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FLIGHT CONTROL PRINCIPLES FOR CONTROL CONFIGURED VEHICLES

EDMUND G. RYNASKI

NORMAN C. WEINGARTEN

CORNELL AERONAUTICAL LABORATORY, INC.

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NORMAN C. WEINGARTEN

CORNELL AERONAUTICAL LABORATORY, INC.

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FOREWORD

The research documented in this report was performed for the Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio by the Flight Research Department of the Cornell Aeronautical Laboratory, Inc. (CAL), Buffalo, New York. This study was performed under Air Force Contract No. F33615-71-C-1238, Project No. 8226, "Advanced studies on the compatibility of maneuver load control and relaxed static stability applied to military aircraft". The Project Officer was Captain Bruce Kujawski (FGC) of the Flight Dynamics Laboratory. The CAL Project Engineer was Mr. Edmund Rynaski.

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This technical report has been reviewed and is approved.

GEORGE L. YINGLING
Chief, Advanced Development and
All-Weather Flying Branch
Flight Control Division

ABSTRACT

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1. An analysis was made of control surface combinations and their effectiveness for maneuver load control when used with an airplane having shortened tail length and reduced tail surface area.

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AR - b^2/S - wing aspect ratio

b - reference wing span, ft

b_H - horizontal tail span, ft

$\bar{c}$ - mean aerodynamic wing chord, ft

$\bar{c}_H$ - horizontal tail chord, ft

g - gravitational constant, 32.17 ft/sec^2

h - altitude, ft

I_{yy} - moment of inertia about y-body axis, slug-ft^2

K_A - $\frac{S_{H_{COV}}}{S_{H_{T-33}}}$ - tail area ratio

K_L - $\frac{\ell_{H_{COV}}}{\ell_{H_{T-33}}}$ - tail length ratio

K_p - proportional constant which relates drag to the square of lift in a parabolic drag polar

ℓ_H - distance from $\frac{\bar{c}}{4}$ to $\frac{\bar{c}_H}{4}$, tail length, ft

m - W/g - aircraft mass, slugs

M_B - wing root bending moment coefficient, ft/lb

n_z - normal acceleration, g's

p - roll rate, rad/sec

$\bar{q}$ - $\rho V^2/2$ - dynamic pressure, lb/ft^2

q - pitch rate, rad/sec

r - yaw rate, rad/sec

S - reference wing area, ft^2

S_H - horizontal tail area, ft^2

ΔV - perturbation velocity along x axis, ft/sec

V_0 - total velocity of airplane, ft/sec

W - aircraft weight, lb

α	- angle of attack, rad
β	- angle of sideslip, rad
δ_a	- aileron deflection, rad
δ_e	- elevator deflection, rad
δ_i	- inboard direct lift flap deflection, rad
δ_o	- outboard direct lift flap (collectively operated ailerons) deflection, rad
δ_r	- rudder deflection, rad
ζ_d	- Dutch roll damping ratio
ζ_{ph}	- phugoid damping ratio
ζ_{sp}	- short-period damping ratio
η	- y/b - wing station
θ	- pitch angle, rad
λ_i	- Lagrange multipliers used for constraints in performance indices
ρ	- air density, slug/ft^3
τ_R	- roll mode time constant, sec
τ_S	- spiral mode time constant, sec
ϕ	- bank angle, rad
ω_{n_d}	- Dutch roll undamped natural frequency, rad/sec
$\omega_{n_{ph}}$	- phugoid undamped natural frequency, rad/sec
$\omega_{n_{sp}}$	- short-period undamped natural frequency, rad/sec

ABBREVIATIONS

CCV	- control configured vehicle
MLC	- maneuver load control
RF	- response feedback
WRBM	- wing root bending moment

SECTION I

INTRODUCTION

1.1 BACKGROUND

The research documented in this report investigates the feasibility of implementation of several fundamental Controlled Configured Vehicle (CCV) concepts. The tail length and area of a typical Air Force inventory airplane (the T-33) were decreased to reduce the static stability and additional, active, wing-mounted control surfaces were used to provide a measure of control of the wing lift distribution. Control system design concepts were then used to evaluate the effectiveness of the maneuver load control concepts and to produce flight control system designs that can take advantage of the additional controllability of the vehicle and at the same time provide for adequate flying qualities. There is very little doubt that the geometry and the controllability of present day vehicles can be significantly improved as soon as fly-by-wire and control augmentation are fully accepted as integral parts of a flight control system.

Once feedback and command augmentation are fully accepted, then the full potential of the use of feedback can be and should be investigated and developed, for this knowledge can have a significant impact on the fundamental design of the airframe. For instance, inherent longitudinal or directional stability need not be built into the airframe if sufficient control surface effectiveness and power exist so that stability can be maintained by feedback. This is generally an easy requirement. A more difficult requirement is that, in addition to stability, the vehicle must satisfy flying qualities requirements. The research documented in this report places special emphasis on the flying qualities of the augmented airplane.

The importance of the use of active controllers in addition to the conventional moment producing devices, i.e., elevator, rudder and aileron, has been amply demonstrated. Operational and experimental aircraft now are using or investigating the proper use of active X-force control devices (auto-throttle systems), Z-force control (direct lift flaps or spoilers) and even Y-force control (side force surfaces, differential throttle or differential drag devices). With this acceptance of additional force and moment generating devices, it becomes important to investigate their use in the attainment of many desirable objectives of airplane design, such as control of the wing lift distribution (to reduce the critical wing root bending moment), structural mode control, ride qualities improvement, gust alleviation, and certainly, the dominant factor in all flight control system design, the flying qualities. The research documented in this report emphasizes the use of active surfaces such as a direct lift flap and collectively acting ailerons in addition to the elevator, to produce good flying qualities in addition to reduced wing root bending moment and minimum drag, both statically (in trim) and dynamically, during maneuvering flight.

In this report, the T-33 airplane was used as the object of the study. A fairly modest CCV treatment was applied to the vehicle: the tail length and size were assumed variable; the flap and ailerons, in addition to the elevator, were assumed to be actively controlled as force and moment generating devices. The object was generally to determine the tail length, size and surface motions that would satisfy the generally conflicting requirements of minimum change in wing root bending moment, minimum maneuver drag and minimum deviation from Level 1 flying qualities. Much more could have been, and eventually should be, done. The usefulness of additional force and moment devices, such as canard surfaces, spoilers, actively controlled thrust, sectioned flap and sectioned ailerons could have been included in the investigation. They were not included for three reasons:

1. It was felt that the configuration selected would demonstrate the feasibility. Feasibility rather than final design was considered to be the major objective of the program.

2. In this area, which represents a fairly radical departure from conventional design and development, careful, measured advances should be made.

3. Time and money for an extensive investigation were not available, and in fact should not have been made available at so preliminary a stage.

A second area in which the study was restricted was the area of more extensive geometrical changes in the airframe or the characteristics of the airframe. The tail area and length of the T-33 were considered changeable; these changes affected the stability derivatives of the airplane. The effects of other geometrical parameters were not investigated but eventually they should be considered in the CCV context.

Therefore, the study was restricted to feasibility rather than a thorough investigation of the potential of CCV concepts. Yet the results are both promising and gratifying. The wing root bending moment can be reduced; the drag can be controlled, and the T-33 airplane can be made to have Level 1 flying qualities with a shorter and smaller tail. Therefore, from the CCV point of view, the T-33 is overdesigned; weight, drag and structural loads can be reduced by using a feedback augmented flight control system. Since the demands of performance right now result in dynamically less well behaved airframes requiring augmentation to bring their dynamic behavior up to acceptability, this report takes the attitude that stability should not impose strong constraints on the geometry of the bare airframe. Feedback augmented flight control systems should provide the required stability. The research documented in this report shows how advanced flight control system design techniques can be used to realize many of the CCV concepts.

1.2 APPROACH AND ORGANIZATION

The systematic study of relaxed static stability, maneuver load control and attainment of good flying qualities was performed in four major steps:

1. The tail size and length were varied and combinations of elevator motion, inboard flap and collectively acting ailerons were used to force the vehicle to fly "exactly" as a flying qualities model. For different combinations of surface usage, the changes in drag, wing root bending moment and measures of control surface effort were obtained as the vehicle maneuvered through a 4 g pullup "exactly" as the flying qualities model. These calculations verified the level of utility of using the additional surfaces. This research is described in Section II.

2. A feedback flight control system was then designed to minimize a weighted measure of incremental wing root bending moment, incremental drag, error in dynamical response between the actual aircraft and the flying qualities model, and control activity. Familiar linear optimal control techniques were used to obtain the solutions. The effect of variations of the weighting of the different elements in the performance index was investigated to some extent. The purpose was to verify that the different weighting could be successfully "juggled" rather than obtaining a final, or "about-to-be-mechanized" design. Section III of the report describes this phase of the study.

3. A direct parameter minimization of the performance index was tried. The purpose was to try to directly obtain values of tail length, size and control deflections that would minimize a weighted measure of incremental wing root bending moment, incremental drag and minimum deviation from Level 1 flying qualities behavior during a 4 g pullup. This effort is detailed in Section IV of the report.

4. Based upon the more than twenty years of experience with feedback fly-by-wire systems at the Cornell Aeronautical Laboratory, maximum attainable feedback gains for the present state of the art of flight control system design were estimated. Checks were made to assure that the feedback designs defined in Section III of the report appeared feasible.

SECTION II

ANALYSIS OF RELAXED STATIC STABILITY AND MANEUVER LOAD CONTROL SURFACE REQUIREMENTS

2.1 INTRODUCTION

The first logical step in the alteration of an existing airplane to achieve the objectives of a Control Configured Vehicle is the determination of the effects of varying the tail length and area on the stability of the vehicle and the ability of the chosen force and moment generating devices to maneuver the airplane properly. If the aerodynamic characteristics of the airframe do not inherently provide the proper forces and moments, then the surfaces must be dynamically moved in a way to compensate for the loss in aerodynamically generated forces and moments. The analysis of geometry changes and surface adequacy was accomplished in the following manner:

1. The equations of motion of the T-33 airplane as a function of tail length and area were developed.

2. The equations of motion of two Level 1 aircraft mathematical models were derived from MIL-F-8785(B).

3. The changes in drag and wing root bending moment were developed as a function of tail length, tail area, and elevator, flap and collectively acting aileron deflections.

4. Combinations of these three surfaces were then deflected in a way that forced the T-33 to respond to a 4 g pullup command as the flying qualities model would respond.

5. Evaluations were then made of the effectiveness and usefulness of the surfaces in reducing maneuver drag, wing root bending moment and relative surface deflection.

2.2 DERIVATION OF T-33 EQUATIONS OF MOTION

<u>Equations of Motion and Dimensional Data</u>

The longitudinal equations of motion for the T-33 airplane used as the example in this CCV study are as follows:

$$\Delta\dot{V} = -g\theta - \frac{\bar{q}S}{m}\left(\left\{\frac{2}{V_0}C_{D_t} + C_{D_{\Delta V}}\right\}\Delta V + C_{D_{0|\delta_e|}}|\delta_e| + C_{D_{0|\delta_i|}}|\delta_i| + C_{D_{0|\delta_o|}}|\delta_o| + K_p\left\{C_{L_\alpha}\alpha + C_{L_{\delta_i}}\delta_i + C_{L_{\delta_o}}\delta_o\right\}^2 + K_p\left\{C_{L_{\delta_e}}\delta_e\right\}^2\right) \quad (1)$$

$$\dot{\alpha} = \dot{\theta} - \frac{\bar{q}S}{V_0 m}\left(\left\{\frac{2}{V_0}C_{L_t} + C_{L_{\Delta V}}\right\}\Delta V + \left\{C_{L_\alpha} + \frac{T_t}{\bar{q}S}\right\}\alpha + C_{L_{\delta_e}}\delta_e + C_{L_{\delta_i}}\delta_i + C_{L_{\delta_o}}\delta_o\right) \quad (2)$$

$$\ddot{\theta} = \frac{\bar{q}S\bar{c}}{I_{yy}}\left(\frac{\bar{c}}{2V_0}C_{m_q}\dot{\theta} + \frac{\bar{c}}{2V_0}C_{m_{\dot{\alpha}}}\dot{\alpha} + C_{m_{\Delta V}}\Delta V + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e + C_{m_{\delta_i}}\delta_i + C_{m_{\delta_o}}\delta_o\right) \quad (3)$$

For some of the calculations the following linearized drag equation was used:

$$\dot{u} = -g\theta - \frac{\bar{q}S}{m}\left(\left\{\frac{2}{V_0}C_{D_t} + C_{D_{\Delta V}}\right\}\Delta V + C_{D_\alpha}\alpha + C_{D_{\delta_e}}\delta_e + C_{D_{\delta_i}}\delta_i + C_{D_{\delta_o}}\delta_o\right) \quad (4)$$

Two flight conditions with different values of n_z/α were investigated, though most of the work was performed with the first flight condition (FC-1). However, the methods used would apply to the second as well.

	FC-1	FC-2
V_0	641 ft/sec	414 ft/sec
h	10,000 ft	20,000 ft
$\bar{q}$	360 lb/ft^2	108 lb/ft^2
W	12,000 lb	15,000 lb
I_{yy}	20,700 slug-ft^2	22,000 slug-ft^2

The dimensional data common to both flight conditions for the CCV-T-33 are:

wing: area, S = 234.8 ft^2

span, b = 37.5 ft

chord, $\bar{c}$ = 6.72 ft

taper ratio = .355

sweepback of quarter cord, $\Lambda_{c/4}$ = 5.0°

AR = 6.

airfoil section = NACA 65_1-213, a = .5

δ_i - It has been assumed during the course of this study that the inboard flap can readily be changed into an active control device, requiring a change from the present split flap arrangement to a simple flap with the following characteristics:

area, each side $S_{\delta_i} = 15.32 \text{ ft}^2$

chord, $\bar{c}_{\delta_i} = 1.8 \text{ ft}$

wing station: 0 ft - 8.5 ft

δ_0 - It has also been assumed that the T-33 ailerons can be made to act collectively to a longitudinal stick command. These outboard direct lift flaps have the following characteristics:

area, each side $S_{\delta_i} = 8.75 \text{ ft}^2$

chord, $\bar{c}_{\delta_i} = 1.17 \text{ ft}$

wing station: 8.5 ft - 16.0 ft

<u>Horizontal tail:</u> area, $S_H = 43.5 \text{ ft}^2$

span, $b_H = 15.58 \text{ ft}$

chord, $\bar{c}_H = 3.12 \text{ ft}$

taper ratio = .32

$AR = 5.6$

horizontal tail length $\left(\frac{\bar{c}}{4} \text{ to } \frac{\bar{c}_H}{4}\right)$, $l_H = 15.9 \text{ ft}$

horizontal tail volume, $V_H = \frac{S_H}{S} \cdot \frac{l_H}{\bar{c}} = .438$

<u>δ_e - elevator</u>

area, each side $S_{\delta_e} = 8.7 \text{ ft}^2$

chord, $\bar{c}_{\delta_e} = .71 \text{ ft}$

<u>vertical tail:</u>

area, $S_V = 22.6 \text{ ft}^2$

vertical tail length $\left(\frac{\bar{c}}{4} \text{ to } \frac{\bar{c}_V}{4}\right)$, $l_V = 16.1 \text{ ft}$

vertical tail volume $V_V = \frac{S_V}{S} \frac{l_V}{b} = .041$

Stability Derivatives of the Basic CCV - T-33

All of the nondimensional stability derivatives, except the δ_i and δ_o control derivatives, for the basic T-33 were obtained from Reference 1. The control derivatives for the inboard and outboard flaps, δ_i and δ_o, were estimated from Air Force Datcom methods (Reference 2) using the dimensional data above. These stability derivatives are presented in Table I.

TABLE I

T-33 STABILITY AND CONTROL DERIVATIVES

Parameter	FC-1	FC-2	Parameter	FC-1	FC-2
C_{D_0}	0.019	0.019	C_{L_0}	0.15	0.15
C_{D_u}	0.117	0.106	C_{L_α}	6.50	5.90
$C_{D_0\|\delta_e\|}$	0.0111	0.0111	$C_{L_{\delta_e}}$	0.362	0.343
$C_{D_{\delta_e}}$	0.0176	0.0173	$C_{L_{\delta_i}}$	1.64	1.64
$C_{D_0\|\delta_i\|}$	0.0408	0.0408	$C_{L_{\delta_o}}$	1.065	1.065
$C_{D_{\delta_i}}$	0.0703	0.0703	C_{m_0}	-0.010	-0.009
$C_{D_0\|\delta_o\|}$	0.0192	0.0192	C_{m_α}	-0.690	-0.590
$C_{D_{\delta_o}}$	0.0384	0.0384	$C_{m_{\dot\alpha}}$	-3.30	-3.10
K_P	0.06	0.06	C_{m_q}	-7.50	-6.90
C_{D_t}	0.019	0.031	$C_{m_{\delta_e}}$	-0.94	-0.90
C_{L_t}	0.142	0.59	$C_{m_{\delta_i}}$	-0.524	-0.524
$C_{D_{\Delta V}}$	0.00	0.00	$C_{m_{\delta_o}}$	-0.199	-0.199
$C_{m_{\Delta V}}$	0.00	0.00	n_z/α	45.00	10.00
$C_{L_{\Delta V}}$	0.00	0.00			

Stability Derivatives in Terms of K_A and K_L

The horizontal tail area and tail length of the T-33 were altered to investigate the various CCV configurations. The following parameters are functions of the tail size:

C_{D_0}	C_{L_α}
$C_{D_0\|\delta_e\|}$	C_{m_α}
$C_{L_{\delta_e}}$	$C_{m_{\dot\alpha}}$
$C_{m_{\delta_e}}$	C_{m_q}

Expressions for these parameters were developed in terms of the horizontal tail area ratio and tail length ratio of the particular CCV configuration under study and the basic T-33 values, where:

$$K_A = \frac{S_{H_{CCV}}}{S_{H_{BASIC\ T\text{-}33}}} \text{, tail area ratio} \tag{5}$$

$$K_L = \frac{\ell_{H_{CCV}}}{\ell_{H_{BASIC\ T\text{-}33}}} \text{, tail area ratio} \tag{6}$$

Methods from Reference 2 were used to estimate the tail contributions to the above derivatives. It was assumed that the dynamic pressure ratio at the tail was 1.0 as the horizontal tail is more than five feet above the wing and should be out of the wake of the wing. However, some of the derivatives were a function of the downwash at the tail which was estimated from Datcom methods as:

$$\frac{\partial \epsilon}{\partial \alpha} = .484\,(K_L)^{-.4} \qquad \text{(Datcom Sec. 4.4.1)}$$

The expressions for the parameters influenced by the tail size are listed below with the corresponding Datcom sections which were used in their derivation:

Parameter	Datcom Section	FC-1	FC-2
C_{D_o}	4.15	$.0177+.0013\,K_A$	$.0177+.0013\,K_A$
$C_{D_{o\|\delta_e\|}}$	just used area and length ratio multiples of basic δ_e derivatives	$.0111\,K_A$	$.0111\,K_A$
$C_{L_{\delta_e}}$		$.362\,K_A$	$.343\,K_A$
$C_{m_{\delta_e}}$		$-.94\,K_A K_L$	$-.90\,K_A K_L$
C_{L_α}	4.5.1.1	$6.0+[.968-.468(K_L)^{-.4}]K_A$	$5.45+[.879-.425(K_L)^{-.4}]K_A$
C_{m_α}	4.5.2.1	$.49-[2.29-1.11(K_L)^{-.4}]K_A K_L$	$.48-[2.08-1.01(K_L)^{-.4}]K_A K_L$
$C_{m_{\dot\alpha}}$	7.4.4.2	$1.96-5.24\,K_A(K_L)^{1.6}$	$1.65-4.75\,K_A(K_L)^{1.6}$
C_{m_q}	7.4.1.2	$3.35-10.85\,K_A(K_L)^2$	$2.95-9.85\,K_A(K_L)^2$

Characteristics of the T-33 as a Function of K_A and K_L

The short period characteristics of the T-33 were calculated for various K_A and K_L values, to see what the bare airframe characteristics were without augmentation. The tail ratios, K_A and K_L, were varied together and kept at identical values. A simultaneous reduction in both tail length and tail size, with corresponding reduction in elevator area, could be considered to be a fairly severe loss of stability as well as control capability of the airplane. However, it was felt that the loss in control effectiveness and control power could be maintained with an all-movable horizontal surface. In addition, it was found that the net effect of inboard flap deflections and outboard flap deflections would generally be in a direction that supplements the pitching moment capability of the elevator during pullup maneuvers. The best values of K_A and K_L investigated separately are, of course, an important CCV concept, but for this study, the simultaneous variation of these parameters vividly demonstrated the problems and design principles. At these values of interest they both had approximately the same effect on the moment derivatives which directly affect the short period mode.

The short period frequency and damping and n_z/α for these configurations were estimated by the following approximations (Reference 3):

$$\omega_{SP} = M_q Z_w - M_\alpha$$

$$\zeta_{SP} = -\frac{(M_q + M_{\dot\alpha} + Z_w)}{2\omega_{SP}}$$

$$\frac{n_z}{\alpha} = -\frac{u_0}{g} Z_w$$

The change in mass and moment of inertia, I_{yy}, for the CCV configuration with K_A and K_L of .5 from the normal T-33 were estimated at a negative three percent and negative ten percent respectively. It is assumed that these vary approximately linearly for intermediate values.

The short period characteristics and n_z/α for the various configurations are listed in Table II and shown in Figure 1.

TABLE II

VARIATIONS IN FLYING QUALITIES PARAMETERS

AS A FUNCTION OF K_A AND K_L

K_A, K_L	FC-1			FC-2		
	ω_{sp} ~ rad/sec	ζ_{sp}	n_z/α	ω_{sp} ~ rad/sec	ζ_{sp}	n_z/α
1.0	4.627	.4155	45.67	2.215	.3167	9.96
.875	3.420	.4538	45.19	1.605	.3473	9.85
.75	1.810	.6932	44.66	.7062	.6207	9.73
				real roots:		
.725	1.325	.9087	44.57	-.1855	-.6513	9.71
	real roots:					
.70	-.1247	-2.183	44.47	.2350	-1.033	9.69
.625	1.192	-3.232	44.19	.8291	-1.522	9.63
0	2.36	-4.05	43.19	1.382	-1.934	9.52

Derivation of Wing Root Bending Moment and Drag During Maneuvers

To determine the wing root bending moment (WRBM) developed in a maneuver it is first necessary to calculate the spanwise lift distribution over the wing and then find the equivalent force and moment arm to which this distribution is equivalent. (It was assumed that the lift distribution for the wing due to angle of attack and the incremental lift distributions due to flap deflections are linear with α, δ_i, and δ_o and are simply additive with no interference effects between control surfaces. This restricts the validity of the solutions to the linear range of C_{L_α} ($|\alpha| \leq 15°$) and $|\delta| \leq 25°$. This is a simplification that was felt to be justifiable for the depth of investigation considered in this study.)

The shapes of the lift distribution curves due to α, δ_i, and δ_o were estimated from methods in Reference 2 (Datcom Section 6.1.5.1). These are shown in Figure 2. The area under each curve can be treated as the total lift increment due to a particular α, δ_i, or δ_o deflection, respectively. The spanwise lift distribution times span position is shown in Figure 3. The area under each curve is equivalent to the incremental WRBM due to a particular α, δ_i, or δ_o deflection. When the areas under the curves in the second set are divided by the areas in the first set, the result is the moment arm, or the position at which an equivalent force will produce the same WRBM as the lift distribution did.

