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Fundamental Geokinetic Considerations in Multiple Position Gyrocompassing

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13. ABSTRACT

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Fundamental Geokinetic Considerations in Multiple Position Gyrocompassing

1. INTRODUCTION

The inertial guidance system is a major subsystem in many Air Force weapon systems. The primary purpose of the guidance system is to provide navigation and other pertinent weapon delivery data to the system operators. To successfully perform this function, it is necessary that the inertial guidance system be accurately initialized. Data such as initial position, velocity and target position are required as inputs to the navigation computations. In addition, the inertial reference platform must be aligned with respect to some known reference. The optical alignment and the self-alignment process are the two particular techniques most frequently used:

(1) In the optical alignment technique, the inertial platform is slaved to a known optical reference. It is essential to have a known stable optical reference, such as an optical cube mounted on a pier, whose azimuth with respect to celestial North has been determined through extensive measurement to an accuracy better than the azimuth required by the inertial reference unit. Additionally, some means must exist to transfer the cube azimuth to the inertial platform.

(2) In the self-alignment process technique, the inertial platform requires no external references. Here the astronomic azimuth of the inertial element is

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determined by making measurements of the Earth's diurnal rotation vector, $\underline{W}_{ie}$, and the Earth's gravity vector $\underline{g}$.

Under Project 7628 (Geophysical and Geokinetic Effects), AFCRL scientists have been investigating the effects of geokinetic disturbances on the performance of inertial instruments and systems. Since inertial instrumentation requires electromechanical components whose function demands high sensitivity to mechanical motions, geokinetic errors sources, if not properly accounted for, will obviously degrade performance of the instrumentation. The purpose of this report is to define the theoretical performance deviations which could be expected in a self-alignment process for inertial instrumentation called "multiple position gyro-compassing." These deviations are a direct result of the fundamental geokinetic motions of tilt and polar wobble.

2. MULTIPLE POSITION GYROCOMPASS ANALYSIS

In multiple position gyrocompassing, a precision rate integrating gyroscope operating in the rate mode is sequenced through a number of different attitudes relative to the Earth rate vector and gravity vector. The rate measurements made in these positions are then differenced and averaged to obtain a bias-free astronomic azimuth of the sensors reference axis that is independent of the gyro drift and bias.

Figure 1 is a pictorial representation of a single degree-of-freedom rate integrating gyroscope. When operating in the rate mode, the gyro performance model is given by:

$$W = W_{IRA} + B + \sigma + \frac{P_{SRA}\, f_{IRA} - P_{IRA}\, f_{SRA}}{H} \qquad (1)$$

where

W_{IRA} = the input reference axis component of the angular velocity of the gyro case with respect to inertial space.

B = the constant portion of the gyro output, bias.

σ = a random uncertainty of the gyro output.

P_{SRA} = pendulosity (mass unbalance times distance) along the spin reference axis.

P_{IRA} = pendulosity along the input reference axis.

f_{IRA} = specific force along the input reference axis.

f_{SRA} = specific force along the spin reference axis.

H = angular momentum of the gyro rotor.

