,(n_{y\,\text{пред}}-1)}{V_{\text{п}}}; \qquad (11)$$

– with attack from below

$$\omega_{\text{п. пред}}=\frac{g\,(n_{y\,\text{пред}}+1)}{V_{\text{п}}}. \qquad (12)$$

Further, from Formula (8), given the maneuvering of the target ($\omega_{\text{ц}}$), we find the point of tangency of the pursuit curve with the boundary of the region of possible attacks with respect to the limiting load factors.

PROBLEM 1.18. Determine the regions of possible attacks and the trajectory of interceptor homing by the "pursuit" method. We assume that the target does not maneuver, the target and interceptor velocities are constant, and motion is in the plane defined by the velocity vectors $V_{\text{п}}$ and $V_{\text{ц}}$. Given: $V_{\text{п}}$ = 930 km/h, $V_{\text{ц}}$ = 800 km/h, $R_{\text{п}}$ = 10, 15, and 20 km.

Solution. According to Fig. 1.4, which shows the mutual position of the interceptor and the target during the "pursuit" method, for attack of the target from the aft hemisphere we have the following equations:

— closing speed

$$\frac{dD}{dt} = V_ц \cos\varphi_п - V_п; \qquad (1)$$

— angular velocity of the sighting line

$$\frac{d\varphi_п}{dt} = -\frac{V_ц \sin\varphi_п}{D}. \qquad (2)$$

From Eq. (2) we obtain the equation for the region $\omega_{п.расп} \leq$ $\leq \omega_{п.потр}$:

$$D = \frac{V_ц \sin\varphi_п}{\omega_{п.расп}}, \qquad (3)$$

where $\omega_{п.расп}$ with respect to interceptor load forces is defined by the familiar relationship

$$\omega_{п.расп} = \frac{g\sqrt{n^2_{п.расп} - 1}}{V_п}. \qquad (4)$$

Now let us derive an equation for the interceptor homing trajectory. We divide (1) by (2); then

$$\frac{\frac{dD}{dt}}{D} = \left(\frac{p}{\sin\varphi_п} - \operatorname{ctg}\varphi_п\right)\frac{d\varphi_п}{dt}. \qquad (5)$$

After integrating (5) we get

$$D = C\frac{(\sin\varphi_п)^{p-1}}{(1+\cos\varphi_п)^p}; \qquad (6)$$

$$p = \frac{V_п}{V_ц},$$

where C is the integration constant, determined from the initial homing conditions characterized by the parameters D_0, $\varphi_{ц0}$, and calculated from the formula

$$C = \frac{D_0(1+\cos\varphi_{ц_0})^p}{(\sin\varphi_{ц_0})^{p-1}}. \qquad (7)$$

Substituting (7) into (6), we obtain a calculation formula for the trajectory of interceptor homing by the "pursuit" method:

$$D = \frac{D_0(1+\cos\varphi_{ц_0})^p(\sin\varphi_п)^{p-1}}{(1+\cos\varphi_п)^p(\sin\varphi_{ц_0})^{p-1}}. \qquad (8)$$

To construct the regions of possible attacks it is important to be able to determine the target relative bearings $\varphi_{цA}$ and $\varphi_{цB}$ for which the interceptor homing trajectory is tangent to and intersects the region $\omega_{п.расп} \leq \omega_{п.потр}$ at the given distance, e.g., at the maximum allowed rocket-firing distance. The coordinates of these points are calculated from the formulas

$$\cos\varphi_{цA} = -\frac{V_п}{2V_ц};$$
$$\sin\varphi_{цB} = \frac{D\omega_{п.расп}}{V_ц}. \qquad (9)$$

For attack of the target from the forward hemisphere the kinematic relationships change somewhat. In this case we have

$$\frac{dD}{dt} = -V_ц\cos\varphi_ц - V_п; \qquad (10)$$

$$\frac{d\varphi_ц}{dt} = \omega_{п.расп} = \frac{V_ц\sin\varphi_ц}{D}. \qquad (11)$$

Dividing (10) by (11) we get

$$\frac{\frac{dD}{dt}}{D} = -\left(\frac{p}{\sin\varphi_ц} + \mathrm{ctg}\,\varphi_ц\right)\frac{d\varphi_ц}{dt}. \qquad (12)$$

After integration we get the following homing equation for collision-course attacks:

$$D = \frac{D_0(\sin\varphi_{ц_0})^{p+1}(1+\cos\varphi_ц)^p}{(1+\cos\varphi_{ц_0})^p(\sin\varphi_ц)^{p+1}}; \qquad (13)$$

Constructing Eqs. (3), (8), (9), and (13) in a polar system of coordinates D, $\varphi_ц$ we get a graphic representation of the regions of possible attacks. Figure 1.18 shows the solution for the numerical conditions of the problem.

PROBLEM 1.19. Determine the regions of possible attacks and the trajectory of interceptor homing by the "pursuit with lead" method, using the assumptions and conditions of Problem 1.18. The lead angle ε is 10 and 40°.

Solution. Interceptor homing by the "pursuit with lead" method is illustrated by Fig. 1.19, according to which we have the following

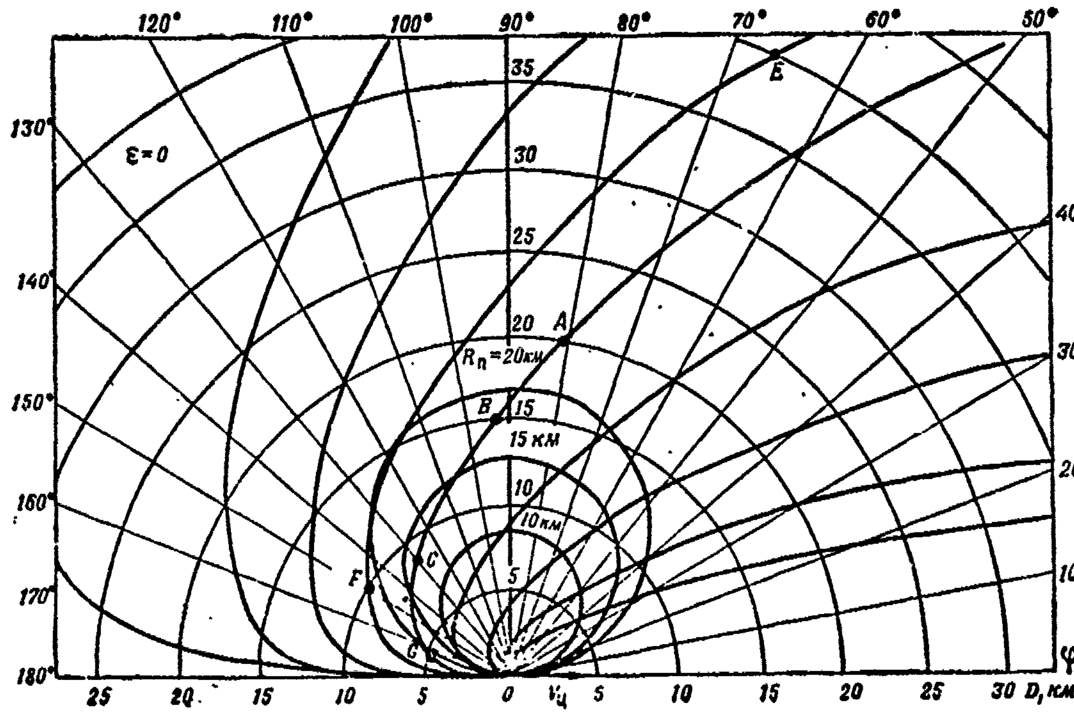


Fig. 1.18.

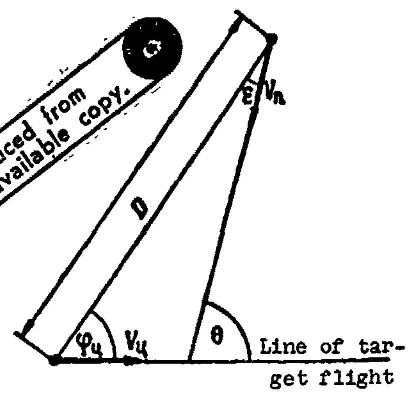


Fig. 1.19.

equations:

— for closing speed

$$\frac{dD}{dt} = V_ц \cos\varphi_п - V_п \cos\varepsilon; \qquad (1)$$

— for angular velocity of the line of sighting (available interceptor angular velocity)

$$\frac{d\varphi_п}{dt} = \omega_{п.\,расп} = \frac{-V_ц \sin\varphi_п + V_п \sin\varepsilon}{D}. \qquad (2)$$

From (2) we get the equation for the regions $\omega_{п.расп} \leq \omega_{п.потр}$:

$$D = \frac{-V_ц \sin\varphi_п + V_п \sin\varepsilon}{\omega_{п.\,расп}}. \qquad (3)$$

Then we divide (1) by (2); we get

$$\frac{\frac{dD}{dt}}{D} = \left(\frac{\cos\varphi_п - p\cos\varepsilon}{-\sin\varphi_п + p\sin\varepsilon}\right)\frac{d\varphi_п}{dt}. \qquad (4)$$

Let us integrate Eq. (4):

$$\int_{D_0}^{D}\frac{dD}{D}=\int_{\varphi_{u_0}}^{\varphi_u}\frac{\cos\varphi_u\,d\varphi_u}{(-\sin\varphi_u+p\sin\varepsilon)}+p\int_{\varphi_{u_0}}^{\varphi_u}\frac{\cos\varepsilon\,d\varphi_u}{(-p\sin\varepsilon+\sin\varphi_u)};$$

$$\ln\frac{D}{D_0}=\ln\frac{\sin\varphi_{u_0}-p\sin\varepsilon}{\sin\varphi_u-p\sin\varepsilon}+ \tag{5}$$

$$+\frac{-p\cos\varepsilon}{\sqrt{1-p^2\sin^2\varepsilon}}\ln\frac{1-p\sin\varepsilon\sin\varphi_u+\cos\varphi_u\sqrt{1-p^2\sin^2\varepsilon}}{-p\sin\varepsilon+\sin\varphi_u}\Bigg|_{\varphi_{u_0}}^{\varphi_u}.$$

With $p^2\sin^2\varepsilon < 1$, which is practically always true, we get the following equation for calculating the trajectory of interceptor homing by the "pursuit with lead" method:

$$D=\frac{D_0\,(1-C\sin\varphi_{u0}-A\cos\varphi_{u0})^B\,(\sin\varphi_u-C)^{B-1}}{(1-C\sin\varphi_u-A\cos\varphi_u)^B\,(\sin\varphi_{u0}-C)^{B-1}}, \tag{6}$$

where D_0 and D are the distances of the beginning and end of homing; $\varphi_{ц0}$ and $\varphi_ц$ are the target relative bearings at the beginning and end of homing;

$$C=p\sin\varepsilon; \tag{7}$$

ε is the constant lead angle;

$$A=\sqrt{1-C^2}; \tag{8}$$

$$B=\frac{p\cos\varepsilon}{A}. \tag{9}$$

The target relative bearings corresponding to the points of tangency and intersection of region $\omega_{п.расп} \leq \omega_{п.потр}$ with the homing trajectory at the given distance are defined by the formulas

$$\cos\varphi_{цA}=\frac{V_п\cos\varepsilon}{2V_ц}; \tag{10}$$

$$\sin\varphi_{цB}=\frac{D\omega_{п.расп}+V_п\sin\varepsilon}{V_ц}. \tag{11}$$

From Eqs. (3) and (6) we construct the regions of possible attacks in polar coordinates D, $\varphi_ц$. Quite often enough we have regions of possible attacks in a relative coordinate system associated with the target, as shown in Figs. 1.20 and 1.21. To construct the homing trajectory in absolute coordinates we must express time t in terms of D and $\varphi_ц$. We can show that

$$t = \frac{1}{V_{п}(p^2 - 1)\cos\varepsilon}\{D_0[p + \cos(\varphi_{ц0} + \varepsilon)] - D[p + \cos(\varphi_{ц} + \varepsilon)]\}. \quad (12)$$

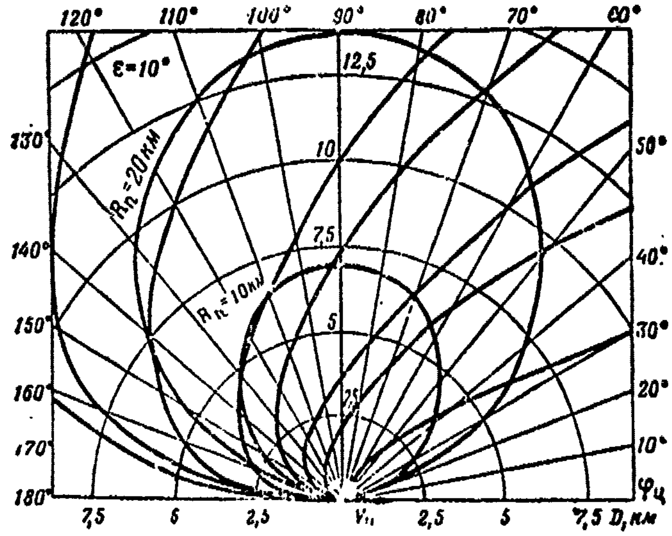


Fig. 1.20.

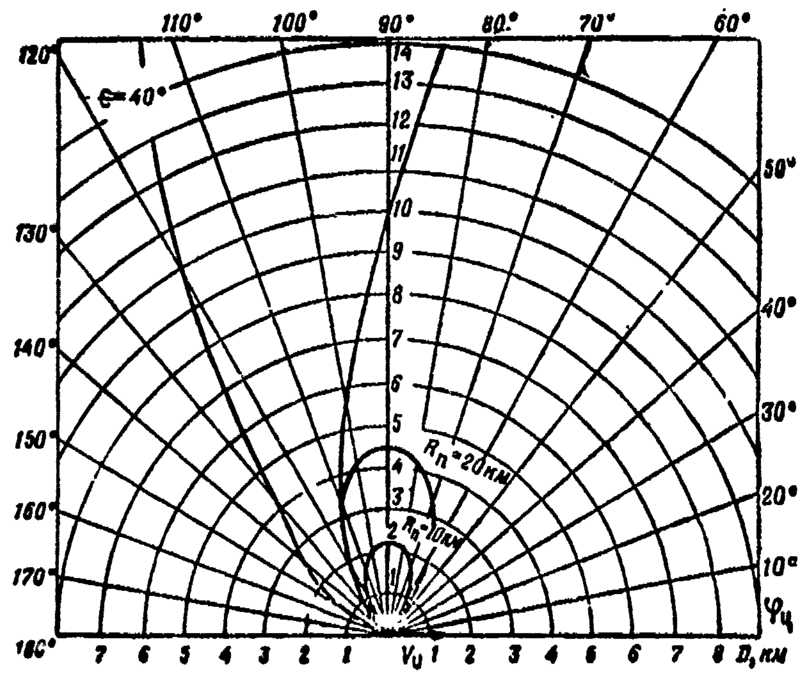


Fig. 1.21.

PROBLEM 1.20. An interceptor, at speed $V_п$ = = 930 km/h, using the pursuit method attacks, in the horizontal plane, a target which is on a constant heading at speed $V_ц$ = 800 km/h. The mutual position of the interceptor and target at the beginning of interceptor homing is described by the initial range D_0 = 20 km and the initial target relative bearing $\varphi_{ц0}$ = 80°. The minimum radius of turn of the interceptor $R_п$ = 10 km. The region of allowed rocket firing from the aft hemisphere is limited by the angles $\theta \leq \pm 50°$ and ranges of 5-15 km (we assume that the firing distance does not depend on the attack aspect angle). The target's defensive weapons do not allow an approach nearer than 5 km. Determine if, under these conditions, a successful attack on the target is possible. At what range and at what target relative bearing is rocket firing possible? How do the attack possibilities change if the interceptor's radius of turn is at least 20 km?

Solution. Let us use the graph calculated when solving Problem 1.18 (Fig. 1.18).

The homing trajectory corresponding to our initial data ($D_0 = 20$ km, $\varphi_{ц0} = 80°$) (point A in the figure) does not intersect the boundary of the region of possible attacks for $R_п = 10$ km. Consequently, from the available interceptor load factor, attack on the target by the pursuit method is possible under the given circumstances. Closing with the target along the homing trajectory, the interceptor is at the maximum allowed firing distance D = 15 km when the target relative bearing is 94° (point B). Since $\theta > 50°$, firing is impossible. As we see from the figure, rocket firing will become possible at distance D = 8.75 km, when $\theta = 50°$ (point C). At point G firing will again become impossible due to the enemy's defensive fire. If the region of possible attacks is limited to a circle corresponding to the interceptor turn radius $R_п = 20$ km, as we see from Fig. 1.18 the homing trajectory beginning at point A will intersect this circle. Consequently, interceptor homing by the pursuit method is impossible in this case.

PROBLEM 1.21. At what target relative bearing $\varphi_ц$ must the interceptor be guided by the surface guidance system if, from the moment of target acquisition by the airborne radar, the interceptor realizes a pursuit trajectory which will assure arrival at the allowed firing range $D_п = 10$ km in the minimum time? The interceptor speed is 930 km/h, the target speed is 800 km/h. The target acquisition range by the airborne radar $D_{обн} = 40$ km. Rocket firing is possible only with the angles $\theta = \pm 40°$ from the aft hemisphere of the target. The available interceptor load factor is $n_п = 2$. We disregard surface-guidance heading errors, i.e., we consider that the guidance system guides the interceptor such that at the moment of target acquisition the velocity vector of the interceptor is directed strictly toward the target.

Solution. As was shown in Problem 1.17, the limiting pursuit trajectory is tangent to a circle corresponding to the available interceptor load factor. Consequently, on this homing curve it is also necessary to find the point where the interceptor must be guided by the surface system to the range of target acquisition by the airborne radar. This point is point E in Fig. 1.18. Its corresponding target relative bearing $\varphi_{ц} = 66°$. Then, as the interceptor closes with the target it is at point F, at which the conditions of allowed rocket firing are satisfied: $D = 10$ km, $\theta = 30°$. The azimuth firing conditions had been satisfied prior to this ($\theta < 40°$), but the range firing conditions were not, since $D > 10$ km.

PROBLEM 1.22. What should the lead angle ε be so as to assure interceptor homing by the "pursuit with lead" method from the point of the end of surface guidance characterized by an initial range $D_0 = 10$ km and target relative bearing $\varphi_{ц0} = 90°$? The interceptor should arrive at the allowed rocket firing range $D = 5$ km with target relative bearing $\varphi_{ц} = 70°$. The minimum interceptor turn radius $R_{п} = 10$ km. How do the conditions change for carrying out an attack by the "pursuit with lead" method if the interceptor turn radius is 20 km?

Solution. Let us use the solution of Problem 1.19. With lead angle $\varepsilon = 10°$ (Fig. 1.20) attack is not assured: the homing trajectory intersects the boundary of the region of possible attacks corresponding to the value $R_{п} = 10$ km. With $\varepsilon = 20°$ the interceptor arrives, from the point of the start of homing, at the allowed firing range $D = 5$ km with a target relative bearing $\varphi_{ц} = 64°$. Attack under these conditions is also possible with a minimum interceptor turn radius $R_{п} = 20$ km. In this case we need only increase lead angle ε to 40°.

PROBLEM 1.23. An interceptor attacks, at low altitude, a nonmaneuvering target using the pursuit method. Given: $V_{ц} = 600$ km/h, $V_{п} = 800$ km/h, available interceptor load force $n_{п.расп} = 1.41$, interceptor load force limited by low-altitude maneuvering-safety

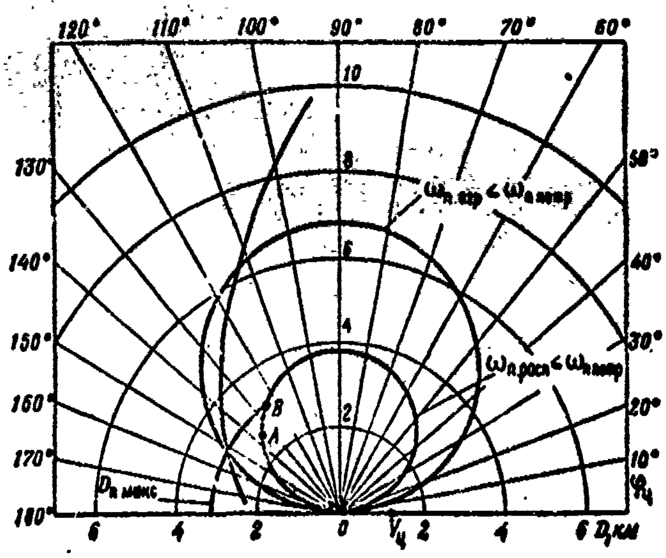


Fig. 1.22.

conditions $n_{п.огр} = 1.15$. Show if attack on a target is possible if at the start of homing the distance "interceptor-target" is 6 km, while the target relative bearing $\varphi_ц = 115°$. How should the interceptor maneuver in order to reach the rocket-firing range of 3.1 km?

Solution. Constructed boundaries of the regions of possible attacks (pursuit-method homing trajectory) show (Fig. 1.22) that with a g-force of 1.15 the interceptor intersects the boundary of $\omega_{п.огр} \leq \omega_{п.потр}$, i.e., errorless flight along the pursuit curve is impossible. To reach the rocket-firing range the load factor must be increased: closing with the target along the pursuit curve for arrival at $D_п = 3.1$ km is already possible with $n_п = 1.25$.

CHAPTER 2

ATTACK ON A TARGET. ROCKET HOMING

In this chapter we derive relationships among the characteristics of the interceptor, the rocket, and the target, calculation formulas for required load factors and the time of controlled rocket flight; these form the basis for constructing the regions of possible rocket firings, which allow us to graphically study the combat capabilities and optimum conditions for attack against an aerial target.

In solving the problems we limit ourselves to the kinematics of rocket homing, i.e., the rocket, interceptor, and target are examined as geometric points coinciding with the centers of mass of these bodies.

Naturally, an actual rocket trajectory differs from a kinematic one: we must consider the inertia of the rocket and of the automatic rocket-flight control system, perturbing influences on the motion of the rocket and the target, instrument errors in the elements of the closed control system, etc. Solution of problems of the combat use of "air-to-air" rockets with consideration of the above factors necessitates the use of mathematical models which can be realized on digital computers. These research methods are outside the scope of this book.

A rocket closes with a target along trajectories which are determined by the homing method used. Therefore, problems devoted to attack on an aerial target begin with the derivation, in general form, of equations which describe rocket-to-target approach (Problems 2.1, 2.2). Then we compare various methods of rocket homing, and determine the dependences for the load factors required for carrying out a given homing method; the time of controlled rocket flight vs target, interceptor, and rocket velocity; and the target relative bearing at the moment of firing. We examine the following methods of rocket homing: "line-of-sight" (Problems 2.3-2.9), "pursuit" (Problems 2.10 and 2.11), "constant-bearing approach" (Problems 2.12-2.17), and "proportional approach" (Problems 2.18-2.20). The dependences obtained allow us to analyze the interceptor's capabilities to attack an aerial target under various combat-use conditions, determine the optimum ranges and aspect angles for firing, and construct the regions of possible firings for given characteristics of the rocket, interceptor, target, and conditions of combat use (Problems 2.6-2.10). By region of possible firings we mean the geometric positioning of the interceptor relative to the target (with respect to range and aspect angle), from which we can assure successful guidance of the rocket to the target.

Of definite practical interest in this chapter are other problems, solution of which allows us to determine:

— the optimum aspect angles of attacks on a maneuvering target and the required load factors for the rocket and interceptor (Problems 2.13, 2.16, and 2.20);

— the firing range, with consideration of errors in guiding the rocket to the lead point at the moment of firing (Problem 2.14);

— restrictions under which an examined homing method cannot be carried out (Problems 2.11, 2.12, 2.14, and 2.19);

— a miss by the rocket as a function of conditions of combat use and firing (Problem 2.17).

PROBLEM 2.1. To derive, in general form, kinematic equations which characterize rocket-to-target approach in the horizontal plane.

Solution. The mutual positions of the target and rocket (Fig. 2.1) at any moment of time after firing are characterized by distance D, target relative bearing $\varphi_{\text{ц}}$, and rocket relative bearing φ_{p}.

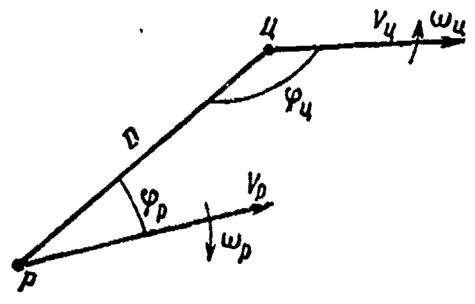


Fig. 2.1.

Rocket-to-target approach is completely described by the rate of change of D and of the target $\varphi_{\text{ц}}$ and rocket φ_{p} relative bearings with time. Obviously, a decrease in D by ΔD in time interval Δt is equal to the sum of the projections of velocities $V_{\text{ц}}$ and V_{p} onto the sighting line, multiplied by Δt:

$$-\Delta D = (V_{\text{p}} \cos\varphi_{\text{p}} + V_{\text{ц}} \cos\varphi_{\text{ц}})\,\Delta t.$$

Passing to the limit, we get a differential equation for rocket-to-target approach:

$$\frac{dD}{dt} = -(V_{\text{p}} \cos\varphi_{\text{p}} + V_{\text{ц}} \cos\varphi_{\text{ц}}). \qquad (1)$$

Since between the angular and linear velocities there exists the familiar dependence

$$\omega = \frac{V}{R},$$

where R is the radius of rotation, the angular velocity of the line of sighting is defined by the equation

$$\omega_{\text{д}} = \frac{-V_{\text{ц}} \sin\varphi_{\text{ц}} + V_{\text{p}} \sin\varphi_{\text{p}}}{D}. \qquad (2)$$

For the case when the target maneuvers with angular velocity $\omega_{\text{ц}}$, the angular velocity of the rocket relative to the target is equal to the sum of $\omega_{\text{ц}}$ and the rate of change of the target relative bearing $\varphi_{\text{ц}}$ with time:

$$\omega_{\text{p}} = \omega_{\text{ц}} + \frac{d\varphi_{\text{ц}}}{dt} = \frac{-V_{\text{ц}} \sin\varphi_{\text{ц}} + V_{\text{p}} \sin\varphi_{\text{p}}}{D}. \qquad (3)$$

Equations (1), (2), and (3) are general equations for any method of rocket guidance.

Naturally, to integrate these equations of motion with specific guidance methods and for given initial conditions we must know the laws of the change in speeds $V_ц$ and V_p and relative bearings $\varphi_ц$ and φ_p with time.

PROBLEM 2.2. For a nonmaneuvering target and with constant rocket and target speeds, let us compare two methods, based on the required rocket-homing time t_p: "constant-bearing approach" and "pursuit." Let us show how time t_p depends on the velocity ratio $V_p/V_ц$ and the attack aspect angle.

Solution. Using Formula (1) of Problem 2.1, and considering that with constant-bearing approach

$$\varphi_p = \text{const} \quad \text{and} \quad \varphi_ц = \text{const},$$

by integrating we get

$$\int_{D_п}^{0} dD = -\int_{0}^{t_p} (V_p \cos\varphi_p + V_ц \cos\varphi_ц)\, dt, \qquad (1)$$

from which the flight time for a rocket from the moment of firing to the moment of impact with the target

$$t_{p.\text{пар}} = \frac{D_п}{V_p \cos\varphi_p + V_ц \cos\varphi_ц}, \qquad (2)$$

where $D_п$ is the firing distance.

For the "pursuit" method, during the entire rocket homing time the rocket relative bearing $\varphi_p = 0$, since the rocket's velocity vector is always directed toward the target. Thus, on the basis of Formulas (1) and (3) of Problem 2.1, the equations of motion of a rocket using the "pursuit" method, for the case of a nonmaneuvering target, assume the following form:

$$\frac{dD}{dt} = -(V_p + V_ц \cos\varphi_ц); \qquad (3)$$

$$\omega_p = \frac{d\varphi_u}{dt} = \frac{V_u \sin\varphi_u}{D}. \quad (4)$$

Multiplying Eq. (3) by the value $\left(\cos\varphi_u - \frac{V_p}{V_u}\right)$, and Eq. (4) by sin φ_u, and subtracting one result from the other, we get

$$\left(\cos\varphi_u - \frac{V_p}{V_u}\right)\frac{dD}{dt} - D\sin\varphi_u \frac{d\varphi_u}{dt} = \frac{V_p^2}{V_u} - V_u. \quad (5)$$

Let us introduce the designation

$$b = \frac{V_p}{V_u}, \quad (6)$$

then

$$(\cos\varphi_u - b)\,dD - D\sin\varphi_u\,d\varphi_u = V_u(b^2-1)\,dt. \quad (7)$$

Let us integrate Eq. (7) by parts:

$$\int_{D_n}^{D}(\cos\varphi_u - b)\,dD - \int_{\varphi_{u0}}^{\varphi_u} D\sin\varphi_u\,d\varphi_u = V_u(b^2-1)\,t; \quad (8)$$

$$\int_{D_n}^{D}(\cos\varphi_u - b)\,dD = uv - \int v\,du;$$

$$u = \cos\varphi_u - b; \quad du = -\sin\varphi_u\,d\varphi_u;$$

$$dv = dD = \dot{D}dt; \quad v = D;$$

$$uv - \int v\,du = (\cos\varphi_u - b)\,D\Big|_{D_n}^{D} + \int_{\varphi_{u0}}^{\varphi_u} D\sin\varphi_u\,d\varphi_u.$$

Let us substitute the obtained result into (8). We then get

$$D(\cos\varphi_u - b) - D_n(\cos\varphi_{u0} - b) + \int_{\varphi_{u0}}^{\varphi_u} D\sin\varphi_u\,d\varphi_u - \int_{\varphi_{u0}}^{\varphi_u} D\sin\varphi_u\,d\varphi_u = -D(b - \cos\varphi_u) + D_n(b - \cos\varphi_{u0}) = V_u(b^2-1)\,t. \quad (9)$$

From Eq. (9) we get a calculation formula for rocket flight time from distance D_n to distance D:

$$t = \frac{D_n(b - \cos\varphi_{u0}) - D(b - \cos\varphi_u)}{V_u(b^2-1)}. \quad (10)$$

The rocket flight time from firing to impact with the target, when D = 0, is defined by the formula

$$t_{\text{р. пог}} = \frac{D_{\text{п}}(b - \cos\varphi_{\text{ц0}})}{V_{\text{ц}}(b^2 - 1)} \tag{11}$$

To compare the "pursuit" and "constant-bearing approach" homing methods it is best to construct the ratio

$$A = \frac{t_{\text{р. пог}}}{t_{\text{р. пар}}}$$

and the dimensionless time difference

$$B = \frac{V_{\text{ц}}}{D_{\text{ц}}}(t_{\text{р. пог}} - t_{\text{р. пар}})$$

as functions of the target relative bearing $\varphi_{ц}$ at the moment of firing for various velocity ratios $b = V_{р}/V_{ц}$. After simple transformations we get

$$A = \frac{t_{\text{р. пог}}}{t_{\text{р. пар}}} = \frac{(b - \cos\varphi_{\text{ц}})\left(\cos\varphi_{\text{ц}} + \sqrt{b^2 - \sin^2\varphi_{\text{ц}}}\right)}{b^2 - 1}; \tag{12}$$

$$B = \frac{V_{\text{ц}}}{D_{\text{п}}}(t_{\text{р. пог}} - t_{\text{р. пар}}) = \frac{b - \cos\varphi_{\text{ц}}}{b^2 - 1} - \frac{1}{\cos\varphi_{\text{ц}} + \sqrt{b^2 - \sin^2\varphi_{\text{ц}}}}. \tag{13}$$

Graphs of these functions are given in Fig. 2.2. For practical purposes it is important to note that the difference in rocket flight times and, consequently, the difference in covered distances and in the corresponding distances of the target intercept lines are maximum when firing occurs with $\varphi_{ц} = 90°$. Time ratio A, however, has its maximum when the target relative bearing at the moment of firing is 60°. As the rocket speed approaches that of the target, the difference and ratio of the rocket-flight times sharply increase. The larger the velocity ratio $b = V_{р}/V_{ц}$, the less difference there is between the homing methods. When $b \geq 1.7$ there is practically no difference whatsoever.

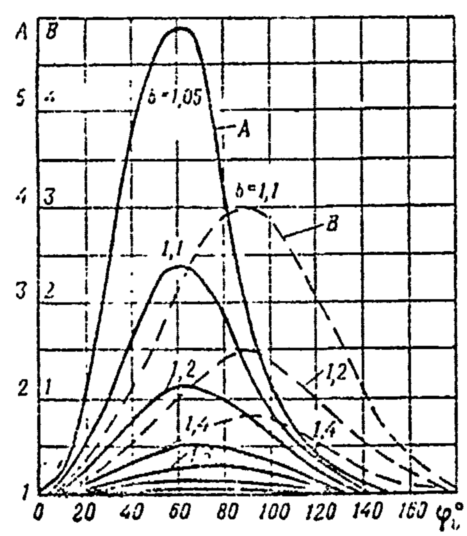


Fig. 2.2

PROBLEM 2.3. The interceptor attacks a nonmaneuvering target flying at the same altitude as the interceptor and at a constant speed. The speed of the interceptor is also constant. After the airborne radar locks on to the target the air-to-air rocket is fired. The rocket is guided to the target by the line-of-sight method, i.e., at any given moment the interceptor, rocket, and target are in a straight line, in the equisignal zone of the airborne radar. Let us assume that during rocket guidance the interceptor heading does not change, while the target is destroyed only by a direct hit by the rocket. The automatic tracking angle for the airborne radar imposes no restrictions on the attack. We are required to show how the rocket load factor, required to carry out the homing procedure, and the rocket flight time from the moment of firing to encounter with the target depend on the speeds of the target, interceptor, and rocket ($V_{ц}$, $V_{п}$, $V_{р}$) and on the target relative bearing $\varphi_{ц}$ ($\varphi_{ц} = \varphi_{ц0}$ — for simplicity we drop the "0" in this example). We are given the following:

$$\varphi_{ц} = 90 - 180°;\ \frac{V_{ц}}{V_{п}} = 1;\ \frac{V_{р}}{V_{п}} = 1 - 3;\ \varphi_{п} = 0 - 15°.$$

The natural speed of the rocket $V_{р.ср}$ is considered to be constant and equal to the average value during controlled flight:

$$V_{р} = V_{р\ ср} + V_{п}.$$

Solution. Figure 2.3 shows the kinematic relationships for rocket guidance using the line-of-sight method for the conditions of our problem. Let us introduce the designations $D_{п}$ — the distance "interceptor-target" at the moment of firing; $d_{р}$ — the current distance "interceptor-rocket"; d — the current distance "interceptor-target."

Then, according to Fig. 2.3 we have the following equations:

$$x = d_{р} \sin \varphi_{п}; \tag{1}$$
$$y = d_{р} \cos \varphi_{п} + y_{п}; \tag{2}$$
$$\operatorname{ctg} \varphi_{п} = \frac{y_{ц} - y_{п}}{x_{ц}}; \tag{3}$$
$$d^2 = x_{ц}^2 + (y_{ц} - y_{п})^2; \tag{4}$$

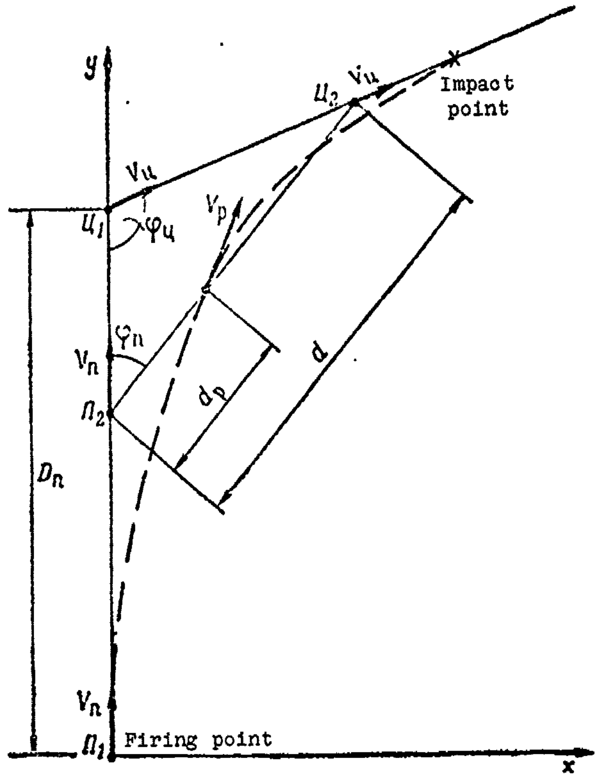


Fig. 2.3.

$$V_{\text{p}}^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2; \qquad (5)$$

$$y_{\text{п}} = V_{\text{п}} t; \qquad (6)$$

$$x_{\text{ц}} = V_{\text{ц}} t \sin\varphi_{\text{п}}; \qquad (7)$$

$$y_{\text{ц}} = V_{\text{ц}} t \cos\varphi_{\text{п}} + D_{\text{п}}. \qquad (8)$$

Differentiating the speed V_p with respect to t, we get a formula for calculating the side (normal) acceleration of the rocket:

$$a_{\text{p}} = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{V_{\text{p}}}. \qquad (9)$$

For solving the problem in the most general form it is convenient to introduce dimensionless, relative characteristics. To do this, let us divide all distances by the value of the firing range $D_{\text{п}}$ and all speeds by the interceptor speed $V_{\text{п}}$, and multiply time t_p by the value $V_{\text{п}}/D_{\text{п}}$. Then Eqs. (1)-(9) become dimensionless. For example, for the dimensionless coordinates of the rocket we have

$$X = \frac{d_{\text{p}}}{D_{\text{п}}} \sin\varphi_{\text{п}} = D_{\text{p}} \sin\varphi_{\text{п}}; \qquad (10)$$

$$Y = \frac{d_{\text{p}}}{D_{\text{п}}} \cos\varphi_{\text{п}} + \frac{V_{\text{п}} t}{D_{\text{п}}} = D_{\text{p}} \cos\varphi_{\text{п}} + \tau, \qquad (11)$$

where τ is dimensionless time.

The dimensionless rocket speed is expressed as follows:

$$v_{\text{p}}^2 = \dot{X}^2 + \dot{Y}^2 = f(\tau),$$
$$\tau = \frac{V_{\text{п}} t_{\text{p}}}{D_{\text{п}}}. \qquad (12)$$

Differentiating Function (12) with respect to τ we get

$$v_p^2 = \left(\frac{dD_p}{d\tau}\sin\varphi_u + D_p\cos\varphi_n\frac{d\varphi_n}{d\tau}\right)^2 +$$

$$+\left(\frac{dD_p}{d\tau}\cos\varphi_n + D_p\sin\varphi_n\frac{d\varphi_n}{d\tau} + 1\right)^2 =$$

$$= \left(\frac{dD_p}{d\tau}\right)^2 + \left(D_p\frac{d\varphi_n}{d\tau}\right)^2 + 2\frac{dD_p}{d\tau}\cos\varphi_n + 2D_p\sin\varphi_n\frac{d\varphi_n}{d\tau} + 1. \tag{13}$$

Solution of the quadratic equation

$$\left(\frac{dD_p}{d\tau}\right)^2 + 2\frac{dD_p}{d\tau}\cos\varphi_n + \left(D_p\frac{d\varphi_n}{d\tau}\right)^2 -$$

$$-2D_p\sin\varphi_n\frac{d\varphi_n}{d\tau} - v_p^2 + 1 = 0 \tag{14}$$

gives us, for the dimensionless current distance "interceptor-rocket" D_p, the following nonlinear first-order differential equation:

$$\frac{dD_p}{d\tau} = -\cos\varphi_n +$$

$$+\sqrt{\cos^2\varphi_n - \left(D_p\frac{d\varphi_n}{d\tau}\right)^2 + 2D_p\frac{d\varphi_n}{d\tau}\sin\varphi_n + v_p^2 - 1}. \tag{15}$$

The dimensionless side acceleration of the rocket is calculated from the formula

$$A = \frac{1}{v_p}\left(\frac{dX}{d\tau}\cdot\frac{d^2Y}{d\tau^2} - \frac{d^2X}{d\tau^2}\cdot\frac{dY}{d\tau}\right), \tag{16}$$

where X and Y are the relative coordinates of the rocket;

$$\frac{d\varphi_n}{d\tau} = -\frac{\sin\varphi_n}{pD^2};$$

$$\frac{d^2\varphi_n}{d\tau^2} = -\frac{2\frac{d\varphi_n}{d\tau}\frac{dD}{d\tau}}{D}. \tag{17}$$

Here

$$p = \frac{V_n}{V_u}$$

and

$$D = \frac{d}{D_u}. \tag{18}$$

From (9) and (12) we have, for the relative acceleration of the rocket,

$$A = \frac{a_p D_n}{V_n^2}. \quad (19)$$

We can obtain numerical values for the acceleration A and time τ required to satisfy the homing law if we solve Eqs. (16) and (12) with initial condition D(0) = 0 and vary the initial given values $\varphi_ц$, V_p, and $V_ц$. Differential Eq. (15) is solved by numerical integration (naturally, it is best to use a computer for this purpose).

The ·sults of calculations, with attack from the aft hemisphere, are shown in Figs. 2.4 and 2.5.

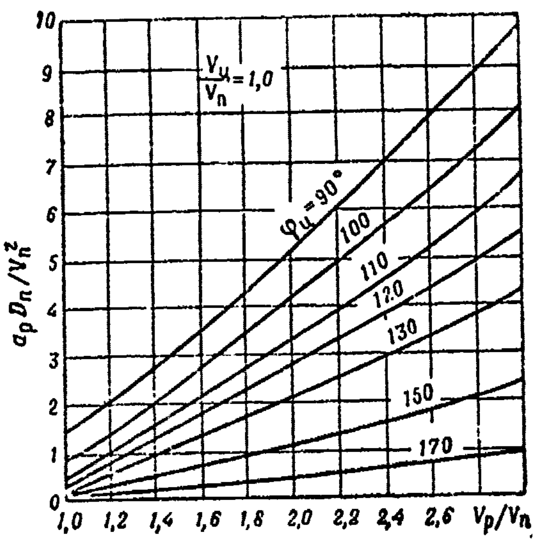


Fig. 2.4.

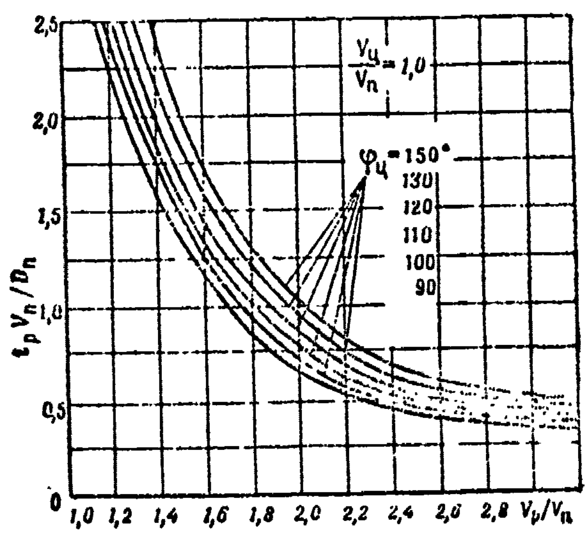


Fig. 2.5.

The results of calculations of A and τ for attack from the forward hemisphere with target relative bearings $\varphi_ц$ = 0-15°, $V_ц/V_п$ = 1 and 0.6, and $V_p/V_п$ = 1-3 are given in Figs. 2.6 and 2.7 on p. 52.

PROBLEM 2.4. How does the required rocket load factor change under the target-attack conditions given in Problem 2.3, if the velocity ratio $V_ц/V_п$ changes within limits 0.5-1.2, $V_p/V_п$ changes from 1 to 3, and the target relative bearing $\varphi_ц$ changes from 150 to 180° at the moment of firing? We determine, for given target speed $V_ц$, the velocity ratios $V_p/V_п$ for which the required rocket load

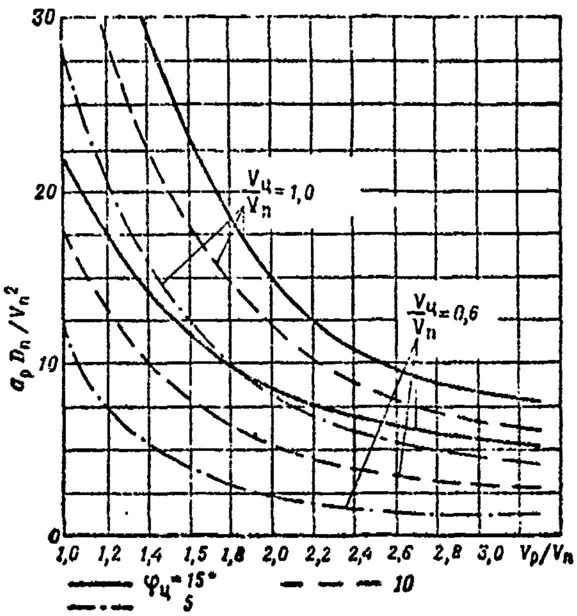


Fig. 2.6.

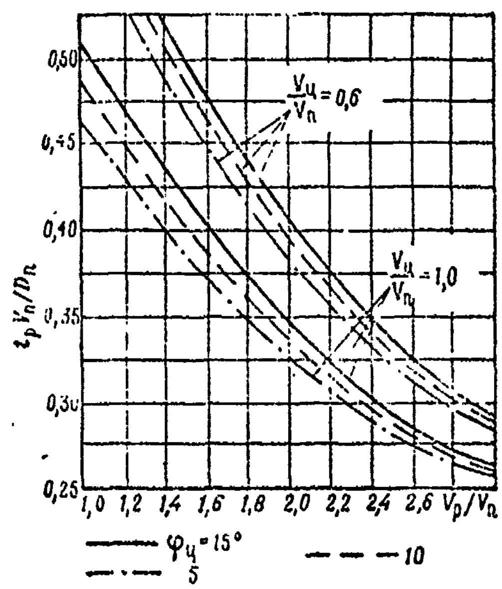


Fig. 2.7.

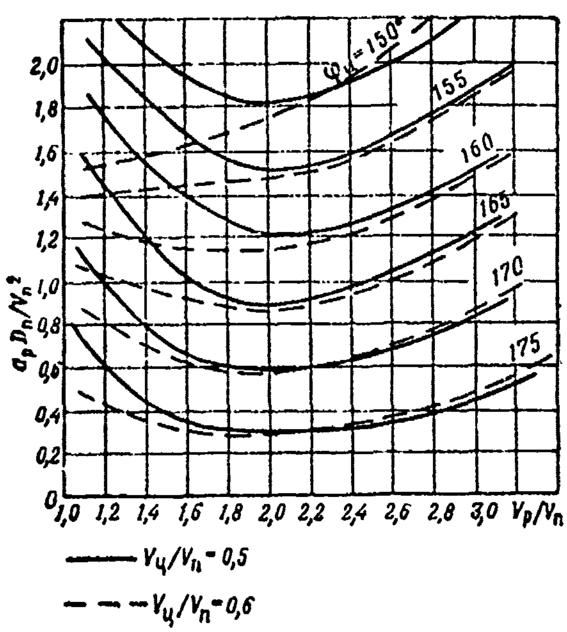


Fig. 2.8.

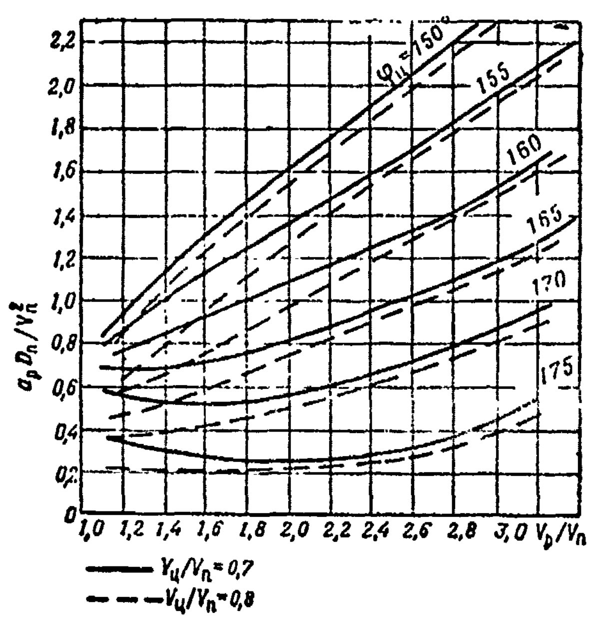


Fig. 2.9.

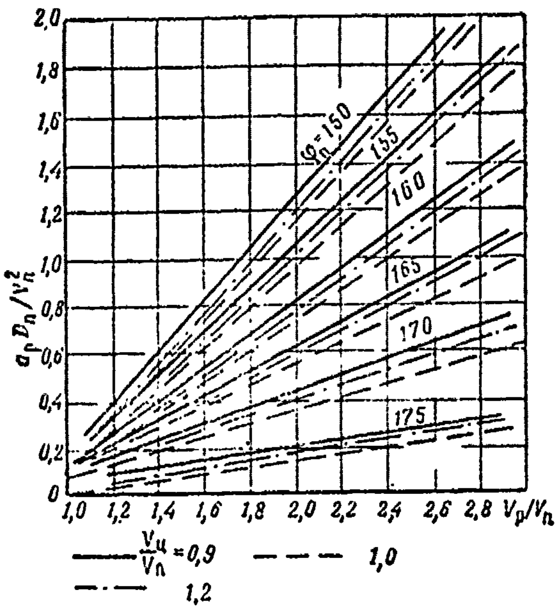


Fig. 2.10.

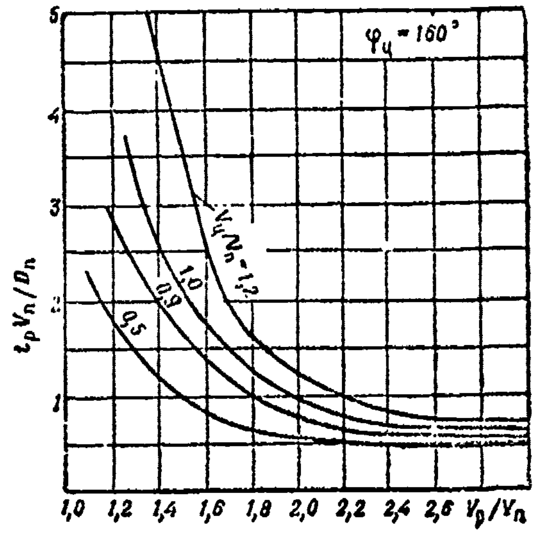


Fig. 2.11.

factor is minimum. Under these combat conditions, how does the relative rocket flight time τ to impact with the target change if the rocket is fired when the target relative bearing $\varphi_ц = 160°$?

Solution. Using the formulas obtained in Problem 2.3, let us calculate the functions $A = f(\varphi_ц, V_p/V_п, V_ц/V_п)$. The results of the calculations are given in Figs. 2.8-2.10. An interesting fact is that at low target speeds $V_ц = (0.5\text{-}0.6)V_п$ there is, throughout the entire range of target relative bearings from 150 to 180°, an optimum value for the rocket speed at which the required rocket load factor is minimum. Beginning at $V_ц = 0.7V_п$ and higher, the required load factor is minimum with minimum rocket speed.

Figure 2.11 shows the results of calculations of relative time τ as a function of $V_p/V_п$ and $V_ц/V_п$ with $\varphi_ц = 160°$.

PROBLEM 2.5. For the target-attack conditions described in Problem 2.3 we are given the absolute average speed of the rocket V_p = 900 m/s, interceptor speed $V_п$ = 400 m/s, target speed $V_ц$ = 400 m/s, target relative bearing at the moment of rocket firing $\varphi_ц = 120°$, and the available rocket load factor $n_p = 5$. We are required to do the following:

1. Determine the distance from which the rocket can be fired.

2. Verify whether or not the rocket can be fired at distance $D_п = 15$ km with target relative bearing $\varphi_ц = 120°$, if the available time of controlled rocket flight $t_p = 20$ s, $V_p = 800$ m/s, $V_п = 300$ m/s, and $V_ц = 300$ m/s. If firing is impossible, determine the target relative bearing or firing distance for $\varphi_ц = 120°$ for which the available rocket flight time is equal to the time from firing to rocket impact with the target.

3. Determine the minimum distance from which the rocket can be fired, if the target relative bearing at the moment of firing $\varphi_ц = 150°$, $V_ц = 250$ m/s, $V_п = 250$ m/s, $V_p = 500$ m/s, and the available rocket load factor $n_p = 10$. How does the minimum rocket-firing distance change if the interceptor speed increases to 500 m/s?

Solution. 1. In Fig. 2.4, for $V_p/V_п = 2.25$ and $V_ц/V_п = 1$ with $\varphi_ц = 120°$ let us determine the relative acceleration $A = 3.4$, and then from Eq. (19) of Problem 2.3 we find the firing range:

$$D_п = \frac{AV_п^2}{a_p} = 11{,}3 \text{ km}.$$

2. With $V_ц/V_п = 1$, $V_p/V_п = 2.66$, and $\varphi_ц = 120°$, according to Fig. 2.5 $\tau_p = 0.49$. The absolute value of the required rocket-flight time to target impact from Eq. (12) of Problem 2.3 is

$$t_p = \frac{\tau D_п}{V_п} = 24{,}5 \text{ s}.$$

Since this required time is more than is available, firing at a distance of 15 km with $\varphi_ц = 120°$ is impossible. The required time can be decreased in this case, if firing occurs at lower target relative bearings. We can easily see that when $\varphi_ц = 90°$ the required time $t_p = 18.5$ s, i.e., shorter than the available time. Consequently, attack must take place from the forward hemisphere of the target, at small target relative bearings, or the rocket must be fired from a shorter range if the target relative bearing cannot be decreased. With $\varphi_ц$ = 120° the permitted firing range

3. According to Fig. 2.4, with $\varphi_ц = 150°$, $V_ц/V_п = 1$, and $V_р/V_п = 2$, we have $a_рD_п/V_п^2 = 1.1$ whence $D_п$ = 1.75 km, while with a doubling of $V_п$, $D_п$ = 6.1 km.

PROBLEM 2.6. Construct the zones of possible firing for the case of rocket homing described in Problem 2.3, if the available rocket load factor $n_р = 7$, the available time of controlled flight $t_р$ = 20 s, target speed $V_ц$ = 1500 m/h, interceptor speed $V_п$ = 1500 km/h, and the average natural speed of the rocket $V_{р.ср}$ = 500 m/s.

Solution. Let us determine, for $V_ц/V_п = 1$ and $V_р/V_п =$ = (500 + 417)/417 = 2.2 for various target relative bearings $\varphi_ц$, the relative values $\tau = t_рV_п/D_г$ and $A = a_рD_п/V_п^2$ (Figs. 2.4-2.7) and then, from Eqs. (12) and (19) of Problem 2.3, the corresponding maximum and minimum firing ranges. The zones of possible firings, constructed with respect to $D_{п.макс}$ and $D_{п.мин}$, are given in Fig. 2.12.

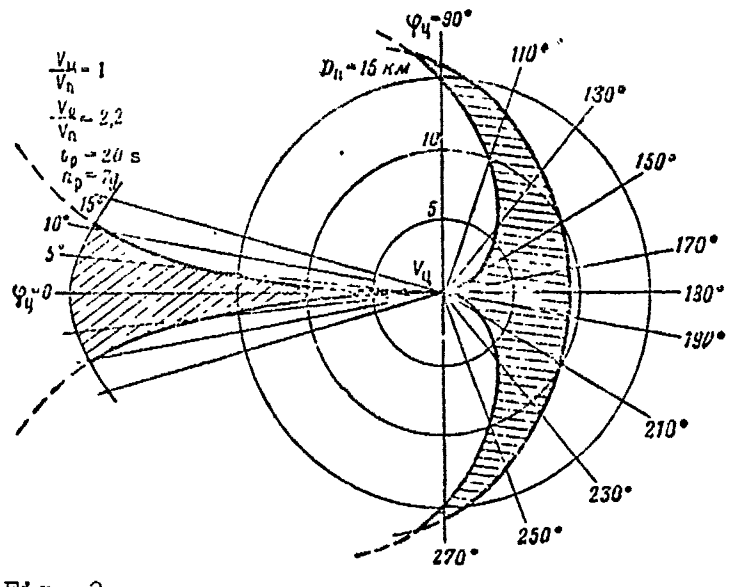


Fig. 2.1

PROBLEM 2.7. The interceptor attacks a nonmaneuvering target, firing its rocket with constant target relative bearing $\varphi_ц = 150°$. The rocket is homed to the target by the line-of-sight method. The given target $V_ц$, interceptor $V_п$, and [illegible] $V_р$ velocities are assumed constant. Let us show how the rang[illegible] [illegible]ing distances $D_{п.мин}-D_{п.макс}$ changes if, with the other characteristics unchanged, we increase: 1) $V_п$ from 200 to 600 m/s, and 2) $V_р/V_п$ from 1.3 to 2.4.

Solution. Using graphs to determine $A = a_р D_п / V_п^2$ and $\tau = t_р V_п / D_п$ for given values of $V_р/V_п$, $V_ц/V_п$, and $_ц$ (Figs. 2.4 and 2.5), let us calculate, for each fixed value of $V_р/V_п$ and $V_ц/V_п$ from the values of A and τ the corresponding minimum and maximum firing distances:

$$D_{п.мин} = A \frac{V_п^2}{a_р}; \quad D_{п.макс} = \frac{t_р V_п}{\tau}.$$

Varying $V_р/V_п$ and $V_п$ within these ranges, let us construct a generalized graph of the change in range of the firing distances with given values of $\varphi_ц$ and $V_ц/V_п$. The graph for $\varphi_ц$ = 150° and $V_ц/V_п$ = 1 is shown in Fig. 2.13. This graph shows that, with the other characteristics unchanged, an increase in $V_п$ reduces the zone of possible firings with respect to distance, while an increase in $V_р/V_п$ expands this zone. With an increase in $V_п$ from 200 to 600 m/s the range of firing distances decreases from 600 m to 0 when $V_р/V_п$ = 1.3. With an increase of $V_р/V_п$ from 1.3 to 2.4 the range of firing distances increases from 600 to 2800 m with $V_п$ = 200 m/s.

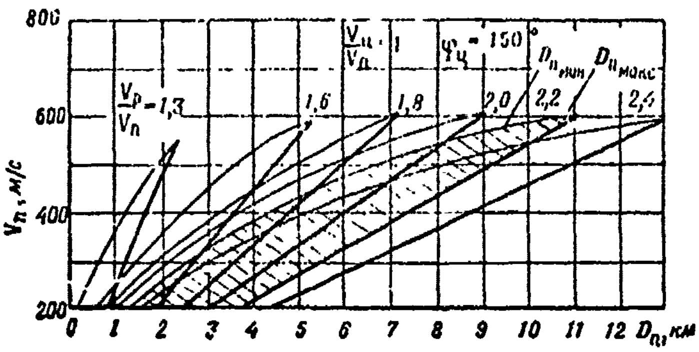


Fig. 2.13.

PROBLEM 2.8. A rocket is homed to a nonmaneuvering target by the line-of-sight method (see Problem 2.3). Show how the required rocket load force at the point of target contact depends on the target relative bearing $\varphi_ц$ and the ratio of rocket to target speed $V_р/V_ц$. Determine the limits within which the minimum rocket firing distance, determined by the available rocket load force, changes. Assume that the rocket and target speeds are constant. Given: $V_р$ = 1000 m/s, $V_ц$ = 500 m/s, $a_р$ = 49 m/s^2, $\varphi_ц$ = 60°.

Solution. To carry out the line-of-sight method the required normal (side) acceleration of the rocket (the acceleration directed perpendicular to the kinematic trajectory) at the point of target impact is defined by the formula

$$a_p = \frac{V_п^2}{D_п}\left(2\frac{V_p}{V_ц}\sin\varphi_ц + \frac{d_п}{D_п}\frac{\sin 2\varphi_ц}{\cos\varphi_p}\right). \quad (1)$$

The angle φ_p is defined by the relationship

$$\sin\varphi_p = \frac{d_п V_ц}{D_п V_p}\sin\varphi_ц, \quad (2)$$

since

$$\frac{\sin\varphi_p}{\sin\varphi_ц} = \frac{V_ц}{V_p} = \frac{d_п}{D_п}, \quad (3)$$

where $d_п$ is the distance covered by the rocket from the time of firing, $D_п$ is the firing distance.

When homing occurs under ideal conditions, the acceleration required for accomplishing the kinematic trajectory is exactly equal to (1). Actually, the required acceleration is greater, since movement of the rocket as fluctuating perturbations act on it is of a variational nature, and in attempting to comply with the given law the actual accelerations should exceed those necessary for precise motion along the kinematic trajectory, not considering these variational processes.

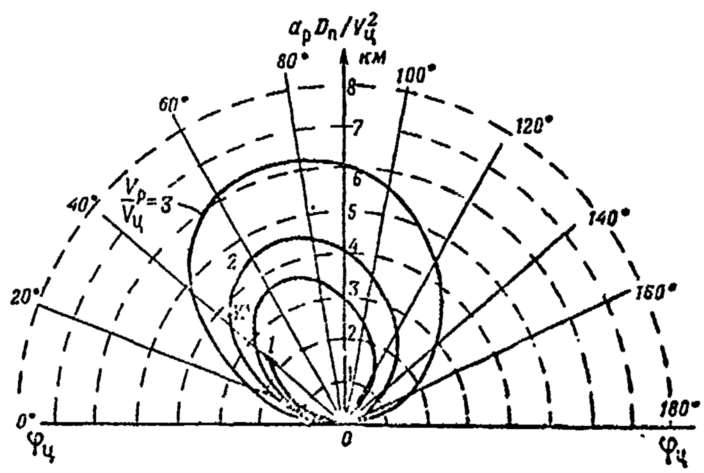


Fig. 2.14.

To determine $D_{п.мин}$ we use Fig. 2.14, which gives the relative (dimensionless) required rocket acceleration $a_p D_п/V_ц^2$ at the point of impact with the target as a function of the target relative bearing $\varphi_ц$ for various ratios $V_р/V_ц$. Knowing the target speed $V_ц$ and the available normal acceleration $a_р$, we determine the value of $a_р D_п / V_ц^2$ from the graph in Fig. 2.14 for the given combat conditions, and then calculate the minimum rocket firing distance $D_{п.мин}$. For the conditions of our problem we have

$$a_р = 49\ m/s^2, \quad \varphi_ц = 60°, \quad \frac{V_р}{V_ц} = 2.$$

From the graph in Fig. 2.14

$$\frac{a_р D_п}{V_ц^2} = 4{,}4.$$

Let us multiply this by

$$\frac{V_ц^2}{a_р} = \frac{500^2}{49}.$$

Then the minimum rocket firing distance $D_{п.мин}$ = 22.4 km.

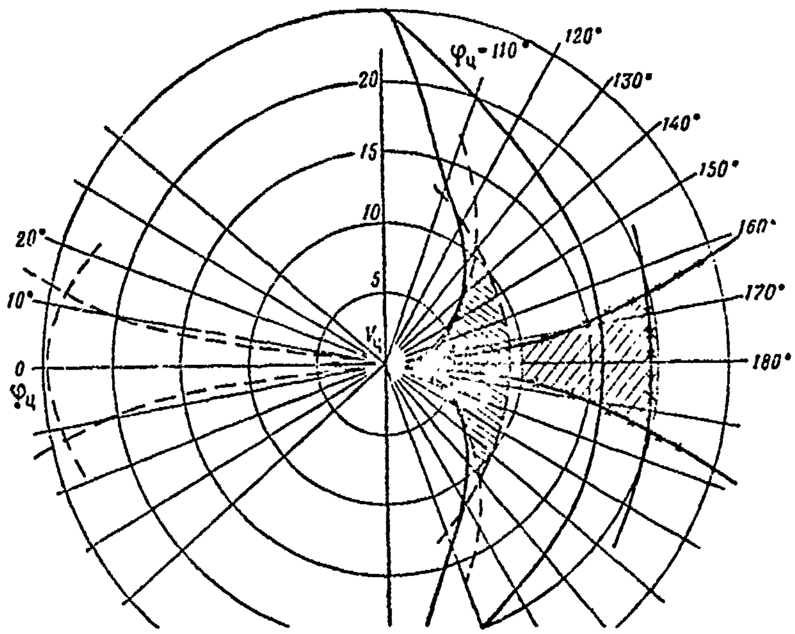
Calculations show that with a velocity ratio $V_р/V_ц \leq 1.2$, attack is limited to a target relative bearing value of the order of $\varphi_ц \approx 60°$. This is explained by the fact that when $V_р/V_ц \leq 1.2$ according to Eq. (2) there are large angles between the rocket's velocity vector and the line of sighting, particularly on the phase of the trajectory before impact with the target, when $d_п/D_п \approx 1$. Thus the graph in Fig. 2.14 in first approximation gives the minimum rocket firing distances for a broad range of combat conditions. Using these data, and having determined the maximum firing distance from the available controlled-flight time, we construct the zones of possible firings.

PROBLEM 2.9. A rocket is homed on a nonmaneuvering target by the line-of-sight method. We are given the target speed $V_ц$ = 500 m/s, the interceptor speed $V_п$ = 500 m/s, and the natural speed of the rocket $V_{р.ср}$ = 500 m/s, the available rocket load factor $n_р$ = 5, and the available rocket controlled-flight time $t_р$ = 20 s. Determine, for these starting data, the zones of possible firings and show how they change if, with the other characteristics unchanged, the target speed is halved, the rocket speed is doubled, and the interceptor speed is doubled.

Solution. By analogy with Problem 2.7, let us determine the appropriate values $A = a_p D_п / V_п^2$ and $\tau = t_p V_п / D_п$ using the graphs in Figs. 2.4-2.11 for given target relative bearings $\varphi_ц$, and then, using Eqs. (12) and (19) of Problem 2.3, calculate the minimum and maximum firing distances (in km):

$$D_{п.мин} = A \frac{V_п^2}{a_р} = 5{,}1A; \quad D_{п.макс} = \frac{t_р V_п}{D_п} = \frac{10}{\tau}.$$

The zones of possible firings are shown in Fig. 2.15.