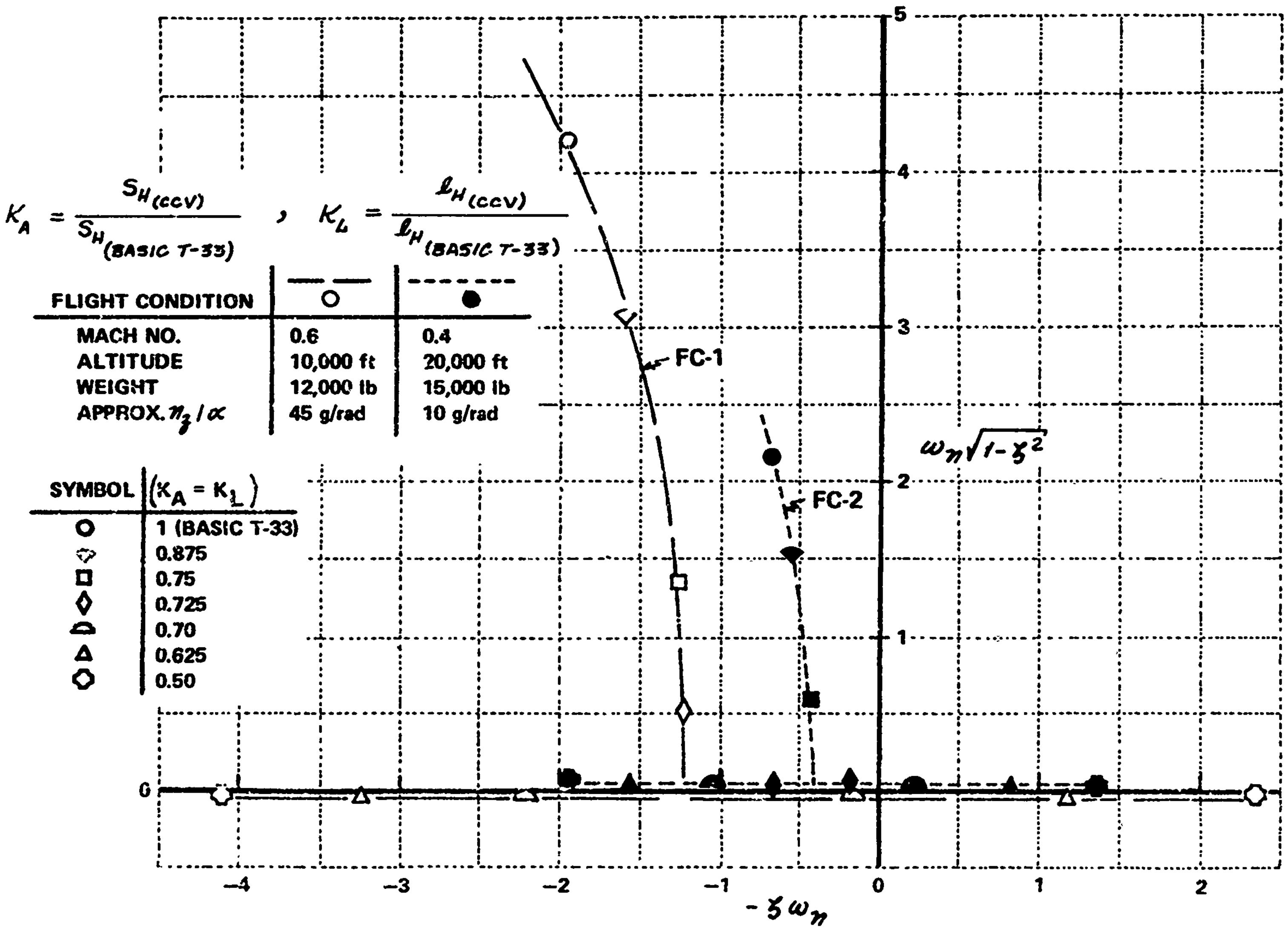


Figure 1 ROOT LOCUS OF CCV-T-33 SHORT PERIOD POLES vs TAIL AREA & LENGTH RATIOS

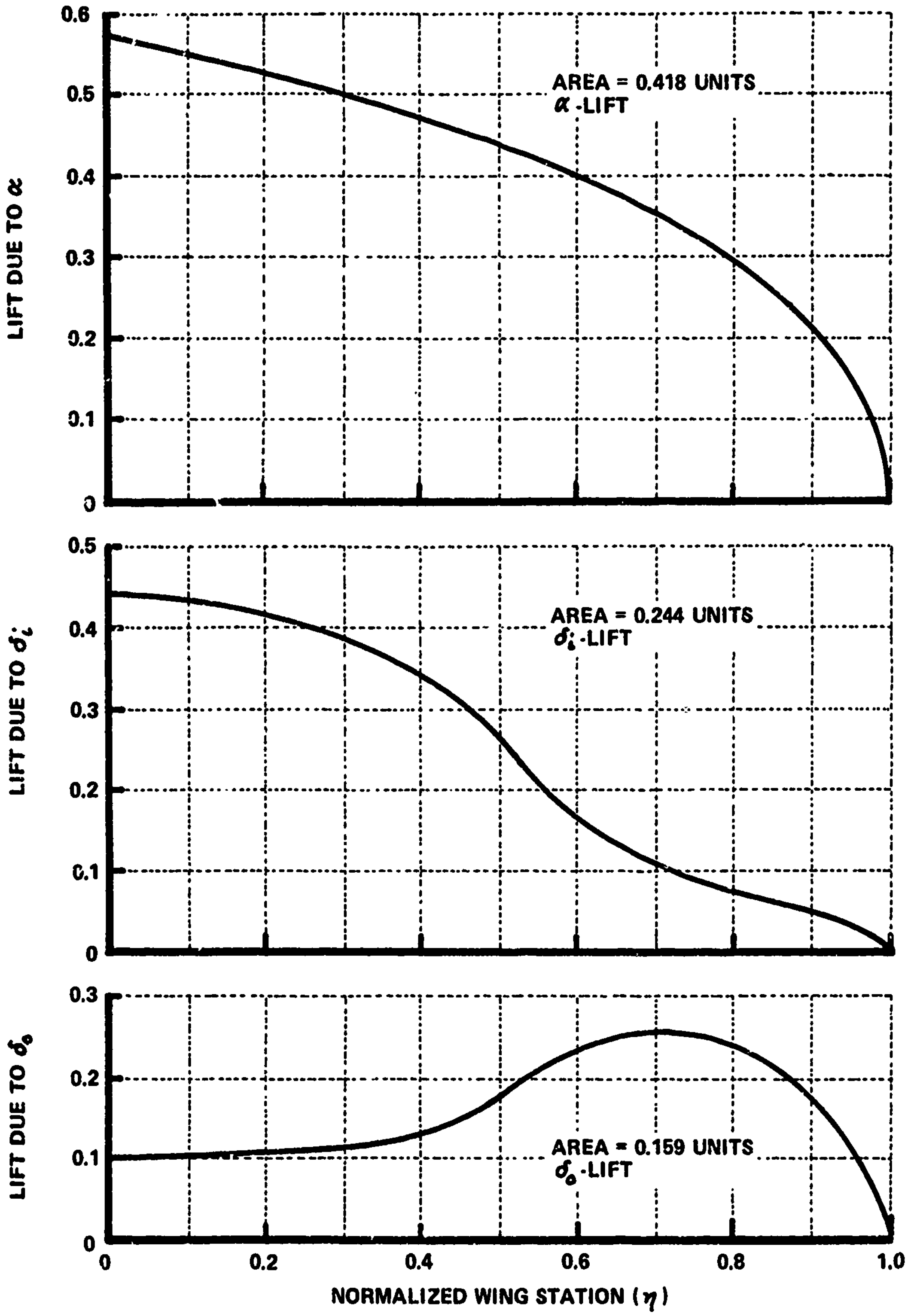


Figure 2 WING LIFT DISTRIBUTIONS FOR α, δ_i, δ_o

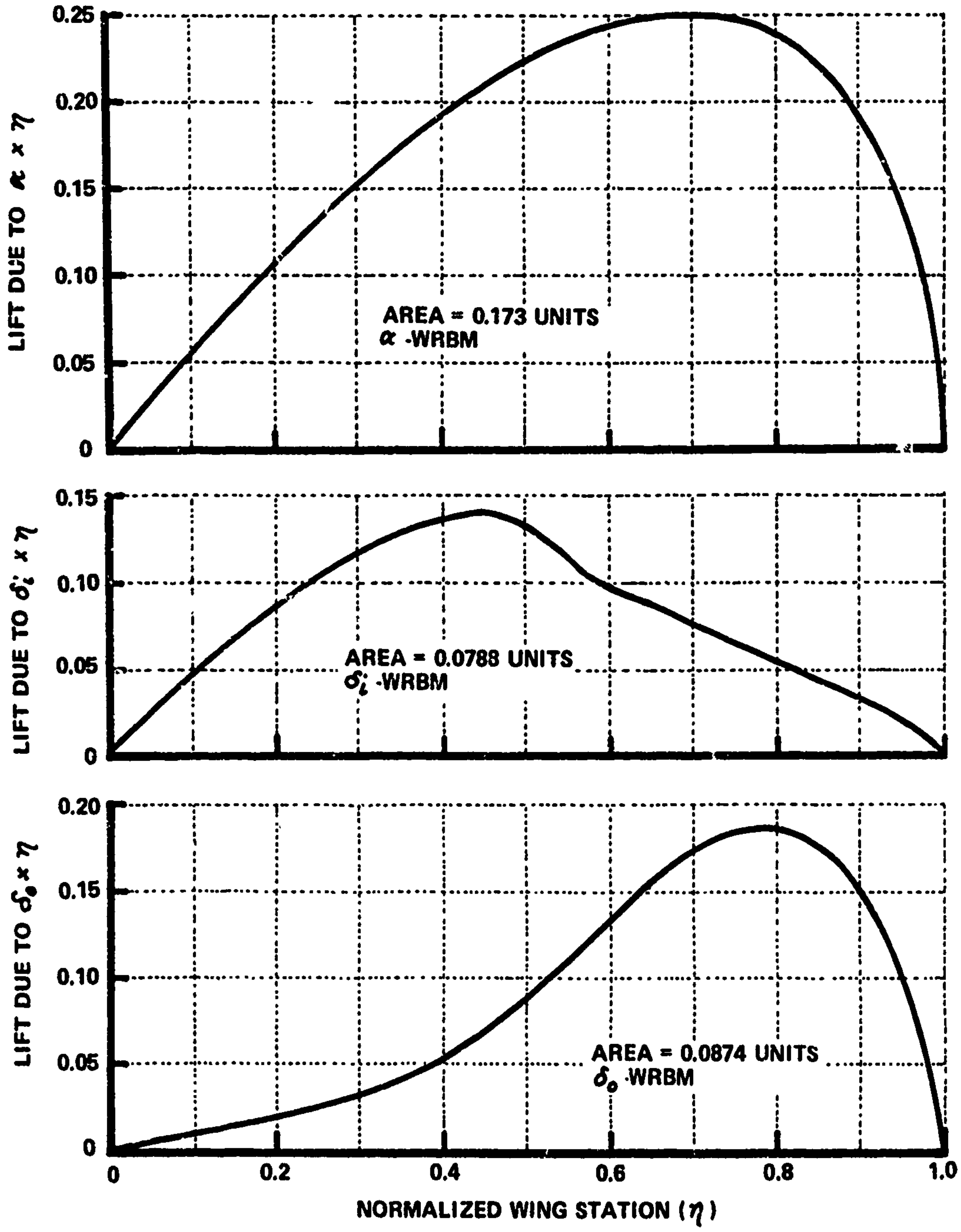


Figure 3 WING ROOT BENDING MOMENT CONTRIBUTION vs WING STATION

The above procedure was carried out on the accompanying figures with a planimeter with the following results:

Lift distribution due to:	Lift area	Lift x span position area (WRBM)	Equivalent WRBM arm
α	.418	.173	.414b = 6.62 ft
δ_i	.244	.0788	.323b = 5.16 ft
δ_o	.159	.0874	.548b = 8.77 ft

The incremental WRBM derivatives for each wing are given below. The moment arm associated with M_{B_0} and M_{B_α} are the same because the lift at $\alpha = 0$ has the same distribution as the lift at $\alpha \neq 0$.

$$M_{B_0} \text{ (zero angle of attack) } = \left(\tfrac{1}{2} C_{L_0} \bar{q} S\right) \times (6.62 \text{ ft})$$
$$M_{B_\alpha} = \left(\tfrac{1}{2} C_{L_\alpha} \bar{q} S\right) \times (6.62 \text{ ft})$$
$$M_{B_{\delta_i}} = \left(\tfrac{1}{2} C_{L_{\delta_i}} \bar{q} S\right) \times (5.16 \text{ ft})$$
$$M_{B_{\delta_o}} = \left(\tfrac{1}{2} C_{L_{\delta_o}} \bar{q} S\right) \times (8.77 \text{ ft})$$

	FC-1	FC-2
M_{B_0}	41,970 ft-lb	12,590 ft-lb
M_{B_α}	1,460,500 ft-lb/rad	397,860 ft-lb/rad
$M_{B_{\delta_i}}$	357,650 ft-lb/rad	107,300 ft-lb/rad
$M_{B_{\delta_o}}$	394,750 ft-lb/rad	118,420 ft-lb/rad

The increment in WRBM developed in a maneuver is then written as

$$\Delta WRBM = M_{B_\alpha}(\alpha - \alpha_{TRIM}) + M_{B_{\delta_i}} \delta_i + M_{B_{\delta_o}} \delta_o \tag{7}$$

The incremental drag developed during a maneuver is defined by the following nonlinear equation:

$$\begin{aligned} \Delta Drag = \bar{q} S \Big(& C_{D_{0_{|\delta_e|}}} |\delta_e| + C_{D_{0_{|\delta_i|}}} |\delta_i| + C_{D_{0_{|\delta_o|}}} |\delta_o| \\ & + K_p \left\{ C_{L_\alpha} \alpha + C_{L_{\delta_i}} \delta_i + C_{L_{\delta_o}} \delta_o \right\}^2_{\text{(from induced drag on wing)}} \\ & + K_p \left\{ C_{L_{\delta_e}} \delta_e \right\}^2_{\text{(from induced drag on horizontal tail)}} \Big) \end{aligned} \tag{8}$$

In some instances it was necessary to use a linear drag equation to simplify the calculations:

$$\Delta\,Drag \cong \bar{q}\,S\left(C_{D_{\delta_e}}\delta_e + C_{D_{\delta_i}}\delta_i + C_{D_{\delta_o}}\delta_o + C_{D_\alpha}\alpha\right) \tag{9}$$

2.3 DERIVATION OF AN IDEAL FLYING QUALITIES MODEL

Good flying qualities, as defined in MIL-F-8785B, is one of the major objectives of this study. Flight control system designs will be derived that will augment the airplane in such a way that the augmented vehicle response will be identical to or a close approximation to a model having "ideal" flying qualities. In this section, the equations of motion of the ideal model, as applied to this particular airframe, the geometrically altered T-33, are derived.

From MIL-F-8785B (Reference 4), an excellent Level 1 longitudinal flying qualities airplane would possess the following short period characteristics:

$$\frac{\omega_{sp}^2}{n_z/\alpha} = 1 \qquad \zeta_{sp} = 0.7 \tag{10}$$

Two models were developed: one for an airplane with a normal lift slope, $C_{L_\alpha} \approx 2\pi$ rad^{-1} (low lift model); and one for an exceptionally high lift airplane, $C_{L_\alpha} \approx 4\pi$ rad^{-1} (high lift model). The drag polar, velocity derivatives, and control derivatives were assumed to be the same as the normal T-33. There are then only four derivatives that need to be found:

$$Z_w\ ,\ M_q\ ,\ M_\alpha\ ,\ M_{\dot\alpha}$$

It was first assumed that $M_q = 2\,M_{\dot\alpha}$ for this is true for the basic T-33 and is a very good approximation for many other aircraft. Then the following equations can be written:

$$\omega_{sp}^2 = 2M_{\dot\alpha}Z_w - M_\alpha \tag{11}$$

$$\zeta_{sp} = \frac{-(3M_{\dot\alpha} + Z_w)}{2\omega_{sp}} \tag{12}$$

$$Z_w = -\frac{\bar{q}\,S}{m u_0}C_{L_\alpha} \tag{13}$$

$$\frac{n_z}{\alpha} = -\frac{u_0}{g}Z_w \tag{14}$$

From the relations $\omega_{sp}^2 = n_z/\alpha$ and $\zeta_{sp} = .7$ for an ideal Level 1 short-period model and the above equations, all of the derivatives of the ideal flying qualities model can be evaluated. Speed stability was not investigated in this study. However, the phugoid roots of the models, which are approximately those of the T-33, were evaluated and were also Level 1.

TABLE III

STABILITY DERIVATIVES AND OTHER CHARACTERISTICS OF TWO LEVEL 1 MODELS

Flight Condition	Low Lift Model		High Lift Model	
	FC-1	FC-2	FC-1	FC-2
C_{L_α}, 1/rad	6.28	6.28	14.32	14.32
n_z/α, g/rad	44.12	10.60	100.7	24.2
ω_{sp}, rad/sec	6.64	3.26	10.03	4.92
ζ_{sp}	.7	.7	.7	.7
Z_w, sec^{-1}	-2.214	-.8237	-5.045	-1.878
M_α, sec^{-2}	-33.661	-8.549	-70.42	-17.93
M_q, $sec^{-1}rad^{-1}$	-4.723	-2.490	-6.00	-3.34
$M_{\dot\alpha}$, sec^{-1}	-2.362	-1.245	-3.00	-1.67
ω_{ph}, rad/sec	.06	.10	.06	.09
ζ_{ph}	.11	.04	.12	.04

The above models are labeled on Figure 4 along with the unaugmented CCV - T-33 configurations. It shows how these configurations compare to MIL-F-8785B short period frequency requirements for Category A Flight Phases (air-to-air combat, weapon delivery, etc.). It can be seen that the normal T-33 ($K_A = K_L = 1$) is close to an ideal model, while the flying qualities deteriorate quickly as K_A and K_L are reduced below .85.

Figure 5 shows digitally computed time histories of both the low lift and high lift model responses. The primary effect of n_z/α can be seen in the transient of the pitch rate response and in the steady state value of $\Delta\alpha$. High n_z/α means higher Z_w which shows up as a smaller overshoot in the pitching rate response to a command elevator input.

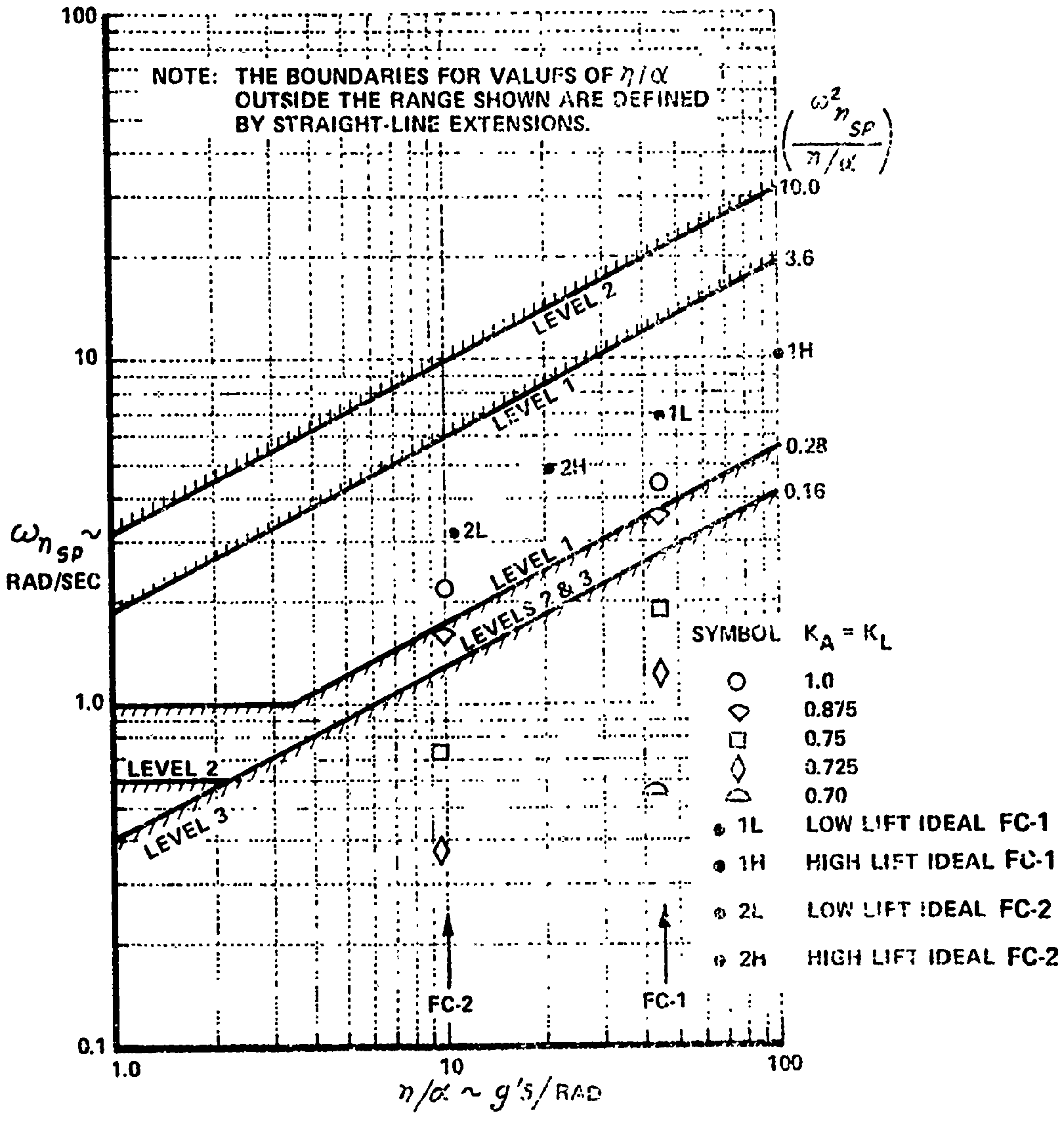
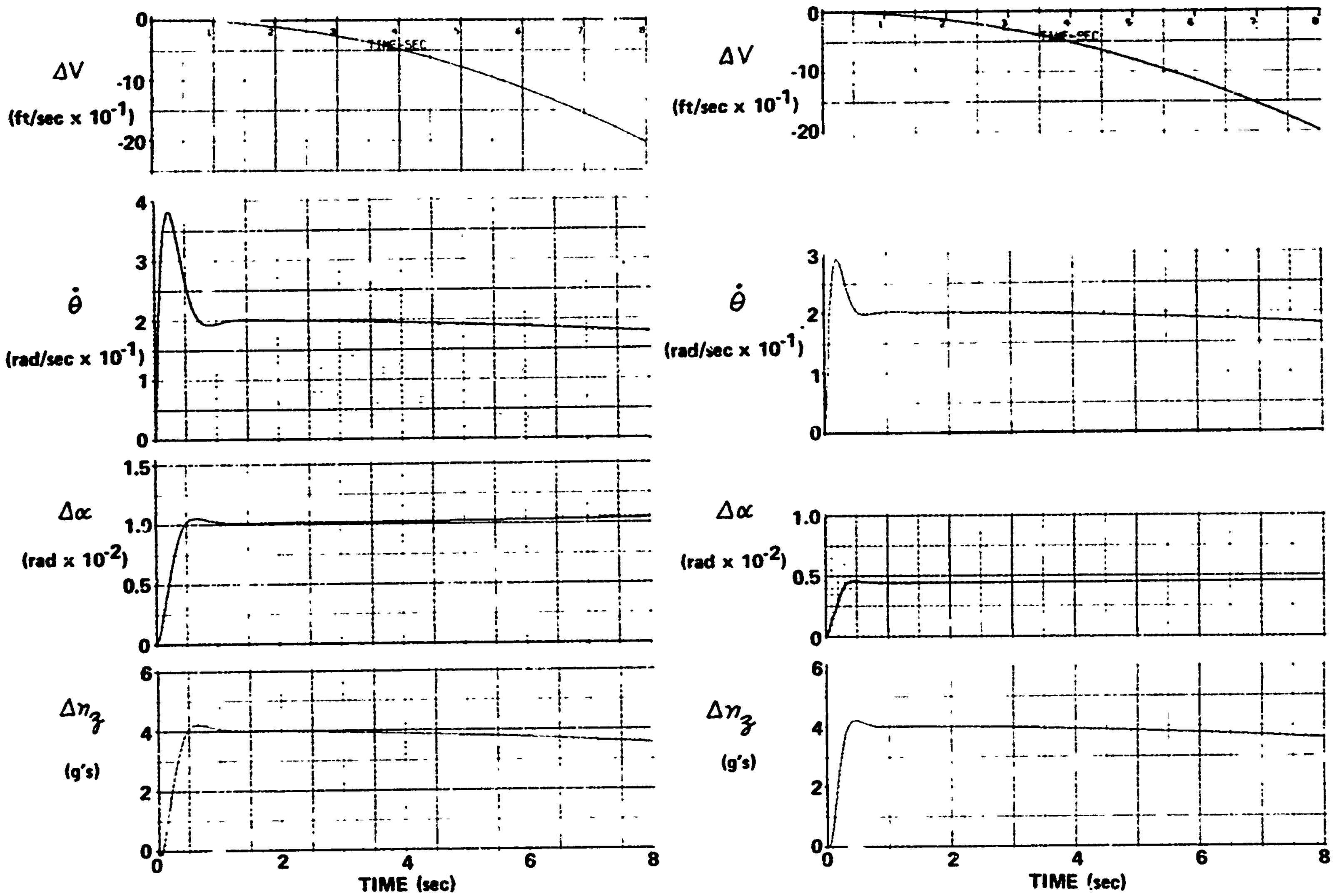


Figure 4 SHORT PERIOD FREQUENCY REQUIREMENTS -- CATEGORY A FLIGHT PHASES



LOW LIFT ($n_z/\alpha = 44.12$)　　HIGH LIFT ($n_z/\alpha = 100.7$)

Figure 5 TRANSIENT RESPONSE OF LOW LIFT AND HIGH LIFT LEVEL 1 MODELS AT FC-1

2.4 DERIVATION OF SURFACE DEFLECTIONS REQUIRED TO MATCH THE MODEL RESPONSE

The flying qualities model defines the equations of motion of a Level 1 airplane. Evaluation of surfaces or other force and moment generating devices is accomplished by obtaining the deflections required to force the CCV - T-33 airplane with altered tail length and size to respond "exactly" as the flying qualities model responds. Three control surfaces: elevator, inboard flap and collectively acting ailerons were used in various combinations to force the altered airplane to respond as the model.