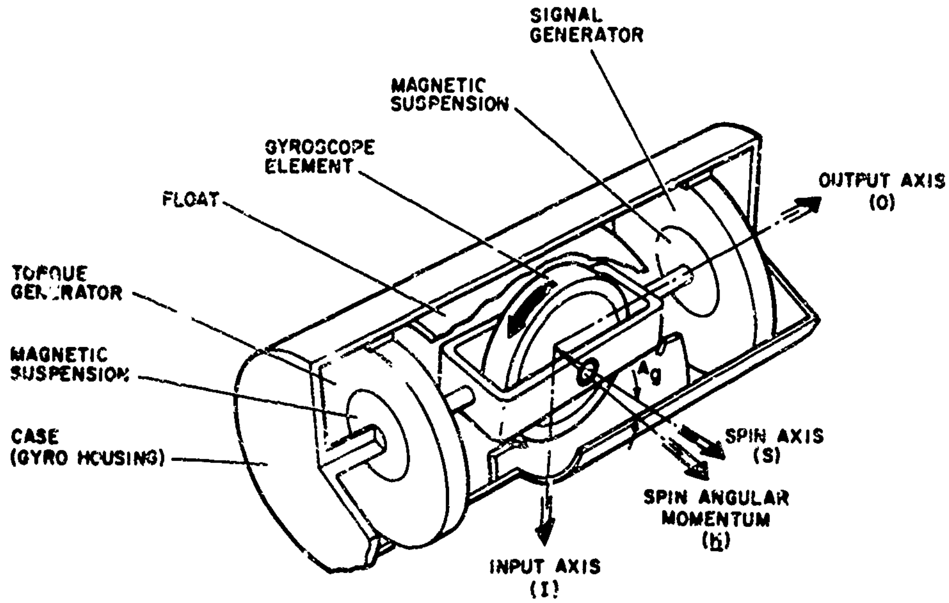

Figure 1. Single Degree-of-Freedom Rate Integrating Gyroscope

If such a gyro is sequenced through four positions given below, it is possible to determine the astronomic azimuth of the sensors spin reference axis from the rates measured by the gyro.

Position 1	Position 2	Position 3	Position 4
IRA East	IRA West	IRA East	IRA West
ORA Down	ORA Up	ORA Up	ORA Down
SRA North	SRA North	SRA South	SRA South

where

Down ≡ along the direction $\underline{g}$.

North ≡ in a plane perpendicular to $\underline{g}$ in the direction of the local horizontal projection of the principle axis of the Earth's figure.

East ≡ forms a right handed orthogonal set.

The nominal position of the gyro frame (g') is defined by the North, East, and Down reference directions as defined above. The actual position of the gyro frame (g) will differ from the nominal gyro frame, because the gimbal system which supports the gyroscope will not be perfectly aligned initially and will continuously change orientation because of changes in the gimbal base with respect to the gravity vector or because of changing environmental conditions. In either case, the attitude of the g frame with respect to the g' frame can be determined from the direction cosine matrix

$$C_{g'}^{g} = \begin{pmatrix} 1 & \epsilon_D & -\epsilon_E \\ -\epsilon_D & 1 & \epsilon_N \\ \epsilon_E & -\epsilon_N & 1 \end{pmatrix}$$

where the tilt angles ϵ_N, ϵ_E, and ϵ_D have been considered to be small enough so that

$$\cos \epsilon_j \approx 1 \qquad \sin \epsilon_j \approx \epsilon_j \qquad \text{and} \qquad \underline{W}_{g'g}^{g} = \begin{Bmatrix} \dot{\epsilon}_N \\ \dot{\epsilon}_E \\ \dot{\epsilon}_D \end{Bmatrix} .$$

With these definitions and the assumption that the gyrocompass is stationary with respect to a point on the surface of the Earth, it is possible to coordinate the specific force vector and the Earth's daily rotation vector in the actual gyro frame.

In the case where nominal motion of the gyroscope with respect to the mass center of the earth is zero, the specific force vector and the gravity vector are identical. Then the specific force can be written as

$$\underline{f}^g = \underline{g}^g = \underline{C}_{g'}^{g}\, g^{g'} = \begin{pmatrix} 1 & \epsilon_D & -\epsilon_E \\ -\epsilon_D & 1 & \epsilon_N \\ \epsilon_E & -\epsilon_N & 1 \end{pmatrix} \begin{Bmatrix} 0 \\ 0 \\ g \end{Bmatrix} = \begin{Bmatrix} -\epsilon_E g \\ \epsilon_N g \\ g \end{Bmatrix} .$$

The Earth's angular velocity can be coordinatized in the Earth centered inertial frame as

$$\underline{W}_{ie}^{e} = \begin{Bmatrix} W_x \\ W_y \\ W_z \end{Bmatrix}$$

where

W_z is the primary component of W_{ie}, and

W_x, W_y are equatorial components ot W_{ie}.

