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ADVISORY GROUP FOR AEROSPACE RESEARCH & DEVELOPMENT

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AGARD LECTURE SERIES No. 52

on

Guidance and Control of Tactical Missiles

NORTH ATLANTIC TREATY ORGANIZATION

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ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT

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AGARD Lecture Series No.52

GUIDANCE AND CONTROL OF TACTICAL MISSILES

C.T.Maney

Lecture Series Director

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PREFACE

This Lecture Series is sponsored by the Guidance and Control Panel and the Consultant and Exchange Program.

New tactical missile requirements are so stringent that weapon subsystem technology must be utilized at the highest possible level consistant with cost, reliability and performance. This is particularly true with the guidance and control subsystems – the "nerve center" of the weapon. As a result of this, there is a continuing requirement for more and better tools for analyzing performance, predicting requirements, determining error sources and selecting suitable concepts.

Due to the extremely high cost of developing and testing prototype concepts for each of the very large number of possible guidance and control concept combinations, the use of simulation through mathematical techniques has become an absolute necessity.

The Lecture Series provides an opportunity for an examination of the utility of modern analysis and evaluation tools and techniques associated with the several commonly used control and guidance concepts. It examines the techniques which are normally employed for error source determination, performance specification, and the use of digital and analogue computers for system performance prediction.

A round table discussion with the participation of all the speakers concludes the Lecture Series presented in three different NATO nations (Norway, Greece and Italy) from 29 May to 6 June 1972.

C.T.Maney
Lecture Series Director

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CONTENTS

I. SOME ASPECTS OF THE SYSTEM DEVELOPMENT PROBLEM

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Gentlemen

It is a real pleasure for us to be here today to discuss some of our views on tactical weapon system development. We plan to discuss in particular, the guidance and control aspects of tactical missiles. We shall analyze how we bound and define problems and how we develop solutions. We will talk primarily methodology; that is, how we identify problems and how we develop solution methodology.

Design engineers must have performance specifications toward which they work to develop designs. The lectures you will hear today and tomorrow will be devoted to discussions of mathematical and laboratory techniques which can be used to determine these performance specifications and to predict whether or not a given design will meet a set of specifications.

Let us begin by reviewing the problem of selection of technology appropriate for a new weapon development and the creation of the preliminary or conceptual design as this is the type of problem normally approached by development planning organizations. Development planners and design engineers labor under a multitude of diverging and sometime violent forces which are related to the general "collapse of time" factor associated with the accelerating technology of 20th century living--I am referring to the ever increasing sophistication of potential enemy threat capability; the growing number of possible applicable technologies; the generally voiced concern over inventory proliferation; and associated increased operator and maintenance personnel training requirements; and above all, the skyrocketing cost of hardware development. Whether we have faced up to it in the past or not, it is now imperative that we in the entire weapon development community devote our primary energies toward the development of quality weapons at lower costs. The demands upon national resources are so great and so diverse that we would be foolish to take any other course. This does not necessarily mean, however, that we have to build cheap systems. What it does mean is that we have to be absolutely sure that the systems we design and develop represent not only the maximum performance for the money invested but also that the type of performance obtained is really needed by the armed forces.

With these constraints upon us then, let us examine the preliminary analysis and review procedures that are often followed in the U. S. leading towards the creation of a new weapon capability.

The operational weapon requirements process begins with identification and assessment of an operational deficiency or need. The primary goals of the initial assessment of an operational need are to establish a clear understanding of the need and to determine its urgency and importance. An operational requirement is normally generated by one of the using Commands and is forwarded to HQ USAF in a document called a Required Operational Capability ROC). Additionally, a ROC can be generated within the Air Staff itself.

HQ USAF/Research Development (RD) receives the ROC and circulates it within the Air Staff and other agencies for review. Study and analysis efforts consider the mission, threat, operational concepts, constraints, resources, and potential alternative proposals. Air Force Systems Command (AFSC) and Air Force Logistics Command (AFLC) are asked to provide inputs regarding technical feasibility, costs, and potential alternative solutions.

The proposal (ROC) is then reviewed by the Requirements Review Group (RRG) comprised of General Officers. The RRG reviews the need for the proposed system or equipment taking into consideration the threat, alternative means of satisfying the ROC, cost estimates, impact on force structure and technical feasibility.

If the ROC is approved at this point, it is considered to be formally validated. A Program Management Directive (PMD) is written by HQ USAF/RD, which represents USAF decision to proceed with system development. The proposal then enters another period of formal study and refinement. AFSC and AFLC are tasked to provide possible alternative proposals, evaluate cost and feasibility and make recommendations.

Though the weapon development process involves extensive additional study, review, hardware prototype building and testing, we, in this short course, are going to concentrate our efforts entirely on an in-depth discussion of the methodology which is used to examine and prepare alternative design solution options during the conceptual phase of development.

Basically what we do in the initial phase of a weapon concept study is to first review the technical and tactical aspects of the immediate threat and then attempt to develop a reasonable prognostication of the particular threat associated with the time frame and mission under investigation. Next we review the present operational capability and prepare an inventory of the available off-the-shelf technology which has potential application to the problem at hand. We also attempt to make an estimation of the required or desired performance parameters which the weapon concept should have. A conceptual design team then is charged with the responsibility for developing a preliminary design with performance capabilities reasonably close to those requested. An operational analysis team is simultaneously charged with ascertaining the effectiveness or utility, i.e. worth, of the performance levels estimated for the preliminary design. An iteration process is thus initiated in which performance goals are modified as preliminary designs are iterated and design effectiveness is estimated. And, of course, by effectiveness I am also including cost; so that really we mean cost effectiveness. The iteration continues until the development planning management concludes that the study has developed the best series of alternative designs that can be accomplished in the time provided. The results of studies of this type become the nucleus of new and quantified specifications for either new weapons or for modifications to existing weapons. The resulting data and rationale are continuously reviewed and examined by appropriate higher level authority until decisions are made for program go ahead or cancellation.

When a tactical missile or other weapon program is approved for development of hardware test items, the system specifications are rather well defined. The several subsystem specifications, however, are less precisely defined. Successively increased definitization of these specifications evolve as tradeoffs are made between performance, cost, risk availability, manufacturing difficulty, environmental limits, and reliability.

Now let us consider our primary interest for this seminar; namely, the conceptual development of tactical missile designs. The several subsystems as usually defined in this weapon include propulsion, warhead, flight control, guidance, and airframe. The guidance system is often further subdivided into mid-course and terminal. This additional subdivision occurs if the device is to be used as a precision type of weapon. An interesting point which will be developed in different ways by later speakers is the tradeoff that is made between mid-course navigation systems and terminal guidance systems.

Another point which will be explored in depth by my colleagues include an evaluation of the types of guidance error and the mathematical tools which exist to evaluate and minimize these errors. We will discuss control system synthesis and design, and mathematical methods for predicting stability and proper system response. We will address methods for determining the sensitivity to overall system performance of various parameters. We plan to discuss guidance analysis methodology and the development of guidance or navigation laws.

Though the real proving ground for theory is the field test, it is well known that the laboratory and the analog/digital computer offer splendid opportunities for testing guidance and control system ideas and designs. We shall examine these tools in depth.

Let me now most briefly summarize what you are about to hear. Our first speaker is Mr Philip Gregory, Manager of Guidance Systems at Martin Marietta Company. He will discuss the evolution of general guidance and control subsystem requirements. He will also be the last speaker this afternoon when he reviews the laboratory evaluation tools used by his engineers.

Our next speaker will be Dr Robert Goodstein of the Boeing Corporation. He is Manager of Guidance & Control Development there. Dr Goodstein will analyze the methodology of development of control system requirements. Dr Goodstein will then return for the opening session tomorrow and present a paper on guidance law applications.

The next member of our team is Mr Duncan Pitman, Senior Staff Engineer for McDonald Douglas Corporation. Mr Pitman will present two papers. The first one on classical control theory and the second on modern theory.

Following Mr Pitman will be Mr E. Heap of the Royal Aircraft Establishment at Farnborough. Mr Heap will present a paper today on numerical analysis and simulation. Tomorrow, he will present another topic; namely, research methodology into certain non-inertial guidance systems.

Tomorrow, in addition to the speakers I have referenced, you will hear Mr Acus, a control system engineer for the U. S. Air Force and Mr Zuerndorfer, Manager of Tactical Systems for the Raytheon Company. Mr Acus will discuss two papers on self-contained guidance systems and Mr Zuerndorfer will review certain aspects of radar guidance.

We would like to encourage all members of the audience to participate with us in this review of design and analysis techniques for the guidance and control of tactical missiles. In order to maximize the efficiancy of this participation, I would like to suggest that you write your questions as they occur to you. We shall collect the questions and present answers to them during the round table discussion tomorrow.

I would like now to introduce to you our first lecturer today--Mr Philip Gregory, Manager of Guidance Systems for the Martin Marietta Company.

II. GENERAL CONSIDERATIONS IN GUIDANCE AND CONTROL TECHNOLOGY

by

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SUMMARY

This paper describes a particular set of mission requirements for an air-to-air missile and an automated design process to synthesize these requirements into the preliminary design of a missile and guidance system. This process makes use of the CAMS (Computer Aided Missile Synthesis) digital computer program which was developed to: synthesize missile configurations including the guidance, controls, secondary power, warhead, and propulsion subsystems; furnish resulting flight performance including trajectories and miss distance; and estimate unit costs.

GUIDANCE AND CONTROL ANALYSIS

The guidance and control analyst is a key figure in the process of developing new tactical missiles. First, the operations analyst studies the battle tactics of opposing forces and poses scenarios which relate the timing and geometry of expected encounters. The guidance and control analyst then interrelates all state-of-the-art missile technologies to synthesize candidate weapons and evaluate their performance. This process is repeated at successive levels of detail each time a successful set of tradeoffs are made, or redone with modified parameters when a failure occurs. A failure is established when state-of-the-art technologies do not satisfy the mission requirements. The process is valuable for identifying areas of research and development requirements, but this is of little solace or comfort to the engineer who is required to solve an immediate problem.

The process of interrelating state-of-the-art missile technologies into candidate systems is both time consuming and costly. It requires the support of specialists in aerodynamics, propulsion, structures, secondary power, flight control, and guidance hardware. In addition, many different combinations are possible requiring the evaluation of a large number of candidate systems.

Computer programs have the potential of reducing the time and cost of this preliminary design process. The Computer Aided Missile Synthesis (CAMS) program has been developed by Martin Marietta under contract F33615-70-C-1753 to the United States Air Force Deputy for Development Planning (ASB), Wright Patterson Air Force Base, Ohio as a missile synthesis and performance prediction tool to aid in rapidly developing large numbers of missile design concepts and in assessing their performance potential and cost. This program permits two functions to be performed during the concept and formulation study phase of any new missile design program: system and subsystem concept evaluation, and subsystem design point tradeoffs.

Without a program such as CAMS, there is normally a problem in effecting integrated missile design (i.e., one where all subsystem interactions are reflected). The CAMS program is constructed so that the system design constraints must be satisfied by all subsystems. This implies a compatible missile design where the various engineering disciplines are matched. In performing design point trades without a CAMS program, normal practice is to isolate a particular subsystem and vary its design parameters to achieve the optimum performance, minimum weight, length, etc. The integrated aspects of the CAMS program ensures a more cost effective answer since the impact on the total system performance may be ascertained by adjusting a single parameter, with design compatibility assured.

All computer programs, however, make use of approximations and the final output should be considered only a preliminary design to be completed in detail if and when the missile program continues.

Before proceeding to a detailed discussion of the CAMS modules, it will be useful to define a typical mission as it might be presented to the guidance analyst.

MISSION SCENARIO

It is desired to develop a more effective weapon for fighter type aircraft to perform dogfights during air superiority, escort, and interdiction missions in the 1975-1985 time frame.

Air superiority missions penetrate hostile territory and seek to engage enemy fighters, inflicting a sufficiently high loss rate to allow subsequent penetration by friendly aircraft. Escort missions provide cover for relatively vulnerable attack or reconnaissance aircraft. The friendly aircraft is assumed to be autonomous, or on his own, whereas the enemy can be either autonomous or ground controlled. Interdiction missions are directed primarily against ground targets. Attack by enemy fighters is generally at their convenience.

Figures 1 through 3 depict the engagement modes desired in the new weapon system. In Figure 1 the friendly aircraft performs a maximum g turn to avoid the enemy's tail attack while launching a weapon. In Figure 2 both aircraft attempt to get in position for a tail attack while the weapon is launched. In Figure 3 the weapon performs in the conventional aggressor role.

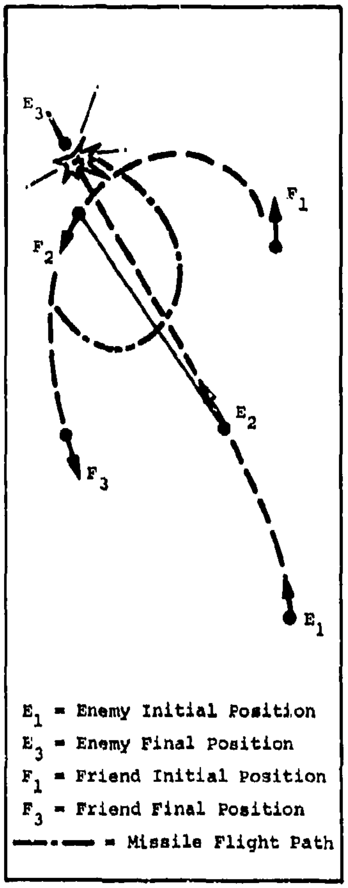


Figure 1. Tail Attack

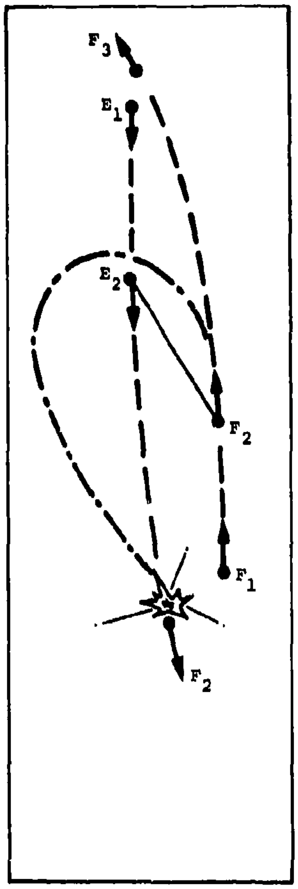


Figure 2. Nose Attack

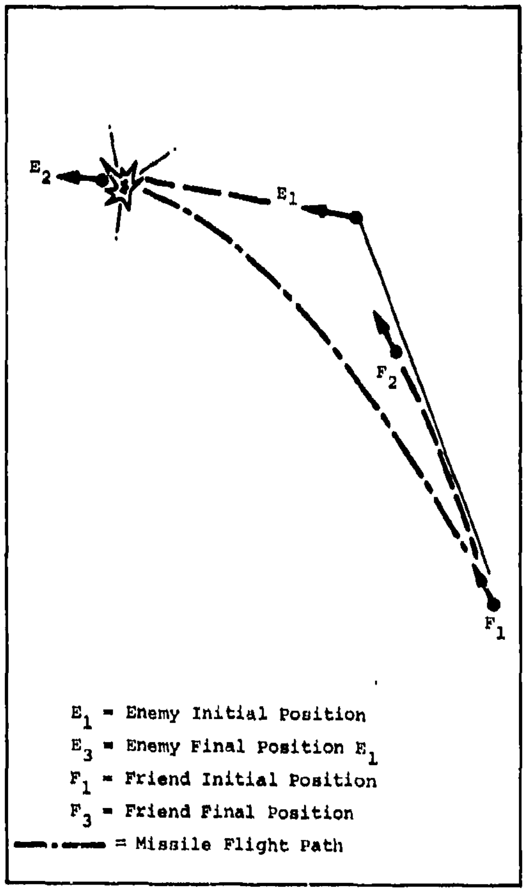


Figure 3. Aggressor

Note that in both Figures 1 and 2 the angle correction capability of the missile would be its most valued feature. If a kill is not achieved on the first pass, then a turn-counterturn dogfight ensues. The participants generally will not be separated by more than their combined turn diameters (about 4 miles at 10,000 feet). A number of passes usually occur before one attains a superior position over the other. This superior position is classically defined as being on the tail of the opponent, and results in shots of the type depicted in Figure 3.

The friendly and enemy aircraft should be assumed to have the design characteristics shown in Figure 4.

To establish missile requirements, some capability must be given to the enemy's weapons. If each side's weapons are equally effective, no advantage can be gained. For this example, assume that in one limiting case the enemy must close in a tail chase to 2 nmi to use his weapons effectively. Given that the new missile will have a 40g axial and lateral acceleration capability, Figure 5 summarizes from geometric considerations the potential tradeoff between missile angular capability and detection range to permit launch and kill before the aggressor closes to within 2 nmi. The friendly aircraft detects the target at the specified detection range, then makes a 3g turn until the missile angular capability can be utilized. If the aircraft does not turn, then 180 degree capability is required.

In a second limiting case the aircraft meet head on, or on slightly displaced trajectories. Figure 6 summarizes the angular requirements for this situation for the idealized candidate missile as a function of time from an initial slant range of 2 nmi.

From a review of these geometries and general practicalities associated with carrying defensive missiles on aircraft, a list of desired system characteristics can be assembled (Table 1). The new missile will require innovations in guidance, propulsion, and control moments as a minimum. It would now be useful to define the CAMS program in more detail and then use it to evolve a solution to this problem (if one exists).

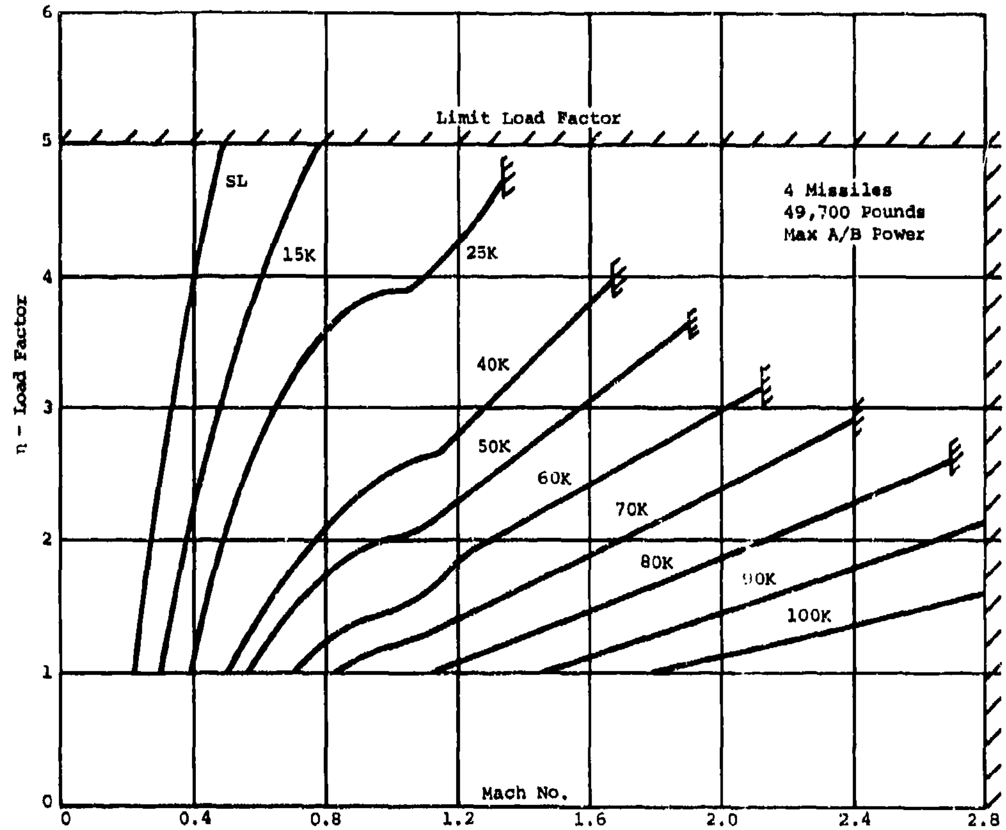


Figure 4. Aircraft Performance

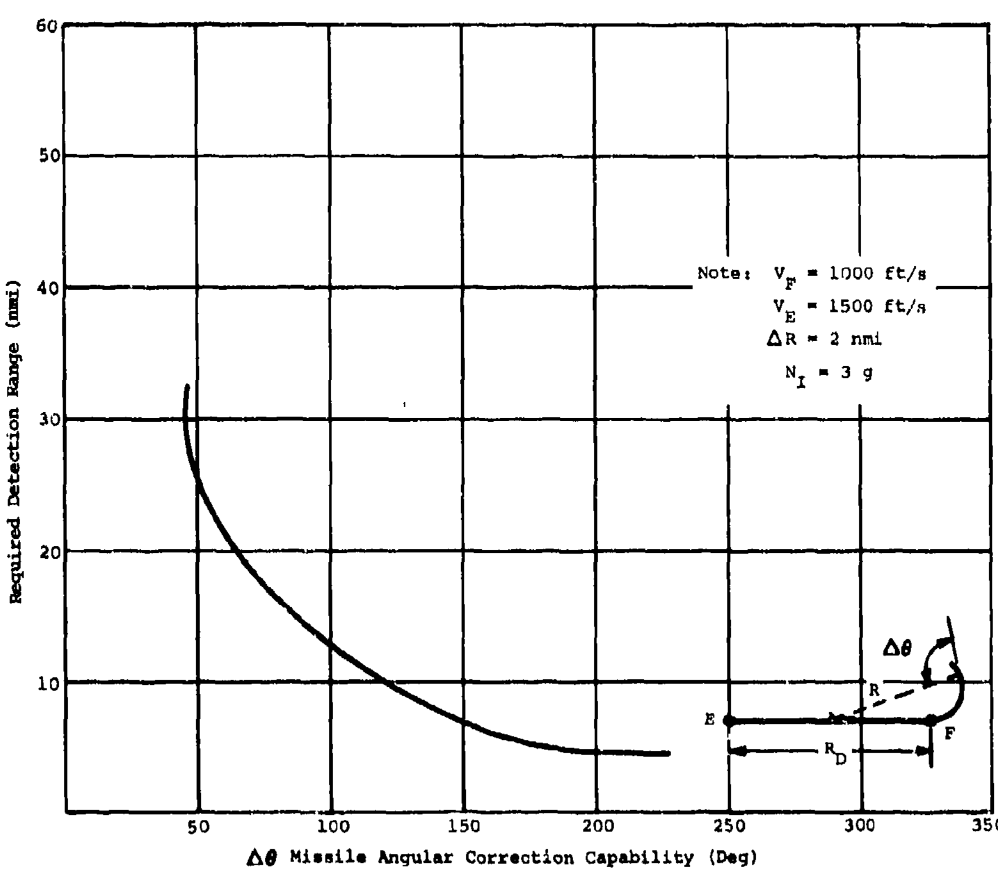


Figure 5. Required Detection Range for Intercepting Enemy Before Launching

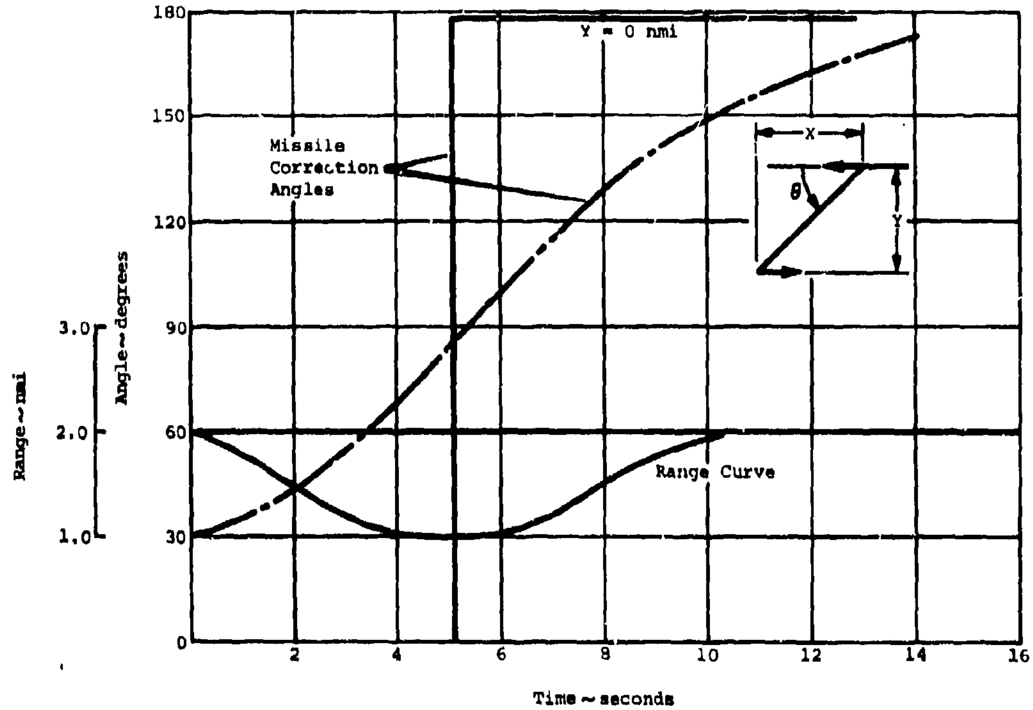


Figure 6. Head-on Engagement Kinematics

TABLE I

Missile Parameters

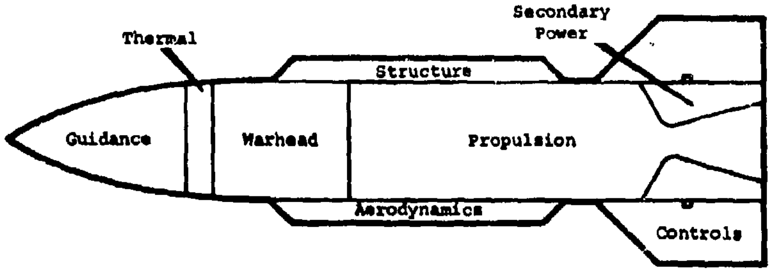
Altitude	Sea level to 100,000 ft
Launch velocity	0.5 to 2.7 mach
Maximum range	6 nmi
Minimum range	1 Kft (V_{CT} 0 ft/s) 4 Kft (V_{CT} 3,000 ft/s)
Angle correction	180°
Weight (maximum)	250 lb
Probability of kill	0.9
Length (maximum)	120 in
Diameter (maximum)	7 in
Axial acceleration	40g
Normal acceleration	40g

CAMS DEFINITION

In any computer program that evaluates system performance, all pertinent subsystems must be considered in sufficient detail to reflect practical nonlinearities. Figure 7 pictorially presents the subsystems and engineering disciplines that are considered in the CAMS design and evaluation process. In general, the subsystem design programs perform the computations and analyses normally performed by the preliminary design engineer either at his desk or through computer programs unique to his discipline. The goal in development of this program is that, at least through the major component level, the design is synthesized based on the physics of the particular design situation, as opposed to an estimate based upon correlation of historical observations of previous systems. This approach enables the user to inspect new technology such as advanced materials properties which could only be addressed subjectively if an observation technique were employed.

In identifying the scope of a program, it is often essential to identify its limitations. From an overall applicability viewpoint, the program encompasses vehicles of from approximately 5 to 30 inches in diameter, and with length to diameter ratio of 3 to 30; these vehicles are single stage, having a single propulsion system (air breathing systems are defined to include the booster) and are basically cylindrical bodies with an ogive forebody. The missiles are assumed to be air launched, although the solution is also generally applicable to ground launched missiles.

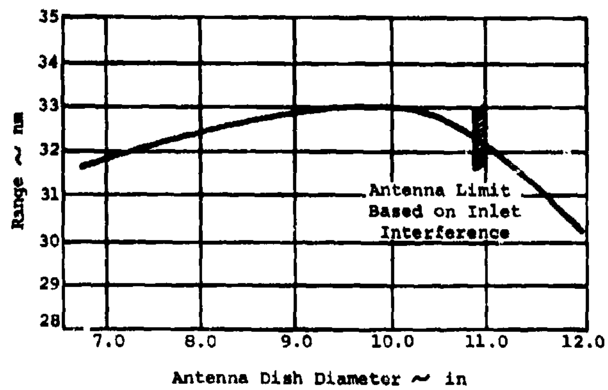
It should be emphasized that CAMS is basically a design (synthesis) program and does not provide an optimum design. An optimum design is the result of many perturbations using the integrated designs resulting from the CAMS program and detailed individual analysis.



Requirement and Evaluation:
Trajectory/Navigation
Miss Distance
Cost
Mass Properties
Configuration

Figure 7. CAMS Scope

The program may be used in two modes: a missile design mode, or as stand-alone disciplinary subprograms. In the latter, it would be employed to screen candidates for a given system against coarse indicators such as weight or volume, or to evaluate the best design conditions to be employed in the missile design mode. The impact of each subsystem on the total vehicle configuration and performance can then be assessed. The results of a guidance tradeoff in the stand-alone mode is shown in Figure 8 which reflects the impact of changing antenna diameter in an active radar system on the performance of an air breathing missile.



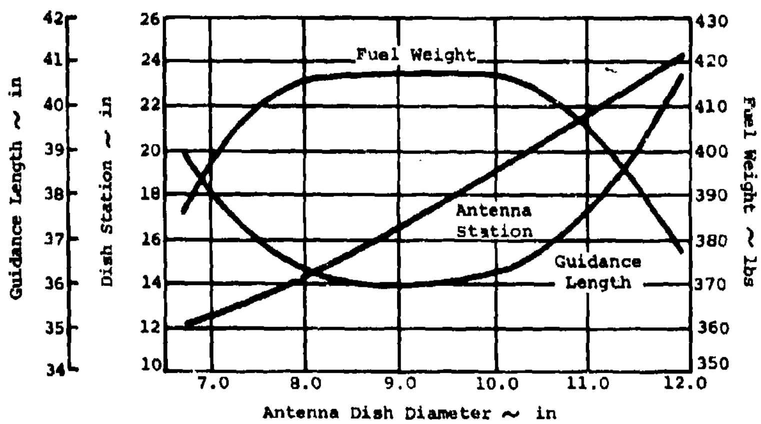


Figure 8. Antenna/Missile Performance Trade

It is possible to evaluate a mix of existing and new subsystems in the study of flight demonstration vehicles, modifications to operational systems, etc. This is accomplished through a by-pass arrangement which permits insertion of the data for an existing subsystem.

In addition, an easily operated stacked-runs capability exists which permits variation of any design parameter so that the most sensitive parameters can be identified when the user is designing a missile which is outside his experience.

The general computation flow is shown in Figure 9. The program starts with the definition of the payload by the guidance, warhead, and packaging subprograms. The propulsion system is then designed followed by the sizing of the control surfaces and the design of the structure and control systems. The weights of

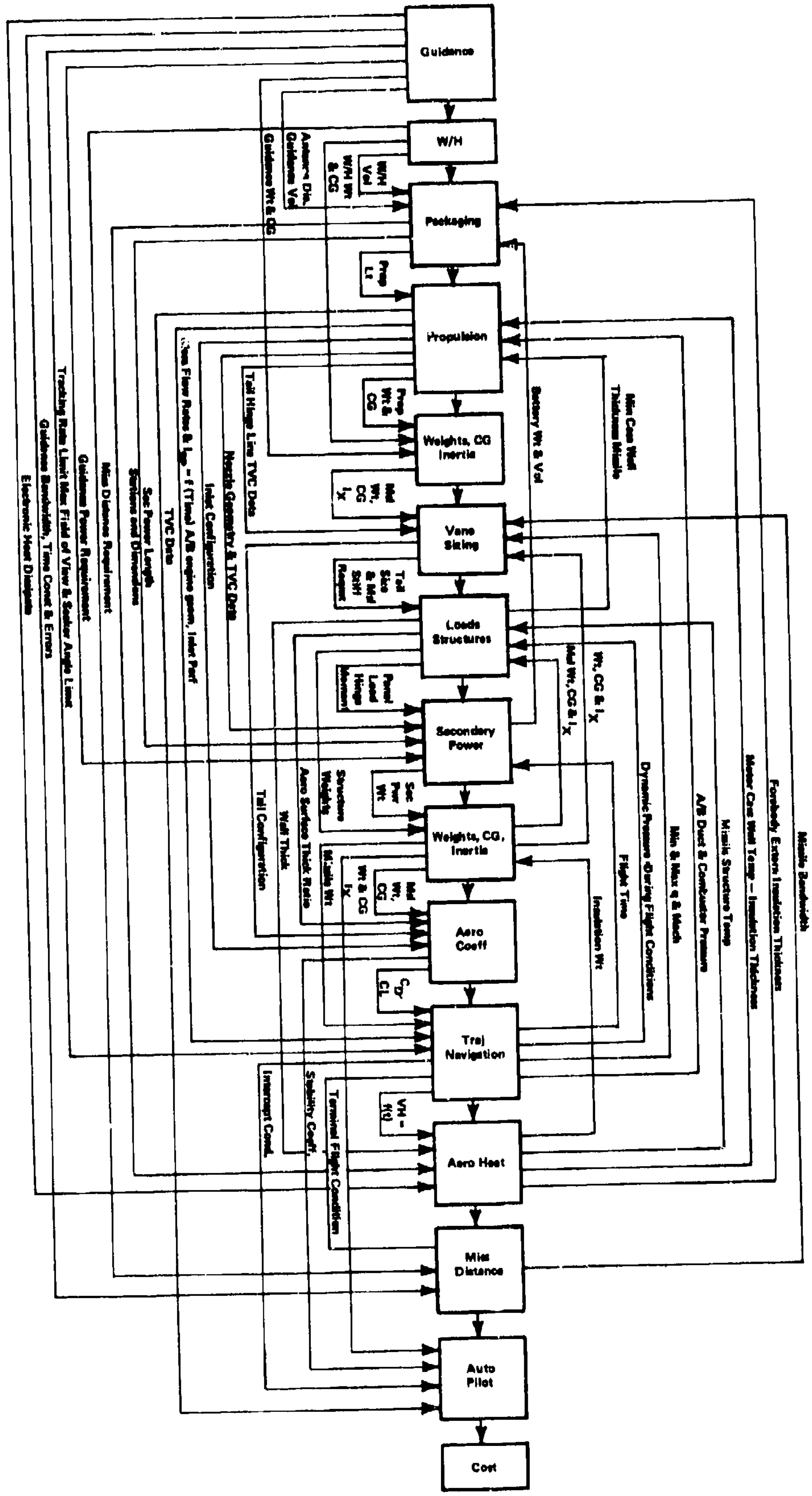


Figure 9. CAMS Subsystem Design Module Interface

the components are summed and compared with the weight at the start of the program; the problem is iterated until there is a satisfactory agreement; performance is ascertained; thermal response is computed; and the airframe material frequencies required to achieve the miss distances are determined. When the desired performance parameters are achieved, the cost evaluation phase is entered with inputs (not shown) from each of the other modules.

The output is derived from the subprograms run during the computation process and from additional subprogram computations. For example, aero data required to perform the computations are limited to drag, lift, and pitching moment coefficients. During the output phase, additional derivatives are computed. In addition, the linear analysis of the autopilot is conducted, the missile cost is estimated, and additional trajectory computations are performed. Selected data are placed on tape for subsequent plotting on off-line devices. Examples are trajectory maps, aerodynamic coefficients, and linear analysis root locus plots. Certain multidimensional results such as the aerodynamic data shown in Figure 10 can be presented as plots; other parameters such as packaging data are printed out in tabular form (Table II).

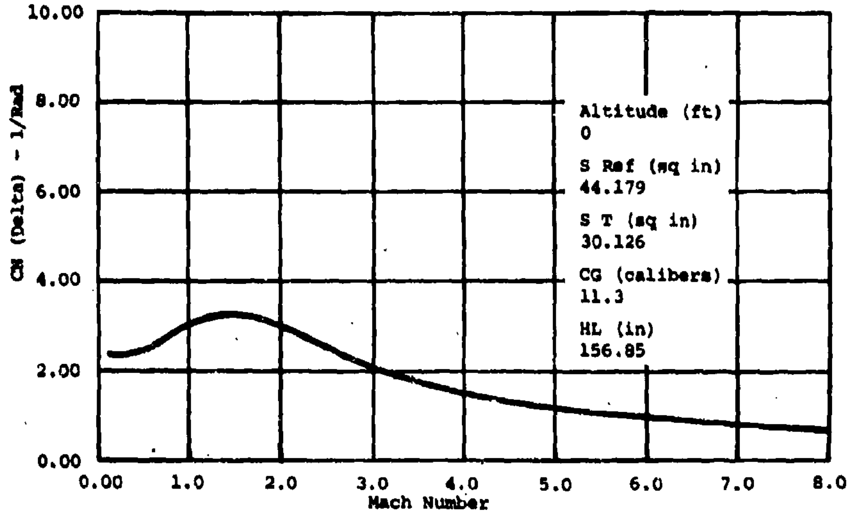


Figure 10. Control Fin Effectiveness, Normal Force Slope

TABLE II

Packaging Data

Computer-Aided Missile Synthesis Program
CAMS Design Review Sample Run
Packaging Data
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Radome length	17.01
Seeker section length	8.93
Electronics section length	7.00
Warhead section length	11.20
Wing control section length	0.0
Roll control section length	0.0
Total non-propulsion length	44.13
Nose bluntness length (XBLUNT)	18.25
Radome length (XRDOME)	17.01
Seeker length (SEEKL)	7.93
Antenna thickness (TDISH)	2.06
Antenna gimbal arm (GLOC)	2.56
Antenna radius (RANTEN)	6.61
Bulkhead thickness (TBLKHD)	1.00
Seeker clearance length (CLSEEK)	1.00
Electronics plus battery length (GL)	6.00
Guidance clearance (CLGL)	1.00
Delta warhead clearance (DLCLWH)	0.0
Warhead length (WHL)	10.20
Warhead aft clearance (CLWH)	1.00
Warhead diameter (DWH)	13.00
Nose bluntness radius (RNOSE)	6.00
Radome thickness (TRDOM)	0.60
Radome weight (WRDOME)	38.08
Bulkhead weight (WBLKHD)	27.80
Missile forebody diameter (DFORE)	16.00
Nose fineness ratio (FINE)	3.00
Warhead volume (VWH)	2000.00
Electronics volume (VOLEL)	895.08
Guidance insulation (TINSG)	0.20
Warhead insulation (TINSW)	0.10
Total missile length (TL)	170.00

Table III presents the subsystem design options available. The user designates the options desired and specifies several trajectories (Figure 11) to start the iterative process. One trajectory, designated the design trajectory, is utilized to establish performance measures and guidance demands and possible axial loading conditions, autopilot design, and thermal environment. The next trajectory is constructed to represent the worst thermal environment, and to generate data for use in the thermal mode. The last three trajectories are optimal and are worst case, used to determine the design conditions for the autopilot and loads. The trajectories shown are representative; the user must input the trajectories for individual problems. If only the design trajectory is input, it will serve as the thermal, load, and autopilot design also.

It is important for the user to appreciate both the size of the program and its running time. In line with the practice of denoting size by the number of boxes of statement cards, CAMS is a 30 box program. It operates with 89 segments on the IBM 360/65 at a core high water mark of 410k bytes. Run times are dependent upon the trajectories employed and so are only indicative. Typical values for a solid rocket are 6 minutes of central processing unit (CPU) time while an air breathing design will take approximately 10 minutes.

The guidance system subroutine will be examined next for a more complete understanding of the operation of the program.

TABLE III

CAMS Subsystem Options

Guidance System	Secondary Power	Propulsion
Active Radar	Control Actuation	Rocket
Semi-Active Radar	Hydraulic Turbo Pump	Solid
Active Laser	Hydraulic Motor Pump	Liquid
Semi-Active Laser	Hydraulic Gas Blowdown	
Electro-Optical	Pneumatic Cold Gas	Air Breathers
Infrared	Electro-Mechanical	(Podded, Integral)
Ultraviolet	D. C. Torquer	Ramjet
Command		Solid Ducted
Inertial Guidance	Electrical Power	
ARH/HOJ	Battery	Thrust Vector
Correlator	Turbine Alternator	Movable Nozzle
	Shaft Extraction	Liquid Injection
		Hot Gas Injection
		Jet Tabs

Control	Aero-Configuration
Autopilot or Torque Balance	Nose Shape (Ogive, Cone, Haack, Power, Hemisphere)
TAIL/body	Nose Blunting
TAIL/wing	Boattail
CANARD/wing	Surface Arrangement (, , ,)
TVC/TAIL/body	Surface Planform
WING/tail	Surface Section (Wedge, Convex, Diamond)
TVC/TAIL/wing	End Plates
Autopilot	
TVC/body	

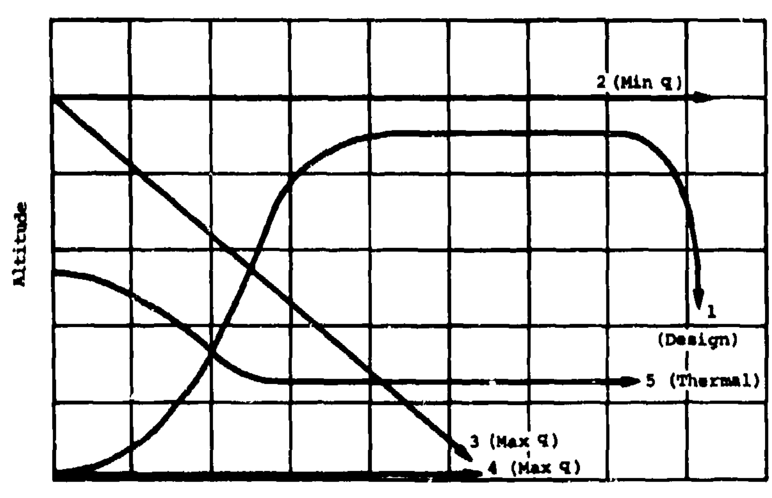

Figure 11. Trajectories

GUIDANCE INFORMATION PROGRAM DESCRIPTION

This program provides a self-sufficient subprogram for synthesis and trade-off analysis of active, semi-active, passive, and autonomous guidance information systems. These candidates are shown in Table III.

The program also includes dual mode guidance of the respective mid-course terminal guidance candidates.

The terminal guidance information subprograms provide three basic options: one to calculate aperture; one to compute threshold, detection, and acquisition ranges; or one to compute transmitter or scene power required. In addition, the program will supply the tracking characteristics, signal-to-noise for detection and acquisition, weight and volume of the guidance system, as well as other guidance characteristics peculiar to each guidance technology.

The midcourse guidance candidates are used to determine the range capability, accuracy, and guidance characteristics of this type guidance when used either as a stand-alone or dual mode system. They also provide the miss distance computation for this type system.

The laser semiactive guidance program is a typical subroutine. The guidance system consists of a laser illuminator which is operative on the ground or in an aircraft and a proportional navigation, laser missile using semiactive guidance. Figure 12 is a block diagram of the basic components of this type of concept divided into two parts: the missile guidance and the laser illuminator.

The missile, its optical seeker head, and electronics constitute a conventional homing missile incorporating proportional navigation. The detector for the seeker would be a solid state device, optimized and spectrally filtered for laser radiation. It could be a four-quadrant type, producing bang-bang or possibly semiproportional error signals. The detector and its associated optics are mounted in a two-axis, gyro-stabilized gimbal system in the nose of the missile. The gimbaled optics and detectors are driven by the torque motors acting in each axis to null out any error signals from the detector.

For the four-quadrant detector, four low-noise solid state video preamplifiers, with optimized input impedance and bandwidth, provide signal amplification prior to the threshold detectors. Four level detector threshold circuits for the four quadrants of the detector establish the seeker's operating sensitivity. The levels of these detectors are adjusted to trigger on signal-to-noise ratio levels which will provide a good probability of detection with a low false alarm rate. Signals which exceed the system threshold are fed to the digital tracker and gating circuit. This circuit provides time correlation gating of the signals to eliminate false alarms due to laser backscatter and background. In performing this function the circuit allows only those signals from the threshold detectors which occur at the laser repetition rate to pass through to the error generators. Synchronizing pulses from the laser illuminator are provided to the missile prior to launch to establish timing of these gating pulses and ensure lock-on to the target only. The time correlation technique increases the immunity to enemy countermeasures.

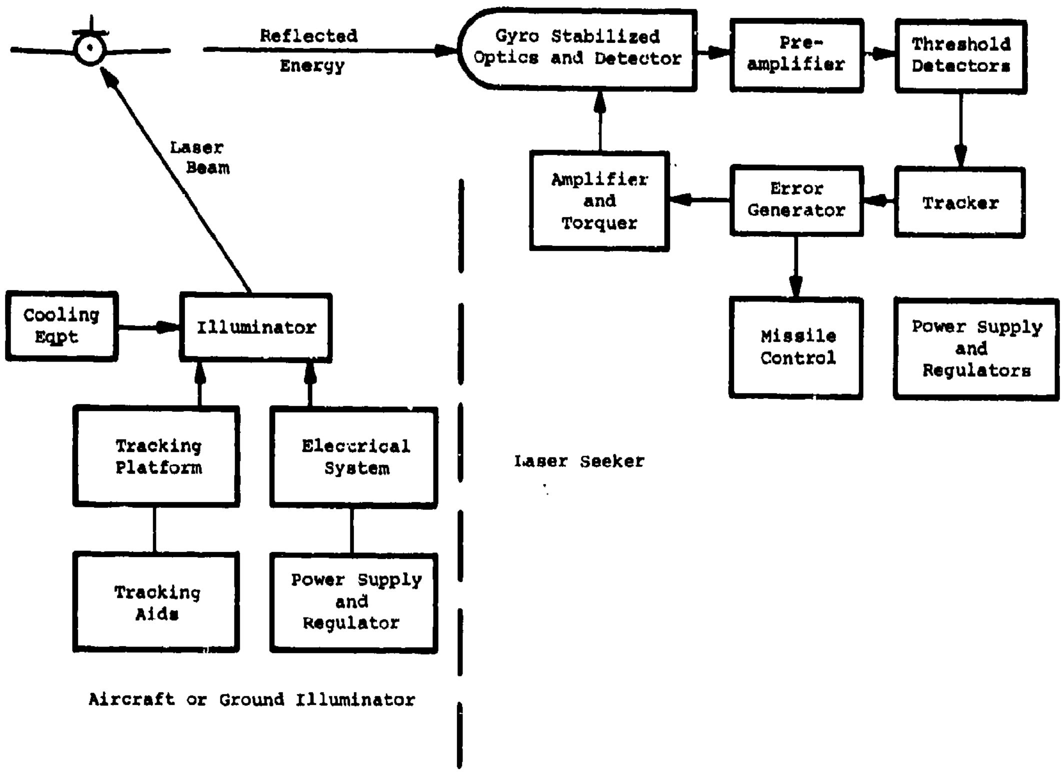

Figure 12. Block Diagram of Semiactive Laser Guidance Technique

Signals from the tracker circuitry are processed in the digital error generators to provide a dc correction signal to the torque motors, thus maintaining the target in the center of the seeker field of view. The digital error generators also sample the position of the gimbaled optics with respect to the missile airframe, and generate correction signals to the missile control section.

The basic components required for illuminating the target are: 1) a Q-switched crystal laser operating in a pulsed mode, to serve as the laser illuminator; 2) a power supply, consisting of either rechargeable batteries or a motor-generator set, to provide the operating power for the laser; 3) the laser electrical equipment, containing the electronic circuitry (storage capacitors, timing circuits, etc.) necessary for controlling the laser's operation; and 4) cooling equipment to provide the necessary cooling (circulated water) for the laser head during its operation.

The laser semi-active subroutine provides a self-sufficient tool for synthesis and tradeoff analysis of a semi-active laser guidance system. The program, at the user's option, will compute the range at which the signal-to-noise ratio is one (1.0), the range at which a probability of tracking is 50 percent, the range at which tracking or acquisition is obtained, and the range at which the system can acquire the target when the detector/seeker is boresighted; the aperture size required for three types of optical systems to obtain a user specified tracking range; or the laser power required to obtain the user's specified range and a given aperture size. In addition, tracking characteristics such as maximum tracking rate, tracking loop time constant, and noise characteristic can be determined. The threshold signal-to-noise ratio and acquisition signal-to-noise is computed based on given illuminator characteristics and false alarm rate. Lastly, the weight, volume, and power requirements representative of this type of hardware are calculated.

