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A RESOURCE ALLOCATION MODEL
FOR A WEAPON SYSTEM MANAGER

by

Hal Bacon Powell

Thesis Advisor: A. W. McMasters

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A Resource Allocation Model

For a Weapon System Manager

by

Hal Bacon Powell
Lieutenant Commander, Supply Corps, United States Navy
B.E.E., Georgia Institute of Technology, 1959

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Author _Hal Bacon Powell_

Approved by: _Alan W. McMasters_
Thesis Advisor

Chairman, Department of Operations Research and
Administrative Sciences

Academic Dean

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I. INTRODUCTION

A weapon system manager is responsible for the development, procurement, operation, maintenance and support of a particular weapon system.

Repair parts and other consumables are currently available to the weapon system operator and maintenance activities through the Navy supply system. Although the weapon system manager is responsible for the support of his weapon system, he does not have control over those resources (people and dollars) necessary to provide supply support [1]. Items with low demand rates are procured and held on hand at operating activities in accordance with allowance lists tailored to meet the requirements of each activity and their associated weapon systems. The range and depth of allowance items carried, however, depend upon "stock funds" made available for this purpose from the inventory manager of the particular "cognizance class" of the material. The allocation of these funds is <u>not</u> made in accordance with the end use of material.

Items with high demand rates are stocked on the basis of usage. The funds made available for stocking these items in the supply system are a function of the demand rate and unit cost of the item. Within a cognizance class, no significance is placed on the end use of the item.

Supply support personnel are normally allocated to support activities on the basis of activity workload by organizational type commanders.

The purpose of this thesis is to propose an analytical model of an information system that will allow a weapon system manager to allocate resources, such as stock funds and supply personnel, to supply support activities so as to attain optimum support for his weapon system. The model takes into account that the supply activity managers are not under the administrative control of the weapon system manager and that their operational objectives may (in general) differ from the weapon system manager goals.

In the hypothetical multi-echelon supply support system as shown in Figure 1:

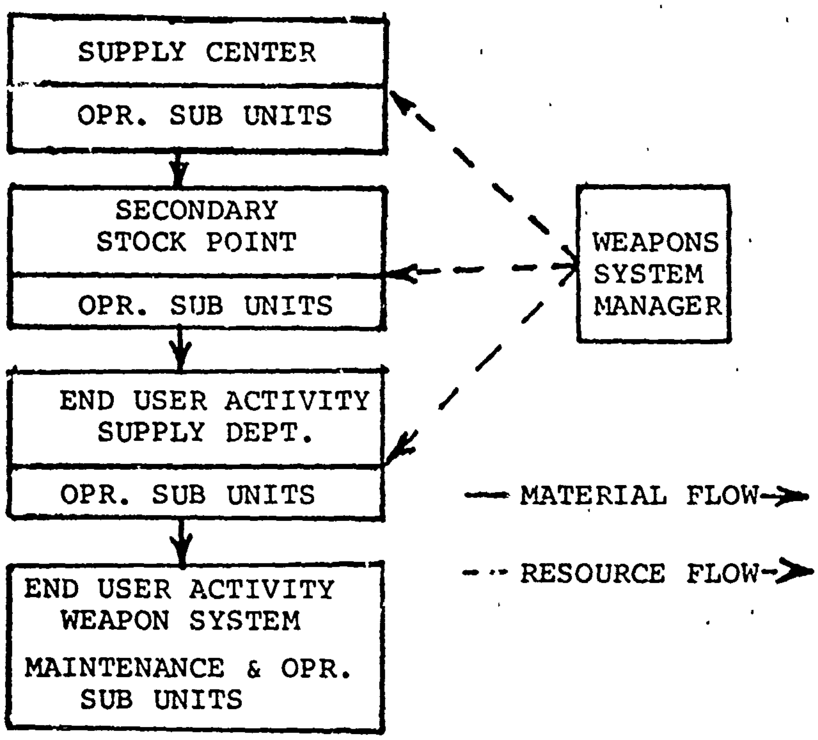

Figure 1.

The Supply Center procures and stocks weapon system material for a geographical sector of the supply system. Its operational sub units perform logistic functions such as determining replenishment requirements, procuring, warehousing and shipping material.

The secondary stock point requisitions material from the supply center with stock fund monies, stores material and ships it to end user activity upon demand. Its operational sub units perform the same logistic functions as above.

The Supply Department of an end user activity stocks, replenishes and issues material to maintenance and weapon system operating sub units of the same activity. The Supply Department operational sub units determine stock requirements, requisition stocks from the Secondary Stock Point, store the material and issue it to end users. The end user activity maintenance sub unit requisitions parts from the Supply Department with operational funds. The weapon system operational sub units requisition consumable material from the Supply Department with operational funds.

The weapon system manager in this hypothetical support system has been given control over stock fund monies and supply support personnel. His problem is to allocate these resources so as to attain optimal support for his weapon system. He must do this even though he does not have administrative control over the support activities.

The Generalized Goal Decomposition (GGD) model was developed by Timothy W. Ruefli [2] for a similar problem--allocating resources under the Program Planning and Budgeting System of the Federal Government. Transformation of the GGD model into the multi-echelon supply weapon system manager model seemed a reasonable approach to the weapon system manager's problem since the "weakness" of the GGD model is precisely what was required in the weapon system support model. The GGD model assumes an administratively weak central manager who sets policies and allocates resources, strong middle managers who drive the system by setting prices internal to the system, and finally, sub unit managers of operational units who have the required information necessary to make optimal management decisions for each of their units, but do not have the necessary information to make optimal solutions for the total system (see Fig. 2).

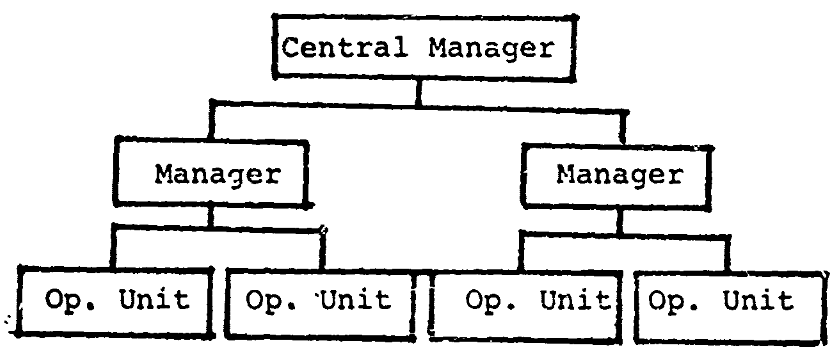

Figure 2.

By turning the model of Fig. 1 "sideways" so that (1) the central manager corresponds to the weapon system manager, (2) the middle managers correspond to the manager of the supply activities, and (3) the operational units correspond to the operation division of the supply managers, the transformation of the support system of Fig. 1 into the organization of Fig. 2 is illustrated by Fig. 3.

