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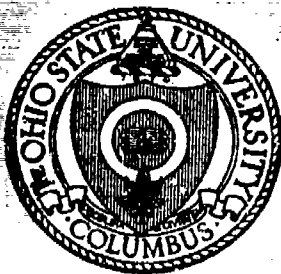
STUDIES IN STATISTICS AND APPLIED  
PROBABILITY WITH APPLICATIONS

AF--(AFOSR-1305-67)

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THE OHIO STATE UNIVERSITY



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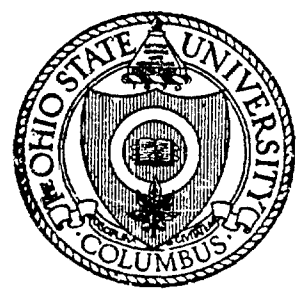
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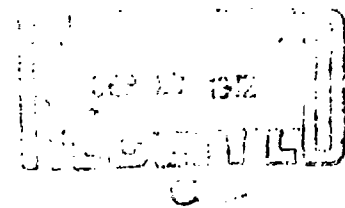
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Final Report.

STUDIES IN STATISTICS AND APPLIED  
PROBABILITY WITH APPLICATIONS

AF--(AFOSR-1305-67)

July 1, 1967 - June 30, 1972



By

J. S. Rustagi and R. C. Srivastava  
Division of Statistics  
The Ohio State University  
Columbus, Ohio 43210

## DOCUMENT CONTROL DATA - R &amp; D

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13. ABSTRACT  The research in problems in applied probability and statistics during the five years of the project was concerned with (a) waiting time distribution of Markov dependent Bernoulli trials, (b) Variational Methods in Statistics, (c) Stochastic Processes and (d) Miscellaneous problems.  Twenty-seven reports were written during the course of the project and many of them have been published in technical journals.			

KEY WORDS

UNIT 1 UNIT 2 UNIT 3

YOLL VS FOLL MS FOLL MS

Waiting time distribution  
Variational Methods  
Pareto distribution  
Stochastic Processes

## INTRODUCTION

With the support of the U.S. Air Force Office of Scientific Research, grant AFOSR-1305-67, research was undertaken to study a collection of problems in statistics and probability with applications to system evaluations. During the period of July 1, 1967 to June 30, 1972, the following areas were investigated and the results obtained were communicated to the agency as well as scientific journals. The details of the scientific reports and various other activities were provided in the annual reports given in the appendix.

The major problems studied are described below with the results obtained and publications arising from thereon.

### I. Waiting time distributions under Markovian dependence.

An investigation was initiated in the study of the waiting time distribution under Markovian dependence. We are concerned here with the number of trials required to obtain exactly a preassigned number of successes when the transition probability matrix for two states, success and failure, is given. The probability distribution has been obtained in terms of transition probabilities and the initial probability of the first trial. The distribution has been studied in detail by Rustagi and Srivastava (1968), Rustagi and Laitinen (1968, 1971). The distribution has been extensively tabulated by Rustagi, Srivastava and Pound (1968).

Bai (1971) has studied the problem of efficient estimation in Markov chains and has obtained various sampling schemes for estimation of transition probabilities. The above distribution plays an important part in the study of one of such schemes. Certain generalizations of the results of DeGroot (1959) for sequential estimation in binomial populations are also obtained.

### References:

1. Rustagi, J. S. and Srivastava, R. C., (1968). "Parameter Estimation in Markov Dependent Firing Distribution," Operations Research, 16, 1222-1227.
2. Rustagi, J. S. and Laitinen, Richard, (1970) "Moment Estimation in Markov Dependent Firing Distribution," Operations Research, 18, 918-923.
3. Rustagi, J. S. and Srivastava, R. C. and Pound, Cynthia, (1969). "Tables of the Markov Dependent Firing Distribution," Report of The Ohio State University. To be published.

4. Degroot, Morris H., (1959). "Unbiased Sequential Estimation for Binomial Populations," Annals of Mathematical Statistics, 30, 80-101.
5. Bai, Do Sun, (1971). "Efficient sampling plans in a two-state Markov chain," Ph.D. Dissertation, The Ohio State University.

