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Nondimensional Steady-State Cable Configurations

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DEFINITION OF TERMS

a	Major buoy radius	(ft)
b	Minor buoy radius	(ft)
B	Total excess buoyancy	(lb)
c	Third principal radius of ellipsoid	(ft)
C	Current velocity at a given point	(ft/sec)
C_{DN}	Cable normal drag coefficient	
C_{DT}	Cable tangential drag coefficient	
d	Cable diameter	(ft)
d*	Nondimensional cable diameter	
E	Cable modulus of elasticity	(lb/ft^2)
F*	Nondimensional cable modulus of elasticity	
E_o	Reference value of cable modulus of elasticity	(lb/ft^2)
L	Total cable length	(ft)
L_o	Reference length	(ft)
s	Unit stretched cable length	(ft)
s*	Dimensionless unit stretched cable length	
s_o	Unit unstretched cable length	(ft)
S_o	Dimensionless unit unstretched cable length	
T	Tension	(lb)
T*	Dimensionless tension	
T_o	Reference tension	(lb)
U	x-velocity component	(ft/sec)
U*	Dimensionless x-velocity component	
V	y-velocity component	(ft/sec)
V*	Dimensionless y-velocity component	
V_o	Reference velocity	(ft/sec)

DEFINITION OF TERMS (Cont'd)

W	z-velocity component	(ft/sec)
W*	Dimensionless z-velocity component	
W_c	Cable weight per foot in water	(lb/ft)
W_c^*	Dimensionless cable weight per foot in water	
w_o	Reference cable weight per foot in water	(lb/ft)
x, y, z	Coordinates	(ft)
Δx	Vertical excursion of buoy from horizontal axis	(ft)
Δy	Horizontal excursion of buoy from vertical axis	(ft)
θ, Φ	Cable angles	(radians)
ρ	Mass density of sea water	$\left(\frac{\text{lb-sec}^2}{\text{ft}^4}\right)$
ρ^*	Dimensionless mass density of sea water	(1)
ρ_o	Reference mass density	$\left(\frac{\text{lb-sec}^2}{\text{ft}^4}\right)$

NONDIMENSIONAL STEADY-STATE CABLE CONFIGURATIONS

INTRODUCTION

The preliminary design of surface or subsurface buoy-cable systems and cable-towed body systems is dependent usually on the following parameters:

a. Cable diameter

b. Cable length

c. Cable weight in sea water

d. Buoy displacement and weight

e. Towed body weight in sea water.

In particular, the design of the AFAR (Azores Fixed Acoustic Range) was dependent on cable parameters. Because specific cable sizes were unknown, it was necessary to investigate the configurations and tensions which would result from various cable diameters, weights, and subsurface buoy sizes. The computational technique developed by Patton[1] was used to compute 27 cases.

The problem was to present these results in a meaningful manner. The steady-state cable equations were nondimensionalized and nondimensional coefficients were generated in order to resolve this problem. The results for the AFAR thermistor array study were then put into nondimensional form and plotted.

Appendixes A and B contain additional comments on further application of the use of the dimensionless steady-state cable equations.

DERIVATION OF NONDIMENSIONAL STEADY-STATE CABLE EQUATIONS

Patton[1] generated steady-state cable equations used to predict equilibrium configurations of moored surface buoys. He began with the cable equations developed by Cristecu[2] and obtained the four equations

$$\frac{dT}{ds_o} = -\frac{1}{2}\rho\, C_{DT}\, d\, V\,|V| + W_c \cos\phi \cos\theta\ , \tag{1}$$

$$\frac{d\phi}{ds_o} = \left(-\frac{1}{2}\rho\, C_{DN}\, d\, V\,|V| - W_c \sin\phi \cos\theta\right)\frac{1}{T}\ , \tag{2}$$

$$\frac{d\theta}{ds_o} = \left(-\frac{1}{2}\rho\, C_{DN}\, d\, W\,|W| - W_c \sin\theta\right)\frac{1}{T\cos\phi}\ , \tag{3}$$

and the auxiliary relation

$$ds = \left(1 + \frac{T}{\frac{E\pi d^2}{4}}\right) ds_o\ . \tag{4}$$

A common method for deriving laws of similarity from differential equations is to express the differential equations in dimensionless form.[3] For the case in question, introduce a characteristic length L_o, a characteristic velocity V_o, a characteristic mass density ρ_o, a characteristic tension T_o, a characteristic cable weight per unit length W_o, and a characteristic modulus of elasticity E_o. Dimensionless variables may be defined as follows:

$$T^* = \frac{T}{T_o}\ , \quad \rho^* = \frac{\rho}{\rho_o}\ , \quad W_c^* = \frac{W_c}{W_o}\ ,$$

$$d^* = \frac{d}{L_o}\ , \quad s^* = \frac{s}{L_o}\ , \quad s_o^* = \frac{s_o}{L_o}\ ,$$

$$U^* = \frac{U}{V_o}\ , \quad V^* = \frac{V}{V_o}\ , \quad W^* = \frac{W}{V_o}\ ,$$

and

$$E^* = \frac{E}{E_o}\ .$$

Here we consider angles already dimensionless.