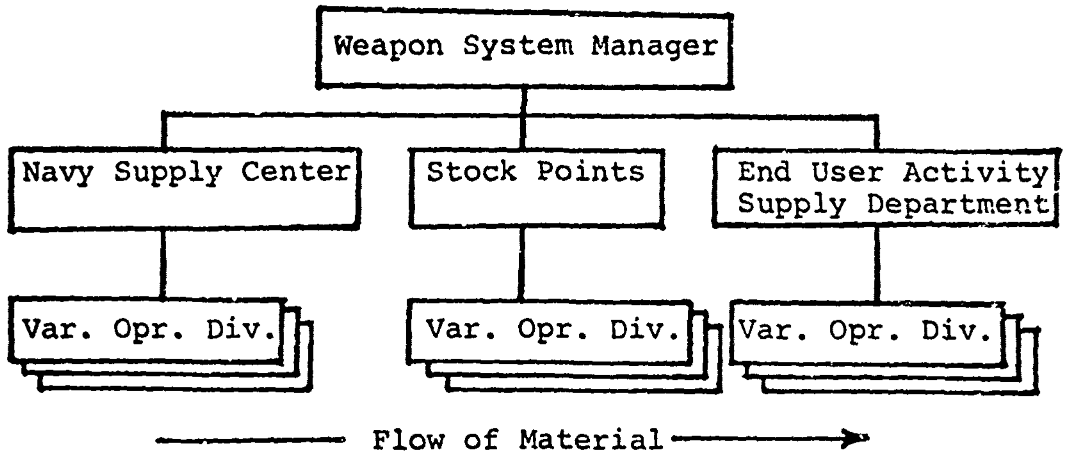

Figure 3.

The solving of the system problem involves interactions among the three levels of Fig. 3. In the Ruefli procedure the solution process begins by the weapon system manager making a preliminary allocation of resources and request for services.

Since each manager of a supply activity has his own opinion (different from other managers) about the relative importance of resources consumed and outputs generated by his activity, he will utilize those resources allocated to him in a manner unique to his activity and his subjective desires. His desires will be affected by the environmental, psychological and political climate at his activity and by his previous experience as a supply manager. He will therefore establish prices for resources and outputs that he alone considers appropriate. The activity manager will not normally have complete information about the sub units under his control. He must rely on proposals from the managers of his sub units to accomplish those goals he considers important. It is assumed that he can develop criteria for trade-offs among resources. Therefore, some measure of importance must be established for each resource consumed and each output produced. This measure of importance will be a function of the internal prices referred to earlier.

Given the set of prices established by the activity manager, the sub unit manager can determine proposals associated with optimal solutions for his unit. As the activity manager receives proposals from each of his sub units, he can better determine the value of the resources and outputs. Thus, he can revise his original prices to agree with the additional information he has received. He will pass these new prices to the weapon system manager with the expectation of receiving a larger allocation for those resources with higher prices and a smaller allocation of those resources which have little value to him.

The weapon system manager will be able to determine from the prices received from the activity manager how his resources should be allocated and how his production goals should be adjusted to allow the system to attain optimal support for his weapon system. Each time the system manager makes a reallocation of goals, the activity manager computes a new set of prices, and the sub unit manager computes a new set of proposals. This process continues until the deviations from the weapon system manager's goals are at a minimum. Neither readjustment of goal level nor modifications of proposals on the part of the supply activity sub units will yield a net decrease in the deviations from the goal levels as a whole. Figure 4 provides a schematic diagram of the solution process.

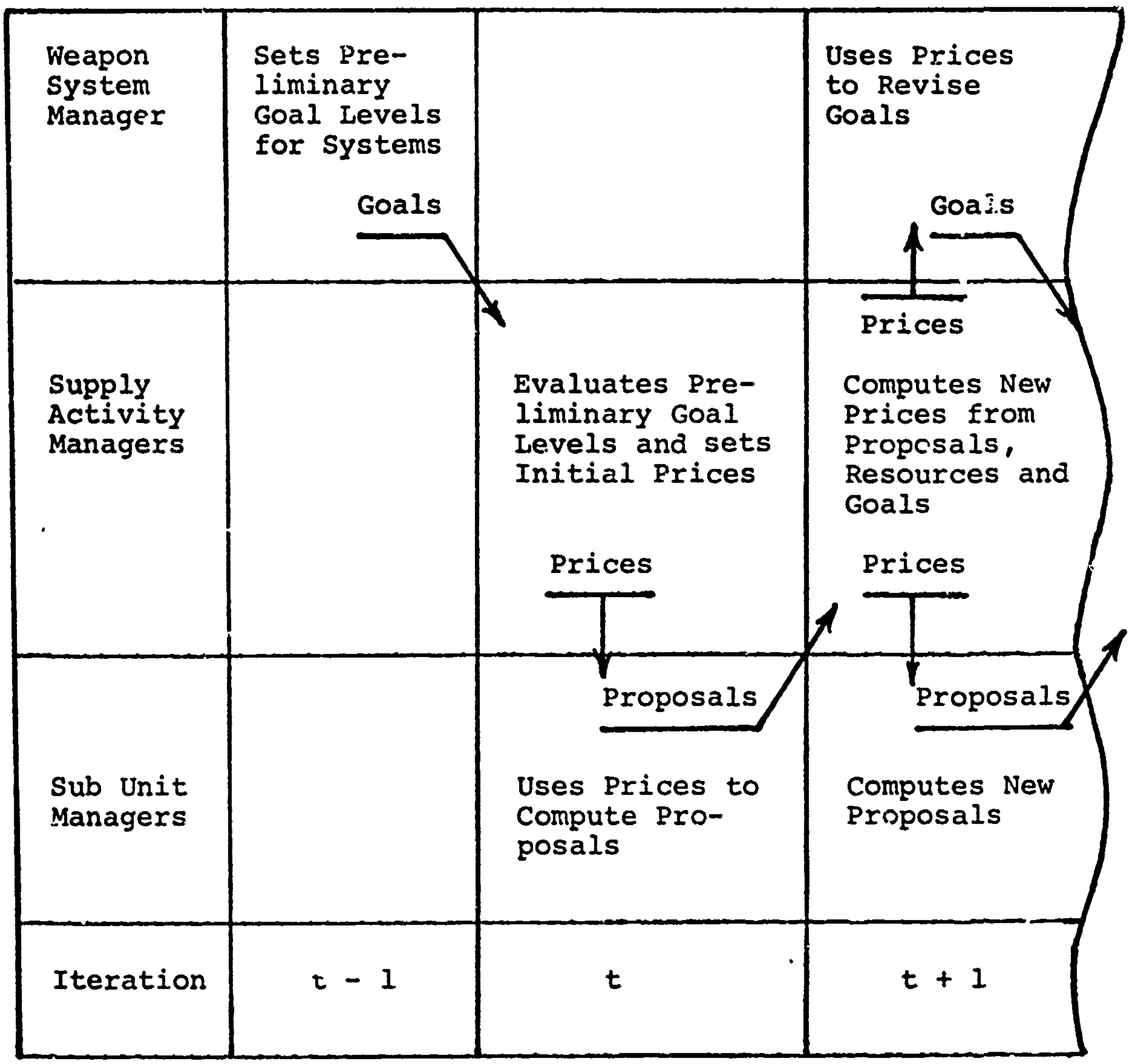

Figure 4.

The GGD model [1] has a linear problem for each level of the system:

1. The Sub Unit Manager's Problem:

Minimize: $\Pi_k^{(t)} P_{kj}^{(t+1)}$,

Subject to: $D_{kj} P_{kj}^{(t+1)} \geq F_{kj}$,

and: $P_{kj}^{(t+1)} \geq 0$.

where:

$$\Pi_k^{(t)} = (\Pi_{k1}^{(t)}, \Pi_{k2}^{(t)}, \ldots \Pi_{km}^{(t)})$$

is a vector of internal prices generated by the supply activity manager k in period t for m resources and requirements.