II. Some variational problems with applications in statistics.

A class of solutions to a moment problem has been considered by Weiss (1956). Integral of this type also occurs in a problem considered by Pitman (1949) in connection with the discussion of the relative efficiency of the Wilcoxon's test. Shannon (1948) considered the problem of maximizing information in connection with his development of a mathematical theory of communication with side conditions on the probability density. In obtaining a lower bound for a probability moment of an absolutely continuous distribution with finite variance, Moriguti (1952) has considered similar problems.

These problems are special cases of a more general problem of optimizing an integral of a known convex (concave) function of a probability density function having a fixed set of moments. Similar problems have been considered by Karlin (1959) and Rustagi (1957, 1961). Utilizing techniques of Rustagi (1957), Isaacson and Rubin (1954) and Karlin (1959), characterization of the solution of the more general problem was accomplished, Sullivan and Rustagi (1970).

References

1. Bellman, Richard, (1957). Dynamic Programming, Princeton University Press, Princeton.
2. Isaacson, S. and Rubin, H., (1954) "On Minimizing an Expectation Subject to Certain Side Conditions," Technical Report N. 25, Applied Mathematics and Statistics Laboratory, Stanford University.
3. Karlin, S., (1959). Mathematical Methods and Theory in Games, Programming and Economics, Vol. 1 and 2. Addison-Wesley Publishing Co., Inc. Reading, Mass.
4. Karlin, S. and Studden, W., (1966). Tchebycheff Systems; With Applications in Analysis and Statistics, Interscience Publishers, New York.
5. Moriguti, S., (1952) "A Lower Bound for the Probability Moment of any Absolutely Continuous distribution with Finite Variance." Annals of Mathematical Statistics, 23, 286-289.
6. Pontryagin, L. S. et al., (1962). The Mathematical Theory of Optimal Processes Interscience Publishers, New York.
7. Rustagi, J. S., (1970). "Selected bibliography on optimizing methods in statistics," Report of The Ohio State University. To be published.

8. Rustagi, J. S., (1971). (Editor) Optimizing Methods in Statistics, Academic Press, New York.
9. Rustagi, J. S., (1957). "On Minimizing and Maximizing a Certain Integral with Statistical Applications," Annals of Mathematical Statistics, 28, 309-328.
10. Rustagi, J. S., (1961). "Bounds for the Variance of Mann-Whitney Statistic," Annals of the Institute of Statistical Mathematics, 13, 119-126.
11. Rustagi, J. S., (1968). "Dynamic Programming Model of Patient Care," Mathematical Biosciences, 3, 141-149.
12. Weirs, L., (1956). "A Certain Class of Solutions to a Moment Problem," Annals of Mathematical Statistics, 27, 851-853.
13. Sullivan, J. and Rustagi J. S., (1970). "On Minimizing an Expectation with Constraints," To be published.
14. Harris, B., (1959) "Determining Bounds on Integrals with Applications to Cataloguing Problems," Annals of Mathematical Statistics, 25, 521-548.

### III. Stochastic Processes

A stochastic model of air interdiction was developed, generalizing the model of Armitage (1970). The probability distribution of the material reaching a given objective under conditions of air interdiction was obtained. Some remarks about the effectiveness of Air interdiction were made. Other areas of inference in Stochastic Processes were studied also.

### References

1. Armitage, J. V., (1970). "Allocation of Sorties in Air Interdiction", Operations Research, 18, 483-496.
2. D. E. Crawford and R. C. Srivastava, (1971). "A Stochastic Model of Air Interdiction", The Ohio State University Technical Report. To be published.

### IV. Miscellaneous problems.

Investigations were initiated on various other problems, some of which are mentioned below.

- (a) Inference in Pareto distributions.
- (b) Estimation of cumulative distribution functions and the study of other non-parametric problems.
- (c) Time-series Analysis
- (d) Characterization of distributions.



ACADEMIC CONTRIBUTIONS

In addition to providing a stimulating research environment in the Division of Statistics, the junior participants earned advanced degrees with the help of research conducted on the project. The following students received M.S. or Ph.D. degrees from The Ohio State University with the partial support of the project.

1969

- 1. Charles Oprian, Ph.D.
- 2. J. Richard Stewart Ph.D.
- 3. Joseph Licing Ph.D.
- 4. Dean Brown, Ph.D.

1970

- 5. James Sullivan Ph.D.
- 6. D. E. Crawford M.S.

1971

- 7. D. S. Bai Ph.D.
- 8. M. G. Phadia Ph.D.
- 9. Y. H. Wang Ph.D.