In terms of the new variables, equations (1) through (4) may be expressed as follows:

$$\frac{d(T^* T_o)}{d(s_o^* L_o)} = -\frac{1}{2}\,(\rho^* \rho_o)(C_{DT})(d^* L_o)(U^* V_o)(|U^* V_o|) + (W_c^* \cdot \quad \cos\phi \cos\theta\ ,$$

$$\frac{d\phi}{d(s_o^* L_o)} = \left[-\frac{1}{2} (\rho^* \rho_o)(C_{DN})(d^* L_o)(V^* V_o)(|V^* V_o|) - (W_c^* w_o) \sin\phi \cos\theta \right] \frac{1}{T^* T_o},$$

$$d(\rho_o^* L_o) = \left[-\frac{1}{2} (\rho^* \rho_o)(C_{DN})(d^* L_o)(W^* V_o)(|W^* V_o|) - (W_c^* w_o) \sin\theta \right] \frac{1}{(T^* T_o \cos\phi)},$$

and

$$d(s^* L_o) = \left[1 + \frac{(T^* T_o)}{(E^* E_o)(\frac{\pi}{4})(d^* L_o)^2} \right] d(s_o^* L_o) \quad .$$

Rearranging gives

$$\frac{dT^*}{ds_o^*} = -\frac{1}{2} \rho^* C_{DT} d^* U^* |U^*| \cdot \left(\frac{\rho_o V_o^2 \cdot L_o^2}{T_o} \right) + W_c^* \cos\phi \cos\theta \cdot \left(\frac{w_o L_o}{T_o} \right),$$

$$\frac{d\phi}{ds_o^*} = \left[-\frac{1}{2} \rho^* C_{DN} d^* V^* |V^*| \cdot (\rho_o L_o^2 V_o^2) - W_c^* \sin\phi \cos\theta \cdot (w_o L_o) \right] \frac{1}{T_o} \cdot \frac{1}{T^*} ,$$

$$\frac{d\theta}{ds_o^*} = \left[-\frac{1}{2} \rho^* C_{DN} d^* W^* |W^*| \cdot (\rho_o V_o^2 L_o^2) - W_c^* \sin\theta \cdot (w_o L_o) \right] \cdot \frac{1}{T_o} \cdot \frac{1}{T^* \cos\phi} ,$$

and

$$ds^* = \left[1 + \frac{T^*}{E^* \frac{\pi}{4} \cdot d^{*2}} \cdot \frac{T_o}{E_o L_o^2} \right] \cdot ds_o^* \quad .$$

The final forms of the dimensionless differential equations after multiplication are:

$$\frac{dT^*}{ds_o^*} = -\frac{1}{2} \rho^* C_{DT} d^* V^* |V^*| \left(\frac{\rho_o V_o^2 L_o^2}{T_o} \right) + W_c^* \cos\phi \cos\theta \cdot \left(\frac{w_o L_o}{T_o} \right), \qquad (5)$$

$$\frac{d\phi}{ds_o^*} = \left[-\frac{1}{2} \rho^* C_{DN} d^* V^* |V^*| \cdot \left(\frac{\rho_o V_o^2 L_o^2}{T_o} \right) - W_c^* \sin\phi \cos\theta \cdot \left(\frac{w_o L_o}{T_o} \right) \right] \frac{1}{T^*}, \qquad (6)$$

$$\frac{d\theta}{ds_o^*} = \left[-\frac{1}{2}\,\rho^* C_{DN}\, d^* \mathbf{V}^* |\mathbf{V}^*| \cdot \frac{\rho_o V_o^2 L_o^2}{T_o} - \mathbf{W}_c^* \sin\theta \cdot \left(\frac{w_o L_o}{T_o} \right) \right] \frac{1}{T^* \cos\phi} ,$$

and the auxiliary relation

$$ds^* = 1 + \frac{T^*}{E^* \frac{\pi}{4} d^{*2}} \cdot \frac{T_o}{E_o L_o^2}\, ds_o^* .$$

If homologous points are considered, the dimensionless variables have the same value for a model and its prototype. Then, for the two systems to be similar, the coefficients $\rho_o V_o^2 L_o^2/T_o$, $w_o L_o/T_o$, and $T_o/E_o L_o^2$ must be the same in each situation.

APPLICATION TO THE AFAR THERMISTOR ARRAY

For the no-current condition, the tension at the buoy approximately equals B. For convenience let the reference tension, T_o, be

$$T_o = B .$$

The quantity $E_o L_o^2$ is related to the tension in the cable. Let

$$E_o L_o^2 = T ,$$

where T is the tension at any point of interest on the cable.

