NRL Report 7448

A Fortran Computer Program to Calculate the Range of a Pulse Radar

L. V. BLAKE

Radar Geophysics Branch
Radar Division

August 28, 1972

NAVAL RESEARCH LABORATORY
Washington, D.C.

59

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

Naval Research Laboratory
Washington, D.C. 20390

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3. REPORT TITLE

A FORTRAN COMPUTER PROGRAM TO CALCULATE THE RANGE OF A PULSE RADAR

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
A final report on one phase of the problem; work is continuing on other phases.

5. AUTHOR(S) (First name, middle initial, last name)

L. V. Blake

6. REPORT DATE
August 28, 1972

7a. TOTAL NO. OF PAGES
58

7b. NO. OF REFS
16

8a. CONTRACT OR GRANT NO.
NRL Problems R02-64 and R02-55A

b. PROJECT NO.
Projects PO-0-0038 and SF 11-141-005-15483

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

NRL Report 7448

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
Department of the Navy
(Naval Ship Systems Command)
Washington, D.C. 20360

13. ABSTRACT

A Fortran computer program to calculate radar maximum range, written for the NRL CDC-3800 computer but adaptable to any computer with a Fortran compiler, is described. The computation follows previously established principles, with the pattern-propagation factors set equal to one, so that the range calculated is for free space in the sense that earth's surface effects are not taken into account. However, the effects of a standard atmosphere are included in the calculation. Reflection-interference effects can be separately described by utilizing the calculated free-space range as an input to computer plotting programs.

The program calculates the range for any specified probability of detection, false-alarm probability, and Swerling fluctuation case by utilizing a slightly modified subroutine written by Fehlner and coworkers of the Johns Hopkins University Applied Physics Laboratory. Postdetection (noncoherent) integration is assumed. The system noise temperature is computed including effects of galactic, cosmic blackbody, solar, and tropospheric noise, and the tropospheric molecular absorption for oxygen and water vapor is calculated for a standard atmosphere. The effect of refraction on the ray path is included in the absorption calculation by ray tracing, assuming a negative-exponential refractivity-height profile. The range of validity of the noise temperature and absorption calculations is approximately 100 MHz to 100 GHz. The computation requires a few seconds with the CDC-3800 computer.

A

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6801

Security Classification

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Radar detection						
Radar maximum range						

DD FORM 1 NOV 65 1473 (BACK)
(PAGE 2)

Security Classification

CONTENTS

Abstract ii
Authorization ii
Problem Status ii

INTRODUCTION 1

COMPARISON WITH OTHER PROGRAMS 3

DATA INPUTS 5

PROGRAM OUTPUT 10

EQUATIONS AND ALGORITHMS 10

- Subroutine ALPHA (FMHZ) 17
- Subroutine ATL0SS (FMHZ, ELEV, ATMP 19
- Subroutine TEMP (FMHZ, ELEV, K, ANF, ALA, ALR, ATMP, TA, TR, TE, TEI, TSYS) 20
- Subroutine PDSN (PDT, PFA, NPULS, KASE, SDB) 22
- Subroutine INVERS (XMIN, XMAX, XL0, XHI, NSIG, LIM, N0I, X, FI, FT, F) 22
- Function PD (SNDB) 23
- Subroutine MARSWR (SNDB, N, FA, KASE, PN) 23

FORTRAN PROGRAM LISTING 24

REFERENCES 53

APPENDIX A — A Form for Listing Data Card Inputs to Program RGCALC 54

ABSTRACT

A Fortran computer program to calculate radar maximum range, written for the NRL CDC-3800 computer but adaptable to any computer with a Fortran compiler, is described. The computation follows previously established principles, with the pattern-propagation factors set equal to one, so that the range calculated is for free space in the sense that earth's surface effects are not taken into account However, the effects of a standard atmosphere are included in the calculation. Reflection-interference effects can be separately described by utilizing the calculated free-space range as an input to computer plotting programs.

The program calculates the range for any specified probability of detection, false-alarm probability, and Swerling fluctuation case by utilizing a slightly modified subroutine written by Fehlner and coworkers of the Johns Hopkins University Applied Physics Laboratory. Postdetection (noncoherent) integration is assumed. The system noise temperature is computed including effects of galactic, cosmic blackbody, solar, and tropospheric noise, and the tropospheric molecular absorption for oxygen and water vapor is calculated for a standard atmosphere. The effect of refraction on the ray path is included in the absorption calculation by ray tracing, assuming a negative-exponential refractivity-height profile. The range of validity of the noise temperature and absorption calculations is approximately 100 MHz to 100 GHz. The computation requires a few seconds with the CDC-3800 computer.

AUTHORIZATION

NRL Problems R02-64 and R02-55A
Project PO-0-0038 and SF 11-141-005-15483

PROBLEM STATUS

A final report on one phase of the problem; work is continuing on other phases.

Manuscript submitted June 1, 1972.

A FORTRAN COMPUTER PROGRAM TO CALCULATE THE RANGE OF A PULSE RADAR

INTRODUCTION

The program to be described performs a calculation of the free-space range of a monostatic pulse radar, following the principles presented in NRL Reports 6930 and 7010 (1). The range calculated is "free space" in the sense that the effects of the earth's surface, such as reflection-interference and below-the-horizon shadow, are not taken into account. However, the absorption and noise of a standard atmosphere, galactic noise, and solar noise are taken into account. For detailed definitions of some of the quantities involved in range calculation and for a discussion of the theory, the reader is referred to those reports. In NRL Report 6930, a Range-Calculation Work Sheet was presented to systematize the calculation and thus to simplify handling the rather large number of quantities and computational steps involved. The use of a computer program represents a still further step toward simplification of the calculation, and also minimizes the possibility of error. The computation requires punching one card with the input data (radar parameters and related quantities). To guard against error in punching this card, the program prints out all of the input data as well as the calculated results. The program has been given the Fortran name RGCALC.

Specifically, this program is a Fortran formulation of Eq. (12) of NRL Report 6930 (Eq. (3) of this report), with the pattern-propagation factors omitted. The signal at the target is therefore assumed to be due solely to direct-path propagation, and if the specified transmitting and receiving antenna gains are those of the beam maxima, the target is assumed to be in the beam maxima. As discussed in NRL Report 6930, the equation is based on the assumption that the detection range is limited by the normal system noise — i.e., that there is no interference from manmade signals or noise, and no clutter signals caused by echoes from extraneous targets, such as rough sea or terrain, rain, or any profusion of other targets in the vicinity of the target whose detection is being considered. The range thus calculated may be called the "basic" detection-range capability of the radar.

The Range-Calculation Work Sheet of NRL Report 6930 requires some auxiliary calculations and the use of some sets of curves to determine the visibility factor (minimum-detectable signal-to-noise ratio); the antenna, transmission-line, receiver, and system noise temperatures; and the atmospheric absorption loss. Calculations equivalent to using these curves and auxiliary calculations are performed within Program RGCALC. The only auxiliary calculation required is that of number of pulses on target, for a scanning radar. It was not considered feasible to do this calculation in the computer program because the number of pulses is sometimes determined by a signal processor rather than by the scanning. However, the calculation is not difficult, in the scanning radar case, as will be discussed later in this report. Postdetection (noncoherent) integration of the pulses is assumed.*

---

*See Ref. 1 (NRL Report 6930), p. 18.

The computer program can also be adapted to calculating the range of a CW radar and of a bistatic radar by suitable redefinition of some of the input parameters. For CW radar, the pulse length τ can be interpreted as the effective sampling time of the detection processor in microseconds; or if such a processor is not used, the parameter τ can be interpreted as the reciprocal of the receiver predetection bandwidth in megahertz, or as the length of time a scanning beam remains on the target, whichever time is the shorter. The bandwidth correction factor can be used as a correction for non-optimum processing or filtering of the signal. The "number of pulses integrated" should be set equal to 1. The transmitter power P_t is defined for this calculation as the average transmitted power in kilowatts.

If the radar is bistatic, the only reinterpretation required is in the meaning of the calculated range number. Instead of representing the monostatic range, it represents the square root of the product of the transmitter-to-target range R_t and the target-to-receiver range R_r. In other words, the calculated number is the geometric mean of R_t and R_r. The transmit and receive antenna gains are already specified separately in the program because they are actually different even for some monostatic radars. The target cross section σ must of course be the bistatic value.

The frequency range over which the program may be used is from about 100 MHz to 100 GHz, but it could be extended downward to about 30 MHz and upward to perhaps 150 GHz without incurring gross errors. (Below 30 MHz the occurrence of ionospheric effects and above 150 GHz the multiplicity of water-vapor absorption resonances invalidate the equations used in the program.)

The program is written with the option of calculating range for either specified detection and false-alarm probabilities, or for a specified signal-to-noise power ratio (expressed in decibels). The latter option is useful in calculating the maximum range of a tracking radar (as distinct from a search or acquisition radar) when the minimum signal-to-noise ratio for successful tracking is known.

Except for the fact that the external noise from celestial and terrestrial sources and the absorption that occurs in the earth's atmosphere are taken into account, the free space range of the radar is calculated. Actually it would be more accurate to call this range a quasi-free-space range because of the inclusion of celestial and terrestrial noise and absorption effects. Because the absorption by the troposphere is dependent on the elevation angle of the ray path, the target elevation angle is one of the input quantities for the range calculation. The radar is assumed to be located at or near the earth's surface — within say a thousand feet of sea level. Range calculations applicable to extraterrestrial locations (e.g., satellite or space-ship radars) can be made by setting the elevation angle to a high value, e.g., 90 degrees. At this elevation angle, the absorption is usually negligible for frequencies appreciably below the 22-GHz water-vapor resonance line. Also, the computed absorption, in decibels, is printed out, so that the computed range can be revised to correct for it if a true free-space range is desired. The ray path for the absorption calculation is computed by a ray-tracing algorithm, assuming an exponential decrease of the refractivity with height (CRPL Exponential Atmosphere), with surface refractivity of 313 N units (2).

The principal non-free-space factor not taken into account is the effect of the reflection and absorption by the earth's surface. These effects may modify the free-space range greatly under some conditions, but they cannot be readily taken into account by a single range calculation. To depict the non-free-space performance of a radar, what is needed is a graphical representation of the detection-range contours in a vertical plane, or, for a target at constant altitude, a plot of the variation of signal strength with range, relative to the signal required for detection.

Programs to produce such plots have been written, and have been described in a separate report (3). One of the inputs required for these programs is the free-space range of the radar. Hence, the present program supplements the plotting programs. They could be combined into a single package. However, this has not been done because ordinarily it is not objectionable to calculate the free-space performance and to plot the non-free-space detection curves as two successive operations.

For radars whose antenna pattern is a narrow elevatable beam, no significant reflection-interference effects occur when the beam is elevated by one beamwidth or more, and the quasi-free-space range calculation then applies directly.

The program to be described has been used and extensively tested over a period of time. Results agree with manual calculations using the range-calculation worksheet of NRL Report 6930 (1). The program was used to compute the ranges of actual Navy radars for a forthcoming NRL Radar Division report.[*] The execution time for a single radar range calculation, for all five Swerling fluctuation cases, is approximately 2 sec on the NRL CDC-3800 computer,[†] not including compilation time which is about 100 sec. (Compilation can of course be avoided by having an "object deck" or machine-language deck punched, and using it instead of the Fortran or "source deck.")

COMPARISON WITH OTHER PROGRAMS

Some years ago, a computer program was developed for the calculation of maximum radar rays by a contractor[‡] for the Scientific and Technical Intelligence Center of the Office of Naval Intelligence (ONI-STIC-50). The work was completed about 1966. This program was based on NRL Report 5868, an earlier edition of NRL Report 6930 (1). It utilized some curves published in that report by reading a finite number of data points into the computer and interpolating between them; this was done for the "visibility factor" (minimum-detectable signal-to-noise ratio), the antenna noise temperature, and the atmospheric absorption loss. Because of this the program was limited to calculating the range for 0.5 probability of detection for a nonfluctuating target, and to the frequency range 100 MHz to 10 GHz.

Another computer program has been described by Boothe (4). This program computes the probability of detection as a function of the range, rather than computing

---

[*] This will be the fifth edition of NRL Report 5637, 4th ed., June 21, 1961, "Navy Radar Systems Survey," R.D. Tompkins.

[†] The execution time depends partly on the number of pulses integrated and on whether or not other immediately preceding calculations have been made for the same number of pulses and the same false-alarm probability.

[‡] The contractor was Control Data Corporation, and the programming was done by Mrs. Irma Wachtel. Consulting assistance on the radar-equation aspects of the work was furnished by the author of this report. The project was initiated and monitored by LCDR William Barron of ONI-STIC. The program is described in an internal ONI document STIC-CW-05-1-66 titled "Computer Program R-50, Radar Maximum Range Calculation," Feb. 23, 1967.

the range for a specified probability. However, the maximum range for specified probability is found by computing probability for decreasing range values until the desired probability is reached. The program also makes use of atmospheric absorption curves from NRL Report 5868, entered as data into the computer; consequently it is limited to the frequency range 100 MHz to 10 GHz, plus perhaps a few "spot" frequencies up to 100 GHz. The report does not state the method used for evaluating antenna noise temperature, and a program listing is not given. The effect of target aspect variation on cross section is taken into account deterministically, rather than statistically using Swerling's theory. The signal-to-noise ratio and the resulting probability of detection are calculated at ranges that decrease in steps corresponding to observation of a target approaching the radar. The target is assumed to be changing aspect according to some known prescription, during this approach, and the corresponding cross-section variation is calculated. (A missile target is assumed in Boothe's analysis.) As the target approaches, when the probability reaches the specified value, the range is printed out or otherwise recorded. Either single-scan or cumulative probability can be specified.

D. M. White has described a comprehensive program (5) to analyze radar performance in a dynamic situation, computing signal-to-noise ratio and detection probability as a function of time and target position, taking into account the effects of multipath interference, clutter echoes from the sea or rain, and target motion. In short, the program simulates in as much detail as is practical the complete radar-target engagement, for a single target. This program utilizes a subroutine written by L. F. Fehlner* of the Johns Hopkins Applied Physics Laboratory, and described by Fehlner in a previous report (6), to calculate the probability of detection for a specified signal-to-noise ratio, false-alarm number, number of pulses integrated, and target fluctuation characteristic. Any one of five fluctuation cases can be specified: the nonfluctuating case and the four Swerling fluctuation cases (1). White mentions other programs that have been written by Kirkwood (7) and by Nolen (8).

Killinger (9) has developed a computer program that calculates the ratio of signal to noise-plus-clutter as a function of target range. Probabilities of detection and false alarm are also computed. Maximum detection range can be found for a specified signal to noise-plus-clutter ratio.

The philosophy of the program to be described in this report is somewhat different from those discussed above. It is not intended to simulate the radar performance in a dynamic situation. Instead, it is intended to provide, for a specified target size, fluctuation model, and detection probability, a single number that will serve as an index of the radar's range capability — a "figure of merit." The geophysical environment is taken into account as realistically as possible except that effects of clutter, rain, and multipath interference are omitted. The factors that are believed to be more realistically or accurately calculated than in other programs are the system noise temperature (or more specifically, the tropospheric, solar, and galactic contributions to the antenna temperature) and the tropospheric absorption loss (due to collision-broadened absorption resonances of the oxygen and water-vapor molecules). The antenna noise temperature and the atmospheric attenuation are computed directly rather than by interpolation using data entered from precalculated curves or tables; consequently, the permissible range of frequency is much greater than for most programs using precalculated temperature and absorption data.

---

*Assisted by R.G. Roll and G.T. Trotter.

The program to be described in this report computes the detection range for either a steady (nonfluctuating) target or for any of the four Swerling fluctuation cases (or for all five cases), for a specified probability of detection and a specified false-alarm probability. Fehlner's subroutine (which he named MARCUM) has been incorporated into the NRL program, with slight modifications, for this purpose. The principal modification has been to provide for calculating on the basis of false-alarm probability, rather than Marcum's false-alarm number. Another modification insures that when successive calls are made to the subroutine with the same false-alarm and number-of-pulses parameters, the bias-level calculation is not repeated. This saves an appreciable amount of computing time in the iterative procedure used to determine signal-to-noise ratio for specified probability. (Subroutine MARCUM actually does the inverse problem of computing probability for a specified signal-to-noise ratio.) Because of these and other changes, the subroutine as actually used in the NRL program has been renamed MARSWR (acronym for Marcum-Swerling); but it is basically Fehlner's MARCUM subroutine. The calculation is made assuming a square-law detector, whereas most radar receivers employ a linear-rectifier detector,* but the difference in performance of the two detector types is about 0.2 dB at most, depending on number of pulses integrated.

DATA INPUTS

The inputs to Program RGCALC for a single radar range calculation are punched on a single data card. The format specifications for this card are as follows. Each of the listed quantities is discussed briefly in the following paragraphs.

