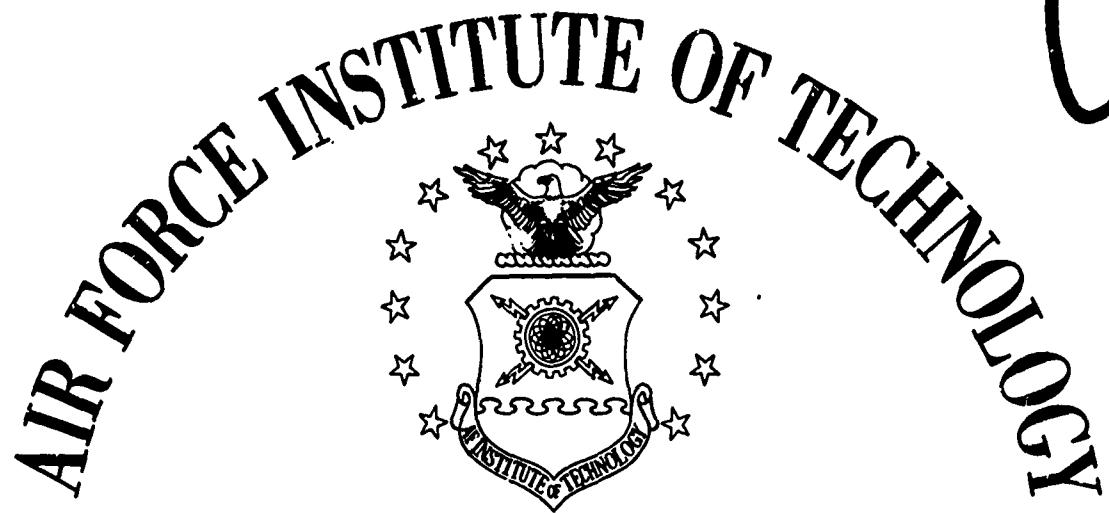


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CONDITIONAL BEST LINEAR INVARIANT
ESTIMATION OF THE LOCATION AND SCALE
PARAMETERS OF THE CAUCHY DISTRIBUTION
BY THE USE OF ORDER STATISTICS

THESIS

GAM/MATH/72-3 Ralph M. Spory, Jr.
 Captain USAF

1972

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CONDITIONAL BEST LINEAR INVARIANT ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY
DISTRIBUTION BY THE USE OF ORDER STATISTICS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Ralph M. Spory, Jr., B.S.
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Graduate Aerospace-Mechanical Engineering
March 1972

Approved for public release: distribution unlimited.

Preface

This thesis is a continuation and extension of previous work by graduate students at the Air Force Institute of Technology in the area of parameter estimation using order statistics of samples from a given distribution. The tables of linear coefficients developed in this report will enable the user to obtain the best linear invariant estimate of the location and scale parameters of the Cauchy distribution for sample sizes of $N=5(1)20$ very efficiently. An attempt was made in the report to provide a clear development of the theory by which these linear coefficients are obtained. In addition, the Fortran program required to calculate and table the linear coefficients is included in Appendix C. The subroutine used to solve the matrix equations is a modification of the Matrix Equation Solver Fortran Subroutine from the Computer Science Center, Wright-Patterson AFB, Ohio.

I also wish to acknowledge my debt to Professor Albert H. Moore, my advisor, for proposing this area of study and for his assistance and encouragement.

Ralph M. Spory, Jr.

Contents

	<u>Page</u>
Preface	ii
List of Figures	v
List of Tables	v
Abstract	vi
I. Introduction	1
Statement of the Problem	1
Objective	1
Definition of Terms	1
Significance	3
Background Information	4
Report Organization	7
Assumptions	7
II. The Cauchy Distribution	8
Introduction	8
Example of the Cauchy Distribution	8
Generation of the Cauchy Density	10
Properties of the Cauchy Distribution	10
III. Order Statistic Theory	12
Introduction	12
Order Statistics	12
Definition	12
Density	12
Expected Values	13
Variance and Covariance	14
Standardized, Cauchy Order Statistics	14
Solution of the Expected Value and Covariance Equations	17
IV. Linear Parameter Estimation	19
Introduction	19
Linear Estimation	19
Mean Square Error	20
Location Parameter	20
Scale Parameter	22
Minimization of the Mean Square Error	23
Matrix Equations	24
Solution of the Matrix Equations	25

Contents (Contd)

	<u>Page</u>
Additional Censoring	26
Censoring from Above	26
Symmetric Censoring	27
V. Use of the Tables	28
Introduction	28
Explanation of Tables I and II	28
Estimation Procedure	28
Examples	29
No Additional Censoring	29
Additional Censoring from Above	31
Simultaneous Estimation	32
VI. Summary	34
Bibliography	36
Appendix A: Table I	38
Appendix B: Table II	70
Appendix C: Computer Programs	87
Vita	97

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List of Figures

<u>Figure</u>	<u>Page</u>
1 The Cauchy Probability Density Function . . .	9

List of Tables

<u>Table</u>		<u>Page</u>
I	Coefficients for Best Conditional Estimation of the Location and Scale Parameters of the Cauchy Distribution (with Additional Censor- ing from Above)	39
II	Coefficients for Best Conditional Estimation of the Location and Scale Parameters of the Cauchy Distribution (with Additional Sym- metric Censoring)	71

Abstract

Linear coefficients which can be applied to sample data from a Cauchy distribution to obtain estimates of the location and scale parameters are developed and tabled. Several previous works have presented such tables for nearly best linear unbiased estimation and best linear unbiased estimation of the parameters. The estimates developed in this paper are best in the sense that they possess minimum mean square error. By using exact values of the means, variances, and covariances of the Cauchy standardized order statistics and minimizing the mean square error function, matrix equations are developed and solved to obtain the required coefficients. These coefficients and values of the MSE are tabled for minimally censored sample sizes of 5 to 20 and for samples which have been additionally censored from above and symmetrically. Procedures for using the tables and several illustrative calculations demonstrate the simplicity of this estimation technique. The Fortran programs required to calculate and table the above values are included in Appendix C.

CONDITIONAL BEST LINEAR INVARIANT ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY
DISTRIBUTION BY THE USE OF ORDER STATISTICS

I. Introduction

Statement of the Problem

Objective. The objective of this thesis is to develop a table of linear coefficients which can be easily applied to sample data from a Cauchy distribution to determine estimates of the location and scale parameters of the distribution. These estimates of the parameters are conditional best linear invariant estimates. These terms and the properties of the estimators are defined in the next section.

Definition of Terms. The Cauchy distribution is a continuous distribution which is frequently introduced to students as an example of a distribution for which the moments do not exist (Ref 6:134). The cumulative distribution function (cdf) is given by

$$F(x) = \frac{1}{2} + \frac{1}{\pi} \operatorname{ARCTAN} \frac{x-t}{s}, \quad -\infty < x < \infty \quad (1)$$

and the probability density function (pdf) is given by

$$f(x) = \frac{s}{\pi[s^2 + (x-t)^2]}, \quad -\infty < x < \infty \quad (2)$$

where s is the scale parameter and t is the location parameter.

Conditional estimation of a parameter is defined as estimation of an unknown parameter when the second parameter is known. In this case the location parameter is estimated conditioned on the value of the scale parameter, and the scale parameter is estimated conditioned on the value of the location parameter.

The best linear invariant estimator is best in the sense that it is a minimum mean-square-error estimator. The mean square error of the estimator is given by

$$\text{MSE} = E[(s^* - s)^2] \quad (3)$$

where s is the true value of the parameter and s^* is the estimated value of the parameter.

For conditional estimation the mean square error is given by

$$\text{MSE} = E[(s^*|t-s)^2] \quad (4)$$

where $s^*|t$ is the estimator of the parameter conditioned on the value of t , and t is the value of the second parameter.

Linear estimation is a technique based on the theory of order statistics, where each ordered sample value is assigned a weight, or linear coefficient, and the coefficients are calculated so as to obtain the best estimate of the parameter. In this case the best estimate is the invariant estimate as defined above. If one uses this method, the estimate of the parameter is given by

$$s^* = \sum_{i=1}^n a_i x_i \quad (5)$$

where the a_i 's are the linear coefficients, the x_i 's are the ordered sample values or the order statistics, and n is the sample size.

The conditional estimator is then given by

$$s^*|t = \sum_{i=1}^n a_i x_i - At \quad (6)$$

where t is the known parameter and A is a constant that is determined by the calculation. The significance of this constant will be explained in Chapter IV.

Order statistic theory, the Cauchy distribution, and the method used to solve for the linear coefficients are developed further in the remaining chapters of this paper.

Significance. When the location and scale parameters of the Cauchy distribution are known, the function is completely defined, and it can be used in a decision-making process where one is working with data which is distributed according to the Cauchy law. The tables of coefficients developed in this thesis will allow the user to estimate the values of these unknown parameters. These estimates can be obtained very efficiently with only the use of a desk calculator. The ordered sample values are simply multiplied by the appropriate coefficients and the results summed to calculate the estimate.

This method of estimation provides a great savings in time in the case of the Cauchy distribution, as the traditional methods of obtaining these estimates either do not apply to the distribution or are very time-consuming. Two of these methods are the method of moments (Ref 11:186) and the method of maximum likelihood (Ref 11:170). The method of moments cannot be applied, since the moments of the Cauchy distribution do not exist, and the maximum likelihood estimate, which is convenient to obtain for some other distributions, requires a great amount of computational effort. Barnett (Ref 1) points out that the frequent occurrence of multiple zeros of the derivatives of the logarithm of the likelihood function requires a complete scan of the likelihood function to locate the maximum which corresponds to the maximum likelihood estimate. The tables of linear coefficients in this thesis will provide the user with an estimate of these parameters for sample sizes of 5 to 20.

Background Information

Work on parameter estimation based on order statistic theory has been carried out by the students at the Air Force Institute of Technology, under the direction of Professor Albert H. Moore, and sponsorship of Dr. H. Leon Harter (ARL, Wright-Patterson AFB), since 1963. These works include parameter estimation of the Cauchy, Weibull, normal, log-normal, logistic, and extreme value distributions. Parameter estimation of the Cauchy distribution includes the works of Chamberlain (Ref 2), Jonson (Ref 9) and Stark (Ref 16).

Chamberlain developed and tabled the coefficients for nearly best linear unbiased estimation of the location and scale parameters for sample sizes 15(1)40. Jonson computed the coefficients for conditional best linear unbiased estimation of the parameters of the Cauchy distribution and compared the efficiency of these estimators with the efficiency of Stark's best linear unbiased estimators. The estimators developed by Jonson and Chamberlain are called nearly best estimators because the approximate values of the covariances of the order statistics given by Blom's approximation were used instead of the exact covariances of the order statistics. Stark's work developed linear coefficients for simultaneous estimation of the location and scale parameters by using the exact values of the means, variances, and covariances of the standardized order statistics.

The works of Chamberlain and Jonson are based largely on the methods presented by Barnett (Ref 1:1205). Barnett tabled the coefficients for the best linear unbiased estimate of the location parameter of the distribution for sample sizes of 5 to 20. The exact values of the means, variances, and covariances of the Cauchy order statistics were calculated to four decimal-place accuracy. The values of the covariances were obtained by numerical integration of expressions containing the joint pdf of the order statistics (see Chapter IV). These functions were integrated over the relevant triangular region by a two-dimensional

extension of the composite Simpson procedure. Although this computation required a large number of steps to obtain the desired accuracy, it did prove feasible.

The median of the sample data has traditionally been used as an estimate of the location parameter of the Cauchy distribution. Cramer (Ref 4:708) states that the variance of this estimator is $\pi^2/4n$ for large samples. Rider (Ref 13:322) shows that this is not a very accurate estimate of the variance of the median for small sample sizes. Rider has tabled the actual variances of the median for small sample sizes.

In 1964, Rothenberg et al. proposed a class of estimators of the location parameter of the Cauchy distribution which is the arithmetic average of a central subset of the sample order statistics. The sample median is a member of this subset, but it was shown that the average of the middle quarter of the ordered samples has a lower asymptotic variance than does the median.

In 1970, Chan (Ref 3:851) proposed a conditional asymptotically best linear estimator of the location and scale parameters based on a few of the order statistics. He has tabled coefficients for $K=1(1)10$ where K is the number of order statistics selected from a large sample. These estimators yield more than 92 percent asymptotic relative efficiency, in the Cramer-Rao sense, for $K \geq 4$.

Report Organization

In order to develop the linear coefficients for the conditional best linear invariant estimation of the parameters of the Cauchy distribution, the distribution and methods by which it is generated will be discussed in Chapter II. Chapters III and IV present order statistic theory and the linear estimation procedure. Chapter V describes the tables of linear coefficients and gives examples of the calculations required to obtain the desired estimators. Tables of the values of the mean-square-error function and the linear coefficients are included in Appendices A and B. Appendix C contains the Fortran programs required to calculate and table the above values.

Assumptions

There are two assumptions made in this report. It is assumed that the sample data are known to come from a Cauchy distributed parent population and that the parameter not being estimated is known, in the case of conditional estimation. In the case of simultaneous estimation it is only assumed that the sample data are known to come from a Cauchy distributed parent population.

II. The Cauchy Distribution

Introduction

The Cauchy distribution is a continuous, symmetric distribution. The cdf and pdf are given by Equations (1) and (2). The plot of the pdf is similar to that of the more familiar normal distribution, except that the curve approaches the axis much more slowly and the tails are thicker. Figure 1, on the following page, is a plot of the density function for three different scale parameters. Feller (Ref 5:57) provides an excellent description of the Cauchy distribution, its peculiarities, and methods by which the distribution is generated.

Example of the Cauchy Distribution

A mirror is arranged parallel to an opposing wall at a distance S from the wall, and the mirror is free to rotate on a vertical axis at A which is located on a line perpendicular to the wall at O . The angle θ is measured from this line to the perpendicular to the surface of the mirror. The mirror reflects a ray of light on the wall at a distance X from the point O . Now, if the angle θ is chosen at random between $-\pi/2$ and $\pi/2$, the random variable, X , is Cauchy distributed and the density of the distribution is given by Equation (2) with a location parameter of zero (Ref 5:57).

