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TRANSITION PROCESSES IN THERMOELECTRIC DEVICES

by

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13. ABSTRACT

Calculation of transitional thermal processes on a thermoelement cold junction under mixed thermoloading is presented.

I

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ENGLISH TITLE: Transition Processes in Thermoelectric Devices

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NOTICE

III

At the present time, semiconductor thermobatteries are used as thermostats in radioelectronic devices, air conditioning, refrigerating of food products, biological preparations and the like. The operation of thermoelectric devices is established by both static characteristics (refrigerating capacity, fixed temperature drop) and by dynamic ones (exit time from a given temperature mode, precision of maintenance of a temperature). Of special interest among possible transitional modes of operation of thermobatteries are transitional processes arising during passage of multistep currents across thermoelement junctions. A number of variants of calculations of temperature movement when thermobattery power is switched on have been published in the literature [1-8]. As a rule, simplified thermobattery models are discussed in the works cited. Either they generally do not take the thermoload on thermobattery junctions into account [3, 4], or they only take into account the heat capacity of the switchplate and the object cooled [1, 2, 5].

Calculation of transitional thermal processes on a thermoelement cold junction under mixed thermoloading is presented below. Subsequently, it is considered that thermobattery junctions have identical electrical and thermal characteristics, and that the side faces of thermobatteries are adiabatically isolated. Then, temperature distribution T of a thermobattery, as a function of coordinate x, directed along the length of thermoelements from the cold junction to the hot, and time t are written in a thermoconductivity equation as

$$\lambda \frac{\partial^2 T}{\partial x^2} = c \frac{\partial T}{\partial t} - j^2 \rho. \qquad (1)$$

Here, λ is the thermoconductivity of thermoelement junctions, c is the volumetric heat capacity of thermobatteries, ρ is the resistivity of thermoelement junctions, j is the current supply density (here and further, the values of physical parameters are related to unit areas of the thermobattery cooling surface).

$$\theta|_{\mathrm{Fo}=0} = \theta_0, \tag{2a}$$

$$\begin{cases} \left.\dfrac{\partial\theta}{\partial\chi}\right|_{\chi=0} = \nu\theta + \tau_1 \dfrac{\partial\theta}{\partial(\mathrm{Fo})} + \mathrm{Bi}_1(\theta - \theta_0) + K_{\mathrm{u}}, \\ \theta|_{\chi=1} = \theta_0. \end{cases} \tag{3a}$$

Using the operator method, it is easy to find the Laplace transform for the temperature of the cold junction

$$\bar{\theta}(p)|_{\chi=0} = \frac{\theta_0}{p} + \frac{\gamma^2 \sqrt{p}\,(\mathrm{ch}\sqrt{p} - 1) - p(\nu\theta_0 - K_{\mathrm{u}})\,\mathrm{sh}\sqrt{p}}{p^2\left[\sqrt{p}\,\mathrm{ch}\sqrt{p} + (\nu + \mathrm{Bi}_1 + \tau_1 p)\,\mathrm{sh}\sqrt{p}\right]}. \tag{4}$$

Transition to the provisional domain is accomplished by the Rieman-Mellin formula and residue theory:

$$\theta(\mathrm{Fo}) = \theta_0 - \frac{\nu\theta_0 - 0{,}5\gamma^2 - K_{\mathrm{u}}}{1 + \nu + \mathrm{Bi}_1} + \sum_{\kappa=1}^{\infty} A_\kappa \exp(-\delta_\kappa^2\,\mathrm{Fo}), \tag{5}$$

where

$$A_\kappa = 2\,\frac{[\delta_\kappa(\nu\theta_0 - K_{\mathrm{u}})\sin\delta_\kappa - \gamma^2(1 - \cos\delta_\kappa)]}{\delta_\kappa^2\left[\delta_\kappa(2\tau_1 + 1)\sin\delta_\kappa + \left(\tau_1\delta_\kappa^2 - 1 - \nu - \mathrm{Bi}_1\right)\cos\delta_\kappa\right]}.$$

The value of δ_K is the positive root of the transcendental equation

$$\mathrm{tg}\,\delta = \frac{\delta}{\tau_1\delta^2 - \nu - \mathrm{Bi}_1}. \tag{6}$$

It follows from formula (5) that the temperature drop $\Delta\theta=\theta_0-\theta$ between thermoelement junctions can be presented in the form

$$\Delta\theta = \Delta\theta_{\text{стац}} - f(\mathrm{Fo}),$$

where

$$f(\mathrm{Fo}) = \sum_{\kappa=1}^{\infty} A_\kappa \exp(-\delta_\kappa^2\,\mathrm{Fo}). \tag{7}$$

Series (7) defines the rate of approach of the temperature of the cold junction to a stable value, in which

$$\Delta\theta_{\text{стац}} = \frac{\nu\theta_0 - (0{,}5 + \xi_к)\,\nu^2 - K_н}{1 + \nu + \mathrm{Bi}_1}. \tag{8}$$

The maximum temperature drop is achieved at a current density ν_T^0, the value of which is equal to

$$\nu_T^0 = \frac{[B^2 + 2\theta_0 B + 2K_н(1 + 2\xi_к)]^{\frac{1}{2}} - B}{1 + 2\xi_к}, \tag{9}$$

in which the value itself of the maximum drop is determined by the following relationship:

$$\Delta\theta_{\text{стац}}^{\text{макс}} = \theta_0 + B - [B^2 + 2\theta_0 B + 2K_н(1 + 2\xi_к)]^{\frac{1}{2}}, \tag{10}$$

here, $B=(1+\mathrm{Bi}_1)(1+2\xi_K)$.

