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THE INTERACTION OF A SHOCK WAVE IN
A PLASTIC MEDIUM WITH AN EDIFICE

by

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В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
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К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

* ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

THE INTERACTION OF A SHOCK WAVE IN A PLASTIC MEDIUM WITH AN EDIFICE

G. M. Lyakhov

(Moscow)

The interaction of a plane shock wave in a plastic medium with a boundary (a plate) made from a nondeformed material was examined in works [1-3]. The dependence of the stress on the deformation $\sigma = \sigma(\varepsilon)$ under a load was approximated by piecewise linear [1, 2] or by power [3] functions, while the unloading was assumed to be originating according to a linear law, different from the loading law [1], or with a constant volume [2, 3].

In work [4] a solution was given for the problem of the interaction of a plane shock wave in an elastic medium with an edifice, which is viewed as a system with three degrees of freedom, which allows us to take into consideration simultaneously the shifting of the edifice on the whole and the deflection of its covering and base. In works [2, 4] a solution is given for the problem of the interaction of a wave with an edifice as with a system with three degrees of freedom in a plastic medium. The dependence $\sigma = \sigma(\varepsilon)$ with a load is nonlinear, and unloading occurs with a constant volume. The effect of the free surface is taken into account.

The results of the calculation are represented in the form of a system of equations, the integration of which is carried out using electronic digital computers. Particular cases are examined, when integration can be done without using computers. A comparison of the movement of the boundary in plastic and in elastic media is made.

1. Let us examine a plastic medium, in which the stress σ and deformation ε during loading are related by the linear law:

$$\sigma = \sigma(\varepsilon), \text{ where } \frac{d\sigma}{d\varepsilon} > 0, \ \frac{d^2\sigma}{d\varepsilon^2} > 0. \qquad (1.1)$$

The unloading and the secondary loading up to a value of the previously achieved stress occurs at constant deformation (Fig. 1).

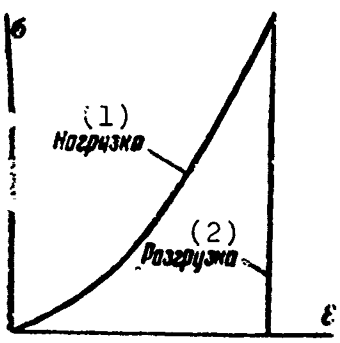

Fig. 1.

KEY: (1) Loading; (2) Unloading.

Similar properties are possessed by nonwater-saturated ground with fair-sized stresses. With the compression of the medium in the plane wave, which corresponds to the uniaxial deformed state, pressure p and specific volume V are determined by expressions:

$$\sigma = -p, \ \varepsilon = \frac{V - V_0}{V_0}; \qquad (1.2)$$

σ is the stress in the direction of movement of the wave.

Transferring to a system of units of p, V, we obtain equation (1.1) with the loading of the medium in the form

$$p = p(V), \ \frac{dp}{dV} < 0, \ \frac{d^2p}{dV^2} > 0; \qquad (1.3)$$

and with unloading

$$\frac{\partial V}{\partial t} = 0. \qquad (1.4)$$

Let us carry out the solution in Lagrange coordinates; mass h, time t.

In the initial section of the medium let h = 0, which we will combine with the free surface; when t = 0 let the pressure increase suddenly from 0 to p_m, and then fall according to the given law:

$$p = f(t). \tag{1.5}$$

Under the effect of this load there is formed in the medium a plane shock wave.

Let there be an edifice in the form of a rectangular parallelepiped in section h*. Its covering and base are viewed as girders, resting freely on a framework (the walls of the edifice). Let us represent the edifice as a system with three degrees of freedom in conformance with the scheme in Fig. 2.

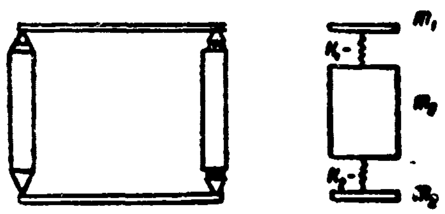

Fig. 2.