$\frac{V_ц}{V_п} = 0{,}5; \frac{V_р}{V_п} = 2$ — $\frac{V_ц}{V_п} = 0{,}5; \frac{V_р}{V_п} = 1{,}5$ — $\frac{V_ц}{V_п} = 1, \frac{V_р}{V_п} = 2$

Fig. 2.15.

PROBLEM 2.10. Determine the zone of possible rocket firings for the case when, from the moment of rocket firing to impact of the rocket with the target, the interceptor flies along a pursuit curve, while the rocket is homed to the target using the equisignal zone created by the airborne radar in the automatic target tracking mode.

Solution. The mutual positions of the interceptor, target, and rocket after firing are illustrated in Fig. 2.16. At any current moment of time the rocket is on the line "interceptor-target."

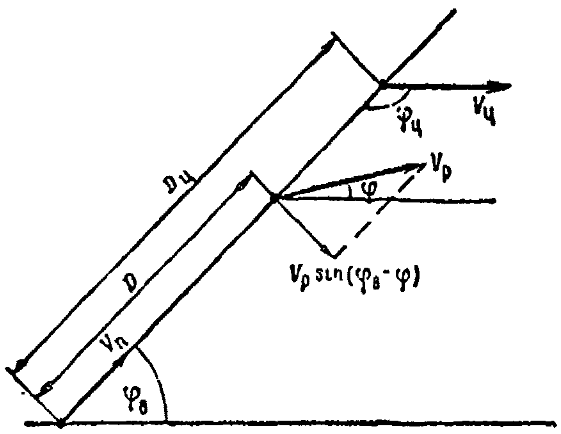


Fig. 2.16.

According to Fig. 2.16 we can write the following kinematic equations for the angular velocity of the line of sighting:

$$\omega_B = \dot\varphi_B = -\frac{V_p \sin(\varphi_B - \varphi)}{D}; \qquad (1)$$

$$\omega_B = -\frac{V_ц \sin\varphi_B}{D_ц} \qquad (2)$$

(the "-" indicates that the angle φ_B decreases).

The rate of departure of the rocket from the interceptor

$$\dot D = V_p \cos(\varphi_B - \varphi) - V_п \qquad (3)$$

Let us differentiate Eq. (1) with respect to t. Then

$$V_p\dot\varphi = \frac{\ddot\varphi_B D + \dot\varphi_B \dot D + V_p \cos(\varphi_B - \varphi)\dot\varphi_B + \dot V_p \sin(\varphi_B - \varphi)}{\cos(\varphi_B - \varphi)}. \qquad (4)$$

Considering that, for all practical purposes, the difference $\varphi_B - \varphi$ is slight and, consequently,

$$\cos(\varphi_B - \varphi) \approx 1; \quad \sin(\varphi_B - \varphi) \approx \varphi_B - \varphi; \quad \dot D \simeq V_p - V_п, \qquad (5)$$

we get

$$V_p\dot\varphi = \ddot\varphi D + 2\dot\varphi_B \dot D + \dot\varphi_B V_п + \dot V_p (\varphi_B - \varphi). \qquad (6)$$

Rocket acceleration perpendicular to D is, on the one hand, equal to $-\ddot\varphi_B D$; on the other hand it is equal to the change in tangential velocity

$$V_p \sin(\varphi_B - \varphi)$$

with time, i.e.,

$$\frac{dV_p}{dt}\sin(\varphi_в - \varphi).$$

Thus, since

$$\dot{V}_p(\varphi_в - \varphi) \simeq \ddot{\varphi}_в D, \tag{7}$$

then

$$V_p\dot{\varphi} = \dot{\varphi}_в(2V_p - V_ц). \tag{8}$$

The kinematic load factor of the rocket is defined by the formula

$$n_p = -\frac{V_p\dot{\varphi}}{g}. \tag{9}$$

Substituting (8) into (9) we get

$$n_p = -\frac{\dot{\varphi}_в(2V_p - V_ц)}{g}. \tag{10}$$

Solving (10) simultaneously with (2), we get a formula for determining the polar coordinates $\varphi_в$, $D_ц$ of the interceptor:

$$\sin\varphi_в = \frac{g n_p D_ц}{V_ц(2V_p - V_ц)}. \tag{11}$$

With target maneuvering with angular velocity $\omega_ц$ Relationship (11) assumes the following form:

$$\sin\varphi_в = \frac{(g n_p - \omega_ц V_ц)}{V_ц(2V_p - V_ц)} D_ц. \tag{12}$$

Then the zones of possible firings are determined in the following sequence. We are given several values of rocket flight time t_p and, from the characteristics of the rocket, we determine the appropriate values of rocket speed V_p, available load factor n_p, and distance $D_ц$. Then from (11) we calculate the corresponding angles $\varphi_в$ which characterize the angular position of the interceptor at the moment the rocket meets the target: points 1, 2, 3, ... in Fig. 2.17. From points 1, 2, 3, ... we construct pursuit curves 1-1', 2-2', 3-3', ... for each time t_p. Obviously, points 1', 2', 3', ... define the position of the interceptor at the moment of rocket firing. Connecting these points and considering the required

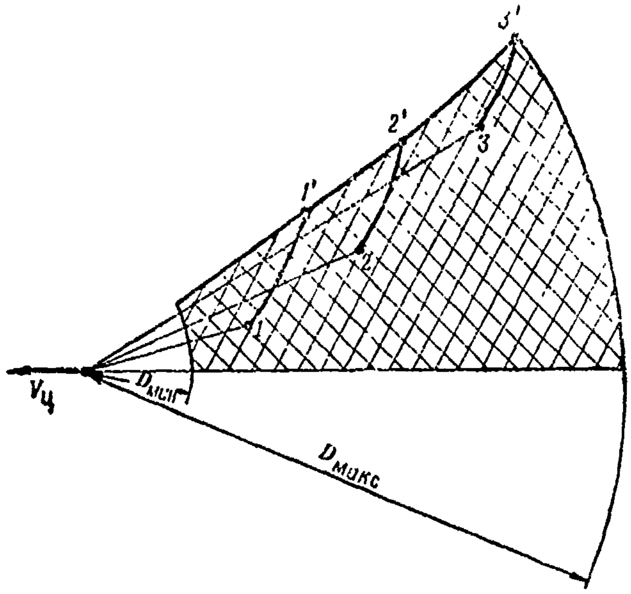


Fig. 2.17.

limitations (e.g., with respect to minimum and maximum distances), we obtain the zone of possible rocket firings.

PROBLEM 2.11. The interceptor and target are at the same altitude and close with constant speeds. After the airborne radar locks on the target the interceptor fires a rocket which is guided to the target by the pursuit method. The mutual positions of the interceptor and target at the moment of firing are described by the firing distance $D_п$ = 0.9 km and the target relative bearing $\varphi_ц$ = 155°. The speed of the rocket as it flies toward the target is assumed to be constant: V_p = 1500 km/h. Determine if the rocket can score a direct hit on the target if the target speed $V_ц$ = 1000 km/h, the available rocket load factor n_p = 10, and the available rocket controlled-flight time t_p = 20 s. Can the rocket hit the target if, under these conditions of rocket firing, the target speed is half as much?

Solution. In the pursuit method, as we know, the rocket's velocity vector is constantly directed toward the target, and the rocket relative bearing is zero. The general formulas for the pursuit method were obtained in Problems 2.1 and 2.2.

To determine the possibility of the rocket's hitting the target we must calculate the rocket load factors, required for this method to be carried out, as it closes with the target, and compare them with the available factor n_p = 10. Then we must calculate the target pursuit time along the pursuit curve, and also compare this with the available rocket controlled-flight time t_p = 20 s. According to Eq. (3) of Problem 2.1, the angular velocity of the rocket required to carry out the pursuit method

$$\omega_p = \frac{V_ц \sin \varphi_ц}{D}. \tag{1}$$

Knowing ω_p, we can easily determine the corresponding rocket load factor n_p. Since the normal rocket acceleration a_p is defined, in terms of angular and linear velocity of the rocket, by the familiar formula

$$a_p = \omega_p V_p, \tag{2}$$

while the centripetal acceleration is connected with load factor n_p by the relationship

$$a_p = g\sqrt{n_p^2 - 1}, \tag{3}$$

then

$$\omega_p = \frac{g\sqrt{n_p^2 - 1}}{V_p}, \tag{4}$$

from which the rocket load factor required for the pursuit method

$$n_p = \sqrt{\frac{\omega_p^2 V_p^2}{g^2} + 1}. \tag{5}$$

For the conditions of our problem, in the first case when $V_ц$ = 1000 km/h = 278 m/s and V_p = 1500 km/h = 416 m/s, for the firing moment we have ω_p = 0.131 rad/s and n_p = 5.5.

The rocket flight time along the pursuit curve, according to Eq. (11) of Problem 2.2, is

$$t = \frac{D_ц\left(\frac{V_p}{V_ц} - \cos\varphi_{ц0}\right)}{V_ц\left(\frac{V_p^2}{V_ц^2} - 1\right)} = \frac{900\,(1{,}5 - \cos 155°)}{278\,(1{,}5^2 - 1)} = 6{,}4\,\text{s}.$$

In this time the target covers a path equal to 6.4·278 = 1780 m, while the rocket travels 6.4·416 = 2680 m. Constructing, for the starting data of the problem, the mutual positions of the interceptor

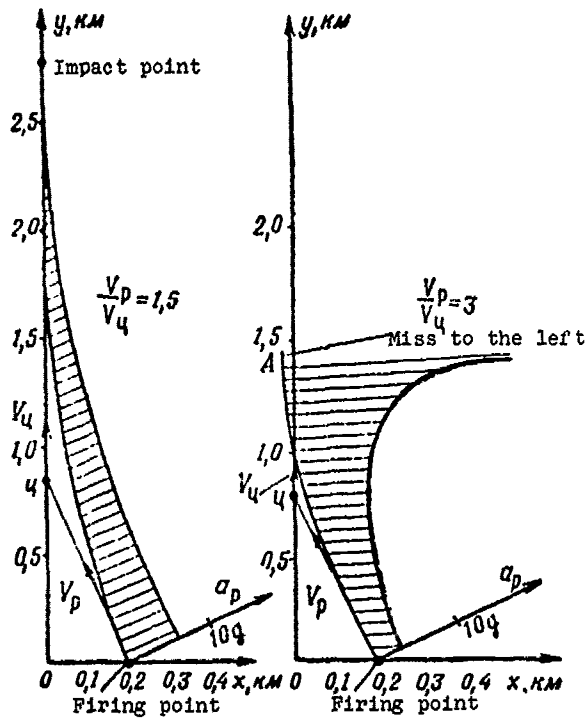


Fig. 2.18.

and target at the moment of firing, and then constructing, as standards, the rocket load factors along its flight trajectory (required for carrying out the pursuit method), we obtain a graphic solution to the problem (for clarity the axes in Fig. 2.18 have different scales). The construction in Fig. 2.18 shows that in the examined case the rocket meets the target ~3 km from the firing point, while the rocket load factor, required to carry out the pursuit method, is maximum at the start of the trajectory but less than that available $n_p = 10$, and then, as pursuit begins at target relative bearing $\varphi_ц = 180°$ it smoothly decreases to zero at the moment the rocket meets the target.

In the second case, when the target speed is half that given above, $V_ц = 500$ km/h = 139 m/s, $b = V_p/V_ц = 3$, $\omega_p = 0.0655$ rad/s, $n_p = 2.55$, and $t_{пор} = 3.2$ s.

In this time the target covers a distance of $3.2 \cdot 139 = 445$ m, while the rocket travels $3.2 \cdot 416 = 1330$ m.

Further calculations show that in this case the required rocket load factor at the moment of firing is minimum, but with time it constantly increases and at point A reaches an infinitely high value. For example, after 2.5 s the distance "rocket-target" $D \approx 100$ m, $\varphi_ц \approx 170°$, $\omega_p = 0.24$ rad/s, and $n_p = 10.3$.

Since the available rocket load factor is limited to the value

n_p = 10, in this case the rocket passes to the left of the target.

The solution to this problem is interesting in that it shows the required boundary conditions for carrying out the pursuit method. It seems that this method is accomplished only when the following condition, necessary for realizing the method, is satisfied: the ratio of absolute rocket velocity to target speed should be within limits from 1 to 2 (it must be greater than 1 in order to overtake the target, and less than 2 so that the load factor not exceed that available).

PROBLEM 2.12. The rocket is guided to a target in the same plane, using the constant-bearing approach method. Given: the target relative bearing $\varphi_ц$, the ratio of the average absolute velocity of the rocket to the target speed $V_p/V_ц$. Determine the rocket relative bearing φ_p for which the homing method is carried out. What velocity relationships should we have in order that the homing method be carried out when the target performs heading maneuvers?

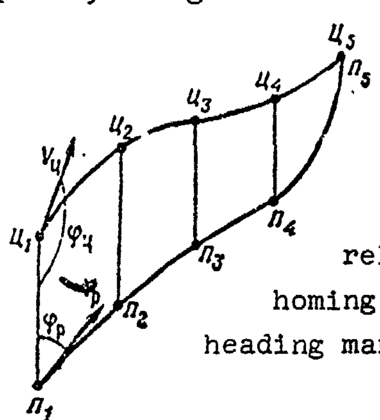


Fig. 2.19.

Solution. For the constant-bearing approach method (Fig. 2.19) it is necessary that the line of sighting remain parallel, i.e., the angular velocity of rotation of the line of sighting should be zero. Consequently, the projections of the vectors of velocities $V_ц$ and V_p onto a perpendicular to the line of sighting should equal one another. For the case of rocket homing in a plane the following condition should be fulfilled:

$$\varphi_p = \arcsin\left(\frac{\sin\varphi_ц}{\frac{V_p}{V_ц}}\right).$$

Obviously, for a nonmaneuvering target and constant target and rocket velocities the rocket relative bearing φ_p = const. As the target changes heading and speed, the rocket can hit the target only when the projections of rocket velocity onto the line of sighting and the perpendicular to it are greater than the corresponding projections of target speed:

$$V_p \sin \varphi_p \geqslant V_ц \sin \varphi_ц \text{ and } V_p \cos \varphi_p > V_ц |\cos \varphi_ц|.$$

PROBLEM 2.13. The rocket is guided to a target by the constant-bearing approach method. Attack on the target is carried out from the forward hemisphere, at target relative bearing $\varphi_ц = 150°$. The mean absolute velocity of the rocket V_p is 1.5-times greater than that of the target $V_ц$. The target maneuvers with lateral acceleration $a_ц = 40$ m/s^2. What should the available rocket load factor be in order to counteract this maneuver? At what aspect angle is it best to attack the target from the standpoint of countering this maneuver? How will the required rocket load factor change if: 1) the velocity ratio $V_p/V_ц$ changes from 1 to ∞, and 2) the target relative bearing changes from 0 to 180°?

Solution. Since in the constant-bearing approach method and with target maneuvering the line of sighting is displaced in parallel, at any moment of time the projections of velocity vectors V_p and $V_ц$ onto the perpendicular to the line of sighting are equal to one another:

$$V_p \sin \varphi_p = V_ц \sin \varphi_ц. \tag{1}$$

When condition (1) is satisfied, for the angular velocity of the line of sighting to be equal to zero during target maneuvering we should have yet another equality:

$$a_p \cos \varphi_p = a_ц \cos \varphi_ц, \tag{2}$$

where $a_ц$ is the target acceleration and a_p is rocket acceleration.

From Eq. (2) we get the calculation formula for the rocket acceleration required to carry out the constant-bearing approach method, when target maneuvering is described by acceleration $a_ц$:

$$a_p = a_ц \frac{\cos \varphi_ц}{\cos \varphi_p}. \tag{3}$$

Let us express the value of cos φ_p in terms of $\varphi_ц$ and the velocity ratio $V_p/V_ц$. According to Eq. (1) we have

$$\varphi_p = \arcsin\left(\frac{\sin\varphi_ц}{\frac{V_p}{V_ц}}\right) \quad (4)$$

and, bearing in mind the properties of the inverse trigonometric functions, we get

$$\cos\varphi_p = \sqrt{1 - \frac{\sin^2\varphi_ц}{\left(\frac{V_p}{V_ц}\right)^2}}. \quad (5)$$

Substituting the given numerical values of $\varphi_ц$ and $V_p/V_ц$ we get

$$\cos\varphi_p = 0{,}943.$$

The required acceleration of the rocket, according to (3),

$$a_p = 36.7 \text{ m/s}^2,$$

while the corresponding rocket load factor

$$n_p = \frac{a_p}{g} = 1{,}15.$$

Naturally, the rocket load factor available to counter the maneuver, i.e., to carry out the method of constant-bearing approach with consideration of target maneuvering, should be at least 4.15.

In order to select the most suitable aspect angle for attack on a maneuvering target, i.e., determine the aspect angle for which the required rocket load factor is minimum, let us construct the dependence of the acceleration ratio $a_p/a_ц$ on the target relative bearing $\varphi_ц$ for the velocity ratio $V_p/V_ц = 1.5$. Substituting (5) into (2) we get

$$\frac{a_p}{a_ц} = \frac{\cos\varphi_ц}{\sqrt{1 - \frac{\sin^2\varphi_ц}{\left(\frac{V_p}{V_ц}\right)^2}}}. \quad (6)$$

A graph of this function is given in Fig. 2.20. Obviously, it is more suitable to attack a maneuvering target with target relative bearings close to 90°; the load factor required of the rocket to counter this maneuver and satisfy the homing requirement is minimum.

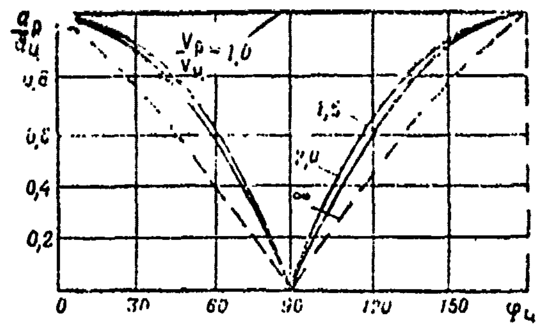


Fig. 2.20.

In Fig. 2.20 we see how the required rocket load factor changes with a change in the velocity ratio $V_p/V_ц$ from 1 to ∞.

PROBLEM 2.14. The rocket is fired from the forward hemisphere of the target. The homing method used is the constant-bearing approach method. The angle of impact of the rocket with the target q = 150° (Fig. 2.21). Determine the minimum permissible firing distance $D_{п.мин}$ if the permissible error in guiding the rocket to the lead point at the moment of firing $\Delta\Gamma$ = 10°, the target speed $V_ц$ = 500 m/s, the mean absolute rocket speed V_p = 1000 m/s, and the maximum available rocket load factor n_p = 20.

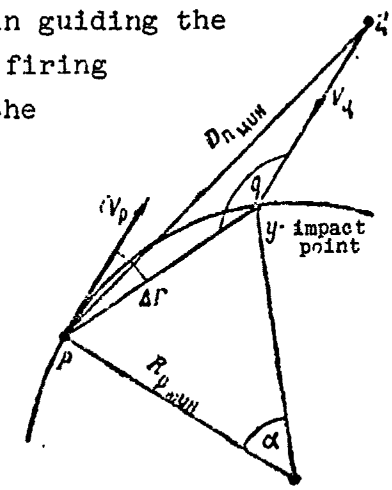


Fig. 2.21.

Solution. The minimum firing distance according to the law of cosines is

$$D^2_{п.мин} = (PУ)^2 + (УЦ)^2 - 2(PУ)(УЦ)\cos q. \qquad (1)$$

Let us determine the distance PУ covered by the rocket from the time of firing to impact with the target. Since the angle ΔΓ is small, the length of arc PУ differs only slightly from a straight line, and then, from the formula for determining the chord length we ha·

$$PУ \approx 2R_{р.мин} \sin\frac{\alpha}{2}.$$

Angle α/2 is equal to angle ΔΓ, as angles with mutually perpendicular sides. Replacing the sine of small angle α/2 with the same angle expressed in radians, we get

$$PУ \approx 2R_{р.мин}\,\Delta\Gamma. \qquad (2)$$

Now let us determine the distance УЦ covered by the target from the moment of firing until it meets the rocket at the lead point. Since for the method of constant-bearing approach

$$\frac{УЦ}{V_ц} = \frac{РУ}{V_р},$$

then

$$УЦ = V_ц \frac{РУ}{V_р},$$

while if we substitute the value for РУ according to (2) we get

$$УЦ = 2\frac{V_ц}{V_р} R_{р.мин} \Delta Г. \qquad (3)$$

Further, let us determine the minimum radius of banked turn of the rocket with respect to the given maximum available load factor:

$$R_{р.мин} = \frac{V_р^2}{g n_р} \approx 5{,}1 \text{ km}.$$

Substituting (2) and (3) into (1) we get a calculation formula for the unknown minimum permissible rocket firing distance

$$D_{п.мин} \approx 2R_{р.мин} \Delta Г \sqrt{1 + \left(\frac{V_ц}{V_р}\right)^2 - 2\left(\frac{V_ц}{V_р}\right)\cos q}. \qquad (4)$$

For the data in our problem we have $D_{п.мин}$ = 2.63 km.

PROBLEM 2.15. Rocket flight by the constant-bearing approach method with constant target and rocket velocities is characterized by the absence of lateral rocket load factor ($n_р$ = 0). With a change in rocket speed on the trajectory (with the other parameters $V_ц$, $\varphi_ц$, and $\varphi_р$ constant) there arises the tangential rocket load factor necessary for restoring the constant-bearing approach method. We express the magnitude of this load factor as a function of the target and rocket speeds and relative bearings. Calculate this load factor if $V_ц$ = 375 m/s, $V_р$ = 750 m/s, $\varphi_ц$ = 30°, tangential acceleration $a_{рт}$ = 100 m/s^2. How does load factor $n_р$ change if the target relative bearing changes from 0 to 90°?

Solution. The value of the lateral load factor $n_р$ is found by solving the following system of equations which describe the kinematic relationships among the parameters of the rocket and the target when rocket motion deviates from the constant-bearing approach method:

$$\left| \begin{array}{l} V_p \sin \varphi_p = V_u \sin \varphi_u, \quad (1) \\ V_p \frac{d\beta}{dt} = -g\sqrt{n_p^2 - 1} \simeq -gn_p, \quad (2) \\ \beta = \varphi_p + \varphi_u. \quad (3) \end{array} \right.$$

Let us differentiate Eq. (1) with respect to time:

$$\frac{dV_p}{dt} \sin \varphi_p + V_p \cos \varphi_p \frac{d\varphi_p}{dt} = V_u \cos \varphi_u \frac{d\varphi_u}{dt} + \frac{dV_u}{dt} \sin \varphi_u. \quad (4)$$

For the constant-bearing approach method we have

$$\frac{d\varphi_u}{dt} = 0.$$

Consequently, Eq. (4) assumes the following form:

$$\frac{dV_p}{dt} \sin \varphi_p + V_p \cos \varphi_p \frac{d\varphi_p}{dt} = 0. \quad (5)$$

Now let us differentiate (3) with respect to time:

$$\frac{d\beta}{dt} = \frac{d\varphi_p}{dt}. \quad (6)$$

From Eq. (2) we have

$$n_p = -\frac{V_p}{g} \frac{d\beta}{dt}, \quad (7)$$

while with consideration of (6)

$$n_p = -\frac{V_p}{g} \frac{d\varphi_p}{dt}. \quad (8)$$

Let us substitute into (8) the expression $d\varphi_p/dt$ as per (5). Then

$$n_p = \frac{1}{g} \frac{dV_p}{dt} \frac{\sin \varphi_p}{\cos \varphi_p}. \quad (9)$$

Let us designate acceleration of the rocket as

$$a_p = \frac{dV_p}{dt},$$

and the velocity ratio V_p/V_u by b. Then fr. . Eq. (1) we have

$$\sin \varphi_p = \frac{1}{b} \sin \varphi_u. \quad (10)$$

Since

$$\cos\varphi_p = \sqrt{1-\sin^2\varphi_p} = \sqrt{1-\frac{1}{b^2}\sin^2\varphi_ц}, \quad (11)$$

substituting (10) and (11) into (9) we get a calculation formula for the unknown tangential rocket load factor:

$$n_p = \frac{a_p}{g}\frac{\sin\varphi_ц}{\sqrt{b^2-\sin^2\varphi_ц}}. \quad (12)$$

For the numerical data in our problem we have b = 2, a_p = 100 m/s^2, $\varphi_ц$ = 30°, and n_p = 2.64.

Now let $\varphi_ц$ = 60°; then n_p = 4.9. When $\varphi_ц$ = 90°, n_p = 5.9.

Thus, with an increase in target relative bearing $\varphi_ц$ from 30 to 90° the lateral rocket load factor increases by a factor of 5.9/2.64 = 2.25.

Let us note that with rocket velocity characteristics $V_p = f(t)$ we can easily find the law of change of rocket acceleration $a_p = dV_p/dt$ and then, using Eq. (12), the law of change of the lateral rocket load factor as a function of flight time.

PROBLEM 2.16. A rocket is guided to a target using the constant-bearing approach method. During the steady-state homing method the target begins to maneuver by banking, with a load factor n_y = 4. In 10 seconds the target comes out of its bank and continues straight-line flight. Determine the required rocket load factor to counter this maneuver. Assumptions: $V_ц$ = const, V_p = const. Given the following conditions: $V_ц$ = 400 m/s, V_p = 800 m/s, $\varphi_{ц0}$ = 30°.

Solution. Let us use Eqs. (1), (2), and (3) from Problem 2.15. Let us differentiate (1) with respect to time t:

$$\frac{dV_p}{dt}\sin\varphi_p + V_p\cos\varphi_p\frac{d\varphi_p}{dt} = \frac{dV_ц}{dt}\sin\varphi_ц + V_ц\cos\varphi_ц\frac{d\varphi_ц}{dt}, \quad (1)$$

while considering V_p = const and $V_ц$ = const we get

$$V_p\cos\varphi_p\frac{d\varphi_p}{dt} = V_ц\cos\varphi_ц\frac{d\varphi_ц}{dt}. \quad (2)$$

The rate of change of the target relative bearing is equal to the angular velocity of the target:

$$\omega_{\text{ц}}=\frac{d\varphi_{\text{ц}}}{dt}. \qquad (3)$$

Considering (3) we get

$$\frac{d\varphi_{\text{р}}}{dt}=\frac{V_{\text{ц}}}{V_{\text{р}}}\frac{\cos\varphi_{\text{ц}}}{\cos\varphi_{\text{р}}}\omega_{\text{ц}}. \qquad (4)$$

Further, let us differentiate (3) of Problem 2.15 with respect to time t:

$$\frac{d\beta}{dt}=\frac{d\varphi_{\text{р}}}{dt}+\frac{d\varphi_{\text{ц}}}{dt}. \qquad (5)$$

From (2) of Problem 2.15 we get

$$n_{\text{р}}=-\frac{V_{\text{р}}}{g}\frac{d\beta}{dt}, \qquad (6)$$

while with substitution of (5),

$$n_{\text{р}}=-\frac{V_{\text{р}}}{g}\left(\frac{d\varphi_{\text{р}}}{dt}+\frac{d\varphi_{\text{ц}}}{dt}\right). \qquad (7)$$

Then, with consideration of (3) we get

$$n_{\text{р}}=-\frac{V_{\text{р}}}{g}\left(\frac{d\varphi_{\text{р}}}{dt}+\omega_{\text{ц}}\right). \qquad (8)$$

Since

$$\cos\varphi_{\text{р}}=\sqrt{1-\sin^2\varphi_{\text{р}}}=\sqrt{1-\frac{V_{\text{ц}}^2}{V_{\text{р}}^2}\sin^2(\varphi_{\text{ц}0}+\omega_{\text{ц}}t)}, \qquad (9)$$

the calculation formula for the unknown rocket load factor takes on the following form:

$$n_{\text{р}}=\omega_{\text{ц}}\frac{V_{\text{р}}}{g}\left[1+\frac{\cos(\varphi_{\text{ц}0}+\omega_{\text{ц}}t)}{\sqrt{b^2-\sin^2(\varphi_{\text{ц}0}+\omega_{\text{ц}}t)}}\right]. \qquad (10)$$

For tne numerical data of our problem we have

$$n_{\text{р}}=\frac{g\sqrt{n_y^2-1}}{V_{\text{ц}}}\frac{V_{\text{р}}}{g}\left[1+\frac{\cos(\varphi_{\text{ц}0}+\omega_{\text{ц}}t)}{\sqrt{b^2-\sin^2(\varphi_{\text{ц}0}+\omega_{\text{ц}}t)}}\right]=$$

$$=m\sqrt{n_y^2-1}\left[1+\frac{\cos(\varphi_{\text{ц}0}+\omega_{\text{ц}}t)}{\sqrt{b^2-\sin^2(\varphi_{\text{ц}0}+\omega_{\text{ц}}t)}}\right]; \qquad (11)$$

$$n_{\text{р}}=2\sqrt{16-1}\left[1+\frac{\cos(30^\circ+0{,}025\cdot 10)}{\sqrt{b^2-\sin^2(30^\circ+0{,}025\cdot 10)}}\right]=10{,}9.$$

PROBLEM 2.17. Determine the amount by which the rocket will miss the target if, in the final phase of controlled flight before encountering the target, the rocket deviates from the kinematic trajectory of the constant-bearing approach method which it is performing. We are given the values of target and rocket speeds and relative bearings: $V_ц = 500$ m/s, $V_p = 1000$ m/s, $\varphi_ц = 30°$, $\varphi_p = 30°$, and the distance "rocket-target" at which the rocket does not satisfy the law of constant-bearing approach (continuing on linear flight) D = 100 m.

Solution. A necessary condition to carry out the constant-bearing approach homing method is that the angular velocity of the line of sighting be equal to zero. When the method is disrupted we have an angular velocity of the line of sighting $\omega_в \neq 0$, and the relative velocity vector $V_{сбл}$ is no longer directed along the line of sighting, as when the method is precisely carried out, but forms a certain angle μ with it (Fig. 2.22). Disruption of the homing method in the phase before target impact also leads to the appearance of miss r whose value equals the shortest distance between the rocket and the target.

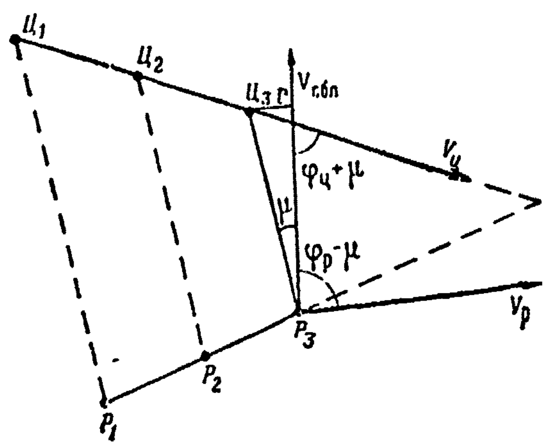


Fig. 2.22.

From Fig. 2.22 we see that miss r, perpendicular to the closing velocity vector $V_{сбл}$, is equal to

$$r = D\sin\mu. \qquad (1)$$

The closing speed in this case will be determined from the relationship

$$V_{сбл} = V_p\cos(\varphi_p - \mu) + V_ц\cos(\varphi_ц + \mu). \qquad (2)$$

e component $V_{сбл}\sin\mu$ is the linear velocity, associated with the angular velocity of the line of sighting by the familiar formula

$$V_{\text{сбл}} \sin \mu = \omega_{\text{в}} D, \qquad (3)$$

from which

$$\sin \mu = \frac{\omega_{\text{в}} D}{V_{\text{сбл}}}, \qquad (4)$$

while considering the fact that angle μ is small

$$V_{\text{сбл}} \approx V_{\text{р}} \cos \varphi_{\text{р}} + V_{\text{ц}} \cos \varphi_{\text{ц}} = \dot{D}. \qquad (5)$$

Then

$$\sin \mu = \frac{\omega_{\text{в}} D}{\dot{D}}. \qquad (6)$$

Substituting (6) into (1), we get the following formula for the rocket miss:

$$r = \frac{D^2 \omega_{\text{в}}}{\dot{D}}, \qquad (7)$$

where

$$\dot{D} = V_{\text{р}} \cos \varphi_{\text{р}} + V_{\text{ц}} \cos \varphi_{\text{ц}}; \qquad (8)$$

$$\omega_{\text{в}} = \frac{V_{\text{ц}} \sin \varphi_{\text{ц}} - V_{\text{р}} \sin \varphi_{\text{р}}}{D}. \qquad (9)$$

Thus the miss of a rocket when the method of constant-bearing approach is not fulfilled is directly proportional to the square of the distance "rocket-target" beginning with which the rocket leaves the kinematic trajectory of the constant-bearing approach method.

For the numerical values of the problem we have

$$\omega_{\text{в}} = \frac{500 \sin 30^\circ - 1000 \sin 30^\circ}{100} = 2{,}5 \text{ rad/s},$$

$$\dot{D} = 1000 \cos 30^\circ + 500 \cos 30^\circ = 1297 \text{ m/s},$$

$$r = \frac{100^2 \cdot 2{,}5}{1297} = 19{,}3 \text{ m}.$$

Let us examine a simplified method for calculating rocket miss with target maneuvering. In this case, when the distance "rocket-target" becomes commensurate with the miss value r, Eq. (7) cannot be used. The error of Eq. (7) can be estimated by assuming that, beginning at this time, the rocket and target move with constant normal load factors and with constant speeds V_p and $V_ц$. If D is not too large, $D/\dot{D}$ is, with sufficient accuracy, equal to the time Δt remaining until the rocket encounters the target. Consequently, the miss

$$r = \Delta t \dot{D}\omega_u.$$

Further, let us estimate the rate of change of instantaneous miss r(t) with time, assuming $\dot{r}$ = const. Let us differentiate (7) with respect to t. We then get

$$\dot{r} = \frac{D}{\dot{D}}(D\dot{\omega}_u + 2\dot{D}\omega_u) = -\Delta t(V_p\dot{\varphi}_p - V_ц\dot{\varphi}_ц). \tag{10}$$

If Δt is small, in this time the instantaneous miss r(t) during maneuvering of the target and rocket with constant load factors changes by the value

$$\Delta r = -\frac{\Delta t^2}{2}(V_p\dot{\varphi}_p - V_ц\dot{\varphi}_ц). \tag{11}$$

Thus, having estimated the rate of change, we can write a more general formula for calculating the rocket miss, taking into account rocket and target maneuvering:

$$r + \Delta r = \left[\dot{D}\omega_u + \frac{1}{2}(V_ц\dot{\varphi}_ц - V_p\dot{\varphi}_p)\right]\Delta t^2. \tag{12}$$

PROBLEM 2.18. A rocket is guided to a target by the method of proportional navigation. The navigation constant k = 4. Construct the rocket flight trajectory for the case when the target flies linearly and with constant speed V = 1500 km/h. The absolute average speed of the rocket $\overline{V}_p$ = 3000 km/h. The mutual positions of the interceptor and the target at the moment of firing are described by the following parameters: distance "interceptor-target" $D_п$, target relative bearing $\varphi_ц$ = 90°. Construct the rocket flight trajectory for interceptor relative bearings at the moment of firing $\varphi_п$ = 30 and 60°. For each case show how the rocket load factor required to carry out the homing method changes as the rocket closes with the target.

Solution. Guidance by the proportional navigation method is characterized by the fact that the rocket flies such that its angular velocity is k-times greater than the angular rate of displacement of the line of sighting, i.e., the control law has the following form:

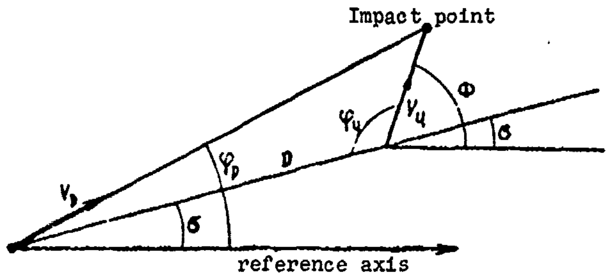


Fig. 2.23.

$$\frac{d\varphi_p}{dt} = k\frac{d\sigma}{dt}.$$

In other words, the rate of change of the rocket relative bearing is proportional to the rate of change of the sighting angle. The coefficient of proportionality k is called the **navigation constant**. In the particular case when k = 1 the method of proportional navigation becomes the pursuit method. Figure 2.24 show the kinematic rocket flight trajectory for the conditions of the problem. The nature of the change of the rocket load factor required to carry out this homing method is shown in Fig. 2.25 as a function of the distance "rocket-target" for various relative bearings of the interceptor $\varphi_П$ at the moment of firing. Solution of the problem illustrates the advantage of the proportional navigation method over other homing methods: the required rocket load factor has decreased by the end of controlled flight.

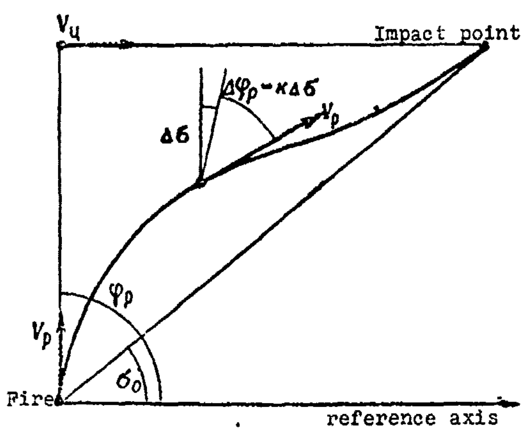


Fig. 2.24.

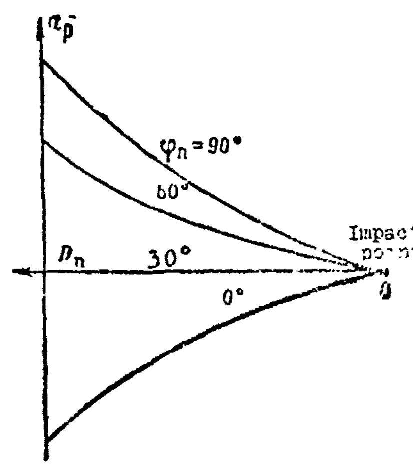


Fig. 2.25.

PROBLEM 2.19. An interceptor attacks a nonmaneuvering target, one which flies at constant velocity and at the same altitude as the interceptor. At the moment of rocket firing the target relative bearing $\varphi_{ц} = 90°$, the interceptor relative bearing $\varphi_{п} = 0$, the distance "interceptor-target" $D_{п} = 10$ km, the interceptor speed $V_{п} = 1400$ km/h, the target speed $V_{ц} = 1960$ km/h, and the natural average rocket speed $V_{p} = 700$ m/s. The rocket is guided by the proportional navigation method. Construct graphically the rocket flight trajectories for cases when the navigation constant k changes from 1 to 20. Show, for these values of k, the nature of the change of the rocket load factor required to carry out this homing method as a function of the time of controlled rocket flight.

Solution. Obviously, the larger the navigation constant the greater the curvature of the rocket's trajectory immediately after firing and, consequently, the greater the rocket load factor required at this moment. As can be seen from the graphic construction

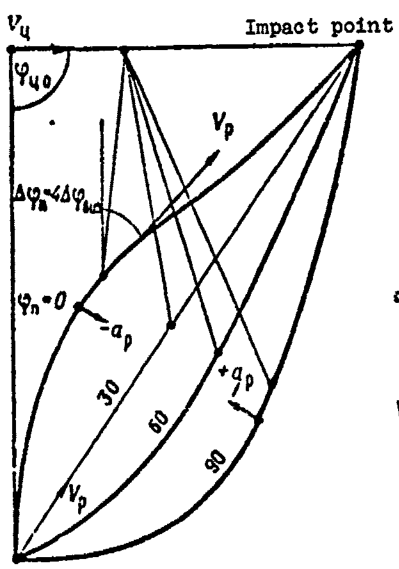


Fig. 2.26.

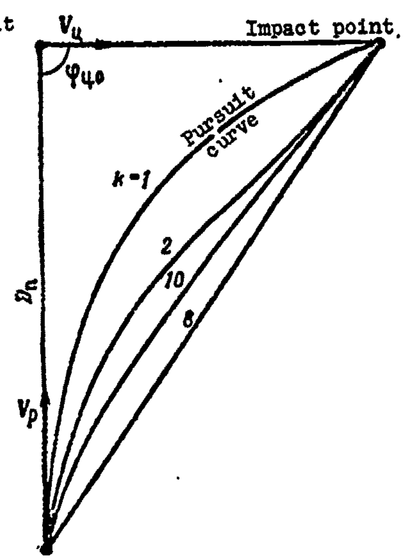


Fig. 2.27.

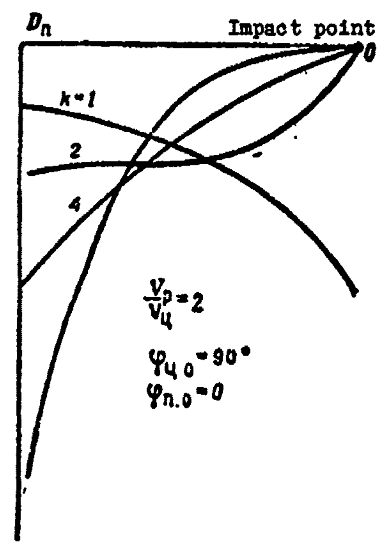


Fig. 2.28.

of the rocket flight trajectories (Fig. 2.26), for any k, after initial curvature of the trajectory, as the rocket closes on the target the trajectory straightens and the required load factor reduces to zero. Solution of the problem shows (Figs. 2.27 and 2.28) that for all practical purposes k can be no higher than 10, since for all k from 10 to ∞ the nature of the flight trajectory is the same.

This and the next problem graphically show that the proportional navigation method makes possible successful attacks on targets at large aspect angles.

PROBLEM 2.20. A rocket is guided by the proportional navigation method to a target which performs heading maneuvers. Show how the relationship between the rocket load factor required to counter the target maneuvering and the target load factor as a function of the distance "rocket-target" changes for various navigation constants k. Find the dependence of rocket miss on target maneuvering. Assumption: the target and rocket speeds are constant.

Solution. According to Fig. 2.23 the equation of proportional guidance is written as follows:

$$\frac{d\varphi_p}{dt} = k\frac{d\sigma}{dt}. \qquad (1)$$

The kinematic equations of motion of the rocket and target can be written in the form of the projection of the closing speed on the line of sighting:

$$\dot{D} = V_{ц}\cos(\Phi - \sigma) - V_p\cos(\varphi_p - \sigma) \qquad (2)$$

and the projection of the closing speed onto the perpendicular to the line of sighting:

$$D\dot{\sigma} = V_{ц}\sin(\Phi - \sigma) - V_p\sin(\varphi_p - \sigma). \qquad (3)$$

Differentiating (3) with respect to t, considering the given condition $V_{ц}$ = const and V_p = const, we get

$$\ddot{\sigma} = -2\frac{\dot{\sigma}\dot{D}}{D} - \frac{V_p\cos(\varphi_p - \sigma)}{D}\dot{\varphi}_p + \frac{a_{ц}}{D}, \qquad (4)$$

where $a_{ц}$ is the projection of target acceleration onto the perpendicular to the sighting line: $a_ц = V_ц \cos(\Phi - \sigma)\dot{\Phi}$.

Equation (4), with consideration of (1), gives a first-order differential equation relative to the angular velocity of rotation of line of sighting $\dot{\sigma}$.

Based on the condition V_p = const we can assume that the closing speed "rocket-target" $\dot{D}$ and the projection of the rocket velocity onto the sighting line $V_p \cos(\varphi_p - \varphi)$ are constant. Then, integration of differential Eq. (4) with $a_ц$ = const gives the following solution relative to the angular velocity of banked turn for the rocket (the rate of change of target relative bearing $\varphi_ц$):

$$k\dot{\sigma} = \dot{\varphi}_p = k\dot{\sigma}_0 \left(\frac{D}{D_н}\right)^{\lambda-2} + \frac{\lambda}{\lambda-2}\left[1 - \left(\frac{D}{D_н}\right)^{\lambda-2}\right]\left[\frac{[illegible]}{V_p \cos([illegible])}\right], \quad (5)$$

where

$$\lambda = -k\frac{V_p \cos(\varphi_p - \sigma)}{\dot{D}}. \quad (6)$$

is the so-called **effective navigation constant**. According to (5), when $\lambda = 2$ the acceleration of the rocket throughout the flight is constant.

With a decrease in λ the rocket load factors required by the end of the flight have increased, which can result in an increase in the misses. When $\lambda \geq 3$ the rocket load factors decrease with closing on the target.

The second term in (5) describes the rocket load factor required to counter the maneuvering of the target.

Figure 2.29 shows the dependences of the ratio of required rocket load factor to target load factor on the relative distance "rocket-target" for various values of λ. These dependences show that with $\lambda = 2$, for interception of a maneuvering target the required load factor increases to infinity at the moment of impact

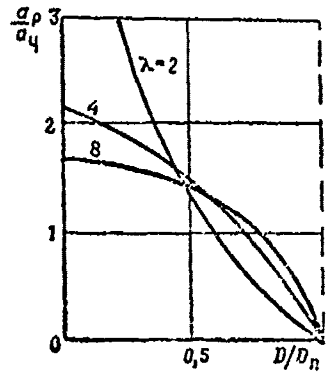


Fig. 2.29.

with the target. Obviously, the optimum value of λ is within the limits of 4-8.

Now let us find how the rocket miss, with homing by the proportional navigation method, depends on target maneuvering. The rocket load factor required to counter a target maneuver on the basis of (5) can be written as follows:

$$n_p = n_ц \frac{\lambda}{\lambda-2}\left[1-\left(\frac{D}{D_п}\right)^{\lambda-2}\right]. \qquad (7)$$

According to Eq. (7) of Problem 2.17 the rocket miss for our example is expressed by the formula

$$r = \frac{\dot{\sigma} D^2}{\dot{D}}. \qquad (8)$$

For the inertialess rocket load factor control circuit the load factor required to carry out the proportional navigation method is connected with the angular velocity of the line of sighting $\dot{\sigma}$ and the closing speed $\dot{D}$ by the dependence

$$n_p = \frac{\lambda}{g}\dot{D}\dot{\sigma}. \qquad (9)$$

Substituting (9) into (8) we get

$$r = \frac{n_p g D^2}{\lambda \dot{D}^2}, \qquad (10)$$

while with consideration of (7) the formula for calculating miss as a function of target load factor assumes the following form:

$$r = \frac{g n_ц D^2}{\dot{D}^2(\lambda-2)}\left[1-\left(\frac{D}{D_п}\right)^{\lambda-2}\right]. \qquad (11)$$

For very long firing distances ($D_п \to \infty$) the required rocket load factor in the final homing stage, to counter the target maneuver, is described by the load factor $n_ц$, which is determined by the limiting value

$$n_{p.макс} = n_ц \frac{\lambda}{\lambda-2}, \qquad (12)$$

while the miss

$$r = \frac{n_{\text{д}} g D^2}{(\Lambda - 2)\dot{D}^2} = \frac{g n_{\text{д}} t_p^2}{\Lambda - 2}, \quad (13)$$

where t_p is the time of rocket homing.

CHAPTER 3

THE TACTICAL-TECHNICAL CHARACTERISTICS OF A FIGHTER-INTERCEPTOR

In this chapter we present problems whose solutions allow quantitative determination of the combat capabilities and combat characteristics of a single fighter-interceptor. First we derive general equations for combat maneuvering and give methods for calculating the ranges (regions) of combat utilization with respect to flight altitudes and speeds (Problems 3.1-3.5). We then establish the interconnections among the basic tactical-technical characteristics (Problems 3.6-3.13). The most important maneuver characteristics (specific excess power, available load factors, turn radius and time, and rate of climb) are calculated for the widely-known F-4C "Phantom" fighter.

From the solutions of these problems we obtained general graphs and nomograms of the dependences among the following:

— the aerodynamic and strength restrictions of the regions of combat maneuvering, flight speed, and flight altitude;

— flight range, takeoff weight, fuel weight, engine efficiency, calorific value of the fuel, and lift-to-drag ratio (LDR);

— flight speed, flight altitude, takeoff weight, thrust-to-weight ratio (TWR), lift coefficient, drag coefficient, and wing area;

— the basic tactical-technical characteristics of the airborne radar, the armament system, and the aircraft.

PROBLEM 3.1. Compile, in general form, equations for the three-dimensional maneuvering of a fighter-interceptor.

Assumptions:

— the fighter-interceptor is examined as a material point;

— thrust P coincides with the longitudinal axis of the plane and with the wing chord (the component of thrust in the direction perpendicular to the trajectory can be disregarded);

— the wing angle of attack is small and can be disregarded.

Let us examine the problem for two cases: 1) without slip, and 2) with slip.

Solution. Let us select the coordinate system associated with the fighter-interceptor (Fig. 3.1): the coordinate origin O is placed at the center of gravity of the aircraft, axis Ox is directed along the velocity vector V_n, axis Oy is perpendicular to Ox in the vertical plane passing through Ox, and axis Oz is perpendicular to plane xOy. Three-dimensional maneuvering of the fighter-interceptor can be described by a system of equations of the forces acting along the axes of the selected selected coordinate system. The sum of the projections of all external forces onto any of the axes is equal to the force of inertia acting along the appropriate axis.

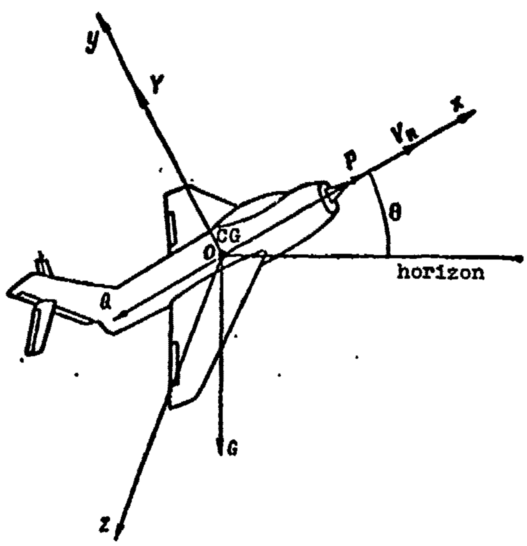


Fig. 3.1.

Let us examine, as per the conditions of the problem, the first case, i.e., when the slip angle β (the angle between vector V_n and the aircraft plane of symmetry) is zero. Then the aerodynamic force can be represented by two vector components: lift Y and drag Q (lateral force Z = 0). For this case, considering the given assumptions, we get the following equations for force projections:

onto axis Ox

$$m\dot{V}_n = P - Q - G\sin\theta; \qquad (1)$$

onto axis Oy

$$mV_n\dot{\theta} = Y\cos\gamma - G\cos\theta; \qquad (2)$$

onto axis Oz

$$-mV_n\dot{\varphi}\cos\theta = Y\sin\gamma, \qquad (3)$$

where

$$\dot{V}_n = \frac{dV_n}{dt}, \quad \dot{\theta} = \frac{d\theta}{dt} \text{ and } \dot{\varphi} = \frac{d\varphi}{dt}. \qquad (4)$$

Here m is the aircraft mass, G is the weight, θ is the trajectory angle (the angle between the velocity vector V_n and the horizon), γ is the bank angle, φ is the relative bearing (the angle between the horizontal projection of the velocity vector V_n and the selected reference direction, e.g., north).

In the second case, i.e., with aircraft slip (β ≠ 0), we have lateral force Z ≠ 0. In this case the equations for the forces acting along the axes of the selected coordinate system Oxyz are written as follows:

$$m\dot{V}_n = (P - Q)\cos\beta + Z\sin\beta - G\sin\theta; \qquad (5)$$

$$mV_n\dot{\theta} = (P - Q)\sin\beta\sin\gamma + Y\cos\gamma - Z\cos\beta\sin\gamma - G\cos\theta, \qquad (6)$$

$$-mV_n\cos\theta\dot{\varphi} = -P\sin\beta\cos\gamma + Q\sin\beta\cos\gamma + Y\sin\gamma + Z\cos\beta\cos\gamma. \qquad (7)$$

System of Eqs. (5)-(7) describes the most general case of three-dimensional maneuvering of the fighter-interceptor. By integrating these systems of differential equations of motion we can calculate any parameters and three-dimensional coordinates for combat maneuvering of a fighter-interceptor.

PROBLEM 3.2. Considering the assumptions made in Problem 3.1, derive the basic equations that describe a combat turn, a change in heading during level flight, and acceleration with straight level flight.

Solution. To calculate the parameters of a combat turn it is necessary to express the nature of the change in flight speed V_n and trajectory angle θ with a change in heading φ. Let us use system of Eqs. (1)-(3) of Problem 3.1. Dividing (1) and (2) by (3), we obtain differential equations for the combat turn:

$$\frac{dV_n}{d\varphi}=-V_n\cos\theta\frac{P-Q-G\sin\theta}{Gn_y\sin\gamma};\qquad(1)$$

$$\frac{d\theta}{d\varphi}=-\frac{n_y\cos\gamma\cos\theta-\cos^2\theta}{n_y\sin\gamma}.\qquad(2)$$

Let us note that when integrating (1) and (2) we must be given the dependence $\gamma = f(\varphi)$, i.e., the nature of the change in bank angle as the combat turn is performed.

2. With a change in heading in level flight we have $\theta = 0$. Then system of Eqs. (1)-(3) of Problem 3.1 assume the following form:

$$m\dot{V}_n=P-Q;\qquad(3)$$

$$mg=Y\cos\gamma;\qquad(4)$$

$$-mV_n\dot{\varphi}=Y\sin\gamma.\qquad(5)$$

3. For the characteristic of fighter-interceptor acceleration with straight level flight we must calculate the dependences

$$V_n=f(t)\text{ and }x=f(t),$$

where x is the distance covered by the fighter-interceptor along axis x. The current velocity value V_n is found by integrating Eq. (1) of Problem 3.1:

$$V_n = \frac{1}{m}\int_0^t (P - Q)\,dt; \tag{6}$$

$$x = \int_0^t V_n(t)\,dt. \tag{7}$$

In aerial combat the movement of a fighter-interceptor is described, as a rule, by a complex three-dimensional trajectory and unstable flight conditions with continuous changes in altitude, speed, and heading. However, the characteristics of steady motion in a single plane are of particularly important practical significance. The following problems are devoted to a quantitative estimate of these characteristics.

PROBLEM 3.3. Calculate the boundaries of the region of combat maneuvering of a fighter-interceptor, those determined by the available engine thrust and by aerodynamic and strength limitations. Construct the boundaries with respect to level-flight speeds and altitudes, i.e., in coordinates M-H. Known: the average flight weight G_{cp}, wing area S, the characteristics of available thrusts as functions of M and H (Fig. 3.2), and the dependence of the aerodynamic coefficients c_{x0} and A on M (Fig. 1.9). In addition to the family of boundaries of combat maneuvering, construct the dependence of the limiting possible power altitude on M for various loads on the wing.

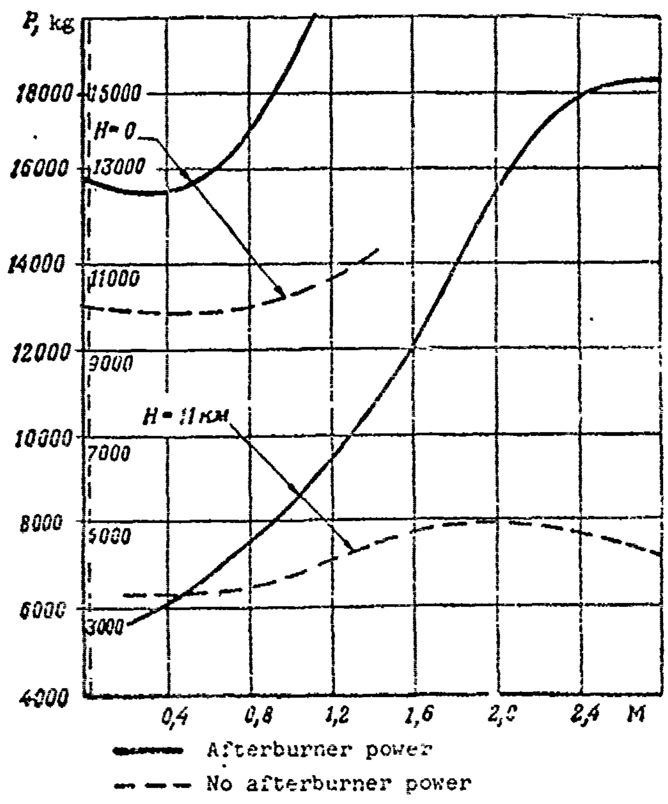


Fig. 3.2.

Solution. On the basis of the general equations of motion (see Problem 3.1) for level flight, from the condition of linearity of the trajectory we have

$$Y + P\sin\alpha - G = 0. \tag{1}$$

From the condition of equilibrium, which keeps the flight speed constant, we have

$$Q - P\cos\alpha = 0. \tag{2}$$

Since the angle of attack $\alpha \approx 0°$, for all practical purposes

$$Y = G;\ Q = P. \tag{3}$$

Let us introduce designations for the coefficients of lift, drag, weight, and thrust:

$$c_y = \frac{Y}{qS};\ c_x = \frac{Q}{qS};\ c_G = \frac{G}{qS};\ c_P = \frac{P}{qS}. \tag{4}$$

Here the velocity head

$$q = \frac{\rho V_п^2}{2}, \tag{5}$$

and the air density

$$\rho = \frac{2q}{V_п^2}. \tag{6}$$

We know that for flights at altitudes from 0 to 11 km the thrust at constant velocity depends somewhat less on altitude than on ρ. Therefore, for altitudes $H \leq 11$ km the thrust coefficient can be calculated from the formula

$$c_{PH<11} = c_{PH=0} + \frac{(c_{PH=11} - c_{PH=0})\,H}{11}. \tag{7}$$

For altitudes $H > 11$ km the thrust decreases almost in proportion to ρ and, consequently, the thrust coefficient is practically independent of altitude and is only a function of velocity $V_п$. Thus, with $V_п$ = const

$$c_{PH>11} = c_{PH=11} = \text{const}. \tag{8}$$

This allows us to conclude that for calculation of the region of combat maneuvering it suffices to know the characteristics of the available thrusts as functions of M for two altitudes (H = 0 and H = 11 km), while Eqs. (7) and (8) are valid for the entire range of altitudes. This assumption greatly simplifies the calculations and has practically no influence on the accuracy of the results.

The aircraft drag polar is represented by the parabolic dependence

$$c_x = c_{x0} + Ac_y^2, \tag{9}$$

where A and c_{x0} are given by the graph in Fig. 1.9.

Let us substitute c_y from (9) into (4). We then get the following formula for the velocity head:

$$q = \frac{G}{c_y S} = \frac{G}{S}\sqrt{\frac{A}{c_{p\,H=11} - c_{x0}}}. \tag{10}$$

Using Formula (10) let us find the boundaries of the region of combat maneuvering with respect to available thrust.

The calculation sequence is as follows:

- given the interceptor flight speed in Mach numbers M;

- find thrust P from the available-thrust characteristics (Fig. 3.2);

- determine the coefficients A and c_{x0} from the graph (Fig. 1.9);

- calculate the velocity head q from Eq. (10);

- calculate the thrust coefficient c_p from Eq. (4);

- find, from Eq. (6), the value of air density ρ which corresponds to the values of q and T_H;

- determine flight altitude H for the value of ρ from ISA (International Standard Atmosphere) tables.

In addition to available-thrust limitations, the region of combat maneuvering is also defined by restrictions on stability and controllability, and also by the strength characteristics of the plane.

The aerodynamic limitation for uniform level flight is defined by the limiting value of the lift coefficient $c_{y\ пред}$, found from the formula

$$V_{\text{в}}=\sqrt{\frac{G}{\frac{\rho S}{2}c_{y\ \text{пред}}}}. \tag{11}$$

For calculations in first approximation we can set $c_{y\ пред} = 1$. Then, substituting this into (11), and given the values of H, from the ISA table we find the corresponding values of ρ and then H.

The restrictions due to aircraft design strength are expressed by the maximum permissible velocity head $q_{пред}$ and by the maximum possible heating temperature $T_{н}$. Knowing q, let us find the values of H and M corresponding to the boundaries of the region of combat maneuvering; we use the formula

$$Ma=V_{\text{п}}=\sqrt{\frac{q_{\text{пред}}}{\rho/2}}. \tag{12}$$

The limitations imposed by heating are defined by the formula

$$M=\frac{V_{\text{п}}}{a}=\sqrt{\frac{T_{\text{н}}-T_0}{0{,}2T_0}}, \tag{13}$$

where $T_{н}$ is the desired temperature with maximum velocity head; T_0 is the temperature of the ambient medium.

Figure 3.3 shows the results of calculations of the level-flight altitude and velocity limitations using Eqs. (12), (13), and (11) with $q_{пр}$ = 5000; 10,000; 15,000; 20,000; and 30,000 kg/m^2, $T_{н}$ = = 100, 150, 200, 250, and 300°C, $c_{y\ пред}$ = 1 for G/S = 200, 400, 600, and 800 kg/m^2.

The same figure gives the family of lines of constant power altitudes:

$$H_{\text{э}}=H+\frac{V_{\text{п}}}{2g}. \tag{14}$$

Along line $H_{э}$ the interceptor has a constant power level from which we can judge as to the power reserve, which is important when estimating three-dimensional maneuvering when one form of energy

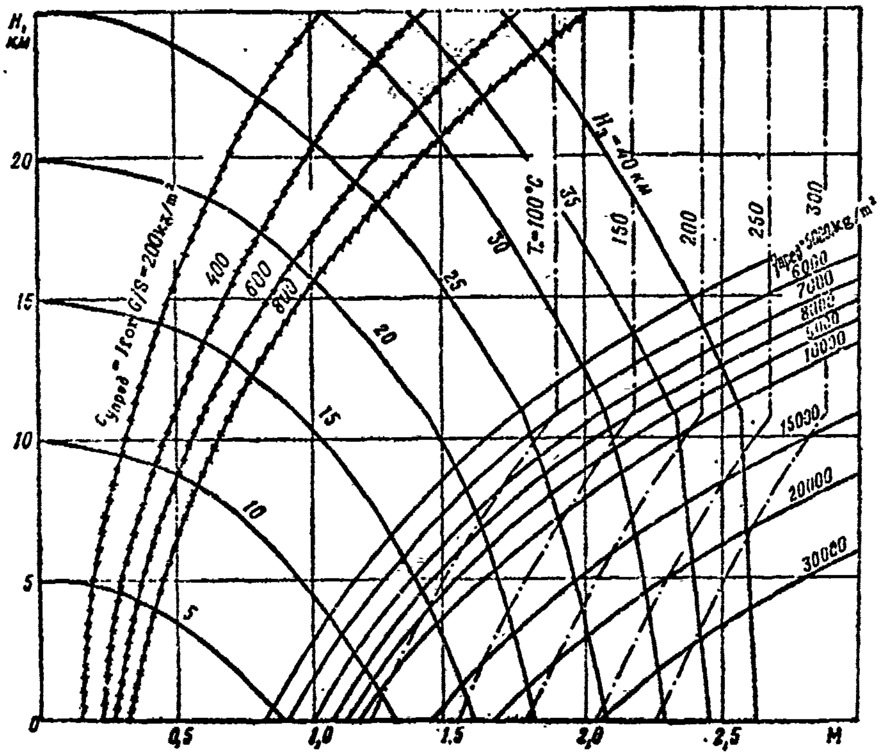


Fig. 3.3.

is converted to another.

Within these obtained regions the fighter-interceptor, using available thrust, can perform various maneuvers. Let us examine a quantitative estimate of the maneuvering capabilities within the regions of combat maneuvering in the next problems.

PROBLEM 3.4. For a fighter-interceptor of the F-4C Phantom type let us calculate the range of combat velocities and altitudes which characterizes its maneuvering capabilities in aerial combat. Given: the aerodynamic and thrust characteristics (Figs. 1.9 and 3.2), the average flight weight G = 17,600 kg, wing area S = 49.2 m^2. Consider the restrictions on maximum M ($M_{пред}$ = 2.1), velocity head ($q_{пред}$ = 9800 kg/m^2), and stability ($c_{y\ пред}$ = 1).

Solution. As the most general criterion characterizing quantitatively the maneuvering capabilities of a fighter-interceptor let us use the specific excess power $N_{уд}$, i.e., the ratio of the rate of change of the resultant power of an aircraft to its weight. A fighter-interceptor at a given altitude and velocity will outmaneuver the enemy only if he has greater specific excess power under the given conditions.

Let us write an equation for the energy level of an aircraft, i.e., the total energy, equal to the sum of the potential and kinetic energies:

$$E = GH + \frac{mV_п^2}{2}, \tag{1}$$

where G, m, H, and $V_п$ are the weight, mass, altitude, and speed of the fighter-interceptor. Dividing (1) by G = mg we get

$$\frac{E}{G} = H + \frac{V_п^2}{2g}. \tag{2}$$

With maneuvering in aerial combat the values of H, $V_п$, and E constantly change. Let us differentiate (2) with respect to t. Then, by definition, the specific excess power is expressed by the formula

$$N_{уд} = \frac{d}{dt}\left(\frac{E}{G}\right) = \dot{H} + \frac{V_п \dot{V}_п}{g}. \tag{3}$$

On the other hand, Eq. (3) is obtained from the condition of balance of forces with flight at angle θ to the horizon:

$$P - Q - G\sin\theta = m\dot{V}_п, \tag{4}$$

where P is the thrust of the powerplant and Q is drag.