Data Item	Format Specification	Card Columns
Transmitter power, kW	F6.0	1-6
Pulse length, μsec	F6.0	7-12
Transmit antenna gain, dB	F4.0	13-16
Receive antenna gain, dB	F4.0	17-20
Target cross section, m^2	F6.0	21-26
Frequency, MHz	F6.0	27-32
Antenna ohmic loss, dB	F4.0	33-36
Receiving line loss, dB	F4.0	37-40
Transmit line loss, dB	F4.0	41-44
Antenna pattern scan loss, dB	F4.0	45-48
Miscellaneous loss, dB	F4.0	49-52
Bandwidth factor, dB	F4.0	53-56
Receiver noise factor, dB	F4.0	57-60
Number of pulses	I5	61-65
Probability of detection	F4.0	66-69
False-alarm exponent	F4.0	70-73
Swerling fluctuation case	I1	74
Target elevation angle, deg	F4.0	75-78
Galactic noise code	I2	79-80

---

*See Ref. 1 (NRL Report 6930), p. 29.

The data items of the preceding list denoted "probability of detection" (Cols. 66-69) and "false-alarm exponent" (Cols. 70-73) are actually so defined only if the "case" parameter (Col. 74) is 0, 1, 2, 3, 4, or 5. If the case parameter is 6 or 7, as will be discussed subsequently in more detail, the range is calculated for a specified signal-to-noise ratio in decibels (Case 6) or ratios (Case 7). If one signal-to-noise ratio is to be specified, it goes in card columns 66-69 in place of the probability of detection. The second signal-to-noise ratio goes in Cols. 70-73, in place of the false-alarm exponent.

Data items shown as having an *F* format specification can be entered as a number including a decimal point, with the number positioned anywhere within the card-column field. If an *F*-specification number happens to be an integer, it can also be entered without a decimal point, but in that case it must be right-adjusted within the card-column field. The specification *F*6.0 means a number of 6 characters or less including decimal point and sign, if any (positive sign is implied if no sign is given). The decimal point can be positioned anywhere in the field. Data items having an I format specification must be integers (no decimal point), and must be right-adjusted in the column field.

This single card contains all the numerical data required for a radar range calculation. However, two data cards are required for each radar calculation; the other (first) card contains any alphanumeric material that may be required to identify the radar. This material is punched anywhere in the 80-column field of the data card, and it will be printed out at the top of the page preceding the listing of input-output quantities.

Calculations for any desired number of radars can be made in one computer run by stacking the data cards in the following manner:

Cards 1, 3, 5, 7, . . . Alphanumeric material identifying the radars

Cards 2, 4, 6, 8, . . . Data cards giving numerical parameters corresponding to the preceding identifier cards.

If it is not desired to provide alphanumeric identifying material, blank cards should be inserted at positions 1, 3, 5, 7, . . . of the data deck. The last card of the deck is an end-of-file card. When this is encountered, the job will be terminated.

In the discussion of definitions that follows, a basic principle should be kept in mind. In any radar system, the partitioning of the system into sections called "antenna," "transmission line," "receiver," and "transmitter" is somewhat arbitrary (see NRL Report 6930, p. 47, Fig. 10, and NRL Report 7010, p. 38, Fig. 5). The points in the system at which this arbitrary partitioning is done determines the numerical values of losses, gains, power, and noise temperatures to be assigned to the factors which will subsequently be identified as L_a, L_r, L_t, P_t, G_t, G_r, T_a, T_r, and T_e ($\overline{NF}$). The range calculation will come out the same no matter how this partitioning is done if the assignment of values to all these quantities is consistent with the partitioning selected. Values of losses in decibels are to be entered on the data card as positive numbers.

Transmitter Power. See NRL Report 6930 (1), p. 11. Symbol P_t. This is the pulse power of the radar in kilowatts.

Pulse Length. See NRL Report 6930, p. 11. The symbol τ is used for the duration between 3-dB points of the transmitted RF pulse, in microseconds. If the radar is of the pulse-compression type, the uncompressed pulse length applies, assuming that P_t is the power of the uncompressed pulse. The basic rule is that the product $P_t \tau$ must equal the transmitted pulse energy. (More specifically, the pulse power in kilowatts times 10^3, multiplied by the pulse length in microseconds times 10^{-6}, must equal the transmitted pulse energy in watt-second.)

Antenna Gain. See NRL Report 6930 p. 12. The power gains of the transmitting antenna (G_t) and receiving antenna (G_r) are in decibels. Power gain G is to be distinguished from directive gain D; these quantities are related by $G = kD$, where k is the radiation efficiency ($k \leq 1$). The radiation efficiency is a measure of ohmic or heat loss in the antenna, and should not be confused with aperture efficiency, which measures the relationship between the directive gain actually obtained and that which would have been obtained if the aperture were uniformly illuminated.

Target Cross Section. See NRL Report 6930, p. 13. The symbol σ is used for the radar cross section of the target in square meters. For comparison of the performance of competing systems, the value $\sigma = 1\ \text{m}^2$ is often used.

Frequency. See NRL Report 6930, p. 14. The symbol f_{MHz} is for the radar frequency in megahertz.

Antenna Ohmic Loss. See NRL Report 6930, p. 48. The symbol L_a is for the ohmic loss of the antenna expressed in decibels. Even though this loss is taken into account by the fact that G represents the power gain rather than the directive gain of the antenna, it must also be entered separately because of its contribution to the system noise. (Its inclusion in the power gain accounts only for its effect on the transmitted and received signal powers.) If there are separate transmitting and receiving antennas, L_a refers to the receiving antenna only. This quantity is negligible for many types of antennas, particularly for parabolic reflector types, for which the approximation $L_a = 0$ dB is usually justifiable. Certain types of array antennas, especially those that employ frequency or phase scanning, may have appreciable ohmic loss.

Receiving Line Loss. See NRL Report 6930, p. 70. The symbol L_r is used for the loss of the receiving transmission line in decibels. This loss usually includes duplexer or circulator losses; the prefatory remark concerning partitioning of the receiving system applies.

Transmitting Line Loss. See NRL Report 6930, p. 70. The symbol L_t is for the loss of the transmitting portion of the transmission line in decibels (not usually identical to L_r). Duplexer loss is usually included. The remark concerning partitioning of the system applies.

Antenna-Pattern Scan Loss. See NRL Report 6930, p. 70. Symbol L_p. This loss reflects the facts that (a) the number of pulses integrated for a scanning radar is somewhat arbitrarily taken to be the number occurring while the target is within the half-power beamwidth of the antenna, and (b) the beam does not have full uniform gain within this beamwidth and zero gain elsewhere. For a nonscanning radar aimed directly at a target, $L_p = 0$ dB. For a simple azimuth-scanning radar, $L_p \approx 1.6$ dB. For a simultaneously azimuth- and elevation-scanning radar, $L_p \approx 3.2$ dB is a reasonable assumption, although this result is based on a crude rather than a sophisticated analysis.

Miscellaneous Loss. See NRL Report 6930, p. 82-84. Symbol L_x. Among the possible losses that may be included here are collapsing loss, signal-processing loss, array-fill-time loss, beam-squint loss, polarization-rotation loss, and rain-absorption loss (if the rainstorm extent is less than the radar-to-target range). The decibel value of this loss is obtained by directly adding the decibel values of individual contributing losses.

Bandwidth Factor. See NRL Report 6930, p. 14. Symbol C_B is for the decibel loss resulting from a mismatch, in the North-filter sense, between the pulse characteristics and the receiver filter transfer characteristic. For a simple pulse radar, this relationship can be analyzed adequately in terms of the pulse length and shape, and the filter bandwidth. For most radars of this type it is reasonable to assume $C_B = 0$ dB, in the absence of specific knowledge to the contrary. For pulse-compression radars, there is usually some loss associated with the compression filter, ranging from perhaps 0.5 dB to several decibels, depending on the technique employed and the compression ratio. As is done with the loss factors, C_B is to be entered as a positive decibel number.

Receiver Noise Factor. (Also called receiver noise figure; although "figure" is perhaps the most common usage, IEEE standards give preference to "factor.") See NRL Report 6930, p. 50. Symbol NF or F_n. The receiver noise factor and receiver noise temperature are alternative ways of expressing the same property of the receiver, but the noise factor has been chosen here because it is the quantity more commonly given in receiver specifications. The decibel value of the noise factor is to be entered on the data card.

Number of Pulses. See NRL Report 6930, pp. 71 and 72. Symbol M. If this number is determined by a signal processor, it must be found by reference to the characteristics of the processor. When it is determined by the scanning action of the radar antenna, and if a simple azimuth scan is employed, the appropriate formula is

$$M = \frac{\varphi \cdot PRF}{6 \cdot RPM \cdot \cos \theta_e}, \tag{1}$$

where φ is the azimuthal half-power beamwidth, degrees; PRF is the pulse repetition frequency, hertz; RPM is the rotation rate of the antenna, revolutions per minute; and θ_e is the elevation angle of the target. (The term $\cos \theta_e$ is significant only when a target is at an elevation angle of about 10 degrees or more. For vertical-fan-beam radars the range is usually calculated at an elevation angle low enough so that $\cos \theta_e \approx 1$.)

For simultaneous azimuth- and elevation-scanning radars, assuming a sawtooth-motion elevation scan and a uniform-speed-rotation azimuth scan, the appropriate formula is

$$M = \frac{\varphi \cdot \theta \cdot PRF}{6\, \omega_v \cdot t_v \cdot RPM \cdot \cos \theta_e}, \tag{2}$$

in which φ, PRF, RPM, and θ_e have the same definitions as before, θ is the vertical beamwidth, ω_v is the vertical scanning speed in degrees per second at the target elevation angle, and t_v is the vertical-scan period in seconds, including the dead time if any.

The number of pulses to be used for radars of other scan types must be analyzed on an individual basis, as discussed in NRL Report 6930.

The number of pulses is entered on the data card as an integer (I-format). Consequently the number must be right-adjusted in the field of columns 61 through 65. If the number calculated from the above formulas is not an integer, it should be rounded off to the nearest integer. (The subroutine that calculates detection probability requires an integer for the number of pulses integrated.)

Probability of Detection. See NRL Report 6930, pp. 18 and 19. Symbol P_d. Probability is here given in the mathematical sense of a number between 0 and 1 (not as a percentage figure). Values larger than 0.99 should not be entered because computational difficulties result. Also, values smaller than the false-alarm probability P_{fa} are meaningless; for practical purposes, P_d should be at least an order of magnitude larger than P_{fa}. (Ordinarily it is many orders of magnitude larger.) Typical values of P_d of interest range from about 0.1 to 0.95.

False-Alarm Exponent. See NRL Report 6930, pp. 18-19. Symbol $-\log_{10} P_{fa}$. Typical values of false-alarm probability range from 10^{-4} to 10^{-12}. The number to be entered on the data card is the positive value of the exponent (power of ten). Thus, for $P_{fa} = 10^{-6}$, enter the number 6.0 on the data card; for $P_{fa} = 2.5 \times 10^{-6}$, enter 5.6.

Swerling Fluctuation Case. See NRL Report 6930, p. 28. Five cases are considered, with 0 representing the nonfluctuating target and integers 1 through 4 representing the 4 Swerling cases as defined in NRL Report 6930 (and elsewhere). If the numbers 0, 1, 2, 3, or 4 are punched on the data card in Column 74, the corresponding fluctuation case will be calculated. If 5 is punched, all 5 cases will be calculated.

Further options are provided by a 6 or a 7 punch in Column 74. A 6 punch signifies that the range is to be calculated for a specified signal-to-noise ratio rather than for specified probabilities of detection and false alarm. For this case, the signal-to-noise ratio, in decibels, is punched in Column 66-69, where probability of detection would ordinarily appear. If a 7 is punched in Column 74, the calculation of range will be made for two different signal-to-noise ratios, one given in Columns 66-69, the other in Columns 70-73. When either 6 or 7 is punched in Column 74, the number-of-pulses entry, Columns 61-65, is ignored. Likewise, if 6 is punched, the false-alarm exponent entry, Columns 70-73, is ignored.

Target Elevation Angle. See NRL Report 6930, pp. 48, 68, 69, and 72 through 80. As mentioned in the Introduction, this factor enters into the "quasi-free-space" range calculation because the effect of the earth's atmosphere on the antenna noise temperature and on absorption loss is taken into account. The elevation angle is to be entered in degrees. If a range calculation applicable in empty space is desired, a close approximation can be obtained, except at frequencies near the water vapor and near or above the oxygen resonances (22 and 60 GHz, respectively) by setting the elevation angle to 90 degrees, because for this setting the absorption is virtually negligible. Also, since the calculated absorption is one of the printed-out quantities, correction for it can be made.

Galactic Noise Code. See NRL Report 6930, p. 49, Fig. 11. As shown in the referenced figure, the noise received from the galaxy to which the solar system belongs varies depending on the part of the galaxy toward which the antenna is pointed. This direction is not usually predictable. Therefore the options of calculating the radar range for three choices of galactic and solar noise levels are provided. The codes are −1 for minimum galactic noise, 0 for average noise, and +1 for maximum galactic noise. (The maximum and mini values are shown by dashed lines in the referenced figure of NRL Report 6930.) The number entered is to be right-adjusted in Columns 79-80

PROGRAM OUTPUT

The output of Program RGCALC is a single printed page for each set of data inputs (two data cards). The alphanumeric material of the first data card is printed at the top of the page. Then the numerical input data are printed, both as a record of the data and to ensure that the data card was correctly punched. Next are printed some intermediate output quantities such as the computed noise temperatures of the system components and of the overall receiving system, and the tropospheric absorption for a two-way path through the entire troposphere at the specified elevation angle. Then, if the number punched in Column 74 of the numerical data input card was 5 or less, the calculated range or ranges are printed for the Swerling case or cases specified. Along with each range figure are also given the calculated signal-to-noise ratio in decibels and the tropospheric attenuation for that range.

If the "case" parameter of Column 74 is 6 or 7, the printed output is modified slightly to reflect the fact that the range has been calculated on the basis of an assumed signal-to-noise ratio rather than for specific probabilities of detection and false alarm and a specific fluctuation model.

Figures 1 through 6 are illustrations of the input data cards and resultant printed output for three different "case" options, namely 1, 5, and 7. Cases 0, 2, 3, and 4 produce output results similar to that shown for Case 1, and Case 6 produces an output similar to that of Case 7, except that the range is then calculated for only one signal-to-noise ratio. The radar parameters of these sample calculations are fictitious.

EQUATIONS AND ALGORITHMS

It has been mentioned that Eq. (12) of NRL Report 6930 (1) is the basis of Program RGCALC. The equation is

$$R_{\mathrm{max}} = 129.2 \left[\frac{P_{t(\mathrm{kW})} \tau_{\mu\mathrm{sec}} G_t G_r \sigma \, F_t^{\,2} F_r^{\,2}}{f_{\mathrm{MHz}}^2 \, T_s \, V_o \, C_B L} \right]^{1/4} . \tag{3}$$

The symbols in this equation have been previously defined in this report except for those that follow:

F_t, F_r — pattern-propagation factors for the radar transmitter-to-target and target-to-receiver paths, respectively. In Program RGCALC, $F_t = F_r = 1$.

T_s — the receiving system noise temperature, kelvins

V_0 — the visibility factor, or predetection signal-to-noise ratio required for the specified probability of detection of the target echo

L — the system loss factor, equal to the product of all the component loss factors.* (In decibels, *L* is the sum of the component decibel losses.)

