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THE ESTIMATION OF IMPULSIVE PRESSURES ARISING IN THE IMPACT OF A DROP AGAINST A SOLID SURFACE

(OTSENKA IMPULSIVNYKH DAVLENII, VOZNIKAYUSHCHIKH PRI UDARE KAPLI O TVERDUYU POVERKHNOST')

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EDITOR'S SUMMARY

In this estimation of impact pressures of a drop with a solid surface, the deformation of the surface of the drop and the onset of post-collisional motion of the liquid are considered. Equations of maximum pressure are developed and their dependence on the extent of wetting of the solid surface shown to be significant. They are not, however, greatly affected by the shape of the free surface formed by the drop on impact. It is also shown how excessive flattening of the drop leads to a reduction of impact pressures.

CONTENTS

The results set out in this article are a further continuation of work[1] which gave a simplified scheme for estimating the impact of a drop against a solid surface.

In that work the calculation was based on the assumption that at the moment of impact the drop assumed the form of a hemisphere. In the present article that constraint is abandoned and the problem of the impact of a drop with a surface of arbitrary shape after impact is considered.

In this case the axisymmetrical problem is replaced by a plane one. This schematisation (as shown in Ref.2) does not have any significant influence on the magnitude of the impulsive pressures obtained.

(1) At the moment of impact of a drop against a solid surface a change in the shape of the surface of the drop must occur as well as motion of the liquid within the drop. Instead of considering this complex unsteady process we shall consider, as in the study of the impact of solid bodies[3], the limiting motion which occurs in step-wise change of velocity of a drop (in a system of coordinates coupled with the liquid, its velocity before impact will be everywhere equal to zero). So as to take into account the change in the surface of the drop on impact we shall consider the flow arising in the drop deformed by impact. Thus, the schematisation of the process of impact of the drop consists in the fact that the continuous process of onset of internal motion in the liquid and deformation of the drop is split into two separate phases: (1) deformation of the surface of the drop; (2) onset of post-collisional motion of the liquid.

Assuming that the liquid is ideal and incompressible we find that the problem of calculation of the motion of the liquid in the drop following impact comes down to determining the velocity potential φ for the following boundary conditions[3]: at the free surface $\varphi = 0$, at the solid wall the normal component of velocity is equal in magnitude and opposite in direction to the velocity of the drop prior to impact.

In what follows we shall consider plane flow, substituting for a drop an infinitely long liquid circular cylinder moving prior to impact in the negative direction of the axis Ox (the axis Oy representing a section of a solid plane wall), see Fig.1.

The problem of calculating this flow will then consist in determining the complex flow potential $w = \varphi + i\psi$ (φ = potential, ψ = stream function) in the region bounded by the plane solid wall AOB and the contour of the free surface ACB for the following boundary conditions:

(1) at AOB the velocity component along the x axis $v_1 = c$ (c = pre-collisional velocity of the drop);

(2) at ACB the potential $\varphi = 0$.

(2) In order to determine the dependence of w on z we shall map the region AOBC on the interior of a unit semicircle in the region of an additional variable $\zeta = \xi + i\eta$ (Fig.2). Let us first find the additional function

$$\Omega = i\zeta \frac{dw}{d\zeta} = i\zeta \frac{dw}{dz}\frac{dz}{d\zeta} . \qquad (2\text{-}1)$$

At the solid wall AOB

$$\frac{dz}{d\zeta} = -i\left|\frac{dz}{d\zeta}\right| = -i\mu(\xi) . \qquad (2\text{-}2)$$

