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Theory of Turbulent Plasma Heating by Anomalous Absorption of Magnetosonic Waves

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13. ABSTRACT

Theory and computer simulation experiments have shown the potential importance of the fluid-like, modified two stream instability as an ion heating mechanism in low β plasmas. In order to take advantage of the strong ion heating which it can produce, we consider inducing the instability with large amplitude waves in order to convert the energy in the wave fields into ion thermal energy. Since the instability is driven by relative electron-ion drifts across a magnetic field $\underline{B}_o$, and has a frequency comparable to the lower hybrid frequency ω_{LH}, a natural choice for the driver wave is a magnetosonic wave which propagates across $\underline{B}_o$ near the Alfvén speed. As a specific example, we demonstrate that for the parameters of the Princeton ST-Tokamak it is theoretically possible to achieve thermonuclear temperatures by anomalously absorbing these waves via the modified two stream instability.

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PROBLEM STATUS

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I. INTRODUCTION:

In the past several years, the topic of plasma instabilities driven by relative drifts of electrons and ions perpendicular to magnetic fields has received considerable attention. Interest in this area of plasma physics is due to the probable importance of these instabilities in plasma applications; e.g., shock wave applications and turbulent heating experiments. The two most extensively treated instabilities of this class are the beam cyclotron instability [1-9] and the modified two stream instability [10-16]. The beam cyclotron instability [1-9] is a short wavelength, micro-instability of the electron cyclotron modes which mainly heats electrons and is now thought to be the anomalous resistance mechanism in subcritical, perpendicular shock waves in which $T_i < T_e$, $U < v_e$ and $\Omega_e \ll \omega_{pe}$. Here U, v_e, Ω_e and ω_{pe} are the electron-ion relative drift speed, electron thermal speed, electron cyclotron frequency and electron plasma frequency respectively. The condition $T_i < T_e$ is imposed since many of the gross features of the instability for $\Omega_e \ll \omega_{pe}$ are similar to those of ion sound [7-9]. In particular $T_i < T_e$ is necessary to overcome the stabilizing influence of ion Landau damping.

The modified two stream instability [10-16] is a longer wavelength ($k\ v_e/\Omega_e \ll 1$ as opposed to $k\ v_e/\Omega_e \gg 1$ for the beam cyclotron instability) instability which is basically nonresonant and thus operates even for $T_i > T_e$. This instability is important since it can lead to very large ion heating and thus is of interest from the standpoint of controlled thermonuclear fusion. In situations where the hardier modified two stream instability is present with other micro-instabilities such as the beam cyclotron and ion acoustic instabilities, it is the modified two stream instability that determines the final state of the system [14] since it stabilizes at a higher level of turbulence. Furthermore, for $\Omega_e \gtrsim \omega_{pe}$, as is the case in many low β fusion experiments, the growth rate of the beam cyclotron instability is very small.

Another cross-field instability can occur in plasmas with more than one species of ion. Since the ion species may have different charge to mass ratios they respond differently to large amplitude driving waves and can develop relative streaming with respect to each other. This streaming can then drive an ion-ion instability.

In this paper we will be concerned with ion heating for fusion applications. Of the possible instabilities, the two most likely to provide the most efficient ion heating are the two-ion species instability [17,19] in the case of a plasma with two ion components (e.g. a deuterium-tritium plasma) and the modified two stream instability. These instabilities operate on the same time scale and in some situations can be expected to coexist with one another. We note however, that for driving wave frequencies appreciably larger than the ion gyro-frequency, the modified two stream instability is possible at lower amplitudes of the driver wave than is the ion-ion instability. Here we direct our attention to heating by the modified two stream instability. This will be the more widely applicable ion heating mechanism since it is an electron-ion instability and does not require two ion species. Further, in the case of two ion species it will be a major factor in the system's development.

We note that if the driver wave has a frequency very close to the lower hybrid frequency then the modified two stream instability and the ion-ion instability become parametric type instabilities [17-19] with properties which are different from the quasi-steady case in which ω_0 is appreciably less than ω_{LH}. In this paper we will be interested in the quasi-steady case.