Calculations according to formula (5) assume knowledge of the solutions of Eq. (6). The values of the two least roots δ_1 and δ_2 were calculated on a digital computer for a wide range of changes in the parameters $\nu+\mathrm{Bi}_1$ and η_1. The results of the calculations are presented in Table 1. The case for $\nu+\mathrm{Bi}_1=0$ and $\eta_1=0$ are limiting. Since the value of δ_K is localized in the corresponding intervals $\delta_K \in ((k-1)\pi, k\pi)$, series (7) quickly converges at sufficiently large values of Fo.

It is evident from the data of Table 1 that the increase in heat capacity η_1 leads to a decrease in the values of δ_K, that is, to an increase in retentivity of thermoelements; increase in heat transfer Bi_1, as well as current density ν, show an opposite effect on the value of δ_K.

Pre-exponential factor A_K also depends on parameters η_1, ν and Bi_1. In addition, the parameters θ_0 and K_H influence their values. The course of changes in coefficients A_1 and A_2, as well as of δ_1^2 and δ_2^2 vs. the ratio of current density ν to its optimum value ν_T^0 for various combinations of parameters Bi_1 and η_1, are presented in Fig. 1, 2, 3 and 4. The relationship of A_1 to current ν is characterized by the presence of a maximum, after which A_1 decreases monotonically, passes through zero and becomes negative. Coefficient A_2 increases monotonically with increase in current ν. With increase in heat transfer in Bi_1, the value of the coefficients of the series decreases. which corresponds to a decrease in the steady drop with increase of Bi_1 (Fig. 1 and 2). Values of δ_1^2 and δ_2^2 increase monotonically together with current strength.

Table 1: Values of δ_1 and δ_2

γ_1 \ $v+Bt_1$	0	0,25	0,5	1,0	1,5	2,0	3,0	4,0	5,0	7,5	10	15	20
0,0	1,571	1,715	1,8[illegible]	2,029	2,175	2,288	2,456	2,570	2,651	2,786	2,863	2,948	2,993
	4,712	4,765	4,816	4,913	5,001	5,087	5,233	5,354	5,454	5,639	5,761	5,908	5,992
0,1	1,429	1,568	1,695	1,896	2,051	2,181	2,370	2,503	2,600	2,754	2,842	2,937	2,987
	4,303	4,351	4,397	4,489	4,582	4,672	4,846	5,003	5,142	5,413	5,599	5,819	5,937
0,25	1,265	1,401	1,520	1,721	1,895	2,022	2,235	2,392	2,509	2,698	2,803	2,919	2,976
	3,935	3,965	3,994	4,057	4,124	4,193	4,338	4,487	4,633	4,972	5,218	5,615	5,816
0,5	1,077	1,199	1,297	1,494	1,652	1,790	2,017	2,197	2,339	2,585	2,730	2,882	2,951
	3,644	3,659	3,673	3,705	3,739	3,776	3,857	3,947	4,016	4,315	4,593	5,092	5,462
1,0	0,860	0,960	1,049	1,[illegible]	1,315	1,467	1,678	1,858	2,012	2,316	2,529	2,777	2,897
	3,426	3,431	3,437	3,448	3,461	3,474	3,503	3,536	3,573	3,686	3,829	4,179	4,553

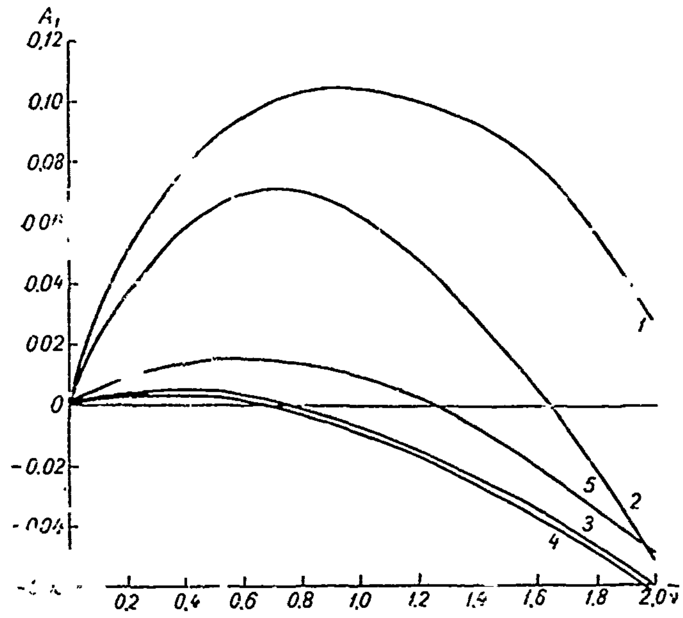

Fig. 1: Coefficient A_1 vs. current density $\overline{\nu}$ (θ_0=0.6)

Key:
1. $Bi_1=0$, $\eta_1=1$
2. $Bi_1=0$, $\eta_1=0$
3. $Bi_1=5$, $\eta_1=0.1$
4. $Bi_1=5$, $\eta_1=0$
5. $Bi_1=5$, $\eta_1=1$

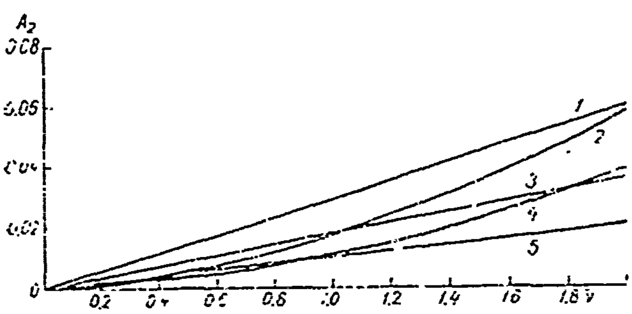

Fig. 2: Coefficient A_2 vs. current density $\overline{\nu}$ (θ_0=0.6)

See Fig. 1 for legend.

