

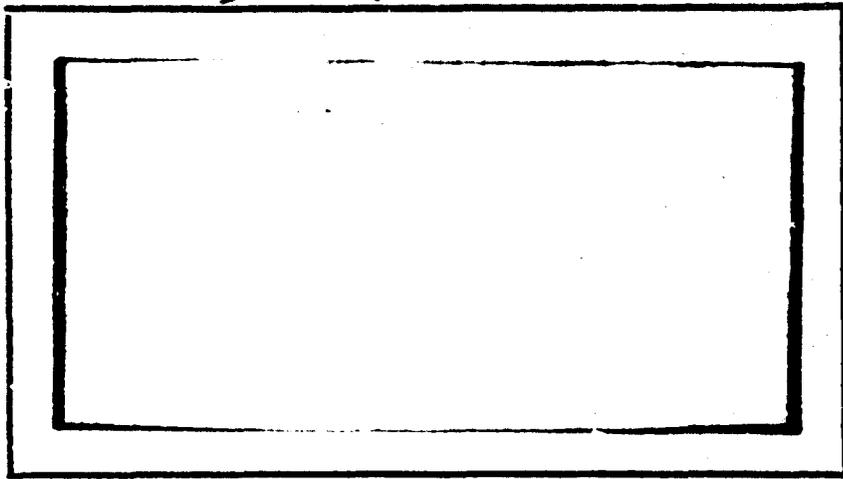
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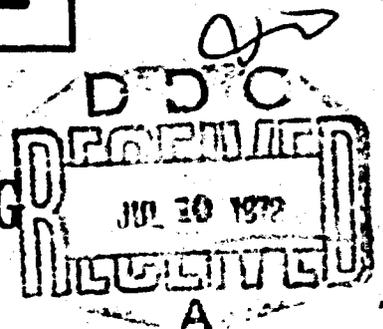


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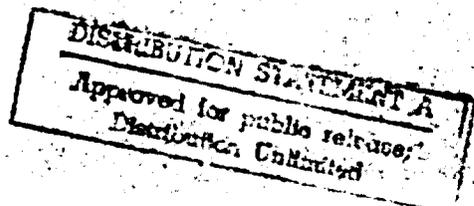


SCHOOL OF ENGINEERING



WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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EXPERIMENTAL STUDY OF  
INSTABILITIES IN THE ELMAX  
THESIS

GEP/PH/72-22

Victor R. Trouy  
Captain USAF

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Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Air Force Institute Of Technology (AFIT-EN) Wright-Patterson AFB, Ohio 45433		2a. REPORT SECURITY CLASSIFICATION Unclassified	
		2b. GROUP	
3. REPORT TITLE  Experimental Study of Instabilities in the ELMAX			
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) AFIT Thesis			
5. AUTHOR(S) (First name, middle initial, last name) Victor R. Trouy Capt USAF			
6. REPORT DATE June 1972		7a. TOTAL NO. OF PAGES 49	7b. NO. OF REFS 13
8a. CONTRACT OR GRANT NO.		9a. ORIGINATOR'S REPORT NUMBER(S) GEP/PH/72-22	
b. PROJECT NO. 7073-00-07			
c. DoD Element 61102F		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)	
DoD Subelement 681501			
10. DISTRIBUTION STATEMENT  Approved for public release, distribution unlimited.			
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY AFIT School of Engineering	
13. ABSTRACT  An experimental search for instabilities in an ELMAX plasma device was made. Plasma generated oscillations of potential and density, with amplitudes several orders of magnitude above background noise, were observed over a wide range of plasma parameters. Measurements of plasma parameters were made with Langmuir probe diagnostic techniques. Frequencies from 3 kHz to 8 kHz were observed with phase velocity in the direction of the electron diammagnetic drift. Ion temperature was not measured and attempted measurements of space potential were not satisfactory. By comparing the observed characteristics of the oscillations with previously identified instabilities, the instabilities responsible for the plasma generated oscillations were tentatively identified as collisional drift waves.			



EXPERIMENTAL STUDY OF  
INSTABILITIES IN THE ELMAX

THESIS

Presented to the Faculty of the School of Engineering  
of the Air Force Institute of Technology

Air University  
in Partial Fulfillment of the  
Requirements for the Degree of

Master of Science

by

Victor R. Trouy, B.S.

Captain                      USAF

Graduate Engineering Physics

June 1972

IV

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Preface

This study is the result of my attempts to experimentally locate instabilities in the ELMAX plasma device. I was fortunate to be given a sophisticated and well designed research tool already in operation. With the exceptions of a few leak-hunts in the vacuum system and the construction of a few probes, no time was used in preparing equipment for the experiment.

Since preparing the equipment was not a part of the study, the equipment and data acquisition system are only briefly described in this paper.

My grateful thanks go to Dr. Gordon Soper, for his patient endurance throughout this study, and to Major Justin Curtis and Drs. John Martin and Merrill Andrews for their many suggestions and hours of assistance.

Many thanks also to Mr. Cliff Van Sickle, Mr. Dennis Grossjean and Mr. Bill Cadwallender, without whom the sophisticated research would never have functioned.

Victor R. Trouy

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Abstract

An experimental search for instabilities in an ELMAX plasma device was made. Plasma generated oscillations of potential and density, with amplitudes several orders of magnitude above background noise, were observed over a wide range of plasma parameters. Measurements of plasma parameters were made with Langmuir probe diagnostic techniques. Frequencies from 3 kHz to 8 kHz were observed with phase velocity in the direction of the electron diamagnetic drift. Ion temperature was not measured and attempted measurements of space potential were not satisfactory. By comparing the observed characteristics of the oscillations with previously identified instabilities, the instabilities responsible for the plasma generated oscillations were tentatively identified as collisional drift waves.

## I. Introduction

The Electrodynamic Magnetoplasma Experiment (ELMAX) is designed to achieve improved plasma containment in a magnetic mirror device. To reduce microinstabilities, an external electric field is applied, and careful design of the plasma injection devices is intended to provide a nearly Maxwellian particle distribution. The remaining instabilities are expected to be macroscopic in nature and therefore susceptible to feedback stabilization. In order to explore the feasibility of feedback stabilization, it was first necessary to determine what instabilities were present in the plasma, and that was the purpose of this study.

Langmuir-probe diagnostic techniques were used to study the plasma. The probe outputs were used as inputs to a spectrum analyzer, an oscilloscope, and a computer analysis system developed by Nunn (Ref 1). A description of the apparatus is given in Section II, and the experimental procedures used are described in Section III.

Since probe theory in a magnetic field is still not completely developed, the results of the computer analysis are open to some question. However, the results proved to be very repeatable and throughout this study they are assumed to be correct. The one exception to this is the measured space potential. As explained in Section III, attempts to use the gradient of space potential to determine the electric field gave ambiguous results. Another important parameter

which was not available during this study was the ion temperature. Although an ion probe had been designed and was being built, it was not available during this study. The lack of these two important parameters made identification of the observed instabilities more difficult and less certain. In particular, it was found that precise knowledge of the electric field would permit one to distinguish the differences between a flute instability and a drift instability. This was particularly significant since the observed instabilities exhibited characteristics common to both.

Another limitation imposed by the apparatus was the frequency range over which measurements were made. Because of high-frequency attenuation in the data-collecting circuits, no attempt was made to observe frequencies above 100 kHz. This limitation is not a severe one since the most useful frequencies for initial experiments with feedback stabilization are lower than 100 kHz.

Section IV includes a comparison of experimental results with theory. If the drift caused by the electric field is assumed to be negligible, it is shown that all of the observed characteristics of the instability conform to those of a drift wave, and that the stability conditions for a drift wave correctly predict the stability of the observed oscillations.

## II. Experimental Apparatus

### Vacuum System

The plasma was produced in a cylindrical vacuum chamber with an internal diameter of 23.5 in. and an internal length of 22.68 in. The chamber had three 7.8 in.-diameter ports in the median plane spaced 90 degrees apart with one centered on the top of the chamber and two on the sides. One side port was used with a quartz window to allow visual observation of the plasma. There were 18 radial ports near each end of the chamber, spaced at 20 degree intervals, approximately 8.5 in. on either side of the median plane. The chamber and other important components of the experimental apparatus are sketched in Fig. 1. The vacuum system included a 10-in. CVC diffusion pump, a liquid-nitrogen-cooled baffle, one 50 cfm and one 12 cfm roughing pump, and pneumatic valves to control the system. Base pressures of  $2 \times 10^{-7}$  torr are obtained using the baffle and  $6 \times 10^{-6}$  torr without the baffle.

### Magnets

As shown in Fig. 1, two magnet coils were mounted at each end of the chamber. The two inboard magnets are 4-in.-thick, 248 turn Magnion coils. They are mounted next to the end of the chamber flange with their centers  $38.60 \pm .05$  cm from the median plane. The two outboard magnets were 5-in.-thick, 122-turn Magnion coils and were mounted with their centers  $63.10 \pm .05$  cm from the median plane. The geometric centers of

