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MISCELLANEOUS PAPER NO. 4-750

CENTER-LINE DEFLECTION OF PNEUMATIC TIRES MOVING IN DRY SAND

by

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November 1955

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U. S. Army Engineer Waterways Experiment Station
CORPS OF ENGINEERS
Vicksburg, Mississippi

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FOREWORD

This paper discusses a study conducted at the U. S. Army Engineer Waterways Experiment Station (WES) under the sponsorship and guidance of the Directorate of Research and Development, U. S. Army Materiel Command, as part of DA Project 1-V-0-21701-A-046, "Trafficability and Mobility Research," Task 1-V-0-21701-A-046-03, "Mobility Fundamentals and Model Studies."

The study discussed was accomplished by personnel of the Army Mobility Research Branch, Mobility and Environmental Division, WES, under the supervision of Messrs. W. J. Turnbull, W. G. Shockley, and S. J. Knight, and Dr. D. R. Freitag. This paper was prepared for publication in The Journal of Terramechanics, the International Society for Terrain-Vehicle Systems.

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Abstract

Tire deflection data were studied from tests performed in the single-wheel test facilities at the U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi. The tests were performed with a smooth 11.00-20, 12-PR tubular tire moving under a 3000-lb load in mortar sand of various strengths and a smooth 9.00-14, 2-PR tubeless tire moving under an 890-lb load in Yuma sand of various strengths. Plots are presented of the path of a point on the center line of the tires relative to a moving and a fixed frame of reference. Representative plots are included to show the effect of slip, soil strength, and inflation pressure on the path and to compare the paths of a point on towed and powered tires.

CENTER-LINE DEFLECTION OF PNEUMATIC TIRES MOVING IN DRY SAND

D. R. Freitag[1] and M. E. Smith[2]

1. INTRODUCTION

The development of a practicable mathematical description of the performance of wheeled vehicles could be enhanced by a better understanding of the forces acting at the tire-soil interface. The configuration of a moving tire in a yielding soil is considered a "stepping stone" to this understanding. Some studies have been made of the distortion of the cross section of a tire,[3, 4] and recently, as described in this paper, a study was made of the deflection of the center line of a tire moving in air-dry sand.[5] Both studies were a part of the mobility research program in progress in the Mobility and Environmental Division of the U. S. Army Engineer

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[3] U. S. Army Engineer Waterways Experiment Station, Deflection of Moving Tires, A Pilot Study on a 12x22.5 Tubeless Tire, Technical Report No. 3-516, Report 1, Vicksburg, Mississippi, July 1959.

[4] McRae, J. L., and Knight, S. J. "The Terrain-Vehicle Programmes of the U. S. Army Engineer Waterways Experiment Station," Journal of Terramechanics, Vol 1, No. 1, 1964, pp. 98 - 107.

[5] U. S. Army Engineer Waterways Experiment Station, Center-Line Deflection Studies Through July 1963, Technical Report No. 3-516, Report 3, Vicksburg, Mississippi, March 1965.

Waterways Experiment Station (WES), Vicksburg, Mississippi. The work was sponsored by the Environmental Sciences Branch, Research Division, Research and Development Directorate, U. S. Army Materiel Command.

The data used in the center-line deflection study were collected in tests performed in the facilities of the Mobility Section at WES. The smooth 9.00-14, 2-PR tubeless tire was tested in Yuma sand, and the smooth 11.00-20, 12-PR tire with inner tube was tested in mortar sand. In these tests, the movement (in one plane) of a point on the center line of the inner tire surface was measured by a linear-angular gage (Fig. 1). Only data obtained on the first pass through the test section were used in the analyses. Detailed description of test facilities and techniques can be found in a WES publication.[6]

To check the adequacy and accuracy of the measurements of the movement or deflection of the inner tire surface and the assumptions associated with their application, movement of points on the outer tire surface was measured by means of a scratch test.[5, 7] A scratch test consisted primarily of carborundum particles scratching a wax-coated paper fastened to an unyielding surface as a loaded tire was towed on the surface. In general, the movement at the center line of the tire was rearward (opposite to the forward motion of the wheel), while points near the edges of the contact area moved inward toward the center line and, to some extent,

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[6] U. S. Army Engineer Waterways Experiment Station, Performance of Soils Under Tire Loads, Test Facilities and Techniques, Technical Report No. 3-666, Report 1, Vicksburg, Mississippi, January 1963.

[7] Wann, R. L. and Reed, I. F. Studies of Tractor Tire Tread Movements, Paper for presentation at the 1961 winter meeting of ASAE, Chicago, Illinois, December 1961.

forward. Furthermore, a composite of the indicated forward and rearward movement across the face of the tire confirmed the assumed zero-slip condition of the wheel.

2. PNEUMOIDS AND PEDOPNEUMOIDS

The path of a point on the tire surface was reproduced by plotting the deflection measurements with respect to both a fixed and a moving frame of reference. A tire shape is defined as the path of a point on a deforming pneumatic tire relative to a moving reference frame, i.e. the axle center line. If the tire does not deform, its shape is merely a circle. If the tire does deform, but the surface over which the tire moves does not deform, the tire shape within the zone of contact must conform to the contour of the surface. If both the tire and the surface over which it moves are deformable, as in the case of a pneumatic tire moving in a yielding soil, the tire shape represents the balance between the resistance to deformation of the tire and that of the surface. In such a case, therefore, the shape is indicative of the resultant force of the soil on the moving tire, insofar as the resistance to deformation is uniform along the major circumference of the tire.

The path, relative to a fixed reference frame, of a point on a deflecting, torodial, pneumatic tire rolling on an unyielding surface is termed "pneumoid." A similar path, but with the tire traveling on a yielding soil, is called "pedopneumoid." The cycloid in Fig. 2 may be considered to be an undeformed tire rolling without slip, which contacts the soil at A in one case and B in another, but in both cases loses contact with the soil at C. The most interesting portion of the path lies between

A and C or B and C, which illustrates the relative motion between the point on the tire and the soil. This relative motion is not a slip path, as slip is normally defined. In fact, in the case shown, the path represents the case of zero slip. Furthermore, these portions of the path are of different lengths. Since no established term has been found that suitably and concisely describes this length of the path over which wheel and soil are in contact and, at the same time, avoids the confusing implication that there is a relation to slip, the term "olistodos" has been assigned to this path. It will be noted in Fig. 2 that the Z component of the olistodos is simply the sinkage of the wheel. For the net X component, a term "olist" has been assigned. The olist is considered positive when the point of soil-wheel separation (C) is forward of the point of initial contact (A or B). The curve in Fig. 3 is similar to a prolate cycloid but also may represent a pedopneumoid of an undeformed tire moving with slip, which contacts the soil at either A or B and loses contact with the soil at C. From Fig. 3, it can be seen that the olists can be either positive (X_B) or negative (X_A), even though the slip may be the same in both instances. In yielding soil, the olist of a point on the center line of the tire will always be positive when the wheel slip is zero or negative. It may be either positive or negative when the wheel slip is positive.

3. REPRODUCTION OF TIRE SHAPES

Plots of the deflection data with respect to any reference frame represent the path of a point on the inner surface of the tire. To reproduce the path of a point on the outer surface of the deflected tire, it was necessary to assume either (a) that the tire surfaces remained the same distance apart in the process of deflecting or (b) that both surfaces deflected radially. Since the major circumference of the undeflected inner surface was 4.96 in. less than that of the undeflected outer surface of the 11.00-20 tire, and since both surfaces, deflected by a vertical load, should have conformed to the shape of a level unyielding surface on which the tire moved, it seemed reasonable to assume that both surfaces in and near the contact region did not deflect radially. Consequently, assuming that the inner and outer tire surfaces remained the same distance apart and that the tire thickness did not change with inflation pressure or intensity of interface pressures, the shape of the outer surface of the deflected tire was derived from the reproduction of the inner surface of the deflected tire (or plot of the deflection data). The derivation was accomplished by simply adding, on each normal to the inner surface, a constant distance equal to the thickness of the unstressed tire. Unless noted, only the paths of a point on the outer surface of the tire are used in this presentation, and only a portion of each is shown so that the most interesting portions could be enlarged.