To include the effect of cable diameter, let

$$L_o^2 = (L_o)(d_o) .$$

Then the following dimensionless coefficients are used in this application:

a. $\dfrac{\rho_o L_o V_o^2 d_o}{B}$, a dimensionless drag to excess buoyancy ratio;

b. $\dfrac{w_o L_o}{B}$, a dimensionless total cable weight to excess buoyancy ratio;

. $\dfrac{B}{T}$, a dimensionless excess buoyancy to tension ratio;

d θ, the cable angle in the x-z plane; and

e ϕ, the cable angle in the y-z plane.

By trial and error it was found that instead of using θ and ϕ, it is more convenient to utilize the dimensionless spatial coordinates x/L_o, y/L_o, and z/L_o, where x, y, and z are the inertial coordinates of any point on the cable. One sees that the dimensionless spatial coordinates are directly related to the angles θ and ϕ, which are determined by suitable trigonometric manipulation.

CASES INVESTIGATED FOR THE AFAR THERMISTOR ARRAY

To determine the validity of the dimensionless coefficients discussed previously, a number of specific cases had to be investigated. The problem at hand was to investigate the two-dimensional buoy configurations of a 2900-ft long subsurface array (figure 1).

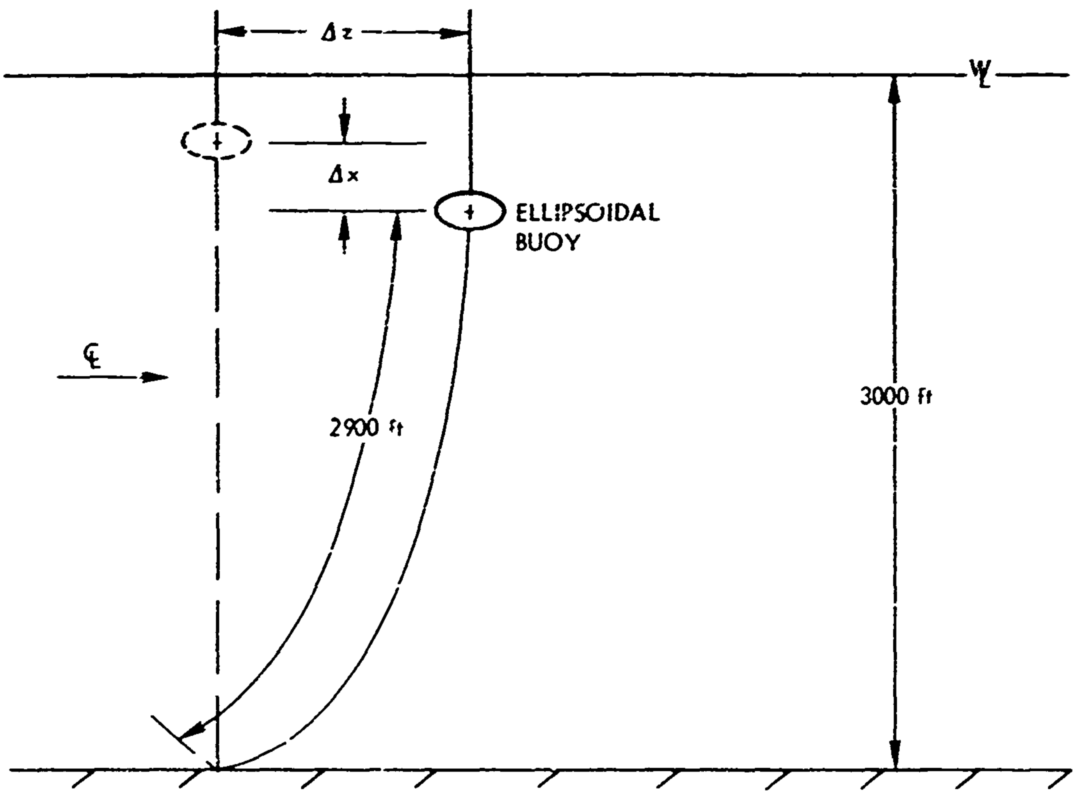

Figure 1. Subsurface Array

The characteristics of the buoy in figure 2 are

a. Excess buoyancy = (1.0 + x) (total cable weight in water), where x is the percentage of extra buoyancy desired.

b. Buoy shape - ellipsoidal and circular in the horizontal plane (a = c) and a = 2b. The buoy is filled with 24 lb/ft^3 syntactic foam.

