NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

KALMAN FILTERING TECHNIQUES
APPLIED TO
AIRBORNE DIRECTION-FINDING
AND EMITTER LOCATION

by

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Kalman Filtering Techniques
Applied to
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and Emitter Location

by

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I. INTRODUCTION

One goal of airborne direction-finding (DF) is to obtain the location of an electromagnetic radiation source in latitude/longitude coordinates on the earth's surface. Over the years many emitter location techniques have been devised, and much research has been devoted to optimizing bearing angle measurements in angle-only direction-finding and navigation systems. Much of the effort has been directed toward physical components such as the directional receiving antenna and the improvement of its characteristics. In airborne DF systems current typical bearing angle-of-arrival (AOA) accuracy is found to be about $\pm 2°$ at microwave frequencies. Despite such accuracy a multiple bearing fix will yield considerable emitter position ambiguity at any significant range between the emitter and the airborne DF system.

To improve this situation a statistical optimal estimate of each received emitter bearing angle was computed using sequential estimation techniques, commonly called Kalman filtering. Additionally, an adaptive gating technique was utilized which selectively filtered out bearing lines which did not closely associate with other bearing lines already filtered and correlated to a distinct emitter location. Smoothing techniques were utilized to improve the unfiltered initial bearing angle, and a smoothed initial bearing angle and filtered final bearing angle were used in a spherical earth triangulation solution to estimate the emitter position. The entire sorting, filtering, smoothing, gating, and triangulation procedure was organized within a software computer program capable of automatically estimating multiple emitter locations given noisy emitter bearing angle data and aircraft navigation data.

II. ANALYSIS OF THE PROBLEM

A. APPLICATION

The objective of this analytical study was to develop an optimal emitter location algorithm which would have application to military aircraft flights in an operational environment. The computer program developed could be utilized in an airborne digital computer system to give real-time analysis of emitter parameters in a multiple emitter environment, or it could be used in a ground-based computer system to give post-flight analysis of large quantities of compiled data..

Since all data sampling and processing was done at discrete time intervals, discrete sequential estimation techniques were utilized. The real facility in using sequential estimation techniques is that each subsequent computation is based only on the new observation and the last previously calculated estimate, and these recursive equation computations require a minimum of computer storage to filter a large quantity of data. This makes such an emitter location algorithm ideal for airborne computer application [1].

To allow generality of application, the software system was developed to process multiple emitter locations, to sort and filter large quantities of emitter data, to filter data at non-uniform sampling intervals, and to estimate emitter locations on any region of the earth's surface. It was initially assumed that no errors existed in the aircraft navigation data; hence these position data together with two or more filtered emitter bearing angles could be used to compute the emitter location (see Figure 1). If a deterministic aircraft navigation error was known to exist, it could be compensated for in the emitter location algorithm. If a random error existed in the aircraft navigation data, it could be minimized by using a non-zero $\underline{Q}$ matrix to account for random excitation noise (see Section III, Part B, page 17) or by Kalman filtering the aircraft navigation data as well as the emitter bearing data. Kalman filtering techniques have also been developed to optimize navigation system accuracy [2].

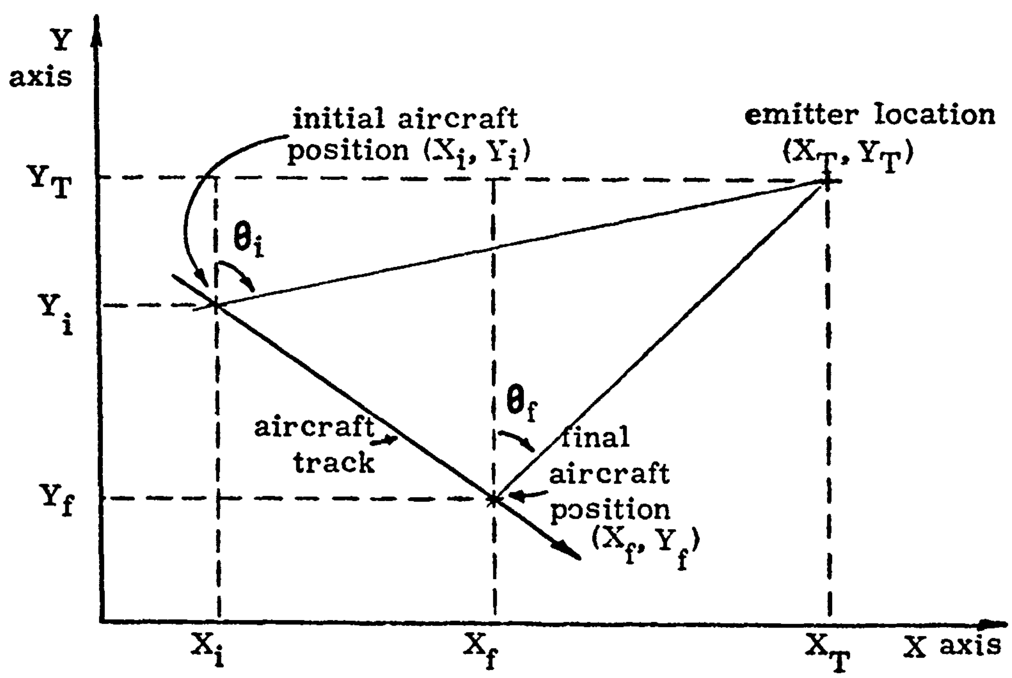


FIGURE 1

GEOMETRY OF EMITTER LOCATION

B. KALMAN FILTER UTILIZATION

A signal $\underline{X}$, measured in the presence of random white Gaussian noise $\underline{V}$, is filtered in an attempt to recover the original signal. The filtered estimate of the state is denoted by $\hat{\underline{X}}$. From Kalman filter theory [3] for discrete systems using first-order difference equations, the kth noisy observation $\underline{Z}(k)$ of the state is given by

$$\underline{Z}(k) = \underline{H}(k)\underline{X}(k) + \underline{V}(k) \qquad k = 1, 2, 3, \ldots \tag{1}$$

where $\underline{H}(k)$ is the observation matrix and $\underline{V}(k)$ is the measurement noise. Since a discrete or sampled system is being considered, k denotes the kth time sample.

The state of a general, discrete system with no deterministic forcing function is given by

$$\underline{X}(k+1) = \underline{\Phi}(k+1,k)\underline{X}(k) + \underline{\Gamma}\,\underline{W}(k) \qquad k = 1, 2, \ldots \tag{2}$$

where $\underline{\Phi}(k+1,k)$ is the state transition matrix, $\underline{\Gamma}$ is a distribution matrix related to the random forcing function, and $\underline{W}(k)$ is a random forcing function to account for random excitation noise. It was assumed that the noise sequences have zero mean and second-order statistics described by

$$E\left[\underline{V}(k)\underline{V}(j)^T\right] = \underline{R}(k)\delta(kj) \tag{3}$$

$$\underline{\Gamma}E\left[\underline{W}(k)\underline{W}(j)^T\right]\underline{\Gamma}^T = \underline{Q}(k)\delta(kj) \tag{4}$$

and $$E\left[\underline{V}(k)\underline{W}(j)^T\right] = \underline{0} \qquad \text{for all } k, j \tag{5}$$

where $\delta(kj) = \begin{cases} 0 & k \neq j \\ 1 & k = j \end{cases}$ is the Kronecker delta.

The Kalman filter recursion equations [1] are summarized below where $\hat{\underline{X}}(k/j)$ denotes the estimate of the state $\underline{X}(k)$ based upon j measurement observations $\underline{Z}(1), \underline{Z}(2), \ldots, \underline{Z}(j)$.

$$\underline{G}(k) = \underline{P}(k/k-1)\underline{H}(k)^T\left[\underline{H}(k)\underline{P}(k/k-1)\underline{H}(k)^T + \underline{R}(k)\right]^{-1} \tag{6}$$

$$\underline{P}(k/k) = \underline{P}(k/k-1) - \underline{G}(k)\underline{H}(k)\underline{P}(k/k-1) \tag{7}$$

$$\underline{P}(k+1/k) = \underline{\Phi}(k+1,k)\underline{P}(k/k)\underline{\Phi}(k+1,k)^T + \underline{Q}(k) \tag{8}$$

$$\hat{\underline{X}}(k/k) = \hat{\underline{X}}(k/k-1) + \underline{G}(k)\left[\underline{Z}(k) - \underline{H}(k)\hat{\underline{X}}(k/k-1)\right] \tag{9}$$

$$\hat{\underline{X}}(k/k-1) = \underline{\Phi}(k,k-1)\hat{\underline{X}}(k-1/k-1) \tag{10}$$

The Kalman filter gains are represented by the matrix $\underline{G}(k)$, and $\underline{P}(k/j)$ represents the error covariance matrix of the various estimates.

For the problem being considered the states are bearing angle-of-arrival and bearing rate, and it is desired to filter the noisy observed bearing angles to obtain optimal estimates of these states. The system dynamics were approximated by a $1/s^2$ plant which results in a state transition matrix given by

$$\underline{\Phi}(k+1,k) = \begin{bmatrix} 1 & T(k+1) \\ 0 & 1 \end{bmatrix}.$$

Since only the bearing angle of the state was observed, $\underline{R}(k)$ and $\underline{W}(k)$ become scalar variance terms and the bracketed terms in equations (6) and (9) become scalars as well. This considerably simplified reducing these matrix equations to scalar form (see Appendix A); however, allowing a non-uniform sampling interval required that the term T(k+1) remain a variable in each of the applicable recursion equations. This required that error covariance terms and Kalman filter gains be computed on line since they were a function of the sampling interval; therefore, these terms could not be precomputed and stored.

Several assumptions are implicit here. Modeling the system dynamics with a $1/s^2$ transfer function gives a linear approximation that may not adequately represent the actual system equations, which are non-linear [4]. It may then become necessary to augment the dynamic model with an additional difference equation which includes the second derivative of the measured bearing angle, or extended Kalman filtering techniques may be employed to more accurately approach the non-linearity of the dynamic system. Extended Kalman filtering techniques are discussed in Section III, Part F, of this paper.

The statistical nature of Kalman filtering techniques requires that the statistics of the measurement noise $\underline{V}(k)$ and the excitation noise $\underline{W}(k)$ be completely specified, or if unknown they must be assumed. If no deterministic errors are known to be coloring these noise sources, the assumption of white noise may be a good one although it may not exactly model the actual noise bearing error. Kalman filtering assumes that the noise is independent from one sampling interval to the next [5]. This may not be an accurate assumption since successive bearing angle observations from a single emitter might have similar error statistics. Any deterministic bias in aircraft antenna bearing angle-of-arrival measurement would further negate this assumption. This difficulty was reported by Reeves [6], who recommended that emitter bearing angle-of-arrival measurement noise be considered to have both random and correlated components. This would require augmenting the state vector equation with an additional variable and further complicate the Kalman filtering process.

C. SMOOTHING TECHNIQUES

Since Kalman filtering techniques sequentially filter the second and subsequent bearing angle observations associated with a single emitter, the first bearing angle associated with a particular emitter remains unfiltered and no optimal estimate exists. Thus a technique of filtering backward in time or smoothing was used to improve the estimate of the initial bearing angle. The smoothing equations are similar to Kalman filter equations in that they are recursive and functions of similar statistical parameters, but they update or smooth previous data based on more recent data that has been optimally estimated.

A general technique proposed by Rauch [7] gives the smoothed estimate of the initial observation after k time intervals as

$$\underline{\hat{X}}(1/k) = \underline{\hat{X}}(1/k-1) + \underline{D}(1/k)\underline{G}(k)\left[\underline{Z}(k) - \underline{H}(k)\underline{\hat{X}}(k/k-1)\right] \quad (11)$$

where $\underline{D}(1/k) = \underline{D}(1/k-1)\underline{P}(k-1/k-1)\underline{\Phi}(k,k-1)^T\underline{P}(k/k-1)^{-1}$ (12)

For a non-uniform sampling interval $\underline{D}(1/k)$ becomes a function of T(k) and must be solved sequentially before the smoothed initial estimate $\underline{\hat{X}}(1/k)$ can be computed. For an analysis of these equations reduced to scalar form, see Appendix B. These scalar smoothing equations were included in the software computer program and yielded a smoothed estimate of the initial bearing angle and bearing rate.

III. COMPUTATIONAL PROCEDURE

A. INITIAL DATA SORT

The problem was posed to accommodate a realistic airborne environment with multiple emitters at various RF frequencies, emitter bearing angle-of-arrival errors, and a constant velocity moving aircraft. Aircraft navigation and emitter target data was sampled and recorded at discrete but time-varying intervals. Since the number of emitters was unknown and large quantitites of signal data were recorded at various frequencies, pulse repetition frequencies (PRF), and pulse widths (PW), it was necessary that the data be initially sorted by frequency and PRF to initially estimate the number of distinct emitter targets. This was accomplished with sufficiently small frequency and PRF gates to separate and associate emitter parameters with a distinct emitter target. Additionally, it was found that multiples of PRF may exist which associate with a single emitter, so a check for PRF multiples had to be included within the PRF sorting subroutine. One set of data processed by this program included PRF multiples given on page 59.

Multiple emitters of the same frequency and PRF could still result in an ambiguous solution; therefore, each bearing angle initially associated with a distinct emitter by the initial data sort was sequentially filtered and gated to determine if it correlated with this emitter. Due to the large quantity of data to be processed, any bearing angle not correlated to a distinct emitter after being filtered and gated was discarded from the data bank. Further investigation and greater computer storage capability could allow such discarded bearing angles to be re-associated with a different emitter; however, this process was not developed within this analysis.

B. KALMAN FILTER INITIALIZATION

Initialization of the Kalman filter equations requires a knowledge of the system dynamics, the statistical properties of the Kalman filter parameters, and some information about the initial state of the system. Since it was necessary to associate each sorted data sample (emitter bearing angle and associated emitter data) with a particular emitter, the notation JSET (I, J) was devised, where I is the emitter target number with which the data sample is associated and J is the jth sample of sequential data associated with that emitter. JSET(I, J) gives the sequential sample number for this data sample when listed together with all other data. See page 63 for a listing of initial JSET data. To initialize the Kalman filter equations, KI was used as the initial JSET value; i.e., JSET(I, 1) = KI for the ith emitter target.

To filter emitter bearing angles-of-arrival, the noisy observation of the state from equation (1) is given by

$$\underline{\theta}(k) = \underline{H}(k)\,\underline{\Theta}(k) + V(k) \tag{13}$$

where $\underline{\theta}(k) = \begin{bmatrix} \theta(k) \\ \dot{\theta}(k) \end{bmatrix}$ = vector of noisy measured observations,

$\underline{\Theta}(k) = \begin{bmatrix} \Theta(k) \\ \dot{\Theta}(k) \end{bmatrix}$ = state vector of exact emitter bearing angle

and bearing rate, and $\underline{H}(k) = [1 \quad 0]$ is the observation matrix for measurement of only noisy bearing angle θ. Since the bearing rate $\dot{\theta}(k)$ was not measured, it must be estimated to initialize the sequential estimation process. This bearing rate depends on the speed and heading of the aircraft with respect to the emitter, the relative bearing angle-of-arrival, and the range r to the emitter. Since the range to an unknown emitter is unknown, it must be estimated. For this program r = 150 nautical miles was assumed and

$$\dot{\theta}(KI) = \frac{v}{r} \sin(BRNG) \times \frac{PIRAD}{3600} \quad \text{degrees/second} \tag{14}$$

where BRNG = relative bearing angle-of-arrival

PIRAD = 57.29578 degrees/radian

v = aircraft velocity in knots.

If an estimated value of range to the emitter is known and different from 150 nautical miles, then this new value should be used in equation (14) and substituted into the computer program.

Since an initial optimal estimate was not available to the filtering process described by equations (9) and (10), the first bearing angle optimal estimate THTD(KI) was assumed equal to the noisy measured bearing angle THETAD(KI), and the first bearing rate estimate TDTD(KI) was assumed equivalent to $\dot{\theta}$(KI). The smoothing process was similarly initialized.

The initial uncertainty of emitter bearing angle and angle rate on filter initialization was accounted for in the initial values of the error covariance matrix $\underline{P}(1/0)$. Since stationary emitter locations were considered, the random forcing function $\underline{W}(k)$ of the dynamic state equation (2) could be set to zero if exact knowledge of the observer's aircraft position were known. However, since the aircraft position may have both random and deterministic errors associated with its position measurement and because of the simplified $\underline{\Phi}$ matrix, a non-zero $\underline{Q}$ matrix was utilized. For $\underline{W}(k)$ having a zero mean and $E\left[\underline{W}(k)^2\right] = 1.0$ degrees^2, from equation (4)

$$\underline{Q}(k) = \begin{bmatrix} \frac{T^2(k)}{2} \\ T(k) \end{bmatrix} \begin{bmatrix} 1.0 \end{bmatrix} \begin{bmatrix} \frac{T^2(k)}{2} & T(k) \end{bmatrix} = \begin{bmatrix} \frac{T^4(k)}{4} & \frac{T^3(k)}{2} \\ \frac{T^3(k)}{2} & T^2(k) \end{bmatrix}. \qquad (15)$$

The value of R(k), the scalar variance of the measurement noise, was assumed to be constant and estimated according to the angle measurement accuracy of the DF system being considered. The initial $\underline{D}$ matrix for the sequential smoothing process was initialized as the identity matrix (see Appendix B).

When utilizing this program for emitter location with different known measurement and covariance statistics, these initialization parameters must be changed in the computer program to ensure optimal solution locations. The only other term which must be modified by the program user is the value of NUM, which is the total number of data samples to be sorted and filtered by this program. Since the data for each emitter must be filtered separately, the filter process occurs once for each distinct emitter target as determined by the initial data sort. Once an optimal estimate of emitter bearing angle and bearing rate have been initialized, the filter process is ready to commence.

C. KALMAN FILTER PROCESS

The Kalman filter equations utilized to filter the states, emitter bearing angle θ and bearing rate $\dot{\theta}$, are those noted on page 11 shown as equations (6) through (10) with $\hat{\underline{X}}$ replaced by $\hat{\underline{\theta}}$. Since these are vector equations and tedious to handle within a computer program, their scalar counterparts were derived (see Appendix A) and utilized within the program. The optimal estimation equation is similar to equations (9) and (10) and is given by

$$\begin{bmatrix} \hat{\theta}(k/k) \\ \hat{\dot{\theta}}(k/k) \end{bmatrix} = \begin{bmatrix} 1 & T(k) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\theta}(k-1/k-1) \\ \hat{\dot{\theta}}(k-1/k-1) \end{bmatrix} + \begin{bmatrix} G1(k) \\ G2(k) \end{bmatrix} E(k) \qquad (16)$$

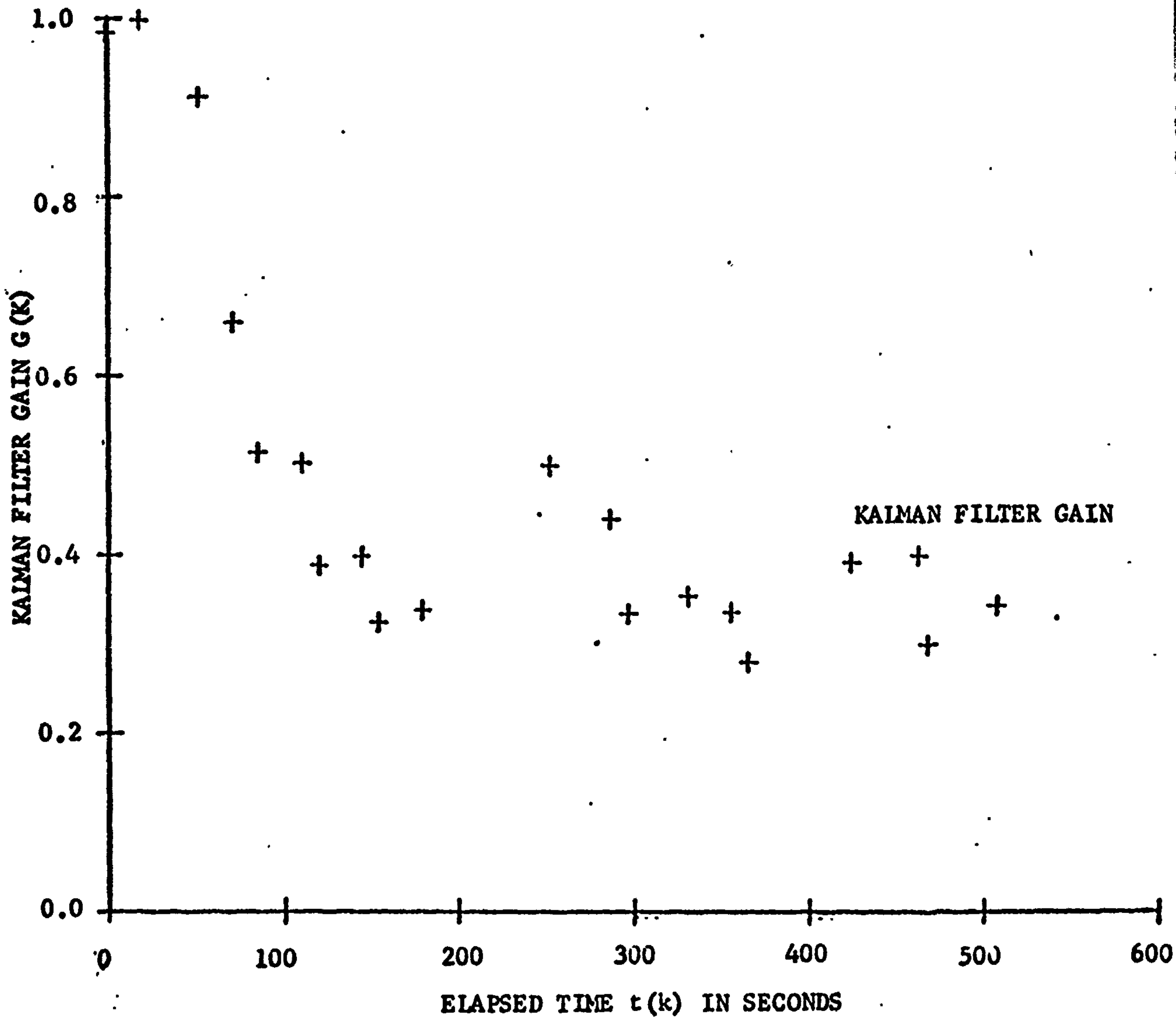
where $$E(k) = \theta(k) - \hat{\theta}(k-1/k-1) - T(k)\hat{\dot{\theta}}(k-1/k-1) \qquad (17)$$

results in a scalar term since only one of the state variables was measured.