The density of the random variable, X , can easily be verified by a method due to Meyer (Ref 10:88-89). In the above example

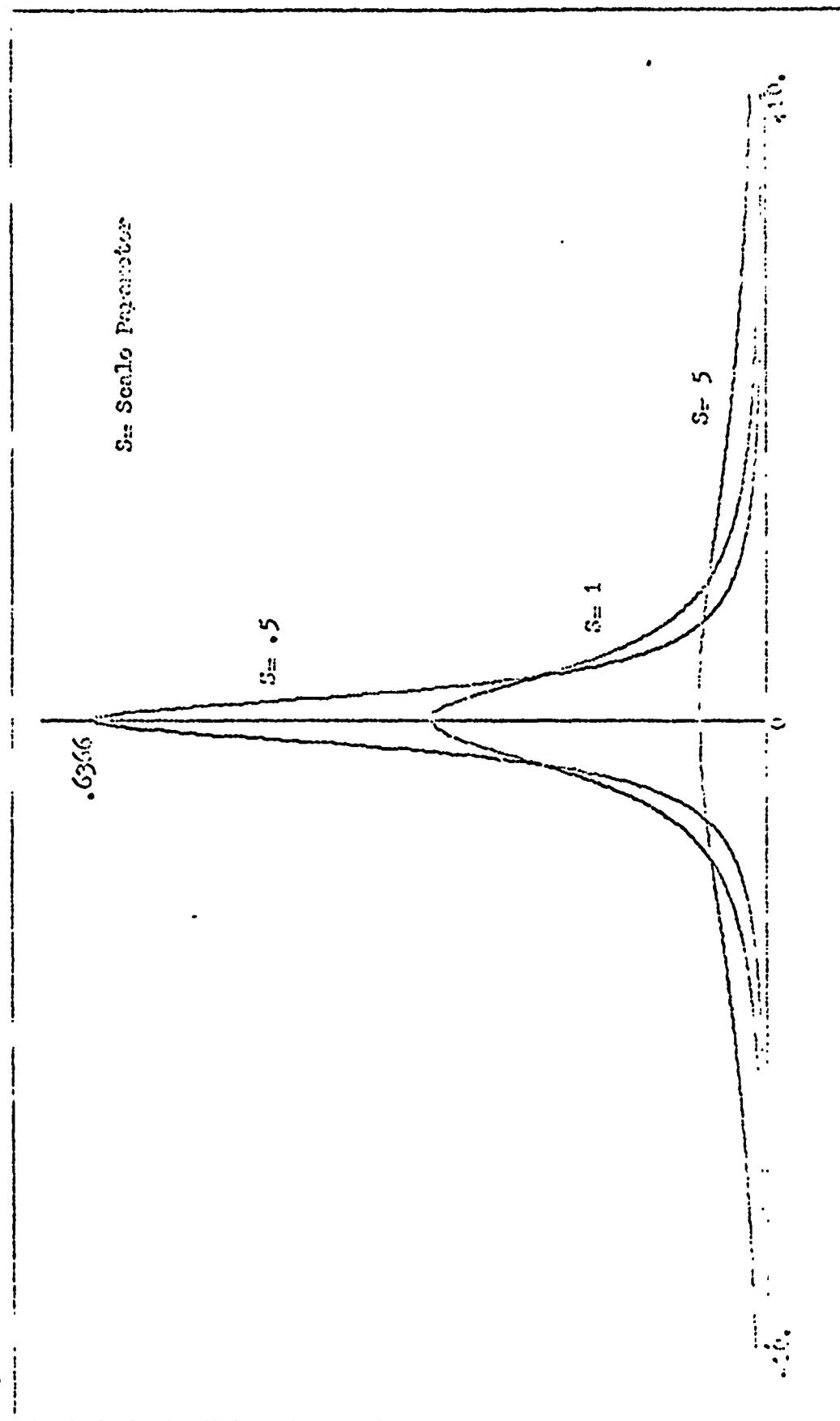


Fig. 1. The Cauchy Probability Density Function

$$f(\theta) = \frac{1}{\pi} \quad \text{for} \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

is a uniform density. Then

$$g(x) = \frac{1}{\pi} \frac{d\theta}{dx} \quad (7)$$

where $\theta = \tan^{-1} \left(\frac{x}{s} \right)$. Therefore

$$g(x) = \frac{1}{\pi} \frac{1}{1+\frac{x^2}{s^2}} \frac{1}{s} \quad (8)$$

$$g(x) = \frac{1}{\pi} \frac{s}{[s^2+x^2]} \quad \text{for } -\infty < x < \infty \quad (9)$$

This is the Cauchy pdf with scale parameter s and location parameter zero.

Generation of the Cauchy Density

The Cauchy density may be generated in many ways. The Student's t density with $n=1$ is a Cauchy density. If x and y are two independent random variables from a standardized normal distribution, the quotient of these random variables is a standardized Cauchy distribution. In addition, if the random variable x is Cauchy distributed with a scale parameter of 1 then $1/x$ has the same density. Once again, these densities may be verified by the method described by Meyer (Ref 5:109-110).

Properties of the Cauchy Distribution

Due to the thick tails of the Cauchy distribution, estimation of its center is very difficult. The moments of the

Cauchy distribution do not exist. Jonson (Ref 9:11) shows the characteristic function of the Cauchy random variable to be

$$C_X(t) = \exp(ict - b|t|) \quad (10)$$

where C is the location parameter and b is the scale parameter, and that the moments do not exist since the partial derivatives of $C_X(t)/i^k$ with respect to t evaluated at t=0 are infinite.

It can also be shown directly, that the first moment about the origin of the Cauchy distribution does not exist (Ref 6:145).

III. Order Statistic Theory

Introduction

To calculate the conditional best linear invariant estimates of the parameters of the Cauchy distribution the exact values of the means, variances, and covariances of the Cauchy order statistics are required. The order statistic theory and application of this theory to the Cauchy distribution will be reviewed in this chapter. Reference 15 is a rather complete collection of contributions to order statistic theory and includes articles up to 1962. The following development follows that of Chapter II from the above reference.

Order Statistics

Definition. If a random sample of size n (x_1, x_2, \dots, x_n) is taken from a population, these independent random variables can be rearranged so that

$$x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$$

When the variables are arranged in order of magnitude, they are called order statistics of the sample. Since these samples are from a continuous population

$$P(x_{(i)} = x_{(j)}) = 0 \quad \text{for all } i \neq j$$

Density. The pdf of the i th order statistic, $x_{(i)}$, is given by

$$g(x_{(i)}) = \frac{n!}{(i-1)!(n-i)!} [F(x_{(i)})]^{i-1} [1-F(x_{(i)})]^{n-i} f(x_{(i)}) \quad (11)$$

where $F(x_{(i)})$ = cdf of x evaluated at $x=x_{(i)}$

$f(x_{(i)})$ = pdf of x evaluated at $x=x_{(i)}$ (Ref 15:12)

The joint distribution of the i th and j th order statistics ($i < j$) is given by

$$g(x_{(i)}, x_{(j)}) = \frac{n!}{(i-1)!(j-i-1)!(n-j)!} [F(x_{(i)})]^{i-1} [F(x_{(j)}) - F(x_{(i)})]^{j-i-1} [1-F(x_{(j)})]^{n-j} f(x_{(i)}) f(x_{(j)}) \quad (12)$$

for $x_{(i)} < x_{(j)}$ (Ref 15:12)

From Equations (11) and (12) expressions for the expected values and covariances of the Cauchy order statistic can be developed.

Expected Values. Let x be a continuous random variable with pdf f , then the expected value of x is given by

$$E[x] = \int_{-\infty}^{\infty} x f(x) dx \quad (\text{Ref 10:121}) \quad (13)$$

Using Equations (13) and (14), one finds that the expected value of the i th order statistic is given by

$$E[x_{(i)}] = \int_{-\infty}^{\infty} x_{(i)} g(x_{(i)}) dx_{(i)} \quad (14)$$

And the expected value of the product of the i th and j th order statistics is given by

$$E[x_{(i)}x_{(j)}] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_{(i)}x_{(j)} g(x_{(i)}, x_{(j)}) dx_{(i)} dx_{(j)} \quad (15)$$

where $g(x_{(i)}, x_{(j)})$ is as defined by Equation (12).

Variance and Covariance. A two-dimensional random variable $(x_{(i)}, x_{(j)})$ possesses a property called the covariance of $x_{(i)}, x_{(j)}$. In this case the random variables are order statistics, and in a sense the covariance is a measure of the dependence between the two values of the order statistics. The covariance of $(x_{(i)}, x_{(j)})$ is formally defined as the product moment about the respective expected values of the order statistics. Meyer (Ref 10:144) defines the covariance of the two random variables as follows:

$$\text{Cov}(x_{(i)}, x_{(j)}) = E\{[x_{(i)} - E(x_{(i)})][x_{(j)} - E(x_{(j)})]\} \quad (16)$$

It can be easily shown from (16) that:

$$\text{Cov}(x_{(i)}, x_{(j)}) = E[x_{(i)}x_{(j)}] - E[x_{(i)}] \cdot E[x_{(j)}] \quad (17)$$

The variance of the i th order statistic may be considered as a special case ($i=j$) of Equation (16) where the $\text{Var}[x_{(i)}]$ is given by

$$\text{Var}[x_{(i)}] = E\{[x_{(i)} - E(x_{(i)})]^2\} \quad (18)$$

With the expressions presented in this chapter, the expected values, variances, and covariances of the Cauchy order statistics can be developed.

Standardized, Cauchy Order Statistics. Let $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ be a set of ordered sample values from a Cauchy

distributed parent population. A new set of standardized order statistics can be developed where

$$U(i) = \frac{x(i)-t}{s} \quad -\infty < U(i) < \infty \quad (19)$$

with $U(1) < \dots < U(i) < \dots < U(n)$

$$\text{If the pdf of } x \text{ is } f(x) = \frac{s}{\pi[s^2 + (x-t)^2]} \quad (2)$$

then the density of U is given by

$$p(u) = \frac{1}{\pi[1+u^2]} \quad -\infty < u < \infty \quad (20)$$

$$\text{and its cdf is given by } P(u) = \frac{1}{2} + \frac{1}{\pi} \text{ ARCTAN } u \quad (21)$$

The pdf of the i th standardized order statistic from Equation (11) is given by

$$q(u(i)) = \frac{n!}{(i-1)!(n-i)!} [P(u(i))]^{i-1} [1-P(u(i))]^{n-i} p(u(i)) \quad (22)$$

and the joint pdf of the standardized order statistics is given by Equation (12) with x replaced by u , g by p , F by P and f by p .

The above expression can be simplified by making the following substitution:

$$\text{Let } \theta(i) = \text{ARCTAN } u(i), \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \quad -\infty < u(i) < \infty \quad (23)$$

$$\text{where } du(i) = (1 + \tan^2 \theta(i)) d\theta_i \quad (24)$$

$$\text{and } \theta(j) = \text{ARCTAN } u(j) \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \quad (25)$$

then from Equation (22)

$$q(u_i) = \frac{n!}{(i-1)!(n-i)! \pi^n} [\theta + \frac{\pi}{2}]^{i-1} [\frac{\pi}{2} - \theta]^{n-i} \frac{1}{(1+\tan^2\theta)} \quad (26)$$

and

$$\begin{aligned} q(u_i u_j) &= \frac{n!}{(i-1)!(j-i-1)!(n-j)! \pi^n} [\theta + \frac{\pi}{2}]^{n-i} \\ &\quad [(\theta + \frac{\pi}{2}) - (\theta + \frac{\pi}{2})]^{j-i-1} \cdot [\frac{\pi}{2} - \theta]^{n-j} \frac{1}{(1+\tan^2\theta)} \frac{1}{(1+\tan^2\theta)} \end{aligned} \quad (27)$$

By substituting these equations into Equations (14), (15), (17), and (18) the desired expressions for the expected values, variances and covariances of the standardized order statistics are obtained.

$$E[u_i] = \frac{n!}{(i-1)!(n-i)! \pi^n} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan\theta [\theta + \frac{\pi}{2}]^{i-1} [\frac{\pi}{2} - \theta]^{n-i} d\theta_i \quad (28)$$

$$\begin{aligned} \text{Cov}[u_i, u_j] &= \frac{n!}{(i-1)!(j-i-1)!(n-j)! \pi^n} \iint_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan\theta \tan\phi [\theta + \frac{\pi}{2}]^{i-1} \\ &\quad \cdot [(\theta + \frac{\pi}{2}) - (\phi + \frac{\pi}{2})]^{j-i-1} [\frac{\pi}{2} - \theta]^{n-j} d\theta d\phi - \frac{(n!)^2}{(i-1)!(n-i)!(j-1)!(n-j)! \pi^n} \\ &\quad \cdot \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan\theta [\theta + \frac{\pi}{2}]^{i-1} [\frac{\pi}{2} - \theta]^{n-i} d\theta \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan\phi [\phi + \frac{\pi}{2}]^{j-1} [\frac{\pi}{2} - \phi]^{n-j} d\phi, i < j \end{aligned} \quad (29)$$

$$\text{Var}[u_i] = \frac{n!}{(i-1)!(n-j)! \pi^n} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^2\theta (i) [\theta(i) + \frac{\pi}{2}]^{i-1} [\frac{\pi}{2} - \theta(i)]^{n-j} d\theta(i) \quad (30)$$

Solution of the Expected Value and Covariance Equations

Equations (28), (29), and (30) cannot be evaluated in terms of elementary functions, but they can be evaluated by numerical integration techniques with the aid of an electronic computer. The integral factor of Equation (28) does not converge for values of $i=1$ or n , and Equations (29) and (30) do not converge for values of $i=1, 2, n$, and $n-1$. Barnett (Ref 1:1209) points out that for these values of i , the $\cos \theta$ and $\cos \emptyset$ factors in the denominator of the respective integrands are dominant as θ and \emptyset tend toward their limiting values, and it is therefore apparent that the means of the first and last order statistics do not exist, and the variances and covariances of the first two and last two order statistics do not exist.

The task of evaluating the remaining quantities is somewhat reduced due to the symmetry of the Cauchy distribution.

For the expected values

$$E[u_{(i)}] = -E[u_{(n+1-i)}] \text{ for even } n, i \neq 1 \text{ or } n \quad (31)$$

$$E[u_{(n+1)/2}] = 0 \text{ for odd } n \quad (32)$$

and for the covariances of the standardized order statistics:

$$\begin{aligned} \text{Cov}[u_{(i,j)}] &= \text{Cov}[u_{(n+2-i,n+1-j)}] = \text{Cov}[u_{(j,i)}] \\ &= \text{Cov}[u_{(n+2-j,n+2-i)}] \\ &\text{for } i < j \text{ and } i \neq 1, 2, n, \text{ and } n-1 \end{aligned} \quad (33)$$

Even with this reduction in the number of quantities to be computed, the integrals must be approximated by an electronic computer. Barnett (Ref 1) calculated these expected values for $n=3(1)20$ and the variances and covariances for $n=5(1)20$ to four-decimal-place accuracy. Although the expected values and variances can be calculated quite efficiently, the double integration in the covariance expression requires extensive computer time. Stark (Ref 16:44-48) calculated and tabled these quantities for sample sizes of $5(1)20$ to six-decimal-place accuracy. The expected values and covariances calculated by Stark were read into the main programs in Appendix C for the calculation of the linear coefficients for conditional best linear invariant estimation of the parameters.

IV. Linear Parameter Estimation

Introduction

The purpose of this thesis is to develop the coefficients required to obtain best linear invariant estimates of the parameters of the Cauchy distribution. This chapter explains linear estimation, develops the estimator with the minimum mean square error for both parameters, and describes the method used to solve the resulting simultaneous equations. The coefficients are calculated for the minimally censored sample, which consists of the ordered sample less the four extreme order statistics (the first two and the last two), and for samples which are additionally censored from above and symmetrically. In addition the values of the mean square error function are computed. All of these quantities are tables in the Appendices.

Linear Estimation

Linear estimation is a form of estimation where the ordered data are assigned weights and the new values are summed to give the estimate of the desired parameter. These weights are the coefficients and they are calculated so as to obtain the best linear estimator of the parameter. In this case the best estimator is the one with the minimum mean square error. The expressions required to obtain these best estimators are developed in the next section.

Mean Square Error

Location Parameter. From Equation (6) the conditional estimate of the location parameter is given by

$$t^*|S = \sum_{i=1}^N a_i x_i - AS \quad (34)$$

where t^* is the estimate of the location parameter

a_i is the coefficient to be computed

S is the scale parameter

A is a constant to be computed

N is the sample size

and from Equation (4) the mean square error of the estimator of the location parameter is given by

$$MSEL = E[(t^*|S - t)^2] \quad (35)$$

where t is the true value of the location parameter. $MSEL$ is the quantity to be minimized.