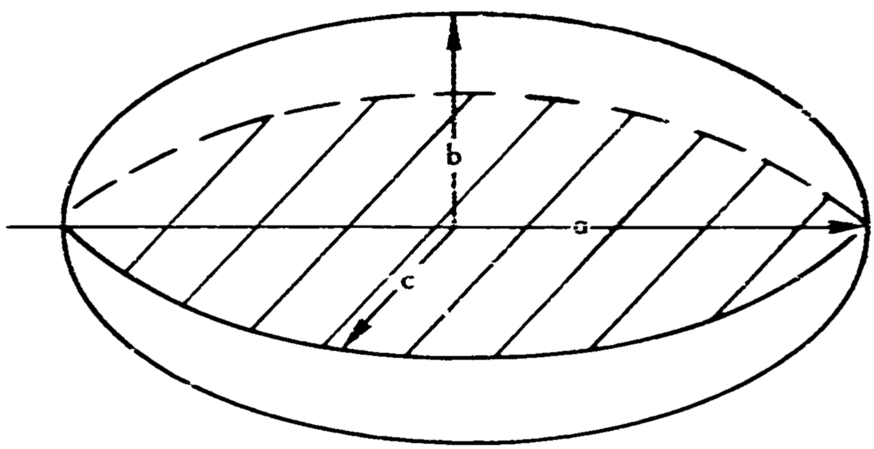

Figure 2. Ellipsoidal Buoy

The cable characteristics are listed in table 1. In all, 27 cases were investigated.

RESULTS

Figure 3 gives total excess buoyancy plotted versus buoy major radius, a, for all cases investigated. For a given cable weight and a percentage of the extra excess buoyancy, one can find the associated buoy dimensions and total excess buoyancy. Figure 4 shows dimensionless vertical excursion of the buoy versus dimensionless drag for constant buoyancy to total cable weight ratio. Figure 5 presents dimensionless horizontal excursion of the buoy versus dimensionless drag for constant buoyancy to total cable weight ratio. In these figures, excursions are the distance the buoy is away from the straight line vertical configuration (i.e., static condition with zero current).

Table 1. Cases Investigated

Cable Diameter (in.)	Weight per ft (lb/ft)	Cable Length (ft)	Excess Buoyancy Above Total Cable Weight (%)	Uniform Current (knots)
1.00	0.3 0.5 0.7	2900.0	20	0.5
1.25	0.3 0.5 0.7	2900.0	20	0.5
1.50	0.3 0.5 0.7	2900.0	20	0.5
1.00	0.3 0.5 0.7	2900.0	30	0.5
1.25	0.3 0.5 0.7	2900.0	30	0.5
1.50	0.3 0.5 0.7	2900.0	30	0.5
1.00	0.3 0.5 0.7	2900.0	40	0.5
1.25	0.3 0.5 0.7	2900.0	40	0.5
1.50	0.3 0.5 0.7	2900.0	40	0.5