Let us designate by m_1 and m_2 the masses of the covering and of the base reduced to the middle of the span, by m_3 - the mass of the framework, by y_1, y_2 and y_3 - the shifts of these masses respectively in a fixed system of coordinates, and by $\varkappa_1$ and $\varkappa_2$ - the rigidities of the covering and of the base. The reduction of the masses is carried out in conformance with work [4].

Let us examine the interaction of a plane shock wave with the edifice. The scheme of the regions of various solutions in plane h, t is represented in Fig. 3. Behind the edifice there is a second plastic medium, for which the dependence p = p*(V) is given. During loading $d^2p/dV^2 > 0$, and during unloading $\partial V/\partial t = 0$. We can disregard the deformation of the framework of the edifice, as well as the edge effects, connected with the flow of the wave around the edifice. In Fig. 3 the edifice in section h = h* is not represented. The movement of the medium

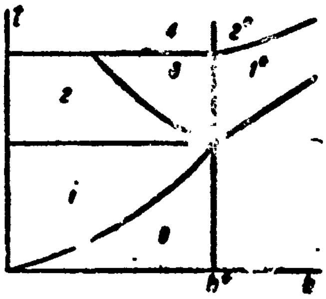

Fig. 3.

in Lagrange coordinates in plane h, t is determined by equations:

$$\partial u/\partial t + \partial p/\partial h = 0,$$
$$\partial u/\partial h - \partial V/\partial t = 0. \qquad (1.6)$$

In regions where unloading or secondary loading of the medium occurs when $\partial V/\partial t = 0$, the solution of equations (1.6) has the form:

$$u = \varphi_i(t),\ p = -h\dot{\varphi}_i(t) + \psi_i(t), \qquad (1.7)$$

where [illegible] the velocity of the particles, and i is the member of the region in plane h, t.

In regions where primary unloading of the medium takes place, and the wave is not a shock wave, the Riemannian solution is valid:

$$u = F_i\left(h - \sqrt{-\frac{dV}{dp}}\, t\right),$$
$$u - \int \sqrt{-\frac{dV}{dp}}\, dp = \beta_i. \qquad (1.8)$$

Functions φ_i, ψ_i, and F_i and the constant value β_i are found from the initial and boundary conditions.

Region 1 in Fig. 3 corresponds to a shock wave propagating through the medium, and the solution has the form of (1.7). The sought-for functions are $\varphi_1(t)$, $\psi_1(t)$; the line of the front is $h_1(t)$ (Fig. 3). From the condition in the initial section we will find

$$\psi_1(t) = P(t). \qquad (1.9)$$

The conditions at the front of the shock wave at coordinates h, t have the form

$$p = h_1^2(t)(V_0 - V),\ u = h_1(t)(V_0 - V). \qquad (1.10)$$

The subscript 0 refers to the medium before the wave front.

Taking into account (1.7), in conformance with (1.10) we obtain a system of two equations with two unknown functions $\varphi_1(t)$ and $h_1(t)$:

$$
\begin{aligned}
-h_1(t)\,\dot{\varphi}_1(t) + \dot{f}(t) &= \dot{h}_1(t)\,\varphi_1(t);\\
\varphi_1(t) &= \dot{h}_1(t)\,\{V_0 - V[-h_1(t)\,\dot{\varphi}_1(t) + f(t)]\}.
\end{aligned}
\qquad (1.11)
$$

The initial conditions are: $h_1(0) = 0$, $f(0) = p_m$, whence

$$\varphi_1(0) = \sqrt{\rho_m(V_0 - V_m)}, \quad V_m = V(p_m).$$

Integrating system (1.11), we will find the solution in region 1. The concrete form of the sought-for functions depends on the form of the given functions $p = p(V)$ and $p = f(t)$.