The user also has the option of selecting the type of target reflectivity. The material that makes up the target may be such that it acts like a diffuse reflector, a semispecular reflector, or a semi-diffuse reflector.

The program performs a tradeoff analysis for a given acquisition range and optical aperture to show the effects of field of view on signal to noise, scan rate, optical T:No, and probability of detection.

The user has the option of selecting one of three basic optical systems. The first is a reflective system; for example, a Cassegrainian optical system. The second is a reflective-refractive optical system which has a refractive primary lens and a reflective secondary element. The third optical system contains purely refractive elements.

Next, the user can select an air-to-air or an air-to-ground mission, which affects the scan rates of the system.

GUIDANCE PROGRAM OUTPUTS

The performance and seeker characteristics are transferred (Figure 13) to other modules, as well as serving as user information.

The guidance volume requirement is divided into seeker volume and electronics volume; however, these two are added to yield the guidance volume, which is sent to the packaging module. The seeker length and dome diameter for the semiactive laser optics system are also transferred to packaging. The dome diameter is used to determine the point on the tangent ogive to place a hemispherical nose.

The guidance weight is transferred to the weights module; however, for user benefit, the guidance write-out specifies the seeker weight and electronics weight. The electronics weight includes the weight of the power supply and regulators; therefore, average power requirements for the guidance system include the electronics/power supply; gimbaled spin motors and torquers; and detector power. These values are transferred to the secondary power module. The field of view, gimbal limit, and maximum tracking rate are used in the trajectory-navigation module as limits which are continuously monitored, and appropriate messages are printed when these limits are exceeded. For example, during the terminal guidance portion of the missile trajectory run, the gimbal angle of the seeker is compared to the limit. If this limit is exceeded, then a message is printed stating so and the program is continued.

The miss distance module assays the accuracy of the missile based on such parameters as guidance bandwidth, reference range, acquisition range, and guidance one-sigma error sources. The reference range is that range at which the signal to noise equals one.

The cost module assesses the type of guidance and optics and prices the semi-active laser system.

To illustrate the program operation, assume a semi-active laser guidance system is desired for the air-to-air missile requirement defined earlier. The system would use an illuminator on the launch aircraft to perform midcourse guidance through the high angle turn using a beamrider technique and conventional terminal guidance. The beamrider alone has very poor accuracy at long ranges, whereas the terminal guidance system cannot be designed to make large angle corrections with a lock-on before launch system.

To achieve the high turning rates, a tail controlled aerodynamic configuration with dual level thrust control is defined (Figure 14) with jet vanes to give the effect of thrust vector control. During the first phase of the flight, for an enemy on tail engagement, the missile is required to turn as fast as permitted. The aero capability is limited to approximately a 30 degree angle of attack. The effect of the jet vanes is to increase the angle of attack to approximately 45 degrees. To assist in turning the missile as fast as possible, the thrust is kept at a low level to keep the missile velocity low during the turn. The vanes tend to erode due to the high temperature exposure; however, they last for approximately 3 to 4 seconds, which is sufficient to allow a 180 degree maneuver. The aero capability of the missile is sufficient with the high thrust level to allow it to overtake the target of interest here.

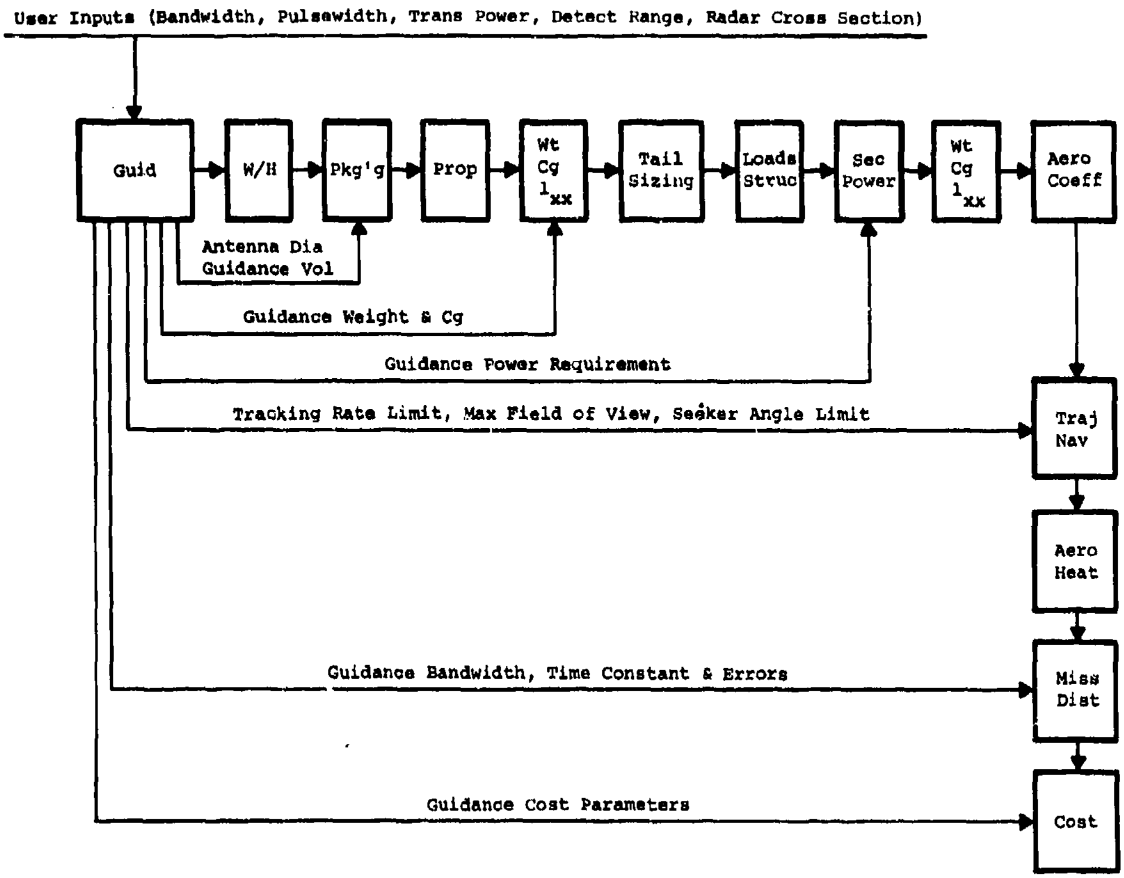

Figure 13. CAMS Guidance Module Interface

Weight 145 lbs

Advantages/Disadvantages

1. Improved Range
2. Good turn capability (α_{max} = 45°)
3. Poor A/C missile interface due to wing span
4. Improved spherical coverage

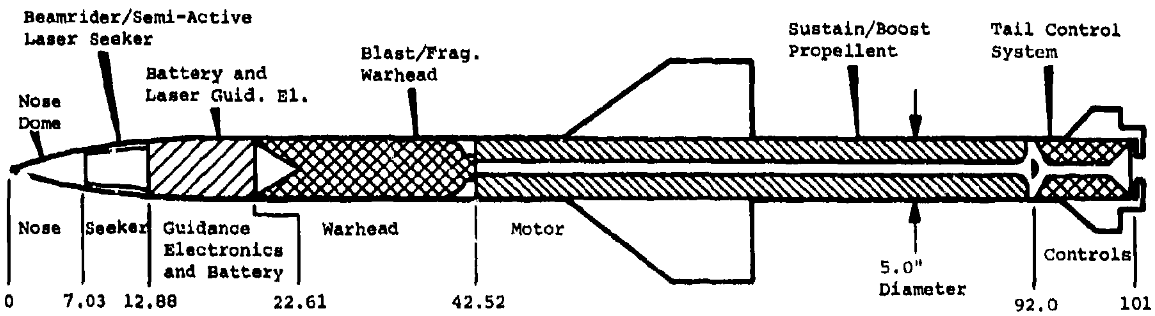

Figure 14. Conceptual Missile System

The missile is launched with the lower thrust profile for the rear hemisphere engagement and commanded to a large angle of attack until the terminal guidance system picks up the reflected energy from the target; then the terminal guidance signal is used to trigger the dual area actuation system which closes the nozzle area and yields a high thrust level. Thus, the propulsion is utilized in the direction of the target and yields an excellent protection footprint.

For the head-on engagement, it is most important that the enemy be killed as soon as possible to prevent him from launching his missile on the friendly aircraft. Thus, if the target is within the terminal field of view at the time of launch, the high thrust level is commanded immediately.

Before ascertaining the detailed characteristics of the guidance system, the CAMS program should be used to develop several trajectory runs with no restrictions upon field of view or gimbal rates to approximate the missile performance and establish that this configuration is in the correct performance range. Figures 15 and 16 illustrate the resulting trajectories which satisfy the original requirements. Table IV summarizes the inputs required for this guidance option of CAMS. All of the values shown are stored in the program and will be used to determine performance unless changed by a new input card. Table V furnishes the output data to establish if the iterative numbers fed to other subroutines have consistent values and that the mission requirements are satisfied.

The output indicates that the signal-to-noise ratio is satisfactory. The optical dome (called RADOME for purposes of common program output) can be built, the acquisition and detection ranges are slightly under those desired, while weight, power, size, etc. are within acceptable limits.

This iteration (conveniently chosen) may convince the guidance analyst of the feasibility of the new missile concept; however, everyone is not so pleased. A great many problems have been transferred from the missile to the aircraft fire control system. Specifically, what is going to keep the laser pointed at the right geometric position in space to provide the beamrider midcourse guidance and then hold it on target for the terminal phase in the face of aircraft motions and maneuvers? Will the propulsion man smile about a dual level thrust system? Truly, our guidance analyst is assisting these technologies by pointing out areas for their research and development. Our guidance analyst has accomplished the first iteration in weapon system evolvement; he has thrown the problem to someone else. If he is part of a systems team, the problem will shortly come back to him with instructions to use single level propulsion and a different guidance technology. Then, back to the drawing board, oops, computer.

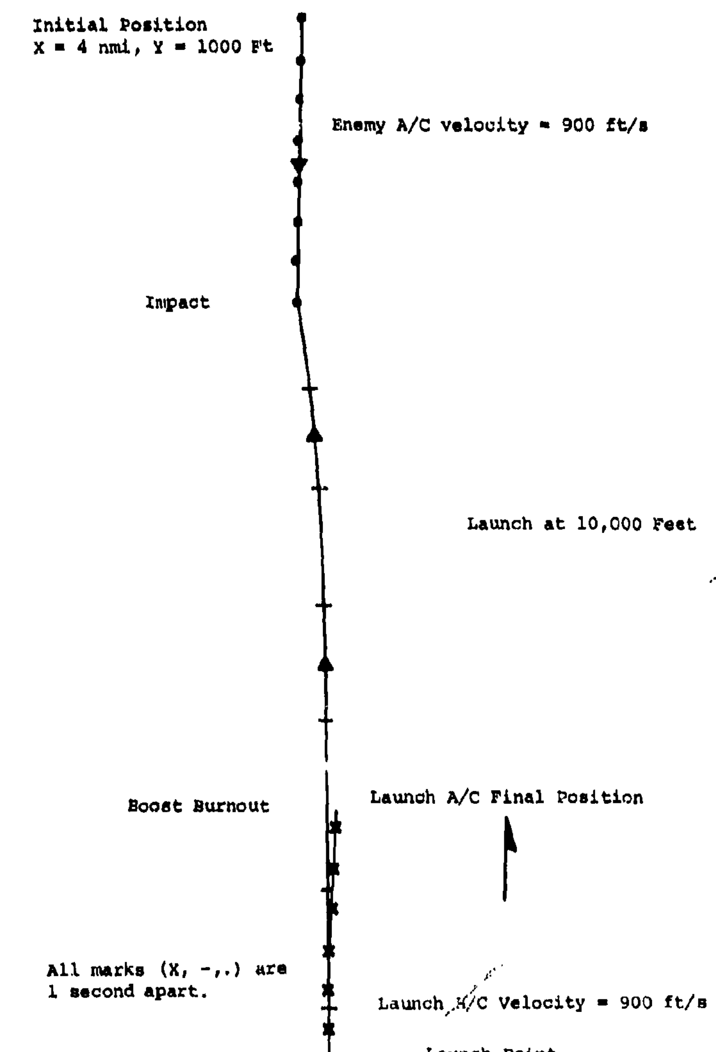

Figure 15. Head-on Engagement (Kill Before Launch)

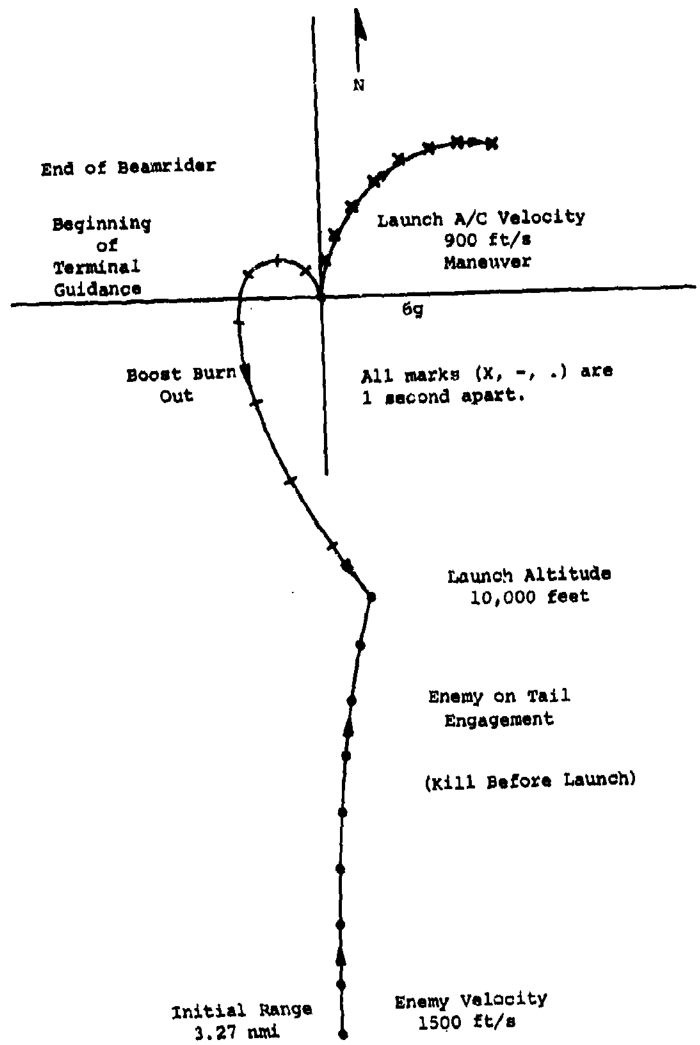

Figure 16. Turn-Counterturn Kill

TABLE IV

Computer Aided Missile Synthesis Program
Semi-Active Laser Guidance
19.21.19 Thu 11.18.71

Input Summary

Air-Air Case
Refractive-Reflective Optics

Diffuse Reflective Target

Air-Air Case
Collocated Semi-Active Laser

Acquisition range (nmi)	1.0000	Detector quantum efficiency	0.30000
Detector dark current %micro amp)	0.10000E-00	Wavelength %meters)	0.10600E-05
False alarm rate	6.0000	Video bandwidth (Hz)	0.11700E-08
Preamplifier noise %NA<	0.60000E-01	Height of target %Kft<	10.000
Solar irradiance %w/sq m/a)	15.000	Height of receiver %Kft<	10.000
Meteorological range %nmi<	60.000	Laser pulse width (μs)	0.15000E-01
Target normal to LOS angle %deg<	0.20000	Boresight error %mr< (tracking)	0.20000
Aiming error %mr< (tracking laser)	0.25000	Target velocity rel to LOS (ft/sec)	900.00
Thickness of dome glass (in)	0.10000E-08	Length from optics to pivot (in)	0.50000
Illuminator peak power (watts)	1.0000	Height of illuminator %Kft)	10.000
Illuminator range %nmi<	0.20000	Aperture %sq in<	6.2500
Beamwidth %mr< (tracking)	1100.0	Optical transmission	0.69000
Optical filter bandwidth %A<	0.70000	Reflectivity of background	0.10000E-00
Reflectivity of target	5.00000	Detector diameter %in<	0.75000
Seeker aft bulkhead diameter (in)	0.90000	Velocity of aircraft/missile %ft/sec<	0.90E-Q3
Detection probability	30.000	Electronics density (lbs/in^3)	0.21750E-01
1/2 scan angle %deg<	10.000	Minimum guidance range (nmi)	0.50000E-01
Pulse rate frequency (C/S)	30.000	No. of quadrant cells	4.0000
Desired field of view (deg)	1.0000	Effective area of the target (ft^2)	10.000
No of quadrant summed before detection			

Compute Range

TABLE V

Computer-Aided Missile Synthesis Program
LAAM Design Case
Semi-Active Laser Guidance
19.21.19 Thu 11.18.71

Output Summary

Signal/Noise

Signal/noise ratio (dB)	8.722	Threshold/noise ratio	6.177
Noise bandwidth %Hz<	0.2933E-08	Fluctuating signal to noise (dB)	0.0
Total signal to noise (ratio)	7.452		

Radome Sizing

Aperture length (inches)	2.9610	Aperture (inches squared)	6.2500
Focal length (optics/antenna) in	1.3995	Thickness of radome	0.25000
Dish length	0.0	Length from pivot to front of dish	0.75000
Gimbal freedom (degrees)	30.0	1/2 scan angle (degrees)	30.0

Range at S/N=1 for Given Aperture

Signal current (amps)	0.18790E-07	Internal noise current (amps)	0.60000E-08
Solar shot noise current (amps)	0.17335E-07	Aperture diameter (in)	2.8209
Aperture (in^2)	6.2500	Actual signal/noise ratio	1.019

Range at S/N = 1 (Kft) 91.61

Range at Detection

Signal current (amps)	0.11353E-06	Internal noise current (amps)	0.60000E-08
Solar shot noise current (amps)	0.17335E-07	Aperture diameter (in)	2.8209
Aperture (in^2)	6.2500	Actual signal/noise ratio	6.155

Detection Range at PD = 0.5 (Kft) 48.94

Acquisition Range

Signal current (amps)	0.13771E-06	Internal noise current (amps)	0.60000E-08
Solar shot noise current (amps)	0.17335E-07	Aperture diameter (in)	2.8209
Aperture (in^2)	6.2500	Actual signal/noise ratio	7.466

Acquisition Range Off Boresight (Kft) 45.94

Boresight Acquisition Range

Signal current (amps)	0.13768E-06	Internal noise current (amps)	0.60000E-08
Solar shot noise current (amps)	0.17335E-07	Aperture diameter (in)	2.8209
Aperture (in^2)	6.2500	Actual signal/noise ratio	7.464

Boresight Acquisition Range (Kft) 29.94

Guidance System Parameters

Solution of FOV versus PD for range (Kft)	29.94	Aperture (sq in)	6.250

Alpha FOV	Signal/ RMS Noise	T number	Focal Length	Scan Rate	Probability of Detection
2.00	21.480	9.1683	21.484	10.000	1.0000
4.00	20.498	4.5828	10.739	20.000	1.0000
6.00	19.125	3.0536	7.1554	30.000	1.0000
8.00	17.596	2.2886	5.3628	40.000	1.0000
10.00	16.081	1.8292	4.2863	50.000	1.0000
15.00	12.828	1.2156	2.8484	75.000	1.0000
2y.00	10.446	0.90760	2.1267	100.000	0.91101
30.00	7.4643	0.59726	1.3995	150.00	0.90107
*40.00	5.7589	0.43969	1.0303	200.00	0.33808
*50.00	4.6787	0.34319	0.80419	250.00	0.67071E-01
37.55	6.1026	0.47076	1.1031	187.75	0.50000

Guidance

Seeker line of sight (deg)	0.0	Threshold noise ratio	6.17662
Acquisition seeker power (w/in^2)	0.70326E-06	Width of scan %Kft<	1816.41846
Effective beamwidth %R<	0.52510E-03	Circle of equal probability (ft)	9.2590
Seeker dynamic range req	10532.	System false alarm (fa/sec)	0.14207E-01
Blind probability	0.36000E-01	Seeker bandwidth (rad/sec)	20.00
Seeker time constant (sec)	0.5000E-01	Detector responsivity (amp/watt)	0.4540E-01
Maximum tracking rate (deg/sec)	75.000	Maximum scan rate (deg/sec)	150.00
Ratio of the energy that falls on the target	0.27867	Seeker weight (lb)	5.6500
		Electronics volume (in^3)	180.00
Seeker volume (in^3)	113.51	Detector power req (watts)	1.0000
Seeker length (in) (reflect-refract)	4.948	Electronics power req (watts)	10.000
		Guidance weight (lb)	9.565
Electronics weight (lb)	3.9150	Seeker white noise PSD (rad^2/Hz)	0.31831E-07
Seeker power req (watts)	2.0000	Glint noise (ft sq/rad/sec)	5.508
Guidance volume (cubic inches)	293.5	Drift rate noise (rad^2/Hz)	0.1000E-11
Dome diameter (in)	4.734	Seeker dome error slope	0.3000E-01
Bandwidth to glint (rad/sec)	12.00	No roll rate requirements	
Gimbal freedom (deg)	22.50		
Bandwidth of drift (rad/Hz)	8.000		
Range noise power sp den (rad^2/rad/sec)	0.0		

Hemispheric nose section required

III. CONSIDERATIONS IN CONTROL TECHNOLOGY

DEVELOPMENT OF CONTROL SYSTEM REQUIREMENTS

by

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SUMMARY

The control system engineer must be apprised of all mission requirements and the alternate solutions proposed by all subsystem engineers. He must conceive preliminary control system concepts which are compatible with the mission requirements. He then participates in preliminary analytical work which leads to the establishment of increasingly firm control system requirements and the tolerance of the control system parameters to changes in other subsystem parameters.

Examples of the process and the different levels of control system requirements definition are given for representative tactical missile situations.

1. INTRODUCTION

During the life cycle of a tactical weapon system, the requirements for the control system are set in the early design phases. From then on, it becomes increasingly more difficult to change the requirements. The difficulty is a matter of cost, schedule, capability, or combinations. The first topic treated below will describe the timing of control system requirements establishment and who sets the requirements.

Following the description of when and who typically sets the requirements, three examples will be presented which show the variety of issues, candidate solutions, and selections of control systems to meet requirements.

The examples are hypothetical in that no actual weapon systems or their requirements and schedule are cited. The ranges of parameter values and the types of control system implementations are, however, realistic. All are covered in text book and open literature sources. The type of weapon system requirement, process of analysis, reasoning on concept selection, are intended to provide, by example, assistance in current and future tactical weapon control system selection.

2. WHEN AND WHO SETS CONTROL REQUIREMENTS?

Market survey studies reveal that many tactical weapon systems are conceived and identified, but few go all the way through production and deployment. For those which do, the time span is on the order of 7 to 10 years for all the events to be completed.

During this lengthy period, the subsystems, including the control subsystem, undergo their development to maturity. With the aid of Figure 1, the period during which the control requirement is established will be identified.

For a typical tactical weapon system, a ten-year span between first identification as a concept, an acronym, or a budget item is postulated. Figure 1 displays typical times associated with Concept Formulation, Design Development and Competition among interested companies or agencies, the Development and Test of weapon system prototypes, and finally the Production and Deployment of the weapon system.

In the Concept Formulation phase, the pace of weapon system and threat definitions is fast. Calculations and analyses are made for feasibility and performance goals, without complete, thorough coverage of flight regimes. All engineering disciplines - propulsion, structures, guidance, system analyses, etc., - need some inputs with which to carry on their own preliminary work. Therefore, the control system engineer can change his ideas in a few minutes in response to desired weapon system characteristics.

After sufficient iterations and more formal specification of the weapon system requirements and threat definition, favored configurations begin to develop. Each engineering discipline engineer becomes more conscious of his design establishment effect on the other engineers. The control subsystem is one of the last to respond to the desires and requirements of the others, who are reaching and homing on the targets assuming perfect control of the vehicle. In the Design Development phase, a few minutes of discussion will now stretch to hours of trade-off activity, and the beginnings of written commitment records concerning control system requirements and design.

A go-ahead on a weapon system to Development and Test, followed by Production and Deployment, brings about formal, contractual, performance requirements and official documentation. As these phases proceed, the length of time to change a requirement or design feature grows very long. Paperwork must be channeled, boards must meet, approvals must be obtained, funds must be made available. The control system requirements selected in the latter part of Design Development and committed in early Development and Test must stand firm to be considered successful. The times shaded in Figure 1 show the period considered most critical for the control system engineer in establishing the control requirements and design for a major tactical missile. The activity during this period will be emphasized in the examples.

The number and type of control system engineers vary in the weapon system time cycle. A typical variation is to start and maintain a very small number of analysis oriented engineers through the concept and design definition phases. These engineers should have some hardware experience. Their main forte should be synthesis and analysis of control concepts, and simulation of complex systems capability. In the later phases of the weapon system, larger numbers of engineers are required to handle hardware flow, formal paperwork, test and field activity, manufacturing, and field delivery activity. Typical numbers of personnel with engineering degrees and control system specialization are shown in Figure 2 for different phases of weapon system development. The shaded area shown between Design Development and Development and Test signifies that during that time period, one to three engineers make the binding, long-term decisions on requirements to be met and a configuration to meet them.

TYPICAL TIMELINE

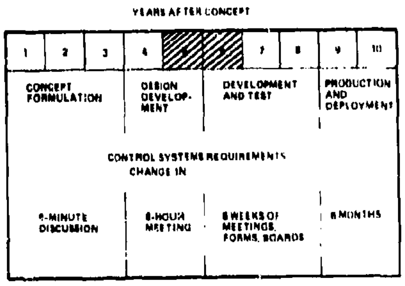

Figure 1

NUMBER OF CONTROL SYSTEM ENGINEERS

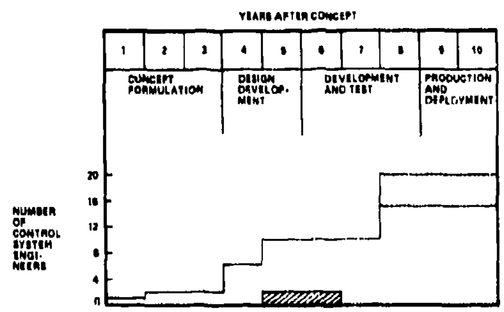

Figure 2

3. EXAMPLES

To illustrate the types of decisions required at the critical time of control system requirement selection, three examples are presented. The relationship between the three situations is that they are different in technical detail but similar in that an engineer, fairly early in the total life of the system, made decisions which were very difficult, from then on, to modify.

3.1 EXAMPLE 1, SURFACE-TO-AIR MISSION, S-A

The control system for a defensive, surface-to-air mission is considered as the first example. Figure 3 shows the general features, and the example is referred to as S-A, for surface-to-air. The total height and range envelope are such that ground commands to the missile are required between launch and acquisition by a terminal homing sensor. The target spectrum is broad, including high and low flying aircraft assumed to be capable of sensing the missile's presence and performing evasive maneuvers, and formation flights designed to counter a one-on-one firm lock-on and track.

In the early phases of study, control system desired features come to light. Simulations show that a midcourse speed loss due to an over-responsive control system inducing too much drag limits the range. High response is required after lock-on to cope with maneuvers. With both midcourse and terminal phases, considerable avionics will be on board, and sharing computing functions is desired. Finally, even though the threat is considered advanced, techniques and hardware proven in the past are desired to minimize program risk. These conflicting desires are indicated pictorially in Figure 4.

These mission desires lead the control engineer to consider alternate solutions to the major control system aspects. Three of the major aspects are tabulated, along with contrasting solutions and their pros and cons, in Figure 5. A roll-to-steer configuration is slower but simpler than a cruciform configuration in carrying out heading change commands. Providing control system gain changes over the wide dynamic pressure regime can be accomplished by ground observation and transmission or by sensing in each missile. Providing the hardware in the missile allows for intermittent ground link loss and increased missile handling capability. The uplink for steering commands places digital equipment in each missile. Therefore the use of digital devices in a computer for autopilot calculations becomes an obvious consideration as an alternate to using conventional analog computing elements. The question of analog versus digital autopilot calculations becomes one of the major trades, with st[illegible]y and cost considerations.

The control system engineer now has to do his analyses, interpretations, and coordination with the other engineers on the program.

EXAMPLE S-A, SURFACE-TO-AIR MISSION

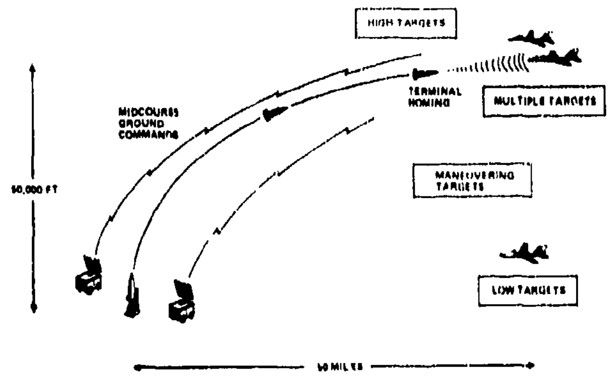

Figure 3

MISSION DESIRES AND CONSTRAINTS, EXAMPLE S-A

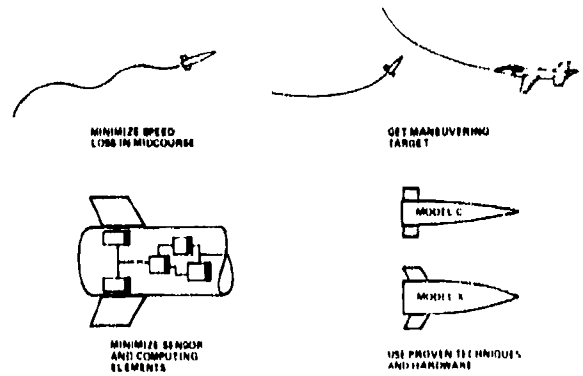

Figure 4

For the issue of roll-to-steer versus a cruciform configuration, a response time requirement needs to be established. The change in miss distance as a function of missile response time is determined from simulation and plotted in the upper left part of Figure 6. A highly responsive missile has small miss distances for 2g maneuvering targets, and has increasing miss distance as the response time increases and the missile becomes sluggish. For 10g maneuvering targets, if the missile is too responsive it loses speed and misses badly. If it is too slow in response, it also misses badly. A regime of best response time is observed. If the roll-to-steer missile response cannot be brought down to this regime, the cruciform configuration will be selected.

MAJOR CONTROL SYSTEM TRADES, EXAMPLE S-A

	CONFIGURATION		GAIN CHANGE		AVIONICS	
	ROLL-TO-STEER	CRUCIFORM	ALL IN UPLINK	SOME ON BOARD	ANALOG	DIGITAL
PRO	BETTER STABILITY FEWER ACTUATION PARTS	SHORT RESPONSE TIME	LOWER COST FLY-AWAY HARDWARE	TOLERATE DATA LINK LOSS	PROVEN INDEPENDENT SUBSYSTEM	UTILIZE CENTRAL DIGITAL COMPUTER
CON	LONG RESPONSE TIME	SIGNAL MIXING REQUIRED MORE ACTUATION PARTS	UPLINK MUST STAY ON COMPLEX GROUND SYSTEM	HIGHER COST	HIGHER COST	UNFAMILIAR TECHNIQUE

Figure 5

SIMULATION RESULTS, EXAMPLE S-A

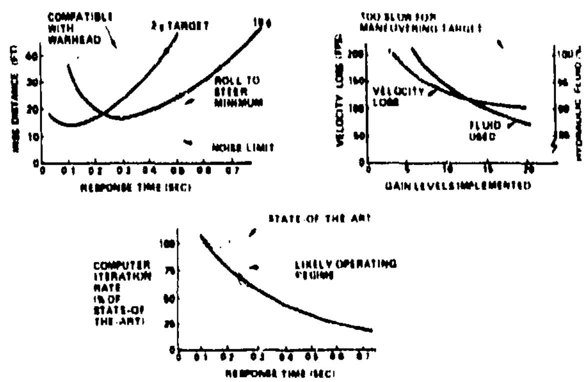

Figure 6

To see whether the control system gain changes should be computed on the ground and transmitted, or should be implemented in each missile, an analysis is performed. The number of gain levels incorporated is varied as the missile passes through the spectrum of dynamic pressures of midcourse flight. The velocity lost due to missile control activity, and the power required to produce this activity, are highest for a small number of gain level changes (non-optimal) and decrease as the number of gain levels increase (near-optimal). Tolerable velocity losses and power levels are marked on the upper right data of Figure 6. The number of gain level changes can be picked. A small number can be implemented easily in each missile; a large number will be too costly to implement in each missile.

The establishment of a nominal response time for the missile assists in determining the feasibility of a digital autopilot. If the computation rates are within state-of-the-art computer capabilities, so that development risk is low, then the cost trades on avionics hardware favor the all-digital computing system. The lower curve of Figure 6 shows computer iteration rate required for analog-like performance as a function of response time. For rapid, near-zero, response times, the computer speeds cannot be met. For slow missile response, or long response time, the computer speeds are well within the state-of-the-art.

The major decisions made by the control system engineer are arrived at after considering the data and the situations. They are, as summarized in Figure 7, 1) roll-to-steer is too slow; go with a cruciform configuration even though the complexity of sorting out the commands to the four actuators is complicated; 2) use the up-link for the numerous gain change commands and give up the ability to ride smoothly through an up-link loss; 3)use the less familiar digital autopilot, since computation rates are within the state-of-the-art and a cost saving can result from advances in digital circuit components.

The final step of the major commitment by the control system engineer is to set down and formalize the control system requirements. He establishes these in the accepted media of the program. Figure 8 illustrates a tabular summary of the typical items to which he commits. The double-ended arrow signifies that, from this point on, changes in any of the subsystems on the right could influence one or more of the control system values and cause possible major impacts. Similarly, if the control system implementation does not meet the committed values, one or more of the subsystems may be impacted in a major way.

CONTROL SYSTEM SELECTION, EXAMPLE S-A

- **CRUCIFORM, ROLL-STABILIZED**
- **GAIN CHANGE ON UP-LINK**
- **DIGITAL AUTOPILOT**

Figure 7

CONTROL SYSTEM REQUIREMENTS, EXAMPLE S-A

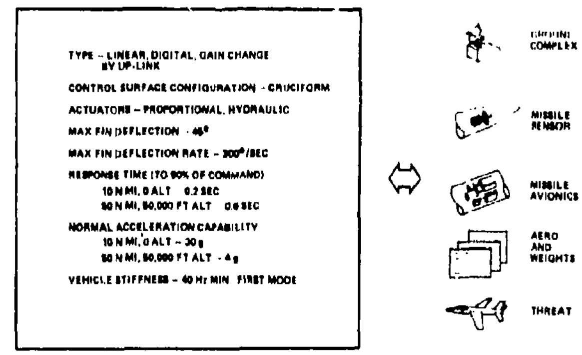

Figure 8

3.2 EXAMPLE 2, SURFACE-TO-SURFACE, S-S

In the second example, some of the same and some different considerations are involved for the control system engineer. The example considered is a rail launched missile, boosted to flying speed, and then powered by a cruise engine on a relatively long subsonic flight. The flight is programmed so that a long high altitude midcourse flight is followed by a low level dash and homing on a specified target. The altitude - range and altitude - speed regimes are shown in Figure 9; and the example is referred to as S-S for surface-to-surface.

Some of the major mission issues which the control system engineer is faced with are shown in Figure 10. Control of the time of arrival at the target is required, for overall mission effectiveness. This requires speed control as an additional loop. A mission requirement which dominates many control system requirements is for terrain clearance. A probability of clobber at a particular average altitude is desired by mission analysts and presented to the control system engineer. The mission planners want no restrictions on their ability to plan and execute flights from weather or geographical location. Finally, the propulsion, aerodynamics, and structure preliminary designs converge on a few possibilities which look favorable provided the control system limits the angles of attack and sideslip so that sufficient air is available to the engine inlets.

EXAMPLE S-S, SURFACE-TO-SURFACE MISSION

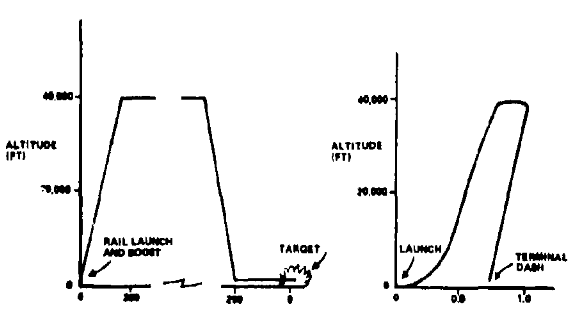

Figure 9

MISSION DESIRES AND CONSTRAINTS, EXAMPLE S-S

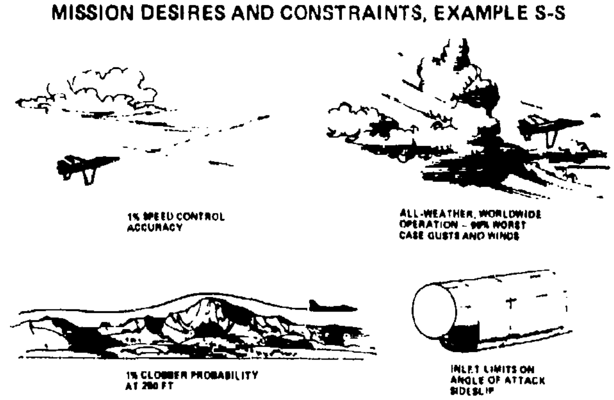

Figure 10

The major trades and decisions required of the control system engineer are brought out by the mission requirements. If speed control, terrain clearance, and no masking of the inlets, can be implemented with two axes of control elements rather than three, a cost savings results. If pre-launch time can be allotted to inputting gain changes to each missile, then the world-wide operation can be effected without an adaptive autopilot. Fast acting hydraulic actuation is more costly than using bleed air from the engines for rotating the control surfaces. A mechanical clutch, powered from the engine, is a risky development but low weight approach. Finally, the trade between an independent control subsystem with analog computing elements versus an integrated control subsystem using a central digital processor appears as an issue. These trades and their pros and cons are displayed in Figure 11.

Decisions are sought by analysis, simulation, and discussion with the other subsystem engineers. The decisions on the major trade items are interrelated with one another and greatly influenced by the other mission constraints, particularly the low altitude and range requirements.

To help decide whether a two axis roll-to-steer configuration can be used, the data of the upper curve of Figure 12 are generated. The ability to recover from substantial gusts inducing sideslip is evaluated for a nominal configuration with two axes and three axes of control. At 5° of induced sideslip, inlet air begins to fall off rapidly. The sideslip angle is determined as a function of yaw axis static stability, which increases as the tail surface increases. Since drag increases with tail surface area, reducing range, a limit is set at which the drag is considered excessive. Designs which hold sideslip below 5° and tail area below the value corresponding to 0.007^{-1} are considered acceptable.

MAJOR CONTROL SYSTEM TRADES, EXAMPLE S-S

	CONFIGURATION				ACTUATION			COMPUTING ELEMENTS	
	AXES CONTROLLED		GAIN CHANGE						
	TWO	THREE	ADAPTIVE	PROGRAMMED	HYDRAULIC	PNEUMATIC	ELECTRO-MECHANICAL	ANALOG	DIGITAL
PRO	MIN COST, WEIGHT VOLUME	BETTER DISTURBANCE CONTROL	EASIEST FOR VARIED CONDITIONS	LOWER COST, LESS HARDWARE	BEST RESPONSE	GOOD SHELF LIFE LOW COST	MEDIUM RESPONSE MEDIUM WEIGHT	PROVEN, LOW COST COMPONENTS	FLEXIBLE
CON	HIGHER RISK	HIGHER COST, WEIGHT, VOLUME	HIGHER COST	MISSION RISKS	EXTRA SUB-SYSTEM LOW SHELF LIFE	POOR RESPONSE	HIGH RISK	CHANGES VERY EXPENSIVE	HIGHER RISK

Figure 11

ANALYSIS AND SIMULATION RESULTS, EXAMPLE S-S

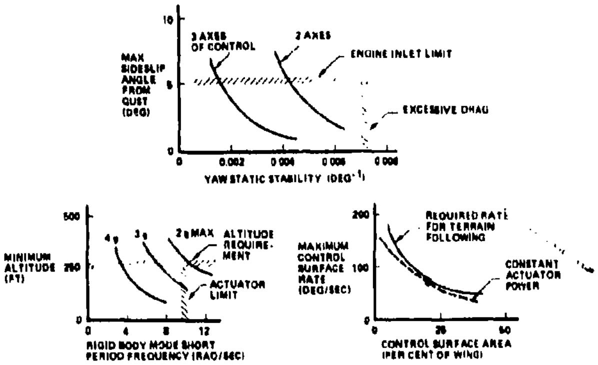

Figure 12

The issue of the autopilot loop gain change implementation is decided by considering mission convenience versus cost. Both adaptive systems and programmed systems will work. The adaptive system, once implemented, is simpler to use than the programmed system for which the pre-launch required program changes with weather and profile.

The actuation system selection is dependent primarily on the low altitude requirement. Missile flight at altitude in mid-course and in the homing phase do not call for large accelerations or rapid response. The lower left curve of Figure 12 shows the variation of minimum altitude with short period frequency for different maximum missile acceleration levels. The higher the frequency the fewer the g's required for terrain avoidance. However, at too high a frequency control surface actuators cannot be obtained. The solid curve in the lower right portion of Figure 12 shows the surface rates required for terrain following as a function of control surface area. Since control surface area is proportional to hinge moment, the peak power required can be determined at each possible operating point. By trial, an operating point can be found at which, for the same peak power, less than the maximum surface rate required for terrain following results. Therefore, the dashed curve of constant actuator power suggests operation at about 25% of the wing area.

The considerations on analog versus digital equipment for the on-board calculations are similar to the adaptive versus programmed autopilot issue. The system will work both ways. Analog is more familiar. Digital is more flexible and can be incorporated into other computing hardware.

A set of configuration decisions is reached and is listed in Figure 13. It is decided to go to the lower cost two-axis control system since sufficient margin appears available to handle the gust problem. With an edge in low cost from the two axis decision, the mission planning convenience of an adaptive system is selected. The rigid body mode, g level selection is made to hold wing area down with a 3g maximum acceleration at about 8 rad/sec. When the peak power required is calculated, it is found that bleed air from the engine can provide sufficient torque at high enough frequency to use pneumatic actuation. Finally, with low cost hardware for the control and actuation devices, the flexibility of digital operation is chosen over the lower risk analog computing elements which are less expensive at the start but very difficult to change.

With the major design decisions made, the control system engineer can now establish a set of requirements to design and build to, similar to the listing of Figure 8 for the surface-to-air missile. As in that case, it becomes very difficult to change the design values of the control subsystem or react to changes in the other subsystems once the requirements are set.

3.3 EXAMPLE 3, AIR-TO-SURFACE, A-S

The third example to be discussed is an air-to-surface mission, referred to as A-S. The mission flight phases include launch over a wide range of airplane speed and altitude conditions, missile midcourse flight over a wide range of speed and altitude conditions, and terminal homing on ground targets. The homing phase does not dominate the flight control design; launch safety and the wide dynamic range of flight conditions are the key items. The mission is illustrated in Figure 14.

CONTROL SYSTEM SELECTION, EXAMPLE S-S

- CONTROL TWO AXES
- ADAPTIVE
- PNEUMATIC ACTUATION
- DIGITAL COMPUTATION

Figure 13

EXAMPLE A-S, AIR-TO-SURFACE MISSION

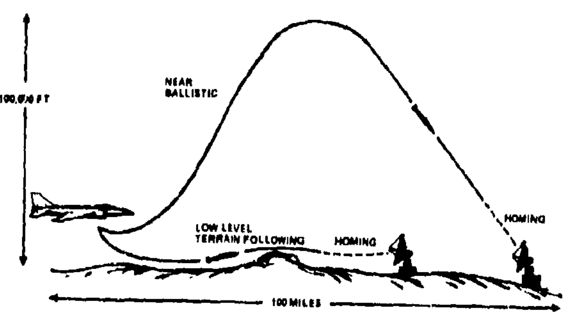

Figure 14

The considerations which the mission systems engineers have great concern for in their discussions with the control system engineers are shown in Figure 15. Missile storage internal to carrier aircraft will be required, so the number and size of the fins, and whether a folding mechanism is required, become key questions. The issue of launch safety combined with the requirement to have the missile fly high, low, to the side, and to the rear, cause a great variety of flight condition stability analyses and simulations to be performed.

The choices for the control surfaces and the data which assist in making the selections and setting the requirements are shown in Figure 16. Folding fins can be larger and provide less risk of unstable flight regimes. Fixed fins provide a simpler, cheaper, design. The upper curve of Figure 16 is basically a drag versus increasing control surface area curve, with the fin span of a three fin configuration used as the parameter. Three fins will be more difficult to store than four. The mission planners and aerodynamicists set a desired upper limit on drag, beyond which range and velocity penalties become too large and can only be allowed if the missile cannot be stabilized. The fixed fin configuration is clearly desirable, pending stability considerations. To decide whether three fins, with less hardware, can be used, the stability of the missile critical flight regimes needs to be assessed for an assumed autopilot capability. Plots similar to the lower one of Figure 16 are made for all the critical conditions. The one shown, for a supersonic turning condition, shows the static stability in yaw as a function of angle of attack. For three fins and high angles of attack, the body masks the fin which is to provide the lateral stabilizing force. The nominal autopilot cannot overcome the de-stabilizing effects with three fins.

MISSION DESIRES AND CONSTRAINTS, EXAMPLE A-S

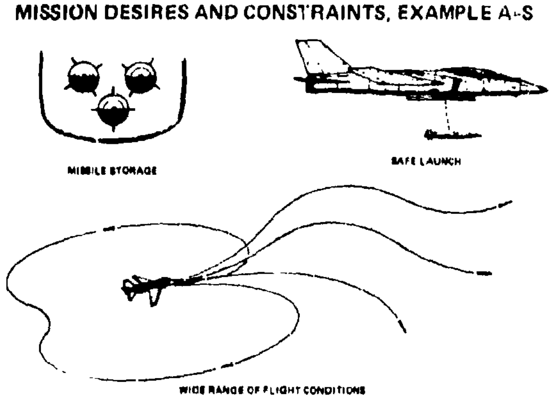

Figure 15

CONTROL SURFACE TRADES AND DATA, EXAMPLE A-S

	STORAGE		NUMBER	
	FIXED	FOLDING	THREE	FOUR
PRO	SIMPLE DESIGN LOW COST	CAN BE STATICALLY STABLE	CHEAPER EASIER TO STORE	LESS FIN EFFECTIVENESS VARIATION
CON	LESS STABLE POOR RESPONSE	MECHANISM UNRELIABLE HIGHER DRAG	LARGE FIN EFFECTIVENESS VARIATION	EXTRA HARDWARE

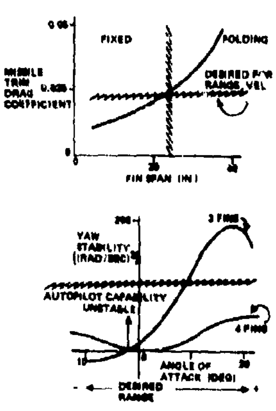

Figure 16

The particulars of the autopilot design must now be set, with nominal autopilot performance used in the control surface trades. The autopilot choices and data are displayed in Figure 17.