Program RGCALC is an executive routine which reads the data cards and then transmits the data to a subroutine named RANGE. This subroutine calls other subroutines which calculate the system noise temperature T_s, the signal-to-noise ratio V_0, and a table

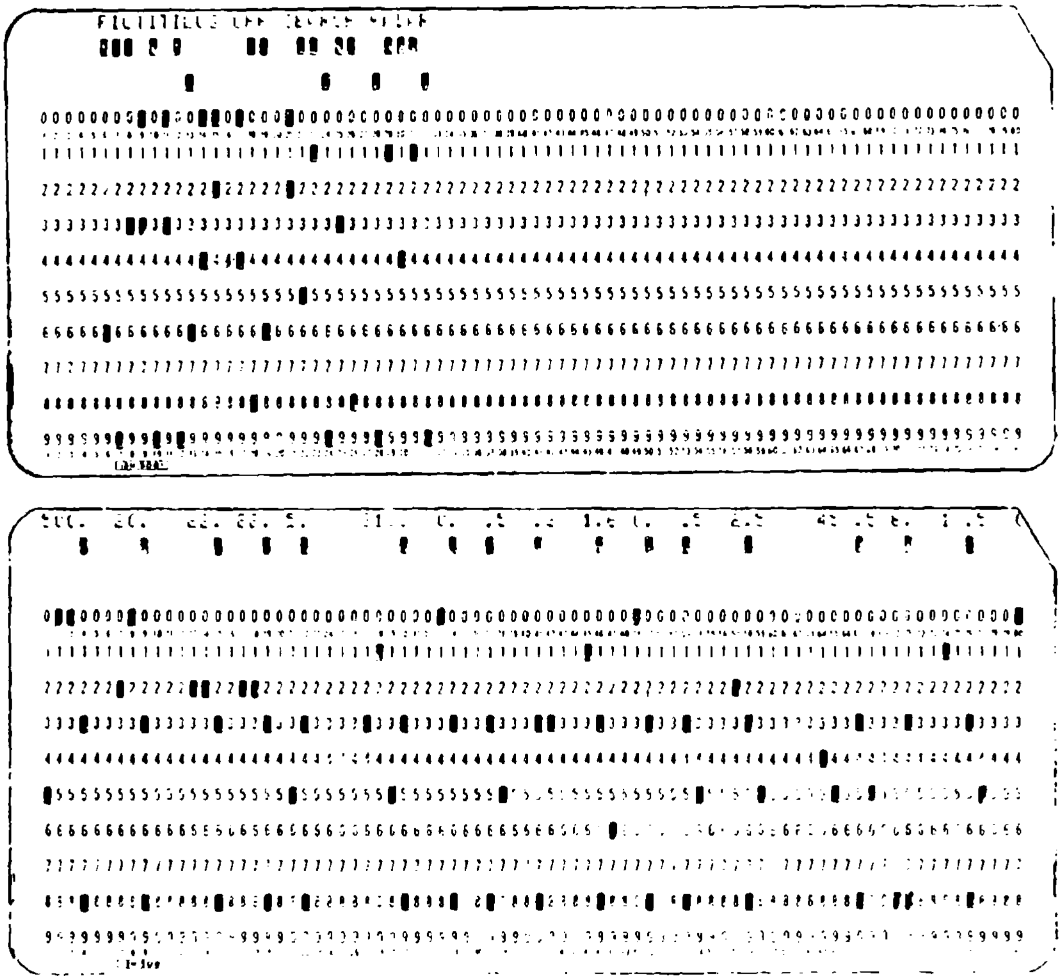

Fig. 1 — Data cards for calculating range of a UHF search radar, Swerling case 1 target fluctuation (1 punch in Col. 74 of second card).

---

*Loss factor is sometimes expressed as a number less than one, in which case it would belong in the numerator of the range equation. Here the correct engineering definition of loss as the reciprocal of gain is employed, so that $L \geq 1$.

```
RADAR NAME OR DESCRIPTION --

FICTITIOUS UHF SEARCH RADAR


RADAR AND TARGET PARAMETERS (INPUTS) --

PULSE POWER, KW .........................................     500.0
PULSE LENGTH, MICROSEC ..................................   20.0000
TRANSMIT ANTENNA GAIN, DB ...............................      22.0
RECEIVE ANTENNA GAIN, DB ................................      22.0
FREQUENCY, MHZ ..........................................     315.0
RECEIVER NOISE FACTOR (FIGURE), DB ......................       2.5
BANDWIDTH CORRECTION FACTOR, DB .........................       0.5
ANTENNA OHMIC LOSS, DB ..................................       0.0
TRANSMIT TRANSMISSION LINE LOSS, DB .....................       0.3
RECEIVE  TRANSMISSION LINE LOSS, DB .....................       0.5
SCANNING-ANTENNA PATTERN LOSS, DB .......................       1.6
MISCELLANEOUS LOSS, DB ..................................       0.0
NUMBER OF PULSES INTEGRATED .............................        45
PROBABILITY OF DETECTION ................................     0.500
FALSE-ALARM PROBABILITY, NEGATIVE POWER OF TEN                  8.0
TARGET CROSS SECTION, SQUARE METERS .....................    5.0000
TARGET ELEVATION ANGLE, DEGREES .........................      0.50
AVERAGE SOLAR AND GALACTIC NOISE ASSUMED
PATTERN-PROPAGATION FACTORS ASSUMED = 1.

     ***********************************

CALCULATED QUANTITIES (OUTPUTS) --

NOISE TEMPERATURES, DEGREES KELVIN --
        ANTENNA (TA) ....................................     251.6
        RECEIVING TRANSMISSION LINE (TR) ................      35.4
        RECEIVER (TE) ...................................     225.7
        TE X LINE-LOSS FACTOR = TEI .....................     253.2
        SYSTEM (TA + TR + TEI) ..........................     540.2
TWO-WAY ATTENUATION THROUGH ENTIRE TROPOSPHERE, DB              1.0


 SWERLING        SIGNAL-         TROPOSPHERIC     RANGE,
 FLUCTUATION     TO-NOISE        ATTENUATION,     NAUTICAL
 CASE            RATIO, DB       DECIBELS         MILES
 -----------     ----------      ------------     --------

      1             1.90            0.97           209.9
```

Fig. 2 — Program RGCALC output for data cards of Fig. 1

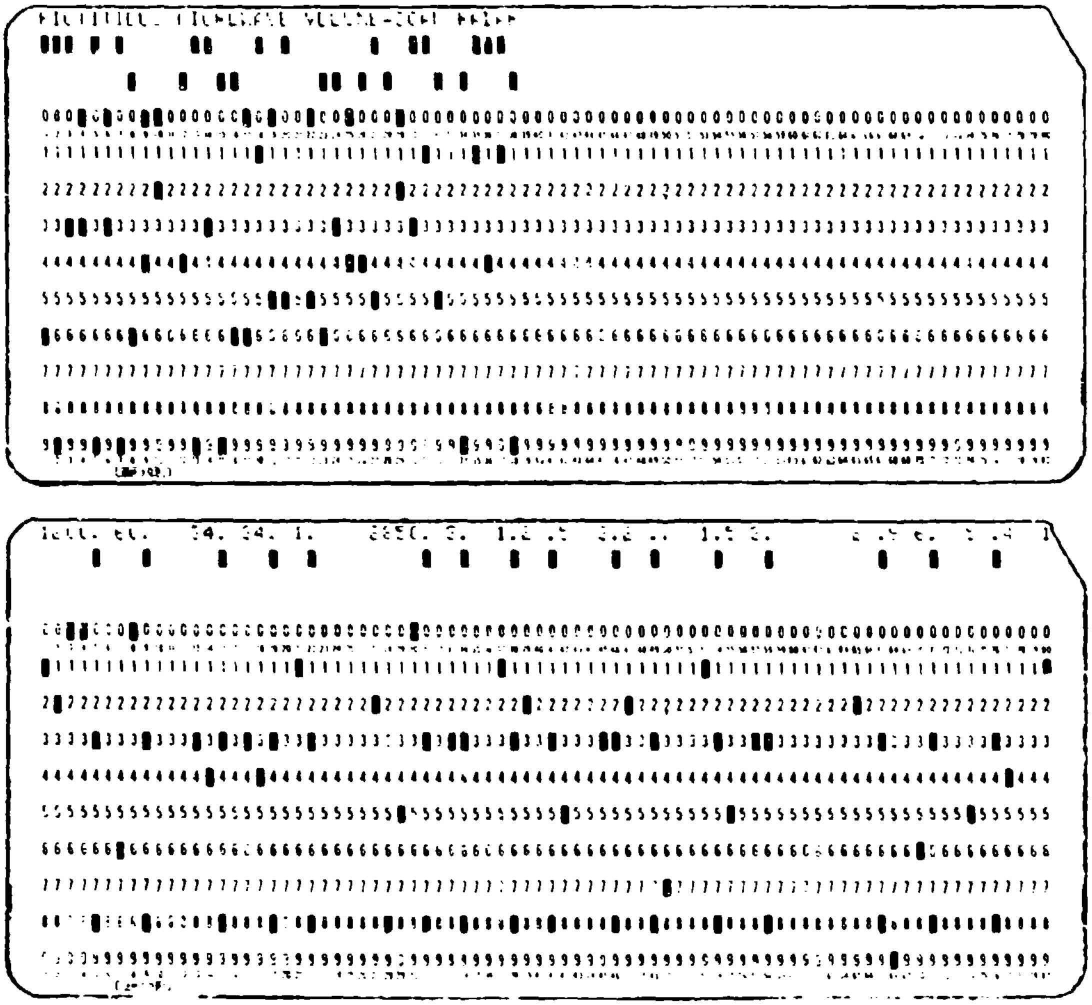

Fig. 3 — Data cards for calculating range of a microwave volume-scanning radar, Swerling cases 0, 1, 2, 3, 4 fluctuation (5 punch in Col. 74 of second card)

(array) of tropospheric absorption losses in decibels at range increments along the (refracted) ray path for the specified elevation angle. Then a system loss factor L, with tropospheric absorption loss omitted, is calculated. The range equation arithmetic is then performed, using the input data and the calculated values of T_s, V_0, and L. Then, in the table of calculated absorption loss values, a value of this loss corresponding to the calculated range is found by interpolation. The range is corrected by a factor corresponding to this loss factor; then the new loss factor corresponding to this corrected range is found, again by interpolation; this iteration is repeated until the last correction corresponds to less than 0.1 dB, in a subroutine named ITERAT.

```
     RADAR NAME OR DESCRIPTION --

FICTITIOUS MICROWAVE VOLUME-SCAN RADAR


     RADAR AND TARGET PARAMETERS (INPUTS) --

     PULSE POWER, KW ......................................     1200.0
     PULSE LENGTH, MICROSEC ...............................    60.0000
     TRANSMIT ANTENNA GAIN, DB ............................       34.0
     RECEIVE ANTENNA GAIN, DB .............................       34.0
     FREQUENCY, MHZ .......................................     2850.0
     RECEIVER NOISE FACTOR (FIGURE), DB ...................        3.0
     BANDWIDTH CORRECTION FACTOR, DB ......................        1.5
     ANTENNA OHMIC LOSS, DB ...............................        3.0
     TRANSMIT TRANSMISSION LINE LOSS, DB ..................        0.5
     RECEIVE  TRANSMISSION LINE LOSS, DB ..................        1.2
     SCANNING-ANTENNA PATTERN LOSS, DB ....................        3.2
     MISCELLANEOUS LOSS, DB ...............................        0.7
     NUMBER OF PULSES INTEGRATED ..........................          2
     PROBABILITY OF DETECTION .............................      0.900
     FALSE-ALARM PROBABILITY, NEGATIVE POWER OF TEN              6.0
     TARGET CROSS SECTION, SQUARE METERS ..................     1.0000
     TARGET ELEVATION ANGLE, DEGREES ......................       0.40
     MAXIMUM SOLAR AND GALACTIC NOISE ASSUMED
     PATTERN-PROPAGATION FACTORS ASSUMED = 1.

             ***********************************

     CALCULATED QUANTITIES (OUTPUTS) --

     NOISE TEMPERATURES, DEGREES KELVIN --
             ANTENNA (TA) .................................      211.1
             RECEIVING TRANSMISSION LINE (TR) .............       92.3
             RECEIVER (TE) ................................      288.6
             TE X LINE-LOSS FACTOR = TEI ..................      380.5
             SYSTEM (TA + TR + TEI) .......................      683.9
     TWO-WAY ATTENUATION THROUGH ENTIRE TROPOSPHERE, DB            3.6


      SWERLING      SIGNAL-       TROPOSPHERIC   RANGE,
      FLUCTUATION   TO-NOISE      ATTENUATION,   NAUTICAL
      CASE          RATIO, DB     DECIBELS       MILES
      ----------    -----------   ------------   --------

          0            10.65           2.88        126.9

          1            18.69           2.17         83.2

          2            14.83           2.52        101.9

          3            14.83           2.52        101.9

          4            12.86           2.69        113.0
```

Fig. 4 – Program RGCALC output for data cards of Fig. 3

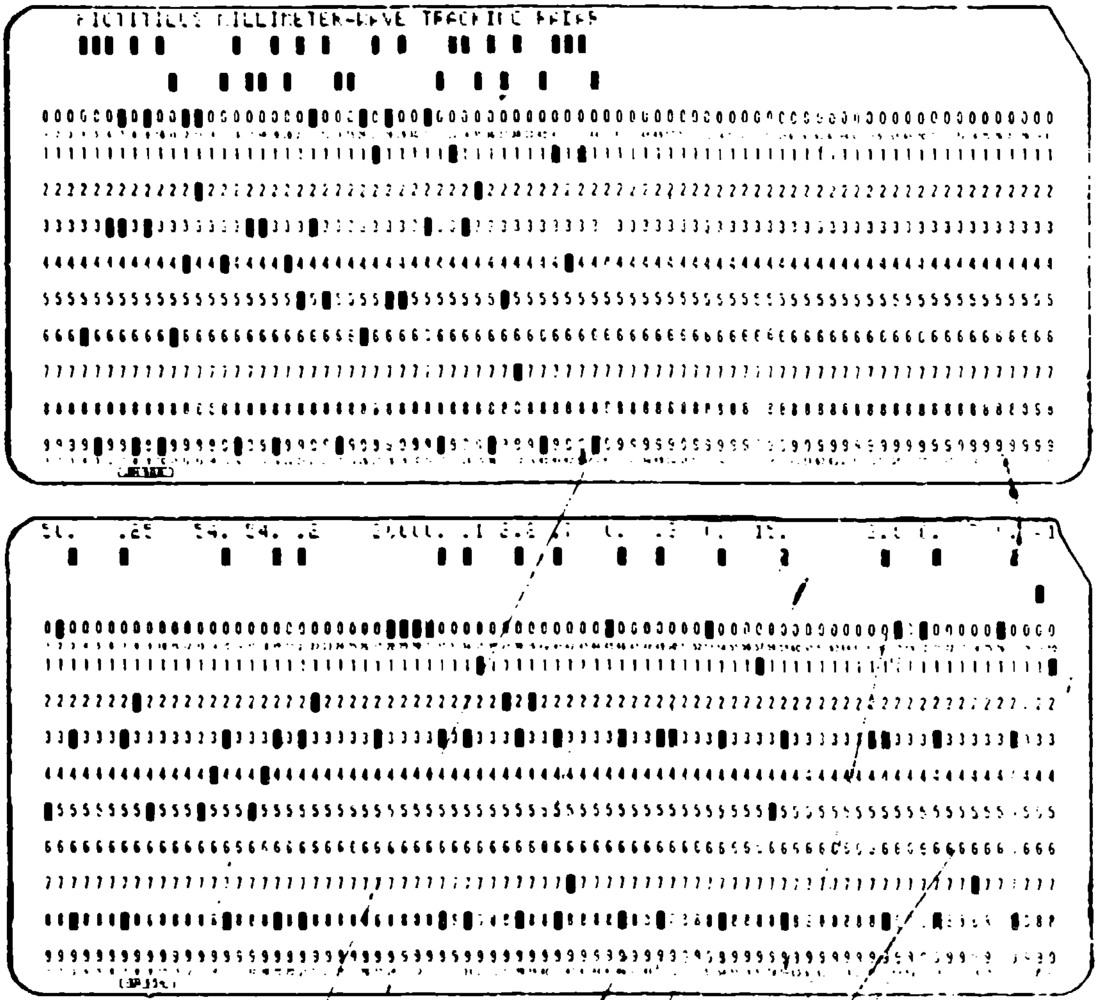

Fig. 5 — Data cards for calculating range of a millimeter-wave tracking radar on a specified signal-to-noise-ratio basis (7 punch in Col. 74 of second card)

```
   RADAR NAME OR DESCRIPTION --

 FICTITIOUS MILLIMETER-WAVE TRACKING RADAR


   RADAR AND TARGET PARAMETERS (INPUTS) --

   PULSE POWER, KW ...................................         50.0
   PULSE LENGTH, MICROSEC ............................       0.2500
   TRANSMIT ANTENNA GAIN, DB .........................         54.0
   RECEIVE ANTENNA GAIN, DB ..........................         54.0
   FREQUENCY, MHZ ....................................      30000.0
   RECEIVER NOISE FACTOR (FIGURE), DB ................         15.0
   BANDWIDTH CORRECTION FACTOR, DB ...................          0.0
   ANTENNA OHMIC LOSS, DB ............................          0.1
   TRANSMIT TRANSMISSION LINE LOSS, DB ...............          0.7
   RECEIVE  TRANSMISSION LINE LOSS, DB ...............          2.2
   SCANNING-ANTENNA PATTERN LOSS, DB .................          0.0
   MISCELLANEOUS LOSS, DB ............................          0.3
   SIGNAL-TO-NOISE RATIO, DB .........................          3.0
   SIGNAL-TO-NOISE RATIO, DB .........................          0.0
   TARGET CROSS SECTION, SQUARE METERS ...............       0.2000
   TARGET ELEVATION ANGLE, DEGREES ...................         0.00
   MINIMUM SOLAR AND GALACTIC NOISE ASSUMED
   PATTERN-PROPAGATION FACTORS ASSUMED = 1.

              ***********************************

   CALCULATED QUANTITIES (OUTPUTS) --

   NOISE TEMPERATURES, DEGREES KELVIN --
              ANTENNA (TA) ...........................        282.6
              RECEIVING TRANSMISSION LINE (TR) .......        191.3
              RECEIVER (TE) ..........................       8880.6
              TE X LINE-LOSS FACTOR = TEI ............      14738.1
              SYSTEM (TA + TR + TEI) .................      15212.0
   TWO-WAY ATTENUATION THROUGH ENTIRE TROPOSPHERE, DB          34.4


   RANGE =       22.1 N. MI.,   TROPOSPHERIC ATTENUATION =    7.27 DB

   FOR SPECIFIED SIGNAL-TO-NOISE RATIO =    3.00 DB


   RANGE =       25.0 N. MI.,   TROPOSPHERIC ATTENUATION =    8.15 DB

   FOR SPECIFIED SIGNAL-TO-NOISE RATIO =    0.00 DB
```

Fig. 6 — Program RGCALC output for data cards of Fig. 5

Figure 7 is a flow chart showing the sequence of events in the computation. As indicated, checks are made at several points which permit certain portions of the calculation to be omitted if more than one range calculation is being made and if results of some steps in the preceding calculation can be used.