In Section II we briefly review the main points of the theory of the modified two stream instability and the conditions under which it is present.

In Sec. III new computer simulation results using an electromagnetic particle simulation code are presented which confirm the theory with respect to the conditions necessary for a strong instability capable of large ion heating.

In Section IV we discuss a means of inducing the instability to take advantage of the desirable property of strong ion heating. Specifically, we consider launching large amplitude, magnetosonic waves with frequency less than the electron-ion lower hybrid frequency. The electrons will respond to the wave fields by moving with their E x B drift velocity in such a situation while the more massive ions will respond differently. This will produce an oscillating relative drift between the electrons and ions across the magnetic field and under appropriate conditions, which we give, the modified two stream instability can develop and lead to anomalous absorption of the magnetosonic waves. A specific application of this method of ion turbulent heating is given for plasmas with the parameters of Princeton's ST-Tokamak. The results show that such a scheme for heating plasmas may provide temperatures sufficient for a controlled thermonuclear reaction.

Section V concludes the paper with a short summary of the results and their implications.

II. Theory of the Modified Two Stream Instability:

In previous publications [13-15] we have presented a detailed theory of the modified two stream instability in conjunction with extensive electrostatic computer simulations which confirm its importance as an ion turbulent heating mechanism. Here we will briefly review the main points of the theory.

The important wave modes associated with the modified two stream instability are basically non-resonant, fluid-like modes with $k\,\rho_i >> 1 >> k\,\rho_e$, where $\rho_{i,e}$ is the ion, electron cyclotron radius. The wave frequency ω obeys $\Omega_i << \omega << \Omega_e$, and thus the ions are effectively unmagnetized while the electrons are tightly bound to the magnetic field lines. Indeed, the most important modes have $\mathrm{Re}(\omega) \sim \mathrm{Im}(\omega) \sim \omega_{pi}\Big/\left(1 + \frac{\omega_{pe}^2}{\Omega_e^2}\right)^{1/2} = \omega_{LH}$, where ω_{LH} is the lower hybrid frequency. Also, for the most important waves $k_{\parallel}/k \sim (m_e/m_i)^{1/2}$, $k_{\parallel}$ being the component of the wave vector parallel to $\underline{B}_o$. The fact that the electrons are free to accelerate only along the magnetic field causes them to behave as though they had an effective mass $\overline{m}_e \gg m_e$. This can be seen from the fluid limit of the electrostatic dispersion relation (we discuss electromagnetic effects below) in the electron frame of reference [10-12]

$$1 + \omega_{pe}^2/\Omega_e^2 - \frac{\omega_{pi}^2}{(\omega-\underline{k}.\underline{U})^2} - \frac{k_\parallel^2}{k^2}\,\frac{\omega_{pe}^2}{\omega^2} = 0. \qquad (1)$$

From Eq. (1) we see that $\overline{m}_e = (k^2/k_\parallel^2)\, m_e$, which implies $\overline{m}_e \sim m_i$ for the most important modes. Note that Equation (1) has the same form as the dispersion relation for the usual Buneman two stream instability. However, the threshold for the modified two stream instability is $U > v_i$ rather than $U > v_e$. While Eq. (1) predicts an increasing growth rate with increasing $k_\parallel/k$, the inclusion of thermal effects causes the maximum growth rate to occur for $k_\parallel/k \sim (m_e/m_i)^{1/2}$. Thus since $\overline{m}_e \sim m_i$ for the fastest growing waves, we might expect the ion heating to be comparable to the electron heating. The ions will be heated nearly perpendicular to $\underline{B}_o$ since the waves propagate mainly across $\underline{B}_o$, while the electrons can be expected to heat parallel to B_o as this is the direction in which they can move freely.