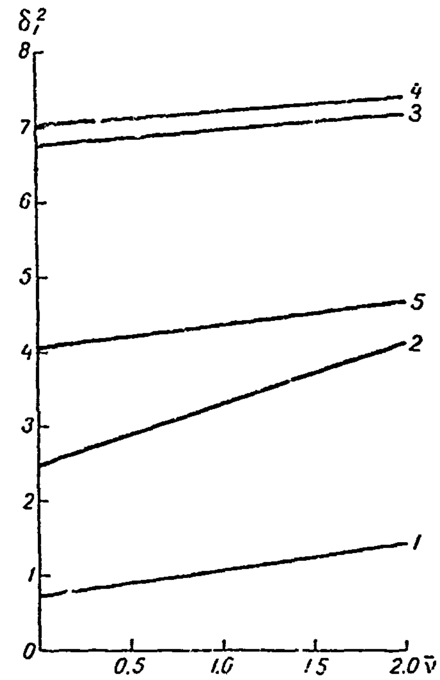

Fig. 3: δ_1^2 vs. current density $\bar{\nu}$ (θ_0=0.6)

See Fig. 1 for legend.

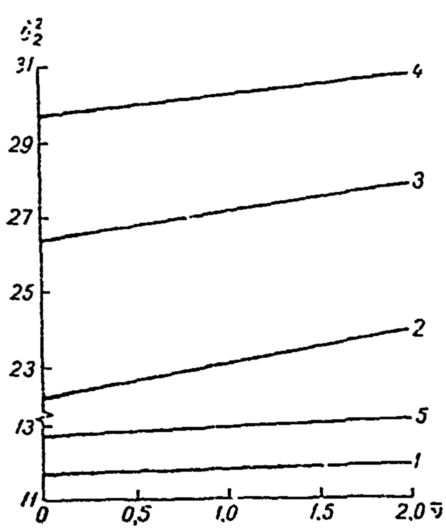

Fig. 4: δ_2^2 vs. current density $\bar{\nu}$ (θ_0=0.6)

See Fig. 1 for legend.

In those cases when the parameters Bi_i and η_1 assume limiting values, close to zero or fairly large, it is casy to determine analytically the corresponding values of δ_K and A_K.

Thus, when $\eta_1 \to 0,\ \nu + Bi_1 \to 0,\ K_п = 0$

$$\delta_к = \frac{\pi}{2}(2k-1), \quad A_к = 8\,\frac{\pi(2k-1)\nu\theta_0 + 2(-1)^k \nu^2}{(2k-1)^3 \pi^3},$$

when $\nu + Bi_1 \gg 1,\ K_п = 0$

$$\delta_к = k\pi, \quad A_к = \frac{2\nu^2\,[(-1)^k - 1]}{\pi^2 k^2 (\nu + Bi_1)},$$

when $\eta_1 \gg 1,\ K_п = 0$

$$\delta_1 = \left[\frac{1 + \nu + Bi_1}{\eta_1}\right]^{\frac{1}{2}}, \quad \delta_к = (k-1)\pi \quad (k = 2, 3, \ldots,),$$

$$A_1 = \frac{\nu\theta_0 - 0{,}5\nu^2}{1 + \nu + Bi_1}, \quad A_к = \frac{2\nu^2\,[1 + (-1)^k]}{\pi^4 (k-1)^4 \eta_1} \quad (k = 2, 3, \ldots,).$$

Correspondingly, temperature changes at sufficiently high values of Fo are determined by the following relationships:

when $\eta_1 \to 0,\ \nu + Bi_1 \to 0,\ K_п = 0$

$$\Delta\theta = \Delta\theta_{стац} - 8\,\frac{\pi\nu\theta_0 - 2\nu^2}{\pi^3} \exp\left(-\frac{\pi^2}{4}\,Fo\right). \tag{11}$$

when $\nu + Bi_1 \gg 1,\ K_п = 0$

$$\Delta\theta = \Delta\theta_{стац} + \frac{4\nu^2}{\pi^2(\nu + Bi_1)} \exp(-\pi^2 Fo), \tag{12}$$

when $\eta_1 \gg 1$

$$\Delta\theta = \Delta\theta_{стац}\left[1 - \exp\left(-\frac{1 + \nu + Bi_1}{\eta_1}\,Fo\right)\right]. \tag{13}$$

The temperature of the cold junction in the transition mode of operation, depending on the current strength, can approach equilibrium from the sides of large or small values, that is, monotonically or passing through a minimum. The condition fixing the limit for these two cases is the equality of coefficient A_1 with zero:

$$\delta_1\left(\theta_0 - \frac{K_п}{\nu}\right)\sin\delta_1 - \nu(1 - \cos\delta_1) = 0. \tag{14}$$

Determination of the current ν_0 which satisfies condition (14) should be accomplished numerically together with the solution of Eq. (6). It is interesting to compare the values of ν_0 obtained here with the optimum ones in the steady-state mode of operation. Data of the calculation of ν_0 for a series of values of the parameters η_1 and Bi_1 (at θ_0=0.6 and K_H=0) are presented in Table 2. When $K_H \to 0$, the value of ν_0 can be found analytically,

$$\nu_0 = \frac{\theta_0 \arccos\left(-\frac{\theta_0}{1+\theta_0}\right)}{\sqrt{1+2\theta_0}}. \tag{15}$$

Since $\nu^0_T = \sqrt{1+2\theta_0} - 1$, then

$$\frac{\nu_0}{\nu^0_T} = \frac{1}{2}\left(1 + \frac{1}{\sqrt{1+2\theta_0}}\right)\arccos\left(-\frac{\theta_0}{1+\theta_0}\right).$$

At any θ_0, the latter ratio exceeds $\frac{\pi}{2}$, that is, it is greater than unity. An increase in the thermocapacitive load η_1 favors an increase in current ν_0, but heat transfer Bi_1 decreases the value of ν_0 (Table 2). In the general case, the ratio $\frac{\nu_0}{\nu^0_T}$ can be both larger and smaller than unity. Thereby, the extreme on the curve of the function θ(Fo), arising at $\nu=\nu_0$, can appear at current values lower than the optimum (at fairly large values of Bi_1) and in excess of the optimum in the steady-state mode of operation. Since $\delta_2 > \delta_1$, the mode of operation in transition from a monotonic temperature drop with time to a temperature change with the extreme, is the most "rapid." Current density ν_0 corresponds to the shortest time for the transition process, and the $\nu=\nu_0$ mode of operation is advisable in conditions when the current ν_0 provides a sufficient lowering of the temperature in the steady-state condition.