From Equation (35)

$$MSEL = E[(t^*|S)^2 - 2t(t^*|S) + t^2] \quad (36)$$

From Ref 10:134

$$\text{Var}[x] = E[x^2] - (E[x])^2 \quad (37)$$

therefore $\text{Var}[t^*|S] = E[(t^*|S)^2] - (E[t^*|S])^2 \quad (38)$

and $MSEL = \text{Var}[t^*|S] + (E[t^*|S] - t)^2 \quad (39)$

Now by the substitution of Equation (34),

$$MSEL = \text{Var} \left[\sum_{i=1}^N a_i x_{(i)} - AS \right] + (E \left[\sum_{i=1}^N a_i x_{(i)} - AS \right] - t)^2 \quad (40)$$

Freund (Ref 6:173-174) shows that for a given set of random variables x_1, x_2, \dots, x_n where $Y = \sum_{i=1}^N a_i x_i$, a linear combination of N random variables,

$$E[Y] = \sum_{i=1}^N a_i E[x_i] \quad (41)$$

$$\text{and } \text{Var}[Y] = \sum_{i=1}^N a_i^2 \text{Var}[x_{(i)}] + 2 \sum_{i < j} \sum a_i a_j \text{Cov}[x_{(i)} x_{(j)}] \quad (42)$$

for $i < j$

Now using relations (41) and (42) Equation (40) becomes

$$\begin{aligned} MSEL = & \sum_{i=1}^N a_i^2 \text{Var}[x_{(i)}] + 2 \sum_{i < j} \sum a_i a_j \text{Cov}[x_{(i)} x_{(j)}] + \\ & \left(\sum_{i=1}^N a_i E[x_{(i)}] - AS - t \right)^2 \end{aligned} \quad (43)$$

The expected values, variances and covariances developed in Chapter III were for standardized order statistics, where the standardized order statistic was defined by Equation (19). Therefore

$$x_{(i)} = S U_{(i)} + t \quad (44)$$

$$E[x_{(i)}] = S E[u_{(i)}] + t \quad (45)$$

$$\text{Var}[x_{(i)}] = s^2 \text{Var}[u_{(i)}] \quad (46)$$

$$\text{Cov}[x_{(i)} x_{(j)}] = s^2 \text{Cov}[u_{(i)} u_{(j)}] \quad (47)$$

Now define the following symbols:

Let

$$u_i = E[u_{(i)}]$$

$$\sigma_i = \text{Var}[u(i)]$$

$$\sigma_{ij} = \text{Cov}[u(i)u(j)]$$

Now by substituting these relations into Equation (43)

$$MSEL = \sum_{i=1}^N a_i^2 s^2 \sigma_{ii} + 2 \sum_{i=1}^N \sum_{j=i+1}^N a_i a_j s^2 \sigma_{ij} + \left[\sum_{i=1}^N a_i (s u_i + t) - A S - t \right]^2 \quad (48)$$

and adding the constraint that $\sum_{i=1}^N a_i = 1$ and applying this equation to a minimally censored Cauchy sample of size N-4 (the two extreme order statistics are removed from each end), the following equation is obtained

$$MSEL = s^2 \left(\sum_{i=3}^{N-2} a_i^2 \sigma_{ii} + 2 \sum_{i=3}^{N-2} \sum_{j=i+1}^{N-2} a_i a_j \sigma_{ij} + \left[\sum_{i=3}^{N-2} a_i u_i - A \right]^2 \right) \quad (49)$$

Scale Parameter. An expression similar to Equation (49) can be developed for the scale parameter where the mean square error of the estimated scale parameter is given by

$$MSSE = E[(S^*|t-S)^2] \quad | \quad (50)$$

where S is the true value of the scale parameter,

S* is the estimate of S, and

t is the true value of the location parameter.

Lct

$$S^*|t = \sum_{i=1}^N d_i x(i) - Dt \quad (51)$$

and substitute into Equation (50) to obtain

$$MSSE = \text{Var} \left[\sum_{i=1}^N d_i x(i) - Dt \right] + (E \left[\sum_{i=1}^N d_i x(i) - Dt \right] - S)^2 \quad (52)$$

And by the use of Equations (42), (45), (46), and (47),
Equation (52) becomes

$$MSES = \sum_{i=1}^N d_i^2 s^2 \sigma_{ii} + 2 \sum_{i=1}^N \sum_{j=i+1}^N s^2 d_i d_j \sigma_{ij} + \left[\sum_{i=1}^N d_i (s \mu_i + t) - D t - S \right]^2 \quad (53)$$

Now by adding the constraint that $\sum_{i=1}^N d_i = D$ and considering
a minimally censored Cauchy sample the mean square error of
the estimate of the scale parameter is given by

$$MSES = s^2 \left(\sum_{i=3}^{N-2} d_i^2 \sigma_{ii} + 2 \sum_{i=3}^{N-2} \sum_{j=i+1}^{N-2} d_i d_j \sigma_{ij} + \left[\sum_{i=3}^{N-2} d_i \mu_{i-1} \right]^2 \right) \quad (54)$$

Equations (54) and (49) are the required expressions to
compute the mean square error, but in this case, it is de-
sired to minimize these quantities.

Minimization of the Mean Square Error

Both expressions for the mean square error contain the
scale parameter squared. At this point, the problem is to
determine the values of the a_i 's, A, d_i 's and D which min-
imize the respective MSE function, and these values will be
the same if the functions are minimized without the (s^2)
term.

Taylor (Ref 17:198) describes Lagrange's method of min-
imizing a function of several variables subject to a con-
straint. In applying this method the original function is
modified by adding the constraint equation multiplied by a
Lagrangian multiplier, and then taking partial derivatives
of the function with respect to each variable and multiplier.

The resulting derivatives are set equal to zero to form a set of simultaneous equations. These equations are then solved for the values of the variables and multipliers which minimize the function.

Matrix Equation. To develop the matrix equations for the calculation of the required coefficients, Equation (49) is modified by adding the constraint, $\sum_{i=3}^{N-2} a_i = 1$, to give

$$L = \sum_{i=3}^{N-2} a_i^2 \sigma_{ii} + 2 \sum_{i=3}^{N-2} \sum_{j=i+1}^{N-2} a_i a_j \sigma_{ij} + \left[\sum_{i=3}^{N-2} a_i \mu_i - A \right]^2 + \lambda (\sum_{i=3}^{N-2} a_i - 1) \quad (55)$$

Now if $N=7$ is the total sample size and $m=3$ is the size of the sample after censoring, application of the Lagrangian method results in the following set of equations.

$$\frac{\partial L}{\partial \lambda} = a_3 + a_4 + a_5 - 1 = 0$$

$$\frac{\partial L}{\partial A} = -a_3 \mu_3 - a_4 \mu_4 - a_5 \mu_5 + \lambda = 0$$

$$\frac{\partial L}{\partial a_3} = a_3 (\sigma_{33} + \mu_3^2) + a_4 (\sigma_{34} + \mu_3 \mu_4) + a_5 (\sigma_{35} + \mu_3 \mu_5) - A \mu_3 + \frac{\lambda}{2} = 0 \quad (56)$$

$$\frac{\partial L}{\partial a_4} = a_3 (\sigma_{43} + \mu_4 \mu_3) + a_4 (\sigma_{44} + \mu_4^2) + a_5 (\sigma_{45} + \mu_4 \mu_5) - A \mu_4 + \frac{\lambda}{2} = 0$$

$$\frac{\partial L}{\partial a_5} = a_3 (\sigma_{53} + \mu_5 \mu_3) + a_4 (\sigma_{54} + \mu_5 \mu_4) + a_5 (\sigma_{55} + \mu_5^2) - A \mu_5 + \frac{\lambda}{2} = 0$$

The above equations in matrix form are:

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & -\mu_3 & -\mu_4 & -\mu_5 \\ 1 & -\mu_3 & (\sigma_{33} + \mu_3^2) & (\sigma_{34} + \mu_3\mu_4) & (\sigma_{35} + \mu_3\mu_5) \\ 1 & -\mu_4 & (\sigma_{43} + \mu_4\mu_3) & (\sigma_{44} + \mu_4^2) & (\sigma_{45} + \mu_4\mu_5) \\ 1 & -\mu_5 & (\sigma_{53} + \mu_5\mu_3) & (\sigma_{54} + \mu_5\mu_4) & (\sigma_{55} + \mu_5^2) \end{bmatrix} \begin{bmatrix} \frac{\lambda}{2} \\ A \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (57)$$

A similar procedure for Equation (54) results in the following matrix equation for the coefficients of the scale parameter.

$$\begin{bmatrix} (\sigma_{33} + \mu_3^2) & (\sigma_{34} + \mu_3\mu_4) & (\sigma_{35} + \mu_3\mu_5) \\ (\sigma_{43} + \mu_4\mu_3) & (\sigma_{44} + \mu_4^2) & (\sigma_{45} + \mu_4\mu_5) \\ (\sigma_{53} + \mu_5\mu_3) & (\sigma_{54} + \mu_5\mu_4) & (\sigma_{55} + \mu_5^2) \end{bmatrix} \begin{bmatrix} d_3 \\ d_4 \\ d_5 \end{bmatrix} = \begin{bmatrix} \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} \quad (58)$$

where $D = d_3 + d_4 + d_5$

The matrix Equations (57) and (58) include matrices of the expected values and covariance of standardized order statistics and the column vectors of the desired variables. These equations can be solved for the column vector of $m + 2$ variables for the location parameter and m variables for the scale parameter.

Solution of the Matrix Equations. The above matrix equations were solved for sample sizes of $N = 5(1)20$ on the CDC 6600 computer. Basically, the main Fortran program reads in the values of the means, variances, and covariances, and

the matrix equations are calculated and then solved by a subroutine to the main Fortran program. This subroutine is a modification of the "Matrix Equation Solver" Fortran extended subroutine due to the Computer Science Center, Wright-Patterson AFB, Ohio. The values of the linear coefficients and constants are tabled for each N and M. In addition, the value of the mean square error function is tabled for each sample of size N and M. The values of the mean square error functions are calculated from Equations (49) and (54).

Equation (49) can be rearranged to give:

$$MSE = \sum_{i=3}^{N-2} \sum_{j=3}^{N-2} a_i a_j \sigma_{ij} + \left[\sum_{i=3}^{N-2} a_i u_i - A \right]^2 \quad (59)$$

and a similar expression can be developed for the MSE of the scale parameter. With the known values of a_i and A from the solution of Equation (57), the MSE is easily computed in a subroutine to the main program.

Additional Censoring

Censoring from Above. The minimally censored sample was defined as the basic ordered sample of size N with the two extreme order statistics censored from each end of the sample. Additional censoring was accomplished by censoring the sample from above so that M (the number of sample values remaining after censoring) decreases from (N-4) to (1) for each sample size (N). The value of the mean square error function was also tabled for each M. The values of the coefficients and MSE for the minimally censored sample and

additional censoring from above are given in Table I, Appendix A.

Symmetric Censoring. Each sample of size $N=5(1)20$ was also censored symmetrically from both ends. In this case M ranges from $N-4$ to 2 for even sample sizes and $N-4$ to 3 for odd samples. The symmetric censoring was terminated at $M=3$ since with $M=1$ the estimate of the location parameter is the median and there is no information available to compute the estimate of the scale parameter.

V. Use of the Tables

Introduction

The results of this thesis are Tables I and II included in the Appendices. With these tables the user is able to calculate the best linear invariant estimates of the location and scale parameters of the Cauchy distribution. This chapter explains these tables, gives the required procedure to obtain the conditional estimates or simultaneous estimates, and provides examples of these calculations.

Explanation of Tables I and II

Both tables include the values of the mean square error function, and the values of the coefficients and constants which are applied to the sample data to obtain the estimates of the parameters. These values are tabled for each N (sample size) and M (sample size after censoring). The coefficients of the order statistics required for estimation of the location parameter are listed in the columns under **LOCATION**, and the same values for the scale parameter are listed in the same manner under **SCALE**. Table I is used for a minimally censored sample or for a sample with additional censoring from above, and Table II is used for a sample which has been censored symmetrically.

Estimation Procedure

Best linear invariant estimation of a parameter consists of the following steps:

1. Obtain the sample data.
2. Order the data and determine N.
3. Minimally censor the two extreme sample values from each end, and additionally censor as desired.
4. Determine N and M and enter the appropriate table to obtain the coefficients and constants.
5. Multiply the sample values by their respective coefficients and sum the result to obtain the estimate of the parameter.
6. If the sample was additionally censored from above (Table I) conditional estimation is required, and the appropriate constant times the known parameter must be summed with the terms in step 5.

Examples

No Additional Censoring. As an example of the use of the tables to determine an estimate of the parameters of the Cauchy distribution, assume that the following data are known to come from a Cauchy distributed parent population. The true value of the location parameter is 8.0 and the true value of the scale parameter is .5.

$x_1 = 5.689853$	$x_6 = 9.222433$
$x_2 = 7.835235$	$x_7 = 8.316519$
$x_3 = 9.641365$	$x_8 = 7.902609$
$x_4 = 7.201119$	$x_9 = 18.926210$
$x_5 = 8.739464$	

When the data are ordered the following order statistics are obtained:

$$\begin{array}{ll}
 x(1) = 5.689853 & x(6) = 8.739464 \\
 x(2) = 7.201119 & x(7) = 9.222433 \\
 x(3) = 7.835235 & x(8) = 9.641365 \\
 x(4) = 7.902609 & x(9) = 18.926210 \\
 x(5) = 8.316519 &
 \end{array}$$

After these statistics are censored the subset $x(3)$ through $x(7)$ remains with $N=9$ and $M=5$. From Table I, the following coefficients are obtained:

$$\begin{array}{ll}
 a_3 = -.067277 & a_6 = .245395 \\
 a_4 = .245395 & a_7 = -.067277 \\
 a_5 = .643765 & A = 0
 \end{array}$$

Now the estimate of the location parameter t^* is

$$t^* = a_3x(3) + a_4x(4) + a_5x(5) + a_6x(6) + a_7x(7) \quad (60)$$

And when this calculation is carried out $t^* = 8.290177$.

To estimate the scale parameter the required coefficients are obtained from the same table.

$$\begin{array}{ll}
 d_3 = -.153945 & d_6 = .369546 \\
 d_4 = .369546 & d_7 = .153945 \\
 d_5 = 0 & D = 0
 \end{array}$$

Now

$$s^* = d_3x(3) + d_4x(4) + d_5x(5) + d_6x(6) + d_7x(7) \quad (61)$$

and when these calculations are carried out $s^* = .522809$.

It can be seen by inspecting the tables that the estimation is conditional estimation when the sample is censored additionally from above. In all other cases the values of A and D are equal to zero.

Additional Censoring from Above. To demonstrate the conditional estimation procedure, consider the data from the previous example. Additional censoring from above will be performed so that N=9 and M=3. Now from Table I:

$$a_3 = -.070500 \quad a_5 = .82517$$

$$a_4 = .245330 \quad A = -.032654$$

and

$$d_3 = -.231235 \quad d_5 = .779717$$

$$d_4 = -.588770 \quad D = -.040287$$

$$t^*|s = a_3x(3) + a_4x(4) + a_5x(5) - As \quad (62)$$

$$s^*|t = d_3x(3) + d_4x(4) + d_5x(5) - Dt \quad (63)$$

When these calculations are carried out, the following estimates are obtained: $t^*|s = 8.265232$ and $s^*|t = .342227$. The same procedures apply to the estimation of the parameters with additional symmetric censoring with the coefficients from Table II. The reader will notice that the value of the MSE of the estimate increases as less information is considered. For the location parameter estimator of the last example the MSE increases from .38655 to .39752 as M decreases from 5 to 3.

Simultaneous Estimation. Due to a technique suggested by Herman for unbiased estimation (Ref 8), it is possible to use Table I for simultaneous estimation of the parameters when neither is known.

If \bar{t} = the simultaneous estimate of the location parameter
 \bar{s} = the simultaneous estimate of the scale parameter
 t^* and s^* are as defined earlier

then
$$\bar{t} = \frac{\sum_{i=3}^{M+2} a_i x(i) - A \sum_{i=3}^{M+2} d_i x(i)}{1-AD} \quad (64)$$

and
$$\bar{s} = \frac{\sum_{i=3}^{M+2} d_i x(i) - D \sum_{i=3}^{M+2} a_i x(i)}{1-AD} \quad (65)$$

As an example of this technique, consider the data from the example with "additional censoring from above".

$a_3 = -.070500$	$d_3 = -.231235$	$x(3) = 7.835235$
$a_4 = .245330$	$d_4 = -.588770$	$x(4) = 7.902609$
$a_5 = .82517$	$d_5 = .779717$	$x(5) = 8.316519$
$A = -.032654$	$D = -.040287$	

$$\bar{t} = \frac{8.248905 + .000651}{.998684} = 8.260428$$

$$\bar{s} = \frac{.019931 + .332324}{.998684} = .352719$$

In summary, it can be seen that although this technique of estimation is conditional estimation, the conditional requirement is only necessary when estimating from sample data

that have been censored additionally from above. The method provides a simple and efficient procedure to estimate the Cauchy parameters.

VI. Summary

The objective of this thesis was to develop a table of linear coefficients which could be easily applied to sample data to obtain the conditional best linear invariant estimates of the location and scale parameters of the Cauchy distribution. These estimates are best in the sense that they process minimum mean square error. The coefficients and constants required to calculate these estimates for minimally censored samples and samples with additional censoring from above are tabled in Appendix A. The same values for samples with additional symmetric censoring from above and below are tabled in Appendix B. In addition the value of the mean square error function for each sample size is also included in these tables. The computer programs required to calculate and table the above data are included in Appendix C.

This paper also includes a brief review of the previous work in estimation of the parameters of the Cauchy distribution from 1961 through 1970. Much of this effort was accomplished at the Air Force Institute of Technology under the direction of Professor Albert H. Moore and sponsorship of Dr. H. Leon Harter. These works are concerned primarily with unbiased estimates of the parameters.