Multiplying both sides of (4) by $V_п/G$ we get

$$\frac{V_п(P - Q)}{G} - V_п \sin\theta = \frac{V_п \dot{V}_п}{g}. \tag{5}$$

The term V sin θ is the change of altitude with time, i.e., the rate of climb:

$$u = \dot{H} = V_п \sin\theta. \tag{6}$$

Consequently, we get

$$\frac{V_n(P-Q)}{G}=\dot{h}+\frac{V_n\dot{V}_n}{g}. \qquad (7)$$

Equating (7) and (3) we can write the specific excess power as

$$N_{yx}=\frac{V_n(P-Q)}{G}. \qquad (8)$$

Further, let us use the following familiar relationships:

drag

$$Q=c_x\frac{\rho V_n^2}{2}S, \qquad (9)$$

drag coefficient

$$c_x=c_{x0}+Ac_y^2. \qquad (10)$$

Considering (9) and (10) we get

$$N_{yx}=\frac{V_n\left[P-(c_{x0}+Ac_y^2)\,qS\right]}{G}. \qquad (11)$$

Let us find a calculation expression for the lift coefficient c_y. Since for flight along a trajectory with angle θ to the horizon

$$Y=G\cos\theta, \qquad (12)$$

then

$$c_y=c_G\cos\theta. \qquad (13)$$

On the basis of (13) and (6) we have

$$c_y^2=c_G^2\cos^2\theta=c_G^2(1-\sin^2\theta)=c_G^2\left(1-\frac{u^2}{V_n^2}\right). \qquad (14)$$

Let us substitute Eq. (10) into the equation

$$P=Q+G\sin\theta \qquad (15)$$

Then

$$c_{x0} + Ac_y^2 - c_P + c_G \sin\theta = 0;$$
$$c_{x0} + Ac_G^2\left(1 - \frac{u^2}{V_n^2}\right) - c_P + c_G\frac{u}{V_n} = 0;$$
$$\frac{Ac_G^2}{V_n^2}u^2 - \frac{c_G}{V_n}u + c_P - c_{x0} - Ac_G^2 = 0; \quad (16)$$

from which

$$u = \frac{V_n}{2Ac_G}\left[1 - \sqrt{1 - 4A(c_P - c_{x0} - Ac_G^2)}\right]. \quad (17)$$

Consequently,

$$c_y^2 = c_G^2\left\{1 - \frac{1}{4A^2c_G^2}\left[1 - \sqrt{1 - 4A(c_P - c_{x0} - Ac_G^2)}\right]^2\right\}. \quad (18)$$

On the basis of the relationships obtained we can propose the following algorithm for calculating the unknown dependences H = f(M) for various values $N_{уд}$ = const:

– given the number M;

– from M find c_{x0} and A (Fig. 1.9);

– collect all possible values of H for which the given values $N_{уд}$ = 0, 25, 50, 100, 150, ... kgm/s·kg are obtained from Eq. (1), for which

– from H find the corresponding values of ρ and a using tables;

– from M and H find thrust P from the graph (Fig. 3.2);

– calculate

$$q = \frac{\rho V_n^2}{2};\ c_G = \frac{G}{qS};\ V_n = Ma;$$

– using Eqs. (7) and (8) of Problem 3.3 calculate c_p;

– having calculated c_y^2 from (18) and having substituted all obtained intermediate values into (11), find $N_{уд}$.

If the obtained value of $N_{уд}$ does not equal the given one then, using the iteration method, repeat the above algorithm for calculating $N_{уд}$ for other values of altitude H until the given integer $N_{уд}$ is

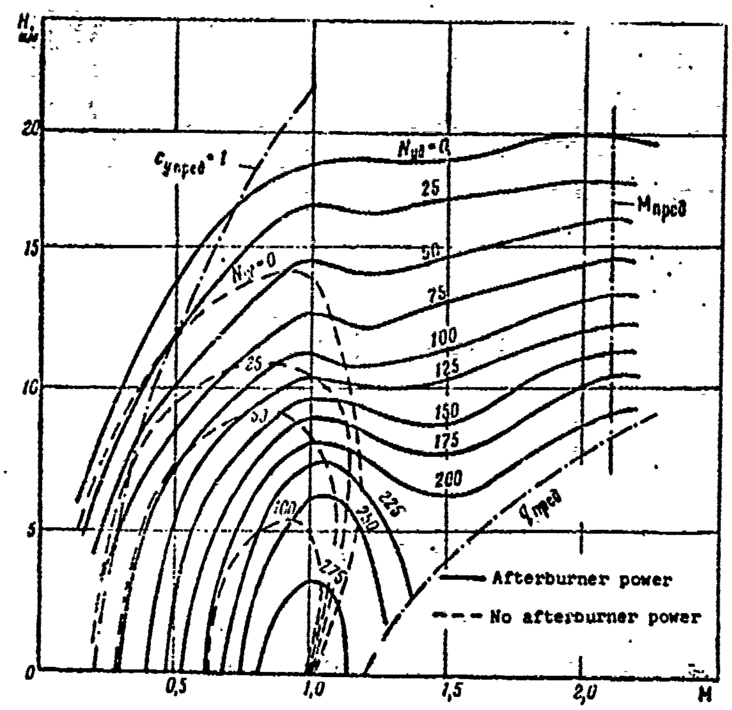


Fig. 3.4.

obtained; then note the obtained pairs of numbers M and H for which Eq. (11) gives the precise given value of $N_{уд}$. Repeat the calculation algorithm for another value of M. Having obtained all pairs of numbers M and H for the given value of $N_{уд}$, calculate the same group of M and H for the next given value of $N_{уд}$. Repeat the calculation algorithm in this sequence for the entire range of M and H for combat maneuvering of the examined fighter-interceptor.

The results of the calculations for the starting data of our problem are given in Fig. 3.4. The restrictions on $q_{пред}$ and $c_{y\ пред}$ are taken from Fig. 3.3. Superimposing the graphs H = f(M) for identical values of $N_{уд}$ for two comparable fighters, we can graphically show the regions M-H in which one fighter is more maneuverable than the other in aerial combat.

PROBLEM 3.5. Calculate, in coordinates M-H, the regions of combat maneuvering for various values of normal load factor n_y, longitudinal load factor n_x, radius of banked turn $R_п$, the time required to turn 180° in the horizontal plane $t_{180°}$, and the vertical rate of climb u with and without afterburner power. Use the data of Problem 3.4 as the starting data.

Solution. Let us examine the banked turn of an interceptor in the horizontal plane. The normal load factor n_y is the ratio of lift Y to weight G. It acts in the plane of symmetry of the aircraft perpendicular to velocity vector $V_п$ and equals

$$n_y = \frac{Y}{G} = \frac{c_y}{c_G}. \quad (1)$$

For the available normal load factor we have

$$n_{y\,pacn} = \frac{Y_{pacn}}{G} = \frac{c_{y\,pacn} q S}{G} = 0{,}7 c_{y\,pacn} M^2 \frac{P_в S}{G}, \quad (2)$$

where $c_{y\ pacn}$ is the available lift coefficient, $P_в$ is air pressure.

For supersonic fighter-interceptors, in first approximation

$$c_{y\,pacn} \approx \frac{1}{\sqrt{M^2-1}}. \quad (3)$$

Substituting (3) into (2) we get

$$n_{y\,pacn} = 0{,}7 \frac{M^2 P_в S}{G\sqrt{M^2-1}}. \quad (4)$$

From the aircraft drag polar

$$c_x = c_{x0} + A c_y^2 \quad (5)$$

we find c_y, which we substitute into (1). Then the calculation formula for the available normal load factor with curved flight in the horizontal plane assumes the following form:

$$n_y = \frac{c_y}{c_G} = \sqrt{\frac{c_P - c_{x0}}{A c_G^2}}. \quad (6)$$

It is just this load factor, created by engine thrust, which is one of the basic characteristics of the maneuvering capabilities of an interceptor.

The algorithm for calculating the boundaries of the regions of combat maneuvering in M-H coordinates for various values of n_y is as follows:

– given the value of M;

– from the graphs in Figs. 3.2 and 1.9 we find the corresponding thrust P and coefficients c_{x0} and A;

— from Eq. (4) of Problem 3.3 we calculate the thrust coefficient c_P;

— for a fixed value of load factor n_y we calculate the weight coefficient c_G by the formula

$$c_G = \frac{1}{n_y} \sqrt{\frac{c_P - c_{x0}}{A}}; \tag{7}$$

— knowing c_G, we find the value of air density ρ from the ratio $c_G = G/qS$;

— from the table we find the flight altitude H corresponding to density ρ.

The boundaries of the regions of combat maneuvering of an F-4C type aircraft, calculated by the described algorithm and corresponding to various values of load factor c_y, are given in Fig. 3.5 for afterburner and no afterburner engine operation.

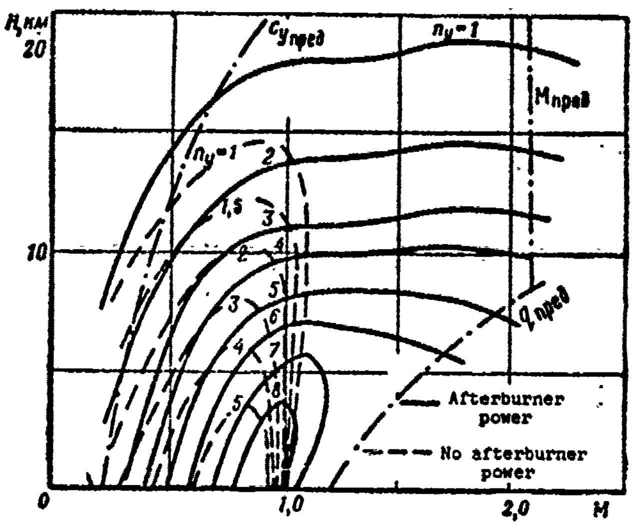


Fig. 3.5.

By a similar method we calculate the boundaries of the regions of combat maneuvering for various values of the longitudinal load factor n_x, the radius of banked turn $R_п$, the time required to turn 180° $t_{180°}$, and the rate of climb u. The results of these calculations for the F-4C are given in Figs. 3.6-3.9. The following quantitative relationships form the basis for these calculations. By definition of longitudinal load factor we have

$$n_x = \frac{\dot{V}_x}{g}. \tag{8}$$

Consequently,

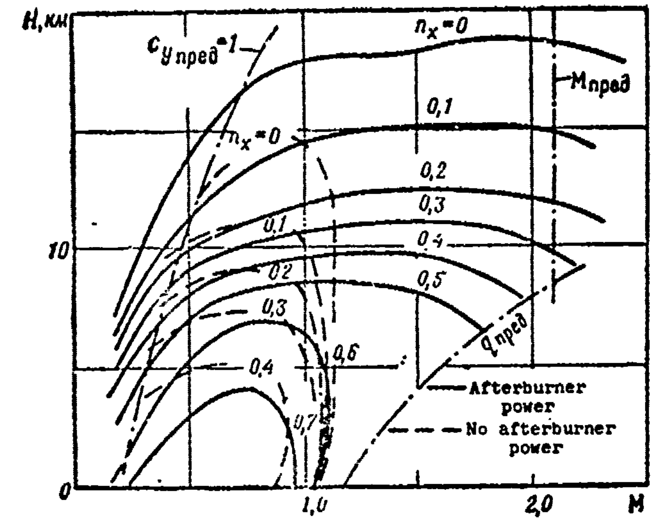

H,км
$c_{y\,пред}=1$
$n_x=0$
$M_{пред}$
0,1
$n_x=0$
0,2
0,3
0,1
0,4
0,2
0,5
0,3
$q_{пред}$
0,6
0,4
0,7
Afterburner power
No afterburner power
10
0
1,0
2,0
M

Fig. 3.6.

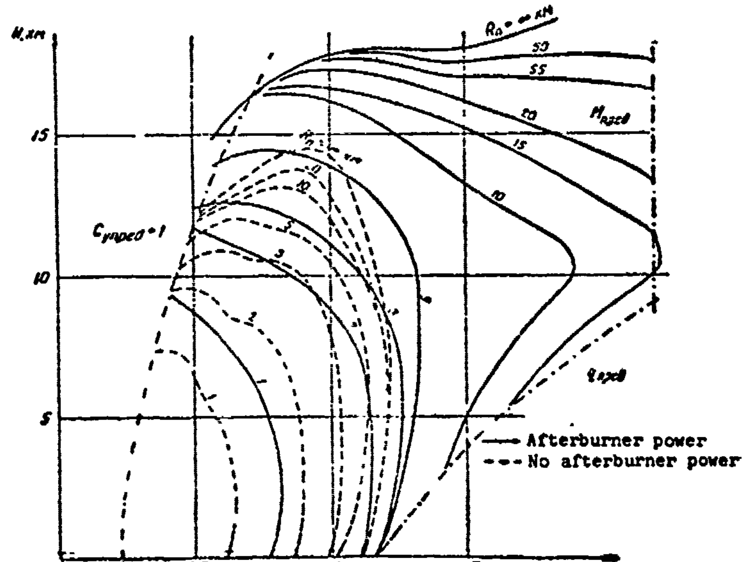
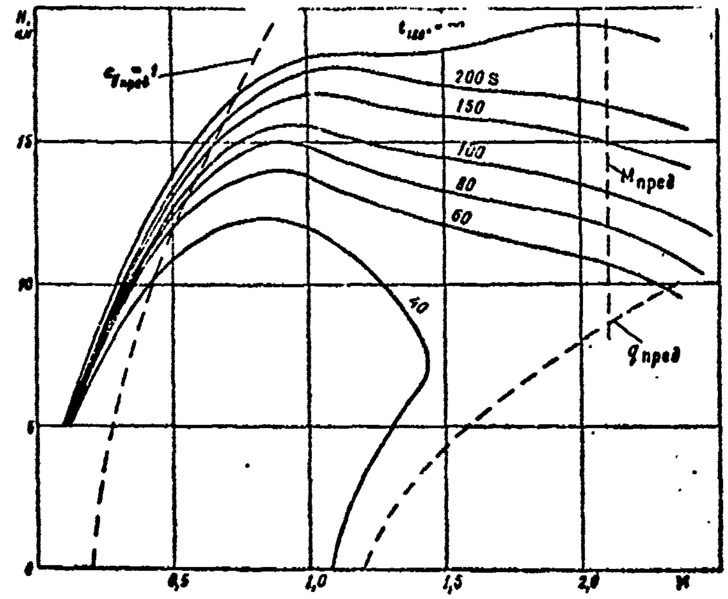
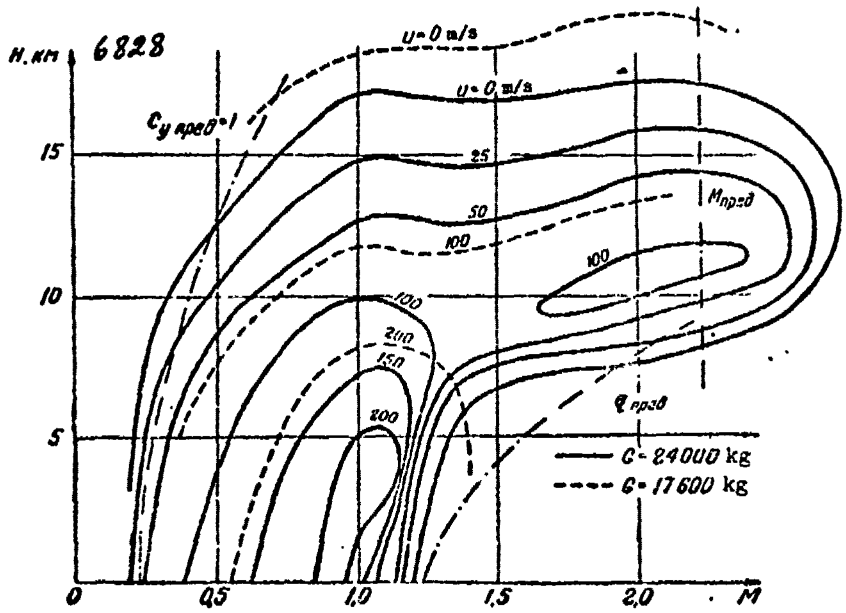

Afterburner power
No afterburner power
15
10
5
0
0,5
1,0
1,5

Fig. 3.7.



200 s
150
100
80
60
40
$M_{пред}$
$q_{пред}$
0,5
1,0
1,5
2,0
M

Fig. 3.8.



H, км
6828
v = 0 m/s
v = 0 m/s
$C_{y\,пред}=1$
25
50
100
100
100
200
150
200
$M_{пред}$
$q_{пред}$
G = 24000 kg
G = 17600 kg
15
10
5
0
0,5
1,0
1,5
2,0
M

Fig. 3.9.

$$n_x = \frac{c_p - c_{x0}}{c_G} - Ac_G \quad (9)$$

or

$$n_x = Ac_G(n_y^2 - 1). \quad (10)$$

Each load-factor value n_y has its own corresponding specific radius of banked turn R_n and time $t_{180°}$ required to turn 180°. Let us find these relationships.

The conditions for balance of forces with turning flight in the horizontal plane are written as follows:

$$Z = Y\sin\gamma; \quad G = Y\cos\gamma, \quad (11)$$

where γ is the bank angle.

For centrifugal force, from mechanics we have the familiar relationship

$$Z = \frac{GV_n^2}{gR_n}, \quad (12)$$

from which the radius of banked turn

$$R_n = \frac{GV_n^2}{gZ}. \quad (13)$$

Substituting the expressions for G and Z into (13) we get

$$R_n = \frac{V_n^2 Y\cos\gamma}{gY\sin\gamma} = \frac{V_n^2}{g\,\mathrm{tg}\,\gamma} = \frac{(Ma)^2}{g\sqrt{n_y^2 - 1}}. \quad (14)$$

Here, according to (11) and (1) we have

$$\cos\gamma = \frac{G}{Y} = \frac{1}{n_y}; \quad \sin\gamma = \frac{1}{n_y}\sqrt{n_y^2 - 1}; \quad \mathrm{tg}\,\gamma = \sqrt{n_y^2 - 1}. \quad (15)$$

Then we find the time required to turn 180°:

$$t_{180°} = \frac{\pi R_n}{V_n}. \quad (16)$$

Having substituted the expression for load factor n_y according to (6), we obtain the final calculation formulas:

$$R_n = \frac{(Ma)^2}{g\sqrt{\frac{c_P - c_{x0}}{Ac_a^2} - 1}}; \quad (17)$$

$$t_{180^\circ} = \frac{\pi Ma}{g\sqrt{\frac{c_P - c_{x0}}{Ac_a^2} - 1}}. \quad (18)$$

When constructing the boundaries of the regions of combat maneuvering, for various values of n_y, R_n, and t_{180° we must take into consideration the aerodynamic restriction on $c_{y\ доп}$ which, by definition

$$n_y = \frac{Y}{G} = \frac{c_y}{c_a}$$

is defined by the inequality

$$n_y c_a \leqslant c_{y\ доп}. \quad (19)$$

In this case the boundary of the region of combat maneuvering is calculated using the formula

$$M = \frac{1}{a}\sqrt{\frac{Gn_y}{S\frac{\rho}{2}c_{y\ доп}}}. \quad (20)$$

Given the values of n_y and M, from (20) we find the air density ρ and then, from the table, the corresponding flight altitude H.

Examining the rate of climb u as a parameter for the region of combat maneuvering, we can write the following relationships:

$$\sin\theta = \frac{\Delta P}{G} = \frac{u}{V_n} = \frac{u}{Ma}, \quad (21)$$

where u is the rate of climb, i.e., the projection of velocity V_n onto the vertical axis; ΔP is the difference between the thrust required to gain altitude and that required for level flight.

Since from (21)

$$u = \frac{\Delta P V_n}{G}, \quad (22)$$

while the normal load factor

$$n_y = \frac{Y}{G} = \cos\theta, \quad (23)$$

for $\alpha \approx 0$ and $P = P_{макс}$ we get

$$u = Ma\frac{P_{макс} - Q}{G}. \quad (24)$$

Using these relationships and Eq. (17) of Problem 3.4, we calculate the function $H = f(M)$ with u = const, using an iteration method similar to that described above.

PROBLEM 3.6. Based on tactical requirements, the range of level flight of a fighter-interceptor should be $L_{г.п}$. Determine the required relative weight of the fuel, if we know the powerplant efficiency $\eta = 0.4$, the calorific value of the fuel δ = 10,000 kcal/kg, and the LDR of the aircraft K. Show how the relative fuel weight $\xi_т$ depends on the LDR for three values of flight range $L_{г.п}$ = = 1000, 2500, and 4500 km. How does the dependence $\xi_т = f(K)$ change if the engine efficiency changes from 0.4 to 0.6? Assume that the entire fuel supply is expended in level flight.

Solution. During level flight, work is accomplished equal to the work of transporting an average flight weight G_{cp} a distance of $L_{г.п}$. This work is numerically equal to $G_{cp}L_{г.п}$ and is balanced by the work of the powerplant with burning of $G_т$ kilograms of fuel, which in turn is equal to $G_т K\delta\eta$. Thus we have

$$G_{cp}L_{г.п} = G_т K\delta\eta. \quad (1)$$

Since the average weight is associated with the takeoff weight by the relationship

$$G_{cp} = G_{взл} - \frac{G_т}{2}, \quad (2)$$

the relative fuel weight is

$$\xi_т = \frac{G_т}{G_{cp}} = \frac{G_т}{G_{взл} + \frac{G_т}{2}}. \quad (3)$$

But according to (1)

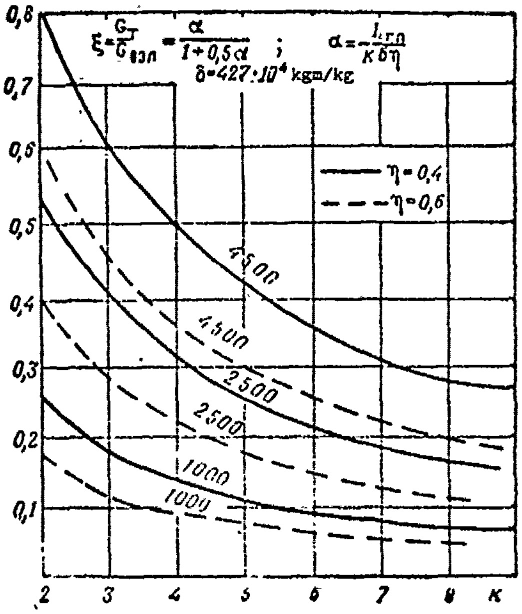


Fig. 3.10.

$$\frac{G_т}{G_{cp}}=\frac{L_{г.п}}{K\delta\eta}. \qquad (4)$$

Consequently, the calculation formula has the following form:

$$\xi_т=\frac{\alpha}{1+0.5\alpha}, \qquad (5)$$

where

$$\alpha=\frac{L_{г.п}}{K\delta\eta}. \qquad (6)$$

Substituting into (6) the value of δ we must first convert kcal into kgm, remembering that 1 kcal = 427 kgm. Varying the value of K from 2 to 9, we find the desired dependence $\xi_т = f(K)$ for the starting data of the problem (the solid lines in Fig. 3.10). With $\eta = 0.6$ the solution is shown by the dashed line.

PROBLEM 3.7. Based on tactical requirements, for an interceptor the range of level cruising flight should be 2000 km. Determine the weight of the fuel if the interceptor takeoff weight $G_{взл}$ = 20 tons, engine efficiency $\eta = 0.4$, the calorific value of the fuel δ = 10,000 kcal/kg, the lift and drag coefficients $c_y = 0.45$ and $c_x = 0.1$. By what value does the flight range decrease if the relative fuel weight is halved?

Solution. Let us designate the range of level flight of the interceptor as $L_{г.п}$. Then the energy expended by the engine when the interceptor covers distance $L_{г.п}$ is equal to the work required to transport the weight of the interceptor over this distance. Since the weight of the interceptor changes within significant limits during the flight, we will use the concept of average weight G_{cp}, considering it to be constant and equal to

$$G_{cp}=G_{взл}-\frac{G_т}{2}, \qquad (1)$$

where $G_{\text{т}}$ is the weight of the fuel.

Considering this quite permissible simplification, the work accomplished when the interceptor flies the distance $L_{\text{г.п}}$ is

$$\frac{G_{cp} L_{\text{г.п}}}{\frac{c_y}{c_x}} = G_{\text{т}} \delta \eta. \qquad (2)$$

The relative fuel weight, according to (1), is defined by the relationship

$$\xi_{\text{т}} = \frac{G_{\text{т}}}{G_{\text{взл}}} = 2\left(1 - \frac{G_{cp}}{G_{\text{взл}}}\right). \qquad (3)$$

From (2) we have

$$G_{\text{т}} = \frac{L_{\text{г.п}} G_{cp}}{K \delta \eta}, \qquad (4)$$

where

$$K = \frac{c_y}{c_x}. \qquad (5)$$

Substituting the value of G_{cp} from (1), we get

$$G_{\text{т}} = \frac{L_{\text{г.п}}}{K \delta \eta}\left(G_{\text{взл}} - \frac{G_{\text{т}}}{2}\right); \qquad (6)$$

$$G_{\text{т}}\left(1 + \frac{L_{\text{г.п}}}{2K \delta \eta}\right) = \frac{G_{\text{взл}} L_{\text{г.п}}}{K \delta \eta}. \qquad (7)$$

Thus the calculation formula for the relative fuel weight assumes the following form:

$$\xi_{\text{т}} = \frac{\frac{L_{\text{г.п}}}{K \delta \eta}}{1 + \frac{L_{\text{г.п}}}{2K \delta \eta}}. \qquad (8)$$

For the numerical values of our example we have $\xi_{\text{т}} = 0.23$.

We should bear in mind that in Eq. (8) we must insert $L_{\text{г.п}}$ in meters, while δ is in kgm/mg, i.e., consider that

1 kcal = 427 kgm.

Thus, having obtained the relative fuel weight, let us determine its absolute weight:

$$G_T = 4.6 \text{ tons.}$$

Then let us determine how $L_{\Gamma.\Pi}$ changes if we decrease ξ_T by a factor of two.

From (8) we have

$$L_{\text{г.п}} = K\delta\eta \frac{\xi_T}{1-\frac{\xi_T}{2}}. \qquad (9)$$

If we cut ξ_T in half, $L_{\Gamma.\Pi}$ decreases by a factor of 2000/940 = 2.13, since

$$L_{\text{г.п}} = 4{,}5 \cdot 427 \cdot 10^4 \cdot 0{,}4 \frac{0{,}115}{1-\frac{0{,}115}{2}} = 940 \text{ km}.$$

For all practical purposes, the flight range of a cruising interceptor will be somewhat less, since we must consider the additional expenditure for takeoff, as well as the emergency fuel supply.

PROBLEM 3.8. Determine the required relative thrust of an interceptor during level flight at altitude H and speed V_Π, if we know the takeoff weight $G_{\text{взл}}$, the drag coefficient c_x, the wing area S, and the takeoff TWR ε.

Solution. The thrust required for level flight is defined by the familiar formula

$$P_{\text{потр}} = c_x \frac{\rho V_\text{п}^2}{2} S, \qquad (1)$$

where ρ is the air density at the examined altitude.

Let us introduce the concept of relative thrust:

$$\psi = \frac{P_{\text{потр}}}{P_{\text{взл}}} = \frac{c_x \rho V_\text{п}^2 S}{2 G_{\text{взл}} \varepsilon}, \qquad (2)$$

where $\varepsilon = P_{\text{взл}}/G_{\text{взл}}$ is the takeoff TWR.

To graphically trace the interconnection among ψ, the takeoff weight, and the flight speed and altitude, let us construct a nomogram (Fig. 3.11) which covers practically the entire range of change of the examined characteristics.

PROBLEM 3.9. Show the limits of change of interceptor thrust required for level flight, if the LDR of the aircraft is doubled. Determine the thrust required for level flight if we know the LDR K = 5, the takeoff weight $G_{взл}$ = = 20 tons, and the fuel weight G_T = 4.6 tons, while the takeoff thrust is 0.8 of the takeoff weight.

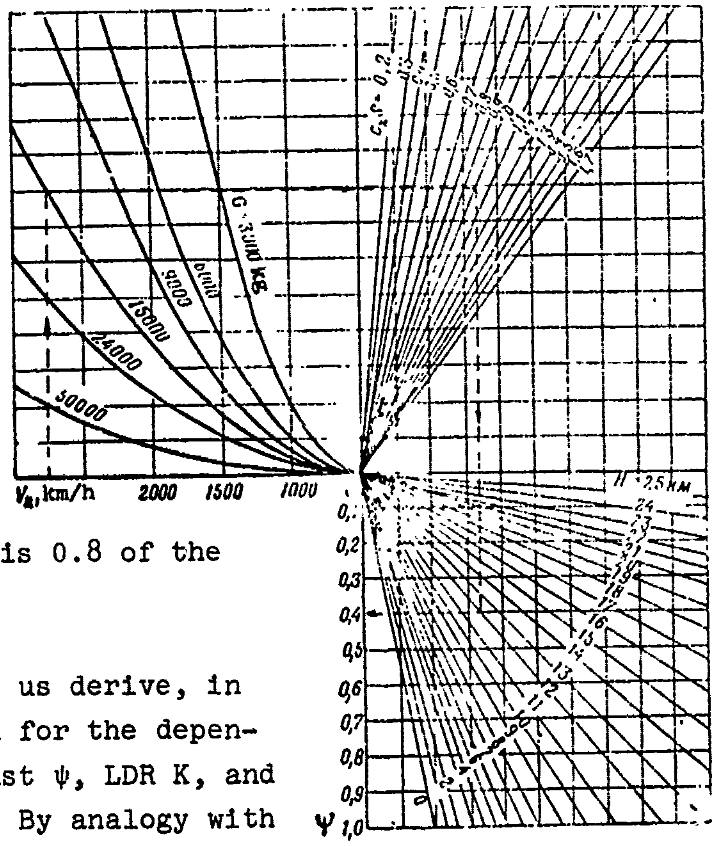


Fig. 3.11.

Solution. First let us derive, in general form, the formula for the dependence among relative thrust ψ, LDR K, and relative fuel weight ε_T. By analogy with the concept of average takeoff weight G_{cp} introduced in Problem 3.7 we can, in first approximation, use the concept of average thrust required for level flight and the average LDR value K_{cp}.

Then

$$P_{\text{потр. ср}} = \frac{G_{ср}}{K_{ср}} \tag{1}$$

or, substituting the expression for G_{cp} from Problem 3.7, we get

$$P_{\text{потр. ср}} = \frac{2G_{взл} - G_т}{2K_{ср}} \tag{2}$$

while introducing the concept of relative fuel weight

$$\xi_т = \frac{G_т}{G_{взл}}, \tag{3}$$

we obtain the following formula for calculating the average thrust required for level interceptor flight:

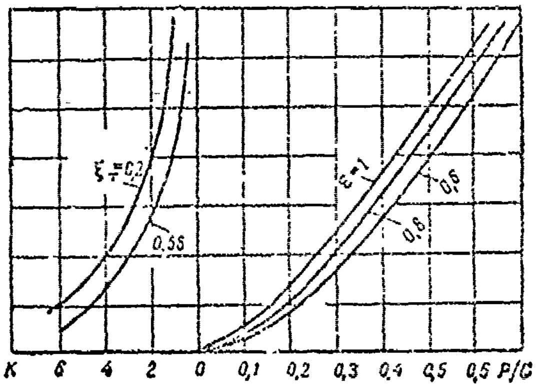


Fig. 3.12.

$$P_{\text{потр. ср}} = \frac{G_{\text{взл}}(2-\xi_{\text{т}})}{2K_{\text{ср}}}. \qquad (4)$$

Let us designate the TWR as

$$\varepsilon = \frac{P_{\text{взл}}}{G_{\text{взл}}}, \qquad (5)$$

then

$$P_{\text{потр. ср}} = \frac{P_{\text{взл}}(2-\xi_{\text{т}})}{2K_{\text{ср}}\varepsilon}. \qquad (6)$$

Thus the average TWR

$$\psi = \frac{P_{\text{потр. ср}}}{P_{\text{взл}}} = \frac{2-\xi_{\text{т}}}{2K_{\text{ср}}\varepsilon}. \qquad (7)$$

The dependence $\psi = f(\xi_{\text{т}}, K)$ is shown in Fig. 3.12. For the numerical data of our problem, with K = 5, $\xi_{\text{т}}$ = 0.23, and ε = 0.8, the required horizontal thrust ψ = (2 - 0.23)/(2·5·0.8) = 0.22. If $K_{\text{ср}}$ is doubled, the required average horizontal TWR is decreased by a factor of two.

PROBLEM 3.10. Establish the basic relationships among the interceptor characteristics during takeoff. Show how the takeoff run depends on the takeoff weight, the TWR, the LDR, air density, and coefficient of wheel friction.

Solution. The takeoff run $L_{\text{р}}$ is the path covered during the uniformly accelerated movement of the interceptor, from speed V = 0 to lift-off speed $V_{\text{отр}}$ in takeoff time $t_{\text{р}}$. Consequently,

$$L_{\text{р}} = V_{\text{ср}}t_{\text{р}} = \frac{V_{\text{отр}}t_{\text{р}}}{2} = \frac{V^2_{\text{отр}}}{2a_{\text{ср}}}, \qquad (1)$$

where $a_{\text{ср}}$ is the average acceleration, a value we assume as constant.

Now let us express $V_{\text{отр}}$ and $a_{\text{ср}}$ in terms of the basic interceptor characteristics and forces acting during takeoff.

At the moment of lift-off, the lift is balanced by the takeoff weight $G_{\text{взл}}$ minus the projection of engine thrust P onto the vertical axis:

$$G_{взл} - P\sin\alpha_{отр} = c_{y\,отр}\frac{\rho V_{отр}^2}{2}S, \qquad (2)$$

from which

$$V_{отр} = \sqrt{\frac{2(G_{взл} - P\sin\alpha_{отр})}{c_{y\,отр}\rho S}}, \qquad (3)$$

where $c_{y\ отр}$ is the coefficient of lift at the moment of lift-off, ρ is air density, S is wing area, and $\alpha_{отр}$ is the angle of attack at the moment of lift-off.

Acceleration a_{cp} is determined from the equation of motion of the interceptor during the take-off run:

$$ma_{cp} = P - Q - F_{тр}, \qquad (4)$$

$$m = \frac{G_{взл}}{g}, \qquad (5)$$

where m is the mass of the interceptor, Q is drag force, and $F_{тр}$ is wheel friction.

From (4), considering (5), we get

$$a_{cp} = \frac{g(P - Q - F_{тр})}{G_{взл}}. \qquad (6)$$

Since forces P, Q, and $F_{тр}$ change during the takeoff run, we must substitute their average values into (6).

We know that the average takeoff thrust is of the order of 0.95 of static thrust $P_{ст}$ (when V = 0), considering the losses in the engine inlet ducts:

$$P_{cp} = 0{,}95P_{ст}. \qquad (7)$$

The average total resistance of the interceptor is

$$(Q + F_{тр})_{cp} \simeq f'G_{взл}, \qquad (8)$$

where f' is the total coefficient of resistance to motion of the interceptor during the takeoff run.

We know that

$$f' = 0{,}5\left(f + \frac{1}{K_{отр}}\right), \quad (9)$$

where f is the coefficient of wheel friction and $K_{отр}$ is the LDR of the interceptor during lift-off.

Substituting (7)-(9) into (6) we get

$$a_{ср} = \frac{g\,(0{,}95P_{ст} - f'G_{взл})}{G_{взл}} = g\,(\varepsilon - f'), \quad (10)$$

where $\varepsilon = 0.95(P_{ст}/G_{взл})$ is the average effective TWR of the interceptor during the takeoff run.

Considering (3) and (10) we get the calculation formula for the length of the takeoff run:

$$L_{р} = \frac{G_{взл} - P\sin\alpha_{отр}}{c_{y\,отр}\rho S g\,(\varepsilon - f')}. \quad (11)$$

These formulas allow us to: calculate the speed of lift-off as a function of ambient temperature and pressure (having determined ρ), takeoff weight, and thrust; determine the length of the takeoff run as a function of these values and also of the runway slope and wind speed and direction.

In the calculations we can use the following values of f': 0.065-0.07 for concrete, 0.075-0.08 for hard ground, and 0.2-0.26 for dry viscous soil.

PROBLEM 3.11. Substantiate the tactical-technical requirements on the basic characteristics of the airborne radar of the interceptor and sighting, if the following are known: interceptor speed $V_{п}$ = = 2000 km/h; target speed $V_{ц}$ = 1800 km/h; range of possible rocket firings $D_{п.макс}-D_{п.мин}$ = 50-15 km; maximum altitude of target above interceptor $\Delta H_{р}$ = 5 km; total time to accomplish all operations of target acquisition and interception, interceptor homing, and preparation of the rockets for firing t_{Σ} = 30 s; interceptor-target homing method — constant-bearing approach.

Solution. The basic tactical-technical characteristics of the airborne radar of the interceptor are the maximum detection range, azimuth and elevation view angle, the accuracy in measuring the target's coordinates, and the range and azimuth resolutions.

Since there must be at least 30 seconds from the moment of target acquisition by the airborne radar to the moment the rockets are fired, while the closure speed is maximum with attack strictly on a head-on course, the maximum target detection range should meet the requirement

$$D_{обн.\,макс} \geqslant D_{п.\,макс} + (V_п + V_ц)\, t_у. \qquad (1)$$

The required azimuth view angle $\psi_п$ is determined by the maximum angle of target sighting with the given interceptor homing method. Since for constant-bearing approach the interceptor relative bearing

$$\varphi_п = \arcsin \frac{V_ц}{V_п} \sin \varphi_ц, \qquad (2)$$

while for all-aspect weapons the target relative bearing $\varphi_ц$ = 0-180°, considering that $\varphi_п$ reaches its maximum when $\varphi_ц$ = 90° we get

$$\varphi_{п.\,макс} = \arcsin \frac{V_ц}{V_п}. \qquad (3)$$

This is the required target sighting angle which, naturally, should be symmetric about the longitudinal axis of the aircraft. Consequently,

$$\psi_п \geqslant 2 \arcsin \frac{V_ц}{V_п}. \qquad (4)$$

For the data in our problem we have: $V_ц/V_п$ = 0.9, $\varphi_{п.макс}$ = = arc sin 0.9 = 64°, azimuth view angle should meet the requirement $\psi_п \geq 128°$. Let us note that when the target speed decreases or the interceptor speed increases, i.e., with a decrease in $V_ц/V_п$, the required azimuth view angle decreases. For example, when $V_ц/V_п$ = = 0.7, $\psi_п \geq 2 \cdot 44.5 = 89°$. The maximum view angle required from the standpoint of elevation is determined from the conditions of interception of the target with maximum vertical separation $\Delta H_р$, assuring destruction of the target by a rocket when fired from minimum range. For level interceptor flight this elevation is arc sin $(\Delta H_p/D_{п.мин})$. Considering the elevation view angle to be symmetric about the longitudinal axis of the aircraft, and considering the error in determining the target altitude $\Delta H_ц$, we find that the elevation view angle should satisfy the requirement

$$\psi_в \geqslant 2 \arcsin \frac{\Delta H_p + \Delta H_ц}{D_{п.мин}}. \quad (5)$$

For the numerical data of the problem and with $\Delta H_ц$ = 1 km we get

$$\psi_в \geqslant 2 \arcsin \frac{5+1}{15} = 47°.$$

The required target coordinate measurement accuracy can be substantiated by calculating the lead angle ψ when rockets are fired from distance $D_п$ to the lead point of target impact. For constant-bearing approach

$$\sin \psi = \frac{D_п \omega_в}{V_p}, \quad (6)$$

where $\omega_в$ is the angular rate of displacement of the target sighting ling and V_p is the average rocket velocity.

Let us differentiate Eq. (6). Then

$$\Delta D_п = \frac{V_p \cos \psi \Delta\psi}{\omega_в}. \quad (7)$$

The maximum error in range measurement occurs when $\cos \psi = 1$. If we allow an error $\Delta\psi = 1°$ in calculating the lead angle and assume that $\omega_в = 5$ °/s, then $\Delta D_п \leq 167$ m when $V_p = 3000$ km/h and $\Delta D_п \leq 16.7$ meters when $\omega_в = 50$ °/s. To substantiate the required resolution of the airborne radar, let us examine the occurrence, in the acquisition sector, of two targets flying at range D from the interceptor with distance d and interval l. Then, obviously, the range resolution should meet the requirement $\Delta D \leq d$, while the azimuth resolution should satisfy the relationship

$$\Delta \varphi_{аз} \leqslant \frac{l}{D}. \quad (8)$$

PROBLEM 3.12. The interceptor is guided, by the direct-intercept method, to a nonmaneuvering target at the same altitude (Fig. 3.13). The interceptor and target speeds are constant. Determine the required angle of automatic target tracking by the airborne radar (azimuth) if the attack should be guaranteed for any aspect angle, while the speed of the intercepted targets changes within the range from 1000 to 3000 km/h. Given:

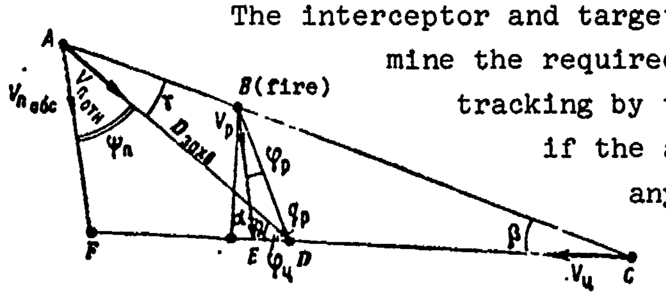


Fig. 3.13.

interceptor speed $V_п$ = 1700 km/h, rocket speed $V_р$ = 500 m/s, range for target lock-on by airborne radar $D_{захв}$ = 70 km, rocket firing range $D_п$ = 20-50 km.

Solution. According to the kinematic scheme for interception (Fig. 3.13) we have

$$\frac{AF}{FC}=\frac{BE}{EC}=\frac{V_п}{V_ц};$$

$$EC=D_р\frac{V_ц}{V_п};$$

$$\frac{V_п}{V_ц}=\frac{\sin q_р}{\sin \varphi_р}. \qquad (1)$$

The path covered by the rocket to encounter with the target

$$BE=D_р=\frac{D_п}{\sin(q_р+\varphi_р)}\sin q_р. \qquad (2)$$

The path covered by the target from the moment of firing to its encounter with the rocket is defined by the formula

$$DE=D_ц=\frac{D_п}{\sin(q_р+\varphi_р)}\sin\varphi_р. \qquad (3)$$

The target relative bearing at the moment of radar lock-on of the target

$$\varphi_ц=\gamma+\beta,$$

where

$$\beta=\operatorname{arc\,tg}\frac{\sin(q_р+\varphi_р)}{\cos(q_р+\varphi_р)+\frac{V_ц}{V_п}}; \qquad (4)$$

$$\gamma = \arc\sin \frac{\sin\beta}{D_{захв}}\left(D_p \frac{V_ц}{V_п} - D_ц\right). \qquad (5)$$

In Fig. 3.13 the target tracking or sighting angle is the angle between the interceptor relative velocity vector $V_{п.отн}$ and the interceptor absolute velocity vector $V_{п.абс}$.

The required angle for automatic target azimuth tracking is, naturally, a function of the attack aspect angle. For the given value of target relative bearing $\varphi_ц$ we have

$$\psi_п = 180° - \alpha - \varphi_ц = \varphi_p + q_p - \varphi_ц. \qquad (6)$$

Thus, to answer the question posed in the problem we must be given, in the range from 0 to 180°, various values of $\varphi_ц$ and calculate the unknown angle $\psi_п$ from known values of $V_ц$, $D_{захв}$, $D_п$, $V_п$, and V_p using Eqs. (1)-(6). The results of the calculations for the conditions stated in our problem are given in Fig. 3.14.

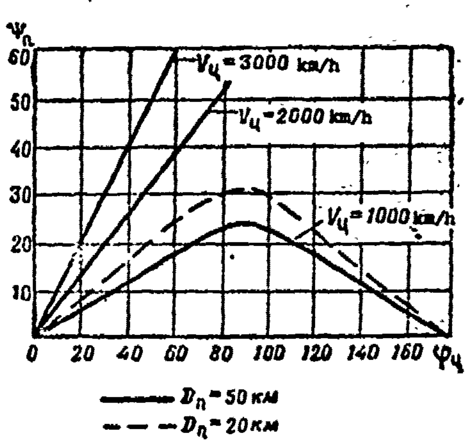


Fig. 3.14.

As the dependence $\psi_п = f(\varphi_ц)$ shows (Fig. 3.14), with relatively slow speeds of the targets to be intercepted the required azimuth angle of automatic tracking is maximum with a target relative bearing $\varphi_ц = 90°$. With target speeds $V_ц \geq 2000$ km/h, angle $\psi_п$ continuously increases with an increase in target relative bearing; if $\psi_п$ is restricted by the design of the antenna system or the interceptor fuselage, target attack is limited to a specific small range of target relative bearings from the forward hemisphere.

PROBLEM 3.13. Determine the required elevation angle of automatic target tracking by the airborne radar, if the target must be intercepted from the forward hemisphere with vertical separation. We consider that the rocket should be "illuminated" from the moment of firing to its impact with the target. Given: interceptor speed

$V_п$ = 1700 km/h, rocket speed $V_р$ = 500 m/s, firing distance $D_п$ = 50 km, vertical separation of target over interceptor at the moment of airborne radar target lock-on ΔH = 8 km, the speed of the intercepted targets varies within the range $V_ц$ = 1000-3000 km/h. How does the elevation angle of automatic target tracking change if the interceptor "zooms" before firing? Here we know the loss in interceptor speed in the "zoom" $\Delta V_п$ = 500 km/h.

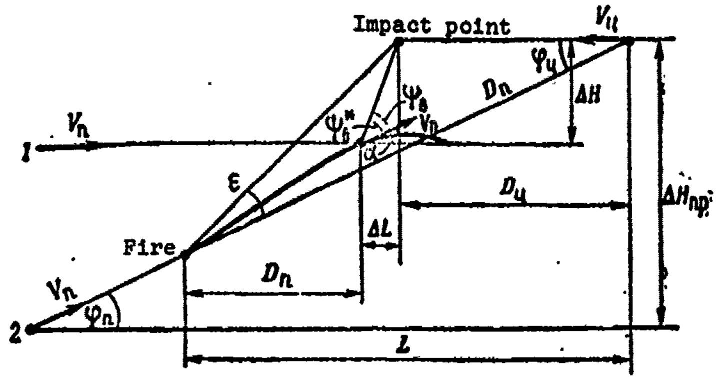


Fig. 3.15.

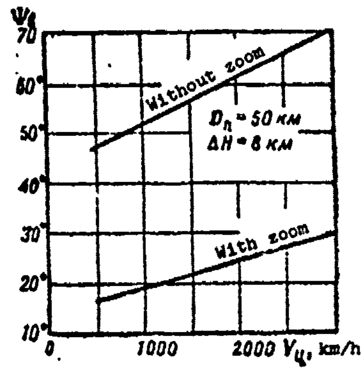


Fig. 3.16.

Solution. According to 1 in Fig. 3.15 we see that the required elevation angle of automatic target tracking without "zooming" is

$$\psi_в = \operatorname{arctg} \frac{\Delta H}{\Delta L}, \tag{1}$$

where

$$\Delta L = L - (D_ц + D_п); \tag{2}$$
$$D_ц = V_ц t; \tag{3}$$
$$D_п = V_п t; \tag{4}$$

t is the time from rocket firing to encounter with the target.

Considering the slope of the interceptor trajectory during the "zoom" we have (position 2 in Fig. 3.15)

$$\psi_в^* = \psi_в - \alpha, \tag{5}$$

where α is the pitch-up angle of the interceptor.

The gain in altitude during the "zoom" is defined by the relationship between kinetic and potential energies:

$$\frac{mV_{\text{п}}^2}{2} = mgH. \qquad (6)$$

Let us differentiate (6); then

$$\Delta H = \frac{V_{\text{п}} \Delta V_{\text{п}}}{g}. \qquad (7)$$

Thus, the altitude of the "zoom" is directly proportional to the interceptor speed $V_{\text{п}}$ and the loss in speed during the "zoom" $\Delta V_{\text{п}}$.

The firing distance is defined by the relationship

$$D_{\text{п}} = V_{\text{ц}} t \cos\varphi_{\text{п}} + (V_{\text{п}} + V_{\text{р}}) t \cos\varepsilon, \qquad (8)$$

from which

$$t = \frac{D_{\text{п}}}{V_{\text{ц}} \cos\varphi_{\text{п}} + (V_{\text{п}} + V_{\text{р}}) \cos\varepsilon}. \qquad (9)$$

Then we have

$$\varphi_{\text{п}} = \arc\sin\frac{\Delta H_{\text{пр}}}{D_{\text{п}}}; \qquad (10)$$

$$\varepsilon = \arc\sin\frac{V_{\text{п}}}{V_{\text{п}} + V_{\text{р}}} \sin\varphi_{\text{п}}. \qquad (11)$$

Formulas (1), (2), (5), (8), and (9) allow us to calculate the unknown angle $\psi_{\text{в}}$ as a function of target speed $V_{\text{ц}}$ and the given values of $V_{\text{п}}$, $V_{\text{р}}$, ΔH, and $D_{\text{п}}$. Figure 3.16 shows the results of the calculations. The graph $\psi_{\text{в}} = f(V_{\text{ц}})$ shows that a "zoom" makes it possible to significantly reduce the requirements on target tracking elevation by the airborne radar.

CHAPTER 4

EVALUATING COMBAT EFFECTIVENESS USING BASIC THEOREMS OF THE PROBABILITY THEORY

Since the outcome of combat between a fighter-interceptor and a target depends on a number of factors and generally is a random event, the quality of carrying out a combat problem can be evaluated quantitatively most completely by probability factors, e.g., the probability that a target will be destroyed (shot down).

In this chapter we give problems which illustrate the possible ways of using the basic theorems and laws of probability theory to evaluate various aspects of the combat efficiency of an interceptor when engaging in aerial combat.

The events of "hitting a target" and "destroying (shooting down) a target" are considered as equivalent in all the problems of this chapter.

If necessary, before examining the problems brush up on the basic assumptions and definitions of probability theory using [2].

PROBLEM 4.1. As a result of inaccurate surface guidance the interceptor finds itself either in the region of permissible rocket firing or in the region of cannon operation. Determine the probability that an aerial target will be downed, if the interceptor attacks once using either a rocket or cannon fire. The probability of downing the target when the interceptor is in the region of permitted rocket firing is 0.5, while that in the cannon-fire region is 0.3.

Solution. Event A – "interceptor entered the region of permitted rocket firing" – and event B – "interceptor entered the region of permitted cannon fire" – are mutually exclusive (entering one region excludes entry into the other). Therefore let us use the theorem of the addition of probabilities: the probability that one of two (or several) mutually exclusive events will occur is equal to the sum of the probabilities of the examined events. The unknown probability

$$P(A+B)=P(A)+P(B)=0,5+0,3=0,8.$$

PROBLEM 4.2. An interceptor armed with two rockets having semiactive homing heads is guided to an aerial target. With reliable functioning of the surface and airborne guidance systems the interceptor reaches the zone of permitted firing with a probability of 0.9. Provided there is failure-free operation of the firing system and the systems of rocket homing and destruction, one fired rocket will destroy a target with probability $P_1 = 0.8$. The interceptor fires both rockets at the target; each rocket is guided independently. The reliability of the surface guidance system $P_2 = 0.9$, that of the airborne radar during rocket control $P_3 = 0.95$, that of the rocket firing and homing system $P_4 = 0.8$, and that of explosion of the rocket warhead $P_5 = 0.9$. Determine the probability of downing a target, considering these reliability indicators.

Solution. The probability of downing a target is equal to the product of three independent probabilities: that of entering the zone of permitted firing, that of failure-free operation of the surface guidance system P_2, and that of destroying the target with two rockets. This latter probability, considering the given reliability indicators, is

$$1-(1-P_1P_3P_4P_6)^2=0{,}794.$$

Consequently, the probability of downing the target is 0.643.

PROBLEM 4.3. With surface guidance to a high-speed bomber, the interceptor finds itself at a certain random distance and random aspect angle relative to the target, resulting in random detection of the bomber. The probability P_1 of detecting the bomber at a distance of more than 100 km is 0.6; the probability P_2 of detecting it at a distance of less than 100 km is 0.4. If the target is detected at a distance of less than 100 km, the interceptor enters the aft hemisphere with probability $P_3 = 0.9$ and downs the bomber with probability $P_4 = 0.8$. If the bomber is detected at a distance of more than 100 km, the interceptor enters the forward hemisphere with probability $P_5 = 0.8$ and downs the bomber with probability $P_6 =$ ^ 7. Determine the probability of destroying the bomber.

Solution. Using the theorems of addition and multiplication of probabilities, we obtain the probability of destroying the bomber:

$$P=P_1P_5P_6+P_2P_3P_4=0{,}624.$$

PROBLEM 4.4. Determine the probability of destroying a target if the interceptor fires its rockets from one of three distances: D_1, D_2, and D_3. The probability of destroying the target with rocket firing at range D_1 is 0.4, at range D_2 – 0.5, and at range D_3 – 0.3.

Solution. Event A_1 – "firing the rocket at distance D_1" – is equivalent to the event $B_1 = A_1\bar{A}_2\bar{A}_3$ (the rocket is fired at range D_1 and not fired at ranges D_2 and D_3). Similarly, event A_2 – "firing the rocket at range D_2" – is equivalent to $B_2 = A_2\bar{A}_1\bar{A}_3$. Finally, event $B_3 = A_3\bar{A}_1\bar{A}_2$. The probability of destroying the target is equal to the probability of destruction with firing from only one of the three ranges, i.e., the probability $P(B_1 + B_2 + B_3)$ of the occurrence of one of the three events B_1, B_2, or B_3. Since events B_1, B_2, and B_3 are mutually exclusive, the theorem of addition of probabilities is applicable:

$$P(B_1+B_2+B_3)=P(B_1)+P(B_2)+P(B_3).$$

Now let us find the probabilities of each of the events B_1, B_2, and B_3. Events A_1, A_2, and A_3, and their opposite events $\bar{A}_1$, $\bar{A}_2$, and $\bar{A}_3$ are independent. Consequently, the theorem of multiplication of probabilities is applicable to them:

$$P(B_1)=P(A_1\bar{A}_2\bar{A}_3)=P(A_1)P(\bar{A}_2)P(\bar{A}_3)=0{,}4(1-0{,}5)(1-0{,}3)=$$
$$=0{,}14;$$
$$P(B_2)=P(A_2\bar{A}_1\bar{A}_3)=P(A_2)P(\bar{A}_1)P(\bar{A}_3)=$$
$$=0{,}5(1-0{,}4)(1-0{,}3)=0{,}21;$$
$$P(B_3)=P(A_3\bar{A}_1\bar{A}_2)=P(A_3)P(\bar{A}_1)P(\bar{A}_2)=$$
$$=0{,}3(1-0{,}4)(1-0{,}5)=0{,}09.$$

Thus, the probability of target destruction

$$P(B_1+B_2+B_3)=0{,}14+0{,}21+0{,}09=0{,}44.$$

PROBLEM 4.5. As the interceptor closes with the target it fires three rockets, one each firing, at three different ranges. The probability of destroying the target with firing at range $D_{макс}$ is $P_1 = 0.7$; at range $D_{ср}$ – $P_2 = 0.8$; at range $D_{мин}$ – $P_3 = 0.6$. To destroy the target it suffices at that least one rocket hit be scored. Determine the probability of destroying the target.

Solution. Event A_1 – "a hit by a rocket fired from range $D_{макс}$," event A_2 – "a hit by a rocket fired from range $D_{ср}$," and event A_3 – "a hit by a rocket fired from range $D_{мин}$" are independent in the aggregate. On the basis of the corollaries of the theorems of addition and multiplication of probabilities we find that the probability of at least one of events A_1, A_2, or A_3 is equal to the difference between one and the product of the probabilities of the opposite events. The probabilities of events opposite to events A_1, A_2, and A_3 (the probability of misses) are, respectively,

$$q_1=1-P_1=1-0{,}7=0{,}3;$$
$$q_2=1-P_2=1-0{,}8=0{,}2;$$
$$q_3=1-P_3=1-0{,}6=0{,}4.$$

Consequently, the probability of target destruction

$$P_{пор} = 1 - q_1 q_2 q_3 = 0{,}976.$$

PROBLEM 4.6. In order to shorten the intercept line, an aerial target is attacked simultaneously, but independently, by several interceptors. The probability that one interceptor will hit the target $P_1 = 0.4$. How many interceptors are required in order that the target be destroyed with a probability of at least 0.9?

Solution. Let us designate by A the following event: in an attack by n interceptors, at least one hits the target. Since the events consisting of hits by the first, second, third, etc. interceptors are independent in the aggregate and have identical probability P_1, then

$$P(A) = 1 - q^n$$

where

$$q = 1 - P_1 = 1 - 0{,}4 = 0{,}6.$$

By stipulation in the problem, $P(A) \geq 0.9$. Consequently, $0.9 \leq \leq 1 - 0.6^n$, or $0.1 \geq 0.6^n$. Taking the logarithm of this latter inequality we get

$$n \geqslant \frac{\lg 0{,}1}{\lg 0{,}6} = 4{,}5.$$

Thus, at least 5 interceptors are required to destroy a target with a probability of at least 0.9 with $P_1 = 0.4$.

PROBLEM 4.7. When attacking a single aerial target the interceptor downs it with a probability $P_1 = 0.9$. The interceptor attacks three separate targets once each. What is the probability that all three targets will be downed?

Solution. Since all three attacks are independent, the desired probability will be

$$0{,}9^3 = 0{,}729.$$

PROBLEM 4.8. An aerial target is attacked by two interceptors. The probability that the target will be destroyed by the first interceptor $P_1 = 0.8$, and by the second – $P_2 = 0.6$. What is the probability that the target will be destroyed by only one interceptor?

Solution. Destruction of a target by only one interceptor means that either the first interceptor successfully attacks the target while the attack of the second interceptor is unsuccessful, or vice versa. The probability of such an event is as follows:

$$P = 0{,}8\,(1 - 0{,}6) + 0{,}6\,(1 - 0{,}8) = 0{,}44.$$

PROBLEM 4.9. There are two interceptors of different types. The probability of destruction of an aerial target by the first interceptor $P_1 = 0.7$, by the second – $P_2 = 0.8$. What is the probability that the target will be destroyed with simultaneous attack by both interceptors?

Solution. The desired probability is equal to the probability P(A + B) of destruction of the target by at least one of the interceptors. The probability of destruction by each interceptor does not depend on the results of attack by the other, and therefore event A ("destruction of the target by the first interceptor") and event B ("destruction of the target by the second interceptor") are independent. According to the theorem of the addition of probabilities of compatible events, the probability P(A + B) is equal to the sum of the probabilities of events A and B minus that of their combined occurrence:

$$P(A+B) = P(A) + P(B) - P(AB) = 0{,}7 + 0{,}8 - 0{,}7 \cdot 0{,}8 = 0{,}94.$$

This problem can also be solved by using the formula

$$P(A+B) = 1 - q(A)\,q(B) = 1 - (1 - P_1)(1 - P_2) = \\ = 1 - (1 - 0{,}7)(1 - 0{,}8) = 0{,}94.$$

PROBLEM 4.10. The interceptor fires four rockets at an aerial target in sequence at various ranges and independently (e.g., with four independent successive attacks). The success in homing the rockets is characterized by the probabilities 0.3, 0.4, 0.5, and 0.3, while the probabilities of target destruction with explosion of the warheads of one, two, three, or four rockets in this case are 0.4, 0.7, 0.8, and 0.9, respectively. Determine the probability of destruction of the target.

Solution. For the solution we must use the formula for total probability. If a certain event A begins with a number of events H_i, called hypotheses and comprising a complete group of mutually exclusive events, the probability that event A will begin is equal to the sum of the products of the probability of each hypothesis times the probability of the onset of event A for the given hypothesis:

$$P(A)=\sum_{i=1}^{n}P(H_i)P\left(\frac{A}{H_i}\right).$$

Let us examine the following five mutually exclusive hypotheses comprising the total group: H_0 — none of the four rockets explodes at the target, H_1 — one rocket explodes at the target, H_2 — two rockets explode at the target, H_3 — three rockets explode at the target, and H_4 — all four rockets explode at the target.

Using the theorems of addition and multiplication of probabilities, we define the probabilities of these hypotheses:

$$P(H_0)=(1-0,3)(1-0,4)(1-0,5)(1-0,3)=0,147;$$
$$P(H_1)=0,3(1-0,4)(1-0,5)(1-0,3)+$$
$$+(1-0,3)0,4(1-0,5)(1-0,3)+(1-0,3)(1-0,4)0,5(1-0,3)+$$
$$+(1-0,3)(1-0,4)(1-0,5)0,3=0,351;$$
$$P(H_2)=0,3\cdot0,4(1-0,5)(1-0,3)+0,3(1-0,4)0,5(1-0,3)+$$
$$+0,3(1-0,4)(1-0,5)0,3+(1-0,3)0,4\cdot0,5(1-0,3)+$$
$$+(1-0,3)0,4(1-0,5)0,3+(1-0,3)(1-0,4)0,5\cdot0,3=0,335;$$
$$P(H_3)=0,3\cdot0,4\cdot0,5(1-0,3)+$$
$$+0,3\cdot0,4(1-0,5)0,3+0,3(1-0,4)0,5\cdot0,3+$$
$$+(1-0,3)0,4\cdot0,5\cdot0,3=0,129;$$
$$P(H_4)=0,3\cdot0,4\cdot0,5\cdot0,3=0,018.$$

The conditional probabilities of event A (target destruction) with realization of the examined hypotheses are as follows:

$$P(A/H_0)=0;\ P(A/H_1)=0,4;\ P(A/H_2)=0,7;\ P(A/H_3)=0,8;\ P(A/H_4)=0,9.$$

Having determined the probabilities of the hypotheses and the corresponding conditional probabilities of target destruction under these hypotheses, let us find, from the total-probability formula, the desired probability of target destruction:

$$P(A)=P(H_0)P(A/H_0)+P(H_1)P(A/H_1)+ \\ +P(H_2)P(A/H_2)+P(H_3)P(A/H_3)+ \\ +P(H_4)P(A/H_4)=0{,}147\cdot 0+0{,}351\cdot 0{,}4+ \\ +0{,}335\cdot 0{,}7+0{,}129\cdot 0{,}8+0{,}018\cdot 0{,}9=0{,}4943.$$

PROBLEM 4.11. Two interceptors independently fire one rocket each at an aerial target. The probability that the target will be destroyed by the rocket from the first interceptor is 0.9, by the rocket from the second – 0.6. As a result of the attack by the interceptors, one rocket hits the target. What is the probability that the target will be downed by the first interceptor?

Solution. The problem is solved using the Bayes formula (the theorem of hypotheses). If A can begin with any of n hypotheses H_i comprising a complete group of mutually exclusive events, and as a result of the experiment event A begins, the probability that event A will begin with the given hypothesis H_i ($i = 1, 2, \ldots, n$) is

$$P(H_i/A)=\frac{P(H_i)P(A/H_i)}{P(A)},$$

where $P(H_i/A)$ is the so-called *a posteriori probability* (probability after the fact), i.e., the probability that hypothesis H_i occurs provided that event A begins; $P(H_i)$ is the probability of hypothesis H_i before the fact (a priori probability); $P(A/H_i)$ is the probability that event A will begin provided that hypothesis H_i obtains; and P(A) is the probability that event A will begin, calculated from the formula for total probability.

Before target destruction (before the fact) we have the following mutually exclusive hypotheses comprising the total group of events: H_1 – neither rocket hits the target; H_2 – the rocket from the first interceptor hits the target, the second misses; H_3 – the rocket from the second interceptor hits the target, the first misses; and H_4 - both rockets hit the target.

The probabilities of these hypotheses are:

$$P(H_1)=(1-0{,}9)(1-0{,}6)=0{,}04;\quad P(H_2)=0{,}9(1-0{,}6)=0{,}36;$$
$$P(H_3)=0{,}6(1-0{,}9)=0{,}06;\quad P(H_4)=0{,}9\cdot 0{,}6=0{,}54.$$

The conditional probabilities of the observed event A (one rocket hits the target) with these hypotheses are

$$P(A/H_1)=0;\quad P(A/H_2)=1;\quad P(A/H_3)=1;\quad P(A/H_4)=0.$$

After event A has occurred (after the fact), hypotheses H_1 and H_4 become impossible. Consequently, the desired probability of hypothesis H_2 according to Bayes' formula is

$$P(H_2/A)=\frac{P(H_2)P(A/H_2)}{P(H_2)P(A/H_2)+P(H_3)P(A/H_3)}=$$
$$=\frac{0{,}36\cdot 1}{0{,}36\cdot 1+0{,}06\cdot 1}=0{,}857.$$

The probability of hypothesis H_3 is

$$P(H_3/A)=\frac{P(H_3)P(A/H_3)}{P(H_2)P(A/H_2)+P(H_3)P(A/H_3)}=$$
$$=\frac{0{,}06\cdot 1}{0{,}06\cdot 1+0{,}36\cdot 1}=0{,}143.$$

PROBLEM 4.12. Entering the region of possible rocket fire, an interceptor fires a rocket which downs a target. The extent of the region of possible firing is 2-10 km. The probability that firing occurs in the range 2-4 km is 0.2; 4-6 km - 0.3; 6-8 km - 0.4; and 8-10 km - 0.1. The probability that the target will be destroyed when the rocket is fired from 2-4 km is 0.3; 4-6 km -- 0.7; 6-8 km - C.8; and 8-10 km - 0.5. Determine the probability that firing will occur in the range of distances 6-8 km.†

Solution. First let us determine the probabilities of the possible hypotheses and the conditional probabilities of target

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† This probability is of practical interest when the pilot does not remember the firing range and the shot must be analyzed after the flight.

destruction under these hypotheses. Before destruction of the target (before the fact) four hypotheses are possible: H_1, H_2, H_3, and H_4, i.e., the rockets are fired from distances of 2-4, 4-6, 6-8, and 8-10 km, respectively.

The probabilities of these hypotheses are, respectively:

$$P(H_1)=0{,}2;\quad P(H_2)=0{,}3;$$
$$P(H_3)=0{,}4;\quad P(H_4)=0{,}1.$$

After target destruction (after the fact, resulting in observance of event A), the conditional probabilities of target destruction under the examined hypotheses are

$$P(A/H_1)=0{,}3;\ P(A/H_2)=0{,}7;\ P(A/H_3)=0{,}8;\ P(A/H_4)=0{,}5.$$

Now, using Bayes' formula, let us calculate the desired probability of hypothesis H_3:

$$P(H_3/A)=\frac{0{,}4\cdot 0{,}8}{0{,}2\cdot 0{,}3+0{,}3\cdot 0{,}7+0{,}4\cdot 0{,}8+0{,}1\cdot 0{,}5}=0{,}5.$$

PROBLEM 4.13. The interceptor fires four rockets independently at a maneuvering aerial target. The probability that each rocket will reach the region of reliable operation of the radio fuse is 0.6. Determine the probability that the radio fuses operate equally for two rockets at the target, while the other two rockets pass by without exploding.

