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TECHNICAL REPORT

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INTEGRITY ANALYSIS OF  
A BAKELITE MONING GLIDING PARACHUTE  
IN A UNIFORM WIND

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WIND TUNNEL LABORATORY

Security Classification

## DOCUMENT CONTROL DATA - R &amp; D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)		2a. REPORT SECURITY CLASSIFICATION	
US Army Natick Laboratories Natick, Massachusetts 01760			
		2b. GROUP	
3. REPORT TITLE			
Trajectory Analysis of a Radial Homing Gliding Parachute in a Uniform Wind.			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates)			
5. AUTHOR(S) (First name, middle initial, last name)			
Arthur L. Murphy, Jr.			
8. REPORT DATE		7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
		35	6
6a. CONTRACT OR GRANT NO.		6b. ORIGINATOR'S REPORT NUMBER(S)	
b. PROJECT NO.			
c.		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
d.			
10. DISTRIBUTION STATEMENT			
Approved for public release; distribution unlimited.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY	
		US Army Natick Laboratories Natick, Massachusetts 01760	
13. ABSTRACT			
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DD FORM 1473

NOV 61

REPLACES DD FORM 1473, 1 JAN 54, WHICH IS OBSOLETE FOR ARMY USE.

I-A

Security Classification

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Wind (Meteorology)	6					
Uniform	0					
Gliding	7					
Parachutes	7					
Automatic Control	7					
Homing	7					
Radar Homing	7					
Equations	8, 10					
Parameters	8, 10					
Accuracy	4					
Impact Prediction	4					

I-B

Abstract continued.

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Technical Report

73-2-AD

TRAJECTORY ANALYSIS OF A RADIAL  
HOMING GLIDING PARACHUTE IN A UNIFORM WIND

by

Arthur L. Murphy Jr.

September 1971

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I-D

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## FOREWORD

This report presents an analytic assessment of one type of steering law used to control the trajectory of gliding decelerators. The analysis is part of a continuing effort directed toward investigating methods which will improve the accuracy and dispersion characteristics of airdrop systems.

This study was conducted under Department of the Army Project No. 1F1 62203 AA33, Drop Zone Dispersion Studies.

## NOMENCLATURE

- r = Magnitude of the radius vector in polar coordinates
- t = Time
- U = Vector component of the parachute's total airspeed vector in the horizontal plane
- u = Magnitude of U
- w = Magnitude of the wind speed vector w
- x = Horizontal space coordinate fixed to earth
- y = Horizontal space coordinate perpendicular to x and fixed to earth
- $\lambda$  = u/w wind penetration parameter
- $\theta$  = Azimuth angle in polar coordinates

### Subscripts

- l = Launch
- m = Minimum

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## ABSTRACT

The two dimensional trajectory of a radial homing gliding parachute in a uniform wind is presented. The kinematics of the motion is discussed generating two first order differential equations which are separated and solved by direct integration. The resulting expression functionally relates the radial position of the parachute to the instantaneous value of the azimuth angle. Utilizing this result, an exact solution for time in terms of space coordinates is then obtained. From geometric considerations, the angular motion along the azimuth direction is found to be stable when the system is flying into the wind. The trajectory equation shows that under the radial guidance constraint, a gliding system without wind penetration ability can never pass directly over the intended point of impact. However, when the system's glide capability is greater than that of the wind, the parachute has the potential of always reaching the target, provided there is sufficient flight time. In this glide region, the total time to the target as a function of launch angle relative to the wind line, generates the plane curve of a limaçon. An optimum launch point is established from this result. When a fixed flight time is specified, the locus of points of equal time to the target is shown to be an ellipse. This determines the release path for the case of multiple delivery from a single launch vehicle.

## INTRODUCTION

The most severe limitation on parachute delivery is its inherent inaccuracy. Descending as a static entity in an environment which cannot be controlled or precisely predicted, the parachuted load impacts essentially where the wind directs it. Investigators, both private and government sponsored, recognizing the constraint imposed by the passive role of the standard decelerator, have developed highly maneuverable gliding canopies capable of penetrating winds in excess of twenty five knots. In addition to their aerodynamic qualities these flexible wings can be stowed and deployed according to standard parachute methodology and thereby retain the desirable packaging feature of conventional designs.

Deceleration systems, employing gliding canopies, have been extensively investigated in research and development programs by both military and space agencies. In general, the utilization of unmanned gliding systems for military airdrop or for providing the terminal stage air transport of re-entry vehicles, requires guidance and control equipment. Radio control guidance systems have been developed for military airdrop applications,<sup>1</sup> and have been proposed for use in sounding rocket payload retrieval.<sup>2</sup>

In these systems the parachute's direction of flight is controlled through a servo mechanism rigged to the suspension lines of the canopy. Left or right constant rate turns are

produced by retracting the appropriate control line. The communication which activates the control and steers the system is provided by a radio transmitter located at the intended impact point, and a receiver with two antennas, positioned on the suspended load. The antennas are physically separated and are located on the load, so as to define a plane which is perpendicular to the horizontal projection of the parachute's total airspeed vector. With this arrangement, signals emanating from the transmitter will appear equal in magnitude to the separate antennas only when the system's velocity vector is aligned with a radial path connecting it and the target. For any other orientation, a disparity in signal strength is perceived which activates a control, producing a rotation of the parachute towards the ground based transmitter. The wind acting in conjunction with some over control, continually disturbs the system from fixing on a straight line course causing the parachute to steadily maneuver as it seeks radial alignment. This motion which effectively produces what might be termed a radial homing maneuver, persists throughout the flight or until the load passes directly over the transmitter. Upon passing over the target the system executes an orbital path about the transmitter until impact.