The flying qualities model represents three degrees of freedom of motion; if fewer than three independent force and moment generating devices are used for control, the response of the flying qualities model cannot be exactly reproduced; only one state variable (and derivatives) per controller can, in general, be made to respond as the flying qualities model would respond. The calculations for the control motions are done in the following way:

The small perturbation equations of motion for an aircraft can be written in the state-vector form

$$\dot{x} = Fx + Gu \qquad (15)$$

where

x is a vector representation of the state variables of the vehicle. For this aircraft, the state vector $x^T = [\alpha, \theta, \dot{\theta}, \Delta V]$ has been chosen.

u is a vector representing the control variables of the vehicle; in this case the control vector elements are defined from elevator deflection, δ_e , flap deflection, δ_i , and collectively acting ailerons, δ_a . $u^T = [\delta_e, \delta_i, \delta_a]$

F is a square (4 x 4) matrix of dimensional stability derivatives of the airplane. A coefficient of this matrix, when multiplied by a state variable and an appropriate inertia or mass, represents a moment or force applied to the vehicle due to the configuration of the vehicle.

G is a matrix of dimensional control derivatives of the vehicle. A coefficient of this matrix, when multiplied by a control surface deflection and an appropriate mass or inertia, represents a force or moment applied to the aircraft by a surface deflection.

The equations of motion (15) are partitioned:

$$\begin{bmatrix} \dot{x}_1 \\ \hline \dot{x}_2 \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ \hline x_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ \hline G_2 \end{bmatrix} u \qquad (16)$$

The partitioning is selected so that F_{22} and G_2 are of the same p x p dimension when p represents the number of independent controllers on the vehicle. The equations of motion of the flying qualities model are similarly partitioned:

$$\begin{bmatrix} \dot{x}_{1_m} \\ \hline \dot{x}_{2_m} \end{bmatrix} = \begin{bmatrix} F_{11_m} & F_{12_m} \\ F_{21_m} & F_{22_m} \end{bmatrix} \begin{bmatrix} x_{1_m} \\ \hline x_{2_m} \end{bmatrix} + \begin{bmatrix} G_{1_m} \\ \hline G_{2_m} \end{bmatrix} u_m \qquad (17)$$

where $x_{2_m}(t)$ represents the motions or state variables of the flying qualities model that we wish the aircraft to reproduce.

The control law is of the following form

$$u(t) = K_1 \dot{x}_{2_m}(t) + K_2 x_{2_m}(t) - K_3 x_1(t) \qquad (18)$$

Substituting Equation 18 into Equation 16 yields

$$\dot{x}_2 = (F_{21} - G_2 K_3)x_1 + F_{22} x_2 + G_2 K_2 x_{2_m} + G_2 K_1 \dot{x}_{2_m} \qquad (19)$$

Taking the Laplace transform of (19) and rearranging yields

$$(Is - F_{22}) x_2(s) = (F_{21} - G_2 K_3) x_1(s) + (G_2 K_1 Is + G_2 K_2) x_{2_m}(s) \qquad (20)$$

The substitution of the following gains

$$\begin{aligned} K_1 &= (G_2^T G_2)^{-1} G_2^T \\ K_2 &= -(G_2^T G_2)^{-1} G_2^T F_{22} \\ K_3 &= (G_2^T G_2)^{-1} G_2^T \end{aligned} \qquad (21)$$

into Equation 20 yields

$$(Is - F_{22}) x_2(s) = (Is - F_{22}) x_{2_m}(s) \qquad (22)$$

and we have the desired result: namely that $x_2(t) = x_{2_m}(t)$.

The system is shown in block diagram form in Figure 6 below.

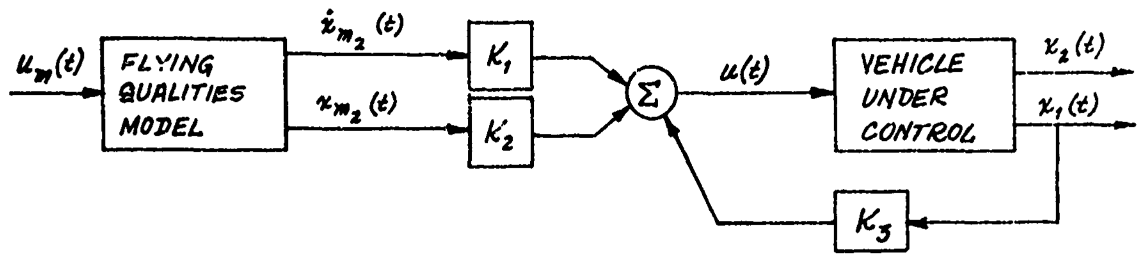


Figure 6 MODEL FOLLOWING SYSTEM

Although the computation to determine the control system deflections, $u(t)$ actually involves the definition of a control law, and incidentally a control system design, the design is generally not a good one and would not normally be mechanized. The feedback gains K_3 can be destabilizing and because the subset $x_2(t)$ of the state vector is not required to be fed back, the system design would be sensitive to poorly known or varying stability derivatives. However, if the same number of controllers as degrees of freedom of motion were available, then the model following design of Figure 6 is practical and any feedback, presumably the feedback that leads to a stable, insensitive system, would form the basis for a model following flight control system design.

2.5 RESULTS OF THE STUDY

The curves of Figures 7, 8, and 9 and Table IV summarize the results of using the model-following technique to obtain the control surface deflections required to perform a 4 g pullup which would be performed by an airplane whose equations of motion were exactly the same as the low lift flying qualities model at FC-1 of Table II. Figure 7 shows the change in drag obtained by using only the elevator, the elevator and the flap, and finally the elevator plus collectively acting ailerons as a function of tail length and size ratios, K_A and K_L . The curve shows that the maneuver drag is reduced both by shortening the tail and reducing the tail size and by using both elevator and flap or ailerons to perform the maneuver. Drag can be reduced by about 13% using two surfaces, and it can be reduced by approximately 8.5% by cutting the tail length and size in half. A maximum reduction of 30% can be obtained by cutting the tail length and size in half and using both elevator and ailerons for maneuvering control. To do this, however, would not be practical, for as shown by Figure 9, the maximum control surface deflection of the elevator would be tripled. Since the maximum elevator deflection of the T-33 is limited to -25.0°, the 4 g pullup would represent the maximum at FC-1, thereby limiting the maneuvering capability of the airplane.

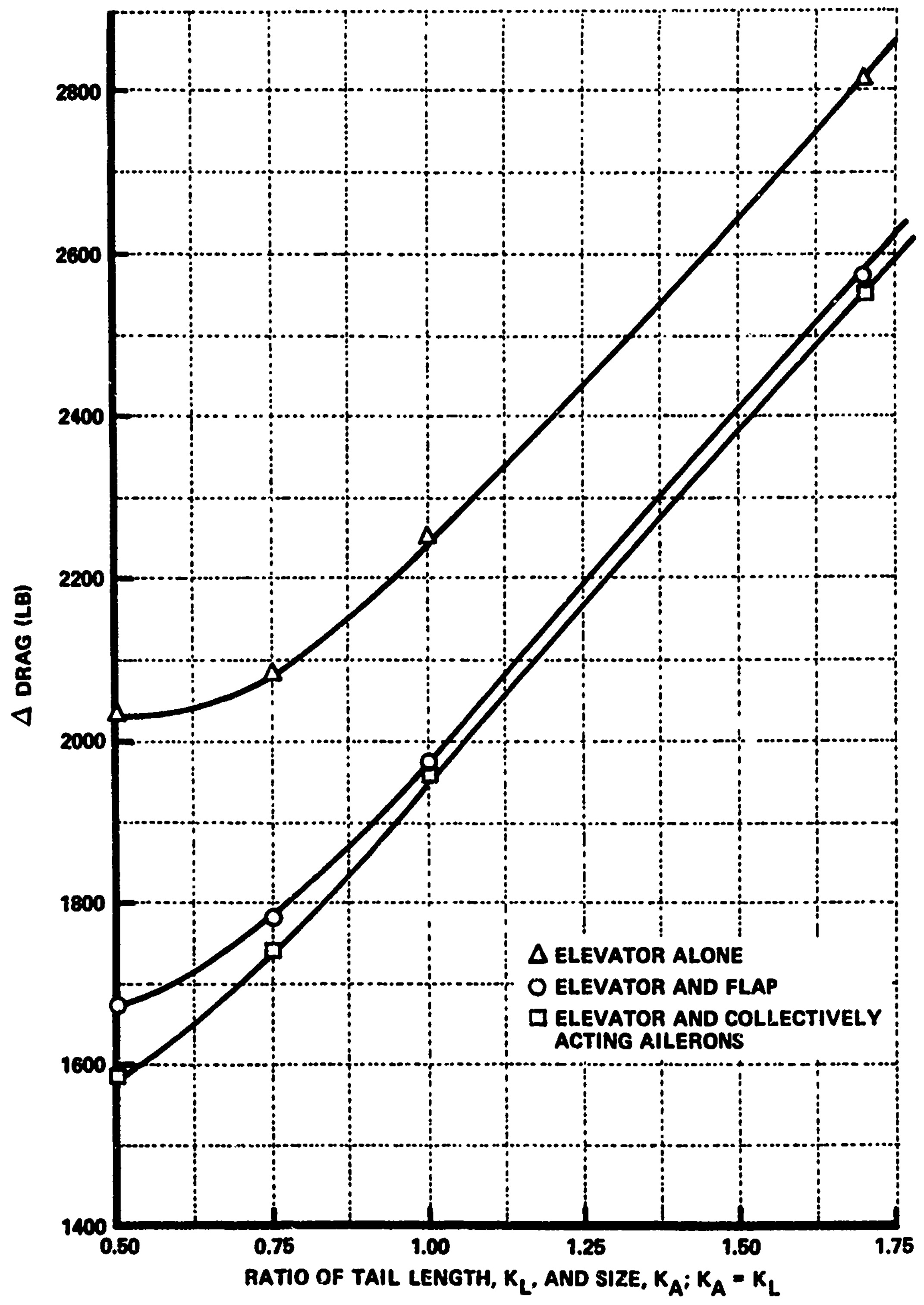


Figure 7 EFFECT OF TAIL LENGTH AND SIZE ON DRAG DURING 4 g PULLUP

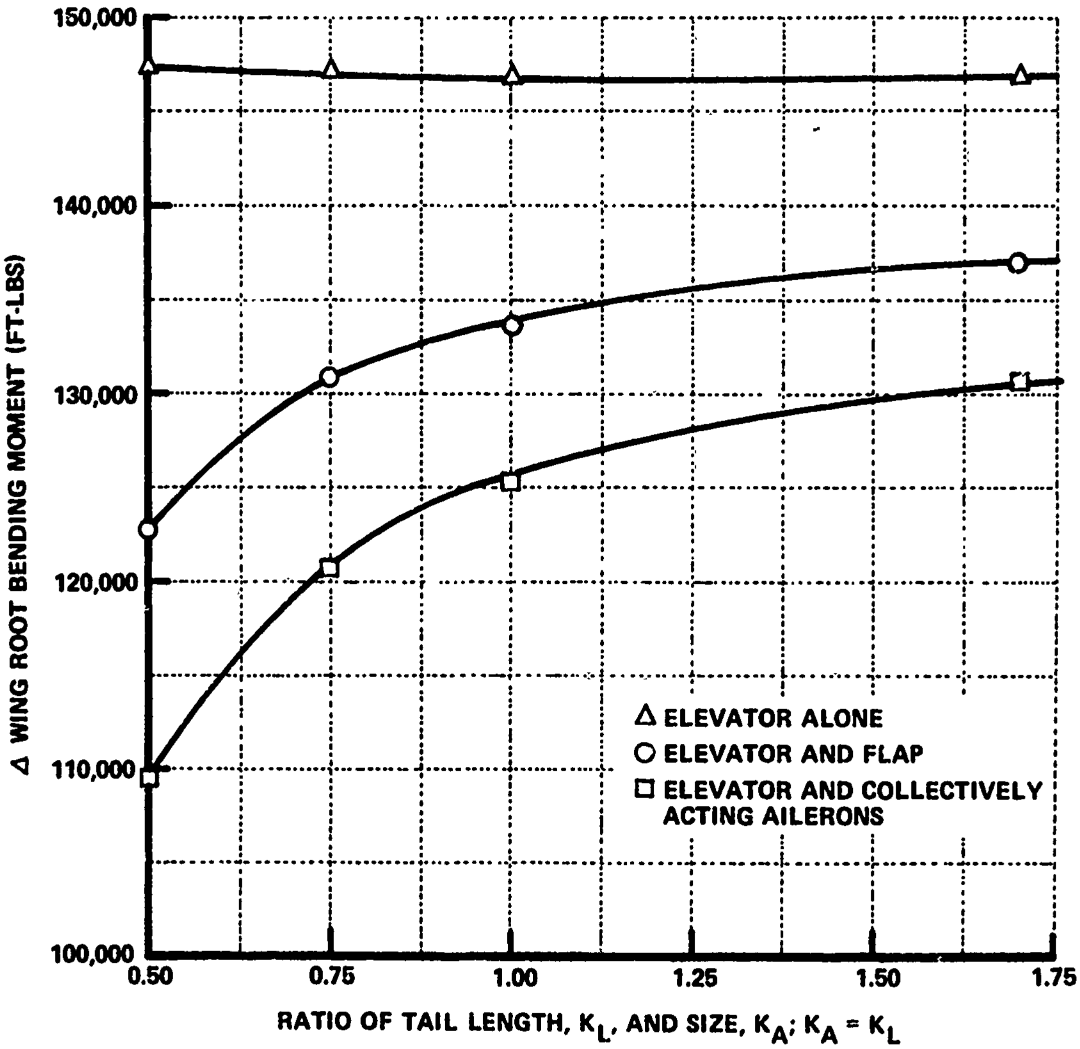


Figure 8 EFFECT OF TAIL LENGTH AND SIZE ON WING ROOT BENDING MOMENT DURING 4 g PULLUP

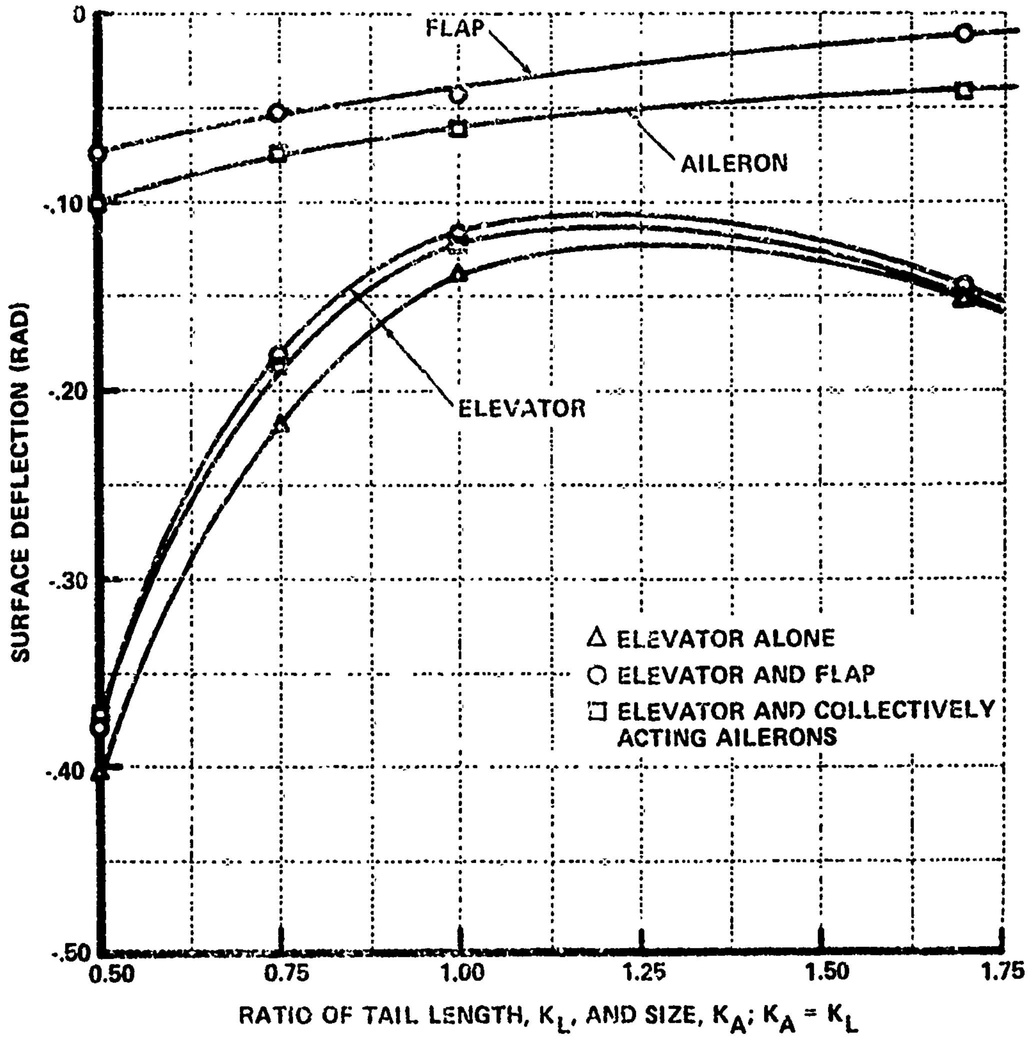


Figure 9 EFFECT OF TAIL LENGTH AND SIZE ON MAXIMUM SURFACE DEFLECTIONS REQUIRED FOR 4g PULLUP

The reduction in the maximum wing root bending moment by using auxiliary surfaces is equally pronounced as compared to changes in drag. Compared with the nominal size tail length and area, the wing root bending moment decreases by approximately 9.2% by the use of flap and elevator and by approximately 15% by using elevator and collectively acting aileron. The change in wing root bending moment also decreases as the tail length and area are reduced but the change, as shown in Figure 8, is not as dramatic as the drag effect.

These improvements in drag and moment changes cannot be obtained without cost. The cost is obviously in the deflections of the control surface required to perform the maneuver. The maximum deflections of the elevator, the flap and the ailerons are shown in Figure 9. As the tail area and length are cut in half, the elevator deflections required to perform the 4 g pullup are tripled, with some decrease in maximum elevator required when either the flaps or the ailerons are used. Because the elevator deflection is reduced rather than increased, the flaps and ailerons produce a beneficial effect; they aid rather than fight the elevator in the generation of the response.

Influence of the Flying Qualities Model

The flying qualities model that dictated the shape of the response during the 4 g pullup has a strong, probably the most important, effect on the results. Table IV below summarizes a few of the more important results using the high n_z/α model, where $n_z/\alpha = 100.7$, as compared to the lower ($n_z/\alpha = 44.12$) model of the previous analysis.

TABLE IV
4 g PULLUP WITH HIGH n_z/α MODEL

Aircraft Configuration	Surface Configuration	Δ Drag (lb)	Δ WRBM (ft-lb)	Max. Surface Deflec. (rad) δ_e	δ_i	δ_o
Base T-33	δ_e only	907	129,100	-.229	--	--
$K_A = K_L = 1.0$	$\delta_e + \delta_i$	2900	139,400	-.157	+.217	--
	$\delta_e + \delta_i + \delta_o$ ($\delta_i = \delta_o$)	2740	158,100	-.137	+.128	+.128
	$\delta_e + \delta_o$	2490	184,600	-.113	--	+.315
$K_A = K_L = 0.5$	δ_e only	814	129,100	-.381	--	--
	$\delta_e + \delta_i$	2970	142,300	-.361	+.228	--
	$\delta_e + \delta_i + \delta_o$ ($\delta_i = \delta_o$)	2800	163,300	-.278	+.132	+.132
	$\delta_e + \delta_o$	2260	179,900	-.182	--	+.315

By comparing the surface deflections required of the high and low requirements (both models, high and low n/α, satisfy the flying qualities requirements in MIL-F-8785B equally well) there is a significant difference. For the low n/α models of Figures 7, 8, and 9, the flap and aileron deflections were negative, which decreased the lift on the wing and significantly reduced the Wing Root Bending Moment. For the high lift model at FC-1 whose characteristics are tabulated in Table III, the n/α of the model was significantly higher than that of the T-33. This means that the airplane performs the 4 g pullup with less pitching motion than it normally would have. In order to do this, the effective slope of the lift curve must be increased. This requirement demanded a positive (downward) deflection of either the flap or the collectively acting ailerons. An increase in wing lift produces an increase in wing root bending moment as vividly demonstrated in Table IV. If elevator alone is used to generate the required change in lift, a larger maximum elevator deflection must be used to perform the 4 g pullup, for $1/\tau_{\theta_2}$ is larger in absolute value with the high n/α model. When two control surfaces are used for maneuvering, both the angle of attack and the pitch rate response of the model are exactly reproduced by the T-33. This demands a different pitching-heaving behavior than is normally obtained in a T-33, causes the positive flap and/or aileron deflections, and increases the wing root bending moment. Thus it can be concluded that relaxed static stability and maneuver load control are incompatible for high lift models.

As the ailerons and flaps are used, the deflection action requirements of the elevator are significantly reduced and the elevator deflections become less strongly a function of tail length and size. This is to be expected because heaving (direct lift) requirements are predominant; pitching motions are not as important in producing lift changes and therefore become a less strong function of tail length and size.

The flying qualities then, have a very pronounced effect on the maneuver load alleviation requirements of a Control Configured Vehicle. The n/α requirements to be satisfied by the CCV should be as low as possible consistent with available elevator power if wing trailing edge lift modulating surfaces are used. This will generally require negative (T.E. up) lifting control surface deflections that will aid the elevator and at the same time redistribute the lift on the wing in a desirable way to reduce the wing root bending moment.

SECTION III

THE SYNTHESIS PROBLEM

3.1 INTRODUCTION

In the previous section, the T-33 airplane was forced to respond as the flying qualities model responds. The tail length and size were varied to determine the effect of these parameters on the drag, wing root bending moment and on the surface deflections. Several combinations of candidate surface configurations inherent to the T-33 airplane that can be considered for maneuvering were considered. These surfaces were the elevator, the flaps (assumed to be a simple flap capable of both positive and negative deflections), and collectively acting ailerons. Other surfaces, such as the addition of spoilers or canard surfaces were briefly considered then discarded, because it is felt that these additional force and moment generating devices should not be added to the airframe unless the existing surfaces prove incapable of doing the job.