AUTOPILOT TRADES AND DATA, EXAMPLE A-S

	FEEDBACK		IMPLEMENTATION		GAIN CHANGE	
	ATTITUDE ATTITUDE RATE	NORMAL ACCEL-ERATION	ANALOG	DIGITAL	PRE-PRO-GRAM	ADAPT-IVE
PRO	SIMPLE	CAN LIMIT ANGLE OF ATTACK	FASTER RESPONSE BETTER GAIN MARGIN	MORE FLEXIBLE EASIER TO CHANGE	SIMPLE	HANDLE FLIGHT CON-DITION CHANGES
CON	NO ANGLE OF ATTACK LIMIT	NEEDS ACCELER-OMETERS	DIFF-ICULT TO CHANGE	LESS GAIN MARGIN	WIDE RANGE OF FLIGHT CONDI-TIONS	MORE COMPLEX

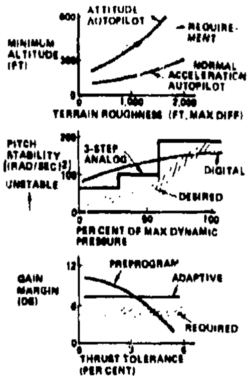

Figure 17

CONTROL SYSTEM SELECTION, EXAMPLE A-S

- CONTROL SURFACES
  - FIXED FINS
  - FOUR FINS
- AUTOPILOT
  - NORMAL ACCELERATION FEEDBACK
  - ANALOG
  - QUASI-ADAPTIVE

Figure 18

The flight conditions for which pitch axis motion are investigated include launch with controls locked, high altitude with high angle of attack, and terrain avoidance with high vertical acceleration requirements. The use of a minimum of flight hardware is always a prime consideration. The use of an attitude autopilot, with no vertical acceleration limit, is easy to implement and does not need accelerometers. The flight condition which is most critical turns out to be the terrain avoidance mode in which rough terrain produces signals calling for high vertical accelerations and high angles of attack. Simulations show that without acceleration sensing leading to angle of attack limiting, the missile will go unstable over rough terrain. The data are shown in the upper curve of Figure 17. Over the full range of dynamic pressure, the use of analog filter elements versus digital computations is investigated. The digital autopilot is more flexible and simpler to change, but the analog autopilot is faster and more capable. The middle curves of Figure 17, showing the capability of each as a function of dynamic pressure, show that the digital autopilot cannot provide the desired stability at high dynamic pressures. Finally, fixed gain scheduling prior to launch would suffice if the velocity-time history was predictable to a certain tolerance. Gain margin as a function of thrust uncertainty is plotted in the lower curves of Figure 17. As the uncertainty increases, the fixed gain technique cannot handle the high dynamic pressure condition of the curves. A gain scheduling technique based on sensed dynamic pressure is not affected by engine thrust variations.

Thus, the main configuration features are formulated by analyses of the sensitivity of key control system items to weapon system parameters. Figure 18 summarizes the choices made in the air-to-surface mission. The data curves used to make the decisions set the values to which the control system engineers commit to numerical requirements, in a format similar to the one of Figure 8 of the surface-to-air, or S-A, example.

In this example, with its exceptional range of dynamic pressure, the presented sequence of trades, data, and selection, represents a great simplification of the design activity. Iteration after iteration was required as aerodynamic data were refined, weight statements made complete, simulations expanded and, most significant, compromises reached between desired and feasible technical goals.

4. CONCLUSION

The last two decades have brought much of the same and a few different aspects of control system configuration and requirement establishment. Every design looks like some thing between DaVinci and some thing in someone's brochure. Any claim to a new idea can be invalidated in someone's archives. Some things which have changed significantly are the analytical ability of our young engineers, the simulation tools now available, and the sensor and computing elements available for on-board missile use. The role of the control system engineer, even with these advances, will continue to be to hear the desires of the mission analysts, the uncertainties of the aerodynamicists, the limitations of the avionics engineers, and the wails of the weights engineers, and then make the missile fly properly.

ADJOINT SOLUTIONS TO INTERCEPT GUIDANCE

by

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SUMMARY

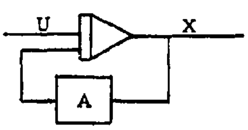
The adjoint equations yielding the error sensitivites of a linear system are explained. The Laplace transforms representing the solutions of the adjoint equations of a linear interceptor guidance system are developed. The solutions for an interceptor, represented by a first-order lag and utilizing proportional navigation, are derived.

ADJOINT SYSTEMS

To approach the solution of a set of linear differential equations

$$\dot{X} = AX + U$$

which may be diagrammed as



let us introduce an as-yet-undefined vector Z and form y, the single-valued inner product of Z and X:

$$y = Z^T X$$

Differentiating, we obtain

$$\begin{aligned} \dot{y} &= \dot{Z}^T X + Z^T \dot{X} \\ &= \dot{Z}^T X + Z^T (AX + U) \\ &= (\dot{Z}^T + Z^T A) X + Z^T U \end{aligned}$$

If we find a Z that makes

$$\dot{Z}^T + Z^T A = 0$$

then

$$\dot{y} = Z^T U$$

$$y = \int Z^T U$$

is a set of integrals giving y.

If we have solved for Z such that at some time T all values of Z are zero except for z_K, then $y(T) = X_K(T)$.

<u>Solution for Z</u>

The differential equations defining Z

$$\dot{Z}^T + Z^T A = 0$$

which might also be written

$$\dot{Z} = -A^T Z$$

are to be solved for boundary conditions defined at some future time, T. Let us, therefore, start the system with these conditions and solve backward in time by changing the independent variable from t to $t' = T - t$.

Comparing the diagrams of this equation with that of the equation in X, it may be noticed that, apart from the deletion of the inputs U, the transposition of A changes a_{ij} from being the gain of the connection between the output of the j^{th} integrator and the input of the i^{th} integrator to being the gain between the output of the i^{th} integrator and the input of the j^{th} integrator.

The diagram of the differential system describing X may, therefore, be changed to that describing Z by deleting all external inputs and reversing all integrators.

The components of Z are referred to most commonly as adjoint solutions, sensitivity coefficients, convolution kernels, Green's functions, or canonical conjugates of X, depending on the use and the user.

INTERCEPT ADJOINT FUNCTIONS

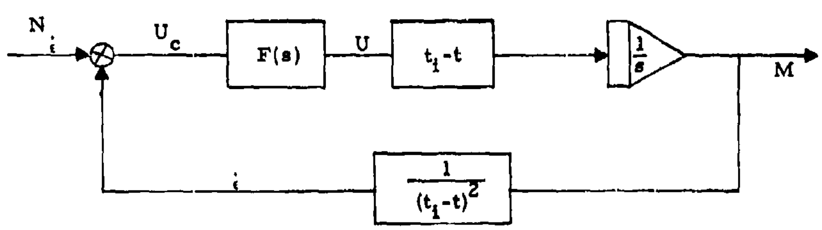
To describe the guidance loop, where acceleration normal to the line of sight is commanded to be some function of the line of sight rate, consider the relationships already discovered:

$$\dot{M} = U(t_1 - t)$$

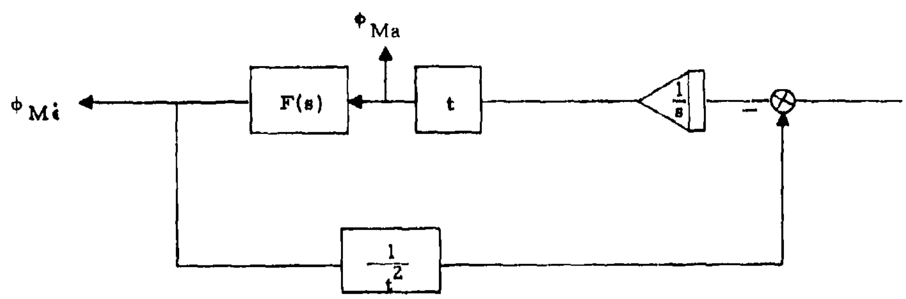
$$\dot{\epsilon} = \frac{M}{-R(t_1 t)^2}$$

$$U = F(s)\,\dot{\epsilon}$$

where F(s) includes missile filter and guidance gain. Choosing our unit of length to be $-\dot{R}$, to simplify, the system is represented as



If F(s) is assumed to have constant coefficients, the adjoint system that yields the convolution kernel giving the miss generated by information noise and target acceleration is



The equation described by the diagram

$$\frac{d}{dt}\left(\frac{M}{Ft}\right) = -\frac{M}{t^2}$$

may be expanded to

$$\frac{1}{t}\frac{d}{dt}\left(\frac{M}{F}\right) - \frac{M}{F\,t^2} = -\frac{M}{t^2}$$

which may be rearranged as

$$t\,\frac{d}{dt}\frac{M}{F} - \frac{M}{F} = -M$$

In the Laplace transform convention, this becomes

$$t \frac{sM}{F(s)} - \frac{M}{F(s)} = -M$$

which develops into

$$-\frac{d}{ds}\frac{sM}{F(s)} - \frac{M}{F(s)} = -M$$

which solves to

$$M(s) = \frac{F(s)\exp\left[\int \frac{ds}{sF(s)}\right]}{s^2}$$

Particular Cases

1. Simple Lag Representation of Missile

$$F(s) = \frac{\lambda}{s+1}$$

$$M(s) = \frac{\lambda s^{\lambda-2}}{(s+1)^{\lambda+1}}$$

e.g., for $\lambda = 3$

$$\phi_{M\dot{\epsilon}} = \frac{-3\dot{R}}{2T^2} t^2 (t-3T) e^{-\frac{t}{T}}$$

2. Critically Damped Second-Order Missile

$$F(s) = \frac{\lambda}{(s+1)^2}$$

$$M(s) = \frac{s^{\lambda-2}}{(s+1)^{\lambda+2}} e^{\frac{\lambda}{s+1}}$$

which may be solved as a series of integrals of

$$\frac{\cos h\, 2\lambda t}{t}$$

multiplied by e^{-t}.

3. Constant True Bearing or Propellant Utilization

$$F(s) = \frac{\lambda(s+1)}{s}$$

$$M(s) = \lambda s^{\lambda-3}(s+1)\, e^{-\frac{\lambda}{s}}$$

For integral values of λ, this represents the function:

$$\phi_{M\dot{\epsilon}} = K\left[\frac{d^{\lambda-2}}{dt^{\lambda-2}} + \frac{d^{\lambda-1}}{dt^{\lambda-1}}\right] J_o\ (2\sqrt{\lambda t})$$

For half integral values of λ, it represents the function:

$$\phi_{M\dot{\epsilon}} = K\left[\frac{d^{\lambda-2.5}}{dt^{\lambda-2.5}} + \frac{d^{\lambda-1.5}}{dt^{\lambda-1.5}}\right] \frac{1}{\sqrt{t}} \cos 2\sqrt{\lambda t}$$

Other Error Sensitivities of a First-Order Missile

1. Angle Rate Error

As previously derived, the miss of a proportional navigation system involving a first-order missile, due to an impulse of line of sight rate, at time t, is

$$\phi_{M\dot{\epsilon}}\,(t_1 - t)$$

which is the inverse Laplace transform of

$$L\left[\phi_{M\dot{\epsilon}}\right] = \frac{\lambda s^{\lambda-2}}{(s+1)^{\lambda+1}}$$

2. Angle Error

An angle error is differentiated to obtain the angle rate error; therefore, we obtain the relationship:

$$\phi_M = \frac{d}{dt}\phi_{M\dot{\epsilon}}$$

$$L\left[\phi_{M\epsilon}\right] = \frac{\lambda s^{\lambda-1}}{(s+1)^{\lambda+1}}$$

3. Target Maneuver

Referring back to the adjoint diagram, we notice

$$\phi_{Ma} = F(s)\,\phi_{M\dot{\epsilon}}$$

Therefore

$$L\left[\phi_{Ma}\right] = \frac{s^{\lambda-1}}{(s+1)^{\lambda}}$$

4. Initial Heading Error

A unit impulse of target acceleration generates a unit of crossing velocity, which would result in a miss of $(t_1 - t_o)$ if no action were taken. The effect is the same as an initial unit-heading error; therefore:

$$\phi_{M\epsilon_o} = \frac{R'}{T}\,\phi_{Ma}$$

The following table presents results of applying some of the above functions.

ADJOINT FUNCTIONS (ERROR SENSITIVITIES) OF A PROPORTIONAL NAVIGATION SYSTEM INCLUDING A FIRST-ORDER LAG

$L(\phi)$	$\lambda = 2$	$\lambda = 3$	$\lambda = 4$
$\phi_{M\epsilon}$ $\dfrac{\lambda s^{\lambda-1}}{(s+1)^{\lambda+1}}$	$\dfrac{-Rt(t-2T)e^{-\frac{t}{T}}}{T^2}$	$\dfrac{\dot{R}t(t^2-6tT+6T^2)e^{-\frac{t}{T}}}{2T^3}$	$\dfrac{-\dot{R}t(t^3-12t^2T+36tT^2-24T^3)e^{-\frac{t}{T}}}{6T^4}$
$\phi_{M\dot{\epsilon}}$ $\dfrac{\lambda s^{\lambda-2}}{(s+1)^{\lambda+1}}$	$\dfrac{\dot{R}t^2 e^{-\frac{t}{T}}}{T}$	$\dfrac{-\dot{R}t^2(t-3T)e^{-\frac{t}{T}}}{2T^2}$	$\dfrac{\dot{R}t^2(t^2-8tT+12T^2)e^{-\frac{t}{T}}}{6T^3}$
ϕ_{Ma} $\dfrac{s^{\lambda-2}}{(s+1)^{\lambda}}$	$\dfrac{te^{-\frac{t}{T}}}{T}$	$\dfrac{-t(t-2T)e^{-\frac{t}{T}}}{T^2}$	$\dfrac{t(t^2-6t-T+6T^2)e^{-\frac{t}{T}}}{2T^3}$
Miss due to:			
Step rate error	$\dfrac{\dot{R}}{T}\left[2T^2-(t^2+2tT+2T^2)e^{-\frac{t}{T}}\right]$,	$\dfrac{\dot{R}t^3 e^{-\frac{t}{T}}}{2T^2}$	$\dfrac{-\dot{R}t^3(t-4T)e^{-\frac{t}{T}}}{6T^3}$
Maximum value	$2\dot{R}T$	$0,67\dot{R}T$	$0,36\dot{R}T$
if			
Applied at t =	∞	3T	2T
Random angle	$\dfrac{\dot{R}\sqrt{NT}}{2}$	$\dfrac{3\dot{R}\sqrt{NT}}{4\sqrt{2}}$	$\dfrac{\dot{R}\sqrt{5NT}}{4}$
Glint	$\dfrac{0,56W}{\sqrt{T}}$	$\dfrac{1,08W}{\sqrt{T}}$	$\dfrac{1,7W}{\sqrt{T}}$
Acceleration			
(target maneuver or autopilot bias)	$aT^2\left(\left(1+\frac{t}{T}\right)e^{-\frac{t}{T}}-1\right)$	$\frac{1}{2}at^2 e^{-\frac{t}{T}}$	$\dfrac{at^2(t-3T)e^{-\frac{t}{T}}}{6T}$
Maximum value	aT^2	$0,27\ aT^2$	$0,13\ aT^2$
if			
Applied at t =	∞	2T	1,27T
Initial heading error	$\dot{R}te^{-\frac{t}{T}}$	$\dfrac{\dot{R}}{2T}t(t-2T)e^{-\frac{t}{T}}$	$-\dfrac{\dot{R}}{6T^2}t(t^2-6tT+6T^2)e^{-\frac{t}{T}}$
Maximum value	$0,37\ \dot{R}T(\gamma_o-\epsilon_o)$	$0,21\ \dot{R}T(\gamma_o-\epsilon_o)$	$0,17\ \dot{R}T(\gamma_o-\epsilon_o)$
If guidance initiation at	T	0,83T	0,41T

N = Power spectral density per cps.
W = Effective width of target.
T = Lumped missile and filter time constants.

OPTIMIZATION

by

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SUMMARY

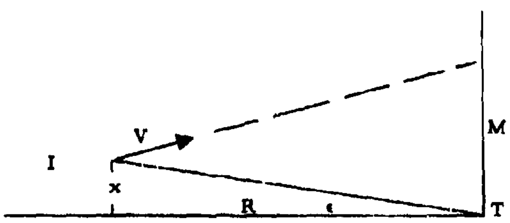
The procedure for optimizing a linear system against a quadratic cost function is developed by the method of completing a square. The optimal intercept guidance law against a nonmaneuvering target when the cost is energy lost to drag is shown to be proportional navigation with a gain of 3.

QUADRATIC COST MINIMUM

The cost of an interceptor missile is, to a considerable degree, associated with its weight. The greater part of the system expenses in effort, equipment, and risk of life is expended in delivery to the point of launch so that the smaller and lighter the vehicle, the more can be launched for the same cost (assuming, of course, equal reliability and effectiveness).

The weight of an interceptor missile and, therefore, its cost is dependent on the size of the warhead it is required to deliver and the amount of propellant required to effect its delivery to the target. The size of warhead required is proportional to the square of the expected miss while the amount of propellant must be sufficient to accelerate the vehicle to an acceptable closing velocity and make up the velocity lost to aerodynamic drag. The cost chargeable to the guidance system designer is, therefore, proportional to the square of the final miss plus the integral of that part of the aerodynamic drag resulting from guidance maneuvers.

To state this cost in the form of an equation, let us define some reasonable descriptors of the intercept geometry:



where

T is a target

I is an interceptor

V is the velocity of I relative to T

Line IT is line of sight (LOS)

The coordinate system (R, X) is set up so that the initial value of x is small

$$V = -\dot{R}$$

Defining t_1 as the time at which R goes to zero assuming V to be constant, we get

$$R = V(t_1 - t)$$

$$M = x + \dot{x}(t_1 - t)$$

$$\dot{M} = \ddot{x}(t_1 - t)$$

i. e., miss can be reduced by application of acceleration.

ϵ is a small angle $\approx \frac{x}{R}$

$$\dot{\epsilon} = \frac{\dot{x}}{R} - \frac{x\dot{R}}{R^2}$$

$$= \frac{\dot{x}(t_1 - t) + x}{V(t_1 t)^2}$$

$$= \frac{M}{V(t_1 - t)^2}$$

i.e., $\dot{\epsilon}$ gives a measure of the present predicted miss, and acceleration $U = \ddot{x}$ normal to IT provides a means of reducing it.

Remembering that aerodynamic drag has a component that is proportional to the square of the lift acceleration U, it is now possible to mathematically express the cost attributable to guidance-commanded maneuvers:

$$J = K_1 M(t_1)^2 + \int_{t_o}^{t_1} K_2 \ddot{x}^2 dt$$

$$J = K_1 M(t_1)^2 + \int_{t_o}^{t_1} K_2 u^2 dt$$

Considering that the first part of the expression looks like the terminal value of an integral,

$$\int_{t_o}^{t_1} \frac{d}{dt}\left[P(t)M^2(t)\right] dt$$

with

$$P(t_1) = K_2$$

and

$$\frac{d}{dt}\left[PM^2\right] = \dot{P}M^2 + 2PM\dot{M} = \dot{P}M^2 + 2PMU(t_1 - t)$$

and wishing to bring into consideration the second part of the cost function, we first add, and then subtract, the integrand from the above:

$$\frac{d}{dt}PM^2 = \dot{P}M^2 + 2PMu(t_1 - t) + K_2u^2 - K_2u^2$$

This may be rewritten as

$$\frac{d}{dt}PM^2 = \frac{1}{K_2}\left[\frac{PM(t_1 - t)}{K_2} + K_2 u\right]^2 - K_2u^2$$

provided

$$\dot{P} = \frac{P^2(t_1 - t)^2}{K_2^2}$$

i. e.,

$$\frac{dP}{P^2} = \frac{(t_1-t)^2}{K_2^2}\, dt$$

$$-\frac{1}{P} = -\frac{(t_1-t)^3}{3} + C$$

Knowing that

$$P(t_1) = K_1$$

$$C = -\frac{1}{K_1}$$

the solution to the equation may then be restated as

$$P = \frac{1}{\frac{1}{K_1} + \frac{t_1-t^{\;3}}{3K_2}}$$

Placing this value of P into the integral, we obtain

$$\int_{t_o}^{t_1} \frac{d}{dt}\left[PM^2\right] dt = \frac{1}{K_2}\int_{t_o}^{t_1}\left[\frac{PM(t_1-t)}{K_2} + K_2 u\right]^2 - K_2\int_{t_o}^{t_1} u^2$$

$$= K_1 M^2(t_1) \cdot P(t_o)M^2(t_o)$$

Rearranging terms, we get

$$K_1 M^2(t_1) + K_2\int_{t_o}^{t_1} u^2 = P(t_o)M^2(t_o) + K_2\int_{t_o}^{t_1}\left[PM(t_1-t) + K_2 u\right]^2$$

$$= J$$

We thus obtain a new expression for the cost. The first term is established at t_o as being the minimal cost. The integral, having a non-negative integrand, has a minimum possible value of zero, which may be achieved by holding the integrand equal to zero:

$$u = -\frac{PM(t_1-t)}{K_2}$$

Considering that K_1, the cost of warhead, is much greater than K_2, and that the cost of maneuver when (t_1-t) is small may be ignored, it is possible to closely approximate P:

$$P \approx \frac{3K_2}{(t_1-t)}3$$

The optimum maneuver can be represented by

$$u_{opt} = -\frac{3M}{t_1-t^{\;2}} = -3V\dot{\imath}$$

GENERAL CASE

Given a system defined by $\dot{x} = Ax + Bu$ and a quadratic cost function

$$J = x^T Px(t_1) + \int x^T Qx + u^T Ru,$$

we desired to find the action u, which minimizes the cost.

If we consider the nonintegral term to be the terminal value of an integral, the equation is

$$\int \frac{d}{dt}\left[x^T Px\right] dt = \int x^T \dot{P}x + \dot{x}^T Px + x^T P\dot{x}$$

$$= \int x^T \dot{P}x + x^T A^T Px + u^T B^T Px + x^T PAx$$

$$+ x^T PBu + x^T Qx + u^T Ru - x^T Qx - u^T Ru$$

which may be written

$$x^T Px\Big]_{t_o}^{t_1} = \int (R^{-1}B^T Px + u)^T R (R^{-1}B^T Px + u) - \int x^T Qx + u^T Ru$$

provided

$$PBR^{-1}B^T P = \dot{P} + A^T P + PA + Q$$

which may be written as

$$-\dot{P} = A^T P + PA + Q - PBR^{-1}B^T P$$

(the Ricatti equation encountered in filter design).

The cost function is represented by

$$J = x^T Px(t_1) + \int x^T Qx + u^T Ru$$

$$= x^T Px(t_o) + \int (R^{-1}B^T Px + u)^T R (R^{-1}B^T Px + u)$$

Because the constant term is established at t_o, the cost of guidance is felt only in the integral, which may be reduced to its minimum, 0, by making

$$u = -R^{-1}B^T PX$$

KALMAN FILTER

by

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SUMMARY

The Kalman filter is developed as a rational application of Gauss' method of least-mean-square error summing, which adds together independent measurements and estimates proportionally to the inverse of the variances of expected errors. The discrete measurement summer is developed into the continuous filter by shortening the time between measurements.

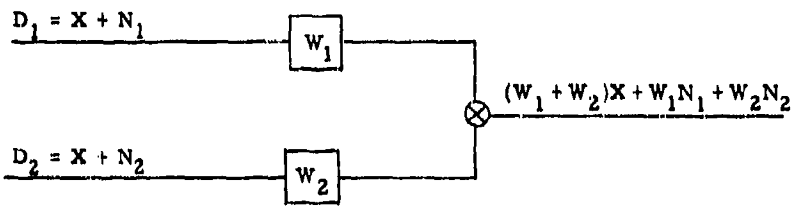
FILTER DESIGN

The decision to apply a filter to a data source must be based upon a knowledge that the data cannot be accepted as a true and exact measure of the system being observed. Implicit in the decision, therefore, is information about the system in addition to that contained in the data.

The design of the filter, then, is based on the question of how best to combine the information content of the observations with the <u>a priori</u> information on how the system should behave.

<u>Gaussian Summing</u>

Given two sources of data on a given quantity X, each containing some expected error, the data may be weighted separately and added together:

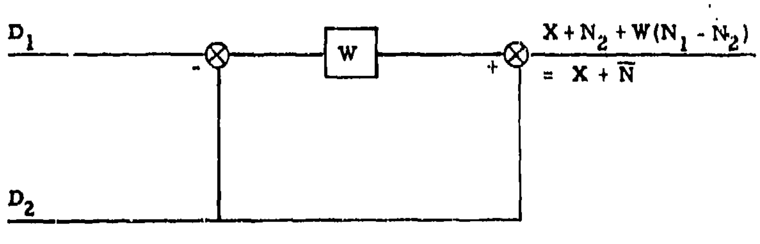


Since the required output is X, we choose $W_1 + W_2 = 1$, denoting

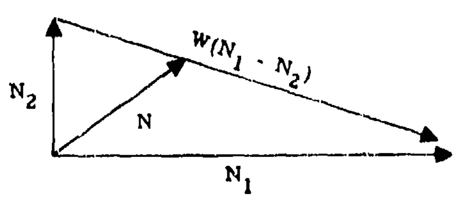
$$W_1 = W$$

$$W_2 = 1 - W$$

The diagram may be rearranged as



If N_1 and N_2 are uncorrelated, they may be considered as vector quantities at right angles:



$\bar{N}$ will be minimal if W is chosen to make $\bar{N}$ perpendicular to $N_1 - N_2$, in which case we get

$$\frac{W(N_1 - N_2)}{N_2} = \frac{N_2}{N_1 - N_2}$$

or

$$W = \frac{N_2^2}{(N_1 - N_2)^2} = \frac{N_2^2}{N_1^2 + N_2^2}$$

Since the expected values of $N_1^2 + N_2^2$ are σ_1^2 and σ_2^2, the best choice of W is

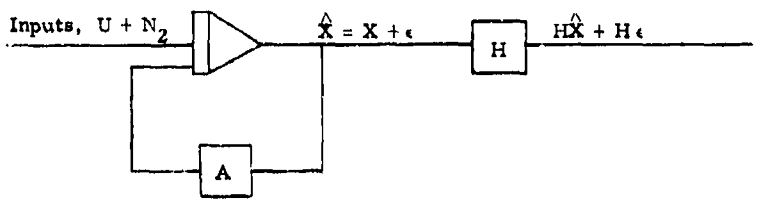
$$W = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

KALMAN FILTER

A measurement on a system is a measurement of some, all, or combinations of the components of the state vector plus errors:

$$D = HX + N_1$$

The *a priori* knowledge of the state of the system may be considered as stored in an analog simulation of the system:



We now have a set of readings and a set of values of what we think these readings should be. These can be summed by Gauss' criterion to get the best combined estimate of the measured quantities:

$$W_o = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$$

(Gauss' summation criterion)

Where the system state is an array of variables (state vector), σ^2 represents the expected mean products of all components with all components:

$$\sigma_2^2 = \overline{H\epsilon\epsilon^T H^T} = H\overline{\epsilon\epsilon^T}\, H^T$$

where

$$\overline{\epsilon\epsilon^T} = \overline{\left|\begin{matrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \cdot \\ \cdot \\ \eta_n \end{matrix}\right| \overline{\begin{matrix} \eta_1 & \eta_2 & \cdots & \eta_n \end{matrix}}}$$

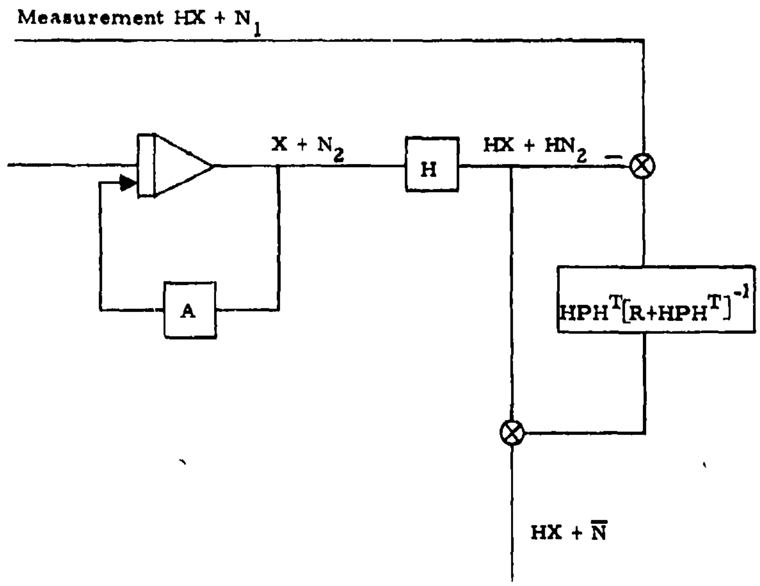
$$= \left|\begin{matrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & & \\ \cdot & & & \\ \cdot & & & \\ \sigma_{n1} & & & \sigma_{nn} \end{matrix}\right|$$

which is called the covariance matrix P. σ_1^2 is a similar matrix of averaged products $\overline{N_1 N_1{}^T} = R$

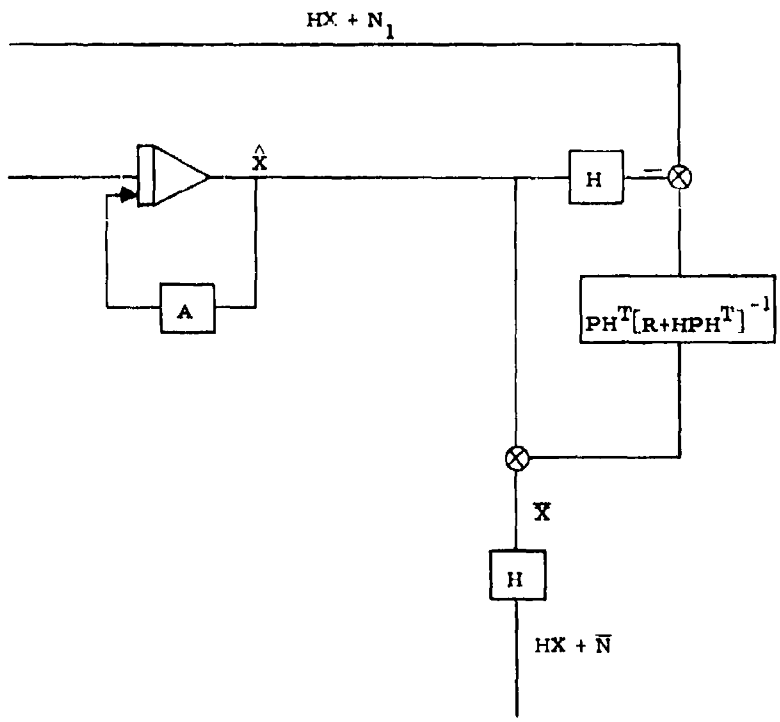
Applying Gauss' criterion to the summing of the measured and known state of a system proceeds as follows:

$$W = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} = HPH^T \left[R + HPH^T\right]^{-1}$$

The complete data summing system may be pictured as



which may be rearranged as

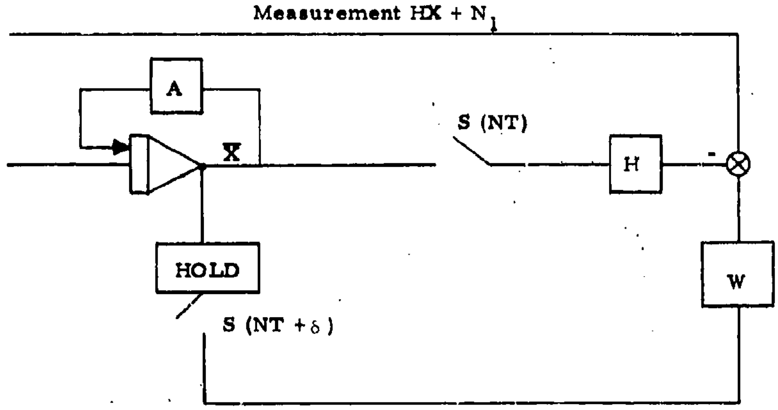


For convenience, we will replace $PH^T \left[R + HPH^T\right]^{-1}$ with W.

If the output of the H multiplier is the best possible estimate of HX, the input must be the best estimate of X. Let us, therefore, replace the state of the estimator with this value:

$$\bar{X} = \hat{X} + W(Y - H\hat{X})$$

Our diagram now becomes



where

$$\bar{X} = X + N_2 + W(N_1 - HN_2)$$

$$= X + N^1$$

$$N^1 = WN_1 + (1 - WH)N_2$$

$$P' = \overline{N^{1^2}} = W\overline{N_1^2}WT + (1 - WH)\overline{N_2^2}(1 - WH)^T$$

$$= (1 - WH)\overline{N_2^2} - \overline{N_2^2}H^TW^T + \underbrace{WH\overline{N_2^2}H^TW^T + W\overline{N_1^2}W^T}_{\underbrace{W(H\overline{N_2^2}H^T + \overline{N_1^2}W^T}_{\overline{N_2^2}H^TW^T}}$$

$$= (1 - WH)\bar{P}$$

If, between samples, the state vector changes from X to θX, where $\theta = e^{AT}$, then

$$\hat{X}_{n+1} = \theta\bar{X}_n + \int_t^{t+\Delta T} N_2 = \theta(X_n + N_{2,n}) + \Delta N_2$$

and

$$P_{n+1} = \theta P'_n\theta^T + C = \theta\left[1 - WH\right]P_n\theta^T + C$$

<u>Summary</u>

P_n is the matrix of correlations of estimation uncertainties of the components of the system state. R_n is the matrix of correlations of expected measurement errors.

The weighting to be given the difference between a measurement and the expected value of the measurement is

$$W_n = P_nH^T\left[R_n + HP_nH^T\right]^{-1}$$

$$P_{n+1} = \theta (1 - WH) P_n \theta^T + C$$

where θ is the matrix defining the expected evolution of the system between measurements, and C is the expected increase in P due to error in the model.

CONTINUOUS FILTERING

As the sampling period approaches zero, the covariance matrix P approaches a continuous function of time, and the above iterative processes approach the differential equations defining it. If the error on the reading R is considered to be the result of a white noise that has been filtered over the sampling time T, then R is inversely proportional to T:

$$\text{i.e.,} \quad R = \frac{R_o}{T}$$

If the model error is considered to be the result of a white noise on the integrator inputs and if T is considerably smaller than any time constant of the model, the increase in P due to the noise over one sampling period T is proportional to T:

$$\text{i.e.,} \quad \Delta P_N = CT$$

If we let $T \rightarrow 0$, then

$$W = PH^T \left[R + HPH^T\right]^{-1} \rightarrow PH^T R_o^{-1} T$$

$$\theta = e^{AT} \rightarrow 1 + AT$$

and

$$P + \Delta P = \theta \left[1 - WH\right] P \theta^T + \Delta P_N$$

$$= \left[1 + AT\right] \left[P - WHP\right] \left[1 + A^T T\right] + CT$$

$$\rightarrow P + APT + PA^T T - PH^T R_o^{-1} HPT + CT$$

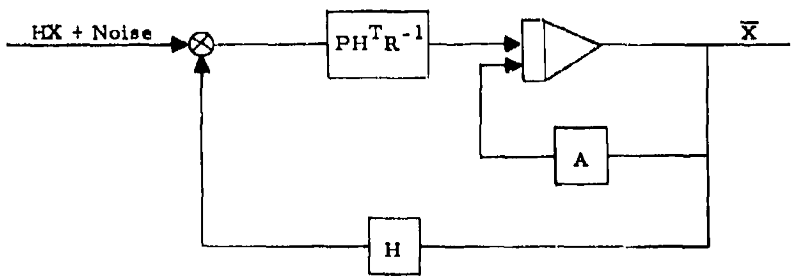
which can be rearranged as

$$\Delta P \rightarrow AP + PA^T - PH^T R_o^{-1} HP + CT$$

$$\frac{\Delta P}{T} \rightarrow \dot{P} = AP + PA^T - PH^T R_o HP + C$$

This equation is called a Ricatti equation, and is solved to give P, which in turn, gives the optimal weighting coefficients $W = PH^T R^{-1} T$.

The correction to the estimate $W \left[M - H\hat{X}\right]$ may be added to the estimate in the infinitesimal time T by placing it continuously on the inputs to the integrators with a gain 1/T. The diagram of the continuous filter is then



SOLVING RICATTI'S EQUATION

The equation $\dot{P} = AP + PA^T - PH^TR_0HP + C$ may be reduced to a form more amenable to solution by introducing as-yet-undefined matrices Z and Y:

$$Z = PY$$

$$\dot{Z} = P\dot{Y} + \dot{P}Y$$

$$= P\dot{Y} + APY + PA^TY - PH^TR_o^{-1}HPY + CY$$

$$= P\dot{Y} + A^TY - H^TR_o^{-1}HZ + AZ + CY$$

Equating the expression in parentheses to zero results in two equations:

$$\dot{Z} = AZ + CY$$

$$\dot{Y} = A^TY + H^TR_o^{-1}HZ$$

which, when solved, yield

$$P = ZY^{-1}$$

NUMERICAL ANALYSIS AND SIMULATION EVOLUTION

by

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SUMMARY

A review is given of the quantitative advantages and disadvantages of digital and analogue computer techniques for the simulation of missile guidance and control, and a methodology of using hybrid simulation is developed. It is shown how a hybrid computer can be used to aid the choice of an acceptable missile system within a wide spectrum of complexity, particularly when many non-linear factors and statistical aspects are involved. Using this facility, mathematical modelling not only helps specific projects in their R & D phases, but it can contribute to management decisions in feasibility studies, eg in the choice of missile instrument combinations and in the specification of their desired standard of performance. It can also safeguard against complex systems being over-designed to the detriment of their cost.

1. INTRODUCTION

It has been the practice for many years to simulate complex processes which are not easily amenable to analytic solutions. Especially in the aero-space field, the use of simulators has now become widespread. They are indispensable tools of research, both as hardware simulation facilities, eg in aircraft flight simulators, or as computer simulations in theoretical systems studies, eg in missile design work. Simulation studies of the latter type, using mathematical models, are the subject of this review. They are normally used throughout the whole range of aerospace industry where dynamic problems involving large numbers of parameters have to be solved. Right at the start of a new concept, simple studies are carried out using computer simulation techniques in order to establish the feasibility of the idea. During the development phase these studies require more sophisticated simulations in order to investigate and clarify the so-called 'grey areas' so that managements can make decisions. Later, in the production phase, and test periods of the completed product, the simulator is used to assist in the final assessment of the device, examining, for instance, the reliability aspects in the presence of environmental disturbances and engineering tolerances. The use and evaluation of statistical methods constitute an important part of any investigation. In systems of this kind the inherent noise of the system, mainly coming from the sensors, gives rise to inaccuracies. Problems of this nature require statistical treatments and therefore a high number of runs. A mathematical model, therefore, can be used with great advantage as an aid to understanding a complex dynamical system. Guided missile systems, in particular, can be studied, evaluated and developed in this manner.

This paper is aimed, therefore, at highlighting modern techniques of numerical analysis and computer simulation, when applied to the research and development of tactical missiles. It compares current techniques of digital and analogue computer simulations on a quantitative basis, and develops a methodology of using hybrid (ie digital/analogue) computers which can combine the benefits of both these separate approaches. It is also shown how hybrid computer simulation aids the parallel development of new hardware. An example is given of a laser guided missile study in an air-to-ground mode, showing how simple guidance and control factors can be assessed according to their contributing effects.

2. MODEL COMPLEXITY

The degree of complexity in a mathematical model of a missile system depends on the stage of development of the project. In pilot studies, for example, a very simplified model may be acceptable, taking into account only the most relevant parameters. In some of these circumstances, and at a very early stage, analytical solutions may be acceptable and may be obtained fairly cheaply. At the other extreme, however, eg in flight trials analysis or at the Service acceptance stage, a complex mathematical model using realistic representations of each of the subsystems involved is usually necessary for a more detailed understanding of the performance of the missile. The cost of evaluating this performance by means of a mathematical model generally increases rapidly with the model complexity. The two extremes therefore require different methodologies. Neither of these extremes are considered here, however, but rather an in-between modelling technique associated with either feasibility studies or the research and development phase of a missile project. In these stages the missile designer desires to make the best use of his computer facilities in order to aid him with critical decisions such as the choice of equipment and its quality, and the selection of good missile design characteristics, etc. Numerical solutions of a comprehensive mathematical model of the system can help him with these decisions. Basically two methods of simulation are available, digital or analogue computing, but they have different advantages and disadvantages when considered for this task. Digital computers can provide, inter alia, good accuracy, whereas analogue computers are easier to programme, but each have different time scales and running costs depending on the numbers of runs eventually required. These aspects are covered in more detail below.

There are also other factors having an impact on computer requirements. For example, there is an increasing emphasis on providing cheaper missile systems to meet future operational requirements. This leads to considerations of sub-optimal designs and simpler guidance and control systems. Sub-optimal considerations demand the study of a wide spectrum of solutions. This might impose a penalty because both the number of mathematical models required increases and, by necessity, the quantity of numerical solutions. Simple devices can also lead to a greater computing complexity, because a system which has become "simple" by ingenious engineering may be more difficult to describe in mathematical terms. Typical examples are a bang-bang control compared with a proportional control or complex cross-coupling phenomenon often met with in simple systems. A linear dynamical system of high order can be represented by a straightforward differential equation, but two or more non-linear factors in a system soon require evaluation by numerical analysis and simulation. The tendency is, therefore, towards greater mathematical complexity, even when considering simple non-linear, or sub-optimal devices separately. A combined requirement for sub-optimal solutions of non-linear systems exercises an even more stringent methodological requirement on the evaluation of future mathematical models. It will be shown that neither analogue nor digital simulations alone can meet this requirement since each simulation has a limited capability, either in degree of representation, accuracy or running time. At this stage the need for hybrid simulation techniques therefore begins to arise, but let us look first of all at the two separate approaches in more detail.

3. COMPUTER TOOLS

The following are some of the advantages and disadvantages of analogue and digital computers.

3.1 Analogue Computers

Advantages

a) The mathematical model is easy to set up, especially for non-linear systems.

b) Design changes are very simple to execute by switching to alternative analogue circuits.

c) Separate blocks in the simulation can be developed to any desired degree of sophistication.

d) It is easy to check component performance by theoretical analysis.

e) Fast running times of up to 100:1 on real time are possible.

f) Hardware inclusion is possible and model matching is relatively easy.

Disadvantages

a) The change of parameter values takes time, and each time a change takes place the calculation should be checked.

b) The analogue computer has limited accuracy within the specified scaling of the problem.

c) The computer is subject to drifts.

3.2 Digital Computers

Advantages

a) Almost any desired accuracy can be obtained, drift free, provided that the correct word length is chosen and computing time is not at a premium.

b) Parameter changes can be programmed easily for extensive numbers of runs.

c) Simulation languages have been developed so as to make programming easy and quick to develop.

Disadvantages

a) It can be difficult to change a digital programme to incorporate design changes. Programme changes are also necessary if extra print-outs are required to give further insight.

b) Step lengths of integration processes have to be reduced in stages until an acceptable repeatability is obtained before production runs are possible.

c) The step length changes of (b) are necessary every time a parameter value is changed.

d) Multiple discontinuities and non-linear aspects are difficult to programme.

e) Existing simulation languages can introduce hidden inaccuracies which are difficult to trace when discontinuities have to be simulated.

f) Extensive simulation programmes have long running times and are therefore costly to run.

3.3 Comparison between Digital and Analogue Simulation

Let us now compare the relative costs of analogue and digital simulation. Figure 1 shows diagrammatically how the costs of simulating a typical missile system vary according to the number of computer runs. Digital programming is generally cheaper and quicker to develop than analogue programmes (eg typically 1 man, 3 months, £1500 digitally, compared with 2 men, 6 months, £6000, for an analogue approach; these figures constitute the starting numbers on Fig 1). For large numbers of runs analogue simulation is favoured because the faster running capability makes the cost per run nearly negligible. The ratio of running costs, or the slopes of the curves, is of the order of 500:1 in favour of analogue modelling, using figures of £30/hr, 3 runs/hr which give £10 per digital run, and £60/hr, 3000 runs/hr which give £0.02 per analogue run. In the chosen example the cross-over occurs at about 4-500 runs.

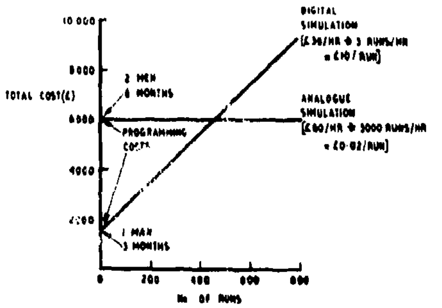

FIG.1 COMPARATIVE COSTS OF SIMULATION v. No OF RUNS

As listed opposite, the development of digital simulation languages has made programming easier, but inaccuracies can be introduced by using integration routines with fixed step lengths for the simulation of systems with discontinuities present. The computation can be made accurate by varying the step lengths and changing the independent variable to ensure that any discontinuities occur at the beginning of a step. The use of longer word lengths also increases digital accuracy. These techniques, however, tend to make the programming more difficult, and this defeats the objective of modern simulation languages which are intended to make digital programming easy and cheaper. Analogue simulations alone have the advantage of meeting the requirement for increased output of computer runs, but they have all the disadvantages listed earlier, particularly reduced accuracy and long times for parameter changes. Hybrid computation enables both of these difficulties to be overcome by means of accurate digital computation where required, and digital control of analogue changes. Before carrying the argument further it would be advisable to consider the overall advantages and disadvantages of hybrid computers when used for simulation purposes.

3.4 Hybrid Computers

Advantages

a) All the above advantages in 3.1 of analogue computation, are available in a hybrid computer, together with:-

b) Increased modelling accuracy, where necessary, by means of digital computation.

c) Parameter changes can be programmed as in purely digital simulations.

d) Statistical analyses of answers can proceed digitally for the reduction of large numbers of quantitative results.

e) Subsystems can be allocated to either digital or analogue computation according to preference, suitability or applicability.

Disadvantages

a) Increased complexity of developing a hybrid computer model implies longer development times.

b) Specialised programming staff are also required for efficient hybrid operation.

c) Analogue components are still subject to computational drifts, but statistical techniques can be used (i) to detect them and (ii) to evaluate the significance of differences between subsystems in the presence of residual computing inaccuracies.

It can be envisaged that the disadvantages as listed will become less important in the future. The increased development time will be reduced by software developments already under consideration, and the increasing use of hybrid computers will ameliorate the staffing difficulty mentioned under item (b). In our experience the advantages outweigh the disadvantages heavily, even at this stage of usage.

4. VALIDATION

4.1 Matching with Current Systems

Many engineers and managers are justifiably suspicious of the usage of extensive mathematical models, so it is necessary to build up a working confidence that the models are truly representative. Strictly speaking a mathematical model, which in the first place was set up to give answers to problems not amenable to be solved by any other method (eg analytical), has no yardstick by which it can be checked. Otherwise this yardstick would have been used in the first place for the study. To model, on the other hand, every detail of a complex machine is a superhuman task and could be uneconomical. Certain assumptions, simplifications and abstractions have to be made, therefore, in writing the programme. The question arises as to how adequate the mathematical model is for the purpose of the envisaged study and its objectives. The user should be aware, therefore, of the limitations of his model, but nevertheless have sufficient confidence in it. The process of establishing this high level of confidence is referred to as model validation. It is most important that any model is acceptably validated before it is used as a management tool. There are a number of methods of carrying out this validation process up to a certain degree. The model might, for example, be matched with test flight results of current missile systems. Alternatively physical laboratory experiments might be conceived which are themselves models of the real world but which are the nearest accessible physical representations. Model calculations can also be checked out by analytical solutions if suitable simplifications can be made. One of the most important methods of validating a model is by relating it to current hardware developments, even to the extent of including a hardware subsystem in the simulation itself.

4.2 Hardware Inclusion

The inclusion of new hardware developments in hybrid simulations serves four purposes (a) to verify parts of the model, (b) to help the design engineer to make component improvements early in the development period, (c) to obtain an indication of the relative merits of various hardware designs and (d) to highlight important factors, introduced by the individual devices, which significantly affect the performance. When this is done, however, the computations have to be run in real time and the advantages of running faster are lost. This is also true if a human operator is included in real time simulations. It is an advantage to revert to speeded up runs when confidence has been established, and the relevant factors revealed by hardware investigations are included in the simulation, so that no facets of the hardware are being lost.

We have, so far, sketched out a skeleton for a simulation methodology which is most easily met by hybrid means. This methodology will now be considered further and shown to be suitable for both non-linear missile systems and sub-optimal evaluations.

5. SIMULATION METHODOLOGY

Figure 2 shows the simulation programme from which the techniques of numerical analysis and computer simulation can provide a spectrum of systems understanding.