Table 2: Relationship between current ν_0 and ν^0_T at various thermal loads on the cold junctions

η_1	0	0,1	0,5	1	∞	0	0	0
Bi_1	0	0	0	0	0	0	1	5
ν^0_T	0,483	0,483	0,483	0,483	0,483	0,483	0,530	0,573
ν_0	0,79	0,83	0,97	1,05	1,20	0,79	0,66	0,37
$\bar{\nu} = \frac{\nu_0}{\nu^0_T}$	1,64	1,74	2,00	2,17	2,48	1,64	1,25	0,65

η_1	0	0	0	0,1	0,5	1	[illegible]
Bi_1	10	∞	5	5	5	5	5
ν^0_T	0,584	0,600	0,573	0,573	0,573	0,573	0,573
ν_0	0,24	0	0,37	0,41	0,55	0,72	1,20
$\bar{\nu} = \frac{\nu_0}{\nu^0_T}$	0,41	0	0,65	0,71	0,97	1,25	2,09

In solving the termal conductivity Eq. (1a) in form (5), a rapid convergence is observed for sufficiently high temporal values of Fo. For plotting the transition process at small temporal values of Fo, the solution of Eq. (1a) should be used in the form of an exponential series Fo. From formula (4), the expansion of $\theta(p)$ by exponents $p^{-1/2}$ can be obtained:

$$\bar{\theta}(p) = \frac{\theta_0}{p} - \frac{\nu\theta_0 - K_н}{\eta_1}\frac{1}{p^2} + \frac{\eta_1\nu^2 + \nu\theta_0 - K_н}{\eta_1^2 p^{\frac{5}{2}}} -$$
$$- \frac{(\nu\theta_0 - K_н)\left[1 - \eta_1(\nu + Bi_1)\right] + \eta_1\nu^2}{\eta_1^3 p^3} +$$
$$+ \frac{(\nu\theta_0 - K_н)\left[1 - 2\eta_1(\nu + Bi_1)\right] + \eta_1\nu^2\left[1 - \eta_1(\nu + Bi_1)\right]}{\eta_1^4 p^{\frac{7}{2}}} - \ldots,$$

which, in the temporal domain, corresponds to the series

$$\theta(Fo) = \theta_0 - \frac{\nu\theta_0 - K_н}{\eta_1}Fo + \frac{4}{3\sqrt{\pi}}\frac{\nu\theta_0 - K_н + \eta_1\nu^2}{\eta_1^2}Fo^{\frac{3}{2}} -$$
$$- \frac{(\nu\theta_0 - K_н)\left[1 - \eta_1(\nu + Bi_1)\right] + \eta_1\nu^2}{2\eta_1^3}Fo^2 + \frac{8}{15\sqrt{\pi}} \times$$
$$\times \frac{(\nu\theta_0 - K_н)\left[1 - 2\eta_1(\nu + Bi_1)\right] + \eta_1\nu^2\left[1 - \eta_1(\nu + Bi_1)\right]}{\eta_1^4}Fo^{\frac{5}{2}} + \ldots \tag{16}$$

Similarly, the expansion of $\theta(Fo)$ by powers of Fo can be found for the case of a negligible heat capacity η_1:

$$\theta(Fo) = \theta_0 - \frac{2}{\sqrt{\pi}}(\nu\theta_0 - K_н)\sqrt{Fo} + \left[(\nu\theta_0 - K_н)(\nu + Bi_1) +\right.$$
$$\left. + \nu^2\right]Fo - \frac{4}{3\sqrt{\pi}}\left[(\nu\theta_0 - K_н)(\nu + Bi_1) + \nu^2\right](\nu + Bi_1)Fo^{\frac{3}{2}} +$$
$$+ \frac{1}{2}\left[(\nu\theta_0 - K_н)(\nu + Bi_1) + \nu^2\right](\nu + Bi_1)^2 Fo^2 - \ldots \tag{17}$$

Comparison of formulas (16) and (17) shows that the presence of a thermal capacitance on the cold junction leads to a slowing down of the rate of cooling in the initial period. The first two terms of expansion (17) coincide with the corresponding terms of the expansion for the case of cooling of the plate by a constant output source, equal to $\nu\theta_0 - K_H$ [9].

To evaluate the accuracy of calculations of temperature by the analytic formulas, using the first terms of the corresponding series, the data of the numerical solution of Eq. (1a) can be used. A graph of change in temperature drop between battery junctions with time for various values of the ratio $\bar{\nu} = \frac{\nu}{\nu_T^0}$ an.' the parameters η_1 and Bi_1 (θ_0=0.6, K_H=0) is presented in Fig. 5. In the case η_1=0, Bi_1=0 (Fig. 5a), large current values correspond to the curves with large angles of ascent at the coordinate origin. Curves for currents less than ν_0 (for the case under consideration, $\nu_0=1.64\nu_T^0$) have a monotonic character and do not intersect when $\nu<\nu_T^0$, but curves corresponding to currents exceeding the value of ν_T^0 begin to intersect the preceding ones. When the current exceeds the value ν_0, the temperature trend has an extreme. Curve 5 ($\nu=1.5\nu_T^0$ and near to the value of ν_0) has the greatest rate of establishment of the steady-state fall for the relationships presented.