The Cauchy distribution and its peculiarities are discussed with a review of several ways in which the distribution may be generated. The order statistic theory required

to obtain the expected values, variance, and covariances of the order statistics is also reviewed. The mean square error function for the estimate of the parameters is developed and minimized by Lagrangian techniques to obtain the matrix equations required to calculate the linear coefficients.

The report is concluded with an explanation of the tables and several examples of the application of these coefficients. A technique of simultaneous estimation of both parameters of the distribution is also presented with an example of the technique.

The method of estimation presented in this paper and the attached tables of linear coefficients provide a simple and efficient method of obtaining either conditional or simultaneous best linear invariant estimates of the parameters of the Cauchy distribution.

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APPENDIX A

Table I

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	Coeff.	MSE	I	Coeff.	
5	1	1.22125	3	1.000000	1.000000	3	0.300000	
			A	0.000000		0	0.000000	
6	2	.86657	2	.500000	.63722	3	-.501421	
			4	.500000		4	.501421	
			A	0.000000		0	0.000000	
6	1	1.69039	3	1.000000	.89285	3	-.296212	
			A	-.361760		0	-.296212	
7	3	.65725	3	.058194	.50082	3	-.394255	
			4	.031111		4	.031111	
			5	.058194		5	.058194	
			A	-.000000		0	0.000000	
7	2	.60985	2	.054557	.64999	3	-.552915	
			4	.044342		4	.044342	
			A	-.034501		0	-.034501	
7	1	1.27736	2	1.000000	.76118	3	-.377247	
			A	-.633782		0	-.377247	
8	4	.47326	3	-.050159	.40849	3	-.245497	
			4	.560059		4	-.342912	
			5	.560059		5	-.342912	
			6	-.030059		6	.245497	
			A	.007109		0	-.007109	
8	3	.47544	3	-.049331	.46637	3	-.297136	
			4	.557989		4	-.423557	
			5	.491242		5	.736492	
			A	.026764		0	.026764	
8	2	.54623	3	-.078387	.66325	3	-.448491	
			4	1.078387		4	.209932	
			A	-.137294		0	-.230559	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CALCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FROM ABOVE)

N	M	MSF	** LOCATION **		** SCALE **		
			I	CCEF.	MSE	I	
8	1	1.56897	3	1.00000	.67689	3	-.373372
			A	-.86538E		C	-.373372
9	5	.38655	3	-.367277	.34138	3	-.153946
			4	.245395		4	-.369546
			5	.847765		5	-.361663
			6	.245795		6	.369546
			7	-.367277		7	.153946
			A	-.987700		C	-.373372
9	4	.39134	3	-.367732	.36208	3	-.156946
			4	.261551		4	-.414873
			5	.657829		5	-.14547
			6	.153829		6	.515526
			A	.332798		C	.073100
9	3	.39792	3	-.372501	.49045	3	-.231235
			4	.245331		4	-.539779
			5	.925171		5	.779717
			A	-.832854		C	-.041287
9	2	.55204	3	-.125203	.62227	3	-.315727
			4	1.125216		4	-.487119
			A	-.763133		C	-.42544
9	1	1.92972	2	1.00000	.62466	3	-.342568
			A	-.976808		C	-.348558
10	6	.72520	3	-.062304	.29371	3	-.100113
			4	.033053		4	-.707114
			5	.478762		5	-.212155
			6	.478652		6	.212155
			7	.093283		7	.317114
			8	-.032004		8	.100113
			A	-.000000		C	0.000000

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COEF.	MSE	I	COEF.	
10	5	.33118	3	-.062798	.30691	3	-.104844	
			4	.136164		4	-.318302	
			5	.488100		5	-.226247	
			6	.489474		6	.214673	
			7	-.070340		7	.450766	
			A	.027727		C	.025046	
10	4	.33110	3	-.062795	.37224	3	-.129212	
			4	.086192		4	-.398429	
			5	.488149		5	-.316693	
			6	.489464		6	.865059	
			A	.027748		C	.030738	
10	3	.36670	3	-.072397	.51050	3	-.185102	
			4	.075618		4	-.591117	
			5	.027681		5	.531655	
			A	-.139110		C	-.193624	
10	2	.61474	3	-.146188	.56628	3	-.214511	
			4	1.146188		4	-.261139	
			A	-.516339		C	-.475647	
10	1	2.34957	3	1.000000	.59786	3	-.320762	
			A	-1.275712		C	-.320762	
11	7	.28197	3	-.052380	.25743	3	-.067613	
			4	.018550		4	-.271566	
			5	.295622		5	-.275726	
			6	.496199		6	-.368116	
			7	.295622		7	.275726	
			8	.629659		8	.231556	
			9	-.052380		9	.067613	
			A	-.000000		C	-.000000	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FROM ABOVE)

** LOCATION **				** SCALE **			
I.	M	MSE	T	Coeff.	MSE	I	Coeff.
11	6	.28618	3	-.163106	.26450	3	-.069538
			4	.009127		4	-.238771
			5	.311133		5	-.284754
			6	.535608		6	-.002774
			7	.312712		7	.279757
			8	-.065558		8	.335071
			4	.321511		0	.019421
11	5	.28727	3	-.053277	.30034	3	-.079526
			4	.009770		4	-.274159
			5	.314193		5	-.333662
			5	.510961		6	-.021317
			7	.228280		7	.755217
			4	.344547		0	.046557
11	4	.28464	3	-.055181	.30377	3	-.106737
			4	.008677		4	-.373581
			5	.312151		5	-.476055
			6	.74629		6	.919502
			4	-.027862		0	-.037231
11	3	.38440	3	-.072501	.50400	3	-.142150
			4	-.014380		4	-.509573
			5	1.036381		5	.296992
			A	-.254588		0	-.352037
11	2	.68974	3	-.157460	.51717	3	-.148759
			4	1.157400		4	-.344726
			4	-.658171		0	-.497405
11	1	2.82441	3	1.000000	.56795	3	-.204753
			A	-1.465011		0	-.294753

TABLE I

 COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
 LOCATION AND SCALE PARAMETERS OF THE CALCHY DISTRIBUTION
 (WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	CCOF.	MSE	I	CCOF.	
12	8	.24810	3	-.043156	.22895	3	-.047234	
			4	-.024229		4	-.174151	
			5	.164418		5	-.266502	
			6	.412968		6	-.138924	
			7	.432968		7	.138924	
			8	.164418		8	.266592	
			9	-.024229		9	.174151	
			10	-.043156		10	.047234	
			A	.000000		C	.000000	
12	7	.25146	3	-.043718	.23299	3	-.048090	
			4	-.024417		4	-.177375	
			5	.157102		5	-.271901	
			6	.410402		6	-.142455	
			7	.438907		7	.139738	
			8	.168750		8	.26957	
			9	-.005635		9	.245441	
			A	.016644		C	.014866	
12	6	.25385	3	-.04474	.25368	3	-.052546	
			4	-.024281		4	-.194254	
			5	.169944		5	-.299573	
			6	.415629		6	-.162238	
			7	.417115		7	.142029	
			8	.065787		8	.611814	
			A	.045201		C	.045231	
12	5	.25440	3	-.044239	.31139	3	-.065373	
			4	-.024741		4	-.247095	
			5	.169502		5	-.381476	
			6	.414241		6	-.225124	
			7	.435777		7	.946081	
			A	.025397		C	.031082	
12	4	.27609	3	-.049964	.41155	3	-.058587	
			4	-.02263		4	-.333771	
			5	.169986		5	-.578536	
			6	.712.41		6	.7996.7	
			A	-.119151		C	-.161253	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE GARCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FROM ABCVF)

N	M	MSE	** LOCATION **			** SCALE **		
			I	Coeff.	MSE	I	Coeff.	
12	3	.37871	3	-.371378	.47849	3	-.106153	
			4	-.265712		4	-.434762	
			5	1.137290		5	.546663	
			A	-.366992		C	-.463562	
12	2	.78122	3	-.164384	.47882	3	-.165116	
			4	1.164384		4	-.378495	
			A	-.791721		C	-.485111	
12	1	.77236	3	1.000000	.55178	3	-.271615	
			A	-1.650322		C	-.271615	
13	9	.22140	3	-.035307	.26804	3	-.033978	
			4	-.077765		4	-.131416	
			5	.031475		5	-.232813	
			6	.291384		6	-.21347	
			7	.411198		7	.005000	
			8	.230784		8	.201347	
			9	.031475		9	.232813	
			10	-.037160		10	.171415	
			11	-.035307		11	.033978	
			A	5.660000		C	-.090660	
13	8	.22463	3	-.035908	.26846	3	-.034358	
			4	-.077441		4	-.133021	
			5	.032645		5	-.235753	
			6	.294295		6	-.204156	
			7	.416738		7	-.090778	
			8	.234958		8	.202741	
			9	.033571		9	.234467	
			10	-.038859		10	.132257	
			A	.012274		C	.011422	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	CORF.		I	CORF.	
13	7	.22698	3	-.035231		.22099	3	-.036523
			4	-.037726			4	-.141427
			5	.024307			5	-.251238
			6	.299769			6	-.219293
			7	.414256			7	-.055401
			8	.301725			8	.205084
			9	-.025198			9	.464179
			A	.040426			0	.539379
13	6	.22696	3	-.036231		.25686	3	-.142739
			4	-.037606			4	-.156044
			5	.024086			5	-.297125
			6	.299362			6	-.255517
			7	.414933			7	-.322553
			8	.274446			8	.847945
			A	.047681			0	.053962
13	5	.22351	3	-.037426		.22890	3	-.055599
			4	-.139923			4	-.217447
			5	.093147			5	-.394815
			6	.720764			6	-.307904
			7	.659497			7	1.011928
			A	-.023931			0	-.037623
13	4	.27132	3	-.044681		.41570	3	-.172011
			4	-.052328			4	-.297591
			5	.080583			5	-.534221
			6	1.046416			6	.579841
			A	-.198543			0	-.303862
13	3	.46790	3	-.070399		.44685	3	-.378000
			4	-.197513			4	-.213357
			5	1.167912			5	-.131810
			A	-.473784			0	-.524168
13	2	.80646	3	-.169347		.44967	3	-.178745
			4	1.169147			4	-.396911
			A	-.917945			0	-.465646

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSUSING FROM APCY)

N	M	MSE	** LOCATION **		** SCALE **	
			I	COEF.	MSE	I
13	1	3.93247	3	1.000000	.57998	3
			A	-1.833325		C
14	10	.19961	3	-.029130	.13722	3
			4	-.040524		4
			5	.031712		5
			6	.195410		6
			7	.342931		7
			8	.342931		8
			9	.195410		9
			10	.031712		10
			11	-.040524		11
			12	-.031712		12
			4	-.031712		C
14	9	.21127	3	-.029426	.18874	3
			4	-.040914		4
			5	.032132		5
			6	.197229		6
			7	.346861		7
			8	.347641		8
			9	.197726		9
			10	.032790		10
			11	-.033458		11
			4	.039475		C
14	8	.21470	3	-.029322	.19665	3
			4	-.041321		4
			5	.032898		5
			6	.202724		6
			7	.352978		7
			8	.363669		8
			9	.202570		9
			10	-.071674		10
			A	.074352		C

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TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSING FROM ABCVF)

N	M	MSE	** LOCATION **			** SCALE **		
			I	Coeff.	MSE	I	Coeff.	
14	7	.21543	3	-.329909	.21956	3	-.329553	
			4	-.741427		4	-.118383	
			5	.133365		5	-.271719	
			6	.232103		6	-.269233	
			7	.755501		7	-.123714	
			8	.355674		8	.139797	
			9	.123648		9	.723623	
			A	.064135		C	.057560	
14	6	.21661	2	-.030138	.26827	3	-.376455	
			4	-.041079		4	-.146551	
			5	.032723		5	-.298443	
			6	.211616		6	-.329541	
			7	.754759		7	-.171684	
			8	.492968		8	1.001392	
			A	.023116		C	.030014	
14	5	.22120	2	-.032564	.34375	3	-.047514	
			4	-.046640		4	-.19209	
			5	.030123		5	-.392009	
			6	.205903		6	-.446757	
			7	.947412		7	.979685	
			A	-.089665		C	-.177774	
14	4	.27538	3	-.041681	.46831	2	-.057335	
			4	-.063835		4	-.237098	
			5	.021299		5	-.469446	
			6	1.034217		6	.370976	
			A	-.293380		C	-.419003	
14	2	.43701	2	-.069677	.41657	3	-.059400	
			4	-.118676		4	-.242495	
			5	1.138273		5	-.246471	
			A	-.575760		C	-.548457	
14	2	1.044.6	2	-.172426	.47747	3	-.059896	
			4	1.172427		4	-.342701	
			A	-.1070652		C	-.442597	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FROM ARCS)

N	M	MSE	** LOCATION **		** SCALE **	
			I	COEF.	MSE	I
14	1	4.56418	3	1.00000	.57116	3
			4	-2.33717	0	-2.27584
15	11	.18201	3	-.02413	.17153	3
			4	-.03971	4	-.07733
			5	.00274	5	-.15742
			6	.12273	6	-.21621
			7	.26931	7	-.18179
			8	.33588	8	-.11111
			9	.26931	9	.17178
			10	.12273	10	.21621
			11	.00274	11	.15742
			12	-.03971	12	-.07733
			13	-.02413	13	-.18179
			14	.00000	14	-.11111
15	10	.18363	3	-.02434	.17253	3
			4	-.04651	4	-.07774
			5	.00281	5	-.14437
			6	.12541	6	-.21787
			7	.27191	7	-.15112
			8	.33919	8	-.11047
			9	.27213	9	.15764
			10	.12542	10	.21773
			11	.00729	11	.14437
			12	-.07543	12	-.15412
			13	.00730	13	.01694
15	9	.18627	3	-.02467	.17766	3
			4	-.04056	4	-.08016
			5	.01702	5	-.15647
			6	.12716	6	-.21425
			7	.27647	7	-.15387
			8	.34556	8	-.11174
			9	.27735	9	.15721
			10	.12876	10	.21400
			11	-.09201	11	.31100
			12	.02860	12	.12750

TABLE T

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	CCEF.	MSE	I	CCEF.	
15	8	.18755	3	-.024339	.19275	3	-.021370	
			4	-.540774		4	-.087292	
			5	.302293		5	-.181475	
			6	.128662		6	-.234450	
			7	.279438		7	-.173487	
			8	.348973		8	-.097174	
			9	.231160		9	.159772	
			10	.024087		10	.599783	
			A	.353400		C	.054471	
15	7	.18762	3	-.024651	.22502	3	-.025002	
			4	-.740812		4	-.162996	
			5	.603282		5	-.214926	
			6	.128554		6	-.279624	
			7	.279180		7	-.212548	
			8	.349615		8	-.221325	
			9	.306171		9	.913932	
			A	.147462		C	.656721	
15	6	.19283	3	-.025543	.28238	3	-.031787	
			4	-.342425		4	-.130950	
			5	.311718		5	-.274741	
			6	.124720		6	-.362567	
			7	.281414		7	-.286215	
			8	.656707		8	1.056739	
			A	-.020750		0	-.030386	
15	5	.21673	3	-.022193	.35072	3	-.042111	
			4	-.049671		4	-.165933	
			5	-.013590		5	-.351461	
			6	.137779		6	-.470708	
			7	.049182		7	.760461	
			A	-.150455		C	-.25041	
15	4	.28561	3	-.032642	.38881	3	-.045210	
			4	-.070389		4	-.137657	
			5	-.019718		5	-.306469	
			6	.1179269		6	.120170	
			8	-.369382		C	-.52045	

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TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABCVS)