Solution. Let us designate the events "operation of the radio fuse of the first, second, third, and fourth rockets" by A_1, A_2, A_3, and A_4, respectively, and the opposite events by $\bar{A}_1$, $\bar{A}_2$, $\bar{A}_3$, and $\bar{A}_4$. Then event B_2 – "the radio fuses of exactly two rockets operate at the target" – can occur in the following six ways:

$$A_1,\ A_2,\ \bar{A}_3,\ \bar{A}_4;\quad \bar{A}_1,\ A_2,\ A_3,\ \bar{A}_4,$$
$$A_1,\ \bar{A}_2,\ A_3,\ \bar{A}_4;\quad \bar{A}_1,\ A_2,\ \bar{A}_3,\ A_4;$$
$$A_1,\ \bar{A}_2,\ \bar{A}_3,\ A_4;\quad \bar{A}_1,\ \bar{A}_2,\ A_3,\ A_4.$$

Since these events are mutually exclusive, while events

$$A_1, A_2, A_3, A_4, \quad \bar{A}_1, \bar{A}_2, \bar{A}_3, \bar{A}_4$$

are statistically independent, then

$$B_2 = A_1A_2\bar{A}_3\bar{A}_4 + \bar{A}_1A_2A_3\bar{A}_4 + A_1\bar{A}_2A_3\bar{A}_4 + \\ + \bar{A}_1A_2\bar{A}_3A_4 + A_1\bar{A}_2\bar{A}_3A_4 + \bar{A}_1\bar{A}_2A_3A_4.$$

Since

$$P(A_1) = P(A_2) = P(A_3) = P(A_4) = P(A) = 0{,}6; \\ P(\bar{A}_1) = P(\bar{A}_2) = P(\bar{A}_3) = P(\bar{A}_4) = P(\bar{A}) = \\ = 1 - P(A) = 1 - 0{,}6 = 0{,}4,$$

then

$$P(B_2) = 6\,[P(A)]^2\,[P(\bar{A})]^2 = 6\cdot 0{,}6^2\cdot 0{,}4^2 = 0{,}346.$$

Here we used only the theorems of addition and multiplication of probabilities. However, it is more convenient to find the solution using the Bernoulli formula (the theorem of the repetition of trials), according to which the probability that event B will occur exactly m times, if n independent trials are realized, in each of which event B appears with probability P, is

$$P_{m,n} = C_n^m P^m (1-P)^{n-m},$$

where C_n^m is the number of combinations of n from m:

$$C_n^m = \frac{n!}{m!\,(n-m)!}.$$

For our example

$$P(B_2) = C_4^2\cdot 0{,}6^2 (1-0{,}6)^{4-2} = \frac{4!}{2!\,(4-2)!}\,0{,}6^2 (1-0{,}6)^2 = 0{,}346.$$

Let us remember that the set of probabilities $P_{m,n}$ is called the *binomial probability distribution*, since numerically the probabilities $P_{m,n}$ are equal to the coefficients of x^m in expansion of the binomial

$$(q + Px)^n$$

in powers of x, where q = 1 - P.

The binomial coefficients C_n^m can be obtained from the Pascal triangle:

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

etc.

In the Pascal triangle, each number is equal to the sum of the two numbers diagonally above it (e.g., 10 = 4 + 6).

PROBLEM 4.14. Two interceptors fire on the same target. The first fires 9 times per second and hits the target with a probability of 0.8; the second fires 10 times a second and hits the target with a probability of 0.7. The target is downed by the fire. Determine the probability that it will be downed by the second interceptor.†

Solution. Let us designate by A_1 and A_2 the events when the first and second interceptors, respectively, fire a round. Since the ratio of the rates of fire 9/10 = 0.9, then

$$P(A_1) = 0{,}9P(A_2).$$

Let us designate by B the event "hit during firing." Then, by the stipulation of the problem, we have

$$P(B/A_1) = 0{,}8$$

and

$$P(B/A_2) = 0{,}7.$$

The desired probability is found from Bayes' formula:

$$P(A_2/B) = \frac{P(A_2)P(B/A_2)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2)} =$$

$$= \frac{0{,}7P(A_2)}{0{,}9P(A_2)\,0{,}8 + 0{,}7P(A_2)} = \frac{0{,}7}{1{,}42} = 0{,}493.$$

PROBLEM 4.15. The probability of an interceptor's intercepting an aerial target in one sortie P_1 = 0.45. For interception, n = 100 sorties are flown. What is the probability P that interception will

† This probability is of practical interest when analyzing aerial gunnery from cannons having different rates of fire.

occur less than 51 times but more than 29 times, and less than 51 but more than 39 times?

Solution. Using the Bernoulli formula we find the desired probability

$$P = \sum_{m=30}^{50} C_{100}^{m} P_1^{m} (1 - P_1)^{100-m} =$$

$$= \sum_{m=30}^{50} \frac{100!}{m!\,(100-m)!} 0{,}45^{m} (1 - 0{,}45)^{100-m}. \qquad (1)$$

Direct calculation of this expression is very unwieldy. Therefore, let us use the approximation, familiar for large n, of the binomial distribution by normal distribution, with the same mathematical expectations and dispersions:

$$M[m] = nP_1 = 100 \cdot 0{,}45 = 45;$$
$$\sigma^2 = nP_1(1 - P_1) = 100 \cdot 0{,}45\,(1 - 0{,}45) = 24{,}75.$$

Then, using integration in place of summation we get

$$P = \frac{1}{\sigma\sqrt{2\pi}} \int_{30}^{50} e^{-\frac{(m - M[m])^2}{2\sigma^2}} dm = \frac{1}{\sqrt{\pi 49{,}5}} \int_{30}^{50} e^{-\frac{(m-45)^2}{49{,}5}} dm. \qquad (2)$$

Introducing the new integration variable

$$x = \frac{m - 45}{\sqrt{49{,}5}}, \qquad (3)$$

we find

$$P = \frac{1}{\sqrt{\pi}} \int_{-\frac{15}{\sqrt{49{,}5}}}^{\frac{5}{\sqrt{49{,}5}}} e^{-x^2} dx. \qquad (4)$$

Using tables for the Laplace function $\Phi(x)$ for the argument $x\sqrt{2}$ and $m/\sigma = 0$ we finally obtain

$$P = \frac{1}{2}\left[\Phi\left(\frac{5}{\sqrt{49{,}5}}\right) + \Phi\left(\frac{15}{\sqrt{49{,}5}}\right)\right] = 0{,}8386.$$

For the second condition of the problem, analogously, we get

$$P = \frac{1}{\sqrt{\pi}} \int_{-\frac{5}{\sqrt{49{,}5}}}^{\frac{5}{\sqrt{49{,}5}}} e^{-x^2} dx = \Phi(0{,}71) = 0{,}68.$$

This problem is instructive mainly from a methodological standpoint. It shows that with a large number of independent events (with large n) it is advisable, in place of unwieldy calculations using the Bernoulli formula, to approximate Eq. (1) by the probability integral and solve using tables. This solution is also valid for other practical problems, e.g.:

1. 100 missiles are fired at a target. The probability of one round's hitting the target $P_1 = 0.45$. Determine the probability that the target will receive less than 51, but more than 29, hits.

2. 100 rockets are stored in a warehouse for one year. During this period the parameters of each rocket will exceed the tolerance limits with probability $P_1 = 0.45$. Determine the probability that during this time it will be necessary to adjust the parameters of less than 51 but more than 29 rockets.

PROBLEM 4.16. An interceptor is armed with two rockets. The rockets can be fired within the limits from $D_{макс}$ to $D_{мин}$; the maximum probability of destruction is somewhere between the two, at some optimum range $D_{опт}$. The interceptor can select one of two tactics:

1) fire the rockets successively at $D_{макс}$ and $D_{опт}$;

2) fire a salvo at $D_{опт}$.

In the second case the target performs an evasive maneuver to break off the attack. Determine the most favorable rocket firing – salvo or successive – if the probability of carrying out the second attack $P_{ат2}$ is 0.5, the probability of target destruction with firing from $D_{макс}$ is $P_1 = 0.2$, and the probability of target destruction with firing at $D_{опт}$ is $P_2 = 0.6$.

Solution. An answer to the problem requires that we solve the inequalities

$$P_1 + (1 - P_1) P_2 + P_1 P_2 \lesseqgtr 1 - (1 - P_2)^2 \qquad (1)$$

or

$$P_1+P_{aт2}P_2(1-P_1)+P_1P_2P_{aт2}\lesseqgtr 1-(1-P_2)^2P_{aт2}. \qquad (2)$$

Inequalities (1) and (2) are written based on use of the theorems of addition and multiplication of probabilities. The resulting probability of target destruction with successive firing of two rockets and provided that the probability of carrying out a second attack $P_{aт2} = 1$ is equal to the sum of the probabilities of three mutually exclusive events:

1) the probability P_1 that the target will be destroyed by the first rocket;

2) the probability $(1 - P_1)P_2$ that the target will not be destroyed by the first rocket but will be destroyed with probability P_2 by the second rocket;

3) the probability P_1P_2 that the target will be destroyed by a combined hit by both rockets.

The resulting probability of target destruction with salvo fire of both rockets, considering the hits as independent events, is $1 - (1 - P_2)^2$. When the probability of carrying out a second attack $P_{aт2} \neq 1$, by analogous reasoning we get Inequality (2).

For the numerical data in our problem we have

$$0{,}2+(1-0{,}2)\,0{,}6+0{,}2\cdot 0{,}6<1-(1-0{,}6)^2;$$
$$0{,}8<0{,}84,$$

i.e., when $P_{aт2} = 1$ the most favorable fire is salvo fire at optimum range.

When $P_{aт2} = 0.5$ we have

$$0{,}2+0{,}5\cdot 0{,}6\,(1-0{,}2)+0{,}2\cdot 0{,}6\cdot 0{,}5>[1-(1-0{,}6)^2]\,0{,}5,$$

i.e., 0.5 > 0.42 and, consequently, sequential fire is more favorable.

Let us note that Inequalities (1) and (2) are also valid when there is more than one target. Then the resulting probabilities, i.e., both sides of the inequalities, represent the mathematical expectations of the number of downed targets, and solution of the appropriate inequality answers the question of which is more favorable: to fire all rockets at the first target or to attack both targets.

PROBLEM 4.17. The interceptor fires four rockets in succession. Each succeeding rocket is fired without analysis of the results of the preceding one; therefore, the target can be destroyed by any of the rockets. The probability of a rocket's hitting the target depends on the firing distance, and is P_1, P_2, P_3, and P_4, respectively, for the first, second, third, and fourth rockets. Determine the resulting probability of destroying the target $P_{\Sigma 4}$, if one hit is enough to destroy it. Generalize the solution for n rockets.

Assumptions: the firings are statistically independent; accumulation of damage is disregarded.

Solution. By stipulation of the problem, the probability of target destruction is none other than the probability that at least one rocket will hit the target, which is defined as the sum of the probabilities that any of the four rockets will hit the target while the other three do not, any two of the four will hit while the others will not, any three of the four will hit while the other will not, and, finally, that all four rockets will hit the target. To explain the specific regularity in the calculation formulas, let us first write expressions for the probabilities $P_{\Sigma 2}$ and $P_{\Sigma 3}$ of target destruction with firing of two and three rockets:

$$P_{\Sigma 2}=P_1(1-P_2)+(1-P_1)P_2+P_1P_2=P_1+P_2-P_1P_2; \quad (1)$$

$$\begin{aligned}P_{\Sigma 3}=&P_1(1-P_2)(1-P_3)+P_2(1-P_1)(1-P_3)+\\&+P_3(1-P_1)(1-P_2)+P_1P_2(1-P_3)+P_1P_3(1-P_2)+\\&+P_2P_3(1-P_1)+P_1P_2P_3=P_1+P_2+P_3-P_1P_2-P_1P_3-\\&-P_2P_3+P_1P_2P_3.\end{aligned} \quad (2)$$

Obviously, by analogy with the structure of Eqs. (1) and (2) we can write a formula to define the probability of target destruction by firing four rockets in succession:

$$\begin{aligned} P_{\Sigma 4} = P_1 + P_2 + P_3 + P_4 - P_1P_2 - P_1P_3 - P_1P_4 - P_2P_3 - \\ - P_3P_4 - P_2P_4 + P_1P_2P_3 + P_1P_2P_4 + P_2P_3P_4 + \\ + P_3P_4P_1 - P_1P_2P_3P_4. \end{aligned} \quad (3)$$

For any number of successive firings, when the probabilities of a hit are different for each firing, by comparing (1)-(3) we get the following general formula for calculating the probability of target destruction:

$$\begin{aligned} P_{\Sigma n} = P_1 + P_2 + \ldots + P_n - P_1P_2 - P_1P_3 - \ldots - P_{n-1}P_n + \\ + P_1P_2P_3 + P_1P_2P_4 + \ldots + (-1)^{n-1} P_1P_2P_3 \ldots P_n, \end{aligned} \quad (4)$$

where P_1P_2, P_1P_3, etc. are the probabilities of double hits; $P_1P_2P_3$, $P_1P_2P_4$, etc. are the probabilities of triple hits; $P_1P_2 \ldots P_n$ are the probabilities that all n rockets will hit.

We can easily see that when the probabilities of each rocket's hitting are equal

$$P_1 = P_2 = P_3 = \ldots = P_n$$

we get the familiar formula

$$P_{\Sigma n} = 1 - (1 - P_1)^n. \quad (5)$$

PROBLEM 4.18. An interceptor designed for two attacks, closing with a bomber at the maximum range of possible firing, downs it with one rocket with the probability $P_1 = 0.4$. During the rocket-homing process the interceptor comes into the bomber's zone of defensive fire and can be downed by this fire with a probability $P_n = 0.1$. If the interceptor is not damaged, in the second attack it goes to the optimum firing distance and downs the bomber with probability $P_2 = 0.7$. What are the probabilities for downing the target and the interceptor?

Solution. The bomber is destroyed in either the first or the second attack. The probability of the second event is defined as the product of three simultaneous events:

1) the bomber remains undamaged after the first attack; this probability is $1 - P_1 = 0.6$;

2) the interceptor remains undamaged after counterfire by the bomber; this probability is $1 - P_\text{п} = 0.9$;

3) the bomber is destroyed in the second attack.

Thus the probability of the interceptor's downing the bomber is

$$P_\text{б} = 0{,}4 + 0{,}6 \cdot 0{,}9 \cdot 0{,}7 = 0{,}778.$$

The probability $P_\text{п}$ of the bomber's downing the interceptor is equal to the probabilitie of two simultaneous events:

1) the bomber remains undamaged after the first attack;

2) the interceptor is downed by bomber counterfire.

Consequently,

$$P_\text{п} = 0{,}6 \cdot 0{,}1 = 0{,}06.$$

PROBLEM 4.19. One rocket hit is enough to destroy a target. Successive attacks are made on the target, with each succeeding rocket being fired only after analysis of the result of the previous one. Determine the average number $N_\text{п}$ of interceptors required to destroy a target if the probability of destruction by a hit from any of the fired rockets is P and each interceptor has m_p rockets.

Solution. The number of firings N needed to get one hit is a random number, having the values N = 1, 2, 3, ..., i, etc., to ∞. As the complete characteristic of this random number we have the distribution density whose ordinate P_i is equal to the probability of hitting a target during the i-th firing provided there were no hits in previous firings. Let us use the following designations: P_1 — the probability that the first rocket will hit the target; P_2 — the probability that the second rocket will hit the target and that the first will not; P_3 – the probability that the third rocket will hit the target and the first and second will not; etc.

The required number of interceptors, by definition of mathematical expectation, is

$$M[N_n] = M\left[\frac{N}{m_p}\right] = \frac{1}{m_p}\sum_{i=1}^{\infty} iP_i = \frac{1}{m_p}(1\cdot P_1 + 2P_2 + 3P_3 + \ldots). \qquad (1)$$

The probability P_i is equal to the probability $(1 - P)^{i-1}$ that in $(i - 1)$ firings there are no hits multiplied by the probability that the rocket hits the target in the i-th firing, i.e.,

$$P_i = P(1-P)^{i-1}. \qquad (2)$$

According to (2)

$$P_1 = P;\ P_2 = P(1-P);\ P_3 = P(1-P)^2\ldots$$

Consequently,

$$M[N_n] = \frac{1}{n_p}[P + 2P(1-P) + 3P(1-P)^2 + \ldots] = \frac{P}{m_p}\frac{1}{[1-(1-P)]^2} = \frac{1}{m_p P}. \qquad (3)$$

PROBLEM 4.20. A low-altitude target, covered with intense radio interference, must be intercepted. For the interception we have a formation of three interceptors; one of these, flying ahead of the other two, serves to relay guidance commands to the pair flying at the same altitude as the target. The target, with probability of 0.3, can cut off the possibility of attack by each of the interceptors with interference jamming either of the airborne radar or the guidance-command radio transmission lines. If any of the interceptors reaches the firing distance it downs the target with probability of 0.5. What is the probability that the target will be downed?

Solution. The probability of downing the target is calculated using the total-probability formula. Downing of the target can occur only in conjunction with one of three events (hypotheses) which form the total group of mutually exclusive events:

1) the target is attacked by all three interceptors;

2) the target is attacked by the relay plane and either of the other two;

3) the target is attacked only by the relay plane.

If the relay plane is jammed with interference, attacks by all three interceptors are cut off.

The probabilities of these favorable (relative to the event "target downing") hypotheses are as follows:

$$P_1 = 0{,}7^3 = 0{,}343; \quad P_2 = 2 \cdot 0{,}7^2 \cdot 0{,}3 = 0{,}294;$$
$$P_3 = 0{,}7 \cdot 0{,}3^2 = 0{,}063.$$

The conditional probabilities of target downing under these hypotheses are

$$P_{\text{пор}1} = 1 - 0{,}5^3 = 0{,}875; \quad P_{\text{пор}2} = 1 - 0{,}5^2 = 0{,}75;$$
$$P_{\text{пор}3} = 0{,}5.$$

The probability of target downing

$$P_{\text{пор}} = \sum_{i=1}^{3} P_i P_{\text{пор}i} = 0{,}343 \cdot 0{,}875 + 0{,}294 \cdot 0{,}75 + 0{,}063 \cdot 0{,}5 = 0{,}54.$$

PROBLEM 4.21. The interceptor is guided to an aerial target which may or may not use interference against the airborne radar, "jamming" its range channel. The command "Fire" is given to the pilot only if, for calculating the lead angle up to the moment of rocket firing, there is determination of the distance to the target at least once every n cycles of three-dimensional scanning of the airborne radar beam. If the target does not use interference, in one cycle the range is determined with probability $P_1 = 0.6$; if interference is used the probability $P_2 = 0.1$. The probability that the target will use interference is constant for any cycle, and equal to $P_n = 0.4$. What is the probability that, in n = 3 cycles of airborne radar scan, the command "Fire" will be given?

Solution. The total probability of determining the distance to the target in one radar scan cycle is

$$P_{д1} = (1 - P_{п}) P_1 + P_{п} P_2 = (1 - 0{,}4)\, 0{,}6 + 0{,}4 \cdot 0{,}1 = 0{,}4.$$

The desired probability for generating the command "Fire" is the probability that the distance to the target is determined at least once every n cycles of airborne radar scan. This probability is

$$P_{в.р} = 1 - (1 - P_{д1})^n = 1 - (1 - 0{,}4)^3 = 0{,}784.$$

PROBLEM 4.22. A group of interceptors "combs" a region of probable target location. Each interceptor has a flight path which it maintains with rms error σ = 5 km. The sector of scan for the airborne radar of each interceptor allows it to detect a frontal target in a sector of width 2b = 50 km (Fig. 4.1). To reduce the probability that a target will penetrate the region where the scan sectors of two adjacent interceptors join, there must be a specific overlap. Determine the necessary size of the overlap of adjacent scan sectors, if the permitted probability of separation due to inaccurate holding of the flight path should not exceed $P_p \leq 0.04$.

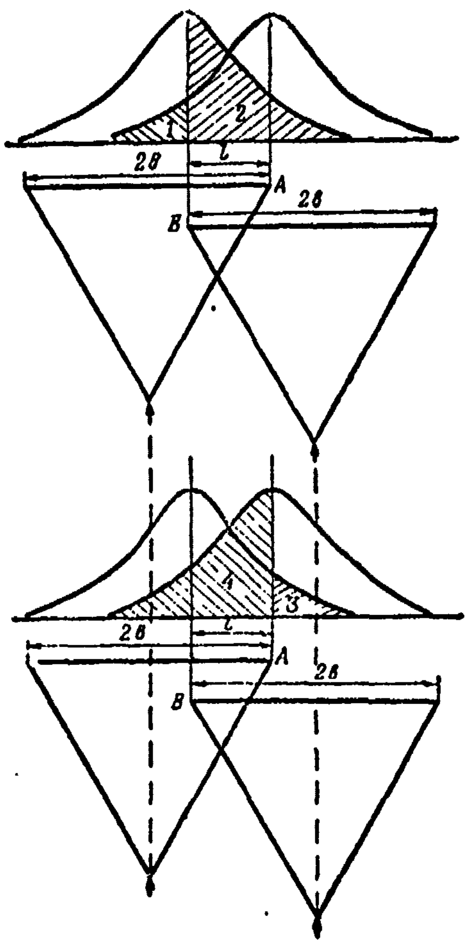

Fig. 4.1.

Solution. According to Fig. 4.1, a break between two adjacent sectors occurs in two cases: 1) point A of the sector of the left-hand interceptor is in region 1 while point B of the sector of the right-hand interceptor remains within region 2; and 2) point A is in region 4 while point B is in region 3.

The probability that there will be a break at the places where the sectors join is equal to the probability of two simultaneous independent events characterizing the two cases above, which in turn are mutually exclusive. Consequently, using the theorem of multiplication and addition of probabilities, we get

$$P_p = 2[0,5 - \Phi(l)]\,0,5 = 0,5 - \Phi(l),$$

where l is the width of the scan sector overlap.

Since by stipulation $P_p \leq 0.4$, then $\Phi(l) \geq 0.46$. From tables of the Laplace functions we find that $\Phi(l) = 0.46$ with $l = 2\sigma$.

Thus, is the overlap of the airborne radar scan sectors is 10 km, the probability of a break and, consequently, the probability of target penetration does not exceed 0.04.

CHAPTER 5

THE EFFECTIVENESS OF SURFACE GUIDANCE AND TARGET SEARCH AND DESTRUCTION

For operational-tactical calculations it is particularly important to have such quantitative indicators of combat efficiency for fighter-interceptors as the probability of surface guidance, the probability of target acquisition during search, and the probability of destroying the attacked target. This chapter is devoted to methods for calculating these probabilities.

To evaluate the effectiveness of surface guidance we establish the dependence between the probability of successful surface guidance, the tactical-technical characteristics of the interceptor, and the accuracy of the radar field of the ground system (Problems 5.1-5.7).

By the term *probability of surface guidance* we mean the probability that errors in the position of the interceptor velocity vector at the end of surface guidance relative to the direction toward the calculated lead point of target impact does not exceed permissible val -. Thus, the probability of surface guidance is defined by the relationships between permitted and random guidance errors. Therefore, in a number of problems we show how to calculate the permitted errors in heading surface guidance and random heading errors in surface guidance that arise due to errors in determining target and interceptor coordinates, speeds, and heading by the ground system.

In Problem 5.8 we show how to convert two-dimensional heading and altitude guidance errors into an equivalent one-dimensional guidance error.

In Problem 5.9 we show the functional change in the probability of surface guidance with a change in target-detection range by the airborne radar and a change in interceptor speed. The proposed solution method can be extrapolated to determining the dependences between relative changes in any parameter that influences the effectiveness and relative changes in the guidance probability.

In Problem 5.10 we obtain calculation formulas for determining the probability of detecting a target when searching a given region.

We show possible methods for calculating combat effectiveness if the deciding factor for the quality with which a combat problem is solved by an interceptor is minimum target penetration within the air defense area (Problem 5.11).

In Problem 5.12 we show how to determine the region of probable location of a target if no data are available on it, and the search region during independent combat operations by interceptors.

The dependence of the probability of destroying an attacked target on its dimensions and rocket miss is established by solving Problem 5.13.

PROBLEM 5.1. Surface guidance of an interceptor to a target is accomplished in the horizontal and vertical planes by transmission of heading and climbing commands. Altitude guidance is when, at detection distance $D_{обн}$, the target is in the elevation scan sector of the airborne radar $\pm\alpha_{в}$. For heading guidance it must be in the region bounded by permissible heading errors $\pm\Delta Q_{доп}$ which the interceptor and rocket can correct during homing. The ground system guides the interceptor with random errors having normal distribution and characterized by rms heading and altitude errors σ_Q and σ_H.

As a result of target heading maneuvers, during guidance there arises a certain dynamic errors in the interceptor heading m_Q, which is the mathematical expectation of the law of heading guidance error distribution. Express in general form the probability of surface guidance of an interceptor in terms of these errors.

Solution. The surface guidance probability is the probability that the resulting guidance error, consisting of random and dynamic errors, does not exceed the permitted error. Since guidance is performed independently in two planes, the guidance probability can be represented as the product of the conditional guidance probabilities in the horizontal and vertical planes:

$$P_{п.н}=P_{г}P_{в}. \qquad (1)$$

On the other hand, surface guidance can be interpreted as the entry of a random point (the interceptor), distributed by the normal law in the plane Q-H (heading-altitude), into a rectangle bounded by permitted heading and altitude errors (Fig. 5.1).

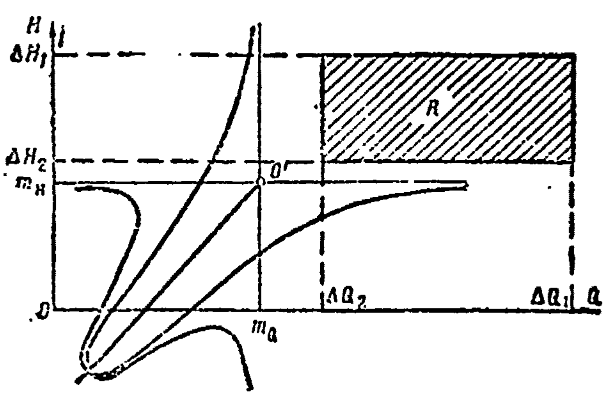

Fig. 5.1.

In the general case the normal density distribution in plane Q-H is written as follows [2]:

$$f(Q,H)=\frac{1}{2\pi\sigma_Q\sigma_H}\cdot e^{-\frac{(Q-m_Q)^2}{2\sigma_Q^2}-\frac{(H-m_H)^2}{2\sigma_H^2}}, \qquad (2)$$

where σ_Q, σ_H are the rms heading and altitude guidance errors; m_Q, m_H are the dynamic errors (constants) as a result of target heading and altitude maneuvering.

Errors m_Q and m_H define the shift in the center of the normal law relative to the center of the rectangle formed by the permitted guidance errors. These permitted guidance errors, measured in degrees, are the permitted deviations of the true interceptor trajectories from the calculated one in positive and negative directions of coordinate axes Q,H. In our example the target performs only heading maneuvers, and $m_H = 0$. To obtain the formula for calculating the guidance probability $P_{н.н}$ we must integrate the distribution density function within limits of the permitted errors:

$$P_{н.н} = \int_{-\Delta Q_{доп}}^{+\Delta Q_{доп}} \int_{-\Delta H_{доп}}^{+\Delta H_{доп}} f(Q, H)\, dQ\, dH =$$

$$= \int_{-\Delta Q_{доп}}^{+\Delta Q_{доп}} \frac{1}{\sigma_Q \sqrt{2\pi}} e^{-\frac{(Q-m_Q)^2}{2\sigma_Q^2}} dQ \int_{-\Delta H_{доп}}^{+\Delta H_{доп}} \frac{1}{\sigma_H \sqrt{2\pi}} e^{-\frac{(H-m_H)^2}{2\sigma_H^2}} dH. \quad (3)$$

The indefinite integral $\int e^{-t^2} dt$ is not expressed in terms of elemental functions, and is calculated using tables. Tables for this function (called the Laplace function or the probability integral) are given, e.g., in the literature [2, 3]. Thus, in the general case the formula for calculating surface guidance probability assumes the following form:

$$P_{н.н} = \frac{1}{4}\left[\Phi\left(\frac{\Delta Q_{доп} + m_Q}{\sigma_Q \sqrt{2}}\right) - \Phi\left(\frac{m_Q - \Delta Q_{доп}}{\sigma_Q \sqrt{2}}\right)\right]\Phi\left[\left(\frac{\Delta H_{доп} + m_H}{\sigma_H \sqrt{2}}\right) - \Phi\left(\frac{m_H - \Delta H_{доп}}{\sigma_H \sqrt{2}}\right)\right], \quad (4)$$

where

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \quad (5)$$

In our example $m_H = 0$; $\pm\Delta H_{доп} = \pm\alpha_в D_{обн}$. Then

$$P_{н.н} = \frac{1}{2}\left[\Phi\left(\frac{\Delta Q_{доп} + m_Q}{\sigma_Q \sqrt{2}}\right) - \Phi\left(\frac{m_Q - \Delta Q_{доп}}{\sigma_Q \sqrt{2}}\right)\right]\Phi\left(\frac{\alpha_г D_{обн}}{\sigma_H \sqrt{2}}\right), \quad (6)$$

where $D_{обн}$ is the airborne radar detection range.

When calculating $P_{н.н}$ it is convenient to use the graph in

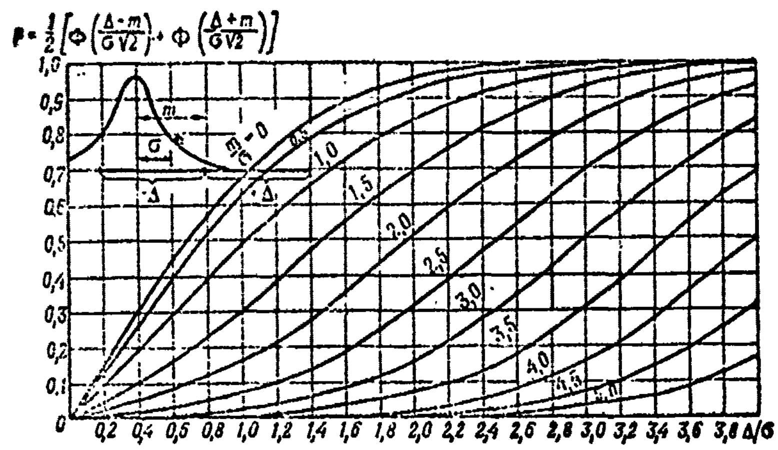

Fig. 5.2.

Fig. 5.2, which shows the dependence for the probability P that a random point, distributed normally with rms error σ and system error m, will fall in linear segment ±Δ.

PROBLEM 5.2. Derive a formula for calculating the rms interceptor relative bearing error arising due to errors in determining the interceptor and target coordinates.

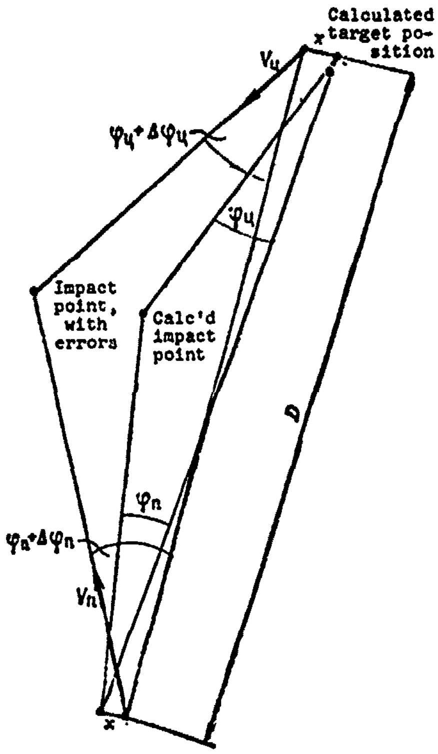

Fig. 5.3.

Solution. The error in interceptor relative bearing due to errors in determining the interceptor and target coordinates, as shown in Fig. 5.3, can be found from the following obvious relationship:

$$\Delta\varphi_{\text{п}} = \frac{\Delta V_{\text{п}}}{V_{\text{п}}}. \qquad (1)$$

Squaring $\Delta\varphi_{\text{п}}$ and taking the mathematical expectation of the square of this value we get

$$M\left[\frac{\Delta V_{\text{п}}^2}{V_{\text{п}}^2}\right] = \sigma^2_{\varphi_{\text{п}}} = \frac{\sigma^2_{V_{\text{п}}}}{V_{\text{п}}^2}. \qquad (2)$$

The interceptor velocity vector with respect to coordinates $x_{\text{н}}$, $x_{\text{к}}$ of the beginning and end of the segment covered by the interceptor in time $T_{\text{н}}$ is determined by the relationship

$$V_{\text{п}} = \frac{x_{\text{к}} - x_{\text{н}}}{T_{\text{н}}}. \qquad (3)$$

The error in speed due to errors in determining the coordinates of the beginning and end of the segment covered in $T_п$ is

$$\Delta V_п = \frac{\Delta x_к - \Delta x_н}{T_н}. \quad (4)$$

Let us square this error and take the mathematical expectation from the result. This will be the dispersion in determining the interceptor's speed:

$$M[\Delta^2 V_п] = \sigma^2_{V_п} = M\left[\frac{\Delta^2 x_к + \Delta^2 x_н - 2\Delta x_н \Delta x_к}{T_н^2}\right] = \\ = \frac{\sigma^2_{x_к} + \sigma^2_{x_н} + 2R_{x_н x_к}}{T_н^2}. \quad (5)$$

Since, because of the independence of errors $\Delta x_н$ and $\Delta x_к$, the correlation moment $R_{x_н x_к} = 0$, while $\sigma_{x_к} = \sigma_{x_н} = \sigma_x$, then

$$\sigma^2_{V_п} = \frac{2\sigma_x^2}{T_н^2}. \quad (6)$$

Substituting (6) into (2), we get the desired calculation formula for the rms error in the interceptor relative bearing:

$$\sigma_{\varphi_п} = \frac{\sigma_x \sqrt{2}}{T_н V_п}. \quad (7)$$

Having determined $\sigma_{\varphi_п}$ and knowing the permitted heading guidance errors we can, using Eq. (6) of Problem 5.1, calculate the probability of heading surface guidance of the interceptor. The results of calculations by Eq. (7) for σ_x = 0.5-5 km, $T_н$ = 5-20 s, and $V_п$ = 500-3000 km/h are given in Fig. 5.4.

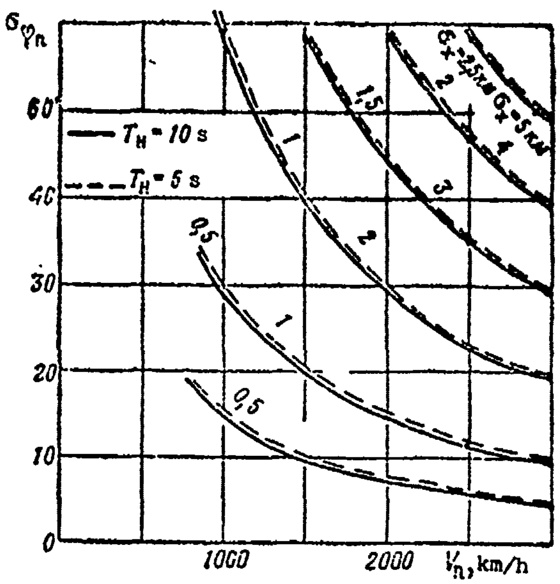

Fig. 5.4.

PROBLEM 5.3. Determine how the rms error in determining the required interceptor heading depends on the error in determining target speed $V_ц$, interceptor speed $V_п$, target relative bearing $\varphi_{\text{ц}}$, and locations of the interceptor and target. The interceptor is guided to a nonmaneuvering target by the constant-bearing approach method.

Solution. As the kinematic relationships show (Fig. 5.3), the lead angle, or interceptor relative bearing, uniquely defined the required interceptor heading which must be developed when solving the problem of interceptor contact with the target, and which must be transmitted aboard in the form of a guidance command. According to the method of constant-bearing approach guidance we have

$$\sin\varphi_{\text{п}} = \frac{V_{\text{ц}}}{V_{\text{п}}}\sin\varphi_{\text{ц}}. \tag{1}$$

The calculated values of $V_{\text{ц}}$, $V_{\text{п}}$, and $\varphi_{\text{ц}}$ which are used to determine the lead angle are random values, and differ from the true values of these parameters by values of the corresponding random measurement errors $\Delta V_{\text{ц}}$, $\Delta V_{\text{п}}$, and $\Delta\varphi_{\text{ц}}$. In addition, the error in determining the interceptor heading introduces inaccurate determination of the locations of the interceptor and target. Considering these errors $\Delta V_{\text{ц}}$, $\Delta V_{\text{п}}$, and $\Delta\varphi_{\text{ц}}$ we have

$$\sin(\varphi_{\text{п}} + \Delta\varphi_{\text{п}}) = \frac{V_{\text{ц}} + \Delta V_{\text{ц}}}{V_{\text{п}} + \Delta V_{\text{п}}}\sin(\varphi_{\text{ц}} + \Delta\varphi_{\text{ц}}). \tag{2}$$

Assuming the errors Δ to be small values, and disregarding values of the second order of smallness, after transformations of the trigonometric expressions in (2) we get

$$\Delta\varphi_{\text{п}} = \frac{V_{\text{ц}}\sin\varphi_{\text{ц}}}{V_{\text{п}}\cos\varphi_{\text{п}}} - \operatorname{tg}\varphi_{\text{п}} - \frac{\operatorname{tg}\varphi_{\text{п}}}{V_{\text{п}}}\Delta V_{\text{п}} + \frac{\sin\varphi_{\text{ц}}}{V_{\text{п}}\cos\varphi_{\text{п}}}\Delta V_{\text{ц}} + \frac{V_{\text{ц}}\cos\varphi_{\text{ц}}}{V_{\text{п}}\cos\varphi_{\text{п}}}\Delta\varphi_{\text{ц}}. \tag{3}$$

Since when generating the heading guidance command we use Eq. (1), the error in determining the interceptor relative bearing is determined only by those terms in (3) which contain errors $\Delta V_{\text{п}}$ and $\Delta V_{\text{ц}}$ in determining interceptor and target speeds and error $\Delta\varphi_{\text{ц}}$ in determining target relative bearing. Consequently,

$$\Delta\varphi_{\text{п}} = \frac{\sin\varphi_{\text{ц}}}{V_{\text{п}}\cos\varphi_{\text{п}}}\Delta V_{\text{ц}} + \frac{V_{\text{ц}}\cos\varphi_{\text{ц}}}{V_{\text{п}}\cos\varphi_{\text{п}}}\Delta\varphi_{\text{ц}} - \frac{\operatorname{tg}\varphi_{\text{п}}}{V_{\text{п}}}\Delta V_{\text{п}}. \tag{4}$$

Here the error in determining the velocity

$$\Delta V=\frac{\Delta x\sqrt{2}}{T}, \tag{5}$$

while the error in determining the target relative bearing

$$\Delta\varphi_{ц}=\Delta\varphi_{ц.Q_{ц}}+\Delta\varphi_{ц.x}, \tag{6}$$

where Δx is the error in coordinate determination, T is the observation time during which the speed is determined (discreteness of coordinate measurement), $\Delta\varphi_{ц.Q_{ц}}$ is the error in determining the target relative bearing as a result of target heading error $Q_{ц}$, and $\Delta\varphi_{ц.x}$ is the error in determining target relative bearing due to error in determining target coordinates.

To determine the dispersion of interceptor relative bearing it is necessary to square both sides of Eq. (4) and find the mathematical expectation from the result, using familiar laws [2]. Having performed these operations we get

$$\sigma^2_{\varphi_{п}}=\frac{1}{V^2_{п}\cos^2\varphi_{п}}\left[\sin^2\varphi_{ц}\sigma^2_{V_{ц}}-2\sin\varphi_{ц}\sin\varphi_{п}R_{V_{п}V_{ц}}+\right.$$
$$+\sin^2\varphi_{п}\sigma^2_{V_{п}}+2V_{ц}\cos\varphi_{ц}\sin\varphi_{ц}R_{\varphi_{ц}V_{ц}}-$$
$$\left.-2V_{ц}\cos\varphi_{ц}\sin\varphi_{п}R_{\varphi_{ц}V_{п}}+V^2_{ц}\cos^2\varphi_{ц}\sigma^2_{\varphi_{ц}}\right]. \tag{7}$$

Now let us determine the rms errors $\sigma_{V_{ц}}$, $\sigma_{V_{п}}$, and $\sigma_{\varphi_{ц}}$ in Eq. (7).

1. The rms error in determining target speed – $\sigma_{V_{ц}}$.

As can be seen from Fig. 5.5, the error in target speed is due to errors in determining the coordinates of two target blips from which the value of $V_{ц}$ is determined. Let us designate the distance between points $Ц_1$ and $Ц_2$ by d, the observation time by T, and the random errors in coordinate determination by Δx_1 and Δx_2. Then the measured value of target speed

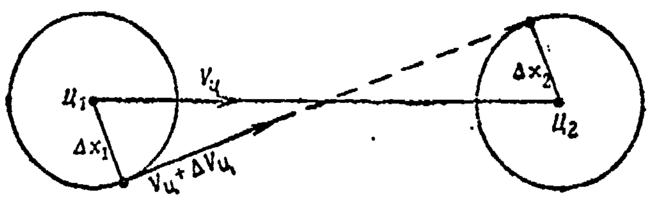

Fig. 5.5.

$$V_{ц} + \Delta V_{ц} = \frac{d}{T} + \frac{\Delta x_1 + \Delta x_2}{T}, \tag{8}$$

while the absolute error in determining target speed

$$\Delta V_{ц} = \frac{\Delta x_1 + \Delta x_2}{T}. \tag{9}$$

To find the dispersion of this error we must square (9) and take the mathematical expectation of the result. Then

$$\sigma^2_{V_{ц}} = \frac{1}{T^2}\left(\sigma^2_{x_1} + \sigma^2_{x_2} + 2R_{x_1 x_2}\right). \tag{10}$$

Because of the statistical independence of errors Δx_1 and Δx_2, correlation moment $R_{x_1 x_2}$ is equal to zero. The random errors in coordinate measurement are constant:

$$\sigma_{x_1} = \sigma_{x_2} = \sigma_x. \tag{11}$$

Consequently, for the dispersion in determining target speed we get the following calculation formula:

$$\sigma^2_{V_{ц}} = \frac{2\sigma^2_x}{T^2}. \tag{12}$$

2. The dispersion in determining interceptor speed is similarly determined, i.e.,

$$\sigma^2_{V_{п}} = \frac{2\sigma^2_x}{T^2}. \tag{13}$$

3. The dispersion in determining target relative bearing is

$$\sigma^2_{\varphi_{ц}} = \sigma^2_{\varphi_{ц Q_{ц}}} + \sigma^2_{\varphi_{ц x}} + 2R_{\varphi_{ц Q_{ц}} \cdot \varphi_{ц x}}, \tag{14}$$

since, on the basis of (6), the error in determining the target relative bearing is composed of the error in target relative bearing as a result of error in target heading $\Delta\varphi_{цQ_{ц}}$ and the error of the target relative bearing due to error in measuring the target coordinates $\Delta\varphi_{ц.x}$. Let us define the calculation expressions for the terms in (14). According to Fig. 5.6 the error in determining the target

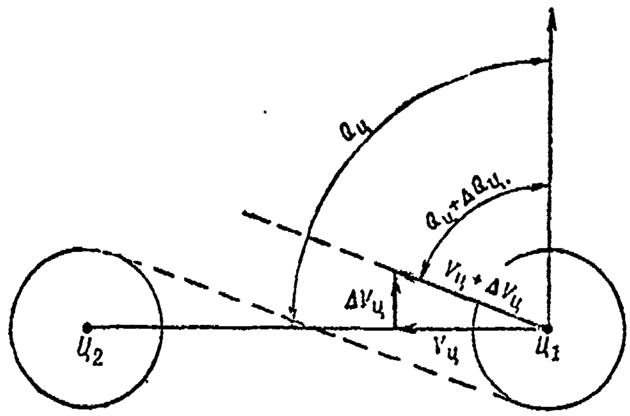

Fig. 5.6.

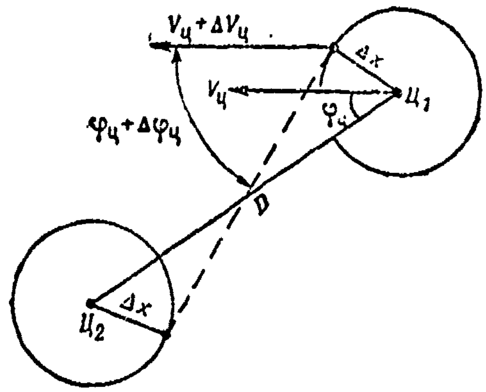

Fig. 5.7.

relative bearing due to inaccurate measurement of target heading

$$\Delta\varphi_{\text{ц}Q_\text{ц}} = \frac{\Delta V_\text{ц}}{V_\text{ц}}. \qquad (15)$$

Consequently, the dispersion of this error

$$\sigma^2_{\varphi_{\text{ц}Q_\text{ц}}} = \frac{\sigma^2_{V_\text{ц}}}{V^2_\text{ц}}. \qquad (16)$$

Substituting the value $\sigma^2_{V_\text{ц}}$ according to (12), we get

$$\sigma^2_{\varphi_{\text{ц}Q_\text{ц}}} = \frac{2\sigma_x^2}{T^2V_\text{ц}^2}. \qquad (17)$$

According to Fig. 5.7, the error in determining target relative bearing due to errors in target coordinate measurement

$$\Delta\varphi_{\text{ц}x} = \frac{\Delta x}{D}, \qquad (18)$$

while the dispersion of this error

$$\sigma^2_{\varphi_{\text{ц}x}} = \frac{2\sigma_x^2}{D^2}. \qquad (19)$$

The correlation moment between the values $\varphi_{\text{ц}Q_\text{ц}}$ and $\varphi_{\text{ц}x}$

$$R_{\varphi_{\text{ц}Q_\text{ц}}\varphi_{\text{ц}x}} = M\left[\Delta\varphi_{\text{ц}Q_\text{ц}}\Delta\varphi_{\text{ц}x}\right] = \frac{\sigma_x^2}{TV_\text{ц}D}. \qquad (20)$$

Thus, after substitution of (17), (19), and (20) into (14) we get the following calculation formula for dispersion in determining the target relative bearing:

$$\sigma^2_{\varphi_\text{ц}} = \frac{2\sigma_x^2}{V_\text{ц}^2 T^2} + \frac{2\sigma_x^2}{D^2} + \frac{\sigma_x^2}{TV_\text{ц} D}. \tag{21}$$

By similar methods we determine the correlation moment between target relative bearing and target speed:

$$R_{\varphi_\text{ц} V_\text{ц}} = 2\sigma_x^2 \left(\frac{1}{DT} + \frac{1}{T^2 V_\text{ц}}\right) \tag{22}$$

and the correlation moment between target relative bearing and interceptor speed:

$$R_{\varphi_\text{ц} V_\text{п}} = -\frac{\sigma_x^2}{DT}. \tag{23}$$

Since speeds $V_\text{п}$ and $V_\text{ц}$ are statistically independent, the correlation moment between these values is zero:

$$R_{V_\text{ц} V_\text{п}} = 0. \tag{24}$$

Substituting (12), (13), (21), (22), and (23) into (7), we get the following calculation formula for the dispersion of the heading error of interceptor surface guidance:

$$\sigma^2_{Q_\text{расч}} = \sigma^2_{\varphi_\text{п}} = \frac{1}{V_\text{п}^2 \cos^2 \varphi_\text{п}} \left[\frac{2 \sin^2 \varphi_\text{п}}{T^2} \sigma_x^2 + \sin^2 \varphi_\text{п} \sigma^2_{V_\text{п}} + 2V_\text{ц} \cos \varphi_\text{ц} \sin \varphi_\text{п} \left(\frac{1}{DT} + \frac{1}{T^2 V_\text{ц}}\right) \sigma_x^2 + 2V_\text{ц} \cos \varphi_\text{ц} \sin \varphi_\text{п} \frac{\sigma_x^2}{DT} + V_\text{ц}^2 \cos^2 \varphi_\text{ц} \left(\frac{2\sigma_x^2}{V_\text{ц}^2 T^2} + \frac{2\sigma_x^2}{D^2} + \frac{\sigma_x^2}{V_\text{ц} TD}\right)\right]. \tag{25}$$

Let us stress that Eq. (25) was obtained under the assumption that the error in determining the required heading is equal to the error in determining the lead angle, while we can disregard the error in determining the position of the sighting line "interceptor-target."

We can show that Eq. (25) is also valid for the case when the interceptor is guided to the target by the direct-intercept method. The results of calculations performed using Eq. (25) for a wide range of initial data are given in Figs. 5.8-5.10.

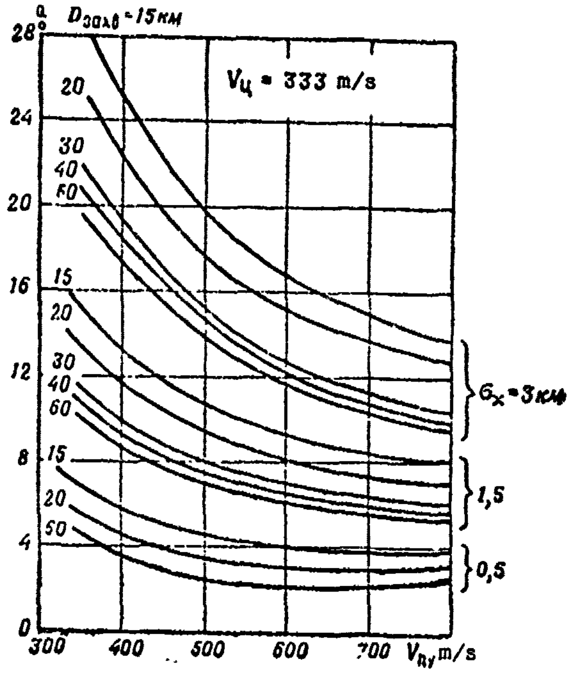

Fig. 5.8.

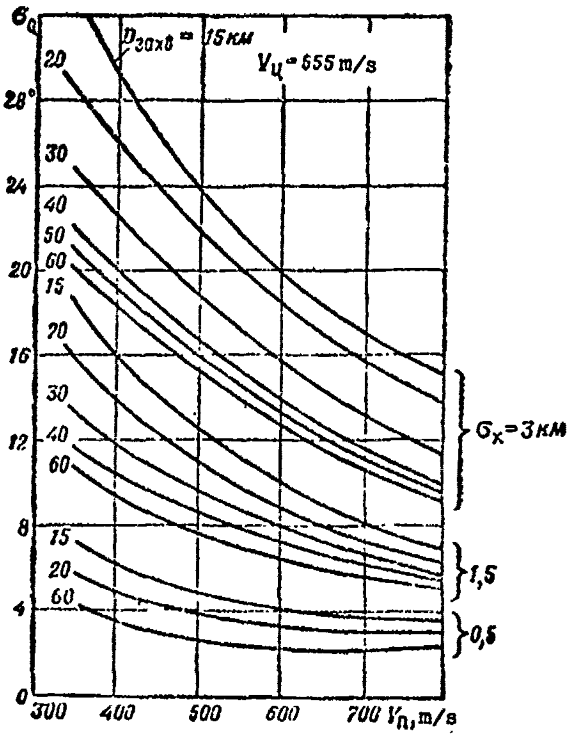

Fig. 5.9.

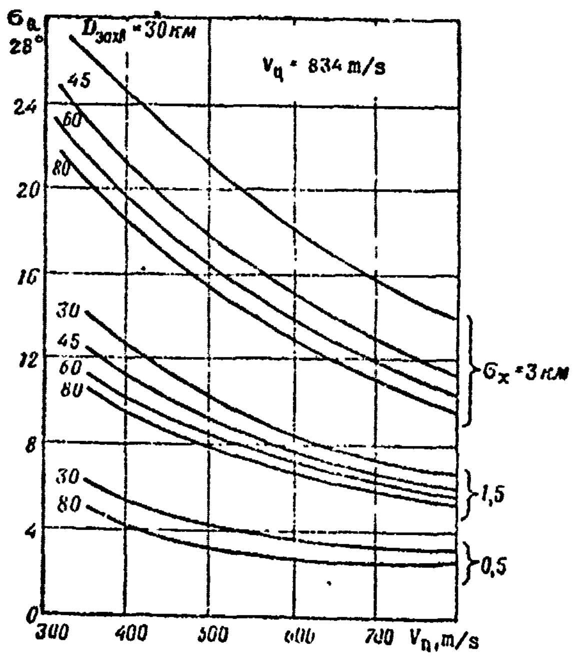

Fig. 5.10.

The calculated values of the rms errors in interceptor heading are the initial values for determining the probability of surface guidance, if for the given conditions of combat operation we know the permitted errors in interceptor heading guidance.

The methods for determining the permitted guidance errors are presented in the following problems.

PROBLEM 5.4. Show how the permitted heading guidance errors depend on the interceptor and rocket characteristics. Assumptions: the interceptor is guided on head-on or pursuit courses at small aspect angles. The heading error is corrected by turning the interceptor and the rocket with constant radii. The delay of the rocket and interceptor in processing the guidance commands is disregarded.

Solution. Let us designate by Δx the linear permitted error in guidance, and by D the distance from the interceptor at the moment of lock-on to the point at which the rocket meets the target. Then when the interceptor is guided, at small aspect angles, to the forward and aft hemispheres, the permitted heading guidance error

$$\Delta Q_{доп} = \frac{\Delta x}{D} \text{ in radians.} \tag{1}$$

The linear permitted error Δx should equal the sum of the projections of the distances covered by the interceptor, rocket, and target during the closing process onto a perpendicular to the direction "interceptor-target" at the moment of lock-on. As the interceptor turns, lateral acceleration occurs, directed toward a decrease in Δx. The interceptor moves with uniform acceleration in this direction, and the distance covered is $a_п t_п^2/2$. Similarly, the distance covered in this direction by the rocket is $a_р t_р^2/2$, and by the target $a_ц(t_п + t_р)^2/2$. Consequently,

$$\Delta x = \frac{a_п t_п^2}{2} + \frac{a_р t_р^2}{2} - \frac{a_ц (t_п + t_р)^2}{2}, \tag{2}$$

where

$$t_п = \frac{D_{max} - D_п}{V_{сбл.\,п-ц}}; \quad t_р = \frac{D_п}{V_{сбл.\,р-ц}}; \quad a = \frac{V^2}{R}; \tag{3}$$

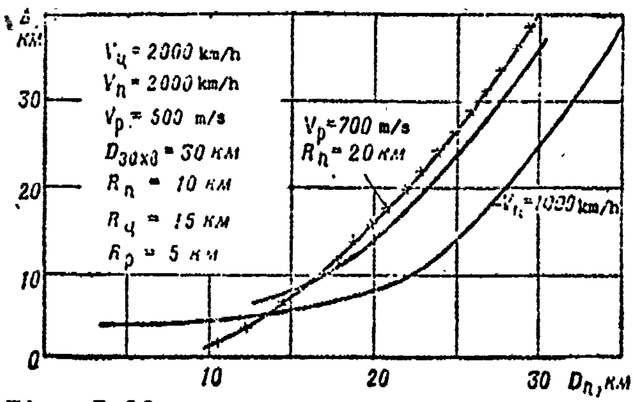

Fig. 5.11.

$t_п$ is the time of interceptor homing up to the time of rocket firing; $t_р$ is the rocket f..ght time to target impact; $D_{захв}$, $D_п$ are the lock-on and firing ranges; $V_{сбл.п-ц}$, $V_{сбл.р-ц}$ are the corresponding closing speeds; R is the radius of turn.

The results of calculations for several values of the characteristics are given in Fig. 5.11.

PROBLEM 5.5. The interceptor is guided to an aerial target by the pursuit method. The rocket is also homed by the pursuit method. Find the dependence of the permitted surface-guidance heading errors $\Delta Q_{доп}$ on the target, interceptor, and rocket speeds, the lock-on and firing ranges, and the turn radii for the interceptor and rocket. We examine two cases: 1) the firing error is zero, and the entire permitted surface-guidance error is corrected by the interceptor itself; 2) surface-guidance errors are corrected by the interceptor and the rocket.

Assumptions: 1) the speeds and turn radii are constant; 2) inertia and lag are disregarded.

Solution. For the case when the surface-guidance errors are corrected only by the interceptor, at the moment of firing the error is zero and the interceptor strictly carries out the pursuit method, while the rocket during the stage of homing until impact with the target also does not deviate from the pursuit method.

According to Fig. 5.12, which shows a kinematic scheme for correcting surface-guidance errors, the interceptor flight time from the lock-on range $D_{захв}$ to firing of the rocket is

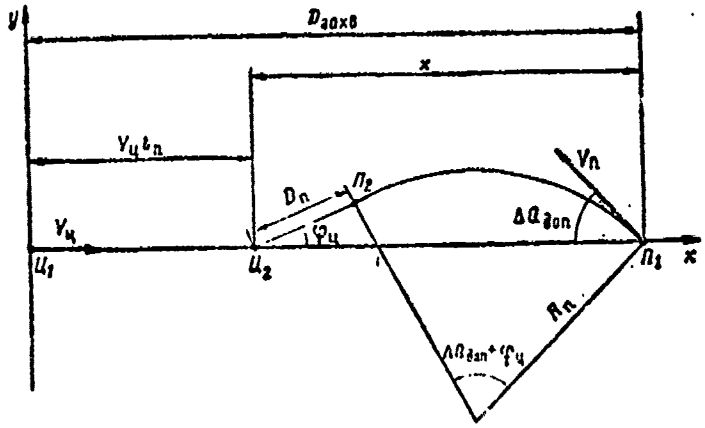

Fig. 5.12.

$$t_п = \frac{\Delta Q_{доп} + \varphi_ц}{\omega_п} = R_п \frac{\Delta Q_{доп} + \varphi_ц}{V_п}, \quad (1)$$

where $\varphi_ц$ is the target relative bearing at the moment of rocket firing; $\omega_п = V_п / R_п$ is the angular velocity of the interceptor.

Let us write the projections onto axes x and y:

$$D_п \cos\varphi_ц = D_{захв} - V_ц t_п - R_п \sin\Delta Q_{доп} - R_п \sin\varphi_ц; \quad (2)$$

$$D_п \sin\varphi_ц = R_п \cos\varphi_ц - R_п \cos\Delta Q_{доп}. \quad (3)$$

Substituting the time $t_п$ into Eq. (2) we get

$$R_п \sin\Delta Q_{доп} = D_{захв} - \frac{R_п}{p}(\Delta Q_{доп} + \varphi_ц) - R_п \sin\varphi_ц - D_п \cos\varphi_ц; \quad (4)$$

$$R_п \cos\Delta Q_{доп} = R_п \cos\varphi_ц - D_п \sin\varphi_ц, \quad (5)$$

where

$$p = \frac{V_п}{V_ц}.$$

Let us introduce the designations

$$A = D_{захв} - \frac{R_п}{p}\varphi_ц - R_п \sin\varphi_ц - D_п \cos\varphi_ц; \quad (6)$$

$$B = R_п \cos\varphi_ц - D_п \sin\varphi_ц. \quad (7)$$

Then Eqs. (4) and (5) are written as follows:

$$R_п \sin\Delta Q_{доп} = A - \frac{R_п}{p}\Delta Q_{доп}; \quad (8)$$

$$R_п \cos\Delta Q_{доп} = B. \quad (9)$$

Let us square these last two equations and combine them. Then we obtain the following quadratic equation for the permitted surface-avoidance heading error:

$$R_п^2 = A^2 - 2A\frac{R_п}{p}\Delta Q_{доп} + \frac{R_п^2}{p^2}\Delta Q_{доп}^2 + B^2 \quad (10)$$

or

$$\Delta Q^2_{зоп} - 2A\frac{p}{R_п}\Delta Q_{зоп} + \frac{A^2 + B^2 - R_п^2}{R_п^2}p^2 = 0. \tag{11}$$

Solution of quadratic Eq. (11) gives the desired dependence

$$\Delta Q_{зоп} = \frac{p}{R_п}[A \pm \sqrt{R_п^2 - B^2}]. \tag{12}$$

It remains to express angle $\varphi_ц$ in terms of the given values. For the pursuit method, when the target does not maneuver, we have the following expression for the angular velocity of turn of the rocket, which in this case is equal to the angular velocity of the sighting line:

$$\frac{V_ц \sin\varphi_ц}{D_ц} = \omega_л = \omega_p = \frac{V_p}{R_p}, \tag{13}$$

where the rocket's radius of turn is defined from the familiar relationship

$$R_p = \frac{V_p^2}{g\sqrt{n_p^2 - 1}}. \tag{14}$$

Thus,

$$\varphi_ц = \arcsin\frac{V_p}{R_p}\frac{D_п}{V_п}. \tag{15}$$

In the particular case of $V_p = V_п$,

$$\varphi_ц = \arcsin\frac{pD_п}{R_p}.$$

Now let us examine the case when part of the surface-guidance heading error is corrected by the rocket. In this case the interceptor turns, as shown in Fig. 5.13, by the angle

$$\Delta Q_{зоп} + \varphi_ц - \Delta Q_p. \tag{16}$$

The angular velocity of turn of the rocket is written in this case as follows:

$$\omega_p = \frac{V_ц \sin\varphi_ц}{D_п} + \frac{V_p \sin\Delta Q_p}{D_п} = \frac{V_p}{R_p}. \tag{17}$$

The maximum possible target relative bearing at the moment of firing of the rocket

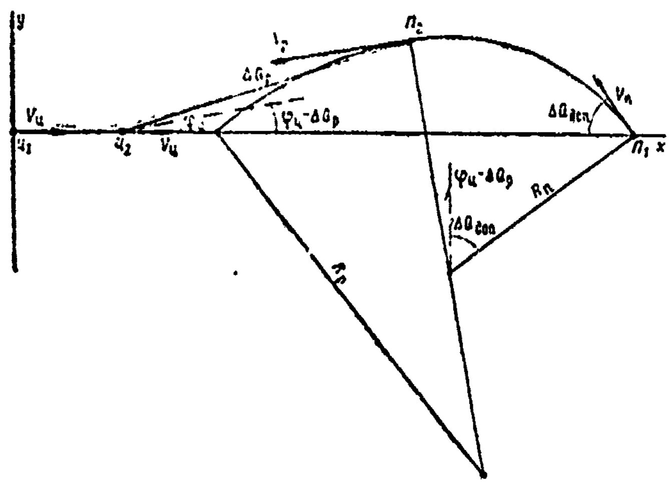

Fig. 5.13.

$$\varphi_{ц.\,макс} = \arcsin \frac{V_р}{R_р}\frac{D_п}{V_ц}. \qquad (18)$$

The target relative bearing at the moment of rocket firing, considering the error corrected by the rocket, is

$$\varphi_ц = \varphi_{ц.\,макс} - \Delta Q_р. \qquad (19)$$

The interceptor flight time from the moment of lock-on to the moment the rocket is fired is

$$t_п = \frac{R_п}{V_п}(\Delta Q_{доп} + \varphi_ц - \Delta Q_р). \qquad (20)$$

Substituting (19) into (17) we get

$$\frac{V_ц \sin(\varphi_{ц.\,макс} - \Delta Q_р) + V_р \sin \Delta Q_р}{D_п} = \frac{V_р}{R_р}. \qquad (21)$$

For small angles $\Delta Q_{доп}$, as given in the problem, we get

$$\Delta Q_р = \frac{\frac{D_п V_р}{R_р} - V_ц \sin \varphi_{ц.\,макс}}{V_р - V_ц \cos \varphi_{ц.\,макс}}. \qquad (22)$$

PROBLEM 5.6. The interceptor is guided to a nonmaneuvering target strictly from the forward hemisphere. The interceptor and target speeds are constant, and $V_п$ = 1200 km/h and $V_ц$ = 1900 km/h, respectively. The ground system assures interceptor guidance accuracy with a heading rms of $\sigma_Q = 5°$. Determine the guidance probability, if the interceptor begins to correct the guidance errors at target lock-on (by the airborne radar) distance $D_{захв}$ = 35 km, maneuvering with constant radius of turn $R_п$ = 10 km to the firing range $D_п$ = 12 km. We assume that the firing error is zero, i.e., the interceptor itself corrects all surface-guidance errors. We also assume that surface-guidance errors $\Delta Q_{доп}$ are small and $V_{рр} \approx V_п$. Under these conditions of surface guidance we are given the probability of target destruction by the rocket of 0.9.

Solution. According to the kinematic scheme for correcting surface-guidance errors (Fig. 5.12) we have

$$\frac{D_{захв}}{V_{сбл}} = \frac{x}{V_п \cos \Delta Q_{доп}} \approx \frac{x}{V_п}. \tag{1}$$

Projecting onto the x axis we get

$$x = \frac{V_п D_{захв}}{V_{сбл}} = D_п \cos \varphi_ц + R_п \sin \varphi_ц + R_п \sin \Delta Q_{доп}. \tag{2}$$

Let us square (2). Then

$$\begin{aligned} \left(\frac{V_п}{V_{сбл}}\right)^2 D^2_{захв} = D^2_п \cos^2 \varphi_ц + R^2_п \sin^2 \varphi_ц + \\ + R_п \sin^2 \Delta Q_{доп} + 2 D_п R_п \cos \varphi_ц \sin \varphi_ц + 2 D_п R_п \cos \varphi_ц \sin \Delta Q_{доп} + \\ + 2 R^2_п \sin \varphi_ц \sin \Delta Q_{доп}. \end{aligned} \tag{3}$$

For the case when the firing error is zero we have

$$\sin \varphi_ц \approx 0; \quad \cos \varphi_ц \approx 1. \tag{4}$$

Then, considering that

$$\sin \Delta Q_{доп} = \Delta Q_{доп},$$
$$\cos \Delta Q_{доп} = 1,$$

while $\sin^2 \Delta Q_{доп}$ is an infinitely small second-order term, we get

$$\left(\frac{V_п}{V_{сбл}}\right)^2 D^2_{захв} = D^2_п + 2 D_п R_п \cdot \Delta Q_{доп}, \tag{5}$$

from which we find the desired dependence of permitted surface-guidance heading error on speeds $V_ц$, $V_п$, and $V_р$, lock-on and firing ranges $D_{захв}$ and $D_п$, and interceptor turn radius $R_п$:

$$\Delta Q_{доп} = \frac{\left(\frac{V_п}{V_{сбл}}\right)^2 D^2_{захв} - D^2_п}{2 D_п R_п}. \tag{6}$$

For the data in our problem we have $\Delta Q_{доп} = 9.8°$. Thus, the guidance probability $P_{н.н} = 0.85$.