This paper treats the approach portion of the trajectory of a radial homing gliding parachute in a uniform wind. Exact solutions completely determining the path in terms of

time and space coordinates are obtained. A detailed analysis of these equations is made leading to a comprehensive assessment of gliding parachute capability when operating in a uniform wind and constrained by radial homing. Previous efforts dealing specifically with wind effects upon gliding systems have been carried out on a numerical basis and are contained in References 3 and 4. An independent analytic treatment similar in scope and applied to determining the time required for an aircraft to execute a round trip in a uniform wind, may be found in Reference 5.

#### EQUATIONS FOR RADIAL GUIDANCE

##### Analysis

Figure 1a depicts the essential geometric aspects of a radial homing gliding parachute in a uniform wind. The wind speed is taken to be steady, to lie entirely in the x-y plane, and to point along the y axis in a negative sense. There is no loss of generality in the arbitrary alignment of the wind line with the y direction, since any other selection merely constitutes a rotation of the resulting trajectory relative to this coordinate axis.

Under equilibrium conditions, the magnitude of the parachute's velocity relative to the air mass remains constant. The lift to drag ratio is fixed, thereby, specifying the vertical and horizontal projections of the total airspeed vector. It is assumed that the physical act of deflecting a

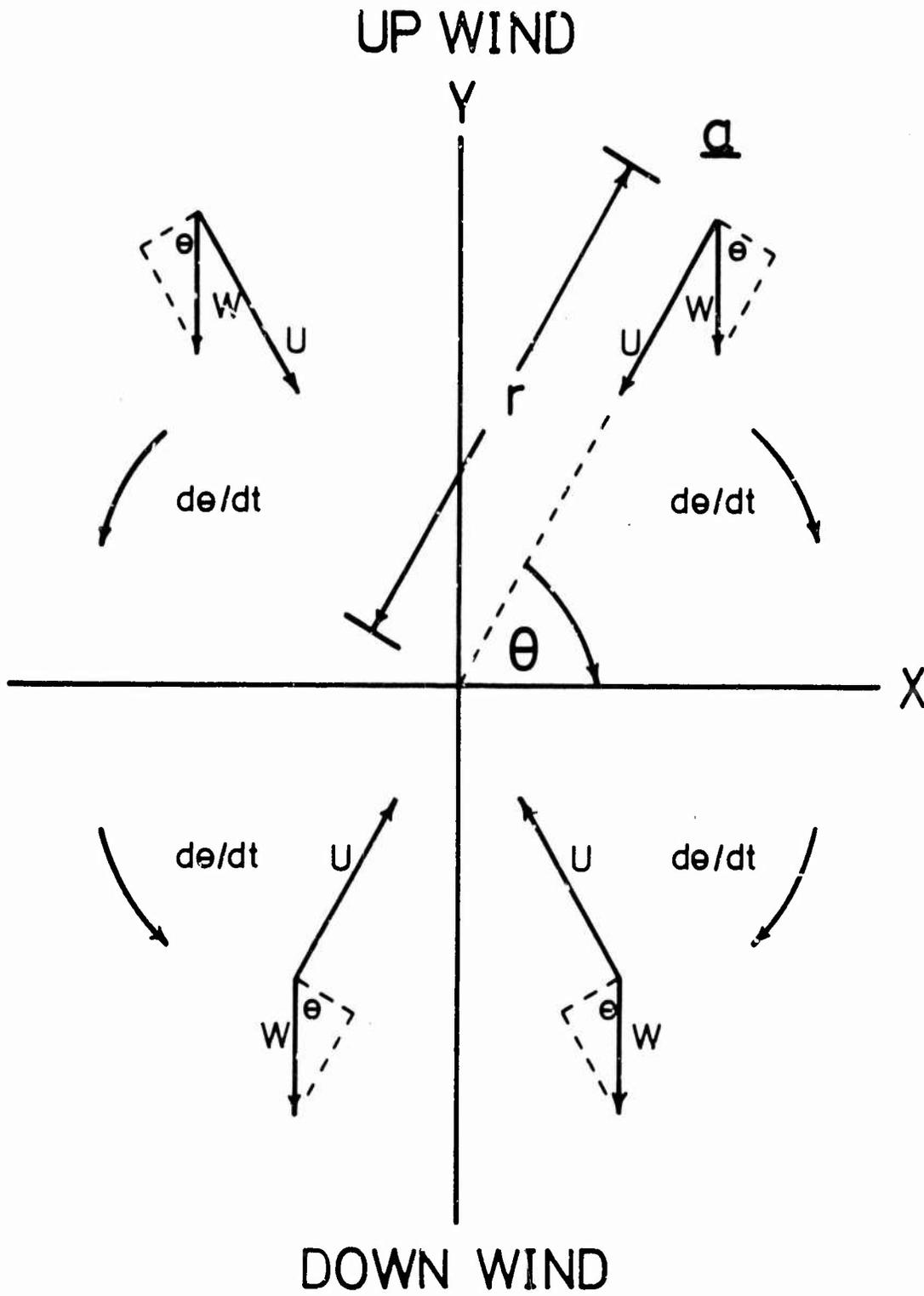


FIGURE 1. THE KINEMATICS OF RADIAL HOMING

control line serves to change only the direction of the horizontal velocity component while leaving the vertical vector undisturbed. The motion is, therefore, separated with the vertical mode one of steady descent. As a consequence, the problem becomes a two-dimensional one where the radial guidance constraint reacting to the presence of the wind, forces the airspeed vector in the horizontal plane, to assume a continuous orientation in the negative radial direction. Operationally, this motion that is idealized by steady modulation of the control, approximates a series of discrete actions combining left and right turns with periods of straight flight.