Then the Earth's angular velocity in the g' frame is given by

$$\underline{W}_{ie}^{g'} = \begin{Bmatrix} -\sin L \cos\lambda W_x - \sin L \sin\lambda W_y + \cos L W_z \\ -\sin\lambda W_x + \cos\lambda W_y \\ -\cos L \cos\lambda W_x - \cos L \sin\lambda W_y - \sin L W_z \end{Bmatrix}$$

where

L is the reduced astronomic latitude

λ is the reduced celestial longitude

$\lambda = \ell + W_z t$

ℓ → reduced terrestrial longitude from Greenwich

t → is time

The earth's angular velocity coordinatized in g frame then becomes

$$\underline{W}_{ie}^{g} = \begin{Bmatrix} \cos L W_z + \sin L W_z \epsilon_E - \sin L \cos\lambda W_x - \sin L \sin\lambda W_y \\ -\sin\lambda W_x + \cos\lambda W_y - \epsilon_D \cos L W_z - \epsilon_N \sin L W_z \\ -\sin L W_z + \cos L W_z \epsilon_E - \cos L \cos\lambda W_x - \cos L \sin\lambda W_y \end{Bmatrix} .$$

From the fundamental law of vector addition of angular velocities, the gyro case angular rate can be written as

$$\underline{W}_{ig}^{g} = \underline{W}_{ie}^{g} + \underline{W}_{eg'}^{g} + \underline{W}_{g'g}^{g}$$

or

$$\underline{W}_{ig}^{g} = \begin{Bmatrix} \cos L W_z + \sin L W_z \epsilon_E - \sin L \cos\lambda W_x - \sin L \sin\lambda W_y + \dot{\epsilon}_N \\ -\sin\lambda W_x + \cos\lambda W_y - \cos L W_z \epsilon_D - \sin L W_z \epsilon_N + \dot{\epsilon}_E \\ -\sin L W_z + \cos L W_z \epsilon_E - \cos L \cos\lambda W_x - \cos L \sin\lambda W_y + \dot{\epsilon}_D \end{Bmatrix} \quad (4)$$

With the relationships Eqs. (4) and (3) substituted into Eq. (1) for the positions stated in Eq. (2), the following four indicated rates are obtained:

$$W_1 = -\sin\lambda W_x + \cos\lambda W_y - \epsilon_D \cos L W_z - \epsilon_N \sin L W_z + B + \sigma_1$$

$$+ \frac{J}{H} (P_{SRA} \epsilon_N g + P_{IRA} \epsilon_E g) + \dot{\epsilon}_E$$

$$W_2 = +\sin\lambda\, W_x - \cos\lambda\, W_y + \epsilon_D \cos L\, W_z + \epsilon_N \sin L\, W_z + B + \sigma_2$$

$$+ \frac{1}{H}(-P_{SRA}\,\epsilon_N g + P_{IRA}\,\epsilon_E g) - \dot{\epsilon}_E$$

$$W_3 = -\sin\lambda\, W_x + \cos\lambda\, W_y - \epsilon_D \cos L\, W_z - \epsilon_N \sin L\, W_z + B + \sigma_3$$

$$+ \frac{1}{H}(+P_{SRA}\,\epsilon_N g - P_{IRA}\,\epsilon_E g) + \dot{\epsilon}_E$$

$$W_4 = +\sin\lambda\, W_x - \cos\lambda\, W_y + \epsilon_D \cos L\, W_z + \epsilon_N \sin L\, W_z + B + \sigma_4$$

$$+ \frac{1}{H}(-P_{SRA}\,\epsilon_N g - P_{IRA}\,\epsilon_E g) - \dot{\epsilon}_E \ .$$

The quantity

$$W = 1/4\,(W_1 - W_2 + W_3 - W_4)$$

is formed and is seen to be equal to

$$W = -\epsilon_D \cos L\, W_z - \epsilon_N \sin L\, W_z - \sin\lambda\, W_x + \cos\lambda\, W_y$$

$$+ P_{SRA}\,\epsilon_N g + \sigma + \dot{\epsilon}_E$$

where

$$\sigma = \frac{\sigma_1 - \sigma_2 + \sigma_3 - \sigma_4}{4} \ .$$

In addition it should be noted that any mass unbalance drift terms, due to the specific force component along the output axis, will also be eliminated by the averaging process above.

The normal calculation made is

$$\epsilon_D = \frac{-W}{\cos L\, W_z}$$

which is the astronomic azimuth of the gyro spin reference axis and is in error by the four following quantities:

$$e_1 = \frac{\sec L\, \dot{\epsilon}_E}{W_z}$$

$$e_2 = \frac{\sec L\, \sigma}{W_z}$$

$$e_3 = \left(\tan L - \sec L \frac{P_{SRA}\, g}{W_z}\right)\epsilon_N$$

$$e_4 = \frac{\sin\lambda}{\cos L}\frac{W_x}{W_z} - \frac{\cos\lambda}{\cos L}\frac{W_y}{W_z}$$

which represent the errors due to the imperfect level of the instrument about the North-South axis, the tilt rate about the West-East axis, the component of the Earth's rotation vector in the reduced equatorial plane, and the statistical behavior of the gyro drift.