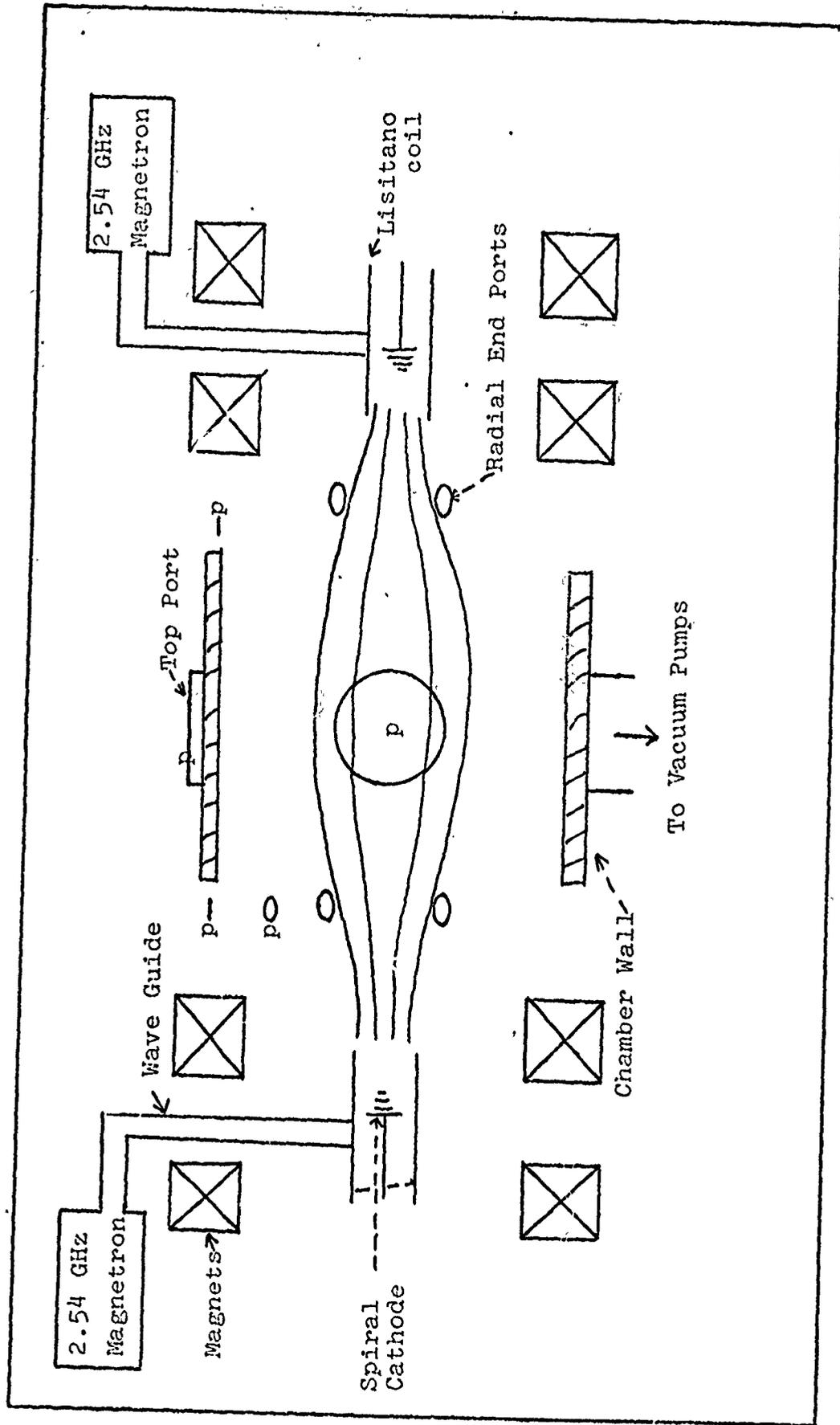


FIG. 1. The ELMAX Chamber

the coils were aligned to within  $\pm 0.05$  cm of the chamber axis. The power supplies were able to provide 250 amps of current to the outboard magnets which were wired in series, and 200 amps total current to the inboard magnets which were wired in parallel. This provided a magnetic field in the chamber of approximately 450 gauss at the chamber axis near the end probes (8.5 in. axially from the median plane) and approximately 235 gauss on axis at the median plane. The maximum magnetic field, occurring at each end of the chamber, is approximately four times the field at the median plane, resulting in a mirror ratio of four to one. At the plasma pressures observed throughout this study, the ratio of plasma pressure to magnetic pressure is very low (Beta much less than one) and the applied magnetic field is assumed to be unchanged by the presence of the plasma.

#### Lisitano Coils and Spiral Cathodes

The plasma sources, located at each end of the chamber between the inboard and outboard magnets, consisted of a 4-in. diameter helical Lisitano coil with a 3.5 in. i. d. quartz liner, and a 2 in. diameter heated spiral. The microwave source for each Lisitano coil was a 1 kw, 2.45 GHz air-cooled magnetron r.f. generator. Both magnetrons were powered by the same 4 kw, 500mA d.c. power supply. The magnetrons were connected to the coils through a wave guide and stub tuner. Each Lisitano coil consisted of a hollow metal cylinder with a continuous slot machined through the cylinder wall in the shape of a helix. The slot served as a transmission line for

the power from the magnetrons, and the net effect was an oscillating electric field applied to the plasma. The direction of the electric field was made to be almost parallel to the applied magnetic field by using coils with small helix angles. The d.c. power supplies for the spiral cathodes supplied approximately 28 volts at 90 amps to the spirals. The polarity of the voltage could be reversed, and the spirals could be operated with either end grounded or with both ends floating. Changing the grounding of the spirals was found to have no useful effect on the plasma but reversing the polarity of the voltage resulted in a reversal of the applied radial electric field. In the experiments described in this study, the voltage is applied to the spirals so that the radial field is directed inward toward the center of the plasma. An electric field pointing outward was found to result in a hollow plasma with maximum density near the outer edge of the spirals rather than on the axis of the chamber.

#### Langmuir Probes

Two radially traversing, Langmuir, single probes at right angles to one another were mounted in the median plane of the chamber. Two other probes were mounted in two of the radial end ports, one on top of the chamber in line with the top center probe, and the second in a side port, 40 degrees from the first. Occasionally, one of the end probes was moved to the top end port at the other end of the chamber to make measurements. This allowed the symmetry of the plasma along

the axis of the chamber to be measured, and also allowed simultaneous observation of any observed oscillations at both ends and at the center of the chamber. This provided information about the volume of the plasma affected by the oscillation and the phase relation of the oscillation at one end of the chamber relative to the center or other end of the chamber. The ports in which probes were installed are marked in Fig. 1 with a "p".

#### Data Collecting Equipment

To observe and record data directly from the various probes, a dual-beam Tektronix oscilloscope, type 551, was used with two plug-in units, a Nelson Ross spectrum analyzer with a range from 0 to 100 kHz, and a four channel, chopped-beam, Tektronix type 1A4 amplifier. A Tektronix oscilloscope camera was used to record the data.

Pressure in the chamber was measured with an MKS baratron pressure meter, type 144. The flow of argon gas into the chamber was measured with flow meters calibrated in standard  $\text{cm}^3$  per sec for argon.

To analyze the data from the probes a computerized system was used. The basic components were a Hewlett Packard 2116C computer, an HP digital voltage source (DVS), and an HP digital voltmeter (DVM), types 6130B and 2116C. The DVS and DVM were both controlled directly by the computer. The DVS was used to impress voltage on a selected probe and the amplified current was then measured and fed directly into the computer. The program used is listed in Appendix A, with a brief discussion of the main features of the program.

### III. Experimental Procedures and Results

#### General Approach

The first step in studying plasma instabilities in the ELMAX was obviously to find some instabilities, that is, to determine what values of the various control parameters available would result in an instability being observed. An integral part of this step was deciding how to observe the instability if it did occur. After the first step had been accomplished, the next step was to determine what characteristics of the instability could be observed, and what plasma parameters could be measured, which would allow the instability to be compared to known types of instabilities and thus to identify it.

#### Control Parameters - Finding the Instabilities

There were four control parameters which were systematically varied during the search for instabilities. These were: magnet current, gas flow rate, current to the magnetrons, and current to the spiral cathodes. In addition to these control parameters, the polarity of the voltage across the spirals could be reversed, the spiral grounding could be changed, and the magnet current could be reversed. Variation of these control parameters cannot be directly related to the resulting variation in plasma parameters because of the very complicated interrelations which are present. For example, variation of the magnet current not only changed the magnetic

field but also changed the density and temperature profiles of the plasma. In the same way, variation of the current of the spirals was found to affect not only the observed values of temperature and density, but also the frequency and radial position of the observed oscillations. For this reason the control parameters and plasma parameters were treated as distinct from one another. Control parameters were varied to change the plasma being studied and then plasma parameters measured as if they were independent variables. To detect any instabilities which might occur, the floating potential from at least two probes, usually the two probes at the center of the chamber, was displayed on the oscilloscope. The oscilloscope was monitored at all times when control parameters were being varied. If an oscillation appeared which might indicate the presence of an instability, pictures of the scope trace and of the spectrum analyzer display were made, and all control parameters were recorded. At first, no useful results were obtained using this technique. Many alternate methods of detecting instabilities were considered but no acceptable alternative was found. Fortunately, during this time, the ELMAX system was being gradually improved. Analysis of data from previous experiments showed that higher magnetic fields than those being used in the present experiment had resulted in a relatively quiet plasma with what appeared to be discrete oscillations sometimes present. This indicated that increasing the current available to the magnets might improve results. The current available was increased by about twenty percent.

It was also noted that the power supplies to both the magnets and the spiral had significant amounts of ripple in their output voltage which might have been obscuring the results of an instability. Filters were installed which significantly reduced these ripples. With these improvements, useful oscillations began to be detected.

#### Plasma Parameters-Recording Instability Characteristics

Having detected useful oscillations the next step was to record as many plasma parameters as possible while the instability was present, and to observe and record whatever other characteristics could be detected. The computer analysis of the probe curve provided the means of measuring the plasma parameters. The parameters which were available from the computer analysis included: electron temperature, ion density, floating and space potential, and Debye length. By varying the probe position by a small amount, usually one-quarter or one-half cm, and repeating the computer analysis, all of these parameters were available as functions of radial probe position. This in turn allows the calculation of the gradients of these parameters. A typical computer output sheet is shown in Appendix A.

The frequency of the oscillation was measured with the spectrum analyzer, and phase relations between the floating potential oscillations at the various probe positions were recorded, using pictures of the scope trace. By biasing the

probe with the proper voltage, it was driven into the ion-saturation portion of the probe curve. Oscillations in the current drawn by the probe are therefore primarily related to ion density fluctuations and allow a measurement of the phase relation between density and potential at that probe. This phase relation is an important characteristic of the oscillation since different types of instabilities will cause different phase relations between density and potential. For example, drift wave instabilities usually have the density wave leading the potential wave, while flute instabilities normally result in oscillations with the opposite density-potential phase relation.

The potential measured in this experiment is the floating potential of the probe. Since the density-potential relationships discussed above are actually derived theoretically using the space potential of the plasma, it would be very helpful to determine the exact phase difference, if any, between the space and floating potential. This phase difference is often assumed to be negligible, and in the paper used to compare experimental results from this study with theoretical predictions no distinction is made between space and floating potential. However, an experiment was devised which would measure the phase relation between density and floating potential and between density and space potential to insure the validity of the assumption that no significant phase difference existed between the two potentials. To accomplish these measurements, a "Boxcar" integrator was used. Although this

device is rather complicated, the principles of operation are straightforward. The boxcar accepts two inputs, the first input being the varying amplitude of a given wave, and the second being a trigger pulse synchronized in some way with the input wave. The "Boxcar" then samples the wave amplitude during some small time interval at a phase point of the incoming wave determined by the trigger pulse. Adjusting the trigger pulse to coincide with consecutive phase points along the incoming wave allows the characteristics of the wave to be determined as a function of phase position. In this experiment, the input wave was the output of the Langmuir probe as the biasing potential was varied to generate the probe curve. The output of the "Boxcar" was then used as the input to the computer which analyzed the probe curve. In this way the various plasma parameters, specifically the temperature, density, and potential were calculated as a function of phase position of the oscillation. By plotting these variables versus the phase position, the relation between these variables could be determined. The results of this experiment were inconclusive for two reasons. First, several hours were required to complete one run through one wavelength of the oscillation. When the beginning of the run was repeated immediately after completing a run, to check the repeatability of the results, different results were obtained. Second, a careful analysis of the output of the "Boxcar" revealed that the output was not maintaining a constant relation to the input amplitude as it should have.