Within the constraints specified then, the fundamental relationship describing the motion can now be stated as a vector equation relating the absolute velocity of the system relative to an earth fixed reference, to the sum of the wind velocity and the horizontal component of the parachute airspeed vector  $U$ . Expressed in polar coordinates the scalar equations obtained from this vector equality are:

$$dr/dt = -(w \sin \theta + u), \quad (1)$$

and

$$(r)d\theta/dt = -w \cos \theta. \quad (2)$$

#### Stability Characteristics of the Wind Line

Before attempting to obtain solutions from equations (1) and (2), some immediate information regarding the nature

of the angular motion of the parachute with respect to the wind axis can be found directly by examination of equation (2) in conjunction with figure 1. From (2) it is seen that  $d\theta/dt < 0$ , thereby, requiring  $\theta$  to be a decreasing variable. The physical ramifications of this observation can be appreciated by noting again from equation (2) that the angular velocity of the system relative to the fixed reference is due entirely to the wind with no contribution from the vector U. At locations then on the upwind side, that is, coordinate positions where  $y$  is a positive, this component of the wind velocity will continually increase any misalignment between the wind line and the vector U. On the down-wind side, ( $y < 0$ ), angular alignment will be reinforced by this action of the wind. Consequently, and as a figure 1 shows, approaches made along the wind axis from a down-wind position will be insensitive to nominal heading disturbances and are, therefore, stable while the converse is true for the upwind case. Hence, a gliding system which is executing a radial steering maneuver in somewhat steady atmospheric conditions naturally seeks and maintains alignment into the wind. This property, that exists at least in theory, enhances the potential application of gliding decelerators, particularly when consideration is given to the problem of reducing and cushioning the horizontal velocity prior to and during impact.

## Trajectory Determination

Turning now to equation (1) and (2) the  $r$  and  $\theta$  variables may readily be separated to give;

$$dr/d\theta = r(\tan\theta + \lambda \sec\theta); \quad (3)$$

Where  $\lambda = u/w$  is defined as the wind penetration parameter. The expression given in (3) can now be integrated directly yielding:

$$r = K \sec\theta (\sec\theta + \tan\theta)^\lambda; \quad (4)$$

Where  $K$ , the constant of integration, is given by:

$$K = r_1 / \sec\theta_1 (\sec\theta_1 + \tan\theta_1)^\lambda. \quad (5)$$

The basic formulation and subsequent solution to (3) assumes initial alignment along a radial at some angular offset from the wind line. To apply (3) the initial or launch position will be selected to lie along a ray between  $\pm 90$  degrees. The situation of perfect alignment along the wind axis at launch is a special case and cannot be handled directly with (3). Evaluation of this particular condition is made directly in (1) by requiring  $\theta$  to be 90 degrees. The resulting solution is the physical case where the parachute either closes with, remains stationary, or departs from the target along a straight line path coincident with the wind line.

A rectangular form for equation (4) can be obtained by defining:

$$p = x/k, \quad (6)$$

and

$$q = y/K. \quad (7)$$

Making the appropriate substitutions in (4) yields:

$$q = (1/2)\{p^{(\lambda+1)/\lambda} - p^{(\lambda-1)/\lambda}\}. \quad (8)$$

For the special case of  $\lambda = 1$  this expression reduces to:

$$q = (1/2)(p^2 - 1). \quad (9)$$

Hence, when the wind speed is equal in magnitude to the parachute's horizontal airspeed component  $u$ , the ground track of a radial homing gliding system will be parabolic. This result will be seen to be of practical significance when trajectories with larger values of  $\lambda$  are examined.

It is now possible to obtain an integral relationship for time by combining equation (2) with the expression in (4). This gives:

$$t = (-K\lambda/u) \int \sec^2 \theta (\sec \theta + \tan \theta)^\lambda d\theta. \quad (10)$$

Equation (10) can be integrated by parts to give the general expression for time;

$$t = \{\lambda/u(\lambda^2-1)\}\{r_1(\lambda-\sin\theta_1)-r(\lambda-\sin\theta)\}. \quad (11)$$

However, for the special case of  $\lambda = 1$ , (11) does not hold requiring evaluation directly from (10). Thus when  $\lambda = 1$ ;

$$t = (-K\lambda/2u)\{\sec\theta(\sec\theta+\tan\theta)+\ln(\sec\theta+\tan\theta)\}\Big|_{\theta_1}^{\theta}. \quad (12)$$

At this point, the trajectory of a radial homing gliding parachute operating in a uniform wind is completely specified by equations (4), (5), (11), and (12).