PROBLEM 5.7. The calculated interceptor heading which assures encounter with an aerial target at zero aspect angle is 70°. As a result of random perturbations in the guidance circuit and pilot errors the interceptor holds the calculated heading with an rms of $\sigma_Q = 5°$. The guidance errors have normal distribution. Determine the permitted maximum interceptor headings that guarantee a guidance probability of at least $P_н = 0.95$.

Solution. The probability of guiding an interceptor to a target is defined numerically as the probability of hitting a random heading Q distributed normally with mathematical expectation $m_Q = 70°$ and an rms of $\sigma_Q = 5°$, in an angular range of permitted heading guidance errors positioned symmetrically relative to the calculated heading m_Q:

$$P_н = P\{m_Q - \Delta Q_{доп} \leqslant Q \leqslant m_Q + \Delta Q_{доп}\} = \Phi\left(\frac{\Delta Q_{доп}}{\sigma_Q}\right), \quad (1)$$

where Φ is the Laplace function.

By stipulation in the problem we have

$$\Phi\left(\frac{\Delta Q_{доп}}{\sigma_Q}\right) = 0{,}95. \quad (2)$$

From tables of the Laplace function we find the value of the argument which satisfies (2):

$$\frac{\Delta Q_{доп}}{\sigma_Q} = 1{,}96,$$

from which the permitted heading guidance errors

$$\Delta Q_{доп} = 1{,}96 \cdot 5 = 9{,}8°.$$

Thus the desired maximum interceptor headings are within the limits of 60.2-79.8°.

PROBLEM 5.8. From known permitted heading $\Delta Q_{доп}$ and altitude $\Delta H_{доп}$ guidance errors find the equivalent one-dimensional permitted guidance error which defines the boundary of the region, the probability of guidance into which is equal to the probability that the interceptor will be within a rectangle bounded by the values $\Delta Q_{доп}$ and $\Delta H_{доп}$.

Solution. As can be seen from Fig. 5.14, the equivalent one-dimensional guidance error $X_{доп}$ is a certain average of segments $\Delta Q_{доп}$ and $\Delta H_{доп}$. The probability of guiding the interceptor into

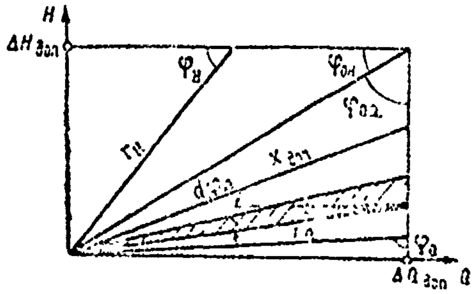

Fig. 5.14.

basic sector $d\varphi_Q$ is proportional to the square of this sector. Therefore the average value of the one-dimensional permitted guidance error $X_{доп}$ is found from the relationship

$$X_{доп}=\frac{\int\limits_{\varphi_{0Q}}^{\frac{\pi}{2}} r_Q d\varphi_Q+\int\limits_{\varphi_{0H}}^{\frac{\pi}{2}} r_H d\varphi_H}{\frac{\pi}{2}}. \qquad (1)$$

Here we divide by π/2 since averaging was done from concepts of symmetry only in the first quadrant.

Since

$$r_Q=\frac{\Delta Q_{доп}}{\sin\varphi_Q}; \qquad (2)$$

$$r_H=\frac{\Delta H_{доп}}{\sin\varphi_H},$$

then

$$X_{доп}=\frac{2}{\pi}\left(\Delta Q_{доп}\int\limits_{\varphi_{0Q}}^{\frac{\pi}{2}}\frac{d\varphi_Q}{\sin\varphi_Q}+\Delta H_{доп}\int\limits_{\varphi_{0H}}^{\frac{\pi}{2}}\frac{d\varphi_H}{\sin\varphi_H}\right). \qquad (3)$$

Solution (3) is obtained using the familiar table integral

$$\int\frac{dx}{\sin x}=\ln \operatorname{tg}\frac{x}{2}. \qquad (4)$$

Thus, the desired value

$$X_{доп}=\frac{2}{\pi}\left(\Delta Q_{доп}\ln \operatorname{tg}\frac{\varphi_Q}{2}\Big|_{\varphi_{0Q}}^{\frac{\pi}{2}}+\Delta H_{доп}\ln \operatorname{tg}\frac{\varphi_H}{2}\Big|_{\varphi_{0H}}^{\frac{\pi}{2}}\right)=$$
$$=-\frac{2}{\pi}\left(\Delta Q_{доп}\ln \operatorname{tg}\frac{\varphi_{0Q}}{2}+\Delta H_{доп}\ln \operatorname{tg}\frac{\varphi_{0H}}{2}\right). \qquad (5)$$

Further, we have

$$\left.\begin{aligned}\varphi_{0Q}&=\operatorname{arc\,tg}\frac{\Delta Q_{доп}}{\Delta H_{доп}};\\ \varphi_{0H}&=\operatorname{arc\,tg}\frac{\Delta H_{доп}}{\Delta Q_{доп}};\end{aligned}\right\} \qquad (6)$$

$$\operatorname{tg}\frac{\varphi}{2}=\frac{1-\cos\varphi}{\sin\varphi}=\frac{(1-\cos\varphi)\sqrt{1+\operatorname{tg}^2\varphi}}{\operatorname{tg}\varphi}=$$
$$=\frac{\left(1-\frac{1}{\sqrt{1+\operatorname{tg}^2\varphi}}\right)\sqrt{1+\operatorname{tg}^2\varphi}}{\operatorname{tg}\varphi}=\frac{\sqrt{1+\operatorname{tg}^2\varphi}-1}{\operatorname{tg}\varphi}. \qquad (7)$$

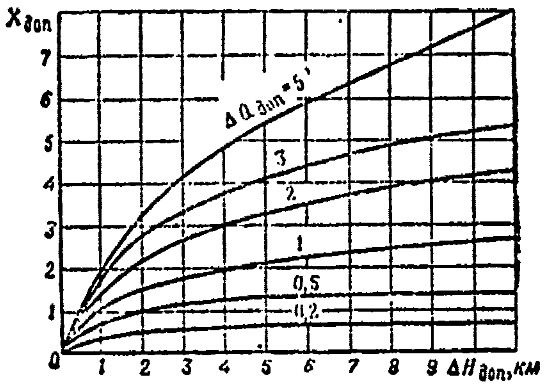

Fig. 5.15.

Having substituted (7) into (5), and considering that tan π/4 = = 1, after simple transformations we get

$$X_{доп} = \frac{2}{\pi}\left(\Delta Q_{доп} \ln \frac{\sqrt{1+\frac{\Delta Q^2_{доп}}{\Delta H^2_{доп}}}-1}{\frac{\Delta Q_{доп}}{\Delta H_{доп}}} + \Delta H_{доп} \ln \frac{\sqrt{1+\frac{\Delta H^2_{доп}}{\Delta Q^2_{доп}}}-1}{\frac{\Delta H_{доп}}{\Delta Q_{доп}}}\right) =$$

$$= \frac{2}{\pi}\left(\Delta Q_{доп} \ln \frac{\Delta Q_{доп}}{\sqrt{\Delta Q^2_{доп} + \Delta H^2_{доп}} - \Delta H_{доп}} + \Delta H_{доп} \ln \frac{\Delta H_{доп}}{\sqrt{\Delta Q^2_{доп} + \Delta H^2_{доп}} - \Delta Q_{доп}}\right). \quad (8)$$

Figure 5.15 shows a graph of the dependence of $X_{доп}$ on $\Delta Q_{доп}$ and $\Delta H_{доп}$. Using the described method we can also show that Eq. (8) is valid for both systematic and rms guidance errors.

PROBLEM 5.9. Show how the surface-guidance probability $P_{н.н}$ changes with a change in target-acquisition distance $D_{обн}$ by the airborne radar, if we know that the permitted and system guidance errors are directly proportional, while the random surface-guidance errors are inversely proportional, to acquisition distance $D_{обн}$. Find the dependence between changes in $P_{н.н}$ and interceptor speed $V_п$ under the same conditions, but with the permitted errors inversely proportional to $V_п$.

Solution. To decrease the number of arguments, in the general formula for calculating surface-guidance probability (Problem 5.1) let us convert from heading and altitude errors to the corresponding equivalent errors calculated by Eq. (8) of Problem 5.8. Considering this, and on the basis of Eq. (6) of Problem 5.1, the surface-guidance probability can be written as follows:

$$P_{н.н} = \frac{1}{2}\Phi\left(\frac{X_{доп}}{\sigma_{экв}\sqrt{2}}\right)\left[\Phi\left(\frac{X_{доп}+X_m}{\sigma_{экв}\sqrt{2}}\right) - \Phi\left(\frac{X_m - X_{доп}}{\sigma_{экв}\sqrt{2}}\right)\right]. \quad (1)$$

where $X_{доп}$, X_m, and $\sigma_{экв}$ are the corresponding equivalent permitted, system, and random surface-guidance errors.

From the stipulations of the problem we have

$$\left.\begin{aligned} X_{доп} &= k_1 D_{обн}; \\ X_m &= k_2 D_{обн}; \\ \sigma_{экв} &= \frac{k_3}{D_{обн}}, \end{aligned}\right\} \quad (2)$$

where k_1, k_2, and k_3 are the proportionality coefficients.

Substituting (2) into (1) we get the dependence of the absolute value of surface-guidance probability on the absolute value of the range of target acquisition by the airborne radar:

$$P_{н.н}(D_{обн}) = \frac{1}{2}\Phi\left(\frac{k_1 D^2_{обн}}{k_3}\right)\left[\Phi\left(\frac{(k_1+k_2)}{k_3}D^2_{обн}\right) - \Phi\left(\frac{(k_2-k_1)}{k_3}D^2_{обн}\right)\right]. \quad (3)$$

To find the dependence between relative values of $\Delta P_{н.н}/P_{н.н}$ and $\Delta D_{обн}/D_{обн}$ it is necessary to differentiate Eq. (3) with respect to $D_{обн}$ and divide the result by $P_{н.н}$. However, this differentiation leads to very unwieldy expressions due to the presence of the probability integral $\Phi(x)$ in (3). To obtain simpler calculation formulas we must approximate the Laplace integral function $\Phi(x)$ by functions which are more convenient for differentiation. Possible approximations of $\Phi(x)$ are given in Fig. 5.16, where $\Phi(x)$ is replaced, at the ends of the range of change of x, by a Maclaurin series; throughout the range of x it is replaced by the hyperbolic tangent [th] tanh x. Using the approximation

$$\Phi(x) \approx \operatorname{th} x,$$

we get

$$P_{н.н}(D_{обн}) = \frac{1}{2}\operatorname{th}\left(\frac{k_1 D^2_{обн}}{k_3}\right)\left[\operatorname{th}\left(\frac{(k_1+k_2)}{k_3}D^2_{обн}\right) - \operatorname{th}\left(\frac{(k_2-k_1)}{k_3}D^2_{обн}\right)\right]. \quad (4)$$

A comparison of Formulas (1) and (4) is shown in Fig. 5.17.

Then let us differentiate (4) with respect to $D_{обн}$. Then

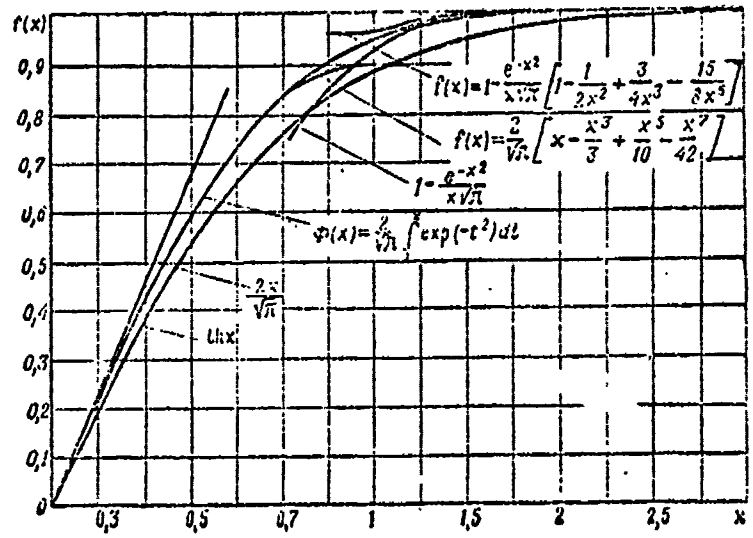

Fig. 5.16.

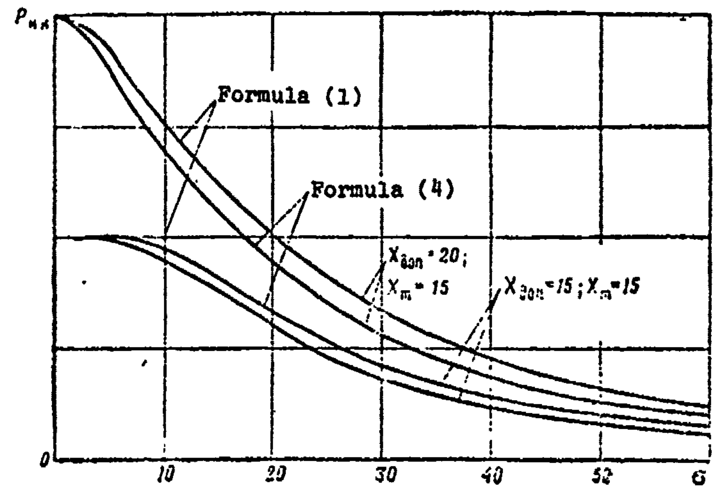

Fig. 5.17.

$$\frac{\Delta P_{н.н}(D_{обн})}{\Delta D_{обн}} = \frac{k_1 D_{обн}}{k_3 \operatorname{ch}^2\left(\frac{k_1 D^2_{обн}}{k_3}\right)}\left[\operatorname{th}\left(\frac{(k_1+k_2)}{k_3}D^2_{обн}\right) - \operatorname{th}\left(\frac{(k_2-k_1)}{k_3}D^2_{обн}\right)\right] + \operatorname{th}\left(\frac{k_1 D^2_{обн}}{k_3}\right)\left[\frac{D_{обн}\left(\frac{k_1+k_2}{k_3}\right)}{\operatorname{ch}^2\left[\frac{(k_1+k_2)}{k_3}D^2_{обн}\right]} - \frac{D_{обн}\left(\frac{k_2-k_1}{k_3}\right)}{\operatorname{ch}^2\left[\frac{(k_2-k_1)}{k_3}D^2_{обн}\right]}\right]. \quad (5)$$

Dividing derivative (5) by the absolute probability value $P_{н.н}(D_{обн})$ and using, in intermediate transformations, the dependences

$$\left.\begin{aligned} \operatorname{sh} 2x &= 2 \operatorname{sh} x \operatorname{ch} x; \\ \operatorname{th} x - \operatorname{th} y &= \frac{\operatorname{sh}(x-y)}{\operatorname{ch} x \operatorname{ch} y}, \end{aligned}\right\} \tag{6}$$

we get

$$\frac{\Delta P_{н.н}}{P_{н.н}} = \frac{4k_1 D^2_{обн}}{k_3 \operatorname{sh} 2\frac{k_1 D^2_{обн}}{k_3}} \left\{ 1 + \frac{(k_1+k_2)\operatorname{ch}\left[\frac{(k_2-k_1)}{k_3} D^2_{обн}\right]}{2k_1 \operatorname{ch}\left[\frac{(k_2+k_1)}{k_3} D^2_{обн}\right]} - \right.$$

$$\left. - \frac{(k_2-k_1)\operatorname{ch}\left[\frac{(k_2+k_1)}{k_3} D^2_{обн}\right]}{2k_1 \operatorname{ch}\left[\frac{(k_2-k_1)}{k_3} D^2_{обн}\right]} \right\} \frac{\Delta D_{обн}}{D_{обн}} = f(D_{обн}) \frac{\Delta D_{обн}}{D_{обн}}. \tag{7}$$

A graph of function $f(D_{обн})$ for various values of coefficients k_1, k_2, and k_3 is gi\ :n in Fig. 5.18. As can be seen from the graph, the following inequality is always satisfied:

$$f(D_{обн}) < 4. \tag{8}$$

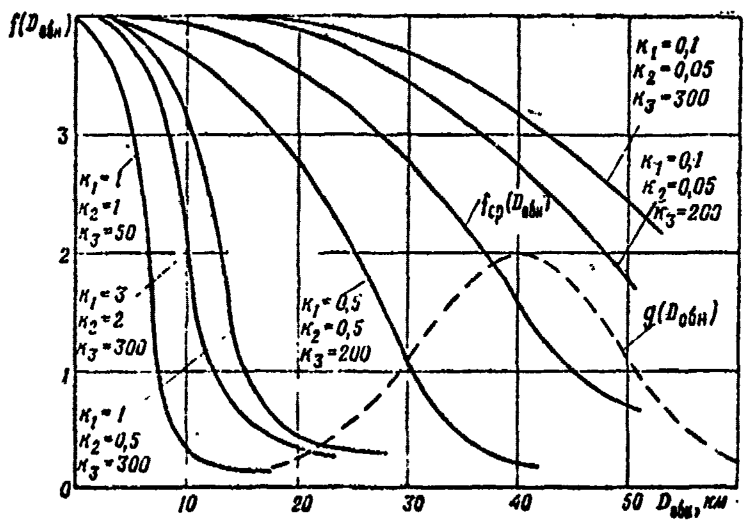

Fig. 5.18.

With short target acquisition ranges, changes in $D_{обн}$ have a strong influence of effectiveness: with a 10% change in $D_{обн}$, $P_{н.н}$ decreases by 40%.

For interceptor speed $V_п$, from the stipulations of the problem we have

$$X_{доп} = \frac{k_4}{V_п};\quad X_m = k_5 V_п;\quad \sigma_{экв} = \frac{k_6}{V_п}. \tag{9}$$

Let us differentiate $P_{н.н}$ with respect to $V_п$; then

$$\frac{\Delta P_{н.н}}{P_{н.н}} = \frac{\sqrt{2}\, k_5 V_п^2 (a^2 - b^2)}{k_4 abc} \frac{\Delta V_п}{V_п}, \tag{10}$$

where

$$a = \mathrm{ch}\left(\frac{k_4 - k_5 V_п^2}{k_6 \sqrt{2}}\right); \tag{11}$$

$$b = \mathrm{ch}\left(\frac{k_4 + k_5 V_п^2}{k_6 \sqrt{2}}\right); \tag{12}$$

$$c = \mathrm{sh}\left(\frac{\sqrt{2}\, k_4}{k_6}\right). \tag{13}$$

PROBLEM 5.10. The target is located at some random point of a region whose area is $S_ц$. The interceptor independently seeks the target in this region. Determine how the acquisition probability depends on the width 2b of the sector scanned by the airborne radar, the interceptor speed $V_п$, and search time $t_п$, if we assume that during the search time the area $S_ц$ remains constant and the target can appear several times at each point in the region, i.e., the target maneuvers such that during the entire search time it has equally probable distribution in area $S_ц$.

Solution. Let us designate by Q the probability that the target is not detected up to moment t. The probability of target acquisition in elemental time span Δt immediately before or after moment t is defined by the ratio of area $\Delta S_ц$ scanned in Δt to the entire area $S_ц$. Thus, the probability of target detection in Δt provided that the target is not detected prior to this is

$$\Delta P_{обн} = \frac{\Delta S_п}{S_ц} Q;$$

$$\Delta (1 - Q) = \frac{\Delta S_п}{S_ц} Q; \tag{1}$$

$$\frac{dQ}{dS_п} = -\frac{Q}{S_ц}. \tag{2}$$

The solution to this first-order differential equation when the initial condition Q = 1 is satisfied, when t = 0, is expressed by the obvious equation

$$Q = \exp\left(-\frac{S_п}{S_з}\right). \tag{3}$$

Consequently, the probability of target acquisition

$$P_{обн} = 1 - \exp\left(-\frac{2bV_п t_п}{S_з}\right), \tag{4}$$

where $V_п$ is interceptor speed and $t_п$ is search time.

PROBLEM 5.11. A target passing through the air-defense zone photographs objects; during this it undergoes damage which is directly proportional to the time the target remains unchallenged in the air-defense zone. An interceptor carries out n attacks in order to down the target. The probability of target downing in one attack $P_1 = 0.4$. The time required for the interceptor to climb and for guidance is $t_н = 15$ min. The time between successive attacks $t_a = 1$ min. The dimensions of the air-defense zone are 100 × 300 km. Target speed $V_ц = 1000$ km/h. The width of the strip photographed by the target is 25 km. As the indicator of effective interceptor action we select the relative length of time the target is in the air-defense zone. Determine the law of change of this effectiveness indicator as a function of time t.

Solution. The time of unchallenged target operation in the air-defense zone is a random magnitude, whose mathematical expectation is

$$T = \sum_{m-1}^{n} (P_m - P_{m-1})\, t_m + (1 - P_n)\, T, \tag{1}$$

where $P_m - P_{m-1}$ is the probability that the target will be downed in the m-th attack; $1 - P_n$ is the probability that the target will be downed in n attacks; T is the maximum time the target remains in the air-defense zone.

Carrying the constant T to the left side and introducing the term $P_n T$ under the summation sign, we get

$$T - T' = \sum_{m=1}^{n} (t_{m+1} - t_m) P_m. \qquad (2)$$

The effectiveness indicator assumes the following form:

$$\frac{T - T'}{T} = \sum_{m=1}^{n} P_m \frac{t_{m+1} - t_m}{T}. \qquad (3)$$

The time interval between successive attacks

$$t_{aT} = \frac{t_n - t_1}{n - 1}, \qquad (4)$$

and then

$$P_m = \frac{t_n - t_1}{T(n - 1)}. \qquad (5)$$

Thus, the formula for calculating the effectiveness indicator assumes the following form:

$$\frac{T - T'}{T} = 1 - \frac{t_1}{T} - \frac{(t_n - t_1)(P_n - P_1)}{P_1 T (n - 1)} - \frac{(T - t_n)(1 - P_n)}{T}, \qquad (6)$$

where P_1 is the probability of downing the target in one attack; t_1 is the minimum time of unchallenged target operation in the air-defense zone, equal to the time required for the first interceptor to take off and for its flight to target encounter; $t_n - t_1$ is the time required to carry out n successive attacks; $T - t_n$ is the time reserve before the next attack.

Formula (6) shows that even with $P_1 = 1$ the effectiveness is always less than 1, i.e.,

$$\frac{T - t_1}{T} < 1.$$

For the conditions of our problem, the total length of the target's flight path for complete photography of the entire air-defense zone is (100/25)300 = 1200 km, and the maximum time the target remains unchallenged in the air-defense zone is T = 1200/1000 = 1.2 hours.

Consequently, the effectiveness (T - T')/T depends on time as follows:

$$\frac{T-T'}{T} = 1 - \frac{0,25}{1,2} - \frac{(0,25 + 4\cdot 0,0167 - 0,25)(1-0,4)}{0,4\cdot 1,2\cdot 3} = 0,764,$$

when n = 4 attacks are carried out, with analysis of the results of each previous attack, and $P_n = 1$. If in this case n = 2, then $P_n \approx$ ≈ 0.8 and the effectiveness is

$$\frac{T-T'}{T} = 1 - \frac{0,25}{1,2} - \frac{(0,25 + 2\cdot 0,0167 - 0,25)}{0,4\cdot 1,2\cdot 1} -$$
$$- \frac{(1,2 - 0,25 - 2\cdot 0,0167)(1-0,8)}{1,2} = 0,57.$$

PROBLEM 5.12. Determine the depth of the region l in which the aerial target appears with probability 0.96, and the distance of the line of start of search $D_{н.п}$, if 50 minutes pass from the time that information appears regarding the target.

Given:

– distance of the warning line $D_{оп}$ = 2000 km;

– accuracy in measuring target coordinates σ_x = 30 km;

– target speed $V_ц$ = 900 km/h;

– limits of target maneuvering, with respect to velocity, $\Delta V_ц$ = = ±150 km/h;

– limits of target maneuvering, with respect to heading, $\Delta Q_ц$ = = ±0-30°.

Let us assume that the coordinates of target position due to speed and heading maneuvers are distributed triangularly (Simpson rule) as shown in Fig. 5.19.

Solution. 1. Let us determine the limits of probable deviation of target-position coordinate due to velocity maneuvering:

$$y = \Delta V_ц t = \pm 125 \text{ km}. \quad (1)$$

2. Let us determine the limits of probable deviation of target-position coordinate due to heading maneuvering:

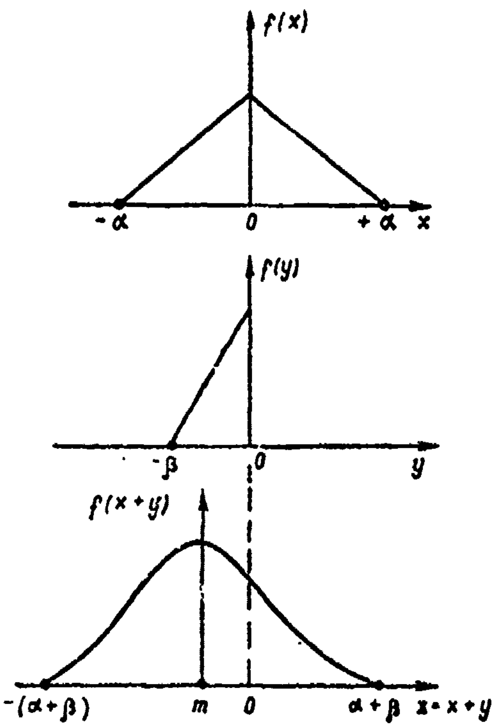

Fig. 5.19.

$$\beta = V_{ц}t - V_{ц}t\cos Q_{ц};\qquad (2)$$
$$\beta_1 = 0;\quad \beta_2 = 100 \text{ km}.$$

3. Let us determine the rms deviation and the mathematical expectation of the total law of distribution of the target-position coordinate due to combined speed and heading maneuvers. The combination of two Simpson rules [2] gives the total distribution law close to normal. The rms deviation of this law is

$$\sigma_{ц} = \frac{2\alpha + \beta}{6},\qquad (3)$$

while the mathematical expectation is determined by solving the quadratic equation

$$m_{ц}^2 + 2m_{ц}\beta + \frac{\beta^2}{2} = 0.\qquad (4)$$

For our example we have $\sigma_{ц1}$ = 42 km, $\sigma_{ц2}$ = 58 km; solution of Eq. (4) gives $m_{ц}$ = 171 km.

4. Let us determine the rms deviation of the resulting law of distribution of the target-position coordinate due to speed and heading maneuvers and due to imprecise measurement of the target coordinates:

$$\sigma_{ц\Sigma} = \sqrt{\sigma_{ц}^2 + \sigma_{x}^2};\qquad (5)$$
$$\sigma_{ц\Sigma 1} = 52 \text{ km};$$
$$\sigma_{ц\Sigma 2} = 65 \text{ km}.$$

5. If the target performs no speed and heading maneuvers, in time t = 50 min it will be at a distance

$$D_{ц} = V_{ц}t = 1250 \text{ km}$$

from the airfield.

6. If the target makes no heading maneuvers, but increases its speed, in t = 50 min it will be

$$D_{\text{оп}} - V_{\text{ц}}t - 2\sigma_{\text{цz1}} = 1146 \text{ km}$$

from the airfield.

7. If the target increases its speed and performs heading maneuvers to its limiting value, in 50 min it will be

$$D_{\text{оп}} - V_{\text{ц}}t + m_{\text{ц}} + 2\sigma_{\text{цz}} = 1551 \text{ km}$$

from the airfield. This is the maximum possible rear boundary of the region of probable location of the target by the time the information is 50 min old, and is thus the terminal search line $\Gamma_{\text{к.п}}$.

Consequently, the depth of the region of probable target location

$$l = D_{\text{к.п}} - D_{\text{н.п}} = 405 \text{ km}.$$

Therefore, in our example, target search should begin at a distance $D_{\text{н.п}}$ = 1146 km. The depth of the region of probable target location at the moment search begins is 405 km. The target is located within these limits with a probability of 0.96.

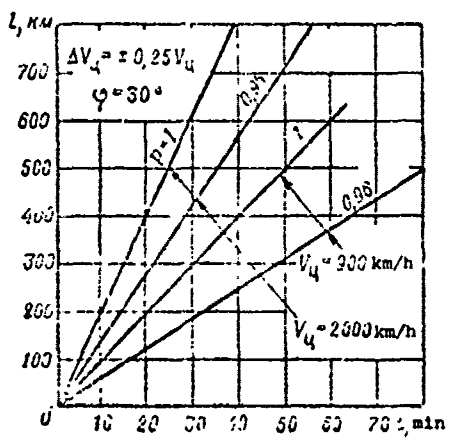

Fig. 5.20.

The dependence of the depth of the region of probable target location l on the age of the information is shown in Fig. 5.20.

PROBLEM 5.13. An aerial target is destroyed by an air-air rocket after the radio fuze has operated at the moment the rocket reaches any point in the target plane not more than R = 7 m from the center of gravity of the target. Errors in rocket homing are described as a miss – the distance of random coordinates of rocket flight in the target plane from the target's center of gravity. In trying to break off attack by the interceptor the target uses, with equal probability, six various types of maneuvers, in which rocket miss is described by the following values for the average and rms homing errors:

Numerical miss characteristics, m	Types of target maneuver					
	1	2	3	4	5	6
Mathematical expectation	2	3,5	2,6	3,7	0,2	3,8
RMS deviation	4,7	5,9	5,1	4,3	3,3	4,9

Show how the probability of target destruction depends on its dimensions and on the values of the systematic and random rocket homing errors. Determine the probability of target destruction for the given conditions, and show how we can calculate the destruction probability for the case when a miss has no systematic component but does have a random one.

Solution. According to the conditions of the problem the destruction zone is a circle of radius R = 7 m whose plane is perpendicular to the direction of rocket flight and whose center coincides with the target's center of gravity. The probability of target destruction can be interpreted as the probability of hitting within this circle.

The process of rocket flight to the target involves continuous correction of rocket homing errors arising for various reasons. When the target maneuvers, the rocket's automatic control system, because of inertia of the system elements, corrects the errors with a slight delay, as a result of which we have a systematic homing error and a systematic rocket miss. Random external perturbations (noise, fluctuating processes in the system elements, random changes in flight conditions) are the causes for these random homing errors and random miss. Dispersion of the rocket is caused by the action of a number of independent factors, and therefore the miss is a random magnitude, distributed normally, while in light of the symmetry of the control system the miss distribution in first approximation is circular. However, when dispersion along coordinates axes x,y is characterized by rms errors which are not identical but which are of the same order ($\sigma_x \approx \sigma_y$), the equivalent circular rms error is defined by the formula

$$\sigma = \sqrt{\sigma_x \sigma_y}. \qquad (1)$$

Derivation of a precise formula for calculating the equivalent circular rms error from known values of rms errors σ_x and σ_y is given in the solution to Problem 5.8. Thus, the miss of a rocket relative to the target's center of gravity can be described by circular normal distribution of the random radius-vector modulus, whose average value is equal to the distance of the dispersion center from the coordinate origin, coincident with the target's center of gravity (with the center of the circle). Consequently, the probability of target destruction is numerically equal to the probability that the rocket, whose coordinates in the target plane are statistically independent and distributed by a circular normal law, falls within a circle of radius R whose center coincides with the target's center of gravity.

If the coordinate origin coincides with the center of the circle, the direction of the axes of the Cartesian coordinates can always be selected such that one of the miss components will not contain a systematic error. Then, assuming that the rocket homing errors are statistically independent and distributed with a circular normal law with mathematical expectation m and rms error σ, the two-dimensional distribution density of coordinates x and y of rocket miss can be represented in the following form:

$$f(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{(x-m)^2 + y^2}{2\sigma^2}}. \qquad (2)$$

To find the miss distribution as the distribution of the radius-vector modulus, let us convert the Cartesian coordinates x,y into polar coordinates ρ,φ using the equations

$$\begin{cases} x = \rho \cos \varphi, \\ y = \rho \sin \varphi. \end{cases} \qquad (3)$$

where ρ is the modulus of the random miss radius-vector.

Let us find the Jacobian of the transform when converting from variable Cartesian coordinates to variable polar coordinates:

$$\frac{\partial(x, y)}{\partial(\rho, \varphi)} = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \varphi} \end{vmatrix} = \begin{vmatrix} \cos\varphi & -\rho\sin\varphi \\ \sin\varphi & \rho\cos\varphi \end{vmatrix} = \rho(\cos^2\varphi + \sin^2\varphi) = \rho. \quad (4)$$

Since the two-dimensional distribution density in the polar coordinate system is defined in terms of the two-dimensional distribution density in Cartesian coordinates by the relationship

$$f(\rho, \varphi) = \rho f(x, y) = \rho f(\rho\cos\varphi, \rho\sin\varphi), \quad (5)$$

where $\rho > 0$ and $0 \leq \varphi \leq 2\pi$, the one-dimensional distribution density of a miss – the radius-vector modulus – is found from the formula

$$f(\rho) = \rho\int_0^{2\pi} f(\rho\cos\varphi, \rho\sin\varphi)\,d\varphi = \frac{\rho}{2\pi\sigma^2}\int_0^{2\pi} e^{-\frac{\rho^2 - 2m\rho\cos\varphi + m^2}{2\sigma^2}}\,d\varphi = \frac{\rho}{2\pi\sigma^2}e^{-\frac{\rho^2+m^2}{2\sigma^2}}\int_0^{2\pi} e^{\frac{m\rho\cos\varphi}{\sigma^2}}\,d\varphi. \quad (6)$$

The integral in Eq. (6) reduces to a zero-order Bessel function of an imaginary argument, since in general form an n-th order Bessel function is represented by the integral

$$I_n(x) = \frac{1}{\pi}\int_0^{\pi} e^{x\cos\varphi}\cos n\varphi\,d\varphi, \quad (7)$$

while when n = 0 we get a zero-order Bessel function:

$$I_0(x) = \frac{1}{\pi}\int_0^{\pi} e^{x\cos\varphi}\,d\varphi. \quad (8)$$

Consequently, Eq. (6) can be rewritten as follows:

$$f(\rho) = \frac{\rho}{\sigma^2}e^{-\frac{\rho^2+m^2}{2\sigma^2}} I_0\left(\frac{m\rho}{\sigma^2}\right). \quad (9)$$

The integral distribution function F(ρ) of miss ρ also gives us the desired probability of a rocket's hitting within a circle of radius R when the center of dispersion of the circular normal law is displaced by the value m from the center of the circle. Thus we have

$$P_{пор}=F(\rho)=\int_0^R \frac{\rho}{\sigma^2} e^{-\frac{\rho^2+m^2}{2\sigma^2}} I_0\left(\frac{m\rho}{\sigma^2}\right) d\rho. \quad (10)$$

Let us introduce new variables — the relative values

$$\overline{R}=\frac{R}{\sigma} \text{ and } \overline{m}=\frac{m}{\sigma}. \quad (11)$$

Then the formula for calculating the probability of destruction is expressed as follows:

$$P_{пор}=\int_0^{\frac{R}{\sigma}} \overline{R}\, e^{-\frac{\overline{R}^2+\overline{m}^2}{2}} I_0(\overline{m}, \overline{R})\, d\overline{R}. \quad (12)$$

The results of the calculation by Eq. (12) are shown in Fig. 5.21.

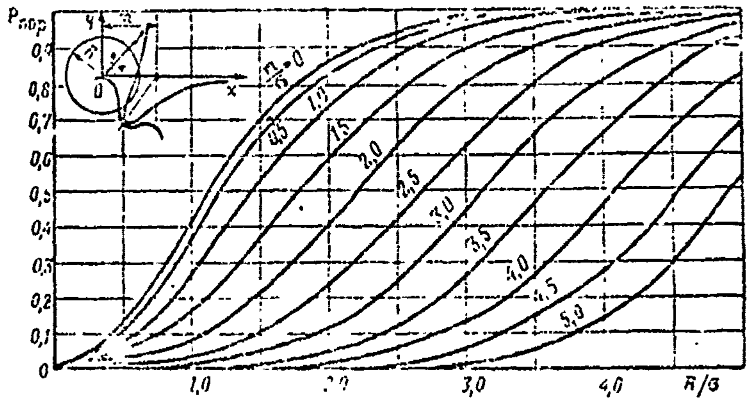

Fig. 5.21.

When m < 1 (small systematic rocket homing error), the probability of destruction can be calculated quite accurately from the formula

$$P_{пор}=1-e^{-\frac{R}{2(1+0,5\overline{m}^2)}}. \quad (13)$$

When the rocket homing systematic error is zero, while random errors are distributed by the circular normal law with rms deviation σ, the probability of destruction, like the probability of the rocket's hitting in a circle of radius R, is

$$P_{пор}=\int_0^R \frac{\rho}{\sigma^2} e^{-\frac{\rho^2}{2\sigma^2}} d\rho = 1-e^{-\frac{R^2}{2\sigma^2}}. \quad (14)$$

This is the so-called Rayleigh law. In this case P_{nop} is defined from the curve m/σ = 0 from the graph in Fig. 5.21.

For the conditions of the problem, using Eq. (12) or the graph in the figure, we obtain the following probabilities of the rocket's hitting within a circle of radius R = 7 m: 0.64, 0.46, 0.57, 0.60, 0.89, and 0.54, respectively, for target maneuvers of types 1-6.

The resulting probability of target destruction is defined from the formula for total probability:

$$P_{\text{пор}\,\Sigma} = \sum_{i=1}^{6} P(H_i)\, P_{\text{пор}}(H_i). \tag{15}$$

where $P(H_i)$ is the probability that the target selects the i-th type of maneuver (hypothesis H_i); $P_{\text{пор}}(H_i)$ is the conditional probability of target destruction under hypothesis H_i.

Since by stipulation in the problem all six types of target maneuver are equally probable,

$$P(H_1) = P(H_2) = \ldots = P(H_6) = \frac{1}{6}$$

and the resulting probability of target destruction $P_{\text{пор}\Sigma} = 0.617$.

CHAPTER 6

THE COMBAT EFFECTIVENESS OF GROUP OPERATIONS OF FIGHTER-INTERCEPTORS

Indicators of the combat effectiveness during operation of a group of interceptors against a group of targets can be calculated only from the known indicator of a single operation against a single target. Therefore, before analyzing the problems of this chapter it would be advisable to become familiar with the problems of Chapter 3.

In addition, we assume that the reader is familiar with the principles of the general efficiency theory, game theory, and the queueing theory. If not, we recommend that the basic postulates and methods of these disciplines be studied in the available literature [3-8, 29-32, 39].

Since the basic purpose of an interceptor is to destroy aerial targets, the basic criterion for the combat effectiveness during group combat operations of interceptors is the number of destroyed (downed) targets. This number is a random one and, consequently, its most complete characteristic is the law of the distribution of this random number. The basic quantitative indicators of the combat effectiveness of interceptor group operations, and the ones most often used in practice, are the numerical characteristics of the law of distribution of the number of downed targets, viz.: its mathematical expectation (that of the number of downed targets), the probability of downing all targets in flight, and the probability of downing at least a given number of targets. Most of the problems in this chapter are devoted to determining these characteristics.

In Problem 6.1 we determine the distribution law of the probabilities of destroying a group target with a group of interceptors.

The most general problems are 6.2, 6.3, and 6.4, whose solution gives us the dependences of the mathematical expectation of the relative number of downed targets on the number of operations against each target, the probability of destroying a target with a single operation against it and with a given quality of solution of the problem of combat control. This latter is taken into account by examining three models of group aerial combat, which define the limits of combat effectiveness.

1. A model of combat in which there is no target distribution, i.e., when each interceptor can attack any target in the sortie, resulting in intermingling of the targets and nonuniform distribution of targets among the interceptors (Problem 6.3).

2. A model of combat with ideal target distribution i.e., when the targets are uniformly distributed among the interceptor and operations against each target are carried out independently of one another without analyzing the results of the previous attacks (Problem 6.2).

3. A model of combat with ideal target distribution, analysis of the results of each attack, and reaiming to another target if the previous one is downed (Problem 6.4).

Obviously, any case of group combat operations which is possible in practice can be evaluated by one of these models. This is illustrated by the solution of a number of problems (6.5-6.7).

Those problems whose solutions give us the dependence of the relative carrying capacity of a multichannel surface-guidance system on the number of guidance channels, the average time for guidance of a single interceptor, the intensity of the target sortie, and the depth of the belt in which interception takes place, are characterized by great generality. Using queueing theory, we examine two models of interceptor combat operations:

1. Interception of targets on a given line, or the Erlang model (Problems 6.8-6.10);

2. Interception of targets in a given belt, or the Barrer model (Problem 6.11).

Classical queueing theory gives analytical formulas for calculating the carrying capacity of multichannel systems only for the case when the intensity of the flow of targets is constant. To expand these conditions we examine Problems 6.13 and 6.14, which show ways for possible calculation of the effectiveness indicators for the most general case when the intensity of the flow of targets changes randomly with time and is characterized by a given distribution law.

A number of problems are devoted to determining the combat effectiveness during independent combat operations, when a group of interceptors independently seeks out a group target and subsequently destroys it, and to determining the required number of interceptors to assure a given effectiveness (Problems 6.15-6.18).

PROBLEM 6.1. $N_п$ interceptors are guided to $N_ц$ targets. Each target is attacked by one interceptor, resulting in each target's being downed with a probability of P_1. Naturally, the number of downed targets is a random number. We must give the complete characteristic of this random number. Given: $N_п = 10$, $N_ц = 10$, $P_1 = 0.7$.

Solution. As the all-inclusive characteristic of the number of downed targets $N_{с.ц}$ we have the distribution law for the probabilities of this random magnitude. If one interceptor is guided to each of $N_ц$ targets, and the interceptor downs the target with probability P_1, the density P_{ci} of the distribution of the number of downed targets is defined by the coefficients of the expansion of Newton's binomial. Let us designate by Q_1 the probability opposite to P_1. Then the desired ordinates of the distribution law are defined by the following type of expansion:

$$(P_1+Q_1)^{N_ц}=P_1^{N_ц}+N_ц P_1^{N_ц-1}Q_1+\frac{N_ц(N_ц-1)}{2!}P_1^{N_ц-2}Q_1^2+\ldots$$

Here $P_1^{N_ц}$ is the probability of the downing of all $N_ц$ targets; $P_1^{N_ц}$ + + $N_ц P^{N_ц-1} Q_1$ is the probability of downing at least $N_ц$ - 1; the first three terms are the probability of downing at least $N_ц$ - 2; etc.

Using this method we find that when each of 10 targets is attacked by an interceptor and P_1 = 0.7, the probabilities that at least 10, 9, 8, ..., 1, 0 targets will be downed are, respectively, 0.03, 0.15, 0.39, 0.63, 0.85, 0.96, 0.99, 0.995, 0.997, 0.998, and 0.999. Using the formula for determining the mathematical expectation, let us not calculate the mathematical expectation of the number of downed targets:

$$M[N_{c.ц}]=\sum_{i=1}^{10} iP_{ci},$$

where P_{ci} is the probability that exactly i targets will be downed; i = 0-10.

$$M[N_{c.ц}]=10\cdot 0{,}03+9\cdot 0{,}12+8\cdot 0{,}24+7\cdot 0{,}24+6\cdot 0{,}22+$$
$$+5\cdot 0{,}11+4\cdot 0{,}03+3\cdot 0{,}005+2\cdot 0{,}002+1\cdot 0{,}001=7,$$

which coincides with the obvious calculation using the formula 0.7•10 = 7. Unlike the distribution function, which gives the probability that at least a given number of targets will be downed, the distribution density gives the probability that exactly a certain number of targets will be downed. For our example, the probabilities that exactly 10, 9, 8, ..., 1, 0 targets will be downed are, respectively, 0.03, 0.12, 0.24, 0.24, 0.22, 0.11, 0.03, 0.005, 0.002, and 0.001.

PROBLEM 6.2. $N_ц$ targets participate in a raid on an object. A flight of $N_п$ interceptors is sent to intercept these targets. At the command point all interceptors are uniformly distributed against all targets, since all targets are of equal value. Each interceptor has $m_р$ rockets which are fired independently at the targets. There is no damage accumulation; each rocket destroys a target with identical probability P_1. Show how the expectation of the number of downed targets $M[N_{с.ц}]$ depends on the number of attacks against each target and the probability P_1 of downing a single target with the firing of one rocket.

Solution. The resulting probability P_Σ of destroying each target after $m_р N_п/N_ц$ operations is equal to the probability of the event consisting of the fact that each target is destroyed after any of $m_р N_п/N_ц$ rocket firings against it. Either it is destroyed by the first rocket, or the first rocket does not destroy it but the second one does; the first two rockets do not destroy it but the third one does; etc. Consequently,

$$P_\Sigma = P_1 + Q_1 P_1 + Q_1^2 P_1 + \ldots = P_1 \sum_{m=0}^{m=\frac{m_р N_п}{N_ц}-1} Q_1^m = P_1 \frac{1-Q_1^{\frac{m_р N_п}{N_ц}}}{1-Q_1} = 1-(1-P_1)^{\frac{m_р N_п}{N_ц}}, \qquad (1)$$

where $\sum_m Q_1^m$ is the sum of the geometric progression whose first term is 1 and whose denominator is Q_1. In practice, it is convenient to use, instead of the absolute value of the expectation of the number of downed targets, the relative value:

$$\frac{M[N_{с.ц}]}{N_ц} = 1-(1-P_1)^{\frac{m_р N_п}{N_ц}}. \qquad (2)$$

The graph of this function is given in Fig. 6.1.

When it is impossible to uniformly spread all operations against all targets, against a certain number of the targets there is one operation more than against all the others. In this case,

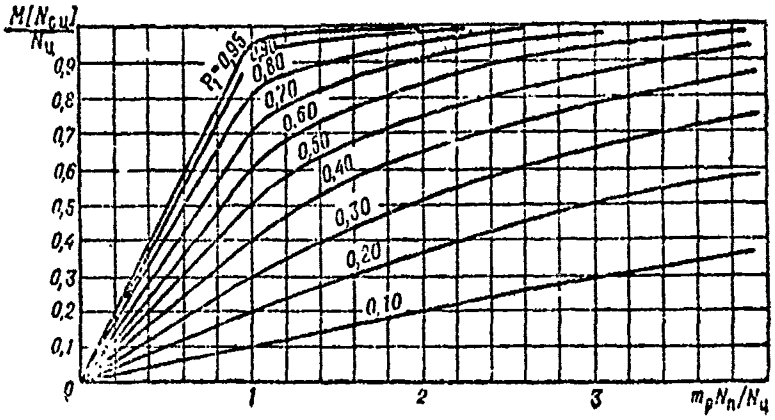

Fig. 6.1.

$$\frac{M[N_{c.\,ц}]}{N_ц} = \left[1-(1-P_1)^{\frac{m_p N_п}{N_ц}}\right](1-\alpha P_1), \qquad (3)$$

where

$$\alpha = \frac{m_p N_п}{N_п} - [\alpha] = 0 - 1, \qquad (4)$$

[a] is the integral quotient of $m_p N_п$ divided by $N_ц$.

We can also use the following formula:

$$\frac{M[N_{c.\,ц}]}{N_ц} = [1-(1-P_1)^{[\alpha]}] + \alpha\{1-(1-P_1)^{[\alpha]+1}], \qquad (5)$$

but calculations are simpler using Eq. (3).

PROBLEM 6.3. A group of $N_п$ interceptors conducts aerial combat with $N_ц$ bombers. The mutual positions of the interceptors and targets are such that each interceptor can attack any target and down it with identical probability P_1. Show the maximum increase in effectiveness if we turn from free aerial combat, i.e., from tactics under conditions of the complete absence of target distribution, to tactics under conditions of ideal target distribution, when all targets are uniformly distributed among the interceptors. The expectation of the number of downed targets is the criterion of combat effectiveness.

Solution. Since with ideal target distribution $N_п/N_ц$ operations are carried out against each target, the probability of downing each target, as shown in Problem 6.2, is

$$1-(1-P_1)^{\frac{N_п}{N_ц}}, \tag{1}$$

i.e., the expectation of the relative number of downed targets

$$\frac{M[N_{c.п}]}{N_ц} = 1-(1-P_1)^{\frac{N_п}{N_ц}}. \tag{2}$$

When there is no target distribution whatsoever, the probability that a certain interceptor will operate against a specific target is $1/N_ц$. The probability that ν interceptors of all $N_ц$ will operate against a specific target is calculated from the formula for the term of the Newton binomial expansion:

$$P(\nu) = C^{\nu}_{N_п}\left(\frac{1}{N_п}\right)^{\nu}\left(1-\frac{1}{N_п}\right)^{N_п-\nu}; \tag{3}$$

$$C^{\nu}_{N_п} = \frac{N_п!}{\nu!\,(N_п-\nu)!}.$$

The probability that the selected target will be undamaged after ν attacks is

$$P(\nu)(1-P_1)^{\nu}.$$

However, the target need not be attacked ν times; it can be attacked 0 times, 1 time, 2 times, ..., or $N_п$ times. Consequently, the resulting probability that a target will be undamaged is equal to the sum of the probabilities of all favorable events:

$$\sum_{\nu=0}^{N_п} P(\nu)(1-P_1)^{\nu} = \left(1-\frac{P_1}{N_ц}\right)^{N_ц}. \tag{4}$$

The expectation of the relative number of downed targets with no target distribution whatsoever is equal to the probability of the opposite event:

$$\frac{M[N_{c.п}]}{N_ц} = \left[1-\left(1-\frac{P_1}{N_ц}\right)^{N_п}\right]. \tag{5}$$

Tables of Function (5) are given in Appendix 1. Thus, the gain in effectiveness is expressed by the formula

$$\Delta \frac{M[N_{\text{с.ц}}]}{N_{\text{ц}}} = (1-P_1)^{\frac{N_{\text{п}}}{N_{\text{ц}}}} - \left(1-\frac{P_1}{N_{\text{ц}}}\right)^{N_{\text{п}}}. \qquad (6)$$

Function (6) depends on P_1, $N_{\text{п}}$, and $N_{\text{ц}}$, and in general form it is unnecessary to construct it as a function of the argument $N_{\text{п}}/N_{\text{ц}}$, since $N_{\text{ц}}$ must be specified. However, for large values of $N_{\text{ц}}$,

$$\left(1-\frac{P_1}{N_{\text{ц}}}\right)^{N_{\text{п}}} \approx e^{-\frac{N_{\text{п}}}{N_{\text{ц}}}}, \qquad (7)$$

since by definition of the Euler number

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

we have

$$\lim_{N_{\text{ц}}\to\infty}\left(1-\frac{P_1}{N_{\text{ц}}}\right)^{N_{\text{п}}} = \lim_{N_{\text{ц}}\to\infty}\left(1+\frac{1}{\frac{N_{\text{ц}}}{(-P_1)}}\right)^{\frac{N_{\text{ц}}(-P_1)}{(-P_1)}\frac{N_{\text{п}}}{N_{\text{ц}}}} = e^{-\frac{N_{\text{п}}P_1}{N_{\text{ц}}}}. \qquad (8)$$

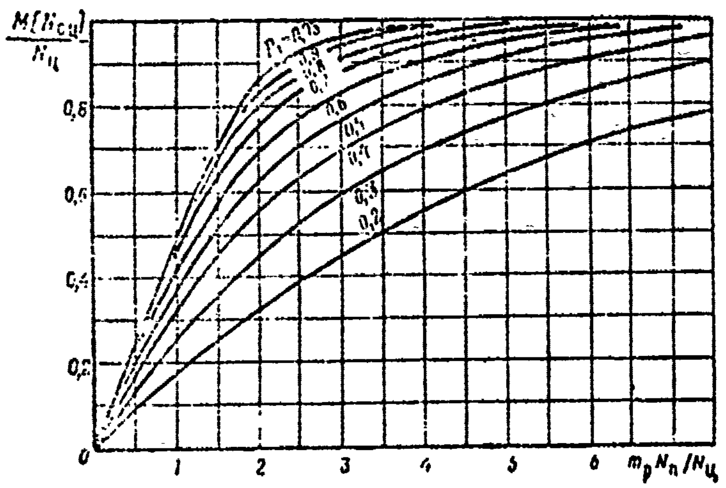

Fig. 6.2.

Calculations from Formula (5) for P_1 = = 0.2-0.95 are given in Fig. 6.2. Using the graphs in Figs. 6.1 and 6.2 we can construct, for any P_1, the effectiveness gain function. For values P_1 = 0.2-0.95 this function is as shown in Fig. 6.3. With small values of $N_{\text{ц}}$ and $N_{\text{п}}$, to construct the function $\Delta(M[N_{\text{с.ц}}]/N_{\text{ц}})$ we must use Eq. (6) directly. Calculations given in the tables of Appendix 1 allow us to judge the rapid convergence of (5) to (8), and determine the limits of validity for constructing the gain function $\Delta(M[N_{\text{с.ц}}]/N_{\text{ц}})$ from Eq. (6). The effectiveness in the absence of target distribution can also be estimated by another

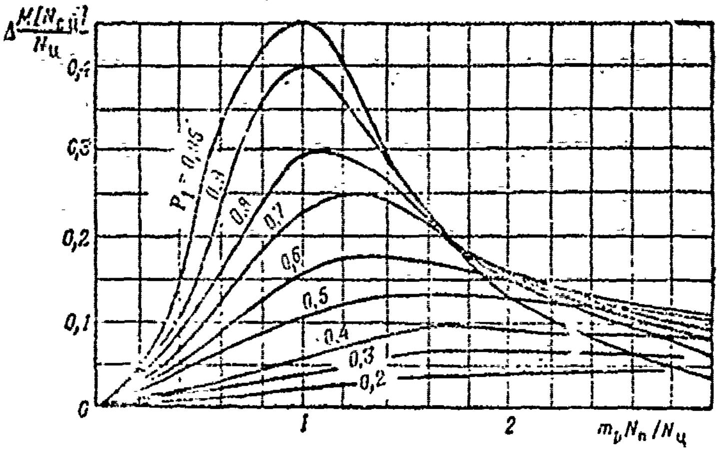

Fig. 6.3.

method. In this case, the number of operations $N_п/N_ц$ carried out against each target is a random number, subject to the Poisson principle. The probability that exactly k operations will be carried out against each target is calculated by the formula

$$P_к = \frac{\left(\frac{N_п}{N_ц}\right)^k_{ср}}{k!} e^{-\frac{N_п}{N_ц}}. \qquad (9)$$

Using the formula for the total probability, we get the probability of the downing of each target:

$$P = \sum_{k=0} P_k [1 - (1 - P_1)^k] = 1 - e^{-\frac{N_п}{N_ц} P_1}.$$

PROBLEM 6.4. Determine the gain in combat effectiveness (the difference in the expectation of the number of downed targets) when turning from tactics under conditions of the lack of any organized target distribution to tactics under conditions of ideal (uniform) target distribution, and then to tactics with reaiming, when each subsequent attack is carried out only after the results of the previous one have been analyzed. Show how the desired difference depends on the probability of interception of a single target when the interceptor fires one rocket.

Solution. Since the number of destroyed (damaged) targets is a random number, the most general characteristic of this number is its distribution law. Consequently, the general method of determining the effectiveness of group interceptor operations against a group of targets is to determine the distribution law for the number of damaged targets, from which we can then calculate such characteristics as the expectation of the number of damaged targets, the probability of damaging all targets, and the probability of damaging at least a given number of targets. The ordinate of the density distribution function for the number of damaged targets is the probability P_i of destroying exactly i targets. When $i < N_ц$, the probability P_i is defined by the binomial distribution law:

$$P_i = C^i_{N_n} P^i_1 (1 - P_1)^{N_n - i} = \frac{N_n!}{i!(N_n - i)!} P^i_1 (1 - P_1)^{N_n - i}. \qquad (1)$$

Thus, for this case the probability of damaging all targets is defined by the formula

$$P_{N_ц} = 1 - \sum_{i=0}^{N_ц - 1} P_i, \qquad (2)$$

since the probability of the oppositve event – the probability of destroying at least one target – is

$$\sum_{i=0}^{N_ц - 1} P_i,$$

i.e., equal to the probability of damaging one, two, three, etc., up to $N_ц$ - 1 targets, but not all $N_ц$ targets. Using the theorem of the separation of sums (integral), Eq. (2) can be written as follows:

$$P_{N_ц} = \sum_{i=0}^{N_n} C^i_{N_n} P^i_1 (1 - P_1)^{N_n - i} - \sum_{i=0}^{N_ц - 1} C^i_{N_n} P^i_1 (1 - P_1)^{N_n - i}.$$

Since

$$\sum_{i=0}^{N_n} C^i_{N_n} P^i_1 (1 - P_1)^{N_n - i} = 1,$$

as the sum of the probabilities of a complete group of events, the formula for calculating the probability of damage to all targets assumes the following form:

$$P_{N_ц} = \sum_{i=N_ц}^{N_n} C^i_{N_n} P^i_1 (1 - P_1)^{N_n - i}. \qquad (3)$$

Analogously, for the probability of damaging at least the given number of targets we have the following formula:

$$P_m = \sum_{i=m}^{N_n} C^i_{N_n} P^i_1 (1 - P_1)^{N_n - i}, \qquad (4)$$

where m is the given number of targets which must be destroyed, i.e., the number of damaged targets can be m or more. The expectation of the number of damaged targets is obtained by using the formula for calculating the expectation as the sum of all possible random numbers of damaged targets multiplied by the probability of the occurrence of these numbers. For this we must multiply each random number of damaged targets i by the ordinate of distribution density (1) and sum these derivatives from 1 to $N_ц$:

$$\sum_{i=1}^{N_ц} iC_{N_п}^{i} P_1^{i} (1-P_1)^{N_п - i}. \tag{5}$$

Then, to sum (5) we must add the sum which takes into account the possibility that the total number of operations $N_п$ carried out by all interceptors can be greater than the number of targets $N_ц$. The formula for calculating the expectation of the number of damaged targets then assumes the following form:

$$M[N_{с.ц}] = \sum_{i=1}^{N_ц} iC_{N_п}^{i} P_1^{i} (1-P_1)^{N_п - i} + \\ + N_ц \sum_{i=N_ц+1}^{N_п} C_{N_п}^{i} P_1^{i} (1-P_1)^{N_п - i}. \tag{6}$$

Graphs of functions (3) and (6) are given in Figs. 6.4 and 6.5. The gain in effectiveness with reaiming is shown in Fig. 6.6.

As comparison of the dependences $M[N_{с.ц}] = f(P_1, N_ц, N_п)$ shows for cases of ideal target distribution, the absence of target distribution, and reaiming, the greatest differences in $\Delta M[N_{с.ц}]$ occur with $P_1 > 0.5$ and $N_п/N_ц = 1\text{-}2$. The dependences $\Delta M[N_{с.ц}]$ can be widely used to estimate the influence of the quality of combat control and the degree of target distribution and reaiming on the effectiveness.

The dependences

$$\frac{M[N_{с.ц}]}{N_ц} = f\left(P_1, \frac{N_п}{N_ц}\right),$$

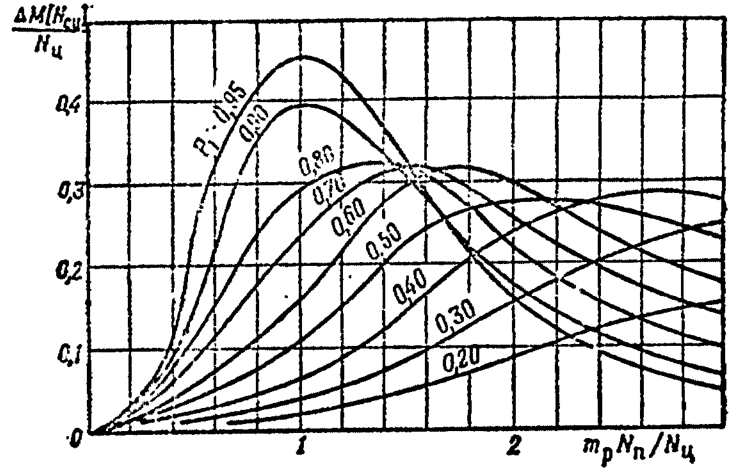

Fig. 6.4.

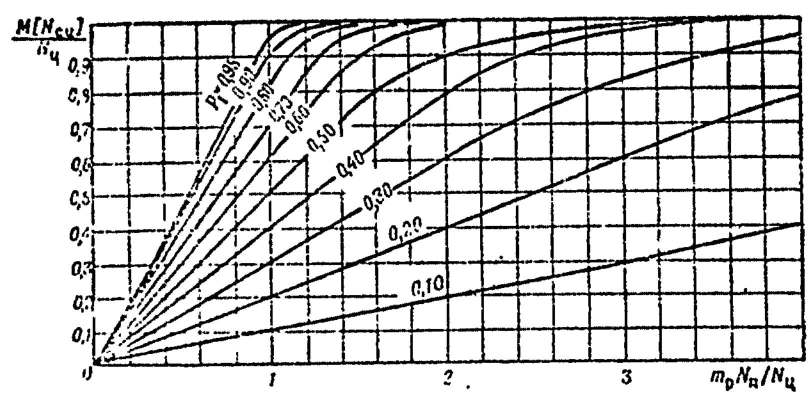

Fig. 5.5.

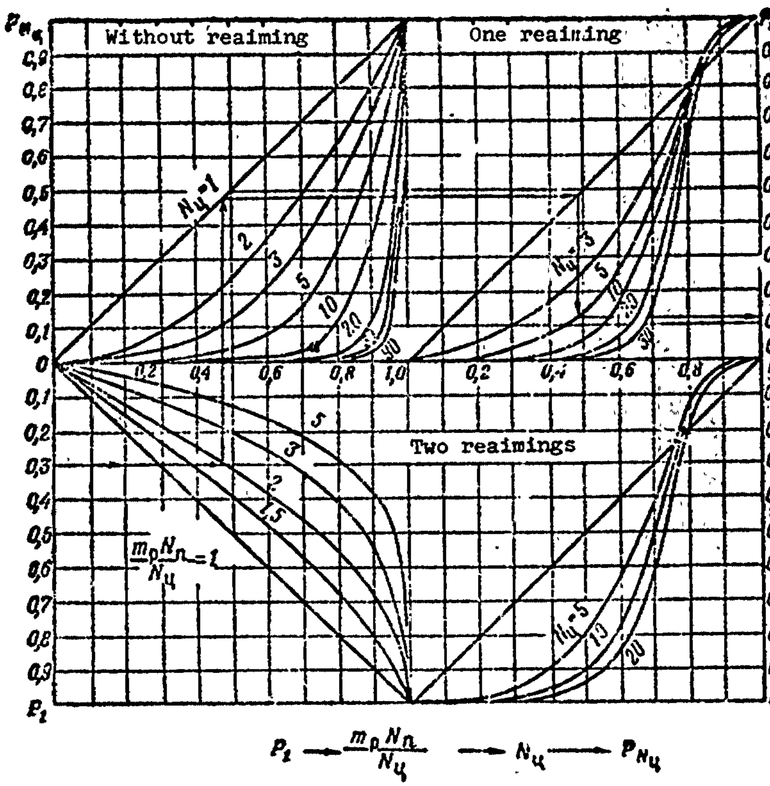

Fig. 6.6.

and also the nomogram for defining $P_{N_ц}$ are all-purpose when a flight of interceptors is used, if we are given quantitatively the effectiveness level. Using these we can also solve the opposite problem: to determine the effectiveness $M[N_{c.ц}]$, $P_{N_ц}$ if we know the number of targets $N_ц$, the designated flight of interceptors $N_п$, and the probability of target destruction in a single operation P_1.

We should remember that in Problems 6.2-6.4, when deriving the calculation formulas we assumed that the interceptor carried out one attack. When each interceptor can carry out m_p single attacks, everywhere in the formulas for these problems we must replace $N_п$ by $m_pN_п$. We must keep in mind a similar substitution when using the graphs in Figs. 6.1, 6.2, and 6.4, and the nomogram of Fig. 6.6.

In addition, we should add that calculations by Eq. (6) of this problem as a function of the values of $N_ц$ and $N_п$ become less unwieldy if we use the formulas

$$M[N_{c.ц}] = P_1 N_п - \sum_{i=N_ц+1}^{N_п} (i - N_ц) C_{N_п}^i P_1^i (1 - P_1)^{N_п - i}; \tag{7}$$

$$M[N_{c.ц}] = N_ц - \sum_{i=0}^{N_ц} (N_ц - i) C_{N_п}^i P_1^i (1 - P_1)^{N_п - i}. \tag{8}$$

Appendix 2 contains the table for function (6).

PROBLEM 6.5. Determine the average percentage of downed targets as a function of the ratio $N_п/N_ц$ between the number of interceptors and the number of targets in the raid, if aerial combat consists of individual battles in which the targets are downed independently of one another with probability P_1, while the interceptors are downed with probability P_2. Let us assume that we have ideal target distribution for both opponents.

Given: P_1 = 0.3, 0.5, 0.7; P_2 = 0.1, 0.3, 0.5, 0.7, 0.9; $N_п/N_ц$ = 0-5.

Solution. The expectation of the relative number of downed targets is defined by the formula

$$\frac{M[N_{\text{с.ц}}]}{N_{\text{ц}}}=P_{\text{ц}}(1-P_{\text{пор.п}}), \tag{1}$$

where $P_{\text{ц}}$ is the probability of downing each target; $P_{\text{пор.п}}$ is the probability of damaging each interceptor.