The results of this particular experiment were therefore not used.

### Experimental Results

Continuous, almost sinusoidal oscillations were observed over a wide range of control parameters. These oscillations had an apparent clockwise phase velocity as looking along the magnetic field and in the direction of the magnetic field. To facilitate discussion, the direction of the magnetic field, which was parallel to the longitudinal axis of the vacuum chamber, is designated as the positive z-direction. Then the r and theta directions are designated as the other two directions, the apparent phase velocity of the oscillations as in the positive theta direction. Wave-length along the z-axis could not be measured exactly, since probes could be inserted at only three points along the z-axis. If the three probes along the z-axis were all adjusted to the same radial position, the center probe showed the largest amplitude; however, if the three probes were adjusted to the radial distance which gave the largest amplitude for that probe, the three amplitudes were approximately the same. For the oscillations which are analyzed in detail in this study, the center probe had the largest oscillation at a radial position of 3 cm. The two end probes showed the maximum oscillations when positioned at 2.5 cm. When the probes were adjusted to these positions, no phase difference between the three probes could be detected. These observations can be explained if a longitudinal standing wave is assumed to exist in the plasma. The boundary conditions for

this standing wave would be the physical characteristics of the chamber. Since the first conducting surface which was in contact with the plasma at each end of the chamber was the spiral cathode, the wavelength of the standing wave must be some integral multiple of the distance between the two spirals. A wavelength equal to this length would result in a minimum amplitude at the center of the chamber. Since this was not observed, the wavelength of the standing wave must equal at least twice the distance between the spirals.

To determine the wavelength of the oscillations in the theta direction, the output of two probes in the same r-theta plane were compared on the oscilloscope, using a copped sweep on the oscilloscope so that the phase relation between the two signals could be seen. When using the center probes, which were positioned 90 degrees apart, a phase relation of either 90 or 180 degrees was observed. The change from one phase relation to the other was always accompanied by an apparent doubling of frequency and was caused by a change in the applied magnetic field. This was interpreted as a mode change. The term "mode" as used here refers to the ratio between the circumference of the plasma column at the radial position at which the oscillation is detected and the wavelength of the oscillation. Since the oscillation is observed to be continuous, the wavelength must be equal to the circumference divided by some integral number. If this were not true, interference would occur at some point around the plasma column and a continuous oscillation

would not be observed. If  $\lambda_{\perp}$  is the wavelength of the oscillation in the theta direction, and  $K_{\perp}$  is the wave number, then

$$K_{\perp} = 2\pi(\lambda_{\perp})^{-1} = 2\pi(2\pi P/M)^{-1} \quad (1)$$

$$K_{\perp} = 2\pi M/2\pi r = M/r \quad (2)$$

where  $M$  is the mode number. When an apparent phase shift of 90 degrees is observed between two probes positioned 90 degrees apart, this is interpreted to mean that the same maximum potential arrives at the second probe one-fourth of a cycle after it reached the first probe. This is called an  $m=1$  mode. If the phase shift appears to be 180 degrees, this means that one probe is sensing a minimum potential while the other is sensing a maximum. This is interpreted to mean that the wavelength of the oscillation is equal to one-half of the circumference, that is,  $m=2$ .

The sketches in Fig. 2 illustrate the results of a sequence of spectrum analyses of the oscillation. The input to the spectrum analyzer was the floating potential from a probe at the center of the chamber, radially positioned to the point of maximum amplitude of the oscillation. The magnetic field was increased after each analysis was made so the sequence illustrates the behavior of the oscillation as a function of magnetic field. Fig. 2(a) shows a mode 1 oscillation with a frequency of 8 kHz. With the magnetic field increased, Fig 2(b) shows a decrease in the frequency of the mode 1 oscillation to 6 kHz and at 12 kHz can be seen the beginning of a mode 2

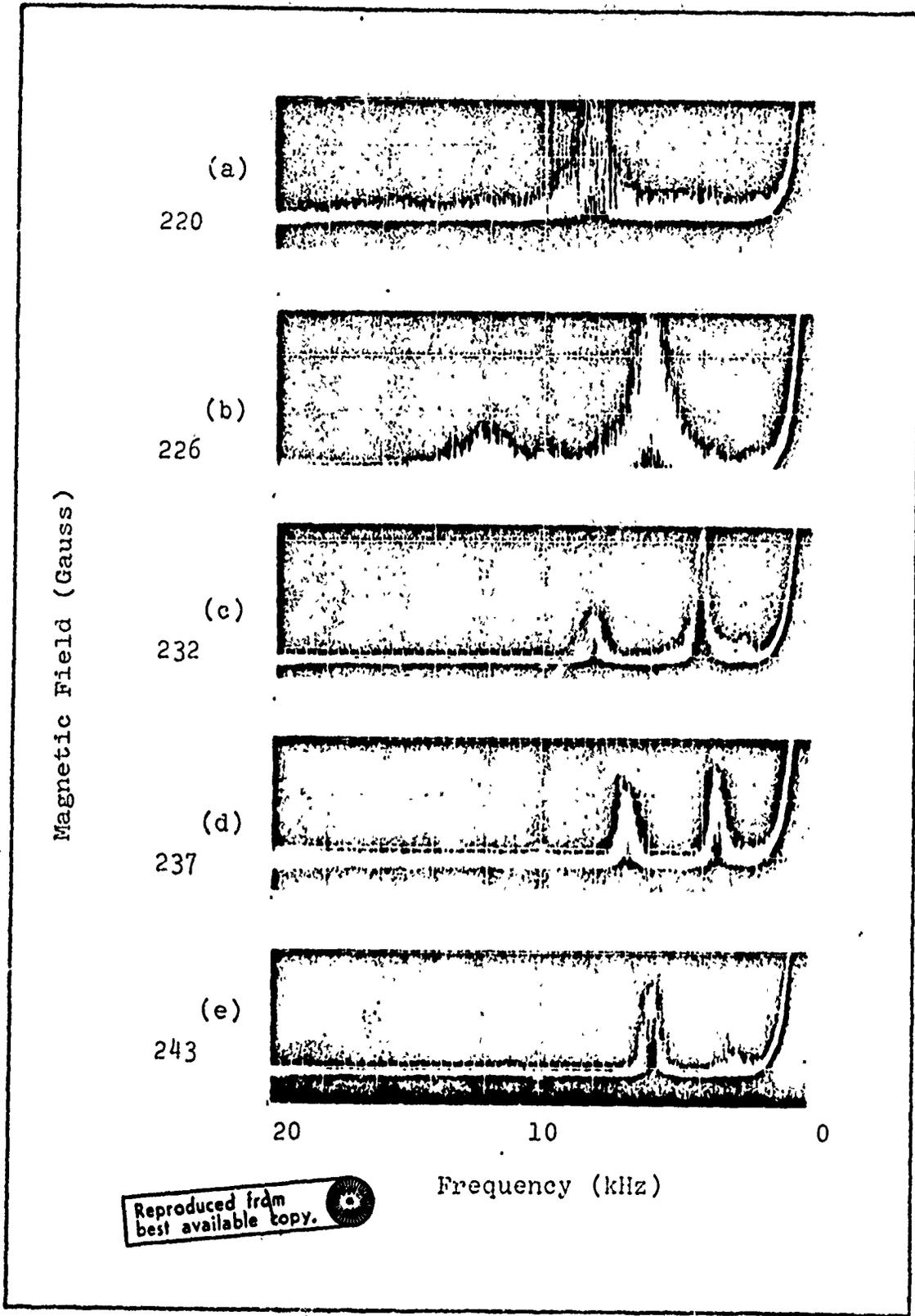


Fig. 2 Spectrum Analyses of Oscillations

oscillation. Fig. 2(c) shows the continuing decrease in frequency of both modes as the magnetic field is increased. The increase in the amplitude of the mode 2 oscillation and the decrease in the amplitude of the mode 1 can clearly be seen in Fig. 2(d) and 2(e). In Fig. 2(e) the mode 1 oscillation has almost completely disappeared, while the mode 2 oscillation has reached its maximum amplitude and continued to decrease in frequency.

To determine the phase relation between the density and potential oscillations, a probe was biased with a negative voltage so that the current drawn by the probe was a measure of the ion density in the plasma surrounding the probe. The current was then amplified and displayed on the oscilloscope to determine the phase relation between density and potential. The density was found to lead the potential by approximately 90 degrees, both in mode 1 and mode 2.

The oscillations were observed over a wide range of control parameters. The limits were usually imposed by equipment limitations rather than the loss of the oscillations. The magnetic field ranged from 200 gauss to 250 gauss in the center of the chamber. No oscillations were observed below 200 gauss and 250 gauss required the maximum magnet current available. Oscillations were found only in the upper twenty percent of the range available in spiral current. This range corresponds to the largest density gradients which were found in plasma density. Pressure could be varied from about 0.4 microns of  $H_g$  to about 2.5 microns without losing the oscillations, but were primarily studied at a pressure between 1.5 to 2.0 microns because

the conditions were the easiest to maintain in this pressure region. Control of the power to the magnetrons, and adjustment of the stub tuners was not used primarily to vary the plasma parameters, but rather were both adjusted so as to insure equal production of plasma at both ends of the chamber with a minimum amount of noise in the magnetron circuits. This was done to minimize the presence of source-generated oscillations in the plasma.

Reversing the current to the magnets resulted in no apparent change in the oscillations, other than a reversal of the theta phase velocity relative to the chamber, as expected.

Reversing the polarity of the spiral voltage resulted in a hollow plasma with maximum density occurring at about 3 cm rather than at the center of the plasma. This plasma configuration was not studied because of the large amount of noise present in the probe outputs and because of the undesirable density profile.

With this knowledge of the general characteristics of the oscillations, a general, qualitative comparison of the instability responsible for the oscillations could now be made. For a quantitative comparison, however, some information on the variation of the plasma parameters was required. To obtain this information, radial scans of the plasma electron temperature, the ion density, and the space potential of the plasma were made. This was done by making a computer run with the probe in a given radial position and obtaining a computer output which gave the desired parameters at that probe position. The probe was then moved to a position a small distance from the first position and another computer run was made. The plots shown in

Fig. 3 through Fig. 8 display the ion density and electron temperature as a function of probe position. Fig. 3 shows the parameters as measured at the top probe with a mode 1 oscillation present in the plasma. Fig. 4 gives the parameters with the magnetic field reduced until the oscillations disappear. Fig. 5 gives the same information as Fig. 4 except that the oscillations were removed by reducing spiral current rather than magnetic field. Fig. 6 through Fig. 8 give the same information, except that the end probe was used rather than the center probe.