#### TARGET LIMITS WITH RADIAL HOMING

##### The Case of $\lambda < 1$ .

Equation (4) is graphically represented in figure (2) for the circumstance where  $\lambda < 1$ . A unit launch radius is assumed and curves are generated for a selected value of the initial azimuth angle. The nature of these curves indicate that  $r$  never becomes zero. Instead, the path lines appear to bend away causing the parachute to pass through what appears to be a minimum radial position relative to the aiming point.

The first of these observations can be verified directly by examining equation (4) when  $r$  is required to be zero. This leads to the expression;

$$0 = \{(1+\sin\theta)^{\lambda-1}/(1-\sin\theta)^{\lambda+1}\}^{1/2}. \quad (13)$$

This relationship cannot be satisfied when  $\lambda < 1$  for values of  $\theta$  in the range,  $-90 < \theta < 90$  degrees. Consequently,  $r$  can never be zero when  $\lambda < 1$ .

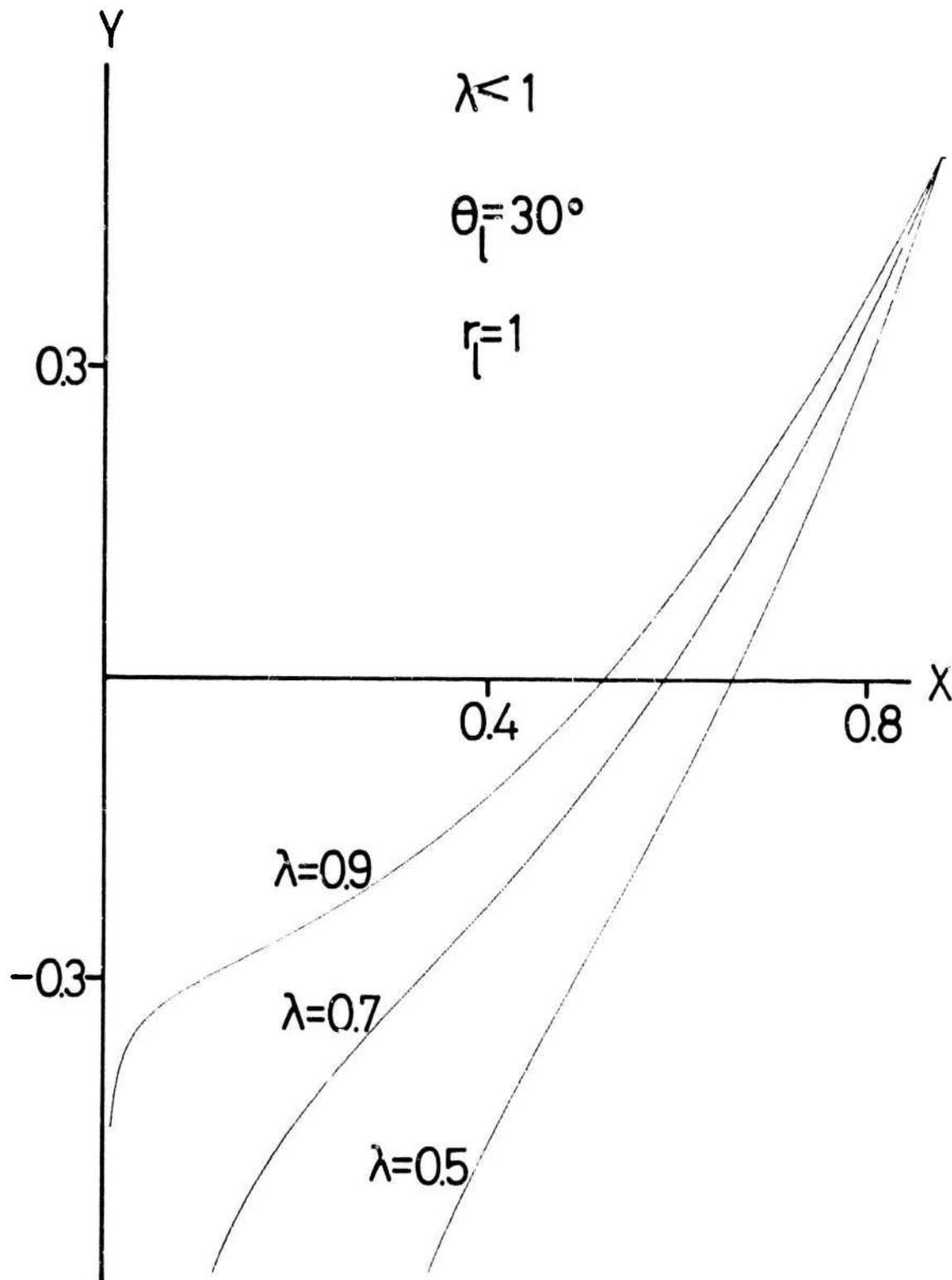


FIGURE 2: RADIAL HOMING TRAJECTORIES WHEN  $\lambda < 1$ .