D_{kj} = Technologic matrix describing the operation of the sub unit j under supply activity k.

F_{kj} = A vector of stipulations which affect the production feasibility of sub unit j under supply activity k.

$P_{kj}^{(t+1)}$ = A proposed solution (mix of resource inputs and production outputs) in period t + 1 for sub unit j under supply activity k. It is a (m x 1) column vector of variables.

2. The Supply Activity Manager's Problem:

Minimize: $W_k^+ Y_k^+ + W_k^- Y_k^-$,

Subject to: $\sum_j \sum_t P_{kj}^{(t)} \lambda_t^{(j)} - I Y_k^+ + I Y_k^- = G_k^{(t)}$,

$$\sum_t \lambda_t^{(j)} = 1 \qquad j = 1, 2, \ldots,$$

and: $\lambda_t^{(j)}, Y_k^+, Y_k^-, \geq 0$.

where:

W_k^+ = a (1 x m) row vector of weights assigned to positive deviations from the goals by supply activity manager k.

W_k^- = a (1 x m) row vector of weights assigned to negative deviations from the goals by supply activity manager k.

$P_{kj}^{(t)}$ = a (m x 1) column vector proposal from sub unit j under supply activity k in period t.

I = (m x m) identity matrix.

$G_k^{(t)}$ = a (m x 1) vector of goals (resources and outputs) allocated to supply activity k for period t.

Y_k^+ = a (m x 1) column vector of positive deviations from the goals by supply activity k.

Y_k^- = a (m x 1) column vector of negative deviations from the goals by supply activity k.

$\lambda_t^{(j)}$ = activity level (as a fraction of the proposed level) of sub unit j proposal made in period t.

3. The Weapon System Manager's Problem:

Maximize: $$\sum_k \pi_k^{(t)} G_k^{(t+1)},$$

Subject to: $$\sum_k R_k G_k^{(t+1)} + S_0 = G_0,$$

and $$G_k^{(t+1)} \geq 0 \ (k = 1, 2, \cdots, m),$$
$$S_0 \geq 0.$$

where $\pi_k^{(t)}$ = a (1 x m) row vector of internal prices generated by supply activity manager k during period t.

R_k = matrix of coefficients relating the goal levels of the supply activities. Provides transformation rates or weights that relate G_k's to G_o .

G_o = a (m x 1) column vector of global (total system) resources and requirements.

$G_k^{(t+1)}$ = a (m x 1) column vector of revised goals to be allocated to supply activity k for period t + 1.

S_o = a (m x 1) column vector of slack variables.

The three problems are solved sequentially in accordance with Fig. 4, where the Goals correspond to $G_k^{(t)}$ of the weapon system managers problem; the prices correspond to the value of $\Pi_k^{(t)}$, the dual variables associated with the goal constraints, of the supply activity manager's problem; finally, (3) the proposals are the same proposals, $P_{kj}^{(t+1)}$ of the sub unit manager's problem. If the goal levels, alternatives (proposals), and shadow prices are generated using the rules of the simplex procedure, the process will terminate in a finite number of iterations [3]. The initial allocation of resources by the weapon system manager and the initial prices assigned by the supply activity manager are done ad hoc.

II. MODEL FORMULATION

A. GENERAL

As observed in the previous section, Ruefli's Generalized Goal Decomposition model is similar in concept to the resource allocation model for a weapon system manager. There are, however, differences that should be noted.

The operating unit problem was represented as a linear program in the GGD model. A linear program would not, in general, adequately describe the behavior of a supply activity sub unit. Further, it is not necessary that a sub unit of a stock point have the same objectives and constraints that a sub unit of an end user activity Supply Department. The sub model used to describe the behavior of the sub units in the resource allocation model may vary from the simplest deterministic lot size model [4] to complex multi-item probabilistic time weighted backorder model [5]. For sake of illustration, a form of both will be used so as to (1) demonstrate the ability of the allocation model to find a solution for a system structure that has diverse objectives among its supply activities, and (2) to illustrate the flexibility of the allocation model concept when applied to multi-echelon supply systems. Since only a two echelon supply system model is necessary to illustrate the concepts, this thesis will be limited to the interactions of (1) stock point with two sub units, (2) an end user activity supply department with two sub units, and (3) a weapon system manager (Fig. 5).

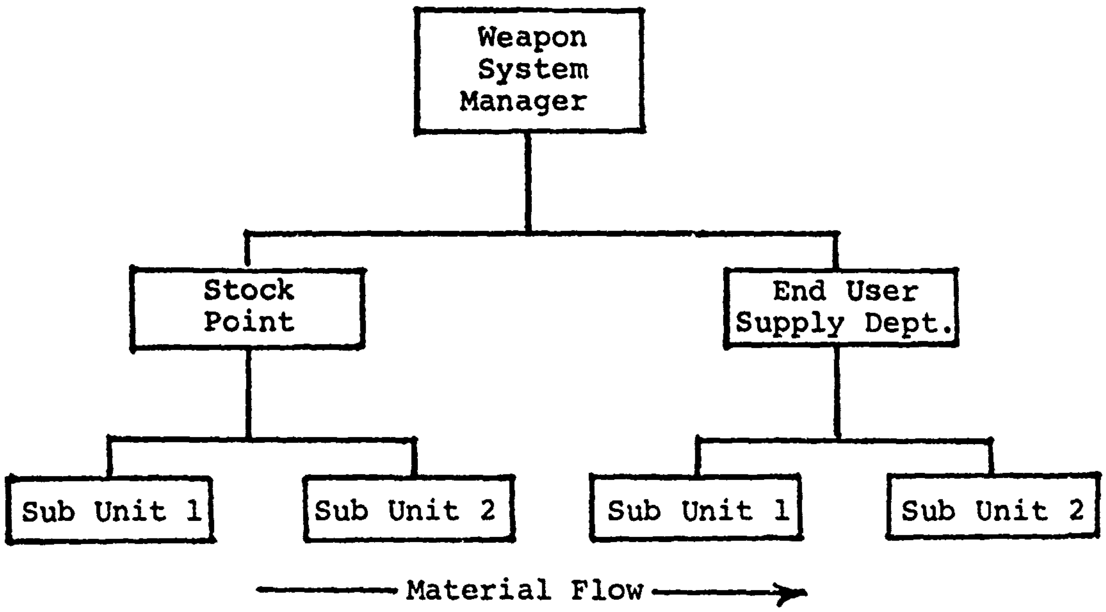

Figure 5.

B. THE STOCK POINT SUB UNIT PROBLEM

Currently, most supply activity in the Naval Supply System use a form of a deterministic lot size model [4] to determine their stock requirements. Therefore, it will be assumed that the Stock Point behavior can be so described. The objective of the stock point is to minimize cost while maintaining enough stock on hand so as to fill all demands. Since demand is assumed deterministic, "perfect" supply availability (no stock outs) can be attained.

It will be assumed for simplicity that each of the sub units stock only one item.

The sub units are assumed to be evaluated by the Stock Point Manager on three points: (1) the total cost for operating their unit, (2) the cost of holding inventory at their unit, and (3) the number of people required to operate their unit.