The significance of the data given in Fig. 3, 4, and 5 can best be seen by comparing the data observed with the oscillations present to each of the two cases with the oscillations not present. The temperature variation is very slight in either case, however the density with oscillations present (Fig. 3) is three times as great as with no oscillations due to reduced magnetic field (Fig. 4) but only half again as large as the density when the oscillations are removed by reducing the current to the spiral cathodes. As is discussed in comparing the observed oscillations with known types of oscillations, the density gradient emerges as the important factor rather than the density itself. The same relations are seen in Fig. 6, 7, and 8 for the data taken from the end probes.

Although space potential is calculated in the program used, it is not plotted because it was found not to be useful in determining the nature of the instability causing the observed

oscillations. Near the end of this study, some improvements in the computer program was made which caused different values to be calculated for space potential. This occurred too late for new runs to be made, however, so no valid information about the space potential is available for this study.

Another plasma parameter which would aid in a quantitative comparison of the observed instabilities with known theory is the ion temperature. No method was available during this study to measure ion temperature.

The frequency observed ranged from 3 kHz to 8 kHz and exhibited a strong dependence on magnetic field. As the magnetic field is increased, the observed frequency decreases until a mode change occurs. At this point, the frequency doubles and then begins to decrease again with increasing magnetic field.

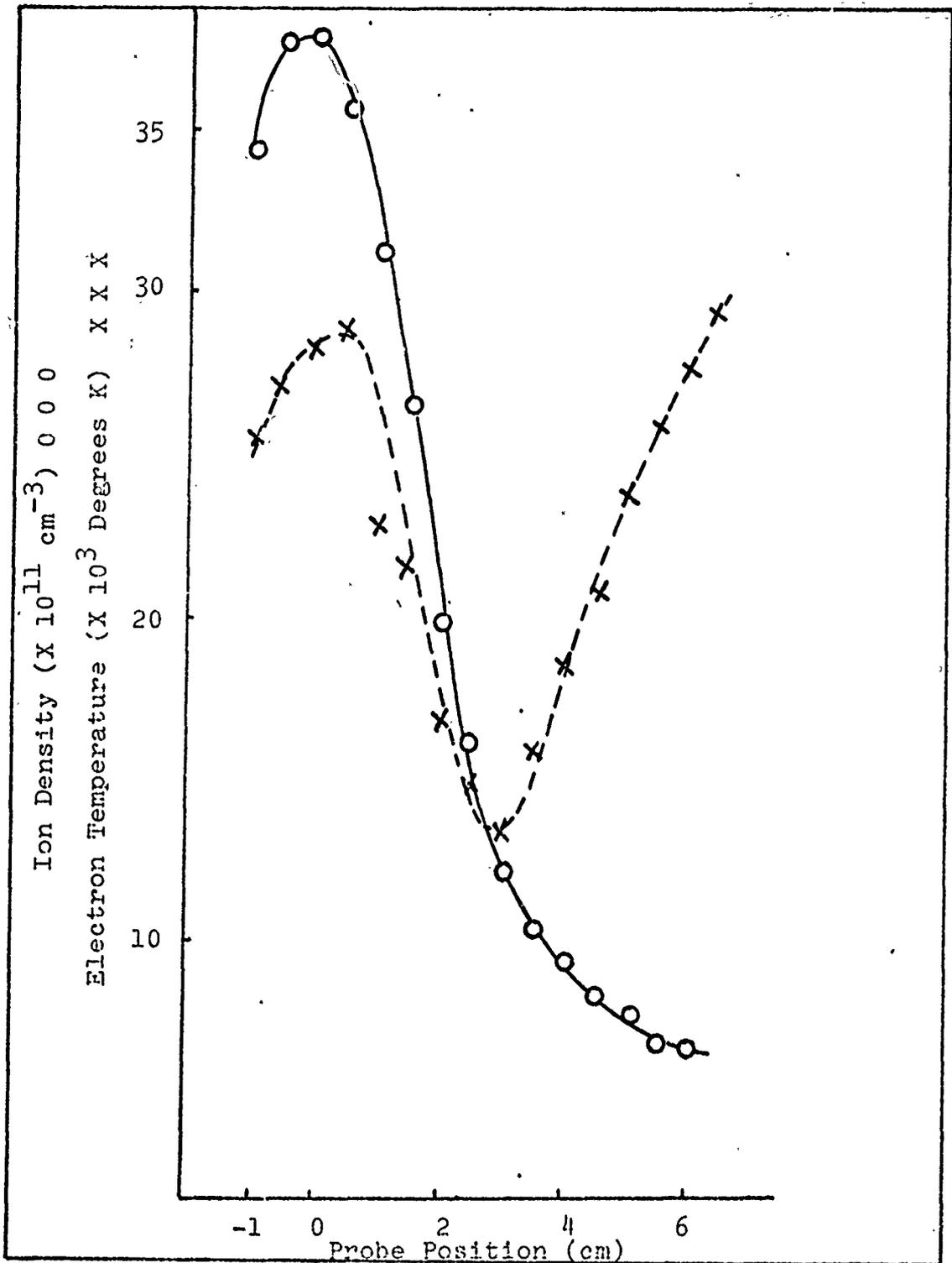


Fig. 3 Center Probe with Mode 1 Oscillations

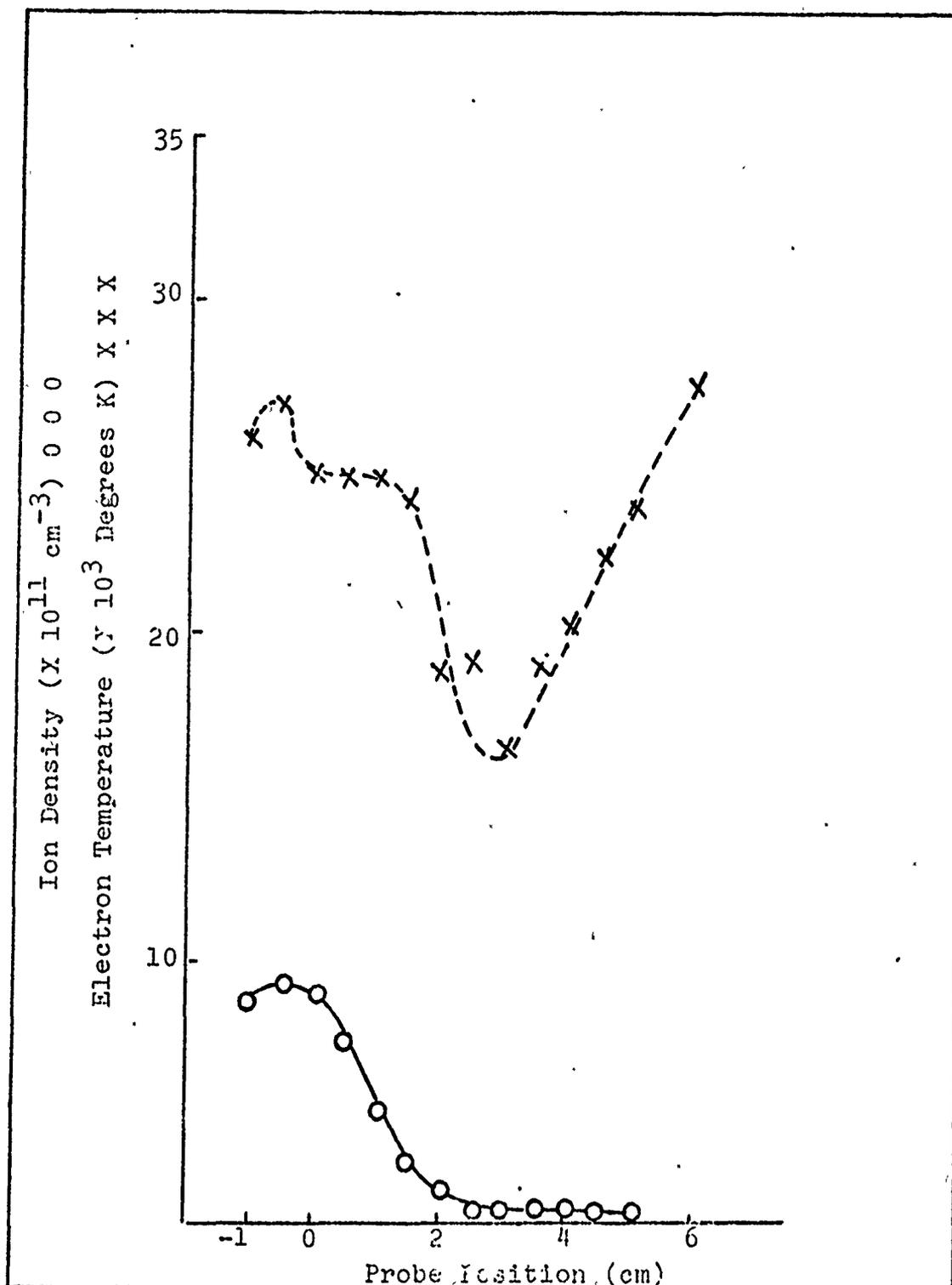


Fig. 4 Center Probe - Reduced Magnetic Field

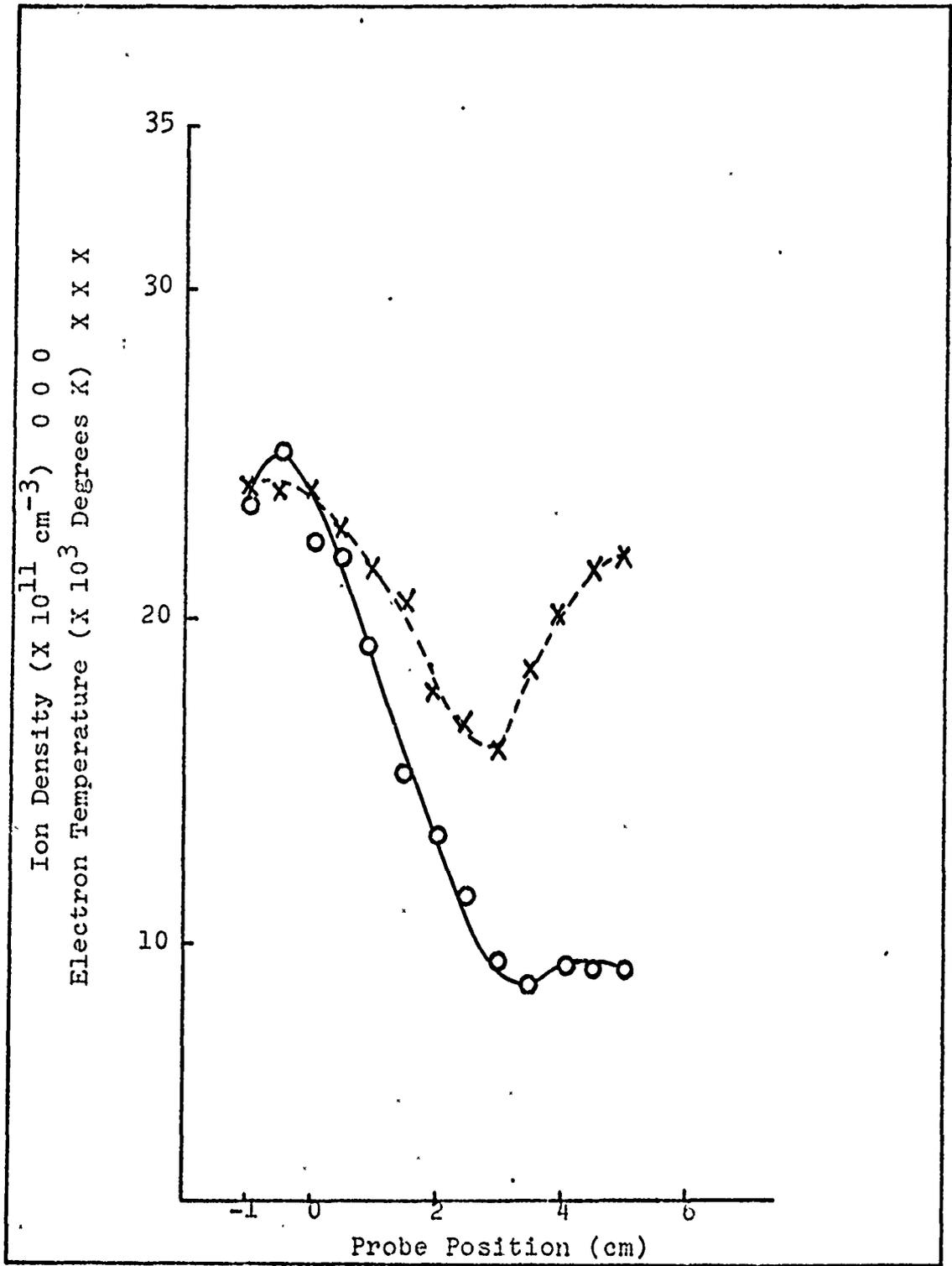


Fig. 5 Center Probe - Reduced Spiral Current

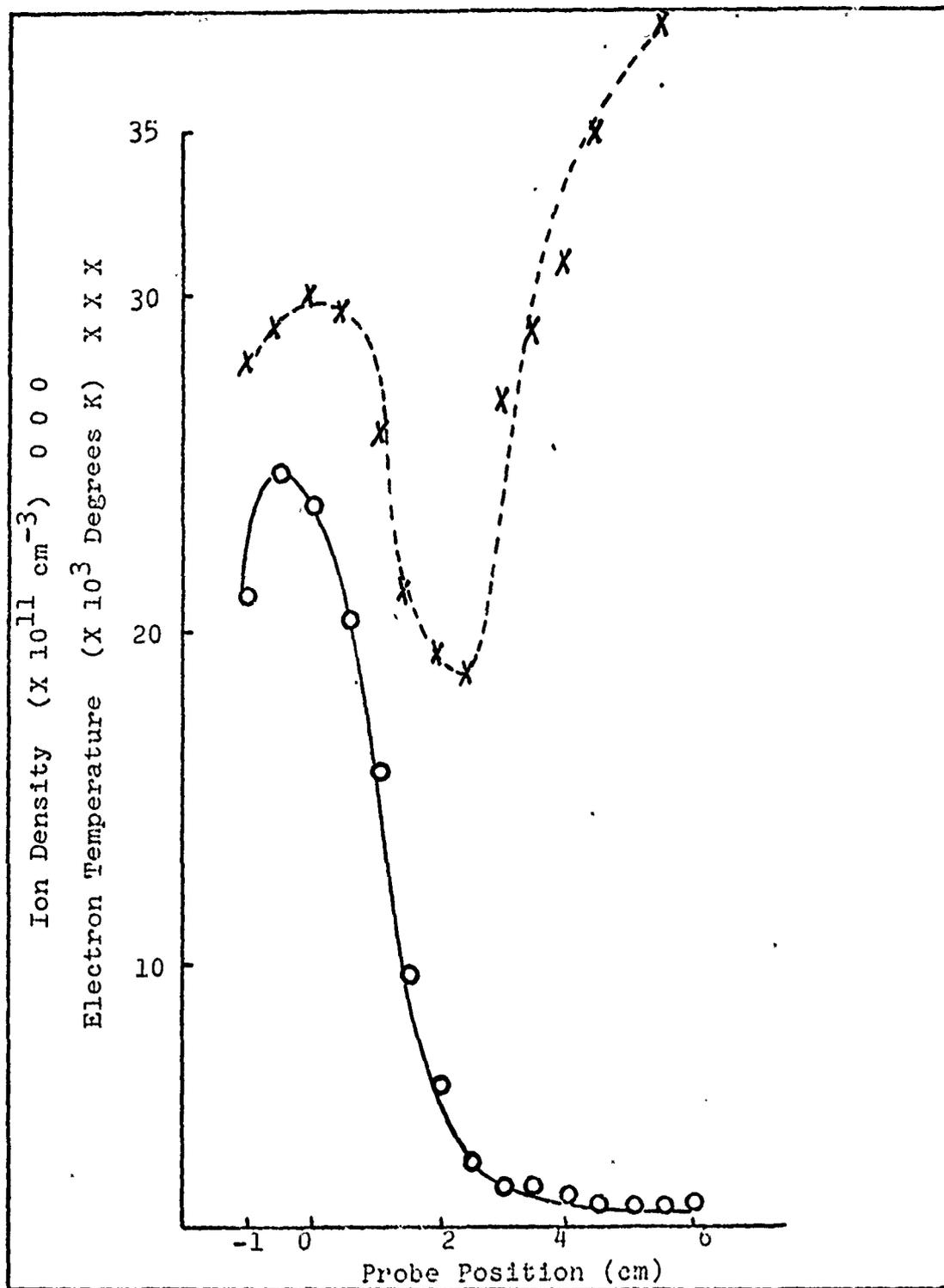


Fig. 6 End Probe With Mode 1 Oscillations

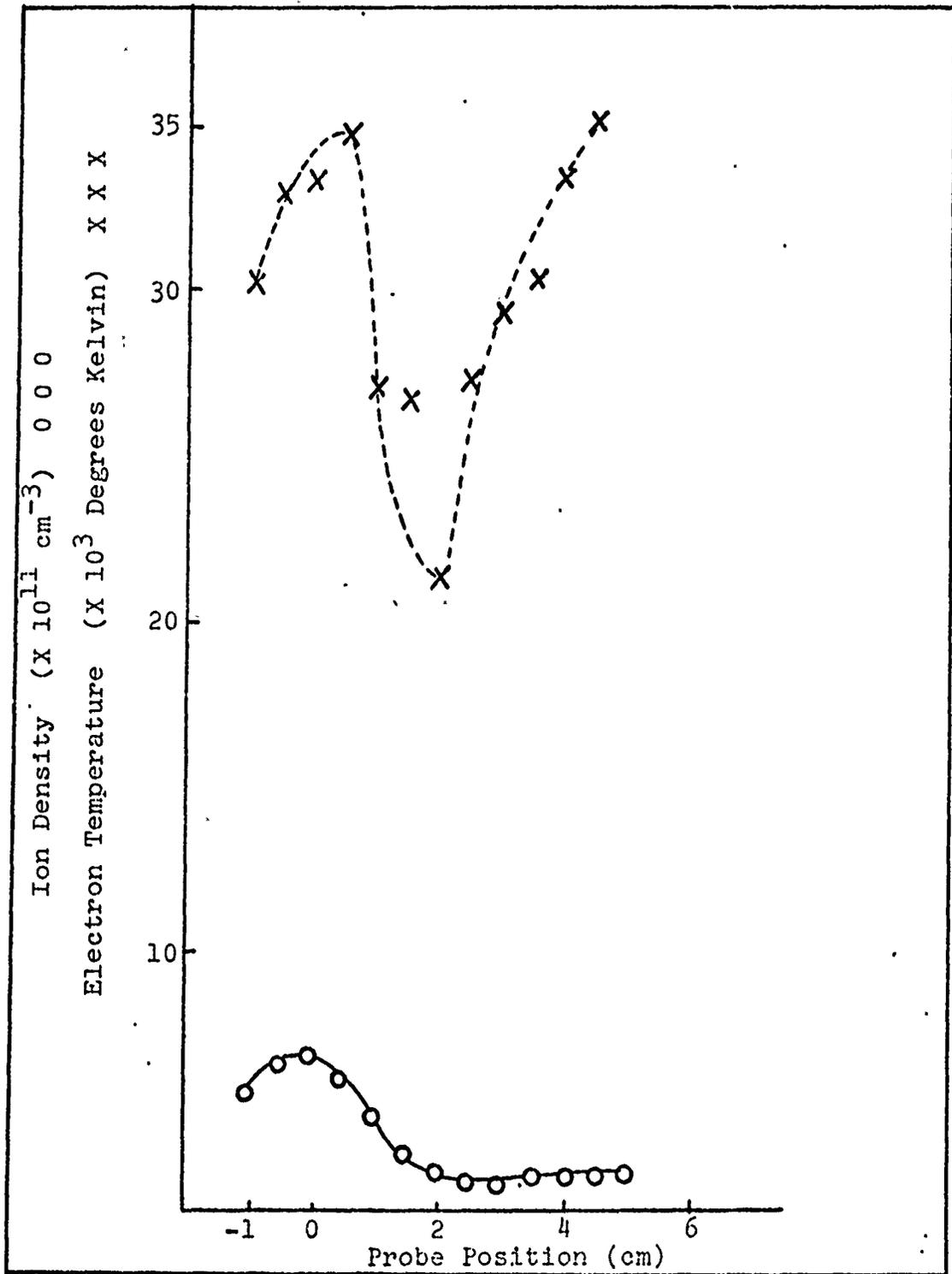


Fig. 7 End Probe - Reduced Magnetic Field

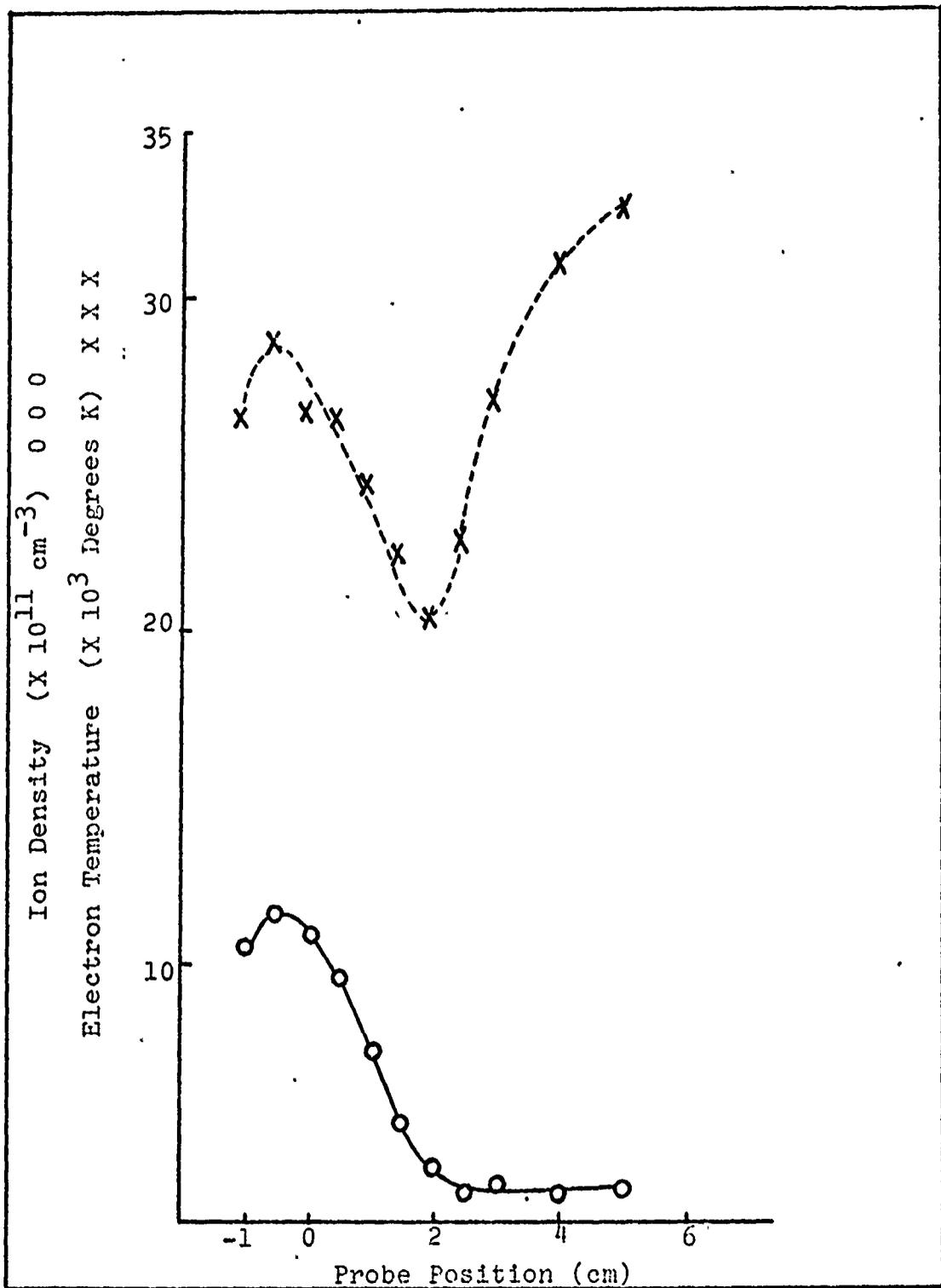


Fig. 8 End Probe - Reduced Spiral Current

IV. Theory

The very large number of known instabilities makes a comparison of a given experimentally observed instability with all known instabilities almost impossible. The literature survey by Auer, et al. proved invaluable in making the comparison. This survey lists 136 known instabilities and cites references for each. Many of the instabilities listed could be eliminated immediately because of some obvious disagreement with the observed characteristics. For example, all of the ion-cyclotron instabilities were eliminated just on the basis of the frequency observed being much too low for this type of instability. (Ref 2)

Many other instabilities would exhibit experimental characteristics which are in agreement with those observed, but can be eliminated on the basis of some one calculated plasma parameter, such as one of the collision frequencies. An example of this type is the Simon-Hoh instability (Ref 3:383) which was eliminated because the ion-neutral collision frequency is too low in the experimental plasma (Ref 4:14).

The Kelvin-Helmholtz instability was considered to be almost identical to the observed instability until the parallel wavelength was measured. Since the Kelvin-Helmholtz wave is characterized by short parallel wavelengths, the observation of long parallel wavelengths allowed elimination of this instability (Ref 5:818).