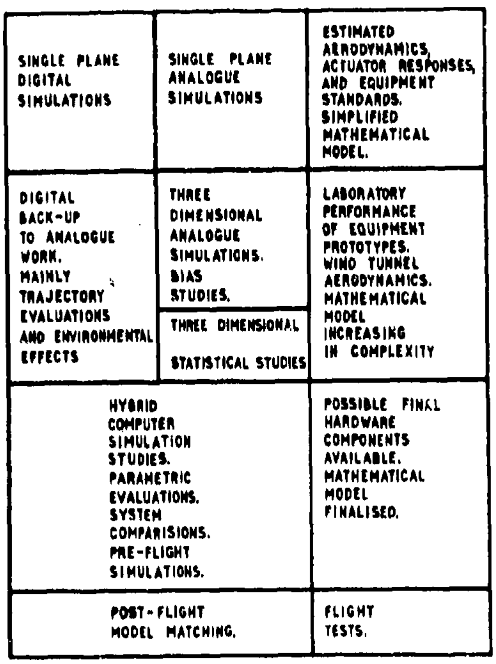

FIG. 2 SIMULATION PROGRAMME

5.1 Model Development

Experience has shown that it is expeditious to build up the model gradually rather than to attempt a comprehensive version at the outset. The more likely course of action in arriving at an acceptable solution is as follows. The development of a typical missile simulation programme starts with either simple digital computer programmes or analogue runs of simplified versions of the mathematical model. At this stage the missile representation may be very much simplified, with only estimated aerodynamics, actuator responses and autopilot characteristics, and it may be that only single plane flight dynamics are represented by the model. A careful note should be made at this stage. It might be obvious that the system is characterised by a high degree of cross coupling, and the effects of, say, a spinning movement, might heavily affect the single plane dynamics. In this case one would have to start off in three dimensions, although the parameters mentioned earlier could still be of an elementary nature.

Only a few digital computer runs are possible because the cost of running them rises rapidly as the model develops. When the model complexity has increased significantly to include non-linear effects the usefulness of an analogue computer becomes more evident. A greater return in understanding follows faster running, which is particularly useful in studies of biases in the parameters represented in the model. As the number of runs is increased the dominant factors and parameters begin to emerge, together with some knowledge of their independence or otherwise. The degree of correlation can be judged on the basis of further computer runs in which two or more parameters are varied. Finally statistical studies can proceed in which combined and interacting variations of many variables can be evaluated, eg in Monte Carlo type sampled simulations. The statistical response of a system in the presence of noise can also be determined. When the mathematical model is finalised, numerical solutions are evaluated most efficiently on a hybrid computer. Non-linear aspects are more suited to analogue evaluation, but care has to be exercised in structuring the model in analogue form so as not to exceed the complement of analogue equipment. It is probably generally true that in complex guidance and control modelling the computer analogue complement is soon used up and a compromise becomes necessary in the system modelling. A good balance needs to be maintained so that all important aspects are modelled to an acceptable depth of understanding. The correct balance can only be learned through hard experience on the job, and by continuous contact with real hardware.

If a missile system is sufficiently well developed and if it is planned to go forward to full scale flight tests then hybrid computations can assist in assessing the many effects of environmental factors in extensive pre-flight simulations, including malfunctions. Autopilot gain settings in feedback controls can be determined to ensure good stability characteristics. Instrument combinations can be selected together with their manufacturing standards. The nominal performance and likely deviations due to statistical uncertainties can be predicted for specific trials conditions. Finally if sufficient records are taken as the flight tests take place, it becomes possible to carry out post flight simulations for model matching purposes. Adjustments can be made, for example, to modify the aerodynamics based on wind tunnel data, if these are inadequate to describe the actual behaviour. Sometimes only minor changes have to be made. The post flight mathematical model then represents the fullest possible theoretical understanding of the weapon system under consideration. In this manner the modelling activity can be used as a management tool, which can contribute to the success of a Research and Development project. This applies not only in the missile field but to any technological system which is complex and dynamic.

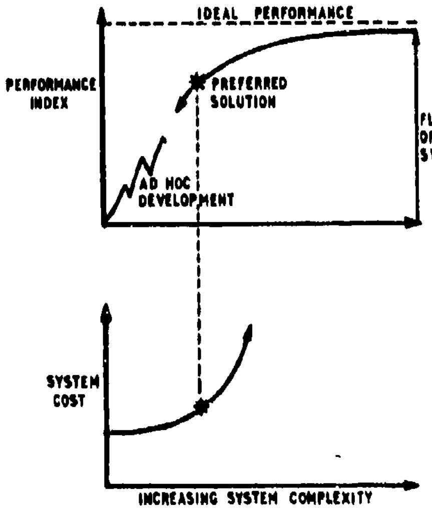

FIG.3 SUB-OPTIMAL SOLUTIONS

5.2 Sub-Optimal Solutions

We have so far discussed the development of a model but we come now to consider how the model is actually used in the most efficient manner. As indicated before, one of the aims of a missile designer is to gain an understanding of simple systems. One method of designing simple missile systems is to consider sub-optimal solutions. Figure 3 illustrates this situation schematically. It shows that as the system complexity increases, say by increasing the degree of sophistication in guidance loop filtering, then the performance criterion generally follows a curve of diminishing returns. The curve is shown incomplete in the upper diagram of Figure 3 to emphasise that ad hoc investigations and developments, indicated at the lower end of the diagram, must be considered to be merely attempts to obtain significant improvements, without necessarily succeeding. At the other extreme of complexity a theoretically optimum solution, say a Kalman filter in the guidance loop, can provide the ideal performance. There is, however, an associated cost curve related to this complexity spectrum, which is shown in the lower part of Figure 3. It indicates that the ideal performance might only be achieved at a premium which is too high for the user, eg the computational costs associated with the Kalman filter could be great. In such a situation it becomes necessary to study the spectrum of sub-optimal solutions which contribute to the upper curve. The bulk of the work is concerned with a withdrawal from the optimum to find a working compromise between acceptable cost and performance degradation due to the departure from the ideal.

The point to be made here is that the simulation techniques being described in this paper enable not only the fully optimum solution to be evaluated technically, but also the exchanges which result from withdrawing from the optimum. Computer techniques now being developed allow the work load to be expanded to cover a wide spectrum of sub-optimal conditions, rather than merely evaluate the design and performance of a fully optimum system.

6. MISSILE GUIDANCE AND CONTROL EXAMPLE

An example will be given now which, although being treated in this paper in a generalised form, could be considered to be typical of problems encountered in missile studies.

6.1 The Problem and Results

The simulation technique will be demonstrated using a mathematical model of non-linear complexity representing a wide spectrum of missiles from a freely falling bomb without either guidance and control, to a laser guided air-to-surface missile of hypothetical design. The first order effects of including simple guidance on a freely falling bomb will be shown. Then the effects of having either continuous or pulsed guidance information available from the sensor are demonstrated. Various degrees of complexity in roll control, from freely rolling weapons to roll rate and roll position stabilisation are added also. Finally a rocket motor is added to show what improved performance arises from increased speed and aerodynamic response in the case considered. An AD4/IBM 1130 hybrid computer was used for this study in which the digital part of the computer was used to control the analogue.

Figure 4 illustrates a laser guided missile attack in which the target is illuminated by a laser source. The reflection of this radiation is detected by a sensor in the head of the missile, and semi-active homing can take place, provided that the missile is fitted with control devices for manoeuvre. A number of non-linear factors have been included in the simulation and the degree of complexity can be judged from the inclusion of:-

a) a laser guidance source, which, in the simulation, can be either continuous or sampled.

b) a laser detector homing head.

c) a proportional navigation homing guidance law.

d) a signal hold device for retaining a laser signal between pulses.

e) a pulse width modulator.

f) variable aerodynamic conditions (non-linear).

g) either bang-bang or proportional control actuation.

h) a segmented roll resolver for transferring demands in space axes to missile actuators for a rolling missile.

i) gravity compensation in the vertical command.

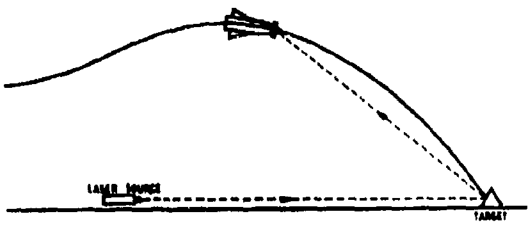

FIG.4 LASER GUIDED MISSILE

(The missile is simple in concept with no autopilot (aerodynamic stabilisation only) ie no rate gyros, accelerometers or shaping networks, and the target is assumed to be stationary.)

Consider first the freely falling bomb from release conditions giving a miss distance of about 120 arbitrary units, (arbitrary units being used since only relative merits are being investigated). Typical strike points of this magnitude are shown in the central part of Figure 5. The dispersion of the sixteen impact points shown arises from imposed variations in altitude and cross-track at release. The central part of Figure 5 also shows what improvement can be obtained when guidance is introduced, pulsed in the first instance, together with bang-bang control. The first order improvement is to about 50 units radial miss distance. Further potential improvements are offered by the introduction of either continuous guidance data or proportional control. If bang-bang control is still used, but the guidance is continuous in form, then the miss distance can be reduced to the order of 40 units. Continuous guidance with proportional control produces better performance. A miss distance of about 15 units can be achieved with a freely rolling missile. A further improvement to 10 units can be obtained when it is roll position stabilised, (see the left hand picture of Figure 5), but it will be noticed that the ballistic travel of unguided roll position stabilised rounds can be greater than when freely rolling. The above figures apply to an unpowered guided weapon whereas in this case the addition of a rocket motor, to increase the speed and aerodynamic response, showed that miss distances of the order of 5 units could be achieved with pulsed guidance, bang-bang controls and a freely rolling missile compared with the 50 units given above for an unpowered version. The exchange rates given above are, however, for illustrative purposes only, to show the value of the systematic approach. The results are not intended to show general characteristics because a complete missile system depends on so many parameters and the interactions between them. For instance they relate to specific aerodynamic properties which have remained unchanged throughout. It could be that significant improvements might also be forthcoming from aerodynamic redesign, an investigation which could also be carried out by hybrid computer simulation to determine theoretically the best aerodynamic properties required.

6.2 Computational Implications

The previous paragraph was intended only to serve as an indication of the problem. In actual fact extensive hybrid computing has been completed with this model. This included multi-parameter changes and statistical runs of the order of a few hundred thousand. Blocks of 100 nominally repeated runs were used as a basis for comparison to cover analogue computing inaccuracies. For each run the co-ordinates of miss distance were recorded together with approach angles at the target. The digital part of the computer then calculated means, standard deviations and rms radial miss distances for each block of runs, and the results were printed out immediately. Arrays of blocks, typically 5 x 5, were then computed automatically by digital controlled analogue simulation, for a range of values of each of two parameters, ie a total of 2500 runs (namely 5 x 5 x 100) in each typical array. Some of the parameters covered were the homing head gains, frictions and biases, the navigation constant of the proportional navigation law, the gravity compensation factor and aerodynamic parameters. One of the arrays of 2500 runs took only about 1 hour of hybrid computing time. If this number of runs had been completed by purely manually operated analogue computation, with the same results being recorded by hand, and potentiometer values being set by hand between blocks, it is estimated that the time on the computer would have been increased at least ten fold, most of the time being taken up by documentation. Considerable additional time would also have been required for off-line analysis of results.

A purely digital simulation of the same problem was also developed in parallel, but it ran into significant difficulties because of the many non-linearities included. It used a conventional simulation language and, when completed, took about 25 mins per run. A five by five array of only one run per block would have taken about 10 hours.

Hybrid computation, therefore, provided the most useful quantitative results in a readily assimilable form. Parameter dynamics were recorded on paper traces, and displayed on oscilloscopes as required. Further software available included sub-routines for statistical analyses of variance of results. Significant savings in both time and effort therefore resulted from operating the model in hybrid form. With imposed and controlled noise studies in the system model, statistical studies are a necessity, thus enhancing the hybrid approach.

7. CONCLUSIONS

It has been shown that either digital or analogue computer techniques alone do not meet all the requirements for obtaining a full understanding of the performance of a complex system using a mathematical model approach. A methodology of using a hybrid computer for simulation purposes has therefore been developed and applied to the study of a missile guidance and control problem.

Using a hybrid computer to simulate rapidly varying, wide bandwidth missile system components it is possible to (a) compare alternative solutions to a given tactical requirement, (b) include many non-linear factors, (c) study sub-optimal solutions, (d) perform large numbers of runs, (e) compute trajectories 100 times faster than real time, (f) explore the effects of manufacturing tolerances and instrument standards, (g) check out parallel hardware developments and (h) develop a missile to meet a cost-effective requirement.

This thorough method of evaluating a future system can include a very elaborate simulation of any part of the system requiring deeper investigation, and it is possible to validate the model by the inclusion of actual developed hardware in the simulation itself.

The example given of a laser guided missile study, in the air-to-surface mode, indicates how this methodology can be applied to safeguard against future tactical missiles being over-designed with expensive sub-systems. Furthermore, the methodology developed is suited to other complex and dynamic technologies, by utilising extensive numerical evaluations of either deterministic parameter variations or statistical uncertainties in a representative mathematical model.

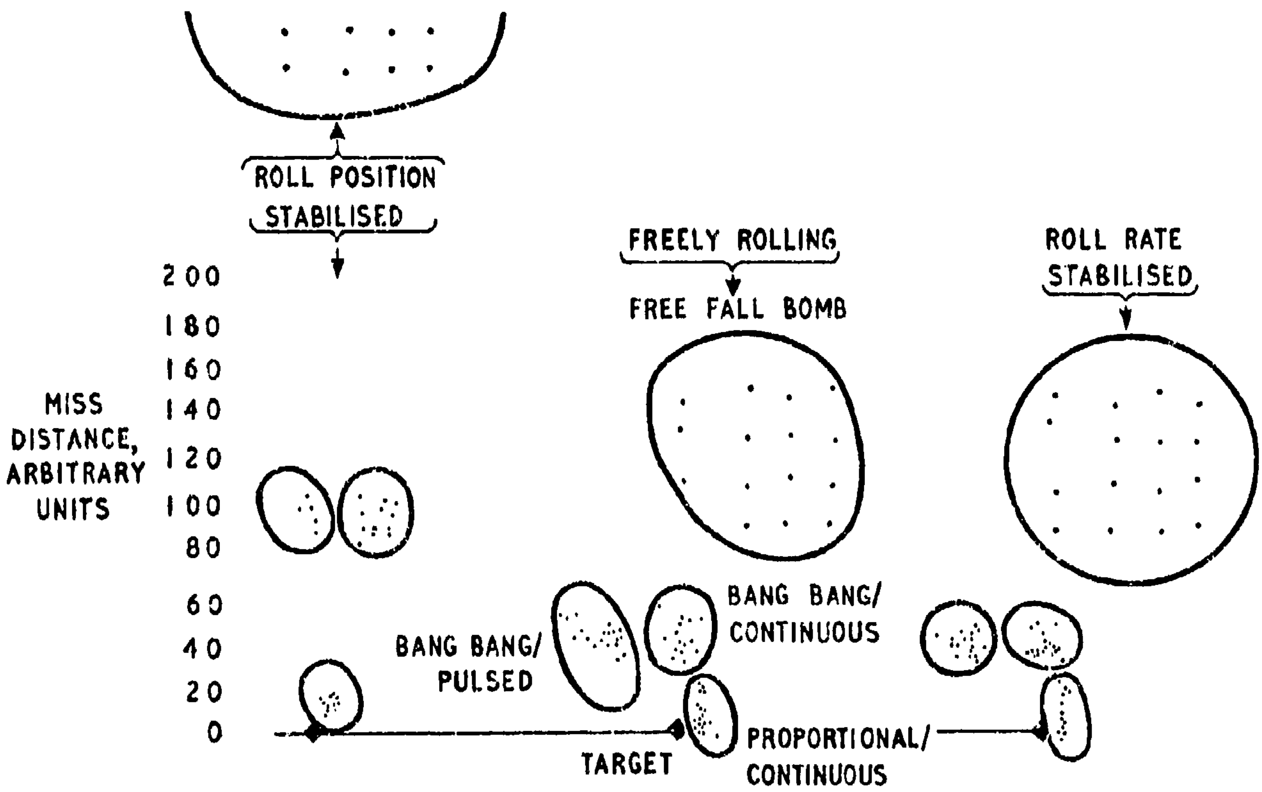

FIG.5 TYPICAL IMPACT POINTS

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LABORATORY TECHNIQUES AND EVALUATION METHODOLOGY

by

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SUMMARY

A discussion of the characteristics of typical electro-optical terminal guidance subsystems including area correlator and gated trackers are furnished to define those parameters (aim and lock-on capability, tracking accuracy, tracking bandwidth, aspect angle capability, sensitivity to target and light level variations, acquisition envelope, and range closure effects) which are important to the system user.

The Guidance Development Center (GDC), a laboratory designed to repeatably measure these properties, is described and a movie shown which illustrates its operation.

Typical area correlator tracker characteristics are furnished and a run schedule defined to evaluate the performance parameters described above. An economic analysis is presented to illustrate the potential cost savings over flight test.

Tactical weapon systems demand increased accuracies at longer ranges and low cost systems to permit virtually unlimited use in combat situations. If these high accuracy (small miss distances) demands are met, the dynamic effects and performance limitations of both the missile and seeker must be taken into account as well as the static characteristics. For effecting low cost and high performance, cost tradeoff studies must be made of the component parts, delineating performance characteristics versus cost.

TERMINAL GUIDANCE SUBSYSTEMS

Terminally guided electro-optic weapons (Figure 1) provide a cost effective solution to point target destruction. Such weapons can be broadly divided into two categories: those which are aimed and locked on the target (ALT) by a pilot and those which search and acquire a target (SAT) after launch. Either system may lock on the signature of the target itself or the signature of the area surrounding it.

Figure 1. Electro-Optic Weapons

The basic components of a guidance system which can perform in either mode (Figure 2) include a set of optics, gimbals, and photomultiplier tube which has been suitably designed by subsystem tradeoffs to permit acquisition, withstand body motions, and yet furnish end game accuracy, a TV camera to permit pilot aiming (ALT) and/or a memory device for lock on after launch (SAT), and an electronic chassis for signal processing, logic functions and power regulation and distribution.

Various modes of release (Figure 3) are important for a SAT guidance system since aircraft survival is improved by maneuvers during delivery and lack of fixed delivery geometric constraints. In all SAT modes the attack is against a prebriefed target with the image of the target area or target stored in memory. The pilot navigates over an initialization point and flies any one of several predetermined maneuvers to launch the weapon so that it will pass through an acquisition basket, acquire the target, and maneuver to it. The SAT guided missile can also be retained on the aircraft, flown through the acquisition basket and then released in the event that the target is not defended. All launch modes are designed to provide a high missile approach angle to the target since this trajectory minimizes signal attenuation in the optical path provided a cloud-free line-of-sight to the target can be established. This trajectory also assists in providing a high warhead function angle which minimizes the effects of aimpoint bias, fuzing height, and bomblet trajectory after burst in the case of dispersion weapons. The warhead function angle can also be improved by biasing the trajectory (Figure 4) using intermediate prestored target views called updates which have been selected and stored in the guidance system. As the missile closes with the target, the reference scene is changed forcing the missile to fly a lofted trajectory to acquire the new scene.

Flexibility of delivery is also important for an ALT guidance system (Figure 5) since targets of opportunity may appear from many geometrics. In this system the pilot acquires the target by viewing it through the missile optics and manually positioning the guidance head on the point of interest. In addition to the obvious advantage of quick response, this system is usually less costly than an SAT since there are little if any memory requirements.

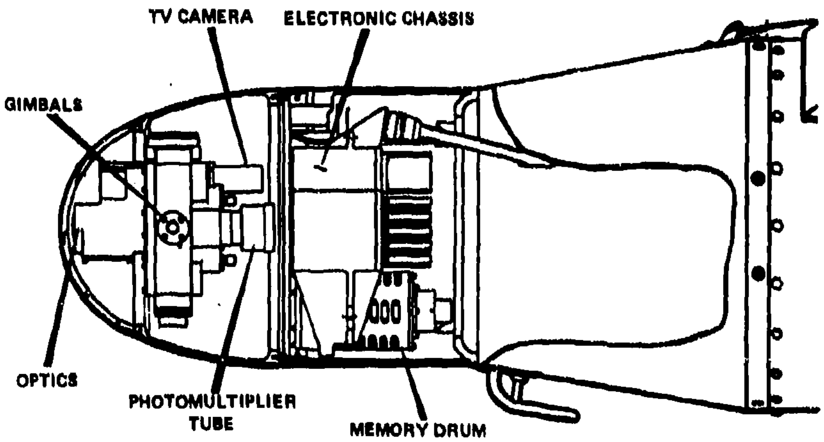

Figure 2. Multi-Mode Guidance System

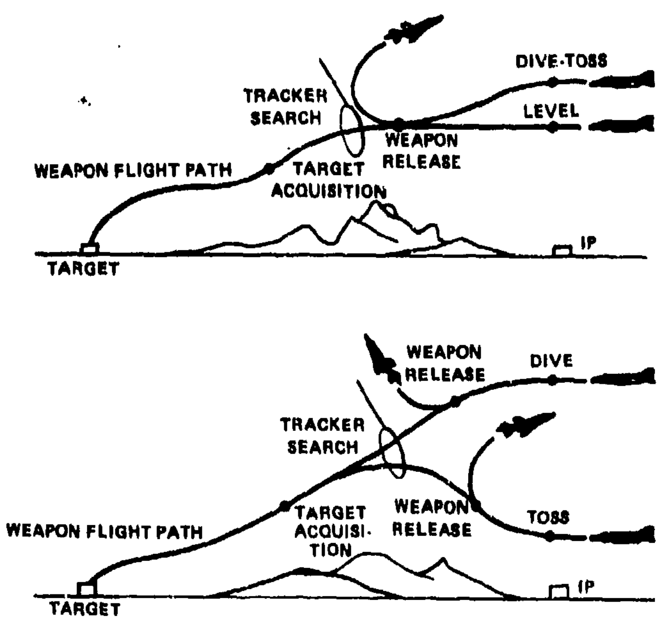

Figure 3. SAT Delivery Options

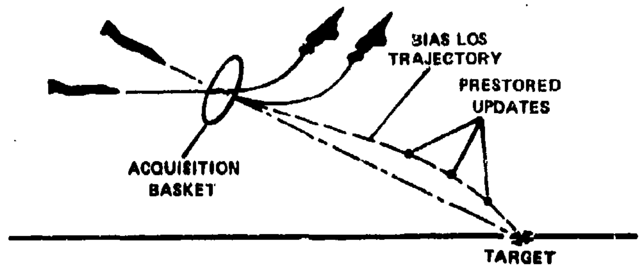

Figure 4. SAT Trajectory Bias

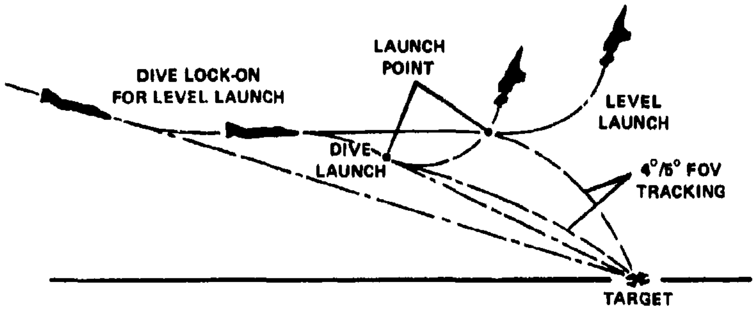

Figure 5. ALT Trajectories

TESTING REQUIREMENTS

By reviewing the above missions, it is possible to define a number of system parameters which must be demonstrated by test to establish that a specific set of guidance hardware will operate in a broad and useful set of tactical conditions:

1. Initial Aiming and Lock-On Accuracy

This test consists of pointing at a target across a range of illuminations from 100 to 10,000 foot-candles and measuring the jump that occurs when the lock on switch is engaged. A typical run schedule (Figure 6) would be repeated at each light level to determine if the effect is statistical in nature and accomplished against two or more targets of varying contrast at different approach trajectory angles. It should be performed at the acquisition range desired, usually about 50,000 feet and some intermediate range.

Run No.	Target	Slant Range (Ft)	Depression Angle (Deg)	Light Level	Video Signature and Light Level Threshold Test
1-1	A	20K	25	Max	X
1-2			25	Min	
1-3			45	Max	
1-4			45	Min	
1-5		48K	25	Max	
1-6				Min	
1-7	B			Max	
1-8				Min	
1-9		20K		Max	X
1-10	B	20K	25	Min	
1-11			45	Max	
1-12			45	Min	
1-13	C		25	Max	X
1-14			25	Min	
1-15			45	Max	
1-16			45	Min	
1-17		48K	25	Max	
1-18		48K	25	Min	

Figure 6. Initial Aiming Run Schedule

2. ASPECT ANGLE SENSITIVITY

In this test the system should be locked on to the target and then translated vertically and horizontally a distance of '25,000 feet to establish the point where lock on is broken. The test should be repeated against different contrast targets.

3. STATIC TRACKING ACCURACY

This test consists of locking the guidance head on the target at acquisition range and measuring the guidance head drift with time in the three rotational degrees of freedom. This test is performed to determine whether fixed biases such as gimbal mass unbalance will disturb dynamic tracking results. The time duration should be typical of that expected in missile flights.

4. TRACKING BANDWIDTH MEASUREMENTS

During this test the guidance head is locked on to a target while the head is rotated in a sinusoidal motion in each axis at a fixed amplitude with the frequency increasing until loss of lock occurs. Range, lighting, and trajectory parameters should also be varied.

5. ACQUISITION ENVELOPE

This test is useful for all SAT systems and some types of ALT devices. It consists of inserting a scene into memory, then moving ''X'' degrees away from the perpendicular line of sight to establish when the device fails to acquire the target. Again the other significant signature parameters should be varied to establish sensitivity.

6. OPEN LOOP RANGE CLOSURE

In this test the guidance unit is moved along the direct line of sight from acquisition (50,000 feet) to loss of lock (usually less than 5,000 feet). The usual parameters are varied in addition to closing speed.

7. CLOSED LOOP RANGE CLOSURE

For final verification of weapon system accuracy, the tracker must be evaluated in a dynamic simulation. In these tests everything is put together and the missile should be flown through the complete range of expected variations.

Approximately 6,000 runs are required to complete the above program. To accomplish all of these tests in the real world would require the development of new test tools, take a long time, and require the expenditure of many missiles. Fortunately, a test chamber, a Guidance Development Center, has been defined which will satisfy these requirements at a reasonable cost.

SIMULATION FACILITY AND CAPABILITIES

To simulate missile problems and test optical tracking techniques, four elements are required.

a) A target against which the missile can fly;

b) Translational and rotational systems to simulate the spatial relationships of the missile relative to the target;

c) Provisions for mounting the guidance seeker;

d) Computers to implement the mathematical models of the missile aerodynamics, kinematics, actuators, and autopilot as well as numerical calculations of accelerations and velocities.

These basic elements properly integrated will provide a laboratory for the development and evaluation of advanced electro-optical guidance systems. It will also provide the resources and capabilities necessary for developing and evaluating missile systems, subsystems and components throughout the RDT&E development cycle.

To provide the first three capabilities, the Guidance Development Center (GDC) has six degrees of freedom, three degrees of translation and three degrees of rotation. The three degrees of rotation are provided by a three axis gimballed flight table (Figure 7, Item 1) in which the seeker can be mounted to the inner roll gimbaled housing (Figure 8). This provides all of the body angular displacements, body rates, and accelerations required for open or closed loop tests. The three degrees of translation are provided by three transport systems. The lateral transport moves the flight table in a horizontal direction through a carriage/rail system (Item 2) mounted on a horizontal beam assembly (Item 3) and in a vertical direction through an end box/rack and rail vertical column system (Item 4) inside the vertical column housing structure. This translation provides all vertical or altitude displacements, rates, and accelerations. These five degrees of freedom are so hardware interfaced that they can be considered as comprising a five degree of freedom assembly. The longitudinal transporter system (Item 5) moves the terrain model through a carriage interfaced rack and rail system toward the flight table. This longitudinal travel provides range closure displacements, rates, and accelerations. The dynamic range ratio of 1000:1 on all drives provides large capability for scaling options. The longitudinal system transports the three dimensional terrain model which provides three dimensional static and dynamic signatures to the seeker. The terrain model (Figure 9) is transported on the longitudinal system and provides a series of straight line contrast areas, bland topography with various contrast targets, and servo-controlled moving target models. The straight line contrast areas with a target located closely to the contrast line exercises the closure shift drift problem experienced in most correlators. The bland area with various contrast targets exercises the acquisition and hold lock capability under different lighting and contrast ratios. The moving targets provide dynamic tracking capability against changing background scenes. The target model can be tilted to an infinite number of positions from 0 to 25 degrees from the horizontal so that various geometries and altitudes can be accommodated. When the target model is horizontal, it can be rotated in azimuth and secured at 90 degree points for presenting different aspect angles to the seeker. The target model and transport system are operable from inside the laboratory under controlled artificial lighting conditions from 10^{-2} to 200 footcandles as well as outside for bright sunshine and natural lighting conditions (Figure 10).

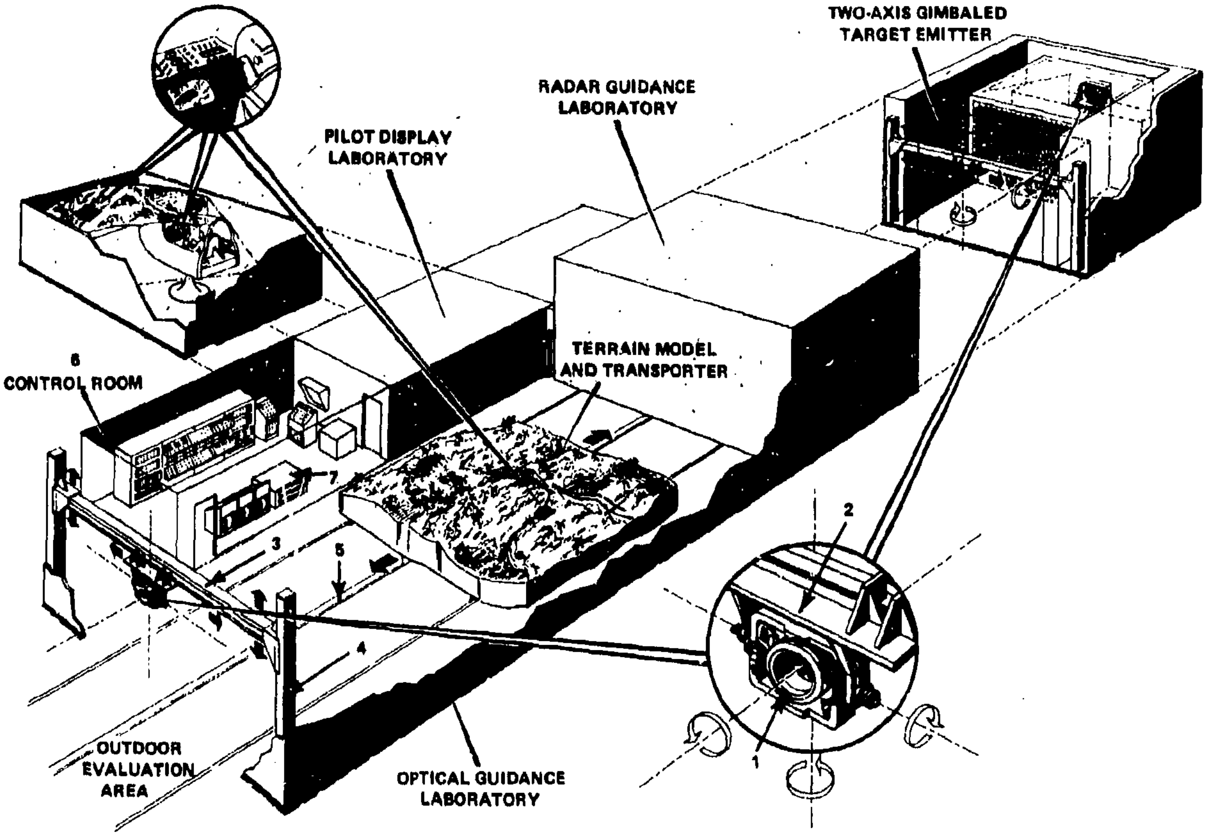

Figure 7. Guidance Development Center Layout

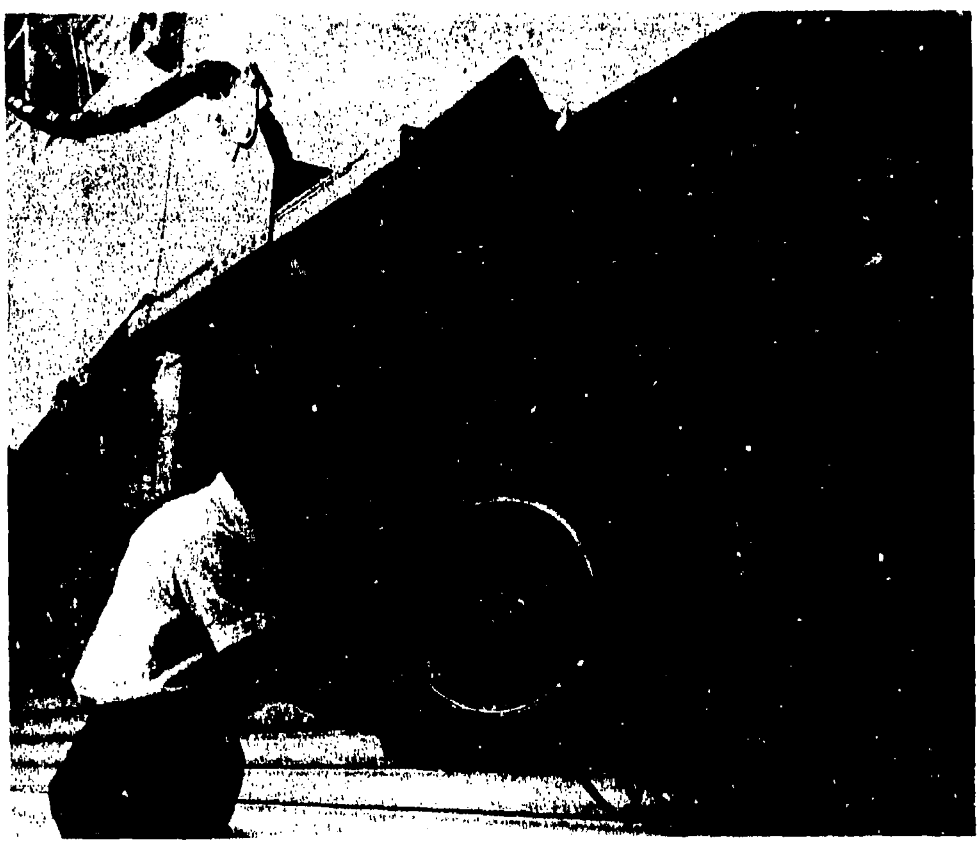

Figure 8. XAGM 79 Seeker Mounted

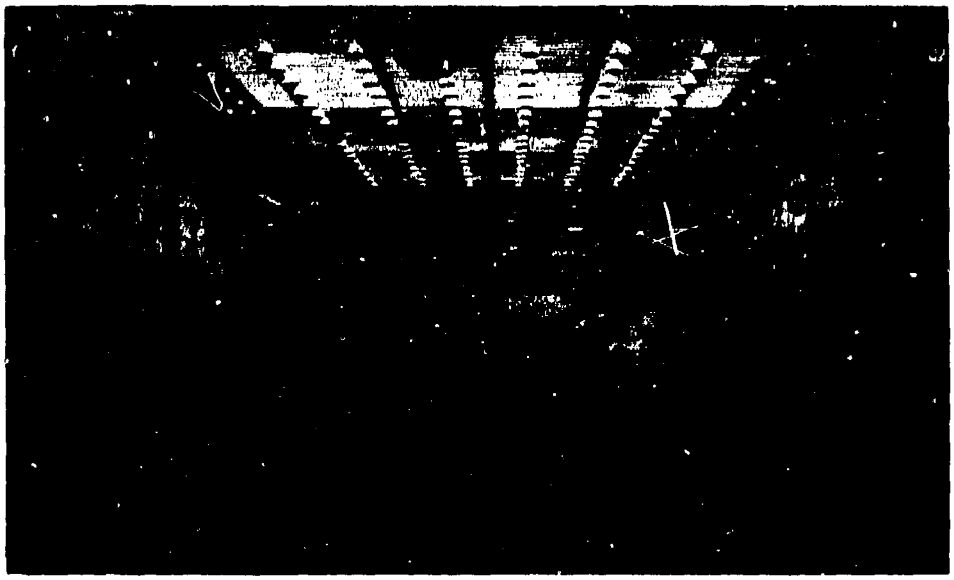

Figure 9. Terrain Model

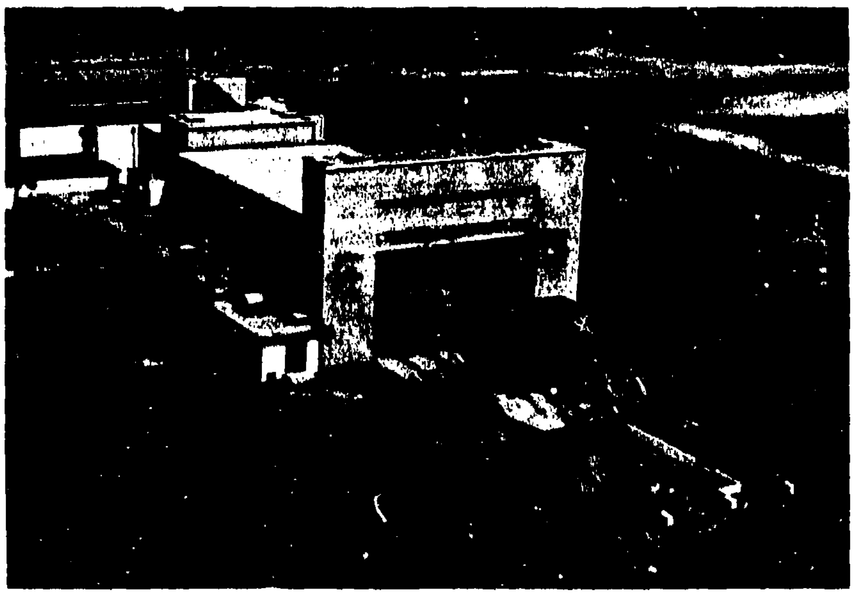

Figure 10. Terrain Model Outside

To accommodate the long focal length lenses (250 mm) on the optical guidance seekers, an autofocusing lens system (Figure 11) must be used for collimating the light in the near field. For the task to be accomplished, the lens system must be essentially a zoom lens working backwards. Hence, as the target model closes range toward the seeker, the focal length changes. The autofocuser transforms light from a scaled model to the seeker optics in the same manner as the light from the same scene in the real world is reflected to the seeker optics.

The control room (Figure 7, Item 6) is the nerve center of the GDC, since all equipment is operated from this point, all data collected and laboratory conditions are monitored here. The control room houses the control consoles (Item 7) from which all rotational and translational drives are operated. The closed circuit television and lens drives are also controlled from consoles in this room. The instrumentation lines from the seeker are brought into an instrumentation console from subsequent rerouting to consoles and data handling and recording equipment.

This physical equipment provides the system geometry and dynamics and when properly driven by a computer simulation provides the system capability shown in Table I. Other simulations can be accommodated by changing the scale factor of the terrain model. Removable features permit variations between 300:1 and 1200:1.

Figure 11. Infinite Focus Lens System

TABLE I

Optical Guidance Simulation
Area Flight Parameters (600:1 scale)

Slant range – 50,000 ft
Altitude – 24,000 ft
Lateral range – to 24,000 ft
Horizontal velocity – to 6,000 ft/s
Vertical velocity – to 3,600 ft/s
Lateral velocity – to 2,400 ft/s
Pitch velocity – to 200 deg/s
Yaw velocity – to 200 deg/s
Roll velocity – to 800 deg/s
Pitch position – ±120 deg
Yaw position – ±45 deg
Roll position – continuous
Sensor system weight – 50 pounds
Sensor system size – 14 in dia

The fourth element required is the computers, which are also located in the control room. The elements of a typical seeker/missile/target simulation relationship are shown in Figure 12 where:

λ is the angle between the seeker axis and the missile centerline;

ϵ is the angle between the line-of-sight and the seeker axis;

θ is the missile attitude angle reference inertially to the x coordinate;

γ is the missile flight path angle.

The missile and seeker angles must be considered as having been derived from vector quantities since all have components in both the x-y and x-z planes. Since the missile roll angle affects the missile error angle, three degrees of rotational freedom between the missile and the reference frame are necessary. In addition, the seeker may have two degrees of freedom with respect to the airframe. If the seeker's third degree of freedom (roll) is activated, a means must be provided to resolve pitch and yaw error components back to the missile pitch and yaw frame of reference. This resolution must be built into the seeker head and after processing, the error signals are the same as though this additional degree of freedom did not exist. For some cases of simulation, where the seeker gimbal hardware is not available, the flight table must have the capability to perform this task. The resolution can be effected through a combination of resolvers on the flight table or computer and computer program for controlling the candards for induced roll.

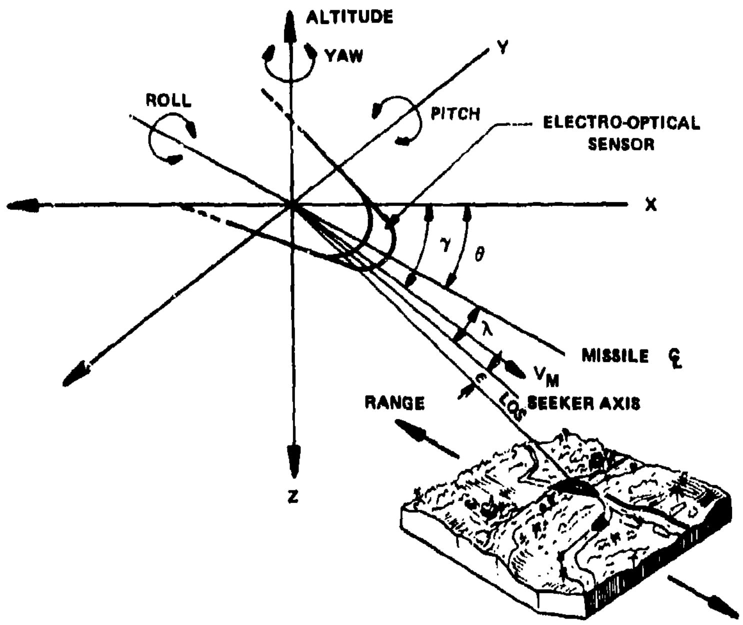

Figure 12. Missile/Seeker/Target Relationship

A block diagram depicting the overall simulation of a missile in flight is shown in Figure 13. It is divided into computer performed simulation, translational and rotational degrees of freedom equipment, a target terrain model, and a guidance seeker package. The flight table provides the missile reference frame from an inertial frame. All forces and moments on the airframe are calculated from this reference frame. Division by mass properties (inertia) then gives the accelerations in the same frame. Integration of these accelerations, then, gives the translational (u, v, and w) and rotational (p, q, and r) velocities in this body frame of reference. These velocities must then be transformed to an inertial frame of reference to be correct for commanding the velocities of the three degrees of translation. Thus, by adding the three degrees of translation, the true dynamic spatial relationship of the missile relative to the target is obtained. By having this angular and rotational interaction, the closed loop simulation permits the computer representation of the aerodynamics, kinematics, autopilot and actuators to present the same dynamic environment that the seeker experiences under actual flight conditions. Even launch dynamics and wind buffeting loads can be simulated with realistic forces being applied to the seeker under test.

The significance of the multi-feedback loops is that it serves to wash out the effects of many of the errors that can occur in implementing the simulation. Some of the non-simulation errors are:

a) Aerodynamic coefficients and characteristics generally between 5 and 25 percent;

b) Normal manufacturing tolerances of the missile system introduce uncertainty in some parameters such as center of gravity and bias errors such as roll torque;

c) As a result of a), the velocity of the missile between 5 and 10 percent.

Although these data may seem gross, dynamic performance characteristics can be determined under a controlled and repeatable environment with the utilization of a precision simulation facility.

Where seeker hardware, physical target, and simulated missile dynamics are operating in a closed loop, an effect is introduced that is not present in open loop guidance. This is the dynamic effect that can cause an uncontrolled oscillation if the phase shift around the loop becomes 180 degrees at the time the loop gain is unity. These phase shifts are inherent in the various pieces of the overall loop and cannot be designed out of the missile/seeker/target tracking task. Generally, the dynamics limit the flight hardware guidance loop bandwidth to 0.5 Hz or less. For good following, the translational velocity loops should be designed for at least 3.0 Hz in the longitudinal, vertical, and lateral directions. The rotational rates and rise times are not adequate to simulate the short period motions of the airframe; however, this is not required since the simulation is not intended to evaluate autopilots and flight control stability. This limitation is accepted to reduce the weight placed on the movable sensor transport. If an autopilot/airframe should have a combined bandpass significantly greater than 0.5 Hz, the problem must be programmed as essentially a trajectory problem. Here only lift and drag curves are utilized to simulate the airframe. Account must be taken of the angle of attack so that the missile centerline will have the correct angle with the velocity vector.

The mechanical elements of the GDC are designed so that all structural vibration frequencies will be above 10 Hz and thus out of the pass band of most guidance sensors. The structural stiffness permits holding the extraneous angular motions due to vibrations to about 0.1 milliradian. Optical sensors presently being designed have linear regions of operation of about 1 milliradian. The stiffness permits the system investigation to separate the inherent difficulties in the guidance sensor from those due to simulator departures from the ideal.

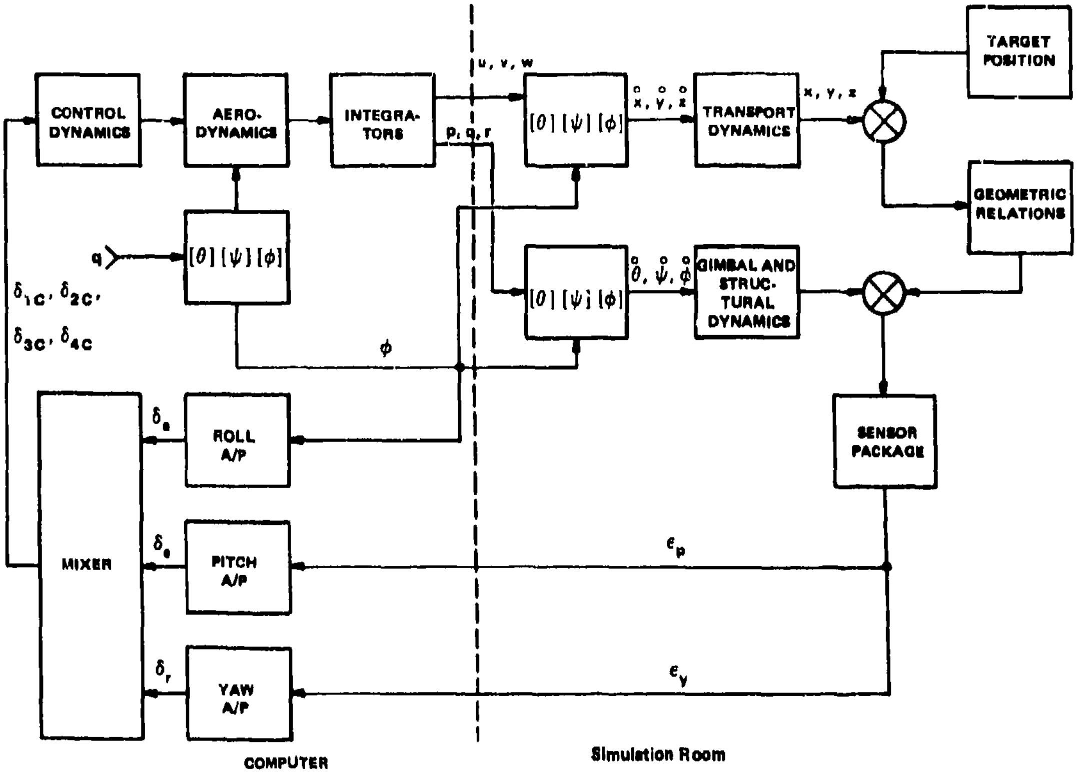

Figure 13. Overall Simulation

With the seeker mounted on the three axis flight table, the steering signals from the seeker hardware are used to drive the mathematical model simulated on the computer. The output of the math model then provides input signals to the translational and rotational drives.