N	M	MSE	** LOCATION **				** SCALE **			
			I	CCEF.	MSE	I	CCEF.	MSE	I	CCEF.
15	3	.47642	3	-.369066	.39032	3	-.045589	.39032	3	-.045589
			4	-.133482		4	-.189476		4	-.189476
			5	1.222547		5	-.315933		5	-.315933
			A	-.572382		C	-.550856		C	-.550856
15	2	1.13325	3	-.174993	.41034	3	-.046584	.41034	3	-.046584
			4	1.174997		4	-.37242		4	-.37242
			A	-1.157473		C	-.419785		C	-.419785
15	1	5.24717	3	1.503111	.52440	3	-.218117	.52440	3	-.218117
			4	-2.181482		C	-.218117		C	-.218117
16	12	.16719	2	-.327142	.15819	2	-.014524	.15819	2	-.014524
			4	-.337187		4	-.69477		4	-.69477
			5	-.017592		5	-.131367		5	-.131367
			6	.373662		6	-.186477		6	-.186477
			7	.219527		7	-.172219		7	-.172219
			8	.236514		8	-.071793		8	-.071793
			9	.296514		9	.177798		9	.177798
			10	.210527		10	.172219		10	.172219
			11	.073562		11	.186477		11	.186477
			12	-.017592		12	.171367		12	.171367
			13	-.037187		13	.061473		13	.061473
			14	-.026142		14	.014524		14	.014524
			A	.060000		C	-.000000		C	-.000000
16	11	.16837	3	-.020996	.15885	3	-.014586	.15885	3	-.014586
			4	-.037344		4	-.061732		4	-.061732
			5	-.013674		5	-.131365		5	-.131365
			6	.374283		6	-.187319		6	-.187319
			7	.212274		7	-.173519		7	-.173519
			8	.238964		8	-.071222		8	-.071222
			9	.290177		9	.073923		9	.073923
			10	.202474		10	.172721		10	.172721
			11	.074581		11	.187175		11	.187175
			12	-.017324		12	.171450		12	.171450
			13	-.066671		13	.032611		13	.032611
			A	.005842		C	.005842		C	.005842

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COEF.	MSE	I	COEF.	
16	10	.17067	3	-.020568	.16233	3	-.014911	
			4	-.037327		4	-.062196	
			5	-.013772		5	-.134932	
			6	.075514		6	-.191654	
			7	.215427		7	-.177287	
			8	.307661		8	-.073539	
			9	.393989		9	.071421	
			10	.216279		10	.175217	
			11	.176763		11	.189661	
			12	-.099438		12	.240615	
			A	.023850		0	.022495	
16	9	.17273	3	-.020759	.17256	3	-.015874	
			4	-.039143		4	-.066138	
			5	-.017747		5	-.147839	
			6	.076603		6	-.204660	
			7	.206198		7	-.190167	
			8	.307359		8	-.080834	
			9	.309314		9	.072382	
			10	.209948		10	.182021	
			11	-.037989		11	.495709	
			A	.048613		0	.048678	
16	8	.17245	3	-.020768	.19589	3	-.018016	
			4	-.039144		4	-.075149	
			5	-.013679		5	-.163766	
			6	.076820		6	-.237952	
			7	.208549		7	-.219615	
			8	.308338		8	-.098325	
			9	.310125		9	.072809	
			10	.169779		10	.801061	
			A	.057497		0	.065047	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COEF.	MSE	I	COEF.	
16	7	.17397	3	-.026982	.23591	3	-.021953	
			4	-.038640		4	-.091766	
			5	-.014272		5	-.200705	
			6	.076477		6	-.289777	
			7	.208592		7	-.275771	
			8	.308432		8	-.133829	
			9	.430397		9	1.041716	
			A	.021116		C	.028499	
16	6	.18459	3	-.022393	.20460	3	-.027751	
			4	-.041681		4	-.116240	
			5	-.017155		5	-.255769	
			6	.076974		6	-.371461	
			7	.214186		7	-.362846	
			8	.793269		8	1.014156	
			A	-.375217		C	-.127443	
16	5	.21736	3	-.026751	.14888	3	-.037756	
			4	-.051320		4	-.140266	
			5	-.025734		5	-.310036	
			6	.079083		6	-.454764	
			7	1.024427		7	.567076	
			A	-.231331		C	-.371305	
15	4	.30946	3	-.037976	.36833	3	-.035637	
			4	-.075517		4	-.150212	
			5	-.049350		5	-.333147	
			6	1.162852		6	-.074632	
			A	-.451610		C	-.553629	
16	3	.52125	3	-.068644	.36844	3	-.035601	
			4	-.144795		4	-.150228	
			5	1.213138		5	-.355550	
			A	-.765624		C	-.541179	
16	2	1.27356	3	-.17734	.39694	3	-.037151	
			4	1.17734		4	-.350354	
			A	-1.273158		C	-.396514	

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TABLE T

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FROM ABOVE)

N	M	MSE	** LOCATION **		** SCALE **	
			I	Coeff.	MSE	I
16	1	5.98125	3	1.000000	.51914	3
			A	-2.353784		0
17	13	.15440	3	-.016948	.14677	3
			4	-.033775		4
			5	-.022361		5
			6	.039373		6
			7	.143717		7
			8	.245754		8
			9	.239481		9
			10	.246750		10
			11	.147717		11
			12	.039373		12
			13	-.022361		13
			14	-.033775		14
			15	-.016948		15
			A	.000000		0
17	12	.15543	3	-.017760	.14723	3
			4	-.077995		4
			5	-.022406		5
			6	.039664		6
			7	.144729		7
			8	.247481		8
			9	.297537		9
			10	.247572		10
			11	.144390		11
			12	.039992		12
			13	-.022239		13
			14	-.058987		14
			A	.004668		C
						.004422

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	MSE	** LOCATION **		** SCALE **		
			I	COEF.	MSE	I	
17	11	.15742	7	-.017275	.14963	3	-.011572
			4	-.034414		4	-.048830
			5	-.022735		5	-.109811
			6	.740287		6	-.167253
			7	.146812		7	-.178946
			8	.251008		8	-.117718
			9	.294774		9	-.030573
			10	.251412		10	.116387
			11	.147567		11	.177536
			12	.041281		12	.165969
			13	-.098682		13	.193391
			A	.019544		0	.018577
17	10	.15920	3	-.017465	.15673	3	-.012134
			4	-.034773		4	-.051213
			5	-.022896		5	-.115222
			6	.040969		6	-.175678
			7	.148893		7	-.138201
			8	.254536		8	-.124689
			9	.299182		9	-.002656
			10	.255471		10	.119465
			11	.150622		11	.183114
			12	-.074439		12	.439684
			A	.043159		C	.042490
17	9	.15968	3	-.017512	.17250	3	-.013338
			4	-.034947		4	-.056545
			5	-.022872		5	-.127365
			6	.041700		6	-.194587
			7	.149717		7	-.230431
			8	.255912		8	-.146899
			9	.307799		9	-.007914
			10	.257755		10	.125372
			11	.072257		11	.639583
			A	.059947		C	.064757

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TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FPCM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COEF.	MSE	I	COEF.	
17	8	.15904	3	-.017547	.20170	3	-.015736	
			4	-.034941		4	-.066547	
			5	-.023022		5	-.150219	
			6	.041117		6	-.230395	
			7	.149504		7	-.250117	
			8	.255620		8	-.172748	
			9	.310354		9	-.019956	
			10	.228916		10	.962805	
			A	.045277		C	.057087	
17	7	.16441	3	-.018186	.24749	3	-.019470	
			4	-.036169		4	-.082504	
			5	-.024400		5	-.185852	
			6	.042648		6	-.288277	
			7	.150923		7	-.316774	
			8	.259506		8	-.226763	
			9	.629096		9	1.093044	
			A	-.010330		C	-.027595	
17	6	.18166	3	-.023022	.30198	3	-.024511	
			4	-.040501		4	-.102164	
			5	-.029017		5	-.272345	
			6	.048019		6	-.361853	
			7	.157725		7	-.402131	
			8	.891902		8	.990870	
			A	-.134389		C	-.224874	
17	5	.22184	3	-.024962	.34019	3	-.027439	
			4	-.051F17		4	-.116824	
			5	-.040919		5	-.266581	
			6	.038007		6	-.41F689	
			7	1.079393		7	.364180	
			A	-.3021F6		C	-.463353	
17	4	.21695	3	-.036865	.34006	3	-.028294	
			4	-.078718		4	-.121619	
			5	-.071319		5	-.275744	
			6	1.136042		6	-.154522	
			A	-.530738		C	-.572169	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	CCEF.	MSE	I	CCEF.	
17	3	.57989	3	-.068335	.35046	3	-.028309	
			4	-.152717		4	-.120576	
			5	1.22152		5	-.376428	
			A	-.855725		0	-.525714	
17	2	1.42458	3	-.178705	.38630	3	-.030092	
			4	1.178705		4	-.345311	
			A	-1.384461		0	-.375393	
			3	1.930603		2	-.192132	
			4	-2.524459		0	-.192132	
18	14	.14349	3	-.014372	.13667	3	-.078997	
			4	-.030309		4	-.038353	
			5	-.026609		5	-.088544	
			6	.016373		6	-.142010	
			7	.039735		7	-.167494	
			8	.0195778		8	-.138745	
			9	.060374		9	-.052684	
			10	.060334		10	.053894	
			11	.0195338		11	.013045	
			12	.090335		12	.0167494	
			13	.016373		13	.0142010	
			14	-.026609		14	.038544	
			15	-.030309		15	.038353	
			16	-.014372		16	.008997	
			A	.000000		0	-.000000	

TABLE T

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	MSF	** LOCATION **		** SCALE **		
			I	Coeff.	MSE	I	
18	13	.14432	3	-.014454	.13719	3	-.009319
			4	-.030571		4	-.038445
			5	-.026755		5	-.088757
			6	.016484		6	-.142355
			7	.009947		7	-.167919
			8	.016617		8	-.139706
			9	.0261915		9	-.054054
			10	.0261946		10	.053938
			11	.016605		11	.178580
			12	.010178		12	.167781
			13	.016659		13	.142224
			14	-.026562		14	.068628
			15	-.051802		15	.051689
			A	.003771		C	.003585
18	12	.14602	3	-.014923	.13839	3	-.009172
			4	-.030923		4	-.038929
			5	-.027742		5	-.099883
			6	.016746		6	-.144186
			7	.011247		7	-.173113
			8	.009148		8	-.147633
			9	.0265724		9	-.055152
			10	.0265458		10	.054184
			11	.0199453		11	.139765
			12	.0101966		12	.169249
			13	.017624		13	.143315
			14	-.094077		14	.156816
			A	.016197		C	.016406

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FROM ABOVE)

** LOCATION **				** SCALE **			
N	M	MSF	I COEF.	MSF	I	CCEF.	
13	11	.14779	3 -.014797	.14393	3 -.009470		
			4 -.031279		4 -.047376		
			5 -.027310		5 -.093249		
			6 .017786		6 -.149553		
			7 .002740		7 -.176740		
			8 .001904		8 -.146407		
			9 .000173		9 -.058120		
			10 .000524		10 .054714		
			11 .002885		11 .143118		
			12 .004254		12 .173431		
			13 -.004180		13 .339527		
			4 .037675		E .036691		
13	10	.14860	3 -.014973	.15519	3 -.010228		
			4 -.031426		4 -.047626		
			5 -.027767		5 -.103023		
			6 .017346		6 -.162051		
			7 .003612		7 -.191764		
			8 .003510		8 -.159890		
			9 .0271372		9 -.065482		
			10 .0271929		10 .055303		
			11 .005102		11 .150025		
			12 .000015		12 .589786		
			A .059111		E .067689		
13	9	.14961	3 -.014973	.17627	3 -.011659		
			4 -.031426		4 -.049770		
			5 -.027368		5 -.115169		
			6 .017344		6 -.185479		
			7 .003608		7 -.221577		
			8 .003505		8 -.185924		
			9 .0271365		9 -.083446		
			10 .0271919		10 .055266		
			11 .005945		11 .862454		
			A .057945		E .069736		

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM AECVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COEF.	MSE	I	COEF.	
18	8	.15026	3	-.015357	.21065	3	-.314023	
			4	-.031868		4	-.059926	
			5	-.027965		5	-.138972	
			6	.015945		6	-.224587	
			7	.103681		7	-.268915	
			8	.204184		8	-.270559	
			9	.272101		9	-.107483	
			10	.478078		10	1.371159	
			A	.019186		C	.026898	
18	7	.15830	3	-.015923	.25751	3	-.017296	
			4	-.033687		4	-.074063	
			5	-.0130358		5	-.172223	
			6	.015887		6	-.271168	
			7	.105751		7	-.377784	
			8	.209735		8	-.295561	
			9	.748794		9	1.37332	
			A	-.065339		C	-.106203	
18	6	.18004	3	-.018267	.30355	3	-.020016	
			4	-.039307		4	-.058445	
			5	-.0136748		5	-.206297	
			6	.013355		6	-.336646	
			7	.112176		7	-.410314	
			8	.969889		8	.733574	
			A	-.194981		C	-.328739	
18	5	.22028	3	-.022614	.32746	3	-.022468	
			4	-.051825		4	-.096545	
			5	-.051758		5	-.225727	
			6	.015666		6	-.360757	
			7	1.12052		7	.104499	
			A	-.371112		C	-.530020	
18	4	.24050	3	-.035909	.32950	3	-.022702	
			4	-.081149		4	-.097603	
			5	-.039122		5	-.228702	
			6	1.205180		6	-.238759	
			A	-.657164		C	-.537454	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	CCE.	MSE	I	CCE.	
18	3	.62501	3	-.058111	.73570	3	-.022287	
			4	-.159259		4	-.099286	
			5	1.227373		5	-.385447	
			4	-.943231		6	-.516921	
18	2	1.58643	3	-.130105	.37773	3	-.024743	
			4	1.190108		4	-.331158	
			A	-1.494767		6	-.355926	
18	1	7.60217	3	1.700060	.51163	3	-.181294	
			A	-2.693790		6	-.181294	
19	15	.13431	3	-.019278	.12821	3	-.072227	
			4	-.037164		4	-.031059	
			5	-.028162		5	-.073208	
			6	.031200		6	-.122162	
			7	.035862		7	-.154747	
			8	.040561		8	-.147151	
			9	.0223692		9	-.090459	
			10	.052657		10	.000000	
			11	.0223692		11	.090459	
			12	.050501		12	.147151	
			13	.065843		13	.154747	
			14	.001216		14	.127162	
			15	-.029188		15	.073208	
			16	-.027164		16	.031059	
			17	-.012279		17	.007227	
			A	-.010000		6	.000000	

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TABLE T

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FROM ARCV)

N	M	MSF	** LOCATION **			** SCALE **		
			I	CCEF.	MSF	I	CCEF.	
19	14	.17468	3	-.012370	.12844	3	-.307241	
			4	-.027329		4	-.031116	
			5	-.028324		5	-.073343	
			6	-.011222		6	-.122389	
			7	.066102		7	-.155038	
			8	.151377		8	-.147427	
			9	.224963		9	-.090653	
			10	.253993		10	-.007043	
			11	.224903		11	.099568	
			12	.151457		12	.147359	
			13	.066303		13	.154959	
			14	.001357		14	.122293	
			15	-.028179		15	.073252	
			16	-.045405		16	.041774	
			A	-.007377		C	.002934	
19	13	.17613	3	-.012471	.12967	3	-.007311	
			4	-.027618		4	-.031417	
			5	-.028612		5	-.074056	
			6	-.001273		6	-.123568	
			7	.066982		7	-.156584	
			8	.153177		8	-.148958	
			9	.227479		9	-.091692	
			10	.256999		10	-.000291	
			11	.227972		11	.091112	
			12	.153569		12	.148375	
			13	.067498		13	.155992	
			14	.001285		14	.122993	
			15	-.027716		15	.128271	
			A	.013483		C	.012843	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FROM ARCFE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COEF.	MSE	I	COEF.	
19	12	.137781	3	-.012622	.133332	3	-.007527	
			4	-.027947		4	-.032323	
			5	-.029929		5	-.076106	
			6	.001376		6	-.127198	
			7	.067977		7	-.161236	
			8	.155306		8	-.153565	
			9	.230710		9	-.004886	
			10	.260732		10	-.001129	
			11	.231101		11	.092629	
			12	.156220		12	.151316	
			13	.069256		13	.159011	
			14	-.103263		14	.242652	
			A	.032613		E	.007155	
19	11	.178895	3	-.012714	.141511	3	-.037691	
			4	-.028140		4	-.034354	
			5	-.029194		5	-.081728	
			6	.001505		6	-.135352	
			7	.068710		7	-.171789	
			8	.156047		8	-.164071	
			9	.232984		9	-.192293	
			10	.263476		10	-.007332	
			11	.233839		11	.095700	
			12	.158459		12	.157572	
			13	-.046870		13	.562131	
			A	.054150		D	.055195	
19	10	.138806	3	-.012723	.15694	3	-.008885	
			4	-.028154		4	-.038217	
			5	-.029091		5	-.008209	
			6	.001571		6	-.150497	
			7	.068889		7	-.191991	
			8	.157173		8	-.184334	
			9	.233486		9	-.116939	
			10	.264227		10	-.028216	
			11	.234499		11	.103624	
			12	.110355		12	.760415	
			A	.067188		E	.071363	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ARCFE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	CCEF.	MSF	I	CCEF.	
19	9	.13943	3	-.012771	.18263	3	-.019306	
			4	-.028279		4	-.044712	
			5	-.029276		5	-.105682	
			6	.071379		6	-.177169	
			7	.068756		7	-.226319	
			8	.157114		8	-.219197	
			9	.233423		9	-.142683	
			10	.263943		10	-.018104	
			11	.345912		11	1.000462	
			A	.042918		C	.156716	
19	8	.14725	3	-.013147	.22813	2	-.012605	
"			4	-.029182		4	-.054331	
			5	-.03458		5	-.128671	
			6	.070588		6	-.216779	
			7	.069114		7	-.277963	
			8	.159140		8	-.272243	
			9	.236468		9	-.183253	
			10	.607029		10	1.120277	
			A	-.016450		C	-.025214	
19	7	.15500	3	-.014292	.26448	3	-.015262	
			4	-.031913		4	-.065967	
			5	-.033031		5	-.156587	
			6	-.001486		6	-.2640	
			7	.071719		7	-.341667	
			8	.166276		8	-.378998	
			9	.344326		9	.934616	
			A	-.116143		C	-.198183	
19	6	.18197	3	-.016932	.29991	3	-.017501	
			4	-.038227		4	-.075061	
			5	-.041991		5	-.179993	
			6	-.006738		6	-.304879	
			7	.075752		7	-.396482	
			8	1.028136		8	.563752	
			A	-.265298		C	-.423763	