The total cost K, for operating the stock point sub unit is the sum of (1) ordering cost equal to $\frac{\lambda}{Q}$ A, where A is the cost of processing an order, λ is the demand per unit time for material and Q is the quantity of material ordered by each order, and (2) holding cost equal to IC $\frac{Q}{2}$, where IC is the inventory carrying cost per unit of inventory and $\frac{Q}{2}$ is the average inventory on hand [7]. Therefore, $K = \frac{\lambda}{Q} A + IC \frac{Q}{2}$.

The cost, g_1, of holding inventory, as above, is IC $\frac{Q}{2}$.

The number of people, g_2, required to operate the sub unit is assumed to be a linear function of the number of orders processed. Therefore, $g_2 = \frac{\lambda}{Qp}$, where $\frac{\lambda}{Q}$ is the number of orders processed per period and p is a factor equal to average number of people required to process them.

The stock point sub unit problem is:

minimize: $$Z = \pi_K K + \pi_1 g_1 + \pi_2 g_2,$$

such that: $$K = \frac{\lambda A}{Q} + \frac{ICQ}{2},$$

$$g_1 = \frac{ICQ}{2},$$

$$g_2 = \frac{\lambda}{Qp},$$

and $$K, g_1, g_2, Q \geq 0.$$

where: π_K = price for an operational cost dollar,

π_1 = price of an inventory holding cost dollar,

π_2 = price per person required to operate the unit.

This problem is nonlinear in the decision variable Q in both the objective function and the constraints. The constraints region is convex as is the objective function; therefore, a solution can be obtained from setting

$$\frac{\partial Z}{\partial Q} = 0$$

and solving for Q.

C. THE SUPPLY DEPARTMENT SUB UNIT PROBLEM:

The Supply Officer of a Naval activity which operates weapon systems is likely to put great importance on the length of time a requisition, not filled by his stock, is held as a backorder on his department. The critical resources necessary for his operation sub units are normally stock fund monies and supply support personnel. Therefore, it is assumed he will evaluate his sub units on: (1) time weighted back-orders outstanding (i.e., No. backorder x length of time out-standing in their unit), (2) the cost of holding inventory at their unit, and (3) the number of people required to operate their unit. For simplicity of illustration, it is assumed each sub unit stocks one item.

The time weighted backorder [5] B, is equal to:

$$\frac{1}{Q}\left[\frac{1}{2}\left[\sigma^2+[r-\mu]^2\right]\Phi\left(\frac{r-\mu}{\sigma}\right)-\frac{\sigma}{2}[r-\mu]\,\phi\left(\frac{r-\mu}{\sigma}\right)\right],$$

where: Q is the quantity per order, r is the reorder level, μ is the expected lead time demand, σ^2 is the variance of the lead time demand; $\Phi(r)$ is $\int_r^\infty \phi(x)dx$, the tail of the normal distribution; and $\phi(x)$ is $\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ the density of the normal distribution.

The cost, g_1, of holding inventory is

$IC[r+\frac{Q}{2}-\mu]$, where IC is the inventory carrying cost and $r+\frac{Q}{2}-\mu$ is the average inventory on hand.

The number of people, g_2, required to operate the sub unit is, as before, $\frac{\lambda}{Qp}$. The Supply Department sub unit problem is:

minimize: $Z = \pi_B B + \pi_1 g_1 + \pi_2 g_2$

such that: $B = \frac{1}{Q}\left[\frac{1}{2}[\sigma^2 + [r-\mu]^2]\,\Phi\left(\frac{r-\mu}{\sigma}\right) - \frac{\sigma}{2}[r-\mu]\,\phi\left(\frac{r-\mu}{\sigma}\right)\right.$,

$$g_1 = IC[r+\frac{Q}{2}-\mu],$$

$$g_2 = \frac{\lambda}{Qp},$$

and $$B, g_1, g_2, Q \geq 0.$$

where:
- π_B = price of a unit backorder per period,
- π_1 = price of an inventory holding cost dollar,
- π_2 = price per person required to operate the unit.

This problem is nonlinear in the decision variables r and Q in both the objective function and constraints. The constraint region is convex as is the objective function [5] but the objective function is not easily differentiated and the calculus cannot be used as before. Reference 5 provides a solution procedure utilizing numerical methods.

D. THE STOCK POINT MANAGER'S PROBLEM

The stock point manager, as stated before, is assumed to consider three measures important in the operation of his activity: (1) the total cost of operating the sub units, (2) the total stock fund monies required to pay for inventory holding cost, and (3) the total number of people required to operate his activity. The operations of the individual sub units are assumed to be independent of each other. Thus, it is reasonable to assume that his objectives will be based on the sum of the two sub units performance measures. It is further assumed that he can weight these objectives "a priori" according to what he considers is their relative worth. It is assumed that the manager receives linear satisfaction returns from each of the performance factors. This assumption is more restrictive but still reasonable for small changes in levels of operations. For example, it may not be true that requiring only half as many people to do a job will double the stock point manager's satisfaction, but it is reasonable that a 10% reduction in personnel requirements will increase his satisfaction approximately 10%. The same is true for the other two measures of performance. Under these assumptions, linearity of objectives and additivity of constraints can be assumed [6].

Therefore, the GGD model for the middle manager will apply in the weapon support system allocation model. Based on his past experience and present environment, it is assumed that the stock point manager can determine the following weights in his objective function:

$W_k^+ = C_k$ when his personal goal for total operational cost is zero. This implies that he considers total operational costs of thi activity important and he would like to reduce these costs as close to zero as possible (W_k^- is meaningless since his activity cannot make a profit).

$W_1^+ = C_1$ when the goal for inventory holding cost is equal to the currently assigned goal from the weapon system manager. Although he feels total operation costs are important, he feels that stock fund monies are C_1/C_k times more important to the system. $W_1^- = 0$ because he feels that if an allocation of stock fund monies is made, there is no utility in not using it.

$W_2^+ = C_2$ because he feels that support personnel are times as important as stock fund monies. $W_2^- = 0$ because not utilizing people assigned, has no worth.

The stock point manager's problem is:

minimize: $$C_k Y_k^+ + C_1 Y_1^+ + C_2 Y_2^+ ,$$

subject to: $$\sum_t \lambda_t^{(1)} P_1^{(t)} + \sum_t \lambda_t^{(2)} P_2^{(t)} - I\, Y_m^+ + I\, Y_m^- = G_1^{(t)},$$

$$\sum_t \lambda_t^{(1)} = 1,$$

$$\sum_t \lambda_t^{(2)} = 1,$$

and: $$\lambda_t^{(1)}, \lambda_t^{(2)}, Y_m^+, Y_m^- \geq 0 \quad t = 1, 2, \cdots$$

where: The $P_i^{(t)}$'s and $\lambda_t^{(i)}$'s are as previously defined, and $G_1^{(t)} = \begin{bmatrix} 0_{(t)} \\ g_1^{(t)} \\ g_2^{(t)} \end{bmatrix}$, $Y_m^+ = \begin{bmatrix} Y_K^+ \\ Y_1^+ \\ Y_2^+ \end{bmatrix}$, $Y_m^- = \begin{bmatrix} Y_K^- \\ Y_1^- \\ Y_2^- \end{bmatrix}$.