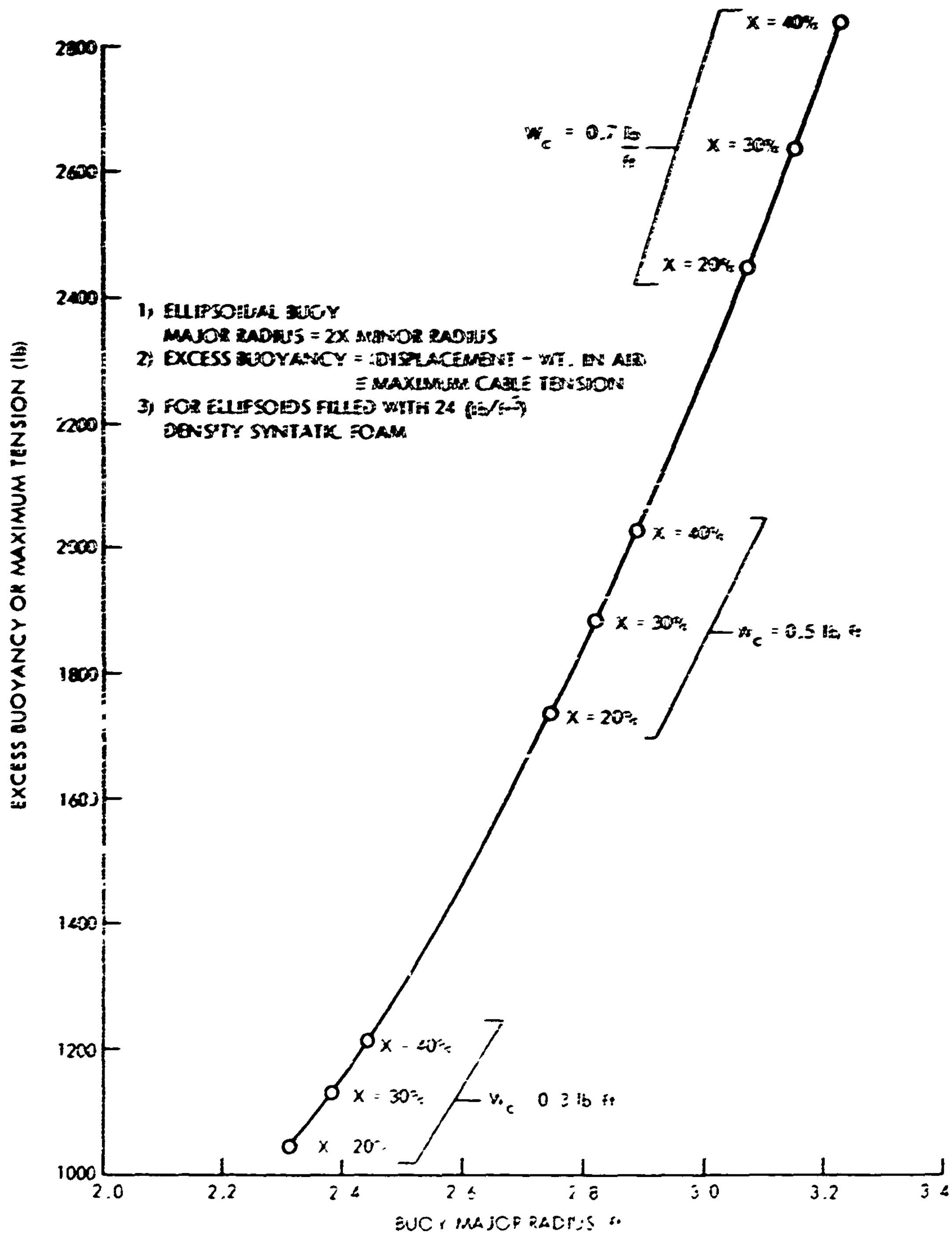

Figure 3. Excess Buoyancy versus Major Radius

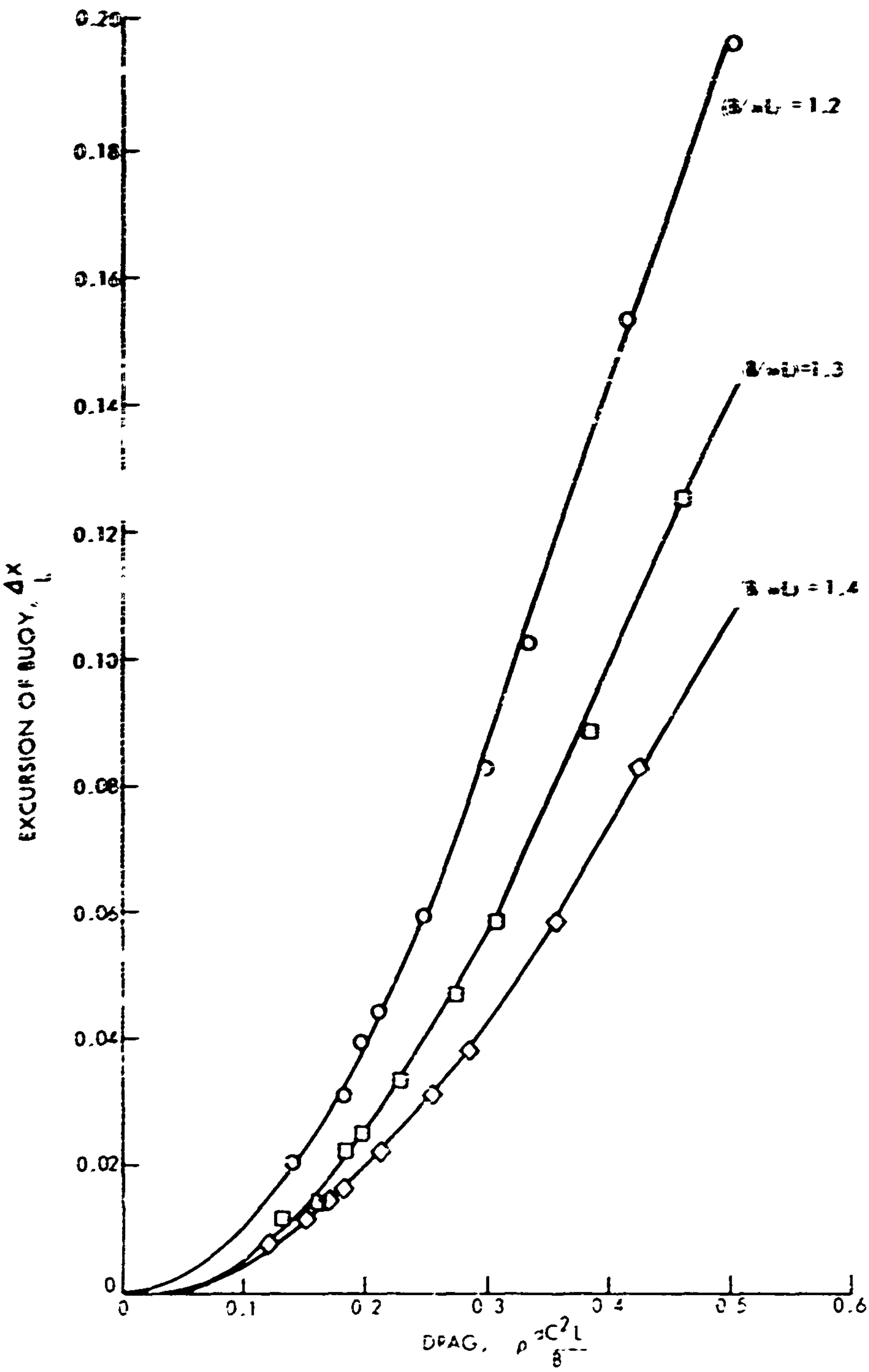

Figure 4. Dimensionless Vertical Excursion versus Dimensionless Drag

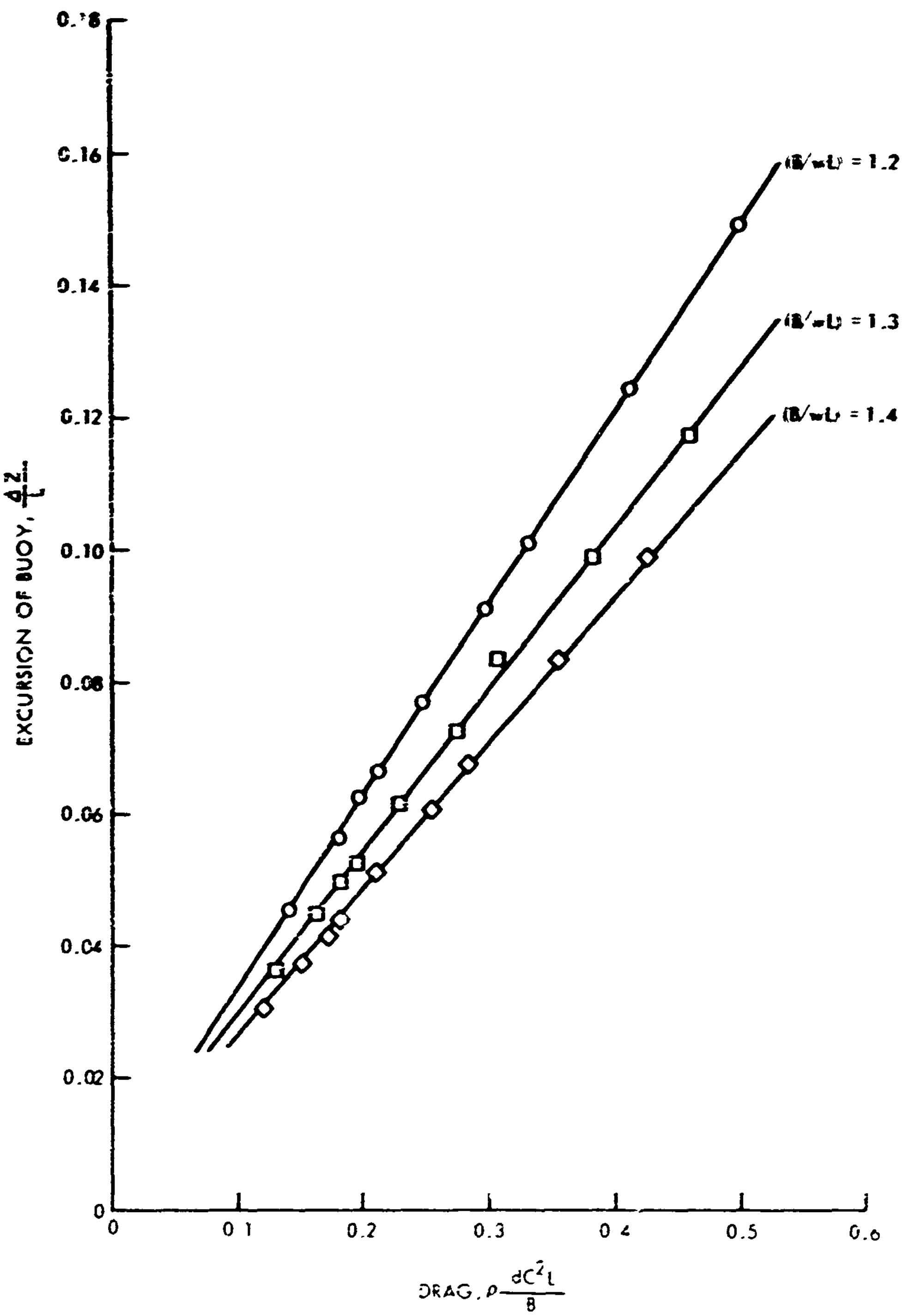

Figure 5. Dimensionless Horizontal Excursion versus Dimensionless Drag

DISCUSSION

Figures 4 and 5 show that the dimensionless parameters discussed previously can be calculated and plotted in a reasonable fashion. Specific application to the 2900-ft-subsurface array is straightforward:

a. Choose a cable diameter, total length, and weight per foot in water, d, L, and w_o.

b. Choose an average uniform current value, C.

c. Choose a buoyancy to total cable weight (in water) ratio, $B/w_o L$.

d. Compute B:

$$B = (w_o L)(1 + x)$$

e. Compute

$$\frac{\rho d C^2 L}{B}$$

f. Go to the curve of B/wL = constant with $\left(\frac{\rho d C^2 L}{B}\right)$ value and find $\frac{\Delta x}{L}$.