In the following sections, the algorithms of the various subroutines of the program will be described. The names in parentheses following the subroutine name are its parameters in the calling sequence.

Subroutine ALPHA (FMHZ)

This subroutine performs the initial step in the computation of tropospheric absorption loss and noise temperature; it computes a set of absorption coefficients in decibels per nautical mile for a set of altitudes above sea level from 0 to 100,000 ft. The first 21 of these altitudes (from 0 to 2000 ft) are at intervals of 100 ft; the next 28, to 30,000 ft, are at intervals of 1000 ft; the next 20, to 70,000 ft, are at intervals of 2000 ft; and the last 6, to 100,000 ft, are at intervals of 5000 ft. This graduation of height intervals reflects the fact that the absorption coefficient changes more rapidly in the lower atmosphere than it does at higher altitudes.

The absorption coefficients are calculated at each of these altitudes for both oxygen and water vapor, and the two coefficients are added to obtain a total absorption coefficient. The resulting array of 75 coefficients, for the frequency FMHZ (first parameter of the calling sequence) is named ALPH (J, 75), J = 3. It is transmitted as output via a COMMON block named PTR.

The computation is done using the theory of Van Vleck as described in NRL Report 7010 (1), except that in the previous version of the subroutine described there, Van Vleck's centroid approximation was used, and it is not valid in the region near the oxygen resonances from about 50 to 70 GHz. The new version of the subroutine, as now used in Program RGCALC, performs a more exact calculation by summing the separate absorption contributions of each of 46 individual oxygen resonance frequencies. Consequently, range calculations can now be made within as well as outside the frequency region 50 to 70 GHz.

The calculations are made for the standard dry atmosphere known as the U.S. Extension to the International Civil Aviation Organization (ICAO) Standard Atmosphere (10). The model for water-vapor content of the atmosphere is based on a humidity profile given by Sissenwine and others (11) as representative of the midlatitude mean humidity. This midlatitude mean, however, has a surface water-vapor content of 5.947 gm/m^3, and it was desired (in accordance with prevailing practice) to adopt the surface water-vapor density value of 7.5 gm/m^3 for the absorption computations. Therefore, the values of the Sissenwine model were all multiplied by the factor 7.5/5.947 = 1.261. The tabulation given in Sissenwine's report is for altitudes at intervals of 2 km in the region of interest here. The values corresponding to the altitudes specified in Subroutine ALPHA were obtained by means of an interpolation technique developed for digital-computer plotting of a smooth curve through a set of data points (12).

The sets of 75 values of pressure, temperature, and water-vapor density values defining this model atmosphere are entered into Subroutine ALPHA in the form of Fortran DATA statements (arrays PP, TT, and RR), thus obviating any necessity of reading them in from

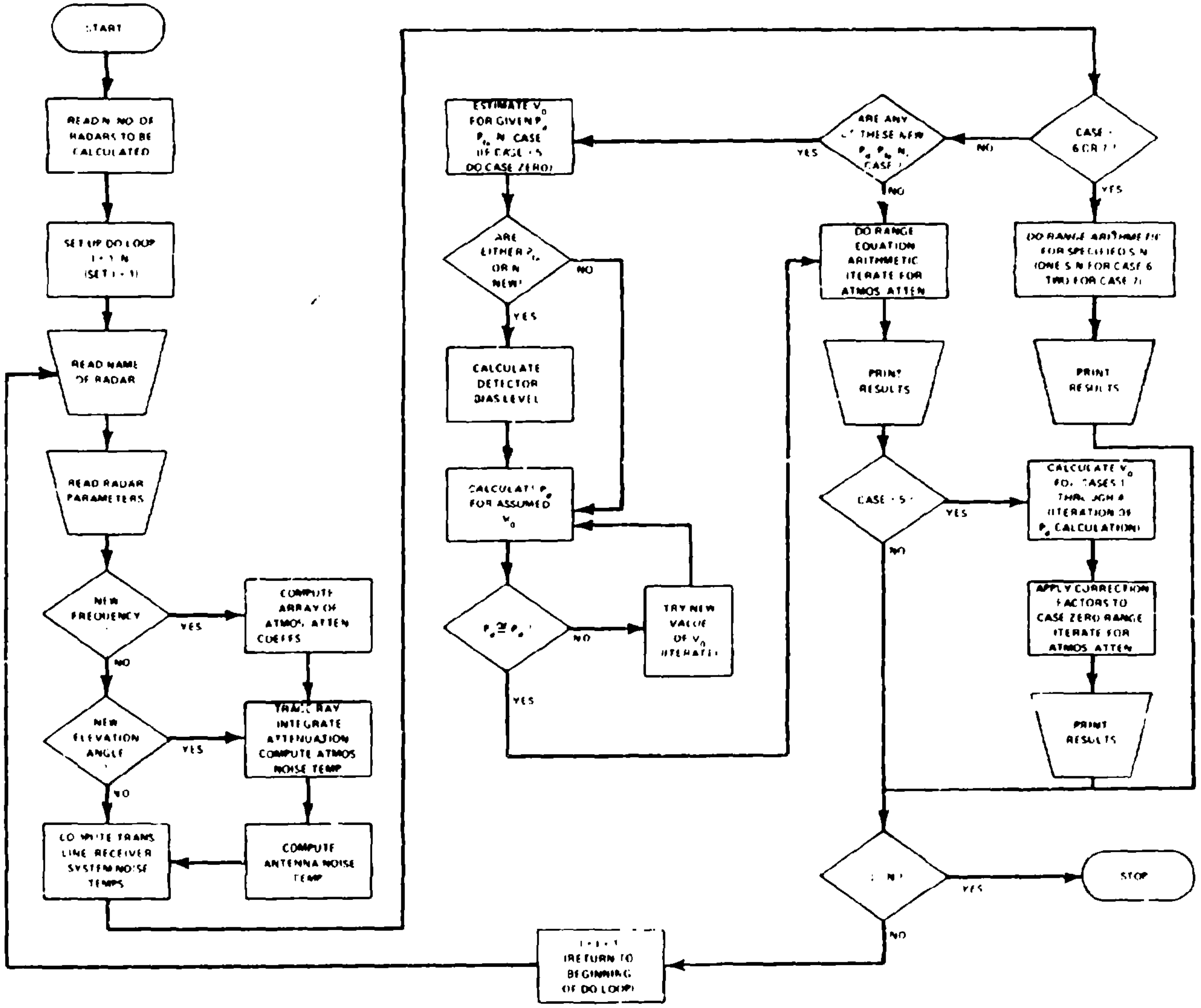

Fig. 7 — Flow chart for Program RGCALC

a separate deck of data cards each time the program is run. The set of 46 oxygen resonance frequencies is similarly entered via DATA statements; these frequencies are separated into two arrays, FTRP and FTRM, corresponding to two classes of quantum-mechanical state transitions of the oxygen molecule. The details of the calculations are described by Meeks and Lilley (13); their formulations of the oxygen absorption equations were employed. (The frequencies FTRP correspond to their symbol ν_{N+}, and FTRM corresponds to ν_{N-}.)

The only deviation from the Meeks and Lilley calculations was the use of a slightly different dependence of line width on altitude. The model of Reber, Mitchell, and Carter (14) was used for this part of the calculation.

Subroutine ATLØSS (FMHZ, ELEV, ATMP)

The input parameters are FMHZ, frequency in megahertz, and ELEV, elevation angle in degrees. The output parameter ATMP is the tropospheric noise temperature computed for the specified frequency and elevation angle. Another output, transmitted via CØMMØN block RGA, is an array of absorption values named ATTN (decibels) corresponding to a set of range values along the ray path at angle ELEV, corresponding to the altitude values of Subroutine ALPHA. The corresponding array of range values RG is similarly transmitted as output.

Subroutine ALPHA is called by Subroutine ATLØSS, and the resulting array of 75 values of absorption coefficients ALPH(J,75), J = 3, is used to calculate cumulative absorption along the ray path at angle ELEV. The ray path in the refracting atmosphere is computed by numerical-integration ray tracing, from the formula (15)

$$R(h_1) = \int_o^{h_1} \left(\frac{dR}{dh}\right) dh \tag{4}$$

in which

$$\frac{dR}{dh} = \frac{n(h)}{\sqrt{1 - \left[\dfrac{n_o \cos \theta_o}{n\,(1+h/r_o)}\right]^2}} . \tag{5}$$

Here R is the radar range corresponding to height h_1 as measured along the ray path of the initial elevation angle θ_o, $n(h)$ is the refractive index height profile, n_o is the value of n at $h = 0$, and r_o is the radius of the earth (more specifically, it is the distance from the earth's center to the initial point of the ray).

The attenuation is then computed along this ray path by numerically integrating the equation

$$A(R_1) = 2 \int_o^{h_1} \gamma(h) \cdot \frac{ds}{dh} \cdot dh, \tag{6}$$

in which $R_1 = R(h_1)$, and $\gamma(h)$ is the absorption coefficient at height h, as given by the array ALPH (J, 75), J = 3. The derivative ds/dh is equal to $[1/n(h)] \cdot dR/dh$; that is, R is the radar range measured along the ray path, and s is the geometric distance along the same path. The derivatives ds/dh and dR/dh are computed in a short subroutine named *DDH(H)*, in which the single parameter is the height H. The derivatives are named *DSDH* and *DRDH* and are transmitted to Subroutine ATLØSS via a CØMMØN block named *DRS*.

The refractive index model used is given by (2)

$$n(h) = 1 + 0.000313\, e^{-kh} \tag{7}$$

where $k = 4.3848 \times 10^{-5}$ if h is in feet. The earth's radius is assumed to be 6370 km, or 2.0899×10^{7} ft.

A special technique is used to perform the integrations in the vicinity of $h = 0$ for the special case $\theta_o = 0$, because in that case dR/dh and ds/dh both become infinite. This technique was described by the author in a paper published in 1968 (15).

The tropospheric noise temperature T_{atm} (Fortran name ATMP) is computed by numerical integration of the equation

$$T_{\text{atm}} = 0.2303 \int_o^{\infty} \gamma(R)\, T_t(R) \exp\left[-0.2303 \int_o^{R} \gamma(r)\, dr\right] dR, \tag{8}$$

where dR is taken along the ray path. T_t is the thermal temperature of the troposphere; its values are transmitted to Subroutine ATLØSS from ALPHA via CØMMØN block TMP.

The previously described modification of Simpson's rule *cannot* be used to perform this integration because it is an integration with respect to R (range) rather than h (height). The h intervals, as described in the section on Subroutine ALPHA, are uniform (over each of the four height regions); however, the corresponding R intervals are not uniform. Another special modification of Simpson's rule was devised to handle this problem; it is embodied in Subroutine INTGRT, which is called by ATLØSS to perform the numerical integration of Eq. (8). Further details of the absorption and noise temperature calculations will be given in a report to be written in the near future, in which curves for absorption and noise temperature as functions of frequency and elevation angle will be presented.

Subroutine TEMP (FMHZ, ELEV, K, ANF, ALA, ALR, ATMP, TA, TR, TE, TEI, TSYS)

The input parameters FMHZ, ELEV, and ATMP are the same as those of Subroutine ATLØSS. The other input parameters are

K — galactic noise code (−1, 0, +1)

ANF — receiver noise factor $\overline{NF}$, dB

ALA — antenna loss factor L_a, dB

ALR — receiving line loss factor L_r, dB.

The output parameters are

TA — antenna noise temperature T_a, kelvins
TR — receiving-transmission-line noise temperature T_r
TE — receiver noise temperature T_e
TEI — product of T_e and L_r
TSYS — system noise temperature T_s.

The antenna noise temperature is computed by use of equations given in NRL Report 6930, p. 49, and Report 7010, pp. 40 through 44 (1). The sky temperature, named TAI, is first computed from the equation

$$T_{sky} = (T_{gal} + T_{bb})/L_{atm} + T_{a(sun)} + T_{atm} \tag{9}$$

in which T_{gal} is the galactic noise temperature, T_{bb} is the cosmic blackbody temperature (2.7 K), L_{atm} is the atmospheric loss factor (expressed as a power ratio ≥ 1), $T_{a(sun)}$ is the solar contribution to the antenna temperature (assuming that the sun is in an average-level sidelobe of the antenna pattern), and T_{atm} is the atmospheric noise temperature (ATMP, obtained from Subroutine ATLØSS). The galactic temperature is given by

$$T_{gal} = T_{100} \cdot (100/f_{MHz})^{2.5}. \tag{10}$$

The quantity T_{100} is the galactic temperature at the frequency f_{MHz} = 100 MHz. Its numerical value depends on the galactic noise code K according to the following prescription.

K	T_{100} (kelvins)
−1	500
0	3050
+1	18,650

The solar contribution to antenna temperature $T_{a(sun)}$ is obtained from a table of values of the solar noise temperature T_{sun} entered via a DATA statement; the table corresponds to frequencies in the range 100 MHz to 10 GHz, and the values are taken from Fig. 6 of Report 7010, p. 43 (1). At frequencies between the tabulated values, T_{sun} is found by linear interpolation. Above 10 GHz, T_{sun} is assumed to have the constant value 10,000 kelvins. The solar temperature T_{sun} is related to $T_{a(sun)}$ by the equation

$$T_{a(sun)} = T_{sun} \times 4.75 \times 10^{-5}/L_{atm}. \tag{11}$$

The numerical factor takes into account the assumed unity-gain average sidelobe level, the ratio of the sun's noise diameter to the total solid angle (4π steradians) viewed by the antenna including its side and back lobes, and the assumption that the sun is on the average ten times noisier than indicated by the referenced curve, which portrays the "quiet sun." Then, $T_{a(sun)}$ is decreased by a factor of 10 if K = −1 and increased by 10 if K = +1, where K is the galactic (and solar) noise code.

The resulting value of sky temperature TAI is multiplied by 0.876 to take into account the fraction of the total antenna pattern subtended by the sky, and to this is added the contribution due to antenna-loss noise, in accordance with Eq. (37) of NRL Report 6930.

The transmission line and receiver noise temperatures T_r and T_e are computed in accordance with Eqs. (40) and (41) of NRL Report 6930, p. 50, and combined to give the system noise temperature by the equation

$$T_s = T'_a + T_r + L_r\, T_e, \tag{12}$$

where L_r is the receiving line loss factor. The product $L_r T_e$ is also reported to Subroutine RANGE as the parameter TEI, and is printed as an intermediate output of the program along with T_a, T_r, T_e, and T_s.

Subroutine PDSN (PDT, PFA, NPULS, KASE, SDB)

The purpose of this subroutine is to find the signal-to-noise ratio required for detection SDB, for the specified probability of detection PDT, false-alarm probability PFA (expressed as a positive number representing the negative power of ten), number of pulses integrated NPULS, and Swerling fluctuation case, KASE. Subroutine PDSN does not perform the calculation; it merely manages it by calling other subroutines. The actual calculation requires an iteration, which it performed by Subroutine INVERS, called by PDSN. Before calling INVERS, PDSN estimates a range of decibel values (lower value DB1, upper value DB2) likely to contain the true value SDB. An empirical formula is used for this purpose. This procedure minimizes the number of iterations required. Subroutine PDSN is called from Subroutine RANGE, and the resultant value of SDB is used as a factor in the range calculation. When the "case" parameter of Subroutine RANGE is 6 or 7 (Col. 74 of the data card), PDSN is not called, since the signal-to-noise ratio is then a direct input and need not be calculated.