E. SUPPLY OFFICER'S PROBLEM:

The Supply Officer, as stated before, is assumed to consider three measures important in the operation of his sub units: (1) time weighted backorders, (2) inventory holding costs for which he requires stock fund monies and (3) the number of people required to operate his sub units. The same assumption about additivity of constraints and linearity of his objective function made in the case of the stock point manager apply to the Supply Officer of the end user activity.

Therefore, the GGD sub model for the middle manager will apply. Based on his past experience and present environment, it is assumed that the supp'y officer has the following weights in his objective function "a priori":

$W_B^+ = C_B$ when his personal goal for total backorders is zero. This implies that he considers total time weighted backorders a very important measure of performance for his activity and would like to reduce backorders as close to zero as possible. (W_B^- is meaningless since his activity cannot have negative backorders.

$W_1^+ = C_1$ when the goal for inventory holding cost equal to the currently assigned goal from the weapon system manager. Although he feels inventory holding costs are important, he feels that suffering a unit period backorder for the weapon system is C_B/C_1 times more costly. $W_1^- = 0$ because he feels if allocated stock funds are not used they have no worth.

$W_2^+ = C_2$ because he feels that support personnel are C_2/C_1 times as important as stock fund monies and $W_2^- = 0$ because not utilizing people assigned has no worth.

The Supply Officer's problem is:

minimize: $C_B Y_B^+ + C_1 Y_1^+ + C_2 Y_2^+,$

subject to: $\sum_t \lambda_t^{(1)} P_1^{(t)} + \sum_t \lambda_t^{(2)} P_2^{(t)} - I\,Y_m^+ + I\,Y_m^- = G_2^{(t)},$

$$\sum_t \lambda_t^{(1)} = 1,$$

$$\sum_t \lambda_t^{(2)} = 1,$$

and $\lambda_t^{(1)},\ \lambda_t^{(2)},\ Y_m^+,\ Y_m^- \geq 0 \quad t = 1, 2, \cdots,$

where: The $P_i^{(t)}$'s and $\lambda_t^{(i)}$'s are as previously defined,

and $G_2^{(t)} = \begin{bmatrix} 0 \\ g_1^{(t)} \\ g_2^{(t)} \end{bmatrix}, \quad Y_m^+ = \begin{bmatrix} Y_K^+ \\ Y_1^+ \\ Y_2^+ \end{bmatrix}, \quad Y_m^- = \begin{bmatrix} Y_K^- \\ Y_1^- \\ Y_2^- \end{bmatrix}.$

F. THE WEAPON SYSTEM MANAGER'S PROBLEM

The weapon system manager wishes to maximize the supply support of his weapon system. He is not in a position to evaluate the support directly; therefore, he must rely on the information he can get from the various activities that support his weapon system and those which are supported. Because the activities which are supported would not be able to determine what would give better support, it is reasonable to assume that only the supporting activities have enough information to be of use. Because the managers of these various activities will value resources and requirements differently, a reasonable approach to the problem would be to set policies for the system and then allocate resources such that deviation from these policies is at a minimum. The GGD model sets forth a formal structure to provide the weapon system manager with the information to determine "what x amount of resources will provide in weapon system support." The use of the Π values (dual variables) in his objective function is the same as considering <u>all</u> the constraints (at all echelons) of the system simultaneously [7]. Therefore, it is reasonable to assume that maximizing the value of resources by allocating them to the strongest need is a linear objective. The weapon system manager redistributes resources of the total system within the system. Thus it is reasonable to assume that his problem constraints are additive. Therefore, the GGD sub model for the central manager will apply.

The weapon system manager's problem is:

maximize: $$\sum_t \left[\pi_{11}^{(t)} G_{11}^{(t)} \gamma_t^{(1)} + \pi_{21}^{(t)} G_{21}^{(t)} \gamma_t^{(1)} + \pi_{12}^{(t)} G_{12}^{(t)} \gamma_t^{(2)} + \pi_{22}^{(t)} G_{22}^{(t)} \gamma_t^{(2)} \right],$$

subject to $$\sum_t \left[\begin{bmatrix} G_{11}^{(t)} \\ G_{21}^{(t)} \end{bmatrix} \gamma_t^{(1)} + \begin{bmatrix} G_{12}^{(t)} \\ G_{22}^{(t)} \end{bmatrix} \gamma_t^{(2)} \right] + \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} G_{10} \\ G_{20} \end{bmatrix},$$

subject to: $\sum_t \gamma_t^{(1)} = 1,$

$$\sum_t \gamma_t^{(2)} = 1,$$

and $\gamma_t^{(1)}, \gamma_t^{(2)}, S_1, S_2 \geq 0 \qquad t = 1, 2, \ldots,$

where: π_{ij} = value of resource i according to supply activity manager j,

G_{i0} = total system quantity of resource i,

$G_{ij}^{(t)}$ = quantity of resource i allocated to supply activity j during iteration t,

and $$G_{ij}^{(t+1)} = \sum_t G_{ij}^{(t)} \gamma_t^{(j)}.$$

It should be noted that the Π values received from the various supply activity managers are affected by his personal scale of values. Therefore, the weapon system manager should normalize the Π values before using them in the problem. When an excess of a particular resource exists at a supply activity, the Π value for that resource will be zero because the constraint will not be binding for that goal. In this case, the supply activity is required to tell the weapon system manager how many units are required of the resource to maintain the present activity level of the supply activity. In a real world situation this statement would be, "The value of more resource i is zero as long as x amount is allocated to this activity." The weapon system manager must then place an additional constraint in his problem assuring the allocation of x units of resource i to the supply activity. This situation can be observed in the example problem.

III. EXAMPLE PROBLEM SOLUTION

To illustrate the solution procedure of the "Resource Allocation Model for a Weapon System Manager," a numerical example is presented. The following parametric values are assumed. The weapon system manager's total resources, G_o, are: G_{10} = \$2000 in stock fund monies and G_{20} = 24 supply support personnel. The values assigned to the resources by the stock point manager are: $C_k = 1$, $C_1 = 10$ and $C_2 = 100$. The values assigned by the Supply Officer of the end use activity are: $C_B = 200$, $C_1 = 1$ and $C_2 = 200$. The sub units of the supply activities are assumed to have the following operational parameters: Stock Point Sub Unit Number One, $\lambda_1 = 200$, $A_1 = 40$, $IC_1 = 10$ and $p_1 = 1$; Stock Point Sub Unit Number Two, $\lambda_2 = 1000$, $A_2 = 100$, $IC_2 = 20$ and $p_2 = 2$; Supply Department Sub Unit Number One, $\lambda_1 = 10$, $IC_1 = 100$, $\mu_1 = 10$, $\sigma_1 = 10$ and $p_1 = 0.5$; and Supply Department Sub Unit Number Two, $\lambda_2 = 100$, $IC_2 = 50$, $\mu_2 = 20$, $\sigma_2 = 20$ and $p_2 = 1$.

The solution procedure begins with the weapon system manager making an ad hoc resource allocation: Stock Point, $G_{11}^{(1)}$ = \$600 and $G_{12}^{(1)}$ = 15 people; Supply Department, $G_{12}^{(1)}$ = \$1400 and $G_{22}^{(1)}$ = 9 people. After receiving the allocation, the stock point manager passes the following prices to his sub units: $\Pi_K^{(1)} = 1$, $\Pi_1^{(1)} = 0$ and $\Pi_2^{(1)} = 0$. (He wants to minimize total operating cost as much as possible.) The Supply Officer passes to his sub units the following prices: $\Pi_B^{(1)} = 200$, $\Pi_1^{(1)} = 1$ and $\Pi_2^{(1)}$ equal to 200.