Therefore, vertical excursion of the buoy, Δx, is known, and the buoy dimensions can be found from figure 3. Horizontal excursions, Δz, can be found in a similar fashion.

SUMMARY

This report shows that the steady-state cable equations (1) through (4) can be nondimensionalized and meaningful dimensionless coefficients generated. These coefficients can then be applied to steady-state buoy-cable configurations and cable-towed body configurations. The resulting dimensionless curves can be of aid to the designer and user of these systems. The discussions in the appendixes show that more investigation of this subject is needed. Hopefully, more will be done in the future.

Appendix A

METHOD OF ISOCLINES APPLIED TO TWO-DIMENSIONAL CASE

Consider the dimensionless steady-state equations (5) through (7) again.

$$\frac{dT^*}{ds_o^*} = -\frac{1}{2}\rho^* C_{DT} d^* U^* |U^*| \cdot \left(\frac{\rho_o V_o^2 L_o^2}{T_o}\right) + W_c^* \cos\phi \cos\theta \cdot \left(\frac{w_o L_o}{T_o}\right) \qquad (5)$$

$$\frac{d\phi}{ds_o^*} = \left[-\frac{1}{2}\rho^* C_{DN} d^* V^* |V^*| \cdot \left(\frac{\rho_o V_o^2 L_o^2}{T_o}\right) - W_c^* \sin\phi \cos\theta \cdot \left(\frac{w_o L_o}{T_o}\right)\right]\frac{1}{T^*} \cdot \qquad (6)$$

and

$$\frac{d\theta}{ds_o^*} = \left[-\frac{1}{2}\rho^* C_{DN} d^* W^* |W^*| \cdot \left(\frac{\rho_o V_o^2 L_o^2}{T_o}\right) - W_c^* \sin\theta \cdot \left(\frac{w_o L_o}{T_o}\right)\right]\frac{1}{T^* \cos\phi} \cdot \qquad (7)$$

Consider the problem in the x-z plane only. Then ϕ becomes 0, and equations (5) through (7) reduce to

$$\frac{dT^*}{ds_o^*} = -\frac{1}{2}\rho^* C_{DT} d^* U^* |U^*| \left(\frac{\rho_o V_o^2 L_o^2}{T_o}\right) + W_c^* \cos\theta \left(\frac{w_o L_o}{T_o}\right) \qquad \text{(A-1)}$$

and

$$\frac{d\theta}{ds_o^*} = \left[-\frac{1}{2}\rho^* C_{DN} d^* W^* |W^*| \left(\frac{\rho_o V_o^2 L_o^2}{T_o}\right) - W_c^* \sin\theta \left(\frac{w_o L_o}{T_o}\right)\right]\frac{1}{T^*} \qquad \text{(A-2)}$$

Note that the dimensionless velocity components V^* and W^* are related to the dimensionless free stream velocity by

$$V^* = \frac{V}{V_o}\sin\theta$$

$$W^* = \frac{V}{V_o}\cos\theta \ .$$

Substitution into (A-1) and (A-2) gives

$$\frac{dT^*}{ds_o^*} = -\frac{1}{2}\rho^* C_{DT} d^* \frac{V}{V_o}\sin\theta \left|\frac{V}{V_o}\sin\theta\right| \cdot \left(\frac{\rho_o V_o^2 L_o^2}{T_o}\right) \cdot W_c^* \cos\theta \left(\frac{w_o L_o}{T_o}\right) \qquad \text{(A-1a)}$$

and

$$\frac{d\theta}{ds_o^*} = \left[-\frac{1}{2}\rho^* C_{Dx} d^* \frac{V}{V_o}\cos\theta \left|\frac{V}{V_o}\cos\theta\right| \cdot \left(\frac{\rho_o V_o^2 L_o^2}{T_o}\right) - w_c^* \sin\theta \left(\frac{w_o L_o}{T_o}\right)\right]\frac{1}{T^*} \quad \text{(A-2a)}$$

The shape of the solution curves of equations (A-1a) and (A-2a) can be investigated using a method of isoclines.[4] One approach would be to assume a value for each slope (dT^*/ds_o^* and $d\theta/ds_o^*$) and solve the two equations simultaneously for T^* and θ, given the values of the dimensionless parameters and assuming suitable values for ρ^*, d^*, and other variables.

