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DETECTION AND ANALYSIS OF MULTIPLE SEISMIC EVENTS

E. A. FLINN
T. J. COHEN
SEISMIC DATA LABORATORY

D. W. McCOWAN
CONSULTANT TO THE SEISMIC DATA LABORATORY

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13 ABSTRACT

We have tested several methods of analyzing short-period seismic records to detect multiple events. The data were from a pair of chemical explosions at a dam site near Alma Ata, recorded at the Mould Bay, Canada, seismic observatory. The methods evaluated included visual analysis of array beams, multichannel matched filtering, autocorrelation analysis, and cepstral analysis. For matched filtering we used the seismogram waveforms from a seismic event which occurred reasonably close to the Alma Ata site.

Measurements using the spectra or the cepstra appear to be more accurate in determining the delay time between the two events. The analysis methods discussed here are generally in good agreement with the announced time between the explosions, and their yield ratio.

14 KEY WORDS

Multiple Explosions

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I. INTRODUCTION

In this study we compare and evaluate various methods for detection and analysis of multiple explosions. We use the term "multiple explosions" to mean only two explosions detonated within a time interval of a few seconds, and do not consider here the problem of detecting more than two explosions. Our main objective is to detect the possible occurrence of a double explosion and to use the P wave arrivals to measure the delay time between the two events. Some of the measurement techniques discussed here also were found to give estimates of the relative amplitude and polarity of the two events studied. The P phases from a double explosion have the same polarity, of course, but the pP reflection usually has polarity opposite to that of the main P phase. The discrimination on seismic records between the case of a multiple shot or of a single shot and its pP phase is our ultimate concern.

The multiple explosion problem is very similar to that of the "bubble pulse" or hydroacoustic reverberation, which has been treated extensively (see, for example, Weston, 1959). The difference here is that in the hydro-acoustic problem the bubble pulse has the same seismic signature as the initial explosion. On the other hand, two nearby underground explosions may not generate the same waveforms, and so the seismic records may not be a simple superposition. If the medium near the sources was not deformed beyond the elastic limit by the first explosion, then the composite signature recorded by a distant receiver should be a superposition of phenomena characteristic of the two single explosions. This assumption of superposition is fundamental to all the analysis we present here.

A similar problem is that of eliminating multiple reflections in reflection seismic exploration work. One technique which has been used is deconvolution (Rice, 1962), which "whitens" or flattens out the peaks in the power spectrum, thus suppressing the multiple reflections (since the multiples occur approximately periodically in time, they appear as peaks in the spectrum). Of course, deconvolution whitens all the other information in the recordings, so it is not easy to predict the ultimate effect of prewhitening. In any case, eliminating a periodic phenomenon is not what is desired in our problem:

on the contrary, we wish to detect and resolve the separate components of any periodic function which may be present.

In any double event the observed P phase delay time is a function of three parameters:

1. the actual delay time incorporated into the firing sequence;
2. the projection of the shot spacing on the shot-to-station ray path;
3. the difference in depth between the two sources.

To determine the actual time delay for a double event requires that corrections be made for shot spacing and depth differences. Determination of the delay time associated with the pP phase for each event (Cohen, 1970) may allow us to estimate event depths and to adjust the P phase delay time observed at a given station. The study of shot spacing, however, requires that we have a network of stations distributed more or less uniformly about the source region (ideally at the same distance). Since we used seismograms recorded at a single station, the P phase delay time we determined do not imply a unique shot geometry or programmed firing sequence. Even if the explosions appeared from the observations to have occurred at the same depth, the observed delay time is a function of azimuth and distance, and generalization to other cases is difficult.

II. DATA

The data used in this study were short-period vertical seismograph records made at the Mould Bay, Canada (NP-NT) observatory, whose element coordinates are given in Table 1. The seismic source we studied was a pair of chemical explosions used to build a dam site near Alma Ata in 1965 (Aptikeyev et al., 1967). We used the P arrival waveform and three minutes of the P coda for most of the analysis procedures; for cepstral analysis we used 51.2 seconds of data following the P onset. The raw data are shown in Figure 1. For the matched filter reference waveform we used the NP-NT records of a simple event which occurred reasonably near the Alma Ata explosion site (Figure 2).

Aptikeyev et al. reported close-in measurements of the ratio of seismic amplitudes of the two events which gave an average ratio of 1.6. Our records show a ratio of 1.7; this implies a seismic scaling power of approximately 0.6, which is consistent with the teleseismic P observations of Carder and Mickey (1962) as summarized by Muller et al. (1962).

Location information for the chemical explosions is given in Table 2.

TABLE 1

Mould Bay (NP-NT) Element Coordinates

Channel	X Coordinate (km)	Y Coordinate (km)
Z1	0.0	-1.0
Z2*	0.0	0.0
Z3	0.0	1.0
Z4	0.0	2.0
Z5	-1.0	0.0
Z6	1.0	0.0
Z7	2.0	0.0

*Z2 is the center element: latitude 76°15'08"N, longitude 119°22'18"W, elevation 0.059 km.

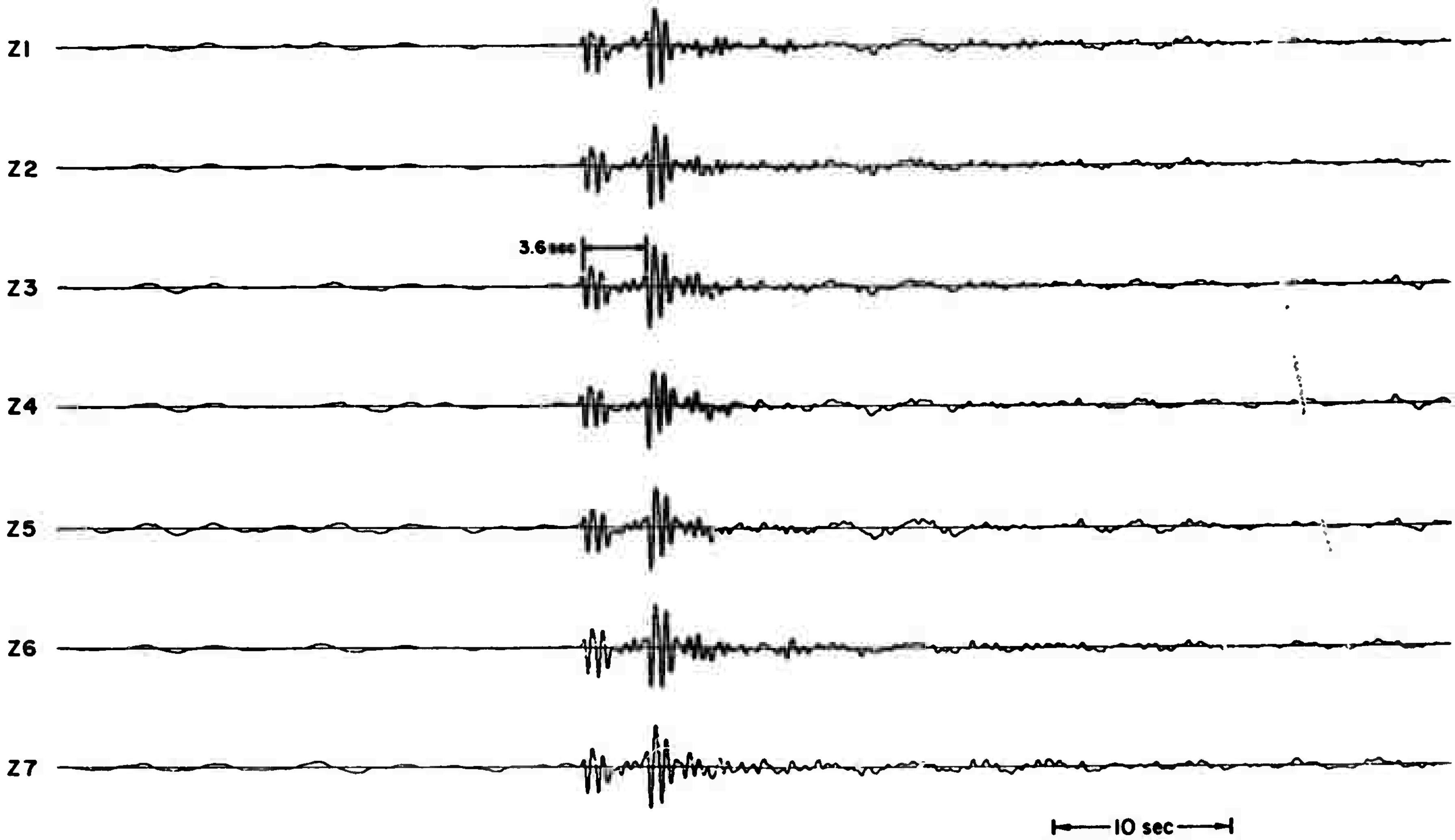

Figure 1. Raw seismograms for the double explosion.