The reasonableness of the assumptions previously stated has been verified by close checks with the completely independent measurements from the scratch tests on the firm surface. Fig. 4 shows a reproduction of the shape assumed by the 11.00-20 tire inflated to 17.5 psi and moving under a 3000-lb load on an unyielding surface. The pneumoid derived from the shape of the outer surface of the deflected tire is shown in Fig. 5 and indicates an olist of 0.07 in. For the same conditions, the scratch test also showed an olist of 0.07 in. Olists of other pneumoids and their corresponding scratch tests also agree admirably.

4. TIRE CONTACT LENGTH ON THE UNYIELDING SURFACE

The data from which the pneumoids were derived show that an arc length of the undeflected tire became much shorter while in contact with the surface. This was corroborated by the observations that the rolling circumference (the forward advance per revolution of the wheel) on the unyielding surface was less in every instance than the length of the major circumference of the undeformed tire and that none of the radii of the deflected tire were greater than the radius of the undeflected tire. While this condition became evident in the study of the pneumoids, it is more clearly shown by the shape of the tire in Fig. 4. According to the assumed relations, the lengths of an inner and outer surface of a tire in contact with an unyielding surface are equal and, in this case, were 13.35 in. The arc lengths of the inner and outer surfaces of the undeflected tire were 13.87 and 14.42 in., respectively, and represented one-ninth of a wheel revolution. Obviously, the amount by which the lengths of the deflected surfaces differed from the lengths of their respective undeflected surfaces (0.52 in. for the inner surface and 1.04 for the outer surface) were not proportional to the radii of either the undeflected or the deflected tire. Furthermore, the deflection of the outer surface was approximately radial, while that of the inner surface was not radial. Although it is not evident in the tire shapes in Fig. 4, the wheel advanced only 13.28 in. during one-ninth of a revolution, which indicates that the center line of the wheel had to slide a distance of 0.07 in. on the unyielding surface. Of equal interest is the observation that a radial projection of points on the inner surface of the tire would require the assumption that the tire thickness significantly changes; and the contact length of the derived surface then would be much greater than 13.35 in. This greater contact length would require that the center line of the wheel slide a distance many times longer than the 0.07 indicated by the scratch test.

Since the external tire movement inferred by the assumed relations of the deflection of the inner and outer tire surfaces agreed with the external measurements of the scratch tests, pedopneumoids and shapes of the tire moving in a yielding sand were reproduced by the same method used in reproducing the pneumoids and shapes of the tire moving on an unyielding surface. Discussion of the pedopneumoids and tire shapes follows.

5. TOWED- VERSUS POWERED-WHEEL TESTS

The shapes shown in Fig. 6 are of the towed and the powered (11.00-20) tire moving under essentially the same conditions of load, inflation pressure, and soil strength but at different conditions of sinkage and slip. Movement vectors have been added to these shapes to illustrate that, although the shapes are similar, the movement of a point on the undeflected tire to its position on the deflected powered tire is unlike the movement of a point to its position on the deflected towed tire.

These vectors indicate that, during the process of tire deflection, the movement of a point on the towed tire has a large radial component and a small tangential component. The direction of the tangential component depends upon the location of its origin with respect to the deformed tire and the center line of the axle. The movement of a point on the powered tire has both radial and tangential components also, but the direction of the tangential component is opposite to the direction of the applied torque (or the wheel rotation).

The data from which the shape of the powered and towed tire were derived also were used to derive the pedopneumoids shown in Fig. 7. The pedopneumoid on the right is that of the towed tire with negative slip; the one on the left is that of a powered tire with positive slip. The original soil surface and the beginning points of the olistodos of both test conditions have been drawn to coincide to emphasize the difference in the two olistodos. For clarity, arrows have been drawn along the pedopneumoids to show the direction the reference point followed in each case. The end of each olistodos, the lowest contact of the reference point with the soil, is shown as the bottom of the rut. It is readily noted that the olist is positive for the towed tire with negative slip, and negative for the powered tire with positive slip. However, the magnitude of the olist is only very roughly proportional to the magnitude of the slip. Not enough information is available to determine if the olist is related to the magnitude of the forces on the tire.

6. EFFECT OF INFLATION PRESSURE ON THE SHAPE OF THE 11.00-20 TIRE

The effect of inflation pressure on the shape of the 11.00-20 tire is illustrated by the tire shapes in Fig. 8. The pedopneumoids (Fig. 9) derived from these tire shapes also exhibit an effect of inflation pressure. This effect cannot be attributed entirely to inflation pressure because the pull, sinkage, and slip were not the same in both tests; however, the load on the tire and the soil strength were essentially the same.

Each tire shape in Fig. 8 is symmetrical about the point of maximum deflection. At 19-psi inflation pressure, the tire was flattened in the contact region, but at 45-psi inflation pressure, the tire was nearly round. As expected, the corresponding pedopneumoid followed the same general pattern, i.e. when the contact region of the tire assumed a relatively flat shape, the olistodos also was relatively flat (Fig. 9).

7. EFFECT OF THE TEST VARIABLES ON THE SHAPE OF THE 9.00-20 TIRE

Data obtained with a 9.00-14, 2-PR smooth tire have been used to show the effect on tire shape of the soil strength, slip, and inflation pressure. For these tests, the tire was operated under approximately a constant load (860 to 880 lb), so the only independent test variables were slip, soil strength, and inflation pressure (or percent deflection, which is the ratio of the maximum deflection on an unyielding surface to the section height of the unloaded tire).

7.1 Slip

The effect of slip on tire shape is demonstrated in Fig. 10 by shapes derived from data collected in a single test in which the slip was programmed to increase linearly with time. In this test, the strength of the sand (25 cone index) and the inflation pressure (17.5 psi) were essentially constant, but the input torque and sinkage increased as slip increased. The tire shapes were superimposed so that the original soil surfaces at each slip condition coincided. These tire shapes indicate that as slip increased, the general shape of the tire in the contact zone changed first from a symmetrical curve that was flat in the central portion to a symmetrical curve that bent upward in the central portion, then to an asymmetrical curve that bent upward in the central portion. However, the tire shapes superimposed in the manner shown in Fig. 11 indicate that the change in shape (caused by the tire yielding to or resisting the applied forces) occurred within approximately the same portion of the deformed tire. It follows that the increase in contact length with an increase in slip was an effect of an increase in sinkage rather than an effect of change in tire shape. It is observed readily in Fig. 10 that the projected contact length increased with an increase in slip. This increase in length was relatively small compared to the increase in sinkage. Therefore, the ratio of sinkage to the projected contact length also increased with an increase in slip. It is of further interest to note from the shapes in Fig. 10 that as slip increased, a smaller portion of the total contact length was behind the center line of the axle; that is, the point of tire-soil separation moved closer to the vertical plane of the axle center line. It is observed also from these tire shapes and all others reproduced so far (including one for a 90% slip condition) that the point of tire-soil separation was always behind the center line of the axle.

7.2 Soil Strength

The tire shapes in Fig. 12 were derived from tests in which the tire inflation pressure was 12.5 psi and the slip was approximately 20%. At this slip condition, both the input torque and the pull developed by the wheel were less in the softer sand than in the firmer sand, but the sinkage developed was more. Since soil strength (25 and 60 cone index) was the only independent variable that was not essentially constant, its effect on tire shapes is clearly indicated. The shape of the tire in the softer sand was symmetrical, and the shape of the tire in the firmer sand was asymmetrical with a greater upward curvature in the central portion of the contact region. With respect to sinkage, this change in symmetry was opposite to the trend in the progression of the tire shapes shown in Fig. 10. However, as noted, not all influencing factors are equal in each case.