Due to the non-uniform sampling interval, the Kalman filter gains G1(k) and G2(k) may vary and not approach a steady state value at a uniform rate as they would with a constant sampling interval. This is shown graphically in Figure 2, which depicts the transient Kalman

FIGURE 2

KALMAN FILTER GAIN SCHEDULE
FOR A NON-UNIFORM SAMPLING INTERVAL
KALMAN FILTER PROCESS



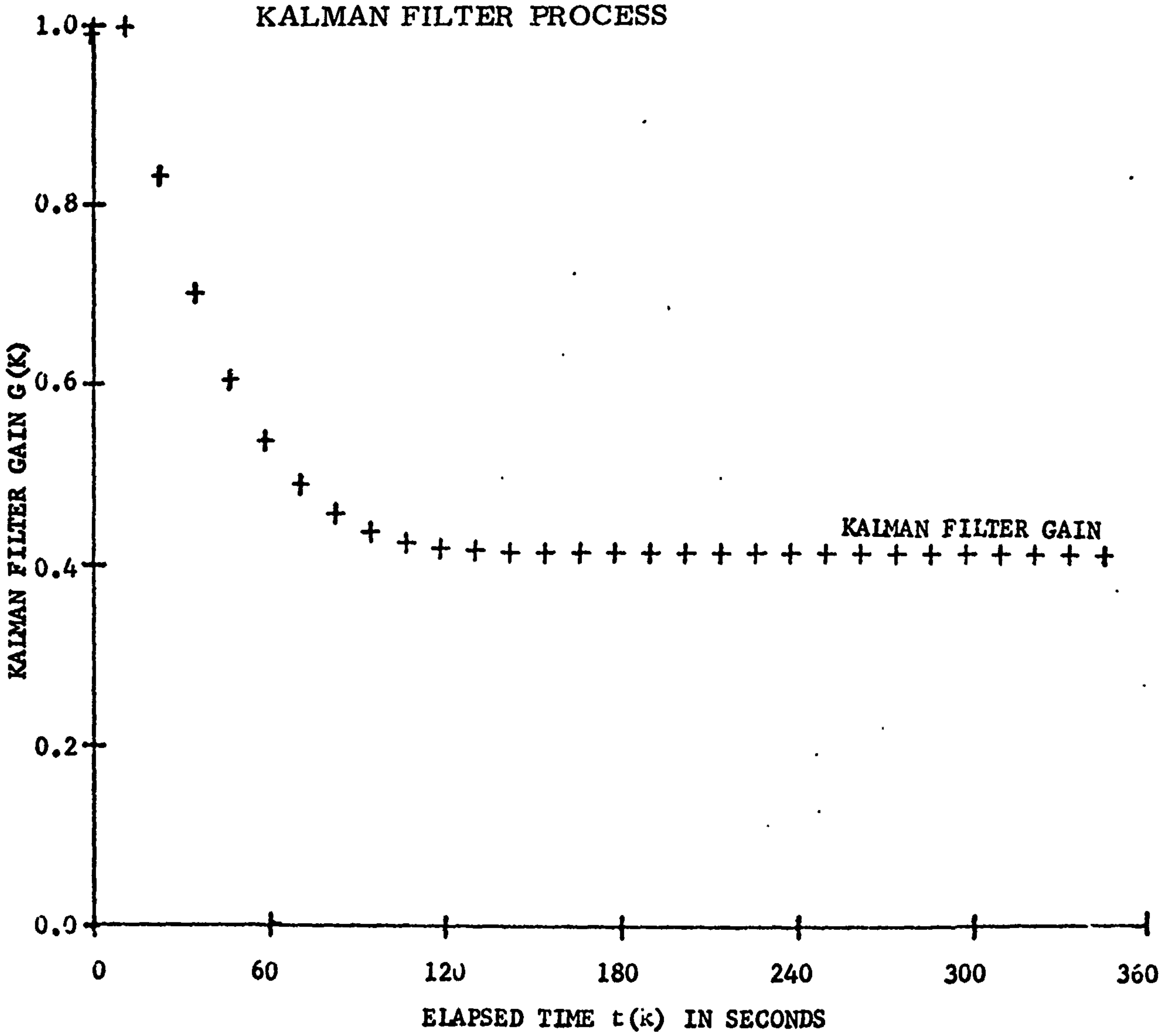
NOTES:

1. Points plotted above represent discrete values of Kalman filter gain G1(K) associated with filtering the bearing angle of the state vector.

2. Since the prediction covariance matrix (P(k+1/k) given by equation (8) is a function of Φ(k+1, k), which contains the sampling interval T(k+1), it can be seen that the Kalman filter gain G(k) will also vary with T(k+1) as in equation (6). As the sampling interval increases, the gain will also increase.

FIGURE 3

KALMAN FILTER GAIN SCHEDULE
FOR A UNIFORM SAMPLING INTERVAL
KALMAN FILTER PROCESS



NOTES:

1. Points plotted above represent discrete values of Kalman filter gain G1(K) associated with filtering the bearing angle of the state vector.

2. After ten sampling intervals the gain has nearly reached a steady state value of 0.417. This asymptote is determined by the non-zero value of the Q matrix.

3. If system random excitation noise (i.e., the Q matrix) were assumed to be zero, then the Kalman filter gain would approach zero upon reaching steady state.

filter gain schedule of a non-uniform sampling interval. Figure 3 shows the Kalman filter gain schedule of a uniform, twelve-second sampling interval computer simulation, which was developed to allow an error analysis to be made on the Kalman filter program.

The adaptive gate was chosen to be a function of the prediction variance term P11(k/k-1), which is a variable function, and the measurement noise variance R(k), which is constant. The adaptive gate was set to selectively compare the filtered bearing angle with previously correlated bearing angles. If a filtered bearing does not fall within the adaptive gate, it is discarded and the sampling interval increases to the last previous correlated bearing. This increases the Kalman filter gain and opens the adaptive gate to a wider allowable value until a subsequent bearing angle is found which correlates to this emitter, then causing the adaptive gate to reduce in size. The adaptive gate was chosen to be

$$\mathrm{GATE}(K) = C \cdot \sqrt{P11(k) + R(K)} \qquad (18)$$

where C is a constant scale factor. Since the prediction covariance $\underline{P}(k/k-1)$ may be defined as

$$\underline{P}(k/k-1) = E\left\{\left[\underline{\Theta}(k) - \underline{\hat{\theta}}(k/k-1)\right]\left[\underline{\Theta}(k) - \underline{\hat{\theta}}(k/k-1)\right]^T\right\}, \qquad (19)$$

which is a function of the mean square error between the true value of the bearing angle and its optimal estimate, it can be seen that as the estimate error increases the adaptive gate will increase and as the optimal estimate approaches the true value the gate size will decrease as is desired. The constant term R(k) prevents the gate from getting so narrow that no emitter bearing angles could correlate to a distinct emitter, and the variable term P11(k) allows the gate size to "adapt" to the transient filter errors as was noted above.

D. INITIAL BEARING ANGLE SMOOTHING PROCESS

In order to obtain a smoothed estimate of the first bearing angle associated with each emitter target, it was necessary to solve equation (11) sequentially within the Kalman filter process since the gain term was recomputed after each sampling interval. The smoothed initial bearing angle estimation equation used in the software program is similar to equation (11) and is given by

$$\begin{bmatrix} \hat{\theta}(1/k) \\ \hat{\dot{\theta}}(1/k) \end{bmatrix} = \begin{bmatrix} \hat{\theta}(1/k-1) \\ \hat{\dot{\theta}}(1/k-1) \end{bmatrix} + \begin{bmatrix} D11(1/k) & D12(1/k) \\ D21(1/k) & D22(1/k) \end{bmatrix} \begin{bmatrix} G1(k) \\ G2(k) \end{bmatrix} E(k), \quad (20)$$

where E(k) is defined by equation (17) and the scalar terms in the $\underline{D}$ matrix are as derived in Appendix B. To compute these terms, both $\underline{\Phi}(k,k-1)^{-1}$ and $\underline{P}(k/k-1)^{-1}$ had to be determined. The inverse of the state transition matrix is obviously given by

$$\underline{\Phi}(k,k-1)^{-1} = \begin{bmatrix} 1 & -T(k) \\ 0 & 1 \end{bmatrix} . \quad (21)$$

Should the error covariance matrix become singular because of numerical inaccuracy in computation, no inverse will exist. In this case corrective action must be taken before attempting to compute the inverse. Several references have discussed this problem, suggested solutions, and Schmidt [8] gives a survey of current techniques in this area. For this program, the IBM 360 subroutine MINV was utilized to compute the error covariance matrix inverse with good numerical accuracy.

E. EMITTER LOCATION ALGORITHMS

Several techniques were investigated to obtain a computationally simple yet accurate algorithm to calculate the emitter locations. Since more than two bearing lines may result in an ambiguous emitter location, it was most logical to use only the smoothed initial bearing angle $\hat{\theta}_i$ and the filtered final bearing angle $\hat{\theta}_f$ to compute the location. Any such location procedure would give most accurate results if the two bearings intersected at right angles; however, infrequent data collection, short-leg flight tracks, or long-range distances from the emitter generally prevent such choice of initial and final bearing angles.

1. Plane Triangulation Solution

The basic "flat earth" triangulation solution was presented by Kayton and Fried [9] and discussed in some detail in Refs. 6 and 10. Figure 1 depicts the geometry of the triangulation method of emitter location solution, and the solution equations are given by

$$\tan \hat{\theta}_i = \frac{X_T - X_i}{Y_T - Y_i} \tag{22}$$

and

$$\tan \hat{\theta}_f = \frac{X_T - X_f}{Y_T - Y_f} \tag{23}$$

where the emitter location coordinates (X_T, Y_T) are unknown and must be determined.

If latitude (L)/longitude (λ) coordinates are substituted for the X/Y axes in the flat earth model, the following equations result:

$$\tan\hat{\theta}_i = \frac{(\lambda_T - \lambda_i)\cos L_T}{L_T - L_i} \quad (24)$$

$$\tan\hat{\theta}_f = \frac{(\lambda_T - \lambda_f)\cos L_T}{L_T - L_f}. \quad (25)$$

These equations gave a more accurate solution but were more complex to program since one of the unknowns (L_T) occurs both as an explicit function and as a cosine function. This difficulty was resolved by solving for an intermediate value of emitter latitude (TILA) based on an aircraft midlatitude cosine function (WAV) which was known and then by recomputing emitter latitude (TLA) based on the cosine function of the intermediate emitter latitude. The resulting equations are given by

$$WAV = \frac{L_i + L_f}{2} \quad (26)$$

$$TILA = \frac{(\lambda_f - \lambda_i)\cos(WAV) + L_i\tan\hat{\theta}_i - L_f\tan\hat{\theta}_f}{\tan\hat{\theta}_i - \tan\hat{\theta}_f} \quad (27)$$

$$TLA = \frac{(\lambda_f - \lambda_i)\cos(TILA) + L_i\tan\hat{\theta}_i - L_f\tan\hat{\theta}_f}{\tan\hat{\theta}_i - \tan\hat{\theta}_f} \quad (28)$$

$$TLO = \lambda_f - (L_T - L_f)\tan\hat{\theta}_f/\cos(TILA) \quad (29)$$

where TLA = emitter target latitude L_T

TLO = emitter target longitude λ_T

L_i = initial aircraft position latitude

λ_i = initial aircraft position longitude

L_f = final aircraft position latitude

λ_f = final aircraft position longitude.

These equations are derived in Appendix C and are equivalent to the "round world" model developed on pages 41-43 of Ref. 6.

2. Spherical Triangulation Solution

Two approaches to a spherical earth solution were taken. The first involved the derivation of a spherical triangulation solution with two equations resulting from two spherical triangles as shown in Fig. 4. These equations are derived in Appendix D and are given by

$$\tan\hat{\theta}_i = \frac{\tan\left[(\lambda_T - \lambda_i)\cos L_T\right]}{\sin(L_T - L_i)} \tag{30}$$

$$\tan\hat{\theta}_f = \frac{\tan\left[(\lambda_T - \lambda_f)\cos L_T\right]}{\sin(L_T - L_f)} \quad . \tag{31}$$

These equations represent exact solutions for emitter location on a spherical earth of constant radius. It is known that the earth is an oblate spheroid so small errors may be introduced at large distances from the emitter with such a spherical earth solution. However, according to Kayton and Fried [9], a constant radius of curvature can be used everywhere on the earth with an error of less than 0.2 percent of distance.

If the emitter location problem is concerned only with microwave frequency emitters, then relatively short distances from the

FIGURE 4

SPHERICAL EARTH TRIANGULATION SOLUTION FOR EMITTER LOCATION

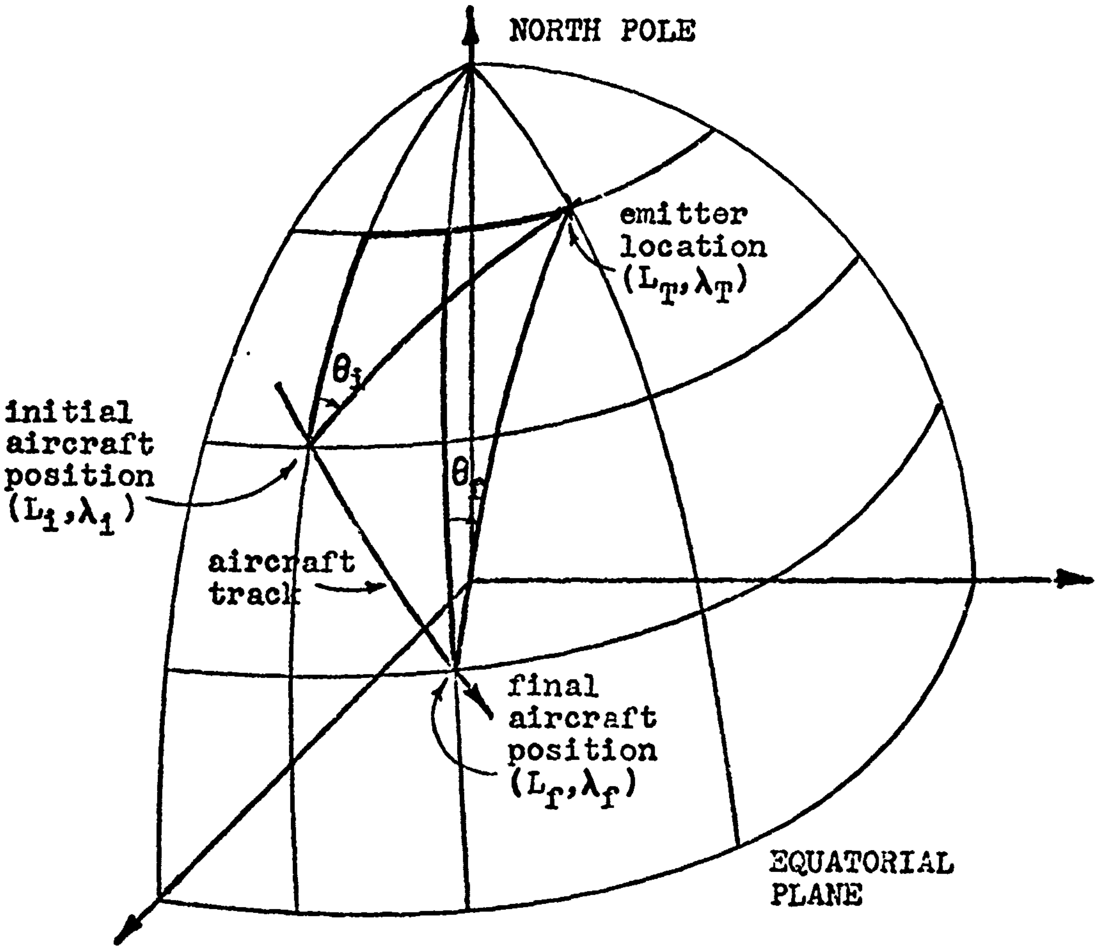


FIGURE 4a
SPHERICAL TRIANGULATION GEOMETRY

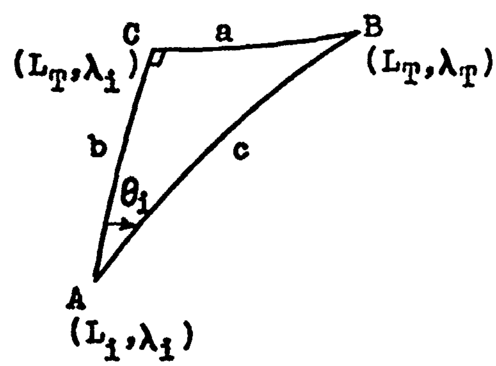


FIGURE 4b
INITIAL SPHERICAL TRIANGLE

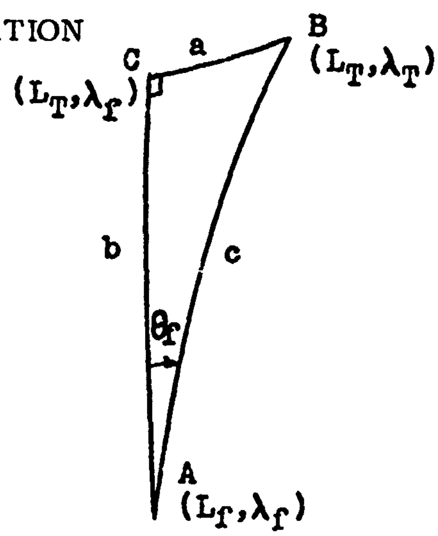


FIGURE 4c
FINAL SPHERICAL TRIANGLE

aircraft to the emitter will exist since microwave electromagnetic (EM) energy propagation approximates a straight line path (line-of-sight) on the earth's surface. Reintjes and Coate [11] show that the maximum distance for direct EM wave propagation to a radar horizon is given by

$$d = \sqrt{2h_a} + \sqrt{2h_e}\,, \tag{32}$$

where d = radar horizon in statute miles

h_a = aircraft altitude in feet

h_e = emitter altitude in feet

assuming a 4/3 effective radius of the earth for standard EM energy refraction characteristics. If h_e is unknown or assumed to be zero, then aircraft altitudes (h_a) up to 80,000 feet should not yield radar horizons greater than 400 statute miles (350 nautical miles). At this distance latitude/longitude differences between the aircraft and emitter are small (less than 6°), and it can be seen that the emitter location algorithm developed from equations (24) and (25) is nearly equivalent to the exact spherical solution from equations (30) and (31) since the tangent and sine functions of small angles are approximately equal to the radian measure of the angles themselves. Thus for ranges up to 360 nautical miles from the emitter, the planar solution will be less than 0.5 percent in error from the spherical solution. For ranges of less than 150 nautical miles the error may be neglected and the plane triangulation solution may be considered sufficiently accurate for emitter location estimation.

If greater accuracy is desired, the spherical triangulation equations (30) and (31) may be solved by an iterative procedure as suggested in Appendix E. It was found to be tedious to attempt a closed-form solution.

3. Earth-Centered Coordinate System Solution

Another approach to the spherical earth solution was considered using tangent plane projections and an earth-centered coordinate system (x_1, x_2, x_3), where the x_3 axis is the polar axis and the x_1, x_2 plane is the equatorial plane of the earth. An observation plane was defined for each emitter-bearing line, which passed through the center of the earth and inscribed a great circle arc on the earth's surface between the aircraft and emitter positions. The procedure for obtaining the emitter location required solving the equations of the observation planes, two at a time, with a constraint equation that the emitter be located on the earth's surface. Assuming a constant earth radius and letting x_3 equal a constant, the emitter position may be computed from the equations of the plane (x_1', x_2', c) where x_3 = constant as shown below.

$$a_{11}x_1' + a_{12}x_2' = -a_{13}c \quad (33)$$

$$a_{21}x_1' + a_{22}x_2' = -a_{23}c \quad (34)$$

These equations were then transformed back to the original coordinate set (x_1, x_2, x_3) by the equation

$$x_i = Ax_i' \qquad i = 1, 2, 3. \quad (35)$$

where $A = Re/(x_1^2 + x_2^2 + c^2)^{\frac{1}{2}}$ (36)

and Re = average radius of the earth.

With proper coordinate rotation, it is possible to have the projection plane ($x_3 = c$) tangent to the earth's surface and centered at the aircraft observation position.

F. EXTENDED KALMAN FILTERING

It may be desirable to directly filter and update the estimates of emitter position (L_T, λ_T) as described by Reeves [6]. If only bearing angle-of-arrival information is available as observable data, however, a non-linear transformation must be utilized to transform observable bearing angles into filtered position estimates.

This was accomplished by extended Kalman filtering techniques utilizing the concepts of small perturbation theory and a Taylor series expansion about an initial point. Equation (1) then becomes

$$\underline{Z}(k) = N\left[\underline{X}(k)\right] + \underline{V}(k) \tag{37}$$

where $\underline{Z}(k)$ represents the observable bearing angles, $\underline{X}(k)$ is the emitter position vector, and N represents the non-linear transformation given by

$$\hat{\theta} = \tan^{-1}\left(\frac{(\lambda_T - \lambda)\cos L_T}{L_T - L}\right). \tag{38}$$

If the position error of the state vector $\underline{X}(k)$ is given by

$$\tilde{\underline{X}}(k) = \underline{X}(k) - \hat{\underline{X}}(k), \tag{39}$$

then the true position $\underline{X}(k)$ may be expanded about the most recent optimal estimate $\hat{\underline{X}}(k)$ in a Taylor series expansion with higher order.

terms neglected as shown below:

$$N\left[X(k)\right] = N\left[\hat{X}(k)\right] + \underline{M}(k)\underline{\tilde{X}} + \ldots\ldots \qquad (40)$$

where $$\underline{M}(k) = \left.\frac{\partial N}{\partial \underline{X}}\right|_{\hat{X}} = \left[\frac{\partial\hat{\theta}}{\partial\lambda_T} \quad \frac{\partial\hat{\theta}}{\partial L_T}\right] \qquad (41)$$

for the emitter location algorithm given by equations (24) and (25).

The Kalman filter recursion equations (6) through (8) may then be rewritten to account for this non-linear observation matrix and are given by

$$\underline{G}(k) = \underline{P}(k/k-1)\underline{M}(k)^T\left[\underline{M}(k)\underline{P}(k/k-1)\underline{M}(k)^T + \underline{R}(k)\right]^{-1} \qquad (42)$$

$$\underline{P}(k/k) = \underline{P}(k/k-1) - \underline{G}(k)\underline{M}(k)\underline{P}(k/k-1) \qquad (43)$$

$$\underline{P}(k+1/k) = \underline{\Phi}(k+1,k)\underline{P}(k/k)\underline{\Phi}(k+1,k)^T + \underline{Q}(k)\ . \qquad (44)$$

Since the non-linear filter is processing estimates of a stationary emitter position, the $\underline{\Phi}$ matrix becomes the identity matrix and the equations are considerably simplified and reduced to scalar form (see Appendix F).

The greatest difficulty in this approach was found in initialization of the extended Kalman filter process. Errors or inaccuracies in initialization may cause the recursive computations to diverge, and it was therefore decided to commence extended Kalman filter processing only after linear filtering and smoothing of the bearing angle observations had been completed and an initial optimal estimate of emitter position obtained. This would allow the extended Kalman filter process to be used as a final correction procedure to further improve the final optimal estimates of emitter position.

IV. PRESENTATION OF RESULTS

A. MONTE CARLO SIMULATION ANALYSIS

Since this Kalman filter program was developed to process actual aircraft flight data, a Monte Carlo simulation was utilized to accomplish program validity checks and error analysis studies. Figure 5 depicts the geometry of the simulation with an aircraft flying due south at 600 knots off the coast of southern California. Two VORTAC stations were chosen as emitters with accurately known positions given as

EMITTER #1	Oceanside VORTAC Channel 100 Frequency 1197 mHz	33.24055° N -117.41694° W
EMITTER #2	San Diego VORTAC Channel 125 Frequency 1212 mHz	32.78222° N -117.22444° W.

Bearing angle-of-arrival information was assumed to arrive every six seconds alternately from each target giving a twelve-second uniform sampling interval for simplicity. Known true bearing angles were computed numerically in the absence of measurement noise and allowed position location within twenty feet of the known locations given above.