With all of these systems integrated, the GDC functions as one of the vital elements in the overall missile system development process. It allows rapid and repeatable testing of guidance components, subsystems and systems under realistic controlled conditions. As a precision laboratory tool, the GDC provides the capability for

a) Evaluating breadboard and brassboard hardware performance,

b) Evaluating design modifications to hardware,

c) Establishing component parameters for optimum performance,

d) Evaluating tracking tasks,

e) Evaluating CEP studies, and

f) Final preflight check for development programs.

LABORATORY OPERATION

Simulation by definition is only an approximation of the real world. It is acceptable only if experience proves it to be so. The laboratory just described was conceived, designed, and built at the risk that it might prove useless. Its design was an extrapolation of the concepts explored by many others and took advantage of the experience, good and bad, of the preceding attempts. Its final validation was accomplished during 1967 when 16 missiles were evaluated in it and then fired to confirm operating characteristics. No missile which has operated through the range of conditions possible to simulate in the GDC has yet failed to perform during terminal trajectory flight conditions. It is being used by all of the U.S. services, other governments, and other commercial contractors and has been in two shift operation almost continuously. This style of testing has proven itself to the point where it can receive its most sincere vote of confidence, it is being duplicated by others.

The final test of a laboratory is the economics of its operation. Two questions arise: Could something less sophisticated and therefore less costly perform the function, and if the investment must be made, will there be savings to permit its recovery? In answer to the first question, it has always been a GDC philosophy that no piece of equipment should be tested in the GDC until it has worked in a dynamic environment against a two dimensional scene. Despite this requirement, no guidance unit, government supplied or private contractor, has ever performed within its design specifications when tested through the range of design parameters in the GDC, including units which have been captive flight tested. Therefore, no simpler simulation has presently been established which can replace the GDC.

The recovery of investment is a more complicated question. To accomplish the tests described in the preceding section requires a program of the type shown in Figure 14. Typically, this effort would cost approximately $50,000 and would establish the performance of one developmental unit or qualify one flight test missile. Each additional missile would be qualified at a cost of $2500. A 20 missile program would therefore cost approximately $100,000, exclusive of the costs of the hardware supplier in maintaining and evaluating his equipment. Since the above program would require four months for completion, these costs would be another $100,000 including data reduction and evaluation for a total cost of $200,000. Total test time on the 20 missiles would be 600 hours. Experience has shown that a captive flight test program run extremely effectively will produce 10 hours of useful data per week, being limited to daytime operation and clear days. Assuming that the costs of maintaining the test range and aircraft are equivalent to those of the GDC (rather conservative), the flight test program will cost $700,000 and take over one year to complete. This implies that if we have both capabilities, 1/2 million dollars can be saved on each program. Since the laboratory requires an investment of approximately 6 million dollars to duplicate, these costs can clearly be recovered in a short time if multiple electro-optical missile systems are being developed.

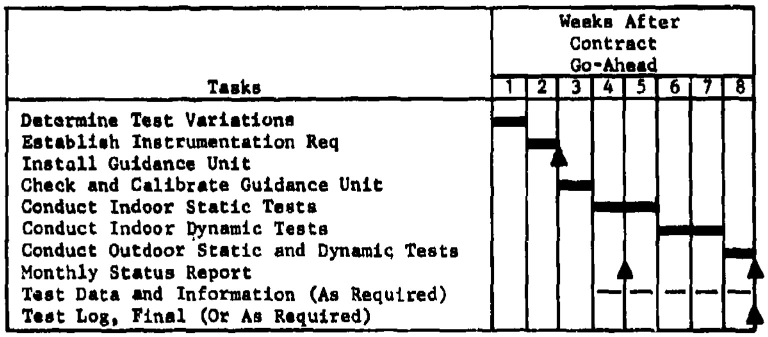

Figure 14. Typical Schedule for 8 Week Evaluation Period

This analysis, however, neglects much larger potential savings. Since the cost of many programs is established by the costs to develop the configuration before flight test, the use of the laboratory as an evaluation tool to reduce development costs will have a much greater return. One recent electro-optical program required an investment of over 300 million dollars in eight years. Of this amount, five years and 160 million dollars were spent before production. Three years were spent in prototype and pilot testing and evaluation at a cost of 100 million dollars. During this time 60 missiles were built and fired. Of these firings, twelve were classed as ''partial successes.'' Had these flight been saved, a cost reduction of over 10 million dollars would have occurred, thus recovering all lab expenses in less than two years.

FUTURE DEVELOPMENTS

The past success of the integrated lab approach to electro-optical terminal guidance system testing will ensure its application to other systems of the future. This approach is basically an extension of inertial guidance testing philosophy and has been applied to other technologies, most notably IR. Efforts are underway to solidify a similar approach to RF systems. The system laboratory with its inherent ability to permit precise variation of system parameters, repeatable and timely testing, and lower development risks and costs has become an accepted and required ingredient in guidance system development.

IV. CONSIDERATIONS IN GUIDANCE TECHNOLOGY

GUIDANCE LAW APPLICABILITY FOR MISSILE CLOSING

by

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SUMMARY

Terminal guidance analysts use a small number of guidance law general types to produce missile steering signals from sensed target information leading to suitably close miss distances.

Miss distance variations for the different guidance laws are displayed for an air target intercept as target and missile characteristics are changed.

A general comparison of guidance law applicability is presented for air and surface targets.

1. INTRODUCTION

The selection of a guidance law or doctrine for terminal homing is in the middle of the action in guidance and control system development.

Between the overall mission concept and the detailed selection of control and avionics hardware, many trades and design decisions are made. A key part of the process is the selection of the analytical formulation for converting sensed target information into missile steering commands. The analyst who selects the formulas must consider mission desires, avionics capabilities, and cost. He must make wise choices in counsel with the other engineers and managers on the program.

The guidance law is a part of the guidance loop shown in Figure 1, which depicts the intimate involvement of many weapon system subsystems. The three basic types of guidance laws will be discussed. The sensitivity of the performance of the guidance laws to guidance loop subsystem parameter variations will be displayed in typical situations. Finally, factors considered and guidance law applicability guidelines are shown for air and ground targets.

The typical sensitivity data and the guidelines are intended to assist analysts in selecting future guidance laws for tactical homing missiles.

2. GUIDANCE LAW TYPES

Three guidance law general categories can be named, into which all other guidance laws can be forced to fit. Several modifiers may be required to name special cases, but much of the literature of the past twenty years refers to Line-of-Sight, Pursuit, and Proportional guidance as the major guidance law types or categories.

The definitions used in Guidance, by Locke and his collaborators (Van Nostrand, 1955) are followed and illustrated in Figure 2 for the three guidance law types.

GUIDANCE LOOP

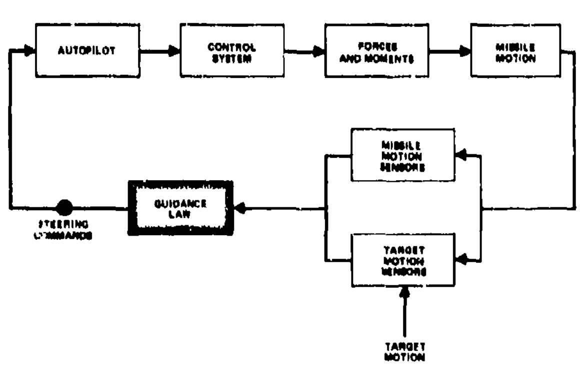

Figure 1

GUIDANCE LAW TYPES

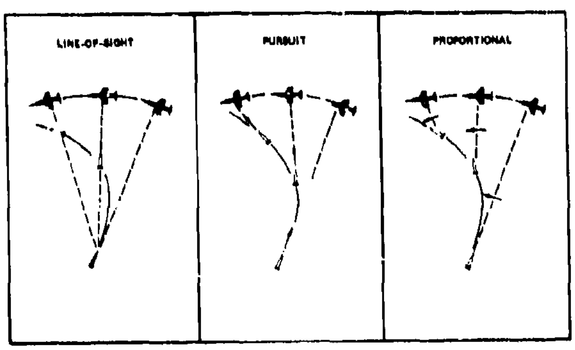

Figure 2

A Line-of-Sight missile guidance law is one in which the missile is intended to remain on the line joining the target and a point of control. The left drawing of Figure 2 shows the missile on the line-of-sight at three positions in the trajectory. A beam, optical or radar, for example, tracking the target could illuminate a receiver in the missile and cause error signals to be created if the missile gets off the beam.

A Pursuit missile guidance law is one in which the missile velocity vector is always directed towards the target. The middle drawing of Figure 2 shows the missile velocity vector aimed at the target in three positions along the trajectory. The direction of the missile velocity vector must be sensed to steer the missile with this guidance law.

A Proportional guidance law is one in which the rate of change of missile heading is made proportional to the rate of rotation of the line-of-sight from the missile to the target. In the right drawing of Figure 2, three positions are shown in which the angular rate symbol on the line-of-sight signifies a missile avionics measurement of the line-of sight rate. Lateral acceleration commands are proportional to the rate. For constant speed of the missile and target, and zero line-of-sight rate, the missile is on a straight line collision course.

In the three drawings of Figure 2, the general shapes of the trajectories are similar and all are intended to look like successful intercepts are being made. They should look similar, since the purpose in all is to steer to intercept the target. To distinguish different features, a closer look is required at the missions, target situations, and avionics required for each.

The line-of-sight mission has the target and missile in view simultaneously, with steering signals proportional to the angle of the missile off the line-of-sight. Missions are generally short range - hundreds of yards to a few miles - since the missile does not track as it closes. A speed advantage for the missile is required, since there is no anticipation or lead in the simpler mechanizations. Target maneuvers will throw large excursions into the missile trajectory. The avionics for the missile are simple. For radar tracking, a rear facing antenna and the electronics to sort out left-right from up-down displacements make up the guidance law implementation required items. For optical viewing from the launcher, an unreeling wire attached directly to the missile can be used to transmit left-right and up-down commands. With these implementations, it is necessary to have the launcher reference system or personnel in the loop continually from launch to impact. Figure 3 summarizes the features of line-of-sight guidance.

With Pursuit guidance, a missileborne tracker is assumed. Missile range can be up to tens of miles if guidance to acquisition is available. Lock-on ranges are tracker limited to a few miles. Once locked on, the missile is on its own and any mid-course guidance system attention can be discontinued. The direction of the velocity vector V of the missile is intended to point at the target, and lateral acceleration a steering commands proportional to the angular deviation θ are issued to bring this about, as shown in Figure 4. Inertial sensors, wind vanes, or angle of attack meters can be used for velocity vector direction establishment. For small angles of attack, a modification of pursuit guidance is to simply point the centerline of the missile, referenced to the tracker, at the target. Non-zero lead angles can be mechanized to assist with relatively fast targets or for proximity or altitude fuzing situations in which the warhead position relative to the target is significant. Even with such mechanizations, the act of simply looking and pointing can frequently be implemented with simple missile processing.

LINE-OF-SIGHT GUIDANCE

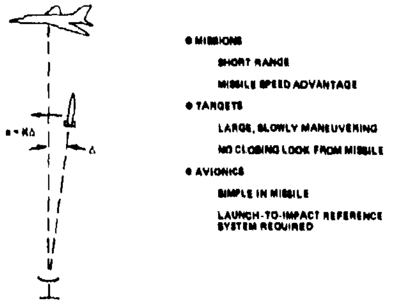

Figure 3

PURSUIT GUIDANCE

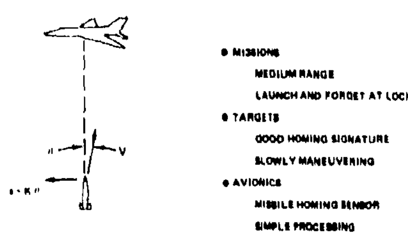

Figure 4

Proportional guidance is the type used in somewhat more difficult guidance situations. The rapid sensing and reaction to target maneuvering makes this law more desirable. As shown in Figure 5, the angular rate $\dot{\theta}$ of the line-of-sight to the target, determined by a missile tracker, must be sensed on the missile. Lateral acceleration a is proportional to the rate. The proportionality constant K is varied to make the missile attain the degree of responsiveness which is compatible with its response, tracker noise, target signal noise, and target maneuver capability. The requirement to measure line-of-sight rate and produce steering commands in anticipated difficult tracking situations generally causes a complex data processing task for the missile avionics.

The three laws described above will be applicable and selected in different situations. Differences in their ability to produce results will be studied next.

3. GUIDANCE LAW SENSITIVITY

The use of a particular guidance law in a particular application depends on whether it can provide small miss distances. Until the miss distances are small enough, cost, complexity, and all the other pertinent parameters do not enter into consideration.

To show how miss distance can vary in a particular situation with guidance law choice, a simulation was performed for an air target. The missile and target models used represent no particular system, and the terminal sensor can be considered as a radar, infra-red, or optical homer. A set of nominal conditions was selected for the start of the final homing phase and different engagement parameters were varied for each of the three guidance laws. In this way, the sensitivity of the guidance laws themselves can be determined and displayed.

The nominal conditions of the engagement are displayed in Figure 6. The interceptor to the left is assumed to have a speed V_M of 2000 ft/sec and be five degrees off the line-of-sight to the target in a top view. The analysis is for the horizontal plane only. The range at simulation start is 10,000 ft. The speed V_T of the target at the right of Figure 6 is assumed to be constant at 1,000 fps, at zero degrees with the line-of-sight, so that the nominal engagement time is about three and a half seconds.

PROPORTIONAL GUIDANCE

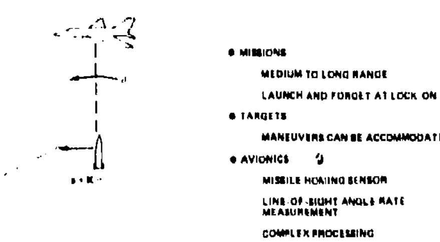

Figure 5

NOMINAL CONDITIONS

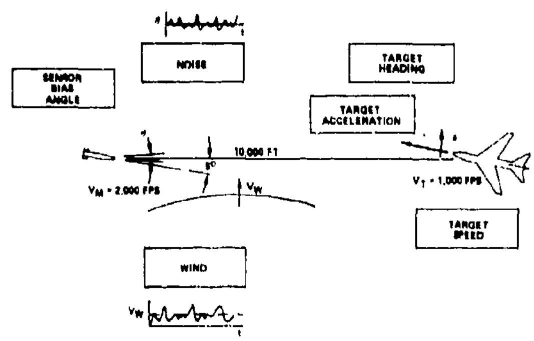

Figure 6

The parameters studied for their effect on miss distance as a function of guidance law are shown in the boxes of Figure 6. They are: initial heading of the target, varied off the line-of-sight; target speed; magnitude of target acceleration for evasion capability, measured as a target turn capability to the target's right; sensor bias in the measurement of the line-of-sight angle; sensor noise from all sensor causes at two different levels of target generated noise; and the average magnitude of a wind gust from one side whose instantaneous magnitude is a random function of time.

The simulation results will now be displayed. To obtain the data, the parameter whose sensitivity was under investigation was varied as indicated in the curves, and several digital computer runs at several parameter values for each guidance law were made. The root-mean-square (RMS) miss distance in the horizontal plane at closest approach of several runs for each condition is plotted. The mean value of the miss distances can be a very significant parameter but is not reported. A tight grouping of miss distances with all misses to the same side can produce far different target kill effects than a scattering of misses with average value zero.

The sensitivity of the guidance laws to target heading is shown in Figure 7. The proportional guidance law use of line-of-sight rate measurement produces larger corrective action earlier than the line-of-sight law or the pursuit law. The latter two respond too late when the missile is not going to come pretty close as the last few seconds begin. For the case of the target at twenty degrees off the line-of-sight, if both missile and target kept on straight line courses, the miss would be about 1500 feet, indicating that control authority is not limiting the reduction of miss distance.

When plotted to the scale of 100 feet for maximum miss distance, the sensitivity of the guidance laws to target velocity over a range between zero and double the nominal speed seems small, as shown in Figure 8. Close examination shows, however, that the proportional guidance law, subject to the errors in angle rate measurement, produces a larger miss distance than the other laws as target speed increases.

GUIDANCE LAW SENSITIVITY TO TARGET HEADING

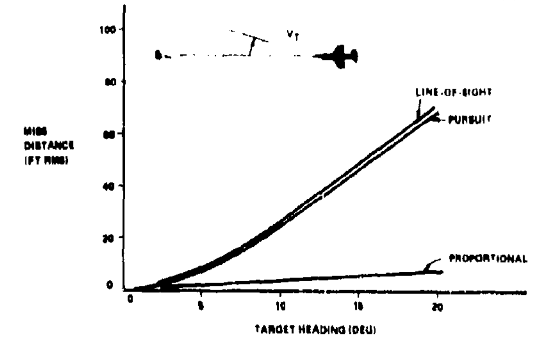

Figure 7

GUIDANCE LAW SENSITIVITY TO TARGET SPEED

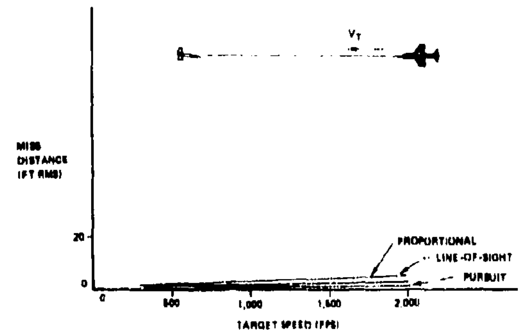

Figure 8

If the target vehicle turns away from the missile, the proportional guidance law is able to command effective response to the target acceleration sooner than the line-of-sight or pursuit laws. Figure 9 shows that as target acceleration increases, all the guidance laws produce commands which lead to similarly increasing miss distance magnitudes However, if the gain for proportional guidance is raised by one-third, the miss distances fall noticeably, as shown in the lowest curve marked as having a higher proportionality constant. Similar changes to the line-of-sight and pursuit laws do not produce similar improvements.

For line-of-sight and pursuit guidance, which depend on steering directly at the target, increasing bias errors in line-of-sight measurement will cause increasing miss distances. Bias errors in the line-of-sight measurement can come from mechanical installation, boresight procedures and changes, radome errors, electronic component changes, and mechanical changes. The proportional guidance law, however, is essentially insensitive to bias errors, since they produce no angle rate error. Figure 10 illustrates the trends.

GUIDANCE LAW SENSITIVITY TO TARGET ACCELERATION

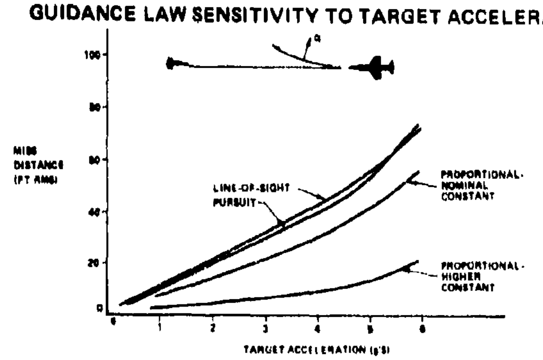

Figure 9

GUIDANCE LAW SENSITIVITY TO SENSOR ANGLE BIAS

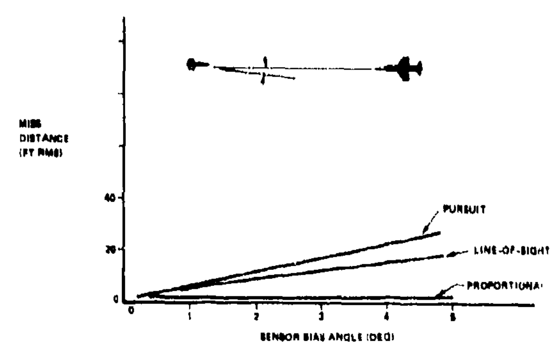

Figure 10

While the proportional guidance law can handle a fixed or bias angle error, it is susceptible to causing erroneous steering signals from noisy angle sensing leading to very noisy angle rate information. Figure 11 displays the effects of sensor noise on miss distance, showing line-of-sight and pursuit guidance to be less affected by the angle noise. The solid curves are for sensor noise with no glint or noise from the target. When target noise is added, which increases in angular effect as the missile closes, the proportional guidance law miss distances are increased substantially, but the angle-only laws are not affected, as shown by the dashed curves.

The slow response of the line-of-sight guidance law to wind gusts is shown in Figure 12. With pursuit guidance, with its sensing of the velocity vector direction, and proportional guidance, in which the line-of-sight rate change is detected quickly when the missile is blown off course, there is less sensitivity to the wind.

The simulation and results described are of preliminary nature, since many more details are added to the simulation of a particular system and several more parameters are varied. However, the trends are useful and can provide guidelines to the applicability of the different guidance laws for air target situations.

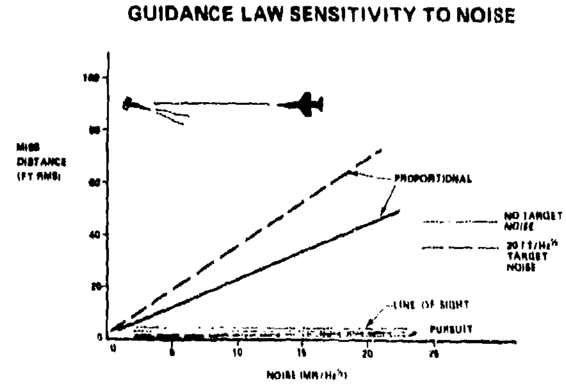

Figure 11

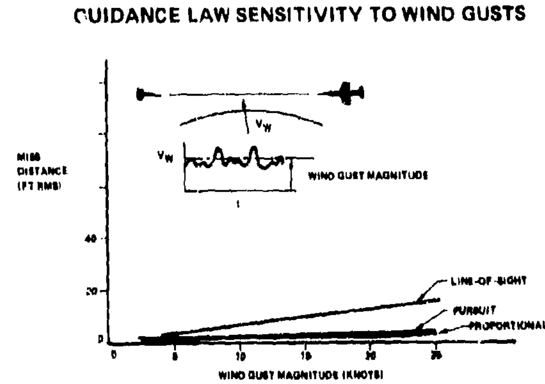

Figure 12

4. GUIDANCE LAW APPLICABILITY

A tabulation of the results of a sensitivity study can lead to guidelines for guidance law applicability. The study results for air targets are displayed in Figure 13, with a performance rating assigned to the sensitivity of each guidance law to the parameters varied.

The line-of-sight and pursuit laws are seen to have poor or average performance in several categories. The avionics equipment cost and complexity is less for these laws than for proportional guidance. The proportional guidance performance is good in all categories except in its response to noise. Angle rate measurement noise, homing sensor noise, and target noise all cause responsiveness which generates steering signals driving the missile all over the sky.

If the guidance constant is set for low gain in proportional guidance, performance is poor against maneuvering targets. For high gain, performance is good against maneuvering targets but poor against noisy targets. The simulation data required for such items as gain setting frequently involves cross-plotting two sensitivity curves or applying more than one parameter change at a time. The combinations are endless. One set of curves which deals with gain constant establishment is shown in Figure 14. The increase in miss distance from noise with increasing gain and the decrease in miss distance from target acceleration with increasing gain suggest a compromise setting for the gain constant.

GUIDANCE LAW TRENDS FOR AIR TARGETS

		TARGET HEADING	TARGET SPEED	TARGET ACCELERATION	SENSOR ANGLE BIAS	NOISE	WIND GUSTS
LINE OF SIGHT	GOOD		✓			✓	
	AVERAGE				✓		✓
	POOR	✓		✓			
PURSUIT	GOOD		✓			✓	✓
	AVERAGE				✓		
	POOR	✓		✓			
PROPORTIONAL	GOOD	✓	✓	✓	✓		✓
	AVERAGE						
	POOR					✓	

Figure 13

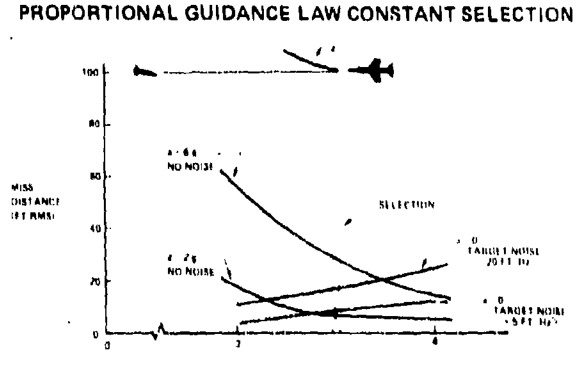

Figure 14

A guidance law analyst should select the lowest cost, simplest, guidance law which can meet miss distance or kill probability requirements. Against air targets, simpler engagement situations can use line-of-sight or pursuit guidance. Proportional guidance is employed for more difficult engagements.

When the straightforward implementation cannot provide sufficient performance, modifications may be required, at the expense of additional equipment and complexity. Two examples of modifications to proportional guidance are shown in Figure 15. There are advantages to adding a bias to the steering signal, as listed. The bias can bring the missile more nearly into a head-on situation, assisting against fast targets and missile loss of speed due to control application. Some targets are more vulnerable to warhead effects from the top or bottom, and a correct bias can make the missile arrive early or late - high or low - for an anticipated target and closing geometry. Another technique in use is to change the guidance constant as a function of time-to-go to intercept. This reduces control activity and, with it, power consumption and speed loss. For high gain at the end, the missile calls for highest accelerations to close on the target. Many other such variations have and will be conceived and used by guidance analysts.

In the same manner as sensitivity studies and applicability trends for guidance laws with air target situations have been studied, studies of surface targets can be performed. Target motion is not involved. The presence of the land or sea can call for warhead detonation above the target in some cases, rather than impact, as an additional consideration.

In Figure 16, a tabulation of general trends of guidance law performance against surface targets is displayed. For line-of-sight steering to the target, the trajectory tends to flatten and, for low approaches, raises the probability of clobber. Sensor angle bias has a similar effect on line-of-sight and pursuit guidance. The over-all assessment leads to pursuit and proportional guidance having similar performance. With smaller avionics costs, pursuit guidance is frequently chosen over proportional for surface targets.

PROPORTIONAL GUIDANCE LAW VARIATIONS

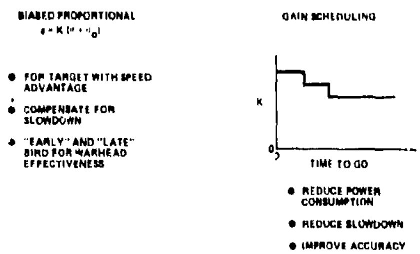

Figure 15

GUIDANCE LAW TRENDS FOR SURFACE TARGETS

		AIR BURST FUZE	SENSOR ANGLE BIAS	NOISE	WIND GUSTS
LINE-OF-SIGHT	GOOD			✓	
	AVERAGE				
	POOR	✓	✓		✓
PURSUIT	GOOD			✓	✓
	AVERAGE	✓			
	POOR		✓		
PROPORTIONAL	GOOD		✓		✓
	AVERAGE	✓			
	POOR			✓	

Figure 16

As with air targets, many variations are used. Applying a bias to the steering signal when still far from the target provides a higher trajectory shown in Figure 17. This leads to less chance of clobber, a better view of the target, and, in some cases, much improved warhead lethality.

PURSUIT GUIDANCE LAW VARIATION

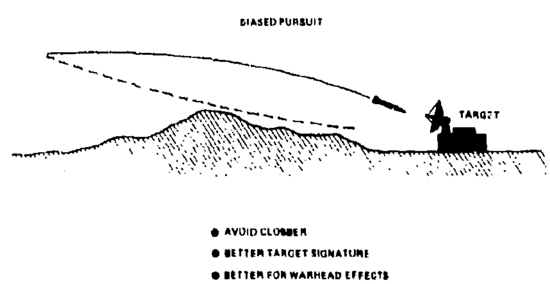

Figure 17

5. CONCLUSION

In a tactical weapon system development, a guidance law must be selected and implemented along with all other hardware and software elements. The guidance law analyst works with all members of the design team. He must be able to work from simple to very complex analyses and simulations, to provide preliminary and firm requirements and designs.

The background of missile system developments of the past twenty-five years has shown certain laws to be applicable in air target, surface target, missile homing, and surface tracking situations. Some of these have been displayed and discussed in a typical example. For any specific weapon system, the details and desires will change but the trends are likely to persist.

The greatest changes in guidance law selection for future systems will likely result from the rapid advances in digital components and devices. Both surface pre-launch data processing and on-board real time functions will be able to be performed with small, low power, reliable computing elements. For the same or smaller power, weight and cost, we will see guidance analysts supplying ever more sophisticated and flexible guidance laws which will provide better missile performance and greater leeway in other subsystem performance.

SELF-CONTAINED GUIDANCE TECHNOLOGY

by

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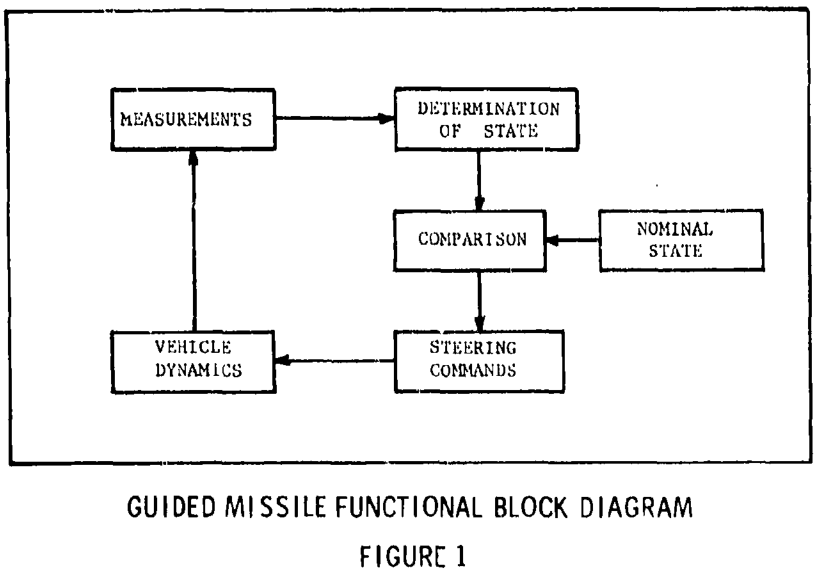
SUMMARY

Inertial technology provides a self-contained guidance capability applicable to tactical air-to-ground missiles. The basic inertial system, which consists of accelerometers, gyros and a computer, is immune to outside interference, and therefore ideally suited to military applications in a hostile environment. Inertial technology has progressed to a point where equipment size and cost are within reason for use with the tactical missile. This paper tutorially presents the basic principles and limitations of inertial guidance, including theory of operation, and physical and analytic coordinate system stabilization. Sources of error, and the propagation of these errors are described. Various methods of alignment, and system mechanization are considered. The state of the art, and the research and development process for inertial systems is discussed. Factors influencing the research and development process are identified along with the relationship between inertial system reliability and cost.

1. INTRODUCTION

Guidance is the art of making modifications in the direction of motion and/or speed of a vehicle, based on an estimate of present position relative to the desired destination. It may be as simple as riding a radio beam or as sophisticated as utilizing the outputs of several measurement sources in an optimal fashion. For military applications, there are obvious reasons for specifying a guidance system which is immune to outside interference, or at least highly resistive to such contamination. A self-contained system, that is, one which does not require externally derived information for its operation, meets this requirement. A guidance system based on measured airspeed, magnetic heading and a clock is an example of a self-contained system. Circumstances often make such a system inadequate or unsatisfactory, and require externally measured quantities to be combined with those of the self-contained system. Although such a system may be less immune to enemy interference, the likelihood of such disturbances occurring can be minimized by the proper selection of externally derived information. As an example, a single radar position fix taken at a low altitude offers little opportunity for an enemy to introduce spurious information into the system. Because of the desirability of immunity to outside interference, the remainder of this discussion will be focused on self-contained systems and selected aided systems appropriate to tactical missile guidance.

Figure 1 is a generalized block diagram showing the functions necessary in missile guidance. Starting with observations or measurements of quantities such as acceleration, range, time, elevation, etc., a particular state or status of the missile can be determined. This state is then compared with the desired or nominal state, and a command is issued to the vehicle reducing deviation between the estimated and nominal states. For all guided missile systems these same functions must be accomplished, i.e. observations, determination of state, deviation from the nominal, and initiation of corrective action. This process can be of a continuous or intermittent nature.



GUIDED MISSILE FUNCTIONAL BLOCK DIAGRAM

FIGURE 1

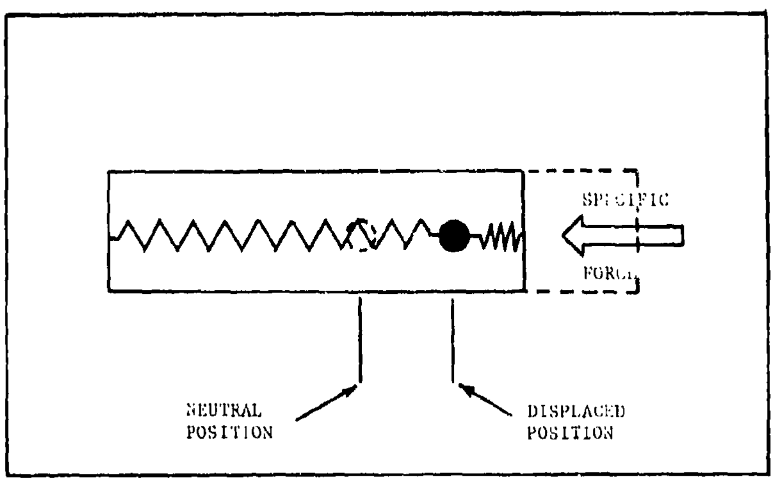
For a completely self-contained guidance system, the common observable quantities available for guidance of missiles are pressure altitude, speed through the air mass, magnetic heading, gyroscopic attitude, linear acceleration, and time. Pressure altitude and air speed require compensation as a result of changes in atmospheric conditions. Magnetic variations exist as a function of geographic location and are of limited use in the polar regions. While guidance systems utilizing these basic measurement quantities can have application to tactical missile guidance, the inherent accuracy limitations of such systems impose strict range and operational constraints. As a result much interest has been focused on inertial sensors which can provide an accurate indication of linear acceleration and vehicle attitude relative to a stable coordinate system. Inertial systems can also place constraints on the missile system and often require externally derived information for satisfactory operation. However, experience has shown inertial components and technology to be vital in a wide variety of missile guidance concepts.

Inertial components have been with us for more than forty years in the form of gyroscopic stabilizers, but it has only been in recent years that this technology has been applied widely to navigation and guidance of airborne vehicles. Today inertial systems are used in commercial airliners, space vehicles, under water and surface ships, and military aircraft and missiles. A review of the basic principles associated with inertial technology will provide a means of highlighting the problems and limitations imposed by the requirements of tactical missiles.

2. PRINCIPLES OF INERTIAL GUIDANCE

An inertial system consists functionally of accelerometers and integrators, gyros, a gravitational computer and a time reference. The accelerometers are the primary sensing devices of an inertial system. Nongravitational accelerations, as sensed by these devices, and gravitational accelerations, as provided by the gravitational computer, are integrated with respect to time to yield velocity and position information. A set of gyros is used to provide a computational frame of reference whose orientation is known with respect to inertial space. Each of these functional elements will be further explained in the following discussion.

The accelerometer can be represented as indicated in Figure 2. It consists of a case or housing and a test mass constrained by springs. A force applied to the case will cause a displacement of the mass from its neutral or zero acceleration position. The magnitude of the displacement is proportional to the force applied. An accelerometer is sensitive to specific force, i.e. the forces resulting from lift, drag and/or thrust. The accelerometer cannot sense gravity acceleration. This point can be visualized by the following example. An ideal accelerometer placed in a free fall environment would indicate zero acceleration, since the only force upon it is that due to mass attraction. This attraction creates an equal acceleration on both the accelerometer case and test mass. On the other hand, an accelerometer held stationary with respect to the earth and having its input axis aligned with the vertical will indicate a one "g" acceleration, i.e. the test mass will become displaced from its neutral position an amount equivalent to one "g". The accelerometer is indicating the specific force or lift required to restrain the accelerometer in a stationary position. Since an accelerometer cannot sense gravitational acceleration, gravitational acceleration must be accounted for in the computation of velocity and position. The magnitude and direction of gravity is calculated and added to an appropriate point in the system. This point will be clarified further in the discussion on mechanizations.

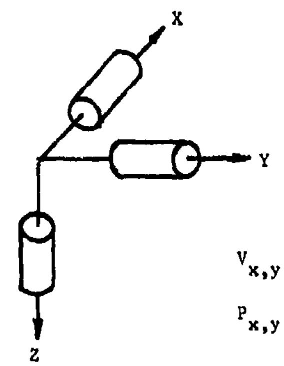


ACCELEROMETER REPRESENTATION

FIGURE 2

Assuming an orthogonal accelerometer arrangement, velocity and position can be calculated as shown in Figure 3, where A is acceleration due to a specific force, g is the gravitational acceleration, V is speed and P is position. The subscript X, Y, and Z identify the particular component while the subscript o defines the conditions at times equal 0. The accelerometers provide the necessary measured data to compute the system's velocity and position in a cartesian coordinate system. Notice that the component values of g must be known, or calculated. This coordinate system must be isolated either physically or analytically from the pitch, roll and yaw motions of the vehicle. Most aircraft and missile inertial navigation systems have used the physical isolation approach and will therefore be discussed first.

The characteristics of rigidity and precession exhibited by a gyro provides a practical method of maintaining this coordinate system isolated from the dynamic behavior of the vehicle. Remembering that a gyro is sensitive to angular rotation, Figure 4 illustrates symbolically a single channel stabilized computational system. The gyro senses rotations induced by the vehicle and provides a signal to the actuator which repositions the platform so that the stable platform and thus the accelerometer input axis remains fixed relative to inertial space. Three gyros placed orthogonally on this platform, along with additional gimbals and actuators, would provide complete isolation of the platform from vehicular motion. The resulting computational coordinate system fixed in inertial space would permit X, Y, and Z components of velocity and position to be calculated. Since the inertial components are essentially rotationally fixed with respect to inertial space they need not be designed to tolerate large angular rotations. This is an important point when comparing gimballed and strapdown systems, as will become evident in later discussions. For convenience, the position and velocity data could be converted to an earth-coordinate system through the solution of transformation equations. Anotner approach would be to control the platform's orientation such that the input axes of two of the accelerometers are maintained in the horizontal plane, while the third remains vertical. In addition, the platform could be rotated about the vertical such that one horizontal accelerometer is always pointed in a north direction. This mechanization provides for direct computation of position in an earth-centered latitude-longitude coordinate system. This mechanization, called a north-seeking local-level system would have advantages and disadvantages relative to the previously described space-oriented mechanization. Before proceeding with the description of the tangent plane mechanization, which is quite appropriate for short range tactical missile applications, several points can be made concerning space-oriented and local-level mechanizations.

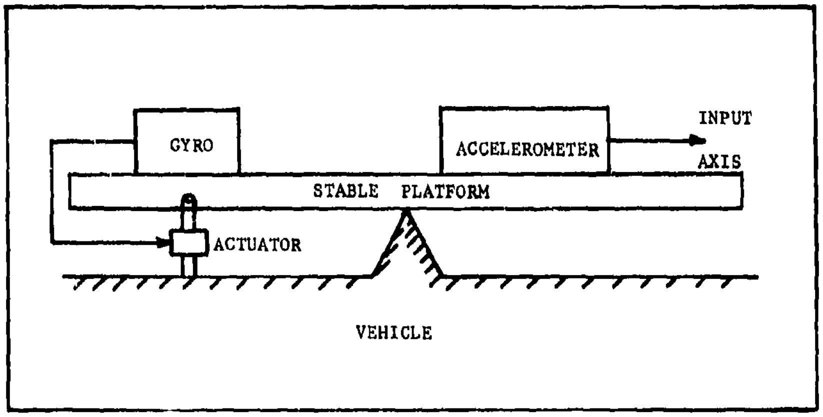


$$V_{x,y,z} = \int_0^t (A_{x,y,z} - g_{x,y,z})\,dt + V_{xo,yo,zo}$$

$$P_{x,y,z} = \int_0^t V_{x,y,z}\,dt + P_{xo,yo,zo}$$

ORTHOGONAL COORDINATE SYSTEM

FIGURE 3



STABILIZED PLATFORM

FIGURE 4

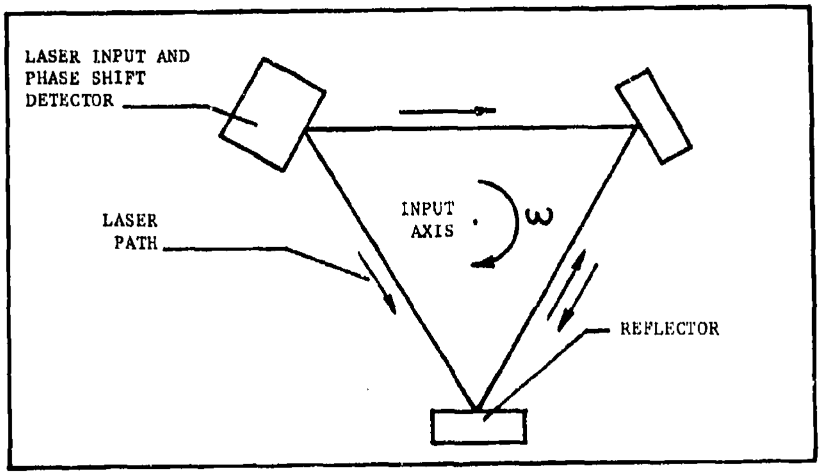
The first point concerns the calculation of acceleration due to earth's gravity. As mentioned previously, gravitational acceleration must be accounted for prior to determining velocity and position. In the local-level mechanization the vertical channel is the only one requiring compensation, since the platform is continuously torqued such that the gravitational vector is coincident with the vertical accelerometer input axis. The space-oriented mechanization on the other hand can experience gravitational acceleration in each of its axes. Therefore this mechanization requires a computer for gravity compensation in all three computational channels. While errors in inertial components have not been discussed as yet, it is appropriate to mention at this time that the rotation of the gravitational vector relative to the components axes also require a more sophisticated inflight calibration procedure to minimize error due to g sensitivity. This source of error, as well as others, will be attended to in a later section.

Let us now return to the tangent plane mechanization. In this system the stable element or platform is maintained in its initial orientation, with respect to an earth fixed point; the launch point. For the tactical missile application this mechanization has as advantages, (1) the target is fixed with respect to the computational reference, (2) the torqueing rates are reduced over those of the local-level system, and (3) the tangent plane system is more compatible than the space-oriented system when being airborne aligned with a locally-level aircraft inertial system. The selection of a mechanization for a particular tactical missile system requires a detailed investigation of the various advantages and limitations of each. Table I provides a comparison of some of these considerations for three common mechanizations. Having reviewed gimballed mechanizations, which provide a physical method of isolating the computational reference system from vehicle motion, it is appropriate at this time to examine analytical isolation or "the strapdown system".

A strapdown inertial system is one in which the basic inertial sensors are mounted directly to the vehicle structure. In this case it is evident that the coordinate system in which measurements are taken is constantly changing due to vehicle motion about its axis systems. In theory this presents no problem. Gyro's mounted orthogonally on the vehicle structure can directly measure vehicle angular rates. This output, when integrated yields vehicle attitude with respect to the initial attitude. Knowing attitude, the accelerometer outputs can be converted to a desired computational reference frame through a coordinate transformation. Several practical problems exist, however, in the implementation of a strapdown system for tactical missiles. As was mentioned in previous discussion, systems which incorporate a stable platform isolate the precision inertial components from vehicle roll, pitch, and yaw. The strapdown system requires its components to function over the full dynamic range of the missile. Angular rates of several hundred degrees per second can be experienced during launch and separation. The conventional strapdown technique would require torqueing of the gyro at vehicle rates to prevent the gyro from exceeding its inner gimbal limits. The error drift rate of these components will increase significantly with increased torqueing requirements. Errors on the order of 2-3 parts per million might well be experienced. Concepts relying on electrostatic gyros and/or laser gyros, which do not require torqueing, may well provide the solution to this problem of high vehicle angular rates.

The electrostatic gyro is basically a spinning sphere electrically suspended in a housing or case. Just as in conventional gyros the spinning mass tends to maintain its position in inertial space. Providing an all-attitude read out to locate the case position relative to the spinning ball yields a direct indication of vehicle attitude and eliminates both torqueing and integration on angular rates. Just as in the space-oriented mechanization discussed earlier, this indicated vehicle attitude is with respect to a coordinate system fixed in space. This same instrument shows promise as a multi-sensor, i.e. it can be designed in such a way as to also provide acceleration information. This could be an attractive concept from a cost standpoint. The laser gyro offers many of the same advantages as the electrostatic gyro, however, it is totally different in concept.

The laser gyro operates on the principal that the apparent path lengths of two counter rotating laser beams will differ in proportion to the inertial rotational rate of the instrument. Figure 5 depicts the basic laser gyro. Two oscillators, one that has energy traveling clockwise, and one that has energy traveling counterclockwise, along with reflectors and light amplifying material comprise the laser gyro. Rotation of the laser assembly about its input axis changes the effective path length for each oscillator; increasing path lengths for the energy traveling in the same direction as the assembly rotation, and decreasing the path length for that energy traveling opposite to assembly rotation. This difference in path length creates a frequency shift in the oscillators. Using phase shift detectors, the direction and magnitude of assembly rotation can be determined. Since there is no torqueing involved, the laser gyro, like the electrostatic gyro, may provide the necessary capability for strapdown inertial guidance of tactical missiles.



LASER GYRO SCHEMATIC

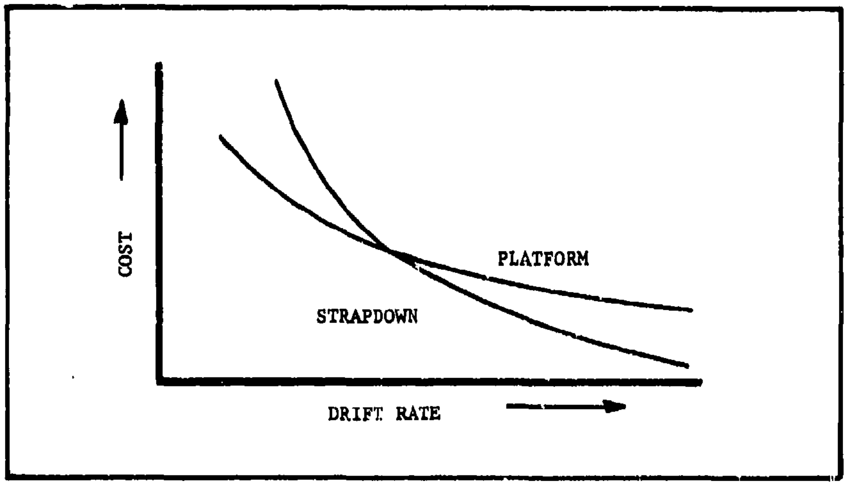
FIGURE 5

Mechanization	Platform Orientation	Gravity Vector Tracking	Correction for Coriolis Acceleration	Direct Availability Of Attitude Relative to Horizontal	Gyro Torquing
Space Oriented	Maintains fixed orientation with respect to inertial space	Yes	No	No	No
Local Level	Maintains horizontal orientation with respect to earth	No	Yes	Yes	Yes (earth rate and vehicle rate)
Tangent Plane	Maintains fixed orientation with respect to launch point	Yes	Yes	No	Yes (earth rate)

PLATFORM MECHANIZATION COMPARISON SUMMARY

TABLE I

Elimination of gimbals through the use of a strapdown system could well result in a reduction in system cost and size. If a crossover point does exist, as suggested by the curves of Figure 6, strapdown systems could well be the system of the future for tactical missiles.



COST TRENDS

FIGURE 6

3. ERROR SOURCES AND PROPAGATIONAL CHARACTERISTICS

As described in a previous section, an inertial system consists of acceleration sensors, integrators, a stabilized coordinate system (physical or analytic) and a gravitational computer. The errors involved in the estimation of present position, velocity and attitude can be divided into two general classifications: instrument or sensor errors and initial condition errors. Figure 7a represents a block diagram of a single-axis inertial system. Figure 7b is the error block diagram for the same system showing instrument and initial condition errors. Initial condition errors represent the uncertainties which exist at the time navigation is initiated. For a ground aligned system, initial condition errors are usually small and relatively unimportant compared with instrument type error sources. A simple error model for the accelerometer is shown in Figure 8. It should be noted that the accelerometer error model is sensitive to accelerations and therefore to the trajectory. The gyro is also sensitive to fixed bias errors and trajectory induced accelerations. Three gyro error sources considered in determining the performance of an inertial system are bias drift, mass unbalance and anisoelasticity.