Possibly the following types of curves (figures A-1 and A-2) could be meaningful, subject to the investigation discussed in the previous paragraph.

An investigation into this approach is planned as a next step.

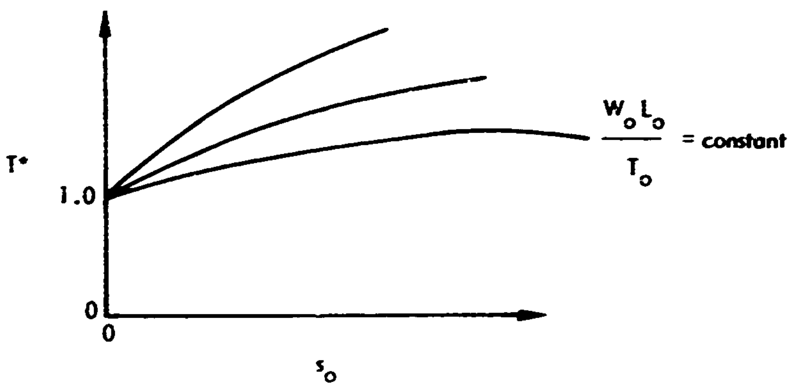

Figure A-1. Dimensionless Tension Solution Curve

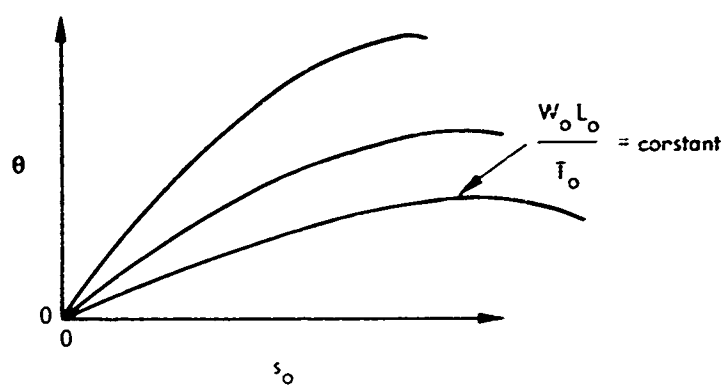

Figure A-2. Dimensionless Angle Solution Curve

Appendix B

TWO SPECIAL APPLICATIONS

An outline of possible meaningful dimensionless parameters as applied to buoy-cable systems and cable-towed buoy systems is presented below.

BUOY-CABLE SYSTEMS

At the buoy for a given cable diameter, the equations are

$$T^* = \frac{T}{T_o} = \frac{T_b}{B} ,$$

where

T_b = tension at the buoy

B = excess buoyancy of the buoy for the zero current condition;

$$\frac{w_o L_o}{T_o} = \frac{w_o L_o}{B} ,$$

where

w_o = weight per unit length of the cable in sea water

L_o = total cable length;

$$\frac{\rho_o V_o^2 L_o^2}{T_o} = \frac{\rho_o V_o^2 L_o^2}{B} ,$$

where

ρ_o = mass density of sea water

V_o = some average current value;

$$\theta = \frac{\Delta z}{L_o} ,$$

where

Δz = radius of buoy watch circle;

$$\phi = \frac{\Delta x}{L_o} ,$$

where

Δx = buoy draft or vertical distance from initial zero current condition.

CABLE-TOWED BODY SYSTEMS

At the ship for a given cable diameter, the equations are as follows:

$$T^* = \frac{T}{T_o} = \frac{T_s}{W_b} ,$$

where

T_s = tension at the ship

W_b = weight of the towed body in sea water;

$$\frac{w_o L_o}{T_o} = \frac{w_o L_o}{W_b} ,$$

where

w_o = weight per unit length of the cable in sea water

L_o = amount of cable paid out from ship;

$$\frac{\rho_o V_o^2 L_o^2}{T_o} = \frac{\rho_o V_o^2 L_o^2}{W_b} ,$$

where

ρ_o = mass density of sea water

V_o = ship speed;

$$\theta = \frac{\Delta z}{L_o} ,$$

where

Δz = distance astern towed body is from fantail;

$$\phi = \frac{\Delta x}{L_o},$$

where

Δx = depth of towed body from ship's fantail.

Note that T_s/W_b is analogous to the often mentioned "depression ratio," and

$$\frac{\dfrac{\Delta z}{L_o} \cdot \dfrac{w_o L_o}{W_b}}{\dfrac{\rho_o V_o^2 L_o^2}{W_b}} = \left(\frac{\Delta z}{L_o}\right) \cdot \left(\frac{w_o}{\rho_o V_o^2 L_o}\right)$$

is analogous to the "normalized body depth."

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