To show the effect of soil strength on the shape of the tire inflated to 40.2 psi, tire shapes were derived from data collected in two tests for which the soil strengths were 26 and 81 cone index, respectively. These tire shapes are shown in Fig. 13. The slip in the soft-sand test was 12.3% and in the firm-sand test was 14.0%. At this relatively high inflation pressure, the tire yielded more readily than did the firm sand and less readily than did the soft sand. As a result, the shape of the tire in the firm sand was somewhat flat in the contact region, and in the soft sand, it was rounded. The shapes of the tire at the lower inflation pressure in Fig. 10 are more flat than rounded in both the soft and the firm sand. This indicates that the shape a tire assumes in the soil represents a balance between the stiffness (as related to inflation pressure) of the tire and the stiffness (as related to strength) of the soil.

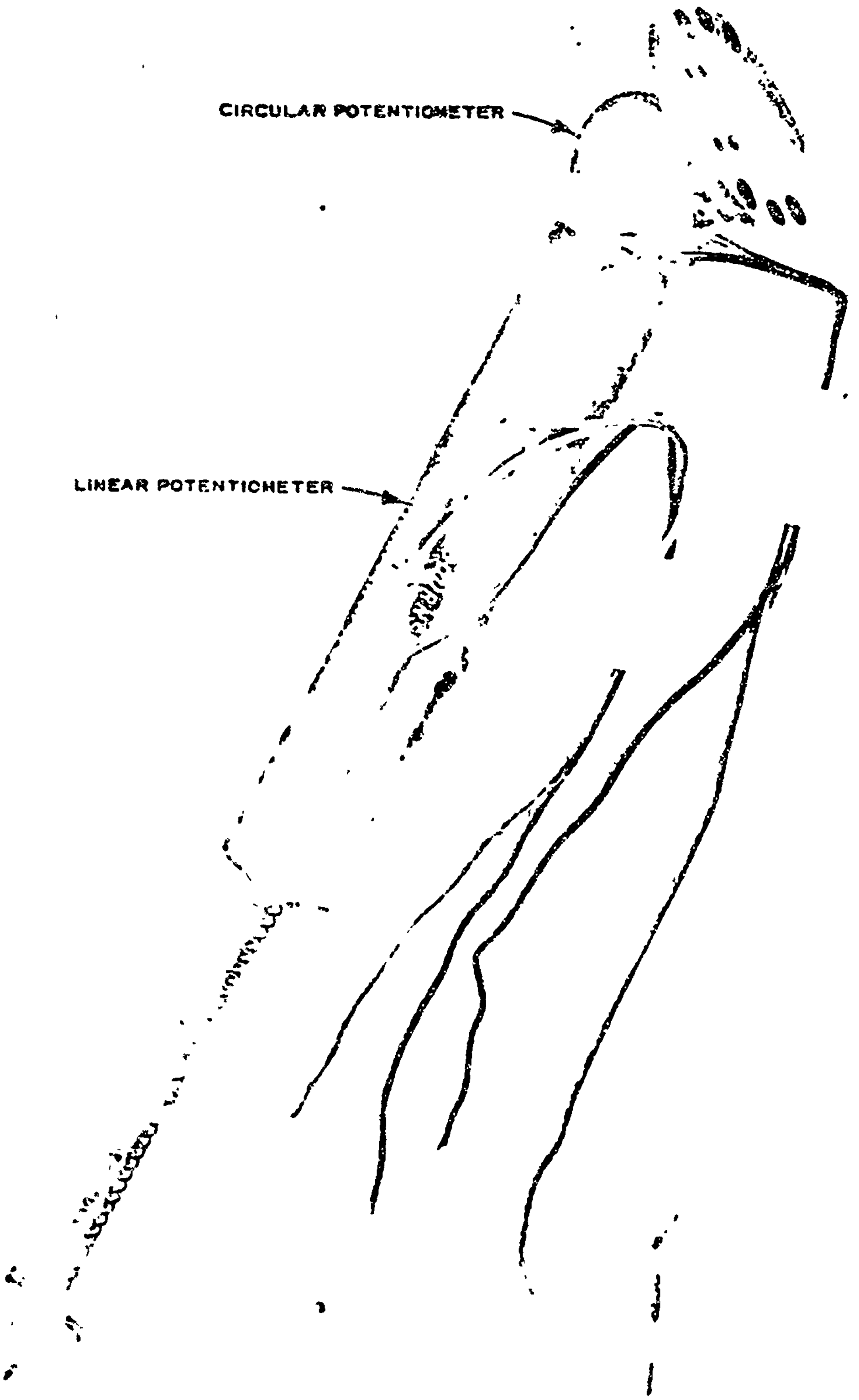
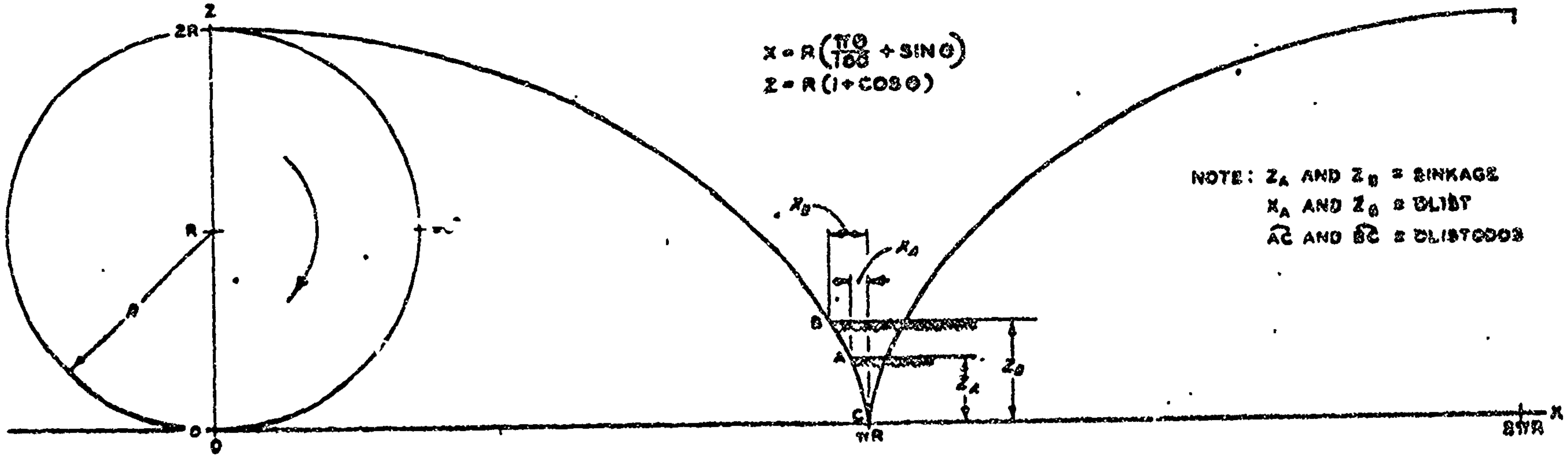


Fig. 1. Linear-angular gage



$X = R\left(\frac{\pi\theta}{180} + \sin\theta\right)$

$Z = R(1 + \cos\theta)$

NOTE: Z_A AND Z_B = SINKAGE
X_A AND Z_B = [illegible]
$\widehat{AC}$ AND $\widehat{BC}$ = [illegible]