In the presence of computer-generated random measurement noise of one degree variance, a large number of simulation runs were executed to obtain a statistically valid error analysis. The results of this analysis are given in Appendix G and summarized below for the two simulation emitters. A range error was computed from each

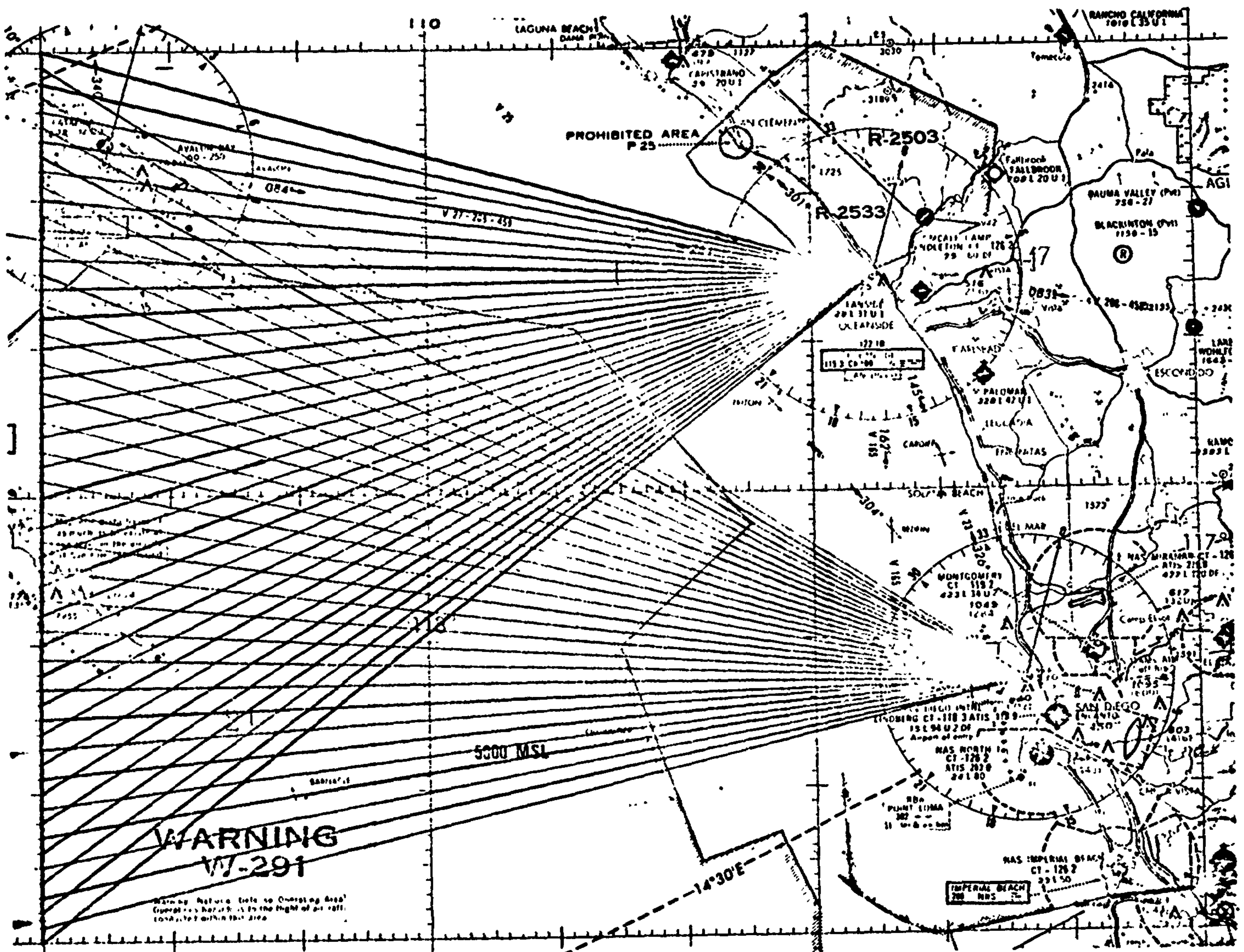


FIGURE 5

PLOT OF MONTE CARLO SIMULATION AIRCRAFT TRACK
AND EMITTER BEARING ANGLES-OF-ARRIVAL

position estimation latitude and longitude error, and all errors were statistically averaged for the total number of runs.

MONTE CARLO SIMULATION ERROR ANALYSIS

average error in nautical miles

emitter	average latitude error	average longitude error	average range error	closest point of approach	normalized estimated position range error
1	0.44852	0.23376	0.53692	54.5	0.985%
2	0.39961	0.23108	0.50758	64.7	0.785%

These results show that the basic plane triangulation solution emitter location algorithm is capable of estimating position within one percent of range at emitter distances of less than 100 nautical miles.

B. AIRCRAFT FLIGHT DATA ANALYSIS

The Kalman filter program was also utilized to process and filter aircraft flight data of the form discussed in Section III, Part A, of this paper. This data processing tested maximum utility of the computer program since certain data samples would not correlate with predicted emitter locations and had to be selectively gated out. Figures 6 through 10 show graphs of noisy unfiltered bearing angle-of-arrival information initially associated with a distinct emitter target. The dotted lines show unfiltered bearing angles and the solid lines show the smoothed initial and filtered final bearing angle. It can be seen that Figures 7, 9, and 10 have erroneous bearing angles which must be excluded from the

Kalman filtering process by the adaptive gating technique. This is shown to occur in the results of the computer output on page 64. Also comparing the final JSET data (page 65) with the initial JSET data (page 63) shows which data samples were not correlated to a distinct emitter target and were discarded.

The emitter location estimation accuracy obtained from the results of processing and filtering flight data was not as good as that obtained from the Monte Carlo simulation; however, it was estimated that the standard deviation of bearing angle measurement noise was approximately 12° and that some aircraft navigation error may have existed as well. Therefore, typical emitter location estimation within 3 percent on an average range from aircraft to emitter target of 100 nautical miles was considered reasonable.

FIGURE 6

PLOT OF NOISY MEASURED BEARING ANGLES ASSOCIATED WITH EMITTER TARGET NTAR = 1

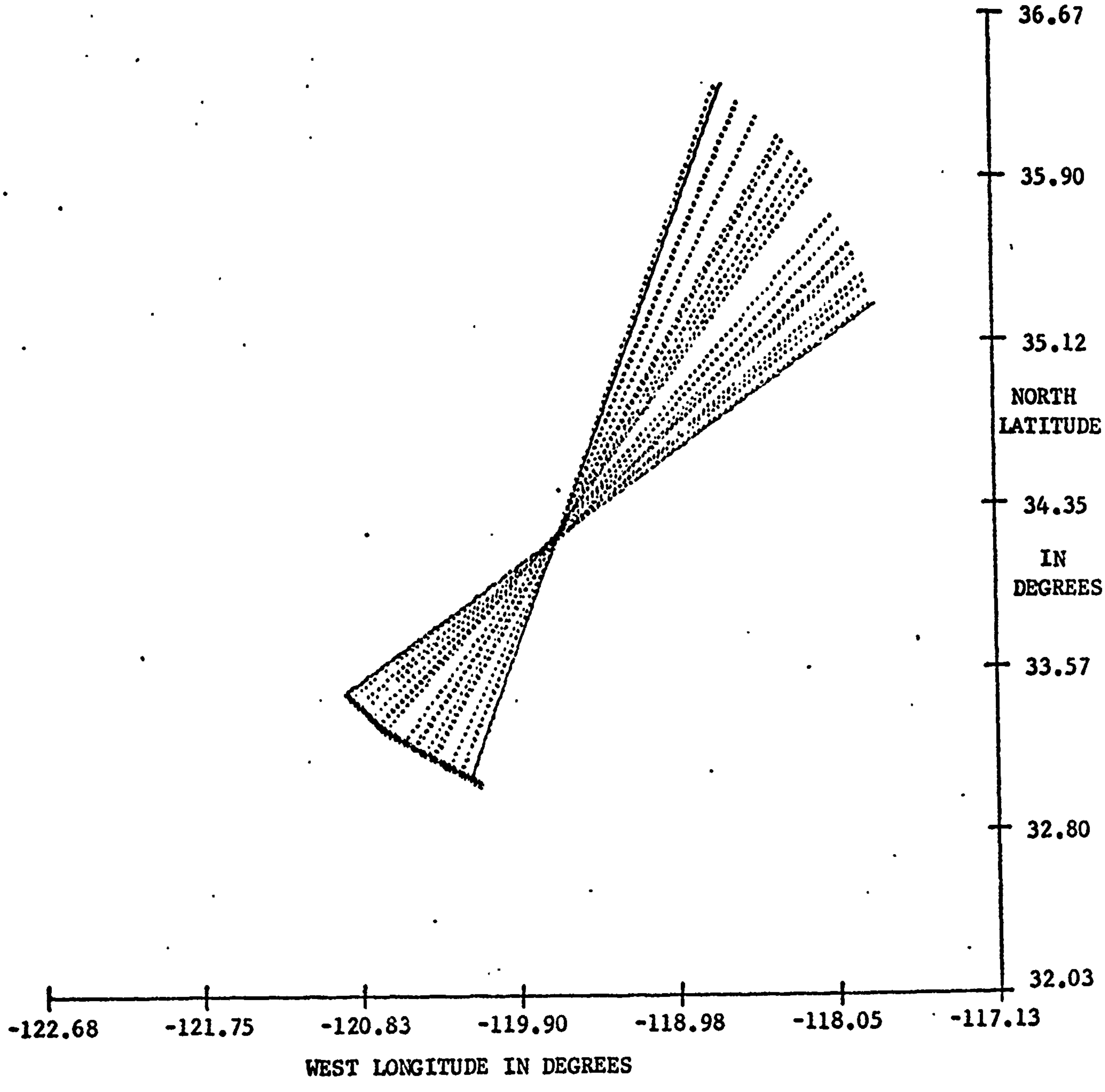


FIGURE 7

PLOT OF NOISY MEASURED BEARING ANGLES ASSOCIATED WITH EMITTER TARGET NTAR = 2

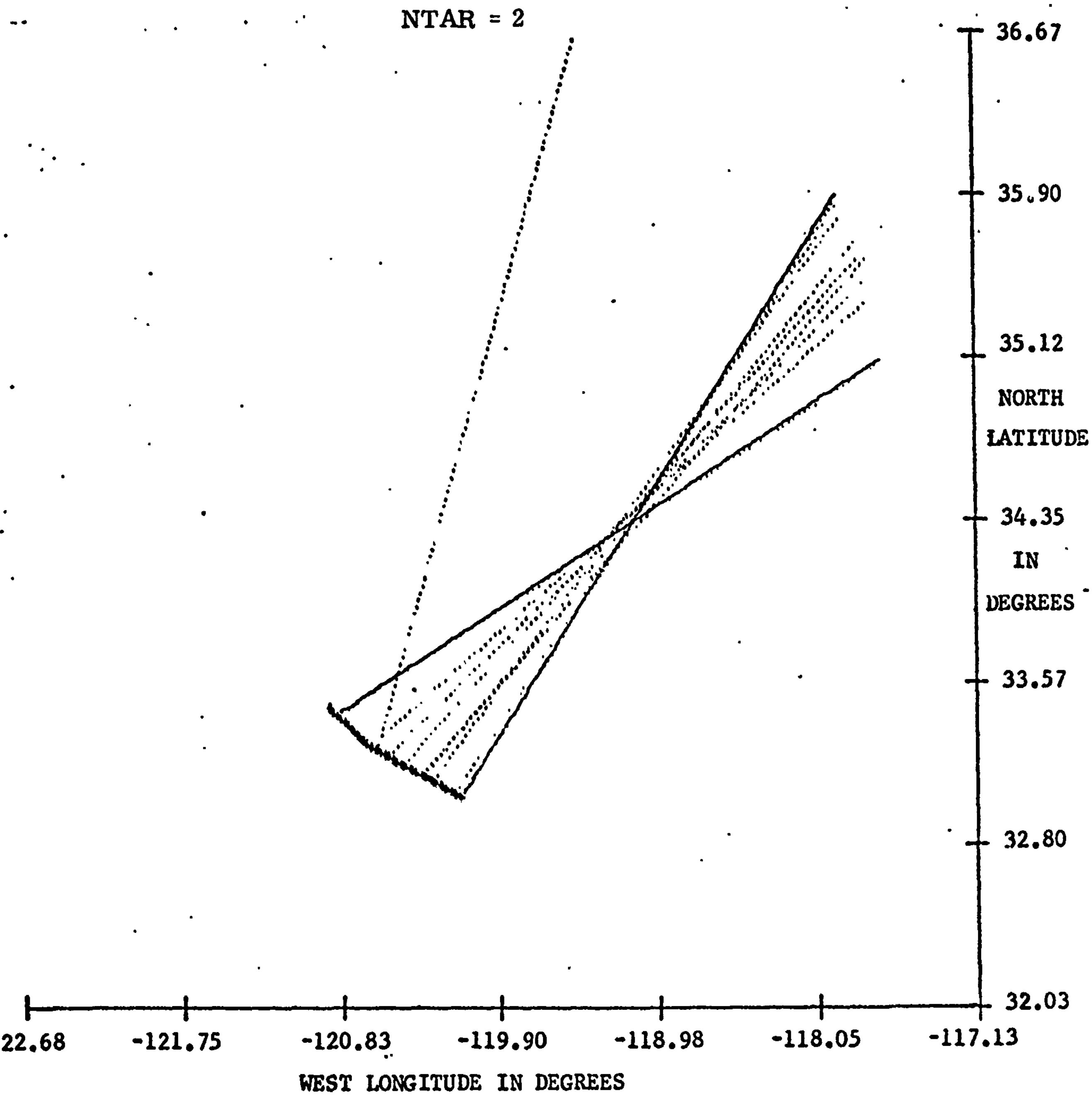


FIGURE 8

PLOT OF NOISY MEASURED BEARING ANGLES
ASSOCIATED WITH EMITTER TARGET
NTAR = 3

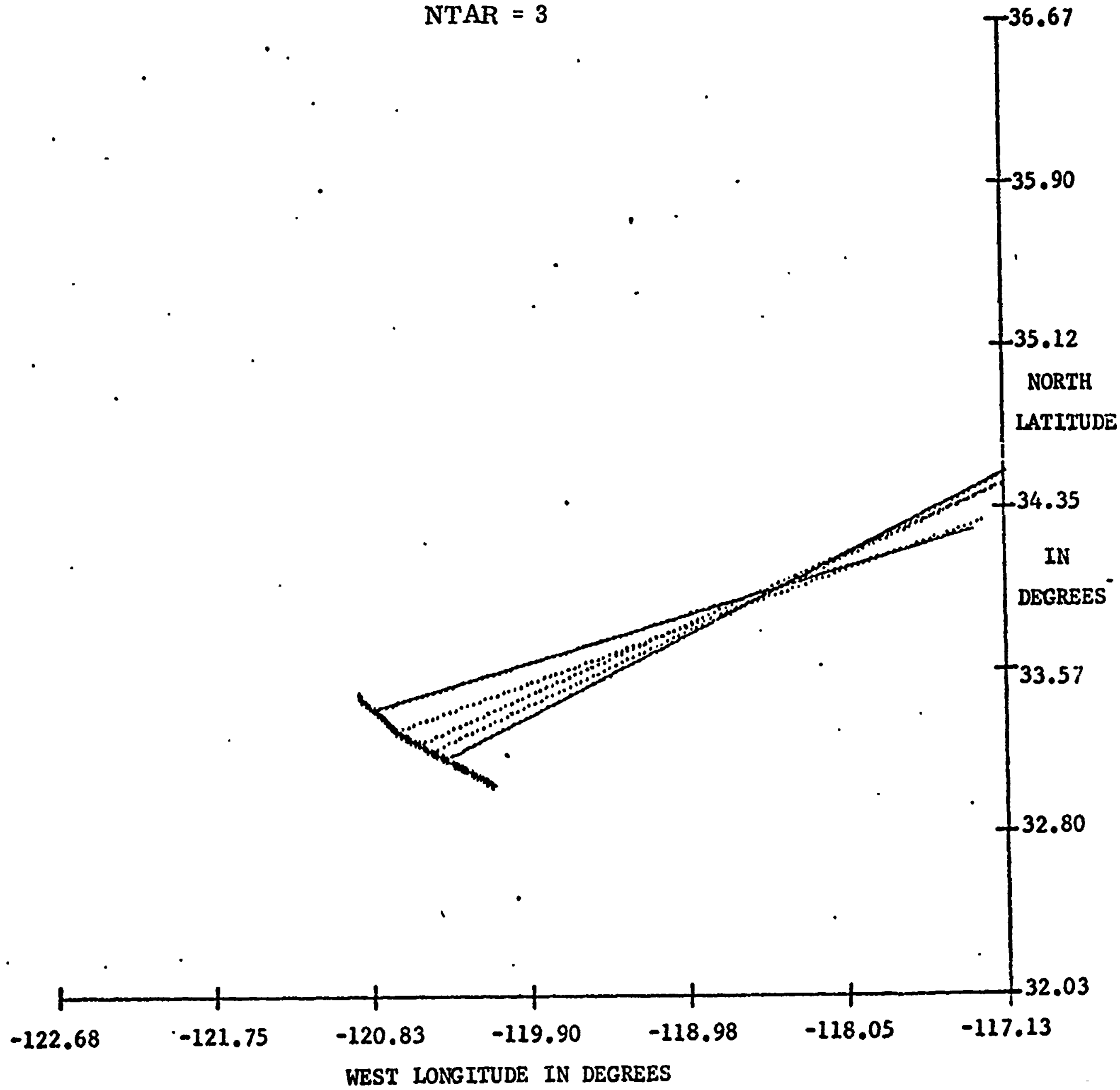


FIGURE 9

PLOT OF NOISY MEASURED BEARING ANGLES
ASSOCIATED WITH EMITTER TARGET
NTAR = 4

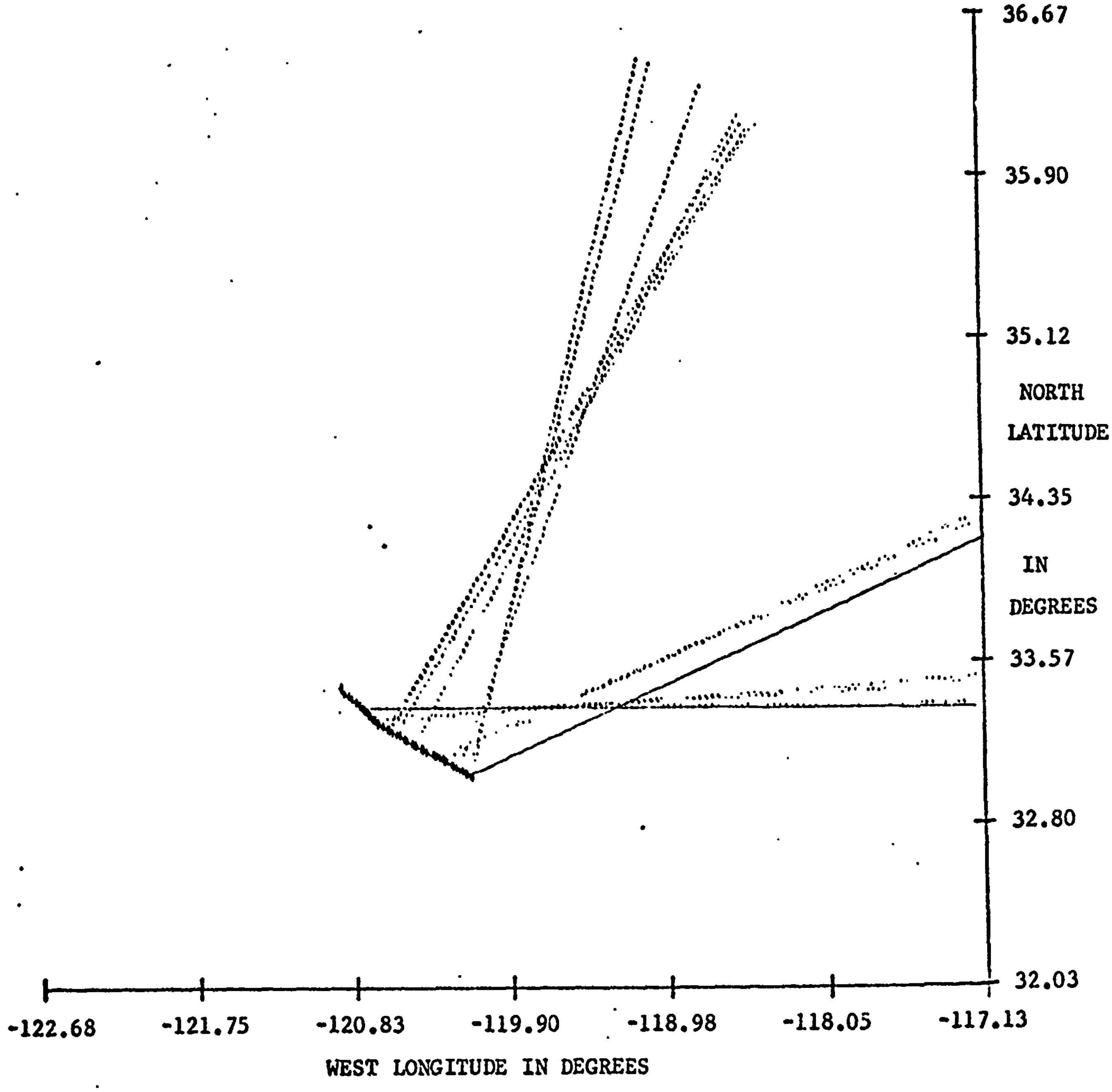
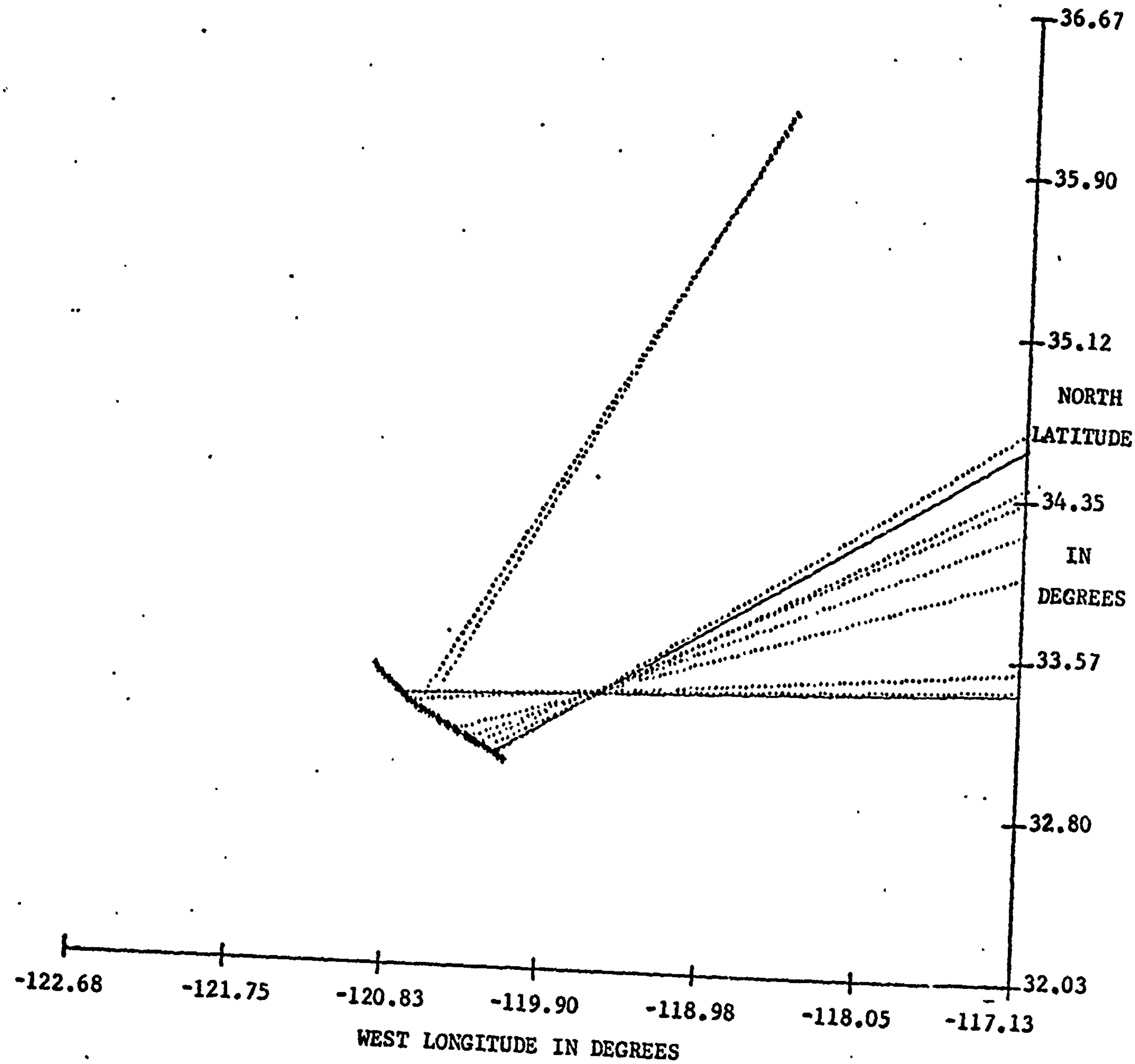


FIGURE 10

PLOT OF NOISY MEASURED BEARING ANGLES
ASSOCIATED WITH EMITTER TARGET
NTAR = 5



V. CONCLUSIONS

It was found that Kalman filtering techniques have useful application in the field of airborne direction-finding and emitter location procedures. Many airborne directional antenna receiving systems have $\pm 10°$ typical bearing accuracies which yield poor results in conventional plotting and triangulation procedures for emitter position fixing. The Kalman filtering process, in effect, statistically filters out random signal error or measurement noise, resulting in an optimal estimate of the true signal.