In the previous section, the approach taken was one of analysis. Parameters were changed and the effects were noted. In this section the problem of synthesis is investigated. Since flying qualities requirements are fairly broad and many different configurations will yield a Level 1 aircraft, the requirement that the airplane respond exactly as the model is much too stringent a requirement. More emphasis should be put on drag and wing root bending moment reduction for this was felt to be actually more important than satisfying the ultimate in flying qualities.

To satisfy the conflicting requirements of minimum drag and wing root bending moment, a performance index was formulated that included quadratic measures of the error in dynamic behavior between the actual aircraft and the flying qualities model, the maneuver drag, the change in wing root bending moment and the control surface motions. The quadratic performance index is an indirect, rather than direct measure of the design objectives. The modeling error, the drag and the wing root bending moments are minimized relative to each other in a way that produces a most useful kind of solution to the problem. The control motions are relatively smooth and well behaved and the control effort and maximum deflections are managed by the judicious choice of weighting parameters in the performance index. The resulting control law is linear for a linearized description of the airplane dynamics, and these control laws are of the type most likely to be actually mechanized on an aircraft. The parameters of the closed loop system will then yield results that will indicate those stability derivatives, such as C_{m_α} and C_{m_q} that a CCV aircraft might inherently possess. The performance index is of the general form

$$2J = \min_{u} \int_{0}^{\infty} \left[\|\dot{x} - Lx\|_Q^2 + \|\Delta D\|_V^2 + \|\Delta WRBM\|_T^2 + \|u\|_R^2 \right] dt \qquad (23)$$

where L = matrix of dimensional stability derivatives of the flying qualities model

$\Delta D = Ex + Hu$ = linearized expression for the change in drag

$\Delta WRBM = Mx + Nu$ = linearized expression for the change in wing root bending moment

u = control deflections; $u^T = [\delta_e, \delta_i, \delta_o]$

x = state vector of the airplane; $x^T = [\Delta V, \Delta\theta, q, \Delta\alpha]$

$\|y\|_Q^2 = y^T Q y$

The solution is constrained by the equations of motion of the T-33 airplane.

$$\dot{x} = Fx + Gu \qquad (24)$$

where F and G are the matrices of dimensional stability and control derivatives of the T-33 airplane given in Table I.

The solution to the problem posed above will yield the motions of the three control surfaces as a function of the state vector $u = -Kx$ that will minimize the performance index. The matrices Q, V, T and R express relative emphasis placed on the requirements to minimize model following errors, drag changes, wing root bending moment changes and control motions. These weighting parameters are adjusted relative to each other, the absolute numbers are comparatively meaningless except in very simple cases.

The problem of minimizing a quadratic performance index is a well established method of flight control system synthesis and literally hundreds of papers and reports have been written on the subject since the technique was formulated and popularized by R.E. Kalman (Reference 7) and S.S. Chang (Reference 8). Later reports, like Reference 9, established the relationships that exist between the performance index form of solution (called linear optimal control) and the more conventional control system synthesis techniques, like root locus methods. Reference 9 gives many examples of realistic flight control applications and the theory is very briefly summarized. The solution to the linear optimal control problem yields the following control law which minimizes the performance index of Equation (23)

$$u = -R^{-1}G'Px \qquad (25)$$

where P is the positive definite symmetric solution to the matrix Riccati equation

$$0 = PF + F^TP - PGR^{-1}G'P + Q\,. \qquad (26)$$

As shown in Appendix I this equation satisfies the general Hamiltonian equations

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} F & -GR^{-1}G^T \\ -Q & -F^T \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} \quad \begin{matrix} x(0) = x_0 \\ \lambda(0) = \lambda_0 \end{matrix} \tag{27}$$

that are necessary and sufficient for the solution of this problem. The mathematics of this problem will not be discussed; however, a brief introduction to one of the more important closed form solution techniques is given in Appendix I. A few characteristics of the solution that will result when this technique is applied are discussed below.

1. Guaranteed Stability - If the bare airframe is unstable, the choice of positive definite weighting matrices Q, V, T and R will always yield stable linear solutions to linear problems. This does not mean that the resultant closed loop system will necessarily be stable. If an error between a model and the actual airplane is minimized, the closed loop aircraft response can be unstable if the model is unstable, but the error between the aircraft and the model will approach zero asymptotically.

2. The solution will generally yield a closed loop system that has a smooth and well behaved response in the state variables (or errors) included in the performance index. These states will generally respond more quickly than the open-loop aircraft and exhibit little overshoot to an initial condition or command input.

3. The solutions as a function of the weighting matrices exhibit no surprises. A series of solutions, which require at most a few moments of digital computation time, quickly establishes the trends of the solutions as a function of the weighting matrices. Engineering judgment based upon the knowledge of the limits, capabilities and flying qualities of the airframe is used to adjust the weighting matrices to rapidly arrive at acceptable solutions. The actual numerical value associated with the performance index is a very poor substitute for knowledge of the airframe stability and control and flying qualities requirements.

3.2 LONGITUDINAL RESULTS

Problem Formulation

The performance index of Equation (23) was reformulated entirely as a model problem, where zero change in drag and wing root bending moment was included in the model formulation as an objective of the solution. The performance index becomes

$$2J = \int_0^\infty \left[(\dot{x} - L_1 x)^T Q'(\dot{x} - L_1 x) + u^T R u\right] dt$$

$$= \int_0^\infty \left[(Yx + Zu)^T Q'(Yx + Zu) + u' R u\right] dt$$

$$= \int_0^\infty \left[x^T Y^T Q' Y x + x^T Y^T Q' Z u + u^T Z^T Q' Y x + u^T (Z^T Q' Z + R) u\right] dt$$

where the relations between X, Y and Z and the original matrices F, G, L, M; N, E, H, Q, R, T and V of Equation (23) are given below

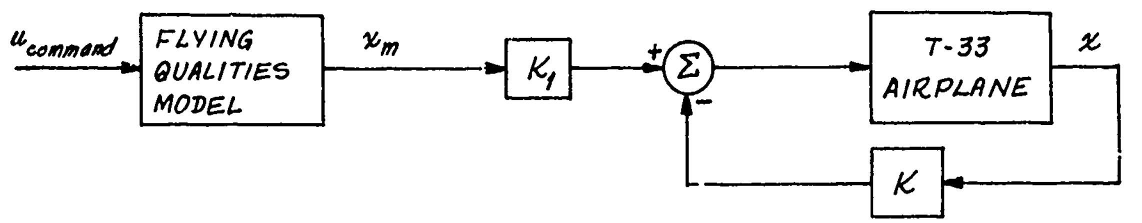
$$Y = \begin{bmatrix} f_{31} - \ell_{31} & 0 & f_{33} - \ell_{33} & f_{34} - \ell_{34} \\ f_{41} - \ell_{41} & 0 & 0 & f_{44} - \ell_{44} \\ 0 & 0 & 0 & e_{14} \\ 0 & 0 & 0 & m_{14} \end{bmatrix} \tag{28}$$

$$Z = \begin{bmatrix} g_{31} & g_{32} & g_{33} \\ g_{41} & g_{42} & g_{43} \\ h_{11} & h_{12} & h_{13} \\ 0 & n_{12} & n_{13} \end{bmatrix} \qquad Q' = \begin{bmatrix} q_3 & 0 & 0 & 0 \\ 0 & q_4 & 0 & 0 \\ 0 & 0 & v & 0 \\ 0 & 0 & 0 & t \end{bmatrix} \qquad \begin{matrix} (29) \\ (30) \end{matrix}$$

$$R = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix} \tag{31}$$

Input Design

The linear optimal control problem described above to obtain the minimum integral of drag change, wing root bending moment change and minimum integral error squared during a transient or set of initial conditions describes only the feedback or regulator part of the solution to the problem. The input or command gains must also be defined. The problem could have been formulated as a model following problem as sketched below:



with a performance index

$$J = \int_0^\infty \left(\|x_m - x\|_Q^2 + \|\Delta D\|_V^2 + \|\Delta WRBM\|_T^2 + \|u\|_R^2 \right) dt \tag{32}$$

but the regulator or closed loop part of the system is not influenced by the model; the model is an uncontrollable part of the system and appears only in the feedforward or command portion of the system. It was felt to be more realistic in terms of an operational system to include the model in the performance index as a restraint on the feedback or regulator part of the system and separately compute the feedforward gains to yield a good quasi-steady state match of the model, at a time $t = 2$ seconds after the applied command. At this time the short period had responded but there was no significant speed change. This was felt to be realistic in terms of the majority of maneuvering requirements of existing fighter aircraft.

Three control surfaces are used to control the three degrees of freedom of motion of the vehicle so the problem can be exactly solved. The command input gains are obtained by solving for the values of the control vector u_c in the following equation

$$\dot{x}_m(t_i) = [F - GK]x_m(t_i) + Gu_c \tag{33}$$

$$u_c = (G^TG)^{-1}G^T\left\{\dot{x}_m(t_i) - [F - GK]x_m(t_i)\right\} \tag{34}$$

where t_i = 2 sec *after the applied pilot input.*

This guarantees that the states of the augmented airplane have the same values (q , α , V , θ , $\dot{q}$, $\dot{\alpha}$, $\dot{V}$) as the model 2 seconds after the command input. The trajectory of the response between t = 0 and t = 2 sec will be different and this difference will be a function of how closely the requirements of minimum dynamic error between the model and the actual aircraft were met compared to the other minimization requirements of the performance index. An alternate way to consider the command part of the system is to connect the stick only to the elevator, but this, as will be shown later, produces a less desirable solution.

Example Solution

A fairly wide range of linear optimal control solutions was run to investigate the feasibility of obtaining less drag and wing root bending moment during dynamic maneuvers. The important considerations were integral of drag or absolute magnitude, and peak wing root bending moment. Additional evaluations were based on maximum control deflections and control action, (integral of the square of the control deflections) as well as evaluations of the time histories of the response of the system.

Solutions were easily obtained that yielded good results and could also be mechanized without difficulty. An example is given by the performance index

$$2J = \int_0^\infty \left\{ \|\dot{x} - Lx\|^2_{q_3 = q_4 = 10^6} + 10^3(\Delta D)^2 + (\Delta WRBM)^2 + \delta_e^2 + \delta_i^2 + \delta_o^2 \right\} dt \tag{35}$$

This performance index weights each portion of the flying qualities error, the drag changes and the wing root bending moment changes in the same order of magnitude and is a very straightforward way to select the weights in the performance index. The solution produces a feedback gain matrix

$$K = \begin{array}{c} \\ \\ \\ \end{array}\begin{matrix} (\Delta V) & (\Delta\theta) & (\dot{\theta}) & (\Delta\alpha) \\ \end{matrix}$$

$$K = \begin{bmatrix} 9.9 \times 10^{-6} & 9.6 \times 10^{-4} & -.25 & -1.03 \\ -4.9 \times 10^{-6} & -4.9 \times 10^{-4} & .124 & -.196 \\ 4.5 \times 10^{-6} & 4.4 \times 10^{-4} & -.112 & 3.88 \end{bmatrix} \begin{matrix} (\Delta\delta_e) \\ (\Delta\delta_i) \\ (\Delta\delta_o) \end{matrix} \tag{36}$$

which indicates that feedback from pitch rate and angle of attack changes to the three control surfaces are the only significant feedback requirements. The gains are quite reasonable and relatively easy to mechanize. The closed loop system matrix becomes

$$F - GK = \begin{bmatrix} -.01424 & -32.17 & +3.82 \times 10^{-4} & 32.17 \\ 0 & 0 & 1.0 & 0 \\ 2.837 \times 10^{-4} & +2.01 \times 10^{-2} & -6.823 & -26.59 \\ -1.569 \times 10^{-4} & +6.19 \times 10^{-6} & .9976 & -1.09 \end{bmatrix} \qquad (37)$$

The feedforward gains obtained from Equation (34) are shown in the sketch below relative to the basic T-33, which requires an elevator deflection of $\delta_e = -.158$ rad to obtain a quasi-static change in normal acceleration of 4 g at the flight condition under study, FC-1.

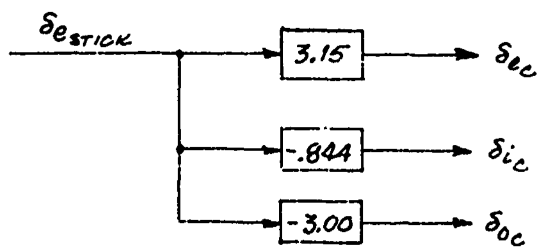


Figure 10 COMMAND INPUT MECHANIZATION

Figure 11 shows the responses of the flying qualities model and the optimal system to a 4 g command input. The response of the augmented vehicle is similar to that of the model and would most likely be considered to have Level 1 flying qualities. The motions of the control surfaces, although initially abrupt because no actuator dynamics were included in the simulation, are not considered excessive.

Table V shows the results of three different solutions with the basic T-33 airplane while Table VI shows the feedforward, and feedback gains and closed loop eigenvalues for the three systems. The tables show that the maneuvering drag and the wing root bending moment can be reduced by a reasonable amount through the reduction of the tail length and area, yet the control system required to still give good Level 1 flying qualities is not overly complex and can be mechanized without difficulty. The only potential problem is that about 4 times as much elevator deflection is required to perform the 4 g pullup when the tail length and size are cut in half. This may be solved by replacing the elevator with an all-movable horizontal surface.

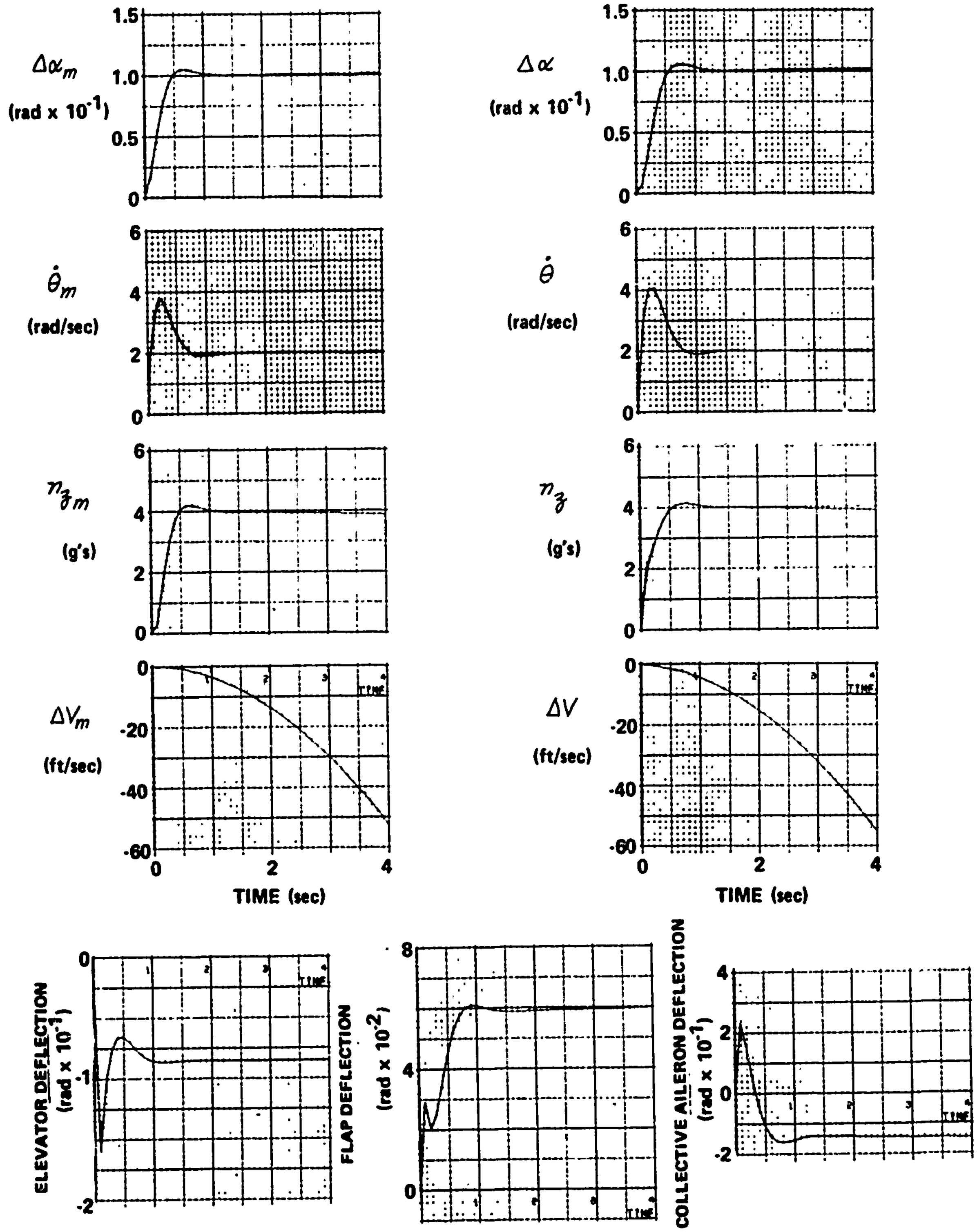


Figure 11 RESPONSE OF CONTROL CONFIGURED VEHICLE AND FLYING QUALITIES MODEL

TABLE V

COMPARISON OF CCV PERFORMANCE FOR 4 g PULLUP

		$K_A = K_L = 1$	$K_A = K_L = 1$	$K_A = K_L = .5$	$K_A = K_L = .5$
Weighting Parameters	Q_3	0	10^6	10^8	10^3
	Q_4	0	10^6	0	0
	V	0	10^3	0	0
	T	0	1	1	10
	R	0	I	$r_2 = r_1 = 10^4$, $r_3 = 10^8$	$10^6 I$
Drag (lb)		2200	2400	1870	1830
Δ WRBM$_{(max)}$		147,000	112,500	127,600	127,200
δ_e		-.0758	-.085	.265	.262
δ_i		---	.059	-.078	-.075
δ_o		---	-.147	.019	.017
$\int_0^2 u^2$		---	.127	.403	.311

TABLE VI

COMPARISON OF CCV DYNAMICS FOR 4 g PULLUP

		$K_A = K_L = 1$	$K_A = K_L = .5$	$K_A = K_L = .5$
	Q_3	10^6	10^8	10^3
Weighting Parameters	Q_4	10^6	0	0
	V	10^3	0	0
	T	1	1	10^{-2}
	R	I	$r_1 = r_2 = 10^4$ $r_3 = 10^8$	$10^6 I$
Significant Feedback Gains	$\delta_e/\dot{\theta}$	-.25	-1.03	-.85
	$\delta_i/\dot{\theta}$	.12	0	-.65
	$\delta_o/\dot{\theta}$	-.11	0	.59
	δ_e/α	-1.03	-12.8	-5.72
	$\delta_o/\dot{\alpha}$	-.20	4.08	-2.61
	δ_o/α	3.9	0	1.06
Feedforward Gains	δ_{ec}/δ_s	3.15	16.1	6.42
	δ_i/δ_s	-.844	-4.18	6.21
	δ_o/δ_s	-3.00	-0.475	-9.86
Short Period	ζ_{sp}	.679	.693	.949
	ω_{sp}	5.83	5.59	7.19
Phugoid	ζ_{ph}	.105	.104	.100
	ω_{ph}	.9651	.069	.071

Effect of Command Input Mechanization

The linear optimal control optimization study of this section produced feedback gains but did not specify the command or feedforward gains and this produces somewhat of a problem.

One obvious way to specify command gains is to design the flight control system as a model following system. A second way is to calculate the feedforward interconnecting gains such that the resulting control effectiveness matrix would be the same as that of the model. A third way would be to command only the elevator, and altering the feedforward gain or the "gear ratio" in such a way that the augmented aircraft maintains a quasi-static 4 g response to the same stick deflection as the basic T-33 airplane. A fourth way of designing the command input portion of the system is given by Equation (34), where the gains are calculated such that the augmented aircraft response has the same state values as the model 2 seconds after the command input is applied by the pilot.

In this section, a comparison is made of two command input designs, command to the elevator alone and command to produce a state match at $t =$ 2 seconds. The feedback used was generated by the third solution to the linear optimal control problem of Table V, in which the weighting parameters were $q_3 = 10^3$, $t = 10^{-2}$, $R = 10^6 I$.

Comparisons of the responses of the systems are shown in Figure 12. The most significant difference between the two system responses is in the control deflection time histories. The design that commanded only the elevator input requires significantly larger peak and steady state control deflections to obtain the 4 g pullup with significantly larger increase in drag during the maneuver. The deflections required of the elevator command system, however, are in the right direction, inboard flap down, outboard flap up, to produce significantly less wing root bending moment change during the maneuver.

There are actually a near-infinite number of ways that the control surfaces can be connected to the stick command input. At least four logical ways are mentioned above, and each has its advantages and disadvantages. Yet all (with the possible exception of the model following arrangement) can be considered to be solutions to the linear optimal control problem solved in this section, for any initial set of control deflections can be thought of as a set of initial conditions of the state vector, since the control law $u = -Kx$ directly relates the control deflections and the state vector.

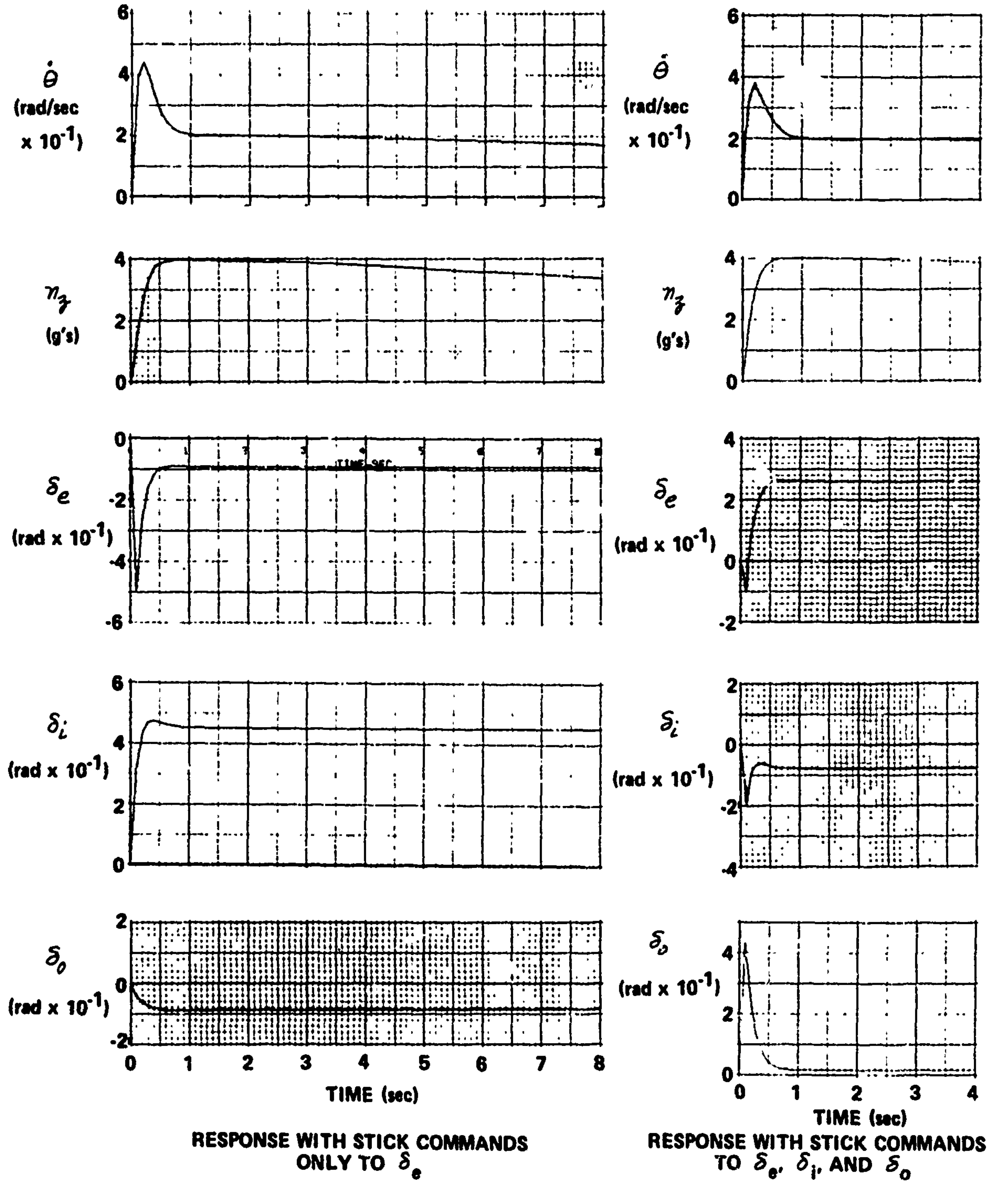


Figure 12 COMPARISON OF RESPONSE WITH TWO DIFFERENT INPUT DESIGNS

Since CCV concepts involve reduced tail lengths and size, using additional control surfaces to aid the resulting less effective elevator, the inputs, i.e., the deflections of the various surfaces required to perform the maneuver, are critical to the CCV design concept. These deflections along with the tail length and area ratios are directly considered and optimized to achieve the desired objective of good model following with minimum change in drag and wing root bending moment.