Electromagnetic effects cause Eq. (1) to be modified to [15]

$$\Omega_a^2/(\omega-\underline{k}.\underline{U})^2 + \Omega_b^2/\omega^2 = 1 \qquad (2)$$

where $\Omega_a^2 = \omega_{pi}^2/\Delta$, $\Omega_b^2 = (k_\parallel^2/k^2)\,\omega_{pe}^2/[\Delta(1 + \omega_{pe}^2/k^2c^2)]$, $\Delta = 1 + (\omega_{pe}^2/\Omega_e^2) . [1 + (1 + \beta_e)^{-1}\,\omega_{pe}^2/k^2c^2]$, and $\beta_e = 8\pi\, N\, T_e/B_o^2$ is the electron beta. Equation (2) has the form of a two stream instability dispersion relation and reduces to Eq. (1) in the limit $c \to \infty$. However, the extra wavenumber dependences introduced by the electromagnetic terms cause stabilization of the modes in certain regions of parameter space. The condition for a strong instability capable of producing large ion heating is [15]

$$1 < U/v_i \gtrsim [\, 2(\, 1 + \beta_e)/\beta_i]^{1/2}, \qquad (3)$$

where $\beta_i = 8\pi\, N\, T_i/B_o^{\,2}$ is the ion beta.

The first condition $U/v_i > 1$ is necessary to overcome ion Landau damping while the other condition is required to overcome the stabilizing electromagnetic effects. Equation (3) is an extremely important condition in considerations of plasma heating by inducing the modified two stream instability with large amplitude waves since, as we shall see, it puts constraints on the amplitude, frequency and wavelength of the driving waves. As can be seen from Eq. (3), this instability has a large range of operation, and therefore can be a strong heating mechanism only for low β plasmas.

The results of our electrostatic computer simulations [13,14] show that the dominant nonlinear effect is particle trapping in the potential troughs of the unstable wave fields. Indeed this is the mechanism responsible for stabilization of the modes, and is responsible for the large plasma heating observed in the simulations. Theory shows that the wave potential energy necessary to begin trapping the electrons is approximately the same as that required to begin trapping the ions. Thus, if the wave fields do not damp appreciably on a time scale associated with the slower of the electron or ion trapping times, both species will eventually trap and heat regardless of which species has the faster trapping time and results in stabilization of the wave fields. This is borne out by our computer simulations. Solutions to the dispersion relation show that the most important waves have phase velocity $V_p \sim U/2$. The trapping width at saturation V_{TR} for the ions perpendicular to $\underline{B}_o$ is approximately the same as that for trapping the electrons parallel to $\underline{B}_o$ and $V_{TR} \sim U/2$. Therefore, trapping arguments predict a final state in which

$$(T_{i\perp}/m_i)^{1/2} \approx (T_{e\parallel}/m_i)^{1/2} \sim U/2, \qquad (4)$$

where U is the initial electron-ion relative drift speed. If initially the drift speed is large compared with the ion thermal speed, Eq. (4) predicts very strong ion heating. The results of the above theory are in good agreement with our electrostatic computer simulation experiments.

III. Electromagnetic Computer Simulation of the Modified Two Stream Instability:

The condition given by Eq. (3) for a strong modified two stream instability capable of producing large ion heating is crucial in considerations of inducing the instability by large amplitude waves. Moreover, it determines in general whether the instability can occur in a given situation. For this reason, we have performed new computer simulations using an electromagnetic particle simulation code [20] to test this very important aspect of the theory. The results of these simulations are presented in Figs. 1 and 2. In both runs spatial variations were allowed only in the x-direction and the ions and electrons were treated electromagnetically. However, the constant magnetic field was not applied to the ions. This is justified since the waves of interest have $\omega >> \Omega_i$ and $k\rho_i >> 1$. The ions were given a drift relative to the electrons, and the initial parameters were $T_e = T_i$, $U/c = 1/10$, $v_e/c = 1/5$, $v_i/c = 1/50$, $m_e/m_i = 10^{-2}$ and $k_\parallel/k = (m_e/m_i)^{1/2}$ (i.e. $\overline{m}_e = m_i$), system size = 256 cells, λ_D = 1 cell, c/ω_{pe} = 5 cells, and 20,480 particles of each species were used. Here, c is the speed of light and λ_D is the electron Debye length. The two runs differred only in the value of Ω_e/ω_{pe}. Run #1, Figs. 1a and 2a, had $\Omega_e/\omega_{pe} = 2$, and Run #2, Figs. 1b and 2b, had $\Omega_e/\omega_{pe} = 1/3$. The run with $\Omega_e/\omega_{pe} = 2$