Fig. 2. Cycloid

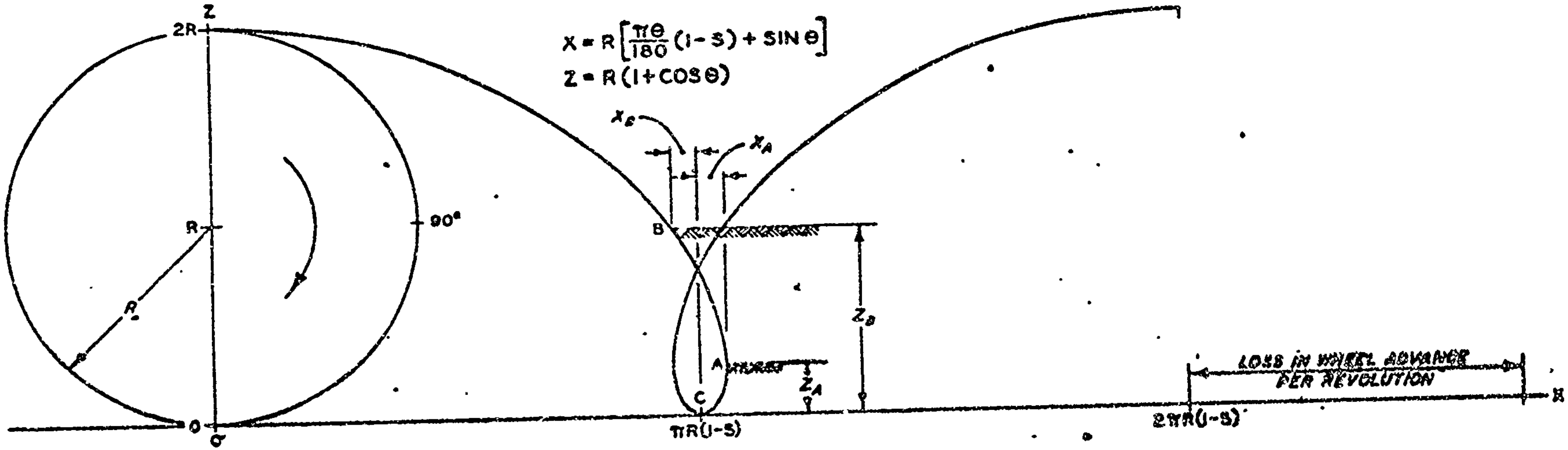


Fig. 3. Prolate cycloid [25.5 percent slip(s)]