The existence of an extremum can now be investigated by applying maxima and minima theory of differential calculus to (4). That is,  $dr/d\theta$  is set equal to zero yielding the requirement that:

$$\text{either } r = 0, \quad (14)$$

$$\text{or } \sin\theta = -\lambda. \quad (15)$$

The first of these results has been previously dealt with thereby designating (15) as the appropriate condition. Since (15) can be satisfied for  $\lambda$  between 0 and 1 the existence of a relative minimum  $r$  has been verified. The magnitude of the minimum radius can now be evaluated as;

$$r_m = \{K/(1-\lambda^2)^{1/2}\} \{1-\lambda/1+\lambda\}^{\lambda/2}; \quad (16)$$

$$\text{When } \theta = \theta_m = \text{Arc sin } (-\lambda). \quad (17)$$

The velocity components at this position are given by;

$$dr/dt = 0, \quad (18)$$

$$\text{and } (r)d\theta/dt = -w(1-\lambda^2)^{1/2}. \quad (19)$$

Equations (16) and (17) completely designate what might be called the perigee of the homing orbit when  $\lambda < 1$ . However, some restrictions are in order concerning the application of (16) regarding the quantity  $K$ . The constant of integration  $K$  is seen, from (5) to be a function of  $\theta_1$  as well as  $\lambda$ .

Intuitively, it would be expected that for fixed values of  $\lambda, r_m$  would tend to increase as  $\theta_1$  takes on values further away from the 90 degree ray. This will be true in (16) up to the point where;

$$\theta_1 = \text{Arc sin } (-\lambda) . \quad (20)$$

Beyond this condition (i.e. launch angles less than those given by equation (20)) the equality required in (15) cannot be met, since,  $\theta$  being a decreasing variable prevents counter clockwise rotations. Physically then, there will be no relative minimum point along the path when  $\theta_1 < \text{Arc sin } (-\lambda)$  and, a system launched at these coordinates will be on a course of ever increasing radius.

From the treatment thus far, some conclusions regarding the accuracy potential of radial homing systems is apparent. A gliding system with  $\lambda < 1$  can never fly directly over the target. Its closest penetration will be given by (16) provided the launch is effected such that  $\theta_1 \geq \text{Arc sin } (-\lambda)$ . If this condition is not met the parachute will be on a divergent path with the launch radius ( $r_1$ ) its minimum point relative to the target.

#### The Case of $\lambda > 1$ .

Much of the previous analysis has laid the foundation for treating this particular case. Returning to equation (13)

it is seen that when  $\theta = -90$  degrees,  $r$  will be zero for all values of  $\lambda > 1$ . The requirement in (14) is now met, thereby establishing the obvious fact that  $r = 0$  is a minimum. Figure (3), which is a plot of equation (4), visually verifies these observations again, for the conditions of a unit launch radius and a selected initial angular coordinate.

From Figure (3) it is noted that the parabolic path (the special case of  $\lambda=1$ ), provides a border to all trajectories with higher values of  $\lambda$  launched from the same position. Extending the tail of this reference parabola effectively produces a boundary containing all paths initiated at the nominal reference radius and at some angular coordinate between the wind line and the selected reference angle. The impact points of all radial homing systems launched under these conditions will be found interior to the envelope defined by the reference parabola and the wind axis. If then, equation (12) is utilized, the above definition of required drop zone area can be further refined.

In summary then, the analysis of the case where  $\lambda > 1$  has demonstrated that a radial homing gliding system with some wind penetration ability, has the potential of always reaching the target provided there is sufficient flight time. It has also been shown, through equation (13), that ideally the system will achieve alignment into the wind the moment it arrives over the target. This, of course, is the