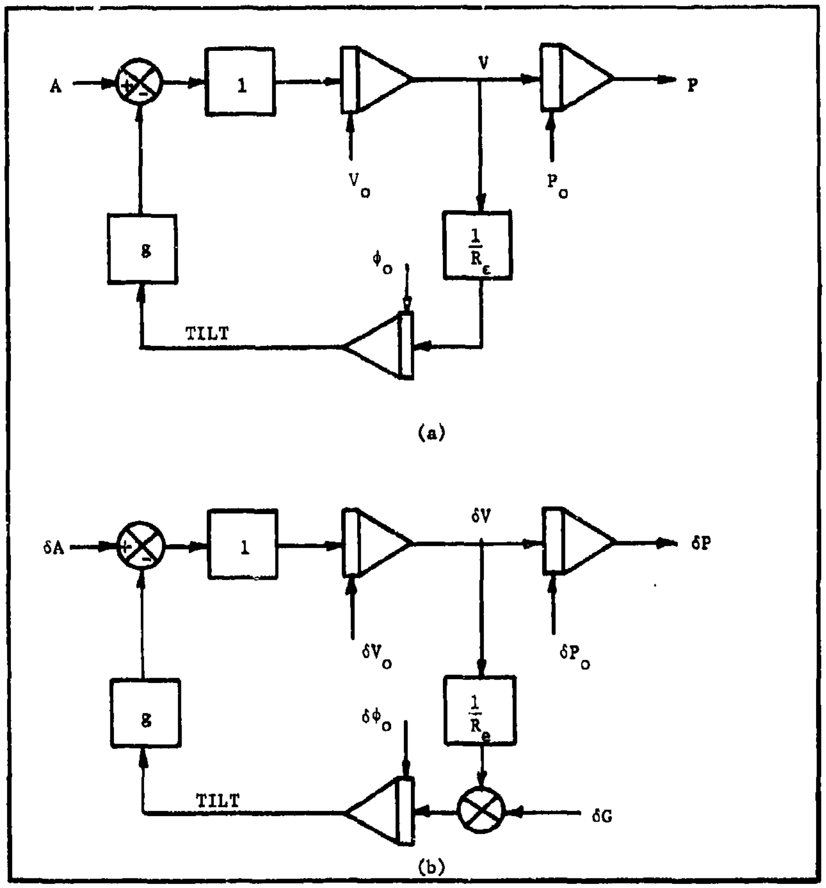
$$\delta\phi_i = R + U_s A_i + U_i A_s + S A_i A_s,$$

where ϕ_i is drift rate about an input axis, A_i, A_s are the components of specific acceleration along the input and spin axes. A is a bias error which exists independent of trajectory. The propagation of error due to the remaining two error sources, mass unbalance, U, and anisoelasticity, S, is dependent on trajectory and has units of degrees per hour per g and degrees per hour per g^2 respectively.

Referring to Figure 9, consider a gyro whose center of mass is displaced along the spin reference axis, SPA, and which is being subjected to an acceleration normal to the output axis, OA. The error torque about the OA resulting from this mass unbalance will vary depending on the acceleration experienced by the instrument. Similarly, a mass unbalance along the input axis, and an acceleration along the spin axis will create an error torque about the output axis. Anisoelastic errors result from a shift in center of mass under the influence of acceleration. Referring to Figure 10, the gyro center of mass is displayed along the SRA by an acceleration along this axis. Acceleration normal to the output axis results in an error torque as discussed for mass unbalance. Again the mass shift could occur due to an acceleration along the input axis, and in this case an error torque would result if an acceleration were experienced along the spin axis. An equal or isoelastic shift along both the spin and input axis would result in a zero error torque. In all cases, an error torque about the OA results in drifting about the IA.

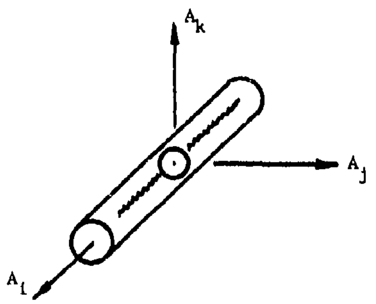
The error models as described here represent but a few of the total errors which contribute to inertial system error propagation. They do not point out the importance of trajectory on component error, and suggest that in the presence of high g's there may be some advantage to preferential orientation of the platform to minimize error. As an example, anisoelastic and mass unbalance error can be eliminated if the gyro is positioned in such a way as to experience acceleration along only its OA. Such an orientation is shown in Figure 11 where two of the three gyros are positioned with OA along the line of acceleration. Considerations of this type become important especially in missiles which experience high g's during boost. Having introduced the major sources of instrument error, it is now appropriate to discuss airborne alignment errors, which are very important in the tactical missile system.

To align an inertial system means to orient or position the coordinate reference system contained within the missile with respect to a known reference system. For a strapdown system the coordinate reference system exists within the computer. For our discussion assume the missile coordinate reference



SYSTEM AND ERROR BLOCK DIAGRAMS

FIGURE 7

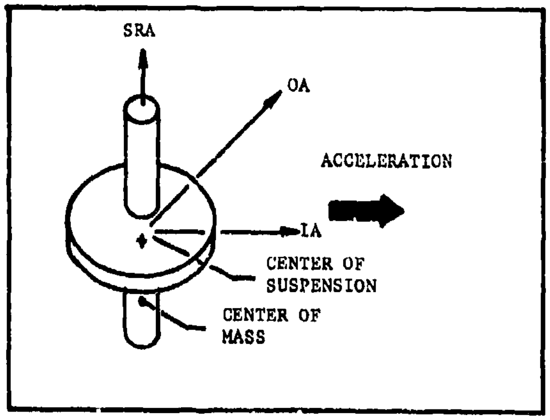


$$\dot{V} = K_o + K_1A_i + K_2A_i^2 + K_3A_i^3 + M_oA_j + M_1A_iA_j + N_oA_k + N_1A_kA_i$$

K_o	bias term
K_1	linear scale factor
K_2, K_3	nonlinearity coefficients
M_o, N_o	sensitivity to cross axis acceleration
M_1, N_1	sensitivity to cross axis acceleration

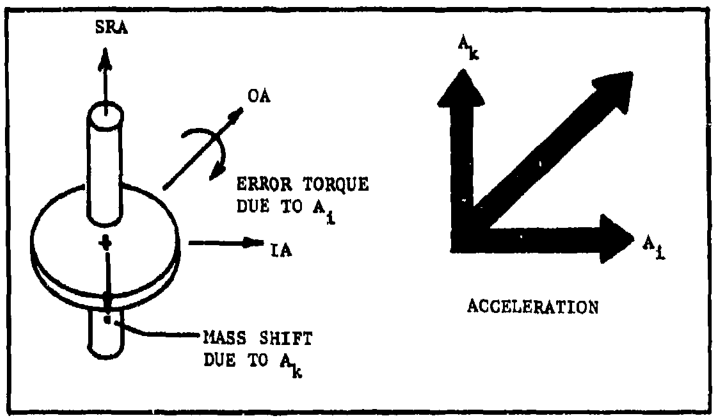
ACCELEROMETER ERROR MODEL

FIGURE 8



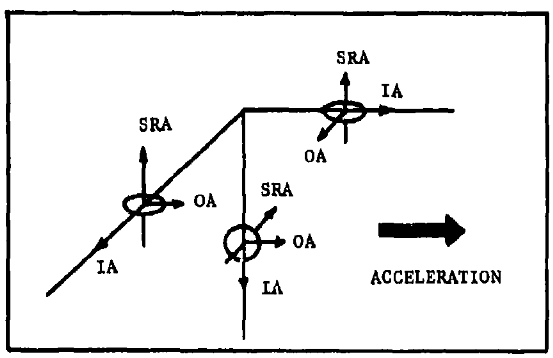
GYRO MASS UNBALANCE

FIGURE 9



GYRO ANISOELASTICITY

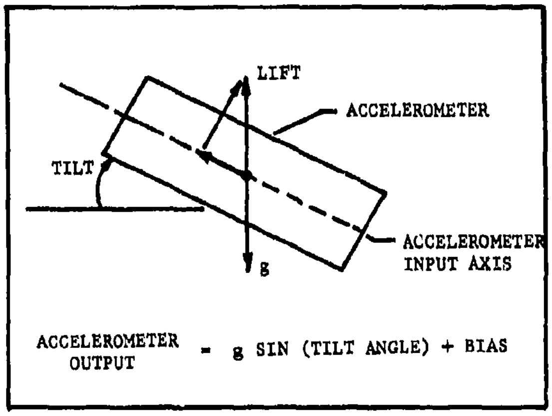
FIGURE 10



AN EXAMPLE OF PREFERRED ORIENTATION

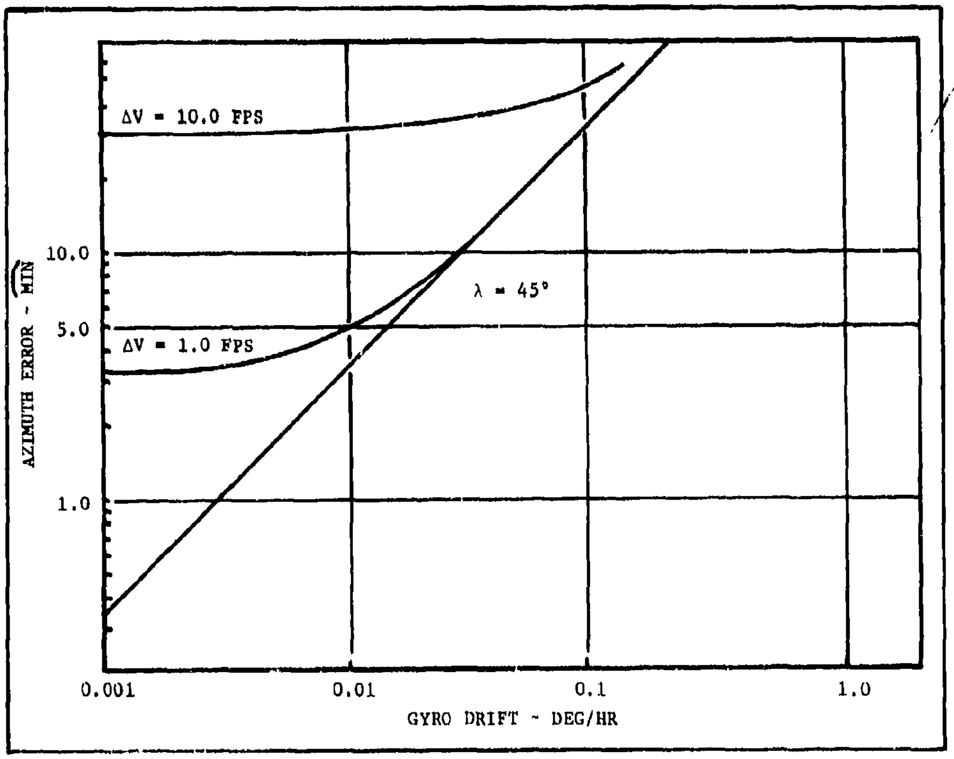
FIGURE 11

system is a stable platform aligned horizontally and having one of its horizontal axes pointing north. An inertial system is capable of being aligned without the benefit of outside sources of information. The horizontal accelerometers can be used to level the platform in the same fashion as a bubble level would be used. The platform is positioned such that the horizontal accelerometers have a zero output. Referring to Figure 12 we see that this condition exists when platform tilt is zero. The steady state angular platform error is essentially determined by the accelerometer bias error. A 10^{-4}g bias error would permit alignment of the platform to about 20 arc seconds, i.e., with the platform tilted 20 arc seconds, the component of the lift vector sensed by the accelerometer would equal the accelerometer bias error, leading to the erroneous conclusion that the platform is level. Having obtained level, the earth's rotation in inertial space can be used to align the platform relative to the earth's spin axis. A gyro mounted on a platform located at the equator and fixed in a level position will experience a rotation with respect to inertial space of approximately 15 degrees per hour (earth rate). This is true only if the input axis of the gyro is coincident with the earth's spin axis, i.e., pointed in a north direction. Thus, the north earth rate component in a level platform can be used to determine the platform's azimuth orientation. This process is called gyrocompassing. Figure 13 depicts the azimuth uncertainty achievable as a function of gyro drift rate and reference velocity error. This figure introduces one of the serious problems of airborne alignment and



ACCELEROMETER LEVELING

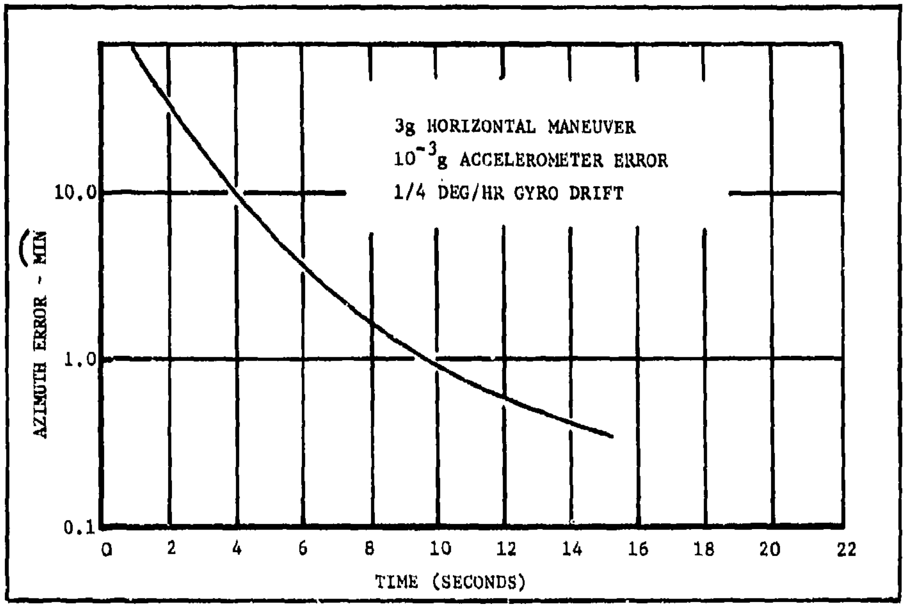
FIGURE 12



AZIMUTH ALIGNMENT ERROR

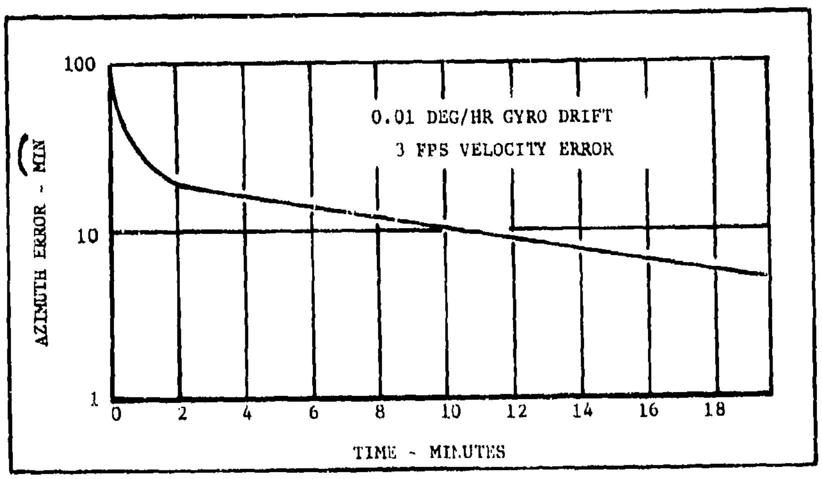
FIGURE 13

that is the errors which result from uncertainties in velocity. Inflight alignment presents a major problem to the missile designer and can be the subject of a long and detailed discussion. For the purpose of this paper three of the more common methods of inflight alignment are listed in Table 2 along with comments. They represent three different qualities of alignment. The most accurate, utilizing a star tracker mounted directly on the missile or immediately adjacent to it, is limited to clear weather operation and carriage such that the tracker is unobscured by the structure of the carrying aircraft. It is also a costly system. Acceleration or velocity matching provides a medium accuracy capability relying on the comparison of outputs from the master (aircraft) and slave (missile) platforms during a vehicle maneuver. Estimation and prediction techniques have improved upon the capability of past matching schemes and have reduced the required time to perform this operation. Figure 14 shows the theoretical uncertainty in determining azimuth orientation using acceleration matching and a 3g maneuver. Applying estimation and prediction techniques to gyrocompassing provides improved performance; however, the time required to achieve a given capability is considerably more than is required for matching techniques. Figure 15 depicts the azimuth error resulting from gyrocompassing using estimation and prediction techniques. Initial position and velocity transfer easily and their accuracy depends upon the external sensors such as a doppler navigator or ground mapping radar.



AZIMUTH ERROR - ACCELERATION MATCHING

FIGURE 14



AZIMUTH ERROR - GYRO COMPASSING

FIGURE 15

METHOD	PRINCIPAL	ADVANTAGES	COMMENT	TYPICAL ACCURACY
Gyro Compassing	Measurement of north component of earth rate	Does not require inertial system in aircraft	Limited by uncertainties in velocity and time available	Greater than ten arc minutes
Acceleration or velocity matching	Comparison between master and slave inertial systems	Internal or external carriage	Limited by component accuracy and cost of software. Requires maneuvering of aircraft	Less than ten arc minutes
Star Tracker	Observation of stars	Very accurate	Weather limitations Method of carriage limited Requires considerable time	Less than one arc minute

AIRBORNE ALIGNMENT TECHNIQUES

TABLE 2

Having introduced the major error sources associated with self-contained inertial systems, the obvious question which arises is "How do these error sources contribute to position uncertainty?" In general, for low g environments such as would be experienced in a transport aircraft, accurate inertial navigation places a more stringent requirement on the gyro performance than on the accelerometer. The high g environments experienced by tactical missiles require higher quality accelerometers with less emphasis on gyro quality.

There are several techniques for determining error propagation for the various sources of error. One method would be through simulation. Equations are written which describe the navigation system. Attitude, velocity and position are computed as a function of time. This computation provides a standard solution which can be compared with results obtained when an error such as gyro drift, is introduced into the computations. Another method involves numerical integration of error equations over the nominal trajectory using errors as forcing functions. A third method uses normalized integrals of acceleration which are derived for the specific mechanization. Each error coefficient is then multiplied by the appropriate normalized integral to obtain the velocity error. Position error can then be approximated by a second integration. Table 3 shows position error sensitivity for two representative air to surface missile trajectories. This table permits a comparison of the influence which each instrument error source has on the positional error at impact. Examples of error propagation will also be provided in the paper entitled, "Application of Inertial Technology to A-G Missiles".

4. STATE OF THE ART

Inertial systems which might be appropriate for tactical missiles exist today in some form with accuracies anywhere from one tenth nautical mile per hour to several hundred miles per hour. For purposes of state of the art discussion I would like to define three qualities of inertial systems; high, medium and low quality.

The high quality inertial unit is characterized by a one tenth nautical mile per hour performance rating. This high quality performance is not easy to come by and as a result is relatively large, costly, slow reacting, and limited in its availability. The medium quality unit is typical of inertial systems used in hundreds of commercial and military aircraft. It is common and readily available from numerous sources. While reduced in size compared with the high quality unit, it remains a relatively expensive item of equipment. The smallest and most recent entry in the inertial field is a series of "low cost" systems which are lumped together into the low quality classification. Most systems which fit into this class are in the early stages of development. Their claim to fame is primarily one of low cost and small size.

Table 4 lists the characteristics of these three classes of equipment and gives a gross type of indication of state of the art. A word of caution! It may be impossible to actually procure a system which meets all of the characteristics defining a particular quality inertial unit. This table is a composite, derived from numerous system descriptions, and as a result represent typical characteristics. It is intended only as a guide in estimating what might reasonably be available for systems of the future. A direct comparison between classes is risky. As an example, the medium class of equipments has been produced in fairly large quantities and thus our estimates of this system are quite reliable. The low quality system, on the other hand, is still in the development stage and may or may not achieve all of its goals. This brings us to our next topic, the Research and Development Process for inertial systems.

5. THE RESEARCH AND DEVELOPMENT PROCESS

The development process for a complex system such as a tactical missile starts considerably before the initiation of effort on the actual missile design. New missiles often incorporate the most recent subsystems and components in their design. These subsystems and components have already gone through a process of Research and Development, R&D, starting with fundamental ideas and proceeding through the analysis, design, fabrication of laboratory models, testing, etc. Their status at the time of their commitment to the missile design will vary greatly. A missile with a requirement for an order of magnitude improvement over other similar systems may have to rely on subsystems and/or components which are relatively unproven and not well understood. The time and resources required to further develop these items sufficiently for use in an operational missile may be larger than that required to modify off the shelf items for a similar design. In the area of inertial guidance we can make coarse estimates concerning the R&D process. The R&D cycle typically will vary from 5 to 10 years in duration, and historically has cost from 5 to 10 million dollars. The R&D cycle as used here terminates with the fabrication and flight test of an engineering model. At this point the inertial unit has been demonstrated in a simulated or actual airborne environment. The testing usually is not extensive, and additional design, testing and product improvement would be required to qualify the inertial unit for inclusion in an operational missile system. The estimate of what is required to develop a particular idea or concept into a working engineering model depends on many factors.

An estimate of R&D schedule and funding is just that; an estimate. Many events can modify these estimates. As an example, inadequate funding at critical points in the development cycle can cause the program to be extended, causing the R&D cost to increase. All programs experience technical problems. However, attempting to achieve too great an advancement in too short a time can result in a more costly development cycle. Changing program requirements during the development phase often increases cost and causes slippage of the schedule. Lack of experience may contribute to inaccurate estimates of anticipated program cost, however, the magnitude of the under estimation due to this cause usually decreases and becomes more realistic as the development process proceeds. One significant contributor to miscalculation of anticipated cost of developing a system is competition.

It is a well known fact that competition in a particular product line will tend to keep the price of that product at a minimum. This phenomenon also holds true for systems "to be developed". The subsystem contractor, or in our case, the inertial hardware vendor, must promise to deliver his system at a price and within a schedule competitive with other vendors in the field. As a result he is forced to be optimistic in estimating development and production cost for his potential system. He may deliberately estimate low, or "buy in" to ensure that he be considered by the customer for future business. This same situation occurs in the organization responsible for the intended weapon system, i.e., the total weapon

ERROR SOURCE	UNITS	42 NM. RANGE (111.5 Sec)			84 NM. RANGE (189.0 Sec)		
		Along Range	Cross Range	Altitude	Along Range	Cross Range	Altitude
X Gyro							
Bias Drift	FT/DEG/HR	--	5.5	--	--	42.6	--
Unbalance Drift	FT/DEG/HR/gx	--	30.0	--	--	79.5	--
	FT/DEG/HR/gz	--	23.0	--	--	93.2	--
Anisoelastic Drift	FT/DEG/HR/g^2	--	42.4	--	--	219.3	--
Y Gyro							
Bias Drift	FT/DEG/HR	-5.5	--	-11.6	-42.6	--	17.8
Unbalance Drift	FT/DEG/HR/gx	--	--	--	--	--	--
	FT/DEG/HR/gz	-23.0	--	31.7	-93.2	--	80.7
Anisoelastic Drift	FT/DEG/HR/g^2	--	--	--	--	--	--
Z Gyro							
Bias Drift	FT/DEG/HR	--	11.6	--	--	-17.8	--
Unbalance Drift	FT/DEG/HR/gx	--	-31.7	--	--	-80.7	--
	FT/DEG/HR/gz	--	--	--	--	--	--
Anisoelastic Drift	FT/DEG/HR/g^2	--	--	--	--	--	--
X Accelerometer							
Bias	FT/10^{-3}g	200.0	--	--	575.0	--	--
Scale Factor	FT/.1 percent	159.8	--	--	369.0	--	--
Y Accelerometer							
Bias	FT/10^{-3}g	--	200.0	--	--	575.0	--
Scale Factor	FT/.1 percent	--	--	--	--	--	--
Z Accelerometer							
Bias	FT/10^{-3}g	--	--	200.0	--	--	575.0
Scale Factor	FT/.1 percent	--	--	193.4	--	--	567.0

MISS DISTANCE SENSITIVITY
TABLE 3

	HIGH QUALITY	MEDIUM QUALITY	LOW QUALITY
Performance Rating (NM/HR)	1/10	1	100
Weight (Pounds)	230	55	15
Volume (Cu Ft)	5	1	1/4
Power Requirements (Watts)	800	300	100
Reaction Time (Hours)	2-1/2	1/4 to 1/2	1/6
Status	Demonstration flights completed	Hundreds of units installed in commercial aircraft	Limited flight testing accomplished with engineering models
Cost (Thousands of Dollars)	350	75	20

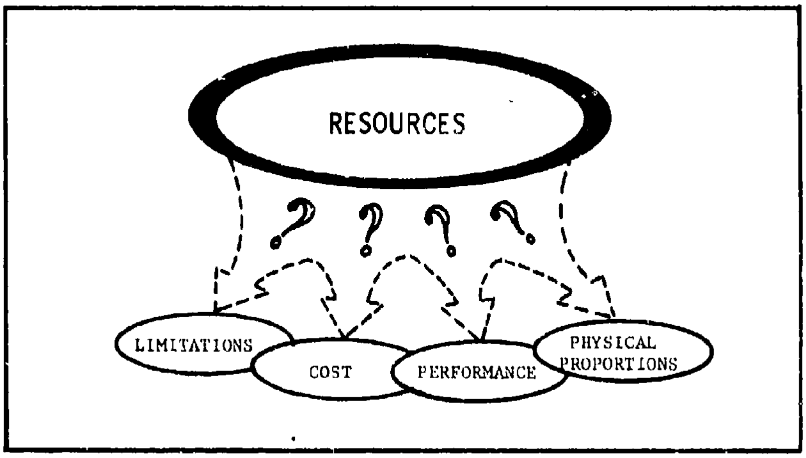
INERTIAL PLATFORM STATE OF THE ART

TABLE 4

system under development must be competitive with other systems already in the inventory or under development. The result is that the projected cost of many systems and subsystems are based on overly optimistic estimates. Unfortunately the real world doesn't always support this optimism. Our record during the last decade is not good. Major U.S. system acquisitions during the 1960's experienced a 40 percent cost growth and a 15 percent schedule slip. While these results are for major system acquisitions, it can be assumed that the guidance subsystem contributed its fair share to this record. Before discussing the cost associated with the ownership of inertial systems, let me introduce some ideas concerning "what is being developed" during an R&D program.

The creation of an inertial system starts with a concept. This concept may be the modification of an existing idea, or a completely new untried principle. This concept has inherent qualities which describe it. These qualities would include the physical proportions, the cost of the hardware, the performance, and the limitations associated with the concept. Time and money can be allotted to the improvement of one or more of these qualities, but in almost all cases limited developmental resources requires emphasis to be placed on one or two specific qualities. Due to the relatively recent emergence of inertial technology for airborne usage, resources have historically been allocated to achieving improvements in performance with a secondary concern for reduction in size. Relaxing the requirements on performance and size permits development of new techniques, procedures and principles which reduce acquisition and maintenance cost. There are efforts in this area currently underway, and results to date have indicated a general trend toward reducing the cost of certain "lower quality" inertial systems having application to missile guidance. Hopefully this trend will continue and also appear in other classes of inertial systems.

Having introduced several ideas or observations concerning the R&D process for inertial systems let me summarize what I consider to be important points. At the beginning of an R&D program, a concept exists and its status is described by a set of qualities which can be identified as performance, physical proportions, limitations, and cost. Limited R&D resources are applied toward the improvement of one or more of these qualities. Historically emphasis has been placed on performance. The resulting system or engineering model represents an improved capability and is ready to be considered for weapon system application. I would like to further identify this engineering model as an "unreliable engineering model". My contention being that because of the limited resources available and the need to be competitive, little real effort can be expended on including reliability into the design. Reliability is introduced at this time because it is an important factor in the consideration of cost of ownership. Past experience has revealed quite painfully that cost of ownership during a single year can exceed the initial unit cost of an inertial navigator. It is important then that we understand and consider this aspect of system cost.



THE APPLICATION OF RESOURCES

FIGURE 16

6. ESTIMATING TOTAL COST

It is obvious that a more reliable system will cost less to maintain and is therefore more cost effective. Or is it? Let us examine the impact of reliability on cost. In most cases there will be a mission reliability requirement imposed on the total weapon system, and therefore on each of its subsystems and all of their components. This mission required reliability can be related to mean time between failure, MTBF, and effective mission duration T, by following simple relationships.

$$R = e^{-(T/MTBF)}$$

Using the reliability requirement, as imposed on the navigation or guidance subsystem by the overall weapon system, and an effective mission duration, a MTBF requirement can be established for the subsystem. How does overall cost vary with this MTBF requirement?

Overall cost is defined as the sum of development, hardware and support cost over the lifetime of the system. Development cost, as used here, consists of three components; cost of developing an "unreliable unit", the cost of developing reliability into the system, and the development cost of aerospace ground equipment, AGE. Hardware cost includes the inertial system's acquisition as well as the AGE equipment acquisition. Support takes into account the cost to repair and the cost of providing spare parts. Reference 5 provides a means for estimating these costs for avionics systems. With modifications to account for a specific avionic system, i.e., an inertial system, the methods of this reference are used to generate cost data as a function of MTBF. Table 5 presents the relationship used for computing the costs which are depicted in Figure 17 for a "buy" of 100 units. It should be noted that for this example a MTBF of about 250 hours yields the lowest overall cost. The sensitivity of cost to underdesigning in terms of MTBF, as compared to overdesigning, can be seen in this plot. Generally speaking, as well as for this example, designing too little reliability into a system is more costly in the long run than overdesigning an equivalent amount. Table 6 sites specific cost for an assumed unreliable system costing twenty thousand dollars and having an effective lifetime of five hundred hours. The shape and magnitude of the cost versus MTBF curve for a specific system can vary drastically from the example of Figure 17 depending on the qualities describing the particular concept, the effective life, size of the buy, repair philosophy, etc.

We have discussed briefly the principles of inertial guidance, errors and their propagation, the state of the art, the development process and the relationship of reliability and cost. With this as background we are now prepared to look at the application of inertial guidance technology to a standoff tactical missile system, which is the title of the paper which follows.

COST ITEM	COST EXPRESSION
DEVELOPMENT	
Unreliable System	100 Cu
Reliable System	.5 (MTBF) Cu
AGE	50 Cu
TOTAL	[150 + .5 (MTBF)] Cu
HARDWARE	
Inertial System	(1 + .003 MTBF) N Cu
AGE	.5 N Cu
TOTAL	(1.5 + .003 MTBF) N Cu
SUPPORT	
Repairing Inertial	$\frac{.2H}{MTBF}$ N Cu
Repairing AGE	$\frac{300}{MTBF}$ N Cu
Spares	$\frac{100 + .3\ MTBF}{MTBF}$ N Cu
TOTAL (H assumed to be 500 hrs)	$\frac{500 + .3\ MTBF}{MTBF}$ N Cu

Cu	Cost of unreliable inertial unit
N	Total number of inertial units
H	Effective lifetime operating hours per system
MTBF	Mean time between failure

COST RELATIONSHIPS

TABLE 5

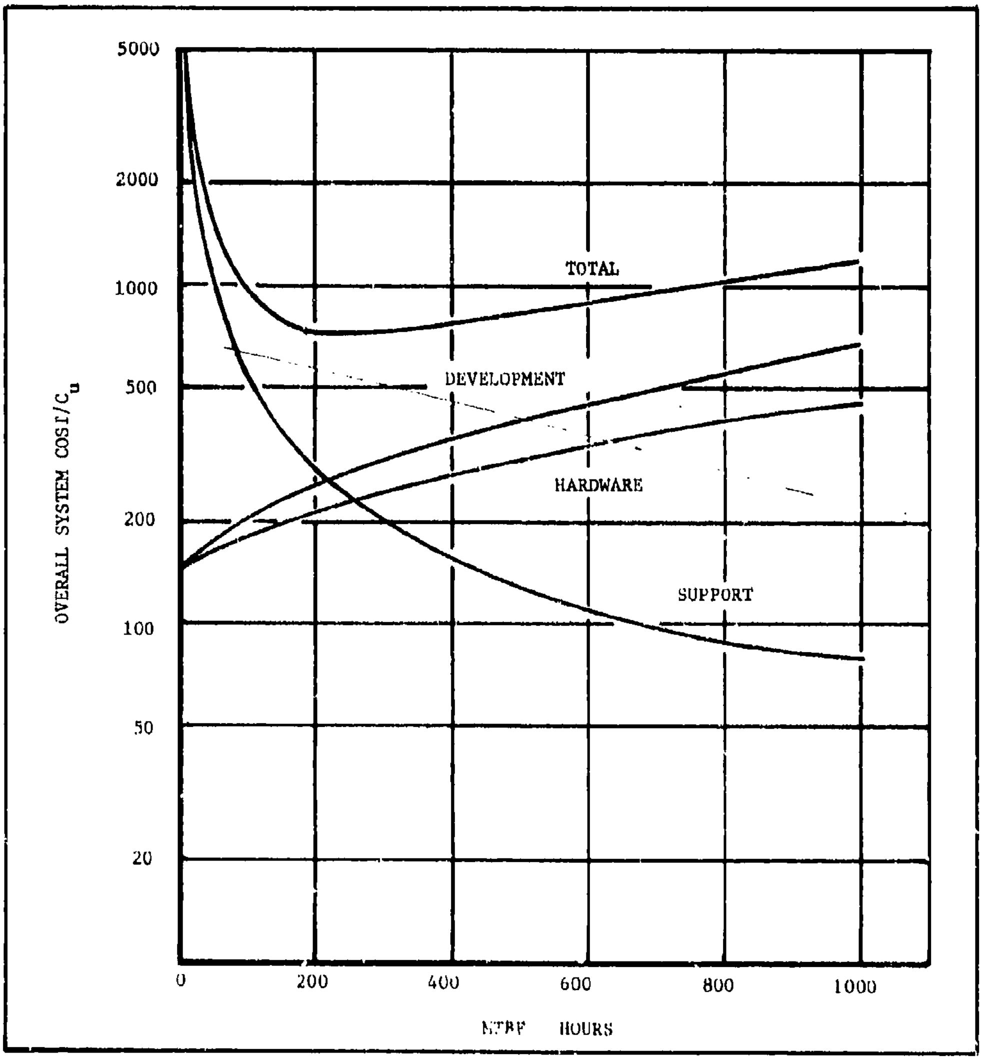
SYSTEM COST EXAMPLE
TABLE 6

COST ITEM	DESIGN MTBF (HOURS) 100	250	400
Development	4.0	5.5	7.0
Hardware	3.6	4.5	6.4
Support	10.6	4.6	3.1
TOTAL (Millions of Dollars)	18.2	14.6	15.5

H = 500 Hours

Cu = $20K

N = 100



NORMALIZED COST VERSUS MTBF

FIGURE 17

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APPLICATION OF INERTIAL TECHNOLOGY TO A-G MISSILES

by

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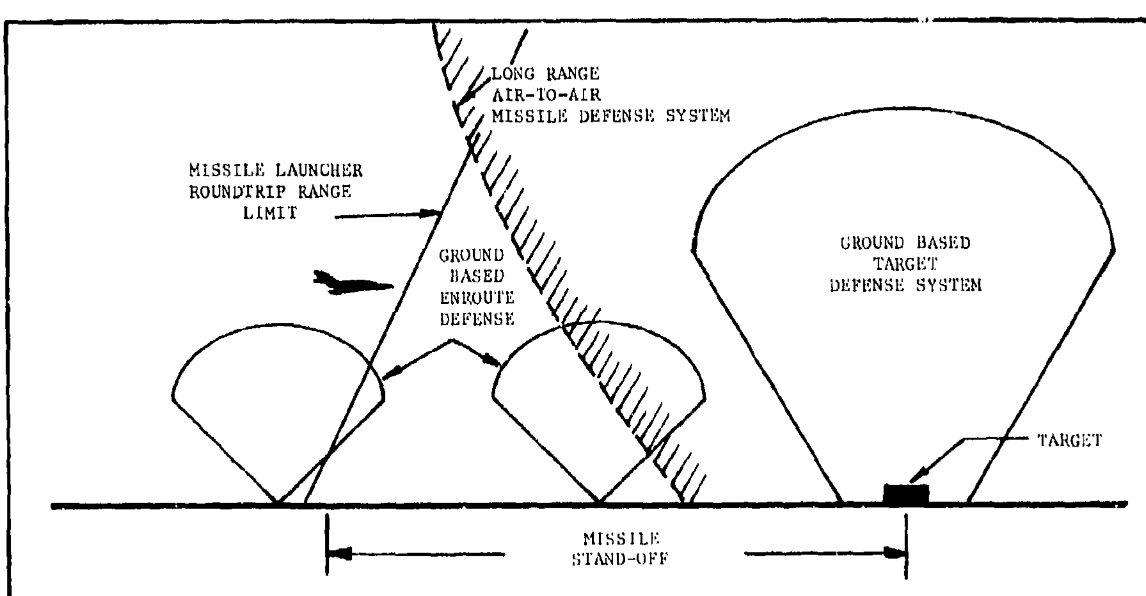
SUMMARY

Inertial technology is particularly attractive for airborne, stand-off tactical weapon systems, both as a midcourse guidance system, and when used in conjunction with a terminal guidance sensor. The capabilities of pure inertial guidance are examined as the midcourse guidance system for a stand-off missile. The relationships between enemy defenses, aircraft capability and missile performance are used to define a hypothetical mission, and a set of guidance system requirements. Error magnitudes are selected, and missile positional error is determined as a function of range. The stand-off range of this particular weapon system is limited by the performance of the midcourse guidance system. Various methods of improving midcourse guidance performance are explored. The advantages and limitations of an aided inertial system are reviewed with emphasis on retaining the advantages of the self-contained system.

The application of inertial technology to the stand-off missile, as discussed here, is not intended to establish present or future capability. The intent is to identify the various factors which influence capability, and suggest those areas in which improvements might be expected.

1. INTRODUCTION

In the preceding paper we reviewed the basic principles of inertial technology, state of the art, the research and development process and some aspects of cost. With this information, as a background, I would like to describe a potential application of inertial guidance technology to a tactical mission. The application is an air-launched stand-off missile. A stand-off missile permits the manned aircraft to launch its weapons without the need for completely penetrating the enemy defenses. This is highly desirable, especially when attacking heavily defended, high value targets. As will become evident, there is a need for some type of terminal guidance system in order to achieve impact errors appropriate for tactical missile applications. Figure 1 depicts some of the considerations necessary to determine the required stand-off range. They include ground and airborne defenses, both enroute and in the vicinity of the target, and range capability of the launch aircraft. An additional consideration would be missile range capability. One of the factors which influences the range over which the missile can operate is guidance system performance. Upon arrival in the vicinity of the target, the stand-off missile's guidance system must satisfy certain requirements as dictated by the terminal guidance sensor and the maneuver characteristics of the missile. These midcourse guidance requirements may be in terms of position, velocity and/or attitude. For the purpose of this illustration assume missile positional uncertainty in the target vicinity to be the dominant guidance requirement.

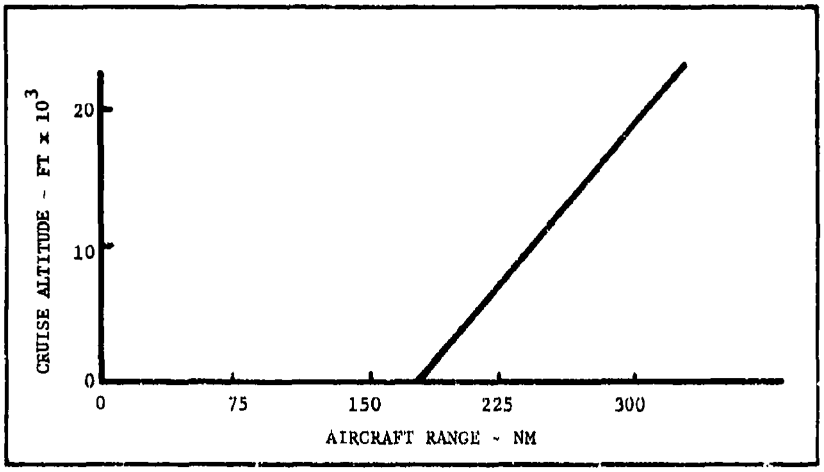


STAND-OFF RANGE CONSIDERATIONS

FIGURE 1

2. THE MISSION

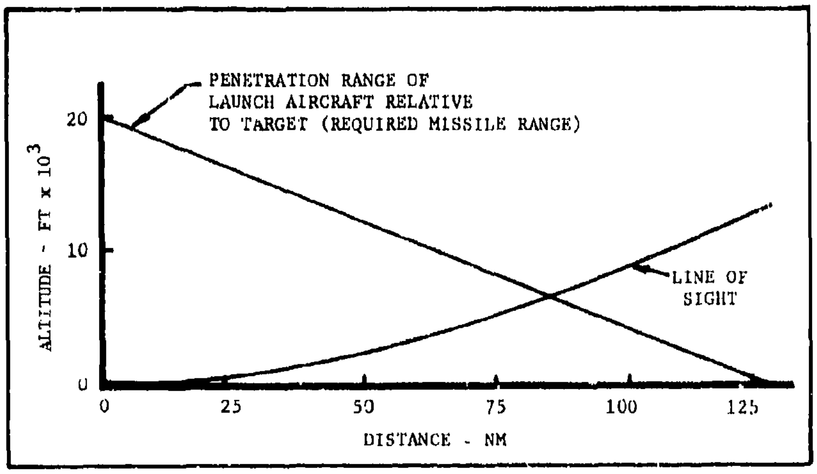
In order to explore the guidance possibilities for a stand-off missile, a sample mission is postulated. Assume a launch aircraft range capability as shown in Figure 2. Further assume the distance from the aircraft takeoff point to the target to be 300 NM. Figure 3 combines the effects of aircraft range with those of remaining below the line-of-sight of ground-based defenses. For this limited situation, we can see from these curves that a stand-off range from zero to 125 NM is required, depending on penetration altitude. As an example, aircraft penetration at five thousand feet altitude would provide undetected flights to within approximately 75 NM of ground based detection systems located along the ground track of the penetration aircraft. Aircraft range at this altitude is limited to approximately 200 NM which would require a missile stand-off capability of about 100 NM. Since the guidance system could conceivably limit the achievable missile stand-off, let us switch our attention to this matter. We must first determine the positional uncertainty requirement placed upon our hypothetical midcourse system. Upon arrival in the vicinity of the target, the terminal guidance system assumes control of the missile. Commands are generated within the missile system to correct missile trajectory errors which may exist. The magnitude of the corrective maneuver achievable by the missile can determine the accuracy requirements of the midcourse guidance system. Assuming that a near vertical terminal trajectory is desired, let the maneuver envelope shown in Figure 4 represent the capability of a stand-off missile capable of operating out to 125 NM from the launch point. The inertial system must be able to provide sufficiently accurate guidance to place the missile within this terminal basket. It is further assumed that the terminal sensor(s) will not impose additional or more stringent requirements on the midcourse guidance system.



LAUNCH AIRCRAFT RANGE

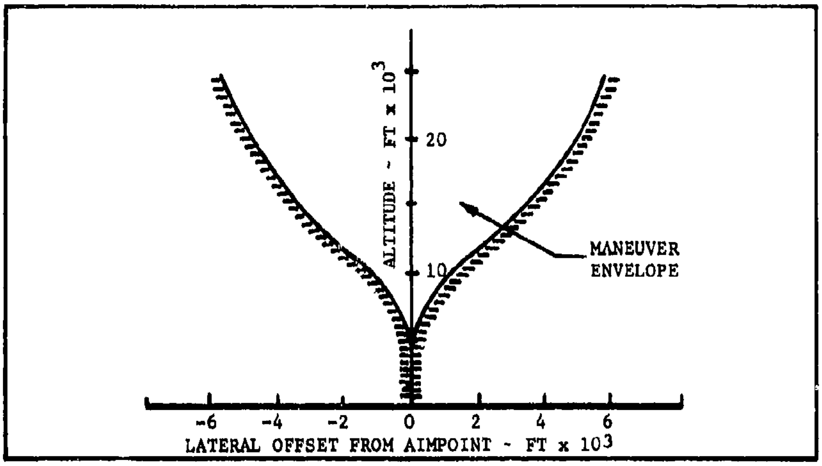
FIGURE 2

We have defined a mission and established a requirement for the midcourse guidance system. We can now apply inertial technology to providing the necessary midcourse guidance system performance. As pointed out in earlier discussions, trajectory, or more accurately acceleration, can greatly influence the performance of inertial components. Therefore, in addition to range and time of flight, the type or shape of the trajectory flown by the stand-off missile will affect the midcourse guidance system's performance. For convenience our considerations will be limited to semi-ballistic missile trajectories.



LINE OF SIGHT AND AIRCRAFT PENETRATION RANGE VERSUS ALTITUDE

FIGURE 3



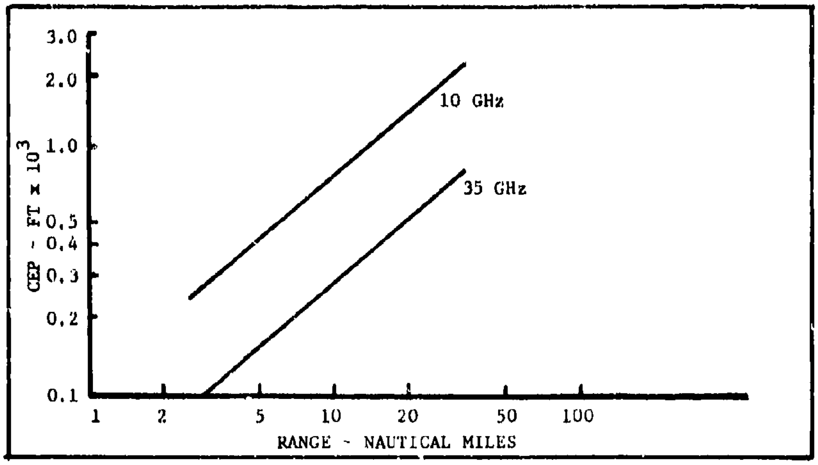
MISSILE MANEUVER CAPABILITY

FIGURE 4

3. ERROR PROPAGATION

The ability to accurately direct an inertially guided air-to-surface missile from its launch point to a point in the vicinity of the target, where terminal guidance is initiated, depends on several factors which are nearly independent of missile inertial guidance system design. These factors are the inherent uncertainties associated with the launch aircraft navigation system, and the operational constraints associated with a deployed system. Specifically, the position, velocity and heading uncertainty of the missile inertial system, at launch, can be no better than that provided by the launch aircraft navigation system. Perfect inertial guidance from missile launch to the terminal acquisition point will not eliminate the error caused by improper initial conditions. Similarly the manner in which the system is used in the field may greatly influence system performance. Ideally, the missile would be launched immediately after a position fix is taken by the launch aircraft, thus avoiding the inevitable build up in launch aircraft position uncertainty after fixing. However, terrain features or other considerations might well make this impractical.

Table 1 lists the major error sources associated with an air-launched missile system, and gives the error magnitude chosen for each. These error magnitudes have been selected as being representative, and also to demonstrate, by example, the trade-offs and interrelationships of missile and aircraft. Figures 5 through 8 offer a few examples of this interrelationship. While not stated explicitly, these examples suggest even further trades involving such considerations as the relative merit of different frequency radars, flight path constraints to assure availability of cultural and terrain features to achieve position fix, aircraft avionics cost, and many others. Let us now examine the impact of these errors in terms of miss distance.