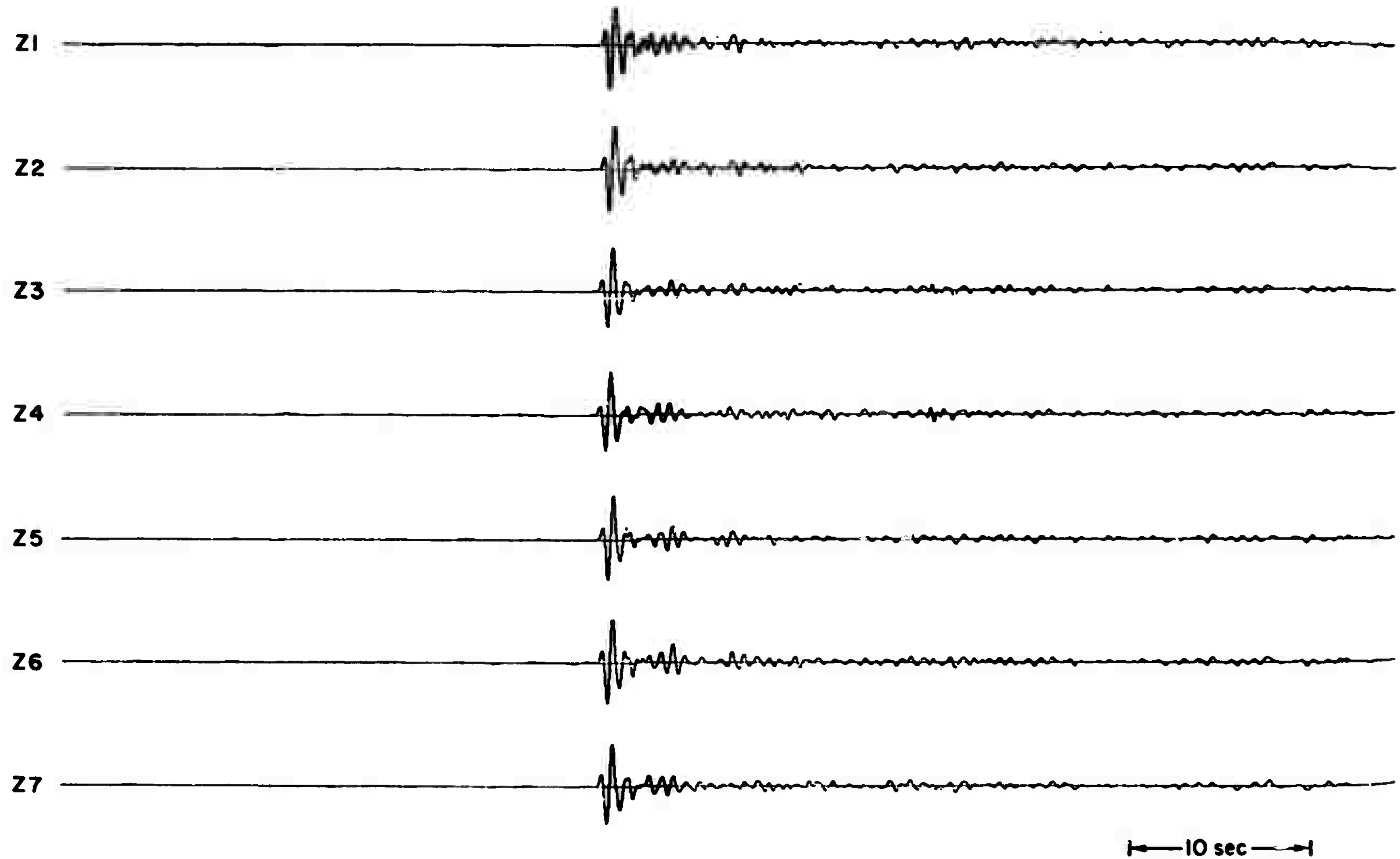

Figure 2. Raw seismograms for the simple reference event.

TABLE 2

Double Event Data

Date	21 October 1966
Latitude	43.2°N
Longitude	77.0°E
Distance from NP-NT	6712 km
Azimuth from NP-NT	4.4° E of N
Yield	(a) 1.7 kT
	(b) 3.6 kT
Origin Time	(a) 04 59 59.1 GMT
	(b) 05 00 02.7

III. ANALYSIS METHODS USED

The methods used here are discussed in increasing order of sophistication:

(1) Direct Measurement from Raw Seismograms

This consists simply of visual recognition of two superimposed time-shifted waveforms on the original raw seismograms and measurement of the time interval between corresponding phases of the two waveforms.

(2) Direct Measurement from Beamed Array Output

The signal/noise ratio can be improved and signal-generated noise diminished by beaming the array before visually examining the data. In the presence of a high ambient noise level it may also be desirable to apply a multichannel filter in addition to beamforming, but for the present study this was not considered necessary.

(3) Detection of Side Peaks in the Autocorrelation

Suppose the record consists of the superposition of a waveform y(t) and the same scaled waveform delayed by T seconds:

$$x(t) = y(t) + a y(t-T), \text{ where } -1 < a < 1. \quad (1)$$

The autocorrelation of x(t) is:

$$r(\tau) = (1+a^2)\, p(\tau) + a[p(\tau+T) + p(\tau-T)] \quad (2)$$

where $p(\tau)$ is the autocorrelation of $y(t)$ alone. Thus the composite autocorrelation has three components: (1) the autocorrelation of the unduplicated signal $y(t)$, with zero-lag amplitude $(1+a^2)$; (2) the same autocorrelation shifted over to peak at lag $\tau=T$, and amplitude scaled by <u>a</u>; (3) the same autocorrelation shifted over to peak at lag $\tau = -T$, with amplitude also scaled by <u>a</u>. Thus it would appear at first sight that the composite autocorrelation should have symmetrically located side peaks, each with peak amplitude $\underline{a}/(1+a^2)$ with respect to the zero-lag peak.

However, the autocorrelation of seismic signals is frequently oscillatory in nature and the apparent side-peak amplitude and lag time measured on the composite autocorrelation are unfortunately distorted because of possible phase differences between $p(\tau)$ and $p(\tau - T)$. It does not appear feasible to attempt a precise determination of a and T from the composite autocorrelation.

On the other hand, we may expect that double events in which <u>a</u> is of order unity will yield composite autocorrelations in which the presence of side peaks is discernible from the envelope of the autocorrelation, and the lag of the side peaks should not be in error by more than one-half cycle of the oscillation period of the autocorrelation.

It T is known from other data, and if the autocorrelation $p(\tau)$ of a single constituent event is known, then a could be determined from the composite autocorrelation function.

(4) Matched Filter Using a Simple Event

The matched filter approach to detection of signals in noise is well known in other scientific fields, and has been successfully applied in seismic problems involving dispersed signals by Alexander and Rabenstine (1967a, 1967b). When the input is the desired signal without noise contamination, the matched filter output is the autocorrelation of the desired signal. Thus if P waves from both a single event and a double event are observed at a given station, and if it is known that the radiated source waveforms are "similar", the single event can be used as a matched filter for the double event. The output should resemble the superposition of the autocorrelation of the single event itself in the same manner as for the previous technique described, to the degree of similarity of the two waveforms.

In order to take advantage of both beamforming and matched filtering techniques, we designed a multichannel matched filter (see Appendix A for details) in which we match filtered the input channels and beamformed the outputs, rather than beamforming and then match filtering the beams. This approach cancels out any travel time anomalies which might otherwise affect matched filter performance.

(5) Effect of Deconvolution on Autocorrelation and Matched Filtering

Autocorrelation and matched filtering depend for their success in separating overlapping signals on the autocorrelation envelope being "narrower" than the signal separation, i.e., the amplitude of the autocorrelation envelope must, by Rayleigh's criterion, have decayed significantly away from the peak in order to be able to resolve two overlapping correlation functions. The error in measuring the overlap separation can frequently be reduced by filtering the original record in such a way that the autocorrelation of the data is contracted or shrunk about its midpoint. This process, called deconvolution in seismic exploration (Rice, 1962), was carried out on the data of this study, and the autocorrelation and matched filtering techniques were applied again to the deconvolved data.

(6) <u>Spectral Nulls</u>

The Fourier transform of a double event is the same as the Fourier transform of the component single events except for a modulating factor involving <u>a</u> and T (Appendix B). This modulating factor is:

$$1 + a^2 + 2a \cos (2\pi fT)^{1/2} \qquad (3)$$

The minima of this function are spaced 1/T Hz apart on the frequency axis. If <u>a</u> is sufficiently large that the modulation is discernible to the eye, the null or trough frequency can be plotted vs. trough number and the slope of the resulting straight line determines T.

The intercept of the resulting straight line at f = 0 will be at a half-integer if <u>a</u> is positive, and at an integer if <u>a</u> is negative. This allows us to determine the relative polarity of the two arrivals.

(7) <u>Cepstral Analysis: The Spectrum of the Spectrum</u>

Bogert et al. (1965) defined the cepstrum*

*To help remember which domain one is talking about, we employ the convention of reversing the consonants of the first syllable of words referring to the spectrum-of-the spectrum: the "cepstrum" is the function itself; its argument is "quefrency" (which has the dimension of time); any filtering we do of the spectrum is "liftering", etc.

of a data segment as the spectrum of the logrithm of the spectrum of the data. Their motivation for studying this function was to detect echoes. If the record consists of a signal y(t) and an undistorted echo of amplitude

$$a\,y\,(t-\tau)$$

at time delay T, then the original record is:

$$z(t) = y(t) + ay(t-T) \tag{4}$$

and the spectrum is:

$$P(f) - |Y(f)|^2[1+a^2 + 2a\cos(2\pi fT)]. \tag{5}$$

The cepstrum is then

$$C(\nu) = |\int_{-\infty}^{\infty} \log P(f)\ \exp[-i2\pi f\nu df]|^2 \tag{6}$$

where

$$\log P(f) = \log|Y(f)|^2 + \log[1+a^2 + 2a\cos(2\pi fT)] \tag{7}$$

When a is small, this becomes:

$$\log P(f) \approx \log|Y(f)|^2 + 2a\cos(2\pi fT) \tag{8}$$

From equation (6) it is apparent that the effect of the echo is to add a sinusoidal ripple to the term which depends only on the signal spectrum. If the signal spectrum is reasonably smooth and if we have some prior idea of the range of delay times T to be expected, we can perform a liftering operation on the spectrum before computing the cepstrum, and identify the echo delay time (hopefully) by a reasonably sharp peak in the quefrency domain.