Temperature-gradient-driven drift instabilities also exhibit characteristics very similar to the observed waves, however, they are stabilized by the presence of a density gradient and one stability criterion is a simple relation between the density and temperature gradients which the experimental plasma does not satisfy (Ref 6:1107).

There were two types of instabilities which were considered which could not be eliminated. These were the collisional drift instability and the flute instability. The term "collisional" as used here refers only to the mechanism which restricts the motion of electrons parallel to the applied magnetic field. As a convenient measure, the drift instability is considered collisional if the mean free path of the electrons between collisions with ions is less than the parallel wavelength of the oscillation. Flute instabilities are almost always considered probable in a mirror-type geometry unless some provision such as Ioffe bars are employed to suppress them. Drift instabilities have been found in almost all kinds of confinement schemes, and are in fact so common that they are often called "Universal Instabilities." The two instabilities are very similar to each other, and in fact the flute is often treated as a special case of the drift instabilities. (Ref 7:154-156)

#### Quantitative Comparison With Flute Instability

The dispersion relation for the flute instability as given by Schmidt (Ref 7:283-294) is used in this analysis. This dispersion relation leads to the stability condition

$$k^2/\Omega^2 (g + u_T^2 \cdot n^1/2n_0)^2 \geq 4g n^1/n_0 \quad (3)$$

where  $k$  is the perpendicular wave number,  $\Omega$  is the ion cyclotron frequency,  $g$  is the effective gravity,  $u_T$  is the thermal speed of the ions and  $n^1/n_0$  is the normalized density gradient. If the plasma is assumed to be nearly thermal, and no radial electric field is considered, then  $g = (\frac{3}{2}) U_T^2/R$ . If the inequality in Egn. 3 is not satisfied, unstable oscillations result. Using data from the top center probe where flute instabilities are most likely to occur and assuming ion temperature to be about 300 degrees Kelvin, the values for the experimental plasma are

$$\begin{aligned} k &= 1 \times 10^3 \text{ m}^{-1} \\ \Omega &= 5.5 \times 10^4 \text{ rad sec}^{-1} \\ U_T^2 &= 6.2 \times 10^4 \text{ m}^2 \text{ sec}^{-2} \\ g &= 1.9 \times 10^4 \text{ m sec}^{-2} \\ n^1/n_0 &= 49 \text{ m}^{-1} \end{aligned}$$

and the inequality is not satisfied. However, solving the dispersion relation for the real part of the frequency gives a value of less than one kHz which does not agree with the observed frequency. In addition, since the drift velocity imparted to the electrons and ions by a flute instability is proportional to the mass of the particles, the drift of the electrons is negligible compared to the ion drift velocity and therefore, the oscillation travels in the direction of the ion

drift velocity. This results in the potential wave leading the density wave which is the reverse of the observed relation. Both of these difficulties can be overcome if a radial electric field is assumed such that the plasma column rotation would give an apparent observed frequency of 4 kHz. With such an electric field, the doppler shift due to the plasma  $E \times B$  drift would dominate the oscillation's phase velocity and the density-potential phase relation would be reversed. In such a case however, the value of  $g$  would be changed by the addition of a term  $V^2/R$  where  $V$  is the tangential velocity of the plasma and  $R$  is the distance from the center of rotation. To give the proper frequency,  $V^2/R = 9.6 \times 10^5 \text{ m} \text{ sec}^{-2}$  and  $g$  becomes  $9.78 \times 10^5 \text{ m} \text{ sec}^{-2}$ . When this new value for  $g$  is inserted into Eqn. 3, the inequality is satisfied and no instability will occur. Justification for the added term for the effective gravity is given by Bhatia (Ref 9:1652). The resulting stability is predicted by Vlasov (Ref 10:486).