Let us use Eq. (2) of Problem 6.2 to calculate these probabilities. For our model of aerial combat with ideal target distribution we have

$$P_{\text{ц}}=1-(1-P_1)^{\frac{N_{\text{п}}}{N_{\text{ц}}}}; \tag{2}$$

$$P_{\text{пор.п}}=1-(1-P_2)^{\frac{N_{\text{ц}}}{N_{\text{п}}}}. \tag{3}$$

Consequently, the desired effectiveness is found from the formula

$$\frac{M[N_{\text{с.ц}}]}{N_{\text{ц}}}=\left[1-(1-P_1)^{\frac{N_{\text{п}}}{N_{\text{ц}}}}\right](1-P_2)^{\frac{N_{\text{ц}}}{N_{\text{п}}}}. \tag{4}$$

The results of the calculations for the initial data of the problem are given in Fig. 6.7.

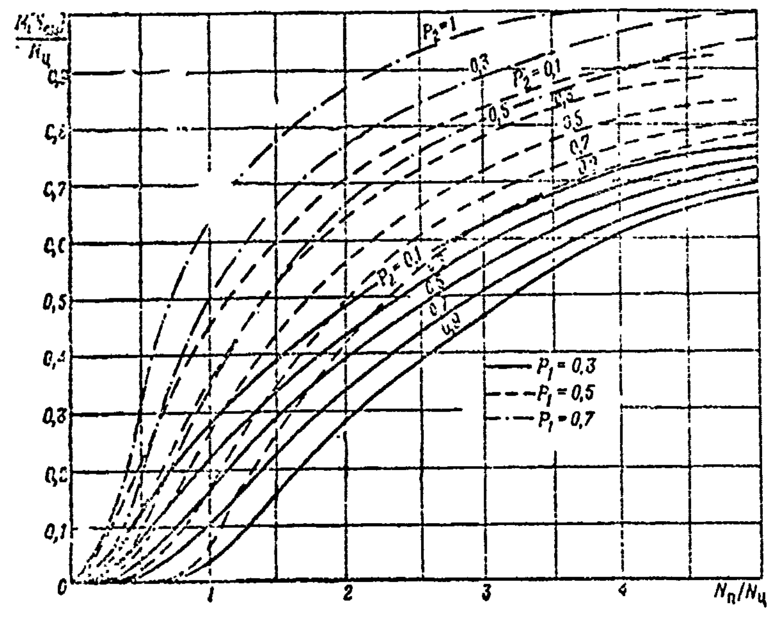

Fig. 6.7.

PROBLEM 6.6. In group aerial combat the number of targets $N_ц$ is equal to the number of interceptors $N_п$. Each interceptor is armed with $m_р$ rockets and can carry out several attacks, destroying in each attack a single target with probability P_1. In turn, due to enemy fire, the interceptors in each attack are damaged with constant probability P_2. Determine the expectation of the relative number of downed targets $M[N_{с.ц}]/N_ц$ for three combat models: no target distribution, ideal target distribution, and ideal reaiming. Determine the optimum number of attacks which each interceptor should carry out to assure maximum effectiveness, remaining undamaged with maximum probability.

Given: $P_1 = 0.7$; $P_2 = 0.1, 0.2, 0.3$.

Solution. By analogy with Problem 6.5 we have

$$\frac{M[N_{с.ц}]}{N_ц} = P_ц (1 - P_{пор.п}), \tag{1}$$

where $P_ц$ is the probability of downing each target which, in accordance with the combat model, is calculated by Eq. (2) of Problem 6.2, (5) of Problem 6.3, and (6) of Problem 6.4; $P_{пор.п}$ is the probability of damage to each interceptor.

Since $N_ц = N_п$, the possible number of attacks on each target with uniform target distribution is

$$n_{ат} = \frac{m_р N_п}{N_ц} = m_р \tag{2}$$

and, consequently,

$$P_{пор.п} = [1 - (1 - P_2)^{n_{ат}}]. \tag{3}$$

Thus, the formula for calculating the effectiveness under the examined conditions assumes the following form:

$$\frac{M[N_{с.ц}]}{N_ц} = P_ц (1 - P_2)^{n_{ат}}. \tag{4}$$

The results of the calculations for the initial data of the problem and $P_1 = 0.7$ are given in Fig. 6.8. Analysis of the graphs

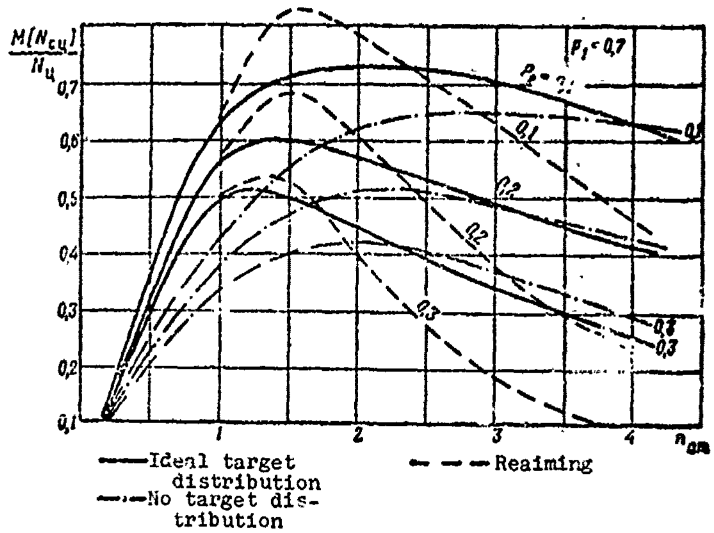

Fig. 6.8.

shows that under the examined conditions there is always a certain optimum number of attacks in which the effectiveness, i.e., the probability of downing each target with no damage to each interceptor, is maximum. This is explained by the fact that with an increasing number of attacks on a target the increased probability of downing each target decreases, while the probability of damage to each interceptor increases in this case.

PROBLEM 6.7. A group target consisting of $N_ц$ aircraft can be attacked by a group of $N_п$ interceptors in the following two ways:

1. The target is attacked by all $N_п$ interceptors in turn; subsequent attacks are carried out after analysis of the results of previous ones; each single interceptor downs any enemy aircraft with probability P_1, and the interceptor in this case is damaged by each enemy aircraft with probability P_2.

2. All interceptors attack a target simultaneously; the probabilities of target and interceptor damage with a single operation against either are also P_1 and P_2.

Determine the dependence of the expectation of the number of downed targets $M[N_{с.ц}]$ on the values of P_1, P_2, $N_ц$, and $N_п$. Considering that the effectiveness depends on the force ratio $N_п/N_ц$, determine the optimum quantitative composition of the group target for which, when attacked by a given number of interceptors, the effectiveness $M[N_{с.ц}]$ is maximum. Show how the effectiveness can be increased if we turn from single to group operations.

Given: $P_1 = 0.5$; $P_2 = 0.5$ and 0.1; $N_ц = 1-5$; $N_п = 1-5$; combat model with ideal target distribution.

Solution. Using Eq. (4) of Problem 6.5 we find the dependence of $M[N_{c.ц}]$ on the given values of P_1, P_2, $N_ц$, and $N_п$ (Fig. 6.9).

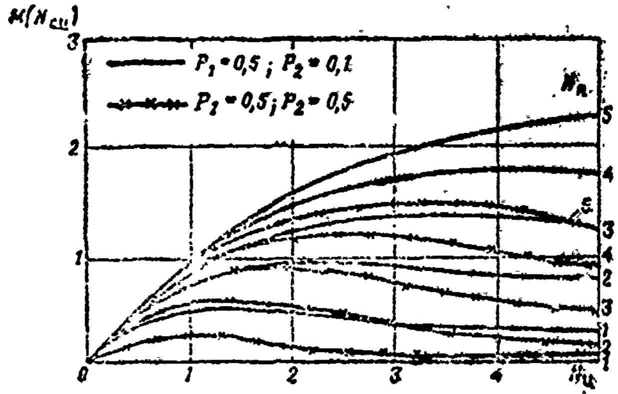

Fig. 6.9.

This dependence shows that we always have optimum quantitative composition of the group target for which the effectiveness, considering counterfire, is maximum with operations by the given number of interceptors. For example, when $P_1 = 0.5$ and $P_2 = 0.5$ it is rational to use a group of five interceptors against 3-4 enemy aircraft, but not against two or five: in the first case the effectiveness is low because of the small number of targets, in the second case — because of the high probability of damage to each interceptor. Using this formula, however, we find the difference in effectiveness for the two given attack methods.

PROBLEM 6.8. The enemy carries out an air raid with an intensity of $\lambda = 10$ targets/hour. The targets cross the line of responsibility of the air-defense region at random moments of time, forming in this case a simple flow. Two guidance stations are set up to organize the interception; each of these stations simultaneously guides a single interceptor to a single target, spending $T_H = 5$ min per guidance. If, at the moment the target crosses the air-defense line, both stations are busy guiding interceptors to previous targets, the given target is unintercepted. How many targets will pass unintercepted if the enemy raid lasts for 10 hours? Let us assume that there are enough interceptors, and only the fact that the stations are busy with guidance results in partial enemy penetration. Determine the expectation of the number of downed targets for $\lambda = 10$ targets/hour, $T_H = 5$ min, if the probability of target downing with the occurring guidance is $P_1 = 0.9$.

Solution. Let us determine the probability P_n that a target, selected at random from the flow, finds both guidance stations busy as it crosses the air-defense boundary. For this let us use the Erlang formula [3, 39]

$$P_n = \frac{\frac{\beta^n}{n!}}{\sum_{k=0}^{n} \frac{\beta^k}{k!}}.$$

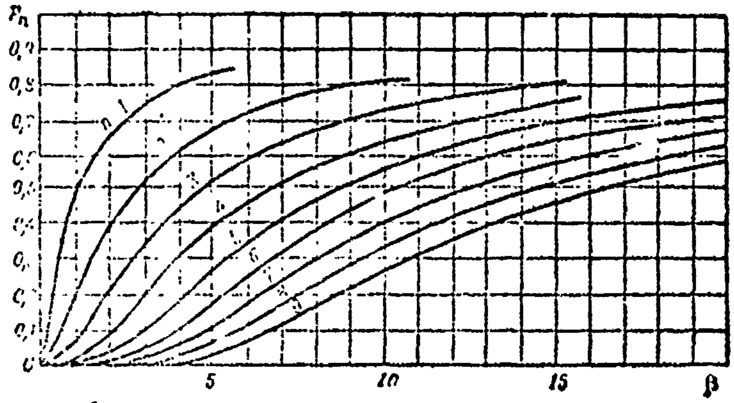

Fig. 6.10.

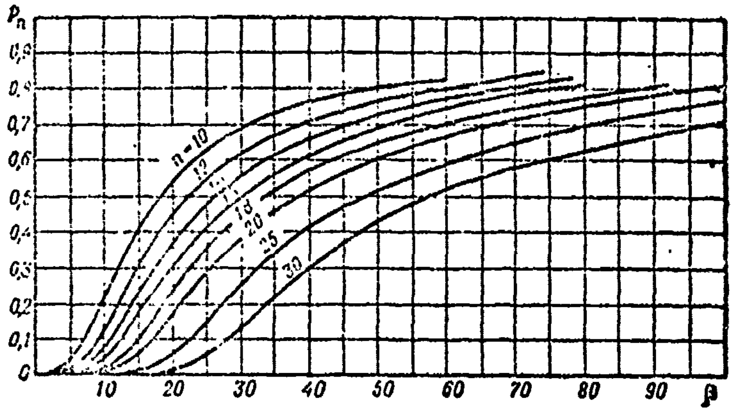

Fig. 6.11.

Calculations made by the Erlang formula are shown in Figs. 6.10 and 6.11 for a great many conditions; the probability P_n that all n channels are occupied is derived as a function of the number of channels and the reduced density:

$$\beta = \frac{\lambda}{\mu} = \lambda T_{w}.$$

For our example we have

$$P_n = \frac{\left(\frac{5}{6}\right)^2}{2! \sum_{k=0}^{2} \frac{\left(\frac{5}{6}\right)^k}{k!}} = 0{,}16.$$

Since under these conditions it is shown that a target, finding both guidance stations occupied, remains unintercepted, P_n = 0.16 expresses the percent of unintercepted targets. In all, in a 10-hour raid there are 10λ = 100 targets; of them, 100·0.16 = 16 targets are unintercepted.

This problem is interesting in that it shows how much the random nature of target appearance can reduce the effectiveness of an air-defense system. Actually, if the targets were to cross the air-defense boundary in various time intervals with the same intensity λ = = 10 targets/hour, i.e., 1 target each 6 minutes, with a guidance time t_H = 5 min all targets would be intercepted even when using only one guidance station. When β = 5/6 and n = 2, from the Erlang formula the probability that all channels are busy $P_n = P_2 = 0.16$. Consequently, the probability that at least one channel is free is $1 - P_n = 0.84$.

The probability that there will be guidance against each target, and the target is in this case downed, is

$$(1-P_n)P_1 = 0{,}84 \cdot 0{,}9 = 0{,}756,$$

while the expectation of the number of downed targets

$$(1-P_n)P_1 N_u = 0{,}84 \cdot 0{,}9 \cdot 100 = 75{,}6.$$

For our example, when the relative carrying capacity of the system is one (i.e., we have a regular flow with 6-minute intervals between targets, and the guidance time is constant, equal to 5 min), the number of downed targets is 100·0.9 = 90. Thus the random nature of the flow of targets and the random guidance time decrease the effectiveness by 16%.

PROBLEM 6.9. The raid is a simple flow of single targets. All targets are intercepted on one given line. The time that the interceptor guidance channels are busy is a random magnitude, with exponential distribution. The average time between successive targets is 5 min; the average time the channels are busy is also 5 min. There are 5 guidance channels.

What are the probability that exactly 0, 1, 2, 3, 4, or 5 channels will be occupied? How do these probabilities change if the average time the channels are busy increases to 10 minutes?

Solution. The answer is obtained using the Erlang formula. The results of the calculations are given in Table 6.1.

The table shows that the probability that all 5 channels will be occupied with β = 2 is ~0.04, i.e., each 25-th target crosses the

Table 6.1

k	$\beta = 1$	$\beta = 2$
0	0,36810	0,13761
1	0,36810	0,27523
2	0,18395	0,27523
3	0,06135	0,18349
4	0,01534	0,09174
5	0,00307	0,03670

intercept line unchallenged; when $\beta = 1$, only each 300-th target is not intercepted.

PROBLEM 6.10. The raid is a simple flow of targets with intensity $\lambda = 10$ targets/hour. The air group has the problem of intercepting at least 90% of the targets. To exclude mutual interference and assure safety among the attacking interceptors, we designate several lines for simultaneous operations against the targets. The number of simultaneous operations does not exceed the number of indicated lines. If there are more targets than lines, they pass unchallenged through the air-defense region. The average time of target intercept on any line $t_{н} = 5$ min. The probability of target damage $P_{пор} = 0.96$. How many intercept lines should be designated so that the probability of target passage does not exceed 0.1? Let us assume that the required number of combat-ready interceptors does not limit the percentage of downed targets, and that the duration of the combat operations is sufficiently long (at least no less than $10t_{н}$), and that the Erlang formulas are applicable for such a steady-state process.

Solution. Let us designate the desired number of intercept lines by $n_{р.п}$. Since the number of simultaneous guidances is equal to the number of designated lines, when using the Erlang formulas the number of channels is the number of lines. Consequently, the probability that all guidance channels are busy is equal to the probability of unchallenged target passage:

$$P_{n_{р.п}} = P_{прон} = \frac{\frac{\beta^{n_{р.п}}}{n_{р.п}!}}{\sum_{k=0}^{n_{р.п}} \frac{\beta^k}{k!}}; \quad \beta = \lambda t_{н}. \qquad (1)$$

The probability that exactly k channels are busy, i.e., the probability that target interception occurs on exactly k lines

$$P_k = \frac{\beta^k}{k!} P_0.$$

when

$$0 < k < n_{p.n}; \qquad (2)$$

$$P_k = \frac{\beta^k}{n_{p.n}!\, n_{p.n}^{k-n_{p.n}}} P_0,$$

when

$$k \geqslant n_{p.n}. \qquad (3)$$

The probability that no channel is used, i.e., the probability that there are no targets at any of the designated lines, is

$$P_0 = \frac{1}{\sum\limits_{n=0}^{n_{p.n}-1} \frac{\beta^k}{k!} + \frac{\beta^{n_{p.n}}}{n_{p.n}!}\left(\frac{n_{p.n}}{n_{p.n}-\beta}\right)}. \qquad (4)$$

Using Formulas (1), (2), and (4), let us calculate the probabilities P_0, P_1, P_2, P_3, ... that there are no targets at any of the designated lines, there are targets at one line, two lines, three lines, etc., and then find the sum

$$\sum_{k=0}^{n_{p.n}} P_k = P_0 + P_1 + P_2 + P_3 + \ldots, \qquad (5)$$

equal to the probability that operations against targets occur on lines whose number does not exceed $n_{p.n}$.

The results of the calculations are summarized in Table 6.2.

Table 6.2

$n_{p.n}$	$\frac{\beta^{n_{p.n}} n_{p.n}}{n_{p.n}!\,(n_{p.n}-\beta)}$	$\sum_{k=0}^{n_{p.n}-1} \frac{\beta^k}{k!}$	P_0	P_1	P_2	P_3	$\sum_{k=0}^{n_{p.n}} P_k$
1	5	1	0,167	0,139	0	0	0,306
2	0,595	1,83	0,412	0,343	0,113	0	0,874
3	0,134	2,18	0,132	0,360	0,150	0,010	0,942

Thus, if we designate 3 lines, $0.942 \cdot 0.96 = 0.905$ targets will be intercepted, and the interceptors' mission will be completed.

PROBLEM 6.11. For the conditions of Problem 6.8, determine the expectation of the number of downed targets if the targets are intercepted not on a line but in a band of width D. The target speed is constant, and equal to $V_ц$.

Solution. Direct solution of this problem is given by the Barrer formula [40], from which we can calculate the probability that all channels of a multichannel system are busy for the case of "impatient" requests distributed by the Poisson law. By an "impatient" target in this case we mean one which, finding all guidance channels busy, waits in the air-defense belt for time $t_{ож} = D/V_ц$, after which it leaves the queueing system. If, however, the target is on queue, it remains in the system until there is no queueing.

For our example, when

$$\alpha = \lambda t_{ож} = \frac{\lambda D}{V_ц}, \quad \beta = \frac{\lambda}{\mu} = \lambda t_ц$$

according to the Barrer formula we have

$$P_n = \frac{(\beta - n)\beta^n e^{\alpha(\beta - n)}}{(\beta - n)\, n! \sum_{k=0}^{n} \frac{\beta^k}{k!} + \beta^{n+1}\left[e^{\alpha(\beta - n)} - 1\right]}. \qquad (1)$$

Analysis of the Erlang and Barrer formulas (Figs. 6.10-6.14) shows that a difference from the simple formulas for regular flow occurs when $\beta = \lambda/\mu \approx n$, i.e., when the reduced target density β is approximately equal to the number of guidance channels. When $\beta = (2-3)n$, there is saturation in the system and $P = n\beta$, while when $\beta \geq 3n$ we can always use the regular-flow formulas:

$$P_n = 1 - \frac{n}{\beta} = 1 - \frac{n}{\lambda t_ц}, \qquad (2)$$

which coincide numerically with the Erlang and Barrer formulas. The Barrer formula must be used when $\beta \approx n$ and $n < 10$-15; in other cases it coincides with Eq. (2).

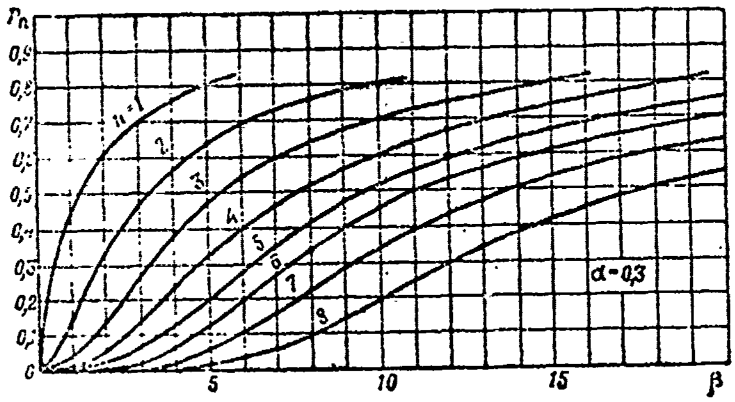

Fig. 6.12.

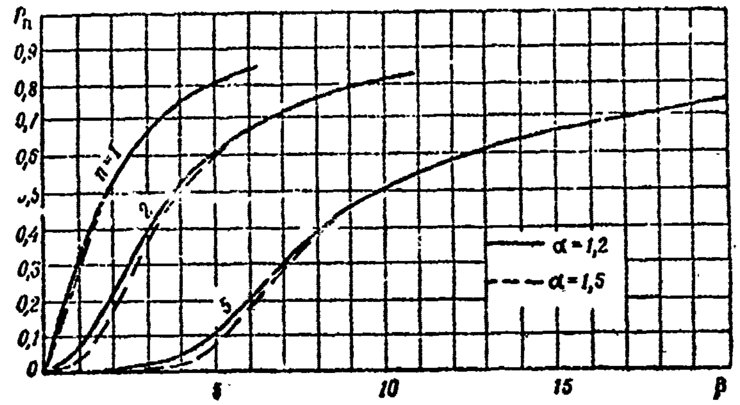

Fig. 6.13.

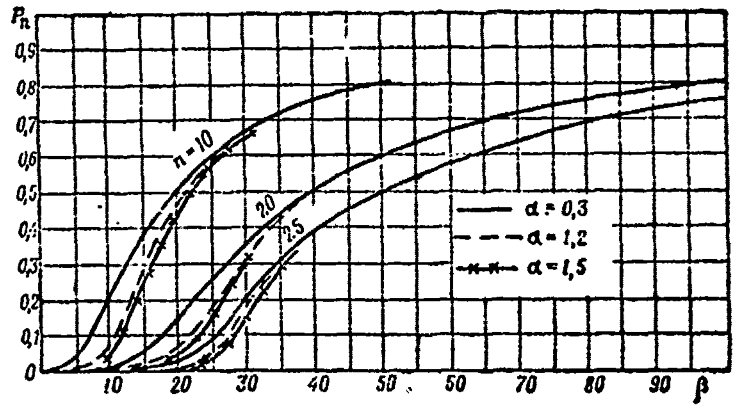

Fig. 6.14.

PROBLEM 6.12. An air raid consists of a random flow of single targets, the intervals T between which are subject to the Poisson law with expectation $T_{ц} = 5$ min. The time during which the guidance channels are busy is a random number, distributed exponentially with expectation $T_{н} = 10$ min. The guidance system operates such that any target enters on "queue" (interceptor guidance to it begins) on any free guidance channel. The combat mission to destroy targets on a given line is set up such that a target arriving at the line at a moment when all $n_{к}$ channels are busy can "wait" no more than two minutes for any channel to become free. Otherwise it crosses the line unintercepted. We must determine the required number of guidance channels $n_{к}$ which will assure an average "expectation" time $T_{ож}$ of no more than two minutes. What will be, in this case, the general average number of targets in the system, and the average number of targets awaiting the start of queueing? Compare the described guidance system with one operating under conditions when $T_{ц}$ and $T_{н}$ are constants and equal to $T_{ц} = 5$ min, $T_{н} = 10$ min.

Solution. The total number of targets in the system is equal to the sum of the number of targets $N_{ц.ож}$ awaiting the start of guidance and the number of targets $N_{ц.н}$ to which the interceptors are being guided:

$$N_{ц} = N_{ц.ож} + N_{ц.н}. \tag{1}$$

From queueing theory we know that for the described multi-channel guidance system the probability that there are $N_{ц}$ targets in the system is calculated from the formulas

$$P_{N_{ц}} = \frac{\rho^{N_{ц}}}{N_{ц}!} P_0, \tag{2}$$

when

$$0 \leqslant N_{ц} \leqslant n_{к},$$

and

$$P_{N_{ц}} = \frac{\rho^{N_{ц}}}{n_{к}!\, n^{N_{ц} - n_{к}}} P_0, \tag{3}$$

when

$$N_{ц} \geqslant n_{к}.$$

Here

$$\beta = \frac{T_{\text{н}}}{T_{\text{ц}}}. \quad (4)$$

From the condition

$$\sum_{N_{\text{ц}}=0}^{\infty} P_{N_{\text{ц}}} = 1 \quad (5)$$

we find that the probability that none of the guidance channels is busy is

$$P_0 = \left[\frac{\beta^{n_{\text{к}}}}{(n_{\text{к}}-1)!\,(n_{\text{к}}-\beta)} + \sum_{N_{\text{ц}}=0}^{n_{\text{к}}-1} \frac{\beta^{N_{\text{ц}}}}{N_{\text{ц}}!} \right]^{-1}. \quad (6)$$

By definition of expectation, the average number of targets in the system is

$$M[N_{\text{ц}}] = \sum_{N_{\text{ц}}=0}^{\infty} N_{\text{ц}} P_{N_{\text{ц}}}. \quad (7)$$

The expectation of the number of targets awaiting the start of guidance is

$$M[N_{\text{ц. ож}}] = \sum_{N_{\text{ц}}=n_{\text{к}}+1}^{\infty} (N_{\text{ц}} - n_{\text{к}}) P_{N_{\text{ц}}}. \quad (8)$$

On the other hand,

$$N_{\text{ц. ож}} = \frac{T_{\text{ож}}}{T_{\text{ц}}}, \quad (9)$$

from which

$$T_{\text{ож}} = N_{\text{ц. ож}} T_{\text{ц}}. \quad (10)$$

Using the formulas derived we can show that

$$T_{\text{ож}} = \frac{\beta^{n_{\text{к}}} T_{\text{н}} P_0}{(n_{\text{к}}-1)!\,(n_{\text{к}}-\beta)^2}. \quad (11)$$

The ratio $T_{\text{ОЖ}}/T_{\text{Н}}$ is determined from the given values of $n_{\text{К}}$, $T_{\text{Н}}$, and T_0 from Fig. 6.15. We also know that for normal functioning of the system, when the targets appear at random time intervals and the guidance time is also a random value the following condition must be satisfied:

$$n_{\text{к}} > \beta. \quad (12)$$

Otherwise, when the number of channels is less than the reduced density β, the line in the system increases endlessly. Consequently, in our example we must examine $n_к > 2$, since

$$\beta = \frac{T_н}{T_п} = \frac{10}{5} = 2.$$

Let $n_к = 3$; then for β = 2 according to (6) we have $P_0 = 0.11$. When $n_к = 4$, $P_0 = 0.13$. Correspondingly, from (11) we get for

$$n_к = 3 \quad T_{ож} = 0{,}45; \quad T_ж = 4{,}5 \text{ min},$$

and for

$$n_к = 4 \quad T_{ож} = 0{,}073; \quad T_ж = 0{,}73 \text{ min}.$$

Thus, to satisfy the condition $T_{ож} \leq 2$ min we must have at least four guidance channels.

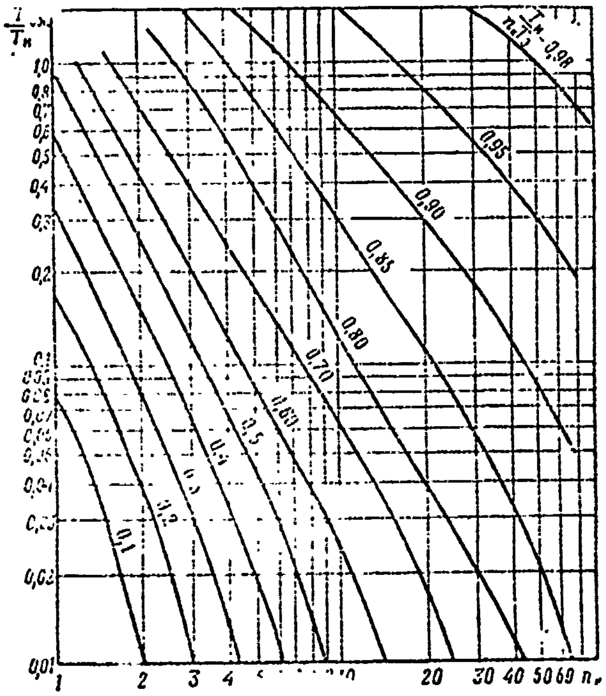

Fig. 6.15.

Now let us determine how many targets $N_ц$, on the average, will be in the system and how many targets $N_{ц.ож}$, on the average, will await the start of guidance. To use Eqs. (7) and (8) we must first calculate, using Eqs. (2) and (3), the probabilities $P_{N_ц}$ that there will be 1, 2, 3, ..., 10, etc. targets in the system. These probabilities are, respectively,

$$P_1 = 0{,}26; \ P_2 = 0{,}26; \ P_3 = 0{,}174; \ P_4 = 0{,}087; \ P_5 = 0{,}044;$$
$$P_6 = 0{,}022; \ P_7 = 0{,}011; \ P_8 = 0{,}005; \ P_9 = 0{,}003; \ P_{10} = 0{,}001; \ P_{11} \approx 0.$$

The average number of targets in the system, according to (7), is $M[N_ц] = 2.16$.

The average number of targets awaiting the start of guidance, according to (8), is $M[N_{ц.ож}] = 0.162$.

During regular flow, when the targets come at equal intervals $T_{ц} = 5$ min and the guidance time is also constant and equal to $T_{H} = 10$ min, two guidance channels assure a relative carrying capacity of one, i.e., in this case all targets will be intercepted, while both channels are continuously occupied. With the random appearance of targets and random guidance time we must have four guidance channels, although the average values of $T_{ц}$ and T_{H} remain unchanged. In this case, on the average, 1.65 guidance channels will be busy, while the probability that all four channels will be busy is 0.348, i.e., there are free channels for almost 65% of the total system operating time. During this time the targets, on the average, await the start of guidance for 0.73 min. The reasons for this are the random clustering and scattering of targets in the total flow.

PROBLEM 6.13. Show how the indicators of the effectiveness of a multichannel guidance system, calculated by the Erlang and Barrer formulas, can be extended to the case when the intensity λ of the incoming flow of targets during a counter-air operation changes by a random law characterized by the distribution function G(λ).

Solution. The probabilities calculated by the Erlang and Barrer formulas for the case when there is the simplest target flow (λ = = cor 'e none other than the average relative time that the guida... ,ystem remains in a certain state E_k. For example, the formula

$$P_n(\lambda=\text{const})=\frac{\frac{(\lambda t_H)^n}{n!}}{1+\frac{\lambda t_H}{1!}+\frac{(\lambda t_H)^2}{2!}+\ldots+\frac{(\lambda t_H)^n}{n!}} \quad (1)$$

defines the average relative time when all n channels are busy with a given target-raid intensity λ and average guidance time t_H.

Obviously, during the counter-air operation the raid intensity λ can change at random. Since it is basically impossible to describe the change in λ by a determinate time function, distribution function G(λ) is the most complete characteristic of intensity as a random value within the examined counter-air operation time T. Now let us designate by $F(\lambda)$, for target flow with intensity λ, the average relative time during which the guidance system is in the state E_k. Since by definition of the distribution function the intensity assumes the value of λ in interval T for time $T \cdot \Delta G(\lambda)$, during this time the guidance system is in the state E_k, on the average, for the time

$$F(\lambda)\, T \cdot \Delta G(\lambda). \tag{2}$$

During the entire counter-air operation, state E_k occurs, on the average, in the time

$$T \int_{\lambda_1}^{\lambda_2} F(\lambda)\, dG(\lambda). \tag{3}$$

Dividing (3) by T, we get the probability that the guidance system is in state E_k.

Thus, to calculate the probability that all channels are busy and the target passes unimpeded across the line, when the target-flow intensity is a random value with distribution function $G(\lambda)$, we must use, in place of Eq. (1), the following relationship:

$$P_n[G(\lambda)] = \int_{\lambda_1}^{\lambda_2} \frac{\frac{(\lambda t_{\text{н}})^n}{n!}\, dG(\lambda)}{1 + \frac{\lambda t_{\text{н}}}{1!} + \frac{(\lambda t_{\text{н}})^2}{2!} + \ldots + \frac{(\lambda t_{\text{н}})^n}{n!}}. \tag{4}$$

The Barrer formula in this case assumes the form

$$P_n[G(\lambda)] = \int_{\lambda_1}^{\lambda_2} \frac{(\lambda t_{\text{н}} - n)(\lambda t_{\text{н}})^n e^{\lambda t_{\text{ож}}(\lambda t_{\text{н}} - n)}\, dG(\lambda)}{(\lambda t_{\text{н}} - n)\, n! \sum_{k=0}^{n} \frac{(\lambda t_{\text{н}})^k}{k!} + (\lambda t_{\text{н}})^{n+1}\left[e^{\lambda t_{\text{ож}}(\lambda t_{\text{н}} - n)} - 1\right]}. \tag{5}$$

PROBLEM 6.14. Propose a general-purpose expression for the distribution density of the intensity of the incoming target flow $g(\lambda)$ which describes the random value of intensity λ during a counter-air operation. Determine the universality such that the simplest target flow, for which λ = const, should be a particular case of the general-purpose law introduced, while their mean values should coincide.

Solution. The following expression corresponds to the universality criteria given in the conditions of the problem:

$$g(\lambda)=\frac{\left(\frac{\lambda}{\delta}\right)^{\left(\frac{\lambda_{cp}}{\delta}-1\right)} e^{-\frac{\lambda}{\delta}}}{\delta\Gamma\left(\frac{\lambda_{cp}}{\delta}\right)}, \tag{1}$$

where λ_{cp} and δ are parameters of the law, while Γ is the gamma-function. Actually,

1) the area beneath the curve described by Function (1), is equal to one, i.e., the expression $g(\lambda)$ corresponds to the property of standardization, as required for density distribution:

$$\int_0^\infty g(\lambda)\,d\lambda=1; \tag{2}$$

2) the expectation of target-raid intensity is

$$\int_0^\infty \lambda g(\lambda)\,d\lambda=\lambda_{cp}; \tag{3}$$

3) dispersion of Law (1) is

$$\int_0^\infty (\lambda-\lambda_{cp})^2 g(\lambda)\,d\lambda=\lambda_{cp}\delta. \tag{4}$$

When $\delta \to 0$ the raid intensity dispersion vanishes, as shown by Eq. (4). This indicates that the probability of any value of λ other than λ_{cp} is zero. Consequently, when $\delta \to 0$ we get the distribution density of the intensity of the simplest flow: $g(\lambda)$ is the delta-function with $\lambda = \lambda_{cp}$. Thus, the parameter δ o. Law (1) is a characteristic of nonsteady target flow. So, if it is necessary to determine the indicators of effectiveness and carrying capacity of the guidance system with a random change in target-raid intensity, we must first determine these indicators for simplest target flow, when the intensity is constant and equal to λ_{cp}, then assign the proposed level of nonstationarity, and, using general-purpose Eq. (1), calculate the desired indicators.

PROBLEM 6.15. An air group consisting of $N_п$ interceptors defends an air-defense line of width B = 1000 km. From the warning band information is received of an air raid consisting of $N_{\text{ц}} = 20$ targets. This allows us to arrange the takeoff of the interceptors. Then, because of the lack of a radar field between the warning band and the object to be defended, the interceptor organizes independent target search, loitering in a band of width B. Because of velocity and heading maneuvers in band B, the targets are distributed randomly by the equal-probability law. How many targets, on the average, will be destroyed on the defense line of width B if the width of target detection by the airborne radar 2b = 50 km, while the probability of target destruction when detected by one interceptor $P_1 = 0.8$?

Solution. The probability of target detection on the defense line is defined by the relationship between the sectors scanned by the airborne radars and the total line width B. Let there be an elemental increment of the scanned sector Δb at some random moment of time t on line B. The probability that a target will be detected by sector Δb is Δb/B. Let us designate by Q(0,b) the probability that up to moment t there are no detected targets. Then the probability that up to moment t there are no detections is

$$dQ(0,b) = -\frac{\Delta b}{B} Q(0,b). \qquad (1)$$

This first-order differential equation satisfies the following initial condition: the probability that there are no detections with Δb = 0 is 1. Considering this condition, the solution, i.e., the probability of no detections, is defined by the formula

$$Q(0,b) = e^{-\Delta b/B}. \qquad (2)$$

Consequently, the probability of target detection by one interceptor is

$$1 - e^{-2b/B}. \qquad (3)$$

The probability of target detection by $N_{\text{п}}$ interceptors is

$$1 - e^{-N_{\text{п}} 2b/B}. \qquad (4)$$

The probability of target destruction by $N_{\text{п}}$ interceptors is

$$1-e^{-N_n 2bP_\text{п}/B}. \qquad (5)$$

The expectation of the number of downed targets

$$M[N_{\text{с. п}}]=N_\text{ц}\left[1-e^{-N_n 2bP_\text{п}/B}\right]. \qquad (6)$$

Substituting the numerical values of our problem we get

$$M[N_{\text{с. п}}]=20\left[1-e^{-20\cdot 50\cdot 0.8/1000}\right]=11.$$

Thus, of 20 targets, only 11 will be downed, on the average. Because of the random nature of destruction of a detected target, an average of 20·0.8 = 16 targets could be destroyed. However, the limited search capabilities of the airborne radars and the great expanse of the line to be defended reduce the expectation of the number of downed targets from 16 to 11.

PROBLEM 6.16. Determine the increase in expectation of the number of downed targets due to optimum flight assignment with independent interceptor combat operations, when it is first necessary to search out the targets in a certain airspace $S_\text{ц}$ and then, after detection, organize attacks to destroy these targets. Let us examine the case when the targets, throughout the entire search time $t_\text{п}$, are distributed with equal probability within a region of area $S_\text{ц}$. To determine this increase we must compare two tactics: 1) each interceptor individually searches out and independently attacks a detected target; 2) targets are sought out by a specially selected search group; after target detection they are destroyed by the search interceptors together with attack interceptors sent to the region of the detected targets. In this case, an optimum flight is designated, determined from the required effectiveness – the probability of downing each target with a given level.

Estimate the increase for the case when bands B = 200 and 400 km are defended; 2b = 50 km.

Solution. For the first tactics, the expectation of the number of downed targets, according to Problem 6.15, is

$$M[N_{c.ц}]_1 = N_ц\left[1 - e^{-\frac{2bV_п t_п N_п P_1}{S_ц}}\right]. \qquad (1)$$

for the second tactics

$$M[N_{c.ц}]_{гр} = N_ц\left[1 - e^{-\frac{2bV_п t_п N_п\left[1-(1-P_1)^{\frac{N_п}{N_ц}}\right]}{S_ц}}\right], \qquad (2)$$

since the probability of downing each target is

$$P_Σ = 1 - (1 - P_1)^{\frac{N_п}{N_ц}}. \qquad (3)$$

The relationship

$$A = \frac{M[N_{c.ц}]_{гр}}{M[N_{c.ц}]_1} = \frac{1 - e^{-\frac{2bV_п t_п N_п\left[1-(1-P_1)^{\frac{N_п}{N_ц}}\right]}{S_ц}}}{1 - e^{-\frac{2bV_п t_п N_п P_1}{S_ц}}}. \qquad (4)$$

gives the desired increase in effectiveness due to optimum flight assignment. Function A vs. flight $N_п$ has a maximum which determines the optimum solution. Calculations show that the tactics of independent search with subsequent call-up of the attack group to the region where the group target is detected allow us to obtain a significant increase in effectiveness. When the ratio of the area scanned by one interceptor to the area of the probable location of the targets η = 0.2-0.05, increase A reaches values of 1.5-2 compared with the case when each interceptor searches and attacks independently with equal probability throughout region $S_ц$.

Figure 6.16 shows Dependence (4) for 2b = 50 km, width of the defense belt B = 200 and 400 km, for various values of P_1.

PROBLEM 6.17. A group of $N_ц$ = 10 targets enter the air-defense zone one after the other. Determine the interceptor flight required to destroy all targets with probability $P_{N_ц}$ = 0.9, if the interceptors attack the same target from the aft hemisphere is sequence, observing the minimum possible safe distances. The depth of target penetration should not exceed $D_{пр}$ = 150 km. The probability of target interception

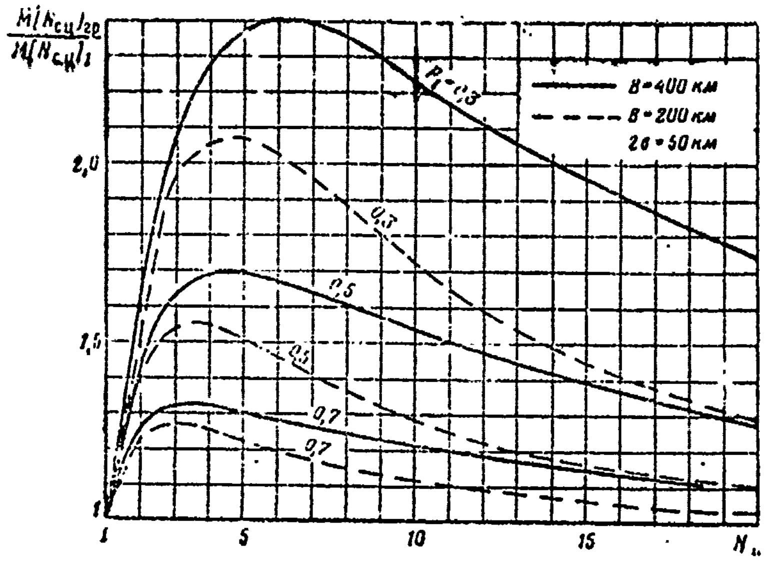

Fig. 6.16.

by a single interceptor is $P_1 = 0.5$. Target speed $V_{ц} = 1000$ km/h, interceptor speed $V_{п} = 1200$ km/h.

Solution. With successive operations against targets, when for safety reasons it is necessary to keep minimum distance between the attacking interceptors $d_{безоп}$, while the depth of target penetration should not exceed $D_{пр}$, the maximum number of successive attacks

$$n_{ат} = \frac{D_{пр}(V_{ц} + V_{п})}{V_{ц} d_{безоп}} + 1, \quad (1)$$

when attack is made from the forward hemisphere, and

$$n_{ат} = \frac{D_{пр}(V_{п} - V_{ц})}{V_{ц} d_{безоп}} + 1, \quad (2)$$

when the attack comes from the aft hemisphere.

In first approximation we can consider that

$$d_{безоп} = V_p t_p, \quad (3)$$

where V_p is the average speed of the rocket, t_p is the maximum time of controlled rocket flight. Let us assume that $d_{безоп} = 30$ km; then $n_{ат} = 2$.

To assure that $M[N_{с.ц}]/N_{ц} = 0.95$ it is necessary, according to Fig. 6.4, that when $P_1 = 0.5$ we have $m_p N_{п}/N_{ц} \approx 2.3$. Thus, to solve the problem the required interceptor flight $N = 2.3 \cdot 30 = 69$.

If the rms error in determining the coordinates of the target and interceptor is σ_x, to increase the assurance of safety during successive attacks by several interceptors the value $d_{безоп}$ should be increased by $3\sigma_x$. E.g., let $\sigma_x = 3$ km. Then $d_{безоп} = 30 + 9 = 39$ km, and when $V_ц = 1000$ km/h, $V_п = 1200$ km/h, and $D_{пр} = 150$ km, $n_{ат} = 1.77$. Then $M[N_{с.ц}]/N_ц \approx 0.83$, i.e., the interceptors cannot carry out their mission.

PROBLEM 6.18. A nonmaneuvering target, consisting of a dense formation of 5-8 aircraft unresolved by the radar station, is attacked in a belt of width B = 160 km. The flight trajectory of this group target forms angle θ with the perpendicular to the belt. To analyze the result of a target-hit by the rockets fired by the interceptors, each subsequent attack occurs after a time equal to the sum of the times of the attack itself (correction of surface-guidance errors, and aiming, firing, and flight of the rocket to impact with the target) and the length of time the damaged target remains in the interceptor's field of view.

Determine the number of successive attacks that can be carried out if the target speed $V_ц = 900$ km/h, the average time for carrying out a single attack $t_{ат} = 2$ min, the average target lifetime $t_{сущ} = 0.5$ min, and angle θ varies within limits of 0 to 80°. How does the desired possible number of attacks change if, with no change in other parameters, the target speed increases to 1500 and 1800 km/h, leading to a corresponding reduction in the operations belt to 120 and 100 km?

Also determine the required number of interceptors to destroy 90% of the targets and a rational rocket-firing regime, if each interceptor carries four rockets, and the probability of damaging a single target with simultaneous firing of two rockets as a function of target speed is as shown in Table 6.3 (p. 206).

Solution. The time the target is located in the belt is

$$\frac{B}{V_ц \cos \theta}.$$

Table 6.3.

$V_ц$, km/h	900	1500	1800
P_1	0,85	0,8	0,7

The time of one interval between two successive rocket firings is $t_{ат} + t_{сущ}$. Consequently, the number of successive attacks which can be carried out against a target in belt S is

$$n_{ат} = \frac{\frac{B}{V_ц \cos\theta} + t_{сущ}}{t_{ат} + t_{сущ}}.$$

The results of calculations using this formula are shown in Fig. 6.17. As can be seen from the graph, the number of possible attacks within limits θ = 0-50° changes only slightly; with a further increase to θ > 50° the value of $n_{ат}$ sharply increases.

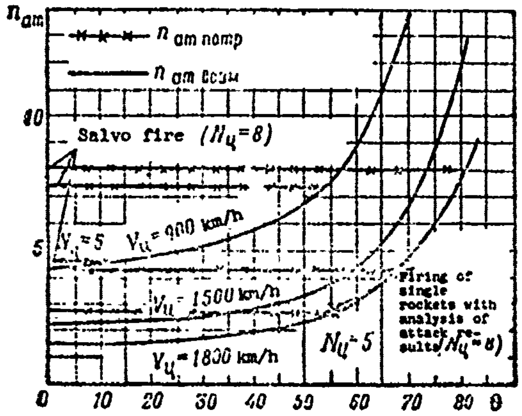

Fig. 6.17.

To determine the rational rocket-firing regime let us compare three combat tactics:

1) the rockets are fired in salvos of two each with the interval between salvos

$$t < t_{ат} + t_{сущ};$$

2) the rockets are fired in salvos of two each with the interval

$$t > t_{ат} + t_{сущ};$$

3) only single rockets are fired, with analysis of the results of each firing.

In the first tactics there is nonuniform distribution of the operations against the targets. Consequently, to calculate the expectation of the relative number of downed targets $M[N_{с.ц}]/N_ц$ we must use Eq. (5) of Problem 6.3. For salvos with time delay $t \geq t_{ат} + t_{сущ}$ and for single firings it becomes possible to distribute the rockets uniformly against the targets and, after analysis of the result of the previous attack, reaim to the undestroyed targets. Consequently, $M[N_{c.ц}]/N_ц$ is calculated from Eq. (6) of Problem 6.4. The results of the calculations are shown in Table 6.4.

Table 6.4.

Combat conditions	Combat tactics								
	1. Independent salvos, two rockets each			2. Salvos of two rockets each in time $t > t_{aт} + t_{сущ}$			3. Single firings with analysis of the results		
$V_ц$, km/h	900	1500	1800	900	1500	1800	900	1500	1800
P_1	0,85	0,8	0,7	0,98	0,96	0,91	0,61	0,55	0,45
Required number of operations against each target	2,7	3	3,4	0,91	0,94	0,98	1,48	1,7	2,2
Required flight of interceptors with $N_ц = 5$	6,75	7,5	8,5	4,5	4,7	4,9	1,85	2,22	2,75
Required flight of interceptors with $N_ц = 8$	10,8	12	13,6	7,2	7,5	7,83	2,96	3,4	4,4

Comparison of the graphs $n_{aт.потр} = f(V_ц)$ and $n_{aт.возм} = f(\theta)$ shows that to assure an effectiveness $M[N_{c.ц}]/N_ц = 0.9$ it is advisable to use the tactic of single firings with analysis of the result of each attack. When $V_ц \geq 1500$ km/h, two-rocket salvo fire with intervals $t > t_{aт} + t_{сущ}$ is advisable only when $\theta > 60°$.

CHAPTER 7

COMBAT READINESS AND RELIABILITY

In addition to combat use, intercept lines, and combat effectiveness, the capabilities of fighter-interceptors are also characterized by combat readiness, i.e., the probability that, by a given time, the interceptor will be ready to carry out its combat mission. In this chapter we examine various quantitative criteria of combat readiness and methods for calculating them.

The most general indicators of the combat readiness of a single interceptor are the coefficients of combat readiness for first and subsequent missions; methods for calculating these are presented in Problems 7.1 and 7.2.

The combat readiness of a group of interceptors (squadron, regiment) is characterized by the probability that at least a given number of available interceptors will be combat ready. This indicator is calculated considering the reliability, the restorability of the aviation materiel, and the number of service personnel and the method of servicing (Problems 7.3-7.6). Problems 7.7-7.10 examine the question of how to take into account the possible failure of aviation materiel when designating a flight of interceptors to carry out a mission and for duty performance.

Problem 7.11 is devoted to methods of figuring the amount of logistics support for combat operations which will guarantee a given level of combat readiness.

Problems 7.12 and 7.13 establish the dependence between the probability of survival and combat readiness during continuous and intense combat operations and the expectation of the number of missions each interceptor can perform under conditions of enemy action. The dependences obtained make it possible to plan the replenishment of the air fleet in time, and determine the combat capabilities of air units based on the number of missions per unit time.

Problem 7.14 establishes the dependence between equipment preparation time and effectiveness.

In Problem 7.15 we determine the dependence between the probability of damage to the enemy during enemy action against an airbase, the damage radius, and the probable deviation and radius of dispersion of the interceptors relative to the enemy's aiming point (explosion epicenter). The nomogram obtained makes it possible to determine, from the permitted excess pressure at the shock-wave front (which the air material undergoes during an explosion) and the TNT equivalent of the explosion, to determine the corresponding radius of damage, and then, from the known probable deviation, to determine unambiguously the radius of dispersion which assures that the probability of interceptor damage will not be above the permitted level.

Problem 7.16 is devoted to estimating the survival probability, while Problem 7.17 discusses monitoring of the combat readiness of the rockets (or other aircraft elements).

PROBLEM 7.1. Combat readiness when carrying out pre-flight preparation is characterized by the probability that all checking of aircraft equipment and systems will be completed by a given time on the interceptor. We call this probability the coefficient of combat readiness for the first mission and designate it by $P_{\text{б.г1}}$. Probability $P_{\text{б.г1}}$ is a function of time: at first, when preparation of the interceptor is just under way, it is zero; after all operations have been completed and if there are no failures during the pre-flight preparation, $P_{\text{б.г1}} = 1$. Since the time for carrying out each operation depends on many random factors (training of the specialists, climatic conditions), the total time $t_{\text{п.п}}$ for preparing the interceptor for the first mission is also a random value. Function $P_{\text{б.г1}}(t_{\text{п.п}})$ characterizes this random time, i.e., $P_{\text{б.г1}}(t_{\text{п.п}})$ is the distribution function for the random time of interceptor pre-flight preparation. Our problem is to express $P_{\text{б.г1}}(t_{\text{п.п}})$ in terms of:

1) the probability $P_{\text{т.г}}$ that all required pre-flight preparation will be finished within the specified time;

2) the probability $P_0(t_{\text{п.п}})$ that no failures will occur during the preparation;

3) the probability $P_{\text{в}}(t_{\text{п.п}})$ that, if a failure is detected, it will be eliminated in the time specified for pre-flight preparation.

Solution. Probability $P_{\text{б.г1}}(t_{\text{п.п}})$ is defined by the probabilities of two favorable and mutually exclusive events:

1) either the interceptor has no failures during preparation time $t_{\text{п.п}}$ and all operations are completed;

2) or the interceptor turns out to have something wrong with it during preparation, but it is repaired in the allotted time and all operations on it are completed.

Consequently,

$$P_{\text{б.г1}}(t_{\text{п.п}}) = P_0(t_{\text{п.п}})P_{\text{т.г}}(t_{\text{п.п}}) + [1 - P_0(t_{\text{п.п}})]P_{\text{в}}(t_{\text{п.п}})P_{\text{т.г}}(t_{\text{п.п}}). \quad (1)$$

If we know the average times of failure-free operation T_0 and recovery $T_{\text{в}}$, then

$$P_0(t_{\text{п.п}}) = e^{-\frac{t_{\text{п.п}}}{T_0}} \quad (2)$$

and

$$P_{\text{в}}(t_{\text{п.п}}) = 1 - e^{-\frac{t_{\text{п.п}}}{T_{\text{в}}}}. \tag{3}$$

PROBLEM 7.2. As the criterion of combat readiness of an interceptor for repeated and subsequent missions we have the probability that the interceptor will be serviced and prepared for a combat mission in the allotted time t. Let us call this probability the coefficient of combat readiness for subsequent missions, and designate it by $P_{\text{б.г2}}(t)$. Find the dependence of $P_{\text{б.г2}}(t)$ on the probabilities that determine the success in carrying out all required operations for mission preparation. Also take into account in this case the possibility of failure occurrence and elimination.

Solution. The probability $P_{\text{б.г2}}(t)$ is defined by the sum of the probabilities of three favorable and mutually exclusive events:

1) the interceptor returned in good condition from a flight, was prepared for the next mission in time $t_{\text{повт}}$, and was not rejected during this time;

2) the interceptor returned in good condition from a flight, but was rejected during preparation for a mission; however it was repaired in time $t_{\text{повт}}$ and during this time was serviced and prepared for the mission;

3) the interceptor returned in poor condition from a flight, but in time $t_{\text{повт}}$ it was repaired and prepared for the next mission.

Consequently,

$$\begin{aligned} P_{\text{б.г2}}(t) &= P_0(t_{\text{пол}})\, P_{\text{т.г}}(t_{\text{повт}})\, P_0(t_{\text{повт}}) + \\ &+ P_0(t_{\text{пол}})\, [1 - P_0(t_{\text{повт}})]\, P_{\text{в}}(t_{\text{повт}})\, P_{\text{т.г}}(t_{\text{повт}}) + \\ &+ [1 - P_0(t_{\text{пол}})]\, P_{\text{в}}(t_{\text{повт}})\, P_{\text{т.г}}(t_{\text{повт}}). \end{aligned}$$

The probabilities P_0 and $P_{\text{в}}$ are calculated from Eqs. (2) and (3) of Problem 7.1, except that in place of time $t_{\text{п.п}}$ we substitute either $t_{\text{повт}}$ or flight time $t_{\text{пол}}$.

PROBLEM 7.3. A squadron of $N_{\text{п}} = 12$ interceptors participates in continuous intense combat operations. At any moment of time, each interceptor can be in only three possible conditions: 1) the interceptor is combat-ready; 2) the interceptor is not combat-ready, but is repaired; 3) the interceptor is not combat-ready and is not repaired, but is awaiting its turn for repair.

Out-of-action interceptors are repaired by special repair crews containing specialists from all services. Repairs are performed in turn; those planes with the oldest damage are repaired first, using any of the free crews. The time of failure-free operation and the repair time for the interceptors are random values subject to exponential distribution law. As criteria for the combat readiness of the squadron we have the probability that at any moment all 12 interceptors are combat ready and the probability that at any moment at least 11 of the 12 are ready (i.e., only 1 or 0 are not combat ready).

Show how these criteria change for ratios of repair time $T_в$ to the average time between failures T_0 of 0.01, 0.025, 0.5, and 0.1, if the number of repair crews varies from 1 to 10. What additional quantitative indicators characterize the combat readiness of an air fleet and the degree to which the specialists are occupied? Solve the problem for $N_п = 40$.

Solution. Since the number of noncombat-ready aircraft is a random value, the most complete characteristic of combat readiness is given by the distribution law for this random number. Knowing the distribution law for the number of noncombat-ready interceptors we can determine any combat-readiness indicator. Our desired criterion for combat readiness in general form is defined as the probability $P_{б.г}$ that of $N_п$ interceptors of the squadron, at least k are combat-ready (in this particular case, at least 12). In other words, we must first determine the probabilities $P_{N_п}(k)$ that of $N_п$, exactly k are not combat-ready ($k = 0, 1, 2, \ldots, N_п - k$), and then sum these probabilities to the given value of $N_п - k$ (in our case, to $12 - 1 = 11$). The probabilities $P_{N_п}(k)$ are the ordinates of the distribution law of the number of noncombat-ready interceptors. Thus,

$$P_{б,г} = 1 - \sum_{k=0}^{N_п - k} P_{N_п}(k). \tag{1}$$

In queueing theory we know the classical problem of using several instruments to service a limited source of requests, which can serve as the basis for solving our problem. According to queueing theory, to determine the probability that of $N_п$ interceptors exactly k will, at any moment of time, be out of order and in the process of repair, we can use the following formulas:

$$P_{N_п}(k) = \frac{N_п!}{k!(N_п - k)!}\left(\frac{\lambda}{\mu}\right)^k P_{N_п}(0), \tag{2}$$

when $1 \le k \le r$, and

$$P_{N_п}(k) = \frac{N_п!}{r^{k-r} r! (N_п - k)!}\left(\frac{\lambda}{\mu}\right)^k P_{N_п}(0), \tag{3}$$

when $k > r$; here r is the number of repair crews; λ and μ are the intensities of failures and repairs.

For exponential time-distribution laws we have

$$\lambda = \frac{1}{T_0}; \quad \mu = \frac{1}{T_в}. \tag{4}$$

Obviously, before calculating the probabilities $P_{N_п}(k)$ from Eqs. (2) and (3), we must determine the value $P_{N_п}(0)$. We find that

$$P_{N_п}(0) = \frac{1}{\sum_{k=0}^{N_п} \frac{P_{N_п}(k)}{P_{N_п}(0)}}, \tag{5}$$

because

$$\sum_{k=0}^{N_п} \frac{P_{N_п}(k)}{P_{N_п}(0)} = \frac{1}{P_{N_п}(0)} \sum_{k=0}^{N_п} P_{N_п}(k) = \frac{1}{P_{N_п}(0)}, \tag{6}$$

since

$$\sum_{k=0}^{N_п} P_{N_п}(k) = 1$$

as a certain event (the sum of the probabilities of all possible states of combat readiness is 1), while the probability $P_{N_п}(0)$ in (6) is removed from the summation sign as a constant.

Thus, the probability $P_{N_п}(0)$, necessary for calculating the probabilities $P_{N_п}(k)$, is determined using Eqs. (2) and (3) by calculating the sums

$$\sum_{k=0}^{N_п} \frac{N_п!}{k!\,(N_п-k)!}\left(\frac{\lambda}{\mu}\right)^k = \sum_{k=0}^{N_п} \frac{P_{N_п}(k)}{P_{N_п}(0)} \qquad (7)$$

and then taking, as per (5), the reciprocal. Having determined the probability $P_{N_п}(0)$ let us find, for all values of k from 1 to r the probability ratios

$$\frac{P_{N_п}(k)}{P_{N_п}(0)}, \qquad (8)$$

and then, by multiplying these ratios by the value $P_{N_п}(0)$, the desired probabilities $P_{N_п}(k)$ for all k = 1-r. Having obtained the values of $P_{N_п}(k)$, let us calculate the coefficient of combat readiness $P_{б.г}$ from Eq. (1).

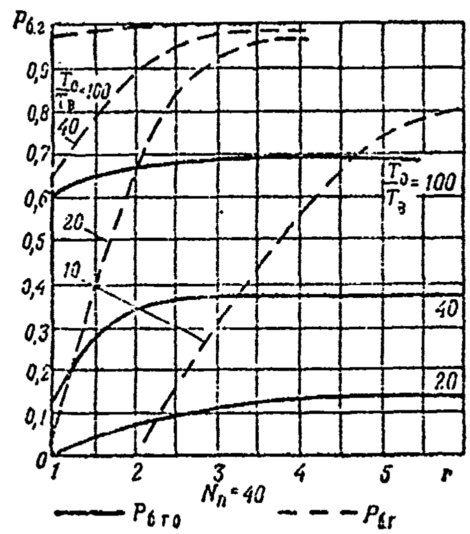

Fig. 7.1.

The results of the calculations for the data in our problem are shown in Fig. 7.1. The dependences $P_{б.г} = f(r, T_0/T_в)$ show that for our data it is advisable to have no more than three repair crews, since when r > 3 the value $P_{б.г}$ is determined completely only by the ratio of T_0 to $T_в$, i.e., there are only two ways of increasing the combat readiness in this case: increase the reliability T_0 or decrease the repair time $T_в$. Increasing the number of repair crews above $r_{опт}$ does not make it possible to increase the combat readiness.

On the basis of Eqs. (2) and (3), which represent the most complete characteristic – the distribution density of the number of repaired interceptors – we obtain other particular quantitative indicators of the combat readiness of an air fleet and the degree to which the repair crews are occupied.

The probability that all r repair teams are free

$$P_0=\left[\sum_{k=0}^{r}\frac{N_n!}{k!\,(N_n-k)!}\left(\frac{\lambda}{\mu}\right)^k+\sum_{k=r+1}^{N_n}\frac{N_n!}{r^{k-r}\,(N_n-k)!\,r!}\left(\frac{\lambda}{\mu}\right)^k\right]^{-1}. \qquad (9)$$

The average number of interceptors awaiting the start of repairs

$$M_1=\sum_{k=r+1}^{N_n}\frac{(k-r)\,N_n!}{r^{k-r}\,r!\,(N_n-k)!}\left(\frac{\lambda}{\mu}\right)^k P_0. \qquad (10)$$

The average number of interceptors being repaired

$$M_2=\left[\sum_{k=0}^{r}\frac{N_n!}{k!\,(N_n-k)!}\left(\frac{\lambda}{\mu}\right)^k+\sum_{k=r+1}^{N_n}\frac{kN_n!}{r^{k-r}\,r!\,(N_n-k)}\left(\frac{\lambda}{\mu}\right)^k\right]. \qquad (11)$$

The average relative number of noncombat-ready interceptors

$$\frac{M_1+M_2}{N_n}. \qquad (12)$$

The average number of free repair teams

$$M_3=\sum_{k=0}^{r}\frac{(r-k)\,N_n!}{k!\,(N_n-k)!}\left(\frac{\lambda}{\mu}\right)^k P_0. \qquad (13)$$

The lost-time coefficient of the repair crews is M_3/r. The probability that the number of interceptors awaiting the start of repairs is greater than some number N ($N \geq r$)

$$P_{>N}=\sum_{k=N+1}^{N_n}P_{N_n}(k)=1-\sum_{k=0}^{N}P_{N_n}(k). \qquad (14)$$

PROBLEM 7.4. Determine the combat readiness of the squadron fleet if there are three repair crews, the average time between the moments of interceptor breakdown is T_0 = 33 hours, while the average repair time T_B = 4.5 hours. For the rest, the conditions of Problem 7.3 are satisfied. Determine the average number $N_{п.б.г}$ of combaready interceptors and the probability that at least 11 of the 12 interceptors will be combat-ready at any random moment of time. How do the desired combat-readiness indicators change if the number of repair crews increases or decreases? What if we increase the number of interceptors $N_П$ to 20?

Solution. Let us use the calculation formulas in Problem 7.3. By definition of expectation we have, from the density-distribution function $P_{N_П}(k)$ for r = 3,

$$M = \sum_{k=0}^{12} k P_{N_П}(k) = 1{,}5.$$

This is the average number of noncombat-ready interceptors. The average number of combat-ready interceptors $N_{П.б.г}$ = 10.5.

The probability that at least 11 of the 12 interceptors will be combat-ready is equal to the probability that either 1 or 0 will not be combat-ready. Using this law, we find that the probability $P_{б.г}$ that at least 11 of 12 interceptors will be combat-ready is equal to the sum of the ordinates with k = 0 or k = 1, i.e., 0.54.

PROBLEM 7.5. We must compare two methods of maintaining the combat readiness of a group of interceptors. In the first method, each aircraft has its own specialist for repairs, and who does not participate in the repair of the other planes. In the second method, a group of specialists services a group of interceptors without being assigned one specific aircraft, i.e., at the moment one interceptor breaks down, if there is at least one free specialist he immediately begins to repair the aircraft. Otherwise, the inoperative interceptor waits until any of r specialists is free. We are given the fact that the reliability and repairability of the interceptors are characterized by the ratio $\lambda/\mu = T_В/T_0 = 0.1$. In the first case, 1 specialist services $N_П$ = 6 aircraft; in the second case, 3 specialists service $N_П$ = 20 aircraft. Show which service method is better.

Solution. Let us use the calculation formulas of Problem 7.3.

When one specialist services n interceptors (instead of r specialists servicing $N_п$ interceptors), where $n \geq N/r$, then

$$P_{N_п}(k) = \frac{N_п!}{(N_п - k)!}\left(\frac{\lambda}{\mu}\right)^k P_0 \qquad (1)$$

where

$$P_0 = \sum_{k=1}^{N_п} P_k = \frac{1}{\sum_{k=0}^{N_п} \frac{N_п!}{(N_п - k)!}\left(\frac{\lambda}{\mu}\right)^k}. \qquad (2)$$

The expectation of the number of interceptors awaiting repairs is

$$M_1 = N_п - \frac{\lambda + \mu}{\mu}(1 - P_0). \qquad (3)$$

When one specialist services 6 aircraft, according to (1) when $\lambda/\mu = 0.1$, $P_0 = 0.48$.

The expectation of the number of noncombat-ready interceptors is $\sum_0^{N_п} kP_{N_п}(k) = 0.85$, while the expectation of the number of combat-ready planes is

$$6 - 0.85 = 5.15.$$

The relative lost-time of one interceptor

$$0.85/6 = 0.14.$$

The relative lost-time for a specialist with $r = 1$ is $P_0 = 0.48$. Now, when three specialists service 20 interceptors, while λ/μ as before is 0.1, $\sum_{k=1}^{20} kP_k = 2.74$, while the number of combat-ready interceptors is 17.26.

The relative lost-time for one interceptor is

$$\frac{2,74}{20} = 0,135; \quad P_{б.г} = 0,865.$$

The relative lost-time for one specialist is

$$\frac{M_3}{3} = 0,401.$$

Thus, the solution shows the advantage of the group servicing method as opposed to the one-specialist-to-one-aircraft method.

Because of the more rational use of specialists at identical strengths, the group servicing method assures a higher level of combat readiness.