The motivation for taking the logarithm is to convert the multiplicative effect of the echo (equation 5) into an additive effect, thus avoiding a convolution in the quefrency domain. That is, the two factors in equation (5) become additive upon taking the logarithm, and the sinusoidal modulation caused by the echo is added to the term depending only on the signal spectrum, rather than multiplying it. If the logarithm were not taken, the cepstrum would contain the convolution of the transforms of the two terms.

If Y(f), the spectrum of the data, were flat and band-limited, the spectrum of log Y(f) would have the form (sin X)/X, where X is proportional to the bandwidth, i.e., we would expect the cepstrum to have a peak at zero quefrency and auxiliary peaks at the side-lobe quefrencies. When Y(f) is not flat but is reasonably smooth, we expect the cepstrum to have the same general form: a peak at zero quefrency and smaller peaks at regularly spaced higher quefrencies. The zero quefrency peak is easily liftered out of the cepstrum.

Taking the logarithm of the spectrum has other consequences: in particular, it has the effect of whitening the spectrum and increasing the relative importance of weaker periodicities. If a is small this can help in the determination of the echo parameters a and T, since the sinusoidal component $2a \cos(2\pi fT)$ is enhanced. However, if more than one echo is present, the inherent complication of the cepstrum (which would contain peaks at each combination of the inter-echo delay times) is further increased and the cepstrum may not be easily interpretable.

When the echo amplitude is large, roughly $|a|>.5$, the scalloping of the spectrum may be caused primarily by interference of the primary phase and its echo. In such cases, taking the logarithm actually weakens the sinusoidal effect and downgrades the cepstral peak we are seeking. In hydroacoustics, Plutchok and Stites (1968) determined bubble pulse periods from the spectrum of the spectrum, but analyses performed using the spectrum of the log spectrum gave unsatisfactory results (R. Stites, personal communication, 1969). In experiments aimed at determining the pP-P interval from short-period records, Cohen (1970) found it necessary to take the spectrum of the spectrum (rather than of the log spectrum) in order to successfully separate the P phase from its surface reflection.

We are led, therefore, to avoid taking the logarithm of the spectrum before computing the cepstrum, since in the absence of prior knowledge we would anticipate that the amplitudes of the two events composing the double event are roughly the same. We thus expect to find that the important cepstral peaks are more pronounced but somewhat broadened by the implicit convolution process described above.

The cepstral analysis techniques used here were described by Cohen (1970). The computer program was written by Plutchok and Stites (1968); it computes autopower spectra, autocorrelation, linear cepstra, and the pseudo-autocorrelation (the inverse Fourier transform of the liftered spectrum) for any number of time series. The program also computes the point-by-point product ("dot product") of the pseudo-autocorrelation and cepstrum.

Because the pseudo-autocorrelation preserves some phase information, a positive echo should cause a coincidence in time of a covariance maximum and a cepstrum maximum. If the dot product of the cepstrum and pseudo-autocorrelation is formed, then the resulting correlation function should exhibit a strong positive peak at the correct delay time.

To enhance the energy in the 2-5 Hz frequency band, we removed the instrument response from the power spectrum; that is, we formed:

$$P'(f) = \frac{f^2\ P(f)}{|H(f)|^2}$$

where P'(f) is the corrected velocity spectrum, P(f) is the original signal spectrum, and H(f) is the system response. The resulting velocity spectrum, enriched with energy in the frequency range 2-5 Hz, provides a longer window within which to search for spectral periodicities.

The programmed instrument response H(f) is shown in Figure 3. The flat response above 1.2 seconds period was adopted to prevent the corrected spectra from blowing up at low frequencies and to prevent the f^2 function from magnifying noise at high frequencies, the spectra are zeroed out above 5 Hz.

In practice we liftered the spectra before transformation. This process consists simply of removing the unwanted low-and high-quefrency components (by multiplication in the quefrency domain), leaving the

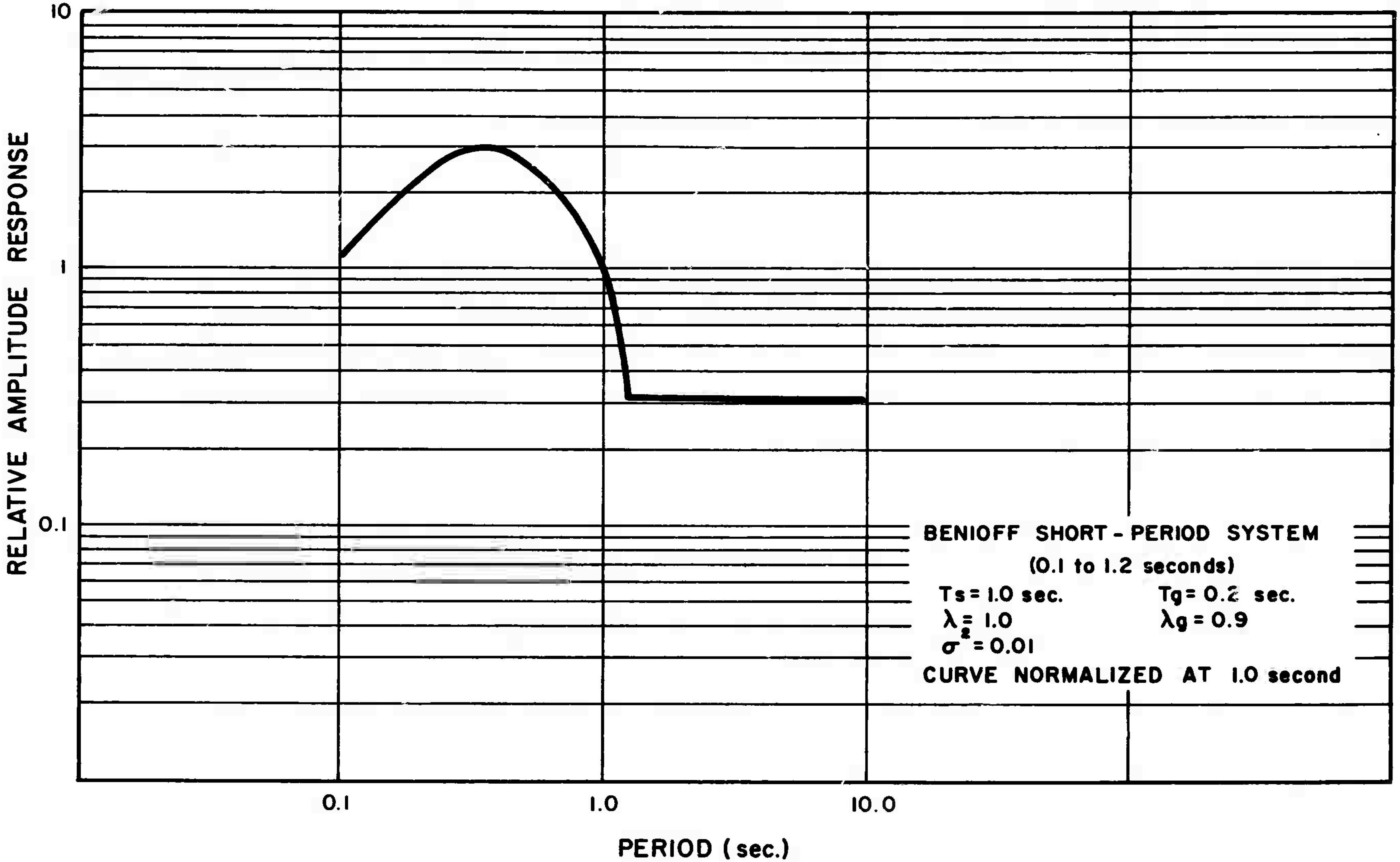

Figure 3. Modified short-period seismometer response.

components of interest intact. A typical lifter response with cutoffs at 0.25 and 6.8 seconds (the 3 dB points) is shown in Figure 4. In the work reported here a short-pass cutoff of 0.1 second and a long-pass cutoff of 35.8 seconds were used.