Regions 2 and 3 correspond to elastic and plastic waves, formed during the interaction of the incident wave with the covering of the edifice. In these regions unloading or a secondary loading of the medium occurs. An elastic wave, as a consequence of the assumption $\partial V/\partial t = 0$ during unloading, propagates with an infinitely large velocity. Simultaneously during the interaction shifting of the edifice occurs as a single unit, and deflections of the covering and of the base appear as well. As a result of the movement of mass m_- behind the edifice the gradual loading of the medium begins and a compression wave is formed (region 1* in Fig. 3), the course in which is determined by the equations in (1.8). The solutions in regions 2, 3 and 1*, as well as the laws of movement of the elements of the edifice are found simultaneously. Let us examine the corresponding initial and boundary conditions.

In section $h = 0$ the change in pressure is given by function (1.9). Hence we obtain

$$\dot{\varphi}_2(t)\big|_0 = f(t). \qquad (1.12)$$

Boundary $h_2(t)$ of regions 2, 3 was previously unknown. In it from the side of region 2 at moment in time t the pressure in each particle reaches a value, which existed previously on the front of incident wave $h_1(t)$ at moment in time $\lambda(t)$. Hence in conformance with (1.7)

$$h_2(t)\frac{d\varphi_2}{dt} + f(t) = h_1(\lambda)\frac{d\varphi_1}{d\lambda} + f(\lambda), \quad h_2(t) = h_1(\lambda). \tag{1.13}$$

On the front of plastic wave $h_2(t)$ the pressure increases abruptly, and then behind the front changes when $\partial V/\partial t = 0$. On line $h_2(t)$ the following relationships are fulfilled:

$$p_3 - p_2 = h_2'^2 [V(p_2) - V(p_3)]; \quad u_3 - u_2 = h_2'(t)[V(p_2) - V(p_3)];$$
$$p_3 - p_2 = h_2'(t)(u_3 - u_2).$$

Hence we obtain two more equations for inclusion into the overall system:

$$\frac{dh_1(\lambda)}{dt}(\varphi_3 - \varphi_2) - h_1(\lambda)(\dot{\varphi}_3 - \dot{\varphi}_2) = \psi_3(t) - f(t);$$
$$\varphi_3 - \varphi_2 = \frac{dh_1(\lambda)}{d\lambda}\frac{d\lambda}{dt}[V(p_2) - V(p_3)], \tag{1.14}$$

where

$$p_3 = -h_2(t)\dot{\varphi}_3 + \psi_3, \quad p_2 = -h_2(t)\dot{\varphi}_2 + f(t).$$

The values with the subscripts 2 and 3 refer to regions 2 and 3.

The front of the wave in the second medium corresponds to a weak separation, and it moves with a constant velocity, equal in coordinates h, t to the acoustic resistance of the medium. Hence we find that in equation (1.8) the constant $\beta = 0$. The pressure and velocity of particles in region 1* are related by the relationship

$$u = \int \sqrt{-\frac{dV}{dp}}\, dp. \tag{1.15}$$

Dependence $p = p^*(V)$ for the second medium is known, hence we find the function

$$p = g(u). \tag{1.16}$$

The velocities of masses m_1 and m_2 are equal to the velocities of the particles adjoining them of the first and, correspondingly, of the second media:

$$\dot{y}_1(t) = \varphi_3(t),\ \dot{y}_2(t) = u(t).$$

Mass m_1 moves under the effect of the difference in forces, acting from the side of the first medium and mass m_3; mass m_2 - under the effect of the difference in forces, applied from the side of mass m_3 and from the side of the second medium; mass m_3 - under the effect of the difference in forces, applied from the side of masses m_1 and m_2.

A system of equations, determining the course in three regions: 2, 3, 1* and the movement of masses m_1, m_2, m_3, includes seven equations with seven unknown functions: $\varphi_2(t)$, $\psi_3(t)$, $h_2(t)$, $\lambda(t)$, $y_1(t)$, $y_2(t)$, $y_3(t)$. These equations correspond to the above considered conditions:

$$\begin{gathered}
h_2(t)\,\frac{d\varphi_2(t)}{dt} + f(t) = h_1(\lambda)\,\frac{d\varphi_1(\lambda)}{d\lambda} + f(\lambda);\\
h_2(t) = h_1(\lambda);\\
\frac{d}{dt}\left[h_1(\lambda)\left(\varphi_3(t) - \varphi_2(t)\right)\right] = \psi_3(t) - f(t);\\
\varphi_3(t) - \varphi_2(t) = \frac{dh_1(\lambda)}{d\lambda}\,\frac{d\lambda}{dt}\left\{V\left[-h_1(\lambda)\dot{\varphi}_2 + f(t)\right] - \right.\\
\left. - V\left[-h_1(\lambda)\dot{\varphi}_3 + \psi_3(t)\right]\right\};\\
(m_1 + h^*)\,\ddot{y}_1(t) = \psi_3(t) - \kappa_1\left[y_1(t) - y_3(t)\right];\\
m_3\ddot{y}_3(t) = \kappa_1\left[y_1(t) - y_3(t)\right] - \kappa_2\left[y_3(t) - y_2(t)\right];\\
m_2\ddot{y}_2(t) = \kappa_2\left[y_3(t) - y_2(t)\right] - g\left[\dot{y}_2(t)\right].
\end{gathered} \tag{1.17}$$

The system is solved with the initial values:

$$\lambda(t^*) = t^*,\ \varphi_2(t^*) = \varphi_1(t^*),\ y_2(t^*) = 0,\ y_3(t^*) = 0,$$
$$y_1(t^*) = 0,\ \dot{y}_1(t^*) = 0,\ \dot{y}_2(t) = 0,\ \dot{y}_3(t) = 0,$$

where t^* is the moment of approach of the wave to the covering of the edifice.

2. Depending on the parameters of the problem - the form of function $p = f(t)$, the properties of both media, the values of masses m_1, m_2, m_3, and the value h^* the system of equations (1.17) can cease to be fulfilled as a result of the fact that the rarefaction wave from the free surface arrives at mass m_1 or in region 1* the compression wave becomes a shock wave and the Riemannian solution will not be valid.

Let us consider the first case. When $t = t^{**}$ as a consequence of the effect of the free surface regions 4 and 2* arise (Fig. 3). In region 4 unloading of the medium takes place and the solution has the form of (1.7). From the condition on the free surface we find one function in region 4:

$$\psi_4(t) = f(t). \qquad (2.1)$$

Depending on the parameters of the problem when $t > t^{**}$ in region 2* both loading and unloading are possible. Let us consider the case of unloading. Then the boundary $h_3(t)$ of regions 1*, 2* is nonrectilinear, previously unknown and must be determined in the course of the solution of the problem. In region 2* the solution has the form of (1.7).

The system of equations determining the flow in two regions - 4 and 2* - and the movement of masses m_1, m_2, m_3, includes five equations with five unknown functions: $\varphi_4(t)$, $y_3(t)$, $y_2(t)$, $h_3(t)$, $\psi_2^*(t)$ (function $\psi_4(t)$ is found, when $y_1(t) = \varphi_4(t)$).

This system corresponds to the equations of motion of masses m_1, m_2, m_3 and the continuity conditions of the pressure and velocity of the particles on line $h_3(t)$ in the second medium:

$$
\begin{aligned}
(m_1 + h^*)\,\ddot{y}_1(t) &= f(t) - \kappa_1\,[y_1(t) - y_3(t)];\\
m_3\,\ddot{y}_3(t) &= \kappa_1\,[y_1(t) - y_3(t)] - \kappa_2\,[y_3(t) - y_2(t)];\\
(m_2 - h^*)\,\ddot{y}_2(t) &= \kappa_2\,[y_3(t) - y_2(t)] - \psi_2^{\bullet}(t);\\
-h_3'(t)\,\ddot{y}_2(t) &= -\psi_2^{\bullet}(t) + p\,[h_3(t), t];\\
\dot{y}_2(t) &= u\,[h_3(t), t].
\end{aligned}
\tag{2.2}
$$