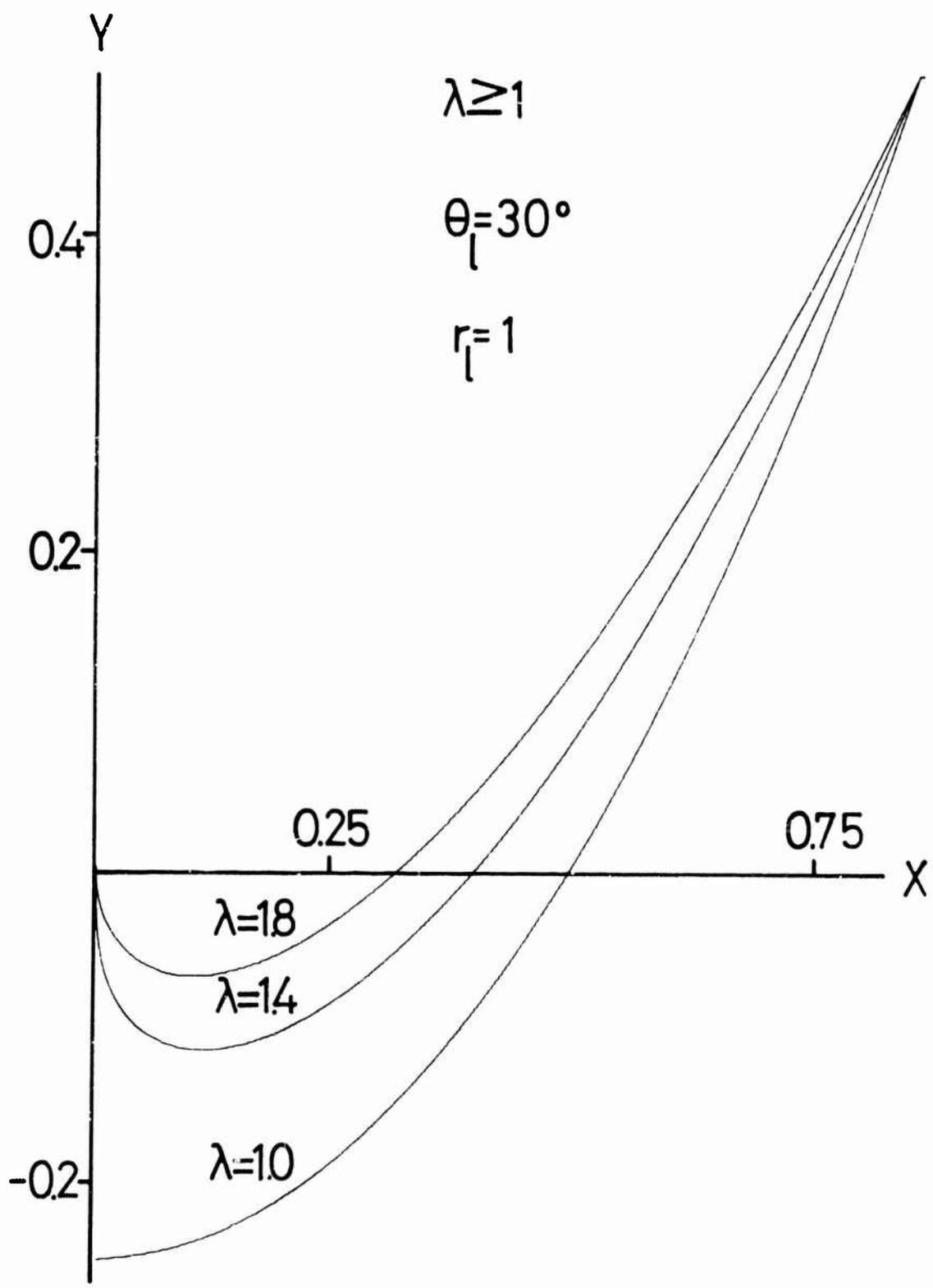


FIGURE 3: RADIAL HOMING TRAJECTORIES WHEN  $\lambda \geq 1$

optimum landing configuration for considerations of impact. The special circumstance where  $\lambda = 1$ , the case of a parabolic path, is seen to provide a tool for estimating the required drop area when certain nominal launch conditions are specified.

### LAUNCH CRITERIA

#### Fixed Radius of Launch

Turning now to equation (11), flight time requirements for radial homing gliding systems will be investigated for the case where  $\lambda > 1$ . Target acquisition implies the physical attainment of coordinates  $r = 0$ , and  $\theta = -90$  degrees.

Imposing these conditions on (11) yields:

$$ut/r_1 \Delta \tau = (\lambda/\lambda^2 - 1)(\lambda - \sin\theta_1). \quad (21)$$

As might be expected, the time necessary to reach the zero radius position is a function of the launch coordinates as well as the penetration ability of the gliding system. When  $\theta_1$  is taken as the independent variable,  $\tau$  a non dimensional time quantity as the radial position coordinate, and  $\lambda$  considered to be a parameter, the plane curve generated by (21) is recognized as a special form of the limaçon of Pascal.

Figure (4) is a graphical presentation of equation (21). Physically, the situation that is being considered, is the case where a circle is imagined to be drawn about the