3. PERFORMANCE DEVIATIONS DUE TO GEOKINETIC EFFECTS

Of the four error sources e_1 through e_4 defined in Section 2, three are due to geokinetic disturbances. Before further investigation of these deviations, however, we will comment on the effect of the random fluctuations of gyro drift.

In general, the statistical description of gyro drift is not agreed upon. Weinstock (1964) has shown large variations in the statistical descriptions of identically designed, high-precision inertial gyros. The model adapted here is in agreement with the majority of analytical attempts to describe the gyro drift. The drift model to be used is

$$\dot{\sigma}' = -1/k\ \sigma' + n(t)$$

where

σ' = the random gyro drift rate.

k = the correlation time of the random process.

$n(t)$ = gaussian distributed white noise.

The gyro drift uncertainty is modeled as a Gaussian Markov process exponentially correlated with correlation time k. The mean of the gyro drift at any time t is given by

$$\langle \sigma'(t) \rangle = e^{-\frac{(t-t_o)}{k}} \langle \sigma'(t_o) \rangle$$

and the variance is given by

$$E_{\sigma'}(t) = \frac{Nk}{2}\left(1 - e^{-\frac{2(t-t_o)}{k}}\right)$$

where N is the constant value of the power spectral density of the noise, n(t). For time periods $(t-t_o)$ small compared to the correlation time, the variance of the gyro drift is given by

$$E_{\sigma'}(t) = (t-t_o)N$$

or the process is similar to a random walk. For time periods longer than the correlation time, the gyro drift is bounded in that

$$E_{\sigma'}(t) = \frac{Nk}{2} .$$

Britting et al (1971) gives the following values for N as a function of when the instrument was designed.

	N in min^2/hr^3
Production:	0.0735
State of the Art:	0.0368
Future:	0.00735

The random fluctuation (σ) which exists at the end of the differencing and averaging process can be determined as a function of the gyro statistical description. The mathematical expectation of σ is given by

$$< \sigma > = 1/4 < \sigma_1 - \sigma_2 + \sigma_3 - \sigma_4 > .$$

If the time interval (t_4-t_o) is small compared to the correlation time k, then

$$< \sigma_1 > \approx < \sigma_2 > \approx < \sigma_3 > \approx < \sigma_4 >$$

and

$$< \sigma > = 0 .$$

This condition is realistic since in a typical multiple position gyrocompass mechanization, the measurement interval $(t_4 - t_o)$ is approximately 10 minutes, while the gyro correlation time is about 24 hours (Britting et al, 1971). The variance of σ is given by

$$E_\sigma = <\sigma^2> = \frac{1}{16} < (\sigma_1 - \sigma_2 + \sigma_3 - \sigma_4)^2 >$$

or

$$E_\sigma = \frac{1}{16} < \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_4^2 >$$
$$+ \frac{1}{8} < \sigma_1\sigma_3 + \sigma_3\sigma_4 - \sigma_1\sigma_2 - \sigma_1\sigma_4 - \sigma_2\sigma_3 - \sigma_3\sigma_4 > .$$

Since the measurement period is small, each value of the drift rate will be independent of the next value. With additional manipulation the variance of σ becomes

$$E_\sigma = \frac{1}{16} \left[E_{\sigma_1} + E_{\sigma_2} + E_{\sigma_3} + E_{\sigma_4} \right] .$$

Then, using the expression for the variance valid for small time intervals

$$E_\sigma = \frac{1}{16} N (t_4 - t_o) .$$

Using the stated values above of N, the variance of the output of the multiple position gyrocompass indication at 45° latitude is given below.

Class of Gyro	E_{e_2}
Production:	9.0 $\widehat{sec}^2$
State of the Art:	4.5 $\widehat{sec}^2$
Future:	0.9 $\widehat{sec}^2$

In all cases it can be seen that the random fluctuations of the gyro drift are a zero mean variable but can contribute significantly to the variance of the system output with the best case (future gyro, 3-standard deviations) having a value of 2.9 sec.