The sub units now have the required information necessary to make their first proposals. Stock Point Sub Unit Number One proposes $P_1^{(1)}$, a vector where: $K_1 = 400$, $g_{11} = 200$ and $g_{21} = 5$. Stock Point Sub Unit Number Two proposes $P_1^{(2)}$, a vector where: $K_2 = 2000$, $g_{12} = 1000$ and $g_{22} = 5$. Supply Department Sub Unit Number One finds a solution to his problem but g_{21} is equal to zero. This implies that it would be more economical to incur backorder cost than to stock the material. If it is assumed, however, that the Supply Officer will not allow the disestablishment of the sub unit, he would change the non-negativity constraint to $g_{21} \geq 1$. The sub unit manager's solution to the modified problem allows him to make his first proposal, $P_1^{(1)}$, a vector where: $B_1 = 4.29$, $g_{11} = 100$ and $g_{21} = 1$. Supply Department Sub Unit Number Two proposes $P_1^{(2)}$, a vector where: $B_2 = 2.87$, $g_{12} = 1450$ and $g_{22} = 4$.

The stock point manager has the following problem in the second iteration:

$$\text{minimize} \quad 1Y_K^+ + 10Y_1^+ + 100Y_2^+,$$

$$\text{subject to} \quad \begin{bmatrix} 400 \\ 200 \\ 5 \end{bmatrix} \lambda_1^{(1)} + \begin{bmatrix} 2000 \\ 1000 \\ 5 \end{bmatrix} \lambda_1^{(2)} - \begin{bmatrix} Y_K^+ \\ Y_1^+ \\ Y_2^+ \end{bmatrix} + \begin{bmatrix} Y_K^- \\ Y_1^- \\ Y_2^- \end{bmatrix} = \begin{bmatrix} 0 \\ 600 \\ 15 \end{bmatrix},$$

$$\lambda_1^{(1)} = 1,$$

$$\lambda_1^{(2)} = 1,$$

and the Y's and λ's non-negative.

The stock point manager is interested in the dual of this problem. The solution: $\Pi_K^{(2)} = 1$, $\Pi_1^{(2)} = 10$ and $\Pi_2^{(2)} = 0$ if $G_2 \geq 10$, gives him his prices to be passed to the sub units and weapon system manager during this, the second iteration. Similarly, the Supply Officer finds his new prices from the dual of his problem:

minimize $\quad 200\, Y_B^+ + 1\, Y_1^+ + 200\, Y_2^+,$

subject to
$$\begin{bmatrix} 4.29 \\ 100 \\ 1 \end{bmatrix} \lambda_1^{(1)} + \begin{bmatrix} 2.87 \\ 1450 \\ 4 \end{bmatrix} \lambda_1^{(2)} - \begin{bmatrix} Y_B^+ \\ Y_1^+ \\ Y_2^+ \end{bmatrix} + \begin{bmatrix} Y_B^- \\ Y_1^- \\ Y_2^- \end{bmatrix} = \begin{bmatrix} 0 \\ 1400 \\ 9 \end{bmatrix}$$
$$\lambda_1^{(1)} = 1$$
$$\lambda_1^{(2)} = 1$$

and Y's and λ's non-negative.

His solution is: $\Pi_B^{(2)} = 200$, $\Pi_1^{(2)} = 1$ and $\Pi_2^{(2)} = 0$ if $G_{22} \geq 5$.

The weapon system manager should now have enough information to make a reallocation for the third iteration but his problem is degenerate. Since supply support personnel are in excess (i.e., $\Pi_2^{(1)} = \Pi_2^{(2)} = 0$ and $G_{12} + G_{22} = 24 \geq 10$), it is reasonable to assume he will normalize ~~the~~ prices on the relative weights the supply activity managers placed on the only other resource of the problem. Therefore the Π values received from the stock point manager will be divided by 10. This makes the objective function the same as the only binding constraint in his problem. Therefore it is degenerate. He is assumed to consider himself as a "tie breaker" in this case and will allocate the resource towards the end use activity since he feels that its objective aligns closer to his than those of the stock point. Therefore his second allocation of resources is: $G_{11}^{(2)} = 400$, $G_{21}^{(2)} = 10$, $G_{12}^{(2)} = 1600$ and $G_{22} = 10$. (Note that the $G_{21} \geq 10$ constraint is met.)

The stock point sub units make their second proposals: $P_2^{(1)}$ where $K_1 = 642$, $g_{11} = 70$ and $g_{21} = 14.2$; and $P_2^{(2)}$ where $K_2 = 3633$, $g_{12} = 300$ and $g_{22} = 16.7$. The supply department sub units second proposals are: $P_2^{(1)}$ where $B_1 = 4.29$, $g_{11} = 100$ and $g_{12} = 1$ (same); and $P_2^{(2)}$ where $B_2 = 2.56$, $g_{12} = 1250$ and $g_{22} = 10$.

The Supply Activity managers solve for their third iteration prices. They are: for the stock point - $\Pi_K^{(3)} = 1$, $\Pi_1^{(3)} = 10$ and $\Pi_2^{(3)} = 100$; for the supply department - $\Pi_B = 200$ $\Pi_1 = 0$ if $G_{12} \geq 1550$ and $\Pi_2 = 0$ if $G_2 \geq 5$. These prices and constraints are passed to the sub units and to the weapon system manager.

The weapon system manager has the problem:

$$\text{maximize} \quad 600\,\gamma_1^{(1)} + 500\,\gamma_2^{(1)} + 1400\,\gamma_1^{(2)} + 0\,\gamma_2^{(2)},$$

$$\text{subject to} \quad \begin{bmatrix} 600 \\ 15 \end{bmatrix}\gamma_1^{(1)} + \begin{bmatrix} 400 \\ 10 \end{bmatrix}\gamma_2^{(1)} + \begin{bmatrix} 1400 \\ 9 \end{bmatrix}\gamma_1^{(2)} + \begin{bmatrix} 1600 \\ 10 \end{bmatrix}\gamma_2^{(2)} + \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 2400 \\ 24 \end{bmatrix},$$

$$\gamma_1^{(1)} + \gamma_2^{(1)} = 1,$$

$$15\gamma_1^{(1)} + 10\gamma_2^{(1)} \geq 10,$$

$$\gamma_1^{(2)} + \gamma_2^{(2)} = 1,$$

$$1400\,\gamma_1^{(2)} + 1600\,\gamma_2^{(2)} \geq 1550,$$

$$9\,\gamma_1^{(2)} + 10\,\gamma_2^{(2)} \geq 5,$$

and γ's non-negative.

The solution: $\gamma_1^{(1)} = .25$, $\gamma_2^{(1)} = .75$, $\gamma_1^{(2)} = .25$ and $\gamma_2^{(2)} = .75$; implies that: $G_{11} = 450$, $G_{21} = 11.75$, $G_{12} = 1550$ and $g_{22} = 9.95$.

During the same iteration (third), the sub units make their proposals. Stock point sub units propose: $P_3^{(1)}$ where $K_1 = 463$, $g_{11} = 115$ and $g_{12} = 8.7$; and $P_3^{(2)}$ where $K_2 = 3073$, $g_{12} = 370$ and $g_{22} = 13.5$. The supply department sub units have no change in their proposals.