#### Quantitative Comparison With Drift Instability

During the literature search for this study, a paper written by Hendel and Politzer (Ref 11) was found which appeared to give almost identical results to the ones obtained during this study. Subsequently, a later paper by Hendel, Chu and Politzer was found which derived a dispersion relation explaining the results of the earlier paper. The observed characteristics of wave number (both parallel and perpendicular), frequency range, density-potential phase

relation, and the dependence of frequency and mode number on magnetic field were all in agreement with the observations made by Hendel, et al. (Ref 12).

The dispersion relation used by Hendel, et al. is derived using linear theory and a slab model for the plasma. In the regime of parameters found in the experimental plasma in this study, the resulting stability condition reduces to

$$K_{11}^2 (n^1/n_0)^{-2} > (m_e/m_i)^{\frac{1}{2}} \quad (4)$$

where  $K_{11}$  is the parallel wave number,  $n^1/n_0$  is the normalized density gradient,  $m_e$  is the electron mass and  $m_i$  is the mass of the ions. Using data from the end probes, this stability condition not only predicts instability when the oscillations are observed, but also predicts stability when the oscillations are eliminated by reducing either spiral current or magnetic field. The correct predictions are considered the best indication that the observed oscillations are indeed caused by a collisional drift instability.

For a numerical analysis, Eq(4) can be rearranged to read

$$K_{11}^2 (m_e/m_i)^{-\frac{1}{2}} > (n^1/n_0)^2 \quad (5)$$

For argon atoms,  $(m_e/m_i)^{\frac{1}{2}}$  equals  $3.69 \times 10^{-3}$ . If  $\lambda_{11}$  is equal to twice the distance between the spiral cathodes,  $K_{11}^2$  equals  $6.85 \times 10^{-4} \text{ cm}^{-2}$ , and the stability condition becomes

$$0.185 \text{ cm}^{-2} > (n^1/n_0)^2 \text{ cm}^{-2} \quad (6)$$

or the equivalent statement, the wave grows if

$$|n^1/n_0| > 0.430 \text{ cm}^{-1} \quad (7)$$

Since the maximum amplitude oscillations were observed at a radial position of 3.0 cm for the center probes and 2.5 cm for the end probes,  $|n^1/n_0|$  is calculated at these positions from the data given in Figs. 3 through 8. For the center probes  $|n^1/n_0|$  equals:

- 0.49 - with oscillations
- 0.12 - with reduced magnetic field  
and no oscillations
- 0.28 - with reduced spiral current  
and no oscillations

For the end probes  $|n^1/n_0|$  equals:

- 0.80 - with oscillations
- 0.23 - with reduced magnetic field  
and no oscillations
- 0.20 - with reduced spiral current  
and no oscillations

The theory agrees with observation in all cases and oscillations are observed only when the inequality in Eqn. (7) is satisfied.

V. Conclusions and Recommendations

The purpose of this study was to find experimentally and to identify, using existing theory, the dominant instabilities in the ELMAX plasma device. The good agreement between the theory developed by Hendel, et. al. was very satisfying. The recommendations which are suggested by the results obtained are simple and obvious. The need to be able to measure ion temperature and parallel wave-length were in fact already known and under consideration before the study began. The study did point out the need for a better way to measure the space potential so that the electric field in the plasma could be calculated.