By computer processing techniques these emitter data may be sorted and sequentially filtered to yield optimal estimates of bearing angle-of-arrival information. Computerized solutions, as suggested within this paper, may then be utilized to compute numerical emitter locations in latitude/longitude coordinates to a high degree of accuracy. For limited-storage airborne computer applications the Kalman filtering process and plane triangulation solution should be readily adaptable to airborne implementation to yield real-time cockpit display outputs with typical estimated emitter position error within three percent of the exact value.

For more precise resolution of emitter location, the spherical earth triangulation solution may be considered; or alternately, the extended Kalman filtering techniques may be utilized to improve or update plane triangulation solution initial estimates of emitter location. These procedures are somewhat more involved, however, and would require greater computer memory and processing time.

APPENDIX A

DERIVATION OF SCALAR KALMAN FILTER RECURSION EQUATIONS

The Kalman filter recursion equations from page 11, equations (6), (7), and (8), are listed below:

$$\underline{G}(k) = \underline{P}(k/k-1)\underline{H}(k)^T\left[\underline{H}(k)\underline{P}(k/k-1)\underline{H}(k)^T + \underline{R}(k)\right]^{-1} \quad (1A)$$

$$\underline{P}(k/k) = \underline{P}(k/k-1) - \underline{G}(k)\underline{H}(k)\underline{P}(k/k-1) \quad (2A)$$

$$\underline{P}(k+1/k) = \underline{\Phi}(k+1,k)\underline{P}(k/k)\underline{\Phi}(k+1,k)^T + \underline{Q}(k) \quad (3A)$$

where in this application $\underline{\Phi}(k+1,k) = \begin{bmatrix} 1 & T(k+1) \\ 0 & 1 \end{bmatrix}$,

$$\underline{H}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

the $\underline{P}$ and $\underline{Q}$ matrices may be considered to have scalar components given by

$$\begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} Q11 & Q12 \\ Q21 & Q22 \end{bmatrix} \quad \text{respectively,}$$

and the $\underline{G}$ matrix is a 2x1 matrix of the form $\begin{bmatrix} G1 \\ G2 \end{bmatrix}$.

Writing equation (1A) in matrix notation yields

$$\begin{bmatrix} G1 \\ G2 \end{bmatrix} = \begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + R \right\}^{-1} ; \quad (4A)$$

and for the observation matrix $\underline{H}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix}$, the inverse term becomes a scalar allowing the gain terms to be computed directly, as is shown below:

$$\begin{bmatrix} G1 \\ G2 \end{bmatrix} = \begin{bmatrix} P11 \\ P21 \end{bmatrix} \begin{bmatrix} P11 + R \end{bmatrix}^{-1} \qquad (5A)$$

which results in the scalar gain equations

$$G1(k) = \frac{P11(k/k-1)}{P11(k/k-1) + R(k)} \qquad (6A)$$

$$G2(k) = \frac{P21(k/k-1)}{P11(k/k-1) + R(k)} \quad . \qquad (7A)$$

From equation (19) which defines the $\underline{P}$ matrix, it can be seen that since $\tilde{\theta}$ and $\tilde{\hat{\theta}}$ are statistically independent, where

$$\tilde{\theta} = \Theta - \hat{\theta} ,$$

then P12 = P21, and the $\underline{P}$ matrix is symmetric. From equation (15) it can be seen that the $\underline{Q}$ matrix is also symmetric. Equation (7A) then becomes

$$G2(k) = \frac{P12(k/k-1)}{P11(k/k-1) + R(k)} \quad . \qquad (8A)$$

To solve for the prediction covariance scalar terms, equation (2A) was substituted into (3A) giving

$$\begin{bmatrix} P11 & P12 \\ P12 & P22 \end{bmatrix}_{k+1/k} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} P11 & P12 \\ P12 & P22 \end{bmatrix}_{k/k-1} - \begin{bmatrix} G1 \\ G2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} P11 & P12 \\ P12 & P22 \end{bmatrix}_{k/k-1} \right) \begin{bmatrix} 1 & 0 \\ T & 1 \end{bmatrix} + \begin{bmatrix} Q11 & Q12 \\ Q12 & Q22 \end{bmatrix}$$

which upon simplifying and writing in scalar form yields equations as noted on page 84 of the Kalman filter program.

APPENDIX B

DERIVATION OF SCALAR SMOOTHING EQUATIONS

The general smoothing equations (11) and (12) are given on page 13 and are repeated below:

$$\underline{\hat{X}}(1/k) = \underline{\hat{X}}(1/k-1) + \underline{D}(1/k)\underline{C}(k)\underline{E}(k) \tag{1B}$$

where $\underline{D}(1/k) = \underline{D}(1/k-1)\underline{P}(k-1/k-1)\underline{\Phi}(k,k-1)^T\underline{P}(k/k-1)^{-1}$ (2B)

and $\underline{E}(k) = \underline{Z}(k) - \underline{H}(k)\underline{\hat{X}}(k/k-1)$. (3B)

In order to solve for a smoothed value of the state $\underline{\hat{X}}(1/k)$, it is first necessary to solve equation (2B) sequentially since the state transition matrix $\underline{\Phi}$ is a function of the variable sampling interval as given by

$$\underline{\Phi}(k,k-1)^T = \begin{bmatrix} 1 & 0 \\ T(k) & 1 \end{bmatrix} . \tag{4B}$$

It is also necessary to obtain the inverse of the error covariance matrix $\underline{P}$ in order to solve equation (2B). On the IBM 360, the matrix inverse routine MINV was used with good numerical accuracy, and the results were programmed as follows:

$$\underline{P}^{-1} = \begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix}^{-1} = \begin{bmatrix} PIN11 & PIN12 \\ PIN21 & PIN22 \end{bmatrix} ; \tag{5B}$$

and letting

$$\underline{P} = \begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix}, \ \underline{D} = \begin{bmatrix} D11 & D12 \\ D21 & D22 \end{bmatrix}, \text{ and } \underline{Q} = \begin{bmatrix} Q11 & Q12 \\ Q21 & Q22 \end{bmatrix}$$

as before. Since the $\underline{P}$ matrix is symmetric (as shown in Appendix A), $\underline{P}^{-1}$ is also symmetric.

To solve for the scalar smoothing equations, equation (8) on page 11 was solved for $\underline{P}(k-1/k-1)$ giving

$$\underline{P}(k-1/k-1) = \underline{\Phi}(k,k-1)^{-1}\left[\underline{P}(k/k-1) - \underline{Q}(k)\right]\underline{\Phi}^T(k,k-1)^{-1}. \qquad (6B)$$

This equation was then substituted into equation (2B) giving

$$\underline{D}(1/k) = \underline{D}(1/k-1)\underline{\Phi}(k,k-1)^{-1}\left[\underline{P}(k/k-1) - \underline{Q}(k)\right]\underline{P}(k/k-1)^{-1}, \qquad (7B)$$

which written in matrix form yields

$$\begin{bmatrix} D11 & D12 \\ D21 & D22 \end{bmatrix}_{1/k} = \begin{bmatrix} D11 & D12 \\ D21 & D22 \end{bmatrix}_{1/k-1} \begin{bmatrix} 1 & -T \\ 0 & 1 \end{bmatrix} \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} Q11 & Q12 \\ Q12 & Q22 \end{bmatrix} \begin{bmatrix} PIN11 & PIN12 \\ PIN12 & PIN22 \end{bmatrix} \right\}.$$

This reduces to

$$\begin{bmatrix} D11 & D12 \\ D21 & D22 \end{bmatrix}_{1/k} = \begin{bmatrix} D11 & D12 \\ D21 & D22 \end{bmatrix}_{1/k-1} \begin{bmatrix} 1 & -T \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 1.0 - QP11 & -QP12 \\ -QP21 & 1.0 - QP22 \end{Bmatrix}$$

where

$$QP11 = Q11(PIN11) + Q12(PIN12)$$

$$QP12 = Q11(PIN12) + Q12(PIN22)$$

$$QP21 = Q12(PIN11) + Q22(PIN12)$$

$$QP22 = Q12(PIN12) + Q22(PIN22)$$

and D(1/0) was initialized as the identity matrix. These are the terms used in the Kalman filter program to solve for sequential values of the $\underline{D}$ matrix as seen on page 84.

APPENDIX C

DERIVATION OF PLANE TRIANGULATION SOLUTION

From Figure 1 it can be seen that

$$\tan\hat{\theta}_i = \frac{X_T - X_i}{X_T - Y_i} \quad (1C)$$

and

$$\tan\hat{\theta}_f = \frac{X_T - X_f}{X_T - Y_f}\,. \quad (2C)$$

Substituting latitude (L)/longitude (λ) coordinates for the X/Y axes of the flat earth model results in angular measurement of distance in the plane. Since equal angles of latitude and longitude yield equal distances of movement only at the equator, a correction factor must be applied to longitude angular measure as either pole is approached from the equator.

At 60° North latitude, for example, one degree of movement measures sixty nautical miles while one degree of movement in longitude only measures thirty nautical miles. This requires that the longitude angular measure be corrected by the cosine of the local latitude coordinate so that the tangent ratio terms will have units of the same dimensional size. This results in the equations given below:

$$\tan\hat{\theta}_i = \frac{(\lambda_T - \lambda_i)\cos L_T}{L_T - L_i} \quad (3C)$$

$$\tan\hat{\theta}_f = \frac{(\lambda_T - \lambda_f)\cos L_T}{L_T - L_f}\,. \quad (4C)$$

These two equations could be solved simultaneously for the two unknowns L_T and λ_T, except that the unknown L_T also appears as a cosine function. The method chosen to resolve this difficulty allowed a known midlatitude function

$$WAV = \frac{L_i + L_f}{2} \tag{5C}$$

to be used in approximating an intermediate value of emitter target latitude TILA where

$$TILA = \frac{(\lambda_f - \lambda_i)\cos(WAV) + L_i \tan\hat{\theta}_i - L_f \tan\hat{\theta}_f}{\tan\hat{\theta}_i - \tan\hat{\theta}_f} \tag{6C}$$

results from substituting cos(WAV) for $\cos L_T$ in a simultaneous solution of equations (3C) and (4C) for L_T. It should be noted that this midlatitude approximation is only valid if the aircraft position latitude is within a few degrees of the emitter location latitude. This approximation would not be valid for HF sky wave radiation transmitted from an emitter at great distances from the receiving aircraft. For situations in which the approximation is valid, equations (28) and (29) give a corrected solution for emitter latitude and longitude based on the intermediate latitude function TILA.

APPENDIX D

DERIVATION OF SPHERICAL EARTH TRIANGULATION SOLUTION

Figure 4a describes the geometry of the spherical triangulation solution, in which the arc length sides of the spherical triangles must be measured from the angle subtended at the center of the sphere. Since latitude (L) and longitude (λ) lines meet at right angles and θ_i and θ_f are known, then two angles of the spherical triangles are known. This allows the side opposite and the side adjacent to the emitter bearing angle θ to be computed.

Using the Law of Sines and the Angle Law of Cosines for spherical triangles, the general solution equations may be derived as follows from Figures 4b and 4c:

Sine Law: $$\frac{\sin(A)}{\sin(a)} = \frac{\sin(B)}{\sin(b)} \text{ or } \sin(A) = \frac{\sin(B)\sin(a)}{\sin(b)} \quad \text{(1D)}$$

Cosine Law: $$\cos(A) = -\cos(B)\cos(C) + \sin(B)\sin(C)\cos(a). \quad \text{(2D)}$$

For angle C = 90° equation (2D) reduces to

$$\cos(A) = \sin(B)\cos(a)\,. \quad \text{(3D)}$$

Dividing equation (1D) by (3D) yields

$$\tan(A) = \frac{\sin(A)}{\cos(A)} = \frac{\tan(a)}{\sin(b)} \quad \text{(4D)}$$

where a and b are angular arcs of the spherical triangles measured at the center of the sphere. From Figures 4b and 4c, b is an arc of latitude and a is an arc of longitude which must be corrected for the local latitude at which it is measured. Therefore,

$$a = (\lambda_T - \lambda)\cos L_T \tag{5D}$$

and $$b = L_T - L\ . \tag{6D}$$

Substituting these values of a and b into equation (5D) results in

$$\tan \theta_i = \frac{\tan\left[(\lambda_T - \lambda_i)\cos L_T\right]}{\sin(L_T - L_i)} \tag{7D}$$

and $$\tan \theta_f = \frac{\tan\left[(\lambda_T - \lambda_f)\cos L_T\right]}{\sin(L_T - L_f)} \tag{8D}$$

which give the exact solution for emitter location on a spherical earth of constant radius.

APPENDIX E

COMPUTER ITERATION SOLUTION TO SPHERICAL TRIANGULATION EQUATIONS

No closed-form solution to the two spherical triangulation equations (30) and (31) could be found since the unknowns (L_T, λ_T) are enmeshed within multiple trigonometric relations. Instead, an iterative approximation computer solution was developed which allowed successive numerical updating of previously computed values. This method is only as good as the initial estimate with which it commences, so it is best described as a spherical earth correction procedure to the plane triangulation solution for emitter location. The procedure is concerned with updating values of the following terms

$$DTLAI = L_T - L_i$$

$$DTLAF = L_T - L_f$$

$$DTLOI = \lambda_T - \lambda_i$$

$$DTLOF = \lambda_T - \lambda_f$$

by sequentially recomputing them from rearranged versions of equations (30) and (31). The procedure is repeated and successive terms are compared until a satisfactory solution is reached. See the computer subroutine on the following page for a further explanation of the iterative process.

APPENDIX F

DERIVATION OF EXTENDED KALMAN FILTER SCALAR EQUATIONS

The extended Kalman filter recursion equations (42), (43), and (44) are similar in form to the basic Kalman filter recursion equations on page 11, except that the observation matrix is a non-linear transformation matrix $\underline{M}$ as defined by equation (41) where

$$M(k) = \frac{\partial \underline{T}}{\partial \underline{X}}\bigg|_{\hat{X}} = \begin{bmatrix} \frac{\partial \theta}{\partial \lambda_T} & \frac{\partial \theta}{\partial L_T} \end{bmatrix} = \begin{bmatrix} DMX & DMY \end{bmatrix} . \quad (1F)$$

Taking partial derivatives of equation (24) yields

$$DMX = \frac{\partial \theta}{\partial \lambda_T} = \frac{(L_T - L_i)\cos L_T}{(L_T - L_i)^2 + (\lambda_T - \lambda_i)^2 \cos^2 L_T} \quad (2F)$$

$$DMY = \frac{\partial \theta}{\partial L_T} = \frac{-(\lambda_T - \lambda_i)\left[(L_T - L_i)\sin L_T + \cos L_T\right]}{(L_T - L_i)^2 + (\lambda_T - \lambda_i)^2 \cos^2 L_T} \quad (3F)$$

These terms are computed numerically and substituted into equations (42) and (43) which are rewritten in matrix form as

$$\begin{bmatrix} GX \\ GY \end{bmatrix} = \begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix} \begin{bmatrix} DMX \\ DMY \end{bmatrix} \left\{ \begin{bmatrix} DMX & DMY \end{bmatrix} \begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix} \begin{bmatrix} DMX \\ DMY \end{bmatrix} + R \right\}^{-1} \quad (4F)$$

and

$$\begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix}_{k+1/k} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P11 & P12 \\ P21 & P22 \end{bmatrix}_{k/k} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} Q11 & Q12 \\ Q12 & Q22 \end{bmatrix} . \quad (5F)$$

```
C  ***************************************************************
C  *                                                             *
C  *            SPHERICAL EARTH SOLUTION SUBROUTINE              *
C  *                                                             *
C  ***************************************************************
C
C   CORRECTIONS TO PLANE TRIANGULATION SOLUTION FOR
C   A SPHERICAL EARTH
C
      DO 40 I=1,NTAR
      NSTA=NST(I)
      KI=JSET(I,1)
      KF=JSET(I,NSTA)
      DO 42 M=1,5
      CTLA=COS(TLA(I))
      DTLAI=TLA(I)-SLA(KI)
      DTLAF=TLA(I)-SLA(KF)
      DTLOI=TLO(I)-SLO(KI)
      DTLOF=TLO(I)-SLO(KF)
      DTLOI=ATAN(TAN(THT1(KF))*SIN(DTLAI))/CTLA
      DTLOF=ATAN(TAN(THT(KF))*SIN(DTLAF))/CTLA
      DTLAI=ARSIN(TAN(DTLOI*CTLA)/TAN(THT1(KF)))
      DTLAF=ARSIN(TAN(DTLOF*CTLA)/TAN(THT(KF)))
      TLAI=DTLAI+SLA(KI)
      TLAF=DTLAF+SLA(KF)
      TLOI=DTLOI+SLO(KI)
      TLOF=DTLOF+SLO(KF)
      TLADI=TLAI*PIRAD
      TLADF=TLAF*PIRAD
      TLODI=TLOI*PIRAD
      TLODF=TLOF*PIRAD
      WRITE(6,44)TLAD(I),TLADI,TLADF,TLOD(I),TLODI,TLODF
   44 FORMAT('0',6F15.5)
      TLA(I)=(TLAI+TLAF)/2.0
      TLO(I)=(TLOI+TLOF)/2.0
      TLAD(I)=TLA(I)*PIRAD
      TLOD(I)=TLO(I)*PIRAD
   42 CONTINUE
   40 CONTINUE
```

Equation (43) is then substituted into (5F); however, the fact that the $\underline{\Phi}$ matrix reduces to the identity matrix for the stationary filtered states (L_T, λ_T) considerably simplifies the recursion equations since they are no longer dependent on the sampling interval T(k). Since the gain terms are computed as a function of the non-linear terms DMX and DMY, it is essential to have the correct units on the error term E(k). In this case the units of gain are not dimensionless but degrees per radian, so the error term must be given in radians to update position estimates given in degrees of latitude and longitude.

These scalar equations may then be written out and programmed sequentially as was done in Appendix A for the linear recursion equations. See page 92 for a listing of these equations within the extended Kalman filter subroutine.

APPENDIX G

MONTE CARLO SIMULATION ERROR ANALYSIS

ESTIMATED EMITTER POSITION ERROR IN NAUTICAL MILES

	EMITTER # 1			EMITTER # 2		
RUN	DLAT1	DLON1	DRNG1	DLAT2	DLON2	DRNG2
1	0.73975	0.98967	1.23559	0.29480	0.08906	0.30796
2	-0.17944	-0.15609	0.23783	0.34882	-0.20192	0.40305
3	0.40009	-0.01287	0.40029	-0.28107	-0.41036	0.49738
4	0.93292	0.40690	1.01780	-0.87250	0.28671	0.91840
5	-0.17761	-0.13885	0.22545	-0.29205	0.22405	0.36809
6	0.33783	-0.09862	0.35193	-1.02905	0.14354	1.03901
7	-0.71136	0.51403	0.87765	-1.19751	0.13350	1.20493
8	-0.60425	-0.53403	0.80642	0.35248	0.27276	0.44569
9	0.07324	0.03655	0.08186	0.06317	0.09631	0.11518
10	-1.38245	0.37886	1.43342	1.10687	-0.43230	1.18829
11	0.61066	0.05379	0.61302	0.30762	-0.20006	0.36695
12	-0.47058	0.07356	0.47630	-0.10986	-0.01153	0.11047
13	-0.31036	0.19127	0.36457	-0.68390	-0.07289	0.68777
14	1.21033	0.20184	1.22704	0.23712	0.14354	0.27718
15	-0.23254	0.17724	0.29239	0.10986	0.32743	0.34537
16	-0.02930	-0.07058	0.07642	0.06500	-0.29322	0.30034
17	0.32593	-0.18253	0.37356	-0.47699	-0.19058	0.51365
18	-0.45135	0.29495	0.53918	0.05768	0.26217	0.26844
19	-0.22797	0.23334	0.32621	0.25726	-0.32259	0.41262
20	0.81482	0.16989	0.83234	-0.86243	-0.36052	0.93475
21	-0.33691	0.27610	0.43559	-0.47424	0.01636	0.47453
22	-0.49713	-0.01287	0.49730	0.33508	-0.06824	0.34196
23	0.40100	0.31886	0.51232	-0.59967	0.01171	0.59978
24	0.08881	0.26115	0.27584	0.00732	0.56970	0.56975
25	-0.44952	0.08000	0.45659	-0.07690	-0.05411	0.09403
26	-0.37354	-0.26644	0.45882	0.24445	-0.35309	0.42945
27	-0.09796	0.36414	0.37709	0.81848	-0.29322	0.86942
28	0.37354	-0.11173	0.38989	0.07965	0.42783	0.43518
29	0.67566	0.35909	0.76515	0.34058	0.07698	0.34917
30	-0.46326	0.26874	0.53556	-0.03387	0.34323	0.34490
31	0.41565	0.06851	0.42126	-1.18652	0.08906	1.18986
32	-0.53192	0.39633	0.66334	-0.01831	-0.36443	0.36489
33	-0.17395	-0.07540	0.18959	0.21790	-0.13071	0.25409
34	-0.25818	0.10161	0.27745	-0.07233	0.15823	0.17398
35	-0.64178	0.29173	0.70498	-0.43030	0.15749	0.45821
36	-0.75256	0.13609	0.76477	-0.16479	0.24748	0.29733
37	0.26825	-0.06483	0.27597	0.65643	-0.80453	1.03835
38	-0.55756	0.16253	0.58076	-0.28198	-0.08739	0.29521
39	-0.60883	0.66208	0.89945	0.26276	0.22405	0.34531
40	0.13092	0.35587	0.37919	-1.25793	-0.15135	1.26701
41	-0.09979	-0.02851	0.10378	1.01257	-0.46632	1.11479
42	-0.40649	-0.04828	0.40935	-0.22614	-0.06154	0.23436
43	-0.06958	0.09081	0.11440	-0.18036	-0.49700	0.52871
44	-1.16730	-0.02598	1.16759	0.35339	-0.12681	0.37546
45	0.04486	0.34024	0.34318	-0.29297	-0.01748	0.29349
46	-0.32043	0.63817	0.71410	-0.25818	-0.33654	0.42416
47	-1.08307	0.35840	1.14083	0.27740	-0.41705	0.50088
48	0.17578	0.39633	0.43356	0.48340	0.29545	0.56654
49	0.12360	0.04874	0.13286	-0.25635	-0.07084	0.26596
50	-0.61523	0.16322	0.63652	-0.07416	-0.16065	0.17694