3.3 LATERAL-DIRECTIONAL RESULTS

Introduction

Although most of this study deals with a CCV for the longitudinal degrees of freedom of motion, it is desirable to check similar vertical tail variations on the lateral-directional behavior of the airplane. A brief investigation was conducted similar to the linear optimal control solution for longitudinal CCV described earlier in this section.

The lateral-directional equations of motion are:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\phi} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} Y_\beta & g/u_0 & Y_p + \alpha & Y_r - 1 \\ 0 & 0 & 1 & 0 \\ L'_\beta & 0 & L'_p & L'_r \\ N'_\beta & 0 & N'_p & N'_r \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p \\ r \end{bmatrix} + \begin{bmatrix} Y_{\delta_a} & Y'_{\delta_r} \\ 0 & 0 \\ L'_{\delta_a} & L'_{\delta_r} \\ N'_{\delta_a} & L'_{\delta_a} \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \quad (38)$$

As was done with the longitudinal modes, it is necessary to derive relationships of the lateral-directional derivatives with the altered vertical tail areas and lengths of the CCV and the normal T-33 derivatives.

The basic T-33 derivatives for the following flight condition were obtained from Reference 1

V_0 = 805 ft/sec

h = 23,000 ft

$\bar{q}$ = 372 lb/ft^2

Using the basic derivatives and methods from Reference 5, the tail effects on the lateral-directional derivatives were evaluated. The lateral-directional equations of motion for the CCV - T-33 can then be written as:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\phi} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -[.134+.057(K_A)] & .04 & 0 & 1. \\ 0 & 0 & 1. & 0 \\ -[13.4+4.4(K_A)^2] & 0 & -2.86 & [.83+.21(K_A)^2(K_L)] \\ -[1.6+7.9(K_A)(K_L)] & 0 & -.044(K_A)^2(K_L) & -[.20+.17(K_A)(K_L)^2] \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p \\ r \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & .052(K_A) \\ 0 & 0 \\ -21.4 & 3.72(K_A)^2 \\ .232 & -7.06(K_A)(K_L) \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \qquad (39)$$

where now:

$$K_A = \frac{S_{V_{CCV}}}{S_{V_{BASIC\ T-33}}} \qquad K_L = \frac{\ell_{V_{CCV}}}{\ell_{V_{BASIC\ T-33}}}$$

and analogous to the longitudinal study, the rudder size was reduced at the same rate as the vertical tail.

Problem Formulation and Results

Similarly to the longitudinal case, it is desired to calculate the response feedback gains necessary for the CCV to have approximately the dynamic characteristics of an ideal model with small control deflections. The ideal model was chosen from Reference 6. The following are its equations of motion and lateral-directional characteristics:

$$\begin{bmatrix} \dot{\beta} \\ \dot{\phi} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -.743 & .04 & 0 & -1. \\ 0 & 0 & 1. & 0 \\ -10.0 & 0 & -4.0 & .865 \\ 5.87 & 0 & .04 & -.507 \end{bmatrix} \begin{bmatrix} \beta \\ \phi \\ p \\ r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -20 & 3.30 \\ 0 & -3.13 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \qquad (40)$$

ω_d = 2.5 rad/sec

ζ_d = .25

τ_R = .25 sec

τ_S = ∞

ϕ/β = .9

The performance index included quadratic measures of the error on the dynamic behavior between the CCV and the model, and the control surface motions:

$$J = \int_0^\infty \left(\| \dot{x} - Lx \|_Q^2 + \| u \|_R^2 \right) dt$$

where both of the weighting matrices were set equal to identity matrices. A more comprehensive study should consider tail loads.

Seven CCV configurations with various values of K_A and K_L were evaluated. The results are listed in Table VII.

Evaluation of Results

All of the modal characteristics, though not identical to the "excellent" model, are still Level 1 according to specifications in Reference 4. However, there are other factors which show that there is a limit to the amount of vertical tail reduction that is allowable.

First of all, β feedback gains much greater than 5. are not realizable, so configuration 7 with a δ_a/β of 8.9 is not realistic. Also the amount of rudder deflection to hold a constant sideslip may be a limiting factor. From MIL-F-8785B, an airplane must be able to hold an approximate 15 degree sideslip in case of an extreme crosswind landing. The amount of control deflection necessary to trim each CCV configuration in a steady level 15 degree sideslip is also listed in the following table. Again, configuration 7 is unrealistic as it would call for a 57 degree rudder deflection.

It can be seen from configurations 2 and 4, and 3 and 6 that the effects of K_A and K_L are almost identical and stability derivatives depend on the value of the product ($K_A \cdot K_L$). The limiting value for the CCV - T-33 for ($K_A \cdot K_L$) is most likely 0.1. From the table it can be seen for this value (configurations 3 and 6) the δ_r/β gains are about 5.0 and the rudder deflection to hold the maximum sideslip is approximately 20 degrees. Though the unaugmented configuration 6 is statically and dynamically unstable:

TABLE VII

RESULTS OF LATERAL-DIRECTIONAL STUDY

Para.	"Excellent Model (Ref. 6)	Basic Unaugmented T-33	Augmented CCV Configurations 1	2	3	4	5	6	7
K_R	--	--	1.0	5.	0.1	1.	.5	.2	.1
K_L	--	--	1.0	1.	0.1	.5	.5	.5	.5
ω_d	2.5	2.2	2.45	2.41	1.90	2.41	2.28	1.87	1.50
ζ_d	.25	.09	.167	.171	.546	.184	.283	.555	.773
τ_R	.25	.3	.250	.250	.248	.250	.249	.249	.249
τ_s	∞	150.	685.	882.	138.	905.	430.	130.	66.4
ϕ/β	.90	1.6	.92	.92	1.28	.92	.99	1.29	1.78
Response Feedback Gains									
δ_a/β	--	--	.356	.252	.167	.534	.325	.202	.159
δ_a/ϕ	--	--	0	0	0	0	0	0	-.001
δ_a/p	--	--	-.051	-.053	-.053	-.051	-.053	-.053	-.054
δ_a/r	--	--	-.008	-.005	0	-.024	-.021	-.013	.003
δ_r/β	--	--	-.044	.964	5.800	.980	2.671	5.727	8.956
δ_r/ϕ	--	--	0	.001	.082	.001	.012	.080	.201
δ_r/p	--	--	.010	.010	-.014	.013	.015	-.020	.031
δ_r/r	--	--	-.039	-.100	-2.447	-.119	-.485	-2.385	-5.548
δ_a for Steady 15° Sideslip	--	-12.4	-12.4	-9.7	-9.3	-10.4	-10.3	-9.6	-9.5
δ_r for Steady 15° Sideslip	--	0.93	0.93	9.5	-19.8	9.3	-1.0	-20.2	-57.3

(the two real Dutch roll roots are -1.7 and +1.05 and

τ_R = .34 sec, τ_S = - 11.4 sec)

with a simple feedback system, this configuration can be made to fly with Level 1 handling qualities. Therefore a corresponding and possibly necessary reduction in vertical tail area and length is compatible with reductions in the horizontal tail. Also with a smaller tail, the vertical tail loads will be reduced, resulting in a possible reduction in structural stiffening and a further weight saving.

SECTION IV

DIRECT OPTIMIZATION

4.1 INTRODUCTION

In Section II of this report, several control surface configurations were investigated in order to obtain measures of change in drag, wing root bending moment and surface deflection as the T-33 airplane, with altered tail length and area, was forced to respond "exactly" as the flying qualities model would respond. This study revealed the effectiveness of surfaces other than the elevator in aiding the elevator to perform the pullup maneuver. Then, in Section III, the control system was obtained that would minimize a measure of the change in drag, wing root bending moment and error between the actual aircraft and the model. Two values of tail length and area were used to show the effect of this parameter on the resulting solutions but no direct effort was made to optimize tail length and area. It was also shown that the input design has a great effect on the resulting system behavior.

In this section, the geometry-dependent characteristics are treated directly. The objective is to determine optimum tail length, tail area and surface deflections that would minimize the trim drag and wing root bending moment and the change in drag, wing bending moment and the model-response error for a 4 g pullup. The problem is open loop in the sense that no feedback will be directly obtained; instead, the geometrical parameters will be optimized, and the control deflections required to minimize drag and wing root bending moment in trim and in maneuvering flight will be treated.

It was felt that the dynamic optimization would provide the optimum tail length and area and once these were obtained the deflections of the surfaces could be calculated to maintain minimum drag and wing root bending moment in trim. Then the required command inputs obtained from the dynamic optimization would complete the design requirements. In actual practice, the two parts, static and dynamic optimization were done separately yet concurrently, so the static optimization did not use the values of K_A and K_L obtained during the dynamic optimization part of the study. It would be a relatively simple matter to repeat the static optimization design for any value of K_A and K_L .

4.2 STATIC TRIM OPTIMIZATION

Formulation of the Problem

It is desired to minimize drag and WRBM while trimming the CCV with the elevator, inboard flap and outboard flap. The following functional, F , is formed containing a measure of drag, WRBM, and Lagrange multipliers times the three longitudinal trim equations (level, 1 g flight with constant thrust and velocity):

$$F = K_1 \cdot (Drag) + K_2 \cdot (WRBM) + \lambda_i \, (Trim\ Equations) \tag{41}$$

where
$$Drag = C_{D_{0_{\delta_i}}} \delta_i + C_{D_{0_{\delta_0}}} \delta_0 + C_{D_{0_{\delta_e}}} \delta_e + K_p \left[C_{L_0} + C_{L_\alpha} \alpha + C_{L_{\delta_i}} \delta_i + C_{L_{\delta_0}} \delta_0 \right]^2 + K_p \left[C_{L_{\delta_e}} \delta_e \right]^2 \tag{42}$$

$$WRBM = M_{B_0} + M_{B_\alpha} \alpha + M_{B_{\delta_i}} \delta_i + M_{B_{\delta_0}} \delta_0 \tag{43}$$

Trim Equations

$$\lambda_1 \left(\frac{thrust}{qS} = C_{D_0} + C_{L_\alpha} \alpha + C_{D_{\delta_i}} \delta_i + C_{D_{\delta_0}} \delta_0 + C_{D_{\delta_e}} \delta_e \right) \tag{44}$$

$$\lambda_2 \left(\frac{weight}{qS} = C_{L_0} + C_{L_\alpha} \alpha + C_{L_{\delta_i}} \delta_i + C_{L_{\delta_0}} \delta_0 + C_{L_{\delta_e}} \delta_e \right) \tag{45}$$

$$\lambda_3 \left(0 = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{\delta_i}} \delta_i + C_{m_{\delta_0}} \delta_0 + C_{m_{\delta_e}} \delta_e \right) \tag{46}$$

Taking derivatives of F with respect to the unknown variables and setting them equal to zero, we obtain seven equations which can be solved simultaneously for a unique solution for each set of weighting constants, K_1 and K_2, on the drag and WRBM.

$$\frac{\partial F}{\partial \alpha} = 0 = K_1 \left[2K_D C_{L_\alpha}^2 + 2K_D C_{L_\alpha} C_{L_{\delta_i}} \delta_i + 2K_D C_{L_\alpha} C_{L_{\delta_0}} \delta_0 + 2K_D C_{L_0} C_{L_\alpha} \right] + K_2 M_{B_\alpha} + \lambda_1 C_{D_\alpha} + \lambda_2 C_{L_\alpha} + \lambda_3 C_{m_\alpha} \tag{47}$$

$$\frac{\partial F}{\partial \delta_e} = 0 = K_1 \left[C_{D_0} \delta_e + 2K_D C_{L_{\delta_e}}^{\;2} \delta_e \right] + \lambda_1 C_{D_{\delta_e}} + \lambda_2 C_{L_{\delta_e}} + \lambda_3 C_{m_{\delta_e}} \tag{48}$$

$$\frac{\partial F}{\partial \delta_i} = 0 = K_1 \left[C_{D_0} \delta_i + 2K_D C_{L_{\delta_i}}^{\;2} \delta_i + 2K_D C_{L_\alpha} C_{L_{\delta_i}} \alpha + 2K_D C_{L_{\delta_i}} C_{L_{\delta_0}} \delta_0 + 2K_D C_{L_0} C_{L_{\delta_i}} \right] + K_2 M_{B_{\delta_i}} + \lambda_1 C_{D_{\delta_i}} + \lambda_2 C_{L_{\delta_i}} + \lambda_3 C_{m_{\delta_i}} \tag{49}$$

$$\frac{\partial F}{\partial \delta_0} = 0 = K_1 \left[C_{D_0} \delta_0 + 2K_D C_{L_{\delta_0}}^{\;2} \delta_0 + 2K_D C_{L_\alpha} C_{L_{\delta_0}} \alpha + 2K_D C_{L_{\delta_i}} C_{L_{\delta_0}} \delta_i + 2K_D C_{L_0} C_{L_{\delta_0}} \right] + K_2 M_{B_{\delta_0}} + \lambda_1 C_{D_{\delta_0}} + \lambda_2 C_{L_{\delta_0}} + \lambda_3 C_{m_{\delta_0}} \tag{50}$$

$$\frac{\partial F}{\partial \lambda_1} = 0 = C_{D_0} + C_{D_{\delta_i}} \delta_i + C_{D_{\delta_0}} \delta_0 + C_{D_{\delta_e}} \delta_e + C_{D_\alpha} \alpha - \frac{thrust}{qS} \tag{51}$$

$$\frac{\partial F}{\partial \lambda_2} = 0 = C_{L_0} + C_{L_{\delta_i}}\delta_i + C_{L_{\delta_o}}\delta_o + C_{L_{\delta_e}}\delta_e + C_{L_\alpha}\alpha - \frac{weight}{\bar{q} S} \qquad (52)$$

$$\frac{\partial F}{\partial \lambda_3} = 0 = C_{m_0} + C_{m_{\delta_i}}\delta_i + C_{m_{\delta_o}}\delta_o + C_{m_{\delta_e}}\delta_e + C_{m_\alpha}\alpha \qquad (53)$$

The seven unknowns are: the trim α , δ_i , δ_o , δ_e ; and the Lagrange multipliers λ_1 , λ_2 , and λ_3 . The values of the latter three parameters have no significance as the equations of motion are just added to insure that the aircraft is trimmed. Only the relative difference in the weighting constants, K_1 and K_2 , is significant, and if both are increased by the same multiple, the resulting solution will be the same except that the λ_i will change by the same multiple. The actual values of K_1 and K_2 have no meaning other than they do cause different minimum drag and WRBM solutions to occur.

Results and Evaluation

The only CCV - T-33 configuration investigated was the one with the normal T-33 tail size, $K_A = K_L = 1$. Other configurations would involve similar results. The significant solutions obtained are summarized in the following table:

TABLE VIII
SUMMARY OF TRIM OPTIMIZATION RESULTS

K_1	K_2	α (rad)	δ_e (rad)	δ_i (rad)	δ_o (rad)	Trim (Level 1 g Flight) (lb)	Trim WRBM (ft-lb)
$10.\times10^{-4}$	1.	.168	-.426	2.069	-4.071	16,313	-580,498
$5.\times10^{-4}$	1.	.020	-.093	.380	-.683	4,278	-52,575
4.4×10^{-4}	1.	.007	-.063	.230	-.381	3,218	-16,312
$4.\times10^{-4}$	1.	-.003	-.041	.117	-.154	2,499	18,316
3.6×10^{-4}	1.	-.015	-.015	-.013	.107	1,953	58,221
$1.\times10^{-4}$	1.	-.079	.130	-.760	1.589	6,956	284,551
$.1\times10^{-4}$	1.	-.110	.200	-1.103	2.294	9,374	392,622
1.	1.	-.113	.207	-1.142	2.373	9,645	404,669
trim with normal T-33 (no δ_i or δ_o available)		-.001	-.010	---	---	1,723	40,998

It can be seen that the trim WRBM can be eliminated completely with inboard and outboard wing flap deflections. However, this is done at the expense of increased trim drag. In fact, it appears as though all of the trim solutions with δ_i and δ_o develop more drag than the basic T-33 solution. Also the trim flap deflections are relatively high for the reduced WRBM solution. For example, the solution which results in an approximate 50% reduction in WRBM ($K_1 = 4.\times10^{-4}$), requires almost 10 degrees of flap deflection. This would severely limit the amount of flap deflection left for maneuvering. Each CCV under study must be evaluated separately to determine, for its particular mission, whether the reduced trim WRBM and the resulting decreased structural weight is worth the increased trim drag and control deflections.

4.3 DYNAMIC OPTIMIZATION

Derivation of Necessary Conditions

The longitudinal small perturbation equations of motion being considered are:

$$\dot{x} = F(\underline{K})\underline{x} + G(\underline{K})\underline{u} \quad , \; x(0) = 0 \tag{54}$$

$$y = A\underline{x} + B\underline{u}$$

$x = (V, \theta, q, \alpha)^T$ - perturbed state vector

$\underline{K} = (K_A, K_L)^T$ - horizontal tail geometrical parameter

$\underline{u} = (\delta_e, \delta_i, \delta_o)^T$ - control perturbation vector

$\underline{y}(t_f) = (\alpha, q, \dot{\alpha})^T$ - measure of state at final time

It is desired to find a step perturbation in the control vector $\underline{u} = \underline{b}$ and the tail parameters $\underline{K}$, such that during the maneuver, which transforms the CCV from the initial state to some final state $y(t_f)$ at a prescribed final time, t_f , the motions of the airplane will be close to an ideal flying qualities model and the drag and WRBM will be minimized. In the following discussions we are concerned only with the drag and WRBM developed during the maneuver so we will use just the linearized incremental expressions:

$$\Delta Drag = \Delta D = \underline{d}_1^T(\underline{K})\,\underline{x} + \underline{d}_2^T(\underline{K})\,\underline{u} \tag{55}$$

$$\Delta WRBM = \Delta m = \underline{m}_1^T \underline{x} + \underline{m}_2^T \underline{u} \tag{56}$$

where $\underline{d}_1^T(\underline{K}) = \left(0,\ 0,\ 0,\ \bar{q}\, S C_{D_\alpha}(\underline{K})\right)$

$$\underline{d}_2^T(\underline{K}) = \left(\bar{q}\, S C_{D_{\delta_e}}(\underline{K}),\ \bar{q}\, S C_{D_{\delta_i}},\ \bar{q}\, S C_{D_{\delta_o}}\right)$$

$$\underline{m}_1^T = \left(0,\ 0,\ 0,\ M_{B_\alpha}\right)$$

$$\underline{m}_2^T = \left(0,\ M_{B_{\delta_i}},\ M_{B_{\delta_o}}\right)$$

From the preceding discussions, the problem previously stated can be recast in the following precise terms:

Find $\underline{K}$ and $\underline{b}$ such that the performance index

$$J' = \int_0^{t_f} \left\{ \|\dot{x} - Lx\|_Q^2 + \|\Delta D\|_v^2 + \|\Delta WRBM\|^2 \right\} d\tau \qquad (57)$$

is minimized subject to the constraints:

$$\dot{x} = F(\underline{K})\underline{x} + G(\underline{K})\underline{b}$$

$$\underline{x}(0) = 0$$

$$y(t_f) = Ax(t_f) + B\underline{b} = \underline{c} = \left[\alpha(t_f),\ q(t_f),\ \dot{\alpha}(t_f)\right]^T \qquad (58)$$

and L is the ideal handling qualities model. The weighting parameters Q, v, t, and $\underline{c}$ and t_f are given.

The Conjugate Gradient Solution

For the conjugate gradient solution, we first introduce a Lagrange multiplier vector $\underline{\lambda} = (\lambda_1, \lambda_2, \lambda_3)^T$ and form a new performance index:

$$J = J' + \sum_{i=1}^{3} \lambda_i y_i^2(t_f)$$

$$= \int_0^{t_f} \left\{ x^T \left[(F-L)^T Q (F-L) + v\underline{d}_1\underline{d}_1^T + t\underline{m}_1\underline{m}_1^T \right] \underline{x} + 2x^T \left[(F-L) QG + v\underline{d}_1\underline{d}_2^T + t\underline{m}_1\underline{m}_2^T \right] \underline{b} \right\} d\tau$$

$$+ t_f \underline{b}^T \left[G^T QG + v\underline{d}_2\underline{d}_2^T + t\underline{m}_2\underline{m}_2^T \right] \underline{b} + \sum_{i=1}^{3} \lambda_i y_i^2(t_f) \qquad (59)$$

The second large term above does not contain an integral sign because all of the terms contained in it are constant for $t \in (0, t_f)$. Also in all of the above expressions, it is to be emphasized that $F, G, A, \mathcal{B}, \underline{d}_1$, and $\underline{d}_2$ are functions of $\underline{K}$, and $\underline{m}_1, \underline{m}_2$, are independent of $\underline{K}$.