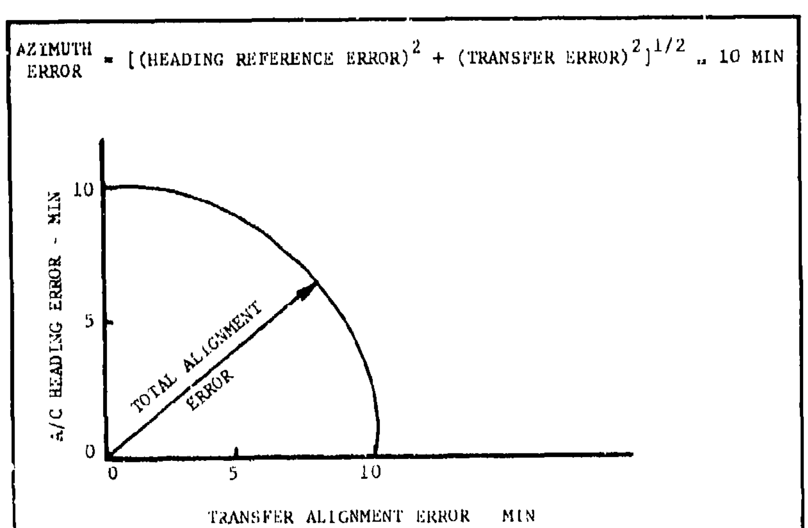
RADAR POSITION FIXING ERROR

FIGURE 5

ERROR SOURCE	ERROR MAGNITUDE (1 Sigma)	COMMENT
Initial Position	680 Ft each axis	X-Band Radar at 10 NM Range
Initial Altitude	200 Ft	Arbitrary
Initial Velocity	2 fps each axis	Doppler Radar
Initial Altitude Rate	2 fps	Arbitrary
Initial Attitude	1 arc Min each axis	Consistent with assumed inertial component quality and prediction and estimation techniques
Initial Azimuth	10 arc Min	Includes A/C Heading Reference and In-Flight Transfer Error
Gyro		
Bias	2°/Hr	
Unbalance	1°/Hr/g	Low Cost State of the Art Inertial System
Anisoelasticity	.0015°/Hr/g^2	
Accelerometer		
Bias	.5 x 10^{-3}g	
Scale Factor	.2 Percent	

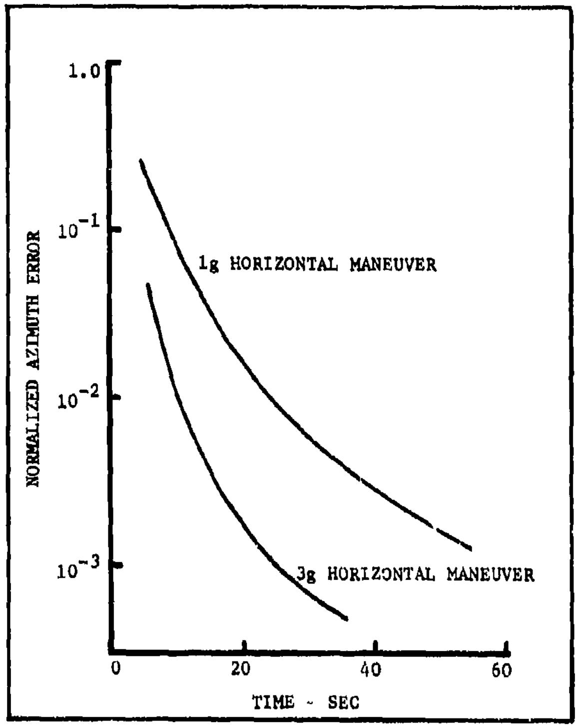
ERROR SOURCES

TABLE 1



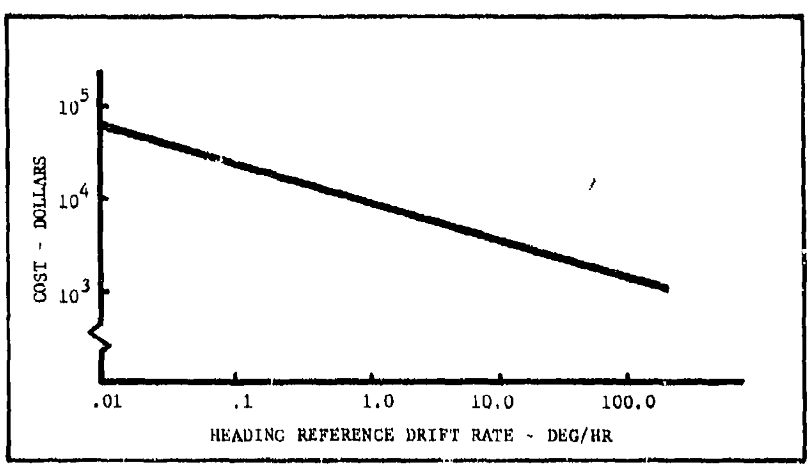
HEADING REFERENCE AND TRANSFER ERRORS

FIGURE 6



AZIMUTH ALIGNMENT

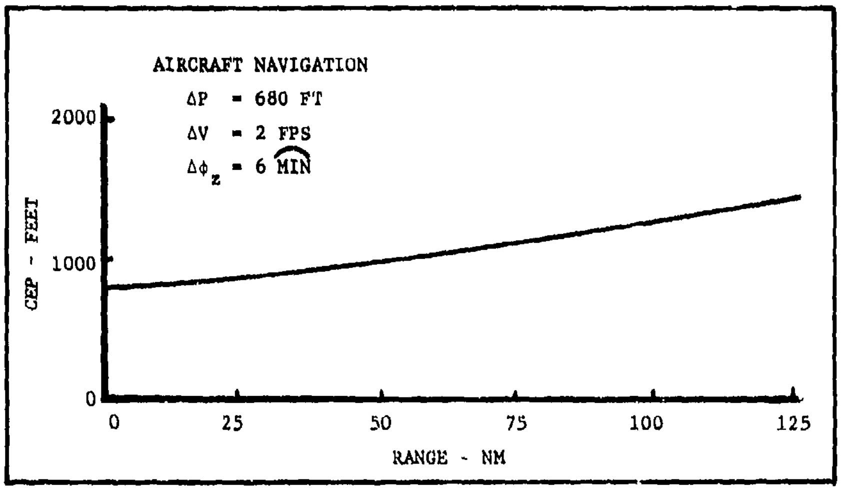
FIGURE 7



HEADING REFERENCE SYSTEM COST

FIGURE 8

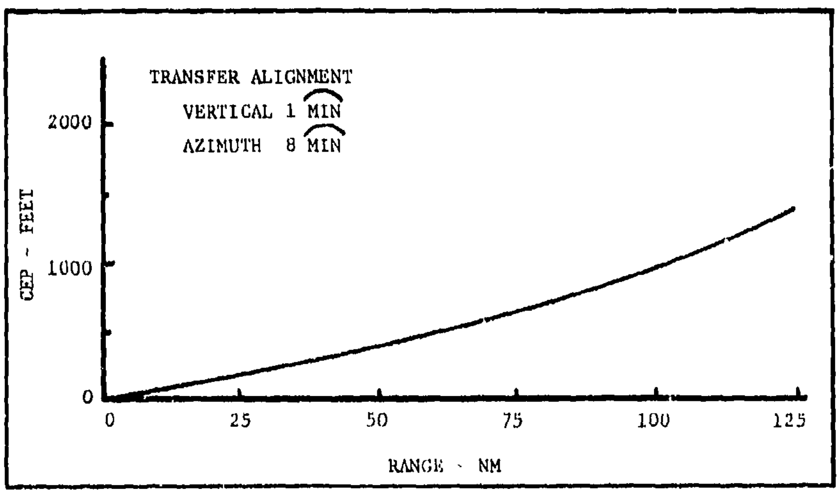
It is appropriate at this time to group the error sources previously described into three categories; aircraft navigation errors, transfer alignment errors, and missile inertial errors. This grouping permits a more perceptive examination of miss distance. Figure 9 displays the positional error which would exist in the vicinity of the target if the aircraft navigation system's error contribution is the only one considered, i.e. attitude transfer and missile inertial navigation are performed perfectly. This positional error, expressed as circular error probable (CEP) is shown as a function of range.



POSITIONAL ERROR ATTRIBUTABLE TO AIRCRAFT NAVIGATION SYSTEM

FIGURE 9

In a similar manner, the aircraft's and missile's contribution to position error is assumed to be zero and only those errors associated with transfer alignment are considered. Figure 10 depicts this situation for the assumed attitude and azimuth errors.

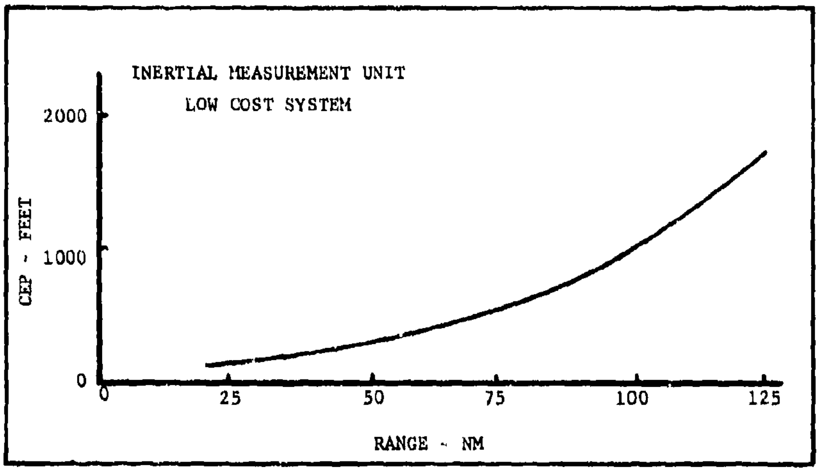


POSITIONAL ERROR ATTRIBUTABLE TO TRANSFER ALIGNMENT

FIGURE 10

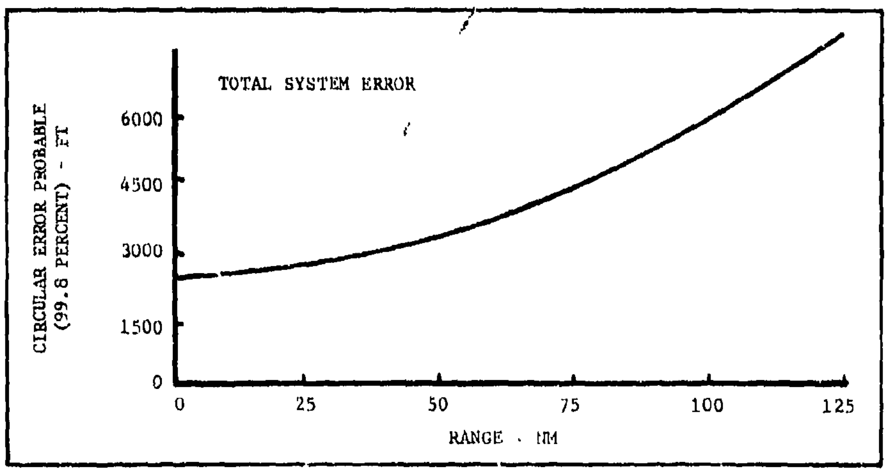
As has been pointed out in our previous discussion, the performance of an inertial system is dependent on the acceleration environment to which it is exposed. For this reason acceleration profiles were generated for semi-ballistic trajectories which are appropriate for terminally guided tactical missiles. Using these trajectories, the error curves of Figure 11 depict the positional error which exists in the vicinity of the target as a result of missile inertial component errors alone.

The positional error, in the vicinity of the target, which results from errors in the aircraft navigation system transfer of attitude information, and inertial instrument errors can be combined statistically. This combined or total position error represents the capability of the midcourse guidance system. Converting this information to a probability circle of 99.8 percent, the total positional error is depicted in Figure 12.



POSITIONAL ERROR ATTRIBUTABLE TO INERTIAL COMPONENT ERROR

FIGURE 11



TOTAL POSITIONAL ERROR IN THE VICINITY OF THE TARGET

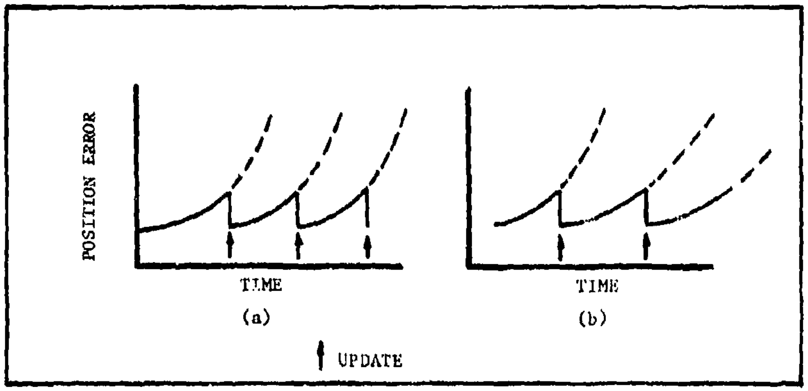
FIGURE 12

Returning to Figure 4 which depicts the terminal positional accuracy requirement for the hypothetical missile system, we see that for initiation of terminal guidance at an altitude of twenty thousand feet, the proposed midcourse guidance system is unacceptable. For ranges in excess of 85 NM, the midcourse guidance error exceeds the terminal maneuver capability of the missile. Several alternatives exist at this point, and will be explored in the following section.

The first and most obvious alternatives would be to review the contribution of each error source and determine which are major contributions. These error sources could then be examined to determine if they can be reduced in magnitude, and, if so, what additional cost, constraint or other penalty would be experienced by the system. A second choice would be to increase the terminal guidance initiation altitude, missile structural limit, or the size of the aerodynamic control surfaces. These changes could increase the terminal maneuver envelope of the missile, but the penalties of doing so must be examined in terms of more stringent terminal sensor requirements, reductions in stand-off range, increased gross weight, etc. The third alternative, and the one of real interest here, is updating the missile guidance system through the use of externally derived information. As we shall see this can result in considerable improvement in performance.

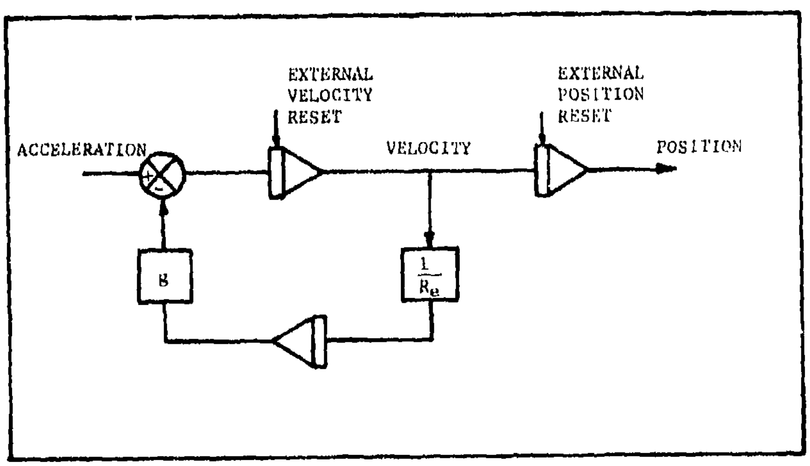
4. AIDED INERTIAL GUIDANCE

An explanation may be due at this time as to just what is meant by updating. Possibly the simplest form of updating would be to re-initialize the position integrators with newly acquired positional information. Assuming the newly acquired information is better than that contained within the navigation system, the immediate result is an improvement in knowledge of position, but no improvement in the rate of error build-up. Figure 13(a) depicts position error of such a brute force updating system as a function of time. If in addition to positional information other forms of sensed or measured information were available, such as velocity, not only would position uncertainties be reduced by updating, but the rate of position error build-up would be reduced somewhat. Refer to Figure 13(b). Simply resetting position and/or velocity as shown in Figure 14 does not make best use of the available information. The application of prediction and estimation techniques can provide significant improvements over the brute force methods. Before exploring these possibilities let us review some potential sources of updating information.



INERTIAL SYSTEM UPDAT...G

FIGURE 13



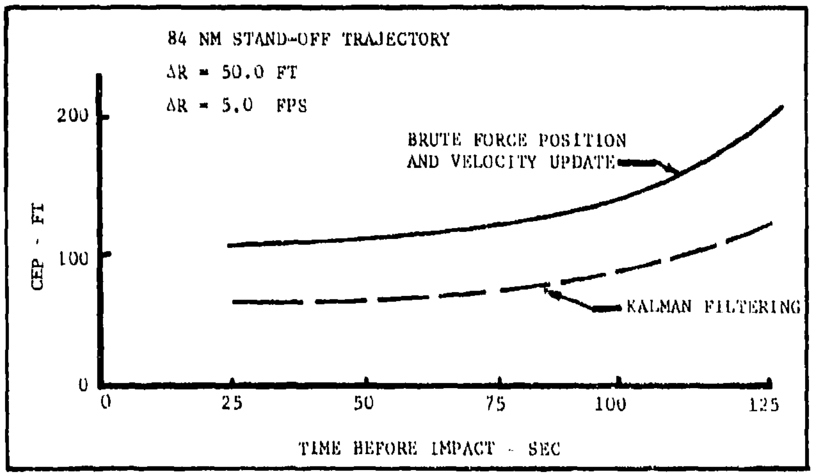
BRUTE FORCE UPDATING

FIGURE 14

Radio navigation lends itself to updating of tactical missile systems. The missileborne equipment can be made relatively small, inexpensive and rugged. The short term sensitivity and accuracy of the inertial system and the long term stability of the radio system are complementary. Line-of-sight frequencies in the 100 to 5000 MHz range can provide accurate position information on demand. Equipment operating in the noisier 100 KHz region provides a substantial increase in coverage area; however, accuracy is degraded somewhat and more time is required for integration or smoothing. World-wide coverage can be obtained with a few stations operating in the 10 KHz frequency region. Transmitting stations located on the ground, in aircraft or in orbital vehicles are possibilities for an aided system. Updating can be a discrete or continuous process over all or only a portion of the missile's flight.

On board sensors can also be used to improve upon the midcourse guidance capability. A doppler radar can measure ground speed. A mapping radar, infrared or television sensor could provide an indication of terrain and cultural features for obtaining a position fix. Such a system could perform all computations and processing on board the missile, or transmit information via a data link to a ground or airborne processing station. The selection of a particular updating concept must include consideration of enemy countermeasures since a heavily defended target would incorporate extensive countermeasure equipment directed at degrading stand-off missile guidance capability. Other important considerations would be the system's vulnerability to attack, and of course the impact on the missile design.

To illustrate the potential improvement using updating, the following example is given. Assum two ground-based distance measuring stations separated by 50 NM, forming a baseline in a range-range positioning system. The ground stations are interrogated by the missile, determining relative range and range rate. This information is processed and used to update the missiles' estimate of position and velocity. The solid line of Figure 15 depicts impact positional error as a function of time to the target at last update. Updating is accomplished by brute force. No attempt is made to calibrate the system or its components. The missile inertial system is similar to the one previously proposed for the unaided midcourse guidance system. The range and range rate errors used in the generation of this error curve are as shown.



POSITION ERROR AT IMPACT

FIGURE 15

The dashed line of this figure is an estimate of the potential achievable using Kalman filter theory. In addition to improving position and velocity through accurate updating, Kalman filtering impro system performance by effectively reducing component errors, thus improving greatly on the short term accuracy of the system. Practical considerations such as computer speed, memory capacity, and mismodelin makes achievement of optimum results unlikely. The design of a Kalman filter requires accurate knowledge of system dynamics, the measurement process and all error co-variances. If the engineering problems asso ciated with the practical implementation of Kalman filtering can be solved for the air-to-surface missile and if the guidance concept is adequately resistive to countermeasures, significant improvements over the attainable with the brute force aided system could be achieved.

A source of error which has not been mentioned up to this point is the uncertainty in knowled of the target's location. A portion of this error can be reduced by the proper selection of the midcours updating method. In the previous paper we discussed the computational coordinate system of the tactical missile. The target's position must be located in this coordinate system. The target is initially locat in the coordinates of the targeting system. This coordinate system may or may not be the same coordin system as used in the tactical missile. As an example, if targeting is accomplished from a vehicle na gating in a radio network and attack is carried out in a geographic coordinate system, then an error exist between these two coordinate systems and will contribute to missile impact error. In the exampl cited previously, inertial system updating is achieved with a ground-based microwave distance measuring system. If target reconnaissance is also conducted in this same measuring system, one important potenti source of error can be greatly reduced. Thus it can be seen that by the selection of a common coordinat system for targeting and strike, the uncertainty of locating the target can be minimized.

5. CONCLUSIONS

Inertial guidance of tactical stand-off missiles can provide a completely self-contained mi course capability for attacking heavily defended targets. This capability is dependent not only on ir component quality, but depends to a large extent, on the quality of aircraft navigation and airborne p form alignment. The aided inertial system can provide significant improvements in performance over that achievable with an unaided system. In addition to this improved performance, enroute updating can provi both targeting and strike in a common coordinate system. These advantages could result in significant reduction in position error, and suggest a potential capability which could eliminate the need for additional terminal guidance. Realization of such a potential is not inevitable. The aided inertial system for tactical stand-off missiles is an attractive concept. As indicated in the preceding paper, a concept has inherent qualities associated with it, and development of one or more of these qualities, as required for a particular air-to-ground application, requires the authorization and expenditure of resources toward this end.

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METHODOLOGY OF RESEARCH INTO COMMAND-LINE-OF-SIGHT AND HOMING GUIDANCE

by

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SUMMARY

This paper reviews a methodology of research into command-to-line-of-sight (CLOS) guidance and semi-active homing missile systems. It discusses the kinematics of various guidance laws from CLOS to pursuit courses and proportional navigation from a fundamental point of view. The interaction between the guidance requirements and the missile system is covered and it is shown that the autopilot and sensor effects need to be considered in hybrid computer simulations. The implications on computer requirements for optimum filtering are also discussed.

1. INTRODUCTION

Missile guidance problems have a number of aspects in common, whether they be related to homing, beam riding or line-of-sight following. Firstly, the basic need is to obtain information on the state variables of the object to be guided, namely the missile, and the destination point, which might be moving, eg the target. Secondly, instruments have to be used to measure this information. These instruments may be limited with regard to what can be measured, the accuracy with which measurements can be made, and where the measurement can take place, ie on the ground or in the missile. Finally the missile has to be manoeuvred in the best possible manner by means of a guidance law, from where it is to where it is desired to be. This brings in not only the kinematics of the movement of the centre of gravity but also the dynamics of the missile about its centre of gravity resulting from its response to a demanded manoeuvre. The guidance problems of command-line-of-sight and homing therefore have some similarities. The target movement and missile response characteristics can be basically similar. They have different guidance laws because in homing the missile itself has to track the target with its self-contained sensors, whereas in command line-of-sight or beam riding an outside reference point is used, say on the ground, for tracking both the missile and the target. In the homing situation the fact that the missile position is not readily available to the missile itself implies that only relative information can be used. In beam riding guidance the measurement of the distance off a line-of-sight from the ground to the target eg a radar beam, is measured in the missile, so it has elements of a mixture of the two basic guidance principles.

This paper reviews current methodology for research into guidance laws of the command line-of-sight and homing types. It is shown, after the basic laws have been discussed, how some aspects can be investigated in a noise-free situation, eg trajectories of flight in homing. It is then shown how it is sometimes important to consider the noise characteristics of the sensors being used, typically the effects of target glint on radar measurements, when miss distances and missile lateral acceleration criteria are chosen. The studies with noise have interacting effects with the kinematics and dynamics of the engagement, ie the navigation law, approach direction and the ratio of missile to target speeds. The necessity of filtering these signals is then discussed, and this leads to the consideration of the statistical optimisation of filters, eg of the Wiener and Kalman types. These theoretical optimisation techniques require the construction of a mathematical model of the engagement and it is necessary to investigate their sensitivity to changes in both the assumptions made in the derivation of the optimum solution, eg on the assumed target glint characteristics, and to changes in the actual system itself, eg unpredictable target manoeuvres. Also sub-optimal solutions may be preferable on a cost/effective basis.

It is shown how the powerful tool of hybrid computer simulation, which was developed in an earlier paper, can be used to evaluate many of these uncertainties in both homing guidance and command line-of-sight following. Although these two guidance laws necessarily require two different simulation models, many of the features of the simulation processes are similar. Illustrations derived from practical experience in one field can be applied directly to the other. The implications on missile autopilot design and computer hardware requirements for the implementation of typical guidance laws are also discussed.

2. BASIC GUIDANCE LAWS

Command line-of-sight (CLOS), homing guidance and beam riding can be consolidated as shown in Table 1 according to the source of the measurements made or parameters estimated and according to whether the computation of the guidance manoeuvre demand is carried out in the missile or on the ground. A typical system state used in CLOS or beam riding is the error off the beam ϵ suitably filtered by a transfer function S(p) to give the demanded lateral acceleration. This filter should contain at least some phase advance for stability reasons, but it can be of sophisticated form if statistically designed with regard to the sight line noise. Beam riding and CLOS are essentially the same dynamically except that the error off the beam is detected in the missile in beam riding, whereas in CLOS ground radar sensors measure it, compute the acceleration demand and transmit this by radio link or wire to the missile. The command link can be regarded as a sophisticated wire connection in a beam riding missile. In the homing guidance laws the relative parameters of sight line spin $\dot{\gamma}_S$, closing speed V_c and look angle L are used together with estimates of missile speed V_m. Basic CLOS and homing guidance laws will now be discussed in more detail separately.

Source of Measurement Estimation and Computation	System States		Lateral Acceleration Demand n_D and Flight Path Rate $\dot{\psi}_F$ required		Guidance Law	
Missile	Sight Line Spin	$\dot{\psi}_S$	$n_D = V_m \dot{\psi}_F$	$\dot{\psi}_F = \dot{\psi}_S$	L = 0: Pure Pursuit	Homing
	Closing Speed	V_c			L ≠ 0: Deviated Pursuit	
	Look Angle	L		$\dot{\psi}_F = K\dot{\psi}_S$	Proportional Navigation	
	Missile Speed	V_m		$\dot{\psi}_F = \dfrac{K_1 V_c \dot{\psi}_S}{V_m \cos L}$	Corrected PN	
	Distance off Beam	ε	$n_D = S(p)\varepsilon$		Beam Riding	
Ground	Missile Speed	V_m	$n_D = S(p)\varepsilon$ + Feed Forward Bias $= S(p)R_m(\theta_T - \theta_m) + f(R_m, V_m, \dot{V}_m, \dot{\theta}_T)$		Command Line of Sight (CLOS)	
	Missile Range	R_m				
	Missile Accn.	$\dot{V}_m$				
	Target Beam Angular Rate	$\dot{\theta}_T$				
	Beam Error Angle	$(\theta_T - \theta_m)$				

TABLE 1 BASIC GUIDANCE LAWS

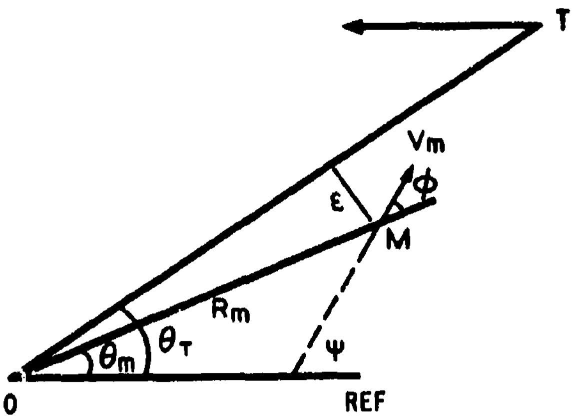


FIG.1 COMMAND LINE OF SIGHT GUIDANCE

2.1 Command-to-line-of-sight

A number of tactical missiles in the surface to air and surface to surface modes are guided on the line-of-sight principle. The missile M of Fig 1 is guided so as to be maintained on the sight line OT. In an actual moving situation the guidance signals transmitted to the missile are the demanded lateral accelerations in two axes at right angles to the beam. These demands are resolved into missile axes within the missile. The demanded acceleration in each plane is split into two terms:-

a) an error compensation term endeavouring to keep the error off the beam ε equal to zero, and

b) feed forward bias terms corresponding to a moving beam.

A simplified guidance loop which combines these two demands is shown in Fig 2. They will now be discussed in more detail. Consider first the error loop.

2.1.1 Basic Guidance Concept

Suppose that the error ε of Fig 1 can be measured either directly or by means of the angular difference between OT and OM, together with some knowledge of missile range R_m, then $\varepsilon = R_m(\theta_T - \theta_m)$. If this error off the beam is used as an acceleration demand n_D, it needs some damping so that good response characteristics are obtained. A dynamic equation of the form $\ddot{\varepsilon} = G_1\varepsilon + G_2\dot{\varepsilon}$ needs to be satisfied, where G_1 and G_2 are constants. This necessity leads immediately to the consideration of a filtered error./ In the presence of noise on the sight-line, and hence on the error ε, such a filter design is not simple and becomes a compromise between requirements for smoothing the noise and giving an adequate response to a demand. Modern techniques allow filters to be designed statistically if some knowledge of the noise characteristics is available or can be assumed. Figure 2 shows the position of such a filter S(p) in the guidance loop. It includes a gain G, and the acceleration demand is $n_D = S(p)\varepsilon = S(p)\,R_m\,(\theta_T - \theta_m)$. The missile transfer function is represented by A(p) and when the achieved acceleration is doubly integrated and divided by R_m it represents a new measure of the missile beam angle θ_m, thus closing the loop when differenced with the target beam angle θ_T.

2.1.2 Feed-forward terms

Consider the situation of Fig 1 in which OM is coincident with OT, but that OT is rotating due to engagement kinematics of the target relative to O, eg due to target manoeuvre. If ϕ is the angle between the missile flight vector and the sight line then $\dot{R}_m = V_m \cos\phi$ and $R_m\dot{\theta}_T = V_m \sin\phi$. The lateral acceleration (latax) which must be applied to the missile for it to stay on the rotating sight-line is

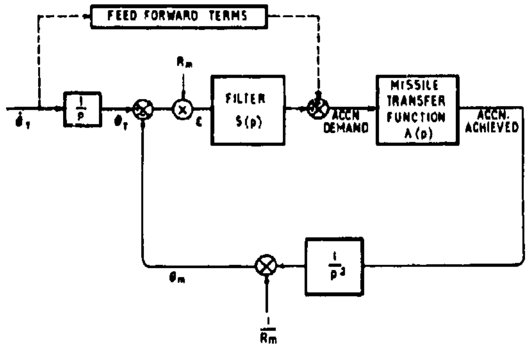


FIG.2 CLOS SIMPLIFIED GUIDANCE LOOP

$$L_o = (R_m\ddot{\theta}_T + 2\,\dot{R}_m\dot{\theta}_T)\cos\phi - (\ddot{R}_m - R_m\dot{\theta}_T^{\,2})\sin\phi$$

$$= (R_m\dot{R}_m\ddot{\theta}_T + 2\,\dot{R}_m^{\,2}\dot{\theta}_T - R_m\ddot{R}_m\dot{\theta}_T + R_m^{\,2}\dot{\theta}_T^{\,3})/V_m$$

Now $\ddot{R}_m = \dot{V}_m \cos\phi - V_m \sin\phi\,\dot{\phi}$ and if ϕ is small $\dot{R}_m \doteq V_m$, $R_m\dot{\theta}_T \doteq V_m\,\phi$ and $\ddot{R}_m \doteq \dot{V}_m - V_m\phi\,\dot{\phi}$, so that

$$L_o \doteq (R_m V_m\ddot{\theta}_T + 2\,V_m^{\,2}\dot{\theta}_T - R_m(\dot{V}_m - V_m\,\phi\,\dot{\phi})\dot{\theta}_T +$$
$$V_m^{\,2}\,\phi^2\dot{\theta}_T)/V_m$$

Line of sight guidance systems are usually used for point defences against air attack, or against slowly moving ground targets, eg tanks, in both of which the sight line rates of rotation are low, hence the angle ϕ is small. The terms in $\phi\,\dot{\phi}$ and ϕ^2 can therefore be neglected and the feed forward terms become

$$L_o \doteq (R_m\ddot{\theta}_T + 2\,V_m\dot{\theta}_T - R_m\dot{V}_m\dot{\theta}_T/V_m)$$

If we write $p\dot{\theta}_T$ for $\ddot{\theta}_T$,

$$L_o \doteq (R_m\,p + 2\,V_m - R_m\dot{V}_m/V_m)\dot{\theta}_T$$

$$= f(R_m,\ V_m,\ \dot{V}_m,\ \dot{\theta}_T) \text{ as shown in Table 1.}$$

This acceleration bias demand is fed forward from $\dot{\theta}_T$ as shown in Fig 2. For its implementation some knowledge is required of the missile range R_m, its velocity V_m and the ratio of acceleration (or deceleration) to the speed ($\dot{V}_m/V_m$). The sight line rate of rotation $\dot{\theta}_T$ and acceleration $p\dot{\theta}_T$ also need to be measured or estimated.

The total acceleration demand is the sum of the error demand and the feed forward terms. Whilst this concept is simple for a CLOS or beam riding guidance situation it is by no means as clear in homing how a guidance law can be devised in the absence of information on missile and target positions. Let us therefore look at what use can be made of relative information.

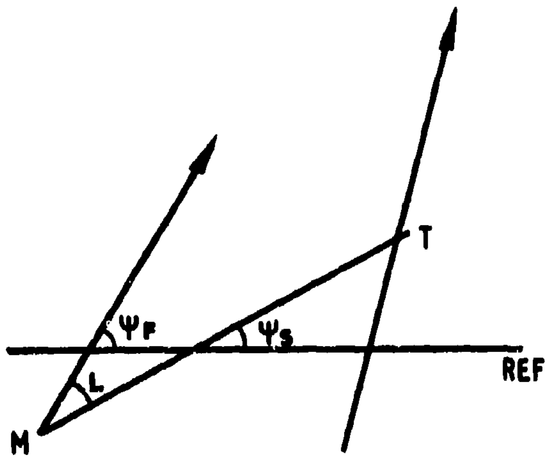
2.2 Homing Guidance

Consider now homing guidance in which one has to formulate the guidance command from information only available in the missile, ie without knowing practically where the target is. It was shown in Table 1 that the traditional homing law called proportional navigation uses the sight line spin rate $\dot{\gamma}_S$ of the target relative to the missile. It is well established as an effective guidance technique for a system which has to derive the basic parameters from measurements made within the missile itself. Fig 3 shows the flight paths of the target and missile relative to a space reference. If this space reference direction is available in the missile by means of a gyroscope, and a homing head can be locked to the target such that its rate of rotation measures $\dot{\gamma}_S$, this spin rate can be factored by K, the navigation constant, to produce a required flight path rate for the missile $\dot{\gamma}_F$. The practical implementation of this law requires that the missile speed should be known or estimated such that the demanded lateral acceleration is $n_D = V_m\dot{\gamma}_F$. L is the look angle, see Fig 3. If zero missile incidence is assumed, the look angle of the homing head is the angle between the missile flight vector and the sight-line to the target.

The simplest attack situation is the so called collision course which arises when missile and target speeds are constant and the target flies on a straight course. The approach direction of the missile is towards a future position of the target in such a way that at the intersection of the two straight courses impact occurs. In this situation it can be shown that $\dot{\gamma}_S$ is zero and the look angle L is constant.

Homing guidance laws are aiming to reach this condition eventually, even when the velocities and target flight paths change. This, of course, will in general demand a change in the guidance parameters used.

Variations in the proportional navigation law can be formulated:- for example (a) when K = 1 we have pursuit courses: pure pursuit when the look angle L is zero, and deviated pursuit if a constant look angle is used, (b) corrections can also be applied to the K factor to allow for the effect of look angle L, closing speed V_c and missile speed V_m. For the above mentioned collision course situation it can be shown that if the acceleration at right angles to the sight-line in the presence of disturbances is chosen to be $K_1V_c\dot{\gamma}_S$, where K_1 is the kinematic gain and V_c is the



$$\dot{\psi}_F = k\,\dot{\psi}_S$$

FIG. 3 PROPORTIONAL NAVIGATION HOMING

closing speed, then this technique tends to minimise the necessary corrective acceleration of the missile, particularly if K_1 is properly chosen. The component of $V_m\dot{\psi}_F$ at right angles to the sight line is $V_m \cos L\, \dot{\psi}_F$ so

$$V_m \cos L\ \dot{\psi}_F = K_1 V_c \dot{\psi}_S$$

$$\text{or } \dot{\psi}_F = \left(\frac{K_1 V_c}{V_m \cos L}\right) \dot{\psi}_S$$

In corrected proportional navigation the lateral acceleration demand to the missile becomes a function of relative parameters only since r[illegible] $= V_m\dot{\psi}_F = \left(\frac{K_1V_c}{\cos L}\right)\dot{\psi}_S$.

Depending on the complexity of instrumentation in the missile, V_c can be either measured or estimated as well as the look angle, L, and spin rate $\dot{\psi}_S$. The kinematic gain K_1 can be selected according to engagement conditions and noise variations on $\dot{\psi}_S$.

3. RESEARCH METHODOLOGY

Having outlined the basic guidance laws the current methodology of research in these fields will now be discussed. This methodology consists of the application to missile guidance problems of the technique, given in an earlier lecture, of hybrid computer simulation. By setting up a mathematical model of either a homing or CLOS situation on a hybrid computer it is possible to study the effects of different guidance parameters and determine which factors in the engagement significantly affect the flight trajectory and miss distance performance. A number of examples from recent studies of a fundamental nature will be given to illustrate the power of these computing techniques. It will be shown, for example, how many statistical engagements can be completed in the form of a laboratory experiment. Noise free runs can give an initial insight into the effects of certain parameters on the missile trajectory, and a significant pay-off arises when noise on the sensors has to be taken into account. As stated before, hybrid computers can be used to simulate noise in a controlled manner and on a known probabilistic basis. For example it is possible to represent target glint by a sequence of white noise signals passed through a filter; the white noise itself being generated by either analogue or digital means and being variable from engagement to engagement, but repeatable from, say, bloc' to block of a number of runs.

Consider first of all some noise-free studies in the homing field, to be followed by investigations with noise.

3.1 Noise-free Studies

As an example of the extent to which noise free runs can be varied parametrically to give considerable insight, many missile trajectories have been evaluated for proportional navigation and other homing laws. The results of some of this work are given in Fig 4, and were obtained by hybrid computation of a digitally controlled analogue model. The trajectories of the missile relative to the target were plotted by computer in sequence as the parameters were changed automatically. Nine diagrams are shown in Fig 4 for each of three speed ratios of missile to target, $\nu = 1.2$, 1.8 and 2.6, and initial look angles L_o of 0, $22\frac{1}{2}^\circ$ and 45°. Each diagram shows relative trajectories from sixteen azimuth directions relative to a non-manoeuvring target flight path, for three navigation constants K = 1, 2 and 4. The target directions of motion are always to the right of the diagrams. Consider the diagram for a speed ratio of 1.8 and zero initial look angle. It can be seen that when K = 1, for pure pursuit, the trajectories all approach the target finally in the tail-on position, whereas for higher values of K, say K = 4, after an initial turn towards the target the relative approach is finally on a constant bearing collision course, shown by straight lines on this relative plot. The diagrams tighten up with increased speed ratio, and when an initial look angle is introduced become non-symmetrical about the flight path of the target. When the initial look angle is large eg $L_o = 45^\circ$, and ν is still 1.8, the curves for K = 1 are now deviated pursuit curves and spiral in to the target. The central diagram for $L_o = 2\frac{1}{2}^\circ$ and $\nu = 1.8$ shows that some initial conditions of azimuth and look angle are fortuitously such that from the outset the missile is on a constant bearing collision course, so no further missile manoeuvre is required. They occur at angles $\frac{\pi}{2} - \sin^{-1}(\nu \sin L_o)$ to the starboard beam of the target. The three diagrams for $L_o = 22\frac{1}{2}^\circ$ show that these angles approach the beam-on condition as the speed ratio increases, being typically 62.7°, 46.5° and 5.8° for $\nu = 1.2$, 1.8 and 2.6 respectively. They finally disappear when $\nu \sin L_o = 1$. This condition occurs, for example, in the left hand diagrams for $L_o = 45^\circ$, between $\nu = 1.2$ and 1.8, when ν would be $1/(\sin 45^\circ) = \sqrt{2} = 1.414$ to be precise.

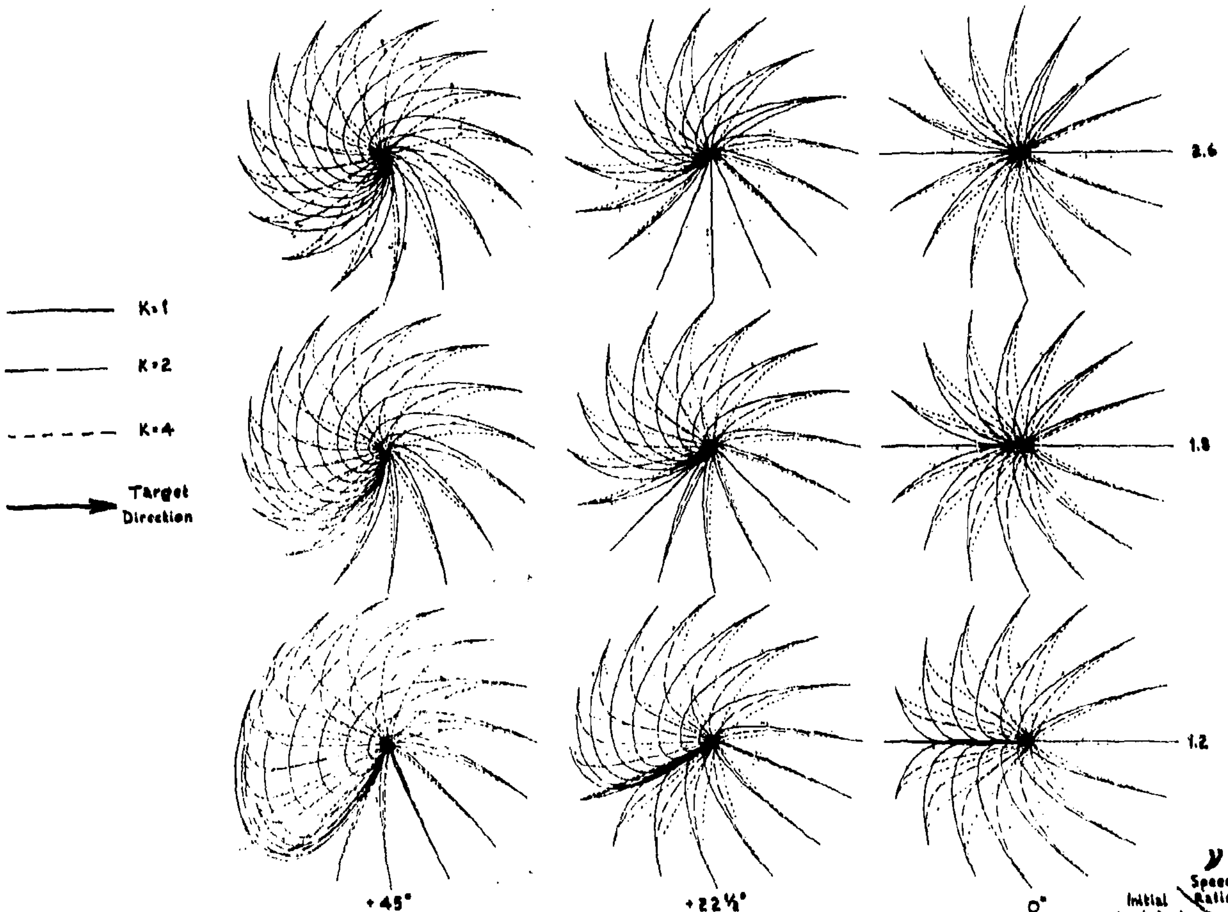


Fig.4. Pursuit courses and Proportional Navigation trajectories

Simultaneous paper trace pen recordings were obtained of the x, y co-ordinates, missile lateral acceleration and look angle during each of the runs, each on a time basis, and could be used for detailed inspection. This technique could be extended easily to study guidance laws in the presence of target manoeuvre, even in the noise free situation, to see if homing head maximum look angles are adequate.

3.2 Studies with Noise

3.2.1 Sample Sizes

When noise must be included in a simulation study one of the first things to establish is the sample size of the number of runs for an adequate statistical set of results. In tail chase homing studies in particular it is necessary for practical reasons, to keep the sample size as low as possible because of the long running times of the simulated engagements. If rms miss distance is the criterion, some preliminary results are required such as those shown in Figure 5 for homing. This figure shows two sets of results for samples of 20 and 100 runs respectively contributing to each rms value. They are plotted for each of three speed ratios, 1.2, 1.8 and 2.6 and eight initial azimuth directions from head-on to tail-on and head-on again. On the basis of these results a sample size of 20 would be acceptable as showing the trends adequately.

3.2.2 Noise sensitivity studies

In studies with noise there is also a necessity to investigate early the sensitivity of the criterion, say rms miss distance again, to the level of noise included in the simulation. An example of this can be seen in Fig 6 where, for one speed ratio of 1.8, the middle curve of the upper diagram of Fig 5 has the noise increased and decreased by 50%.

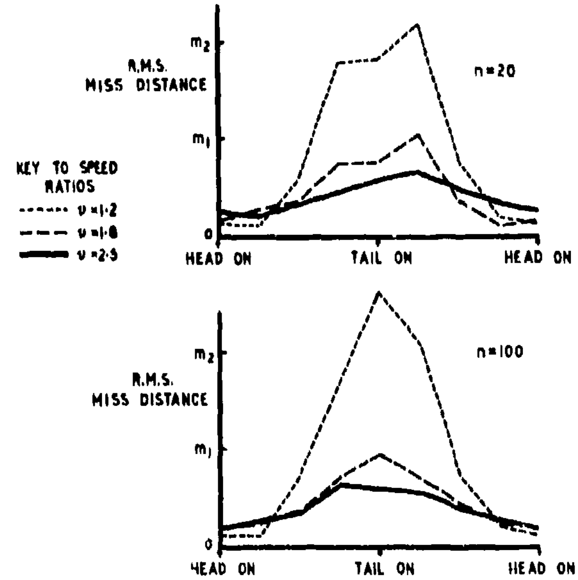


FIG.5 SAMPLE SIZE EFFECTS FOR PROPORTIONAL NAVIGATION

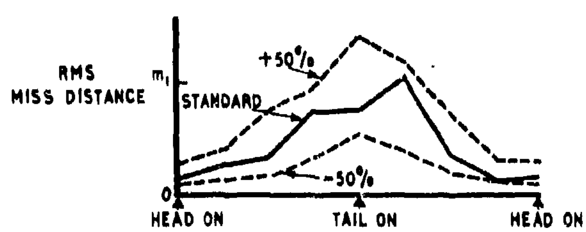


FIG.6 NOISE SENSITIVITY STUDIES

3.2.3 Parametric Variations

Preliminary parametric variations were obtained during the sample size and noise sensitivity studies. For example Fig 5 shows the effects of speed ratios and azimuth directions for homing. It also shows how the miss distances for constant K increase in the tail-on approaches especially at low speed ratios. The results were obtained for only one guidance law, proportional navigation with a navigation constant of 3. At this stage it might be interesting to know whether the same characteristics would be obtained with other parameter values. Having established a reasonable confidence in the simulation it becomes possible to pursue this in extensive parametric studies. Single and multiple parameter changes are relatively easy in hybrid computer simulation. For example we can investigate the possible interacting effects between say eight azimuths, three speed ratios, six K factors either with or without sight-line noise. Taking a sample size of 20 for each combination the results of such a study would be as shown in Fig 7. This diagram serves the purpose of illustrating how extensive numbers of statistical runs can be reduced to an assimilable form. It contains the results of $8 \times 3 \times 6 \times 2 \times 20 = 5760$ computer runs, usefully summarised. The upper diagram of Fig 5 can be seen in Fig 7 as one of the family of characteristic curves, at K = 3. Each small diagram has the same key and axes as Fig 5, viz rms miss distance v azimuth direction. From the more extensive parametric diagrams such as those in Fig 7 significant interactions can begin to be discriminated. For example it turns out that the effect of noise depends on both the K factor and the speed ratio. When K = 1 and the speed ratio large, for example, the miss distances are large near to head-on for kinematic reasons, irrespective of the noise, but in tail-on conditions K = 1 is advantageous whatever the speed, both with and without noise. High values of K are advantageous in noise free situations for any speed ratio, but with noise the advantages are only shown for head-on approaches, ie the left and right hand sides of each sub-diagram. Tail-on approaches with noise show various effects with speed ratio, the miss distances increasing with K factor when the speed ratio is low.