The graph in Fig. 5b (η_1=1, Bi_1=0) makes it possible to follow the effect of thermocapacitive loading on the course of cooling. An extreme is not observed in the curve $\Delta\theta$=f(Fo), even at $\bar{\nu}$ 2 (for this case, $\nu_0=2.17\nu_T^0$. The fastest rate of establishing the steady-state condition among the curves presented corresponds to $\bar{\nu}$=2.

For greater values of the heat transfer coefficient (Fig. 5c, Bi_1=5, η_1=0), the value of $\bar{\nu}$ at which an extreme temperature trend appears is less than unity, and the curve $\Delta\theta$(Fo) for $\nu=\nu_T^0$ has an extreme. The temperature drop at the extreme points of the curve $\Delta\theta$(Fo) exceeds the steady-state one.

The case of Bi_1=5 and η_1=1 (Fig. 5d) is characterized by mutual compensation of the effects of heat transfer and thermocapacitive load, as a result of which the extreme on the curves arises at $\bar{\nu}$=1.25

The data from calculation of function $\Delta\theta$(Fo) according to formula (5) are noted in circles in Fig. 5a, b, and c, and by formulas (16) or (17), by triangles. A comparison with curves plotted according to the results of calculation of function $\Delta\theta$(Fo) by numerical methods on a digital computer shows that, for Fo $\in$ (0; 0.05), good precision of the approximation is insured by three terms of series (8) or (9), and at Fo>0.04, sufficient accuracy is achieved with the calculation of two terms of series (5).

The formulas obtained above for calculation of the cooling processes can be used in discussion of the transition processes in the case of heating (if the temperature of the cold junction is held constant here), substituting $-\nu$ for the value of ν in the corresponding formulas. The parameters δ_K, correspondingly, will be the roots of the equation

$$\operatorname{tg}\delta = \frac{\delta}{\eta_1\delta^2 - Bi_1}.$$

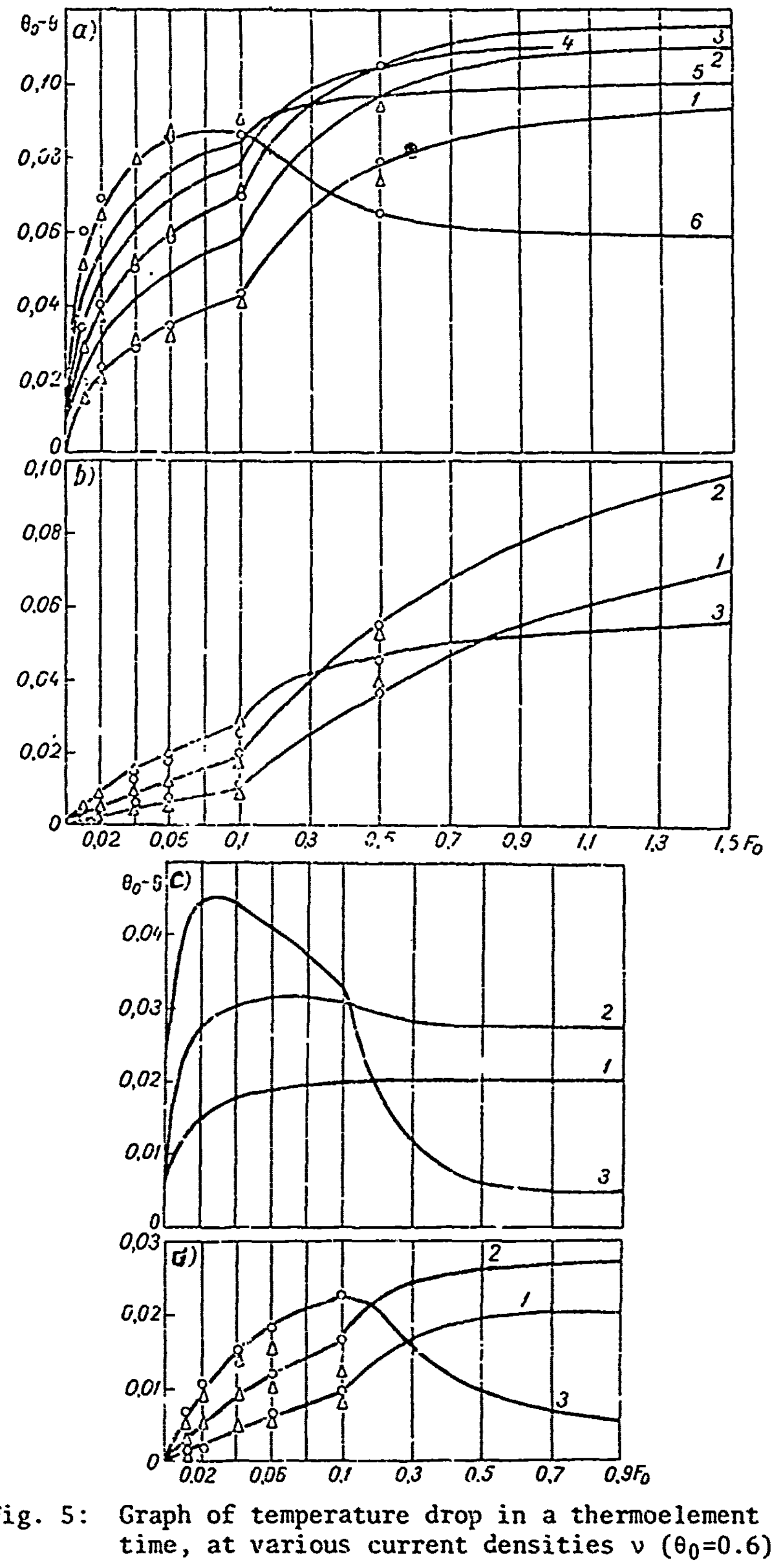

Fig. 5: Graph of temperature drop in a thermoelement vs. time, at various current densities ν (θ_0=0.6)