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TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	CCEF.		MSE	I	CCEF.
19	5	.27908	3	-.022581		.31367	3	-.018412
			4	-.051964			4	-.079684
			5	-.059701			5	-.189857
			6	-.017907			6	-.322359
			7	1.152113			7	.026786
			A	-.437972			8	-.573527
19	4	.76471	3	-.035212		.31316	3	-.018433
			4	-.082915			4	-.079783
			5	-.101308			5	-.190121
			6	1.219724			6	-.296745
			A	-.630935			7	-.584682
19	3	.68337	3	-.067951		.32254	3	-.018780
			4	-.164529			4	-.041173
			5	1.232487			5	-.387021
			A	-1.028578			6	-.486974
19	2	1.75868	3	-.101301		.37675	3	-.021622
			4	1.181301			4	-.317307
			A	-1.603411			5	-.339010
19	1	8.45886	3	1.000000		.51892	3	-.171584
			A	-2.862028			4	-.171584

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CALCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM APCVF)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COEF.	MSE	I	COEF.	
20	16	.12560	3	-.012562	.12057	3	-.005874	
			4	-.024268		4	-.025467	
			5	-.028212		5	-.060584	
			6	-.038612		6	-.104810	
			7	.04153		7	-.143091	
			8	.113176		8	-.146714	
			9	.195798		9	-.112563	
			10	.231607		10	-.042347	
			11	.231604		11	.042339	
			12	.195800		12	.112574	
			13	.113181		13	.145715	
			14	.041260		14	.140589	
			15	-.018612		15	.104810	
			16	-.028212		16	.060584	
			17	-.024268		17	.025467	
			18	-.010562		18	.005874	
			A	.010562		C	.000000	
20	15	.12624	3	-.010562	.12174	3	-.005882	
			4	-.024273		4	-.025443	
			5	-.028323		5	-.060571	
			6	-.038634		6	-.104961	
			7	.041244		7	-.143205	
			8	.113691		8	-.146571	
			9	.196642		9	-.112744	
			10	.232665		10	-.042428	
			11	.232677		11	.042765	
			12	.196668		12	.112691	
			13	.113765		13	.146872	
			14	.041329		14	.140238	
			15	-.008529		15	.104896	
			16	-.028212		16	.060507	
			17	-.040321		17	.034119	
			A	.012535		C	.002424	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CALCHY DISTRIBUTION
(WITH ADDITIONAL CENSURING FROM ABOVE)

N	M	MSF	** LOCATION **			** SCALE **		
			I	Coeff.	MSE	I	Coeff.	
20	14	.12748	3	-.913711	.12164	3	-.005926	
			4	-.024618		4	-.025635	
			5	-.023590		5	-.061433	
			6	-.008695		6	-.105760	
			7	.041694		7	-.141376	
			8	.114887		8	-.143289	
			9	.198592		9	-.113684	
			10	.235116		10	-.042993	
			11	.235177		11	.042499	
			12	.188792		12	.113290	
			13	.115204		13	.147685	
			14	.042102		14	.143958	
			15	-.08217		15	.105734	
			16	-.097744		16	.105954	
			A	.011265		C	.019768	
20	13	.12903	3	-.019845	.12437	3	-.006559	
			4	-.024910		4	-.026212	
			5	-.028914		5	-.062222	
			6	-.008747		6	-.108169	
			7	.042307		7	-.144640	
			8	.116455		8	-.151592	
			9	.191154		9	-.116552	
			10	.238347		10	-.044329	
			11	.238522		11	.042789	
			12	.191676		12	.115019	
			13	.117286		13	.150053	
			14	.043368		14	.143277	
			15	-.105714		15	.236547	
			A	.028131		C	.027107	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSUSING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	Coeff.	MSE	I	Coeff.	
20	12	.13619	3	-.019826	.12039	3	-.005360	
			4	-.025113		4	-.027517	
			5	-.029140		5	-.065065	
			6	-.039743		6	-.113531	
			7	.042841		7	-.152042	
			8	.117770		8	-.159505	
			9	.193271		9	-.123144	
			10	.241441		10	-.047752	
			11	.241761		11	.043277	
			12	.194240		12	.119683	
			13	.119287		13	.155168	
			14	-.075879		14	.428287	
			A	.049338		D	.349418	
20	11	.13652	3	-.01982	.14186	3	-.005371	
			4	-.025106		4	-.030571	
			5	-.029183		5	-.071059	
			6	-.039897		6	-.124050	
			7	.043185		7	-.166249	
			8	.119287		8	-.174981	
			9	.194191		9	-.135985	
			10	.242094		10	-.054844	
			11	.242530		11	.043834	
			12	.195384		12	.125147	
			13	.078526		13	.665765	
			A	.067824		D	.069366	
20	10	.13658	3	-.01988	.16115	3	-.007900	
			4	-.025184		4	-.034212	
			5	-.029215		5	-.092133	
			6	-.039760		6	-.141792	
			7	.043022		7	-.130470	
			8	.119303		8	-.201413	
			9	.193979		9	-.158349	
			10	.241936		10	-.057265	
			11	.242722		11	.043245	
			12	.234640		12	.910571	
			A	.066916		D	.070242	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION (WITH ADDITIONAL CENSORING FROM APCVF)

N	P	MSE	** LOCATION **			** SCALE **		
			I	Coeff.	MSE	I	Coeff.	
20	9	.12224	3	-.011118	.19335	3	-.009379	
			4	-.025559		4	-.043654	
			5	-.029745		5	-.097727	
			6	-.03205		6	-.169661	
			7	.042924		7	-.227901	
			8	.118672		8	-.242595	
			9	.194653		9	-.193806	
			10	.243150		10	-.088218	
			11	.476218		11	1.094773	
			A	.117609		0	.025346	
20	8	.13055	3	-.011679	.22860	3	-.011345	
			4	-.026932		4	-.049237	
			5	-.031607		5	-.118533	
			6	-.031022		6	-.205611	
			7	.043200		7	-.278345	
			8	.121509		8	-.298787	
			9	.200297		9	-.247101	
			10	.715925		10	1.129707	
			A	-.057776		0	-.095326	
20	7	.15392	3	-.013045	.26756	3	-.013390	
			4	-.030266		4	-.058175	
			5	-.036110		5	-.140335	
			6	-.013945		6	-.244106	
			7	.044132		7	-.332036	
			8	.120322		8	-.359379	
			9	.920211		9	.854776	
			A	-.168407		0	-.292604	
20	6	.18592	3	-.015915	.29250	3	-.014765	
			4	-.037206		4	-.064222	
			5	-.045674		5	-.155175	
			6	-.021222		6	-.271593	
			7	.045620		7	-.369521	
			8	1.174467		8	.370427	
			A	-.314526		0	-.494841	

TABLE I

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL CENSORING FROM ABOVE)

N	M	MSE	** LOCATION **			** SCALE **		
			I	Coeff.	MSF	I	Coeff.	
20	5	.25685	3	-.021772	.29862	3	-.015159	
			4	-.351823		4	-.065472	
			5	-.065855		5	-.159556	
			6	-.037466		6	-.278620	
			7	1.176411		7	-.079225	
			A	-.532787		C	-.598533	
21	4	.39137	3	-.034662	.29903	3	-.015149	
			4	-.084106		4	-.065418	
			5	-.111398		5	-.159379	
			6	1.238754		6	-.374719	
			A	-.752676		C	-.575154	
20	3	.74581	3	-.067847	.31345	3	-.015613	
			4	-.160462		4	-.067840	
			5	1.236712		5	-.387947	
			A	-1.112198		C	-.467397	
20	2	1.94137	3	-.182339	.26530	3	-.017382	
			4	1.182339		4	-.334260	
			A	-1.719671		C	-.321432	
20	1	9.42631	3	1.030669	.50671	3	-.162840	
			A	-3.029322		C	-.162840	

APPENDIX B

Table II

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSURING)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COEF.	MSE	I	CCEF.	
5	1	1.22125	3	1.000000	1.000000	3	0.000000	
			A	0.000000		0	0.000000	
6	2	.86053	3	.500000	.63722	3	-.501421	
			4	.500000		4	.501421	
			A	0.000000		0	0.000000	
7	3	.6.725	3	.059194	.50082	3	-.394255	
			4	.059194		4	.000000	
			5	.059194		5	.394255	
			A	-.000000		0	.000000	
8	4	.47326	3	-.050050	.40649	3	-.245497	
			4	.050050		4	-.342910	
			5	.050050		5	.342910	
			6	-.050050		6	.245497	
			A	.000000		0	-.000000	
8	2	.47756	4	.500000	.55065	4	-.913836	
			5	.500000		5	.913836	
			A	0.000000		0	.000000	
9	5	.38655	3	-.067277	.34138	3	-.153946	
			4	.245755		4	-.369546	
			5	.543785		5	-.060110	
			6	.245755		6	.369546	
			7	-.067277		7	.153946	
			A	-.000000		0	.000000	
9	3	.39619	4	.163962	.39972	4	-.678215	
			5	.672776		5	-.000000	
			6	.163962		6	.678215	
			A	-.000000		0	.000000	

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSORING)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COEF.	MSE	I	COEF.	
10	6	.32626	3	-.062004	.29371	3	-.100113	
			4	.083953		4	-.303114	
			5	.478052		5	-.212155	
			6	.478152		6	.212155	
			7	.083953		7	.303114	
			8	-.062004		8	.100113	
			A	-.000000		C	0.000000	
10	4	.33622	4	.000300	.2141	4	-.483478	
			5	.499700		5	-.229589	
			6	.499700		6	.229589	
			7	.000300		7	.483478	
			A	-.000000		C	.000000	
10	2	.33622	5	.500000	.51426	5	-1.288071	
			6	.500000		6	1.288071	
			A	0.000000		C	.000000	
11	7	.28197	3	-.052383	.25743	3	-.367613	
			4	.008650		4	-.231556	
			5	.295622		5	-.275726	
			6	.496199		6	-.000000	
			7	.295622		7	.275726	
			8	.008650		8	.231556	
			9	-.052383		9	.067613	
			A	-.000000		C	-.000000	
11	5	.20947	4	-.066116	.27198	4	-.345120	
			5	.308287		5	-.289100	
			6	.515459		6	.000000	
			7	.308287		7	.289100	
			8	-.066116		8	.345120	
			A	-.000000		C	.000000	
11	3	.20292	5	.236954	.36217	5	-.932166	
			6	.526093		6	.000000	
			7	.236954		7	.932166	
			A	-.000000		C	.000000	

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSORING)

				** LOCATION **				** SCALE **			
N	M	MSE	I	CDEF.		MSF	I	CDEF.		MSF	I
12	8	.24810	3	-.043156		.22895	3	-.047234			
			4	-.024229			4	-.174151			
			5	.164418			5	-.266602			
			6	.472588			6	-.138924			
			7	.402988			7	.138924			
			8	.164419			8	.266602			
			9	-.024229			9	.174151			
			10	-.043156			10	-.047234			
			A	.000000			C	.000000			
12	6	.25491	4	-.037650		.23717	4	-.253041			
			5	.171131			5	-.274964			
			6	.416519			6	-.143749			
			7	.416519			7	.143749			
			8	.171131			8	.274964			
			9	-.037650			9	.253041			
			A	.000000			C	.000000			
12	4	.25982	5	.069716		.28479	5	-.693568			
			6	.430284			6	-.169750			
			7	.430284			7	.169750			
			8	.069716			8	.693568			
			A	.000000			C	.000000			
12	2	.26101	6	.500000		.50528	6	-1.643493			
			7	.500000			7	1.643493			
			A	.000000			C	.000000			
13	9	.22140	3	-.035397		.20604	3	-.033979			
			4	-.037060			4	-.171415			
			5	.091475			5	-.232818			
			6	.290384			6	-.201347			
			7	.431196			7	.000000			
			8	.290384			8	.201347			
			9	.091475			9	.232818			
			10	-.037060			10	.171415			
			11	-.035397			11	.033979			
			A	.000000			C	.000000			

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSURING)

N	M	MSE	** LOCATION **			** SCALE **		
			I	Coeff.	MSE	I	Coeff.	
13	7	.22673	4	-.089454	.21095	4	-.184533	
			5	.084779		5	-.237471	
			6	.298968		6	-.205602	
			7	.412411		7	.050000	
			8	.298968		8	.235602	
			9	.084779		9	.237471	
			10	-.089454		10	.184533	
			A	-.000000		0	-.000000	
13	5	.23261	5	-.024512	.23849	5	-.524726	
			6	.310761		6	-.220134	
			7	.427501		7	.000000	
			8	.310761		8	.229134	
			9	-.024512		9	.524726	
			10	.000000		0	.000000	
13	3	.23276	6	.285408	.34462	6	-1.172345	
			7	.420163		7	.000000	
			8	.285408		8	1.172345	
			9	-.000000		0	.000000	
14	10	.19981	3	-.029130	.18722	3	-.025067	
			4	-.040524		4	-.100183	
			5	.031712		5	-.194902	
			6	.195010		6	-.215077	
			7	.342931		7	-.096717	
			8	.342931		8	.096717	
			9	.195010		9	.215077	
			10	.031712		10	.194902	
			11	-.040524		11	.100183	
			12	-.029130		12	.025067	
			A	-.000000		0	-.000000	

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSOPING)

N	M	MSE	** LOCATION **			** SCALE **		
			I	Coeff.	MSE	I	Coeff.	
14	8	.20397	4	-.084293	.19029	4	-.138664	
			5	.033235		5	-.197523	
			6	.210305		6	-.218998	
			7	.351053		7	-.098143	
			8	.351053		8	.098143	
			9	.210305		9	.218998	
			10	.033235		10	.197523	
			11	-.084293		11	.138664	
			A	-.000000		0	-.000000	
14	6	.21987	5	-.072811	.20711	5	-.402873	
			6	.209616		6	-.275925	
			7	.364205		7	-.155917	
			8	.364205		8	.155917	
			9	.209616		9	.235925	
			10	-.072811		10	.402873	
			A	.000000		0	-.000000	
14	4	.21137	6	.130185	.29638	6	-.887192	
			7	.369815		7	-.133033	
			8	.369815		8	.133033	
			9	.130185		9	.887192	
			A	.000000		C	.000000	
14	2	.21398	7	.500000	.49828	7	-1.988347	
			8	.500000		8	1.988347	
			A	0.000000		C	.000000	

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSORING)