AVERAGE ERROR IN NAUTICAL MILES

RUNS	AVERAGE LATITUDE ERROR	AVERAGE LONGITUDE ERROR	AVERAGE RANGE ERROR	AVERAGE LATITUDE ERROR	AVERAGE LONGITUDE ERROR	AVERAGE RANGE ERROR
50	0.44852	0.23376	0.53692	0.39961	0.23108	0.50758

AIRCRAFT FLIGHT DATA COMPUTER OUTPUT

LISTING OF BEARING ANGLES-OF-ARRIVAL AND AIRCRAFT NAVIGATION DATA

N	HDGD(N)	SLAD(N)	SLOD(N)	BRNGD(N)	THETAD(N)	THETA(N)
1	135.52739	33.35472	-120.90094	283.69995	59.22729	1.03371
2	135.64830	33.33582	-120.87630	282.10010	57.74829	1.00790
3	135.48329	33.31691	-120.85220	286.89990	62.38306	1.08879
4	135.41739	33.30316	-120.83391	280.80005	56.21729	0.98118
5	135.31882	33.28426	-120.80925	300.99976	76.31836	1.33201
6	135.30740	33.28426	-120.80925	279.90015	55.20752	0.96355
7	135.00026	33.26993	-120.79089	278.49976	53.50000	0.93375
8	135.06616	33.25504	-120.77144	314.80005	89.86621	1.56846
9	135.28560	33.24701	-120.75996	277.30005	52.58545	0.91779
10	135.51651	33.23727	-120.74796	275.89966	51.41602	0.89738
11	135.61504	33.22752	-120.73532	312.79980	88.41479	1.54313
12	135.56006	33.22353	-120.73019	277.30005	52.86011	0.92258
13	135.49416	33.21779	-120.72276	312.29956	87.79370	1.53229
14	135.43913	33.21378	-120.71758	275.40015	50.83911	0.88731
15	135.38472	33.20461	-120.70555	273.79980	49.18433	0.85843
16	135.30740	33.19888	-120.69812	311.60010	86.90747	1.51682
17	135.16473	33.19431	-120.69238	311.29980	86.46436	1.50909
18	134.45079	33.18570	-120.68091	299.49976	73.95044	1.29068
19	131.33047	33.18112	-120.67462	276.00000	47.33032	0.82607
20	122.29950	33.16908	-120.65343	291.79980	54.09912	0.94421
21	120.58583	33.16507	-120.64539	275.69971	36.28540	0.63330
22	120.73991	33.15591	-120.62419	283.30005	44.03979	0.76864
23	120.76172	33.15477	-120.62250	276.30005	37.06177	0.64685
24	120.76172	33.15247	-120.61676	258.39990	19.16162	0.33443
25	120.87172	33.14847	-120.60812	275.19995	36.07153	0.62957
26	120.98116	33.14159	-120.59326	274.19971	35.18066	0.61402
27	121.36617	33.13127	-120.57091	275.09985	36.46582	0.63645
28	121.44293	33.12840	-120.56517	280.89990	42.34277	0.73902
29	121.57474	33.12154	-120.55086	308.69971	70.27441	1.22652
30	121.58560	33.12154	-120.55026	289.60010	51.18555	0.89336
31	120.76172	33.10606	-120.51649	287.60010	48.36182	0.84407
32	120.89346	33.10434	-120.51305	279.69995	40.59326	0.70849
33	121.27792	33.09747	-120.49373	278.29980	39.57764	0.69076
34	119.94865	33.08716	-120.47638	309.39990	69.34839	1.21036
35	120.68495	33.08028	-120.46147	287.29980	47.98462	0.83749
36	120.70670	33.07971	-120.45976	272.19995	32.90649	0.57433
37	121.12437	33.07397	-120.44655	276.50024	37.62451	0.65667
38	121.17940	33.05965	-120.41562	315.00000	76.17920	1.32958
39	120.91527	33.05794	-120.41278	283.39966	44.31470	0.77344
40	120.72844	33.05679	-120.40988	275.00000	35.72827	0.62358
41	120.66260	33.04991	-120.39502	306.99976	67.66235	1.18093
42	120.65167	33.04935	-120.39442	274.30005	34.95166	0.61002
43	119.66331	33.03215	-120.35721	287.19946	46.86255	0.81791
44	119.27831	33.02585	-120.34229	285.69995	44.97803	0.78502
45	118.99298	33.02184	-120.33426	312.79980	71.79272	1.25302
46	118.83884	33.01840	-120.32623	312.00000	70.83862	1.23637
47	118.71851	33.01553	-120.31995	276.50024	35.21875	0.61468
48	118.74031	33.00923	-120.30502	272.19995	30.94019	0.54001
49	118.72943	33.00522	-120.29590	311.09985	69.82910	1.21875
50	118.82796	33.00293	-120.28957	282.99976	41.82764	0.73003
51	119.64102	32.99892	-120.28099	309.00000	68.64087	1.19801
52	120.40988	32.99261	-120.26721	271.89990	32.30957	0.56391
53	120.64079	32.98230	-120.24486	267.10010	27.74072	0.48417
54	120.65167	32.97887	-120.23804	267.00000	27.65161	0.48261
55	120.88258	32.97485	-120.22940	306.10034	66.98291	1.16907
56	121.12437	32.96855	-120.21568	266.80005	27.92432	0.48737
57	121.22295	32.96683	-120.21164	276.59985	37.82275	0.66013
58	120.21225	32.96111	-120.19908	264.00000	24.21216	0.42258
59	120.75079	32.95137	-120.17842	263.39990	24.15063	0.42151
60	120.77260	32.95079	-120.17728	301.39966	62.17212	1.08511
61	121.01439	32.94391	-120.16356	277.89990	38.91406	0.67918
62	121.15704	32.94048	-120.15607	298.39966	59.55664	1.03946
63	121.15704	32.93646	-120.14749	257.09985	18.25684	0.31864
64	119.75102	32.92902	-120.13258	257.00000	16.75098	0.29236
65	119.79510	32.92615	-120.12630	278.09985	37.89478	0.66139

NTAR = 1

UNFILTERED EMITTER TARGET DATA INITIALLY ASSOCIATED TO NTAR = 1
EMITTER BEARING LINES OF SIMILAR FREQUENCY AND PRF OR PRF MULTIPLE

NUM	FREQ	PRF	PW	HDGD	BRNGD	THETAD
1	2872.0	320.0	2.40	135.52739	283.69995	59.22729
2	2872.0	322.0	2.50	135.64830	282.10010	57.74829
4	2872.0	320.0	2.50	135.41739	280.80005	56.21729
6	2872.0	318.0	2.40	135.30740	279.90015	55.20752
7	2872.0	320.0	2.60	135.00026	278.49976	53.50000
9	2872.0	319.0	2.20	135.28560	277.30005	52.58545
10	2872.0	321.0	2.50	135.51651	275.89966	51.41602
14	2872.0	320.0	2.60	135.43913	275.40015	50.83911
15	2872.0	320.0	2.20	135.38472	273.79980	49.18433
19	2872.0	320.0	2.40	131.33047	276.00000	47.33032
28	2872.0	319.0	2.60	121.44293	280.89990	42.34277
32	2872.0	319.0	2.50	120.89346	279.69995	40.59326
33	2872.0	320.0	2.50	121.27792	278.29980	39.57764
37	2872.0	320.0	2.40	121.12437	276.50024	37.62451
40	2872.0	320.0	2.50	120.72844	275.00000	35.72827
42	2872.0	320.0	2.50	120.65167	274.30005	34.95166
48	2872.0	319.0	2.30	118.74031	272.19995	30.94019
53	2872.0	323.0	2.80	120.64079	267.10010	27.74072
54	2872.0	320.0	2.50	120.65167	267.00000	27.65161
59	2872.0	322.0	2.50	120.75079	263.39990	24.15063

NTAR = 2

UNFILTERED EMITTER TARGET DATA INITIALLY ASSOCIATED TO NTAR = 2
EMITTER BEARING LINES OF SIMILAR FREQUENCY AND PRF OR PRF MULTIPLE

NUM	FREQ	PRF	PW	HDGD	BRNGD	THETAD
3	2876.0	306.0	2.40	135.48329	286.89990	62.38306
20	2878.0	304.0	2.40	122.29950	291.79980	54.09912
24	2876.0	305.0	2.40	120.76172	258.39990	19.16162
30	2878.0	306.0	2.30	121.58560	289.60010	51.18555
35	2876.0	306.0	2.30	120.68495	287.29980	47.98462
43	2876.0	306.0	2.40	119.66331	287.19946	46.86255
44	2876.0	306.0	2.30	119.27831	285.69995	44.97803
50	2876.0	306.0	2.50	118.82796	282.99976	41.82764
61	2876.0	306.0	2.30	121.01439	277.89990	38.91406
65	2876.0	306.0	2.40	119.79510	278.09985	37.89478

NTAR = 3

UNFILTERED EMITTER TARGET DATA INITIALLY ASSOCIATED TO NTAR = 3
EMITTER BEARING LINES OF SIMILAR FREQUENCY AND PRF OR PRF MULTIPLE

NUM	FREQ	PRF	PW	HDGD	BRNGD	THETAD
5	2846.0	370.0	2.30	135.31882	300.99976	76.31836
18	2846.0	370.0	2.30	134.45079	299.49976	73.95044
29	2846.0	370.0	2.40	121.57474	308.69971	70.27441
34	2846.0	370.0	2.40	119.94865	309.39990	69.34839
41	2846.0	370.0	2.40	120.66260	306.99976	67.66235

NTAR = 4

UNFILTERED EMITTER TARGET DATA INITIALLY ASSOCIATED TO NTAR = 4
EMITTER BEARING LINES OF SIMILAR FREQUENCY AND PRF OR PRF MULTIPLE

NUM	FREQ	PRF	PW	HDGD	BRNGD	THETAD
8	5556.0	320.0	1.00	135.06616	314.80005	89.86621
13	5554.0	640.0	1.10	135.49416	312.29956	87.79370
16	5556.0	320.0	1.00	135.30740	311.60010	86.90747
23	5556.0	640.0	1.00	120.76172	276.30005	37.06177
25	5556.0	320.0	1.10	120.87172	275.19995	36.07153
27	5554.0	640.0	1.10	121.36617	275.09985	36.46582
36	5554.0	640.0	1.10	120.70670	272.19995	32.90649
46	5554.0	640.0	0.90	118.83884	312.00000	70.83862
49	5554.0	320.0	1.00	118.72943	311.09985	69.82910
58	5556.0	640.0	1.00	120.21225	264.00000	24.21216
63	5556.0	640.0	1.00	121.15704	257.09985	18.25684
64	5556.0	320.0	1.10	119.75102	257.00000	16.75098

NTAR = 5

UNFILTERED EMITTER TARGET DATA INITIALLY ASSOCIATED TO NTAR = 5
EMITTER BEARING LINES OF SIMILAR FREQUENCY AND PRF OR PRF MULTIPLE

NUM	FREQ	PRF	PW	HDGD	BRNGD	THETAD
11	5506.0	640.0	0.60	135.61504	312.79980	88.41479
17	5504.0	640.0	0.60	135.16473	311.29980	86.46436
21	5504.0	640.0	0.60	120.58583	275.69971	36.28540
26	5504.0	640.0	0.60	120.98116	274.19971	35.18066
38	5504.0	640.0	0.60	121.17940	315.00000	76.17920
45	5506.0	640.0	0.70	118.99298	312.79980	71.79272
51	5504.0	642.0	0.60	119.64102	309.00000	68.64087
55	5504.0	640.0	0.60	120.88258	306.10034	66.98291
60	5504.0	642.0	0.60	120.77260	301.39966	62.17212

NTAR = 6

UNFILTERED EMITTER TARGET DATA INITIALLY ASSOCIATED TO NTAR = 6
EMITTER BEARING LINES OF SIMILAR FREQUENCY AND PRF OR PRF MULTIPLE

NUM	FREQ	PRF	PW	HDGD	BRNGD	THETAD
12	2876.0	359.0	2.20	135.56006	277.30005	52.86011
22	2876.0	359.0	2.20	120.73991	283.30005	44.03979
47	2876.0	359.0	2.40	118.71851	276.50024	35.21875
52	2874.0	359.0	2.40	120.40988	271.89990	32.30957
56	2876.0	359.0	2.20	121.12437	266.80005	27.92432
62	2874.0	359.0	2.40	121.15704	298.39966	59.55664

NTAR = 7

UNFILTERED EMITTER TARGET DATA INITIALLY ASSOCIATED TO NTAR = 7
EMITTER BEARING LINES OF SIMILAR FREQUENCY AND PRF OR PRF MULTIPLE

NUM	FREQ	PRF	PW	HDGD	BRNGD	THETAD
31	9268.0	500.0	0.20	120.76172	287.60010	48.36182
39	9268.0	499.0	0.20	120.91527	283.39966	44.31470
57	9270.0	500.0	0.30	121.22295	276.59985	37.82275

INITIAL JSET DATA

```
JSET(1, 1)=  1
JSET(1, 2)=  2
JSET(1, 3)=  4
JSET(1, 4)=  6
JSET(1, 5)=  7
JSET(1, 6)=  9
JSET(1, 7)= 10
JSET(1, 8)= 14
JSET(1, 9)= 15
JSET(1,10)= 19
JSET(1,11)= 28
JSET(1,12)= 32
JSET(1,13)= 33
JSET(1,14)= 37
JSET(1,15)= 40
JSET(1,16)= 42
JSET(1,17)= 48
JSET(1,18)= 53
JSET(1,19)= 54
JSET(1,20)= 59
JSET(2, 1)=  3
JSET(2, 2)= 20
JSET(2, 3)= 24
JSET(2, 4)= 30
JSET(2, 5)= 35
JSET(2, 6)= 43
JSET(2, 7)= 44
JSET(2, 8)= 50
JSET(2, 9)= 61
JSET(2,10)= 65
JSET(3, 1)=  5
JSET(3, 2)= 18
JSET(3, 3)= 29
JSET(3, 4)= 34
JSET(3, 5)= 41
JSET(4, 1)=  8
JSET(4, 2)= 13
JSET(4, 3)= 16
JSET(4, 4)= 23
JSET(4, 5)= 25
JSET(4, 6)= 27
JSET(4, 7)= 36
JSET(4, 8)= 46
JSET(4, 9)= 49
JSET(4,10)= 58
JSET(4,11)= 63
JSET(4,12)= 64
JSET(5, 1)= 11
JSET(5, 2)= 17
JSET(5, 3)= 21
JSET(5, 4)= 26
JSET(5, 5)= 38
JSET(5, 6)= 45
JSET(5, 7)= 51
JSET(5, 8)= 55
JSET(5, 9)= 60
JSET(6, 1)= 12
JSET(6, 2)= 22
JSET(6, 3)= 47
JSET(6, 4)= 52
JSET(6, 5)= 56
JSET(6, 6)= 62
JSET(7, 1)= 31
JSET(7, 2)= 39
JSET(7, 3)= 57
```

LISTING OF KALMAN FILTER PARAMETERS

I	J	KK	K	G1(K)	G2(K)	T(K)	E(K)	GATE(K)
1	1	0	1	0.98580	0.0	0.0		
1	2	1	2	0.99996	0.05132	19.48511	-0.54543	974.29150
1	3	2	4	0.91370	0.01972	34.17871	1.06322	20.42409
1	4	4	6	0.66077	0.01124	19.57300	0.15733	10.30163
1	5	6	7	0.51469	0.00771	14.74170	-0.87030	8.61279
1	6	7	9	0.50206	0.00621	24.46582	0.12813	8.50278
1	7	9	10	0.38820	0.00452	9.74414	-0.52989	7.67091
1	8	10	14	0.39830	0.00402	24.43945	0.60146	7.73504
1	9	14	15	0.32591	0.00314	9.80640	-0.71368	7.30789
1	10	15	19	0.33848	0.00292	24.56860	-0.82887	7.37700
1	11	19	28	0.50143	0.00304	73.93848	-0.82394	8.49747
1	12	28	32	0.44026	0.00291	34.57788	0.12999	8.01970
1	13	32	33	0.33461	0.00230	9.83130	-0.29539	7.35553
1	14	33	37	0.35500	0.00256	34.44116	0.14192	7.47088
1	15	37	40	0.33730	0.00245	24.61328	-0.17593	7.37042
1	16	40	42	0.28037	0.00206	9.84668	-0.23737	7.07287
1	17	42	48	0.39363	0.00255	58.93970	-0.22783	7.70516
1	18	48	53	0.40188	0.00270	39.44409	-0.66829	7.75814
1	19	53	54	0.30055	0.00205	4.93677	-0.14583	7.17417
1	20	54	59	0.34579	0.00244	39.42358	-0.85219	7.41809
2	1	0	3	0.98580	0.0	0.0		
2	2	3	20	1.00000	0.00638	156.82422	-0.86391	7841.21484
2	3	20	24	0.61665	0.00505	24.59717	-33.63820	9.69064
2	3	20	30	0.78886	0.00532	69.00879	0.73169	13.05779
2	4	30	35	0.66823	0.00511	59.16089	-0.15179	10.41681
2	5	35	43	0.65760	0.00448	68.85083	2.24968	10.25372
2	6	43	44	0.42984	0.00306	9.78320	-0.72655	7.94606
2	7	44	50	0.42162	0.00325	34.46094	-2.12259	7.88938
2	8	50	61	0.58649	0.00358	83.76660	-0.05772	9.33059
2	9	61	65	0.44875	0.00309	24.76538	0.16925	8.08118
3	1	0	5	0.98580	0.0	0.0		
3	2	5	18	1.00000	0.00972	102.83667	1.98647	5141.83984
3	3	18	29	0.82455	0.00552	88.72388	-1.63304	14.32434
3	4	29	34	0.62591	0.00437	49.30444	0.36708	9.80986
3	5	34	41	0.57737	0.00393	54.13794	0.09885	9.22935
4	1	0	8	0.98580	0.0	0.0		
4	2	8	13	0.99999	0.02562	39.03418	-0.70366	1951.72705
4	3	13	16	0.71699	0.01466	19.63843	0.15646	11.27851
4	4	16	23	0.79911	0.00917	54.08618	-47.05379	13.38674
4	4	16	25	0.83155	0.00864	63.97461	-47.54167	14.61884
4	4	16	27	0.88655	0.00742	89.63501	-45.89462	17.81369
4	4	16	36	0.95376	0.00498	162.59375	-45.69676	27.90096
4	4	16	46	0.97787	0.00351	251.06714	-3.27011	40.32851
4	5	46	49	0.56372	0.00543	19.68750	0.14413	9.08385
4	6	49	58	0.74102	0.00787	64.11743	-41.61142	11.79007
4	6	49	63	0.84422	0.00683	98.58325	-45.44742	15.20187
4	6	49	64	0.86349	0.00651	108.48877	-46.34418	16.23961
5	1	0	11	0.98580	0.0	0.0		
5	2	11	17	0.99999	0.02916	34.29346	-0.70633	1714.69336
5	3	17	21	0.83310	0.01454	34.41162	-48.22182	14.68663
5	3	17	26	0.92856	0.01037	68.95825	-47.36174	22.44804
5	3	17	38	0.98610	0.00478	187.19775	0.36156	50.88951
5	4	38	45	0.69876	0.00599	54.06836	-1.39986	10.93197
5	5	45	51	0.57013	0.00525	34.47925	-1.38312	9.15131
5	6	51	55	0.51924	0.00459	34.46753	0.18741	8.65340
5	7	55	60	0.48515	0.00395	34.52881	-2.30612	8.36201
6	1	0	12	0.98580	0.0	0.0		
6	2	12	22	1.00000	0.01272	78.58911	-4.96594	3929.46289
6	3	22	47	0.95323	0.00368	201.91919	13.84093	27.74507
6	4	47	52	0.61332	0.00567	34.48828	-0.14958	9.64890
6	5	52	56	0.56416	0.00613	34.49512	-2.30110	9.08843
6	6	56	62	0.56612	0.00549	39.41577	33.63329	9.10896
7	1	0	31	0.98580	0.0	0.0		
7	2	31	39	1.00000	0.01452	68.88379	-0.75569	3444.19727
7	3	39	57	0.91990	0.00521	132.89380	1.31592	21.20036