The follow-on experiment is already under way and some initial success with suppression of the observed oscillations lend some support to the conclusions reached. In particular, suppression of the oscillations using suppressor probes located near the end of the chamber, where the observed density gradient is the largest and where, therefore, the oscillations are predicted to be most likely to grow, lends support to the conclusions reached.

The real conclusion of this study must wait until the experiments on feedback stabilization have been completed.

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## Appendix A

Computer Program Used In Analysis

The computer program used in the calculation of electron temperature and ion density is listed in FORTRAN in this appendix, immediately following fig. 9. Fig. 9 shows a typical computer output from the program. The program is designed to analyze the characteristics of a probe curve, which is the plot of probe current versus probe voltage. The program is written to use a variable number of points in the analysis but normally 250 points were used. In Fig. 9 the first numerical entry is simply entering the number of points to be used. To generate the probe curve, a minimum and maximum voltage are selected by trial and error to include both ion saturation current and electron saturation current. In the run which resulted in Fig. 9, these values are -23 volts and +1 volt. The next entries tell the computer which probe is being used and the radial position of the probe. The last entry refers to the gain on an amplifier used in the probe current circuit.

Since the probe current is the sum of the electron and ion current, a straight line approximation of the ion current is made, and then subtracted from the total current to give electron current. The approximation used is the next item given in Fig. 9. Next follows a listing of selected currents and probe voltages with the "INDEX" showing at which of the 250

points they occurred. This read-out helps in the initial selection of the maximum and minimum voltages used and is also helpful in analyzing the operation of the computer.

To calculate the electron temperature, a non-dimensional potential is defined and the electron current equation is written as an exponential function of this potential. Since the defined potential is a function of electron temperature, a plot of the natural log of electron current versus potential yields a straight line, the slope of which determines the electron temperature.

Since the current at ion saturation is a function of the ion density, a value for density can be calculated once the ion current is known. In this program the density is calculated several ways and four different densities are shown on the computer output. In this study the one selected is calculated using theory developed by Laframboise assuming ion temperature to be zero (Ref 13). The selection of this particular density is not too important since in this study the absolute value of the density is not important, but rather the density gradient. All of the densities given by the program yield approximately the same gradients. The electron temperature is shown in Fig. 9 as "TE at (IPL)" and is given in degrees Kelvin. The density appears as the last entry under "DENSITY."

ENTER RUN NUMBER

1

ENTER: NUMBER OF POINTS

VMIN, VMAX

PROBE NUMBER, POSITION (CM)

CONVERTER GAIN

250

-23,1

2,-1

.1

ION CURRENT APPROXIMATION ( 51.9 INCREMENTS)

CI = .1593E+01 \* V + -.4557E+02      EMAX = .8600E+01

ZERO ION CURRENT AT 28.60 VOLTS

	INDEX	V	CP	CES
MIN	1	-.1533E+02	-.6445E+02	
MI	106	-.5205E+01	-.5675E+02	
MM1	113	-.4530E+01	-.5120E+02	.3101E+01
MM3	120	-.3855E+01	-.4980E+02	.3196E+01
IFL	160	.0000E+00	.1200E+01	.6176E+02
ISP	203	.4145E+01	.2260E+04	.2446E+04
MM4	236	.7325E+01	.5225E+04	.4669E+04
MM2	243	.8000E+01	.5321E+04	.4858E+04
MAX	250	.8675E+01	.5594E+04	

TE AT V(IFL) = 12515. DEG K      EMAX = .4339E-01

SHEATH CALCULATIONS

14.07 VOLTS BELOW V(ISP) (146.0 INCREMENTS)

BETA2 = .1978E+01

SHEATH THICKNESS = .7032E-04

RADIUS = .1145E-03 METERS

LENGTH = .9786E-03 METERS

RP/RS = .3859E+00

ION NUMBER DENSITY =

.12419E+13 AT V(IFL)

.50325E+12 AT V(IFL)-10 (USING SHEATH)

DEB	TI/TE	RP/LD	VPF	DENSITY
.869E-03	1	.512E+01	4.98	.789E+12
.773E-03	0	.574E+01	5.24	.999E+12

PAUSE

PAUSE

Fig. 9 Typical Computer Output

## Appendix A

```

0001 FTN,B
0002 PROGRAM DATA
0003 COMMON V(250),CBS(250),CE(250),A(8),B(8),C(8),ALCES(250)
0004 *TS(250),SDCES(250),D(2),TR(2),TXL(2)
0005 CALL CLPS
0006 NUL=8
0007 E=1.6021E-19
0008 XK=1.3805E-23
0009 TENG=1.E-06
0010 A(1)=0.15113
0011 A(2)=0.14661
0012 A(3)=0.13303
0013 A(4)=0.11041
0014 A(5)=0.078733
0015 A(6)=0.038009
0016 A(7)=-0.011765
0017 A(8)=-0.070588
0018 B(1)=0.0
0019 B(2)=3.5714E-3
0020 B(3)=7.1429E-3
0021 B(4)=1.0714E-2
0022 B(5)=1.4286E-2
0023 B(6)=1.7857E-2
0024 B(7)=2.1429E-2
0025 B(8)=0.025
0026 C(1)=-9.0493E-3
0027 C(2)=-8.5650E-3
0028 C(3)=-7.1105E-3
0029 C(4)=-4.6865E-3
0030 C(5)=-1.2928E-3
0031 C(6)=3.0705E-3
0032 C(7)=3.4034E-3
0033 C(8)=1.4706E-2
0034 XM=40.
0035 XMP=1.6725E-27
0036 XME=9.1091E-31
0037 PI=3.14159
0038 EPS=8.8553E-12
0039 WRITE(2,1000)
0040 1000 FORMAT (7/"ENTER: DATE"/7X,"DIAMETER AND LENGTH (IN IN
0041 *CHRS) OF PROBE 1"/7X,"DIAMETER AND LENGTH OF PROBE 2"/
0042 *7X,"VIS,VFP,VSP")
0043 READ(1,1030)NUM1A,NUM1B
0044 1030 FORMAT(2I3)
0045 DO 180 I=1,2
0046 READ (1,*) TR(I),TXL(I)
0047 TR(I)=TR(I)*2.54E-2/2.
0048 180 TXL(I)=TXL(I)*2.54E-2
0049 READ(1,*)VIS,VFP,VSP
0050 CALL LEADR(8,20)
0051 10 PAUSE 1
0052 WRITE(2,1010)
0053 1010 FORMAT (5/"ENTER RUN NUMBER")
0054 READ(1,*)NUM2
0055 40 WRITE (2,1020)
0056 1020 FORMAT (/ "ENTER: NUMBER OF POINTS"/7X,"VMIN,VMAX"/7X,
0057 * "PROBE NUMBER, POSITION (CM)"/7X,"CONVERTER GAIN")
0058 READ (1,*) NE
0059 READ (1,*) V(1),V(NL)

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0060      HEAD (1,*) IP,CM
0061      READ (1,*) GAIN
0062      R=TH(IP)
0063      XL=TXL(IP)
0064      DV=(V(NW)-V(1))/FLOAT(NW-1)
0065      DO 20 I=2,NW
0066 20    V(I)=V(I-1)+DV
0067      VS=V(NW)-V(1)
0068      CALL SWR (14,ID)
0069      IF (ID) 31,32
0070 31    WRITE (2,7100)
0071 7100  FORMAT (//"ANALYTIC GENERATION"/"ENTER X1,X2 FOR RANGE
0072      *OF X")
0073      READ(1,*)X1,X2
0074      DX=(X2-X1)/FLOAT(NW)
0075      X=X1
0076      DO 710 I=1,NW
0077      CPEL=5000.*(1.+TANH(X))
0078      CPIO=5.*X-10.
0079      CPHIE=20.*(1.+TANH(3.*(X+2.)))
0080      CE(I)=CPEL+CPIO+CPHIE
0081 710   X=X+DX
0082      DO 25 I=1,NW
0083 25    CE(I)=CE(I)/GAIN
0084      GO TO 5
0085 32    CONTINUE
0086      DO 33 I=1,NW
0087 33    CE(I)=0.
0088      GAIN=10.*GAIN
0089      NCODE=4
0090      WRITE (10) NCODE
0091      DO 30 I=1,NW
0092      CALL DVS (V(I))
0093      DO 35 J=1,10
0094      READ (10,1040) IS,TCE
0095 1040  FORMAT (1X,11,E10.0)
0096      IS=IS+1
0097      GO TO (35,2,3,3),IS
0098 2     TCE=-TCE
0099      GO TO 35
0100 3     WRITE (2,1041) I
0101 1041  FORMAT (/"DVS OVERLOAD AT DATA POINT",14)
0102      CALL DVS (V(I))
0103      GO TO 40
0104 35    CE(I)=CE(I)+TCE
0105      IF (10.-ABS(CE(I))) 1,4
0106 1     NCODE=3
0107      WRITE (10) NCODE
0108 4     CE(I)=CE(I)/GAIN
0109      CALL RDPS (IERR)
0110      IF (IERR-1) 37,8
0111 8     WRITE (2,1045) I
0112 1045  FORMAT (/"DVS ERROR AT DATA POINT",14)
0113      GO TO 40
0114 30    CONTINUE
0115      CALL DVS (V(1))
0116 5     CALL SWR(0,ID)
0117      IF (ID) 7,9
0118 7     CALL PLOT (V,CE,NW)
0119      CALL SWR(0,ID)