Expected in 1972

- 10. Richard Laitinen Ph.D.
- 11. R. S. Gupta Ph.D.

PUBLICATIONS

Under the auspices of the project, the reports and papers published are given in the appendix.

APPENDIX

Annual Status Report  
on  
"Waiting Time Distribution Under  
Markovian Dependence with Applications"  
(AFOSR - 1305 - 67)  
for the period  
July 1, 1967 to June 30, 1968

J. S. Rustagi  
and  
R. C. Srivastava

## I. Personnel

During the year, the following persons were supported.

### (a) Faculty

J. S. Rustagi, Principal Investigator 25 percent

R. C. Srivastava, Co-principal Investigator 25 percent

### (b) Graduate Research Assistants

Darral Clarke, summer quarter, half time

R. S. Gupta, summer quarter, quarter time

Richard Laitinen, half time

Charles Oprian, summer quarter, half time

Frank Samaniego, summer quarter, half time

## II. Research Reports

The following papers have been prepared under the sponsorship of the project.

1. J.S. Rustagi and R. C. Srivastava, "Parameter Estimation in Markov Dependent Firing Distribution." Accepted for publication in Operations Research
2. R. C. Srivastava, "Estimation of Probability Density Function Based on Random Number of Observations with Applications." Submitted for publication.
3. J. S. Rustagi and Richard Laitinen, "Moment Estimation in the Waiting Time Distribution under Markovian Dependence." To be published.

### III. Other Activities:

1. The tables of the waiting time distribution are being compiled and put in a suitable form for publication. Computer outputs will be utilized directly to reproduce the tables.
2. An intensive research study seminar on non-parametric methods in statistics was conducted during summer, 67 in which the graduate research assistants participated.
3. J. S. Rustagi spent part of the summer quarter, 1967 at Stanford University, Department of Statistics. He gave a paper in the Houston Symposium on biomathematics and was chairman of a session on "Non-parametric methods" in the Second International Symposium on Multivariate Analysis supported by the Office of Aerospace Research, U. S. Air Force.
4. R. C. Srivastava spent the year at Stanford University, Department of Statistics where he was working on the project supported by the U. S. Air Force as well as on a project supported by the U. S. Department of Transportation.

Annual Status Report  
on  
"Waiting Time Distribution Under  
Markovian Dependence with Applications"  
(AFOSR - 1305 - 67)  
for the period  
July 1, 1968 to June 30, 1969

J. S. Rustagi  
and  
R. C. Srivastava  
The Ohio State University

Ah

## I. Personnel

During this period, the following persons were supported.

### (a) Faculty:

J. S. Rustagi, Principal Investigator 25 percent,

R. C. Srivastava, Co-principal Investigator 25 percent,

J. Singh, Assistant Professor 11.1 percent.

### (b) Graduate Research Associates and Assistant:

Richard Laitinen, whole year, half time,

James Sullivan, whole year, half time,

Joseph Liang, eight months, full time,

Dean Brown, three months, full time.

## II. Research Reports

The following papers have been prepared under the sponsorship of the project during the period.

1. J. S. Rustagi, R. C. Srivastava and Cynthia Foud: "Tables of the Markov Dependent Firing Distribution." To be published.
2. J. S. Rustagi and T. W. Loub: "Optimum Distribution of Armor." To be published.
3. J. Singh: "On a Model of Contagious Distributions." To be published.
4. J. S. Rustagi and Richard Laitinen: "On the Efficiency of Moment Estimates in the Markov Dependent Firing Distribution."
5. James Sullivan: "Estimating of Probability Density Functions."

### III. Other Activities

1. J. S. Rustagi and Richard Laitinen gave a paper at the Annual Meetings of the Institute of Mathematical Statistics, Madison, Wisconsin, September, 1968.
2. R. C. Srivastava spent the summer, 1968, at Stanford University, Department of Statistics, where he worked on this project and a project supported by the U. S. Department of Transportation.
3. J. S. Rustagi and R. C. Srivastava attended the Spring Meeting of the Institute of Mathematical Statistics and Biometric Society held at Iowa City, Iowa March, 1969.

### IV. Publications:

J. S. Rustagi and R. C. Srivastava: Parameter Estimation In Markov Dependent Firing Distribution, Operations Research, 1968: 1222- 1227.