PROBLEM 7.6. To repair interceptors which have been put out of commission in combat it is necessary to create a repair team. The delivery of unusable interceptors for repair is in the nature of a random flow with an average interval between successive deliveries T_0 = 2 hours. The repair time is also a random value, subject to exponential law, with expectation $T_в$ = 5 hours. We must determine how many repair crews r are required so that the average noncombat-ready time of the interceptors, i.e., the sum of the times spent awaiting the start of repairs T and the actual repair times $T_в$, not exceed $T_{неб.г}$ = 6 hours. The duration of the planned combat operations is 10 days. For round-the-clock functioning of the repair system there must be, at each working site, two crews, each working 12 hours. The repair of damaged interceptors is set up as follows. Any free team begins to repair any on-line interceptor. When the number of interceptors $N_п$ in the repair system is less than the number of teams, all interceptors are repaired simultaneously, and not all teams are busy. If the number of interceptors $N_п$ is greater than or equal to r, all teams are simultaneously busy and some of the damaged interceptors await the start of repairs.

Solution. The probability of having exactly N interceptors, under steady-state conditions, in the service system (awaiting repair and being repaired) is calculated from the formulas

$$P_{N_п} = P_0 \frac{\rho^{N_п}}{N_п!}, \quad (1)$$

when

$$1 \leqslant N_п < r;$$

$$P_{N_п} = P_0 \frac{\rho^{N_п}}{r!\, r^{N_п - r}}, \quad (2)$$

when

$$N_n \geq r.$$

Here

$$\beta = \frac{T_n}{T_0};$$

$$P_0 = \left[\frac{\beta^r}{r!\left(1 - \frac{\beta}{r}\right)} + \sum_{i=0}^{r-1} \frac{\beta^i}{i!} \right]^{-1} \qquad (3)$$

The average number of interceptors in line for repairs is

$$N_{\text{п. неб. г}} = \frac{\beta^{r+1}}{r \cdot r!\left(1 - \frac{\beta}{r}\right)^2} P_0. \qquad (4)$$

The result of dividing the average number of interceptors in the service system $N_{\text{п.неб.г}}$ by the average number of repairs per unit time $r/T_{\text{в}}$ gives us the interceptor repair and waiting time:

$$T_{\text{неб. г}} = \frac{\beta^r T_n}{r \cdot r!\left(1 - \frac{\beta}{r}\right)^2} P_0. \qquad (5)$$

The average relative time the interceptors are in the service system while awaiting the start of servicing is calculated from the formula

$$\frac{T_{\text{неб. г}} - T_n}{T_{\text{в}}} = \frac{T_{\text{ож}}}{T_{\text{в}}} = \frac{\beta^r P_0}{r \cdot r!\left(1 - \frac{\beta}{r}\right)^2}. \qquad (6)$$

Calculating from (3) the value P_0, and knowing T_0, $T_{\text{в}}$, and r, let us find $T_{\text{ож}}/T_{\text{в}}$ from (6) and then, multiplying by $T_{\text{в}}$, we obtain the absolute time of interceptor noncombat-readiness $T_{\text{неб.г}}$. For our data $\beta = T_{\text{в}}/T_0 = 5/2 = 2.5$. Since when $\beta > r$ the line increases endlessly (the waiting time is finite only when $\beta < r$), let us set r = 3. Then, according to the graph in Fig. 6.15, $T_{\text{ож}}/T_{\text{в}} = 1.4$. Further, $T_{\text{неб.г}} = 1.4 \cdot 5 + 5 = 12$ hours. When r = 4, $\beta/r = 2.5/4 = 0.625$, $T_{\text{ож}}/T_{\text{в}} = 0.2$; $T_{\text{неб.г}} = 0.2 \cdot 5 + 5 = 6$ hours. Thus, there should be at least 4 teams.

Let us estimate the degree of occupation of the crews. In 10 days (10·24)/2 = 120 interceptors arrive for repairs. Their repair takes 120·5 = 600 hours, i.e., each crew works 600/(4·10) = 15 hours per day.

PROBLEM 7.7. To carry out a combat mission we must designate a flight of $N_{п.бг} = 10$ interceptors. We know that if the interceptors take off without pre-flight preparation the probability of armament-system failure $P_{отк} = 0.1$. How many interceptors should be sent up to carry out the mission even in the case of possible failures of certain parts of the interceptors?

Solution. Let us designate by $N_п$ the number of interceptors available for scramble. If the probability of failure of each interceptor is $P_{отк}$, the probability of failure occurring in precisely k of the $N_п$ interceptors is

$$P_{N_п}(k) = C^k_{N_п} P^k_{отк} (1 - P_{отк})^{N_п - k}, \tag{1}$$

where $C^k_{N_п}$ is the number of combinations of $N_п$, k at a time:

$$C^k_{N_п} = \frac{N_п!}{k!\,(N_п - k)!}. \tag{2}$$

For guaranteed completion of the combat mission it is necessary that the probability of air failure greater than $N_п - N_{п.бг}$ (in our example $N_п = 10$) be sufficiently small, e.g., 0.05. Then, with a probability of 0.95 we can state that at least $N_{п.бг} = 10$ interceptors will complete the mission. Expression (1) is none other than the ordinate of the distribution density function of a random number of failed interceptors. Obviously, in practice there may be $k = 0$ failures, or $k = 1, 2, 3, \ldots,$ up to $k = N_п$ failures. We are interested in the probability $P\,(k < N_п - N_{п.бг})$ that there will be no more than $N_п - N_{п.бг}$ failed interceptors. Consequently, to find this probability we must summ all ordinates of the distribution density (1) from $k = 0$ to $k = N_п - N_{п.бг}$:

$$P(k < N_п - N_{п.бг}) = \sum_{k=0}^{N_п - N_{п.бг}} C^k_{N_п} P^k_{отк} (1 - P_{отк})^{N_п - k}. \tag{3}$$

Equating this probability to 0.95 we can, for known values of $N_{п.бг}$ and $P_{отк}$, using Eq. (3), calculated the desired number $N_п$.

For the data in our problem, when $P_{OTK} = 0.1$ the required number of interceptors available for scramble is 14. Then, with a probability of 0.95 the mission will be completed by at least 10 interceptors.

PROBLEM 7.8. At a forward dispersal airfield, 5 interceptors are on round-the-clock combat alert. From the conditions for completing a combat mission it is required that at least 4 of the 5 interceptors be combat-ready at any given time. If one of the interceptors is out of action it is replaced with a ready one from the main airbase. How many replacements should be planned for the monthly alert of these 5 planes if the reliability of each of them is characterized by an average time $T_0 = 75$ hours between successive failures? We assume the failure flow to be simple.

Solution. From reliability theory we know that for simple flow the probability of failures is subject to Poisson's law, for which the expectation of the number of failures occurring in time t is calculated from the formula

$$M[N_{OTK}] = \frac{t}{T_0}.$$

In our example $T_0 = 75$ hours and, in the course of one month, the average number of expected failures per interceptor is $M[N_{OTK}] =$ $= 9.6$.

Then, from the distribution function of Poisson's law let us calculate the maximum possible number of failures per interceptor in one month provided that the expectation of this distribution law $M[N_{OTK}] = 9.6$. With a guarantee of 83% the maximum number of failures per alert interceptor is 12 per month. Thus, for each interceptor at the dispersal airfield we should plan on 12 replacements, while for 5 interceptors – $12 \cdot 5 = 60$. Since with the requirement of having at least 4 combat-ready interceptors of the 5 it is not necessary to replace each failed plane, with a planned 60 replacements the guarantee increases to 90%.

PROBLEM 7.9. An interceptor is armed with two rockets having radar semi-active homing heads, fired in salvo at a target. The probability of target destruction by one rocket is P_1. Besides P_1, the effectiveness of the attack is influenced by the reliability of the armament system and the radio-control system P_0 and the reliability of the rockets P_p. Determine the decrease in the resulting probability of target destruction, considering the reliability, compared with the case when the interceptor is absolutely reliable. Let us assume $P_0 = 0.9$, $P_p = 0.9$. How does the difference in effectiveness change, with and without consideration of reliability, if P_1 varies from 0.2 to 0.99?

Solution. The effectiveness with consideration of reliability

$$P_\Sigma = P_0[1-(1-P_pP_1)^2]. \quad (1)$$

without consideration of reliability

$$P'_\Sigma = [1-(1-P_1)^2], \quad (2)$$

while their ratio

$$\frac{P_\Sigma}{P'_\Sigma} = P_0P_p\frac{2-P_pP_1}{2-P_1}. \quad (3)$$

When $P_0 = P_p = 0.9$ we have (Table 7.1):

Table 7.1.

P_1	0,2	0,4	0,6	0,8	1,0
$\frac{P_\Sigma}{P'_\Sigma}$	0,819	0,830	0,845	0,864	0,891
$0{,}9\cdot P_\Sigma$	0,265	0,473	0,637	0,746	0,802

PROBLEM 7.10. Two interceptors perform independent target search along an 180-km wide front. The target can, at any random moment of time, be in any sector of the front with equal probability. The interceptors are located along the front such that the airborne radar acquisition sectors overlap: the first interceptor examines a region 0-110 km along the front, the second – 70-180 km. The

effectiveness of search depends on the degree of overlap of the radiation patterns. The probability of target detection by one interceptor $P_{01} = 0.9$, while in the overlap region $P_{02} = 1 - (1 - P_{01})^2 = 0.99$.

Determine the resultant probability of target detection with the described independent target search, if the probability of failure-free operation of both radar stations is $P_1 = P_2 = 0.95$. The reliability of all other systems in the interceptors is assumed to be 1.

Solution. The search effectiveness, i.e., the probability of target detection, depends on the reliability of the complex detection system consisting of two on-board radars and which can be in one of the following four states, comprising a complete group of mutually exclusive events:

S_0 – failure-free operation of both stations;

S_1 – the first station fails, the second operates;

S_2 – the second station fails, the first operates;

$S_{1,2}$ – both stations fail.

Of these four states, only the first three are favorable.

According to the formula for total probability, the probability of target detection is equal to the sum of the probabilities of all favorable events:

$$P_0 = \Phi H_0 + \Phi_1 H_1 + \Phi_2 H_2,$$

where Φ_i ($i = 0, 1, 2$) is the probability of target detection, provided that the target is located in the action zone of the i-th state of the system; H_i is the probability that the system is, at an arbitrary moment of time, in one of these four states.

The probability that the system is in state S_0

$$H_0 = P_1 P_2 \approx 0.9.$$

The conditional probability that the target will be detected when both stations operate failure-free $\Phi_0 = 0.92$.

Multiplying H_0 by Φ_0 we get the first term in the formula for total probability: $H_0\Phi_0 = 0.828$.

For state S_1, analogously, we have

$$H_1 = P_2(1 - P_1) = 0{,}05;$$
$$\Phi_1 = 0{,}55 \text{ and } \Phi_1 H_1 = 0{,}028.$$

Since states S_1 and S_2 are identical,

$$\Phi_2 H_2 = 0{,}028.$$

Thus the desired probability of target detection $P_0 = 0.884$.

This problem shows that possible failure of the airborne radars operating, in this case, as a unified system, considerably reduce the effectiveness of independent target search.

PROBLEM 7.11. Depending on the intensity of the combat action, one or two engines per month may be required to restore the combat readiness of the squadron. The probability that one engine will be required is 0.5; the probability that two will be required is 0.3; the probability that no engines will be required is 0.2. One engine a month is delivered to the squadron. Determine the probability of the need arising for any number of engines, if we consider the process of engine requesting and the process of sufficiently long stockpiling (e.g., several months). The maximum possible number of requests for engines does not exceed four per month.

Solution. The expectation of the number of engines required is $M[n_{\text{дв}}] = 1 \cdot 0.5 + 2 \cdot 0.3 = 1.1$, i.e., greater than the monthly average. Obviously, with prolonged combat operations characterized by the indicated rates of required replacement and stockpiling, the combat readiness of the air fleet of the squadron can be described by the probabilities that 0, 1, 2, 3, or 4 engines will be required. Let us designate by $n_{дв}$ the number of unsatisfied requests for engines, and examine the state of the requests in any current k-th and (k+1)-th months. The transition from the k-th to the (k+1)-th month is described by the following matrix of probabilities P for the required number of engines (Table 7.2):

Table 7.2.

(k+1)-th month \ k-th month	—4	—3	—2	—1	0
—4	0,8	0,3	0	0	0
—3	0,2	0,5	0,3	0	0
—2	0	0,2	0,5	0,3	0
—1	0	0	0,2	0,5	0,3
0	0	0	0	0,2	0,5
1	0	0	0	0	0,2

The matrix is filled on the basis of the following logic considerations. Let us assume, e.g., that up to the k-th month the number of unsatisfied requests $n_{дв} = 0$ (the last column in the matrix). Since one engine is delivered during the month, by the start of the (k+1)-th month the number of unsatisfied requests can be either 1 or 0 (since there were 0, 1, or 2 requests); in addition, 1 engine may be unnecessary (the engine arrived without being ordered). The probabilities of these events are equal to 0.3, 0.5, and 0.2. In a similar manner we fill the remaining matrix columns. From the matrix it is easy to write formulas for calculating the desired probabilities that immediately before the k-th month the number of requests $n_{дв} = -4$ (or there are $n_{дв} = 4$ unclaimed engines):

$$P_{-4}(k+1) = 0{,}8P_{-4}(k) + 0{,}3P_{-3}(k) = 0{,}8\cdot 0{,}8 + 0{,}3\cdot 0{,}3 = 0{,}73.$$

For all $n_{дв} \geq -3$ we have

$$P_{n_{дв}}(k+1) = 0{,}2P_{n_{дв}-1}(k) + 0{,}5P_{n_{дв}}(k) + 0{,}3P_{n_{дв}+1}(k);$$
$$P_{>-3}(k+1) = 0{,}2\cdot 0{,}2 + 0{,}5\cdot 0{,}5 + 0{,}3\cdot 0{,}3 = 0{,}38.$$

PROBLEM 7.12. The combat operations of a fighter group have the following nature. Each interceptor can be in only one of two states comprising a complete group of mutually exclusive events: 1) the interceptor completes its mission of intercepting an aerial target; 2) the interceptor is at the airfield and is being prepared for subsequent missions. Both these states involve a certain enemy counteraction. In the first case, in aerial combat the interceptor is subject to enemy fire; in the second case, on the ground the interceptor can be damaged by enemy action against the airfield. Both states are characterized by the specific probability of interceptor survival after each enemy operation. The random nature of survival has the result that the number of missions of each interceptor during combat operations is a random value.

We are required to determine the expectation of this random magnitude, if we know that the probability of survival before the first mission (the probability of interceptor survival if the enemy strikes the airfield before the first mission) is P_1, the probability of survival in aerial combat is P_2, and the probability of survival and combat readiness on the ground between successive missions is P_3. What is the expectation of the number of missions carried out by each interceptor if the probability of survival after any mission is constant and equal to $P_{\text{выж}} = 0.95$?

Solution. By definition of the expectation of a random number, the number of missions carried out by each interceptor, we have

$$M[N_{\text{вып}}] = P_1P_2 + P_1P_2^2P_3 + \ldots + P_1P_2^nP_3^{n-1}, \quad (1)$$

where n is the maximum possible number of missions which can generally be accomplished by one interceptor under the examined combat conditions. The first term in (1) is the probability of survival after the first mission, the second term is the probability of survival after the second mission, etc., and the last term is the probability of survival after the n-th mission. Removing P_1P_2 from the parentheses we get

$$M[N_{\text{вып}}] = P_1P_2(1 + P_2P_3 + \ldots + P_2^{n-1}P_3^n). \quad (2)$$

The expression in parentheses in (2) is the sum of the geometric progression with denominator P_2P_3 and a first term of 1.

Using the formula for the sum of a geometric progression we get

$$M[N_{вып}] = \frac{P_1P_2}{1-P_2P_3}[1-(P_2P_3)^n]. \quad (3)$$

If we consider that each interceptor participates in combat operations until it is destroyed, the probability of survival after n missions is zero, and the expression

$$[1-(P_2P_3)^n] \quad (4)$$

is identical with 1, as the sum of the probabilities of mutually exclusive events forming a complete group.

Consequently,

$$M[N_{вып}] = \frac{P_1P_2}{1-P_2P_3}. \quad (5)$$

For the data in our example we are given that

$$P_1P_2 = P_2P_3 = 0{,}95.$$

If the survival probability up to each forthcoming mission is constant, i.e., the conflict situation with continuous enemy counteraction repeats cyclically,

$$P_1P_2 = P_2P_3 = P_{выж} = \text{const}. \quad (6)$$

Then

$$M[N_{вып}] = \frac{P_{выж}}{1-P_{выж}}. \quad (7)$$

When $P_{выж} = 0.95$, each interceptor carries out, on the average,

$$M[N_{вып}] = \frac{0{,}95}{1-0{,}95} = 19 \text{ missions}.$$

By solving the examined problem we can draw a very important conclusion: with continuous enemy opposition, both in the air and on the ground, with high survivability between successive missions the expectation of the number of missions carried out by each interceptor is relatively low.

PROBLEM 7.13. The expectation $M[N_{выл}]$ of the number of interceptor missions is defined by the combat-operation model described in Problem 7.12. Knowing the value of $M[N_{выл}]$, determine the probability that an interceptor will not be damaged and will be combat ready after any random (given) number of missions.

Solution. From the conditions of the problem it is obvious that before the start of combat operations, when $M[N_{выл}] = 0$, $P_{выж} = 1$. The probability of interceptor damage during the accomplishment of a certain number of missions $\Delta N_{выл}$ is $\Delta N_{выл}/M[N_{выл}]$. The probability that the interceptor will be damaged during $\Delta N_{выл}$ missions, provided that up to this moment its state is characterized by a certain survival probability $P_{выж}$, is

$$\Delta P_{пор} = \Delta(1 - P_{выж}) = P_{выж} \frac{\Delta N_{выл}}{M[N_{выл}]} \quad (1)$$

or

$$\Delta P_{выж} = -\frac{\Delta N_{выл}}{M[N_{выл}]} P_{выж},$$

$$\frac{dP_{выж}}{dN_{выл}} = -P_{выж} \frac{1}{M[N_{выл}]}, \quad (2)$$

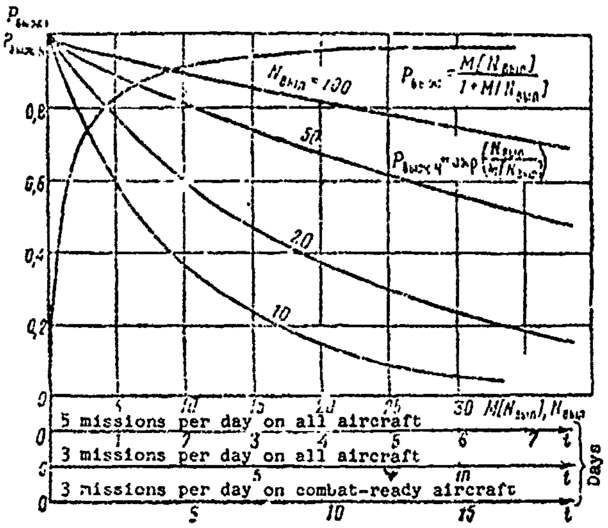

Fig. 7.2.

from which the desired survival probability after any $N_{выл}$-th mission is obtained as the solution of the latter first-order differential equation:

$$P_{выж.N} = e^{-\frac{N_{выл}}{M[N_{выл}]}} \quad (3)$$

A graph of Dependence (3) is given in Fig. 7.2.

PROBLEM 7.14. With prolonged intense combat operations the combat effectiveness of a group of interceptors is determined not only by the probability of intercepting a single target by a single interceptor, but also by the number of missions which the interceptors can carry out per unit time. Let us compare, from the criterion of mission intensity, two air squadrons, each consisting of $N_{п} = 12$ interceptors of various types, characterized by the following times that they are in various states (Table 7.3):

Table 7.3.

Combat-readiness time characteristics	First type of interceptor	Second type of interceptor
$t_{пр}$ – average time the interceptors are in preliminary preparation	5 hours each every 4 days	4 hours each every 3 days
$t_{п.п}$ – average time the interceptors are in pre-flight training	1 hour every day	1 hour, 10 minutes every day
$t_{повт}$ – average time to prepare the interceptors for subsequent missions	30 minutes	40 minutes
$t_{пол}$ – average time of combat mission	1 hour	1 hour, 20 minutes

In addition, we are given the total duration of combat operations: $t_{б.д} = 10$ days. Let us assume that up to the start of combat operations, preliminary preparations have been made for all interceptors, and that during $t_{б.д} = 10$ days all interceptors conduct missions on a round-the-clock basis.

Compare the maximum combat capabilities of the squadrons based on the expectation of the number of downed targets, if the probability of intercepting one target by one interceptor is $P_1 = 0.7$ for the first type and $P_2 = 0.8$ for the second type.

Solution. The average time of the cycle between two successive combat missions is equal to the sum of the average flight times and the times for preparing the plane for the next mission:

$$t_{ц} = t_{пол} + t_{повт}. \tag{1}$$

If the total time during which the interceptors are not busy with preparatory and pre-flight preparations is

$$t_{\Sigma} = t_{б.д} - t_{пр} - t_{п.п}, \tag{2}$$

division of t_{Σ} by $t_{ц}$ gives an expression for the average number of missions which each interceptor can carry out:

$$\frac{t_{\text{б. д}} - t_{\text{пр}} - t_{\text{п. п}}}{t_{\text{пол}} + t_{\text{повт}}}. \tag{3}$$

The average number of squadron missions is

$$\frac{t_{\text{б. д}} - t_{\text{пр}} - t_{\text{п. п}}}{t_{\text{пол}} + t_{\text{повт}}} N_{\text{п}}, \tag{4}$$

while the mission intensity, i.e., the number of missions per unit time,

$$n = \frac{(t_{\text{б. д}} - t_{\text{пр}} - t_{\text{п. п}})}{(t_{\text{пол}} + t_{\text{повт}})} \frac{N_{\text{п}}}{t_{\text{б. д}}}. \tag{5}$$

A squadron equipped with the first type of interceptor can carry out $n_1 = 7.35$ missions per hour, while one having the second type can carry out $n_2 = 5.4$.

The expectations of the number of downed targets during combat operations

$$M[N_{\text{с. ц}}]_1 = 1240;$$
$$M[N_{\text{с. ц}}]_2 = 1030.$$

Thus, although the second type of interceptor is considerably better than the first with regard to interception probability, based on the average number of downed targets during the entire combat operation period a squadron with the first type of interceptor has 1.2-times greater success than one having the second type, for which the probability is considerably higher.

PROBLEM 7.15. The enemy uses nuclear weapons against an interceptor base. We must determine the dispersion radius $R_{\text{рассp}}$, if we know the damage radius of the weapon used $R_{\text{п}}$ and the probable dissipation deviation of this weapon E (or the dissipation radius R_{p}), and we are given the permissible probability of interceptor destruction $P_{\text{пор}}$. Let us assume that the interceptors are uniformly distributed within a circle with radius $R_{\text{рассp}}$ and the center of this circle serves as the enemy's aiming point.

Solution. Using the methodology of estimating the effectiveness of damage to an area target, we can write the following formula for calculating the probability of damage, up to a given level, to interceptors located at distance R_{paccp} from the epicenter of the explosion:

$$P_{\text{пор}} = \frac{R_{\text{пор}}^2}{R_{\text{пор}}^2 + R_p^2 + R_{\text{paccp}}^2}. \quad (1)$$

The destruction radius, as we know, is either determined by instructions or calculated from the given values of the TNT equivalent q of the weapon used and the excess pressure ΔP_{ϕ} at the shock-wave front which an aircraft of the examined design can withstand:

$$R_{\text{пор}}\,[\text{km}] = \sqrt[3]{q\,[\text{kT}]}.$$

Having determined, from q and ΔP_{ϕ} the destruction radius $R_{п}$ and given the various values of probable deviations $E = 0.477R_p$, let us calculate from Eq. (1) the destruction probability. The results of the calculations for a broad range of initial data are given in Fig. 7.3 (p. 232). Using this nomogram we can also determine the required dispersion radius for any specific conditions.

If the dispersion area is rectangular, it is convenient to determine, from the known sides of the rectangle, the equivalent radius of the circle, using Eq. (8) of Problem 5.8, and then determine the damage probability from the nomogram in Fig. 7.3.

With complex configuration of the dispersion area we must use a scatter grid [2].

PROBLEM 7.16. There are 18 interceptors at the airfield. To assure high combat readiness and survivability with enemy action against the base airfield, all interceptors are dispersed uniformly throughout the airfield and, in addition, there are 18 dummy interceptors. The enemy raids the airfield and bombards, at random, half of the aircraft (18 of the total of 36 real and dummy airplanes). During the bombardment, each bombarded interceptor is damaged with

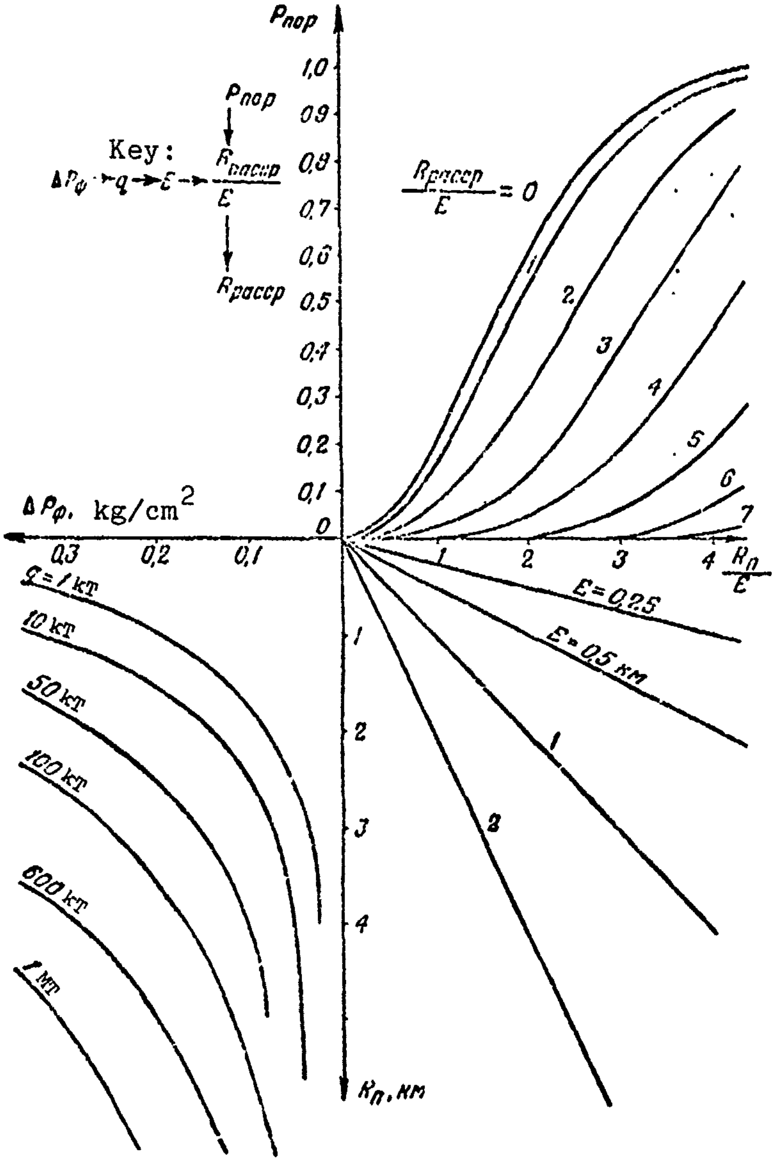

Fig. 7.3.

a probability of 0.2.

Determine the expectation of the number of damaged interceptors. How does this number decrease if we double the number of dummies?

Solution. The enemy selects, with equal probability, any 18 of the 36 targets for bombardment. The number of such cases is equal to the number of combinations of 36, 18 at a time:

$$C_{36}^{18} = \frac{36!}{18!\,(36-18)!}. \quad (1)$$

Here there can be 0, 1, 2, ..., 18 real interceptors. To calculate the desired expectation of the number of damaged interceptors we must know the probability P_m that among the selected 18 there are exactly m real interceptors. Probability P_m is defined by the ratio of the number of cases favorable to the enemy to the total number of cases, C_{36}^{18}. The number of favorable cases is

$$C_{18}^{m} \cdot C_{36-18}^{18-m}, \quad (2)$$

since when selecting m of 18 interceptors, dummies can be selected in C_{18}^{18-m} various ways. Consequently,

$$P_m = \frac{C_{18}^{m}\, C_{36-18}^{18-m}}{C_{36}^{18}}. \quad (3)$$

Having calculated the values of probabilities P_m for m = 1-18, let us find the expectation of the number of bombarded interceptors from the formula

$$M[N_{\text{и.об}}] = \sum_{m=1}^{18} m P_m, \quad (4)$$

and, multiplying this number by 0.2, we find the expectation of the number of damaged interceptors.

Here, when calculating the factorials of large numbers it is convenient to use the Stirling asymptotic formula

$$n! \approx \sqrt{2\pi n}\, n^n e^{-n}. \quad (5)$$

The results of the calculations are given in Table 7.4.

Table 7.4.

m	0—3	4	5	6	7	8	9	10	11	12	13	14	14—18
P_m	0	0,001	0,008	0,038	0,112	0,211	0,260	0,211	0,112	0,038	0,008	0,001	0

Thus, the average number of lost interceptors is

$$0{,}2\sum_{m=1}^{13} mP_m = 0{,}2\,(4\cdot 0{,}001 + 5\cdot 0{,}008 + 6\cdot 0{,}038 + 7\cdot 0{,}112 + 8\cdot 0{,}211 + 9\cdot 0{,}260 + 10\cdot 0{,}211 + 11\cdot 0{,}112 + 12\cdot 0{,}038 + 13\cdot 0{,}008 + 14\cdot 0{,}001) = 1.8.$$

If the number of dummy targets is doubled, we get the following results (Table 7.5):

Table 7.5.

m	0	1	2	3	4	5	6	7	8	9	10	11	12—18
P_m	0	0,002	0,012	0,047	0,12	0,204	0,24	0,197	0,115	0,047	0,014	0,002	0

The expectation of the number of damaged interceptors is

$$0{,}2\,(1\cdot 0{,}002 + 2\cdot 0{,}012 + 3\cdot 0{,}047 + 4\cdot 0{,}12 + 5\cdot 0{,}204 + 6\cdot 0{,}24 + 7\cdot 0{,}197 + 8\cdot 0{,}115 + 9\cdot 0{,}047 + 10\cdot 0{,}014 + 11\cdot 0{,}002) = 1{,}2,$$

i.e., a decrease by a factor of 1.8/1.2 = 1.5.

PROBLEM 7.17. Verify the combat readiness of rockets stored in a warehouse. From experience in previous calculations we know that the probability of rocket inoperability is 0.03. Of the group to be checked we select 100 rockets at random. What is the probability that exactly three rockets will be inoperable? What is the probability that no more than three rockets will be inoperable?

Solution. Using the binomial distribution formula we have the following for the probability that of 100 rockets, exactly three will be inoperative:

$$P(m_{р.н}=3)=C_{100}^{3}0{,}03^{3}(1-0{,}03)^{100-3}=\frac{100!}{3!\,(100-3)!}0{,}03^{3}\cdot 0{,}97^{97}=$$
$$=161\,700\cdot 0{,}03^{3}\cdot 0{,}97^{97}=0{,}227.$$

The probability that of 100 rockets no more than three will be inoperative is

$$P(m_{р.н}\leqslant 3)=\sum_{k=0}^{3}P(m_{р.н}=k)=P(m_{р.н}=0)+P(m_{р.н}=1)+$$
$$+P(m_{р.н}=2)+P(m_{р.н}=3)=C_{100}^{0}0{,}03^{0}(1-0{,}03)^{100-0}+$$
$$+C_{100}^{1}0{,}03^{1}(1-0{,}03)^{100-1}+C_{100}^{2}0{,}03^{2}(1-0{,}03)^{100-2}+$$
$$+C_{100}^{3}0{,}03^{3}(1-0{,}03)^{100-3}=0{,}0475+0{,}147+0{,}2251+0{,}2274=0{,}647.$$

CHAPTER 8

PROBLEMS OF OPTIMIZATION

The determination of combat capabilities and combat effectiveness is never an end in itself, but serves merely as a basis for seeking optimum regimes. For example, when designing a fighter-interceptor we optimally synthesize its basic tactical-technical characteristics, obtaining an aircraft of given effectiveness at minimum cost. Calculation of the effectiveness of a given fighter-interceptor under various combat conditions, as a rule, has the aim of substantiating optimum tactics and methods of combat utilization.

The use of a limited number of means to complete a specific combat mission also makes it necessary to seek optimum solutions. Identical combat missions can be completed with the same effectiveness, but with various expenditures of forces and equipment. Therefore, optimum solutions are found by using methods employing complex criteria of effectiveness, a combination of the quality of completion of the mission and the cost of the forces and equipment necessary to carry it out.

As an example of a complex effectiveness criterion we have the ratio of the expectation of the number of destroyed targets to the total cost of the equipment used in the given operation. This criterion is called the specific combat effectiveness. We can also use the reciprocal of the specific effectiveness — the cost of destroying one target.

The criterion of the ratio of effectiveness to cost makes it possible to select the optimum of several types of fighter-interceptors designed to carry out identical missions. This solution is given in Problem 8.1.

Problem 8.2 discusses selection of the optimum type of interceptor based on the criterion of the cost of target destruction.

Of practical interest are problems whose solutions allow us to determine the reasonable makeup of a combination fire unit, the optimum regime and sequence of rocket firing, and distribution of the units of fire by the ranges of firing and subsequent attacks (Problems 8.3 and 8.4).

Problems 8.5 and 8.6 are devoted to a determination of the optimum makeup of an air group consisting of various types of interceptors and assuring a given effectiveness at minimum cost.

Substantiation of optimum interceptor and target tactics is examined in Problems 8.7-8.9.

The method of designating an optimum flight when the quantitative breakdown of a group target is unknown is shown in Problem 8.10.

In Problem 8.11 we determine the optimum type of interceptor armament.

In Problem 8.12 we optimize the carrying capacity of a detection and guidance system.

Problems 8.13-8.17 give optimum solutions when determining the composition of the air fleet of various types of air groups, and its location by zones of combat operations and base points. We estimate the gain in effectiveness due to optimum designation of a flight of various types of interceptors against various types of group targets.

Problem 8.18 shows how to determine the optimum dispersion radius, if an increase in radius decreases the damage to interceptors from enemy action, but increases the mission time.

Problem 8.19 allows us to substantiate the optimum supply of rockets or any other units and parts, assuring a given level of interceptor combat effectiveness.

PROBLEM 8.1. In model combat operations of interceptors with fighter-bombers at low altitudes we have the characteristics x_i, which determine the combat effectiveness, and their importance coefficients m_i which show how the interceptor effectiveness changes if the given characteristic x_i changes by Δx_i. As specifying characteristics we have the following:

$V_п$ – combat ground speed, Mach;

$\Delta M/\Delta D$ – decrease in M by ΔM with a decrease in flight range by ΔD, caused by installing cannons and radio countermeasure devices aboard the interceptor;

$S_{пор}$ – effective area of interceptor damage with enemy fighter counterfire;

Ω – area bounded by the graph of the dependence "velocity-altitude" of interceptor flight with load factor n = 1;

n – maximum interceptor load factor;

t_p – time to accelerate from cruising speed to maximum low-level speed;

d_1 – relative change in ground flight range under maximum-range conditions as compared with maximum ground speed conditions;

d_2 – relative change in flight range at reference altitude under maximum range conditions as compared with maximum ground speed conditions;

$\bar{t}_6$ – relative change in time of loitering under cruise conditions at the ground as compared with minimum speed conditions;

$\Delta L_p/\Delta G$ – relative change in takeoff run vs. takeoff weight.

Five types of interceptors have been developed, whose specifying characteristics are shown in Table 8.1. All characteristics are given in the form of relative values, with the appropriate characteristics of interceptor No. 5 taken as unity. The table also shows the importance coefficients for each specifying characteristic.

Table 8.1.

Interceptor No.	Characteristics										
	V_n	$\frac{\Delta M}{\Delta D}$	S_{nop}	Ω	n	d_1	d_2	$\bar{t}_6$	$\frac{\Delta L_p}{\Delta G}$	t_p	C
1	1,08	0,87	0,85	0,54	0,58	0,[illegible]		0,67	0,85	1,51	0,87
2	0,93	0,80	1,0	0,79	0,74	0,91	0,94	0,71	0,87	1,09	0,89
3	0,99	0,69	0,97	0,98	0,77	0,91	0,92	0,67	1,0	1,05	0,95
4	0,94	0,68	1,28	0,92	0,93	0,97	1,13	0,79	1,0	1,09	0,98
5	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0
m_i	1,0	1,0	0,45	0,1	1,0	1,0	0,1	0,15	1,5	0,5	1,0

We are required to determine, from the criterion of specific effectiveness (the effectiveness/cost ratio), which of these interceptors is the best.

Solution. Let us examine any one of these characteristics x. If we compare two types of interceptors using this characteristic and the corresponding effectiveness values w_x, we can write

$$\frac{w_{x_1}}{w_{x_2}}=\left(\frac{x_1}{x_2}\right)^m. \tag{1}$$

Using the Newtonian binomial formula we have

$$\left(1+\frac{x_1-x_2}{x_2}\right)^m=(1+\Delta x)^m=1+m\cdot\Delta x+\frac{m(m-1)}{2!}\Delta x^2+ + \frac{m(m-1)(m-2)}{3!}\Delta x^3+\ldots, \quad (2)$$

where Δx is the difference between the characteristics x of these interceptors, related to the value of characteristic x of one of the comparable interceptors. In practice, greatest complexity in comparison occurs when the comparable characteristics x_1 and x_2 differ only slightly from one another. In this case the value of Δx is small compared with x_1 and x_2, and in first approximation we can limit ourselves to the first two terms of the binomial expansion. Then

$$\frac{w_{x_1}}{w_{x_2}}=1+m\cdot\Delta x, \quad (3)$$

which immediately reflects the physical essence of the importance coefficient: m shows how much effectiveness w changes if the specifying characteristic x changes by Δx. For example, if m = 3, this means that the effectiveness increases by 30% when x increases by 10%, since

$$w_{x_1}=w_{x_2}(1+m\Delta x)=w_{x_2}(1+3\cdot 0{,}1)=1{,}3w_{x_2}.$$

Since an improvement of one characteristic of the interceptor can be compensated, in the effectiveness indicator, by a deterioration of another characteristic, comparison of interceptors with consideration of all characteristics which specify the combat effectiveness can be done by the formula

$$w=\frac{w_1}{w_2}=\left(\frac{x_1}{x_2}\right)^{m_x}\left(\frac{y_1}{y_2}\right)^{m_y}\left(\frac{z_1}{z_2}\right)^{m_z}\ldots, \quad (4)$$

since the resulting effectiveness

$$w_1=w_{x_1}w_{y_1}w_{z_1}\ldots=x_1^{m_x}y_1^{m_y}z_1^{m_z}\ldots \quad (5)$$

Comparison indicator w unambiguously shows how much one interceptor is better or worse than another.

The formula for specific useful output (the ratio of the resulting effectiveness to the cost) for the characteristics given in this problem has the following form:

$$w = \frac{1}{C}\left[\frac{V_{n}^{m_1}}{\left(\frac{\Delta M}{\Delta G}\right)^{m_2} S_{\text{пор}}^{m_3}}\right]\left[\frac{\Omega^{m_4} n^{m_5}}{t_{p}^{m_6}}\right]\left[d_1^{m_7} d_2^{m_8} \bar{t}_{6}^{m_9} \left(\frac{\Delta L_{p}}{\Delta G}\right)^{m_{10}}\right]. \qquad (6)$$

Qualitatively, the nature of the influence of the characteristics on efficiency w is obvious, while the quantitative dependence is determined by the values of the importance coefficients m_i, found using model combat operations and given in Table 8.1. The bracketed characteristics can arbitrarily be called, respectively, the relative survivability Ж, maneuverability B, and effectiveness R along the intercept lines. For example, the maneuverability of interceptor No. 1 is

$$B_1 = \frac{0{,}54^{0{,}1}\ 0{,}58}{1{,}51^{0{,}6}} = 0{,}44$$

of that of interceptor No. 5, i.e., the maneuverability of the first is 1/0.44 = 2.28 times poorer.

The results of the calculations are compiled in Table 8.2, from which we can draw an unambiguous conclusion for the problem.

Table 8.2.

Interceptor No.	Characteristics				
	Ж	B	R	ЖBR	ЖBR/C
1	1,33	0,44	0,67	0,392	0,45
2	1,16	0,69	0,70	0,560	0,63
3	1,23	0,75	0,85	0,785	0,825
4	1,24	0,85	0,713	0,745	0,76
5	1	1	1	1	1

Thus, the best interceptor is No. 5, then No. 3, and, finally, the worst is No. 1.

PROBLEM 8.2. We must destroy each of 10 aerial targets with probability $P_{\Sigma} = 0.99$. The commander has three types of interceptors capable of operating under given conditions and characterized by probability P_1 of the downing of a target by one interceptor and cost C_1 of one attack (Table 8.3).

Table 8.3.

Type of interceptor	P_1	C_1*
1	0,6	96
2	0,7	110
3	0,8	175

*Cost expressed in arbitrary units

Cost C_1 includes the cost of one mission and the cost of the rockets fired. Determine which interceptor is best used from the standpoint of assuring the given probability of target destruction P_Σ at minimum expense. Let us assume that each interceptor in one mission makes one attack against a single target.

Solution. To destroy each target with a higher probability P_Σ compared with P_1 of downing a target with one attack by one interceptor, we must carry out n independent attacks against each target. Then

$$P_\Sigma = 1 - (1 - P_1)^n \quad (1)$$

After taking the logarithm of (1) we get a formula for determining the required number of missions with known P_1 and P_Σ:

$$n = \frac{\lg(1 - P_\Sigma)}{\lg(1 - P_1)} \quad (2)$$

The resulting cost of destroying one target, if we know n and C_1, is defined by the formula

$$C_\Sigma = C_1 n = C_1 \frac{\lg(1 - P_\Sigma)}{\lg(1 - P_1)}. \quad (3)$$

The results of the calculations are compiled in Table 8.4.

Table 8.4.

Type of interceptor	Number of missions required to destroy one target with probability 0.99	Cost C_1 of destroying one target	Number of missions required to destroy 10 targets
1	5.025	482.4	50.25
2	3.83	422	38.3
3	2.8	490	28

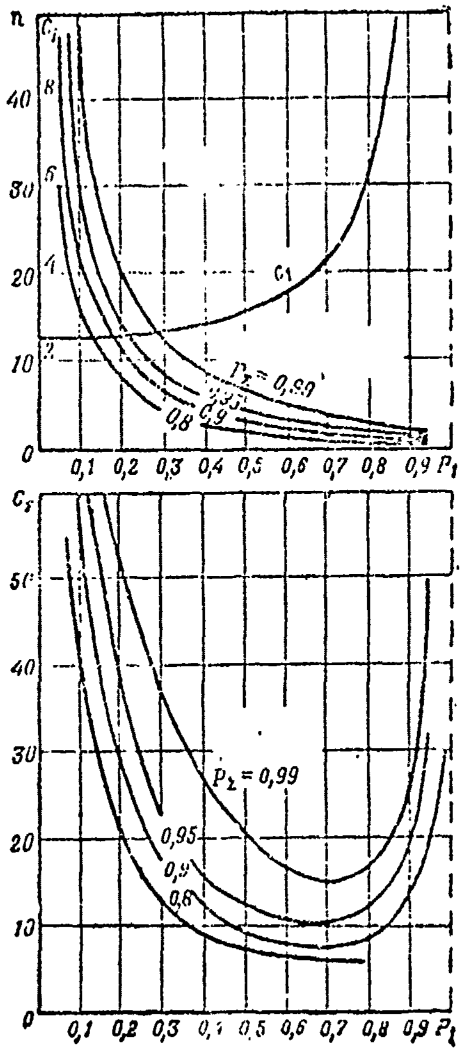

Fig. 8.1.

Comparison of the results of the calculations allows us to conclude that under the given conditions the second type of interceptor is most rational, although its P_1 is not the highest.

In this problem it is interesting that in the case when each higher value of P_1 has a correspondingly higher cost, there is always a certain P_1 which is optimum. With $P_{1опт}$ the resulting probability of target downing P_Σ is achieved at minimum cost. This problem is solved in general form in Fig. 8.1, where the dependence between effectiveness and cost of one attack is expressed by the formula

$$C_1 = a + \frac{b}{1-P_1} + \frac{c}{(1-P_1)^2}. \qquad (4)$$

where a, b, and c are constants.

This formula graphically reflects the position that even with $P_1 = 0$ we have a certain initial cost C_1, while with high values of P_1 great expenses are required in order to increase the effectiveness.

PROBLEM 8.3. An interceptor armed with four rockets can attack two targets in one mission. The probability that the first target will be attacked is $P_{a1} = 0.7$. The probability of carrying out the second attack is $P_{a2} = 0.3$. The conditional probability of destroying a target with one rocket during an attack is $P_1 = 0.6$. If the first target is not attacked, the interceptor expends its entire unit of fire against the second target. We must determine the optimum regime for firing at a target, i.e., how many rockets there should be in the salvo against the first target, so that the expectation of the number of downed targets in two attacks will be maximum.

Solution. The probability of downing the first target is

$$P_{1i} = P_{a1}[1-(1-P_1)^i]. \quad (1)$$

The conditional probability of downing the second target, provided the first target has been attacked, is

$$P_{2(m_p-i)} = P_{a2}[1-(1-P_1)^{m_p-i}]. \quad (2)$$

The conditional probability of downing the second target, provided the first target has not been attacked, is

$$P_{2m_p} = P_{a2}[1-(1-P_1)^{m_p}]. \quad (3)$$

In Eqs. (1), (2), and (3), m_p is the number of rockets in the unit of fire; i is the desired number of rockets fired at the first target.

The expectation of the number of targets downed in two attacks

$$M_i = P_{a1}[1-(1-P_1)^i] + P_{a1}P_{a2}[1-(1-P_1)^{m_p-i}] + + (1-P_{a1})P_{a2}[1-(1-P_1)^{m_p}]. \quad (4)$$

The maximum of function M_i is achieved when

$$\frac{dM_i}{di} = 0,$$

while

$$\frac{d^2M_i}{di^2} < 0.$$

Let us find the first derivative

$$\frac{dM_i}{di} = P_{a1}\ln(1-P_1)[-(1-P_1)^i + P_{a2}(1-P_1)^{m_p-i}] = 0,$$

from which

$$i = \frac{1}{2}\left[\frac{\lg P_{a2}}{\lg(1-P_1)} + m_p\right]. \qquad (5)$$

The second derivative

$$\frac{d^2M_i}{di^2} = -P_{a1}[\ln(1-P_1)]^2[(1-P_1)^i + P_{a2}(1-P_1)^{m_p-i}]$$

is negative; consequently, Eq. (5) corresponds to the maximum expectation M_i. Substituting the numerical values of the problem, according to (5) we get

$$i = \frac{1}{2}\left[\frac{\lg 0{,}3}{\lg(1-0{,}6)} + 4\right] = 2{,}65.$$

Rounding off to the nearest whole number, we have i = 3. For verification we can substitute the numerical values directly into (4). In this case, for values of i = 1, 2, 3, and 4 we get $M_1 =$ $= 0.704$, $M_2 = 0.853$, $M_3 = 0.853$, and $M_4 = 0.77$.

Thus, in this case as well the maximum M_i is reached when i = 3.

PROBLEM 8.4. A unit of fire consisting of m rockets can include m_p rockets with radar homing heads (RHH) and m_T rockets with heat-seeking heads (HSH). In place of any rocket with an HSH we can use a rocket with an RHH. The probability of target damage by a rocket with an RHH is P_p; with an HSH it is P_T. Rockets with RHH can always be used, while those with HSH's can be used only with probability P_a. The value of the probability P_a depends on the attack aspect angle, the flight altitude, and meteorological conditions. We are required to determine the optimum ratio of rockets with RHH's and HSH's for which the probability of target damage P by a combined unit of fire would be maximum. Determine for what values of P_a the use of rockets with HSH's becomes inadvisable. Find the maximum ratio m_T/m_p for two cases:

1) $P_a = 0{,}9$ and $P_p = 0{,}4 - 0{,}5$.
2) $P_a = 0{,}7$ and $P_p = 0{,}3 - 0{,}4$.

Solution. The probability of target damage by a combination unit of fire of rockets with independent firings is

$$P = 1 - (1 - P_p)^{m_p} \{1 - P_a [1 - (1 - P_T)^{m_T}]\} \quad (1)$$

Consequently, the problem reduces to finding the maximum of Function (1) with observance of the following conditions:

$$\left.\begin{array}{l} m_T + m_p = m; \\ m_T \geqslant 0; \quad m_p \geqslant 0. \end{array}\right\} \quad (2)$$

Let us differentiate (1) with respect to m_p and equate the derivative to zero:

$$\frac{dP}{dm_p} = -\{-(1-P_p)^{m_p} P_a (1-P_T)^{m-m_p} \ln(1-P_T) + \\ + [(1-P_a) + P_a(1-P_T)^{m-m_p}](1-P_p)^{m_p} \ln(1-P_p)\} = 0. \quad (3)$$

From Eq. (3) we get

$$m_p = \frac{\lg\left[\frac{\lg(1-P_T)}{\lg(1-P_p)} - 1\right] - \lg\left(\frac{1}{P_a} - 1\right) + m \lg(1-P_T)}{\lg(1-P_T)} \quad (4)$$

Analogously, we get a formula for calculating the number of rockets with HSH's:

$$m_T = \frac{-\lg\left[\frac{\lg(1-P_T)}{\lg(1-P_p)} - 1\right] + \lg\left(\frac{1}{P_a} - 1\right)}{\lg(1-P_T)}. \quad (5)$$

From (4) and (5) it follows that when the condition

$$P_T \leqslant 1 - (1 - P_p)^{P_a} \quad (6)$$

is satisfied, the use of rockets with HSH's becomes inadvisable, i.e., in this case the effectiveness will be higher if only rockets with RHH's are used.

The solution is shown in Fig. 8.2 (p. 247).

PROBLEM 8.5. A regiment is assigned a band of responsibility 300 km wide along the front. It must repel a raid by 30 targets flying column formation at the lowest possible altitude and with no ground radar guidance field. The method for combat use of interceptors is as follows: at the warning command a search group takes off which, after detection of the target, calls in an attack group for joint target destruction (Fig. 8.3). There are two types of

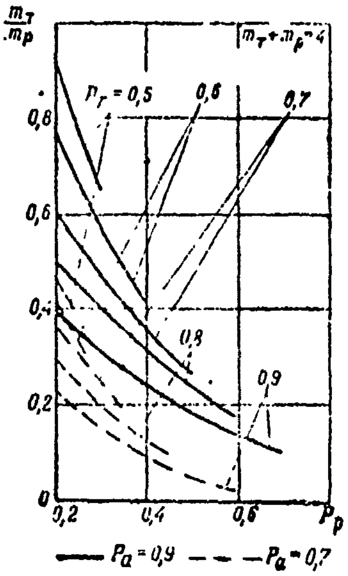

Fig. 8.2.

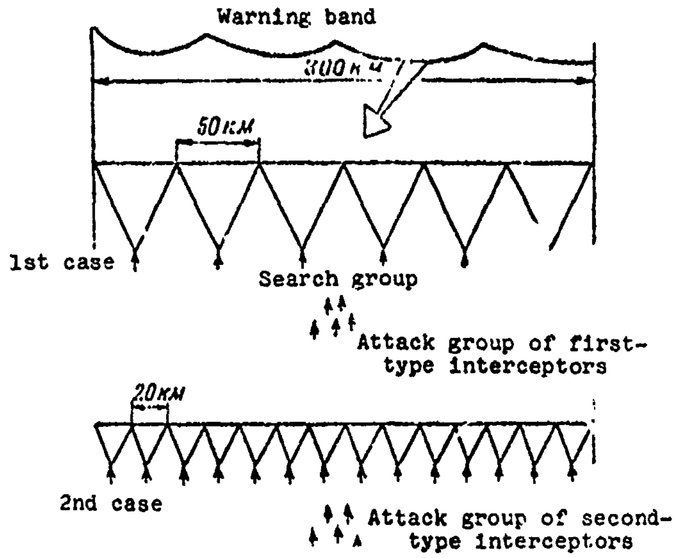

Fig. 8.3.

interceptors with the following ch.racteristics (Table 8.5).

Table 8.5.

Characteristics	Type of interceptor	
	1	2
Number of rockets	4	2
Probab ity of destroying one target with one rocket	0.5	0.3
Width of band of possible detection and att.ck (2b), km	50	20
Expenditures for one mission, arbitrary units	2.5	0.5

With interaction of both types of interceptors, when each interceptor of the second type is given a target indication from the first-type interceptor which detects the target, the effectiveness of the second-type interceptors increases to $P_1 = 0.5$.

We are required to determine the optimum makeup of an air-defense air group consisting of type 1 and 2 interceptors and capable of destroyi 90% of the targets in a raid at minimum cost. We must compare t. . sible combat variants:

1) the ... are resolved by the airborne radar, and the result

of firing each rocket is analyzed, with subsequent reaiming to undestroyed targets;

2) search is conducted for dense enemy combat formations unresolved by the airborne radar.

Let us assume that of the search group, only three type 1 interceptors or only five type 2 interceptors participate in the attack. The remaining interceptors of the search group, because of their remoteness from the point of detection of the target column and because of the short duration of the raid, cannot participate in the attack.

Solution. The mission of the air group can be carried out with various combinations of both types of interceptors. If we use only type 1 interceptors, to obtain the given level of effectiveness $M[N_{c.ц}]/N_{ц} = 0.9$ it is necessary, according to Fig. 6.4 of Problem 6.4, to have $m_p N_п / N_ц = 2$ during reaiming, from which $N = 15$. To this number we must add three interceptors from the search group; then the total cost is $C_\Sigma = 18 \cdot 2.5 = 45$ arbitrary units. Analogously, when using only type 2 interceptors we have $m_p N_п / N_ц = 3.65$, $N_п = 55.5$, and the expenses $C_\Sigma = (55.5 + 10) \cdot 0.5 = 32.75$.

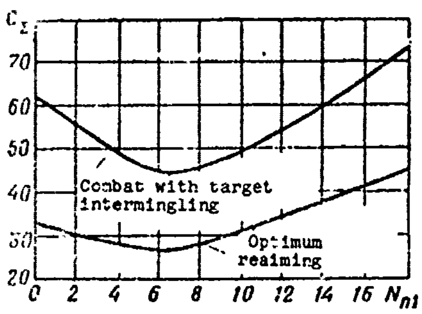

Fig. 8.4.

A graph of the dependence $C_\Sigma = f\left(N_{п_1}\right)$ is given in Fig. 8.4. Obviously, the optimum solution is to use type 1 interceptors in the search group and type 2 interceptors in the attack group. The optimum ratio of the number of interceptors is 1:4, with reaiming after analysis of the results of each individual attack, and 1:10 in the case of combat with target intermingling, when a group target unresolved by the airborne radar is attacked.

PROBLEM 8.6. An air-defense group has the mission of destroying at least 75% of $N_{ц} = 100$ targets in a massed raid. To solve this problem we have two control systems with varying warning and guidance equipment and, consequently, varying control quality, in particular the quality of target distribution.

The first system indicates to each interceptor its own target, and all targets are identically and uniformly fired upon, i.e., there is ideal target distribution. In the second system, each interceptor selects its own target and there is no target distribution by the system; it merely leads the interceptor to the region of target location, allowing it to operate independently. When using the first system the cost of one operation against a target is double that when using the second system.

We must determine, from the criterion of minimum cost of target downing, which control system it is advisable to use. The probability of one interceptor's intercepting a single target is $P_1 = 0.5$ and 0.4 when using the first and second systems, respectively.

Solution. To assure an expectation of the relative number of downed targets $M[N_{с.ц}]/N_{ц} = 0.75$, it is necessary, when using the first control system, that when $P_1 = 0.5$, $N_{п}/N_{ц} \approx 2$, i.e., the given effectiveness is assured if $N_{п} = 200$ interceptors operate. When using the second control system the given effectiveness is assured when $N_{п}/N_{ц} = 3$. Consequently, the ratio of the number of actions required to achieve a given level of effectiveness is 2/3, while the cost ratio is 4/3.

Thus, it is more favorable to use the second system, although it has less equipment for solving target-distribution problems and assures a lower effectiveness of the interceptors. In return, the low cost of the second system allows a greater number of interceptors at identical total cost, and assures lower cost of target downing.

PROBLEM 8.7. An interceptor searches for a target in a certain region. The greater the search time $t_{п}$, the lower the survival probability $P_{\text{вын}}$ of the interceptor (e.g., the probability of surviving enemy action if search occurs over a territory which can be reached by the enemy's destructive weapons). On the other hand, the greater the search time, the greater the probability $P_{\text{обн}}$ that the interceptor will detect the target. Determine the optimum search time, knowing that

$$P_{\text{вын}} = P_1^{\frac{t_{\text{п}}}{t_1}},$$

while $P_{\text{обн}}$ depends on $t_{\text{п}}$ by the law obtained in Problem 5.12.

Solution. The probability of successful completion of a combat mission P is equal to the product of probabilities $P_{\text{вын}}$ and $P_{\text{обн}}$. Obviously, with optimum search time probability P should be maximum. Having solved the equation

$$\frac{d\,(P_{\text{вын}}P_{\text{обн}})}{dt_{\text{п}}} = 0 \tag{1}$$

relative to $t_{\text{п}}$, we get

$$t_{\text{п. опт}} = \frac{1}{k}\ln\left(\frac{\ln P_1 - k}{\ln P_1}\right) \tag{2}$$

where

$$k = \frac{2bV_{\text{п}}}{S_{\text{п}}}, \tag{3}$$

or

$$t_{\text{п. опт}} = \frac{1}{k}\left(1 - \frac{2bV_{\text{п}}}{S_{\text{п}}\ln P_1}\right) \tag{4}$$

The maximum effectiveness with $t_{\text{п.опт}}$

$$P_{\text{макс}} = (P_{\text{обн}}P_{\text{вын}})_{\text{макс}} = \left(1 - e^{-\frac{kt_{\text{п. опт}}}{S_{\text{п}}}}\right)P_1^{\frac{t_{\text{п.опт}}}{t}} \tag{5}$$

PROBLEM 8.8. An interceptor carries out aerial combat with a fighter-bomber using two possible tactics: attack from the forward hemisphere and attack from the aft hemisphere ($п_1$ and $п_2$). The fighter-bomber uses three possible countering tactics: it jams the airborne radar, jams the rockets' homing heads, and conducts cannon fire (u_1, u_2, and u_3, respectively). The effectiveness of the interceptor as a function of the tactics used is shown in Table 8.6 (in arbitrary units).

Table 8.6.

u_i \ u_j	u_1	u_2
u_1	3	10
u_2	4	5
u_3	8	2

Dete mine the optimum tactics of the interceptor and the optimum counter-tactics of the fighter-bomber.

Solution. We have a typical conflict situation examined in the theory of games [29].

The three possible fighter-bomber tactics have three corresponding equations of straight lines in the coordinate system ν, ξ, where ν is the value of the game and ξ is the number which defines the probable combination of pure interceptor tactics.

For fighter-bomber tactic u_1

$$\nu = 3\xi + 10(1-\xi) = 10 - 7\xi; \tag{1}$$

for tactic u_2

$$\nu = 4\xi + 5(1-\xi) = 5 - \xi; \tag{2}$$

for tactic u_3

$$\nu = 8\xi + 2(1-\xi) = 2 + 6\xi. \tag{3}$$

Having constructed the straight lines (Fig. 8.5), we get the region of possible solution to the game (cross-hatched in the figure). The solid line corresponds to minimum effectiveness. The guaranteed effectiveness for the interceptor is determined by the value of the maximum with $\xi = 0.43$.

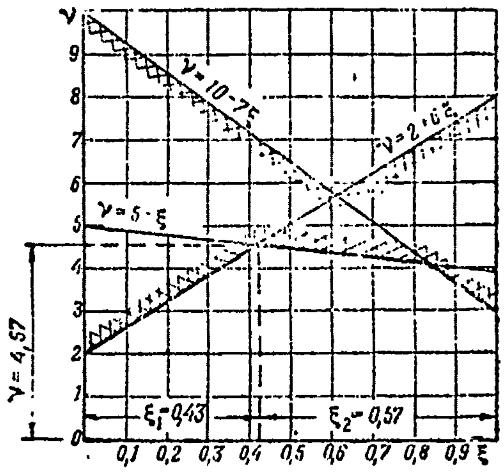

Fig. 8.5.

Thus, the interceptor should alternate its tactics at random with the following frequencies: attack from the forward hemisphere with frequency $\xi_1 = 0.43$, and attack from the aft hemisphere with frequency $\xi_2 = 0.57$. Then the effectiveness for any fighter-bomber tactic will be at least $\nu = 4.57$ arbitrary units.

Solving the system of equations

$$\left.\begin{aligned} 3\eta_1 + 4\eta_2 + 8\eta_3 &= 4{,}57; \\ 10\eta_1 + 5\eta_2 + 2\eta_3 &= 4{,}57; \\ \eta_1 + \eta_2 + \eta_3 &= 1, \end{aligned}\right\} \qquad (4)$$

we find

$$\eta_1 = 0, \ \eta_2 = 0{,}86 \ \text{and} \ \eta_3 = 0{,}14. \qquad (5)$$

i.e., the fighter-bomber can prevent an increase in interceptor effectiveness if it randomly alternates its tactics with frequencies (5). This fighter-bomber mixed tactic (jamming of airborne radar of the interceptor with frequency $\eta_1 = 0$, jamming the rocket homing heads with frequency $\eta_2 = 0.86$, cannon fire with frequency $\eta_3 = 0.14$) is optimum for it since it guarantees it damage of no more than $\nu = 4.57$.

PROBLEM 8.9. An interceptor intended for combat against bombers using the three types of counteraction can be armed with air-to-air guided missiles, unguided rocket projectiles, and cannons.

The expectation of the number of downed bombers in one mission by one interceptor as a function of bomber counteraction and the unit of fire of the interceptor is given in Table 8.7.

Determine the best interceptor armament and the most favorable type of bomber counteraction.

Table 8.7.

Bomber counteraction	Interceptor armament		
	air-to-air guided missiles	cannons	unguided rocket projectiles
Long-range maneuver	2	0	1
Abrupt short-range maneuver	1	2	0
Radio jamming of airborne radar	0	1	2

Solution. We have a zero-sum game with two players. For brevity, let us designate the pure tactics of the interceptors by $п_1$, $п_2$, and $п_3$, and the pure tactics of the bombers by $и_1$, $и_2$, and $и_3$. Then the payoff matrix is written as follows (Table 8.8.):

Table 8.8.

$A =$

$и_i$ \ $п_i$	$п_1$	$п_2$	$п_3$
$и_1$	2	0	1
$и_2$	1	2	0
$и_3$	0	1	2

The selection of move $п_i$ indicates selection of the i-th column, while selection of move $и_i$ indicates selection of the i-th row in matrix A. Solution of the game consists of the mixed tactics of each player. We find the solution by the iteration method.

The method for determining the result of such a game is to play a number of steps (iterations) with pure tactics. Each player notes the tactics used by the enemy in the last step (enemy steps Nos. 1, 2, 3, ..., n - 1), and selects, at step No. n, that tactic which would be best, considering all previous enemy moves.

The tactics of the first move can be selected at will. Let us select, e.g., $п_1$ and $и_1$. In the second move the interceptor acts as if the bomber were to reselect tactic $и_1$. Since the interceptor strives to maximize the results of the game, he again selects tactic

n_1, since the largest number of the first row in matrix A is in the first column. Correspondingly, the bomber selects tactic u_3, since the smallest number in the first column is in the last row.

To determine move No. 3 of the interceptor we calculate the average value from rows 1 and 3 (corresponding to the last tactics u_1 and u_3 selected by the bomber). The maximum of this average value is in column 3. Thus, n_3 is that tactic which is best for the interceptor in this case. The bomber correspondingly responds with tactic u_3.

Let us assume that the bomber in the first (n - 1) moves selected tactics u_{ik} (k = 1, 2, 3, ..., n - 1). For the interceptor we then calculate the average value from rows i_k, select the largest of these average values, and use the number of this maximum for the next move. The bomber then acts in a similar manner. Instead of the average values of the rows (or columns) we can take their sums.

Table 8.9 gives 30 iterations (moves) of a game. The first column in the table is the number of the iteration. The column corresponding to the interceptor tactics is given as a three-digit number in the fourth column of the table. The fifth column shows the sum of all previous columns and indicates, by boldface numbers, the minimum which determines the bomber's next tactic. The sixth column in the table shows the selected row, while the last (seventh) column gives the sum of all previous rows. The boldface numbers are the maximum, showing the next interceptor tactic.

The value of the game ν after n moves is located between the minimum of the column sums and the maximum of the row sums:

$$0{,}8 \leqslant \nu_{10} \leqslant 1{,}5;$$

$$\frac{17}{20} = 0{,}85 \leqslant \nu_{20} \leqslant 1{,}35 = \frac{27}{20};$$

$$\frac{23}{30} = 0{,}77 \leqslant \nu_{30} \leqslant 1{,}13 = \frac{34}{30}.$$

Table 8.9.