If in region 2* the load is continued, then the boundary of region 2* and 1* is rectilinear and parallel to boundary 1*, 0. The system of equations determining movement of the elements of the edifice and the flow in regions 4 and 2* assume the form:

$$
\begin{aligned}
(m_1 + h^*)\,\ddot{y}_1(t) &= f(t) - \kappa_1\,[y_1(t) - y_3(t)];\\
m_3\,\ddot{y}_3(t) &= \kappa_1\,[y_1(t) - y_3(t)] - \kappa_2\,[y_3(t) - y_2(t)];\\
m_2\,\ddot{y}_2(t) &= \kappa_2\,[y_3(t) - y_2(t)] - g(\dot{y}_2(t));
\end{aligned}
\tag{2.3}
$$

$g(\dot{y}_2(t))$ is a known function, determining the connection of p and u in regions 1* and 2*. The integration of the obtained systems of equations must be carried out using electronic digital computers.

3. Let us make an analysis of the obtained regularities for the simplest case, when the deflection of the girders in the covering and base can be disregarded and when the edifice is represented in the form of a boundary (a plate) with mass m. The loading of the media above the plate and below it is determined by the linear dependences:

$$
\begin{aligned}
p &= -A^2 V + B, \quad A_1 B = \mathrm{const};\\
p &= -A^{*2} V + B^*, \quad A^*, B^* = \mathrm{const}.
\end{aligned}
\tag{3.1}
$$

Unloading takes place when $\partial V/\partial t = 0$.

In section $h = 0$ when $t = 0$ the pressure increases abruptly from 0 to p_m and henceforth maintains this value. In this case boundary 1, 0 is rectilinear, its equation is $h = At$, and the solution in regions 1 and 2 has the form:

$$
\psi_2 = \psi_1 = p_m, \quad \varphi_1 = \varphi_2 = \frac{p_m}{A}, \quad p = p_m.
\tag{3.2}
$$

The pressure and velocity of the particles are constant.

Let us introduce a new variable $\tau = t - h^*/A$. The boundary of regions 2 and 3 is rectilinear. From the conditions on this boundary we obtain

$$\dot{\psi}_3 = 2p_m - A\varphi_3 + h\dot{\varphi}_3 = 2p_m - A\varphi_3 + (h^* - A\tau)\dot{\varphi}_3. \tag{3.3}$$

In region 1* loading of the medium occurs. With a linear dependence of the pressure on volume (3.1) the solution of the equations of (1.6) in the region of loading (see, for example, [4]) has the form:

$$p = F_1(h - A\tau) + F_2(h + A\tau),\ Au = F_1(h - A\tau) - F_2(h + A\tau). \tag{3.4}$$

Functions F_1 and F_2 we will find from the initial and boundary conditions. From the conditions of the front of the wave 1*, 0 we find that in region 1* the pressure is connected with the velocity of the particles by the relationship

$$p(\tau) = A^* u(\tau). \tag{3.5}$$

Taking into account (3.3) and (3.5), we get the equations of motion of the boundary:

$$\begin{gathered} m\dot{\varphi}_3 = -h^*\dot{\varphi}_3 + \dot{\psi}_3 - A^*\varphi_3, \\ (m + A\tau)\dot{\varphi}_3 + (A + A^*)\varphi_3 = 2p_m. \end{gathered} \tag{3.6}$$

Integrating and taking into account that when $\tau = 0$ we have $\varphi_3 = 0$, we find the velocity of the boundary and the velocity of the particles in region 3:

$$u = \varphi_3(\tau) = \frac{2p_m}{A + A^*}\left[1 - \left(\frac{m}{m + A\tau}\right)^{\frac{A+A^*}{A}}\right]. \tag{3.7}$$

Hence the pressure in region 3

$$p = -h\dot{\varphi}_3 + \dot{\psi}_3 = 2p_m\left[\left(\frac{m + h^* - h}{m + A\tau} - \frac{A^*}{A + A^*}\right)\left(\frac{m}{m + A\tau}\right)^{\frac{A+A^*}{A}} + \frac{A^*}{A + A^*}\right]. \tag{3.8}$$

On the boundary of regions 2, 3 with the approach through section $h = 0$ the pressure falls. It is determined by expression

$$p = \frac{2Ap_m}{A+A^*}\left[\left(\frac{m}{m+h^*-h}\right)^{\frac{A+A^*}{A}} + \frac{A^*}{A}\right]. \qquad (3.9)$$