corresponds to $U/V_A = 1/2$, where V_A is the Alfvén speed, and Eq. (3) predicts an instability $[V_A = (2/\beta_i)^{1/2}\ v_i]$. Note the exponential growth of energy in the unstable wave fields, Fig. 1a, accompanied by strong electron and ion heating, Fig. 2a. The electrons heat along B_o and the ions heat comparably perpendicular to B_o in agreement with theory. The characteristics of this simulation are similar to those of our electrostatic simulations reported previously [13,14]. Most importantly the final state is in good agreement with the prediction of Eq. (4).

The second run, with $\Omega_e/\omega_{pe} = 1/3$, corresponding to $U/V_A = 3$ shows no signs of instability as predicted by Eq. (3). Note that the instability of Fig. 1a is fully developed by time $t = 250\ \omega_{pe}^{-1}$, while the stable situation has been run out to $t = 1000\ \omega_{pe}^{-1}$. Thus, we conclude from the computer simulations that Eq. (3) must be satisfied for a modified two stream instability, and that when Eq. (3) is satisfied, large ion heating can follow the onset of instability leading to the final state of Eq. (4).

IV. Turbulent Heating by Anomalous Absorption of Magnetosonic Waves:

In this section we discuss a mechanism for inducing the modified two stream instability to take advantage of the large ion heating it can produce for fusion purposes. Specifically, we consider propagating large amplitude waves in the plasma perpendicular to the magnetic field to drive cross field currents. A natural choice for the driver wave is a large amplitude, magnetosonic wave with phase and group velocity comparable to the Alfvén speed V_A and frequency ω_o satisfying $\omega_o < \omega_{LH} \ll \Omega_e$. These waves are governed by the following dispersion relation

$$\omega_o = k_o V_A (1 - \omega_o^2/\omega_{LH}^2)^{1/2}, \qquad (5)$$

where k_o is the wavenumber of the wave. We see that for $\omega_o^2 \ll \omega_{LH}^2$, the waves are approximately dispersionless and $\omega_o \approx k_o V_A$. We will be interested here in frequencies $\omega_o^2 \ll \omega_{LH}^2$ so that the currents driving the modified two stream instability are reasonably steady on the instability growth time. Thus we expect the theory and simulations we have given here and in Refs. 13-15, where we have assumed d.c. drifts, to be applicable. The alternating magnetic field of the large amplitude, magnetosonic wave will create cross magnetic field currents. The electrons will respond to the wave fields by moving with their $\underline{E} \times \underline{B}$ velocity, while the more massive ions will respond differently. This will produce an oscillating relative drift across $\underline{B}_o$ and under appropriate conditions, to be given, the modified two stream instability can develop and absorb energy from the driving waves. We mention here that Babykin, et al. [21] have performed turbulent heating experiments on plasmas in a magnetic mirror machine. Surrounding the contained plasma are radio frequency tank coils which set up an alternating wave magnetic field $B_\sim$ which is superimposed on the mirror field. We suggest that either a coil arrangement similar to that in Ref. 21 or some other method (e.g. microwave horns) can be used for launching large amplitude magnetosonic waves radially into low β magnetically confined plasmas (e.g. Tokamak and magnetic mirror plasmas) and that these waves should be able to cause large ion heating via the modified two stream instability. The waves must be launched so that Eq. (3) is satisfied. This puts a restriction on the amplitude and wavelength of the driving waves. The alternating magnetic field gradient in the wave is related to the current flow in the wave by Ampere's Law, $c \frac{\partial B_\sim}{\partial r} = -4\pi Ne U$. Thus we have

$$U = V_A \; (B_\sim/B_o) \; (k_o c/\omega_{pi}), \qquad (6)$$

where $B_\sim$ is the magnetic field of the radially propagating wave. Substituting Eqs. (5) and (6) into Eq. (3) and assuming $\omega_o^2 \ll \omega_{LH}^2$, we obtain

$$\frac{\Omega_i^2}{\omega_o^2} < \frac{B_\sim^2}{4\pi N T_i} \lesssim \frac{2(1+\beta_e)}{\beta_i} \frac{\Omega_i^2}{\omega_o^2} . \tag{7}$$