Subroutine INVERS (XMIN, XMAX, XL0, XHI, NSIG, LIM, N0I, X, FI, FT, F)

This subroutine performs an iteration to determine the value of the argument X of a function F(X) which will, within a specified accuracy, cause F(X) to equal FT, a specified value of the function. The accuracy parameter is NSIG — the number of significant figures to which agreement is desired between F(X) and FT. LIM specifies a limit on the number of iterations permitted, and N0I (output parameter) reports the number of iterations actually performed. F1 is the actual final value of F. It is required that F be a monotonic function of X within the permissible range of variation of X, which is from XMIN to XMAX, and that the value FT exists within this range. The input parameters XL0 and XHI define a region in which it is guessed that the desired value of X will be found. If no knowledge exists by which to estimate this region, XL0 and XHI can be set equal to XMIN and XMAX; however, the more narrowly the region is defined, the fewer will be the iterations required.

If the slope dF/dX becomes nearly zero in some part of the range from XMIN to XMAX, and if this region is contained between XL0 and XHI, the iteration will converge very slowly, or conceivably not at all. It is for this reason, as mentioned earlier, that values of probability of detection greater than 0.99 should not be specified. Evidently Subroutine MARSWR, which is involved in the iteration, does not define well the slope of the "function" in this region.

Function PD (SNDB)

In order to define a function on which INVERS can operate, the Fortran FUNCTION PD is used, with signal-to-noise ratio (dB) as the argument SNDB. This function merely calls Subroutine MARSWR, which calculates the probability of detection.

Subroutine MARSWR (SNDB, N, FA, KASE, PN)

As has been discussed, this subroutine is basically the subroutine of Fehlner (6) which he named MARCUM. It was renamed MARSWR after a few changes in it were made to adapt it to the requirements of Program RGCALC. The input parameters are SNDB, the signal-to-noise ratio in decibels; N, the number of pulses integrated; FA, the false-alarm exponent; and KASE, the Swerling fluctuation case.

As has been mentioned, Fehlner's original subroutine calculates the probability of detection PN for a specified value of Marcum's false-alarm number rather than on the basis of false-alarm probability. The power-of-ten exponent of the false-alarm number is named FAN in the subroutine. The relation between FAN and the false-alarm probability P_{fa} is

$$\mathrm{FAN} = \log_{10}\left[\frac{\log_e 0.5}{\log_e(1 - P_{fa})}\right]. \tag{13}$$

This relationship is used in Subroutine MARSWR to convert the input parameter FA to the internal parameter FAN.

In using Subroutine MARCUM, it was found that an appreciable portion of the computing time is spent in computing the bias level YB. If succesive calls to MARCUM are made with the same values of N and FA (but with different values of SNDB and KASE), it is not necessary to repeat the bias-level calculation. Therefore a provision for omitting that part of the calculation, when successive calls to MARSWR are made with the same values of N and FA, has been added to the subroutine.

Functions named DGAM, DEVAL, GAM, and SUMLØG are part of the MARCUM subroutine, which was originally written in Fortran II for use with an IBM computer. These functions are also incorporated into MARSWR. The only changes made in them were those necessary to adapt them for use on the NRL CDC-3800 computer. (Some of these adapting changes were made by Stanley Gontarek, of the Naval Air Systems Command.) A further slight change was made in Function SUMLØG; the array named A therein was given a dimension 1000 rather than 200 as in the original MARCUM subroutine. This increased dimensioning saves computing time if successive calls to SUMLØG are made with values of N greater than 200.

FORTRAN PROGRAM LISTING

The Fortran program, subroutines, and functions are listed on the following pages. The names of the listed routines and their computer lengths (number of locations required) are as follows:

Name	Locations Required	
	Octal	Decimal
Program RGCALC	232	154
Subroutine RANGE	2030	1048
Subroutine TEMP	362	242
Subroutine ATLØSS	756	494
Subroutine DDH	112	74
Subroutine ALPHA	520	336
Subroutine INTGRT	211	137
Subroutine ITERAT	220	144
Subroutine PDSN	230	152
Function PD	63	51
Subroutine INVERS	714	460
Subroutine MARSWR	3123	1619
Function DGAM	216	142
Function DEVAL	106	70
Function GAM	206	134
Function EVAL	111	73
Function SUMLØG	4203	2179
Totals	16525	7509

```
                                                     06/20/72
      SUBROUTINE RANGE(PTKW,TAUMS,GTDB,GRDB,SIGSM,FMHZ,ALADB,ALRDB,
     1  ALTDB,ALPDB,ALXDB,CBDB,ANFDB,NPULS,PD,PFA,KASE,ELEV,NOISE)
      COMMON/RGA/ RG(75), ATTN(3,75)
      IF (KASE .EQ. 5) 10,11
   10 IKASE = 0
      GO TO 12
   11 IKASE = KASE
   12 CALL TEMP(FMHZ,ELEV,NOISE,ANFDB,ALADB,ALRDB,ATMP,TA,TR,TE,TEI,
     1   TSYS)
      IF (KASE .GE. 6) 60,61
   60 SNDB=PD
      GO TO 62
   61 CALL PDSN(PD,PFA,NPULS,IKASE,SNDB)
   62 FACDB=GTDB+GRDB-CBDB-ALTDB-ALPDB-ALXDB-SNDB
      FAC=10.**(FACDB*.1)
      RNG0=129.2*(PTKW*TAUMS*SIGSM*FAC/ (FMHZ*FMHZ*TSYS))**.25
      RNGA=RNG0
      CALL ITERAT(RNGA,ATT)
      PRINT 1
      PRINT 100, PTKW
      PRINT 101, TAUMS
      PRINT 102, GTDB
      PRINT 103, GRDB
      PRINT 104, FMHZ
      PRINT 105, ANFDB
      PRINT 106, CBDB
      PRINT 108, ALADB
      PRINT 109, ALTDB
      PRINT 110, ALRDB
      PRINT 111, ALPDB
      PRINT 112, ALXDB
      IF (KASE .GE. 6) 63,64
   63 PRINT 117, SNDB
      IF (KASE .EQ. 7) PRINT 117, PFA
      GO TO 65
   64 PRINT 107, NPULS
      PRINT 113, PD
      PRINT 114, PFA
   65 PRINT 115, SIGSM
      PRINT 116, ELEV
      IF (NOISE) 50,51,52
   50 PRINT 55
      GO TO 53
   51 PRINT 56
      GO TO 53
   52 PRINT 57
   53 PRINT 58
      PRINT 60
      PRINT 2
      PRINT 120
      PRINT 121, TA
      PRINT 122, TR
      PRINT 123, TE
      PRINT 1123, TEI
      PRINT 124, TSYS
      PRINT 125, ATTN(3,75)
```

```
                                                          05/12/72

      PROGRAM RGCALC
      DIMENSION NAME(10)
    2 READ 3, NAME
      IF (EOF,60) 10,11
   10 STOP
   11 PRINT 22
   22 FORMAT(15X,*RADAR NAME OR DESCRIPTION -- */)
      PRINT 30, NAME
      READ 4,     PT,TAU,GT,GR,SIG,FM,ALA,ALR,ALT,ALP,ALX,CB,ANF,NP,PD,
     1   FA,KA,EL,NS
      CALL RANGE(PT,TAU,GT,GR,SIG,FM,ALA,ALR,ALT,ALP,ALX,CB,ANF,NP,PD,
     1   FA,KA,EL,NS)
      PRINT 5
      GO TO 2
    1 FORMAT(6X,I4)
    3 FORMAT(10A8)
   30 FORMAT(10X,10A8//)
    4 FORMAT(2F6.0,2F4.0,2F6.0,7F4.0,I5,2F4.0,I1,F4.0,I2)
    5 FORMAT(1H1)
      END
```

```
      IF (KASE .GE. 6) 66,67
   66 PRINT 166,RNGA,ATT
      PRINT 167, SNDB
      IF (KASE .EQ. 7) 170, 171
  170 SNDB1 = PFA
      DIFF = SNDB-SNDB1
      RNGI = RNGO*10.**(DIFF*.025)
      CALL ITERAT(RNGI,ATT)
      PRINT 166, RNGI, ATT
      PRINT 167, SNDB1
  171 RETURN
   67 PRINT 150
      PRINT 151
      PRINT 152
      PRINT 153
      PRINT 14,IKASE,SNDB,ATT,RNGA
      IF (KASE .EQ. 5) 20,21
   20 DO 30 I=1,4
      CALL PDSN(PD,PFA,NPULS,I,SNDB1)
      DIFF=SNDB-SNDB1
      FAC=10.**(DIFF*.025)
      RNGI=RNGO*FAC
      CALL ITERAT(RNGI,ATT)
      PRINT 14, I,SNDB1,ATT,RNGI
   30 CONTINUE
   21 RETURN
    1 FORMAT (15X*RADAR AND TARGET PARAMETERS (INPUTS) --*/)
  100 FORMAT(15X*PULSE POWER, KW ................................*,F11.1)
  101 FORMAT(15X*PULSE LENGTH, MICROSEC .........................*,F11.4)
  102 FORMAT(15X*TRANSMIT ANTENNA GAIN, DB ......................*,F11.1)
  103 FORMAT(15X*RECEIVE ANTENNA GAIN, DB .......................*,F11.1)
  104 FORMAT(15X*FREQUENCY, MHZ .................................*,F11.1)
  105 FORMAT(15X*RECEIVER NOISE FACTOR (FIGURE), DB .............*,F11.1)
  106 FORMAT(15X*BANDWIDTH CORRECTION FACTOR, DB ................*,F11.1)
  107 FORMAT(15X*NUMBER OF PULSES INTEGRATED ....................*,6X,I5)
  108 FORMAT(15X*ANTENNA OHMIC LOSS, DB .........................*,F11.1)
  109 FORMAT(15X*TRANSMIT TRANSMISSION LINE LOSS, DB ............*,F11.1)
  110 FORMAT(15X*RECEIVE  TRANSMISSION LINE LOSS, DB ............*,F11.1)
  111 FORMAT(15X*SCANNING-ANTENNA PATTERN LOSS, DB ..............*,F11.1)
  112 FORMAT(15X*MISCELLANEOUS LOSS, DB .........................*,F11.1)
  113 FORMAT(15X*PROBABILITY OF DETECTION .......................*,F11.3)
  114 FORMAT(15X*FALSE-ALARM PROBABILITY, NEGATIVE POWER OF TEN *,F11.1)
  115 FORMAT(15X*TARGET CROSS SECTION, SQUARE METERS ............*,F11.4)
  116 FORMAT(15X*TARGET ELEVATION ANGLE, DEGREES ................*,F11.2)
  117 FORMAT(15X,*SIGNAL-TO-NOISE RATIO, DB .....................*F11.1)
   55 FORMAT(15X*MINIMUM SOLAR AND GALACTIC NOISE ASSUMED*)
   56 FORMAT(15X*AVERAGE SOLAR AND GALACTIC NOISE ASSUMED*)
   57 FORMAT(15X*MAXIMUM SOLAR AND GALACTIC NOISE ASSUMED*)
   58 FORMAT(15X*PATTERN-PROPAGATION FACTORS ASSUMED = 1.*)
   60 FORMAT(/22X,35H********************************** /)
    2 FORMAT(15X*CALCULATED QUANTITIES (OUTPUTS) --*/)
  120 FORMAT(15X*NOISE TEMPERATURES, DEGREES KELVIN -- *)
  121 FORMAT(15X*        ANTENNA (TA) ...........................*,F11.1)
  122 FORMAT(15X*        RECEIVING TRANSMISSION LINE (TR) .......*,F11.1)
  123 FORMAT(15X*        RECEIVER (TE) ..........................*,F11.1)
 1123 FORMAT(15X*        TE X LINE-LOSS FACTOR = TEI ............*,F11.1)
```

```
                                                                    06/20/72
124 FORMAT(15X*        SYSTEM (TA + TR + TE) ...................*,F11.1)
125 FORMAT(15X*TWO-WAY ATTENUATION THROUGH ENTIRE TROPOSPHERE, DB *,
   1     F7.1//)
150 FORMAT(15X* SWERLING     SIGNAL-      TROPOSPHERIC    RANGE,*)
151 FORMAT(15X* FLUCTUATION  TO-NOISE     ATTENUATION,    NAUTICAL*)
152 FORMAT(15X* CASE         RATIO, DB    DECIBELS        MILES*)
153 FORMAT(15X* ----------   ----------   -----------     ---------*/)
 14 FORMAT(20X,I1,9X,F6.2, 9X,F6.2,5X,F8.1/)
166 FORMAT(/15X*RANGE = *,F8.1,* N. MI.,  TROPOSPHERIC ATTENUATION = *
   1   ,F6.2,* DB*/)
167 FORMAT(15X,*FOR SPECIFIED SIGNAL-TO-NOISE RATIO = *, F6.2,* DB*//)
    END
```

```
                                                        06/20/72
      SUBROUTINE TEMP (FMHZ,ELEV,K,ANF,ALA,ALR,ATMP,TA,TR,TE,TEI,TSYS)
C     PARAMETER K DETERMINES WHAT NOISE CONDITIONS ARE ASSUMED. K=-1 IS
C     FOR QUIET SUN AND LOWEST GALACTIC NOISE (HIGH GAIN ANTENNA LOOKING
C     IN DIRECTION OF GALACTIC POLE), K=0 IS FOR AVERAGE GALACTIC NOISE
C     (GEOMETRIC MEAN OF LOWEST AND HIGHEST TEMPERATURES), AND SUN NOISE
C     TEN TIMES THE QUIET LEVEL.  K=1 IS FOR MAXIMUM GALACTIC NOISE
C     (GALACTIC CENTER, NARROW BEAM) AND SUN NOISE 100 TIMES QUIET LEVEL.
      DIMENSION T100(3)
      COMMON/RGA/ RG(75), ATTN(3,75)
      DIMENSION FR(8),TS(8)
      DATA(FR=100.,200.,300.,400.,500.,1000.,3000.,10000.),(TS=1.1E6,
     1   1.3E6,1.5E6,1.1E6,1.0E6,2.0E5,3.0E4,1.1E4)
      DATA (T100 = 500.,3050.,18650.)
      DATA (TBLKBY=2.7)
      DATA (FLAST=0.),(ELAST=100.)
      IF (FMHZ.EQ.FLAST .AND. ELEV.EQ.ELAST) GO TO 50
      FLAST=FMHZ
      ELAST=ELEV
      CALL ATLOSS(FMHZ,ELEV,ATMP)
      DO 10 I=2,8
      IF (FMHZ-FR(I))20,30,10
   20 J=I-1
      TSUN = (FMHZ-FR(J))* (TS(I)-TS(J))/ (FR(I)-FR(J)) + TS(J)
      GO TO 40
   30 TSUN = TS(I)
      GO TO 40
   10 CONTINUE
      TSUN=1.0E4
   40 ATT=10.**(-ATTN(3,75)*.05)
   50 TASUN=4.75E-5*TSUN*(10.**K) * ATT
      TA1=(T100(K+2)*(100./FMHZ)**2.5+TBLKBY)*ATT+TASUN*ATMP
      ALAA=10.**(-ALA*.1)
      TA= (.876*TA1 - 254.)*ALAA + 290.
      ALRR=10.**(ALR*.1)
      TR=(ALRR-1.)*290.
      ANFF=10.**(ANF*.1)
      TE=(ANFF-1.)*290.
      TEI=TE*ALRR
      TSYS = TA + TR + TEI
      END
```

```
                                                  06/20/72
      SUBROUTINE ATLOSS(FMHZ,ELEV,ATMP)
      COMMON /PTR/ PP(75), TT(75), RR(75),ALPH(3,75)
      COMMON/RGA/ RG(75),ATTN(3,75)
      DIMENSION I66(4), DELH(4)
      COMMON /RRG/REFO,RAD,GRAD,U
      COMMON/DRS/ DSDH3,DRDH3,AN
      DATA(RG(1)=0.),(ATTN(1,1)=0.),(ATTN(2,1)=0.),(ATTN(3,1)=0.)
      DATA (REFO=.000313),(RAD=20898950.13),(GRAD=.00004385)
      DATA(FLAST=0.),(ELAST=100.),(CONST=.2302585)
      DATA (I66=10,14,10,3),(DELH=100.,1000.,2000.,5000.)
      ATT1(YY)=FAC2*(1.25*Y1+2.*Y2-.25*YY)
      ATT2(YY)=FAC2*(-.25*Y1+2.*Y2+1.25*YY)
      RG1(DR)=FAC1*(1.25*DRDH1+2.*DRDH2-.25*DR)
      RG2(DR)=FAC1*(-.25*DRDH1+2.*DRDH2+1.25*DR)
      IF (FMHZ.EQ.FLAST.AND.ELEV.EQ.ELAST) RETURN
      ELAST = ELEV
      ISIG=1
      THETA=ELEV/57.2957795
      SN=SIN(THETA)
      CS=COS(THETA)
      SS=SN*SN
      RP1=1.+REFO
      U=(RP1*SN)**2 + 2.*REFO + REFO*REFO
      IF (FMHZ.EQ. FLAST) GO TO 55
      CALL ALPHA(FMHZ)
      FLAST=FMHZ
   55 H=0.
      RNG=0.
      ATTEN1=ATTEN2=0.
      K=-1
      HMIN =0.
      IF (ELEV.EQ.0.) HMIN=1.E-9
      CALL DDH(HMIN)
      DRDH1=DRDH3
      DSDH1=DSDH3
      AN1=AN
      TP1=ALPH(3,1)*TT(1)
      TEMP = 0.
      Y1=ALPH(1,1)*DSDH1
      Y11=Y1
      Y12=ALPH(2,1)*DSDH1
      DO 60 J=1,4
      FAC1=DELH(J)/(3.*6076.1155)
      FAC2=2.*FAC1
      IMAX= I66(J)
      DO 61 I=1,IMAX
      K=K+2
      H=H+DELH(J)
      H1=H
      CALL DDH(H)
      DRDH2=DRDH3
      DSDH2=DSDH3
      AN2=AN
      H=H+DELH(J)
      CALL DDH(H)
      Y2=ALPH(1,K+1)*DSDH2
```

```
                                                          06/20/72
      Y21=Y2
      Y22=ALPH(2,K+1)*DSDH2
      Y3=ALPH(1,K+2)*DSDH3
      Y32=ALPH(2,K+2)*DSDH3
      AN3=AN
      IF (ELEV .LT. 1. .AND. H .LT. 201.) 5, 6
    5 CC=CS*CS*(1./RAD-REFO*GRAD/RP1)
      CC1=1./(CC*6076.1155)
      PROD=2.*CC*H1
C     FOLLOWING IS APPROXIMATION REQUIRED NEAR THETA=0 AND H=0 FOR RANGE
C     CALCULATION.  RANGE IS CALCULATED THUS FOR H = 100 AND H= 200 WHEN
C     ELEVATION ANGLE IS LESS THAN 1 DEGREE.
      RNG=CC1*PROD/(SQRTF(PROD+SS)+SN)
      DS1=RNG
C     APPROXIMATE ATTENUATION IS RANGE (TWO-WAY) TIMES AVERAGE VALUE OF
C     GAMMA IN THE RANGE INTERVAL.
      ATTEN1=RNG*(ALPH(1,1) + ALPH(1,2))
      ATTEN2=RNG*(ALPH(2,1) + ALPH(2,2))
C     RADAR RANGE IS GEOMETRIC RANGE TIMES AVERAGE REFRACTIVE INDEX.
      RNG=RNG*(RP1 + 1. + REFO*EXP(-GRAD*H1)) * .5
      ISIG=2
      GO TO 7
    6 DS=RG1(DRDH3)
      RNG=RNG+DS
      DS1= DS/((AN1+AN2)*.5)
      ATTEN1=ATTEN1+ATT1(Y3)
      Y1=Y12
      Y2=Y22
      ATTEN2=ATTEN2 + ATT1(Y32)
    7 RG(K+1)=RNG
      ATTN(1,K+1)= ATTEN1
      ATTN(2,K+1)=ATTEN2
      ATTN(3,K+1)=ATTEN1 + ATTEN2
      GO TO (10,11) ISIG
   11 PROD=2.*CC*H
      RNG=CC1*PROD/(SQRTF(PROD+SS)+SN)
      DS2=RNG-DS1
      ATTEN1=RNG*(ALPH(1,1) + ALPH(1,3))
      ATTEN2=RNG*(ALPH(2,1) + ALPH(2,3))
      RNG=RNG*(RP1 + 1. + REFO*EXP(-GRAD*H )) * .5
      ISIG=1
      GO TO 12
   10 DS=RG2(DRDH3)
      RNG=RNG+DS
      DS2=DS/((AN2+AN3)*.5)
      Y1=Y11
      Y2=Y21
      ATTEN1= ATTEN1 + ATT2(Y3)
      Y1=Y12
      Y2=Y22
      ATTEN2= ATTEN2 + ATT2(Y32)
   12 RG(K+2)=RNG
      ATTN(1,K+2)=ATTEN1
      ATTN(2,K+2)=ATTEN2
      ATTN(3,K+2)=ATTEN1 + ATTEN2
      ALOSS=10.**(-ATTN(3,K+2)*.05)
```

```
                                                                      06/20/72
      TP3=ALPH(3,K+2)*TT(K+2)*ALOSS
      DS1 = (RG(K+1)-RG(K))/((AN1+AN2)*.5)
      DS2 = (RG(K+2)-RG(K+1))/((AN2+AN3)*.5)
      IF (K.EQ.1) 70,71
C     APPROXIMATION EMPLOYED IN PLACE OF FIRST INTEGRATION STEP,
C     IFIED TO GIVE VALID RESULTS IN HIGH-ATTENUATION CASES.  ANALYTIC
C     APPROXIMATION STARTS AT STATEMENT 70.
   70 CEX=0.5*CONST*(ALPH(3,1)+ALPH(3,2))
      CEY=0.5*CONST*(ALPH(3,2)+ALPH(3,3))
      ALOS1=EXPF(-CEX*DS1)
      ALOS2=EXPF(-CEY*DS1)
      ALOS3=EXPF(-CEY*(DS1+DS2))
      DTEMP=(0.5/CONST)*((TT(1)+TT(2))*(1.-ALOS1) + (TT(2)+TT(3))*(ALOS2
     1   -ALOS3))
      GO TO 72
   71 ALOSS=10.**(-ATTN(3,K+1)*.05)
      TP2=ALPH(3,K+1)*TT(K+1)*ALOSS
      CALL INTGRT(DS1,DS2,TP1,TP2,TP3,DTEMP)
   72 TEMP = TEMP+DTEMP
      DRDH1=DRDH3
      TP1 = TP3
      Y1=Y11=Y3
      Y12=Y32
      AN1 = AN3
   61 CONTINUE
   60 CONTINUE
      ATMP=TEMP*CONST
      END
```