J. S. Rustagi and Richard Laitinen: Moment Estimation in Markov Dependent Firing Distribution. Abstract, Annals of Mathematical Statistics, 1968: 1781.



Annual Status Report

on

"Waiting Time Distributions under  
Markovian Dependence with Applications"

(AFOSR-1305-67)

for the period

July 1, 1969 to June 30, 1970

J.S. Rustagi and R.C. Srivastava

Division of Statistics

The Ohio State University

July 27, 1970

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1. Personnel

During the period, the following persons were supported:

a. Faculty

J. S. Rustagi, Principal Investigator, 25 percent

R. C. Srivastava, Co-principal Investigator, 25 percent

b. Graduate Research Associates and Assistants

Yao Hung Wang, one quarter half time

## II. Research reports

The following papers have been prepared under the sponsorship of the project during the period.

1. R. C. Srivastava and A.B.L. Srivastava: On a characterization of the Poisson Distribution, to appear in Journal of Applied Probability.
2. J. S. Rustagi: A selected bibliography of optimizing methods in statistics. To be published.
3. Richard Laitinen: Parameter estimation under a sampling scheme in a Markov dependent firing system. To be published.
4. Charles Oprian: On The Exact Power Functions Of Some Rank Tests For The Trend Alternative. To be published.
5. R. C. Srivastava: On a Characterization of the Poisson Distribution, II. To be published.

### III. Other activities

- a. J. S. Rustagi spent the summer 1969 at the Indian Institute of Agricultural Research Statistics and gave lectures at the following:
  - i) Department of Mathematics, University of Delhi
  - ii) Department of Mathematics, Indian Institute of Technology
  - iii) Department of Mathematics, Indraprastha College, Delhi
  - iv) Department of Statistics, Rajasthan University
  - v) Indian Institute of Agricultural Research Statistics
  - vi) Department of Mathematics, University of Punjab
  
- b. R. C. Srivastava gave an invited talk at the statistics section, Indian Science Congress on 'Some Multivariate Stochastic Models for Virus Populations', Jan. 1970 and presented a paper at the Annual meetings of Institute of Mathematical Statistics meetings in New York, August 1969.
  
- c. J. S. Rustagi attended the Houston Symposium on Bio-mathematics and Computer Science in the life sciences.

Publication:

J. S. Rustagi and T. W. Doub: Optimum distribution of Armor,  
Operations Research, (1970). 559-562.

R. C. Srivastava and A.B.L. Srivastava: On a Characterization  
of the Poisson distribution. Abstract, Annals of Mathematical  
Statistics, 1969: 1874.

Annual Status Report

on

"Studies in Statistics and Applied  
Probability with Applications

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for the period

July 1, 1970 - June 30, 1971

by

J. S. Rustagi and R. C. Srivastava

Division of Statistics

The Ohio State University

Columbus, Ohio 43210

I. Personnel

During the period, the following persons were supported:

(a). Faculty

J. S. Rustagi, Principal Investigator, 25 percent

R. C. Srivastava, Co-principal Investigator, 25 percent

(b) Graduate Research Associates and Assistants

(i) J. Richard Stewart, one quarter half-time

(ii) Richard Oprian, one quarter half-time

(iii) Y. H. Wang, four quarters, half-time

(iv) E. G. Phadia, one quarter, half-time

(v) Richard Laitinen, two quarters, half-time

(vi) Chun Choon Lam, one quarter, half-time

(c) Clerk - typist

Karen St. Andre, typist, partime

## II. Research Reports

The following papers have been prepared under the sponsorship of the project during the period.

1. J. Richard Stewart: Principal Component Analysis of Time Series
2. James A. Sullivan: On minimizing an expectation with Constraints
3. E. G. Phadia: Minimax Estimation of a cumulative distribution function
4. D. E. Crawford and R. C. Srivastava: A Stochastic Model for air Interdiction
5. R. C. Srivastava and Y. H. Wang: On some characterizations of the exponential and related distributions
6. Y. H. Wang: On a characterization of the normal and the Poisson distribution
7. Y. H. Wang: Estimation of a parameter of the Pareto distribution
8. Y. H. Wang: Sequential estimation of the scale parameter of the Pareto distribution