The error due to the tilt about the North-South direction, as defined above, has two contributors one of which is due to the imperfect performance of the gyro. Note that this error could be tuned out, at least in theory, by fabricating a rate integrating gyro with the spin mass unbalance satisfying the following relationship:

$$P_{SRA} = \frac{W_z \sin L \, H}{g} .$$

This, however, is impractical for several reasons. The contribution due to this error source is small in that for a typical large mass unbalance, the error is on the order of 10 meru/g*. Hence at 45° latitude

$$e'_3 = 0.014$$

which is second order at the chosen latitude. Hence the major error source associated with North-South tilt is due to the primary geometric source, tan L. At 45° latitude, every arc sec of tilt results in a deviation of one arc sec in the gyrocompass output. Note that tilt, as defined here, is a rotation of the case of the gyrocompass with respect to **g**, the gravity vector.

The deviation in the gyrocompass output due to the tilt rate about the East-West line has a large sensitivity in that, at 45° latitude,

$$e_1 = 292\ \dot{\epsilon}_E\ \widehat{\sec}$$

where the East tilt rate is given in meru. The largest tilt rates recorded at AFCRL's Haskell Observatory were 4 $\widehat{\sec}$/day, or 3.6×10^{-3} meru (Cabaniss, 1972). The corresponding deviation in gyrocompass output would be

$$e_1 = 1.08\ \widehat{\sec}$$

which again is small. However, since the sensitivity of the output to East axis tilt rate is relatively large, provision should be made to monitor the East axis tilt rate and compensate the output if required.

The remaining error term is due to the physical nature of the rotation of the Earth. With a simple model of the Earth's polar motion, the equatorial components of the Earth rotation vector can be written as

$$W_x = W_e \cos (W_{st} + \Phi)$$

$$W_y = W_e \sin (W_{st} + \Phi)$$

where

W_e → magnitude of the equatorial component of $\underline{W}_{ie}$.

W_s → angular rate of rotation of the equatorial components about the principal Z axis of the Earth.

Φ → is phase angle relating position of instantaneous pole at $t = t_o$.

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* 1 meru = $\frac{1}{1000}$ earth rate = 0.015°/hr

With this representation – which assumes mean pole position of the principal axis of the Earth – the gyrocompass error due to polar motion can be expressed as

$$e_4 = \frac{W_e}{\cos L W_z} \cos (W_{st} + \Phi_1)$$

or

$$e_4 = - \alpha \sec L \cos (W_{st} + \Phi_1).$$

where Φ_1 differs from Φ by the terrestrial longitude, and α is the distance of the instantaneous pole from the mean pole expressed in terms of central angle ($1 \overset{\frown}{\sec} \approx$ 100 ft). At 45° latitude the maximum deviation observed due to polar motion effects would be

$$e_3(\max) = \alpha \sqrt{2} .$$

This functional dependency has been derived for a multiple position gyrocompass, but can be shown to be valid for all other types of inertial system gyrocompassing.

Additional geodetic errors may exist, but are only due to incorrect input data, analysis, or operation, rather than the correct definition of the physical vectors being measured. For instance, if the geographic latitude is used in the calculation of the azimuth data, rather than the reduced astronomic latitude, an additional error will exist equal to

$$e_5 = \epsilon_D \tan L \delta$$

where δ is the difference between the geographic latitude and the reduced astronomic latitude expressed in radians. This again is an extremely small effect. The major geokinetic disturbance, which has not been discussed here, is seismic motions. In general, these motions will have an effect on the indicated azimuth in that these motions degrade the gyro such that the gyro performance model given above is not completely applicable. The explicit relationships between seismic motions and gyroscope performance is the subject of continuing AFCRL research and will be reported on in the future.

4. SUMMARY

It has been shown that the inertial process of multiple position gyrocompassing is subject to deviations attributable to the geokinetic motion of North axis tilt, East axis tilt rate, and polar motion. The process has highest sensitivity to East axis tilt rate. The tilt about the North axis effects the output directly with a sensitivity of tan L. Deviations in performance due to polar motion are small, but probably within the measurement capability of today's instrumentation.

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