Fig. 5: Continued

Key:

a. $\eta_1=0$, $Bi_1=0$
1. $\bar{\nu}=0.5$
2. $\bar{\nu}=0.75$
3. $\bar{\nu}=1.0$
4. $\bar{\nu}=1.25$
5. $\bar{\nu}=1.5$
6. $\bar{\nu}=2.0$

b. $\eta_1=1$, $Bi_1=0$
1. $\bar{\nu}=0.5$
2. $\bar{\nu}=1.0$
3. $\bar{\nu}=1.5$

c. $\eta_1=0$, $Bi_1=5$
1. $\bar{\nu}=0.5$
2. $\bar{\nu}=1.0$
3. $\bar{\nu}=1.5$

d. $\eta_1=1$, $Bi_1=5$
1. $\bar{\nu}=0.5$
2. $\bar{\nu}=1.0$
3. $\bar{\nu}=2.0$

The values of δ_K in the case of heating are lower than in cooling at the same values of ν, η_1 and Bi_1. For determination of the value of δ_1 and δ_2, the data of Table 1 can be used. The difference $Bi_1-\nu$ should be taken here as the input parameter.

Let us consider further a nonequilibrium thermal process in a thermoelement, when the temperature changes with time on the hot side, and account is taken of heat transfer from the hot side Bi_2 and heat capacitance of the hot junctions η_2. In this case, the boundary conditions for Eq. (1a) have the form ($K_H=0$):

$$\left.\frac{\partial\theta}{\partial L}\right|_{L=0} = \left[\nu\theta + \eta_1\frac{\partial\theta}{\partial(\mathrm{Fo})} + \mathrm{Bi}_1(\theta-\theta_0)\right]\Bigg|_{L=0},$$
$$\left.\frac{\partial\theta}{\partial L}\right|_{L=1} = \left[\nu\theta - \eta_2\frac{\partial\theta}{\partial(\mathrm{Fo})} - \mathrm{Bi}_2(\theta-\theta_0)\right]\Bigg|_{L=1}. \tag{18}$$

In solving the problem by the operator method, let us use the Laplace transform for the temperature of the cold junction:

$$\bar{\theta}(p) = \frac{\theta_0}{p} + \frac{\sqrt{p}\,[p(\nu\theta_0-\eta_2\nu^2)+(\nu-\mathrm{Bi}_2)\nu^2](\mathrm{ch}\sqrt{p}-1) - p\nu^2\,\mathrm{sh}\sqrt{p} - p\nu\theta_0(\nu-\mathrm{Bi}_2-\eta_2 p)\,\mathrm{sh}\sqrt{p}}{p^2\{[(\nu+\mathrm{Bi}_1+\eta_1 p)(\nu-\mathrm{Bi}_2-\eta_2 p)-p]\,\mathrm{sh}\sqrt{p} - \sqrt{p}\,[\mathrm{Bi}_1+\mathrm{Bi}_2+(\eta_1+\eta_2)p]\,\mathrm{ch}\sqrt{p}\}}. \tag{19}$$

Expression (19) contains a pole at point p=0 and poles corresponding to solutions of equation

$$[(\nu+\mathrm{Bi}_1+\eta_1 p)(\nu-\mathrm{Bi}_2-\eta_2 p)-p]\,\mathrm{sh}\sqrt{p} - \sqrt{p}\,[\mathrm{Bi}_1+\mathrm{Bi}_2+(\eta_1+\eta_2)p]\,\mathrm{ch}\sqrt{p} = 0. \tag{20}$$

If the thermoelement parameters are such that Eq. (20) has only purely imaginary radicals, converting to the provisional domain, we obtain the relationship

$$\theta(\mathrm{Fo}) = \theta_0 - \frac{(\mathrm{Bi}_2 - \nu)(\nu\theta_0 - 0.5\nu^2) - \nu^2}{(\mathrm{Bi}_1 + \mathrm{Bi}_2) + (\mathrm{Bi}_1 + \nu)(\mathrm{Bi}_2 - \nu)} +$$

$$+ 2\sum_{\kappa=1}^{\infty} \frac{\sin\delta_\kappa \left[C_{\kappa_1}(1 - \cos\delta_\kappa) - C_{\kappa_2}\delta_\kappa \sin\delta_\kappa\right]}{\delta_\kappa^2 \left[C_{\kappa_3}\delta_\kappa \sin^2\delta_\kappa - C_{\kappa_4}\delta_\kappa \cos^2\delta_\kappa + C_{\kappa_5}\sin 2\delta_\kappa\right]}, \tag{21}$$

where

$$C_{\kappa_1} = (\nu - \mathrm{Bi}_2)\nu^2 - \delta_\kappa^2(\nu\theta_0 - \eta_2\nu^2);$$

$$C_{\kappa_2} = \nu^2 + \nu\theta_0(\nu - \mathrm{Bi}_2) + \delta_\kappa^2\eta_2\nu\theta_0;$$

$$C_{\kappa_3} = 2 + (\nu + \mathrm{Bi}_1 - \eta_1\delta_\kappa^2)(2\eta_2 + 1) - (\nu - \mathrm{Bi}_2 + \eta_2\delta_\kappa^2)(2\eta_1 + 1);$$

$$C_{\kappa_4} = \mathrm{Bi}_1 + \mathrm{Bi}_2 - \delta_\kappa^2(\eta_1 + \eta_2);$$

$$C_{\kappa_5} = \delta_\kappa^2 + \frac{3}{2}\delta_\kappa^2(\eta_1 + \eta_2) - \frac{1}{2}(\mathrm{Bi}_1 - \mathrm{Bi}_2) +$$

$$+ (\nu + \mathrm{Bi}_1 - \eta_1\delta_\kappa^2)(\nu - \mathrm{Bi}_2 + \eta_2\delta_\kappa^2);$$

δ_K is the positive root of the equation

$$\operatorname{tg}\delta = \delta \frac{\mathrm{Bi}_1 + \mathrm{Bi}_2 - \delta^2(\eta_1 + \eta_2)}{\delta^2 + (\eta_1\delta^2 - \nu - \mathrm{Bi}_1)(\mathrm{Bi}_2 - \eta_2\delta^2 - \nu)}. \tag{22}$$

It is interesting to compare the transition processes defined by formulas (21) and (22) with curves which satisfy relations (5) and (6) obtained earlier. It should be noted that, as $\mathrm{Bi}_2 \to \infty$ or $\eta_2 \to \infty$, the solution of (21) and Eq. (22) approach the limits of (5) and (6).