FINAL JSET DATA

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JSET(1, 1)=  1
JSET(1, 2)=  2
JSET(1, 3)=  4
JSET(1, 4)=  6
JSET(1, 5)=  7
JSET(1, 6)=  9
JSET(1, 7)= 10
JSET(1, 8)= 14
JSET(1, 9)= 15
JSET(1,10)= 19
JSET(1,11)= 28
JSET(1,12)= 32
JSET(1,13)= 33
JSET(1,14)= 37
JSET(1,15)= 40
JSET(1,16)= 42
JSET(1,17)= 48
JSET(1,18)= 53
JSET(1,19)= 54
JSET(1,20)= 59
JSET(2, 1)=  3
JSET(2, 2)= 20
JSET(2, 3)= 30
JSET(2, 4)= 35
JSET(2, 5)= 43
JSET(2, 6)= 44
JSET(2, 7)= 50
JSET(2, 8)= 61
JSET(2, 9)= 65
JSET(3, 1)=  5
JSET(3, 2)= 18
JSET(3, 3)= 29
JSET(3, 4)= 34
JSET(3, 5)= 41
JSET(4, 1)=  8
JSET(4, 2)= 13
JSET(4, 3)= 16
JSET(4, 4)= 46
JSET(4, 5)= 49
JSET(5, 1)= 11
JSET(5, 2)= 17
JSET(5, 3)= 38
JSET(5, 4)= 45
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JSET(6, 1)= 12
JSET(6, 2)= 22
JSET(6, 3)= 47
JSET(6, 4)= 52
JSET(6, 5)= 56
JSET(7, 1)= 31
JSET(7, 2)= 39
JSET(7, 3)= 57
```

NTAR = 1

FILTERED AND SMOOTHED EMITTER TARGET DATA CORRELATED TO NTAR = 1

NUM	FREQ	PRF	PW	THETAD	THTD	THTD1
1	2872.0	320.0	2.40	59.22729	59.22729	59.22729
2	2872.0	322.0	2.50	57.74829	57.74831	59.22726
4	2872.0	320.0	2.50	56.21729	56.12552	59.07391
6	2872.0	318.0	2.40	55.20752	55.15414	59.04889
7	2872.0	320.0	2.60	53.50000	53.92235	59.18648
9	2872.0	319.0	2.20	52.58545	52.52164	59.16264
10	2872.0	321.0	2.50	51.41602	51.74019	59.24341
14	2872.0	320.0	2.60	50.83911	50.47720	59.14085
15	2872.0	320.0	2.20	49.18433	49.66541	59.24278
19	2872.0	320.0	2.40	47.33032	47.87863	59.36894
28	2872.0	319.0	2.60	42.34277	42.75356	59.51895
32	2872.0	319.0	2.50	40.59326	40.52049	59.50125
33	2872.0	320.0	2.50	39.57764	39.77419	59.53006
37	2872.0	320.0	2.40	37.62451	37.53296	59.51884
40	2872.0	320.0	2.50	35.72827	35.84486	59.52922
42	2872.0	320.0	2.50	34.95166	35.12247	59.53961
48	2872.0	319.0	2.30	30.94019	31.07832	59.54620
53	2872.0	323.0	2.80	27.74072	28.14043	59.55690
54	2872.0	320.0	2.50	27.65161	27.75360	59.55846
59	2872.0	322.0	2.50	24.15063	24.70815	59.56096

SMOOTHED INITIAL BEARING ANGLE = 59.56096
FILTERED FINAL BEARING ANGLE = 24.70815

PLANE TRIANGULATION SOLUTION OF EMITTER LOCATION
EMITTER LATITUDE = 33.98666N
EMITTER LONGITUDE =-119.60387W

NTAR = 2

FILTERED AND SMOOTHED EMITTER TARGET DATA CORRELATED TO NTAR = 2

NUM	FREQ	PRF	PW	THETAD	THTD	THTD1
3	2876.0	306.0	2.40	62.38306	62.38306	62.38306
20	2878.0	304.0	2.40	54.09912	54.09912	62.38306
30	2878.0	306.0	2.30	51.18555	51.03105	62.29199
35	2876.0	306.0	2.30	47.98462	48.03497	62.32124
43	2876.0	306.0	2.40	46.86255	46.09224	61.92072
44	2876.0	306.0	2.30	44.97803	45.39227	62.00427
50	2876.0	306.0	2.50	41.82764	43.05530	62.22186
61	2876.0	306.0	2.30	38.91406	38.93793	62.22676
65	2876.0	306.0	2.40	37.89478	37.80147	62.21768

SMOOTHED INITIAL BEARING ANGLE = 62.21768
FILTERED FINAL BEARING ANGLE = 37.80147

PLANE TRIANGULATION SOLUTION OF EMITTER LOCATION
EMITTER LATITUDE = 34.12234N
EMITTER LONGITUDE =-119.00533W

NTAR = 3

FILTERED AND SMOOTHED EMITTER TARGET DATA CORRELATED TO NTAR = 3

NUM	FREQ	PRF	PW	THETAD	THTD	THTD1
5	2846.0	370.0	2.30	76.31836	76.31836	76.31836
18	2846.0	370.0	2.30	73.95044	73.95042	76.31836
29	2846.0	370.0	2.40	70.27441	70.56093	76.49266
34	2846.0	370.0	2.40	69.34839	69.21106	76.45313
41	2846.0	370.0	2.40	67.66235	67.62057	76.44316

SMOOTHED INITIAL BEARING ANGLE = 76.44316
FILTERED FINAL BEARING ANGLE = 67.62057

PLANE TRIANGULATION SOLUTION OF EMITTER LOCATION
EMITTER LATITUDE = 33.81570N
EMITTER LONGITUDE =-118.15642W

NTAR = 4

FILTERED AND SMOOTHED EMITTER TARGET DATA CORRELATED TO NTAR = 4

NUM	FREQ	PRF	PW	THETAD	THTD	THTD1
8	5556.0	320.0	1.00	89.86621	89.86621	89.86621
13	5554.0	640.0	1.10	87.79370	87.79370	89.86620
16	5556.0	320.0	1.00	86.90747	86.86319	89.84418
46	5554.0	640.0	0.90	70.83862	70.91100	90.19023
49	5554.0	320.0	1.00	69.82910	69.76620	90.18146

SMOOTHED INITIAL BEARING ANGLE = 90.18146
FILTERED FINAL BEARING ANGLE = 69.76620

PLANE TRIANGULATION SOLUTION OF EMITTER LOCATION
EMITTER LATITUDE = 33.25166N
EMITTER LONGITUDE =-119.49638W

NTAR = 5

FILTERED AND SMOOTHED EMITTER TARGET DATA CORRELATED TO NTAR = 5

NUM	FREQ	PRF	PW	THETAD	THTD	THTD1
11	5506.0	640.0	0.60	88.41479	88.41479	88.41479
17	5504.0	640.0	0.60	86.46436	86.46436	88.41478
38	5504.0	640.0	0.60	76.17920	76.17416	88.38852
45	5506.0	640.0	0.70	71.79272	72.21440	88.49161
51	5504.0	642.0	0.60	68.64087	69.23541	88.57054
55	5504.0	640.0	0.60	66.98291	66.89281	88.56223
60	5504.0	642.0	0.60	62.17212	63.35942	88.63921

SMOOTHED INITIAL BEARING ANGLE = 88.63921
FILTERED FINAL BEARING ANGLE = 63.35942

PLANE TRIANGULATION SOLUTION OF EMITTER LOCATION
EMITTER LATITUDE = 33.25288N
EMITTER LONGITUDE =-119.45715W

NTAR = 6

FILTERED AND SMOOTHED EMITTER TARGET DATA CORRELATED TO NTAR = 6

NUM	FREQ	PRF	PW	THETAD	THTD	THTD1
12	2876.0	359.0	2.20	52.86011	52.86011	52.86011
22	2876.0	359.0	2.20	44.03979	44.03979	52.86009
47	2876.0	359.0	2.40	35.21875	34.57146	52.04729
52	2874.0	359.0	2.40	32.30957	32.36740	52.05992
56	2876.0	359.0	2.20	27.92432	28.92722	52.26216

SMOOTHED INITIAL BEARING ANGLE = 52.26216
FILTERED FINAL BEARING ANGLE = 28.92722

PLANE TRIANGULATION SOLUTION OF EMITTER LOCATION
EMITTER LATITUDE = 33.99094N
EMITTER LONGITUDE =-119.53415W

NTAR = 7

FILTERED AND SMOOTHED EMITTER TARGET DATA CORRELATED TO NTAR = 7

NUM	FREQ	PRF	PW	THETAD	THTD	THTD1
31	9268.0	500.0	0.20	48.36182	48.36182	48.36182
39	9268.0	499.0	0.20	44.31470	44.31470	48.36180
57	9270.0	500.0	0.30	37.82275	37.71735	48.22450

SMOOTHED INITIAL BEARING ANGLE = 48.22450

FILTERED FINAL BEARING ANGLE = 37.71735

PLANE TRIANGULATION SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 34.14626N

EMITTER LONGITUDE =-119.10936W

MONTE CARLO SIMULATION COMPUTER OUTPUT

LISTING OF BEARING ANGLES-OF-ARRIVAL AND AIRCRAFT NAVIGATION DATA

N	HDGD(N)	SLAD(N)	SLOD(N)	BRNGD(N)	THETAD(N)	THETA(N)
1	180.00000	33.50000	-118.50000	105.98349	105.22139	1.83646
2	180.00000	33.48332	-118.50000	123.17577	122.24252	2.13353
3	180.00000	33.46666	-118.50000	104.01610	104.94077	1.83156
4	180.00000	33.45000	-118.50000	121.91071	122.29161	2.13439
5	180.00000	33.43332	-118.50000	102.01436	102.36406	1.78659
6	180.00000	33.41666	-118.50000	120.60915	120.92984	2.11062
7	180.00000	33.39999	-118.50000	99.98351	98.88550	1.72588
8	180.00000	33.38332	-118.50000	119.27164	119.94717	2.09347
9	180.00000	33.36667	-118.50000	97.92696	99.29208	1.73297
10	180.00000	33.34999	-118.50000	117.69893	119.21582	2.08071
11	180.00000	33.33333	-118.50000	95.84862	95.14084	1.66052
12	180.00000	33.31667	-118.50000	116.49051	117.06377	2.04315
13	180.00000	33.29999	-118.50000	93.75478	94.13567	1.64298
14	180.00000	33.28333	-118.50000	115.04587	116.65103	2.03594
15	180.00000	33.26666	-118.50000	91.65182	92.50012	1.61443
16	180.00000	33.25000	-118.50000	113.56721	112.34737	1.96083
17	180.00000	33.23332	-118.50000	89.54355	89.34970	1.55945
18	180.00000	33.21666	-118.50000	112.05394	112.10175	1.95654
19	180.00000	33.20000	-118.50000	67.43718	88.07285	1.53716
20	180.00000	33.18332	-118.50000	110.50731	109.82207	1.91676
21	180.00000	33.16666	-118.50000	85.33707	85.69966	1.49574
22	180.00000	33.14999	-118.50000	108.92969	108.74931	1.89803
23	180.00000	33.13332	-118.50000	83.24927	83.28149	1.45354
24	180.00000	33.11667	-118.50000	107.32155	105.82967	1.84708
25	180.00000	33.09999	-118.50000	81.18033	81.26472	1.41834
26	180.00000	33.08333	-118.50000	105.68427	106.09219	1.85166
27	180.00000	33.06667	-118.50000	79.13434	79.86276	1.39387
28	180.00000	33.04999	-118.50000	104.02003	104.02385	1.81556
29	180.00000	33.03333	-118.50000	77.11496	77.43318	1.35146
30	180.00000	33.01666	-118.50000	102.33228	100.94479	1.76182
31	180.00000	33.00000	-118.50000	75.12862	73.85582	1.28903
32	180.00000	32.98332	-118.50000	100.62157	101.37944	1.76940
33	180.00000	32.96666	-118.50000	73.17722	72.90050	1.27235
34	180.00000	32.95000	-118.50000	98.89243	98.43813	1.71807
35	180.00000	32.93332	-118.50000	71.26514	72.60910	1.26727
36	180.00000	32.91666	-118.50000	97.14586	96.18474	1.67874
37	180.00000	32.89999	-118.50000	69.39635	67.66759	1.18102
38	180.00000	32.88332	-118.50000	95.38602	96.01830	1.67583
39	180.00000	32.86667	-118.50000	67.57230	65.77895	1.14806
40	180.00000	32.84999	-118.50000	93.61652	93.04437	1.62393
41	180.00000	32.83333	-118.50000	65.79401	66.04028	1.15262
42	180.00000	32.81667	-118.50000	91.84023	90.44980	1.57865
43	180.00000	32.79999	-118.50000	64.06415	64.31984	1.12259
44	180.00000	32.78333	-118.50000	90.05968	90.87807	1.58612
45	180.00000	32.76666	-118.50000	62.38416	62.62923	1.09309
46	180.00000	32.75000	-118.50000	88.27945	87.80681	1.53252
47	180.00000	32.73332	-118.50000	60.75365	61.49768	1.07334
48	180.00000	32.71666	-118.50000	86.50214	85.91864	1.49956
49	180.00000	32.70000	-118.50000	59.17412	59.35091	1.03587
50	180.00000	32.68332	-118.50000	84.73131	84.47925	1.47444
51	180.00000	32.66666	-118.50000	57.64409	58.88116	1.02767
52	180.00000	32.64999	-118.50000	82.97147	82.71225	1.44360
53	180.00000	32.63332	-118.50000	56.16422	55.42735	0.96739
54	180.00000	32.61667	-118.50000	61.22481	81.17096	1.41670
55	180.00000	32.59999	-118.50000	54.73453	53.34729	0.93109
56	180.00000	32.58333	-118.50000	79.49355	81.11581	1.41574
57	180.00000	32.56667	-118.50000	53.35355	53.39183	0.93186
58	180.00000	32.54999	-118.50000	77.78148	77.89078	1.35945
59	180.00000	32.53333	-118.50000	52.01973	52.08127	0.90899
60	180.00000	32.51666	-118.50000	76.09212	75.29662	1.31417
61	180.00000	32.50000	-118.50000	50.73331	51.08221	0.89155

NTAR = 1

UNFILTERED EMITTER TARGET DATA INITIALLY ASSOCIATED TO NTAR = 1

EMITTER BEARING LINES OF SIMILAR FREQUENCY AND PRF OR PRF MULTIPLE

NUM	FREQ	PRF	PW	HDGD	BRNGD	THETAD
1	1197.0	150.0	3.50	180.00000	105.98349	105.22139
3	1197.0	150.0	3.50	180.00000	104.01610	104.94077
5	1197.0	150.0	3.50	180.00000	102.01436	102.36406
7	1197.0	150.0	3.50	180.00000	99.98351	98.88550
9	1197.0	150.0	3.50	180.00000	97.92696	99.29208
11	1197.0	150.0	3.50	180.00000	95.84862	95.14084
13	1197.0	150.0	3.50	180.00000	93.75478	94.13567
15	1197.0	150.0	3.50	180.00000	91.65182	92.50012
17	1197.0	150.0	3.50	180.00000	89.54355	89.34970
19	1197.0	150.0	3.50	180.00000	87.43718	88.07285
21	1197.0	150.0	3.50	180.00000	85.33707	85.69966
23	1197.0	150.0	3.50	180.00000	83.24927	83.28149
25	1197.0	150.0	3.50	180.00000	81.18033	81.26472
27	1197.0	150.0	3.50	180.00000	79.13434	79.86276
29	1197.0	150.0	3.50	180.00000	77.11496	77.43318
31	1197.0	150.0	3.50	180.00000	75.12862	73.85582
33	1197.0	150.0	3.50	180.00000	73.17722	72.90050
35	1197.0	150.0	3.50	180.00000	71.26514	72.60910
37	1197.0	150.0	3.50	180.00000	69.39635	67.66759
39	1197.0	150.0	3.50	180.00000	67.57230	65.77895
41	1197.0	150.0	3.50	180.00000	65.79401	66.04028
43	1197.0	150.0	3.50	180.00000	64.06415	64.31984
45	1197.0	150.0	3.50	180.00000	62.38416	62.62923
47	1197.0	150.0	3.50	180.00000	60.75365	61.49768
49	1197.0	150.0	3.50	180.00000	59.17412	59.35091
51	1197.0	150.0	3.50	180.00000	57.64409	58.88116
53	1197.0	150.0	3.50	180.00000	56.16422	55.42735
55	1197.0	150.0	3.50	180.00000	54.73453	53.34729
57	1197.0	150.0	3.50	180.00000	53.35355	53.39183
59	1197.0	150.0	3.50	180.00000	52.01973	52.08127
61	1197.0	150.0	3.50	180.00000	50.73331	51.08221

NTAR = 2

UNFILTERED EMITTER TARGET DATA INITIALLY ASSOCIATED TO NTAR = 2
EMITTER BEARING LINES OF SIMILAR FREQUENCY AND PRF OR PRF MULTIPLE

NUM	FREQ	PRF	PW	HDGD	BRNGD	THETAD
2	1212.0	250.0	3.00	180.00000	123.17577	122.24252
4	1212.0	250.0	3.00	180.00000	121.91071	122.29161
6	1212.0	250.0	3.00	180.00000	120.60915	120.92984
8	1212.0	250.0	3.00	180.00000	119.27164	119.94717
10	1212.0	250.0	3.00	180.00000	117.89893	119.21582
12	1212.0	250.0	3.00	180.00000	116.49051	117.06377
14	1212.0	250.0	3.00	180.00000	115.04587	116.65103
16	1212.0	250.0	3.00	180.00000	113.56721	112.34737
18	1212.0	250.0	3.00	180.00000	112.05394	112.10175
20	1212.0	250.0	3.00	180.00000	110.50731	109.82207
22	1212.0	250.0	3.00	180.00000	108.92969	108.74931
24	1212.0	250.0	3.00	180.00000	107.32155	105.82967
26	1212.0	250.0	3.00	180.00000	105.68427	106.09219
28	1212.0	250.0	3.00	180.00000	104.02003	104.02385
30	1212.0	250.0	3.00	180.00000	102.33228	100.94479
32	1212.0	250.0	3.00	180.00000	100.62157	101.37944
34	1212.0	250.0	3.00	180.00000	98.89243	98.43813
36	1212.0	250.0	3.00	180.00000	97.14586	96.18474
38	1212.0	250.0	3.00	180.00000	95.38602	96.01830
40	1212.0	250.0	3.00	180.00000	93.61652	93.04437
42	1212.0	250.0	3.00	180.00000	91.84023	90.44980
44	1212.0	250.0	3.00	180.00000	90.05968	90.87807
46	1212.0	250.0	3.00	180.00000	88.27945	87.80681
48	1212.0	250.0	3.00	180.00000	86.50214	85.91864
50	1212.0	250.0	3.00	180.00000	84.73131	84.47925
52	1212.0	250.0	3.00	180.00000	82.97147	82.71225
54	1212.0	250.0	3.00	180.00000	81.22481	81.17096
56	1212.0	250.0	3.00	180.00000	79.49355	81.11581
58	1212.0	250.0	3.00	180.00000	77.78148	77.89078
60	1212.0	250.0	3.00	180.00000	76.09212	75.29662

INITIAL JSET DATA

```
JSET(1, 1)=  1
JSET(1, 2)=  3
JSET(1, 3)=  5
JSET(1, 4)=  7
JSET(1, 5)=  9
JSET(1, 6)= 11
JSET(1, 7)= 13
JSET(1, 8)= 15
JSET(1, 9)= 17
JSET(1,10)= 19
JSET(1,11)= 21
JSET(1,12)= 23
JSET(1,13)= 25
JSET(1,14)= 27
JSET(1,15)= 29
JSET(1,16)= 31
JSET(1,17)= 33
JSET(1,18)= 35
JSET(1,19)= 37
JSET(1,20)= 39
JSET(1,21)= 41
JSET(1,22)= 43
JSET(1,23)= 45
JSET(1,24)= 47
JSET(1,25)= 49
JSET(1,26)= 51
JSET(1,27)= 53
JSET(1,28)= 55
JSET(1,29)= 57
JSET(1,30)= 59
JSET(1,31)= 61
JSET(2, 1)=  2
JSET(2, 2)=  4
JSET(2, 3)=  6
JSET(2, 4)=  8
JSET(2, 5)= 10
JSET(2, 6)= 12
JSET(2, 7)= 14
JSET(2, 8)= 16
JSET(2, 9)= 18
JSET(2,10)= 20
JSET(2,11)= 22
JSET(2,12)= 24
JSET(2,13)= 26
JSET(2,14)= 28
JSET(2,15)= 30
JSET(2,16)= 32
JSET(2,17)= 34
JSET(2,18)= 36
JSET(2,19)= 38
JSET(2,20)= 40
JSET(2,21)= 42
JSET(2,22)= 44
JSET(2,23)= 46
JSET(2,24)= 48
JSET(2,25)= 50
JSET(2,26)= 52
JSET(2,27)= 54
JSET(2,28)= 56
JSET(2,29)= 58
JSET(2,30)= 60
```

LISTING OF KALMAN FILTER PARAMETERS

I	J	KK	K	G1(K)	G2(K)	T(K)	E(K)	GATE(K)
1	1	0	1	0.99010	0.0	0.0		
1	2	1	3	0.99993	0.08332	12.00000	-0.25723	360.02466
1	3	3	5	0.83379	0.04173	12.00000	-2.29614	7.35863
1	4	5	7	0.70240	0.02544	12.00000	-2.42987	5.49924
1	5	7	9	0.60715	0.01761	12.00000	1.85547	4.78637
1	6	9	11	0.53905	0.01348	12.00000	-1.64236	4.41870
1	7	11	13	0.49142	0.01125	12.00000	0.28345	4.20671
1	8	13	15	0.45945	0.01007	12.00000	0.51604	4.08039
1	9	15	17	0.43927	0.00949	12.00000	-0.92641	4.00630
1	10	17	19	0.42756	0.00924	12.00000	0.25426	3.96513
1	11	19	21	0.42147	0.00916	12.00000	-0.20528	3.94418
1	12	21	23	0.41871	0.00915	12.00000	-0.49200	3.93481
1	13	23	25	0.41768	0.00916	12.00000	-0.20381	3.93134
1	14	25	27	0.41740	0.00917	12.00000	0.60071	3.93040
1	15	27	29	0.41737	0.00917	12.00000	-0.02434	3.93028
1	16	29	31	0.41737	0.00917	12.00000	-1.53360	3.93028
1	17	31	33	0.41734	0.00917	12.00000	0.37788	3.93019
1	18	33	35	0.41729	0.00917	12.00000	2.11394	3.93004
1	19	35	37	0.41724	0.00916	12.00000	-1.75706	3.92987
1	20	37	39	0.41720	0.00916	12.00000	-0.76671	3.92973
1	21	39	41	0.41718	0.00916	12.00000	2.04466	3.92964
1	22	41	43	0.41716	0.00916	12.00000	1.47659	3.92957
1	23	43	45	0.41715	0.00916	12.00000	1.01303	3.92955
1	24	45	47	0.41715	0.00916	12.00000	1.19057	3.92953
1	25	47	49	0.41714	0.00916	12.00000	0.14793	3.92953
1	26	49	51	0.41714	0.00916	12.00000	1.20099	3.92953
1	27	51	53	0.41714	0.00916	12.00000	-1.30133	3.92953
1	28	53	55	0.41714	0.00916	12.00000	-1.24301	3.92953
1	29	55	57	0.41714	0.00916	12.00000	1.05223	3.92953
1	30	57	59	0.41714	0.00916	12.00000	0.91925	3.92953
1	31	59	61	0.41714	0.00916	12.00000	1.05219	3.92953
2	1	0	2	0.99010	0.0	0.0		
2	2	2	4	0.99993	0.08332	12.00000	0.06944	360.02466
2	3	4	6	0.83379	0.04173	12.00000	-1.41083	7.35863
2	4	6	8	0.70240	0.02544	12.00000	-0.55978	5.49924
2	5	8	10	0.60715	0.01761	12.00000	-0.06970	4.78637
2	6	10	12	0.53905	0.01348	12.00000	-1.33647	4.41870
2	7	12	14	0.49142	0.01125	12.00000	0.03041	4.20671
2	8	14	16	0.45945	0.01007	12.00000	-3.23311	4.08039
2	9	16	18	0.43927	0.00949	12.00000	-0.54752	4.00630
2	10	18	20	0.42756	0.00924	12.00000	-1.07857	3.96513
2	11	20	22	0.42147	0.00916	12.00000	-0.06239	3.94418
2	12	22	24	0.41871	0.00915	12.00000	-1.32112	3.93481
2	13	24	26	0.41768	0.00916	12.00000	1.27428	3.93134
2	14	26	28	0.41740	0.00917	12.00000	0.31332	3.93040
2	15	28	30	0.41737	0.00917	12.00000	-1.29135	3.93028
2	16	30	32	0.41737	0.00917	12.00000	1.42955	3.93028
2	17	32	34	0.41734	0.00917	12.00000	-0.51846	3.93019
2	18	34	36	0.41729	0.00917	12.00000	-0.90848	3.93004
2	19	36	38	0.41724	0.00916	12.00000	1.05110	3.92987
2	20	38	40	0.41720	0.00916	12.00000	-0.73004	3.92973
2	21	40	42	0.41718	0.00916	12.00000	-1.30644	3.92964
2	22	42	44	0.41716	0.00916	12.00000	1.52113	3.92957
2	23	44	46	0.41715	0.00916	12.00000	-0.49646	3.92955
2	24	46	48	0.41715	0.00916	12.00000	-0.43472	3.92953
2	25	48	50	0.41714	0.00916	12.00000	0.09782	3.92953
2	26	50	52	0.41714	0.00916	12.00000	0.06985	3.92953
2	27	52	54	0.41714	0.00916	12.00000	0.27159	3.92953
2	28	54	56	0.41714	0.00916	12.00000	1.84546	3.92953
2	29	56	58	0.41714	0.00916	12.00000	-0.60997	3.92953
2	30	58	60	0.41714	0.00916	12.00000	-1.34320	3.92953

FINAL JSET DATA