06/20/72

```
SUBROUTINE DDH(H)
COMMON /RRG/REF0,RAD,GRAD,U
COMMON/DRS/ DSDH3,DRDH3,AN
EX=REF0*EXP(-GRAD*H)
AN=1.+EX
V=EX*(2.+EX)
W1=H/RAD
W=W1*(2.+W1)
DSDH3=AN*(1.+W1)/SQRTF(U*V+W+V*W)
DRDH3=AN*DSDH3
END
```

```
                                                              06/29/72

 SUBROUTINE ALPHA(FMHZ)
 COMMON /PTR/ PP(75), TT(75), RR(75),ALPH(3,75)
 COMMON/H2O/HHEFAC
 DIMENSION FTRP(23),FTRM(23),DELH(4),IMAX(4)
 DATA(DELH=100,,1000,,2000,,5000,),(IMAX=21,28,20,6)
 DATA(AL2=,1),(AL3=,27),(H1=26000,),(H2=82000,),(PHR=,5)
 DATA(CSTH2O= 4,693  E-3),(FRH2O=22,235),(DELZEPO=17,99)
 DATA(CONST=3,7148),(CONST2=0,2960E),(CVP=,750064),(SMXI=,20846)
 DATA(FTRP=56,2648,58,4466,59,5910,60,4348,61,1506,61,8002,62,4112,
162,9980,63,5685,64,1272,64,6779,65,2240,65,7626,66,2978,66,8313,
267,3627,67,8923,68,4205,68,9478,69,4741,70,0000,70,5249,71,0497)
 DATA(FTRM=118,7505,62,4863,60,3061,59,1642,58,3239,57,6125,56,9082
1,56,3634,55,7839,55,2214,54,6728,54,1294,53,5960,53,0695,52,5458,
252,0259,51,5091,50,9949,50,4830,49,9730,49,4648,48,9582,48,4530)
 DATA (PP=
11.01325E+3,1.00959E+3,1.00595E+3,1.00231E+3,9,98689E+2,9,95076E+2,
19,91473E+2,9,87880E+2,9,84299E+2,9,80728E+2,9,77167E+2,9,73617E+2,
19,70077E+2,9,66548E+2,9,63029E+2,9,59521E+2,9,56023E+2,9,52536E+2,
19,49058E+2,9,45591E+2,9,42135E+2,9,08130E+2,8,75129E+2,8,43109E+2,
18,12047E+2,7,81921E+2,7,52710E+2,7,24391E+2,6,96943E+2,6,70347E+2,
16,44581E+2,6,19625E+2,5,95459E+2,5,72065E+2,5,49422E+2,5,27513E+2,
15,06319E+2,4,85822E+2,4,66003E+2,4,46846E+2,4,28334E+2,4,10449E+2,
13,93176E+2,3,76497E+2,3,60398E+2,3,44862E+2,3,29874E+2,3,15420E+2,
13,01484E+2,2,75110E+2,2,50441E+2,2,27969E+2,2,07148E+2,1,88230E+2,
11,71043E+2,1,55428E+2,1,41241E+2,1,28352E+2,1,16641E+2,1,05000E+2,
19,63319E+1,8,75472E+1,7,95650E+1,7,23112E+1,6,57212E+1,5,97323E+1,
15,42901E+1,4,93447E+1,4,48506E+1,3,53282E+1,2,78707E+1,2,19421E+1,
11,73765E+1,1,38270E+1,1,10533E+1)
 DATA (TT=
12,88160E+2,2,87962E+2,2,87764E+2,2,87566E+2,2,87368E+2,2,87169E+2,
12,86971E+2,2,86773E+2,2,86575E+2,2,86377E+2,2,86179E+2,2,85981E+2,
12,85783E+2,2,85585E+2,2,85387E+2,2,85188E+2,2,84990E+2,2,84792E+2,
12,84594E+2,2,84396E+2,2,84198E+2,2,82217E+2,2,80237E+2,2,78256E+2,
12,76276E+2,2,74296E+2,2,72316E+2,2,70337E+2,2,68357E+2,2,66378E+2,
12,64399E+2,2,62420E+2,2,60442E+2,2,58463E+2,2,56485E+2,2,54507E+2,
12,52529E+2,2,50551E+2,2,48574E+2,2,46597E+2,2,44620E+2,2,42643E+2,
12,40666E+2,2,38689E+2,2,36713E+2,2,34737E+2,2,32761E+2,2,30785E+2,
12,28809E+2,2,24859E+2,2,20909E+2,2,16960E+2,2,16660E+2,2,16660E+2,
12,16660E+2,2,16660E+2,2,16660E+2,2,16660E+2,2,16660E+2,2,16660E+2,
12,16660E+2,2,16660E+2,2,16660E+2,2,16660E+2,2,16660E+2,2,16660E+2,
12,16660E+2,2,16660E+2,2,16660E+2,2,16660E+2,2,16660E+2,2,19069E+2,
12,23602E+2,2,28134E+2,2,32664E+2)
 DATA(RR= 7,50000E  0, 7,43586E  0, 7,37179E  0, 7,30779E  0,
1 7,24387E  0, 7,18004E  0, 7,11630E  0, 7,05267E  0, 6,98915E  0,
1 6,92575E  0, 6,86247E  0, 6,79933E  0, 6,73633E  0, 6,67347E  0,
1 6,61077E  0, 6,54822E  0, 6,48585E  0, 6,42364E  0, 6,36162E  0,
1 6,29976E  0, 6,23814E  0, 5,63358E  0, 5,05440E  0, 4,50515E  0,
1 3,98936E  0, 3,50702E  0, 3,04673E  0, 2,62039E  0, 2,23620E  0,
1 1,90459E  0, 1,61971E  0, 1,38092E  0, 1,18128E  0, 1,01091E  0,
1 8,64635E -1, 7,38447E -1, 6,29176E -1, 5,34311E -1, 4,52105E -1,
1 3,83598E -1, 3,23824E -1, 2,75746E -1, 2,31430E -1, 1,91717E -1,
1 1,56023E -1, 1,23607E -1, 9,46502E -2, 7,08040E -2, 5,23315E -2,
1 2,88489E -2, 1,70220E -2, 1,03901E -2, 6,45199E -3, 4,01284E -3,
1 2,38249E -3, 1,46690E -3, 1,05299E -3, 9,02707E -4, 6,40642E -4,
1 7,91078E -4, 7,14460E -4, 6,37648E -4, 5,79962E -4, 5.51107E -4,
```