The difference in the steady-state values of the temperatures found by (21) and (5) is equal to

$$\delta\theta = \frac{0.5\nu^2 + \nu\theta_0(\mathrm{Bi}_1 + \nu)}{(1 + \nu + \mathrm{Bi}_1)\left[\mathrm{Bi}_1 + \mathrm{Bi}_2 + (\nu + \mathrm{Bi}_1)(\mathrm{Bi}_2 - \nu)\right]}$$

and is always positive when the inequality

$$\nu^2 + \nu(\mathrm{Bi}_1 - \mathrm{Bi}_2) - (\mathrm{Bi}_1 + \mathrm{Bi}_2 + \mathrm{Bi}_1\mathrm{Bi}_2) < 0 \tag{23}$$

is observed.

It can be shown that condition (23) is necessary for the limiting solution of Eq. (1a) in limiting conditions (18). Consequently, in accomplishing limitations on the current feed (23), the finiteness of the magnitude of heat transfer leads to a reduction in the effectiveness of thermobatteries in the steady-state mode of operations in comparison with the case when $\mathrm{Bi}_2 \to \infty$.

To evaluate the effect of values of Bi_2 and η_2 on the rate of the transition processes, the values of the roots of Eq. (22) and (5) should be compared. At sufficiently high values of Bi_2 and η_2 it turns out that increase in value of heat transfer Bi_2 from the hot junctions leads to an increase in the radical δ_K, that is, a decrease in the time for the transition processes. An increase in the magnitude of the thermocapacitance η_2 effects the magnitude of δ_K similarly under the condition

$\sqrt{\frac{\nu + Bi_1}{\eta_1}} < \frac{\pi}{2}$ and has the opposite effect when $\sqrt{\frac{\nu + Bi_1}{\eta_1}} > \frac{\pi}{2}$.

In the general case, the expression for a nonequilibrium temperature trend with a multistage current can be presented only in the form of a series; this circumstance strongly hampers its analysis. In particular, the question of the relationship of the extreme value of the temperature drop in a nonequilibrium cooling process and the current supply strength, and the relationship between this value and the maximum temperature drop in a steady-state mode of operation remains unexplained.

In order to obtain the expression $\Delta\theta$ as a function of time, it is convenient to use the model of a thermoelement with an infinite junction length. Here, it should be taken into account that, in a sufficiently short duration of the cooling process after switching on the current, the temperature field at the hot junction still has not begun to affect the magnitude of the temperature of the cold junction. As long as the Fourier number $Fo = \frac{at}{d^2}$ is much less than unity, the temperature trend on the cold junction does not depend on the length of the thermoelement junction. Therefore, it can be considered infinite. In this case, the boundary conditions of the heat conductivity equation (1)-(6) are:
at x=0

$$\lambda \frac{\partial T}{\partial x} = ejT - \alpha_1 (T_0 - T) + g_1 \frac{\partial T}{\partial t} - j^2 \rho_K - q_n, \qquad (24)$$

at $x \to \infty$

$$\frac{dT}{dx} = 0. \qquad (25)$$

The boundary condition on the cold junction takes into account the Peltier effect, heat transfer to the surrounding medium, heat capacity of the cooling body, the Joule effect, separable at the contacts, and the output of the constant heat source.

First of all let us consider the case when the cold junction is isolated from the surrounding medium [10]; then at x=0

$$\lambda \frac{\partial T}{\partial x} = ejT.$$

Let us reduce Eq. (1) to dimensionless form, during which we use the relation $d_э=\frac{\lambda}{ej}$ as the equivalent length. Here,

$$\frac{\partial^2\theta}{\partial\chi^2}+1=\frac{\partial\theta}{\partial(\mathrm{Fo}_э)}, \qquad (26)$$

$$\theta\big|_{\mathrm{Fo}_э=0}=\theta_0. \qquad (27)$$

$$\left.\begin{aligned} &\text{when } \chi=0 \quad \frac{\partial\theta}{\partial\chi}=\theta,\\ &\text{when } \chi\to\infty \quad \frac{\partial\theta}{\partial\chi}=0. \end{aligned}\right\} \qquad (28)$$

Here $\chi=\frac{x}{d_э}=\frac{ejx}{\lambda}$; $\mathrm{Fo}_э=\frac{at}{d_э^2}=\frac{e^2j^2at}{\lambda^2}$.

Eq. (26) is easily solved by the operator method. As a result of temperature change of the cold junction with time, the following is obtained:

$$\Delta\theta(\mathrm{Fo})=(1+\theta_0)(1-\exp\mathrm{Fo}_э\,\mathrm{erfc}\sqrt{\mathrm{Fo}_э})-2\sqrt{\frac{\mathrm{Fo}_э}{\pi}}. \qquad (29)$$

In this manner, temperature change of the cold junction depends only on parameter $\mathrm{Fo}_э$, that is, on the product j^2t. Expression (29) has a maximum at a specific value of the argument $\mathrm{Fo}_э=\mathrm{Fo}_э^*$ determined by the equation

$$\sqrt{\pi\mathrm{Fo}_э^*}\,\exp\mathrm{Fo}_э^*\,\mathrm{erfc}\sqrt{\mathrm{Fo}_э^*}=\frac{\theta_0}{1+\theta_0}. \qquad (30)$$