Equation (7) is a very important constraint on the amplitude of the driving magnetosonic wave. Two additional requirements are

$$(B_\sim/B_o)^2 \ll 1, \tag{8}$$

and

$$k_o \, r_o \gtrsim \pi, \tag{9}$$

where r_o is the plasma radius. If Eq. (8) is not satisfied, then the magnetosonic wave can be expected to perturb the confinement properties of the main magnetic field. Equation (9) simply requires that r_o be large enough to allow the wave to fit in the device. Combining Eq. (9) with Eq. (5) we find

$$\omega_o/\omega_{LH} \gtrsim \pi \, (1 + \Omega_e^2/\omega_{pe}^2)^{1/2}/(r_o \, \omega_{pe}/c). \tag{10}$$

Equations (7), (8) and (10) put requirements on $B_\sim$ and ω_o which must be satisfied in order that the modified two stream instability be a feasible method of ion heating.

As an illustration, we apply Eqs. (7), (8) and (10) to some typical parameters for the Princeton ST-Tokamak [22]: $r_o = 7$ cm , $B_o = 40$ Kg, $N = 2 \times 10^{13}$ cm^{-3}. Using these parameters, we wish to see if the modified two stream instability can heat the ions to fusion temperatures, $T_i \simeq 10$ KeV. We therefore use $T_i = 10$ KeV (i.e. $\beta_i = 0.003$) in Eq. (7). The lower limit on Eq. (7) implies that $v_i < U$. Since our theory and computer simulations [13,14] show that the instability saturates when $v_i \sim U$, this means that if Eq. (7) is satisfied with T_i set equal to 10 KeV, then this temperature should be attained. Equation (10) gives $\omega_o/\omega_{LH} \gtrsim 0.1$ which is a fairly weak condition. Equation (7) yields

$$4 \times 10^{-5} < (B_{\sim}/B_o)^2 \lesssim 4 \times 10^{-2} \qquad (11)$$

for $\omega_{LH}^2 = 10\ \omega_o^2$. Equation (11) can be easily satisfied consistent with Eq. (8). Thus there is a wide range of choices for the wave strength $B_{\sim}$ which can satisfy Eqs. (7), (8), and (10). We, therefore, conclude that requirements (7), (8), and (10) are easily met and it should be possible to attain fusion temperatures by anomalously absorbing large amplitude magnetosonic waves by using them to drive the modified two stream instability. This expectation is reinforced by noting from Eq. (11) that the energy density in the magnetic field of the magnetosonic wave can be a sizeable fraction of the energy density of 10 KeV ions. Therefore, since the growth time is order ω_{LH}^{-1}, which is small compared with energy loss times, and the absorption length is of the order of several centimeters, we expect efficient absorption of large wave energy capable of producing thermonuclear temperatures. Similar calculations show the theoretical feasibility of using the modified two-stream instability to achieve the ignition temperature in fully scaled fusion Tokamaks.

V. Conclusion:

In this paper we have examined the possibility of heating plasmas for fusion purposes by propagating large amplitude, magnetosonic waves to induce the modified two stream instability. The condition of Eq. (3) for a strong modified two stream instability able to cause large ion heating was checked with an electromagnetic simulation code. This condition together with Amperes law and the dispersion relation for magnetosonic waves applied to low β plasma parameters typical of the Princeton ST-Tokamak show the theoretical possibility of achieving thermonuclear temperatures (this is given only as an example and the arguments also apply for fully scaled fusion Tokamaks) in this way. We hope that the discussion presented here will stimulate interest in RF heating experiments, appropriate to the topic of this paper, on low β, magnetically confined plasmas.