06/29/72

```
     1 5.44362E -4, 5.50855E -4, 5.64024E -4, 5.87972E -4, 6.21432E -4,
     1 7.06726E -4, 8.13642E -4, 9.09155E -4, 7.32574E -4, 5.65102E -4,
     1 4.40224E -4)
      DATA(RHOFAC=1.)
      FGHZ=FMHZ*1.E-3
      FGHZ2=FGHZ*FGHZ
      FRATIO=FGHZ/FRH20
      FSUM2=(FGHZ+FRH20)**2
      FDIF2=(FGHZ-FRH20)**2
      I=0
      H=-100.
      DO 100 J=1,4
      IM=IMAX(J)
      DO 100 K=1,IM
      I=I+1
      RO    =RR(I) * RHOFAC
      PPW=RO    *TT(I)/288.75
      PR=PP(I) + PPW/CVP
      TSQ=TT(I)**2
      H=H+DELH(J)
      IF (H.LE.H1)  10,11
  10  AL1=.64
      GO TO 15
  11  IF (H .GT. H2) 12,13
  12  AL1=1.357
      GO TO 15
  13  AL1=.64 + .717*(H-H1)/54000.
  15  HAL1=CONST2*AL1*PP(I)/TT(I)
      HSA1=HAL1*HAL1
      FO=HAL1/(FGHZ2+HSA1)
      SUM=0.
      DO 50 M=1,23
      AN=2.*M-1.
      AN1=AN+1.
      UNP2=AN*(2.*AN+3.)/AN1
      UNM2=AN1*(2.*AN-1.)/AN
      UN02=2.*(AN*AN+AN+1.)*(2.*AN+1.)/(AN*AN1)
      FNP=HAL1/((FTRP(M)-FGHZ )**2+HSA1)+HAL1/((FTRP(M)+FGHZ)**2+HSA1)
      FNM=HAL1/((FTRM(M)-FGHZ )**2+HSA1)+HAL1/((FTRM(M)+FGHZ)**2+HSA1)
      TERM=(FNP*UNP2+FNM*UNM2+FO*UN02)*EXPF(-2.06844*AN*AN1/TT(I))
      SUM=SUM+TERM
  50  CONTINUE
      ALPHO2 = CONST*(PR    /TT(I)**3)*FGHZ2*SUM
      ALPH(1,I)=ALPHO2
      TRO=300./TT(I)
      DELF=DELZERO*(PPW*TRO+ PP(I)*CVP      *SMYI*TRO**.63)*1.E-3
      DELF2=DELF*DELF
      FFR2=FRATIO*DELF*(1./(FSUM2+DELF2)+1./(FDIF2+DELF2))
      ALPH22=CSTH2O*FGHZ*PPW*TRO**3.5*EXPF(2.144*(1.-TRO))*FFR2
      ALFRES=1.361E-2 * FGHZ2 * RO * PR * TT(I)**(-2.5)
      ALPH(2,I) = ALPH22 + ALFRES
C     ALPH(1,I) IS ABS. COEFF. FOR OXYGEN, ALPH(2,I) WATER VAPOR.
C     ALPH(3,I) IS TOTAL ABSORPTION COEFFICIENT
      ALPH(3,I)= ALPH(1,I) + ALPH(2,I)
 100  CONTINUE
      END
```

```
                                                                06/20/72
 SUBROUTINE INTGRT (H1,H2,Y1,Y2,Y3,AREA)
 H12=H1*H1
 H22=H2*H2
 HH=H1*H2
 HPH=H1+H2
 AFAC=(Y1*H2 + Y3*H1-Y2*HPH)/(HH*HPH)
 AREA = (AFAC/3.)*(H22*H2 + H12*H1) + (Y3-Y2-AFAC*H22)*(H22-H12)/
1   (2.*H2)  + Y2*HPH
 END
```

```
                                                                  06/20/72
      SUBRØUTINE ITERAT(RMX,ATT)
C     GIVEN RADAR MAXIMUM RANGE IN NØNABSØRBING SPACE, RMX, THE
C     SUBRØUTINE FINDS THE ATMØSPHERIC ATTENUATIØN ATT, FRØM A TABLE
C     SUPPLIED BY ANØTHER SUBRØUTINE VIA THE CØMMØN STATEMENT,
C     IT THEN CØRRECTS THE RANGE RMX BY A FACTØR BASED ØN ATT.  THIS
C     PRØCEDURE IS ITERATED UNTIL SUCCESSIVE ATT VALUES DIFFER BY LESS
C     THAN 0.1 DECIBEL.
      CØMMØN/RGA/ RG(75), ATTN(3,75)
      ATTL=0.
    1 DØ 10 I=2,75
      IF (RG(I) - RMX) 10,9,11
   11 J=I-1
      ATT=(ATTN(3,I)-ATTN(3,J))*(RMX-RG(J))/(RG(I)-RG(J)) + ATTN(3,J)
      GØ TØ 12
    9 ATT=ATTN(3,I)
      GØ TØ 12
   10 CØNTINUE
      ATT=ATTN(3,75)
   12 DIFF = ATTL - ATT
      RMX = RMX * 10.**(DIFF*.025)
      IF (ABSF(DIFF) .LT. 0.1) RETURN
      IF (RMX .GE. RG(75)) RETURN
      ATTL = ATT
      GØ TØ 1
      END
```

```
                                                        05/12/72

      SUBROUTINE PDSN(PDT,PFA,NPULS,KASE,SDB)
      EXTERNAL PD
      DIMENSION DB0(5),SLOPE(5),PDFAC(5)
      COMMON /PDS/ PFLAST,NLAST,KSLAST
      DATA (PFLAST=0.),(NLAST=0),(KSLAST=-1)
      DATA (DB0=12.5,14.,14.,13.2,13.2),(SLOPE=6.,7.,8.,7.,8.),
     1    (PDFAC=4.8,20.,20.,13.,13.)
      DATA(DBMIN=-30.),(DBMAX=50.),(DB1=0.),(DB2=6.)
      IF (PDT.EQ.PDLAST) 1,20
    1 IF (PFA.EQ.PFLAST) 2,20
    2 IF (NPULS.EQ.NLAST) 3,20
    3 IF (KASE.EQ.KSLAST) RETURN
   20 PDLAST=PDT
      PFLAST=PFA
      NLAST=NPULS
      KSLAST=KASE
      K=KASE + 1
      PULS=NPULS
      DB1=DB0(K)-SLOPE(K)*ALOG10(PULS)+(PDT-.5)*PDFAC(K)+(PFA-8.)*.4-1.
      DB2=DB1+2.
      CALL INVERS(DBMIN,DBMAX,DB1,DB2,4,15,NOI,SDB,PD1,PDT,PD)
      END
```

```
                                                 05/12/72
FUNCTION PD(SNOB)
COMMON/PDS/FA,N,KASE
NP=N
FAN=FA
KAS=KASE
CALL MARSWR (SNOB,NP,FAN,KAS,PD1)
PD=PD1
END
```

```
                                                                 05/12/72
      SUBROUTINE INVERS(XMIN,XMAX,XLO,XHI,NSIG,LIM,NOI,X,F1,FT,F)
C     THIS IS A MODIFICATION (APRIL 1970) OF A PREVIOUS SUBROUTINE NAMED
C     INVERT. THIS VERSION HAS ADDITIONAL PARAMETERS. L. V. BLAKE, NRL.
C  THIS SUBROUTINE FINDS VALUE OF X THAT RESULTS IN F(X) = FT, BY
C  ITERATION BASED ON LINEAR INTERPOLATION/EXTRAPOLATION FROM PREVIOUS
C  TWO TRIALS.  FUNCTION F MUST BE MONOTONIC.
      TEST = 10.**(-NSIG)
      FD = FT
      IF (FT .EQ. 0.) FD = 1.
      NOI = 1
      DELTA = XHI - XLO
      X1=XLO
      X2=XHI
      F1=F(X1)
      F2=F(X2)
      SLOPE=(F2-F1)/(X2-X1)
      IF (SLOPE .EQ. 0.) 10,21
   10 FMAX=F(XMAX)
      FMIN=F(XMIN)
      SLOPE = (FMAX-FMIN)/(XMAX-XMIN)
   21 IF((F2-FT)*SLOPE .LT. 0.) 22,23
   22 X1=X2
      F1=F2
      X2=X2+DELTA
      IF (X2 .GT. XMAX) X2=XMAX
      F2=F(X2)
      GO TO 21
   23 IF((FT-F1)*SLOPE .LT. 0.) 24,25
   24 X2=X1
      X1=X1-DELTA
      IF (X1 .LT. XMIN) X1=XMIN
      F2=F1
      F1=F(X1)
      GO TO 23
   25 XA=X1
      XB=X2
      FA=F1
      FB=F2
      IF (ABSF(F2-FT) .LT. ABSF(F1-FT)) 7,6
    7 F22=F2
      F2=F1
      F1=F22
      X22=X2
      X2=X1
      X1=X22
      GO TO 6
    1 F1=F(X)
      X1=X
      TEST1 = ABS((F1-FT)/FD)
      IF (TEST1 .LE. TEST) 2,6
    2 RETURN
    6 IF (NOI .GE. LIM) 12,13
   12 PRINT 40
      PRINT 41, LIM
      PRINT 42, XMIN, XMAX, XLO, XHI, NSIG, LIM, NOI, X, F1, FT
      RETURN
```

```
                                                  05/12/72
13 IF (F1 .EQ. F2) 15,16
15 IF (F1 .EQ. FB) 17,18
17 XB=X1
   X2=XB
   GO TO 19
18 XA=X1
   X2=XA
19 X=(XA+XB)*.5
   GO TO 1
16 X=(X1-X2)*(FT-F2)/(F1-F2) + X2
   IF (X .LT. XA) X=XA
   IF (X .GT. XB) X=XB
   NOI = NOI + 1
   F2=F1
   X2=X1
   GO TO 1
40 FORMAT (//*   MESSAGE FROM SUBROUTINE INVERS -- *//)
41 FORMAT (*   FUNCTION INVERSION NOT ACCOMPLISHED WITHIN SPECIFIED *
  1  ,I3, * ITERATIONS.*/)
42 FORMAT (/*   INVERS PARAMETERS WERE *,4(E10.3,2X),3(I3,2X),E10.3,
  1  2(2X,E10.3) //)
   END
```

```
                                                   05/12/72
      SUBROUTINE MARSWR (SNDB,N,FA,KASE,PN)
C
C     INPUTS ARE -- SNDB, SIGNAL-TO-NOISE POWER RATIO IN DECIBELS --
C     N, NUMBER OF PULSES INTEGRATED --
C     FA, FALSE ALARM PROBABILITY, EXPRESSED AS ABSOLUTE VALUE OF POWER
C     OF TEN (E.G., FA = 8. MEANS 10.**(-8.) FALSE ALARM PROBABILITY --
C     KASE, SWERLING FLUCTUATION MODEL, WITH KASE = 0 FOR NONFLUCTUATING
C     OUTPUT PN IS PROBABILITY OF DETECTION
C
C     BASED ON PROGRAM WRITTEN AT JHU APPLIED PHYSICS LABORATORY, NAMED
C     SUBROUTINE MARCUM.  MODIFIED AT NRL BY L. V. BLAKE.  THIS VERSION
C     DATED APRIL 1971
C     APL VERSION DEFINED FA AS FALSE ALARM NUMBER (MARCUM CONCEPT).
C     NRL MOD CHANGED THIS TO FALSE ALARM PROBABILITY (AS DEFINED ABOVE)
C     SOME OTHER CHANGES ALSO.
C
      DOUBLE PRECISION ENPR,YBPR,GAMPR,PYB,W,V0,E0,V1,E1,STEP,Y0
      DOUBLE PRECISION DGAM, DEVAL, SUMLOG, SUM, FAN, EN
C
C     COMPUTE MARCUM-SWERLING DETECTION PROBABILITIES
C
C     CONVERT S/N IN DB TO NUMERICAL S/N RATIO
C
      SNR = 10.**(SNDB*.1)
C
C     TO CONVERT THIS SUBROUTINE TO INTERPRET FA AS THE MARCUM FALSE-
C     ALARM NUMBER, CHANGE THE NEXT STATEMENT TO READ -- MODE = 0
C
      MODE=1
C     IF MODE IS 1, CONVERT FA  TO MEAN EXPONENT OF FALSE-ALARM
C     PROBABILITY RATHER THAN MARCUM FALSE-ALARM NUMBER
C
      IF (MODE) 800, 800, 900
  900 FAN =DLOG10(DLOG(.5)/DLOG(1.-(10.D0)**(-FA)))
      GO TO 905
  800 FAN = FA
C
C     TEST INPUTS
C
  905 IF(N) 99,99,2
    2 IF(FA)99,99,3
    3 IF(KASE) 99,4,4
    4 IF(KASE-4) 5,5,99
C
C     ESTIMATE BIAS LEVEL
C
    5 ENPR = 0.
    6 ENPR = FAN
      EN = N
      YBPR = 0.
      IF (NPREV .EQ. N .AND. FAPREV .EQ. FA) GO TO 777
      IF(N-17) 7,7,8
    7 YBPR = EN*(1.+2.2*ENPR/EN**((2.0/3.0)+.015*ENPR))
      GO TO 11
    8 YBPR = EN*(1.+1.3*ENPR/EN**(.5+.011*ENPR))
C
```

```
                                                05/12/72
C     COMPUTE BIAS LEVEL
C
   11 ENPR = 10.**ENPR
      GAMPR = DGAM(YBPR,N-1)
      PYB = .5**(1./ENPR)
      SUML = SUMLOG(N-1)
      IF(GAMPR-PYB) 10,12,12
   10 H = .01
      GO TO 14
   12 H = -.01
   14 Y0 = YBPR
      E0 = DEVAL(Y0,N-1,SUML)
   16 Y1 = Y0+H
      E1 = DEVAL(Y1,N-1,SUML)
      STEP = GAMPR + H*(E0+E1)/2.
      IF(SIGNF(1.,STEP-PYB)-SIGNF(1.,H)) 18,20,18
   18 Y0 = Y1
      E0 = E1
      GAMPR = STEP
      GO TC 16
   20 IF(H) 22,24,24
   22 YB = Y1 - H*(PYB-STEP)/(GAMPR-STEP)
      GO TO 30
   24 YB = Y0 + H*(PYB-GAMPR)/(STEP-GAMPR)
   30 BIAS = YB
  777 YB = BIAS
      NPREV = N
      FAPREV = FA
C
C     SELECT N-S CASE
C
      X = SNR
      K = KASE+1
      GO TO (100,200,300,400,500), K
C
C     CASE 0
C
  100 SUM = 0.
      P = EN*X
      IF(YB-P-EX) 150,102,102
  102 KS = -(EN+1.)/2. + SQRTF(((EN+1.)/2.)**2+P*YB)
      KS = XMAX0F(KS,0)
      GS = 1.-GAM(YB,KS+N-1,TN)
      TS = EVAL(P,KS)*GS
      G = GS
      K = KS
      TERM = TS
      TL = TN
  110 TEMP = SUM+TERM
      IF(SUM-TEMP) 112,116,116
  112 SUM = TEMP
      IF(K) 116,116,114
  114 TERM = TERM*FLOATF(K)*(G-TL)/(P*G)
      G = G-TL
      K = K-1
      TL = TL*FLOATF(K+N)/YB
```

```
                                                              05/12/72
      GO TC 110
  116 TL = TN*YB/FLOATF(KS+N)
      K = KS+1
      G = GS+TL
      TERM = TS*P*G/(GS*FLOATF(K))
  120 TEMP = SUM+TERM
      IF(SUM-TEMP) 122,190,190
  122 SUM = TEMP
      TL = TL*YB/FLOATF(K+N)
      K = K+1
      TERM = TERM*P*(G+TL)/(G*FLOATF(K))
      G = G+TL
      GO TC 120
  150 KS = -1. - EN/2. + SQRTF(EN**2/4.+P*YB)
      KS =XMAXOF(KS,0)
      GS = GAM(YB,KS+N-1,TN)
      IF(GS) 174,174,155
  155 TS = EVAL(P,KS)*GS
      G = GS
      TERM = TS
      K = KS
      TL = TN
  160 TEMP = SUM+TERM
      IF(SUM-TEMP) 162,166,166
  162 SUM = TEMP
      IF(K) 166,166,164
  164 TERM = TERM*FLOATF(K)*(G+TL)/(P*G)
      G = G+TL
      TL = TL*FLOATF(K+N-1)/YB
      K = K-1
      GO TC 160
  166 TL = TN*YB/FLOATF(KS+N)
      K = KS+1
      G = GS-TL
      TERM = TS*P*G/(GS*FLOATF(K))
  170 TEMP = SUM + TERM
      IF(SUM-TEMP) 172,174,174
  172 SUM = TEMP
      TL = TL*YB/FLOATF(K+N)
      TERM = TERM*P*(G-TL)/(G*FLOATF(K+1))
      G = G-TL
      K = K+1
      GO TC 170
  174 SUM = 1.-SUM
  190 PN = SUM
      GO TC 90
C
C     CASE 1
C
  200 IF(N-1) 210,210,220
  210 PN = EXPF(-YB/(1.+X))
      GO TC 90
  220 TEMP = 1. + 1./(EN*X)
      PN = 1. - GAM(YB,N-2,DUM) + EXPF((EN-1.)*LOGF(TEMP)-YB/(1.+EN*X))
     1                             *GAM(YB/TEMP,N-2,DUM)
      GO TC 90
```

```
                                                  05/12/72
C
C     CASE 2
C
  300 IF(N-1) 310,310,320
  310 PN = EXPF(-YB/(1.+X))
      GO TO 90
  320 PN = 1. - GAM(YB/(1.+X),N-1,DUM)
      GO TO 90
C
C     CASE 3
C
  400 IF(N-2) 410,420,430
  410 PN = (1.+2.*X*YB/(X+2.)**2)*EXPF(-2.*YB/(2.+X))
      GO TO 90
  420 PN = (1.+YB/(1.+X))*EXPF(-YB/(1.+X))
      GO TO 90
  430 C = 2./(2.+EN*X)
      D = 1.-C
      IF(YB*D-EN) 440,450,450
  440 SUM = 0.
      TERM = 1.
      J = N
  442 TEMP = SUM+TERM
      IF(SUM-TEMP) 444,446,446
  444 SUM = TEMP
      TERM = TERM*YB*D/FLOATF(J)
      J = J+1
      GO TO 442
  446 PN = 1. - GAM(YB,N-2,DUM) + C*YB*EVAL(YB,N-2)
     1          + D*EVAL(YB,N-1)*(1.+C*YB-(EN-2.)*C/D)*SUM
      GO TO 90
  450 PN = 1. - GAM(YB,N-3,DUM) + YB*EVAL(YB,N-3)*C/D
     1          + EXPF(-C*YB-(EN-2.)*LOGF(D))*(1.+C*YB-(EN-2.)*C/D)
     2           *GAM(YB*D,N-3,DUM)
      GO TO 90
C
C     CASE 4
C
  500 SUM = 0.
      C = 2./(2.+X)
      D = 1.-C
      Q = C/D
      P = C*YB
      KS = (3.*EN+(YB*D))/2.-SQRTF((EN-1.+(YB*D))**2/4.+(YB*D)*(EN+1.))
      KS = XMINOF(KS,N)
      KS = XMAXOF(KS,0)
      K = KS
      J = N-KS
      FKS = KS
      K = XMINOF(KS,N)
      IF(YB-EN*(1.+D)) 550,501,501
  501 GS = 1. - GAM(P,2*N-1-KS,TN)
      IF(GS) 526,526,502
  502 TS = EXPF(FKS*LOGF(C)+(EN-FKS)*LOGF(D)+SUMLOG(N)-SUMLOG(KS)
     1          -SUMLOG(J)+LOGF(GS))
      G = GS
```

```
                                                          05/12/72
      TERM = TS                                                         00020200
      TL = TN                                                           00020300
  510 TEMP = SUM+TERM                                                   00020400
      IF(SUM-TEMP) 512,516,516                                          00020500
  512 SUM = TEMP                                                        00020600
      IF(K) 516,516,514                                                 00020700
  514 TL = TL*P/FLUATF(2*N-K)                                           00020800
      TERM = TERM*FLOATF(K)*(G+TL)/(Q*FLOATF(N-K+1)*G)                  00020900
      G = G+TL                                                          00021000
      K = K-1                                                           00021100
      GO TC 510                                                         00021200
  516 IF(KS-N) 518,526,526                                              00021300
  518 TERM = TS*Q*FLOATF(N-KS)*(GS-TN)/(FLOATF(KS+1)*GS)                00021400
      G = GS-TN                                                         00021500
      TL = TN*FLOATF(2*N-1-KS)/P                                        00021600
      K = KS+1                                                          00021700
  520 TEMP = SUM+TERM                                                   00021800
      IF(SUM-TEMP) 522,526,526                                          00021900
  522 SUM = TEMP                                                        00022000
      IF(K-N) 524,526,526                                               00022100
  524 TERM = TERM*Q*FLOATF(N-K)*(G-TL)/(FLOATF(K+1)*G)                  00022200
      G = G-TL                                                          00022300
      TL = TL*FLOATF(2*N-1-K)/P                                         00022400
      K = K+1                                                           00022500
      GO TC 520                                                         00022600
  526 PN = SUM                                                          00022700
      GO TC 90                                                          00022800
  550 GS = GAM(P,2*N-1-KS,TN)                                           00022900
      IF(GS) 576,576,552                                                00023000
  552 TS = EXPF(FKS*LOGF(C)+(EN-FKS)*LOGF(D)+SUMLOG(N)-SUMLOG(KS)       00023100
     1             -SUMLOG(J)+LOGF(GS))                                 00023200
      G = GS                                                            00023300
      TERM = TS                                                         00023400
      TL = TN                                                           00023500
  560 TEMP = SUM+TERM                                                   00023600
      IF(SUM-TEMP) 562,566,566                                          00023700
  562 SUM = TEMP                                                        00023800
      IF(K) 566,566,564                                                 00023900
    4 TL = TL*P/FLOATF(2*N-K)                                           00024000
      TERM = TERM*FLOATF(K)*(G-TL)/(Q*FLOATF(N-K+1)*G)                  00024100
      G = G-TL                                                          00024200
      K = K-1                                                           00024300
      GO TC 560                                                         00024400
  566 IF(KS-N) 568,576,576                                              00024500
  568 TERM = TS*Q*FLOATF(N-KS)*(GS+TN)/(FLOATF(KS+1)*GS)                00024600
      G = GS+TN                                                         00024700
      TL = TN*FLOATF(2*N-1-KS)/P                                        00024800
      K = KS+1                                                          00024900
  570 TEMP = SUM+TERM                                                   00025000
      IF(SUM-TEMP) 572,576,576                                          00025100
  572 SUM = TEMP                                                        00025200
      IF(K-N) 574,576,576                                               00025300
  574 TERM = TERM*Q*FLOATF(N-K)*(G+TL)/(FLOATF(K+1)*G)                  00025400
      G = G+TL                                                          00025500
      TL = TL*FLOATF(2*N-1-K)/P                                         00025600
      K = K+1                                                           00025700
```

```
                                                      05/12/72
      GO TO 570
  576 PN = 1.-SUM
      GO TO 90
C
C     SET PROBABILITY
C
   90 IF(PN) 91,94,92
   91 PN = 0.
      GO TO 94
   92 IF(PN-1.) 94,94,93
   93 PN = 1.
   94 RETURN
C
C     ERROR MESSAGE FOR BAD INPUTS
C
   99 WRITE (61,9)  N,FA ,SNR,KASE
    9 FORMAT (1H0 /50H UNREASONABLE CALL SEQUENCE TO MARCUM. ZERO RESULT
     1              7HS GIVEN //4H N = I8,5X,5HFA  = E16.8,5X,5HSNR =
     2             E16.8,5X,6HKASE = I8)
      PN = 0.
      BIAS = 0.
      RETURN
      END
```

```
                                                                   05/12/72

      FUNCTION DGAM(B,N)                                              00028100
      DOUBLE PRECISION  SUM,  TERM,TEMP,FJ,DGAM, DEVAL, B, SUML,SUMLOG  00028200
C             INTEGRAL = 1-(SUM, J=0 TO N, OF EXPF(J*LOGF(B)-B-LOGF(NFAC(((28300
      SUM = 0.                                                        (((28400
      K = B                                                           (((28500
      IF(K-N) 100,200,200                                             (((28600
  100 J = K+1                                                         (((28700
      SUML = SUMLOG(J)                                                (((28800
      TERM = DEVAL(B,J,SUML)                                          (((28900
   10 TEMP = SUM+TERM                                                 (((29000
      IF(SUM-TEMP) 15,20,20                                           (((29100
   15 SUM = TEMP                                                      (((29200
      J = J+1                                                         (((29300
      FJ = J                                                          (((29400
      TERM = TERM*B/FJ                                                (((29500
      GO TO 10                                                        (((29600
   20 DGAM = SUM                                                      (((29700
      RETURN                                                          (((29800
  200 J = N                                                           (((29900
      SUML = SUMLOG(J)                                                (((30000
      TERM = DEVAL(B,J,SUML)                                          (((30100
   30 TEMP = SUM+TERM                                                 (((30200
      IF(SUM-TEMP) 35,40,40                                           (((30300
   35 SUM = TEMP                                                      (((30400
      IF(J-1) 40,36,36                                                (((30500
   36 FJ = J                                                          (((30600
      TERM = TERM*FJ/B                                                (((30700
      J = J-1                                                         (((30800
      GO TO 30                                                        (((30900
   40 DGAM = 1.-SUM                                                   (((31000
      RETURN                                                          (((31100
      END                                                             (((31200
```