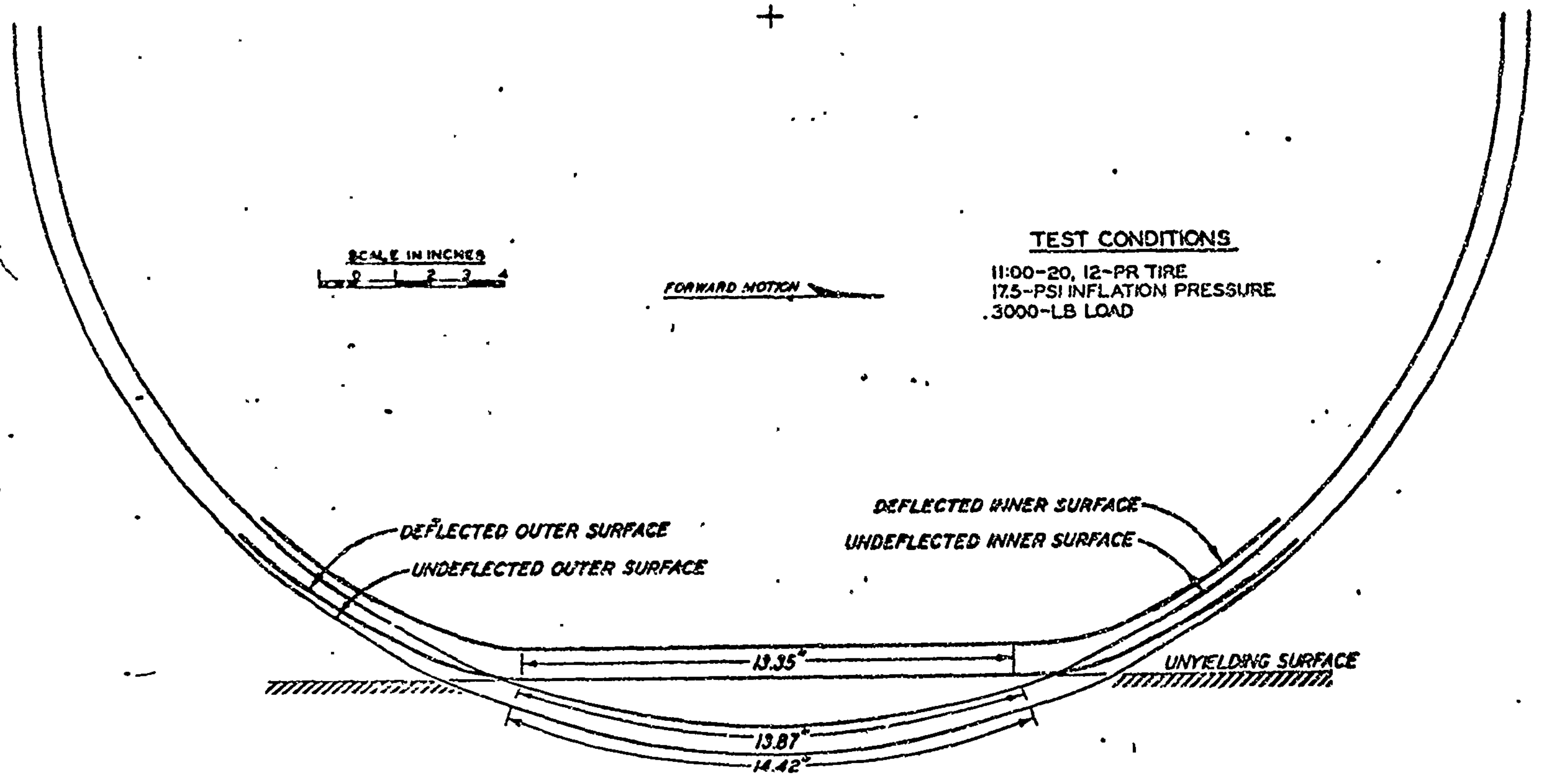


Fig. 4. Shape of a moving tire, unyielding surface

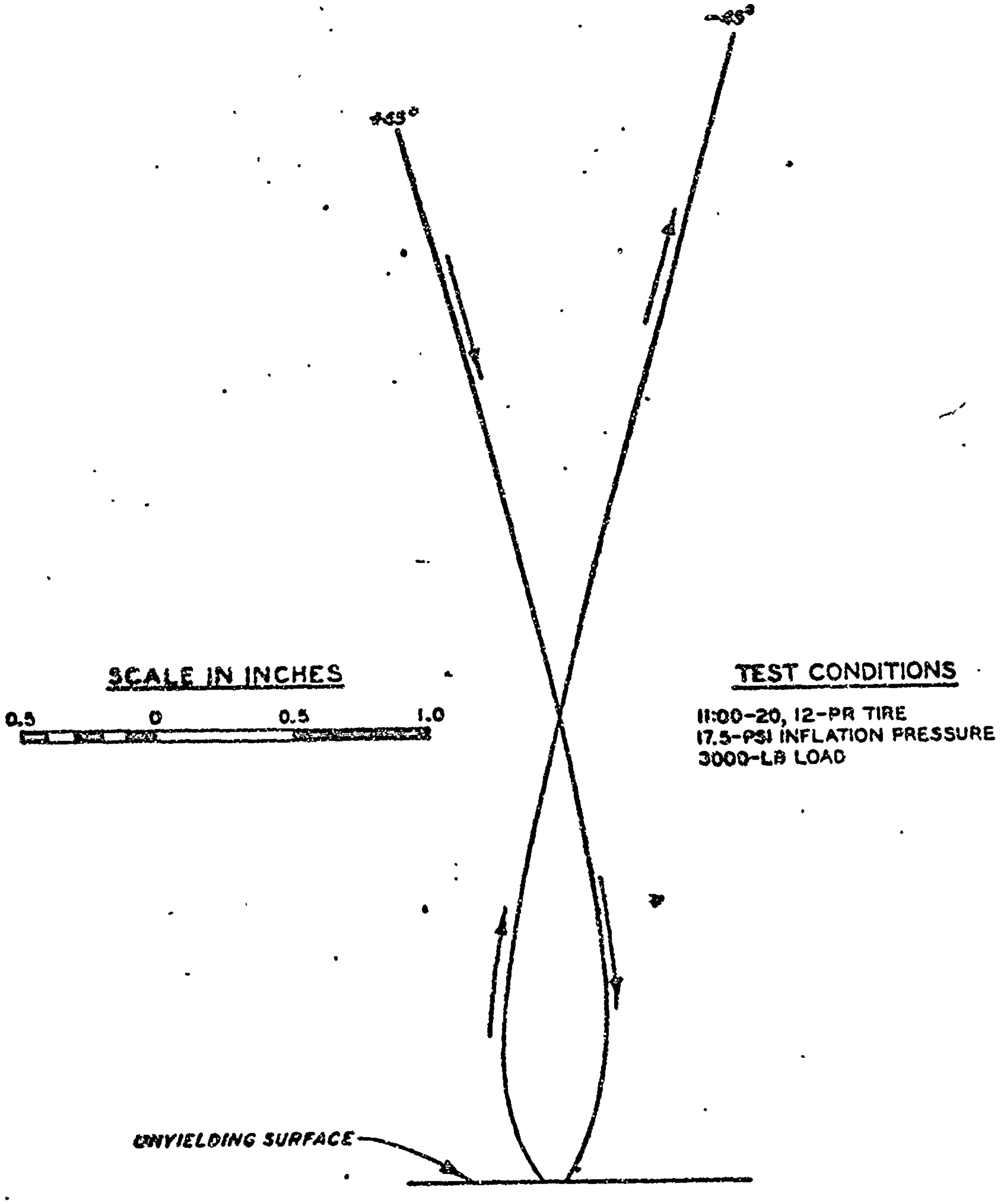


Fig. 5. Pneumoid

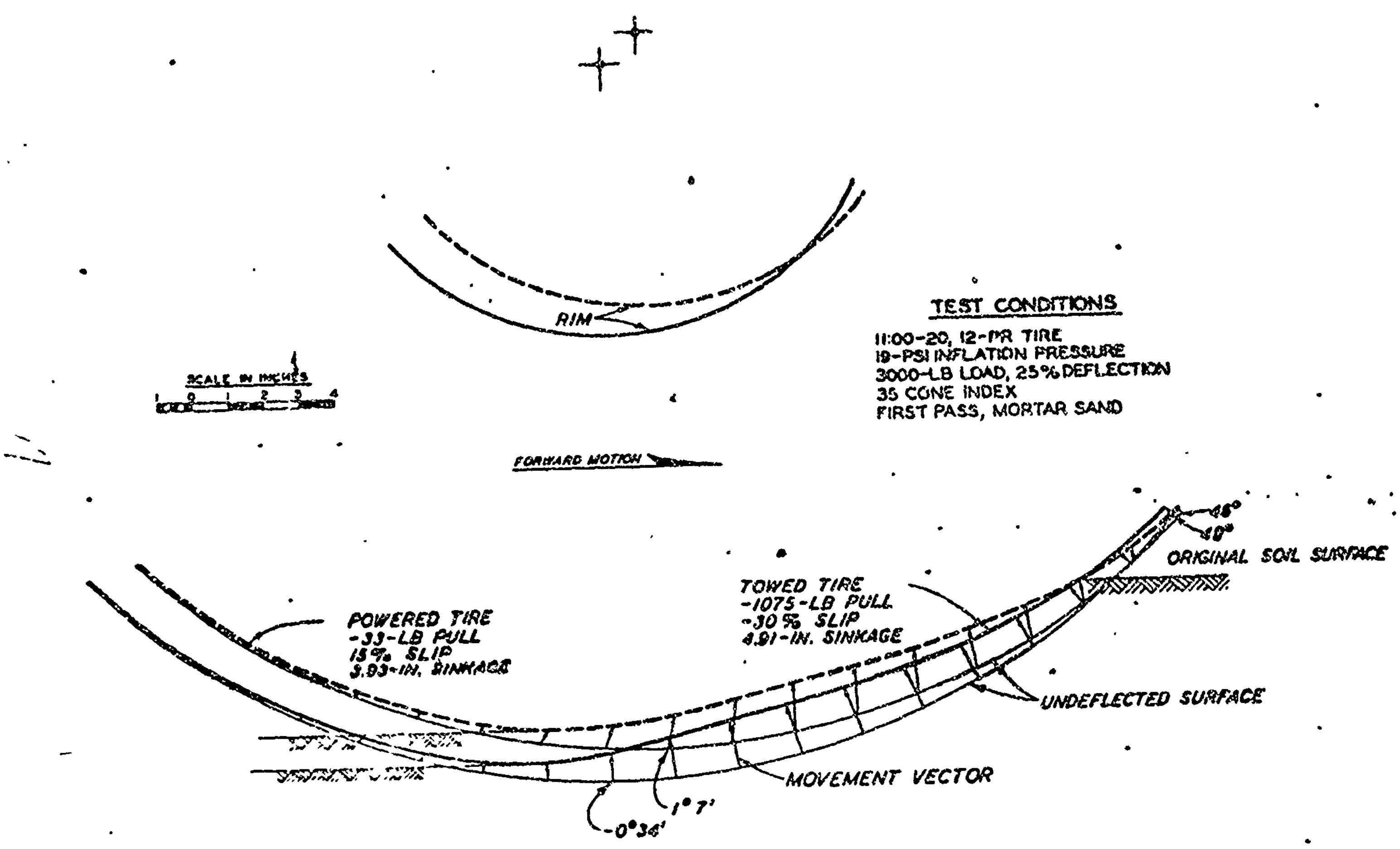


Fig. 6. Shape of a moving tire, powered and towed wheel