Depending on the parameters of the problem the pressure on line 2, 3 can become exhausted and become equal to the pressure in region 2, (i.e., p_m) or remain greater than p_m along the entire path up to $h = 0$. If the pressure does not fall to p_m, then the boundary reaches the initial section when $\tau^* = h^*/A$. If the jump is exhausted, then this takes place when

$$h^{**} = h^* - \frac{(1-\varkappa)\,m}{\varkappa}, \quad \tau^{**} = \frac{(1-\varkappa)\,m}{A\varkappa},$$

$$\text{where } \varkappa = \left(\frac{A-A^*}{2}\right)^{\frac{A}{A+A^*}}. \qquad (3.10)$$

In the first case when $\tau = \tau^{**}$, and in the second when $\tau = \tau^*$ regions 4 and 2* are formed (Fig. 4). Let us designate the moment of their formation (in both cases) by τ^{**}.

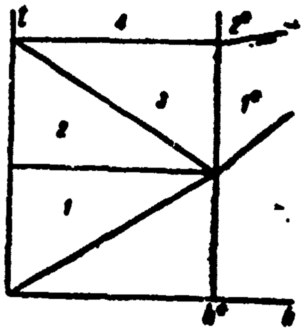

Fig. 4.

In region 4 unloading of the medium takes place, while in region 2* both loading and unloading are possible. In the case of loading in region 2*, by integrating the equation of motion of the boundary

$$m\dot{\varphi}_4 = -h\dot{\varphi}_4 + p_m - A^*\varphi_4, \qquad (3.11)$$

we will find the velocity of the boundary in region 4:

$$\varphi_4 = \frac{p_m}{A^*}\left[\varphi_3(\tau^{**}) - \frac{p_m}{A^*}\right]e^{-\frac{A(\tau-\tau^{**})}{m+h^*}}, \qquad (3.12)$$

where $\varphi_3(\tau^{**})$ is determined from equation (3.7).

Let us consider the case of $A > A^*$. The velocity of the boundary increases and when $\tau \to \infty$ strives to the limit $u \to p_m/A^*$. Without taking into account the effect of the free surface

$u \to 2p_m/A + A^*$. The pressure on the boundary p_4 and p_2^* from the side of the first and second media, respectively, is equal to:

$$p_4 = -h^* \varphi_4 + p_m = p_m - \frac{h^*}{m+h^*}[p_m - A\varphi_3(\tau^{**})]\, e^{-\frac{h^*(\tau-\tau^{**})}{m+h^*}}; \qquad (3.13)$$

$$p_2^* = A\varphi_4 = p_m - [p_m - A\varphi_3(\tau^{**})]\, e^{-\frac{A^*(\tau-\tau^{**})}{m+h^*}}. \qquad (3.14)$$

With the course of time the pressure tends toward p_m, and without consideration of the effect of the surface toward $(2A^*/A + A^*)p_m$.

Let us consider the case of $A < A^*$; then the velocity of the boundary falls, while in region 2* unloading of the medium occurs. The boundary of regions 1*, 2* is unknown. Let us designate it by $h = h(\tau)$. In region 1* from the relationships on the front of the wave we will find

$$p_1^*(\tau) = A^* u_1^*(\tau).$$

Hence because of (3.4) we obtain $F_2 = 0$. On the boundary $F_1(h^* - A\tau^*) = A^*\varphi_3(\tau)$. Let us designate

$$h^* - A^*\tau = \xi.$$

Then

$$F_1(\xi) = A^*\varphi_3\left(\frac{h^*-\xi}{A^*}\right).$$

Hence we will find the first function in region 1* in the form

$$F_1(h - A^*\tau) = A^*\varphi_3\left(\frac{h^*-h+A^*\tau}{A^*}\right). \qquad (3.15)$$