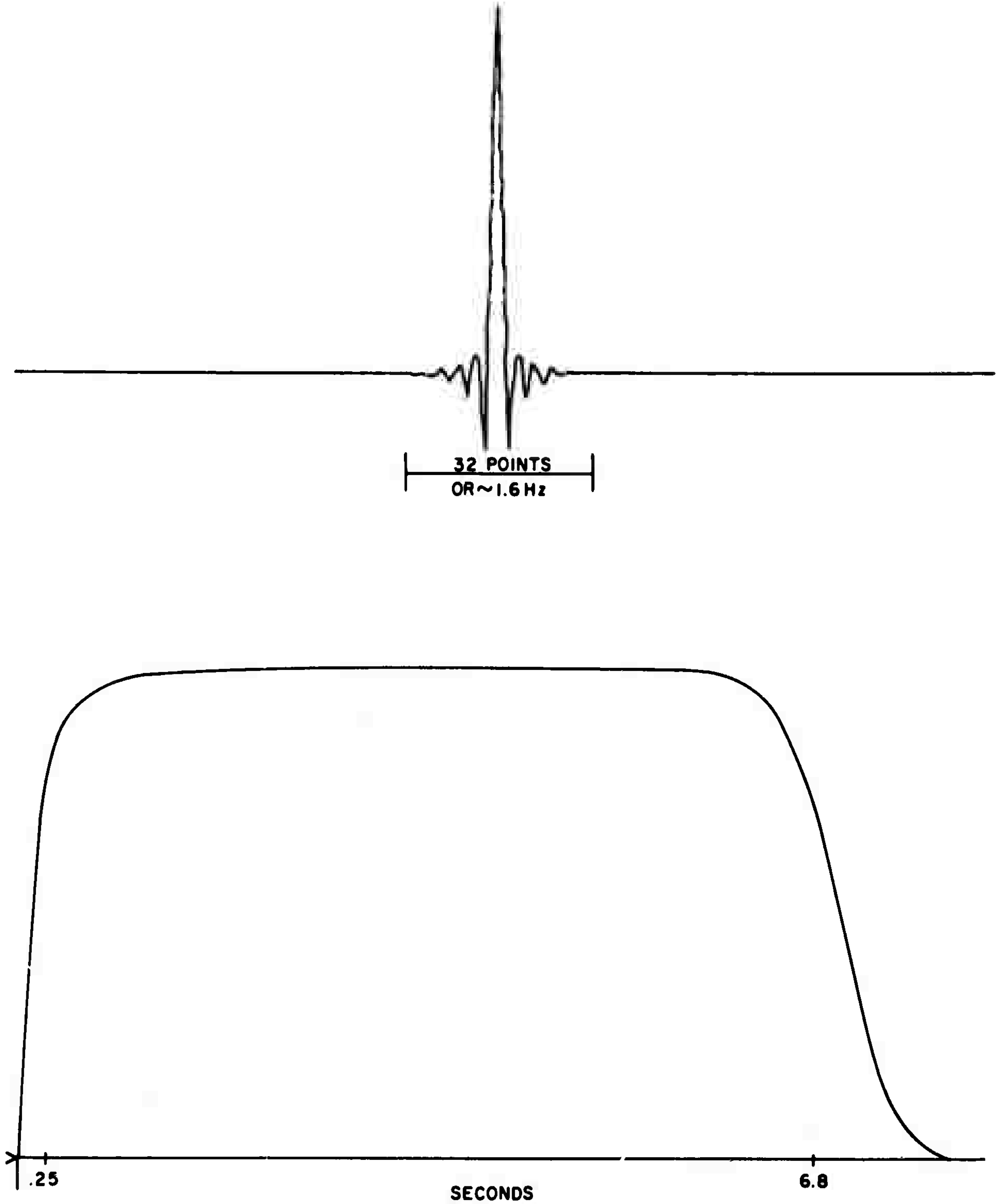

Figure 4. Typical lifter response used in cepstral analysis.

IV. BASIC DATA AND RESULTS

The basic data are summarized in Table 3 and the figures.

(1) Visual Examination

Figure 1 shows the records used for the crudest analysis method, i.e., direct time measurement from the raw data. An interval of 3.6 seconds between arrivals is evident in Figure 1; we estimate the accuracy of this interval to be about 0.2 seconds.

Figures 5 and 6 are high-resolution frequency-wavenumber spectra of the two events studied, computed at a frequency of 1.25 Hz. The azimuth and phase velocity of the peak of each spectrum were read from these figures, and these parameters were used to beamsteer the array. The phased sum for the two events is shown in Figure 7.

Visual examination of the phased sum yields the same conclusion as examination of the raw seismograms. We estimate the delay time to be 3.6 $\pm$ 0.2 seconds.

(2) Autocorrelation Analysis

The autocorrelation functions for the reference event and the double explosion are shown in Figure 8 for the entire P records, and in Figure 9 for the P codas only. The autocorrelation function for the coda of the double

TABLE 3

Results of Time Separation Measurements on the Double Event

Method	
1. By eye, using raw data	3.6 ± 0.2*
2. By eye, using phased sum	3.6 ± 0.2
3. Autocorrelation	3.6 ± 0.2
4. Matched filter	3.6 ± 0.3
5. Matched filter (deconvolved data)	3.6 ± 0.5
6. Spectral troughs	3.7 ± 0.1
7. Cepstral analysis	3.65 ± 0.1
8. Eyewitness report (Aptikayev et al., 1967)	3.56

*estimated errors

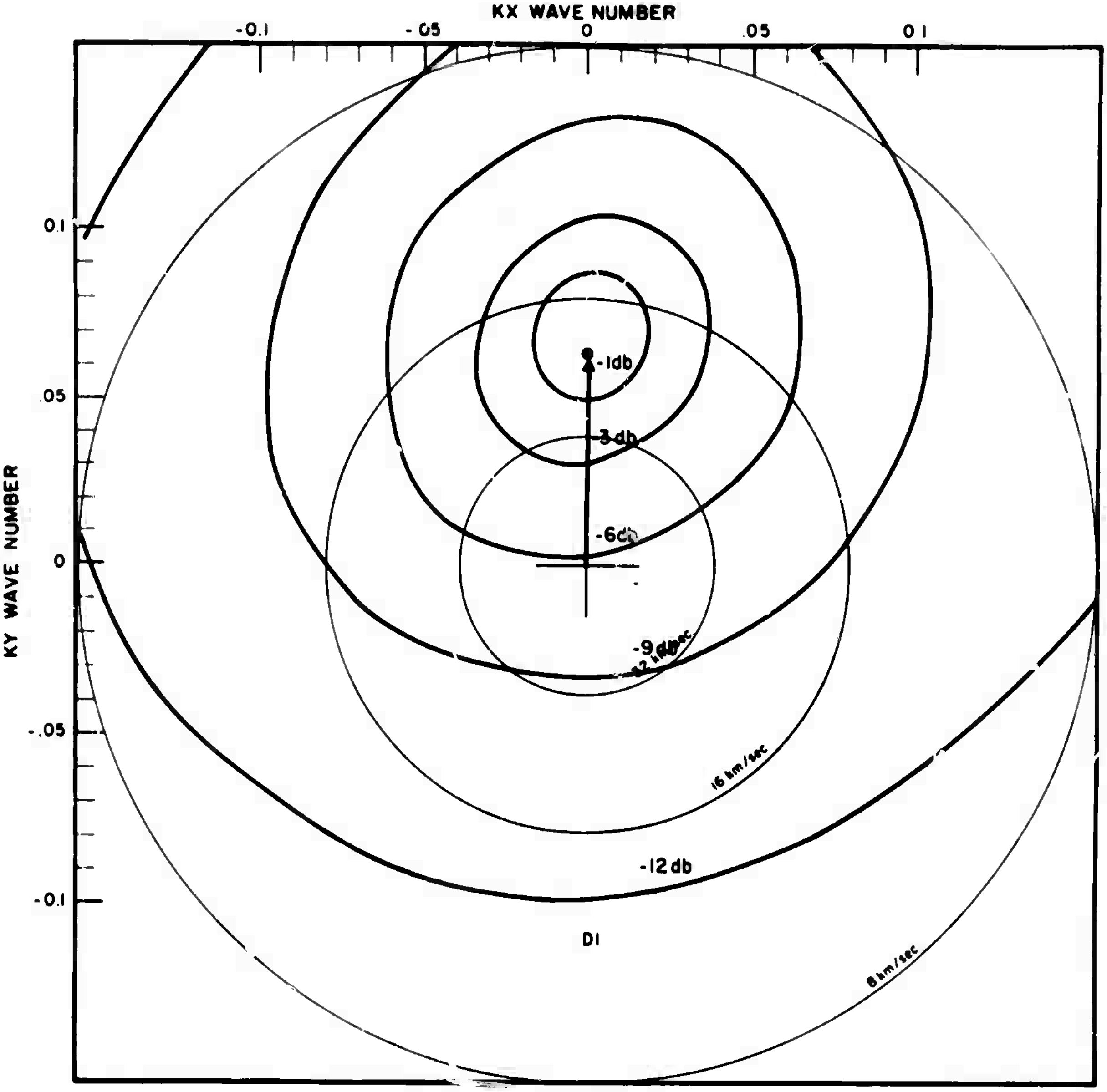

Figure 5. High resolution frequency-wavenumber spectrum of the double explosion, at 1.25 Hz. The peak power occurs at a velocity of 19 km/sec and azimuth 1° east of north.