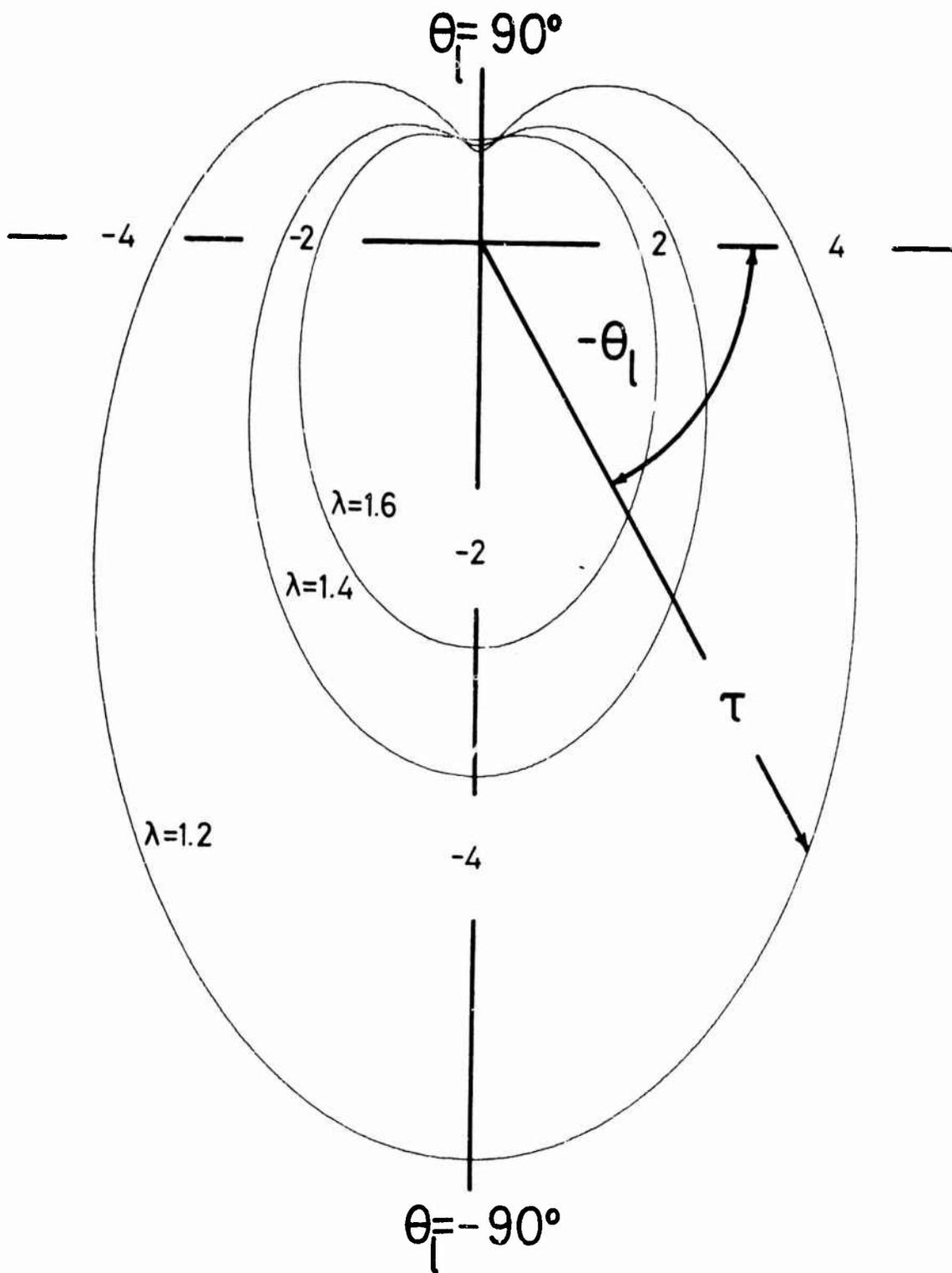


FIGURE 4: FLIGHT TIME REQUIREMENTS FROM POSITIONS ON A UNIT CIRCLE CENTERED AT THE IMPACT POINT.

intended impact point and the required flight time from various locations on the circumference is to be determined. In this discussion it is assumed, of course, that the launch is a premeditated one, and that the question of where to initiate the trajectory is being addressed. The form of (21) is a convenient one for handling this analysis since the quantities capable of independent variation will represent changes in wind heading and intensity. In practice it may be conceivable to exercise tight control over the in-flight positioning of the launch platform, as well as the sequence involving the deployment and inflation of the gliding device. However, control over environmental conditions obviously is beyond the realm of practical consideration. All that can reasonably be expected concerning the wind, is knowledge of its nominal magnitude and direction at some time close to the actual launch. The determining factor then regarding accuracy, will be the wind velocity. It is, therefore, extremely desirable if possible to select an initial launch position which is relatively insensitive to tolerable fluctuations in wind speed or direction.

From Figure (4) the manner in which the time of flight curves tend to flatten and collect near the 90 degree ray leads to the tentative conclusion that this is the optimum azimuth position. To substantiate this observation analytically, the change in  $\tau$  due to independent variations in  $\lambda$  and  $\theta_1$  is investigated through equation (21). Consider then;

$$d\tau = (\partial\tau/\partial\lambda)d\lambda + (\partial\tau/\partial\theta_1)d\theta_1; \quad (22)$$

Where,

$$(\partial\tau/\partial\lambda) = \{(\lambda^2+1)\sin\theta_1 - 2\lambda\}/(\lambda^2-1)^2. \quad (23)$$

and,

$$(\partial\tau/\partial\theta_1) = -\lambda\cos\theta_1/(\lambda^2-1). \quad (24)$$

The most desirable circumstance in (22) would be for  $d\tau$  to be zero for all allowable variations in  $d\lambda$  or  $d\theta_1$ . This requires both (23) and (24) to vanish simultaneously. However, for  $\lambda > 1$  there is no unique launch azimuth which will satisfy this condition. The objective then will be to locate the value of  $\theta_1$  which makes the respective coefficients of  $d\lambda$  and  $d\theta_1$  as close to zero as possible. In this manner the absolute value of  $d\tau$  is in a sense, minimized. To determine  $\theta_1$  the algebra of vectors and vector space will be useful. From equation (22)  $d\tau$  can be thought of as a scalar quantity generated by the dot product of two vectors. These numerical vectors are derived in turn from the sensitivity coefficients  $\partial\tau/\partial\lambda$  and  $\partial\tau/\partial\theta_1$ , and the error terms  $d\lambda$  and  $d\theta_1$ . Utilizing this notion, the Schwartz inequality criterion<sup>6</sup> can be applied directly to equation (22) yielding the requirement that;

$$|d\tau| \leq \{(\partial\tau/\partial\lambda)^2 + (\partial\tau/\partial\theta_1)^2\}^{1/2} \{(d\lambda)^2 + (d\theta_1)^2\}^{1/2}. \quad (25)$$

From this result then it is seen that in order for  $|d\tau|$  to remain small, while the direction and intensity of the wind is allowed to vary in an independent and uncontrollable fashion, the value of  $\{(\partial\tau/\partial\lambda)^2 + (\partial\tau/\partial\theta_1)^2\}^{1/2}$  must be minimum. This condition is met when  $\theta_1 = 90$  degrees for all positive  $\lambda \leq 100$ .