N	M	MSE	** LOCATION **				** SCALE **			
			I	CCEF.	MSE	I	CCEF.	MSE	I	CCEF.
15	11	.18201	3	-.024130	.17150	3	-.018901			
			4	-.039711		4	-.077336			
			5	.002746		5	-.160423			
			6	.123831		6	-.206228			
			7	.269319		7	-.150096			
			8	.335888		8	-.000000			
			9	.269319		9	.150096			
			10	.123831		10	.206228			
			11	.002746		11	.160423			
			12	-.030711		12	.077336			
			13	-.024130		13	.018901			
			A	.000000		C	.000000			
15	0	.18528	4	-.076003	.17350	4	-.106028			
			5	.003373		5	-.161909			
			6	.126662		6	-.208321			
			7	.274780		7	-.151670			
			8	.342655		8	.000000			
			9	.274780		9	.151670			
			10	.126662		10	.208321			
			11	.003373		11	.161909			
			12	-.076003		12	.106028			
			A	.000000		C	.000000			
15	7	.19065	5	-.094496	.18429	5	-.313609			
			6	.132317		6	-.219540			
			7	.284058		7	-.160124			
			8	.354641		8	.000000			
			9	.284058		9	.160124			
			10	.132317		10	.219540			
			11	-.094496		11	.313609			
			A	.000000		C	-.000000			
15	5	.19346	6	.026726	.22037	6	-.697011			
			7	.291384		7	-.188073			
			8	.362781		8	.000000			
			9	.291384		9	.188073			
			10	.026726		10	.697011			
			A	.000000		C	-.000000			

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSORING)

N	M	MSE	** LOCATION **		** SCALE **	
			I	COEF.	MSE	I
15	3	.19351	7	.319305	.33535	7
			8	.361698		8
			9	.319305		9
			A	-.900000		0
16	12	.16769	3	-.020142	.15819	3
			4	-.037067		4
			5	-.013593		5
			6	.073662		6
			7	.200627		7
			8	.296514		8
			9	.296514		9
			10	.200627		10
			11	.173062		11
			12	-.013593		12
			13	-.037067		13
			14	-.020142		14
			A	.000000		0
16	10	.16967	4	-.067479	.15952	4
			5	-.013403		5
			6	.075214		6
			7	.204147		7
			8	.391521		8
			9	.331521		9
			10	.204147		10
			11	.075214		11
			12	-.013403		12
			13	-.067479		13
			A	.000000		0

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSORING)

N	M	MSE	** LOCATION **			** SCALE **		
			I	CCEF.	MSE	I	CCEF.	
16	8	.17440	5	-.101441	.16670	5	-.247285	
			6	.078711		6	-.195068	
			7	.211302		7	-.183517	
			8	.311428		8	-.374253	
			9	.311428		9	.074253	
			10	.211302		10	.180517	
			11	.078711		11	.195068	
			12	-.101441		12	.247285	
			4	.000000		C	-.333000	
16	6	.17788	5	-.038067	.18991	6	-.549245	
			7	.217910		7	-.202856	
			8	.320144		8	-.067822	
			9	.320144		9	.383822	
			10	.217910		10	.202856	
			11	-.038067		11	.549245	
			4	.000000		C	0.000000	
16	4	.17812	7	.178364	.25614	7	-1.071163	
			8	.321636		8	-.110000	
			9	.321636		9	.110000	
			10	.178364		10	1.071163	
			4	.000000		C	0.000000	
16	2	.18145	8	.500000	.49454	8	-2.326576	
			9	.500000		9	2.326576	
			4	.000000		C	.000000	

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSORING)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COFF.	MSE	I	COFF.	
17	13	.15448	3	-.016048	.14677	3	-.011348	
			4	-.037775		4	-.047878	
			5	-.022361		5	-.177651	
			6	.039373		6	-.163919	
			7	.143717		7	-.175151	
			8	.245754		8	-.115015	
			9	.288481		9	.062000	
			10	.245754		10	.115015	
			11	.143717		11	.175151	
			12	.039373		12	.163919	
			13	-.022361		13	.177651	
			14	-.037775		14	-.047878	
			15	-.016048		15	.011348	
			A	.000000		D	.000000	
17	11	.15646	4	-.059372	.14769	4	-.064002	
			5	-.022374		5	-.18151	
			6	.040189		6	-.164791	
			7	.145925		7	-.176132	
			8	.249322		8	-.115673	
			9	.292620		9	-.050000	
			10	.249322		10	.115673	
			11	.145925		11	.176132	
			12	.040189		12	.164791	
			13	-.022374		13	.18151	
			14	-.059372		14	.064003	
			A	.000000		D	.000000	
17	9	.16055	5	-.100548	.15260	5	-.197332	
			6	.042350		6	-.169417	
			7	.150763		7	.181354	
			8	.256378		8	-.119172	
			9	.301313		9	-.050000	
			10	.256378		10	.119172	
			11	.150763		11	.181354	
			12	.042350		12	.169417	
			13	-.100548		13	.197332	
			A	.000000		D	.000000	

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CALCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSORING)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COEF.		MSE	I	COEF.
17	7	.16420	6	-.076144		.16819	6	-.440794
			7	.156165			7	-.197742
			8	.264815			8	-.130199
			9	.310328			9	.000188
			10	.264815			10	.130199
			11	.156165			11	.197742
			12	-.076144			12	.440794
			A	-.000000			C	-.000000
17	5	.16523	7	.074985		.26976	7	-.847435
			8	.269073			8	-.159293
			9	.313362			9	.000000
			10	.269073			10	.159293
			11	.074985			11	.847435
			A	-.000000			C	-.000000
17	3	.16587	8	.343482		.33968	8	-1.632487
			9	.713137			9	-.332487
			10	.343482			10	1.632487
			A	-.070000			C	.000000
18	14	.14240	3	-.014372		.13687	3	-.008597
			4	-.020390			4	-.038357
			5	-.026509			5	-.088544
			6	.016373			6	-.142010
			7	.090335			7	-.167494
			8	.195338			8	-.138745
			9	.260334			9	-.053884
			10	.260334			10	.053884
			11	.195338			11	.138745
			12	.099334			12	.167494
			13	.016373			13	-.142010
			14	-.026509			14	.088544
			15	-.030399			15	-.038357
			16	-.014372			16	-.008597
			A	-.000000			C	-.000000

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSORING)

N	M	LOC.	LOCATION **		** SCALE **	
			I	COEF.	MSE	I
18	12	.14515	4	-.052698	.13752	4
			5	-.026708		5
			6	.016772		6
			7	.103695		7
			8	.197798		8
			9	.263545		9
			10	.263545		10
			11	.197798		11
			12	.103695		12
			13	.016772		13
			14	-.026708		14
			15	-.197798		15
			A	.103695		0
18	10	.14984	5	-.035717	.14097	5
			6	.017022		6
			7	.103855		7
			8	.207302		8
			9	.270627		9
			10	.270627		10
			11	.207302		11
			12	.103855		12
			13	.017022		13
			14	-.035717		14
			A	.000000		0
18	8	.15235	6	-.096677	.15179	6
			7	.107906		7
			8	.219871		8
			9	.270960		9
			10	.270960		10
			11	.219871		11
			12	.107906		12
			13	-.096677		13
			A	.000000		0

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE GELDZY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSURING)

N	M	MSE	** LOCATION **			** SCALE **		
			I	Coeff.	MSE	I	Coeff.	
18	6	.15486	7	.002392	.17941	7	-.685789	
			8	.217078		8	-.176919	
			9	.232730		9	-.069161	
			10	.287732		10	.069111	
			11	.213478		11	.176919	
			12	.012392		12	.685789	
			A	.000000		C	.000000	
18	4	.15476	8	.216717	.25003	8	-1.249093	
			9	.293667		9	-.094011	
			10	.287683		10	.094011	
			11	.216317		11	1.249093	
			A	.000000		C	.000000	
18	2	.15769	9	.510000	.40249	9	-2.660444	
			10	.510000		10	2.660444	
			A	.000000		C	.000000	
19	15	.15461	3	-.012278	.12821	3	-.007227	
			4	-.027194		4	-.031059	
			5	-.028188		5	-.073218	
			6	.001206		6	-.122162	
			7	.065813		7	-.154747	
			8	.150500		8	-.147151	
			9	.227692		9	-.090450	
			10	.252657		10	.060000	
			11	.223692		11	.090459	
			12	.151501		12	.147151	
			13	.065043		13	.154747	
			14	.001206		14	.122162	
			15	-.028188		15	.073218	
			16	-.027194		16	.031059	
			17	-.012278		17	.007227	
			A	.000000		C	.000000	

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TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSURING)

N	M	MSE	** LOCATION **			** SCALE **		
			I	Coeff.	MSF	I	Coeff.	
19	13	.13534	4	-.345722	.12868	4	-.641851	
			5	-.328316		5	-.973387	
			6	.001374		6	-.122525	
			7	.065656		7	-.155242	
			8	.152251		8	-.147637	
			9	.226185		9	-.291762	
			10	.255742		10	.000000	
			11	.226085		11	.093762	
			12	.152251		12	.147637	
			13	.065656		13	.155242	
			14	.001374		14	.122525	
			15	-.028316		15	-.973387	
			16	-.045722		16	.041851	
			A	.000000		C	.000000	
19	11	.17872	5	-.089193	.13115	5	-.129770	
			6	.001985		6	-.124442	
			7	.069772		7	-.157865	
			8	.156134		8	-.150217	
			9	.231582		9	-.292763	
			10	.261481		10	-.000000	
			11	.231582		11	.092368	
			12	.156134		12	.150217	
			13	.068672		13	-.157865	
			14	.001985		14	.124442	
			15	-.029393		15	.129770	
			A	.000000		C	.000000	
19	9	.14183	6	-.106726	.13866	6	-.294711	
			7	.071541		7	-.165944	
			8	.161279		8	-.159179	
			9	.238611		9	-.097336	
			10	.269269		10	.000000	
			11	.238611		11	.097336	
			12	.161279		12	.159179	
			13	.071541		13	.165944	
			14	-.106726		14	.294711	
			A	.000000		C	.000000	

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSORING)

N	M	MSE	** LOCATION **			** SCALE **		
			I	Coeff.	MSE	I	Coeff.	
19	7	.14403	7	-.04651?	.15798	7	-.562861	
			8	.165211		8	-.177759	
			9	.243915		9	-.109575	
			10	.274972		10	.000000	
			11	.247815		11	.129575	
			12	.165211		12	.177759	
			13	-.046512		13	.562861	
			A	.000000		C	-.000000	
19	5	.14427	8	.116949	.20322	8	-.997319	
			9	.244964		9	-.173291	
			10	.276175		10	.000000	
			11	.244964		11	.133291	
			12	.116949		12	.997319	
			A	.000000		C	.000000	
19	3	.14531	0	.362333	.32697	9	-1.356886	
			10	.275934		10	-.660000	
			11	.362333		11	1.356886	
			A	.000000		C	.000000	
23	16	.12569	3	-.011562	.12657	3	-.025874	
			4	-.024269		4	-.025407	
			5	-.029212		5	-.063884	
			6	-.038562		6	-.148103	
			7	.04153		7	-.140191	
			8	.117176		8	-.146714	
			9	.185798		9	-.112563	
			10	.231617		10	-.042343	
			11	.231614		11	.742339	
			12	.185911		12	.112574	
			13	.113101		13	.146715	
			14	.041049		14	.143899	
			15	-.018612		15	.148103	
			16	-.028212		16	.063884	
			17	-.024269		17	.025407	
			18	-.011562		18	.025874	
			A	.000000		C	.000000	

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSORING)

N	M	MSE	** LOCATION **			** SCALE **		
			I	COEF.	MSE	I	COEF.	
20	14	.12679	4	-.040196	.12091	4	-.034167	
			5	-.028333		5	-.060594	
			6	-.038560		6	-.125047	
			7	.041626		7	-.141436	
			8	.114281		8	-.147189	
			9	.187536		9	-.112861	
			10	.233747		10	-.042454	
			11	.233742		11	.042449	
			12	.187538		12	.112867	
			13	.114281		13	.147189	
			14	.041622		14	.140424	
			15	-.038561		15	.105047	
			16	-.028333		16	.060594	
			17	-.040196		17	.034167	
			A	.000000		C	.000000	
20	12	.12971	5	-.081887	.12272	5	-.106763	
			6	-.038301		6	-.106207	
			7	.042772		7	-.142463	
			8	.116959		8	-.149089	
			9	.191667		9	-.114617	
			10	.233791		10	-.047048	
			11	.233787		11	.043243	
			12	.191668		12	.114423	
			13	.116954		13	.149282	
			14	.042768		14	.142261	
			15	-.038301		15	.106297	
			16	-.081887		16	.106763	
			A	.000000		C	.000000	

TABLE II

COEFFICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE
LOCATION AND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION
(WITH ADDITIONAL SYMMETRIC CENSURING)

N	M	MSE	** LOCATION **				** SCALE **			
			I	CCEF.	MSE	I	CCEF.	MSE	I	CCEF.
20	10	.13254	6	-.19460	.12833	6	-.244336			
			7	.044721		7	-.147875			
			8	.126757		8	-.155197			
			9	.197330		9	-.119219			
			10	.245671		10	-.044659			
			11	.245627		11	.044854			
			12	.197372		12	.119214			
			13	.126763		13	.155199			
			14	.044717		14	.147873			
			15	-.198479		15	.244336			
			A	.000000		B	.000000			
20	8	.17581	7	-.078118	.14199	7	-.457638			
			8	.124255		8	-.169691			
			9	.212274		9	-.171772			
			10	.261479		10	-.049269			
			11	.261479		11	.049269			
			12	.212277		12	.131779			
			13	.124261		13	.169692			
			14	-.378712		14	.467623			
			A	.000000		B	.000000			
20	6	.17571	8	.041861	.17267	8	-.816675			
			9	.204381		9	-.156576			
			10	.253809		10	-.059064			
			11	.253808		11	.058978			
			12	.204328		12	.156621			
			13	.041865		13	.816658			
			A	.000001		B	.000001			
20	4	.17584	9	.246572	.24624	9	-1.422961			
			10	.253428		10	-.082729			
			11	.253428		11	.082729			
			12	.246572		12	1.422961			
			A	.000000		B	.000000			
20	2	.17951	10	.500000	.40175	10	-2.991259			
			11	.500000		11	2.991259			
			A	.000000		B	.000000			

APPENDIX C
Computer Program

PROGRAM COEFFS (INPUT, OUTPUT)

C THIS PROGRAM COMPUTES AND TABLES THE COEFFICIENTS FOR BEST
C LINEAR ESTIMATION OF THE LOCATION AND SCALE PARAMETERS OF
C THE CAUCHY DISTRIBUTION FOR SAMPLE SIZES OF 5(1)20 WITH
C ADDITIONAL CENSORING FROM ABOVE. MUST READ IN THE EXPECTED
C VALUES AND COVARIANCES OF THE ORDER STATISTICS FORMAT (6F12.6).
C DIMENSION EXP(20,20),COV(20,20),A(20,20),B(20),X(20),RL(20),
C 1SE(20,20),AS(20,20),ES(20),XS(20),RKC(20),SES(20)
C NN=MAXIMUM SAMPLE SIZE MM=MINIMUM SAMPLE SIZE
C NN=20
C MM=5
C DO 5 N=MM,NN
C NI=N-2
C NR=N/2
C NM=(N+1)/2
C READ 101,(EXP(N,I),I=3,NM)
C DO 4 I=3,NR
4 EXP(N,N+1-I)=EXP(N,I)
5 CONTINUE
DO 100 N=MM,NN
NI=N-2
DO 8 I=3,NI
8 READ 101,(COV(I,J),J=I,NI)
DO 10 I=3,NI
DO 10 J=I,NI
10 COV(J,I)=COV(I,J)
C FILL THE A MATRIX
A(1,1)=0.0
A(1,2)=0.0
A(2,1)=0.0
A(2,2)=1.0
DO 15 I=3,NI
A(I,1)=1.0
15 A(I,2)=-EXP(N,I)
DO 16 J=3,NI
A(I,J)=1.0
16 A(2,J)=-EXP(N,J)
DO 17 I=3,NI
DO 17 J=3,NI
17 A(I,J)=COV(I,J)+EXP(N,I)*EXP(N,J)
C FILL THE B MATRIX
B(1)=1.0
DO 18 I=2,NI
18 B(I)=0.0
C COMPUTE THE COEFFICIENTS OR CENSOR AND COMPUTE THE COEFFICIENTS
N4=N-4
DO 30 ICEN=1,N4
M=N-3-ICEN
NP=X+2
CALL MTXEL(NP,A,B,X)
DO 20 I=3,np
20 RL(I)=X(I)
RK=X(2)
CALL SEL(N,M,np,RK,RL,EXP,COV,SS)