The function $\mu(\xi)$ which is positive everywhere within the segment AOB is determined by the shape of the contour, and

$$\frac{dw}{dz} = c - iv_2 . \qquad (2\text{-}3)$$

Here v_1 and v_2 are the flow velocity components along the x and y axes. Thus

$$\mathrm{Re}\,\Omega = c\xi\mu(\xi) \quad \text{at AOB} . \qquad (2\text{-}4)$$

Since $\varphi = 0$ on the arc ACB, on this arc

$$\frac{dw}{d\zeta} = \pm\left|\frac{dw}{d\zeta}\right| e^{-i \arg \zeta} , \qquad \Omega = \pm i\left|\zeta \frac{dw}{d\zeta}\right|$$

and hence

$$\mathrm{Re}\,\Omega = 0 . \qquad (2\text{-}5)$$

Making use of this we can continue the function Ω analytically through the arc ACB on to the whole upper half-plane of the region of change of ζ. After this we get for Re Ω in the interval $(-1, -\infty, \infty, +1)$ of the axis $O\xi$

$$\mathrm{Re}\,\Omega(\xi) = -\,\mathrm{Re}\,\Omega(\xi^{-1}) = -\frac{c}{\xi}\,\mu(\xi^{-1}) \quad . \tag{2-6}$$

Knowing the value of Re Ω over the whole of the axis $O\xi$ we can determine Ω by means of the Schwartz formula[4]

$$\Omega = -\frac{c}{\pi i}\left\{\int_{-\infty}^{-1} \frac{\mu(\tau^{-1})}{\tau(\tau - \zeta)}\,d\tau + \int_{1}^{\infty} \frac{\mu(\tau^{-1})}{\tau(\tau - \zeta)}\,d\tau - \int_{-1}^{1} \frac{\tau\mu(\tau)}{\tau - \zeta}\,d\tau\right\} + iC \tag{2-7}$$

(C is a real constant which requires to be determined). Substituting for Ω the expression (2-1) we get

$$\frac{dw}{dz} = -\frac{c}{\pi\zeta}\left\{\int_{-1}^{1}\left[\frac{\tau}{\tau - \zeta} + \frac{1}{\zeta(\tau - \zeta^{-1})}\right]\mu(\tau)d\tau\right\} + \frac{C}{\zeta} \quad . \tag{2-8}$$

For $\zeta = \xi C(-1, +1)$ we obtain from (2-8) by means of the Sokhotski formula[4]

$$\frac{d\varphi}{d\xi} = -\frac{c}{\pi\xi}\left\{\int_{-1}^{1}\left[\frac{\tau\mu(\tau) - \xi\mu(\xi)}{\tau - \xi} + \frac{\mu(\tau)}{\xi\tau - 1}\right]d\tau + \xi\mu(\xi)\,\ln\frac{1 - \xi}{1 + \xi}\right\} + \frac{C}{\xi} \ . \tag{2-9}$$

Since

$$\frac{d\varphi}{d\xi} = \frac{d\varphi}{dy}\left|\frac{dz}{d\zeta}\right| \quad \text{on AOB} , \qquad \frac{d\varphi}{dy} = v_2 = 0 \quad \text{at the point } 0$$

then

$$\frac{d\varphi}{d\xi} = 0 \quad \text{when } \xi = 0 \ .$$

From this it follows that $C = 0$.

Assuming that $w = 0$ at point A and integrating (2-8) we obtain for the complex potential at an arbitrary point within the liquid

$$w = -\frac{c}{\pi}\int_{-1}^{\zeta}\frac{1}{\zeta}\int_{-1}^{1}\left[\frac{\tau}{\tau-\zeta}+\frac{1}{\zeta\tau-1}\right]\mu(\tau)d\tau d\zeta = -\frac{c}{\pi}\int_{-1}^{1}\mu(\tau)\ \ln\frac{1-\zeta\tau}{\tau-\zeta}\,d\tau. \quad (2\text{-}10)$$

If the map of the region (z) on the semicircle (ζ) is known then the expression (2-10) together with the function $z = z(\zeta)$ will give the dependence of w on z in parametric form.