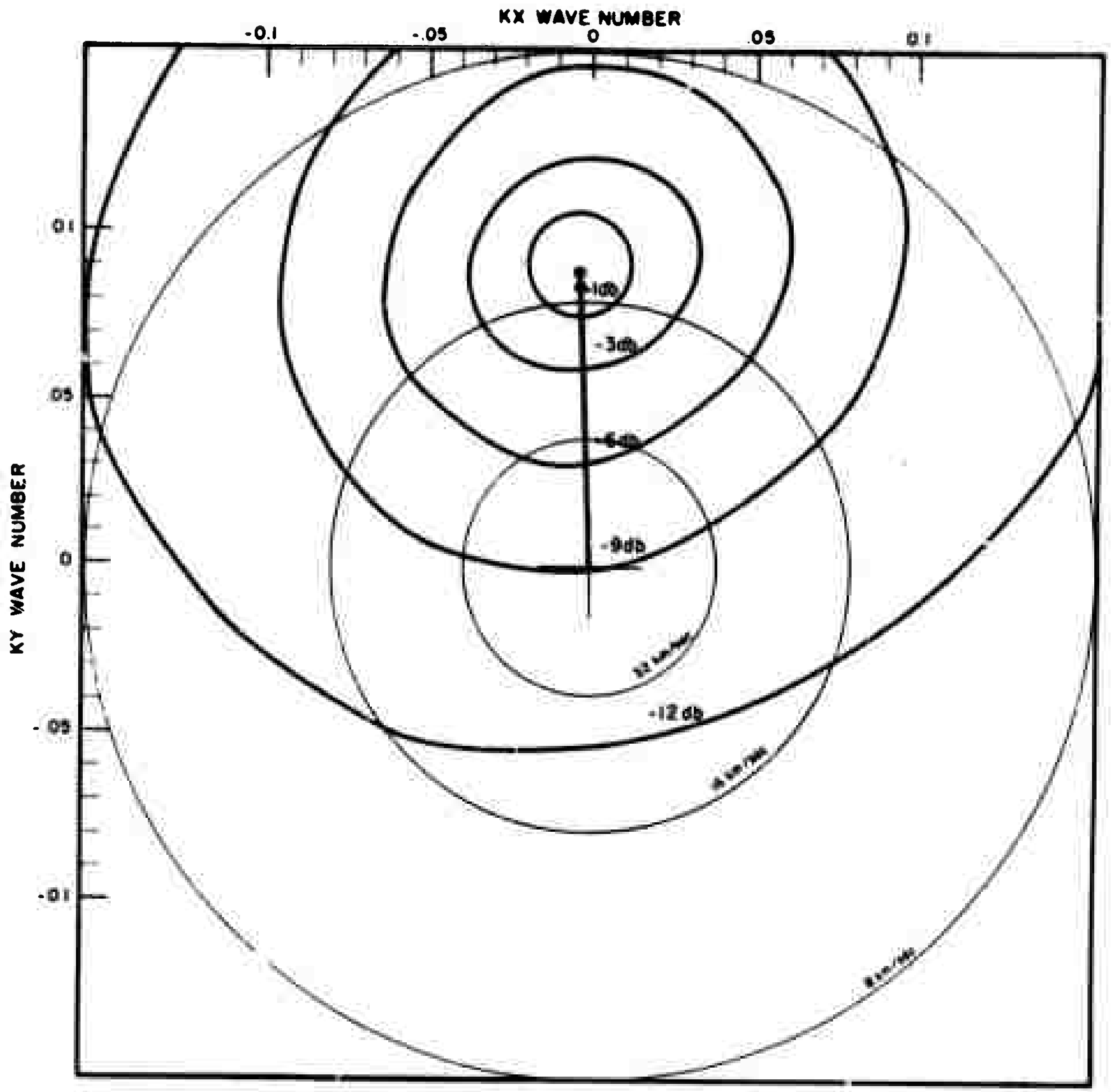

Figure 6. High resolution frequency-wavenumber spectrum of the simple reference event at a frequency of 1.25 Hz. The peak power occurs at a velocity of 13.8 km/sec and an azimuth of 353° east of north.

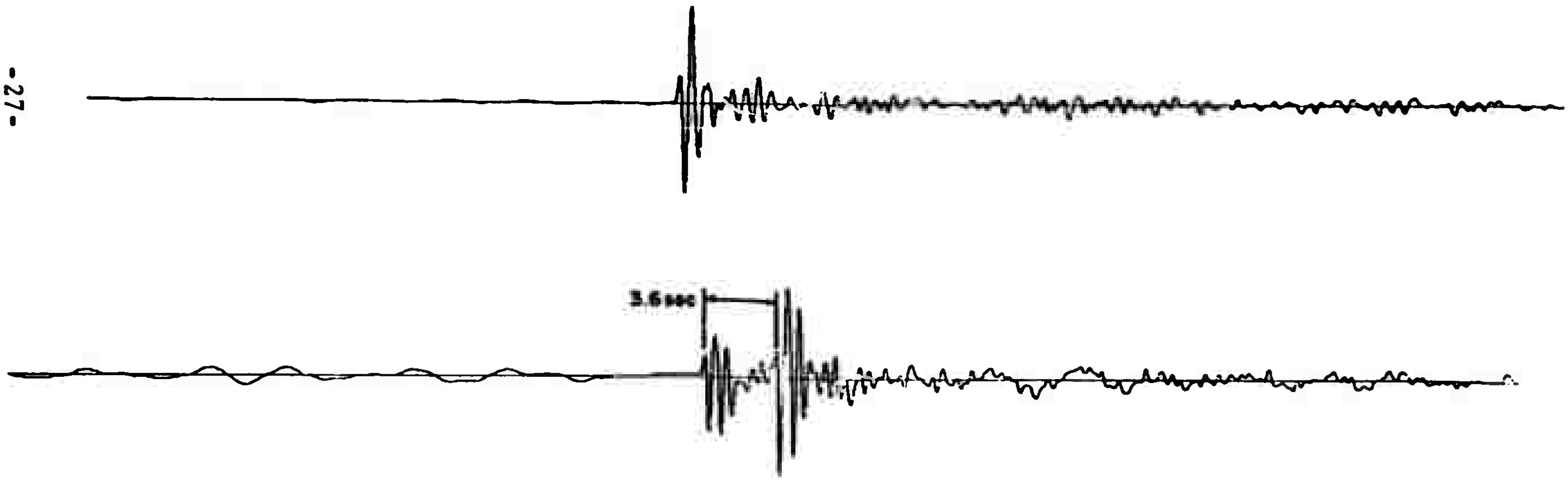

Figure 7. Phased sums: (a) reference event; (b) double explosion.

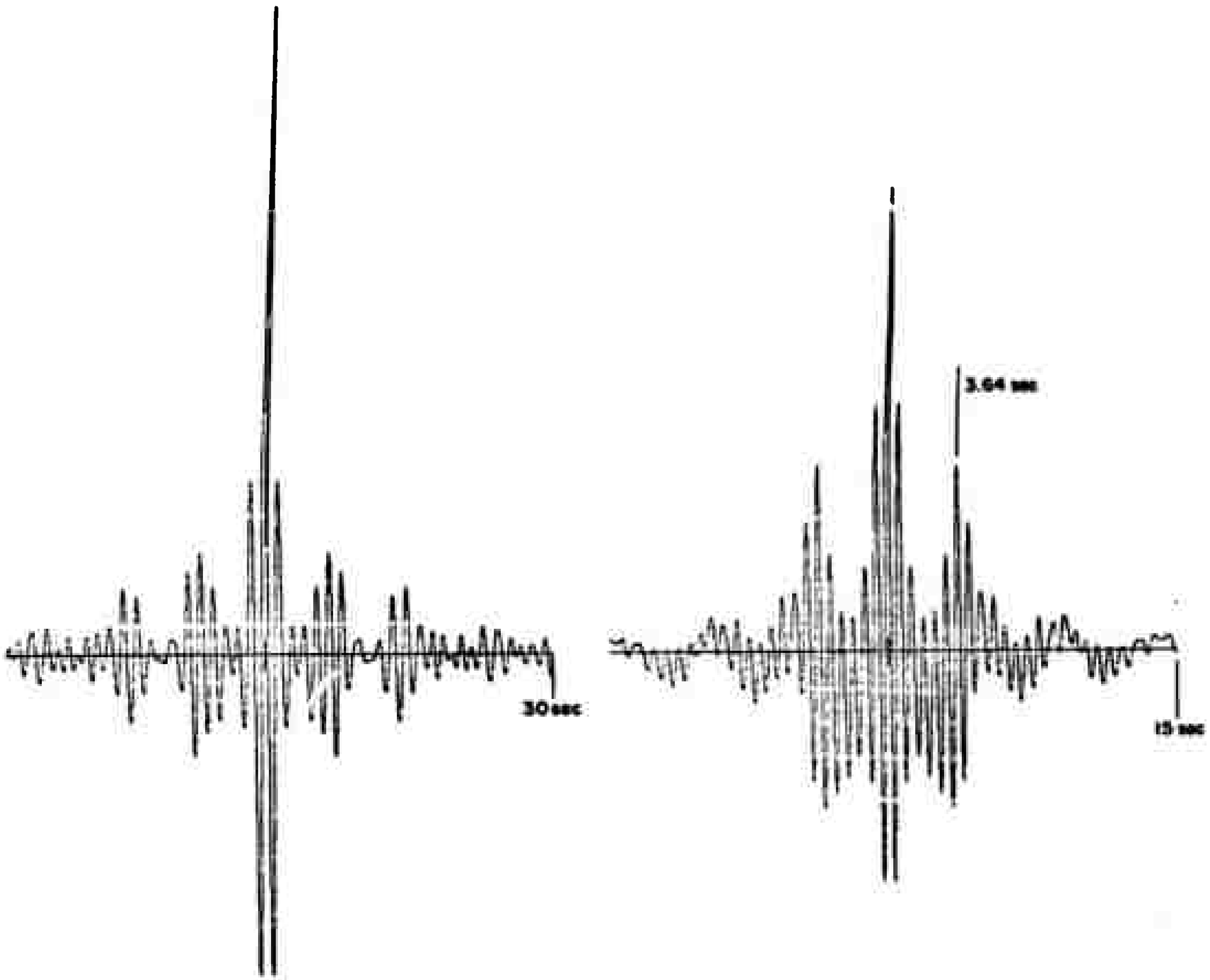

Figure 8. Autocorrelations, total P signal: (a) reference event; (b) double explosion.