The system of equations determining the movement in region 4 has the form

$$\begin{aligned} -h(\tau)\ddot{y}(\tau) + \dot{\varphi}_2(\tau) &= A^*\varphi_3\left(\frac{h^*-h(\tau)+A^*\tau}{A^*}\right);\\ \dot{y}(\tau) &= \varphi_3\left(\frac{h^*-h(\tau)+A^*\tau}{A^*}\right);\\ m\ddot{y}(\tau) &= p_m - \dot{\varphi}_2(\tau). \end{aligned} \qquad (3.16)$$

The first two equations express the continuity of the pressure and velocity of the particles on line $h(\tau)$, dividing regions 1* and 2*, while the third corresponds to the equation of motion of the boundary. Solving the system relative to $h(\tau)$, we obtain

$$[m + h(\tau)]_3 = p_m - A^*\varphi_3, \tag{3.17}$$

where

$$\varphi_3[h(\tau), \tau] = \frac{2p_m}{A + A^*}\left\{1 - \left[\frac{A^*m}{A^*m + A(h^* + A\tau - h(\tau))}\right]^{\frac{A+A^*}{A}}\right\}.$$

The integration may be carried out numerically.

In conclusion, let us consider in more detail a case where the properties of the media on both sides of the boundary are identical, $A^* = A$. Then regions 4 and 2* are formed when $\tau^* = h^*/A$. When $\tau < \tau^*$ in region 3 the pressure on the boundary in conformance with (3.8) is determined by the expression

$$p = p_m\left[1 + \frac{m^2(m - A\tau)}{(m + A\tau)^3}\right], \tag{3.15'}$$

$$\frac{dp}{d\tau} = \frac{2Am^2(A\tau - 2m)p_m}{(m + A\tau)^4}. \tag{3.16'}$$

From (3.15') and (3.16') it follows that the pressure drops, while when $A\tau^{(1)} = m$ it is reduced to p_m. When $\tau^{(2)} = 2\tau^{(1)}$ it reaches a minimum, then increases and tends toward p_m. The pressure on the boundary on the side of the second medium is

$$p^* = A^*\varphi_3(\tau). \tag{3.17'}$$

As the calculations show, irrespective of the dependence of the value of the mass

$$p(\tau^{(2)}) = \frac{26}{27}p_m, \quad p^*(\tau^{(1)}) = \frac{5}{9}p_m, \quad p^*(\tau^{(2)}) = \frac{8}{9}p_m.$$

Henceforth $p^* \to p_m$, remaining less than the pressure from the boundary on the side of the first medium. The acceleration of the boundary is continually positive:

$$\dot{\varphi}_3(\tau) = \frac{2p_m}{m}\left(\frac{m}{m+A\tau}\right)^3. \tag{3.18}$$

In region 3 the acceleration with the passage of time tends towards zero. Here

$$\dot{\varphi}_3(\tau^{(1)}) = \frac{\dot{\varphi}_3(0)}{8}, \quad \dot{\varphi}_3(\tau^{(2)}) = \frac{\dot{\varphi}_3(0)}{64},$$

where $\dot{\Phi}_3(0) = 2p_m/m$ is the acceleration of the boundary at the moment of the approach of the incident wave to it, i.e., when $\tau = 0$.

Let us assume that $h^* = 4$ m, i.e., that the rarefaction wave, proceeding from the free surface and regions 4 and 2*, is formed when $\tau^* = h^*/A = 4\tau^{(1)}$. In region 2* when $A^* = A$ the load is continued and the pressure on the boundary on the side of the first and of the second media is determined by expressions (3.13) and (3.14). Calculations show that when $\tau = \tau^*$ the acceleration of the boundary falls abruptly from $(1/125)\dot{\Phi}_3(0)$ to $(1/250)\dot{\Phi}_3(0)$, while the pressure falls from $(122/125)p_m$ to $(121/125)p_m$. Here $p^* = (120/125)p_m$. Henceforth in region 4 we have $p \to p_m$, $p^* \to p_m$, $\dot{\Phi}_4 \to 0$.