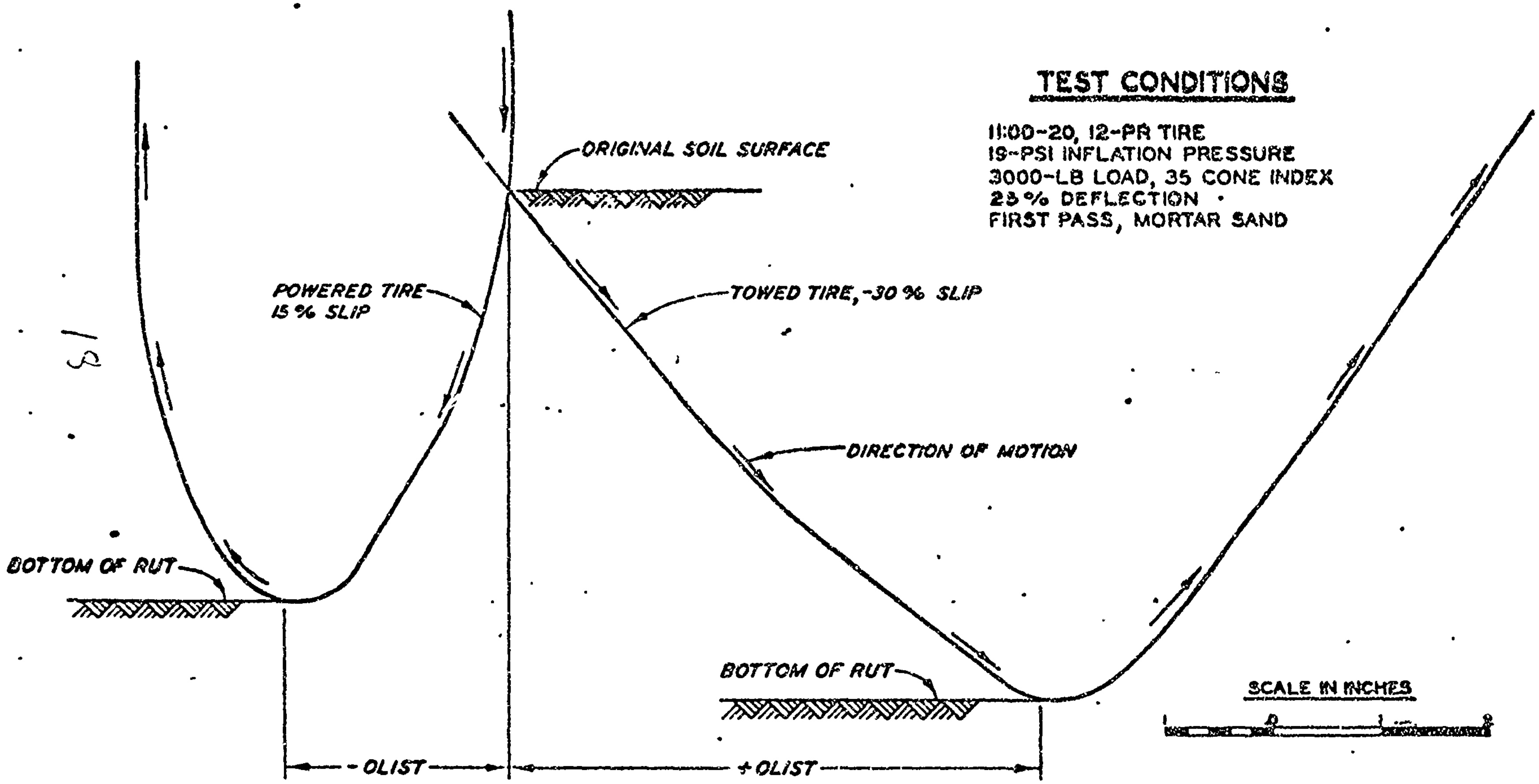


Fig. 7. Pedopneumoids, powered and towed wheel

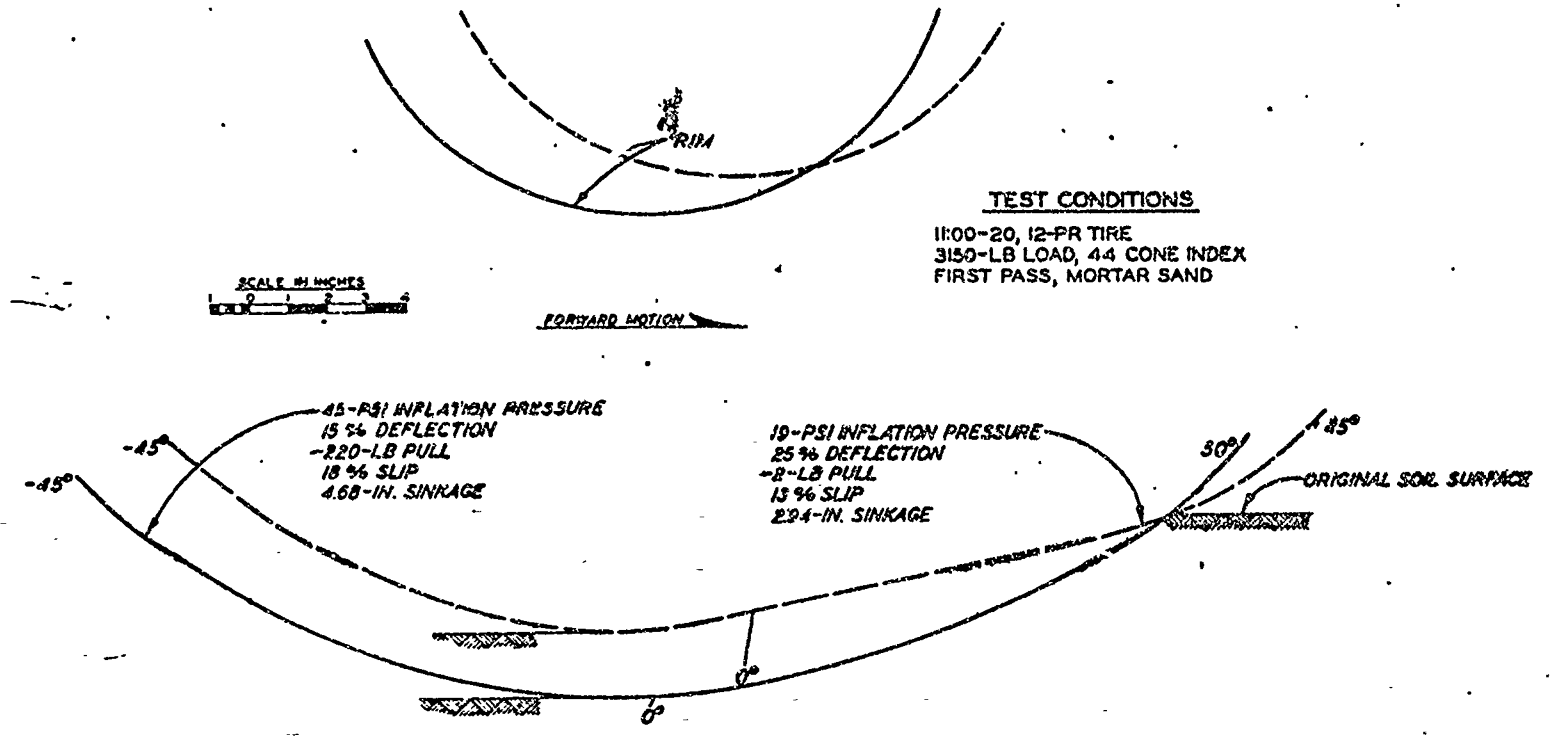


Fig. 8. Shape of a moving tire, powered wheel

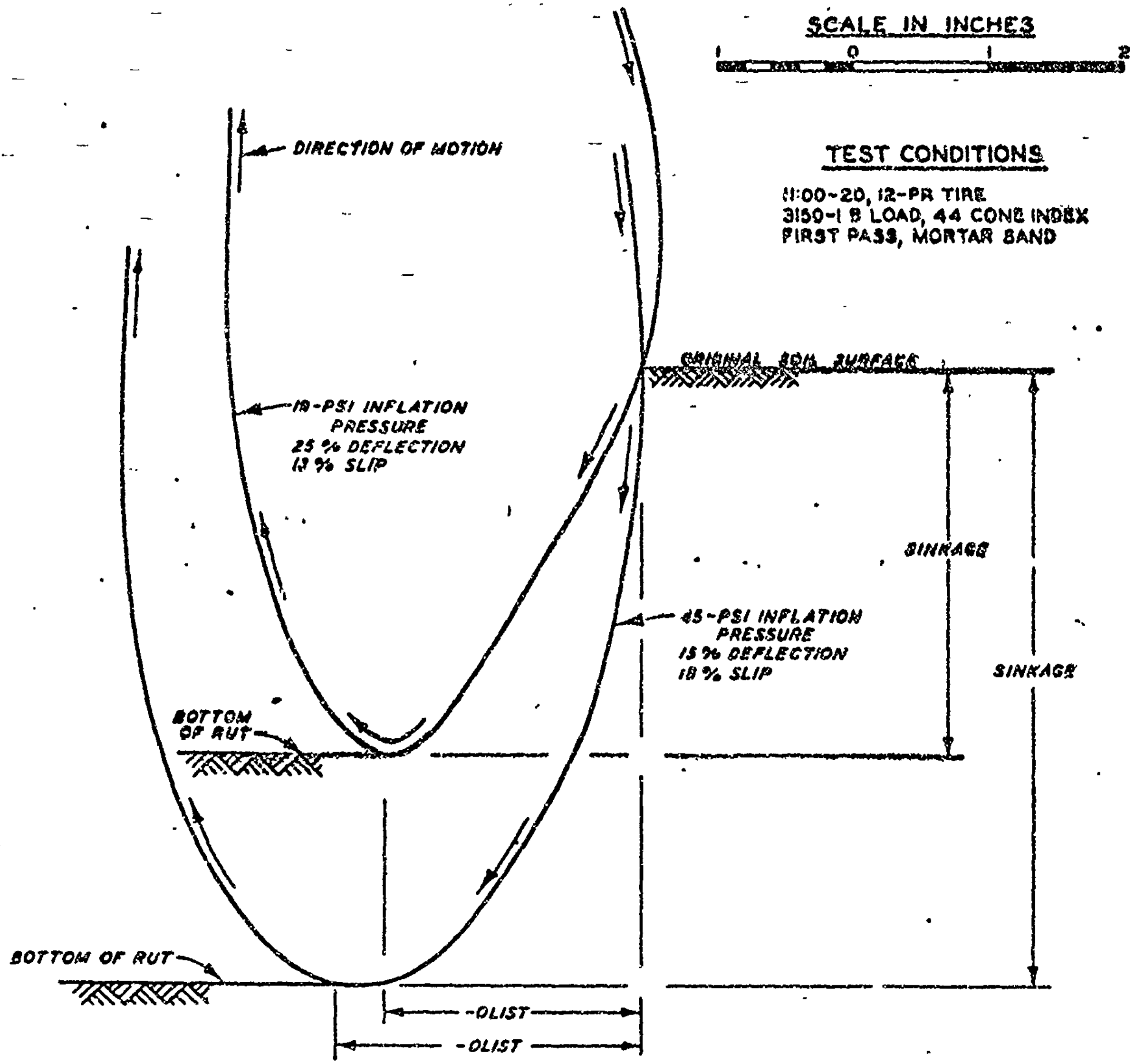


Fig. 9. Pedopneumoids, powered wheel