```
JSET(1, 1)=  1
JSET(1, 2)=  3
JSET(1, 3)=  5
JSET(1, 4)=  7
JSET(1, 5)=  9
JSET(1, 6)= 11
JSET(1, 7)= 13
JSET(1, 8)= 15
JSET(1, 9)= 17
JSET(1,10)= 19
JSET(1,11)= 21
JSET(1,12)= 23
JSET(1,13)= 25
JSET(1,14)= 27
JSET(1,15)= 29
JSET(1,16)= 31
JSET(1,17)= 33
JSET(1,18)= 35
JSET(1,19)= 37
JSET(1,20)= 39
JSET(1,21)= 41
JSET(1,22)= 43
JSET(1,23)= 45
JSET(1,24)= 47
JSET(1,25)= 49
JSET(1,26)= 51
JSET(1,27)= 53
JSET(1,28)= 55
JSET(1,29)= 57
JSET(1,30)= 59
JSET(1,31)= 61
JSET(2, 1)=  2
JSET(2, 2)=  4
JSET(2, 3)=  6
JSET(2, 4)=  8
JSET(2, 5)= 10
JSET(2, 6)= 12
JSET(2, 7)= 14
JSET(2, 8)= 16
JSET(2, 9)= 18
JSET(2,10)= 20
JSET(2,11)= 22
JSET(2,12)= 24
JSET(2,13)= 26
JSET(2,14)= 28
JSET(2,15)= 30
JSET(2,16)= 32
JSET(2,17)= 34
JSET(2,18)= 36
JSET(2,19)= 38
JSET(2,20)= 40
JSET(2,21)= 42
JSET(2,22)= 44
JSET(2,23)= 46
JSET(2,24)= 48
JSET(2,25)= 50
JSET(2,26)= 52
JSET(2,27)= 54
JSET(2,28)= 56
JSET(2,29)= 58
JSET(2,30)= 60
```

NTAR = 1

FILTERED AND SMOOTHED EMITTER TARGET DATA CORRELATED TO NTAR = 1

NUM	FREQ	PRF	PW	THETAD	THTD	THTD1
1	1197.0	150.0	3.50	105.22139	105.22139	105.22139
3	1197.0	150.0	3.50	104.94077	104.94078	105.22136
5	1197.0	150.0	3.50	102.36406	102.74568	105.59840
7	1197.0	150.0	3.50	98.88550	99.60863	106.07678
9	1197.0	150.0	3.50	99.29208	98.56316	105.71794
11	1197.0	150.0	3.50	95.14084	95.89789	106.00923
13	1197.0	150.0	3.50	94.13567	93.99150	105.96504
15	1197.0	150.0	3.50	92.50012	92.22116	105.89708
17	1197.0	150.0	3.50	89.34970	89.86916	105.99577
19	1197.0	150.0	3.50	88.07285	87.92729	105.97491
21	1197.0	150.0	3.50	85.69966	85.81842	105.98711
23	1197.0	150.0	3.50	83.28149	83.56749	106.00682
25	1197.0	150.0	3.50	81.26472	81.38341	106.01180
27	1197.0	150.0	3.50	79.86276	79.51279	106.00418
29	1197.0	150.0	3.50	77.43318	77.44736	106.00429
31	1197.0	150.0	3.50	73.85582	74.74934	106.00293
33	1197.0	150.0	3.50	72.90050	72.68031	106.00438
35	1197.0	150.0	3.50	72.60910	71.37729	106.01534
37	1197.0	150.0	3.50	67.66759	68.69151	106.00587
39	1197.0	150.0	3.50	65.77895	66.22577	106.00211
41	1197.0	150.0	3.50	66.04028	64.84860	106.01045
43	1197.0	150.0	3.50	64.31984	63.45921	106.01512
45	1197.0	150.0	3.50	62.62923	62.03877	106.01743
47	1197.0	150.0	3.50	61.49768	60.80374	106.01921
49	1197.0	150.0	3.50	59.35091	59.26468	106.01933
51	1197.0	150.0	3.50	58.88116	58.18115	106.01985
53	1197.0	150.0	3.50	55.42735	56.18584	106.01968
55	1197.0	150.0	3.50	53.34729	54.07178	106.01976
57	1197.0	150.0	3.50	53.39183	52.77852	106.01956
59	1197.0	150.0	3.50	52.08127	51.54547	106.01935
61	1197.0	150.0	3.50	51.08221	50.46893	106.01910

SMOOTHED INITIAL BEARING ANGLE = 106.01910

FILTERED FINAL BEARING ANGLE = 50.46893

PLANE TRIANGULATION SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 33.24187N

EMITTER LONGITUDE =-117.42503W

NTAR = 2

FILTERED AND SMOOTHED EMITTER TARGET DATA CORRELATED TO NTAR = 2

NUM	FREQ	PRF	PW	THETAD	THTD	THTD1
2	1212.0	250.0	3.00	122.24252	122.24252	122.24252
4	1212.0	250.0	3.00	122.29161	122.29160	122.24252
6	1212.0	250.0	3.00	120.92984	121.16432	122.47420
8	1212.0	250.0	3.00	119.94717	120.11375	122.58440
10	1212.0	250.0	3.00	119.21582	119.24319	122.59787
12	1212.0	250.0	3.00	117.06377	117.67981	122.83490
14	1212.0	250.0	3.00	116.65103	116.63556	122.83015
16	1212.0	250.0	3.00	112.34737	114.09503	123.25594
18	1212.0	250.0	3.00	112.10175	112.40875	123.31427
20	1212.0	250.0	3.00	109.62207	110.43947	123.40269
22	1212.0	250.0	3.00	108.74931	108.78540	123.40640
24	1212.0	250.0	3.00	105.82967	106.59761	123.45934
26	1212.0	250.0	3.00	106.09219	105.35014	123.42816
28	1212.0	250.0	3.00	104.02385	103.84129	123.42418
30	1212.0	250.0	3.00	100.94479	101.69717	123.42987
32	1212.0	250.0	3.00	101.37944	100.54652	123.43112
34	1212.0	250.0	3.00	98.43813	98.74020	123.42911
36	1212.0	250.0	3.00	96.18474	96.71411	123.42439
38	1212.0	250.0	3.00	96.01830	95.40575	123.43004
40	1212.0	250.0	3.00	93.04437	93.46983	123.42645
42	1212.0	250.0	3.00	90.44980	91.21239	123.42110
44	1212.0	250.0	3.00	90.87807	89.99149	123.42590
46	1212.0	250.0	3.00	87.80681	88.09616	123.42477
48	1212.0	250.0	3.00	85.91864	86.17201	123.42412
50	1212.0	250.0	3.00	84.47925	84.42223	123.42419
52	1212.0	250.0	3.00	82.71225	82.67152	123.42422
54	1212.0	250.0	3.00	81.17096	81.01265	123.42426
56	1212.0	250.0	3.00	81.11581	80.04016	123.42412
58	1212.0	250.0	3.00	77.89078	78.24629	123.42422
60	1212.0	250.0	3.00	75.29662	76.07950	123.42451

SMOOTHED INITIAL BEARING ANGLE = 123.42451

FILTERED FINAL BEARING ANGLE = 76.07950

PLANE TRIANGULATION SOLUTION OF EMITTER LOCATION

EMITTER LATITUDE = 32.78056N

EMITTER LONGITUDE =-117.23346W

```
C
C
C
C     **************************************************
C     *                                                *
C     *  KALMAN FILTER PROGRAM TO ESTIMATE EMITTER LOCATION  *
C     *        FROM FILTERED BEARING ANGLES-OF-ARRIVAL       *
C     *                                                *
C     **************************************************
C
C
C
C
C     THIS FORTRAN PROGRAM IS DESIGNED TO ANALYZE AND SORT
C     ELECTRONIC EMITTER PARAMETERS AND AIRCRAFT NAVIGATION
C     DATA, TO FILTER EMITTER DATA USING KALMAN FILTER TECH-
C     NIQUES TO MINIMIZE BEARING ANGLE-OF-ARRIVAL MEASUREMENT
C     NOISE, TO SMOOTH INITIAL UNFILTERED BEARING ANGLES,
C     AND TO PREDICT EMITTER LOCATIONS USING TRIANGULATION
C     METHODS
C
      DIMENSION ALT(100),BRNG(100),BRNGD(100),D11(100),
     1D12(100),D21(100),D22(100),E(100),FREQ(100),G1(100),
     2G2(100),GATE(100),HDG(100),HDGD(100),JSET(30,50),LW(2)
     3MODEN(100),MODET(100),MW(2),NST(100),P11(100),P12(100)
     4P22(100),PIN(2,2),PIN11(100),PIN12(100),PIN22(100),
     5PITCH(100),PRF(100),PW(100),Q11(100),Q12(100),Q22(100)
     6QP11(100),QP12(100).QP21(100),QP22(100),P(100),RB(100)
     7ROLL(100),SLA(100),SLAD(100),SLO(100),SLOD(100),T(100)
     8TT(100),TAT(100),TIMEN(100),TIMET(100),TDT(100),
     9TDTD(100),TDTD1(100),THT(100),THTD(100),THT1(100),
     0THTD1(100),TPT(100),TPTD(100),THETA(100),THETAD(100),
     1TILA(10),TLA(10),TLAD(10),TLO(10),TLOD(10),VEL(100),
     2VELE(100),VELN(100),WAV(100)
C
C     SUBROUTINE TO READ EMITTER TARGET AND AIRCRAFT
C     NAVIGATION DATA FROM CARD DATA DECK
C     DATA SEQUENCE MUST BE OF FORMAT TGT/NAV
C
      NUM=65
      DO 1 I=1,NUM
C     TYPE=11.0   EMITTER TARGET DATA
      READ(5,51)TGT,TIMET(I),BRNG(I),PRF(I),MODET(I),PW(I),
     1FREQ(I)
C     TYPE=7.0    AIRCRAFT NAVIGATION DATA
      READ(5,52) NAV,TIMEN(I),MODEN(I),ALT(I),SLA(I),SLO(I)
      READ(5,56)HDG(I),VELE(I),VELN(I),ROLL(I),PITCH(I)
C
C     SUBROUTINE TO COMPUTE AIRCRAFT VELOCITY
C
      VELS=VELE(I)**2+VELN(I)**2
      VEL(I)=SQRT(VELS)
    1 CONTINUE
C
C     SUBROUTINE TO CHANGE ANGLES AND LATITUDE/LONGITUDE
C     FROM RADIANS TO DEGREES + TENTHS OF DEGREES
C
      WRITE(6,57)
      DO 5 J=1,NUM
      PIRAD=57.29578
      HDGD(J)=HDG(J)*PIRAD
      BRNGD(J)=BRNG(J)*PIRAD
      SLAD(J)=SLA(J)*PIRAD
      SLOD(J)=SLO(J)*PIRAD
      THETA(J)=BRNG(J)+HDG(J)
      IF(THETA(J).GT.6.283186) THETA(J)=THETA(J)-6.283186
      THETAD(J)=BRNGD(J)+HDGD(J)
      IF(THETAD(J).GT.360.0) THETAD(J)=THETAD(J)-360.0
      WRITE(6,58)J,HDGD(J),SLAD(J),SLOD(J),BRNGD(J),
     1THETAD(J),THETA(J)
    5 CONTINUE
C
```

```
C     PROGRAM TO SORT EMITTER TARGET DATA AND ESTIMATE NUMBER
C     OF DISTINCT EMITTER TARGETS
C     DATA IS INITIALLY SORTED BY FREQUENCY AND PRF
C
      NTAR=0
      DO 10 I=1,NUM
      IF(I.EQ.1) GO TO 19
      L=I-1
      DO 18 KQ=1,L
      M=I-KQ
      DELTF=FREQ(I)-FREQ(M)
      DELTPR=PRF(I)-PRF(M)
      IF(ABS(DELTF).LE.4.0.AND.ABS(DELTPR).LE.4.0) GO TO 10
C
C     CHECK FOR MULTIPLES OF PRF
C
      IF(ABS(DELTF).GT.4.0) GO TO 18
      DO 11 N=2,5
      RK=N*PRF(M)
      SK=PRF(I)
      TK=PRF(M)/FLOAT(N)
      CK=ABS(RK-SK)
      DK=ABS(TK-SK)
      FK=N*4.0
      IF(CK.LT.FK.OR.DK.LT.FK) GO TO 10
   11 CONTINUE
   18 CONTINUE
   19 NTAR=NTAR+1
      WRITE(6,59)NTAR
      WRITE(6,61)NTAR
      NJ=1
      JSET(NTAR,NJ)=I
      DO 13 J=1,NUM
      IF(J.EQ.I) GO TO 13
      DELF=FREQ(J)-FREQ(I)
      DELPRF=PRF(J)-PRF(I)
      IF(ABS(DELF).LE.4.0) GO TO 4
      GO TO 13
C
C     CHECK FOR SUBSEQUENT MULTIPLES OF PRF
C
    4 DO 12 N=1,5
      RK=N*PRF(J)
      SK=PRF(I)
      TK=PRF(J)/FLOAT(N)
      CK=ABS(RK-SK)
      DK=ABS(TK-SK)
      FK=N*4.0
      IF(CK.LT.FK.OR.DK.LT.FK) GO TO 14
   12 CONTINUE
      GO TO 13
   14 NJ=NJ+1
      JSET(NTAR,NJ)=J
   13 CONTINUE
      NST(NTAR)=NJ
      NSTA=NST(NTAR)
      IF(NSTA.EQ.1.0) GO TO 17
C
C     SUBROUTINE TO CHECK FOR AND ELIMINATE 360 DEGREE
C     VARIATION IN EMITTER BEARING ANGLE THETA
C
      DO 9 J=2,NSTA
      JM=J-1
      KK=JSET(NTAR,JM)
      K=JSET(NTAR,J)
      IF(THETAD(KK).LT.10.0.AND.THETAD(K).GT.350.0) GO TO 15
      IF(THETAD(KK).GT.350.0.AND.THETAD(K).LT.10.0) GO TO 16
      GO TO 9
   15 DO 6 MM=1,JM
      M=JSET(NTAR,MM)
      THETAD(M)=THETAD(M)+360.0
      THETA(M)=THETA(M)+6.283186
```

```
 6     CONTINUE
       GO TO 17
16     DO 7 MN=J,NSTA
       M=JSET(NTAR,MN)
       THETAD(M)=THETAD(M)+360.0
       THETA(M)=THETA(M)+6.283186
 7     CONTINUE
       GO TO 17
 9     CONTINUE
C
C     PRINT UNFILTERED BEARING LINES OF SIMILAR
C     FREQUENCY AND PRF
C
17     DO 8 J=1,NSTA
       K=JSET(NTAR,J)
       WRITE(6,60)K,FREQ(K),PRF(K),PW(K),HDGD(K),BRNGD(K),
      1THETAD(K)
 8     CONTINUE
10     CONTINUE
C
C     SUBROUTINE TO COMPUTE INITIAL EMITTER TARGET JSET DATA
C
       WRITE(6,62)
       DO 25 I=1,NTAR
       NSTA=NST(I)
       WRITE(6,63)(I,J,JSET(I,J),J=1,NSTA)
25     CONTINUE
C
C     PROGRAM TO FILTER NOISY MEASURED BEARING ANGLE THETAD(K)
C     AND AIRCRAFT NAVIGATION DATA FOR EACH SUSPECTED EMITTER
C     TARGET OF SIMILAR FREQUENCY AND PRF
C
C     INITIALIZATION OF KALMAN FILTER EQUATION PARAMETERS
C
       WRITE(6,50)
       DO 20 I=1,NTAR
       KI=JSET(I,1)
       P11(KI)=10000.0
       P12(KI)=0.0
       P22(KI)=10000.0
       Q11(KI)=0.0
       Q12(KI)=0.0
       Q22(KI)=0.0
       D11(KI)=1.0
       D12(KI)=0.0
       D21(KI)=0.0
       D22(KI)=1.0
       R(KI)=144.0
       G1(KI)=P11(KI)/(P11(KI)+R(KI))
       G2(KI)=P12(KI)/(P11(KI)+R(KI))
       THTD(KI)=THETAD(KI)
       THTD1(KI)=THETAD(KI)
       TDTD(KI)=VEL(KI)*SIN(BRNG(KI))*PIRAD/600000.0
       TDTD1(KI)=TDTD(KI)
       T(KI)=0.0
       K2=0
       K3=1
       WRITE(6,64)I,K3,K2,KI,G1(KI),G2(KI),T(KI)
C
C     START OF KALMAN FILTER PREDICTION PROBLEM
C
       NSTA=NST(I)
       DO 21 J=2,NSTA
22     K=JSET(I,J)
       KK=JSET(I,J-1)
       TKM1=TIMET(KK)
       T(K)=TIMET(K)-TKM1
       TT(K)=T(K)/1000.0
       Q11(K)=TT(K)**4/4.0
       Q12(K)=TT(K)**3/2.0
       Q22(K)=TT(K)**2
       R(K)=144.0
```

```
C
C     START OF KALMAN FILTER RECURSION EQUATIONS
C
      P11(K)=P11(KK)*(1.0-G1(KK))+2.0*P12(KK)*T(K)-(P12(KK)*
     1G1(KK)+P11(KK)*G2(KK))*T(K)+(P22(KK)-P12(KK)*G2(KK))*
     2T(K)**2+Q11(K)
      P12(K)=P12(KK)*(1.0-G1(KK))+(P22(KK)-P12(KK)*G2(KK))*
     1T(K)+Q12(K)
      P22(K)=P22(KK)-P12(KK)*G2(KK)+Q22(K)
      G1(K)=P11(K)/(P11(K)+R(K))
      G2(K)=P12(K)/(P11(K)+R(K))
      TPTD(K)=THTD(KK)+TDTD(KK)*T(K)
      E(K)=THETAD(K)-TPTD(K)
      THTD(K)=TPTD(K)+G1(K)*E(K)
      TDTD(K)=TDTD(KK)+G2(K)*E(K)
C
C     SMOOTHING EQUATIONS FOR FIRST BEARING LINE ESTIMATE
C
      PIN(1,1)=P11(K)
      PIN(1,2)=P12(K)
      PIN(2,1)=P12(K)
      PIN(2,2)=P22(K)
      CALL MINV(PIN,2,DET,LW,MW)
      PIN11(K)=PIN(1,1)
      PIN12(K)=PIN(1,2)
      PIN22(K)=PIN(2,2)
      QP11(K)=Q11(K)*PIN11(K)+Q12(K)*PIN12(K)
      QP12(K)=Q11(K)*PIN12(K)+Q12(K)*PIN22(K)
      QP21(K)=Q12(K)*PIN11(K)+Q22(K)*PIN12(K)
      QP22(K)=Q12(K)*PIN12(K)+Q22(K)*PIN22(K)
      D11(K)=D11(KK)*(1.0-QP11(K))-D12(KK)*QP21(K)+D11(KK)*
     1QP21(K)*T(K)
      D12(K)=D12(KK)*(1.0-QP22(K))-D11(KK)*QP12(K)-D11(KK)*
     1(QP22(K)-1.0)*T(K)
      D21(K)=D21(KK)*(1.0-QP11(K))-D22(KK)*QP21(K)+D21(KK)*
     1QP21(K)*T(K)
      D22(K)=D22(KK)*(1.0-QP22(K))-D21(KK)*QP12(K)+D21(KK)*
     1(QP22(K)-1.0)*T(K)
      THTD1(K)=THTD1(KK)+(D11(K)*G1(K)+D12(K)*G2(K))*E(K)
      TDTD1(K)=TDTD1(KK)+(D21(K)*G1(K)+D22(K)*G2(K))*E(K)
C
C     CORRELATION GATING SCHEME TO ESTIMATE WHETHER FILTERED
C     BEARING ANGLE THTD(K) IS AN EMISSION FROM EMITTER TARGET
C     NTAR = I OR A SPURIOUS EMISSION
C
      GATE(K)=SQRT(P11(K)+R(K))/2.0
      WRITE(6,65)I,J,KK,K,G1(K),G2(K),T(K),E(K),GATE(K)
      IF(ABS(E(K)).LT.GATE(K)) GO TO 21
C
C     THTD(K) IS A SPURIOUS BEARING LINE AND IS DISCARDED
C
   28 NST(I)=NST(I)-1
      NSTA=NST(I)
      IF(NSTA.LT.J) GO TO 20
      DO 27 L=J,NSTA
      JSET(I,L)=JSET(I,L+1)
   27 CONTINUE
      GO TO 22
   21 CONTINUE
C
C     FILTERED BEARING ANGLE THTD(K) IS CORRELATED TO EMITTER
C     TARGET NTAR = I
C
   20 CONTINUE
C
C     SUBROUTINE TO COMPUTE FINAL EMITTER TARGET JSET DATA
C
      WRITE(6,66)
      DO 30 I=1,NTAR
      NSTA=NST(I)
      WRITE(6,63)(I,J,JSET(I,J),J=1,NSTA)
   30 CONTINUE
```

```
C     PROGRAM TO COMPUTE TARGET POSITION BASED ON A PLANE
C     TRIANGULATION SOLUTION
C
      DO 35 I=1,NTAR
      WRITE(6,59)I
      WRITE(6,67)I
      NSTA=NST(I)
      KI=JSET(I,1)
      KF=JSET(I,NSTA)
      DO 45 J=1,NSTA
      K=JSET(I,J)
      WRITE(6,60)K,FREQ(K),PRF(K),PW(K),THETAD(K),THTD(K),
     1THTD1(K)
   45 CONTINUE
      THT(KF)=THTD(KF)/PIRAD
      THT1(KF)=THTD1(KF)/PIRAD
      TAT(I)=TAN(THT1(KF))-TAN(THT(KF))
      WAV(I)=(SLA(KI)+SLA(KF))/2.0
      TILA(I)=((SLO(KF)-SLO(KI))*COS(WAV(I))+SLA(KI)*
     1TAN(THT1(KF))-SLA(KF)*TAN(THT(KF)))/TAT(I)
      TLA(I)=((SLO(KF)-SLO(KI))*COS(TILA(I))+SLA(KI)*
     1TAN(THT1(KF))-SLA(KF)*TAN(THT(KF)))/TAT(I)
      TLAD(I)=TLA(I)*PIRAD
      TLO(I)=SLO(KF)+(TLA(I)-SLA(KF))*TAN(THT(KF))/
     1COS(TILA(I))
      TLOD(I)=TLO(I)*PIRAD
      WRITE(6,68)THTD1(KF),THTD(KF)
      WRITE(6,70) TLAD(I),TLOD(I)
   35 CONTINUE
C
   50 FORMAT('1',14X,'LISTING OF KALMAN FILTER PARAMETERS',
     1//,'     I    J KK    K       G1(K)        G2(K)        T(K)        ',
     2'E(K)',6X,'GATE(K)',/)
   51 FORMAT(7F11.5)
   52 FORMAT(6F13.5)
   53 FORMAT('1',36X,'LISTING OF EMITTER TARGET DATA AND ',
     1'AIRCRAFT NAVIGATION DATA',///,13X,'TARGET PARAMETERS'
     2,37X,'AIRCRAFT PARAMETERS',//,' TYPE        TIME',4X,
     3'FREQ      BRNG     PRF     PW   MO / TIME       HDG         ALT',
     46X,'LAT        LONG        N.VEL      E.VEL      ROLL        PITCH'
     5,2X,'MO',//)
   54 FORMAT(' TGT',I3,1X,F9.3,1X,F6.1,1X,F7.5,1X,F6.1,1X,
     1F4.2,1X,I1)
   55 FORMAT(' NAV',43X,F9.3,1X,F7.5,1X,F7.1,1X,F8.5,1X,F8.5
     11X,F8.3,1X,F8.3,1X,F8.5,1X,F8.5,1X,F3.0,/)
   56 FORMAT(5F13.5)
   57 FORMAT('1 LISTING OF BEARING ANGLES-OF-ARRIVAL AND ',
     1'AIRCRAFT NAVIGATION DATA',///,'    N    HDGD(N)       ',
     2'SLAD(N)    SLOD(N)      BRNGD(N) THETAD(N) THETA(N)',//)
   58 FORMAT(' ',I3,4F11.5,F10.5,F9.5)
   59 FORMAT('1',28X,'NTAR =',I3,//)
   60 FORMAT('0',I3,2F8.1,F7.2,3F12.5)
   61 FORMAT('0 UNFILTERED EMITTER TARGET DATA INITIALLY ',
     1'ASSOCIATED TO NTAR =',I3,//,' EMITTER BEARING LINES',
     2' OF SIMILAR FREQUENCY AND PRF OR PRF MULTIPLE',////,
     3' NUM    FREQ      PRF       PW       HDGD         BRNGD',
     4'        THETAD',/)
   62 FORMAT('1  INITIAL JSET DATA',//)
   63 FORMAT('    JSET(',I1,',',I2,')=',I3)
   64 FORMAT('0',4I3,4F10.5)
   65 FORMAT(' ',4I3,4F10.5,F12.5)
   66 FORMAT('1  FINAL JSET DATA',//)
   67 FORMAT('0FILTERED AND SMOOTHED EMITTER TARGET DATA ',
     1'CORRELATED TO NTAR =',I2,///,' NUM    FREQ      PRF',
     2'      PW         THETAD          THTD          THTD1',/)
   68 FORMAT('0',//,'  SMOOTHED INITIAL BEARING ANGLE =',
     1F10.5,//,'      FILTERED FINAL BEARING ANGLE =',F10.5//)
   70 FORMAT('0 PLANE TRIANGULATION SOLUTION OF EMITTER ',
     1'LOCATION',//,'              EMITTER LATITUDE =',F10.5,'N'
     2//,'             EMITTER LONGITUDE =',F10.5,'W')
      STOP
      END
```