We will require the gradients of J with respect to $\underline{K}$, $\underline{b}$, and λ_i.

$$\nabla_{\lambda_i} J = y_i^2(t_f) \; ; \; i = 1, 2, 3 \tag{60}$$

$$\begin{aligned}
\nabla_{K_i} J = & \left[\sum_{i=1}^{3} \lambda_i \frac{\partial y_i}{\partial K_i}(t_f) \cdot 2y_i\right] + 2t_f \underline{b}^T \left[\frac{\partial G^T}{\partial K_i} QG + v \frac{\partial \underline{d}_2}{\partial K_i} \underline{d}_2^T\right] \underline{b} \\
& + 2\int_0^{t_f} \frac{\partial x^T}{\partial K_i}\left[(F-L)^T Q (F-L) + v\underline{d}_1\underline{d}_1^T + t\underline{m}_1\underline{m}_1^T\right] \underline{x}\, d\tau \\
& + 2\int_0^{t_f} x^T \left[\frac{\partial F^T}{\partial K_i} Q(F-L) + v \frac{\partial \underline{d}_1}{\partial K_1} \underline{d}_1^T\right] \underline{x}\, d\tau + 2\int_0^{t_f}\left[\frac{\partial x^T}{\partial K_i}(F-L)QG + v\underline{d}_1\underline{d}_2^T + t\underline{m}_1\underline{m}_2^T\right]\underline{b}\, d\tau \\
& + 2\int_0^{t_f} x^T\left[\frac{\partial F}{\partial K_i} QG + (F-L)^T Q \frac{\partial G}{\partial K_i} + v\frac{\partial \underline{d}_1}{\partial K_i}\underline{d}_2^T + v\underline{d}_1 \frac{\partial \underline{d}_2^T}{\partial K_i}\right]\underline{b}\, d\tau
\end{aligned} \tag{61}$$

$$\begin{aligned}
\nabla_{b_j} J = & \left[\sum_{i=1}^{3} \lambda_i \frac{\partial y_i}{\partial b_j}(t_f) \cdot 2y_i\right] + 2t_f \underline{e}_j^T \left[G^T QG + v\underline{d}_2\underline{d}_2^T + t\underline{m}_2\underline{m}_2^T\right]\underline{b} \\
& + 2\int_0^{t_f} \frac{\partial x^T}{\partial b_j}\left[(F-L)^T Q(F-L) + v\underline{d}_1\underline{d}_1^T + t\underline{m}_1\underline{m}_1^T\right]\underline{x}\, d\tau \\
& + 2\int_0^{t_f} \frac{\partial x^T}{\partial b_j}\left[(F-L)QG + v\underline{d}_1\underline{d}_2^T + t\underline{m}_1\underline{m}_2^T\right]\underline{b}\, d\tau \\
& + 2\int_0^{t_f} x^T\left[(F-L)^T QG + v\underline{d}_1\underline{d}_2^T + t\underline{m}_1\underline{m}_2^T\right]\underline{e}_j\, d\tau
\end{aligned} \tag{62}$$

where $b_j = \delta_e, \delta_i, \delta_o$ (input steps)

$\mathcal{B} = (\underline{B}_1, \underline{B}_2, \underline{B}_3)$

$\underline{e}_1 = (1,0,0)^T \quad \underline{e}_2 = (0,1,0)^T \quad \underline{e}_3 = (0,0,1)^T$

Also required in the above expressions are the sensitivities: $\partial x/\partial K_i$ and $\partial x/\partial b_j$ which are the solution of:

$$\frac{d}{dt}\left(\frac{\partial x}{\partial K_i}\right) = F\frac{\partial x}{\partial K_i} + \frac{\partial F}{\partial K_i}x + \frac{\partial G}{\partial K_i}\underline{b} \quad , \quad \frac{\partial x}{\partial K_i}(0) = 0$$

$$\frac{d}{dt}\left(\frac{\partial x}{\partial b_j}\right) = F\frac{\partial x}{\partial b_j} + Ge_j \quad , \quad \frac{\partial x}{\partial b_j}(0) = 0 \, , \; G = (\underline{g}_1, \underline{g}_2, \underline{g}_3) \tag{63}$$

The above expressions are reduced somewhat in the calculations because many of the matrices contain many zero locations. The function dependence of the stability derivatives were reduced to linear relations in K_A and K_L. This gave the following matrices for the CCV - T-33 at Flight Condition 1:

$$F = \begin{bmatrix} -.00062(K_A)-.01362 & -32.17 & 0 & -2.2154(K_A)+7.9421 \\ 0 & 0 & 1. & 0 \\ .000136(K_A)(K_L)-.000062 & 0 & -2.743(K_A)(K_L)+1.191 & -35.0883(K_A)(K_L)+17.228 \\ -.000157 & 0 & 1. & -.1299(K_A)(K_L)-2.1691 \end{bmatrix}$$

$$G = \begin{bmatrix} -3.978(K_A) & -15.8886 & -8.6788 \\ 0 & 0 & 0 \\ -25.734(K_A)(K_L) & -14.1077 & -5.2802 \\ -.1276(K_A) & -.5782 & -.3755 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -.000157 & 0 & 0 & -2.1619-.1299(K_A)(K_L) \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -.1276(K_A) & -.5782 & -.3755 \end{bmatrix}$$

$$\underline{d}_1(K) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 9061.2+828.6(K_A) \end{bmatrix} \qquad \underline{d}_2(K) = \begin{bmatrix} 1487.8(K_A) \\ 5942.3 \\ 3245.9 \end{bmatrix}$$

$$m_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1,460,500 \end{bmatrix} \qquad m_2 = \begin{bmatrix} 0 \\ 357,650 \\ 394,750 \end{bmatrix}$$

Letting the parameter vector be defined as:

$$\underline{P} = (K_A, K_L, \delta_e, \delta_i, \delta_o, \lambda_1, \lambda_2, \lambda_3)^T$$

the conjugate gradient algorithm can be written as follows:

(i) Guess an initial value for $\underline{P}$ and call this $\hat{P}_1$

(ii) Compute J and $\nabla_P\, J(\hat{P}_1)$ using the previous expressions and let $a_1 = \nabla_P\, J(\hat{P}_1)$

(iii) Determine the constant multiplier α_i of the corrections a_i by a one dimensional search:

$$\min_{\alpha} J(\hat{P}_i - \alpha_i a_i)$$

(iv) Define the new parameter vector: $\hat{P}_{i+1} = \hat{P}_i - \alpha_i a_i$

(v) Evaluate the updated J and test to see if it has converged to a minimum value. If it has, a solution has been obtained. If it has not converged, proceed to the next step

(vi) Compute the gradient $\nabla_P\, J(\hat{P}_i)$ and new corrections:

$$a_i = \nabla_P\, J(\hat{P}_i) + \frac{\|\nabla_P\, J(\hat{P}_i)\|^2}{\|\nabla_P\, J(\hat{P}_{i-1})\|^2}\, a_{i-1}, \quad i \geq 2$$

(vii) Repeat steps (iii) through (vii) until a minimum J has been reached.

4.4 RESULTS

Results of a few representative runs of the conjugate gradient program are presented in Table IX. Two different initial value sets were run with various weighting constants:

Set 1: initial $K_A = 1.0$, $K_L = .5$

Set 2: initial $K_A = .01$, $K_L = .01$

The maneuver performed was a 4 g pullup in 2 seconds with $q(t_f) = 2$ rad/sec, $\dot{\alpha}(t_f) = 0$ rad/sec.

As can be seen in the table of results, the solutions are not too promising. The conjugate gradient method as formulated here yields many local minima. In fact, for each different set of initial guesses for K_A, K_L, δ_e step, etc., a different solution for the minimum of the performance index, J, was obtained. Also the size of the control step inputs relative to each other had a much more pronounced effect on the solution that was obtained (the initial guess in each case was a $\delta_e = .15$ rad step) than to changes in tail area or tail length. In many solutions the tail parameters increased in size. For Set 2 all of the solutions remained with the smallest tail and even though the conditions at 2 seconds were matched, the airplane was still unstable, and was diverging rapidly at that time.

However, the conjugate gradient method may still be of some use. One remedy to the non-uniqueness problem may be to use a fewer number of unknown variables. By reducing the number of control step inputs as variables, there would most likely be fewer local minimums and more importantly, the solution would have to be reached by the changing of the tail parameters, K_A and K_L rather than through the use of control deflections.

More research in this area is necessary and perhaps a different minimization technique less prone to local minima should be investigated, such as quasilinearization or Kalman filtering. A better choice of cost function may also be found.

TABLE IX

SELECTED RESULTS OF THE CONJUGATE GRADIENT METHOD

4 g pullup maneuver, t_f = 2 seconds, $\nu = 10^{-6}$, $t = 10^{-8}$

Initial Input = $\delta_{e\,step}$ = -.15 rad, $\delta_i = \delta_o = 0$.

	FINAL ITERATED VALUES								
	λ_i	Q	K_A	K_L	δ_e	δ_i	δ_o	ΔWRBM	Δ Drag
Set 1: Initial	10^4	1	1.6	1.6	-.17	.15	-.59	37,440	4,750
K_A = 1.0	10^4	0	1.7	1.7	-.17	.52	.83	11,500	7,260
K_L = .5	0	10^4	1.03	.57	-.52	.58	-.12	132,600	4,850
Set 2: Initial	10^4	1	.0107	.0107	.00005	-.019	.047	56,300	540
K_A = .01	0	1	.0106	.0106	-.0002	.084	-.21	49,200	1,560
K_L = .01	0	10^4	.0107	.0107	.0018	-.75	1.9	402,500	10,630

TABLE X

ATTAINABLE FEEDBACK GAINS

Variable	Surface				
	Elevator	Flap	Other Wing Surface	Rudder	Aileron
ΔV	---	---	---	---	---
$\Delta\alpha$ rad/rad	±5	±5	±4	---	±4
$\dot{\theta}$ rad/(rad/sec)	±3	±3	±2	---	±3
θ rad/rad	±5	±4	±2	---	---
β rad/rad	---	---	---	±3	±8
p rad/(rad/sec)	---	---	---	±2	±3
ϕ rad/rad	---	---	---	---	±4
r rad/(rad/sec)	---	---	---	±3	---

SECTION V

CONTROL SYSTEM MECHANIZATION

5.1 INTRODUCTION

In previous sections, the need for adequate control power was stressed. If the tail size were reduced, including the elevator size, it is fundamental that larger elevator surface deflections would be required to maneuver the airplane. Flap and collectively acting ailerons help the elevator produce the required pitching moments but not to a completely compensating amount, so there is a limit to the extent that the elevator size can be reduced. Control surface power is a fundamental limitation associated with a CCV.

A second fundamental limitation associated with a CCV is the physical limitation associated with feedback control mechanization. Sensor and amplifier noise and structural flexibility limit the amount of feedback that can be applied to an airplane. Table X summarizes some of the regularly attainable feedback gains that are used during flight investigations involving the AF/CAL T-33 and AF/CAL C-131 (TIFS) airplanes. The gains listed in the table are not necessarily the maximum that can be achieved; these gains are regularly and easily obtained without special provision for sensor and amplifier noise, servo bandwidth, structural dynamics and other corrupting or limiting influences. The gain values represent day-to-day state of the science and are conservatively estimated. If differences exist in the maximally allowed feedback among aircraft, the more conservative number is always chosen. It is nevertheless important to note that large differences among individual aircraft do occur and these differences are due mainly to variations in structural flexibility. Because a CCV may be highly flexible, the gains listed below may even be too optimistic without specific structural mode control provisions. If feedback is provided to augment the fundamental, rigid body dynamical behavior and the attainment of this feedback requires extensive structural mode control, then the problems of reliability and aircraft parameter identification are doubly critical. Advanced, accurate methods of vehicle parameter identification are only now being developed (Reference 11) and it will likely be another decade or more before adequate tools are available to identify the structural mode and flutter parameters of an airplane, in addition to the rigid body stability derivatives.

Two general comments can be made, then, about maximally usable feedback gains. First, as mentioned above, the smaller and more rigid the airframe, the higher the attainable feedback gains. Second, the lower the degree of the derivatives of the state variable used for feedback, the higher the feedback gain that can be attained. This limitation is attributable as much to the sensor characteristics as to the structural flexibility. Pitch accelerometers generate more noise than pitch rate gyros which in turn tend to be less noise-free than attitude gyros. There are exceptions, particularly when the good and bad features of angle of attack vanes and accelerometers are evaluated in their ability to alter the short period natural frequency of the airplane and compared to the noise output level of these sensors, but in general, the state derivative rule holds.

As discussed in a previous section, the control system of the T-33 CCV airplane requires mainly pitch rate and angle of attack (or normal acceleration) feedback in the longitudinal plane of motion with very little speed or attitude augmentation. There is no reason to believe that other vehicles would be significantly different unless they possess highly objectionable speed stability or phugoid characteristics, so the major effort was placed upon estimation of the pitch rate and angle of attack gains, with little emphasis on attitude and none at all on velocity. Table X reflects this emphasis.

5.2 CCV BARE AIRFRAME STABILITY DERIVATIVES

Using the attainable feedback gains given in Table X, it is a relatively straightforward computation to estimate the minimum dimensional stability derivatives that must inherently be possessed by the bare airframe of a Control Configured Vehicle, assuming that sufficient surface effectiveness and power is available to augment the derivatives if they are found to be acceptable.

The equations of motion of the bare airframe CCV and the flying qualities model can be expressed as

$$\dot{x} = Fx + Gu \tag{64}$$

$$\dot{x}_m = F_m x_m + G_m u_m \tag{65}$$

The feedback control law is of the form

$$u = -Kx \tag{66}$$

so a fully augmented aircraft can be described by the equation

$$\dot{x} = (F - GK)x + G_m u \tag{67}$$

If the augmented CCV is to fly as the flying qualities model flies, then

$$F_m = F - GK \tag{68}$$

and the matrix of dimensional stability derivatives of the bare airframe are restricted to the range

$$F_{bare\ CCV} = F_m \pm G|K_{max}| \tag{69}$$

If the matrix of stability derivatives of the model, Table III and the matrix of gains (assuming three controllers) were substituted into Equation (69), the longitudinal requirements become:

$$
F_{bare\ CCV} = \begin{bmatrix} -.01424 & -32.17 & 0 & 5.70 \\ 0 & 0 & 1 & 0 \\ .0037 & 0 & -7.085 & -28.4 \\ -.000157 & 0 & 1 & -2.225 \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ 0 & 0 & 0 \\ g_{31} & g_{32} & g_{33} \\ g_{41} & g_{42} & g_{43} \end{bmatrix} \begin{bmatrix} - & \pm 5 & \pm 3 & \pm 5 \\ - & \pm 4 & \pm 3 & \pm 5 \\ - & \pm 2 & \pm 2 & \pm 4 \end{bmatrix}
$$

$$
= \begin{bmatrix} -.01424 \pm - & -32.17 \pm 5g_{11} \pm 4g_{21} \pm 2g_{13} & \pm 5g_{11} \pm 3g_{12} \pm 2g_{13} & 5.7 \pm 5g_{11} \pm 5g_{12} \pm 4g_{13} \\ 0 & 0 & 1.0 & 0 \\ .0037 \pm - & \pm 5g_{31} \pm 4g_{32} \pm 2g_{33} & \pm 3g_{31} \pm 3g_{32} \pm 2g_{33} & -28.4 \pm 5g_{31} \pm 5g_{32} \pm 5g_{33} \\ -.000157 \pm - & \pm 5g_{41} \pm 4g_{42} \pm 2g_{43} & \pm 3g_{41} \pm 3g_{42} \pm 2g_{43} & -2.225 \pm 5g_{41} \pm 5g_{42} \pm 4g_{43} \end{bmatrix} \tag{70}
$$

where the symbol — means that the gain is not considered and has been shown to be negligible.

It becomes clear then, that surface effectiveness plays a very strong role in the augmentation possible. If this aircraft possessed the surface effectiveness of the elevator, flap and ailerons of the T-33, the CCV bare airframe requirements become

$$
F_{\substack{bare\\ CCV}} = \begin{bmatrix} - & -32.17 \pm 101 & \pm 77.0 & 5.7 \pm 134 \\ - & 0 & 1.0 & 0 \\ - & \pm 196 & -7.085 \pm 130 & -5.28 \pm 220 \\ - & \pm 3.7 & 1 \pm 2.87 & -2.225 \pm 5 \end{bmatrix} \tag{71}
$$

which could quickly lead one to the conclusion that the bare airframe stability derivatives are meaningless to a CCV if augmentation is to be fully realized and if the control surfaces have sufficient effectiveness to do the job.

If sufficient control effectiveness is not provided, then it will be difficult to realize CCV objectives. Consider, for instance, the T-33 airplane with shortened tail and reduced horizontal tail surface $K_A = K_L = 0.5$. The matrices of stability and control derivatives become:

$$F_{K_A,K_L=.5} = \begin{bmatrix} -.0139 & 32.17 & 0 & 6.834 \\ 0 & 0 & 1.0 & 0 \\ -.000028 & 0 & .505 & 8.456 \\ -.000157 & 0 & 1.0 & -2.202 \end{bmatrix}, G_{K_A,K_L=.5} = \begin{bmatrix} -2.02 & -15.8 & -8.7 \\ 0 & 0 & 0 \\ -7.3 & -14.1 & -5.3 \\ -.06 & -.58 & -.38 \end{bmatrix} \quad (72)$$

If elevator alone is used for augmentation purposes the aircraft can be augmented to the extent

$$F_{aug} = \begin{bmatrix} -.0139 \pm - & 32.17 \pm 10 & \pm 6.06 & 6.83 \pm 10.1 \\ 0 & 0 & 1.0 & 0 \\ -.000028 \pm - & \pm 36.5 & .505 \pm 21.9 & 8.456 \pm 36.5 \\ -.000157 \pm - & \pm .30 & 1.0 \pm .18 & -2.202 \pm .30 \end{bmatrix} \quad (73)$$

If the matrix of Equation (73) is compared to that of the model, it is seen that two very important terms in the matrix $f_{33} = M_q + M_{\dot{\alpha}}$ and $f_{34} = M_\alpha + M_{\dot{\alpha}} Z_w$, which are used to approximate short period damping and ω_{SP}^2, are just barely attainable. If only elevator is used for augmentation, then the stability derivatives cannot be independently altered. Once a value of feedback gain is selected, each column of the F matrix is fixed, or another way to express this constraint is that since the 1st, 3rd, and 4th rows of the F matrix contain the coefficients of a separate degree of freedom of motion of the vehicle.

The flap and the aileron of the T-33 in this example can just barely be considered independent control devices, because they produce pitching moments, Z and X forces almost, but not quite, proportional to each other. Therefore, normally the two surfaces would not be considered efficient for dynamic augmentation purposes, but they do provide a good measure of controllability of the wing lift distribution and therefore wing root bending moment control.

5.3 CONTROL (FEEDFORWARD) AUGMENTATION

Feedforward, or control augmentation is less restrictive than feedback. If the Control Configured Vehicle does not exhibit the required control effectiveness to produce good handling qualities, then other surfaces can be used to augment the vehicle.

The matrix of control effectiveness terms of the flying qualities model is given by

$$G_m = \begin{bmatrix} -3.98 \\ 0 \\ -270 \\ -.14 \end{bmatrix} \tag{74}$$

using just the elevator to produce the required control forces and moments. The T-33 with reduced tail length and area but using the flap and ailerons is given by

$$G_{T-33_{K_A}.K_L=.5} = \begin{bmatrix} -2.02 & -15.8 & -8.7 \\ 0 & 0 & 0 \\ -7.3 & -14.1 & -5.3 \\ -.06 & -.58 & -.38 \end{bmatrix} \tag{75}$$

The effectiveness of the other surfaces, i.e., the flap and ailerons, can be used to augment the elevator as shown in the following sketch:

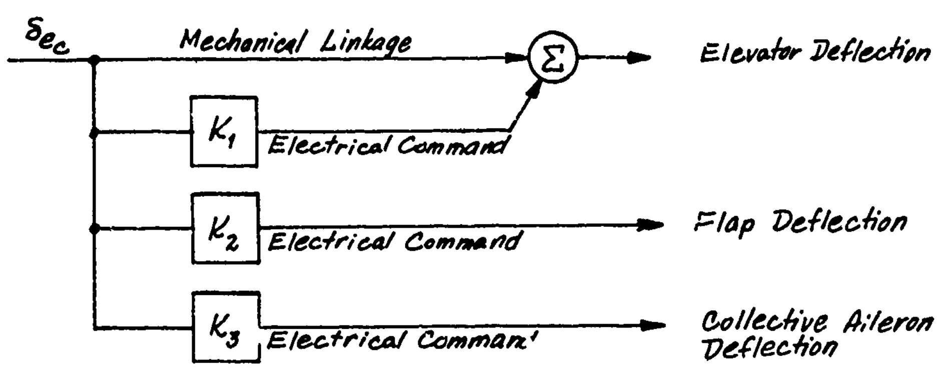


Figure 13 FEEDFORWARD OR COMMAND AUGMENTATION

The gains are computed from

$$\begin{bmatrix} 1+K_1 \\ K_2 \\ K_3 \end{bmatrix} = \begin{bmatrix} G_{T-33,\,K_A,\,K_L=.5} \end{bmatrix}^{-1} \begin{bmatrix} G_m \end{bmatrix}$$

$$= \begin{bmatrix} -2.02 & -15.8 & -8.7 \\ 0 & 0 & 0 \\ -7.3 & -14.1 & -5.3 \\ -.06 & -.58 & -.38 \end{bmatrix}^{-1} \begin{bmatrix} -3.98 \\ 0 \\ -27.0 \\ -.14 \end{bmatrix} \tag{76}$$

or

$$K_1 = 3.71\,, \quad K_2 = -0.895\,, \quad K_3 = 0.995$$

and the system can be mechanized without difficulty.

The investigations and results of this section have shown that stability and flying qualities should have very little influence on the geometry of a Control Configured Vehicle. If sufficient control power is available, the vehicle can have almost any shape and stability augmentation, within the present state of the art, can alter the flying qualities to the desirable Level 1 behavior.

5.4 WEIGHT CONSIDERATIONS

The T-33 configuration with $K_A = K_L = .5$ is about the maximum tail reduction possible for the CCV to still have the ability to obtain a Level 1 airplane with a realizable feedback system. This corresponds to a change in static margin of about -15%. The entire aft section of the fuselage of the T-33, which includes the tail assembly and exhaust extension past the engine, is estimated by CAL personnel to weigh approximately 700 pounds. With the .5 K_A and K_L this can be reduced to 350 pounds. Further weight savings from a lighter wing structure for reduced wing loads due to the MLC system of wing flaps is conservatively estimated at 150 pounds. This 500 pounds is 5% of the normal T-33 dry weight.

This estimate, however does not take into account the weight of the reliable control system that would have to be added to the existing airplane to allow the vehicle to fly with the geometrical configuration assumed in this report. Overall it is believed that the total weight reduction along with the reduced wing root bending moment and drag will significantly improve the performance or payload capacity of the T-33.

SECTION VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

The purpose of the research reported in this document is to investigate CCV design and control system concepts in a general way and to apply these concepts to a T-33 airplane in as realistic a way as possible within the limited scope of the program. The study accomplished this goal in a positive sense and the following conclusions are drawn from the results. These conclusions are tentative; not all of the aspects of the problem we e considered and further effort may modify some of the results but probably not significantly alter the fundamental principles. The major conclusions are summarized below:

1 Relaxed static stability, maneuver load control, and good flying qualities can be made to be compatible if adequate numbers of independent force/moment producing devices with adequate effectiveness and power are provided.

2. Because the geometry and surface configurations are generally fixed, the application of CCV concepts after the fact, i.e., on a presently existing airplane, will yield only limited success. Control Configured Vehicle concepts, to be most effective, should be incorporated into the preliminary design stages of a new airplane.

3. The T-33 configuration with $K_A = K_L = .5$ is about the maximum reduction possible. This corresponds to a change in static margin of about -15%. Total structural weight reductions may amount to about 500 pounds or approximately 5% of the normal T-33 dry weight. This can be interpreted to mean a 10% increase in fuel capacity.

4. Flying qualities have a significant effect on the control system configuration. Flying qualities are very flexibly or broadly defined and can be selected to benefit the maneuver load control objectives of the CCV. Flying qualities requirements can be chosen to restrict or enhance the application of maneuver load control.

5. The present state of the art of feedback control allows for augmentation of an extremely wide variety of bare airframe characteristics and therefore, geometrical shapes of the airframes. Stability constraints, such as static margins, have little or no importance if sufficient control effectiveness and power are available to provide for good flying qualities and maneuverability.

6. A more comprehensive effort, considering many aspects not included in this study would be necessary to optimize the results, but feasibility has been demonstrated.

6.2 RECOMMENDATIONS

This study has only investigated a few CCV concepts: 1) reduced static stability through reduced tail area and tail length, 2) maneuver load control in a pullup through the addition of inboard and outboard direct lift flaps on the wing, and 3) the constraint of flying qualities requirements. The feasibility of this type of vehicle has been demonstrated. However, in the early design of any new CCV, a more extensive study should be carried out. With a precise knowledge of mission and performance criteria, more specific candidate controllers should be investigated. Canards, ventral fins, split wing flaps, and various wing positions should be evaluated to optimize the design to achieve desired characteristics. Torsional wing bending moments must be evaluated, as this may become a problem with the additional wing flaps.

The concepts of relaxed static stability and maneuver load control represent evolutionary, rather than revolutionary advances in aircraft stability and flight control practice. Feedback to augment damping is already in full operational use, so additional feedback and command augmentation to improve static stability is only a step beyond present design procedures. Wing surfaces designed to alter the magnitude and symmetry of the lift along the wing have been in use for fifty years or more. The use of flaps, ailerons or other wing surfaces to alter the lift distribution on the wing during either trimmed or transient flight is also a logical extension of present practices, so maneuver load control is also feasible.

Feasibility only has been demonstrated by the results presented in this report. A more comprehensive study and simulation program is needed before actual mechanization and flight testing could be undertaken. In general, control surfaces are sized and located on aircraft only after extensive analysis and model testing has been done. To be most effective, CCV concepts should be included in the preliminary design stage of an airplane. The application of CCV concepts after the fact of the airplane design will likely be not as effective and the modifications will probably be costly.