### III. Other Activities

- (a) J. S. Rustagi and James Sullivan gave a paper on 'Application of Hahn - Banach theorem to a moment problem' at the Annual meetings of the Institute of Mathematical Statistics held at Laramie, Wyoming, in August , 1970.
- (b) J. S. Rustagi attended the meetings of the American Mathematical Society, held in Atlantic City, January, 1971, meetings of the Institute of Mathematical Statistics held at College Park, in April, 1971, and the Houston Symposium on biomathematics and computer science in the life sciences, March, 1971.
- (c) J. S. Rustagi was elected Fellow of the American Statistical Association at Annual meeting of the Association in Detroit, 1970.
- (d) J. S. Rustagi was chairman of the Organizing Committee of the Symposium on Optimizing Methods in Statistics, held at The Ohio State University in June, 1971, with the support of the Air Force Office of Scientific Research.
- (e) R. C. Srivastava attended the meetings of the Institute of Mathematical Statistics held at Laramie, Wyoming in August, 1970.
- (f) R. C. Srivastava and Y. H. Wang gave a paper on 'Characterization of the exponential and related distributions' at College Park meetings of the Institute of Mathematical

III. (cont.)

Statistics, April, 1971.

- (g) E. G. Phadia gave a paper on 'Minimax estimation of a distribution function at the meeting of Institute of Mathematical Statistics held at College Park in April, 1971.

#### IV. Publication

1. J. S. Rustagi and Richard Laitinen: Moment estimation in Markov - dependent Firing distribution, Operations Research, (1970) : 918 - 923.
2. R. C. Srivastava and A. B. L. Srivastava : On a characterization of Poisson Distribution, Journal of Applied Probability (1970): 497 - 501.
3. J. S. Rustagi and James Sullivan: Application of Hahn - Barach Theorem to a moment problem, Abstract, Annals of Mathematical Statistics, 1970.
4. R. C. Srivastava and Y. H. Wang: On characterization of the Exponential and related distributions, Abstract, Annals of Mathematical Statistics, 1971.

Annual Report  
"Studies in Statistics and Applied  
Probability with Applications  
to System Evaluations

(AFOSR - 1305 - 67)  
for the period  
July 1, 1971 - June 30, 1972

by  
J. S. Rustagi and R. C. Srivastava  
Division of Statistics  
The Ohio State University

I. Personnel

During the year, the following persons were supported

(a) Faculty

J. S. Rustagi, Principal Investigator, 25 percent

R. C. Srivastava, Co-principal Investigator, 25 percent

Jagbir Singh, Research Associate, 25 percent

E. G. Phadia, Research Associate, 50 percent

D. S. Bai, Research Associate, 50 percent

Y. H. Wang, Research Associate, 50 percent

(b) Graduate Research Assistants

R. S. Gupta, summer quarter, half time

(c) Clerk-typist

Dianne Marting, 25 percent

## II. Research Reports

The following papers have been prepared under the sponsorship of the project.

1. Jagbir Singh, "Two-stage Inference for a Simple Linear Model using Prior Beliefs."
2. Jagbir Singh, "On a Convolution of Poisson Distributions with the Zero Class missing."
3. Do Sun Bai, "Efficient Estimation of Transition Probabilities in a Markov Chain."
4. Y. H. Wang, "A Note in Homogeneous Processes with Independent Increments."
5. Y. H. Wang, "On Characterization of Certain Probability Distributions."
6. Jagbir Singh, "Paired Comparison Rankings by Linear Programming."

### III. Other Activities

1. J. S. Rustagi visited the Department of Statistics, Stanford University during the Academic year 1971-72, and worked on a textbook on Statistical Theory in Medical Research.
2. J. S. Rustagi attended the regional meetings of the Institute of Mathematical Statistics and the American Statistical Association at Ames, Iowa during April, 1972.
3. R. C. Srivastava, E. G. Phadia, Y. H. Wang and D. S. Bai attended the meetings of the Institute of Mathematical Statistics in Blacksburg, Virginia during March, 1972.
4. J. S. Rustagi was named on the International Editorial Board of the Journal on Communications in Statistics.
5. J. S. Rustagi was appointed a member of the Prize Committee of Technometrics.
6. Jagbir Singh visited the Department of Statistics, University of Missouri.
7. J. S. Rustagi gave a seminar talk in the Department of Statistics, Stanford University on "Waiting Time Distribution under Markovian Dependence" in April, 1972.
8. R. C. Srivastava visited the Indian Statistical Institute, New Delhi during summer, 1971.