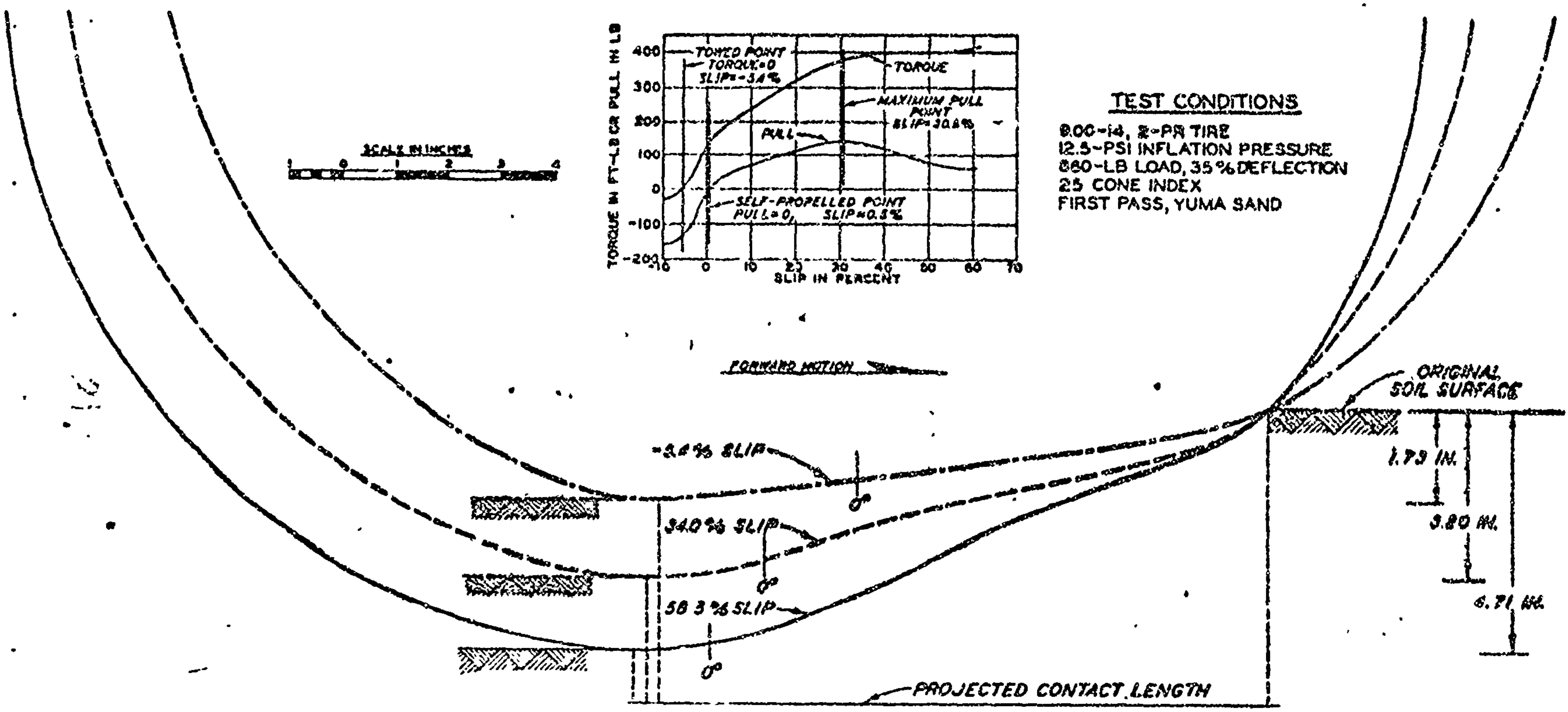


Fig. 10. Shape of a moving tire, effect of slip

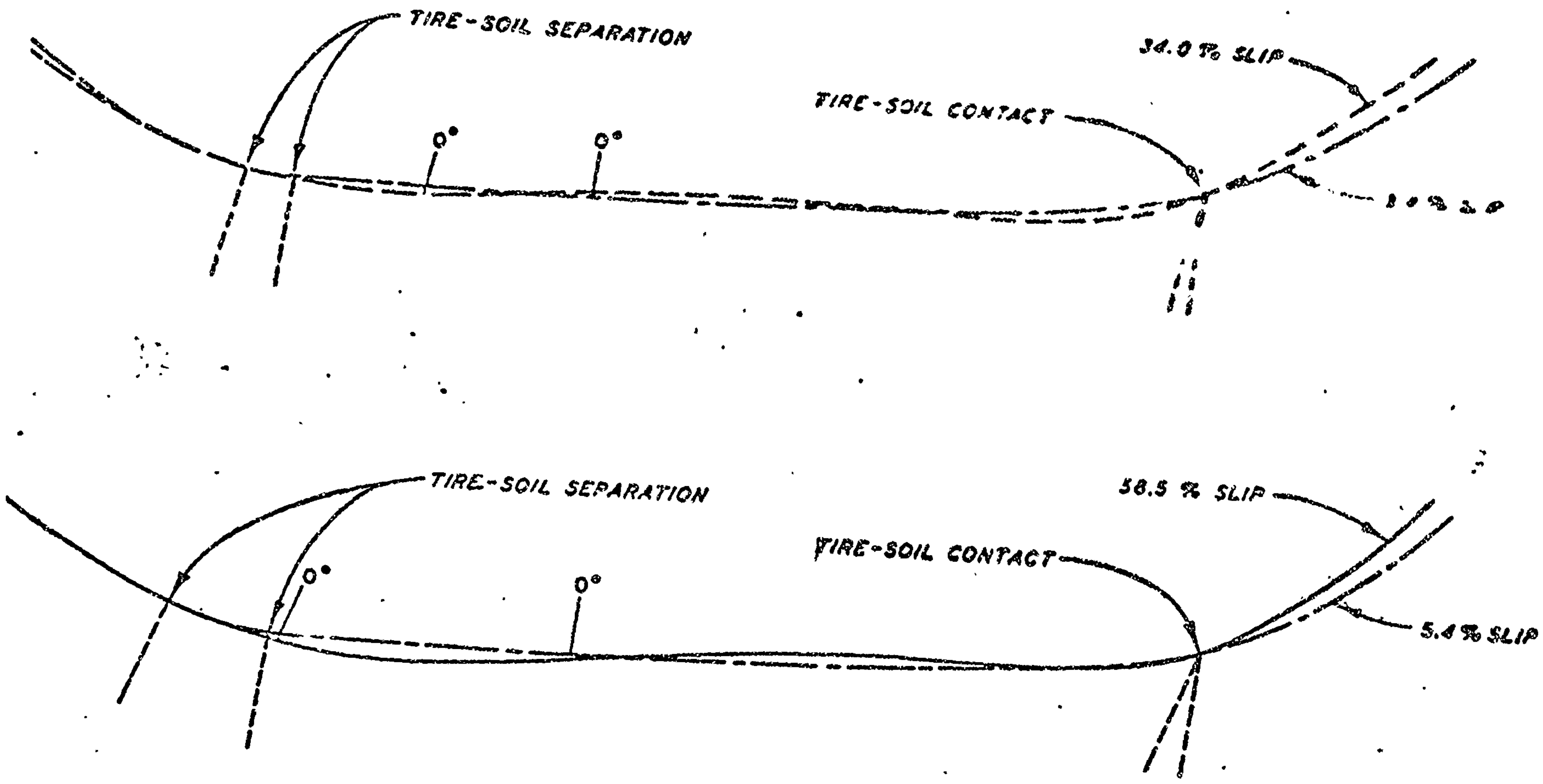


Fig. 11. Superimposed tire shapes

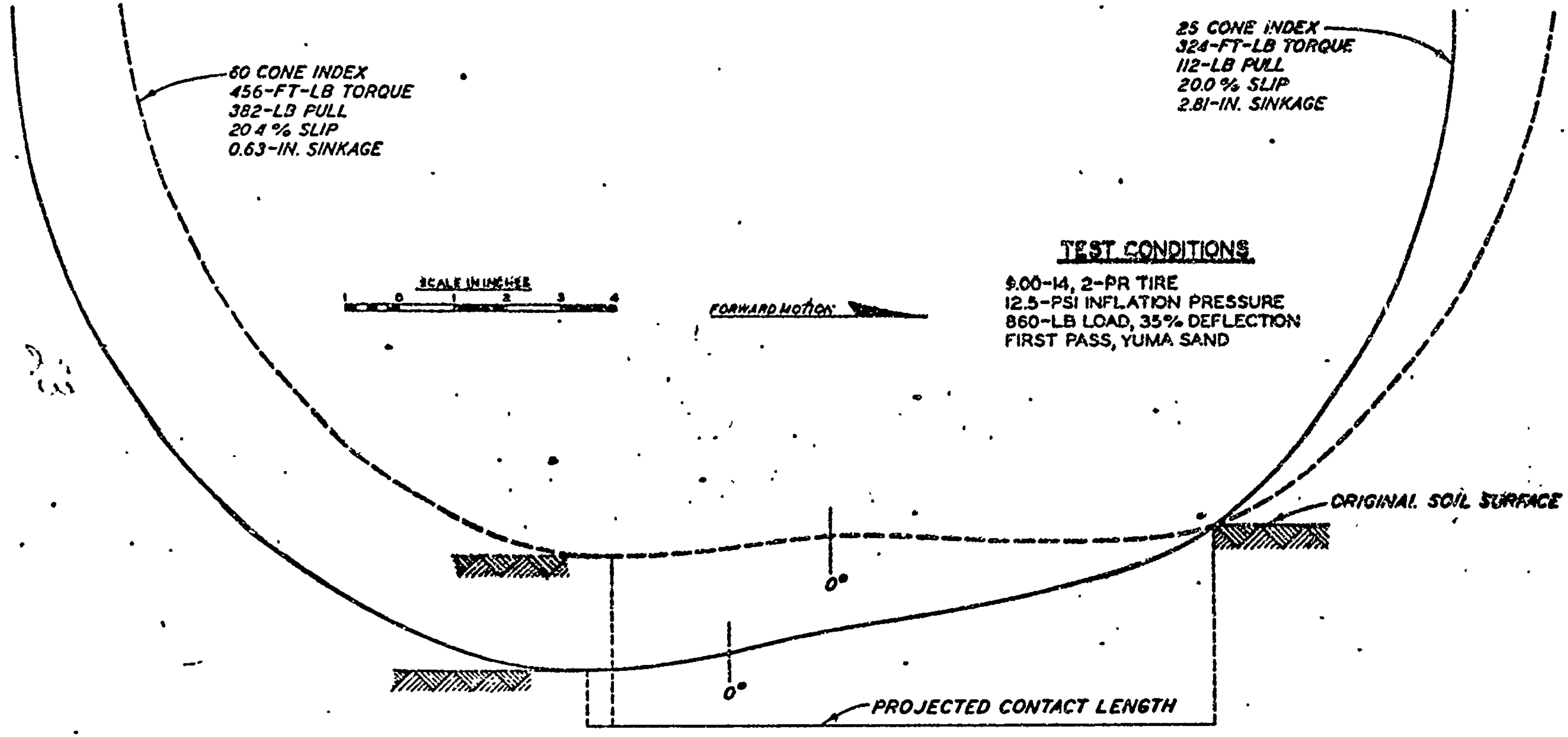