Flying qualities requirements will play an important role in the establishment of CCV airframe designs and augmentation configurations. Two flying qualities models were used in the present study. The flying qualities parameter n/α was shown to have a strong effect on the use of flap and elevator surfaces to simultaneously obtain lower wing root headings and good flying qualities. In addition, relaxed static stability of the bare airframe will require relatively large surface deflections and surface rates to artificially produce the stability characteristics demanded by flying qualities. However, flying qualities requirements are broad, and it appears possible to be able to satisfy both flying qualities requirements and make optimum use of CCV concepts at the same time. The mutual overlap of these requirements should be carefully defined.

It would be very important to consider the effects of geometrical alterations throughout the entire flight envelope of operation of the vehicle. It is one thing to design a flight control system that will give good flying qualities at a single flight condition but an entirely different matter to design a simple system for the entire range of operation of the vehicle. The bare airframe dynamics and the optimum flying qualities model changes as a function of flight condition. Although all the elements that make up a Level 1 airplane are broad in range at one flight condition, a minimum complexity control system that satisfies all elements for a Level 1 airplane at all flight conditions represents a formidable challenge to the Control Configured Vehicle designer.

APPENDIX I

LINEAR OPTIMAL CONTROL

The linear optimal control problem treated in Section III of this report is a variation of the general problem using the performance index

$$J = \int_0^{\infty} \left(x^T H^T Q H x + u^T R u\right) dt \tag{I-1}$$

subject to the constraint of the differential equations of motion

$$\dot{x} = Fx + Gu \tag{I-2}$$

$$y = Hx \tag{I-3}$$

The solution to this problem requires that the Euler-Lagrange equations be satisfied

$$\begin{bmatrix} \dot{x}(t) \\ \dot{\lambda}(t) \end{bmatrix} = \begin{bmatrix} F & -GR^{-1}G^T \\ -H^T Q H & -F^T \end{bmatrix} \begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} \qquad \begin{matrix} x(0) = x_0 \\ \lambda(0) = \lambda_0 \end{matrix} \tag{I-4}$$

subject to the boundary conditions on the state vector $X(0) = X_0$ and the Lagrange multiplier $\lambda(0) = \lambda_0$. The basic problem is to determine the boundary condition $\lambda(0)$ as a function of the state vector, thereby eliminating the two-point boundary value aspects of the problem which will then yield a closed form solution.

It has been shown by R. E. Kalman (Reference 7) and others that $\lambda(0)$ and $X(0)$ are related by the equation

$$\lambda(0) = P(\infty)\, X(0) \tag{I-5}$$

where $P(\infty)$ is the positive definite symmetrical solution to the matrix Riccati equation

$$0 = PF + F^T P - PGR^{-1}G^T P + H^T Q H \tag{I-6}$$

It has been shown (Reference 10, for instance) that the eigenvalues of the Hamiltonian system of Equation I-4 consist of the eigenvalues of the stable optimal closed-loop system with negative real parts Λ and the eigenvalues of the unstable "adjoint" system $-\Lambda$ with positive real parts.

If we transfer Equation I-4 into the diagonal canonical form using a linear transformation

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$$\begin{bmatrix} x \\ \lambda \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad \text{(I-7)}$$

we would have

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}^{-1} \begin{bmatrix} F & -GR^{-1}G' \\ -H'QH & F' \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$= \begin{bmatrix} \Lambda & 0 \\ 0 & -\Lambda \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad \begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}^{-1} \begin{bmatrix} x(0) \\ \lambda(0) \end{bmatrix} \qquad \text{(I-8)}$$

where Λ is an $n \times n$ diagonal matrix with (distinct) negative real parts. The response can be then written

$$\begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} e^{\Lambda t} & 0 \\ 0 & e^{-\Lambda t} \end{bmatrix} \begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix} \qquad \text{(I-9)}$$

or

$$\begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} e^{\Lambda t} & 0 \\ 0 & e^{-\Lambda t} \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}^{-1} \begin{bmatrix} x(0) \\ \lambda(0) \end{bmatrix} \qquad \text{(I-10)}$$

For the time being, write the inverse of the transformation as

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}^{-1} = \begin{bmatrix} K & L \\ M & N \end{bmatrix} \qquad \text{(I-11)}$$

then

$$\begin{bmatrix} x(t) \\ \lambda(t) \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} e^{\Lambda t} & 0 \\ 0 & e^{-\Lambda t} \end{bmatrix} \begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} x(0) \\ \lambda(0) \end{bmatrix} \qquad \text{(I-12)}$$

from which we can obtain

$$\lambda(t) = T_{11} e^{\Lambda t} K + T_{11} e^{\Lambda t} L \lambda(0) + T_{12} e^{-\Lambda t} M x(0) + T_{12} e^{-\Lambda t} N \lambda(0) \tag{I-13}$$

The optimal solution for $x(t)$, which is stable, cannot contain terms in $e^{-\Lambda t}$. Therefore, $x(0)$ and $\lambda(0)$ must be related by the expression.

$$\lambda(0) = -N^{-1} M x(0) \tag{I-14}$$

Substituting this expression for $\lambda(0)$ in Equation I-13 yields

$$x(t) = T_{11} e^{\Lambda t} (K - L N^{-1} M) x(0) \tag{I-15}$$

From the identity

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} K & L \\ M & N \end{bmatrix} = \begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{I-16}$$

We can obtain, among others, the following relationships

$$M = -N T_{21} T_{11}^{-1} \quad \text{(a)} \qquad K = -L T_{22} T_{12}^{-1} \quad \text{(c)}$$

$$N = (T_{22} - T_{21} T_{11}^{-1} T_{12})^{-1} \quad \text{(b)} \qquad L = (T_{21} - T_{22} T_{12}^{-1} T_{11})^{-1} \quad \text{(d)} \tag{I-17}$$

Substituting these expressions for K, L, M, and N in Equation I-15 yields

$$x(t) = T_{11} e^{\Lambda t} T_{11}^{-1} x(0) \tag{I-18}$$

Comparing Equation I-5 and I-14 we find that the steady-state solution to the Riccati Equation is given by

$$P(\infty) = -N^{-1} M \tag{I-19}$$

or, from a substitution in Equation I-19a we have

$$P(\infty) = T_{21} T_{11}^{-1} \tag{I-20}$$

This result could have been shown in another way. From Equation I-8 we have

$$\begin{bmatrix} K & L \\ M & N \end{bmatrix} \begin{bmatrix} F & -GR^{-1}G' \\ -H'QH & -F' \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \Lambda & 0 \\ 0 & -\Lambda \end{bmatrix} \tag{I-21}$$

Expanding the left-hand side of Equation I-21 yields

$$\left[\begin{array}{c|c} KFT_{11} - LH'QHT_{11} - KGR^{-1}G'T_{21} - LF'T_{21} & KFT_{12} - LH'QHT_{12} - KGR^{-1}G'T_{22} - LF'T_{22} \\ \hline MFT_{11} - NH'QHT_{11} - MGR^{-1}G'T_{21} - NF'T_{21} & MFT_{12} - NH'QHT_{12} - MGR^{-1}G'T_{22} - NF'T_{22} \end{array}\right]$$

$$= \begin{bmatrix} \Lambda & 0 \\ 0 & -\Lambda \end{bmatrix} \tag{I-22}$$

From the lower left-hand part of the partitioned matrix of Equation I-22 we have that

$$MFT_{11} - NH'QHT_{11} - MGR^{-1}GT_{21} - NF'T_{21} = 0$$

Substituting $M = -NT_{21} T_{11}^{-1}$, and post multiplying the entire equation by $+T_{11}^{-1}$ and pre-multiplying by $-M^{-1}$ yields

$$T_{21} T_{11}^{-1} F + F^{-1} T_{21} T_{11}^{-1} - T_{21} T_{11}^{-1} GR^{-1} G' T_{21} T_{11}^{-1} + H'QH = 0 \tag{I-23}$$

If we compare Equation I-23 with the Riccati Equation

$$PF + F'P - PGR^{-1}G'P + H'QH = 0 \tag{I-24}$$

we have the same result as in Equation I-20, namely that

$$P(\infty) = T_{21} T_{11}^{-1} = (T_{11}^{-1})' T_{21}' \tag{I-25}$$

A similar development involving the upper-right partitioned matrix of Equation I-22, i.e., that

$$KFT_{12} - LH'QHT_{12} - KGR^{-1}T_{22} - LF'T_{22} = 0 \tag{I-26}$$

will yield another result. Substituting $K = -L T_{22} T_{12}^{-1}$ post multiplying the entire equation by $-L^{-1}$ and pre multiplying by $+T_{12}^{-1}$ yields

$$T_{22}T_{12}^{-1}F + F'T_{22}T_{12}^{-1} - T_{22}T_{12}^{-1}GR^{-1}G'T_{22}T_{12}^{-1} + H'QH = 0 \tag{I-27}$$

or

$$P(\infty) = T_{21}T_{11}^{-1} = T_{22}T_{12}^{-1} \tag{I-28}$$

Therefore, once we know the eigenvectors of the Hamiltonian system of Equation I-4 we can directly compute the optimal feedback control law. The technique shown in this section is independent of the order of the system and the number of controllers of the dynamic system. Equations I-18 and I-20 show that the feedback gains and the regulator transient response are directly related to the eigenvectors of the system. These eigenvectors are a function of the weighting matrices Q and R in the performance index.

Example

Consider the two controller, second order system described by the equations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{I-29}$$

It is desired to find the optimal control law that will satisfy the performance index

$$2V = \min_{u} \int_0^{\infty} (q_1 x_1^2 + q_2 x_2^2 + r_1 u_1^2 + r_2 u_2^2)\, dt \tag{I-30}$$

where $q_1 = q_2 = r_1 = r_2 = 1$

The Hamiltonian system for this example is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\lambda}_1 \\ \dot{\lambda}_2 \end{bmatrix} = \begin{bmatrix} -2 & 2 & -1 & 0 \\ 1 & -3 & 0 & -1 \\ 1 & 0 & +2 & 1 \\ 0 & 1 & 2 & +3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \lambda_1 \\ \lambda_2 \end{bmatrix} \tag{I-31}$$

whose characteristic polynomial is given by

$$\begin{aligned} \Delta(s)\tilde{\Delta}(s) &= (s^2 + 5.55\,s + 5.89)(s^2 - 5.55\,s + 5.89) \\ &= (s \pm 4.115)(s \pm 1.44) \end{aligned} \tag{I-32}$$

The left half plane roots, i.e., the eigenvalues of the closed-loop optimal system, are given by

$$s_{1,2} = -1.44\,,\; -4.115$$

and it is necessary to find the two eigenvectors of Equation I-31 associated with these two eigenvalues

The eigenvector transformation is found to be

$$T = \left[\begin{array}{cc|cc} 1 & 1 & \cdot & \cdot \\ .467 & -.934 & \cdot & \cdot \\ \hline .369 & .147 & \cdot & \cdot \\ .271 & -.093 & \cdot & \cdot \end{array}\right] = \left[\begin{array}{c|c} T_{11} & T_{12} \\ \hline T_{21} & T_{22} \end{array}\right] \tag{I-33}$$

where the two blank columns represent the eigenvectors associated with the eigenvalues $s_{3,4} = +1.44, +4.115$ From Equation I-25 , we have

$$P(\infty) = + T_{21} T_{11}^{-1} = + \begin{bmatrix} .369 & .147 \\ .277 & -.098 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ .467 & -.984 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} +.299 & +.153 \\ +.153 & +.255 \end{bmatrix} \qquad \text{(I-34)}$$

which is a positive definite symetric matrix. The optimal control law becomes

$$u = -R^{-1} G' P x = - \begin{bmatrix} +.299 & +.153 \\ +.153 & +.256 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

and the closed-loop optimal regulator description is

$$\dot{x} = (F - GK)\, x$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \left\{ \begin{bmatrix} -2 & 2 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} .299 & .153 \\ .153 & .256 \end{bmatrix} \right\} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2.299 & 1.848 \\ .848 & -3.256 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \text{(I-36)}$$

whose characteristic polynomial is given by

$$\Delta(s) = s^2 + 5.55s + 5.81 \qquad \text{(I-37)}$$

as predicted by Equation I-32. The regulator transient response is given by Equation I-18.

$$x(t) = T_{11}\, e^{\Lambda t}\, T_{11}^{-1}\, x(0)$$

which, for our example becomes

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ .467 & -.984 \end{bmatrix} \begin{bmatrix} e^{-1.44t} & 0 \\ 0 & e^{-4.115t} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ .467 & -.984 \end{bmatrix}^{-1} \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix}$$

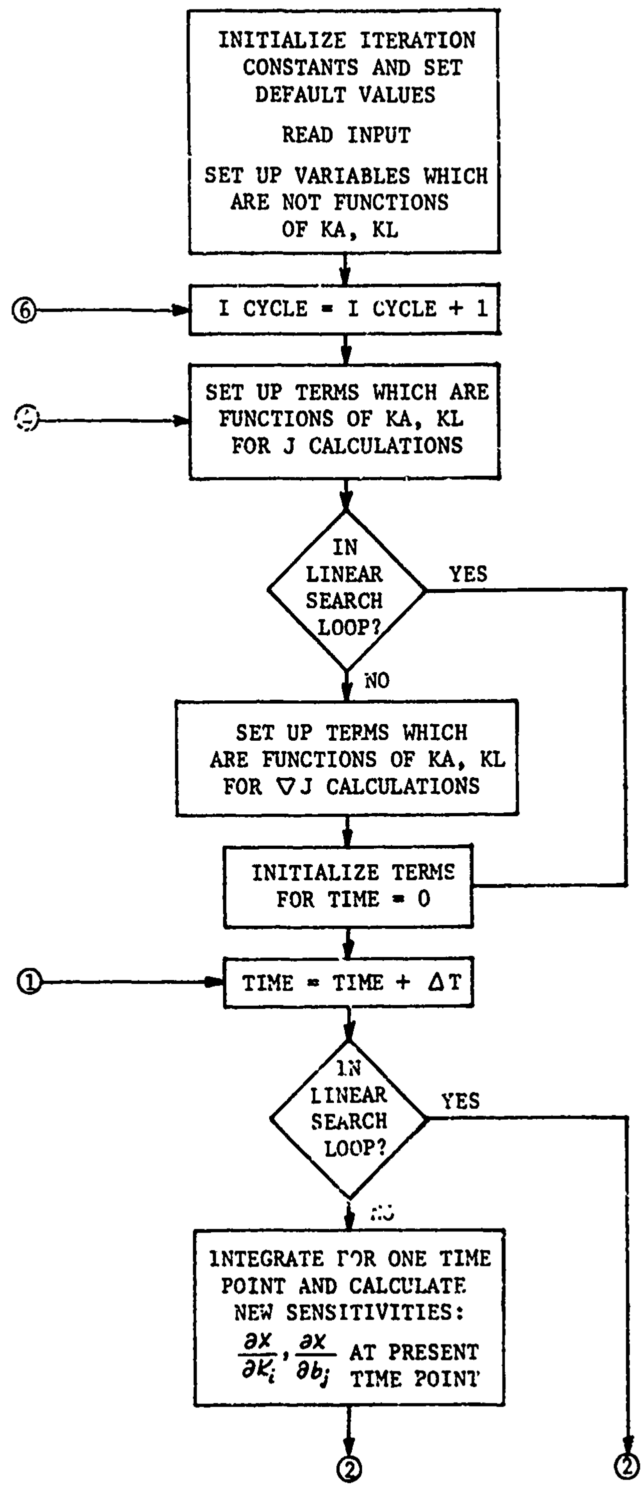
or

$$x_1(t) = (.697\,e^{-1.44t} + .322\,e^{-4.115t})\,x_1(0) + .690\,(e^{-1.44t} - e^{-4.115t})\,x_2(0)$$
$$x_2(t) = (.326\,e^{-1.44t} - .317\,e^{-4.115t})\,x_1(0) + (.322\,e^{-1.44t} + .679\,e^{-4.115t})\,x_2(0)$$

(I-38)

APPENDIX II
CONJUGATE GRADIENT COMPUTER PROGRAM

The flow chart and listing for the conjugate gradient program used in Section IV of the report are given below.



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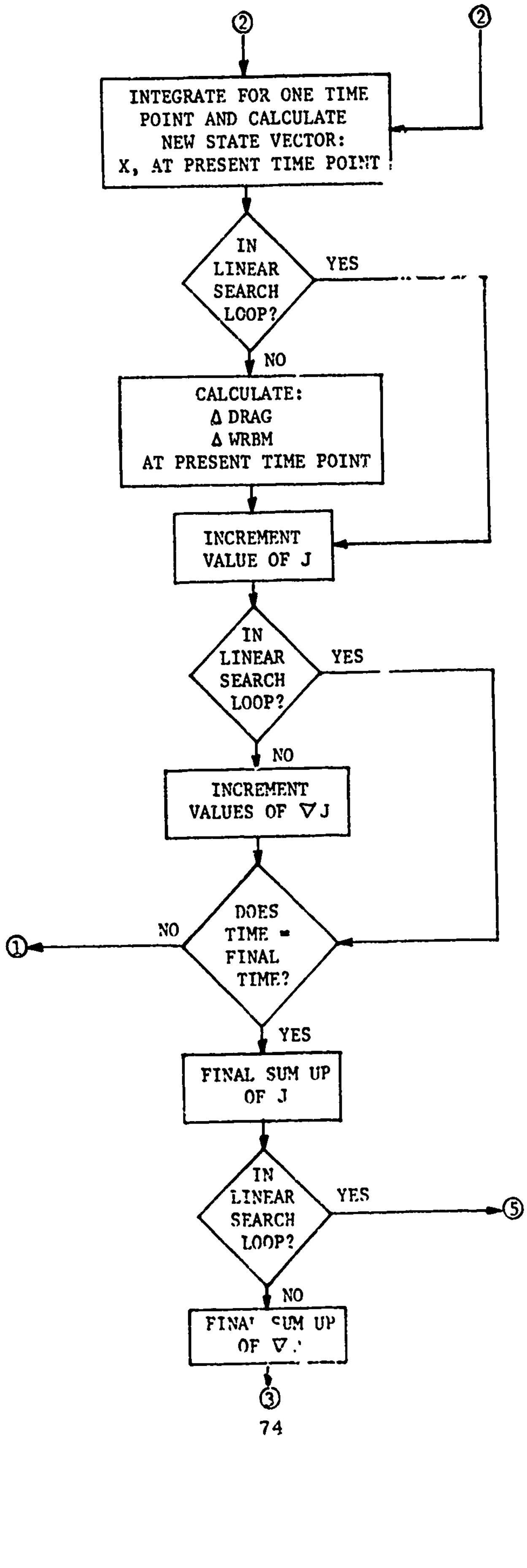
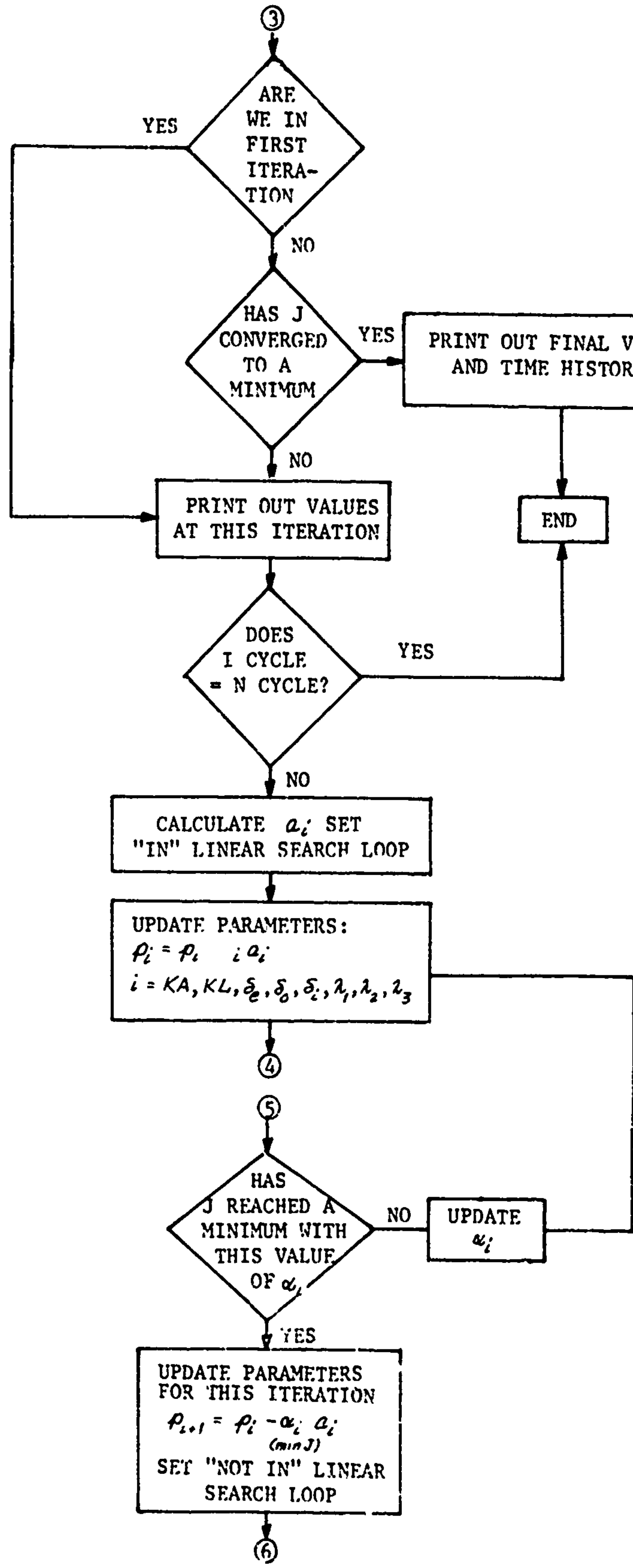

②
②
INTEGRATE FOR ONE TIME POINT AND CALCULATE NEW STATE VECTOR: X, AT PRESENT TIME POINT
IN LINEAR SEARCH LOOP?
YES
NO
CALCULATE: Δ DRAG Δ WRBM AT PRESENT TIME POINT
INCREMENT VALUE OF J
IN LINEAR SEARCH LOOP?
YES
NO
INCREMENT VALUES OF ∇J
DOES TIME = FINAL TIME?
NO
①
YES
FINAL SUM UP OF J
IN LINEAR SEARCH LOOP?
YES
⑤
NO
③

continued . . .


3
ARE WE IN FIRST ITERATION
YES
NO
HAS J CONVERGED TO A MINIMUM
YES
PRINT OUT FINAL VALUES AND TIME HISTORIES
NO
PRINT OUT VALUES AT THIS ITERATION
END
DOES I CYCLE = N CYCLE?
YES
NO
CALCULATE a_i SET "IN" LINEAR SEARCH LOOP
UPDATE PARAMETERS:
$P_i = P_i \quad {}_i a_i$
$i = KA, KL, \delta_e, \delta_o, \delta_i, \lambda_1, \lambda_2, \lambda_3$
4
5
HAS J REACHED A MINIMUM WITH THIS VALUE OF α_i
NO
UPDATE α_i
YES
UPDATE PARAMETERS FOR THIS ITERATION
$P_{i+1} = P_i - \alpha_{i\,(min\,J)}\, a_i$
SET "NOT IN" LINEAR SEARCH LOOP
6

INPUT

All input is read in NAMELIST form:

$u(3)$ = initial input steps of $\delta_e, \delta_i, \delta_o$
$L(4\times4)$ = model F matrix
$Q(4)$ = weighting matrix on states
$XL(3)$ = initial values on $\lambda_1, {}_2, {}_3$
$TITLE$ = (any 80 characters)
$Z(40)$ = constants for F, G, d_1, d_2, m_1, m_2 matrices:

$$F = \begin{bmatrix} Z(1)K_A + Z(2) & -32.17 & 0 & Z(3)K_A + Z(4) \\ 0 & 0 & 1 & 0 \\ Z(5)K_A \cdot K_L + Z(6) & 0 & Z(7)K_A \cdot K_L + Z(8) & Z(9)K_A \cdot K_L + Z(10) \\ Z(11) & 0 & 1 & Z(12)K_A \cdot K_L + Z(13) \end{bmatrix}$$

$$G = \begin{bmatrix} Z(14) \cdot K_A & Z(15) & Z(16) \\ 0 & 0 & 0 \\ Z(17)K_A \cdot K_L & Z(18) & Z(19) \\ Z(20) & Z(21) & Z(22) \end{bmatrix}$$

$$d_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Z(23) + Z(24)K_A \end{bmatrix} \quad d_2 = \begin{bmatrix} Z(25)K_A \\ Z(26) \\ Z(27) \end{bmatrix} \quad m_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Z(28) \end{bmatrix} \quad m_2 = \begin{bmatrix} 0 \\ Z(29) \\ Z(30) \end{bmatrix}$$

DT = Δ time increment

NTP = number of time points

V = weight on drag

T = weight on WRBM

CA = initial constant for α_i (defaulted to 1.)