Based on this final set of proposals there are no changes to the activity manager's prices, or the weapon system manager's allocation. The present solution is optimal. A summary of the normalized manager's cost (dissatisfaction) is:

ITERATION	STOCK POINT	SUPPLY DEPT.	TOTAL
1	840	1582	2422
2	1040	1370	2410
3	836	1370	2206

IV. DISCUSSION OF THE MODEL

A. ASSUMPTIONS

The resource allocation model for a weapon system manager was developed to show how the Generalized Goal Decomposition model concept could be used in a multi-echelon supply support structure. The use of the deterministic lot size model and the time weighted backorder model to describe the behavior of the supply activity sub units was to illustrate an application of the concept. In a "real world" application, the assumptions of the two sub models would be too restrictive to give precise results.

The deterministic lot size model (or Economic Order Quantity Model – EOQ) assumes no stock outs. Most supply activities currently use a modification of the EOQ model where the effects of uncertain demand and procurement lead time are offset by a variable safety level model. The model presented by this study does not account for this added complexity.

The Time Weighted Backorder model assumes normally distributed lead time demand. Many conflicting opinions prevail about this assumption. The author, at this writing, has no personal opinion as to its use except to state that it is an assumption of the model and it is a way to incorporate some stochastic influence into the structure.

Each sub unit of the supply activity was assumed to manage one item. This was purely a simplifying assumption and was not reasonable for a "real world" inventory system. Reference 5 gives a specific formulation of a multi-item inventory problem with time weighted backorder objective function. The reference suggests the use of Sequential Unconstrained Minimizaticn Technique (SUMT) for solving the problem. The computational complexity invoked by using the multi-item problem would more than offset analytical gains in this conceptual study.

The number of people required to run the supply activity sub units was assumed to be a linear function of the number of orders processed by the sub unit. "A priori" this is a reasonable assumption as long as the activity levels of the sub units remain near the current operating level. An investigation into increasing and decreasing returns to scale would be necessary to determine for what span of activity levels the linearity assumption is valid.

The time weighted backorder objective function for the Supply Officer is not the complete answer to his problem. He and the weapon system manager want to maximize the number of operational weapon system units by minimizing the number of units not operational for back ordered repair parts. Since one weapon system unit may have <u>many</u> different parts required for its repair or many units may require <u>only one part</u> for their repair, the time weighted backorder formulation is not a complete answer.

The Stock Point Manager's and the Supply Officer's problem assume linear returns of satisfaction. As stated in the formulation, this would be only an approximation to "real world" manager's utility functions. The argument presented by Ref. 6 holds for only small deviations from the manager's goals.

The weapon system manager's objective function was assumed to be linear. The validity of this assumption would depend upon the structure of the system being modeled. However, the use of the Π values as a force to drive the system, has considerable appeal since the Π values are generated through consideration of all the system's constraints simultaneously. In light of this, the linear assumption should yield a good "first" solution to the weapon system manager's problem.

B. GENERALIZATIONS

The relationship of the information structure to the organizational structure in a system affects the performance of the system. The development of the resource allocation model for a weapon system manager was to illustrate how an information structure — as it relates to the organization structure — could be modeled. The model developed is simplistic in nature, but illustrates the concept which is to be illustrated. The organization of the model is composed of a series of information systems. If the tasks associated with those systems are interdependent, it is necessary to consider the interdependences among the information systems. The model deals with this problem because it permits, in part, a representation of the relation between different information structures and the organization structure of the system. The model involves two types of decentralization – the decentralization of the resource allocation process and the decentralization of alternative generation processes in the supply activity sub units. Only the goal setting function of the weapon system manager is centralized. The relationship among the supply activities and the weapon system manager is conceptually similar to the GGD model. The relationship between the supply activity managers and their sub units are similar to the Dantzig-Wolfe decomposition models [8]. As in the GGD model, the weapon system manager's model assumes the manager achieves coordination through goal-setting rather than price setting. Prices are used in the model, but they are generated by the supply activity managers. Therefore, the weapon system manager can be interpreted as a policy setting entity and the supply activity managers as administrative entities. If computation difficulties are acceptable in the supply activity sub unit model, the constraint space need only be convex. Therefore, probabilistic and nonlinear relationships may be utilized [9]. In the example problem, textbook formulas were used to represent the sub units operation. This sterile approach was not necessary for the utilization of the model. Input–output, regression or rule of thumb models could have provided satisfactory results consistent with their ability to track the required relationships. The use of the textbook models in the example was to illustrate an upper bound on the real world system's effectiveness (i.e., the EOQ model assumes no stockouts, steady state, perfect forecasting, no obsolescence, no mistakes, no coffee breaks, etc).

The supply activity manager's models need be linear only in the constraints. The objective function of the manager may be formulated as a quadratic loss function (i.e.,

, but the resultant computation problems are increased. Reference 6 illustrates explicit uses of linear approximation to nonlinear objective (utility) functions when deviation from a central operating point is not extreme. This would normally be the case where analysis is made on a presently operating system. If a quadratic loss function is necessary to obtain the desired results, the quadratic program can be transformed into a linear program using Kuhn-Tucker conditions. Reference 7, pages 575-580 explains this transformation.

The resource allocation model for a weapon system manager assumes he has no management or directive control over the supply activity managers or their sub units. He must attain his desires by coordination, through allocation of resources and requirements.

Allocation of resources is straightforward but allocation of requirements without directional control needs clarification. By way of example: The support effectiveness of an end user will necessarily be a function of the support effectiveness of the next higher echelon of support. Since the model assumes that the various activities are in competition for system resources, allocating most of the resources to the end user in order to increase his support effectiveness may decrease it because of poor performance of the higher echelons due to their lack of resources. If "support effectiveness at activity k" is considered a resource to the end user and a requirement to supply activity k, the end user must make a trade-off analysis as to how much the "support effectiveness of supply activity k" is worth in terms of other resources. This is what the GGD model does by establishing goals as variables and determining the value of each goal to the manager. The relative worth of the various goals to the supply activity manager is the value of the dual variables of his minimization problem. References 7 and 8 discuss the theory and appropriateness of using the values of the dual variables as production shadow prices (i.e., value of resources internal to a production system).

The strength of this model lies in its ability to describe the economic behavior of various supply activity managers when their objective functions do not align with the total system objective function. In fact, there is no total system objective function. Each activity manager values resources and outputs differently than his counterparts at other activities. The model requires the manager to assign his personal values "a priori." The solution to the dual of the supply manager's problem provides the Π value that establishes the relative values of resources and requirements based on what he feels is important <u>and</u> what is important to the system.

V. EXTENSIONS OF THE MODEL

A. THE n-LEVEL STRUCTURE

The Resource Allocation model for a weapon system manager may be extended to cover the general n-level organization. If resources and requirements are to be allocated through more than three (say n) levels of managers, the model can be applied successively to three levels at a time starting (after the preliminary goal distribution) with the three lowest (including the operational units) levels. As prices are generated the model is applied to successively higher levels until the prices reach the uppermost level (the manager with the total resources). He then makes a revised allocation of the resources and requirements and the model is applied to three levels at a time but to successively lower levels until alternate proposals are made. The process continues until an optimum (in the goal programming sense) is reached. A possible Navy application of the n-level model is shown in Fig. 6.