GAM/MATH/72-3

```
C      FILL THE AS MATRIX
DO 25 I=1,N4
DO 25 J=1,N4
25 AS(I,J)=COV(I+2,J+2)+EXP(N,I+2)*EXP(N,J+2)
C      FILL BS MATRIX
DO 26 I=1,N4
26 BS(I)=EXP(N,I+2)
C      COMPUTE THE COEFFICIENTS FOR THE SCALE PARAMETER
NT=M
CALL MTXEL(NT,AS,BS,XS)
DO 27 I=1,NT
27 RKC(I+2)=XS(I)
CALL SEL(N,M,NP,1.0,RKC,EXP,COV,SES)
SUMD=0.0
DO 28 ID=3,NP
28 SUMD=SUMD+RKC(ID)
RD=SUMD
30 CALL PRINT1(N,M,SE,RL,SES,RKC,RK,3,RD)
100 CONTINUE
101 FORMAT(6F12.6)
STOP
END
```

```

SUBROUTINE PRINT1 (N,M,SEL,COFL,SES,COFS,A,I3,D)
DIMENSION SEL(20,20),COFL(20),SES(20,20),COFS(20)
C      THIS ROUTINE PRINTS THE COEFFICIENTS AND MSE FOR SINGLE
C      CENSORING FROM ABOVE.
C      N=SAMPLE SIZE
C      M=SAMPLE SIZE AFTER CENSORING
C      SEL=MSE OF LOCATION ESTIMATOR
C      COFL= COEFFICIENTS FOR THE LOCATION ESTIMATE
C      SES = MSE OF THE SCALE ESTIMATOR
C      COFS= COEFFICIENTS FOR SCALE ESTIMATE
C      A= CONSTANT FROM LOCATION ESTIMATE
C      D= CONSTANT FROM SCALE ESTIMATE
IF(N.GT.5)GO TO 5
IPAGE=39
PRINT 20
PRINT 21,N,M,SEL(N,M),I3,COFL(3),SES(N,M),I3,COFS(3)
M2=M+2
IF(M.EQ.1)GO TO 4
PRINT 22,(I,COFL(I),ICOFS(I),I=4,M2)
4 PRINT 23,A,D
K=37
RETURN
5 RM=M
RK=K
RLINE=RM+2.0
IF(RLINE.GE.)GO TO 10
8 PRINT 21,N,M,SEL(N,M),I3,COFL(3),SES(N,M),I3,COFS(3)
M2=M+2
IF(M.EQ.1)go to 9
PRINT 22,(I,COFL(I),I,COFS(I),I=4,M2)
9 PRINT 23,A,D
K=K-M2
GO TO 16
10 ISKIP=K+3
DO 11 IS=1,ISKIP
11 PRINT 24
PRINT 25,IPAGE
IPAGE= IPAGE+1
PRINT 20
K=41
15 GO TO 8
16 IF(N.EQ.20)GO TO 17
RETURN
17 IF(M.EQ.1)GO TO 18
RETURN
18 ISKIP=K+3
DO 19 IS=1,ISKIP
19 PRINT 24
PRINT 25,IPAGE
RETURN

```

```

20 FORMAT(1H1,15X,13HGAM/MATH/72-3,///,41X,7HTABLE I,/,19X,51'COFFI
1CIENTS FOR BEST CONDITIONAL ESTIMATION OF THE,/,17X,56HLOCATION AN
2D SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION,/,25X,38H(WITH ADDIT
3IONAL CENSORING FROM ABOVE),/,15X,59(*.*),/,28X,14H**LOCATION**
4,16X,11H** SCALE **,/,16X,1HN,3X,1HM,5X,3HMSE,5X,1HI,4X,5HCOEF.,10
5X,3HMSE,5X,1HI,4X,5HCOEF.,/,15X,59(*.*))
21 FORMAT(1HO,14X,I2,I4,F10.5,I4,F11.6,F13.5,I4,F11.6)
22 FORMAT(3CX,I2,F11.6,15X,2,F11.6)
23 FORMAT(34X,1HA,F11.6,16X,1HD,F11.6)
24 FORMAT(1H )
25 FORMAT(42X,I3)
END

```

SUBROUTINE SEL(N,M,NP,RK,RL,EXP,COV,SE)

DIMENSION RL(20),EXP(20,20),COV(20,20),SE(20,20)

C THIS ROUTINE COMPUTES THE NSE FOR THE LOCATION AND SCALE

C ESTIMATES

SUM1=0.0

SUM3=0.0

DO 10 I=3,NP

SUM2=0.0

DO 5 J=3,NP

5 SUM2=RL(I)*RL(J)*COV(I,J)+SUM2

SUM1=SUM1+SUM2

10 SUM3=RL(I)*EXP(N,I)+SUM3

FA=SUM3-RK

FA2=FA*FA

SE(N,M)=SUM1+FA2

RETURN

END

SUBROUTINE MTXEL(NP,A,B,X)

DIMENSION A(20,20),B(20),X(20),PIV(20),C(38,38)

C THIS ROUTINE IS A MODIFIED VERSION OF MTXEQ-MATRIX EQUATION

C SOLVER SUBROUTINE, COMPUTER SCIENCE CENTER, WRIGHT-PATTERSON

C AFB, OHIO

C TO SOLVE THE LINEAR SYSTEM AX=B

DO 10 J=1,NP

DO 10 I=1,NP

10 C(I,J)=A(I,J)

NPJ=NP+1

DO 20 I=1,NP

20 C(I,NPJ)=B(I)

NP1=NP+1

NPK=NP+1

DO 120 I=1,NP

IP1:I+1

ATPE=0.0

DO 40 J=1,NP

IF (ARS(C(J,I))-ATPE) 40,30,30

30 ATPE=ABS(C(J,I))

IPIV=J

```
40 CONTINUE
  IF (ATPE) 210,210,50
  50 DO 60 J=IP1,NPK
  60 PIV(J)=C(IPIV,J)/C(IPIV,I)
    IFROM=NP
    ITO=NP
  70 IF (IFROM-IPIV) 80,100,80
  80 RM=-C(IFROM,I)
    DO 90 J=IP1,NPK
  90 C(ITO,J)=C(IFROM,J)+RM*PIV(J)
    ITO=ITO-1
100 IFROM=IFROM-1
  IF (IFROM-I) 110,70,70
110 DO 120 J=IP1,NPK
120 C(I,J)=PIV(J)
  I=NP
130 IP1=I
  I=I-1
  IF (I) 160,160,140
140 DO 150 J=NPI1,NPK
  DO 150 L=IP1,NP
150 C(I,J)=C(I,J)-C(I,L)*C(L,J)
  GO TO 130
160 NPJ=NPK+1
  DO 170 I=1,NP
170 X(I)=C(I,NP)
180 RETURN
210 PRINT 1001
1001 FORMAT(37HODEP(A)=0 IN CALL TO SUBROUTINE MTXSL)
  RETURN
END
```

PROGRAM COEFFD(INPUT,OUTPUT)

C THIS PROGRAM COMPUTES THE COEFFICIENTS FOR THE CONDITIONAL
C BEST LINEAR INVARIANT ESTIMATION OF THE LOCATION AND SCALE
C PARAMETERS OF THE CAUCHY DISTRIBUTION FOR SAMPLE SIZES 5(1)20
C WITH ADDITIONAL SYMMETRIC CENSORING. MUST READ IN THE EXPECTED
C VALUES AND COVARIANCES OF THE ORDER STATISTICS, FORMAT(6F12.6)
C NN=MAXIMUM SAMPLE SIZE MM=MINIMUM SAMPLE SIZE

NN=20
M=5
DO 5 N=MM,NN
NI=N-2
NR=N/2
NM=(N+1)/2
READ 101,(EXP(N,I),I=3,NM)
DO 4 I=3, NR
4 EXP(N,N+1-I)--EXP(N,I)
5 CONTINUE
DO 100 N=MM,NN
NI=N-2
DO 8 I=3,NI
8 READ 101,(COV(I,J),J=I,NI)
DO 10 I=3,NI
DO 10 J=I,NI
10 COV(J,I)=COV(I,J)
C FILL THE A MATRIX
A(1,1)=0.0
A(1,2)=0.0
A(2,1)=0.0
A(2,2)=1.0
DO 15 I=3,NI
A(I,1)=1.0
15 A(I,2)--EXP(N,I)
DO 16 J=3,NI
A(1,J)=1.0
16 A(2,J)--EXP(N,J)
DO 17 I=3,NI
DO 17 J=3,NI
17 A(I,J)=COV(I,J)+EXP(N,I)*EXP(N,J)
C FILL THE B MATRIX
B(1)=1.0
DO 18 I=2,NI
18 B(I)=0.0
C COMPUTE THE COEFFICIENTS FOR THE BASIC CENSORED SAMPLE
N=N-4
NP=142
CALL MTXEL(NP,A,B,X)
DO 19 I=3,NP
19 RL(I)=X(I)
RK=X(2)
CALL SELD(N,M,RK,RL,EXP,COV,SED)
C FILL THE AS MATRIX
N4=N-4
DO 25 I=1,N4
DO 25 J=1,N4

```

DO 38 I=1,M
38 RKC(I+2+ICEN)=XS(I)
CALL SELD(N,M,1.0,RKC,EXP,COV,SES)
IB=3+ICEN
IT=IB+M-1
SUMD=0.0
DO 41 ID=IB,IT
41 SUMD=SUMD+RKC(ID)
RKD=SUMD
50 CALL PRINT(N,M,SEL,RL,SES,RKC,RK,IB,RKD)
100 CONTINUE
101 FORMAT(6F12.6)
STOP
END

```

```

SUBROUTINE SELD(N,M,RK,RL,EXP,COV,SED)
DIMENSION RL(20),EXP(20,20),COV(20,20),SED(20,20)
SUM1=0.0
SUM2=0.0
NN1=(N-M)/2+1
NN2=N-NN1+1
DO 10 I=NN1,NN2
DO 10 J=NN1,NN2
10 SUM1=SUM1+RL(I)*RL(J)*COV(I,J)
DO 12 I=NN1,NN2
12 SUM2=RL(I)*EXP(N,I)+SUM2
FNA=SUM2-RK
FNA2=FNA*FNA
SED(N,M)=SUM1+FNA2
RETURN
END

```

```

SUBROUTINE PRINT(N,M,SEL,COFL,SES,COFS,A,I3,D)
DIMENSION SEL(20,20),COFL(20),SES(20,20),COFS(20)
C N=SAMPLE SIZE
C M=SIZE AFTER CENSORING
C SEL=MSE LOCATION
C COFL=COEFFICIENTS FOR LOCATION ESTIMATE
C SES=MSE SCALE
C COFS=COEFFICIENTS FOR SCALE ESTIMATE
C A=CONSTANT FROM LOCATION ESTIMATE
C D=CONSTANT FROM SCALE ESTIMATE
IF(N.GT.5) GO TO 5
IPAGE=71
PRINT 20
PRINT 21,N,M,SEL(N,M),I3,COFL(3),SES(N,M),I3,COFS(3)
PRINT 23,A,D
K=38
RETURN
5 RK=5
RK=K
RLINE=RK+2
IF(RLINE.GE.RK)GO TO 10

```

```

25 AS(I,J)=COV(I+2,J+2)+EXP(N,I+2)*EXP(N,J+2)
C   FILL THE BS MATRIX
DO 26 I=1,N4
26 BS(I)=EXP(N,I+2)
C   COMPUTE THE COEFFICIENTS
NT=M
CALL MTXEL(NT,AS,BS,XS)
DO 27 I=1,NT
27 RKC(I+2)=XS(I)
CALL SELD(N,M,1.0,RKC,EXP,COV,SES)
SUMD=0.0
DO 40 ID=3,NP
40 SUMD=SUMD+RKC(ID)
RKD=SUMD
CALL PRINT(N,M,SED,RL,SES,RKC,RK,3,RKD)
IF(N.EQ.5) GO TO 100
IF(N.EQ.6) GO TO 100
IF(N.EQ.7) GO TO 100
C   CENSOR AND COMPUTE THE COEFFICIENTS
GO TO 29
C   IF IT IS DESIRED TO CENSOR TO SIZE M=1, REMOVE PRECEEDING CARD
IF((N/2)*2-N)28,29,28
28 NC=(N/2)-2
GO TO 30
29 NC=N/2-3
30 DO 50 ICEN=1,NC
NPC=N+1
NP=N+2
M=M-2
DO 31 JD=3,NPC
31 A(2,JD)=A(2,JD+1)
DO 32 ID=4,NP
DO 32 JD=3,NPC
32 A(ID,JD)=A(ID,JD+1)
DO 33 ID=3,NPC
DO 33 JD=2,NPC
33 A(ID,JD)=A(ID+1,JD)
NP=M+2
CALL MTXEL(NP,A,B,X)
DO 35 I=3,NP
35 RL(I+ICEN)=X(I)
RK=X(2)
CALL SELD(N,M,RK,RL,EXP,COV,SED)
C   COMPUTE THE SCALE COEFFICIENTS FOR CENSORED SAMPLE
NTS=M+1
NT=M+2
DO 36 IS=2,NT
DO 36 JS=1,NTS
36 AS(IS,JS)=AS(IS,JS+1)
DO 37 IS=1,NTS
DO 37 JS=1,NTS
BS(IS)=BS(IS+1)
37 AS(IS,JS)=AS(IS+1,JS)
CALL MTXEL(M,AS,BS,XS)

```

```

8 PRINT 21,N,M,SEL(N,M),I3,COFL(I3),SES(N,M),I3,COFS(I3)
M2=I3+M-1
M4=I3+1
IF(M.EQ.1)GO TO 9
PRINT 22,(I,COFL(I),I,COFS(I),I=M4,M2)
9 PRINT 23,A,D
K=K-M-2
GO TO 16
10 ISKIP=K+3
DO 11 IS=1,ISKIP
11 PRINT 24
PRINT 25,IPAGE
IPAGE=IPAGE+1
PRINT 20
K=41
15 GO TO 8
16 IF(N.EQ.20)GO TO 17
RETURN
17 IF(M.EQ.1)GO TO 18
RETURN
18 ISKIP=K+3
DO 19 IS=1,ISKIP
19 PRINT 24
PRINT 25,IPAGE
RETURN
20 FORMAT(1H1,15X,13HGAM/MATH/72-3,//,41X,8HTABLE II,/,19X,51HCOMPF
1ICIENTS FOR BEST CONDITIONAL ESTIMATION OF THE,/,17X,56HLOCATION A
2ND SCALE PARAMETERS OF THE CAUCHY DISTRIBUTION,/,26X,37H(WITH ADDI
3TIONAL SYMMETRIC CENSORING),/,15X,59(*.*),/,23X,14H** LOCATION **
416X,11H** SCALE **,/,16X,1HN,3X,1HM,5X,3HMSE,5X,1HI,4X,5HCOEF.,10
5X,3HMSE,5X,1HI,4X,5HCOEF.,/,15X,59(*.*))
21 FORMAT(1HO,14X,I2,I4,F10.5,I4,F11.6,F13.5,I4,F11.6)
22 FORMAT(33X,I2,F11.6,15X,I2,F11.6)
23 FORMAT(34X,1HA,F11.6,16X,1HD,F11.6)
24 FORMAT(1H )
25 FORMAT(42X,I3)
END

```

Vita

Ralph Merle Spory, Jr. was born 22 April 1940 at New Florence, Pennsylvania, the son of Ralph M. Spory and Mollie I. Spory. After graduating in 1958 from Laurel Valley Joint High School, Bolivar, Pennsylvania, he entered the United States Air Force Academy. He graduated from the Air Force Academy in 1962 with a Bachelor of Science degree and a commission as Second Lieutenant in the United States Air Force. In 1963 he graduated from Pilot Training and spent the next seven years in various flying assignments. He entered the Air Force Institute of Technology in June 1970. He is married to the former Karen Ann Benson of New York City, New York.

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