05/12/72

```
      FUNCTION DEVAL (Y,N,SUML)
      DOUBLE PRECISION  XPON,EN,DEVAL,  Y,SUML
      XPON = -Y
      IF(N) 20,20,10
   10 EN = N
      XPON = XPON+EN*DLOG(Y)-SUML
   20 DEVAL = DEXP(XPON)
      RETURN
      END
```

05/12/72

```
      FUNCTION GAM(B,N,TN)                                        (((32200
C                 SINGLE PRECISION VERSION OF DGAM                (((32300
      SUM = 0.                                                    (((32400
      K = B                                                       (((32500
      IF(K-N) 100,200,200                                         (((32600
  100 J = N+1                                                     (((32700
      TERM = EVAL(B,J)                                            (((32800
      TN = TERM*FLOATF(J)/B                                       (((32900
   10 TEMP  = SUM+TERM                                            (((33000
      IF(SUM-TEMP) 15,20,20                                       (((33100
   15 SUM = TEMP                                                  (((33200
      J = J+1                                                     (((33300
      FJ = J                                                      (((33400
      TERM = TERM*B/FJ                                            (((33500
      GO TO 10                                                    (((33600
   20 GAM = SUM                                                   (((33700
      RETURN                                                      (((33800
  200 J = N                                                       (((33900
      TERM = EVAL(B,J)                                            (((34000
      TN = TERM                                                   (((34100
   30 TEMP = SUM+TERM                                             (((34200
      IF(SUM-TEMP) 35,40,40                                       (((34300
   35 SUM = TEMP                                                  (((34400
      IF(J-1) 40,36,36                                            (((34500
   36 FJ = J                                                      (((34600
      TERM = TERM*FJ/B                                            (((34700
      J = J-1                                                     (((34800
      GO TO 30                                                    (((34900
   40 GAM = 1.-SUM                                                (((35000
      RETURN                                                      (((35100
      END                                                         (((35200
```

```
                                                              05/12/72
                                                                                    (((35300
      FUNCTION EVAL(Y,N)                                                            (((35400
      XPON = -Y                                                                     (((35500
      IF(N) 20,20,10                                                                (((35600
   10 FN = N                                                                        (((35700
      XPON = XPON+FN*LOGF(Y)-SUMLOG(N)                                              (((35800
   20 EVAL = EXPF(XPON)                                                             (((35900
      RETURN                                                                        (((36000
      END
```

```
                                                    05/12/72
   FUNCTION SUMLOG(N)
   DOUBLE PRECISION A, B, SUMLOG
   DIMENSION A(1000)
   DATA(DUMA = 0.),(DUMB = 0.)
   NMAX=1000
   IF(DUMA-DUMB) 20,10,20
10 DUMA = 1.
   DUMB = 0.
   NLAST = 1
   A(1) = 0.
20 NN = XABSF(N)
   IF(NN-1) 30,30,40
30 SUMLOG = 0.
   RETURN
40 IF(NN-NLAST) 50,50,60
50 SUMLOG = A(NN)
   RETURN
60 K = NLAST+1
   IF(NN-NMAX) 70,70,80
70 DO 72 I=K,NN
72 A(I) = A(I-1) + DLOG(FLOATF(I))
   NLAST = NN
   GO TO 50
80 IF(NLAST-NMAX) 82,90,90
82 DO 84 I=K,NMAX
84 A(I) = A(I-1) + DLOG(FLOATF(I))
   NLAST = NMAX
90 B = A(NMAX)
   K = NMAX+1
   DO 92 I=K,NN
92 B = B + DLOG(FLOATF(I))
   SUMLOG = B
   RETURN
   END
```

REFERENCES

1. Blake, L.V., "A Guide to Basic Pulse-Radar Maximum-Range Calculation," Part 1, NRL Report 6930, and Part 2, NRL Report 7010, Dec. 1969.
2. Bean, B.R., and Dutton, E.J., "Radio Meteorology," National Bureau of Standards Monograph 92, U.S. Government Printing Office, Washington, D.C., 1966.
3. Blake L.V., "Machine Plotting of Radio/Radar Vertical-Plane Coverage Diagrams," NRL Report 7098, June 1970.
4. Boothe, R.R., "A Digital Computer Program for Determining the Performance of an Acquisition Radar through Application of Radar Detection Probability Theory," U.S. Army Missile Command Report No. RD-TR-64-2, Dec. 1964.
5. White, D.M., "Radar Simulation and Analysis by Digital Computer," Johns Hopkins University Applied Physics Laboratory Report TG-952, Jan. 1968.
6. Fehlner, L.F., "Marcum and Swerling's Data on Target Detection by a Pulsed Radar," Johns Hopkins University Applied Physics Laboratory Report TG-451, July 1962.
7. Kirkwood, P.K., "Radar Cumulative Detection Probabilities for Radial and Nonradial Target Approaches," Rand Corporation Report RM-4643-PR, Sept. 1965.
8. Nolen, J.C., and others, "Statistics of Radar Detection," Bendix Corporation, Baltimore, Md., Feb. 1966.
9. Killinger, D.K., "Computer Aided Radar Design (CARD)," Naval Avionics Facility Interim Report TR 1461, 12 Aug. 1969.
10. Minzner, R.A., Ripley, W.S., and Condron, T.P., "U.S. Extension to the ICAO Standard Atmosphere," U.S. Dept. of Commerce Weather Bureau and USAF ARDC Cambridge Research Center, Geophysics Research Directorate, U.S. Government Printing Office, Washington, D.C., 1958.
11. Sissenwine, N., Grantham, D.D., and Salmela, H.A., "Humidity up to the Mesopause," Air Force Cambridge Research Laboratories Report AFCRL-68-0550 (Air Force Surveys in Geophysics, No. 206), Oct. 1968.
12. Blake, L.V., "A Modified Simpson's Rule and Fortran Subroutine for Cumulative Numerical Integration of a Function Defined by Data Points," NRL Memorandum Report 2231, Apr. 1971.
13. Meeks, M.L., and Lilley, A.E., "The Microwave Spectrum of Oxygen in the Earth's Atmosphere," J. Geophys. Res. 68 (No. 6):1683-1703 (Mar. 15, 1963).
14. Reber, E.E., Mitchell, R.L., and Carter, C.J., "Attenuation of the 5-mm Wavelength Band in a Variable Atmosphere," IEEE Transactions AP-18 (No. 4):472-479 (July 1970).
15. Blake, L.V., "Radio Ray (Radar) Range-Height-Angle Charts," NRL Report 6650, Jan. 22, 1968, and Microwave J. 11 (No. 10):49-53 (Oct. 1968).
16. Blake, L.V., "Ray Height Computation for a Continuous Nonlinear Atmospheric Refractive-Index Profile," Radio Science 3 (new series) (No. 1):85-92 (Jan. 1968).

Appendix A

A FORM FOR LISTING DATA CARD INPUTS TO PROGRAM RGCALC

The following form has been devised for listing the input quantities needed on data cards for Program RGCALC. The form provides a convenient reference for punching the data cards, since it indicates the card columns for each input quantity. The "case" options (1 through 7) are also described in a footnote on the form.

DATA CARD FORMATS FOR PROGRAM RGCALC
(Reference: NRL Report 7448)

Cards 1, 3, 5, 7 . . . : Name of radar (Format 10A8). Cols. 1-80. (Any other descriptive alphanumeric material can be entered in these columns.)

Cards 2, 4, 6, 8 . . . : Radar data as below. (Note: A quantity with F format can be punched without decimal point if it has an integer value and is right adjusted. Non-integer numbers must be punched with decimal point; right adjustment then unnecessary. Integer-format quantities must be right-adjusted in specified column field.)

DATA ITEM	FORMAT SPEC.	COLS.	RADAR NAME	RADAR NAME	RADAR NAME
Transmitter power, kW (P_t)	F6.0	1-6			
Pulse length, μsec (τ)	F6.0	7-12			
Transmit antenna gain, dB (G_t)	F4.0	13-16			
Receive antenna gain, dB (G_r)	F4.0	17-20			
Target cross section, sq m (σ)	F6.0	21-26			
Frequency, MHz (f)	F6.0	27-32			
Antenna ohmic loss, dB (L_a)	F4.0	33-36			
Receiving line loss, dB (L_r)	F4.0	37-40			
Transmit line loss, dB (L_t)	F4.0	41-44			
Antenna-pattern scan loss, dB (L_p)	F4.0	45-48			
Miscellaneous loss, dB (L_x)	F4.0	49-52			
Bandwidth/shape factor, dB (C_B)	F4.0	53-56			
Receiver noise factor, dB (NF)	F4.0	57-60			
Number of pulses (M) See Report 6930, Eqs. (71) and (72)	I5	61-65			
Cases 0-5: Probability of detection Cases 6-7: Signal-to-noise ratio, dB	F4.0	66-69			
Cases 0-5: False alarm exp't ($-\log_{10} P_{fa}$) Case 6: Blank; Case 7, Signal/noise, dB	F4.0	70-73			
Case (0 to 7)*	I1	74			
Target elevation angle, degrees	F4.0	75-78			
Galactic-noise-level code (−1, 0, +1) (minimum, average, maximum)	I2	79-80			

*Cases are 0 to 4 – range is calculated for corresponding Swerling fluctuation case, 5 – calculated for all five Swerling cases (0, 1, 2, 3, 4), 6 – calculated for the S/N value in Cols 66-69, 7 – calculated for two S/N values, one in 66-69, other in 70-73.

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) AVCO EVERETT RESEARCH LABORATORY 2385 REVERE BEACH PARKWAY EVERETT, MASSACHUSETTS	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

THE USE OF A LASER FOR ARPA MILITARY GEOPHYSICS PROGRAM (ROCK MECHANICS AND RAPID EXCAVATION)

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final Technical Report

5. AUTHOR(S) (First name, middle initial, last name)

Avco Everett Research Laboratory

6. REPORT DATE October 1972	7a. TOTAL NO. OF PAGES 83	7b. NO. OF REFS 5
8a. CONTRACT OR GRANT NO. H0210039 b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY ARPA, Dept. of Defense monitored by Bureau of Mines, Dept. of Interior

13. ABSTRACT

This is a study of the penetration and cracking of rocks using lasers available at the Avco Everett Research Laboratory. The objectives are: 1) to obtain data on the rate of rock damage for various laser conditions and 2) to present an analytical program to predict the temperature and stress in rocks for pointed and annular beams of radiation.

The laser power outputs used thus far was from 1 to 17 kW, CW, 10.6 microns and pulsed laser power up to 1000 joules in 20 microseconds. Data was taken with sharply focused as well as defocussed beams. Data is presented for continuous irradiation as well as pulsed, pointed beams and annular radiation patterns. Three types of hard rock were tested namely: quartzite, a Rhode Island granite, and Dresser basalt. Hole penetration energy per cm of penetration was found to be independent of laser intensity over 6 orders of magnitude. The specific energy for rock removal was found to depend on the fifth root of laser intensity over the same range.

A computer program was developed using the finite element method. It takes a radiation pattern, and delivers curves for thermoelastic strain and stress in any direction. This program is capable of working with high intensity laser radiation because it includes nonlinear heat conductivity, a temperature cut-off at the boiling point, anisotropic elastic conditions and a special iterative procedure to avoid computation instabilities resulting from too rapid heating. This program predicts that efficient cracking of rocks will be achieved by directing the radiation in a large annulus. The beam is moved so that the temperature of any point stays below the melting point. However, thermoelastic stress builds up in depth.

These predictions have been verified by experiments. However, the experiments to spall away basalt and quartzite consume more energy than predicted from the thermoelastic computer program. This is believed to be because the computer predicts the condition of crack generation whereas the experiments show the complete separation of rock material when the cracks are extended to complete spall.

DD FORM 1 NOV 65 1473

UNCLASSIFIED
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
1. Rock Drilling						
2. Tunnelling						
3. Rapid Rock Removal						
4. Laser						
5. Thermoelastic Stress						
6. Pulsed Laser						
7. Granite						
8. Basalt						
9. Quartzite						