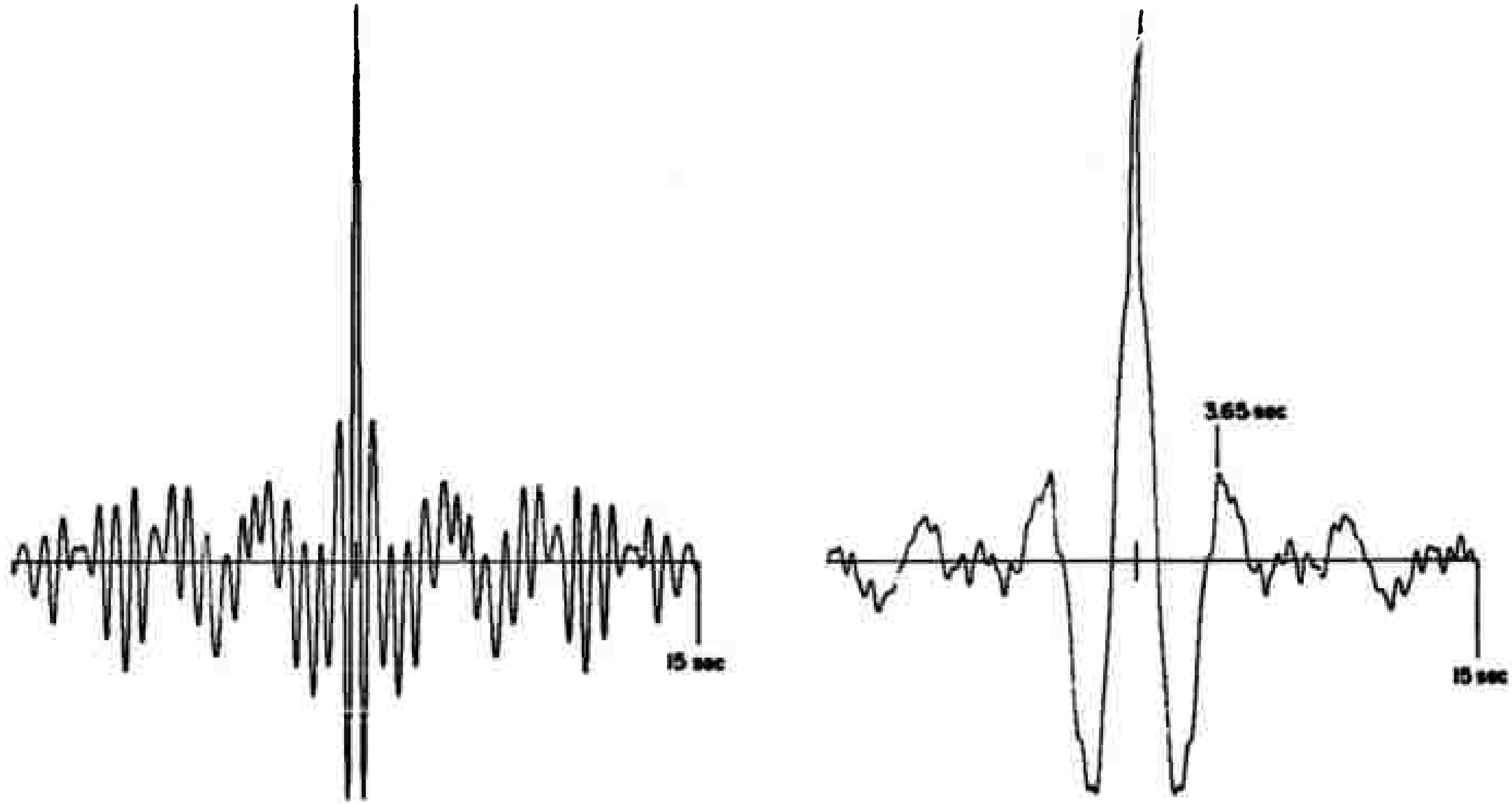

Figure 9. Autocorrelations, P coda only: (a) reference event; (b) double explosion.

explosion shows no doubling effect, but as with the other methods, a doubling interval of 3.6 seconds clearly evident in the autocorrelation of the entire P signal.

(3) Matched Filter using a Reference Event

We chose a simple event which occurred reasonably close to the event under study to evaluate the results of applying a new type of multichannel matched filtering (see Appendix A). The results are shown in Figure 10. A time separation of 3.6 seconds is evident.

We also deconvolved both the matched filter and the data in order to decrease the width of the autocorrelation functions in the matched filter output, and thus improve the resolution. A fifty-point time-domain least-squares inverse operator was used, with the cross-correlation between the inputs and the desired output set to unity for twenty time points. Running the matched filter program on the deconvolved data produced the results shown in Figure 11. It is clear that here the output has indeed been whitened, but unfortunately the signal peaks now look much the same as the noise. It is possible to measure a time separation, but certainly not uniquely. We can .

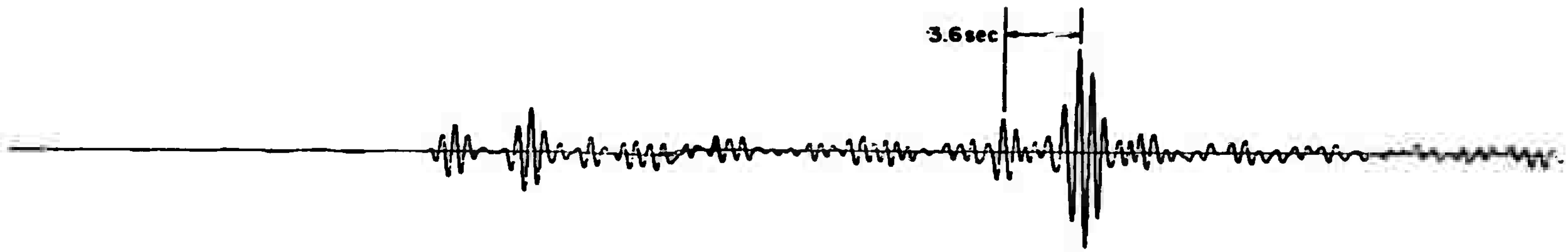

Figure 10. Result of matched filtering.

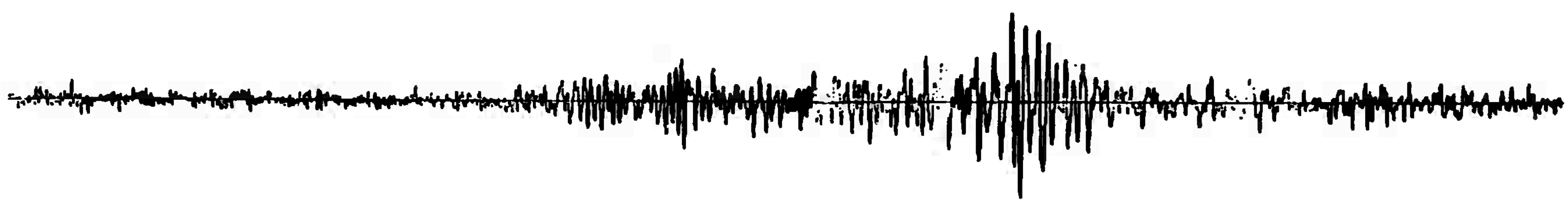

Figure 11. Result of match-filtering the deconvolved data.

find the peak which corresponds to the first shot only because we know the arrival time of the signal on the original records; the peak due to the second explosion could also be identified only by knowing approximately where to look for it. A value of 3.6 seconds for the shot interval could be determined in this way, but the measurement is inaccurate and the agreement with the results of other measurement techniques must be regarded as fortuitous.

(4) Cepstral Analysis and Spectral Nulls

Figure 12 shows cepstral analysis for the reference event. All the secondary cepstral peaks are small, although there is a suggestion of an arrival around 3.7 seconds after the main arrival. This small cepstral peak corresponds to a negative excursion of the pseudo-autocorrelation function, which shows a greater dot product value (negative sign) at 3.75 seconds lag than is found at 3.4 seconds lag (positive sign).

A plot of the null frequencies observed in the spectrum of the reference event is shown in Figure 13. This further demonstrates the negative phase of the