Fig. 12. Shape of a moving tire, 12.5-psi inflation pressure, effect of soil strength

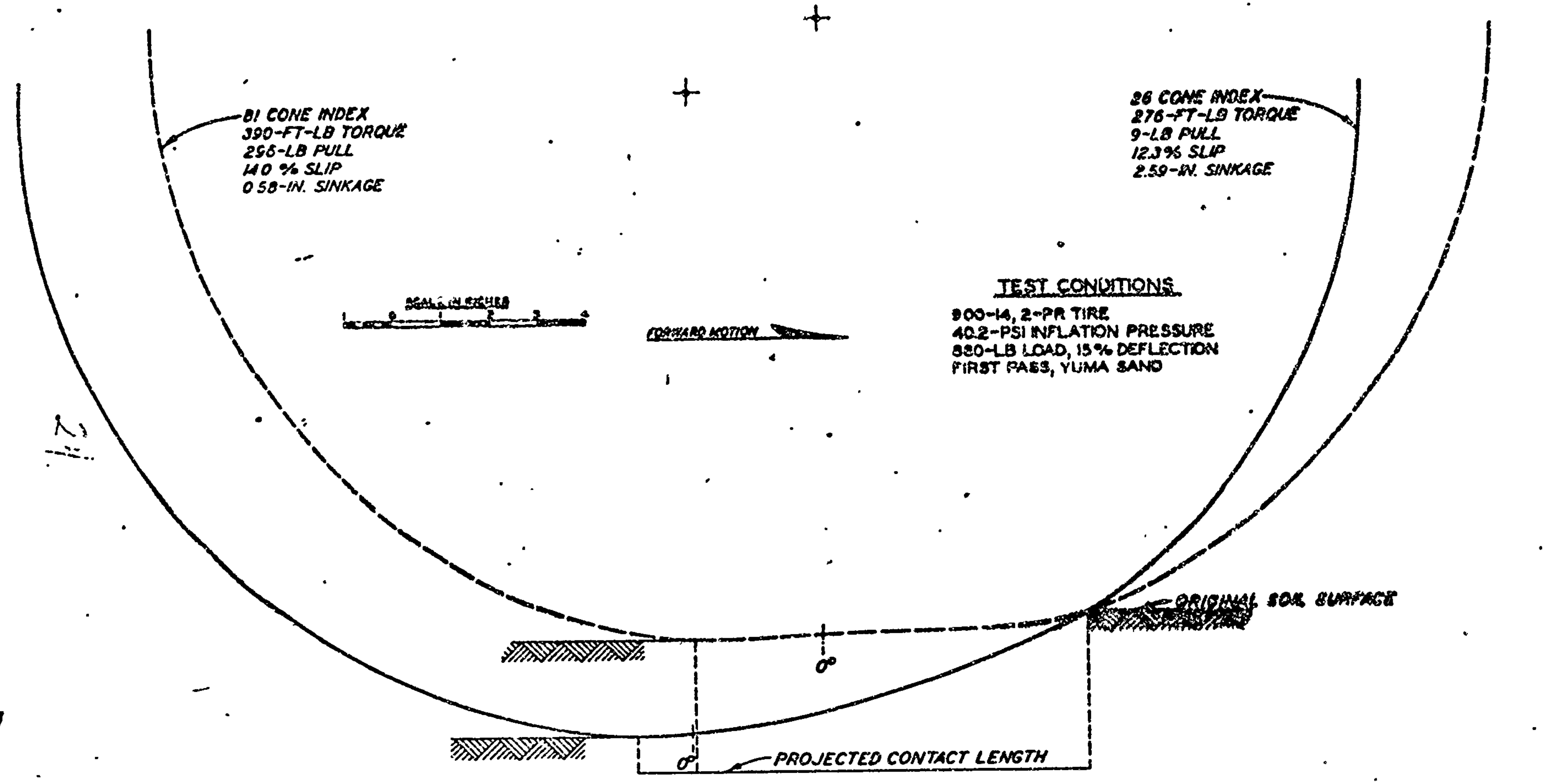


Fig. 13. Shape of a moving tire, 40.2-psi inflation pressure, effect of soil strength

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