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REFERENCES

* On leave of absence from Cornell University, Ithaca, N. Y.

[1] H. V. WONG, Phys. Fluids 13, (1970) 757.

[2] S. P. GARY, J. J. SANDERSON, J. Plasma Phys. 4, (1970) 739.

[3] M. LAMPE, J. B. McBRIDE, J. H. ORENS, R. N. SUDAN, Phys. Letters 35A, (1971) 131.

[4] C. N. LASHMORE-DAVIES, J. Phys. A3 (1970), 140-45.

[5] D. W. FORSLUND, R. L. MORSE, C. W. NIELSON, Phys. Rev. Letters 25, (1970) 1266.

[6] D. W. FORSLUND, R. L. MORSE, C. W. NIELSON, in Plasma Physics and Controlled Thermonuclear Fusion Research (Proc. 4th Int. Conf. Madison, 1971), 2 IAEA, Vienna, (1971) 277.

[7] M. LAMPE, W. M. MANHEIMER, J. B. McBRIDE, J. H. ORENS, R. SHANNY, R. N. SUDAN, Phys. Rev. Letters 26 (1971) 1221.

[8] M. LAMPE, W. M. MANHEIMER, J. B. McBRIDE, J. H. ORENS, K. PAPADOPOULOS, R. SHANNY, R. N. SUDAN, Phys. Fluids 15, (1972) 662.

[9] M. LAMPE, W. M. MANHEIMER, J. B. McBRIDE, J. H. ORENS, Phys. Fluids (to be published).

[10] K. N. STEPANOV, Zh. Tekh. Fiz. 34, (1964) 2146 [Sov. Phys. Tech. Phys. 9 (1965), 1653].

[11] A. B. MIKHAILOVSKII, V. S. TSYPIN, Zh. Eksp. Teor. Fiz., Pisma Red 3 (1966) 247 [JETP Letters 3, (1966) 158].

[12] N. A. KRALL, P. C. LIEWER, Phys. Rev. 4, (1971) 2094.

[13] E. OTT, J. B. McBRIDE, J. H. ORENS, J. P. BORIS, Phys. Rev. Letters 28 (1972) 88.

[14] J. B. McBRIDE, E. OTT, J. P. BORIS, J. H. ORENS, Phys. Fluids (to be published).

[15] J. B. McBRIDE, E. OTT, Physics Letters A (to be published).

[16] S. P. GARY (to be published)

[17] P. K. KAW, Y. C. LEE, Princeton University, Plasma Physics Laboratory Report No. MATT-827.

[18] J. KINDEL, H. OKUDA oral presentation at Sherwood Theory Meeting, Los Alamos, New Mexico (23-24 March 1972).

[19] E. OTT, J. B. McBRIDE, J. H. ORENS, Phys. Fluids (to be published).

[20] I. HABER, C. WAGNER, J. BORIS, J. M. DAWSON, in Proceedings of the Fourth Conference on Numerical Simulation of Plasmas, Washington, D. C. (1970) edited by R. SHANNY and J. BORIS (U.S. GPO, Washington, D. C.).

[21] M. V. BABYKIN, P. P. GAVRIN, E. K. ZAVOISKII, L. I. RUDAKOV, V. SKORYUPIN, G. V. SHOLIN, Zh. Eksp. Teor. Fiz. 46 (1964) 511 [Sov. Phys. JETP. 19 (1964) 349].

[22] D. DIMOCK, D. ECKHARTT, H. EUBANK, E. HINNOV. L. C, JOHNSON, E. MESERVEY, E. TOLNAS, D, GROVE, in Plasma Physics and Controlled Thermonuclear Research (Proc. 4th Int. Conf. Madison, 1971) 1 IAEA, Vienna (1971) 451.