Move No.	Tactics		Column	Sum of columns			Row	Sum of rows		
	interceptor	bomber								
1	2	3	4	5			6	7		
1	1	1	210	2	1	0	201	2	0	1
2	1	3	210	4	2	0	012	2	1	3
3	3	3	102	5	2	2	012	2	2	5
4	3	3	102	6	2	4	012	2	3	7
5	3	2	102	7	2	8	120	3	5	7
6	3	2	102	8	2	10	120	4	7	7
7	3	2	102	9	2	12	120	5	9	7
8	2	2	021	9	4	13	120	6	11	7
9	2	2	021	9	6	14	120	7	13	7
10	2	2	021	9	8	15	120	8	15	7
11	2	2	021	9	10	16	120	9	17	7
12	2	1	021	9	12	17	201	11	17	8
13	2	1	021	9	14	18	201	13	17	9
14	2	1	021	9	16	19	201	15	17	10
15	2	1	021	9	18	20	201	17	17	11
16	2	1	021	9	20	21	201	19	17	12
17	1	1	210	11	21	21	201	21	17	13
18	1	1	210	13	22	21	201	23	17	14
19	1	1	210	15	23	21	201	25	17	15
20	1	1	210	17	24	21	201	27	17	16
21	1	1	210	19	25	21	201	29	17	17
22	1	1	210	21	26	21	201	31	17	18
23	1	3	210	23	27	21	012	31	18	20
24	1	3	210	25	28	21	012	31	19	22
25	1	3	210	27	29	21	012	31	20	24
26	1	3	210	29	30	21	012	31	21	26
27	1	3	210	31	31	21	012	31	22	28
28	1	3	210	33	32	21	012	31	23	30
29	1	3	210	35	33	21	012	31	24	32
30	3	3	102	36	33	23	012	31	25	34

The optimum tactics are determined by the frequencies with which individual pure tactics are selected. For n = 10 iterations the optimum interceptor tactics are defined by the frequencies ξ = 0.1, 0.4, 0.5; the optimum bomber tactics – by frequencies η = 0, 0.7, 0.3; for n = 20: ξ = 0.3, 0.45, 0.25; η = 0.5, 0.35, 0.15; for n = 30: ξ = 0.466, 0.3, 0.233; η = 0.4, 0.233, 0.366.

Thus, the best interceptor armament variant is one which uses rockets, cannons, and unguided rocket projectiles with frequencies 0.466, 0.3, and 0.233, respectively. The most suitable type of bomber counteraction is a random alternation of the types of counteraction (long-range maneuvering, abrupt maneuvering at short range, and jamming of the airborne radar) with frequencies 0.4, 0.233, and 0.367, respectively.

PROBLEM 8.10. An enemy can use, in a raid, three types of bombers at random; the commander of the air unit can oppose these targets with two types of interceptors, each of which can successfully act against each target. The effectiveness of the interceptors is characterized by a matrix (Table 8.10), in which the letter u_i expresses the enemy tactics and n_i the air-defense tactics.

Table 8.10.

n_i \ u_i	u_1	u_2	u_3
n_1	0.1	0.3	0.5
n_2	0.5	0.3	0.1

Determine the optimum variant for assigning a flight of interceptors to destroy the bombers, and the optimum variant for bomber use by the enemy.

Solution. The air-defense has two pure tactics (n_1 and n_2), while the enemy has three (u_1, u_2, and u_3). Which tactic is best for the air defense? If the air defense knew in advance the enemy tactics, response would be very simple. However, the enemy tactics are not know beforehand: the air defense does not know what type of target will be used. Therefore, for the air defense it is more suitable to use not pure but mixed tactics, which is the probability distribution (probability mixture) of pure tactics.

To determine the mixed tactics, the air defense selects a certain number ξ_1 $(0 \leq \xi_1 \leq 1)$, and pure tactics η_1 are selected with probability ξ_1 while pure tactics η_2 are selected with probability $1 - \xi_1$. The gain in this case is determined from the formula for calculating the expectation. If the enemy selects tactics u_1, the gain for the air defense is

$$v = 0{,}1\xi_1 + 0{,}5\,(1 - \xi_1) = 0{,}5 - 0{,}4\xi_1; \quad (1)$$

if, however, the enemy selects tactics u_2, then

$$v = 0{,}3\xi_1 + 0{,}3\,(1 - \xi_1) = 0{,}3. \quad (2)$$

while with tactics u_3

$$v = 0{,}5\xi_1 + 0{,}1\,(1 - \xi_1) = 0{,}1 + 0{,}4\xi_1. \quad (3)$$

Expressions (1), (2), and (3) are straight-line equations (Fig. 8.6). The solid line shows how much the air defense gains in any case with selected tactics ξ_1, even if the enemy knows the value ξ_1. When $\xi_1 = 0.5$ we have maximum air-defense gain (maximum expectation of the number of downed targets):

$$v = 0{,}3.$$

This maximum has the following properties: with no air-defense tactics can the gain be more than $\nu = 0.3$, while with any tactics differing from $\xi_1 = 0.5$ the air defense gains $\nu < 0.3$.

Now let us determine the optimum enemy tactics. Mixed enemy tactics are determined by three numbers ξ_1, η_1, and η_2, where

$$\xi_1 + \eta_1 + \eta_2 = 1 \text{ and } \xi_1, \eta_1, \eta_2 \geqslant 0.$$

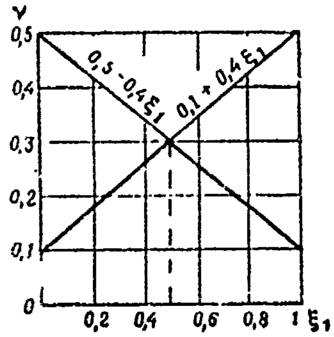

Fig. 8.6.

When using pure tactics η_1 we have the following gain for the air defense:

$$\nu = 0{,}1\xi_1 + 0{,}3\eta_1 + 0{,}5\eta_2 = 0{,}1\xi_1 + 0{,}3\eta_1 + 0{,}5(1 - \xi_1 - \eta_1) = = 0{,}5 - 0{,}4\xi_1 - 0{,}2\eta_1 \quad (4)$$

For pure tactics η_2, respectively,

$$\nu = 0{,}5\xi_1 + 0{,}3\eta_1 + 0{,}1(1 - \xi_1 - \eta_1) = 0{,}1 + 0{,}4\xi_1 + 0{,}2\eta_1. \quad (5)$$

Expressions (4) and (5) are equations of planes, shown in Fig. 8.7. Let us find the equation of the straight line along which these planes intersect. Let us subtract (5) from (4). We get

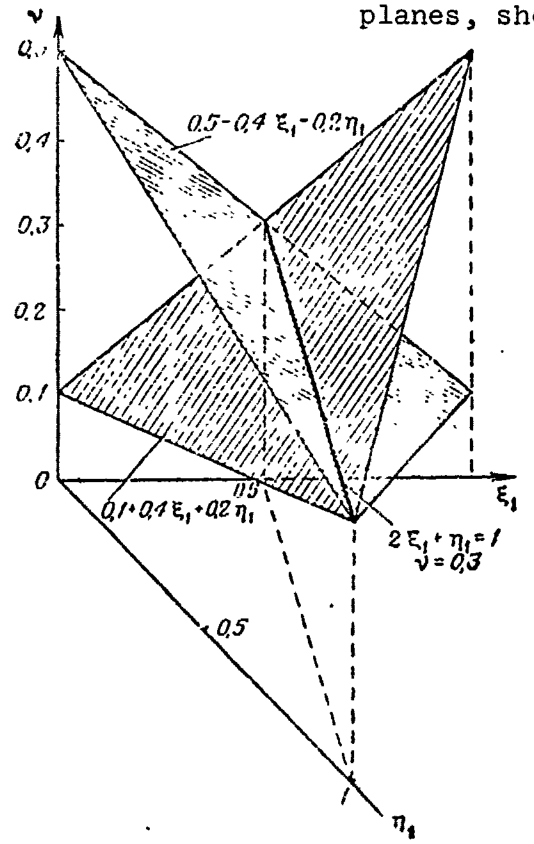

Fig. 8.7.

$$2\xi_1 + \eta_1 = 1. \quad (6)$$

Substituting (6) into (4) we get

$$\nu = 0{,}5 - 0{,}2(2\xi_1 + \eta_1) = 0{,}5 - 0{,}2 \cdot 1 = 0{,}3.$$

This is the distance of line (6) from plane ξ_1, η_1. Any mixed enemy tactics for which (6) is not satisfied give an enemy loss $\nu >$ > 0.3. Such a loss will occur when the air defense knows this tactic. Thus, the value of the game $\nu = 0.3$.

Tactics $\xi_1 = 0.5$ are the optimum air-defense tactics, while tactics ξ_1, η_1, and η_2 when $2\xi_1 + \eta_1 = 1$ are the optimum enemy tactics.

PROBLEM 8.11. To repel a massed raid by bombers covered by escort fighters we use two types of interceptors armed with air-to-air guided missiles, unguided rocket projectiles, and cannons. To carry out a combat mission we require at least 40 missiles, 100 unguided rockets, and 1000 cannon rounds.

The unit of fire for one of the given types of interceptors is characterized by Table 8.11.

Table 8.11.

Unit of fire	Type of interceptor	
	1	2
Cannon projectiles	200	50
Unguided rocket projectiles	4	20
Rockets	2	4

The cost of using the first type of interceptor $C_1 = 2$ arbitrary units; that of the second type $C_2 = 5$.

Determine the optimum number of both types of interceptors for which the mission will be completed with minimum expense. Show how the optimum solution changes with a change in the cost ratio C_1/C_2.

Solution. Let us designate the number of type 1 interceptors by $N_{п1}$, and the number of type 2 interceptors by $N_{п2}$. Then the expenses for carrying out the mission

$$C = 2N_{п1} + 5N_{п2}. \quad (1)$$

To carry out the mission, by stipulation of the problem the following inequalities should be satisfied:

$$\begin{cases} 200N_{п1} + 50N_{п2} \geqslant 1000; \\ 4N_{п1} + 20N_{п2} \geqslant 100; \\ 2N_{п1} + 4N_{п2} \geqslant 40. \end{cases} \quad (2)$$

Thus, in terms of the theory of linear programming the problem is formulated as follows: we must minimize the linear form of (1) with satisfaction of restrictions (2) imposed on the desired variables $N_{п1}$ and $N_{п2}$, where $N_{п1} \geq 0$ and $N_{п2} \geq 0$.

Thanks to the small number of variables, the problem can be solved graphically. All possible variants of combinations of $N_{\text{п}1}$ and $N_{\text{п}2}$ are shown in Fig. 8.8., where the equations of lines (2) are constructed. Line (1) for obtaining minimum expenditures must be located as far to the left and as far to the bottom as possible in Fig. 8.8, but such that it has at least one point in the region of possible values of $N_{\text{п}1}$ and $N_{\text{п}2}$. The dashed line thus obtained at the point of intersection of lines

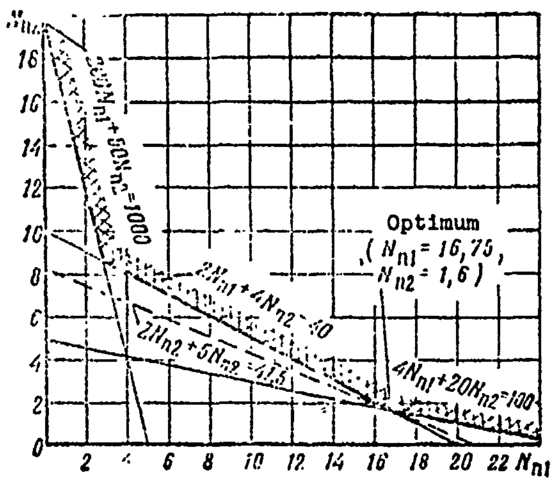

Fig. 8.8.

$$\begin{cases} 4N_{\text{п}1} + 20N_{\text{п}2} = 100; \\ 2N_{\text{п}1} + 4N_{\text{п}2} = 40 \end{cases} \quad (3)$$

gives the optimum solution to the problem: $N_{\text{п}1}$ = 16.75 and $N_{\text{п}2}$ = = 1.6. The expenses here are minimum: C = 2·16.75 + 5·1.6 = 41.5 arbitrary units. Practically speaking, $N_{\text{п}1}$ = 17 and $N_{\text{п}2}$ = 2; C = 44 arbitrary units.

From Fig. 8.8 we can determine how the cost ratio C_1/C_2 influences the optimum solution. If the steepness of line

$$\frac{C_1}{C_2} N_{\text{п}1} + N_{\text{п}2} \quad (4)$$

is greater than that of line

$$4N_{\text{п}1} + 20N_{\text{п}2} = 100, \quad (5)$$

it is advisable to use one type 1 interceptor for the mission. If, however, the steepness of (4) is within limits between the steepness of line (3) and that of the line

$$2N_{\text{п}1} + 4N_{\text{п}2} = 40,$$

we always have a certain optimum relationship between $N_{\text{п}1}$ and $N_{\text{п}2}$. For example, the optimum occurs

for

$$\frac{C_1}{C_2} \geqslant \frac{20}{5} = 4 \text{ when } N_{n1} = 0 \text{ and } N_{n2} = 20;$$

for

$$4 \geqslant \frac{C_1}{C_2} \geqslant \frac{10}{20} = \frac{1}{2} \text{ when } N_{n1} = 2{,}79 \text{ and } N_{n2} = 8{,}28;$$

for

$$\frac{1}{2} \geqslant \frac{C_1}{C_2} \geqslant \frac{1}{5} \text{ when } N_{n1} = 16{,}75 \text{ and } N_{n2} = 1{,}58;$$

for

$$\frac{1}{5} \geqslant \frac{C_1}{C_2} \geqslant 0 \text{ when } N_{n1} = 25 \text{ and } N_{n2} = 0.$$

PROBLEM 8.12. A column of bombers, conducting a low-level raid, is covered by a column of escort fighters at high altitude. The raid is repelled by the operation of two ground radar stations for target detection and two computers which solve the interceptor guidance problems. The radar stations are spaced apart in the direction of the raid, and information on the aerial targets is fed from the first radar to the second; from the output of the second station the bomber coordinates are fed to the first computer, while the coordinates of the escort fighters are fed to the second computer.

The radar stations have scanning sectors which differ in the vertical plane, and their transmission capacities depend on the elevation of the axis of the antenna radiation pattern.

Depending on the elevation of the scan sector, the first radar can simultaneously give coordinates of no more than 40 bombers or 20 escort fighters, or a certain combination of a number of bombers and escort fighters. The second radar, depending on the elevation of the scan sector, can simultaneously give coordinates of no more than 30 bombers or 25 escort fighters, or some combination thereof.

The absolute transmission capacities of the computers are also limited: the first computer can solve guidance problems for no more than 23 bombers; the second - for no more than 18 escort fighters.

The damage inflicted on the enemy by the downing of one bomber is estimated at 2 arbitrary units; that due to the downing of one escort fighter — 1 arbitrary unit.

We must determine the optimum absolute transmission capacity of the detection and guidance system, assuring maximum enemy damage. Let us assume that the probability of guiding the interceptors to the aerial targets is 1.

Solution. Optimum absolute transmission capacity of the detection and guidance system is assured by installing the radar antenna such that $N_{ц1}$ bombers and $N_{ц2}$ escort fighters will be handled simultaneously in the system, while the enemy loss, equal to

$$2N_{ц1} + N_{ц2}, \tag{1}$$

will be maximum. We are required to find the values of $N_{ц1}$ and $N_{ц2}$. Since 1/40 of the transmission capacity of the first radar will be occupied with determining the coordinates of one bomber, for $N_{ц1}$ bombers we require $N_{ц1}/40$ of the transmission capacity of the first radar. Correspondingly, $N_{ц2}/20$ of the transmission capacity of the first radar is required for $N_{ц2}$ escort fighters. Since the relative transmission capacity cannot be greater than one,

$$\frac{N_{ц1}}{40} + \frac{N_{ц2}}{20} < 1 \tag{2}$$

or

$$N_{ц1} + 2N_{ц2} < 40. \tag{3}$$

Analogously we find the following inequality for the transmission capacity of the second radar:

$$\frac{N_{ц1}}{30} + \frac{N_{ц2}}{25} < 1 \tag{4}$$

or

$$2{,}5N_{ц1} + 3N_{ц2} < 75. \tag{5}$$

For the transmission capacity of the first computer, by stipulation of the problem we have

$$N_{ц1} \leqslant 23, \tag{6}$$

for that of the second computer

$$N_{ц2} \leqslant 18. \tag{7}$$

We have the following typical problem of linear programming:

we must maximize the linear form (target function)

$$L = 2N_{ц1} + N_{ц2} \tag{8}$$

with the restrictions imposed on the desired variables $N_{ц1}$ and $N_{ц2}$ in the form

$$\left\{\begin{aligned} N_{ц1} + 2N_{ц2} &\leqslant 40; \\ 2{,}5N_{ц1} + 3N_{ц2} &\leqslant 75; \\ N_{ц1} &\leqslant 23; \\ N_{ц2} &\leqslant 18; \\ N_{ц1} \geqslant 0;\ N_{ц2} &\geqslant 0. \end{aligned}\right. \tag{9}$$

The problem is solved graphically. Let the solution be as follows. In a plane in coordinates $N_{ц1}$, $N_{ц2}$ we construct a region bounded by Inequality (9) and find in this region those values of $N_{ц1}$ and $N_{ц2}$ which determine the maximum of linear form (8). The construction is done in Fig. 8.9. The cross-hatched area in the figure represents the set of variants of the solution. The optimum solution is found as follows. Let us write linear form (8) in the

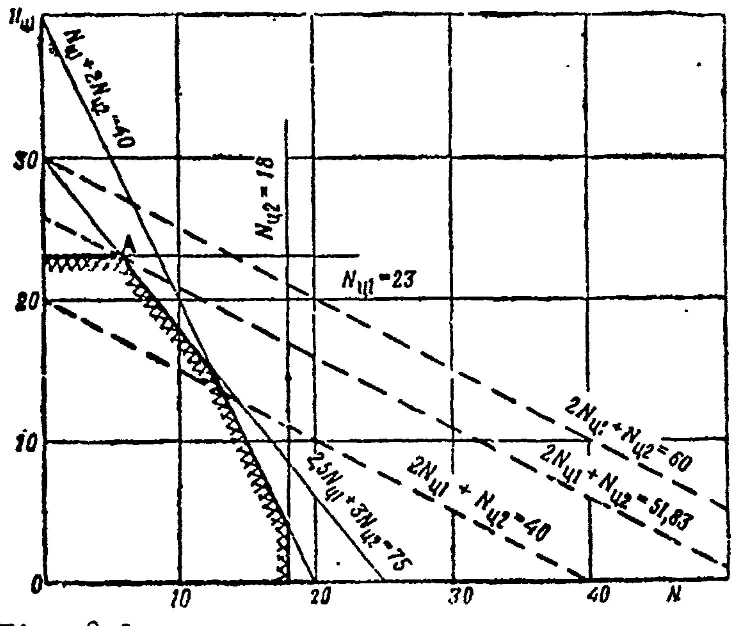

Fig. 8.9.

form

$$\frac{N_{\text{ц}1}}{L/2}+\frac{N_{\text{ц}2}}{L}=1. \qquad (10)$$

This line intersects axis $N_{\text{ц}1}$ at point L/2 and axis $N_{\text{ц}2}$ at point L.

Let L = 60. Then the coordinates of the points of intersection of axes $N_{\text{ц}1}$ and $N_{\text{ц}2}$ are 30 and 60, respectively, i.e., line (10), as can be seen from the figure, passes beyond the limits of the shaded region and there is no unique solution. When L = 40 the line passes through the shaded area, but the optimality requirements are not satisfied. To find the optimum we must move line (10) in parallel up and to the right until it touches the shaded region at one point (point A in Fig. 8.9). This is the point that determines the optimum solution: values $N_{\text{ц}1}$ = 23 and $N_{\text{ц}2}$ = 5.83, for which (8) is maximum.

Thus, if the radar antenna is set such that 23 bombers and 5.83 escort fighters can be "serviced" multaneously, damage to the enemy will be maximum, equal to 2·23 + 5.83 = 51.83 arbitrary units.

Here it is interesting that the transmission capacities are not identically used. With the optimum solution the transmission capacity of the first radar is

$$N_{\text{ц}1}+2N_{\text{ц}2}=23+2\cdot 5{,}83=34{,}67,$$

i.e., it is used only (34.67/40)·100 = 86.8%.

For the second radar we have

$$2{,}5N_{\text{ц}1}+3N_{\text{ц}2}=2{,}5\cdot 23+3\cdot 5{,}83=75$$

and it is occupied (75/75)·100 = 100%.

The first computer is used

$$\frac{23}{23}100=100\%,$$

while the second is used

$$\frac{5{,}83}{18}100=32{,}4\%.$$

PROBLEM 8.13. An enemy in a raid through an air-defense belt can use four types of targets. The air group defending the air-defense belt has four types of interceptors. The effectiveness of each interceptor against each target is characterized by an effectiveness matrix in which u_i designates the enemy tactics and n_i the air-defense tactics (Table 8.12).

Table 8.12.

n_i \ u_i	u_1	u_2	u_3	u_4
n_1	4	-2	-1	-2
n_2	0	2	3	2
n_3	3	0	0	1
n_4	2	2	1	1

The payoff function is the expectation of the number of targets downed by one interceptor in one mission. Negative numbers indicate interceptor losses ("-2" is a total irrevocable loss, "-1" is a temporary breakdown with subsequent recovery). We are required to determine what types and how many interceptors there must be to assure a maximum guaranteed effectiveness level for any enemy tactics, i.e., when the enemy uses the various types of targets in combinations which are random and arbitrary for the air defense.

Solution. Let us use the iteration method. On the basis of the payoff matrix we play a game with a sufficiently large number of steps. In each iteration the enemies want to obtain maximum gain under the condition that the other side in its next move will try to minimize its loss, countering the enemy with a move which is most disadvantageous for it. The start of the iterations can be arbitrary. For example, let us select row n_1. Let us rewrite this row downward and indicate the first minimum with boldface **-2**. Since this minimum is in the second column, let us rewrite the second column to the right of the matrix and also indicate the first maximum **2**. Since

Table 8.13.

n_i \ n_j	n_1	n_2	n_3	n_4	$p = \frac{2}{1} = 2$	$\frac{4}{2} = 2$	$\frac{6}{3} = 2$	$\frac{8}{4} = 2$	$\frac{9}{5} = 1,8$	1,67	1,71	1,625	1,55	1,6	1,54	1,5	1,54	1,5	1,47	1,5	1,47	1,44	1,47	1,45	1,43
n_1	4	−2	−1	−2	−2	−1	−6	−2	−4	−6	−2	−4	−6	−2	−4	−6	−2	−4	−6	−2	−4	−6	−2	−4	−8
n_2	0	2	3	2	2	4	6	6	8	10	10	12	14	14	16	18	18	20	22	22	24	26	26	28	30
n_3	3	0	0	1	0	0	0	3	4	5	8	9	10	13	14	15	1[illegible]	19	20	23	24	25	28	29	30
n_4	2	2	1	1	2	4	6	8	9	10	12	13	1[illegible]	16	17	18	20	21	22	24	25	26	28	29	30

B →

n_i \ n_j	n_1	n_2	n_3	n_4
$Q = \frac{2}{1} = 2$	4	−2	−1	−2
$\frac{0}{2} = 0$	4	0	2	0
$\frac{2}{3} = 0,667$	4	2	5	2
$\frac{4}{4} = 1$	4	4	8	4
$\frac{5}{5} = 1$	6	6	9	5
$\frac{6}{6} = 1$	8	8	10	6
$\frac{8}{7} = 1,14$	8	10	13	8
1,125	10	12	14	9
1,111	12	14	15	10
1,2	12	16	18	12
1,18	14	18	19	13
1,16[illegible]	16	20	20	14
1,23	16	22	23	16
1,215	18	24	24	17
1,21	20	26	25	18
1,25	20	28	28	20

A ↓

A ↓

ν	8/30	3/30	0/30	19/30
1,23	22	30	29	21
1,22	24	32	30	22
1,26	24	34	33	24
1,25	27	34	33	25
1,21	30	34	33	26
1,27	30	36	36	28
1,3	30	38	30	30
1,29	33	38	39	31
1,32	33	40	42	33
1,31	36	40	42	34
1,29	39	40	42	35
1,285	42	40	42	36
1,31	42	42	45	38
1,333	42	44	48	40

B →

									ξ_i
1,45	1,48	1,46	1,44	1,5	1,48	1,46	1,45	1,47	
−10	−6	−[illegible]	−10	−6	−8	−10	−12	−14	0/30
32	32	34	36	36	38	40	42	44	13/30
31	34	35	36	39	40	41	42	43	7/30
32	34	35	36	38	39	40	41	42	10/30

$1,33 < \nu < 1,43$

this maximum is in the second row, let us combine the second row with the row beneath the matrix and indicate the first minimum **0**. Then, since this minimum is located in the second column, let us combine this column with the column written to the right in the matrix, and indicate the maximum **4**. This maximum tells us to combine the second row with the resulting row written for the matrix. The iteration process continues until there is a smooth convergence of the upper and lower bounds of the value of the game p and q ($p \to q \to 0$).

To exclude ambiguity in selecting the next iteration when the row or column contain several identical numbers we must establish a specific rule of choice. Let us stipulate that when identical numbers appear in a row or column we select the first of them: in a row – the first minimum; in a column – the first maximum. Our example is solved by the described method as shown in Table 8.13.

Usually, to obtain sufficiently precise solutions, i.e., to small differences between the lower and upper bounds, within which the value of the game lies, we must perform 20-30 iterations.

After each k-th iteration the value, indicated by boldface, in the row or column is divided by k, which gives the value of the upper and lower bounds p and q of the value of the game, while division of the number of indicated digits by k gives the corresponding statistical frequencies η and ξ, which are approximate values of the optimum tactics of the sides. Obviously, the greater the value of k, the more precise the answer. After 10 steps in our example we find that the value of the game lies within the limits $1.20 \leq \nu \leq 1.43$. Since the approximate solution for tactics η_1 and u_3 gives zero frequencies ($\xi_1 = 0$, $\eta_3 = 0$), these tactics can be discarded. Then the matrix becomes a 3×3 matrix and its solution can be found by a precise method (Table 8.14).

Let us subtract the second and third rows from the first; then

y_1	y_2	y_3
−3	2	1
−2	0	1

Table 8.14

u_j / n_i	u_1	u_2	u_3
n_2	0	2	2
n_3	3	0	1
n_4	2	2	1

Cancelling out in turn the columns of the next matrix, calculating the difference of the products of the numbers along the diagonal, and dividing this difference by the total sum, we get the desired frequencies of the optimum tactics for the enemy: $\eta_1 = 2/7$, $\eta_2 = 1/7$, and $\eta_3 = 4/7$.

Analogously we find the optimum air-defense tactics: $\xi_1 = 3/7$, $\xi_2 = 2/7$, and $\xi_3 = 2/7$. Thus, if the air defense maintains its optimum mixed tactics, attempting to maximize its minimum possible gain, i.e., uses types of interceptors with frequencies

$$\xi_1 = 0;\ \xi_2 = 0{,}43;\ \xi_3 = 0{,}285;\ \xi_4 = 0{,}285,$$

we are assured a guaranteed effectiveness which is numerically equal to the value of the game

$$v = \frac{10}{7} = 1{,}43,$$

regardless of what tactics the enemy prefers.

These optimum mixed tactics are the solution to the problem: they determine what type of interceptor to use and with what frequency the available interceptors should be selected to obtain maximum expectation of the minimum possible number of downed targets.

The most suitable tactics for the enemy are optimum mixed tactics characterized by the following frequencies of random alternation of the four types of targets:

$$\eta_1 = 0{,}285;\ \eta_2 = 0{,}143;\ \eta_3 = 0;\ \eta_4 = 0{,}572.$$

With these frequencies for alternation of types of targets the enemy guarantees himself minimum damage according to the minimax theorem. When assigning a flight of interceptors we must begin from the position that the enemy uses his resources (bombers) with relative frequencies η_1, η_2, η_3, and η_4.

PROBLEM 8.14. A massed raid of air-to-surface missile carriers is to be repulsed by an air-defense air group. Since air-to-surface missiles can be fired at distances of from 1000-2000 km from the interceptor base, two types of interceptors (A and B) can be used. Type A interceptors are used to destroy the carriers on lines 1500-2000 km from the takeoff field; the probability of target destruction in one action $P_{A1} = 0.835$. Type B interceptors operate only 1000-1500 km from the airfield, and the probability of destruction in a single action $P_{B1} = 0.75$.

Each carrier can, with probability $P_I = 0.6$, be in the action zone of type A interceptors, and with probability $P_{II} = 0.4$ be in the zone of type B interceptors.

Each interceptor carries out two actions against the targets; type A interceptors carry these out statistically independently, while type B interceptors carry them out after analysis of the results of the previous attack. The cost of one type A interceptor $C_A = 3$ arbitrary units, while the required number of service personnel per interceptor $r_A = 1.5$. For type B interceptors we have $C_B = 1$ and $r_B = 1$.

Determine the optimum composition of an air-defense air group consisting of types A and B interceptors, assuring destruction of 90% of a raid of $N_ц = 40$ targets when the restrictions on total cost and required number of service personnel C = 75 arbitrary units and r = 65, respectively.

Solution. In general form the solution can be represented by a graph in coordinates N_A, N_B (Fig. 8.10), where N_A is the number of

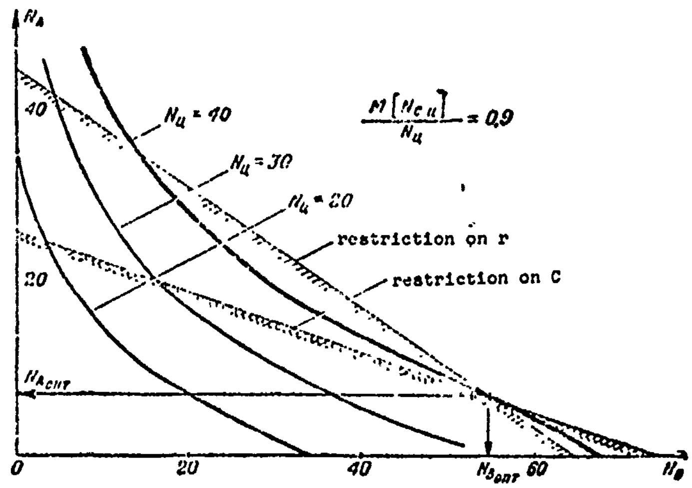

Fig. 8.10.

type A interceptors and N_B is the number of type B interceptors.

First we construct curves of the equal effectiveness of the group consisting of various combinations of N_A and N_B. The effectiveness of single interceptors in operations against single targets is characterized by probabilities P_A and P_B of destroying a target, provided it is in one of the appropriate belts. From the conditions in the problem we have

$$P_A = 0{,}835 \cdot 0{,}6 = 0{,}5 \text{ and } P_B = 0{,}75 \cdot 0{,}4 = 0{,}3.$$

Since by stipulation the relative expectation of the number of downed targets $M[N_{с.ц}]/N_ц = 0.9$, we must determine those pairs of N_A and N_B for which 36 of 40 targets will be destroyed, 18 of 20 targets, etc.

Having constructed the curves of equal effectiveness, we must plot on the graph the restrictions on cost and number of service personnel.

The cost of the interceptors is expressed by the equation

$$C = C_A N_A + C_B N_B, \qquad (1)$$

while the number of service personnel

$$r = r_A N_A + r_B N_B. \qquad (2)$$

By stipulation of the problem we have

$$3N_A + N_B \leqslant 75; \qquad (3)$$
$$1{,}5N_A + N_B \leqslant 65. \qquad (4)$$

Inequalities (3) and (4) graphically represent the half-planes passing through N_A, N_B coordinate axes and bounded on the right by the straight lines

$$\frac{N_A}{\frac{C}{C_A}} + \frac{N_B}{\frac{C}{C_B}} = 1; \qquad (5)$$

$$\frac{N_A}{\frac{r}{r_A}} + \frac{N_B}{\frac{r}{r_B}} = 1. \qquad (6)$$

Thus, writing (3) and (4) as equations of lines in segments, we obtain the coordinates of the points of intersection of these lines with N_A, N_B coordinate axes (considering the sign).

Moving lines (1) and (2) along the family of curves of equal effectiveness until they intersect at one point with the curve of given effectiveness, we find the optimum solution, i.e., that pair of N_A and N_B which assures the given effectiveness level and for which restrictions (3) and (4) on the cost and the number of service personnel are satisfied.

PROBLEM 8.15. Two types of interceptors are used against two types of targets. Show the gain in effectiveness which can be achieved when using optimum tactics compared with the equally-probable alternation of all possible tactics. Determine the gain for the following two effectiveness matrices:

$$A_1 = \begin{Vmatrix} 3 & 2 \\ -4 & 3 \end{Vmatrix}; \quad A_2 = \begin{Vmatrix} -1 & 6 \\ 2 & -1 \end{Vmatrix}.$$

Solution. Let the effectiveness matrix have the following general form:

$$A = \begin{Vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{Vmatrix} = \begin{Vmatrix} a & b \\ c & a \end{Vmatrix}. \qquad (1)$$

For our example $a_{11} = a_{22} = a$, $a_{12} = b$, $a_{21} = c$. As we know, if there is no saddle point, the value of the game is

$$v = \frac{a_{11}a_{22} - a_{12}a_{21}}{N}. \qquad (2)$$

where $N = a_{11} + a_{22} - a_{12} - a_{21}$.

The optimum tactics for interceptor utilization are mixed tactics ξ (ξ_1, ξ_2), where

$$\xi_1 = \frac{a_{22} - a_{21}}{N};$$

$$\xi_2 = \frac{a_{11} - a_{12}}{N};$$

$$\xi_1 + \xi_2 = 1. \tag{3}$$

The optimum enemy tactics are also mixed tactics η (η_1, η_2), where

$$\eta_1 = \frac{a_{22} - a_{12}}{N};$$

$$\eta_2 = \frac{a_{11} - a_{21}}{N};$$

$$\eta_1 + \eta_2 = 1. \tag{4}$$

If all tactics are equally probable, the expectation of the number of downed targets is equal to the average value of the matrix elements:

$$\nu_{cp} = \frac{a_{11} + a_{12} + a_{21} + a_{22}}{4}. \tag{5}$$

Now, assuming that the enemy will alternate his tactics in a manner most unfavorable for us, let us determine the gain in the value of the game relative to the value ν_{cp} which we can guarantee by using our optimum tactics. Let us express this gain in terms of the relative difference

$$\omega = \frac{\nu - \nu_{cp}}{\nu_{cp}}. \tag{6}$$

Let us introduce the designations

$$\alpha = \frac{a}{c}; \quad \beta = \frac{b}{c}.$$

Substituting α and β into the general formula for the value of the game we get

$$\nu = \frac{c(\alpha^2 - \beta)}{2\alpha - \beta - 1}. \tag{7}$$

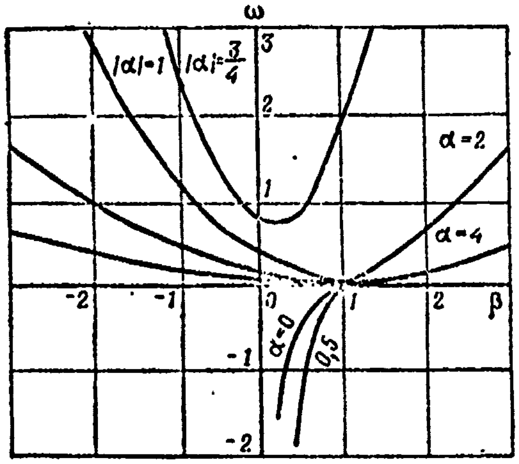

Fig. 8.11.

The average value of the matrix is equal to

$$\nu_{cp} = \frac{c(2\alpha + \beta + 1)}{4}. \qquad (8)$$

For

$$A_1 = \begin{vmatrix} 3 & 2 \\ -4 & 3 \end{vmatrix}$$

we have

$$\alpha = -\frac{3}{4}; \quad \beta = -\frac{1}{2}.$$

Since $\nu_{cp} = 1$ and $\nu = 2.12$, then $\omega = 1.12$, i.e., such a matrix is unfavorable for an air-defense enemy, since the use of optimum tactics compared with equally-probable tactics gives a gain of 12%. The matrix

$$A_2 = \begin{vmatrix} -1 & 6 \\ 2 & -1 \end{vmatrix}$$

is favorable for an air-defense enemy since

$$\alpha = -\frac{1}{2}; \quad \beta = 3; \quad \omega = -0{,}266 < 0,$$

although the value of the game

$$\nu = 1{,}1 > 0.$$

The dependence of the gain in effectiveness on matrix parameters α and β is given in Fig. 8.11.

PROBLEM 8.16. An aviation group consists of three types of interceptors: Π_1, Π_2, and Π_3. The enemy uses three types of fighter-bombers in a raid: $Ч_1$, $Ч_2$, and $Ч_3$. Each type of interceptor can fight each type of fighter-bomber, damaging it with the probability shown in the effectiveness matrix (Table 8.15).

In turn, the fighter-bombers damage the interceptors, respectively, with the probabilities shown in the interceptor-damage matrix (Table 8.16).

Economic expenditures when using interceptors Π_1, Π_2, and Π_3

Table 8.15.

n_i \ u_j	u_1	u_2	u_3
n_1	0,8	0,3	0,4
n_2	0,5	0,7	0,6
n_3	0,4	0,3	0,9

Table 8.16.

$U =$

n_i \ u_j	u_1	u_2	u_3
n_1	0,4	0,2	0,3
n_2	0,5	0,4	0,7
n_3	0,3	0,5	0,2

Table 8.17.

$C =$

n_i \ u_j	u_1, u_2, u_3
n_1	9
n_2	14
n_3	4

are characterized by the values (regardless of the type of target) shown in Table 8.17.

We must determine the most advisable ratio for the use of the interceptors to assure maximum effectiveness (the maximum expectation of the number of downed targets) when the following conditions are satisfied:

1) the probability of damage to the interceptors should not exceed 0.4;

2) the economic expenses should not exceed 10 arbitrary units.

Solution. In the case when the success of an operation is characterized not by one but by several criteria, from all the criteria we select the most important one and require that it become maximum. The auxiliary criteria in this case should satisfy the limiting conditions. As a result of such a representation the problem reduces to one of linear programming. In our case we must maximize the expectation of the number of downed targets while simultaneously satisfying the limiting conditions with respect to damage to interceptors and costs.

Let us transform matrices U and C, subtracting from them $U_0 = 0.4$ and $C_0 = 10$, respectively. Then the matrix for damage to interceptors assumes the following form (Table 8.18):

Table 8.18.

$U' =$

n_l \ u_l	u_1	u_2	u_3
n_1	0	−0,2	−0,1
n_2	0,1	0	0,3
n_3	−0,1	0,1	−0,2

The cost matrix is given in Table 8.19.

Table 8.19.

$C' =$

n_l \ u_l	u_1, u_2, u_3
n_1	−1
n_2	4
n_3	−6

The limiting condition relative to the criterion of damage to interceptors can then be written in the form

$$\left.\begin{aligned} 0\cdot x_1 + 0{,}1x_2 - 0{,}1x_3 - u_1 &= 0; \\ -0{,}2x_1 + 0\cdot x_2 + 0{,}1x_3 - u_2 &= 0; \\ -0{,}1x_1 + 0{,}3x_2 - 0{,}2x_3 - u_3 &= 0. \end{aligned}\right\} \qquad (1)$$

The limiting condition relative to expenditures is represented as follows:

$$x_1 - 4x_2 + 6x_3 + v_1 = 0. \qquad (2)$$

The conditions relative to the expectation of the number of downed targets is written thusly:

$$\left.\begin{aligned} 0{,}8x_1 + 0{,}5x_2 + 0{,}4x_3 - z_1 &= 1; \\ 0{,}3x_1 + 0{,}7x_2 + 0{,}3x_3 - z_2 &= 1; \\ 0{,}4x_1 + 0{,}6x_2 + 0{,}9x_3 - z_3 &= 1. \end{aligned}\right\} \qquad (3)$$

Here it is necessary that the linear form L be minimized:

$$L = x_1 + x_2 + x_3 = \frac{1}{v_1}. \tag{4}$$

Thus, the problem reduces to finding numbers x_1, x_2, and x_3 that satisfy Eqs. (1), (2), and (3).

Omitting the rather unwieldy, but simple, calculations, let us give the final result of solving the problem. The optimum tactics for an air group, consisting of the optimum quantitative ratio among the three types of interceptors, are mixed tactics — the interceptors should be used with frequencies $\xi_1 = 0.146$, $\xi_2 = 0.527$, and $\xi_3 = 0.327$. In other words, the group should contain 14.6% type 1 interceptors, 52.7% type 2 interceptors, and 32.7% type 3 interceptors. When this condition is satisfied, the value of the game (the minimum guaranteed level of the probability of intercepting any target during the raid) is $\nu = 0.511$. Thus, using mixed tactics (ξ_1, ξ_2, ξ_3) we assure maximum combat effectiveness with observance of the limiting conditions with respect to interceptor damage and economic expenditures.

PROBLEM 8.17. To repel a massed raid at low, middle, and high altitudes we must scramble simultaneously 50 type 1 interceptors, 30 type 2 interceptors, and 45 type 3 interceptors. For the required number of interceptors we use two airfields, 1 and 2. The average takeoff time, in seconds, for one interceptor of a given type from the respective airfields is given in Table 8.20.

Table 8.20.

Airfield No.	Type of Interceptor		
	1	2	3
1	4	10	10
2	6	8	20

How should the interceptors be located, by airfield, so that the takeoff time of the entire flight required to carry out the mission is minimum?

Solution. Let us designate by $N_{\text{п}ij}$ the desired number of type i interceptors located at the j-th airfield.

In our example, i = 1, 2, and 3; j = 1 and 2. Thus, $N_{\text{п}1,1}$ is the number of type 1 interceptors at airfield 1, $N_{\text{п}1,2}$ is the number of type 1 interceptors at airfield 2, $N_{\text{п}2,1}$ is the number of type 2 interceptors at airfield 1, etc.

From the conditions of the problem we have

$$\left.\begin{aligned} N_{\text{п}1,1}+N_{\text{п}1,2}&=50;\\ N_{\text{п}2,1}+N_{\text{п}2,2}&=30;\\ N_{\text{п}3,1}+N_{\text{п}3,2}&=45.\end{aligned}\right\} \qquad (1)$$

The takeoff time for interceptors at the first airfield is

$$t_1=4N_{\text{п}1,1}+10N_{\text{п}2,1}+10N_{\text{п}3,1}, \qquad (2)$$

while that for interceptors at the second airfield is

$$t_2=6N_{\text{п}1,2}+8N_{\text{п}2,2}+20N_{\text{п}3,2}. \qquad (3)$$

Obviously, the minimum resulting time for sequential takeoff of all interceptors occurs when $t_1 = t_2$. Thus, it is necessary to find those values of $N_{\text{п}ij} \geq 0$ for which the linear form

$$L=4N_{\text{п}1,1}+10N_{\text{п}2,1}+10N_{\text{п}3,1}=6N_{\text{п}1,2}+8N_{\text{п}2,2}+20N_{\text{п}3,2} \qquad (4)$$

is minimized, with restrictions (1) imposed on the desired variables $N_{\text{п}ij}$.

The problem is solved by methods of linear programming. The solution is given by the optimum plan for interceptor location by airfields (Table 8.21).

The takeoff time for all interceptors with optimum location by airfields is

$$4\cdot 9+1\cdot 0+10\cdot 45=6\cdot 41+8\cdot 30+20\cdot 0=486 \text{ s}.$$

Table 8.21.

Airfield No.	Type of interceptor		
	1	2	3
1	9	0	45
2	41	30	0

Any other variants of interceptor location by airfields increase the reaulting takeoff time required for the flight to carry out its mission.

PROBLEM 8.18. When an enemy operates against an interceptor airbase an increase in the dispersion radius $R_{расср}$ decreases, on the one hand, the probability of damage to the interceptors $P_{пор}$, and increases, on the other hand, the time $t_{б.в}$ required for towing from the dispersion zones and the mission.

Determine the optimum dispersion radius for maximum intensity of a mission by undamaged and combat-ready interceptors. Interceptor damage is determined by action of the enemy against the airfield by air-to-surface missiles carrying nuclear charges with TNT equivalents q = 100 kT. The probable deviation which characterizes the scatter of the rocket is E = 1 km. After an enemy strike the interceptors are towed from the dispersion zones to the runway at a rate $V_{букс}$ = 0.5 km/min.

Solution. The intensity of missions by undamaged interceptors is defined by the expression

$$\mu_в = \frac{N_п (1 - P_{пор})}{t_{б.в}}, \qquad (1)$$

where $P_{пор}$ and $t_{б.в}$ are functions of the dispersion radius $R_{рассР}$. To find the maximum of the function $\mu_в = f(R_{рассР})$ let us first construct, using the nomogram in Fig. 7.3, the dependence $(1 - P_{пор}) = f(R_{рассР})$ and then, on this figure, draw lines for the time required for towing and takeoff of the interceptors as a function of $R_{рассР}$. Then, let us construct, using Eq. (1), the departure intensity as a function of R_{paccp}.

Calculations show that for the initial data the optimum value of R_{paccp} for which the departure intensity reaches its maximum value is ~4 km.

PROBLEM 8.19. To assure combat readiness of airborne radars for the squadron there must be a supply of modules A. With a large supply, some of these modules are excess. The cost of each excess module C_1 = 500 units. If the supply is low, if several modules break down they must be replaced with emergency units flown in, which raises the cost of each module to C_2 = 10,000 units.

We must determine:

1) the optimum number of space modules A if, using the known frequency of breakdowns, we calculate the probability P(r) that exactly r modules A are required for the period of planned combat operations (Table 8.22):

Table 8.22.

r	0	1	2	3	4	5	
s	0	1	2	3	4	5	6 & >
$P(r)$	0,9	0,05	0,02	0,01	0,01	0,01	0
$P(r < s)$	0,9	0,95	0,97	0,98	0,99	1,0	1,0

2) what will be the maximum cost of a module A with emergency air shipment so that the supply of s = 3 modules will be optimum?

Solution. With a supply of s modules, when this supply is greater than the actually required modules r, the expenses are $C_1(s - r)$. When there are not enough modules, i.e., r < s, the expenses are $C_2(r - s)$. Although we do not actually know beforehand exactly how many modules A are required to repair the radars, the probabilities P(r) are known precisely from statistical data on the reliability of the radar operation. Obviously, the resulting expenses C(s) with a constant supply of modules equal to s is calculated as the sum of the particular expenditures for each r multiplied by the corresponding probabilities P(r):

$$C(s)=C_1\sum_{r=0}^{s}{}' P(r)(s-r)+C_2\sum_{r=s+1}^{\infty}{}' P(r)(r-s). \tag{1}$$

The optimum supply s_0 of modules A for which the expected expenses are minimum should satisfy the following inequalities [31]:

$$P(r\leqslant s_0-1)<\frac{C_2}{C_1+C_2}<P(r\leqslant s_0). \tag{2}$$

For our example

$$\frac{C_2}{C_1+C_2}=\frac{10\,000}{500+10\,000}=0{,}952.$$

From Table 8.22 we find that value of s which satisfies the condition

$$P(r\leqslant s-1)\leqslant 0{,}952<P(r\leqslant s).$$

This value will be s = 2. Thus the expenses are minimum if the number of spare modules A is two.

Let us determine what might be the cost of a module with emergency shipment so that a supply of three modules is optimum. Let us substitute s_0 = 3 into (2). Then

$$P(r\leqslant 2)<\frac{C_2}{C_1+C_2}<P(r\leqslant 3).$$

$$0{,}97<\frac{C_2}{500+C_2}<0{,}98.$$

The minimum cost C_2 is

$$\frac{C_2}{500+C_2}=0{,}97;\quad C_2=16\,167.$$

The maximum cost C_2 is

$$\frac{C_2}{500+C_2}=0{,}98;\quad C_2=24\,500.$$

Consequently, a supply of three modules will be optimum if the expenses for one module with emergency replacement are within limits

$$16\,167\leqslant C_2\leqslant 24\,500.$$

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APPENDIX 1

TABLES FOR DETERMINING THE RELATIVE EXPECTATION OF THE NUMBER OF DESTROYED TARGETS WITH RANDOM TARGET DISTRIBUTION

$P_1 = 0{,}1$

N_n \ N_c	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	100
1	0,10[illegible]	0,050	0,0[illegible]	[illegible]	0,020	0,017	0,014	0,012	0,011	0,010	0,007	0,006	0,005	0,003	0,002	0,001
2	0,190	0,098	0,0[illegible]	0,[illegible]50	0,039	0,033	0,028	0,025	0,022	0,020	0,013	0,010	0,007	0,005	0,004	0,002
3	0,27[illegible]	0,143	0,097	0,073	0,059	0,049	0,042	0,037	0,033	0,030	0,020	0,015	0,010	0,007	0,006	0,003
4	0,344	0,185	0,127	0,096	0,078	0,065	0,056	0,049	0,044	0,039	0,026	0,020	0,013	0,010	0,008	0,004
5	0,410	0,226	0,156	0,119	0,096	0,081	0,069	0,061	0,054	0,049	0,033	0,025	0,017	0,012	0,010	0,005
6	0,469	0,265	0,184	0,141	0,114	0,096	0,083	0,073	0,065	0,059	0,039	0,030	0,020	0,015	0,012	0,006
7	0,522	0,302	0,211	0,162	0,132	0,111	0,096	0,084	0,075	0,068	0,046	0,034	0,023	0,017	0,014	0,007
8	0,570	0,337	0,238	0,183	0,142	0,126	0,109	0,096	0,086	0,077	0,052	0,039	0,026	0,020	0,016	0,008
9	0,613	0,370	0,263	0,204	0,166	0,140	0,121	0,107	0,096	0,086	0,058	0,044	0,030	0,022	0,018	0,009
10	0,651	0,401	0,288	0,224	0,183	0,155	0,134	0,118	0,106	0,096	0,065	0,049	0,033	0,025	0,020	0,010
15	0,794	0,537	0,399	0,316	0,261	0,223	0,194	0,172	0,154	0,140	0,095	0,072	0,049	0,037	0,030	0,015
20	0,878	0,642	0,492	0,397	0,332	0,285	0,250	0,222	0,200	0,182	0,125	0,095	0,065	0,049	0,039	0,020
30	0,958	0,785	0,638	0,532	0,455	0,396	0,350	0,314	0,285	0,260	0,182	0,140	0,095	0,072	0,058	0,030
50	0,995	0,923	0,816	0,713	0,636	0,568	0,513	0,468	0,428	0,395	0,284	0,222	0,154	0,118	0,095	0,049
60	0,998	0,954	0,869	0,781	0,702	0,635	0,578	0,530	0,488	0,453	0,331	0,260	0,182	0,139	0,113	0,058
100	1,0	0,994	0,966	0,920	0,867	0,814	0,763	0,716	0,673	0,634	0,488	0,394	0,284	0,221	0,181	0,095

$P_1 = 0,3$

N_a / N_n	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	100
1	0,300	0,150	0,100	0,075	0,060	0,050	0,042	0,037	0,033	0,030	0,020	0,015	0,010	0,008	0,006	0,003
2	0,510	0,278	0,190	0,144	0,116	0,097	0,083	0,074	0,066	0,059	0,040	0,030	0,020	0,015	0,012	0,006
3	0,657	0,386	0,271	0,209	0,169	0,143	0,123	0,108	0,097	0,087	0,059	0,044	0,030	0,022	0,018	0,009
4	0,760	0,478	0,344	0,268	0,219	0,185	0,161	0,142	0,127	0,115	0,078	0,059	0,039	0,030	0,024	0,012
5	0,832	0,556	0,409	0,323	0,266	0,226	0,197	0,174	0,156	0,141	0,096	0,073	0,049	0,037	0,030	0,015
6	0,882	0,623	0,469	0,374	0,310	0,264	0,231	0,205	0,184	0,167	0,114	0,087	0,059	0,044	0,035	0,018
7	0,918	0,679	[illegible]	0,421	0,352	0,302	0,264	0,235	0,211	0,192	0,132	0,100	0,068	0,051	0,041	0,021
8	0,942	0,728	0,570	0,464	0,390	0,337	0,296	0,263	0,238	0,216	0,149	0,114	0,077	0,058	0,047	0,024
9	0,960	0,768	0,613	0,504	0,427	0,370	0,326	0,291	0,263	0,240	0,166	0,127	0,086	0,066	0,053	0,027
10	0,972	0,803	0,651	0,541	0,461	0,401	0,355	0,318	0,288	0,263	0,183	0,140	0,095	0,073	0,058	0,030
15	0,995	0,913	0,794	0,689	0,605	0,537	0,482	0,436	0,399	0,367	0,261	0,203	0,140	0,107	0,086	0,044
20	0,999	0,961	0,878	0,790	0,710	0,642	0,584	0,534	0,492	0,456	[illegible]	0,261	0,182	0,140	0,113	0,058.
30	1,0	0,992	0,958	0,904	0,844	0,783	0,731	0,682	0,632	0,599	0,455	0,365	0,260	0,202	0,165	0,086
50	1,0	0,999	0,995	0,980	0,955	0,923	0,888	0,852	[illegible]	[illegible]	0,636	0,530	0,395	0,314	0,260	0,139
60	1,0	1,0	0,998	0,991	0,976	0,954	0,928	0,899	0,869	0,839	0,702	0,596	[illegible]	0,363	0,303	0,165
100	1,0	1,0	1,0	1,0	0,998	0,994	0,987	0,978	0,966	[illegible]	0,857	0,779	0,634	0,529	0,452	0,260

$P_1 = 0,5$

N_a / N_n	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	100
1	0,500	0,250	0,167	0,125	0,100	0,083	0,071	0,062	0,055	0,050	0,033	0,025	0,017	0,012	0,010	0,005
2	0,750	0,437	0,306	0,234	0,190	0,140	0,138	0,121	0,108	0,097	0,066	0,049	0,033	0,025	0,020	0,010
3	0,875	0,576	0,421	0,330	0,271	0,221	0,199	0,176	0,158	0,140	0,097	0,073	0,049	0,037	0,030	0,015
4	0,938	0,684	0,518	0,414	0,344	0,291	0,234	0,228	0,204	0,185	0,127	0,096	0,065	0,049	0,039	0,020
5	0,969	0,760	0,598	0,487	0,409	0,353	0,300	0,276	0,249	0,226	0,156	0,119	0,081	0,061	0,049	0,030
6	[illegible]	0,822	0,665	0,551	0,469	0,407	0,360	[illegible]	0,290	[illegible]	[illegible]	0,141	0,096	0,073	0,059	0,025
7	0,992	0,867	0,721	0,607	0,522	0,456	0,405	[illegible]	0,330	0,302	0,211	0,162	0,111	0,084	0,068	0,034
8	0,996	0,900	0,767	0,656	0,570	0,501	0,447	0,403	0,367	0,337	0,238	0,183	0,126	0,096	0,077	0,039
9	0,998	0,925	0,806	0,699	0,613	0,543	0,487	0,443	0,402	0,370	0,263	0,204	0,140	0,107	0,086	0,044
10	0,999	0,944	0,838	0,737	0,651	0,581	0,523	0,476	0,435	0,401	0,287	0,221	[illegible]	[illegible]	0,096	0,049
15	1,0	0,987	0,935	0,863	0,794	0,729	0,671	0,620	0,576	[illegible]	0,399	0,316	0,222	[illegible]	[illegible]	0,072
20	1,0	0,997	0,974	0,931	0,878	0,825	0,773	0,725	0,681	0,642	0,492	0,397	[illegible]	0,222	0,182	0,095
30	1,0	1,0	0,996	0,982	0,958	0,926	0,892	0,856	0,820	0,785	0,638	0,532	0,396	0,311	0,260	0,140
50	1,0	1,0	1,0	0,996	0,995	0,987	0,975	0,959	0,913	0,925	0,816	0,718	0,563	0,467	0,395	0,222
60	1,0	1,0	1,0	1,0	0,998	0,995	0,988	0,979	0,968	0,951	0,869	0,781	0,635	0,530	0,453	0,260
100	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,998	0,997	[illegible]	0,960	0,920	0,814	0,716	0,631	0,394

$P_t = 0{,}7$

N_a \ N_n	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	100
1	0,700	0,350	0,233	0,175	0,140	0,117	0,100	0,087	0,078	0,070	0,047	0,035	0,023	0,018	0,014	0,007
2	0,910	0,577	0,412	0,319	0,260	0,220	0,190	0,167	0,150	0,135	0,091	0,069	0,046	0,035	0,028	0,014
3	0,973	0,725	0,547	0,438	0,364	0,311	0,271	0,240	0,216	0,196	0,134	0,101	0,068	0,052	0,041	0,021
4	0,992	0,821	0,655	0,537	0,453	0,391	0,323	0,307	0,277	0,252	0,174	0,133	0,090	0,068	0,055	0,028
5	0,997	0,881	0,735	0,618	0,532	0,462	0,410	0,367	0,333	0,301	0,213	0,163	0,111	0,081	0,068	0,035
6	0,999	0,925	0,797	0,685	0,595	0,525	0,469	0,423	0,385	0,353	0,230	0,192	0,132	0,101	0,081	0,041
7	1,0	0,952	0,844	0,740	0,652	0,580	0,522	0,473	0,433	0,398	0,249	0,221	0,152	0,116	0,094	0,048
8	1,0	0,968	0,881	0,785	0,701	0,629	0,570	0,519	0,477	0,410	0,318	0,248	0,172	0,132	0,107	0,055
9	1,0	0,979	0,908	0,823	0,743	0,673	0,613	0,561	0,517	0,450	0,350	0,274	0,191	0,147	0,119	0,061
10	1,0	0,987	0,929	0,851	0,779	0,711	0,651	0,599	0,555	0,516	0,380	0,300	0,219	0,162	0,132	0,068
15	1,0	0,998	0,981	0,941	0,896	0,844	0,794	0,747	0,703	0,663	0,512	0,414	0,298	0,233	0,191	0,100
20	1,0	1,0	0,995	0,979	0,951	0,916	0,878	0,840	0,802	0,766	0,616	0,509	0,376	0,297	0,246	0,131
30	1,0	1,0	1,0	0,997	0,989	0,976	0,958	0,936	0,912	0,887	0,762	0,657	0,508	0,411	0,345	0,190
50	1,0	1,0	1,0	1,0	0,999	0,998	0,994	0,989	0,983	0,973	0,908	0,831	0,693	0,566	0,506	0,296
60	1,0	1,0	1,0	1,0	1,0	0,999	0,998	0,996	0,992	0,987	0,943	0,862	0,757	0,653	0,571	0,344
100	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,999	0,992	0,972	0,906	0,829	0,756	0,505

$P_t = 0{,}9$

N_a \ N_n	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	100
1	0,900	0,450	0,300	0,225	0,180	0,150	0,129	0,112	0,100	0,090	0,060	0,045	0,030	0,022	0,018	0,009
2	0,990	0,698	0,510	0,399	0,328	0,278	0,241	0,212	0,190	0,172	0,116	0,088	0,059	0,044	0,036	0,018
3	0,999	0,834	0,657	0,535	0,449	0,386	0,338	0,301	0,271	0,246	0,169	0,129	0,087	0,066	0,053	0,027
4	1,0	0,908	0,759	0,639	0,548	0,478	0,423	0,380	0,344	0,314	0,219	0,168	0,115	0,087	0,070	0,036
5	1,0	0,950	0,832	0,720	0,629	0,556	0,497	0,449	0,410	0,376	0,260	0,206	0,141	0,108	0,087	0,044
6	1,0	0,972	0,882	0,783	0,696	0,623	0,562	0,511	0,469	0,432	0,310	0,241	0,167	0,128	0,103	0,053
7	1,0	0,985	0,918	0,832	0,751	0,679	0,618	0,566	0,522	0,483	0,352	0,276	0,192	0,147	0,119	0,061
8	1,0	0,992	0,942	0,870	0,796	0,728	0,667	0,615	0,570	0,530	0,390	0,308	0,216	0,166	0,135	0,070
9	1,0	0,995	0,960	0,899	0,832	0,768	0,710	0,652	0,613	0,572	0,427	0,339	0,240	0,185	0,151	0,078
10	1,0	0,997	0,972	0,913	0,863	0,803	0,747	0,697	0,651	0,611	0,461	0,369	0,263	0,204	0,166	0,086
15	1,0	1,0	0,995	0,978	0,949	0,913	0,873	0,833	0,794	0,757	0,605	0,499	0,367	0,289	0,238	0,127
20	1,0	1,0	0,999	0,994	0,981	0,961	0,936	0,908	0,878	0,848	0,710	0,602	0,456	0,366	0,305	0,165
30	1,0	1,0	1,0	1,0	0,997	0,992	0,984	0,972	0,958	0,941	0,644	0,749	0,599	0,495	0,420	0,238
50	1,0	1,0	1,0	1,0	1,0	0,999	0,998	0,997	0,995	0,991	0,955	0,930	0,782	0,679	0,597	0,361
60	1,0	1,0	1,0	1,0	1,0	1,0	0,999	0,999	0,998	0,997	0,975	0,937	0,839	0,745	0,664	0,419
100	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,998	0,990	0,952	0,897	0,837	0,505

APPENDIX 2

TABLES FOR DETERMINING THE RELATIVE EXPECTATION OF THE NUMBER OF DESTROYED TARGETS WITH UNIFORM TARGET DISTRIBUTION AND SUBSEQUENT ANALYSIS OF THE RESULT OF EACH ATTACK, AND REAIMING

$P_1 = 0{,}1$

N_u / N_n	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50
1	0,100														
2	0,190	0,100													
3	0,271	0,149	0,100												
4	0,344	0,198	0,133	0,100											
5	0,410	0,245	0,167	0,125	0,100										
6	0,469	0,291	0,200	0,150	0,120	0,100									
7	0,522	0,336	0,232	0,175	0,140	0,117	0,100								
8	0,570	0,378	0,265	0,200	0,160	0,133	0,114	0,100							
9	0,613	0,419	0,297	0,225	0,180	0,150	0,129	0,112	0,100						
10	0,651	0,459	0,328	0,250	0,200	0,167	0,143	0,125	0,111	0,100					
15	0,794	0,623	0,476	0,371	0,299	0,250	0,214	0,187	0,167	0,150	0,100				
20	0,878	0,743	0,603	0,486	0,397	0,333	0,256	0,250	0,222	0,200	0,133	0,100			
30	0,958	0,887	0,783	0,679	0,578	0,494	0,427	0,375	0,333	0,300	0,200	0,150	0,100		
50	0,995	0,981	0,950	0,900	0,834	0,759	0,683	0,613	0,551	0,500	0,333	0,250	0,167	0,100	
60	0,938	0,992	0,977	0,948	0,905	0,848	0,783	0,716	0,652	0,594	0,400	0,300	0,200	0,150	0,100

$P_1 = 0{,}3$

$N_н$ \ $N_в$	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50
1	0,300														
2	0,510	0,300													
3	0,657	0,436	0,300												
4	0,760	0,551	0,397	0,300											
5	0,832	0,652	0,489	0,374	0,300										
6	0,882	0,731	0,573	0,447	0,360	0,300									
7	0,918	0,794	0,647	0,517	0,419	0,350	0,300								
8	0,942	0,844	0,712	0,582	0,477	0,400	0,343	0,300							
9	0,960	0,882	0,767	0,643	0,534	0,449	0,386	0,337	0,300						
10	0,972	0,911	0,813	0,698	0,588	0,498	0,428	0,375	0,333	0,300					
15	0,995	0,980	0,944	0,884	0,804	0,717	0,633	0,560	0,500	0,450	0,300				
20	0,999	0,996	0,985	0,962	0,922	0,866	0,798	0,727	0,659	0,598	0,400	0,300			
30	1,0	0,999	0,999	0,997	0,992	0,980	0,960	0,930	0,890	0,842	0,599	0,400	0,300		
50	1,0	1,0	1,0	1,0	0,999	0,999	0,999	0,999	0,997	0,993	0,914	0,715	0,500	0,300	
60	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,999	0,999	0,999	0,975	0,868	0,600	0,450	0,300

$P_1 = 0{,}5$

$N_н$ \ $N_в$	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50
1	0,500														
2	0,750	0,500													
3	0,875	0,687	0,500												
4	0,937	0,812	0,646	0,500											
5	0,969	0,891	0,760	0,61.	0,500										
6	0,984	0,937	0,844	0,719	0,597	0,500									
7	0,992	0,965	0,901	0,801	0,686	0,582	0,500								
8	0,996	0,980	0,339	0,863	0,763	0,660	0,571	0,500							
9	0,928	0,990	0,963	0,909	0,827	0,731	0,640	0,562	0,500						
10	0,999	0,994	0,978	0,940	0,877	0,794	0,702	0,624	0,555	0,500					
15	1,0	0,999	0,998	0,995	0,984	0,961	0,923	0,870	0,808	0,742	0,500				
20	1,0	1,0	0,999	0,999	0,999	0,995	0,988	0,973	0,948	0,912	0,667	0,500			
30	1,0	1,0	1,0	0,999	0,999	0,999	0,999	0,999	0,999	0,997	0,928	0,748	0,500		
50	1,0	1,0	1,0	1,0	1,0	1,0	0,0	1,0	1,0	1,0	1,0	0,994	0,829	0,500	
60	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,949	0,750	0,500

$P_1 = 0,7$

$N_п$ \ $N_п$	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50
1	0,700														
2	0,910	0,700													
3	0,973	0,879	0,700												
4	0,992	0,954	0,853	0,700											
5	0,998	0,983	0,935	0,833	0,700										
6	0,999	0,994	0,973	0,916	0,816	0,700									
7	1,0	[illegible]	0,989	0,960	0,898	0,803	0,700								
8	1,0	1,0	0,996	0,982	0,947	0,881	0,792	0,700							
9	1,0	1,0	0,998	0,992	0,974	0,933	0,866	0,782	0,700						
10	1,0	1,0	0,999	0,997	0,988	0,965	0,919	0,853	0,775	0,700					
15	1,0	1,0	1,0	1,0	1,0	0,999	0,997	0,991	0,978	0,952	0,700				
20	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,999	0,998	0,907	0,700			
30	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,999	0,971	0,700		
50	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,997	0,700	
60	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,984	[illegible]

$P_1 = 0,9$

$N_п$ \ $N_п$	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50
1	0,900														
2	0,990	0 900													
3	0,999	0,983	0,900												
4	1,0	0,998	0,981	0,900											
5	1,0	1,0	0,997	0,977	0,900										
6	1,0	1,0	1,0	0,996	0,974	0,900									
7	1,0	1,0	1,0	0,999	0,994	0,970	0,900								
8	1,0	1,0	1,0	1,0	0,999	0,993	0,967	0,900							
9	1,0	1,0	1,0	1,0	1,0	0,998	0,981	0,964	0,900						
10	1,0	1,0	1,0	1,0	1,0	1,0	0,998	0,989	0,961	0,900					
15	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,999	0,900				
20	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,999	0,900			
30	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,900		
50	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,900	
60	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0,900