It has been established from the above discussion, that flight time requirements as determined from equation (21) for  $\theta_1 = 90$  degrees will be insensitive to minor wind anomalies. Since wind changes cannot be controlled or anticipated in the manner that other variables may be influenced, a release position accurately located according to this criterion will result in the smallest dispersions at the impact point.

#### Fixed Time of Flight

Again considering the case of a premeditated launch where  $\lambda > 1$ , the circumstance very often occurs where it is desirable to deliver multiple loads in some sequential fashion from a single launch vehicle. If a gliding decelerator homing according to some steering law, is employed for this application phasing of the individual deployments becomes critical. This problem can be addressed, that is in an analytical sense, by attempting to determine for the particular guidance routine being used, the locus of points about the target which have equal flight time. For the particular case of radial guidance, this curve can be generated by

requiring both  $t$  and  $\lambda$  to be fixed in (21) and solving for the launch radius  $r_1$  in terms of  $\theta_1$ .

When this is done, it is observed that;

$$r_1 = J / (1 - (1/\lambda) \sin \theta_1) ; \quad (26)$$

Where,

$$J \triangleq (ut)(\lambda^2 - 1) / \lambda^2 . \quad (27)$$

Since  $1/\lambda$  is always positive and less than 1 for all  $\lambda > 1$ , and  $J$  is a positive constant, equation (26) is identified as the polar equation of an ellipse. The focus of the curve becomes the target and the wind line the major axis.

Equation (26) is a surprising and potentially useful result. Radial homing gliding systems whose trajectories are initiated at different points along this elliptical path will have equal flight time requirements relative to reaching their common aiming point. Consequently, when this launch profile is executed the time increment between successive items discharged from an aircraft in motion no longer becomes a significant factor affecting the dispersal of individual loads at the impact point.

#### CONCLUSIONS

The attributes of one type of steering technique used for guiding aerodynamic decelerators capable of independent motion through the air has been discussed by examining the close form solutions to the equations of motion when a constant

wind velocity is assumed. Analysis of the trajectory equation shows that except for perfect alignment along the wind line, a gliding system maneuvering according to a radial aiming scheme, must have a horizontal airspeed component greater than the wind speed in order for it to reach the homing transmitter prior to impact. This result essentially defines the lift to drag requirements for the radial homing system designed to function in some specified nominal or extreme wind environment.

The parabolic ground track which occurs for the special case where the magnitude of the parachute's intrinsic velocity is precisely matched by the wind speed, is an interesting result which may be useful in establishing physical requirements for a target area. That is, assuming that a particular gliding system will only be employed in conditions where it can penetrate the wind and if tolerance limits are then assigned to positioning errors at the release point, along with estimates of wind variability in the time span surrounding the mission, a region with high impact probability can be determined from the trajectory of a reference parabola derived from calculations based on the extreme launch points.

Impact, particularly with high horizontal velocities, is a problem of concern with any airdrop system. A very desirable feature of the radial guidance scheme regarding its

potential for attenuating velocities imparted by the wind is the apparent stabilizing influence of the wind line. That is, the interaction between the radial constraint and the wind velocity essentially forces the system to seek alignment into the wind, and resist disturbances from this position once it is established. In effect, a natural mechanism is provided which attempts to minimize the ground speed while restricting the direction of the net horizontal motion to one dimension.

Launch criteria, for radial homing systems have been identified through analysis of the time solution derived from basic considerations. Two developments have been presented. The first deals with individual load accuracy, and demonstrates that in order to minimize errors at the impact point due to variations in wind velocity, the release point should be located up-wind along the nominal wind axis. Since no physical control over the wind velocity is possible, this result establishes the optimum launch position. The second development, addresses the circumstance requiring the delivery of multiple loads which are individually discharged from a single vehicle at discrete points along its flight track. In this case, the problem is expanded from the determination of a single release point to the determination of a release path. For the radial steering procedure an elliptical path,

derived from nominal wind conditions will define the locus of points from which the flight time necessary to reach the target is constant. That is, the ability to land at the designated impact point is independent of the launch position along this curve. From this result a flight program can be developed for the accurate delivery of multiple loads.

## REFERENCES

1. Slayman, R. A., Bair, H. Q., Rathbun, T. W., "500-Pound Controlled Airdrop Cargo System", GER-13801, Sep, 70 Goodyear Aerospace Corp., Akron, Ohio.
2. Knapp, C. F., Barton, W. R., "Controlled Recovery of Payloads at Large Glide Distances, Using the Para-Foil, Journal of Aircraft, Vol. 5, No. 2, March - April 1968, pp 112-118.
3. Goodrick, T. F., "Estimation of Wind Effect on Gliding Parachute Cargo Systems Using Computer Simulation", AIAA Aerodynamic Deceleration Systems Conference 1970, Paper No. 70-1193.
4. Goodrick, T. F., "Wind Effect on Gliding Parachute Systems With Non-Proportional Automatic Homing Control", TR-70-28-AD, Nov. 69, USA Natick Laboratories, Natick, Ma.
5. Klamkin, M. S. Newman, D. H., "Flying in a Wind Field", American Mathematical Month, Vol 76, No. 9, November 1969, pp 1013 - 1019.
6. Hildebran, F. B., "Methods of Applied Mathematics", 2nd ed., Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1965, pp 23-26.