B. STRUCTURE ANALYSIS

The Resource Allocation model for a weapon system manager has a useful feature not included in other decomposition models. Its formulation implies that the solution reached depends on the structure of the organization being modeled. Other decomposition models yield optimal solutions which are independent of the nature of the decentralization. This is true because the purpose of classic decomposition models has

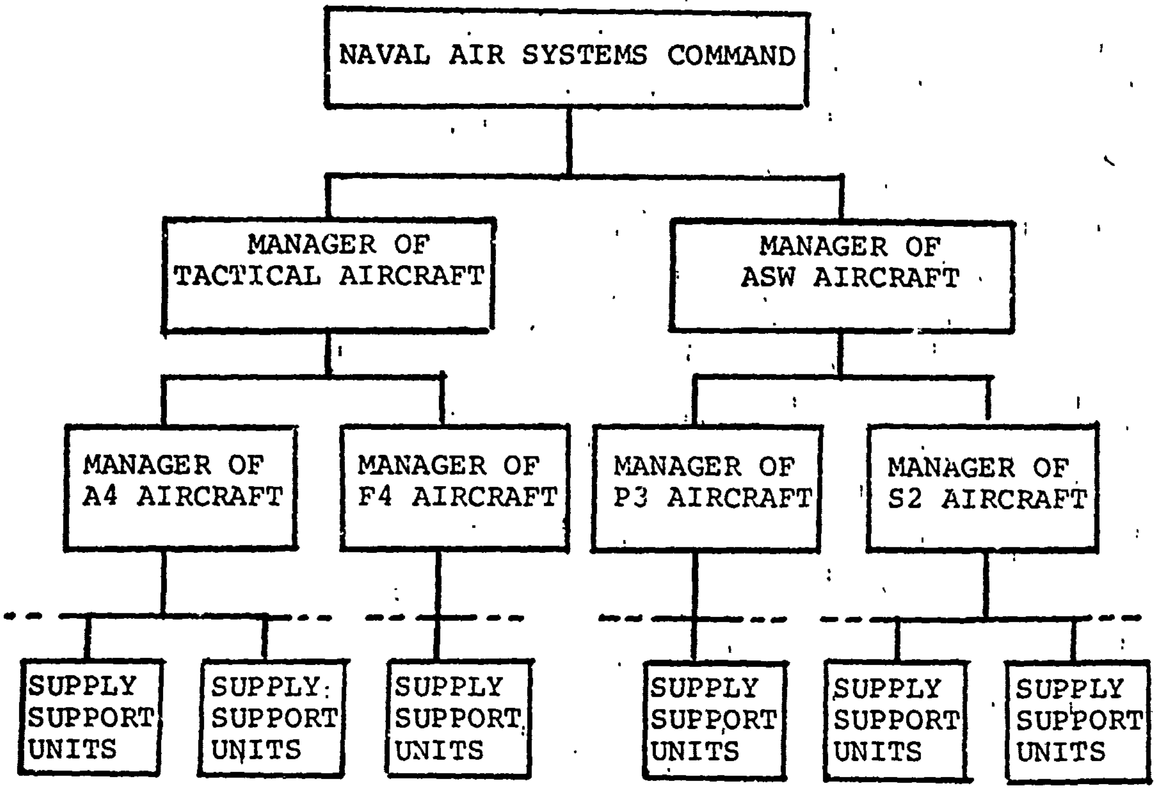

Figure 6

been to find a technique which would find the same solution by a decentralized model that would be found by a centralized model. This is a reasonable objective if the organization is trying to attain the objectives of a strong central manager. This model depends on sub-optimization by middle manager in response to policies set by an administratively weak central manager. If the sub-optimization is ruled out as a possibility, then the dimensions of the organization are relevant only to the mechanics of reaching a solution. Therefore, the effects of the organization are eliminated. However, the weapon system manager's allocation model is sensitive to organization structure. Therefore, it can be used to analyze alternative organizations to determine which structure yields the best benefits. Typical examples would be to combine, omit, and add echelons of supply support to determine how many levels provide the best support for a particular weapon system.

C. MULTI-WEAPON SYSTEM MANAGER'S PROBLEM

Figure 6 illustrates a further area which could be studied by an approach similar to the Allocation model for a weapon system manager. If the supply support units receive resources from many different weapon system managers, the formulation of the model becomes even more complex. The presence of items of support which are common to many weapon systems would not allow the sub units to be segmented into separate support groups.

VI. SUMMARY

A Resource Allocation model for a weapon system manager was synthesized from several sub programs within the structure of the Generalized Goal Decomposition Model. The weapon system allocation model describes the interaction of (1) a weapon system manager who allocates resources, (2) a stock point manager who desires to minimize cost by application of the Economic Order Quantity model at his two sub units, and (3) a Supply Officer of an activity that provides direct weapon system support. The Supply Officer's objective is to minimize time weighted backorders at each of his two sub units.

The concepts of the Generalized Goal Decomposition approach are used to model the information system that permits the weapon system manager to allocate stock fund monies and supply support personnel among the supply activities to attain an optimal system solution which minimizes the supply activity managers' dissatisfaction.

The model takes into account the personal objectives of the supply activity managers. These objectives are, in general, different from other supply managers. The managers are required to assign values (prices) to the resources "a priori." It is these prices that drive the system.

An example problem is presented which illustrates the iterative solution technique required to find the optimal system solution. The solution procedure utilizes Wolf-Dantzig Decomposition procedures between the activity managers and their sub units. The value of the dual variables of the supply activity manager's solution to his own activity decomposition problem is passed to the weapon system manager as an indication of what each resource is worth to the supply activity. The weapon system manager is then able to reallocate his resources to the greatest system need.

The model presented makes many restrictive assumptions for sake of simplicity; however, the purpose of the presentation is to illustrate an analytical approach to a problem involving the personal opinions of the system managers. Thus, the value of the presentation lies more in its concepts than as a model of a real world system.

LIST OF REFERENCES

1. Lowell, W. B., Commander, SC, USN, "Budgeting on a Weapon System Basis," Navy Supply Corps Newsletter, v. 34, No. 10, p. 9-12, October 1971.

2. Ruefli, Timothy W., A Generalized Goal Decomposition Model, Working Paper, WP 70-7, Graduate School of Business of the University of Texas, Austin, Texas, June 1969.

3. Ruefli, Timothy W., Planning in Decentralized Organizations, ph.D. Thesis, Carnegi-Mellon University, 1969.

4. Hadley, G. and Whitin, T. M., Analysis of Inventory Systems, p. 29-40, Prentice-Hall, 1963.

5. Schrady, D. A. and Choe, U. C., Models for Multi-Item Continuous Review Inventory Policies Subject to Constraints, Report NPS 55507104lA, Naval Postgraduate School, p. 2-5, April 1971.

6. Shelly, Maynard W. and Bryan, Glenn L., Human Judgments and Optimality, p. 217-242, Wiley, 1964.

7. Hillier, Fredrick S. and Lieberman, Gerald J., Introduction to Operations Research, p. 520-530, Holden-Day, 1967.

8. Dantzig, George B., Linear Program and Extensions, p. 448-470, Princeton University Press, 1963.

9. Ruefli, Timothy W., "A Generalized Goal Deconponsition Model," Management Science, v. 17, No. 8, p. B505-B518, April 1971.