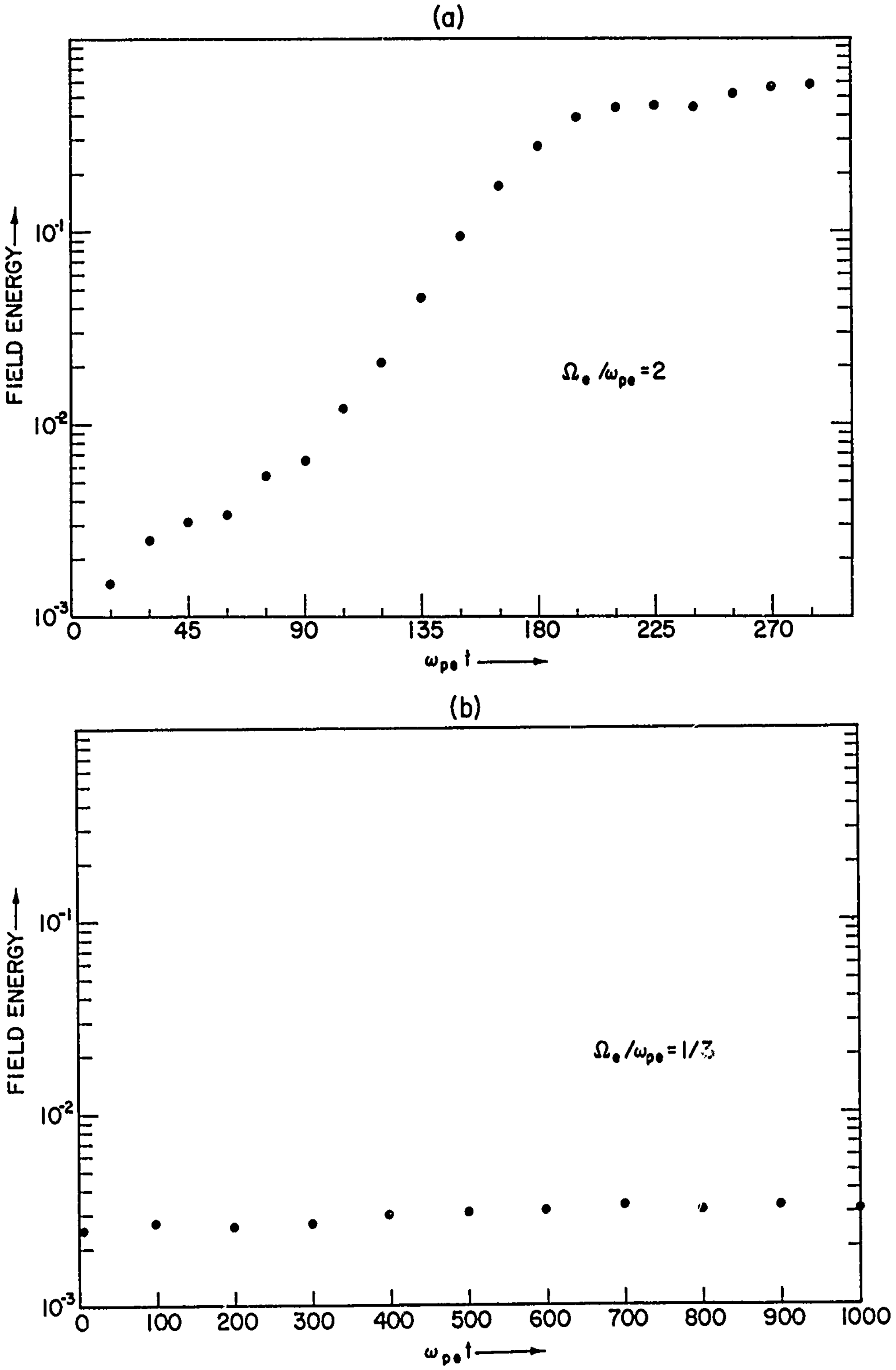

Fig. 1 - Energy in the wave fields $(E^2 + B^2)/8\pi$, versus time. Figure 1a is for $\Omega_e/\omega_{pe} = 2$ and Fig. 1b is for $\Omega_e/\omega_{pe} = 1/3$. Energy units are arbitrary.