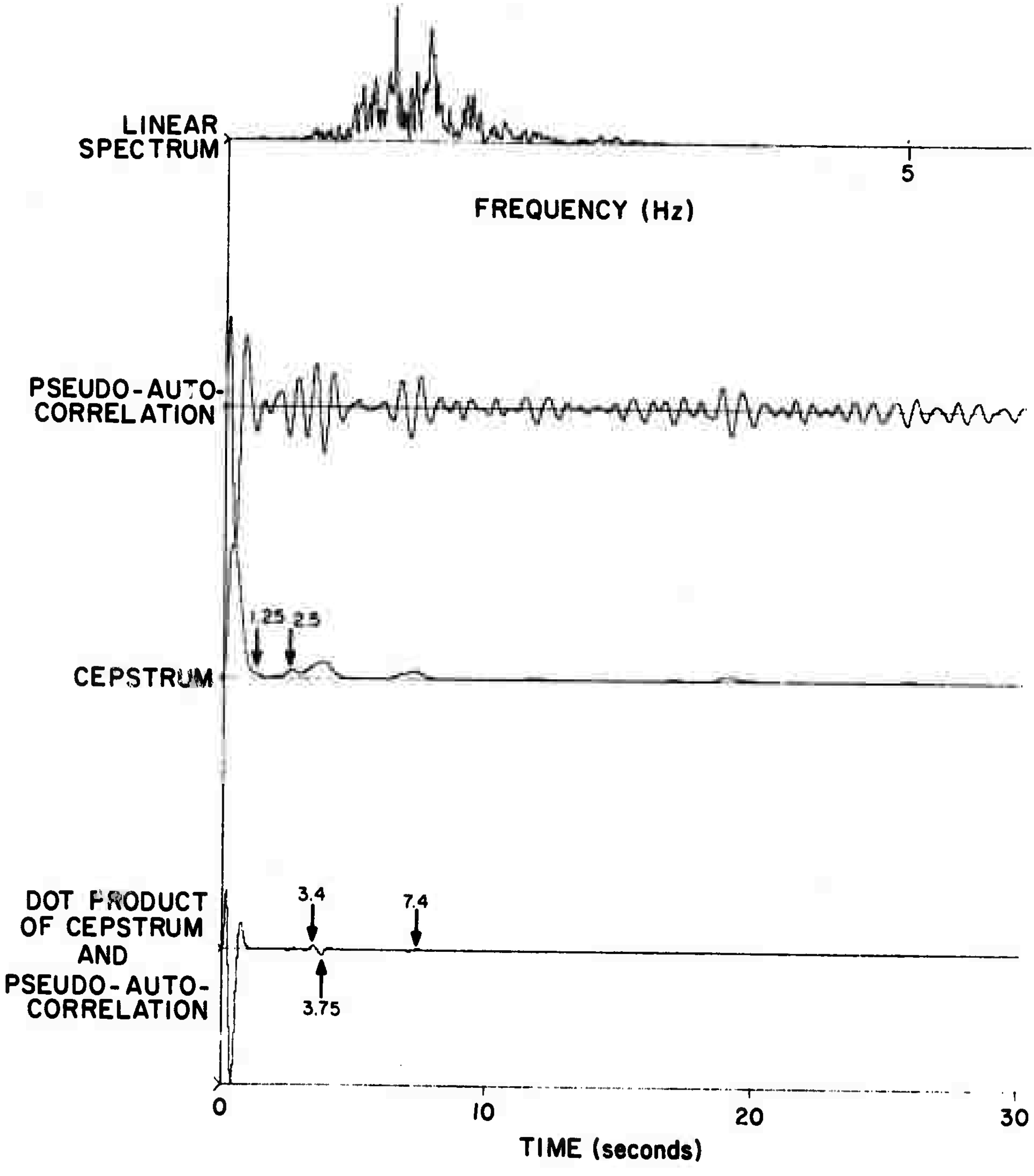

Figure 12. Cepstral analysis, reference event.

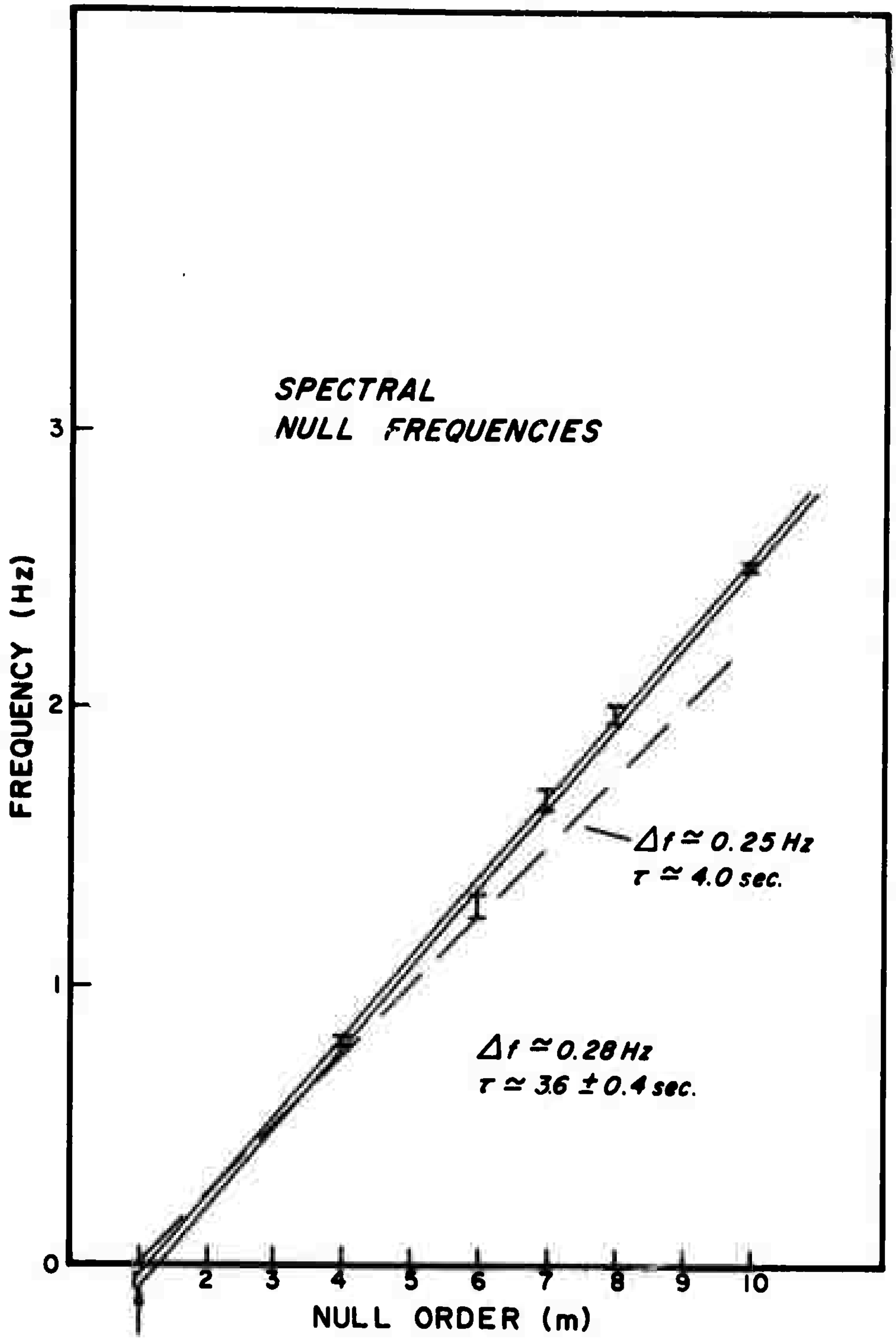

Figure 13. Spectral null frequencies, reference event.

secondary arrival with respect to the first arrival. The plot of null frequency vs. null order suggests a frequency intercept on the whole integer, and thus a difference in polarity of the two possible constituent events.

We conclude that in this reference event a weak secondary arrival with negative phase (relative to the first arrival) may possibly have occurred about 3.75 seconds after the first arrival. We conclude on this evidence that the reference event is probably not a double event, because of the negative polarity of the second arrival.

On the other hand, the results obtained for the double explosion (Figure 14) strongly suggest that this was a multiple event. The most striking features are the coincident positive peaks in the pseudo-autocorrelation and the cepstrum; the dot product of these two functions peaks at 3.65 ± 0.1 seconds, in good agreement with the known delay time of the second explosion.

The plot of null frequency vs. null order (Figure 15) demonstrates that the phase of the second arrival relative to the first is positive. The line of null frequencies is well determined in the frequency range

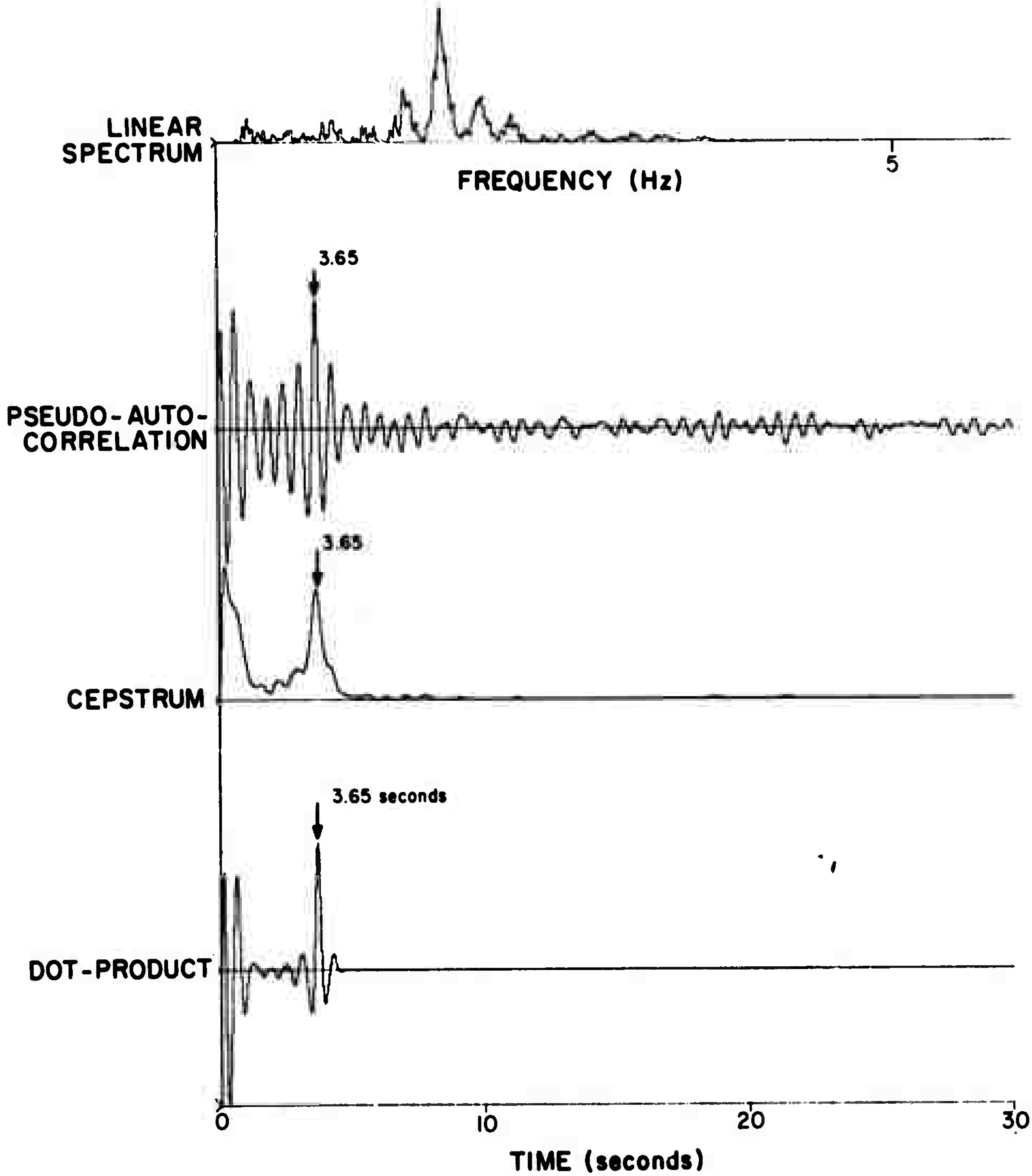

Figure 14. Cepstral analysis, double explosion.