In Fig. 5 curves 1, 2, and 3 correspond to the acceleration of the boundary $\dot{\Phi}(\tau)$, and to the pressures $p(\tau)$, $p^*(\tau)$. It can be seen from the graphs that the boundary up to the arrival of the rarefaction wave is set in motion together with the medium and the arrival of the rarefaction wave has virtually no effect on the movement of the boundary and the value of the loads experienced by it. The jumps $\dot{\Phi}$ and p when $\tau = \tau^*$ are so small, that they are not represented in this figure.

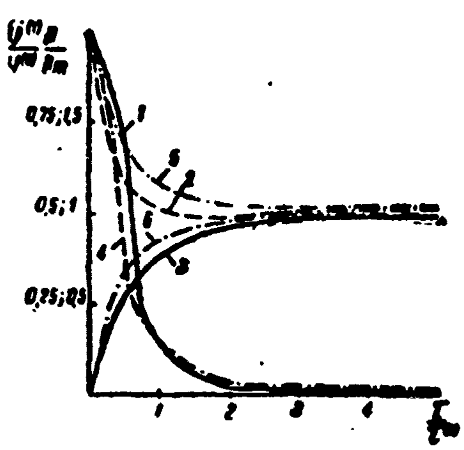

Fig. 5.

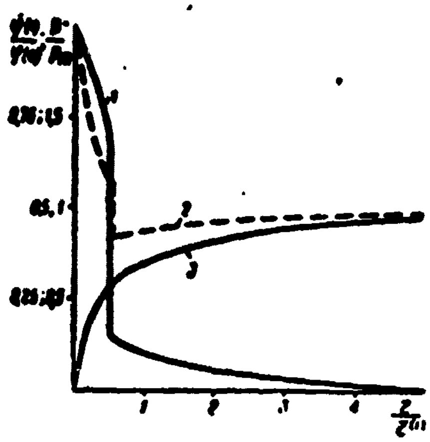

Fig. 6.

In Fig. 6 curves 1, 2, and 3 correspond to $\phi(\tau)$, $p(\tau)$, and $p^*(\tau)$. In contrast to the graphs in Fig. 5 it is assumed that $h^* = m/2$, i.e., the rarefaction wave arises eight times earlier, than when $h^* = 4m$. The arrival of the rarefaction wave leads to an abrupt drop in the acceleration and pressure p. Henceforth with the rise in time $\phi \to 0$, $p \to p_m$, $p^* \to p_m$.

From a comparison of the graphs in Figs. 5 and 6 it follows that with the reduction in distance h^* from the free surface to the boundary the effect of the rarefaction wave on the movement of the boundary increases. However, in the second case also this effect is not great.

With the passage of time, irrespective of the dependence of the distance h^* from the free surface to the boundary, the pressure on the boundary on both sides tends toward p_m, and the acceleration toward zero. The lower the mass of the boundary and the greater the acoustic resistance of medium A, the sooner stationary conditions set in.

Figure 5 shows curves 4, 5, and 6, corresponding to the acceleration and pressures p and p^*, calculated from the formulas of work [4] for the case of interaction of a stationary wave with a boundary of mass m in a linearly elastic medium. A comparison of curves 1, 2, 3 and 4, 5, 6 shows that the differences, correspondingly, between accelerations $\dot{\phi}$ and pressures p and p^* on the boundary in plastic and elastic media up till the arrival of the rarefaction wave from the free surface are not great. The effect of the free surface in an elastic medium is exerted twice as long afterward as in a plastic, and gives rise to an abrupt drop in pressure to a negative value. The difference in values

$\dot{\phi}$, p and p* in the region, where the influence of the free surface is exerted, and in elastic and plastic media is substantial.

Experiments show that in soils the effect of the free surface does not lead to pressure drops to a negative value. This indicates that nonwater-saturated soils behave as plastic media.

Thus, the application of models of elastic and plastic media in the solution of problems of the interaction of waves with the elements of edifices produces close results. However, this is true only in the region where the influence of the free surface is not exerted. In a region formed after the arrival of the rarefaction wave from the free surface, it is essential to take into consideration the plastic properties of the medium.

In concluding the report, the author expresses his graditude to N. I. Polyakova for her discussion of the work.

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