Thus, at θ_0=0.6 the value of $\mathrm{Fo}_э^*$=0.081 and $\Delta\theta_{max}$=0.0895. An increase in current j causes a corresponding decrease in the time of arival of the maximum, and vice versa. The value of the maximum $\Delta\theta$ does not depend on the current. The greatest temperature drop on the element in the steady-state mode of operation is

$$\Delta\theta'_{макс}=1+\theta_0-\sqrt{1+2\theta_0}\text{, when } \theta_0=0.6 \quad \Delta\theta'_{макс}=0.117.$$

The maximum temperature drop in the nonequilibrium mode of operation is less than the maximum temperature drop under steady-state conditions. Thus, at θ_0=0.6, the ratio between these values equals 0.766. The data shown are correct for a thermoelement with finite junction length d if the Fourier number $\frac{at}{d^2} \ll 1$. The maximum temperature drop on cold junctions for a semi-infinite element approaches at $Fo_э = Fo^*_э = \frac{e^2 j^2 a t^*}{\lambda^2}$, that is, at $\frac{at^*}{d^2} = \left(\frac{\lambda}{ejd}\right)^2 Fo^*_э$.

In this manner, condition $\frac{at}{d^2} \ll 1$ leads to the inequality $\nu \gg \sqrt{Fo^*_э}$. As was shown above [Table 2, formula (15)], the minimum current value ν_0 at which an extreme appears amounts to 0.791, and $\sqrt{Fo^*} = 0{,}28$.

Beginning at the value $\nu=\nu_0$, the values of the temperature drop at the extreme point, considered for models with junctions of finite and semi-infinite lengths, practically coincide. The trend of temperature change $\Delta\theta_{стац}$ vs. ν (curve 1) and the values of $\Delta\theta$ at the extreme point of the function $\Delta\theta(Fo)$ (curve 2) for a thermoelement with a finite junction length are shown in Fig. 6. At $\nu=\nu_0$, the maximum appears as $t\to\infty$; with increase in ν, the value of Fo^* decreases, and as $\nu\to\infty$, $Fo^*\to 0$.

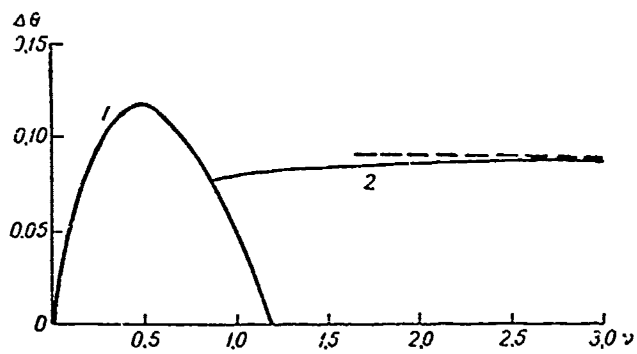

Fig. 6: Temperature drop on a thermoelememt with thermally isolated cold junction vs. current density (θ_0=0.6)

Key: 1. In the steady-state mode of operation
2. Maximum in time

For the general case, when the limiting condition on the cold junction is determined by relationship (24), it also is not difficult to obtain an analytical expression, giving $\Delta\theta$ vs. time. However, here, $\Delta\theta$ already is not only a function of dimensionless parameters θ_0 and Fo, but of the dimensionless complexes

$$\mathrm{Bi}_э = \frac{\alpha d_э}{\lambda} = \frac{\alpha}{ej}; \quad \eta_1 = \frac{g_1}{cd_э} = \frac{g_1 ej}{c\lambda};$$

$$\xi_к = \frac{\rho_к}{\rho d_э} = \frac{\rho_к ej}{\rho\lambda}; \quad K_н = \frac{q_н z d_э}{\lambda} = \frac{q_н z}{ej}.$$

The effect of thermoresistances on a junction decreases with increase in current, and the effect of heat capacity and electrical resistance increases.

Fig. 7 presents $\Delta\theta_{стац}$ (curve 1) and $\Delta\theta$ at the maximum point (curve 2) vs. current ν, for the case $Bi_1=5$ ($\eta_1=0$, $\xi_K=0$, $K_H=0$, $\theta_0=0.6$). The minimum current value ν_0 at which a maximum appears amounts to 0.37, and here $\nu_0<\nu_T^0$. In the case being considered, the temperature drop at the maximum point is a little more than at the steady-state. As $\nu\to\infty$, the value of $\Delta\theta$ coincides with the temperature drop at Bi=0. Curve 2 is plotted from data calculated on a digital computer; the results of the calculation according to the formula for a thermoelement with a semi-infinite junction gives practically identical results, starting with $\nu\approx 1$. The presence of thermocapacitance and contact resistance, as has already been demonstrated, comes down to the fact that the value of $\Delta\theta$ at the maximum point falls with increase in current. In the case of the presence of resistance, the extreme occurs only at sufficiently small currents when $\xi_K<\theta_0$ or $j<\frac{\rho\lambda\theta_0}{\rho_K e}$,

when the magnitude of the Peltier heat exceeds the Joule effect on the cold junction contacts at the initial moment of time.

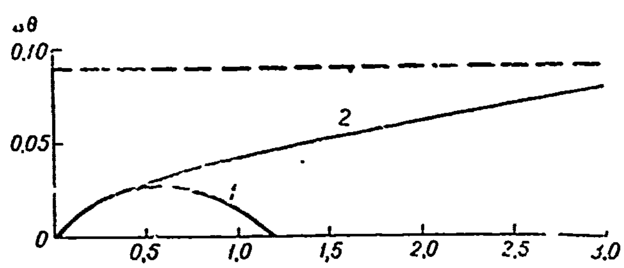

Fig. 7: Temperature drop on a thermoelement vs. current density, taking heat transfer on the cold junction into account ($\theta_0=0.6$, $Bi_1=5$)

Key: 1. In the steady-state mode of operation
2. Maximum in time

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