```
C
C    ******************************************************
C    *                                                    *
C    *  MONTE CARLO SIMULATION RUN OF KALMAN FILTER PROGRAM *
C    *                                                    *
C    ******************************************************
C
C     THIS FORTRAN PROGRAM IS DESIGNED TO ANALYZE AND SORT
C     ELECTRONIC EMITTER PARAMETERS AND AIRCRAFT NAVIGATION
C     DATA, TO FILTER EMITTER DATA USING KALMAN FILTER TECH-
C     NIQUES TO MINIMIZE BEARING ANGLE-OF-ARRIVAL MEASUREMENT
C     NOISE, TO SMOOTH INITIAL UNFILTERED BEARING ANGLES,
C     AND TO PREDICT EMITTER LOCATIONS USING TRIANGULATION
C     METHODS
C
      DIMENSION ALT(100),BRNG(100),BRNGD(100),D11(100),
     1D12(100),D21(100),D22(100),E(100),FREQ(100),G1(100),
     2G2(100),GATE(100),HDG(100),HDGD(100),JSET(5,40),LW(2),
     3MODEN(100),MODET(100),NW(2),NST(100),P11(100),P12(100)
     4P22(100),PIN(2,2),PIN11(100),PIN12(100),PIN22(100),
     5PITCH(100),PRF(100),PW(100),Q11(100),Q12(100),Q22(100)
     6QP11(100),QP12(100),QP21(100),QP22(100),R(100),RB(100)
     7ROLL(100),SLA(100),SLAD(100),SLO(100),SLOD(100),T(100)
     8TT(100),TAT(100),TIMEN(100),TIMET(100),TDT(100),
     9TDTD(100),TDTD1(100),THT(100),THTD(100),THT1(100),
     0THTD1(100),TPT(100),TPTD(100),THETA(100),THETAD(100),
     1TILA(5),TLA(5),TLAD(5),TLO(5),TLOD(5),VEL(100),
     2VELE(100),VELN(100),WAV(100)
C
C     SUBROUTINE TO READ EMITTER TARGET AND AIRCRAFT
C     NAVIGATION DATA FROM CARD DATA DECK
C     DATA SEQUENCE MUST BE OF FORMAT TGT/NAV
C
      NUM=61
      IX=1257
      STDEV=1.0
      AVE=0.0
      WRITE(6,48)
      DO 1 I=1,NUM
C     READ EMITTER TARGET DATA
      READ(5,51)TGT,TIMET(I),BRNGD(I),PRF(I),PW(I),FREQ(I)
C     READ AIRCRAFT NAVIGATION DATA
      READ(5,51)HDGD(I),SLAD(I),SLOD(I),ALT(I),VELN(I),
     1VELE(I)
C
C     GENERATE RANDOM NOISE TO ADD TO KNOWN EMITTER BEARING
C     ANGLES OF ARRIVAL
C
      CALL GAUSS(IX,STDEV,AVE,V)
      THETAD(I)=BRNGD(I)+V
    2 WRITE(6,49)I,TIMET(I),FREQ(I),PRF(I),PW(I),BRNGD(I),
     1V,THETAD(I),HDGD(I),SLAD(I),SLOD(I),ALT(I),VELN(I),
     2VELE(I)
C
C     SUBROUTINE TO COMPUTE AIRCRAFT VELOCITY
C
      VELS=VELE(I)**2+VELN(I)**2
      VEL(I)=SQRT(VELS)
    1 CONTINUE
C
C     SUBROUTINE TO CHANGE ANGLES FROM DEGREES TO RADIANS
C
      WRITE(6,57)
      DO 5 J=1,NUM
      PIRAD=57.29578
      HDG(J)=HDGD(J)/PIRAD
      BRNG(J)=BRNGD(J)/PIRAD
      SLA(J)=SLAD(J)/PIRAD
      SLO(J)=SLOD(J)/PIRAD
      THETA(J)=THETAD(J)/PIRAD
      RB(J)=BRNG(J)-HDG(J)
      IF(THETA(J).GT.6.283186) THETA(J)=THETA(J)-6.283186
```

```
      IF(THETAD(J).GT.360.0) THETAD(J)=THETAD(J)-360.0
      WRITE(6,58)J,HDGD(J),SLAD(J),SLOD(J),BRNGD(J),
     1THETAD(J),THETA(J)
    5 CONTINUE
C
C     PROGRAM TO SORT EMITTER TARGET DATA AND ESTIMATE NUMBER
C     OF DISTINCT EMITTER TARGETS
C     DATA IS INITIALLY SORTED BY FREQUENCY AND PRF
C
      NTAR=0
      DO 10 I=1,NUM
      IF(I.EQ.1) GO TO 19
      L=I-1
      DO 18 KQ=1,L
      M=I-KQ
      DELTF=FREQ(I)-FREQ(M)
      DELTPR=PRF(I)-PRF(M)
      IF(ABS(DELTF).LE.4.0.AND.ABS(DELTPR).LE.4.0) GO TO 10
C
C     CHECK FOR MULTIPLES OF PRF
C
      IF(ABS(DELTF).GT.4.0) GO TO 18
      DO 11 N=2,5
      RK=N*PRF(M)
      SK=PRF(I)
      TK=PRF(M)/FLOAT(N)
      CK=ABS(RK-SK)
      DK=ABS(TK-SK)
      FK=N*4.0
      IF(CK.LT.FK.OR.DK.LT.FK) GO TO 10
   11 CONTINUE
   18 CONTINUE
   19 NTAR=NTAR+1
      WRITE(6,59)NTAR
      WRITE(6,61)NTAR
      NJ=1
      JSET(NTAR,NJ)=I
      DO 13 J=1,NUM
      IF(J.EQ.I) GO TO 13
      DELF=FREQ(J)-FREQ(I)
      DELPRF=PRF(J)-PRF(I)
      IF(ABS(DELF).LE.4.0) GO TO 4
      GO TO 13
C
C     CHECK FOR SUBSEQUENT MULTIPLES OF PRF
C
    4 DO 12 N=1,5
      RK=N*PRF(J)
      SK=PRF(I)
      TK=PRF(J)/FLOAT(N)
      CK=ABS(RK-SK)
      DK=ABS(TK-SK)
      FK=N*4.0
      IF(CK.LT.FK.OR.DK.LT.FK) GO TO 14
   12 CONTINUE
      GO TO 13
   14 NJ=NJ+1
      JSET(NTAR,NJ)=J
   13 CONTINUE
      NST(NTAR)=NJ
      NSTA=NST(NTAR)
      IF(NSTA.EQ.1.0) GO TO 17
C
C     SUBROUTINE TO CHECK FOR AND ELIMINATE 360 DEGREE
C     VARIATION IN EMITTER BEARING ANGLE THETA
C
      DO 9 J=2,NSTA
      JM=J-1
      KK=JSET(NTAR,JM)
      K=JSET(NTAR,J)
      IF(THETAD(KK).LT.10.0.AND.THETAD(K).GT.350.0) GO TO 15
      IF(THETAD(KK).GT.350.0.AND.THETAD(K).LT.10.0) GO TO 16
```

```
      GO TO 9
 15   DO 6 MM=1,JM
      M=JSET(NTAR,MM)
      THETAD(M)=THETAD(M)+360.0
      THETA(M)=THETA(M)+6.283186
  6   CONTINUE
      GO TO 17
 16   DO 7 MN=J,NSTA
      M=JSET(NTAR,MN)
      THETAD(M)=THETAD(M)+360.0
      THETA(M)=THETA(M)+6.283186
  7   CONTINUE
      GO TO 17
  9   CONTINUE
C
C    PRINT UNFILTERED BEARING LINES OF SIMILAR
C    FREQUENCY AND PRF
C
 17   DO 8 J=1,NSTA
      K=JSET(NTAR,J)
      WRITE(6,60)K,FREQ(K),PRF(K),PW(K),HDGD(K),BRNGD(K),
     1THETAD(K)
  8   CONTINUE
 10   CONTINUE
C
C    SUBROUTINE TO COMPUTE INITIAL EMITTER TARGET JSET DATA
C
      WRITE(6,62)
      DO 25 I=1,NTAR
      NSTA=NST(I)
      WRITE(6,63)(I,J,JSET(I,J),J=1,NSTA)
 25   CONTINUE
C
C    SUBROUTINE TO CHECK MONTE CARLO SIMULATION ACCURACY IN
C    THE ABSENCE OF RANDOM NOISE CONTAMINATION OF EMITTER
C    BEARING ANGLES
C
      IF(V.EQ.0.0) GO TO 23
      GO TO 24
 23   WRITE(6,46)
      DO 26 I=1,NTAR
      NSTA=NST(I)
      KI=JSET(I,1)
      KF=JSET(I,NSTA)
      THTD1(KF)=BRNGD(KI)
      THTD(KF)=BRNGD(KF)
 26   CONTINUE
      GO TO 29
C
C    PROGRAM TO FILTER NOISY MEASURED BEARING ANGLE THETAD(K)
C    AND AIRCRAFT NAVIGATION DATA FOR EACH SUSPECTED EMITTER
C    TARGET OF SIMILAR FREQUENCY AND PRF
C
C    INITIALIZATION OF KALMAN FILTER EQUATION PARAMETERS
C
 24   WRITE(6,50)
      DO 20 I=1,NTAR
      KI=JSET(I,1)
      P11(KI)=100.0
      P12(KI)=0.0
      P22(KI)=100.0
      Q11(KI)=0.0
      Q12(KI)=0.0
      Q22(KI)=0.0
      D11(KI)=1.0
      D12(KI)=0.0
      D21(KI)=0.0
      D22(KI)=1.0
      R(KI)=1.0
      G1(KI)=P11(KI)/(P11(KI)+R(KI))
      G2(KI)=P12(KI)/(P11(KI)+R(KI))
      THTD(KI)=THETAD(KI)
```

```
      THTD1(KI)=THETAD(KI)
      TDTD(KI)=VEL(KI)*SIN(RB(KI))/500000.0
      TDTD1(KI)=TDTD(KI)
      T(KI)=0.0
      K2=0
      K3=1
      WRITE(6,64)I,K3,K2,KI,G1(KI),G2(KI),T(KI)
C
C     START OF KALMAN FILTER PREDICTION PROBLEM
C
      NSTA=NST(I)
      DO 21 J=2,NSTA
 22   K=JSET(I,J)
      KK=JSET(I,J-1)
      TKM1=TIMET(KK)
      T(K)=TIMET(K)-TKM1
      TT(K)=T(K)/1000.0
      Q11(K)=TT(K)**4/4.0
      Q12(K)=TT(K)**3/2.0
      Q22(K)=TT(K)**2
      R(K)=1.0
C
C     START OF KALMAN FILTER RECURSION EQUATIONS
C
      P11(K)=P11(KK)*(1.0-G1(KK))+2.0*P12(KK)*T(K)-(P12(KK)*
     1G1(KK)+P11(KK)*G2(KK))*T(K)+(P22(KK)-P12(KK)*G2(KK))*
     2T(K)**2+Q11(K)
      P12(K)=P12(KK)*(1.0-G1(KK))+(P22(KK)-P12(KK)*G2(KK))*
     1T(K)+Q12(K)
      P22(K)=P22(KK)-P12(KK)*G2(KK)+Q22(K)
      G1(K)=P11(K)/(P11(K)+R(K))
      G2(K)=P12(K)/(P11(K)+R(K))
      TPTD(K)=THTD(KK)+TDTD(KK)*T(K)
      E(K)=THETAD(K)-TPTD(K)
      THTD(K)=TPTD(K)+G1(K)*E(K)
      TDTD(K)=TDTD(KK)+G2(K)*E(K)
C
C     SMOOTHING EQUATIONS FOR FIRST BEARING LINE ESTIMATE
C
      PIN(1,1)=P11(K)
      PIN(1,2)=P12(K)
      PIN(2,1)=P12(K)
      PIN(2,2)=P22(K)
      CALL MINV(PIN,2,DET,LW,MW)
      PIN11(K)=PIN(1,1)
      PIN12(K)=PIN(1,2)
      PIN22(K)=PIN(2,2)
      QP11(K)=Q11(K)*PIN11(K)+Q12(K)*PIN12(K)
      QP12(K)=Q11(K)*PIN12(K)+Q12(K)*PIN22(K)
      QP21(K)=Q12(K)*PIN11(K)+Q22(K)*PIN12(K)
      QP22(K)=Q12(K)*PIN12(K)+Q22(K)*PIN22(K)
      D11(K)=D11(KK)*(1.0-QP11(K))-D12(KK)*QP21(K)+D11(KK)*
     1QP21(K)*T(K)
      D12(K)=D12(KK)*(1.0-QP22(K))-D11(KK)*QP12(K)+D11(KK)*
     1(QP22(K)-1.0)*T(K)
      D21(K)=D21(KK)*(1.0-QP11(K))-D22(KK)*QP21(K)+D21(KK)*
     1QP21(K)*T(K)
      D22(K)=D22(KK)*(1.0-QP22(K))-D21(KK)*QP12(K)+D21(KK)*
     1(QP22(K)-1.0)*T(K)
      THTD1(K)=THTD1(KK)+(D11(K)*G1(K)+D12(K)*G2(K))*E(K)
      TDTD1(K)=TDTD1(KK)+(D21(K)*G1(K)+D22(K)*G2(K))*E(K)
C
C     CORRELATION GATING SCHEME TO ESTIMATE WHETHER FILTERED
C     BEARING ANGLE THTD(K) IS AN EMISSION FROM EMITTER TARGET
C     NTAR = I OR A SPURIOUS EMISSION
C
      GATE(K)=3.0*SQRT(P11(K)+R(K))
      WRITE(6,65)I,J,KK,K,G1(K),G2(K),T(K),E(K),GATE(K)
      IF(ABS(E(K)).LT.GATE(K)) GO TO 21
C
C     THTD(K) IS A SPURIOUS BEARING LINE AND IS DISCARDED
C
```

```
 28    NST(I)=NST(I)-1
       NSTA=NST(I)
       IF(NSTA.LT.J) GO TO 20
       DO 27 L=J,NSTA
       JSET(I,L)=JSET(I,L+1)
 27    CONTINUE
       GO TO 22
C
 21    CONTINUE
C
C   FILTERED BEARING ANGLE THTD(K) IS CORRELATED TO EMITTER
C   TARGET NTAR = I
C
 20    CONTINUE
C
C   SUBROUTINE TO COMPUTE FINAL EMITTER TARGET JSET DATA
C
       WRITE(6,66)
       DO 30 I=1,NTAR
       NSTA=NST(I)
       WRITE(6,63)(I,J,JSET(I,J),J=1,NSTA)
 30    CONTINUE
C
C   PROGRAM TO COMPUTE TARGET POSITION BASED ON A PLANE
C   TRIANGULATION SOLUTION
C
 29    DO 35 I=1,NTAR
       WRITE(6,59)I
       WRITE(6,67)I
       NSTA=NST(I)
       KI=JSET(I,1)
       KF=JSET(I,NSTA)
       DO 34 J=1,NSTA
       K=JSET(I,J)
       WRITE(6,60)K,FREQ(K),PRF(K),PW(K),THETAD(K),THTD(K),
      1THTD1(K)
 34    CONTINUE
       THT(KF)=THTD(KF)/PIRAD
       THT1(KF)=THTD1(KF)/PIRAD
       TAT(I)=TAN(THT1(KF))-TAN(THT(KF))
       WAV(I)=(SLA(KI)+SLA(KF))/2.0
       TILA(I)=((SLO(KF)-SLO(KI))*COS(WAV(I))+SLA(KI)*
      1TAN(THT1(KF))-SLA(KF)*TAN(THT(KF)))/TAT(I)
       TLA(I)=((SLO(KF)-SLO(KI))*COS(TILA(I))+SLA(KI)*
      1TAN(THT1(KF))-SLA(KF)*TAN(THT(KF)))/TAT(I)
       TLAD(I)=TLA(I)*PIRAD
       TLO(I)=SLO(KF)+(TLA(I)-SLA(KF))*TAN(THT(KF))/
      1COS(TILA(I))
       TLOD(I)=TLO(I)*PIRAD
       WRITE(6,68)THTD1(KF),THTD(KF)
       WRITE(6,70) TLAD(I),TLOD(I)
C
 35    CONTINUE
C
 46    FORMAT('0 THIS MONTE CARLO SIMULATION HAS NO EMITTER',
      1' BEARING ANGLE MEASUREMENT NOISE',//,' EXECUTE A ',
      2'CALIBRATION CHECK FOR SIMULATION ACCURACY IN ABSENCE'
      3' OF RANDOM NOISE')
 48    FORMAT('1',36X,'LISTING OF EMITTER TARGET DATA AND ',
      1'AIRCRAFT NAVIGATION DATA',///,13X,'TARGET PARAMETERS'
      2,37X,'AIRCRAFT PARAMETERS',//,' JSET TIMET    FREQ     '
      3'PRF      PW       BRNGD       V        THETAD       HDGD     '
      4'SLAD        SLOD        ALT       VELN     VELE',//)
 49    FORMAT(' ',I3,2X,F5.1,2X,F6.1,2X,F5.1,2X,F4.2,2X,F9.5,
      12X,F8.5,2X,F9.5,2X,F5.1,2X,F8.5,2X,F10.5,2X,3F7.1)
 50    FORMAT('1',14X,'LISTING OF KALMAN FILTER PARAMETERS',
      1//,'    I   J KK   K      G1(K)        G2(K)         T(K)        ',
      2'E(K)',6X,'GATE(K)',/)
 51    FORMAT(7F11.5)
 52    FORMAT(6F13.5)
 57    FORMAT('1 LISTING OF BEARING ANGLES-OF-ARRIVAL AND ',
      1'AIRCRAFT NAVIGATION DATA',///,'    N    HDGD(N)         ',
```

```
      2'SLAD(N)    SLOD(N)     BRNGD(N) THETAD(N) THETA(N)',//)
58     FORMAT(' ',I3,4F11.5,F10.5,F9.5)
59     FORMAT('1',28X,'NTAR =',I3,//)
60     FORMAT(' ',I3,2F8.1,F7.2,3F12.5)
61     FORMAT('0 UNFILTERED EMITTER TARGET DATA INITIALLY ',
      1'ASSOCIATED TO NTAR =',I3,//,' EMITTER BEARING LINES',
      2' OF SIMILAR FREQUENCY AND PRF OR PRF MULTIPLE',////,
      3' NUM   FREQ      PRF      PW      HDGD         BRNGD',
      4'        THETAD',/)
62     FORMAT('1  INITIAL JSET DATA',//)
63     FORMAT('    JSET(',I1,',',I2,')=',I3)
64     FORMAT('0',4I3,4F10.5)
65     FORMAT(' ',4I3,4F10.5,F12.5)
66     FORMAT('1  FINAL JSET DATA',//)
67     FORMAT('0FILTERED AND SMOOTHED EMITTER TARGET DATA ',
      1'CORRELATED TO NTAR =',I2,///,' NUM   FREQ     PRF',
      2'     PW      THETAD        THTD        THTD1',/)
68     FORMAT('0',//,'  SMOOTHED INITIAL BEARING ANGLE =',
      1F10.5,//,'     FILTERED FINAL BEARING ANGLE =',F10.5//)
70     FORMAT('0 PLANE TRIANGULATION SOLUTION OF EMITTER ',
      1'LOCATION',//,'               EMITTER LATITUDE =',F10.5,'N'
      2//,'          EMITTER LONGITUDE =',F10.5,'W')
       STOP
       END
```

```
C     ***********************************************************
C     *                                                         *
C     *      EXTENDED KALMAN FILTER COMPUTER SUBROUTINE         *
C     *                                                         *
C     ***********************************************************
C
C      EMITTER LOCATION SOLUTION USING EXTENDED KALMAN
C      FILTERING TECHNIQUES
C      EXTENDED KALMAN FILTER INITIALIZATION
C
        DO 20 I=1,NTAR
        KI=JSET(I,1)
        P11(KI)=2.0
        P12(KI)=0.5
        P22(KI)=1.0
        R(KI)=1.0
        XTD(KI)=-117.42503
        YTD(KI)=33.24187
        XT(KI)=XTD(KI)/PIRAD
        YT(KI)=YTD(KI)/PIRAD
        AT1=(XT(KI)-SLO(KI))*COS(YT(KI))
        AT2=YT(KI)-SLA(KI)
        TX(KI)=ATAN2(AT1,AT2)
        ER(KI)=THETA(KI)-TX(KI)
        DM=(YTD(KI)-SLAD(KI))**2+(XTD(KI)-SLOD(KI))**2
       1*(COS(YT(KI)))**2
        DMX(KI)=(YTD(KI)-SLAD(KI))*COS(YT(KI))/DM
        DMY(KI)=-(XTD(KI)-SLOD(KI))*((YTD(KI)-SLAD(KI))*
       1SIN(YT(KI))+COS(YT(KI)))/DM
        DG=P11(KI)*DMX(KI)**2+2.0*P12(KI)*DMX(KI)*DMY(KI)
       1+P22(KI)*DMY(KI)**2+R(KI)
        GX(KI)=(P11(KI)*DMX(KI)+P12(KI)*DMY(KI))/DG
        GY(KI)=(P12(KI)*DMX(KI)+P22(KI)*DMY(KI))/DG
C
C      START OF KALMAN FILTER PREDICTION PROBLEM
C
        NSTA=NST(I)
        DO 21 J=2,NSTA
 22     K=JSET(I,J)
        KK=JSET(I,J-1)
        Q11(K)=0.10
        Q12(K)=0.0
        Q22(K)=0.10
        R(K)=1.0
C
C      START OF EXTENDED KALMAN FILTER RECURSION EQUATIONS
C
        P11(K)=P11(KK)*(1.0-GX(KK)*DMX(KK))-P12(KK)*GX(KK)*
       1DMY(KK)+Q11(K)
        P12(K)=P12(KK)*(1.0-GX(KK)*DMX(KK))-P22(KK)*GX(KK)*
       1DMY(KK)+Q12(K)
        P22(K)=P22(KK)*(1.0-GY(KK)*DMY(KK))-P12(KK)*GY(KK)*
       1DMX(KK)+Q22(K)
        DM=(YTD(KK)-SLAD(K))**2+(XTD(KK)-SLOD(K))**2
       1*(COS(YT(KK)))**2
        DMX(K)=(YTD(KK)-SLAD(K))*COS(YT(KK))/DM
        DMY(K)=-(XTD(KK)-SLOD(K))*((YTD(KK)-SLAD(K))*
       1SIN(YT(KK))+COS(YT(KK)))/DM
        DG=P11(K)*DMX(K)**2+2.0*P12(K)*DMX(K)*DMY(K)+
       1P22(K)*DMY(K)**2+R(K)
        GX(K)=(P11(K)*DMX(K)+P12(K)*DMY(K))/DG
        GY(K)=(P12(K)*DMX(K)+P22(K)*DMY(K))/DG
        AT1=(XT(KK)-SLO(K))*COS(YT(KK))
        AT2=YT(KK)-SLA(K)
        TX(K)=ATAN2(AT1,AT2)
        ER(K)=THETA(K)-TX(K)
        XTD(K)=XTD(KK)+GX(K)*ER(K)
        YTD(K)=YTD(KK)+GY(K)*ER(K)
        XT(K)=XTD(K)/PIRAD
        YT(K)=YTD(K)/PIRAD
 21     CONTINUE
 20     CONTINUE
```

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