```



```

0120      IF (ID) 40,9
0121  9    IFL=1
0122      DO 60 I=2,NW
0123      IF (ABS(CE(I))-ABS(CE(IFL))) 11,60
0124  11   IFL=I
0125  60   CONTINUE
0126      CPF=CE(IFL)
0127      VPF1=V(IFL)
0128      CALL DVS (VPF1)
0129      DO 70 I=1,NW
0130  70   V(I)=V(I)-VPF1
0131      DVIS=VIS/DV
0132      IDV=DVIS
0133      IF (IDV-3) 12,13
0134  12   IDV=3
0135  13   IF (IFL-(IDV+1)) 14,15
0136  14   WRITE (2,1060)
0137  1060 FORMAT ("V(MIN) TO VIS INCLUDES V(IFL)")
0138      GO TO 10
0139  15   CALL LILSQ (V(I),CE(I),IDV,D,ER1,ER2)
0140      SL=D(1)
0141      CPO=D(2)
0142      VPC=-CPO/SL
0143      WRITE (2,1070) DVIS,SL,CPO,ER1,ER2,VPC
0144  1070 FORMAT ("ION CURRENT APPROXIMATION (" ,F5.1," INCREMEN
0145      *TS)/2X,"CI =" ,E10.4," + J +",E11.4,4X,"EMAX =" ,E10.4,
0146      *3X,"ERMS =" ,E10.4/2X,"ZERO ION CURRENT AT",F6.2," VOLTS":
0147      IMAX=IFL-1
0148      DO 80 I=IDV,IMAX
0149      J=IFL-I
0150      CIC=0.99*(CPO+SL*V(J))
0151      IF (CIC-CE(J)) 80,21
0152  80   CONTINUE
0153      WRITE (2,1030)
0154  1030 FORMAT ("STRAIGHT LINE APROX NOT VALID")
0155      GO TO 10
0156  21   NI=J
0157      DO 90 I=NI,NW
0158      IF (V(I)-VPC) 17,18
0159  17   TS(I)=CE(I)-(CPO+SL*V(I))
0160  18   IF (TS(I)-0.01) 19,90
0161  19   TS(I)=0.01
0162  90   CONTINUE
0163      INC=NUL-1
0164      MM1=NI+INC
0165      MM2=NW-INC
0166      DO 110 I=MM1,MM2
0167      S1=A(I)*TS(I)
0168      DO 100 J=1,INC
0169      K=J+1
0170      IP=I+J
0171      IM=I-J
0172      SUM=TS(IM)+TS(IP)
0173  100   S1=S1+A(K)*SUM
0174      IF (S1-0.01) 101,102
0175  101   S1=0.01
0176  102   CES(I)=S1
0177  110   ALGES(I)=ALOG(S1)
0178      MM3=MM1+INC
0179      MM4=MM2-INC

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```

0180      DO 130 I=MM3,MM4
0181      S2=0.
0182      S3=C(1)*CES(1)
0183      DO 120 J=1,INC
0184      K=J+1
0185      IP=I+J
0186      IM=I-J
0187      S2=S2+B(K)*(ALCES(IP)-ALCES(IM))
0188 120   S3=S3+C(K)*(CES(IM)+CES(IP))
0189      S2=S2/DV
0190      IF (-S2) 121,122
0191 122   S2=100.*E/KK
0192 121   TS(1)=E/(S2*KK)
0193      S3=S3/(DV*DV)
0194 130   SDCES(1)=S3
0195      MM4A=MM4-5
0196      DO 140 I=IFL,MM4A
0197      IF (SDCES(1)) 131,140
0198 131   IF (SDCES(I+5)) 132,140
0199 132   ISP=I
0200      IF (SDCES(I-1)+SDCES(I)) 133,150
0201 133   ISP=ISP-1
0202      GO TO 150
0203 140   CONTINUE
0204      ISP=IFL
0205      WRITE (2,1207)
0206 1207  FORMAT ("DATA DOES NOT EXTEND TO PLASMA POTENTIAL")
0207 150   CONTINUE
0208      WRITE (2,1090)
0209 1090  FORMAT (/6X,"INDEX",7X,"V",12X,"CP",10X,"CES")
0210      WRITE (2,1100) V(1),CE(1)
0211 1100  FORMAT ("MIN",6X,"I",1X,2E13.4)
0212      WRITE (2,1110) MI,V(MI),CE(MI)
0213 1110  FORMAT ("MI",16,1X,2E13.4)
0214      WRITE (2,1120) MM1,V(MM1),CE(MM1),CES(MM1)
0215 1120  FORMAT ("MM1",17,1X,3E13.4)
0216      WRITE (2,1130) MM3,V(MM3),CE(MM3),CES(MM3)
0217 1130  FORMAT ("MM3",17,1X,3E13.4)
0218      WRITE (2,1140) IFL,V(IFL),CE(IFL),CES(IFL)
0219 1140  FORMAT ("IFL",17,1X,3E13.4)
0220      WRITE (2,1150) ISP,V(ISP),CE(ISP),CES(ISP)
0221 1150  FORMAT ("ISP",17,1X,3E13.4)
0222      WRITE (2,1160) MM4,V(MM4),CE(MM4),CES(MM4)
0223 1160  FORMAT ("MM4",17,1X,3E13.4)
0224      WRITE (2,1170) MM2,V(MM2),CE(MM2),CES(MM2)
0225 1170  FORMAT ("MM2",17,1X,3E13.4)
0226      WRITE (2,1180) NE,V(NE),CE(NE)
0227 1180  FORMAT ("MAX",17,1X,2E13.4)
0228      DVFP=VFP/DV
0229      IDV=DVFP
0230      IF (IDV-3) 151,152
0231 151   IDV=3
0232 152   MST=IFL+IDV
0233      CALL LILSQ (V(MST),ALCES(MST),IDV,D,ER1,ER2)
0234      IF (-D(1)) 153,154
0235 154   D(1)=100.*E/KK
0236 153   TES=E/(KK*D(1))
0237      WRITE (2,1190) TES,ER1,ER2
0238 1190  FORMAT (/ "TE AT V(IFL) =" ,F7.0, " DEG K",5X, "EMAX =" ,E10
0239      * ,4,3X, "ERMS =" ,E10.4)

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```

0240      VDL=V(ISP)+D(1)
0241      CI=- (CPO+SL*V(IFL))
0242      DENS2=0.
0243      DENS2A=0.
0244      DENS4=0.
0245      DENS1=CI*SQRT(KK*XMP/(KK*TES))*1.E-12/(1.13*E*PI*R*XL)
0246      DVSC=10.0/DV
0247      IDV=DVSC
0248      ICB=IFL-IDV
0249      IF (-ICB) 156,155
0250 155   ICB=1
0251 156   CP=CE(ICB)
0252      IF (ICB-MI) 172,171
0253 171   CP=CPO+SL*V(ICB)
0254 172   IF (CP) 157,153,157
0255 157   VPICB=ABS(V(ISP)-V(ICB))
0256      IF (ISP-IFL) 159,160,159
0257 160   VPICB=VPICB+5.*TES*KK/E
0258 159   DVSC=VPICB/DV
0259      DELA=SQRT(E*(2.*VPICB)**3/(KK*XMP))
0260      DELB=4.*PI*EPS*XL/(9.*R*CP+1.E-6)
0261      DELC=SQRT(E*VPICB/(KK*TES))
0262      BETA2=ABS(DELA*DELB*(1.+2.66/DELC))
0263      IF (0.5-BETA2) 161,162
0264 162   DELS=R*(9.5547E-2*BETA2+1.001*SQRT(BETA2)-5.7212E-5)
0265      GO TO 165
0266 161   IF (5.0-BETA2) 163,164
0267 164   DELS=R*(7.81E-2*BETA2+1.0315*SQRT(BETA2)-1.388E-2)
0268      GO TO 165
0269 163   DELS=R*(6.894E-2*BETA2+1.0791*SQRT(BETA2)-7.4135E-2)
0270 165   RS=R+DELS
0271      XS=XL+DELS
0272      RD=R/RS
0273      WRITE (2,2001) VPICB,DVSC,BETA2,DELS,RS,XS,RD
0274 2001  FORMAT (/ "SHEATH CALCULATIONS" /5X,F5.2," VOLTS BELOW
0275      *V(ISP) (" ,F5.1," INCREMENTS" /5X," BETA2 =" ,E10.4/5X,"
0276      *SHEATH THICKNESS =" ,E10.4/5X," RADIUS =" ,E10.4," METERS
0277      *"/5X," LENGTH =" ,E10.4," METERS" /5X," RP/RS =" ,E10.4)
0278      IF (14.-BETA2) 168,167
0279 167   DENS4=- (CP*SQRT(KK*XMP/(KK*TES))*1.E-12)/(1.13*E*
0280      *PI*RS*XS)
0281 168   WRITE (2,1210) DENS1,DENS4
0282 1210  FORMAT (/ "ION NUMBER DENSITY =" /5X,E11.5," AT V(IFL)" /
0283      *5X,E11.5," AT V(IFL)-10 (USING SHEATH)" )
0284 158   CONTINUE
0285      BK=KK*1.27
0286      RP=R*100.
0287      EV=KK*TES/E
0288      CGE=4.8022E-10
0289      WRITE (2,501)
0290 501   FORMAT (/3X,"DEB",4X,"TI/TE",4X,"RP/LD",5X,"VPP",5X,
0291      * "DENSITY")
0292      RT=SQRT(XM2/(KK*XMP))
0293      DEB1=SQRT(BK*TES/(4.*PI*CGE*CGE*DENS1))
0294      JP=0
0295      VLT=10.0
0296 299   JPASS=0
0297      DEB=DEB1
0298      RPLD=RP/DEB

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0300 300 GO TO (301,303,310),JP
0301 301 ITITE=1
0302 CII=2.E-4*RPLD*RPLD-0.031752*RPLD+2.8366
0303 GO TO 305
0304 303 ITITE=0
0305 CII=1.6E-4*RPLD*RPLD-0.02848*RPLD+2.2570
0306 305 VPN=-ALOG(RT*CII)
0307 VPF=VPN*EV
0308 IVS=IFL+IFIX(VPF/DV)
0309 IVN=VLT*EV/DV
0310 IVI=IVS-IVN
0311 VIVI=V(IVI)
0312 IF (-IVI) 307,306
0313 306 VIVI=V(1)+FLOAT(IVI-1)*DV
0314 307 CPP=CPO+SL*VIVI
0315 DENS3=-CPP*SQRT(XM*XMP/(2.*PI*XK*TES))*1.E-12/(CII*R*XL*E)
0316 DEB3=SQRT(BK*TES/(4.*PI*CGE*CGE*DENS3))
0317 DEED=ABS(DEB-DEB3)
0318 IF (DEED-0.01*DEB1) 309,314
0319 314 DEB=DEB3
0320 RPLD=RP/DEB
0321 JPASS=JPASS+1
0322 IF (9-JPASS) 309,300
0323 309 WRITE (2,502) DEB3,ITITE,RPLD,VPF,DENS3
0324 502 FORMAT (E3.3,15,E12.3,F8.2,E11.3)
0325 IF (9-JPASS) 311,299
0326 311 WRITE (2,500) DEED
0327 500 FORMAT ("**DEED =",F6.4)
0328 GO TO 299
0329 310 CONTINUE
0330 PAUSE 4
0331 CALL SWR (1,ID)
0332 IF (ID) 22,23
0333 22 N=ISP-MM3+5
0334 CALL PPLOT (V(MM3),TS(MM3),N)
0335 23 CALL SWR (2,ID)
0336 IF (ID) 24,26
0337 24 WRITE (4,1050) NUM1A,NUM1B,NUM2,CX,TES,VFP1,V(IVS),DENS3
0338 *,DENS4
0339 1050 FORMAT (313,6E10.4)
0340 26 CONTINUE
0341 GO TO 10
0342 END
0343 ENDS

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\*\*END-OF-TAPE

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/E

Vita

Victor Robert Trouy was born on 13 September 1936 in Helena, Arkansas. He received the degree of Bachelor of Science in physics from Christian Brothers College in Memphis, Tennessee in 1962. He was commissioned a Lieutenant in the Air Force upon completion of Officers Training School in 1962. He received the aeronautical rating of pilot in 1965, and served flying tours in Hawaii, Southeast Asia and Florida.

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