(3) The impulsive pressure on a solid surface can be determined from the formula[3]

$$p = -\rho\varphi \quad . \qquad (3\text{-}1)$$

Passing to the limit in (2-10) when $\zeta \to \xi$ we obtain for the complex potential at a solid surface

$$w = -\frac{c}{\pi}\int_{-1}^{1}\mu(\tau)\ \ln\frac{1-\xi\tau}{|\xi-\tau|}\,d\tau + ic(y-a) \qquad (3\text{-}2)$$

Substituting in (3-1) the value of the potential φ taken from this formula and the value of $\mu(\tau)$ from (2-2) we define the impulsive pressure at any point of a solid surface as

$$p = \frac{\rho c}{\pi}\int_{-1}^{1}\frac{dy}{d\tau}\ln\frac{|\xi-\tau|}{1-\xi\tau}\,d\tau \quad . \qquad (3\text{-}3)$$

It follows from this expression that at the boundaries of a wetted surface the impulsive pressure is equal to zero. The maximum pressure which arises at the centre of the drop will on the basis of (3-3) be equal (substituting $\xi = 0$ in (3-3) and carrying out partial integration) to

$$p_* = \frac{2\rho c}{\pi}\int_{0}^{1}\frac{|y|}{\tau}\,d\tau \quad . \qquad (3\text{-}4)$$

(4) Let us consider how the magnitude of the maximum impulsive pressure depends on the shape of the drop after impact and the magnitude of the wetted surface. For this we calculate the maximum impulsive pressure p_* for several different contours of the free surface following impact: (1) arc of a circle; (2) ellipse; (3) inversion of ellipse relative to a circle passing through its

minor semi-axis. Each of these contours is determined by two parameters, one of which is the half-width of the wetted surface a.

The second parameter is defined such that the area of section of the liquid cylinder remains constant before and after impact (condition of incompressibility). As the second parameter we shall take the maximum width of the drop in the direction of the abscissa axis b (Fig.1). For the magnitude of the pressure p_* we get the following formulae respectively (if the radius of section of the liquid cylinder before impact is denoted by R):

(1) Arc of circle

$$p_* = \frac{2\rho c}{\pi}\int_0^1 \frac{1 - [(1-\tau)/(1+\tau)]^{2\theta/\pi}}{1 + [(1-\tau)/(1+\tau)]^{2\theta/\pi}}\frac{d\tau}{\tau} = \frac{4\rho c}{\pi}\int_0^1 \frac{1 - t^{2\theta/\pi}}{(1 + t^{2\theta/\pi})(1 - t^2)}\,dt \;. \quad \text{.... (4-1)}$$

Here θ is the angle between a tangent to the arc and the solid surface at their point of intersection measured from within the liquid (boundary angle). Its relation to a is given by

$$a = R\sqrt{2}\sin\theta\sqrt{\frac{\pi}{2\theta - \sin 2\theta}} \;. \quad (4\text{-}2)$$

(2) Ellipse (a and b = major and minor semi-axes of the ellipse respectively)

$$p_* = \frac{\rho c}{\pi}\int_\kappa^1 \left[(a+b)e^{-B} + (a-b)e^{B}\right]\frac{d\tau}{\tau} + \frac{2\rho c\sqrt{a^2 - b^2}}{\pi}\int_0^\kappa \frac{\cos B}{\tau}\,d\tau \quad (4\text{-}3)$$

$$\kappa = \frac{1 - k'}{k}\;, \qquad b = \frac{2R^2}{a} \quad (4\text{-}4)$$

$$B = \frac{\pi}{2K(k)}F\left(k, \arcsin\frac{\sqrt{(1+\tau^2)^2k^2 - 1}}{k(1-\tau^2)}\right) \qquad \text{when } \tau \leqslant x \quad (4\text{-}5)$$

$$B = \frac{\pi}{2K(k)}F\left(k', \arcsin\frac{1-\tau^2}{k'(1+\tau^2)}\right) \qquad \text{when } \tau \geqslant x \;. \quad (4\text{-}6)$$

Here K and F are respectively the complete and incomplete elliptic integrals of the first kind, and k and k' the module and additional modulus of this integral, the modulus k being determined from the major semi-axis of the ellipse by the relationship

$$\frac{K(k')}{K(k)} = \frac{1}{2\pi} \ln\left(\frac{a^2 + 2R^2}{a^2 - 2R^2}\right) . \qquad (4\text{-}7)$$

We should observe that even when $(a/\sqrt{2}\,R) > 1.5$ the difference between $(1 - k')/k$ and 1 is very small and $k' \ll 1$. Thus in (4-3) the second integral can be discarded and the expression for B simplified.