$REDUCE$ = reduction multiple for α_i update (defaulted to .1)

$LOOP$ = max number of loops in α_i updates (defaulted to 10)

$NCYCLE$ = max number of conjugate gradient iterations (defaulted to 10)

$XJSTOP$ = J convergence test: stops if

$$|J_{i-1} - J_i| \leq |XJSTOP \cdot J_i|$$

(defaulted to .005)

$IPRINT$ = to print intermediate interations set to 1 if not set to 0. (defaulted to 1)

KA = initial tail area ratio

KL = initial tail length ratio

OUTPUT

The output form is:

input data:
- Title Date
- Final values wanted for states: $q, \alpha, \dot{\alpha}$
- Model 1 matrix
- Q
- V
- T

for each iteration:

- Title — Date
- Icycle — J
- Time histories of:
  - Time, ΔV, θ, q, α, Δ Drag, Δ WRBM
  - Drag
  - WRBM
- $\int \Delta DRAG$
- $\int WRBM$
- KA
- KL
- δ_e
- δ_i
- δ_o
- $\lambda_1, \lambda_2, \lambda_3$
- F matrix
- Linear search values for α_i, J

```
C  CONJUGATE GRADIENT METHOD FOR CCV ,TO FIND TAIL AREA,LENGTH,AND CONSTANT
C  CONTROL INPUTS DE,DI,DO FOR A 4G PULL UP
      DIMENSION X(4,1),XOUT(4,51),F(4,4),G(4,3),U(3),L(4,4),C(3),Q(4),
     *XL(3),TITLE(20),FFDM(4,4),FGDM(4,3),GGDM(3,3),DFFA(4,4),DFFL(4,4),
     *DFGA(4,3),DFGL(4,3),DGGA(1,3),DGGL(1,3),D2(3,1),M2(3,1),FML(4,4),
     *SB(4,3),PHI1(4,4),PHI2(4,4),Z(40),TIME(51),DFA(4,4),DFL(4,4),
     *   DGA(4,1),TM2M2(3,3),TEMP1(4,4),TEMP2(4,4),TEMP3(4,4),FMLTQ(4,4)
     *,DFATQ(4,4),DFLTQ(4,4),SKA(4,1),SKL(4,1),PHI2G(4,3),PHI2GU(4,1),
     *DRAG(51),WPBM(51),XT(1,4),TEMP4(4,4),SKAT(1,4),SKLT(1,4),SBT(3,4),
     *A(8),APPEV(8),ULAST(3),XLLAST(3),DELJ(8)
      REAL KA,KL,L,M1,M2,KALAST,KLLAST
      REAL*8 TDATE
      NAMELIST/INPUT/U,L,C,Q,XL,TITLE,Z,DT,NTP,V,T,CA,REDUCE,LOOP,NCYCLE
     *,XJSTOP,IPRINT,KA,KL
      CALL CLEAR(CA,DELJ(8))
      CA=1.
      XJSTOP=.005
      REDUCE=.1
      LOOP=10
      NCYCLE=10
      IDELJ=1
      IPRINT=1
      CALL DATE(TDATE)
    1 READ(5,INPUT,END=9999)
      WRITE(6,20)TITLE,TDATE,(C(I),I=1,3),((L(I,J),J=1,4),I=1,4)
   20 FORMAT(1H1,2X,20A4,10X,A8,//,2X,'MODEL Q,ALPHA,ALPHA-DOT AT FINAL
     *TIME ARE: ',3E17.6,//,2X,'MODEL L MATRIX IS:',/,(4E16.6))
      WRITE(6,21) (Q(I),I=1,4),V,T
   21 FORMAT(//,2X,'Q = ',4F17.6,/,2X'V = ',F17.6,/,2X,'T = ',F17.6)
      ISTOP=0
      ICYCLE=0
C  SET UP VARIABLES WHICH ARE NOT FUNCTIONS OF KA,KL
      F(1,2)=-32.17
      F(2,3)=1.
      F(4,1)=Z(11)
      F(4,3)=1.
      G(1,2)=Z(15)
      G(1,3)=Z(16)
      G(3,2)=Z(18)
      G(3,3)=Z(19)
      G(4,2)=Z(21)
      G(4,3)=Z(22)
      D2(2,1)=Z(26)
      D2(3,1)=Z(27)
      M1=Z(28)
      DD1A=Z(24)
      DD2A=Z(25)
      M2(2,1)=Z(29)
      M2(3,1)=Z(30)
      DFA(1,1)=Z(2)
      DFA(1,4)=Z(4)
      DGA(1,1)=Z(14)
      DGA(4,1)=Z(20)
      DO 30 I=1,3
   30 TEMP1(1,I)=M2(I,1)*T
      CALL MATMPY(M2,TEMP1,TM2M2,3,1,3,3,4,3)
   35 ICYCLE=ICYCLE+1
   36 CONTINUE
```

```
C   SET UP TERMS WHICH DEPEND ON KA,KL NEEDED FOR J CALCULATIONS
      F(1,1)=Z(1)+Z(2)*KA
      F(1,4)=Z(3)+Z(4)*KA
      F(3,1)=Z(5)+Z(6)*KA*KL
      F(3,3)=Z(7)+Z(8)*KA*KL
      F(3,4)=Z(9)+Z(10)*KA*KL
      F(4,4)=Z(12)+Z(13)*KA*KL
      G(1,1)=Z(14)*KA
      G(3,1)=Z(17)*KA*KL
      G(4,1)=Z(20)*KA
      D1=Z(23)+Z(24)*KA
      D2(1,1)=Z(25)*KA
      CALL MATEXP(4,DT,F,PHI1,PHI2,TEMP1,TEMP2,4,4,4,4,4,LT)
      IF(LT.EQ.0)GO TO 40
      WRITE(6,37)LT,DT
   37 FORMAT(1H0,3X,'MATEXP FAILED, LT = 'I2,' DT = ',F8.5)
   38 WRITE(6,39)((F(I,J),J=1,4),I=1,4)
   39 FORMAT(1H0,3X,'F MATRIX',/,(4E16.6))
      GO TO 1
   40 DO 41 I=2,NTP
   41 TIME(I)=TIME(I-1)+DT
      CALL MATMPY(PHI2,G,PHI2G,4,4,3,4,4,4)
      CALL MATMPY(PHI2G,U,PHI2GU,4,3,1,4,3,4)
      CALL MATADD(F,L,FML,4,4,4,4,4,1)
C   SET UP MATRICES WHICH DEPEND ON KA,KL , FOR J CALCULATIONS
      DO 45 I=1,3
   45 TEMP1(1,I)=D2(I,1)*V
      CALL MATMPY(D2,TEMP1,TEMP2,3,1,3,3,4,4)
      CALL MATADD(TEMP2,TM2M2,TEMP1,3,3,4,3,4,0)
      DO 46 I=1,4
      DO 46 J=1,3
   46 TEMP2(J,I)=G(I,J)*Q(I)
      CALL MATMPY(TEMP2,G,TEMP3,3,4,3,4,4,4)
      CALL MATADD(TEMP3,TEMP1,GGDM,3,3,4,4,3,0)
      DO 47 I=1,4
      DO 47 J=1,4
   47 FMLTQ(I,J)=FML(J,I)*Q(J)
      CALL MATMPY(FMLTQ,FML,FFDM,4,4,4,4,4,4)
      FFDM(4,4)=FFDM(4,4)+V*D1**2+T*M1**2
      CALL MATMPY(FMLTQ,G,FGDM,4,4,3,4,4,4)
      FGDM(4,1)=FGDM(4,1)+V*D1*D2(1,1)
      FGDM(4,2)=FGDM(4,2)+V*D1*D2(2,1)+T*M1*M2(2,1)
      FGDM(4,3)=FGDM(4,3)+V*D1*D2(3,1)+T*M1*M2(3,1)
      IF(IDELJ.EQ.0) GO TO 60
C    SET UP MATRICES FOR DELJ CALCULATIONS
      DGA(3,1)=Z(17)*KL
      DGL=Z(17)*KA
      DFA(3,1)=Z(6)*KL
      DFA(3,3)=Z(8)*KL
      DFA(3,4)=Z(10)*KL
      DFA(4,4)=Z(13)*KL
      DFL(3,1)=Z(6)*KA
      DFL(3,3)=Z(8)*KA
      DFL(3,4)=Z(10)*KA
      DFL(4,4)=Z(13)*KA
      DO 50 I=1,4
   50 TEMP1(1,I)=DGA(I,1)*Q(I)
      CALL MATMPY(TEMP1,G,DGGA,1,4,3,4,4,1)
```

```
      DO 51 I=1,3
   51 DGGA(1,I)=DGGA(1,I)+V*DD2A*D2(I,1)
      DO 52 I=1,3
   52 DGGL(1,I)=DGL*Q(3)*G(3,I)
      DO 53 I=1,4
      DO 53 J=1,4
   53 DFATQ(I,J)=DFA(J,I)*Q(J)
      CALL MATMPY(DFATQ,FML,DFFA,4,4,4,4,4,4)
      DFFA(4,4)=DFFA(4,4)+V*DD1A*D1
      DO 54 I=3,4
      DO 54 J=1,4
   54 DFLTQ(J,I)=DFL(I,J)*Q(I)
      CALL MATMPY(DFLTQ,FML,DFFL,4,4,4,4,4,4)
      CALL MATMPY(DFATQ,G,DFGA,4,4,3,4,4,4)
      CALL MATMPY(FMLTQ,DGA,TEMP1,4,4,1,4,4,4)
      DO 55 I=1,4
   55 DFGA(I,1)=DFGA(I,1)+TEMP1(I,1)
      DFGA(4,1)=DFGA(4,1)+V*DD1A*D2(1,1)+V*D1*DD2A
      DFGA(4,2)=DFGA(4,2)+V*DD1A*D2(2,1)
      DFGA(4,3)=DFGA(4,3)+V*DD1A*D2(3,1)
      CALL MATMPY(DFLTQ,G,DFGL,4,4,3,4,4,4)
      DO 56 I=1,4
   56 DFGL(I,1)=DFGL(I,1)+FMLTQ(I,3)*DGL
   60 CONTINUE
C        INITIALIZE FOR TIME = 0.
      XJ=0.
      DO 61 I=1,8
   61 DELJ(I)=0.
      DO 62 I=1,4
      X(I,1)=0.
      SKA(I,1)=0.
      SKL(I,1)=0.
      DO 62 J=1,3
   62 SB(I,J)=0.
      SUMD=0.
      SUMW=0.
      DO 100 IT=2,NTP
      IF(IDELJ.EQ.0) GO TO 70
C     CALCULATE SENSITIVITIES
      CALL MATMPY(DFA,X,TEMP1,4,4,1,4,4,4)
      DO 65 I=1,4
   65 TEMP1(I,1)=TEMP1(I,1)+DGA(I,1)*U(1)
      CALL MATMPY(PHI2,TEMP1,TEMP2,4,4,4,4,4,4)
      CALL MATMPY(PHI1,SKA,TEMP1,4,4,1,4,4,4)
      CALL MATADD(TEMP1,TEMP2,SKA,4,1,4,4,4,0)
      CALL MATMPY(DFL,X,TEMP1,4,4,1,4,4,4)
      TEMP1(3,1)=TEMP1(3,1)+DGL*U(1)
      CALL MATMPY(PHI2,TEMP1,TEMP2,4,4,1,4,4,4)
      CALL MATMPY(PHI1,SKL,TEMP1,4,4,1,4,4,4)
      CALL MATADD(TEMP1,TEMP2,SKL,4,1,4,4,4,0)
      CALL MATMPY(PHI1,SB,TEMP1,4,4,3,4,4,4)
      CALL MATADD(TEMP1,PHI2G,SB,4,3,4,4,4,0)
   70 CALL MATMPY(PHI1,X,TEMP1,4,4,1,4,4,4)
      CALL MATADD(TEMP1,PHI2GU,X,4,1,4,4,4,0)
      IF(IDELJ.EQ.0) GO TO 75
      DO 72 I=1,4
   72 XOUT(I,IT)=X(I,1)
      DRAG(IT)=D1*X(4,1)+D2(1,1)*U(1)+D2(2,1)*U(2)+D2(3,1)*U(3)
```

```
      SUMD=SUMD+DRAG(IT)
      WRBM(IT)=M1*X(4,1)+M2(2,1)*U(2)+M2(3,1)*U(3)
      SUMW=SUMW+WRBM(IT)
   75 DO 76 I=1,4
   76 XT(1,I)=X(I,1)
C     INCREMENT J
      CALL MATMPY(FFDM,X,TEMP1,4,4,1,4,4,4)
      CALL MATMPY(FGDM,U,TEMP2,4,3,1,4,3,4)
      DO 77 I=1,4
   77 TEMP3(I,1)=2.*TEMP2(I,1)
      CALL MATADD(TEMP3,TEMP1,TEMP4,4,1,4,4,4,0)
      CALL MATMPY(XT,TEMP4,TEMPJ,1,4,1,1,4,1)
      XJ=XJ+TEMPJ*DT
      IF(IDELJ.EQ.0) GO TO 90
C     INCREMENT DELJ
      DO 80 I=1,4
      SKAT(1,I)=SKA(I,1)
      SKLT(1,I)=SKL(I,1)
      DO 80 J=1,3
   80 SBT(J,I)=SB(I,J)
      CALL MATADD(TEMP1,TEMP2,TEMP3,4,1,4,4,4,0)
      CALL MATMPY(SKAT,TEMP3,TEMPJA,1,4,1,1,4,1)
      CALL MATMPY(SKLT,TEMP3,TEMPJL,1,4,1,1,4,1)
      CALL MATMPY(SBT,TEMP3,TEMP4,3,4,1,3,4,4)
      CALL MATMPY(DFFA,X,TEMP1,4,4,1,4,4,4)
      CALL MATMPY(DFGA,U,TEMP2,4,3,1,4,3,4)
      CALL MATADD(TEMP1,TEMP2,TEMP3,4,1,4,4,4,0)
      CALL MATMPY(XT,TEMP3,TEMP,1,4,1,1,4,1)
      DELJ(4)=DELJ(4)+(TEMPJA+TEMP)*2.*DT
      CALL MATMPY(DFFL,X,TEMP1,4,4,1,4,4,4)
      CALL MATMPY(DFGL,U,TEMP2,4,3,1,4,3,4)
      CALL MATADD(TEMP1,TEMP2,TEMP3,4,1,4,4,4,0)
      CALL MATMPY(XT,TEMP3,TEMP,1,4,1,1,4,1)
      DELJ(5)=DELJ(5)+(TEMPJL+TEMP)*2.*DT
      CALL MATMPY(XT,FGDM,TEMP1,1,4,3,1,4,4)
      DO 85 I=1,3
   85 DELJ(5+I)=DELJ(5+I)+(TEMP4(I,1)+TEMP1(1,I))*2.*DT
   90 CONTINUE
  100 CONTINUE
C     FINAL SUM UP OF J
      CALL MATMPY(GGDM,U,TEMP1,3,3,1,3,3,4)
      XJ=XJ+TIME(NTP)*(U(1)*TEMP1(1,1)+U(2)*TEMP1(2,1)+U(3)*TEMP1(3,1))
      XADD=XL(1)*(X(3,1)-C(1))**2+XL(2)*(X(4,1)-C(2))**2
      ADD=(Z(11)*X(1,1)+X(3,1)+F(4,4)*X(4,1)+G(4,1)*U(1)+G(4,2)*U(2)+
     *G(4,3)*U(3)-C(3))**2*XL(3)
      XJ=XJ+XADD+ADD
      IF(IDELJ.EQ.0) GO TO 190
C     FINAL SUM UP OF DELJ
      DELJ(1)=(X(3,1)-C(1))**2
      DELJ(2)=(X(4,1)-C(2))**2
      DELJ3 = Z(11)*X(1,1)+X(3,1)+F(4,4)*X(4,1)+G(4,1)*U(1)+G(4,2)*U(2)
     $+G(4,3)*U(3)-C(3)
      DELJ(3)=DELJ3**2
      XLADD=0.
      XAADD=0.
      DO 105 I=1,3
      XAADD=XAADD+2.*TIME(NTP)*U(1)*DGGA(1,I)*U(I)
  105 XLADD=XLADD+2.*TIME(NTP)*U(1)*DGGL(1,I)*U(I)
```

```
      DELJ(4)=DELJ(4)+XAADD+XL(1)*SKA(3,1)*2.*(X(3,1)-C(1))+XL(2)*
     *SKA(4,1)*2.*(X(4,1)-C(2))
      DELJ(4)=DELJ(4)+XL(3)*(Z(11)*SKA(1,1)+SKA(3,1)+F(4,4)*SKA(4,1))*
     *2.*(DELJ3)
      DELJ(4)=DELJ(4)+XL(3)*(DFA(4,4)*X(4,1)+DGA(4,1)*U(1))*2.*DELJ3
      DELJ(5)=DELJ(5)+XLADD+XL(1)*SKL(3,1)*2.*(X(3,1)-C(1))+XL(2)*
     *SKL(4,1)*2.*(X(4,1)-C(2))
      DELJ(5)=DELJ(5)+XL(3)*(Z(11)*SKL(1,1)+SKL(3,1)+F(4,4)*SKL(4,1))*
     *2.*DELJ3
      DELJ(5)=DELJ(5)+XL(3)*DFL(4,4)*X(4,1)*2.*DELJ3
      CALL MATMPY(GGDM,U,TEMP1,3,3,1,3,3,4)
      DO 110 I=1,3
  110 DELJ(I+5)=DELJ(I+5)+2.*TIME(NTP)*TEMP1(I,1)
      DO 115 I=1,3
  115 DELJ(I+5)=DELJ(I+5)+XL(1)*SB(3,I)*2.*(X(3,1)-C(1))+XL(2)*SB(4,I)*
     *2.*(X(4,1)-C(2))
      DO 120 I=1,3
      DELJ(I+5)=DELJ(I+5)+XL(3)*(Z(11)*SB(1,I)+SB(3,I)+F(4,4)*SB(4,I) +
     $G(4,I))*2.*DELJ3
  120 CONTINUE
      WRITE(6,125)TITLE,TDATE,ICYCLE,XJ
  125 FORMAT(1H1,2X,20A4,10X,A8,//,5X,'ICYCLE=',I3,4X,'J= ',F13.6,//)
      IF(ICYCLE.EQ.1) GO TO 135
C     TEST FOR J CONVERGENCE
      IF(ABS(XJLAST-XJ).LE.ABS(XJSTOP*XJLAST)) GO TO 140
      IF(ICYCLE.GE.NCYCLE) GO TO 130
      IF(IPRINT.EQ.1) GO TO 135
      GO TO 150
  130 ISTOP=1
      ITER=ICYCLE-1
      WRITE(6,131)ITER
  131 FORMAT(1H0,2X,'J FAILED TO SATISFY CONVERGENCE TEST AFTER ',I3,' I
     $TERATIONS. FINAL TIME HISTORIES ARE:',//)
      GO TO 145
  135 ITER=ICYCLE-1
      WRITE(6,136)ITER
  136 FORMAT(1H0,2X,'AFTER ',I3,' ITERATIONS, TIME HISTORIES ARE:',//)
      GO TO 145
  140 ISTOP=1
      ITER=ICYCLE-1
      WRITE(5,141)ITER
  141 FORMAT(1H0,2X,'J SATISFIED CONVERGENCE TEST AFTER ',I3,' ITERATION
     $S. FINAL TIME HISTORIES ARE:',//)
  145 WRITE(6,146)
  146 FORMAT(3X,'TIME',7X,'DELTA V',10X,'THETA',13X,'Q',13X,'ALPHA',12X,
     *'DRAG',12X,'WRBM',/)
      WRITE(6,147) (TIME(I),(XOUT(J,I),J=1,4),DRAG(I),WRBM(I),I=1,NTP)
  147 FORMAT(1X,F6.2,6E16.6)
      WRITE(6,148) SUMD,SUMW,KA,KL,(U(I),I=1,3),(XL(I),I=1,3)
  148 FORMAT(1H0,' INTEGRAL OF DRAG = ',E13.6,/,' INTEGRAL OF WRBM = ',
     *E13.6,//' KA = ',E13.6,/,' KL = ',E13.6,//,' DF = ',E13.6,/,'
     *DI = ',E13.6,/,' DO = ',E13.6,//,' LAMBDA = ',3E17.6)
      WRITE(6,39) ((F(I,J),J=1,4),I=1,4)
      IF(NCYCLE.EQ.1) GO TO 1
      IF(ISTOP.EQ.1) GO TO 1
  150 CONTINUE
C     CONJUGATE GRADIENT UPDATE LOOP
      BD=BN
```

```
      BN=0
      DO 155 I=1,8
  155 BN=BN+DELJ(I)**2
      IF(ICYCLE.EQ.1) GO TO 161
      BETA=BN/BP
      DO 160 I=1,8
  160 A(I)=DELJ(I)+BETA*APREV(I)
      GO TO 165
  161 DO 162 I=1,8
  162 A(I)=DELJ(I)
C     ONE DIMENSIONAL SEARCH LOOP FOR ALPHA
  165 IDELJ=0
      ALPHA=CA/SQRT(BN)
      XJLAST=XJ
      KALAST=KA
      KLLAST=KL
      DO 168 I=1,8
  168 APREV(I)=A(I)
      DO 169 I=1,3
      ULAST(I)=U(I)
  169 XLLAST(I)=XL(I)
  170 CONTINUE
      DO 210 ILOOP=1,LOOP
  175 KA=KALAST-ALPHA*A(4)
      KL=KLLAST-ALPHA*A(5)
      DO 180 I=1,3
      U(I)=ULAST(I)-ALPHA*A(5+I)
  180 XL(I)=XLLAST(I)-ALPHA*A(I)
      WRITE(6,185) ILOOP,ALPHA
  185 FORMAT(////,2X,'IN ONE DIM. SEARCH LOOP, ILOOP = ',I3,' ALPHA = ',
     *E13.6)
      GO TO 36
  190 WRITE(6,195) XJ
  195 FORMAT(4X,'J = ',E13.6)
      IF(ILOOP.EQ.1) GO TO 200
      IF(XJ.GE.XJLOOP) GO TO 230
      GO TO 205
  200 IF(XJLAST.LT.0.) GO TO 201
      IF(XJ.LT.1.00001*XJLAST) GO TO 205
      GO TO 203
 201  IF(XJ.LE..999999*XJLAST) GO TO 205
  203 ALPHA=ALPHA*REDUCE
      IF(ALPHA.LT.1.E-20) GO TO 1
      GO TO 175
  205 XJLOOP=XJ
      ALPHA=ALPHA*2.
  210 CONTINUE
  215 KA=KALAST-ALPHA*A(4)
      KL=KLLAST-ALPHA*A(5)
      DO 220 I=1,3
      U(I)=ULAST(I)-ALPHA*A(5+I)
  220 XL(I)=XLLAST(I)-ALPHA*A(I)
      GO TO 235
  230 ALPHA=(ALPHA/2.)*(XJ-4.*XJLOOP+3.*XJLAST)/(2.*XJ-4.*XJLOOP+2.*XJLA
     *ST)
      GO TO 215
  235 WRITE(6,240) ALPHA
  240 FORMAT(////,2X,'COMPLETED ONE DIM. SEARCH LOOP, ALPHA = ',E13.6)
```

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      IDELJ=1
      GO TO 35
 9999 STOP
      END
```

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