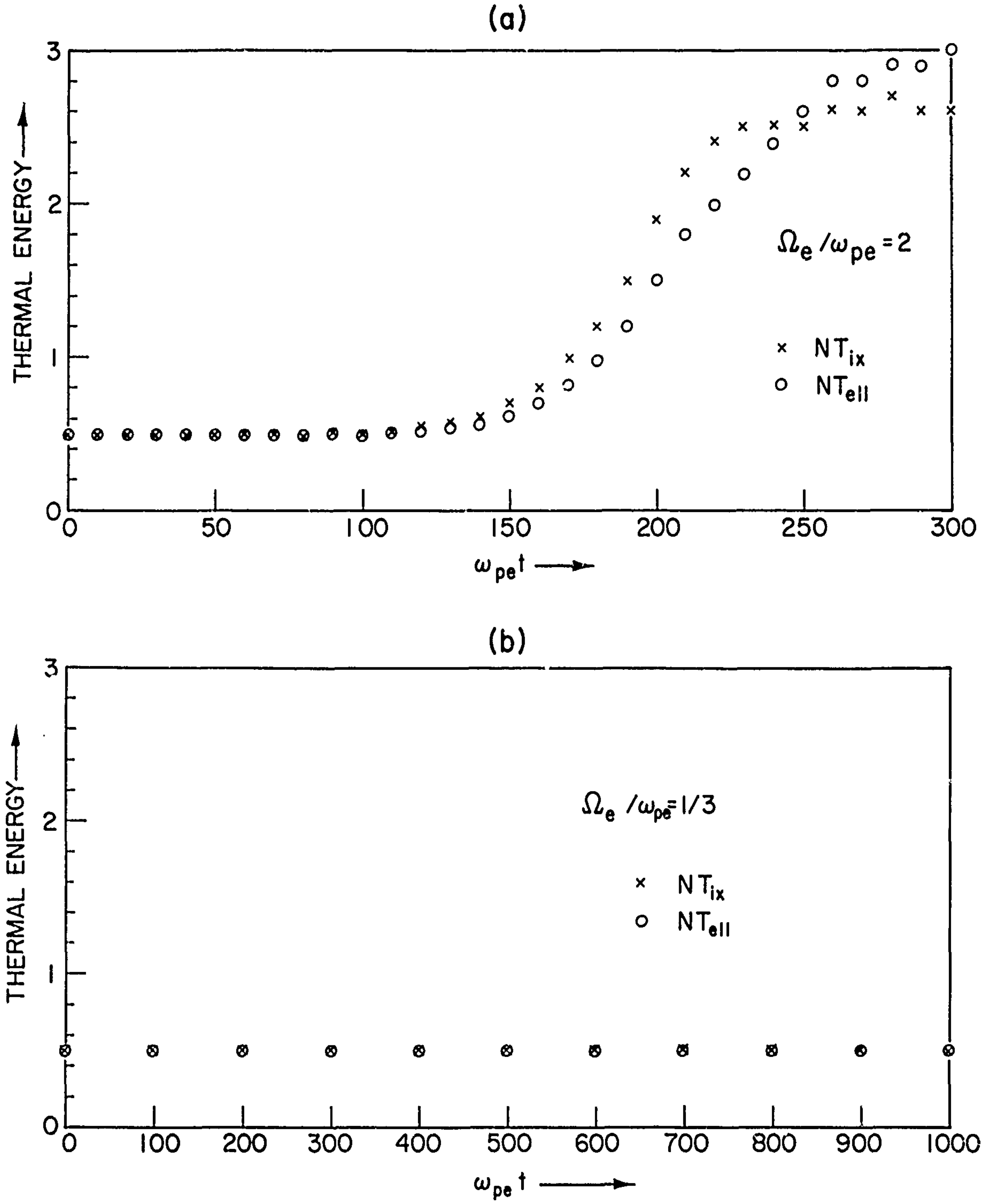

Fig. 2 - Ion thermal energy perpendicular to $\underline{B}_o$ and electron thermal energy parallel to $\underline{B}_o$ in arbitrary units. Figure 2a is for $\Omega_e/\omega_{pe} = 2$ and Fig. 2b is for $\Omega_e/\omega_{pe} = 1/3$.