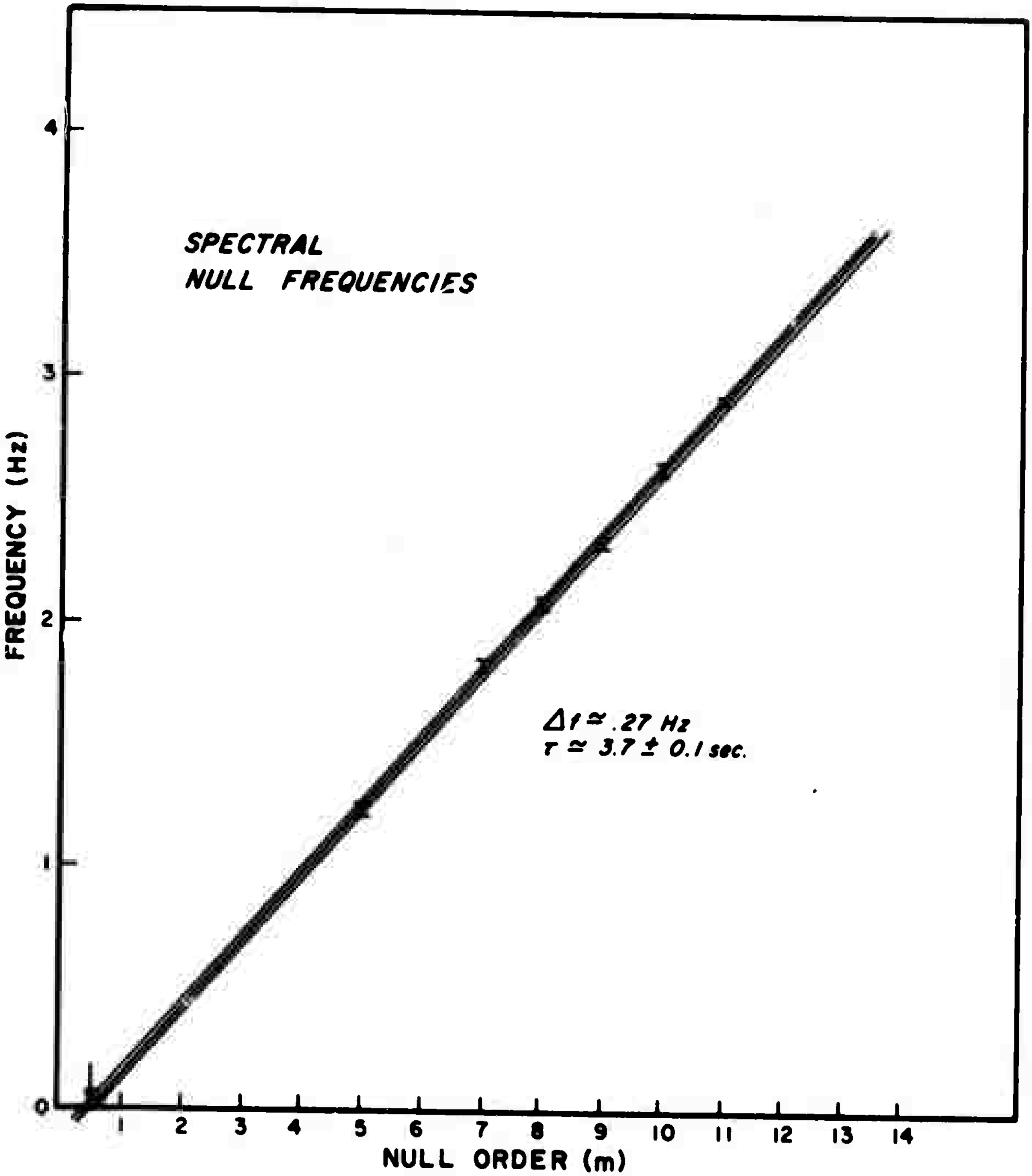

Figure 15. Spectral null frequencies, double explosion.

where there is significant signal power, and this line unambiguously specifies a half-integer intercept, which implies coincident polarity of the two sources. The time delay derived from the intercept is 3.7 ± 0.1 seconds.

Aptikayev et al. (1967) indicate that a 150-meter tunnel led from the auxiliary charges to the main charge. If we take this distance to be the inter-shot spacing and assume that the charges are at the same elevation, the maximum component of delay time (at 54° epicentral distance) caused by shot spacing is about 10 milliseconds, so a correction for shot spacing is unnecessary.

V. CONCLUSIONS

Measurements made from the cepstra of short-period P phases recorded at teleseismic distances appear to give the most accurate estimate of the time separation between components of a double seismic event. The time separation estimated by all the techniques studied agrees well with the published value, as does the amplitude ratio of P waves from the two events.

Deconvolution of the data does not increase the resolution of matched filter outputs; the deconvolution process tends to whiten the signal excessively and renders the signal waveforms quite similar to noise pulses. Modification of the deconvolution filter characteristics might possibly improve the situation, but previous experience suggests that the problem is inherent in data recorded by narrow-band systems (Claerbout, 1964).

Measurements made from the autocorrelation functions of the double explosion, either including or excluding the P coda, are only slightly better than those made from raw data, and are definitely inferior to any frequency-domain technique studied here. The reason appears to be that the short-period P wave autocorrelation functions are as broad in extent as the original waveforms.

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APPENDIX A

MULTICHANNEL MATCHED FILTERING

The multichannel convolution algorithm is:

$$y(t) = \sum_{i=1}^{N} \sum_{\tau} f_i(\tau)x_i(t-\tau) \qquad \text{(A1)}$$

where $y(t)$ is the output, $f_i(t)$ is the filter for the i'th channel, and $x_i(t-\tau)$ is the i'th input. For the purpose of this study the inputs are assumed to be double events recorded by a seismic array. The array is defined by the time shifts δ_i which when applied to the data will align the signals. We assume that both components of the double event require the same shifts for alignment: the inputs are thus assumed to be

$$x_i(\tau) = s_1(t+\delta_i) + s_2(t-T+\delta_i) \qquad \text{(A2)}$$

where T is the time separation between the two events. Matched filtering gives the best results when s_1 and s_2 are the same waveform within a possible scale factor a. This waveform we will simply call s, giving the following model for the input channels:

$$x_i(t) = s(t+\delta_i) + as(t-T+\delta_i) \qquad \text{(A3)}$$

The multichannel filter to be used on these data is a single event from approximately the same location as the double event, recorded by the same array, so that it has the same time shifts as the double event. If it also has the same waveform within a scale factor b, the filter is:

$$f_i(\tau) = bs(-\tau-\delta_i) \qquad (A4)$$

Substituting these into the filtering algorithm gives:

$$y(t) = \sum_{i=1}^{N} \sum_{\tau} bs(-\tau-\delta_i)\ s(t-\tau\delta_i)+a\ s(t-T-\tau-\delta_i) \qquad (A5)$$

Putting $u = -\tau-\delta_i$ and evaluating the second sum gives

$$y(t) = \sum_{i=1}^{N} \sum_{u} b\, s(u)\ s(u+t)+a\ s(u+t-T) \qquad (A6)$$

From the definition of the signal correlation R(p)

$$R(p) \equiv \sum_{v} s(v)\ s(v+p) \qquad (A7)$$

this expression can be written:

$$y(t) = \sum_{i=1}^{N}\ b\ R(t)+ab\ R(t-T) \qquad (A8)$$

Thus the output is the sum of the correlation functions. The noise should tend to cancel out in this process, thereby improving the signal-to-noise ratio in the output. The advantage, of course, is that the actual time shifts δ_i are never needed. As long as a single event is available from a nearby location, no phasing is ever done.

APPENDIX B

POWER SPECTRUM OF A DOUBLE EVENT

Suppose the received signal is the superposition of two identical single event P waves, the second arriving T seconds later and having amplitude a with respect to the first:

$$z(t) = y(t) + a\, y(t+T) \tag{B1}$$

If the Fourier transform of y(t) is Y(f)

$$Y(f) = \int_{\infty}^{\infty} e^{-2\pi ift}\, y(t)dt \tag{B2}$$

then the Fourier transform of z(t) is

$$Z(f) = Y(f) + ae^{2\pi ift}\, Y(f) = Y(f)\,[1+ae^{2\pi ift}] = Ae^{i\phi}\, Y(f) \tag{B3}$$

$$\text{where } A = [1+a^2 + 2a\pi\cos 2\ fT]^{\frac{1}{2}} \tag{B4}$$

$$\text{and } \tan\phi = \sin 2\pi fT/[1+a\cos 2\pi ft] \tag{B5}$$

From equation (B4), it is clear that the spectrum is modulated by a sinusoidal function whose minima are spaced 1/T Hz apart on the frequency axis.