Then

$$p_* = \frac{2\rho c\sqrt{a^2 + b^2}}{\pi} \int_0^{1-k'} \cos\left\{\frac{\pi}{2K(k)} F\left(k, \text{arc cos } \frac{2\tau k'}{1 - \tau^2}\right)\right\} \frac{d\tau}{\tau} . \qquad (4\text{-}8)$$

In this case

$$k' = 4e^{-K(k)} , \quad K = \pi^2 \left[2 \ln \frac{a^2 + 2R^2}{a^2 - 2R^2}\right]^{-1} , \quad K(k') = \frac{\pi}{2} \quad \text{when } \frac{a}{\sqrt{2}\,R} > 2 .$$

(3) Inversion of ellipse

$$p_* = \frac{4c\rho ab}{\pi} \int_0^1 \frac{d\tau}{(b - a)\tau^2 + (b + a)} = \frac{4c\rho ab}{\pi\sqrt{b^2 - a^2}} \text{arctg} \sqrt{\frac{b + a}{b - a}} \qquad (4\text{-}9)$$

where

$$b = \sqrt{4R^2 - a^2} . \qquad (4\text{-}10)$$

When $a = b$ all the contours considered become a semicircle of radius $\sqrt{2}\,R$ and for the maximum pressure we get the value given in[1]

$$p_{**} = \frac{2\rho ca}{\pi} = \frac{2\sqrt{2}\,R\rho c}{\pi} . \qquad (4\text{-}11)$$

(5) The dependence of the maximum pressure on the width of the wetted part of the solid surface for the types of contours considered is shown in Fig.3 where the graphs for the change of the relation $\sigma = p_*/p_{**}$ are shown as a function

of $\alpha = a/\sqrt{2}\,R$. These graphs show that the magnitude of the impulsive pressure is considerably influenced by the magnitude of the wetted solid surface; the shape of the free surface formed by the drop on impact does not greatly affect the magnitude of the impulsive pressure. At the same time the results of these calculations show that excessive flattening of the drop and also insufficient flattening of it lead to reduction of the impact pressures. The magnitude of p_{**} as determined from (4-11) gives the value of the upper limit of the impulsive pressures arising on impact of the drop.

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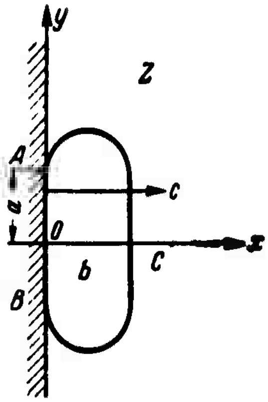

Fig.1

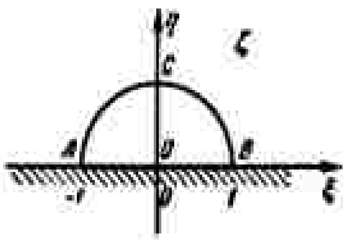

Fig.2

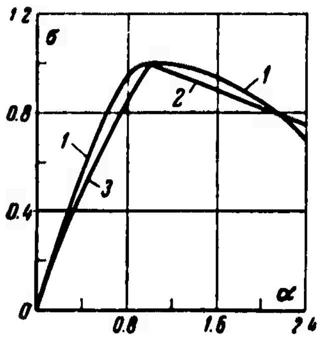

Fig.3

Fig.1-3