It can be seen, therefore, that diagrams such as Fig 7 can be used to extract interacting effects which require deeper investigations in further computer runs. The results of Fig 7 refer to the use of a constant K factor, which would not be the case for current systems. The diagram is given for illustrative purposes only. The study could be taken further using the above computing techniques for other navigation laws, eg those with varying K factors.

Instead of pursuing this further for homing guidance it will be shown how similar parametric studies can be carried out in the context of CLOS with sight-line noise. For example the combined effects of target glint and manoeuvre on rms miss distance and missile lateral acceleration are illustrated in Fig 8. The guidance loop simulated is that of Fig 2 without the feed forward terms, but with a number of non-linear elements in a hypothetical missile representation. For practical purposes the filter S(p) simulated was a double phase advance, which itself had been previously optimised by extensive parametric variations. Fig 8 shows the parametric effects of (a) zero, 50% and 100% of a nominal glint rms value, and (b) zero, 0.5, 1.0, 1.5, 5 and 10g rms target manoeuvre. There is a double line drawn at 1.5g where the manoeuvre values change scale in the carpet plot. At each point in the carpet the rms miss distance is that of 100 separate noisy runs of combined target glint and manoeuvre. It was possible to increase the sample size from 20 to 100 for the CLOS studies because a short, fixed flight time of 5 secs was simulated whereas in the previous homing runs, particularly with tail chases, much longer running times were involved. The basic glint noise characteristics were different from run to run but repeatable from block to block of 100 runs, and scaled in glint rms amplitude from point to point in the carpet. The manoeuvre of the target was constant within each engagement run but the 100 runs represented a Gaussian distribution with the rms value quoted. The diagram therefore contains the results of 100 runs for each of three glints and six manoeuvres, or 1800 simulated engagements. Many more variations were found to be possible since the computer was able to run at 100 times real time, thus completing the amount of information shown in Fig 8 in about half an hour. Typical parameter values which could be changed easily were the glint bandwidth and missile parameters, for example.

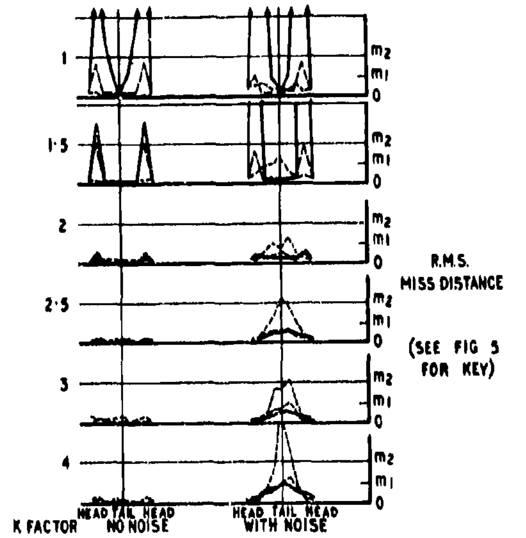
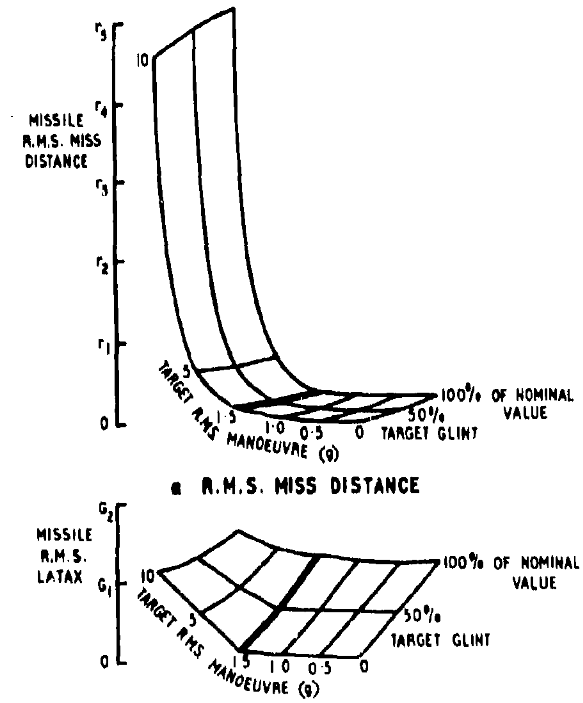


FIG. 7 NOISE EFFECTS FOR PROPORTIONAL NAVIGATION



a R.M.S. MISS DISTANCE

b R.M.S. ACHIEVED LATERAL ACCELERATION

FIG. 8 CARPETS OF R.M.S. MISS DISTANCE AND R.M.S. ACHIEVED LATERAL ACCELERATION

The curves in the upper diagram of Fig 8 show, incidentally, that when the target manoeuvre is low, say less than 5g rms then the variations in both miss distance and latax due to glint are more significant than those due to manoeuvre. When the manoeuvre is greater than 5g rms, however, the miss distances increase significantly, much more than the variation due to target glint. When the target manoeuvre is large, therefore, the missile manoeuvre capability in this example is almost all used up to satisfy the feed forward requirements of a high target beam rate, leaving little latax in hand to reduce the errors off the beam. In designing a missile to a given latax limit, therefore, the limit must be sufficiently large to include a capability to meet the feed forward terms, which must be satisfied first, together with a residual capability for dealing with the errors off the beam. The specification of this residual acceleration will then form the basis for the design of an optimum statistical filter $S(p)$ to replace the phase advance filter used in the above example.

We shall therefore proceed to consider the research methodology further by investigating Wiener and Kalman filters in missile guidance loop design.

4. OPTIMUM FILTERING

The theory of statistical filtering is complicated and has been treated extensively in the literature and will therefore not be given here. Instead a simple example will be used to illustrate how the technique can be applied to CLOS missile guidance systems. Without going into great detail the process is as follows. When both the target glint noise and manoeuvre can be specified on a statistical basis, and linear assumptions can be made about both the missile transfer function and the kinematics then linear theory enables an optimum filter to be designed so as to give a minimum rms miss distance subject to an assumed limit to the available lateral acceleration. Instead of a hard limit to the 'g' capability it is also necessary to assume a statistical distribution, say Gaussian, with a further assumption that the given limit is only exceeded on a low percentage of occasions, typically 5%. Wiener filter theory can cover steady state conditions in the region of a given missile range R_m. If the aerodynamic or kinematic conditions change during flight a series of Wiener filters can be conceived in which the parameters are sequentially switched so as to be optimum throughout the engagement. This could be done by an analogue computing technique in an adaptive manner. The more complex Kalman filter can cover time varying conditions more easily since it is expedient to compute it digitally and recursively, to combine estimates and measurements optimally. Wiener and Kalman filters can be shown to be identical in stationary conditions so Wiener filters can be considered to be forms of sub-optimal Kalman filters if used in their place. There is a distinct advantage in carrying out research with Wiener filters, however, because of the ability to compute by analogue means. The advantage is that the statistical output of the simulations can be significantly increased compared with digital computations. This is the methodology being recommended in this paper for application to missile guidance filtering research. With Kalman filtering research it has been found necessary to slow down the digital simulations to real time, or slower, and the statistical output of the research programme is reduced. It is recognised that ultimately the practical implementation of the optimum, or sub-optimum filter will be by digital computer but there is still so much to learn about design processes that multivariate aspects of guidance problems require a statistical output which cannot yet be met by a purely digital approach. The use of a hybrid computer for research studies enables a smooth transition to take place between analogue or hybrid investigations and digital implementation. Computer hardware can be included in the simulation during development, for example.

Design processes are not always clear cut, as can be seen from an example that arises with Wiener filters. Some optimum filters of this type have long time constants if only guidance along a CLOS beam is considered. This may not be practically conducive to the gathering phase when a missile has to be brought into a guidance beam quickly after launch dispersion. On the other hand this characteristic may not be general, but may differ from missile to missile, and the solution may only be obtained after extensive parametric studies of a compromising nature. There are many such compromise situations arising in guidance and control problems, eg conflicting requirements for operating at high or low altitudes.

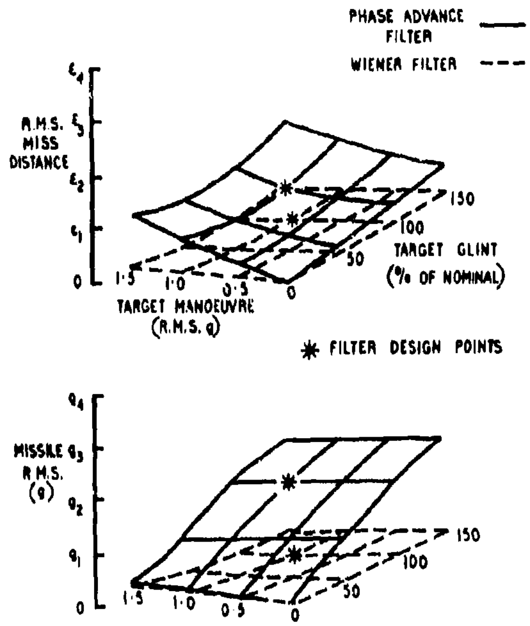


FIG.9 C.L.O.S. : WIENER c.f. PHASE ADVANCE FILTERS

As an example of the use of this methodology when applied to filtering research we can consider further the CLOS situation discussed earlier for glint and manoeuvre variations. Fig 9 shows the results of further simulations comparing two types of filter S(p) for CLOS. The order of improvement in miss distance and latax which can be obtained by using an optimum Wiener filter instead of a phase advance type is shown. In the complex simulation of the CLOS missile guidance loop only the filter S(p) was changed whilst extensive numbers of runs were repeated over the same glint and manoeuvre variations. The design points for the phase advance and Wiener filters are shown by the asterisks at nominal (100%) glint and (1g) manoeuvre conditions. The differences between the two asterisks in each diagram show the improvements in rms miss distance and rms latax respectively which are offered by the Wiener filter compared with phase advance. For system changes in glint and manoeuvre of 0, 50%, 100% and 150% and 0, 0.5, 1.0 and 1.5g respectively, carpets are plotted for miss and latax. Only the carpets for the Wiener filter are labelled, the pattern being identical for phase advance. These overall carpets show that the nominal improvements at the design points continue to be achieved for a constant filter design over a wide variation in glint and manoeuvre, different from the nominal values assumed for each filter synthesis. This type of diagram, therefore, provides a means of comparing the sensitivity of each of two filters to system changes. In this example the Wiener filter comes out with a distinct advantage.

By extensions of the simulation technique it is possible to measure experimentally the sensitivity of the system to changes in the filter design parameters. The simulation model of the system is kept constant whilst one or other of the filter values is altered. As an example of this type of sensitivity analysis Fig 10 shows the results of varying each parameter defining a Wiener filter. The lower part of the diagram shows an analogue form of a Wiener filter consisting of four integrators and nine potentiometers (a) to (i). The small upper diagrams show sensitivity curves for both rms miss distance and rms latax which were obtained for blocks of 100 repeatable noisy runs in which the potentiometers were changed one at a time by up to ± 50% about their nominal design values. It can be seen that, although it was designed to be optimum on a linear basis, it is not fully optimum when modelled in a non-linear guidance and control simulation of the real system. Some further refinements are possible, eg an increase in the value of potentiometer (b) to approach the minimum point of the miss sensitivity curve. It is possible from this type of display to assess how complex the filter synthesis needs to be and what penalties are incurred by neglecting and simplifying certain parts of the design. Using this technique the above filter had already been reduced from sixth order to fourth order, for example, without significant effect.

It would not have been possible to carry out this quantity of statistical design work in the purely digital mode, say for Kalman filtering, unless considerable expense was incurred in production running. There is therefore still much to be gained by executing preliminary research programmes on guidance filtering in the purely analogue or hybrid computing mode, reverting to the digital formulation later on when the system performance is more clearly understood and the practical implementation of the selected design is required to be digital.

Before leaving the subject of optimum filtering a brief comment needs to be made in relation to its application to homing guidance. In homing the situation is slightly different from CLOS. The equations for the kinematics and dynamics might be linearised only with time varying coefficients so that optimum statistical filtering may not apply if based on linear theory.

5. AUTOPILOT IMPLICATIONS

Although trajectory evaluations and optimum filters can be derived using low order mathematical models of the missile dynamics, it is necessary to test the theoretical filters or guidance laws for real systems in a simulation which includes much more detail of the missile and sensors. In particular, important non-linearities in the missile autopilot, homing head or ground radar representations should be simulated. For example the missile transfer function used for the Wiener filter derivation was that of a

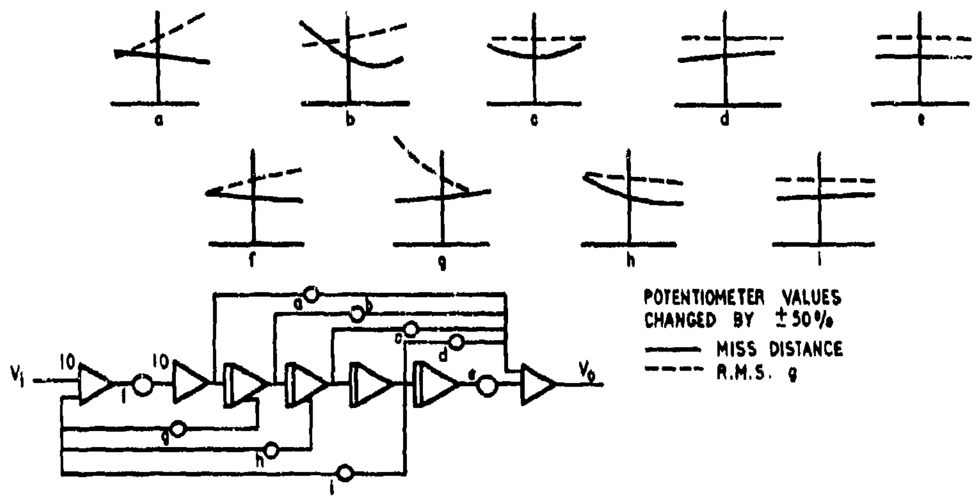


FIG.10 SENSITIVITY OF A WIENER FILTER

quadratic lag, whereas a more detailed representation typical of a number of missile autopilots would include lateral acceleration demand limits, fin limits, instrument feedback gains and suitable shaping networks.

6. COMPUTER IMPLICATIONS

Microminiaturisation techniques are leading towards the possibility of utilising digital computers in a variety of guidance and control applications. Flexibility in mode of operation would then arise from software variations only. Missiles with multiple roles might be developed in which significant changes would be made only by different computer programmes in the guidance system. These techniques might lead to a widening of missile component production tolerances with an associated saving in missile cost. The cost of the computer, however, may be influenced by the storage size and accuracy requirements. Limited word length aspects are already known to cause instabilities. Any computer limitation will therefore affect the performance of the overall system and should be considered in its own right as a sub-system demanding appropriate study.

7. CONCLUSIONS

A methodology of applying hybrid computer simulation techniques to the study of missile guidance laws of the CLOS and homing types has been outlined. Basic guidance laws have been reviewed and simulation studies have indicated that an understanding can be obtained on their effectiveness in both noise free and noisy situations. It has been shown how linearised models can be used to devise statistically optimum guidance filters for simulation in non-linear missile systems. These simulations can then lead to a choice of particular characteristics for engineering design.

In optimisation procedures, however, it should be remembered that theoretically designed filters should always be tested experimentally by simulation techniques to see if they are robust enough to be used in environments which are less certain than the assumptions used in their design. Stringent filters of the Wiener or Kalman types can give good optimum solutions for the assumed model conditions but should be checked for sensitivity variations. It could be that sub-optimal solutions are more acceptable.

In the early stages of research into filter design an analogue approach using stationary Wiener filters is preferable to a digital Kalman filter approach because of the increased statistical output from simulated engagements. In the later stages a digital implementation may be preferred and hybrid simulation should continue to be used to evaluate the dominant error contributions in all fields of missile guidance and control, thus leading to good cost-effective solutions being selected at the feasibility stage.

* * * * * * * *

PULSE DOPPLER MISSILE GUIDANCE - REPRESENTATIVE PARAMETERS AND ASSOCIATED FIRE CONTROL CONSIDERATIONS

by

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SECTION I

This paper will be concerned with the principal problems and solution options available to the designer when addressing the all-weather attack of small tactical targets. At the outset, it should be noted that this is a nearly unexplored area of weapons technology and the considerations given herein are generalizations to some extent. Although but few examples of operational systems of this type now exist, there is little doubt that the advances in microelectronics and data processing devices will shortly bring about a marked increase in the utilization of such techniques.

The requirement for all-weather operation against relatively small and possibly mobile targets dictates the utilization of a microwave terminal guidance sensor. More specifically, it is generally accepted that the optimum region of the electromagnetic spectrum for this purpose is in the X to K_u band vicinity (wavelength of 0.1 to 0.05 feet respectively). (Some special purpose devices have been used at higher and lower frequencies but these are of limited value in the general context of this paper). There exists, of course, a wealth of data describing the theory of operation, performance parameters, and means of implementation of radars operating in these frequency bands, but this information is almost exclusively devoted to the problem of airborne targets. As such, the historical radar problem has been the maximum range detection of isolated reflectors limited primarily by radar-generated noise and available power considerations. The tactical target poses a new problem element, in that this target is usually immersed in a background of unwanted, but usually strong reflectors with sometimes very similar radar reflection characteristics. This paper therefore will largely skirt the more conventional radar considerations and instead will stress the peculiar problem aspects associated with the detection and subsequent tracking of clutter (unwanted reflector) submerged targets on the earth's surface.

In order to contain this extended area of discussion to tractable limits, this paper will first examine the problems of detection and, in the second half, the tracking and fire control considerations associated with the attack of three generic types of tactical targets. For convenience all numerical examples will be treated for X band only.

The mission, for purposes of this discussion, is defined to be the attack of either mobile elements (tanks, trucks, etc.), or stationary, relatively small objects such as buildings, bridges, etc. It is further assumed that a reasonable missile-to-target location uncertainty exists at the initiation of the terminal guidance phase. Hence, in all instances, the system must first facilitate a search over a finite "acquisition window" and then provide means for the designation of the target with a high degree of confidence in its identity.

The circumstances under which target search is initiated may vary considerably depending on the nature of the attack vehicle. This vehicle could be a semi-ballistic or a cruise type missile, it could operate at subsonic or supersonic speeds and be either ground or air launched. As such the area of target location uncertainty, and consequently, the minimum detection range is a function of these specifics coupled with the dynamic or response limits of the systems. Still, experience indicates that most problems will ultimately pose a minimum acquisition range requirement of approximately 10 nautical miles.

The mission, therefore, requires the radar to search over or map the suspect target area at relatively long range. The question then is whether the resulting imagery is adequate to permit detection. This question can be quantized in terms of the diffraction or resolution limit of the sensor as illustrated in Figure 1-1. The diffraction limit of a conventional (non-pulsed doppler) radar is defined by the inherent focusing capability of the radar aperture and its range discrimination capability (ΔR). Thus, the minimum resolution cell dimensions achievable with a beamwidth (β in radians) is βR feet in azimuth, where R is the imaging range in feet and ΔR feet in range. Hence, the surface area within which the radar cannot differentiate between echo sources is $\beta R \Delta R$ ft^2.

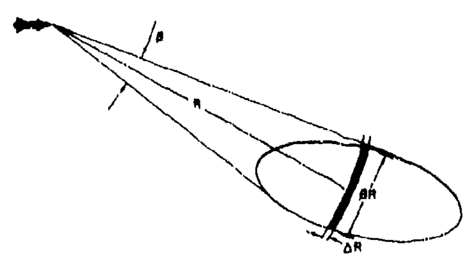

Figure 1-1. Conventional Radar Resolution Area

To illustrate the point, consider that the beamwidth for a practical missile radar aperture of 1-foot is about 6° or 0.1 radian, and the range gate width (ΔR) may be on the order of 100 feet (determined by practical transmitted power limitations). In context of a 10 nautical mile range requirement, the elemental resolution cell is a 6000 X 100 foot rectangle or a 6 X 10^5 ft^2 surface area. The aggregate of unwanted returns (clutter) from such a resolution cell must be small compared to the target return or at least differentiable from its return if target detection is to be accomplished. Thus to detect a target, it is necessary that the radar be able to measure specific physical characteristics of the surface complex and discriminate against unwanted returns. Size, shape and/or motion may be used as discriminants of the target relative to clutter.

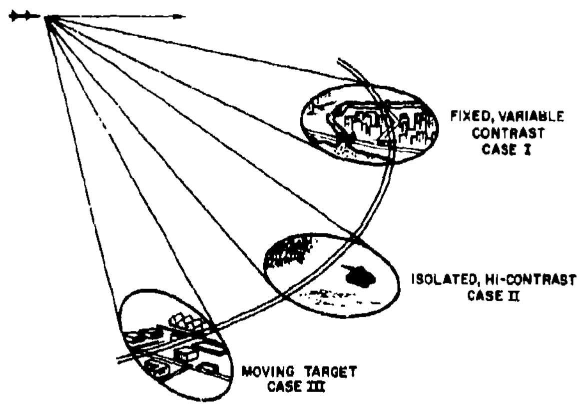

Figure 1-2. Generic Surface Targets

Specifically, the problem can be considered in the context of the three generic surface target cases illustrated in Figure 1-2.

CASE I, is a fixed, high contrast surface reflector of small spatial extent. This tactical target could be a bridge, building or parked aircraft. Importantly, such a target is usually surrounded by similar objects (other buildings, aircraft, water towers, etc.) which are of equal or greater radar reflectivity.

CASE II, is a high contrast target which exists in a field of relatively low radar reflectivity. A single tank in a field is a characteristic example.

CASE III, is a small, moving surface target, again in a background of scatterers with relatively high reflectivity. This situation is represented by vehicular traffic such as a truck.

When this target array is examined in terms of conventional radar technology the detection problems become readily evident. Take the Case I target as a parked aircraft. If the resolution cell is 6000 feet by 100 feet, it is apparent that the cell width of 6000 feet will realistically include other aircraft, hangars, and buildings and land background whose total radar area could be orders of magnitude larger than the target desired. The target is undetectable principally because of the relatively poor spatial resolution in the cross-range direction afforded by the radar antenna focusing capability. Even if the range gate, ΔR, were reduced well below the 100 foot example the same limitation exists. To detect the Case I target, the cell size must first be reduced to separate other nearby cultural background reflectors. In addition, the order to identify the desired target its shape must be somewhat discernible versus other objects in the radar imagery. A subjective quantity but a useable rule of thumb for shape discrimination is that the cell dimension be no larger than 1/10 to 1/3 of the target's "descriptive physical" dimension. Further, the resolution cell should be of equal azimuth and range resolution to be independent of target aspect relative to the radar. Case I target detection capability then is a function of the geometric resolution of the radar.

The Case II target may be a tank whose radar area could be as small as 100 ft^2. This target's shape need not be determined, but its radar return must be significantly larger than the return in any other resolution cell in the searched zone. These radar returns from clutter are a function of the clutter's reflectivity constant, $\sigma°$ (which is a function of depression angle, thus altitude, at the chosen 10 nmi. range) and the resolution cell size. Representative $\sigma°$ variations as a function of altitude are shown in Figure 1-3. For the illustrative resolution cell at 2000 feet altitude $\sigma°$ may be as high as 10^{-3} resulting in radar echo area of 6000 ft^2. Considering also that automatic detection requires signal to clutter ratios of the order of 10:1, it is clear that the missile attack of this target is realizable only by reducing the resolution cell size. The additional clutter returns from vertical surfaces such as rain further limits the effectiveness of the attack again resulting in reduction of cell area. Lower contrast targets would required reduced cell size, thus improved resolution.

The Case III target (a truck, for instance) can be detected only on the basis of motion since its reflection characteristics are assumed to be indifferentiable from other ground scatters. A change in the return from the resolution cell as the target moves through it could be observed. Thus a successive subtraction of fixed returns to cancel clutter could be used to find a moving target, but this approach is usually impractical for all but very large, very fast moving targets. A much better solution is one which senses velocity directly and does not depend on spatial resolution for movement detection.

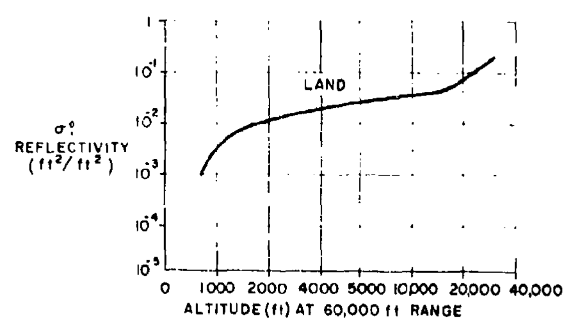

Figure 1-3. Typical Reflectivity Variation

Summarizing the detection capabilities of conventional radars to the requirements of the sample generic targets, it becomes apparent that only in specialized instances can a target be reliably detected in the presence of unwanted clutter returns. For general effectiveness of small aperture radars, significant reductions in the resolution cell's azimuthal extent must be obtained and a direct method of velocity differentiation must be introduced. These can be obtained by using a coherent radar on a missile.

A coherent radar differs from normal pulse radars in that the phase of the transmitted signal is a systematically time varying, repeatable, and well-known quantity. Hence, as shown in Figure 1-4 the phase change between transmitted and received energy corresponds to a constant time delay if the radar and target are stationary. ($\phi_{REC} = A \sin \omega (t + \tau)$ (τ = target range round trip time). If, however, the radar-pathlength is a changing parameter the transmitted-received phase characteristics will similarly reflect this time-varying path length change. When constant speeds are involved, this becomes the well-known doppler shift effect. Before leaving this subject, it should also be noted that if the transmitted waveform has a pulsed-coherent nature the radar is capable of measuring the incremental phase shifts occurring at any stipulated time delay and therefore range.

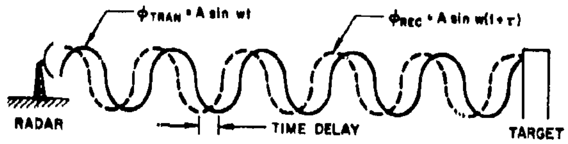

Figure 1-4. Phase Coherent CW Waveform

The coherent radar is therefore able to measure and record the phase and amplitude of the energy returned from an ensemble of points located within a given range slice and bounded by the radar's aperture (ordinarily the diffraction limit). In turn, these recorded phases can be analyzed or filtered after a sufficient number of samples have been obtained thereby providing a means of differentiating between elements lying within the diffraction limits of non-coherent radars.

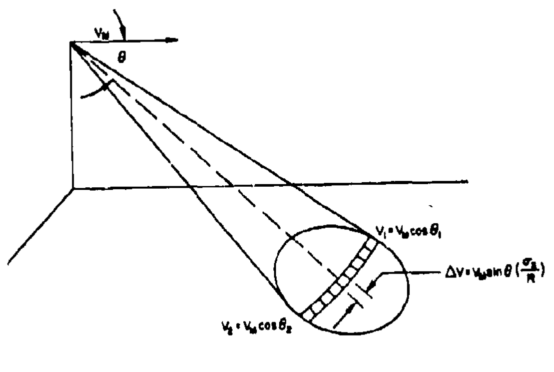

Figure 1-5. Beam Splitting Using Doppler

With the target-oriented discrimination criteria in mind then, consider an array of fixed points on the ground as seen from a moving platform such as a missile (Figure 1-5). When these points lie at the same range, the velocity magnitude to each element is $V = V_m \cos \theta$ hence a function of its angular displacement from the missile velocity vector. In fact, total equivalence between angular displacement and approach velocity exists such that a co-range spatial extent σ_x, on the surface can be described by a velocity interval, $\Delta V \approx V_m \sin \theta (\sigma_x/R)$. This inferred velocity - space relationship is fully determinate when only a single point (the vehicle) is moving. If the target is also moving, a relative "shift" of clutter and target in velocity-space takes place. Both of these properties are exploited in a coherent radar to achieve the detection properties required for surface target attacks.

The velocity increment $\Delta V = V_m \sin \theta (\sigma_x/R)$ of an element of azimuthal extent (σ_x) equates to a doppler bandwidth

$$\Delta f_D = \frac{2 V_m \sin \theta}{\lambda R} \sigma_x \text{ (Hz) at a center frequency of } f_d = \frac{2 V_m \cos \theta}{\lambda}$$

(assuming a doppler unambiguous wave form). The radar must therefore accumulate a sufficient number of return samples to facilitate a search over the return spectrum for the purposes of isolating returns at the range of interest in terms of bandwidth and center frequency.

This is accomplished by accumulating return samples for a minimum period T_i seconds where T_i is equal to $1/\Delta f$ (Δf expressed in Hz), and by subsequently passing this sample history through a bank of contiguous filters, each of which is tuned to a bandwidth Δf.

The additional dimension, bandwidth or frequency increment, is depicted in Figure 1-6. As indicated the returns from a given range are spectrally analyzed in the filter banks as shown at the bottom of this figure. Thus, these are essentially orthogonal to the range dimension. The return from a target of narrow bandwidth will "pile up" in a single (or few doppler cells) while spatially distributed or wideband returns will distribute their returns among many filters. The filtering process then is one of velocity and velocity difference (frequency and bandwidth), but its significance in detection depends on the source of such velocity distributions.

The result of this spectrum analysis is to separate spatial returns from fixed points whose azimuthal extent is

$$\sigma_x = \frac{\lambda R}{2 V_m \sin \theta T_i} \text{ feet.}$$

This spectrum analysis "splits" the radar's physical beamwidth by a factor equal to

$$\frac{\beta R}{\sigma_x} = \frac{\beta}{\lambda} \cdot 2\, V_m \sin\theta\, T_i \,.$$

This can achieve a beam sharpening effect of several orders of magnitude.

Another frequently used term to describe this beam splitting effect is the term "synthetic aperture" or β_s. This is defined to be the physical aperture necessary to achieve the same spatial resolution. The value of β_s is

$$\frac{\sigma_x}{R} \quad \text{or} \quad \frac{\lambda}{2\, V_m \sin T_i}$$

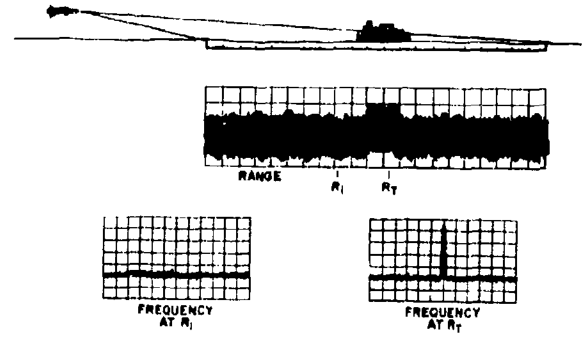

Figure 1-6. Coherent/Non-Coherent Comparison

and since the physical beamwidth is defined as $\beta = \lambda/D$ (where D is the physical aperture diameter) the "diameter equivalence" of synthetic apertures is $D_s = 2\, V \sin\theta\, T_i$. As shown in Figure 1-7 a synthetic aperture is twice the straight line path distance flown normal to the target line of sight. The same relationship can be established from an aperture rather than a velocity point of view, the multiplier 2 is then attributable to the fact that transmission and receipt of energy is conducted over spatially isolated points rather than the whole ensemble.

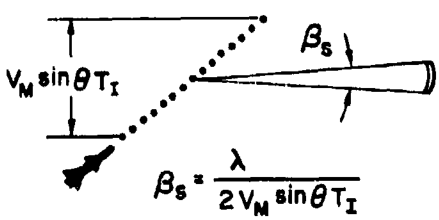

Figure 1-7. Synthetic Aperture

The ground scatterers were considered fixed in the foregoing, if a scatterer Point A moves over the ground it therefore has an additional radial velocity component toward the radar, thus its center frequency is shifted with respect to the ground. As a result, a moving point will appear either in competition with the return of a spatially offset ground resolution cell or, if its radial velocity component exceeds the velocity increment enclosed by the radar's physical beam, the target will appear free of clutter.

Further, if the elements of a desired resolution cell exhibit relative motion, such as ocean waves, they may occupy more of the doppler spectrum that that due to radar motion only. Another example of this "doppler spreading" is the roll motion of a ship which will be enlarged upon later.

Thus, the coherent pulse doppler resolution technique potentially has the attributes necessary to detect surface elements in a much more general sense than the conventional radar.

This ability to resolve elements several hundred times smaller in azimuth than a physical beam and the sensing of surface motion will now be applied to the generic targets to project detection performance.

The general fixed ground target (Case I) demands that the radar separates points of varying contrast which exist in close proximity in one range gate. Thus, spatial discrimination is the important parameter; σ_x should be as small as possible, since shape identification may require that an object be broken into 3-10 elements. There is no theoretical limit to reducing σ_x, but practical considerations resulting from reasonable size and cost do come into consideration. These will be discussed later.

Figure 1-8 is an example of radar imagery as seen by the spectrum analyzer. Here two targets lie in close vicinity to each other (much less than a beamwidth) and are co-range. As expected, they appear separated in doppler since they are slightly angularly displaced with respect to the mapping radar's velocity.

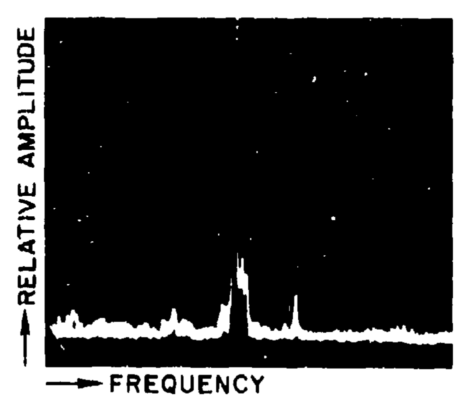

Figure 1-8. Co-range Targets

Case I requires that the radar produces a continuous radar map of sufficient resolution to facilitate the detection of a target. Such a map is produced by spectrally analyzing one or more range slices and to fly the radar over the suspect area such that each ground element passes through the range slices under investigation as the vehicle progresses down range.

Figure 1-9 is a small section of such a radar map. This sample has a spatial resolution of approximately 50 feet, it was produced from an aircraft at approximately 200 knots and with a squint angle of about 30°. (The quality of the imagery shown is constrained by the limited dynamic range used in the recording). This is a coherent radar "map" in that a fixed range interval from an aircraft platform was moved over the surface as shown

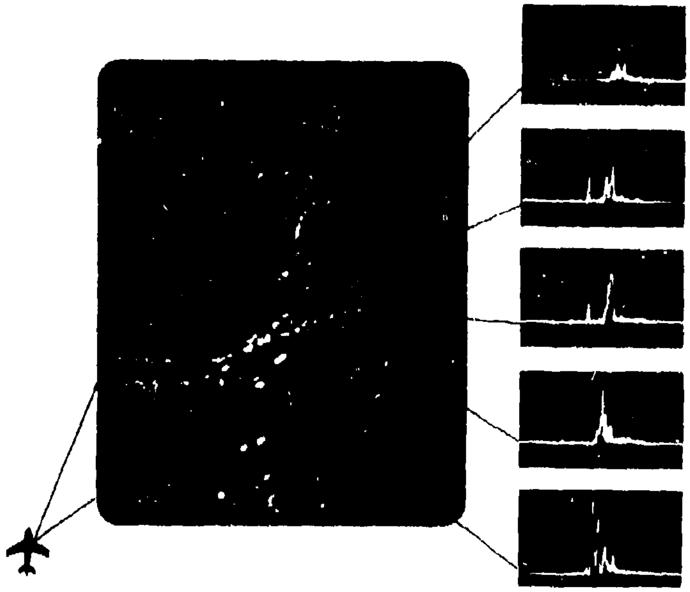

Figure 1-9. Thames River

by the geometry at the left of the imagery. The abcissa is doppler frequency and the energy in each doppler filter is presented as an intensity modulation. Contiguous range "slices" were displayed and the doppler spectrum in selected slices are shown on the right. The area mapped is the shore of the Thames River in Connecticut. The upper spectrum on the right shows water return and the abrupt increase in reflectivity at the land boundary. The second and third spectra show a boat in a cove in closely spaced range slices. Of interest is the predominance of the reflector over the low level water background and two different slices of shore indicating the relative complexity and reflectivity of ground. Two parallel bridges are seen in the map and two representative range gates in the lower two frames.

Further examples of water/land boundaries are shown in Figure 1-10 (Boston), and finally an example of truly high resolution is shown in Figure 1-11. These samples are included to demonstrate the potential of the coherent radar and also indicate the inadequacy of incoherent approaches for the purpose of ground target detection. It must be remembered that this imagery is largely contained in a single physical beam and thus is not resolvable in straight pulse devices.

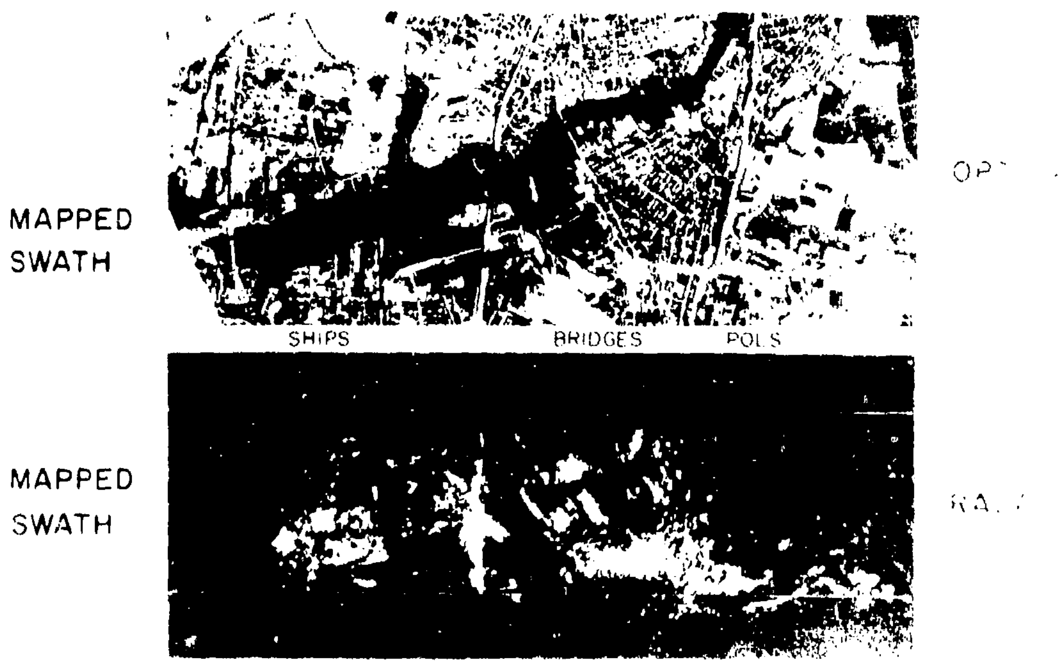

Figure 1-10. Boston

Figure 1-11. High Resolution Radar Map

The second of the generic class of targets (Case II, a tank in a field) requires as a minimum that the return from the target's cell clearly exceeds returns from other clutter sources. Thus, the cell resolution size need not be smaller than the target of interest. On the contrary, little is gained by "splitting" the target's energy causing several resolution cells, rather the radar should be designed to include the target's full extent in azimuth and range, and most importantly be wide enough to include the tank's natural bandwidth. By mapping the target at a reasonable squint angle, it is possible to split or spread clutter (of large spatial extent) among many filters while the target remains effectively in one filter. Examples of clutter as measured along the radar's velocity vector 0° bearing) versus squinted clutter (30° bearing) are shown in Figure 1-12. The zero bearing spectrum shows that the natural bandwidth of rain clutter is considerable and of the order of 150 Hz (at X Band), targets, on the other hand, rarely exceed bandwidths of the order of tens of cycles. Realizing that at higher speeds and squint angles the clutter spectrum can be spread over kilocycles (resolution cell size reduced) as shown in the 30 bearing spectrum one concludes that the amount of clutter energy a target must compete with can be reduced by several hundred times in this fashion.

Figure 1-12. Clutter vs Aspect Angle

Figure 1-13 shows a small target located in clutter. This process allows the detection of even very small targets at significant ranges and depression angles; however, the range gate or detection pattern must be tailored to the clutter suppression characteristics offered by the coherent approach.

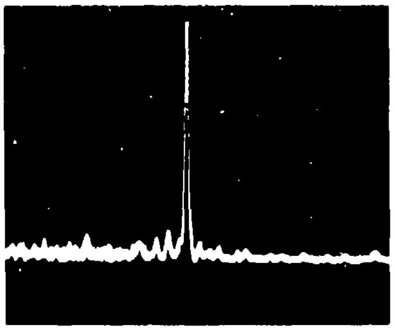

Figure 1-13. Small Target

The moving target situation of Case III is illustrated in Figure 1-15. The velocity of the target observable along the radar line of sight has caused a shift of its doppler frequency designation. As mentioned earlier, if the ground speed of the target is larger than the bandwidth of the physical beam, the target will be shifted out of "main lobe clutter". This condition is shown in the figure. The radar generated noise (normally much lower than surface clutter)is the only contaminant of the target energy at the target's doppler frequency (assuming closing targets) so that the target has high contrast. In this case, it is a relatively simple matter to filter all clutter from the detection process.

In Case III then, velocity resolution as opposed to spatial resolution is the important parameter. If the target is a truck which exists in the presence of high contrast clutter land, sufficient motion must be presented to remove its coherent signature from that of the physical beam.

The preceding is intended to provide some "feel" for the measurements which a coherent radar makes available for the detection process. This is further elaborated in the following.

The problem of directing the missile to begin its terminal attack on a specific target entails the search process as previously outlined, detection of the target from its background, and the requirement to mark or address the target to the missile so that the terminal phase may begin. This marking of the target can be done automatically or through an operator interface depending on the target type as shown in Figure 1-16.

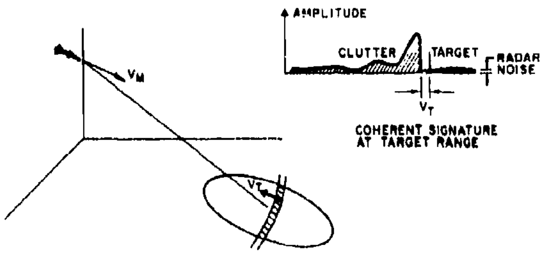

Figure 1-15. Moving Target Detection

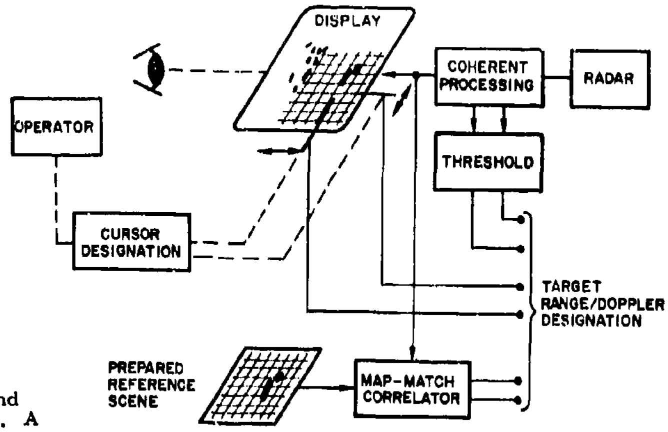

Figure 1-16. Target Designation

In the case of a target (Case II) which exists as a high contrast point over the background target marking can be performed by thresholding. A number of cells (the doppler filters in several range gates) are examined to determine a background level. By the knowledge of the relative contrast expected, a threshold level can be set to assure a high probability of detection. Usually this "alarm" can be evaluated several times (in successive integration periods) to assure that a spurious identification does not occur. All this is performed in the missile since the processing requirements are not restrictive in size and cost. When the target is verified, its range and doppler coordinates are designated to the radar tracking circuitry and terminal homing begins.

In the case of a complex land target (Case I), the simple threshold is inadequate since non-target ground objects can provide a larger signal return than the target of interest. This implies that a spatial signature of the target must be used. This can be performed automatically by inserting into the missile an a priori representation of the target and its surroundings which serves as a reference for map match correlation of the sensed target scene. Manually, an operator can observe the missile generated radar map of the target area and by manipulation of cursors designate the target to the automatic tracking circuits of the missile. Operator designation, of course requires that the map data be available at the launch vehicle, inferring a data transfer link. The processing could be performed on the launch vehicle and the cursor designation is sent back to the missile as a single range-doppler address.

Case III targets having sufficient approaching motion to displace their doppler signature from clutter (discussed previously) are high in contrast since radar noise is the only contaminant. In this case automatic threshold detection techniques can be used aboard the missile, once the clutter return is removed by doppler filtering. Observe that automatic detection and designation allows launch-and-forget operational capability.

Independent of the method of target designation, it results in providing the doppler and range coordinates of the target as they existed when the target was sensed, thus the process is not real time. If threshold detection is used the time delay may be very short, but for operator designation the delay may be tens of seconds, thus the target coordinates have, in fact, changed. This problem is solved by using the mid-course inertial instruments to up-date the target coordinates for use in the attack phase which will be discussed in Section II. The next paragraphs are intended to provide some insight into the limitations of current coherent radars.

As has been implied by the foregoing paragraphs a pulse doppler radar can vary from an extremely simple device to a relatively complex device. This is a function of the target and background for which the radar must be capable of providing discriminants. Basically frequency resolution (thus spatial resolution) and search area are the fundamental radar parameters affecting sensor complexity. Three areas which can present limitations to a missile borne radar are transmitter power, motion compensation, and doppler processing capacity.

The transmitter power required for any application arises from the necessity to have sufficient power on each resolution cell of the search area to detect the target at the required detection range. This can be calculated using the conventional radar range formula but taking into consideration the coherent processing gain of a pulse doppler radar. In so doing the formula can be expressed in the form

$$P_{AV} = K \frac{RS^2}{\sigma_x} \text{ watts.}$$

Thus it can be seen that the average power required is directly proportional to detection range (R) and the square of the search swath width (S) and inversely proportional to spatial resolution. The constant of proportionality is a function of the signal to noise ratio required, target echo area, missile speed and squint angle and radar constants such as noise figure. Figure 1-17 is a plot of average power versus spatial resolution for some typical missile radar search parameters. Presently available transmitter tubes suitable for missile application are capable of 50 watts average power.

Missile motions which cause acceleration along the radar to target line-of-sight cause phase distortions of the return radar signal, thus if these motions are not compensated in the radar system, a blurring or loss of resolution in the desired radar map can occur. These accelerations are caused by missile acceleration resolved along the line-of-sight and centripetal acceleration.

$$a = \frac{dV_m}{dt} \cos\theta - \frac{(V_m \sin\theta)^2}{R}$$

The first term is measured by an accelerometer while the second depends on knowledge of the velocity vector. Errors in these parameters then can be expressed in terms of phase errors and resolutions. If $\lambda/16$ phase error (22 1/2°) is allowed, the acceleration measurement accuracy and velocity undertainty are

$$\partial a = \frac{2V_m^2 \sin^2\theta \sigma_x^2}{\lambda R^2}, \ fps^2$$

$$\partial V = \frac{V_m \sin^2\theta \sigma_x^2}{\lambda R}, \ fps.$$

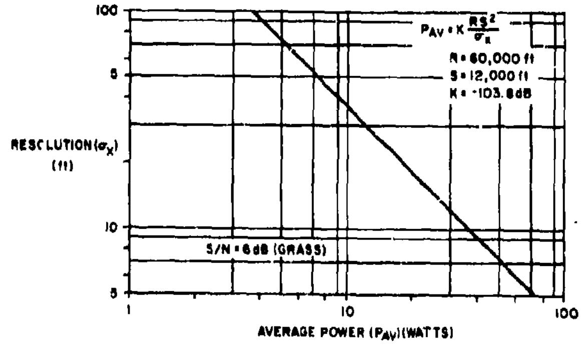

Figure 1-17. Coherent Resolution Power Requirements

The allowable variations of these terms are shown in Figure 1-18 as a function of spatial resolution for typical missile parameters. Low cost accelerometers are available better than 1×10^{-3}g. The velocity accuracy available from low cost inertial platforms are of course a function of the time of flight of the missile and its acceleration profile. Limitations on the doppler processing capacity could be expressed in various terms depending on the method of processing (digital, hybrid or analog). A rather common basis is in terms of memory size required. In order to attain a doppler resolution (corresponding to a spatial resolution) each resolution cell must be sampled for a period T_i seconds. Two samples for each resolution cell are required. This necessitates that a memory for these samples be implemented such that subsequent doppler filtering can be performed. Figure 1-19 shows the plan geometry as may be used to generate a search map.

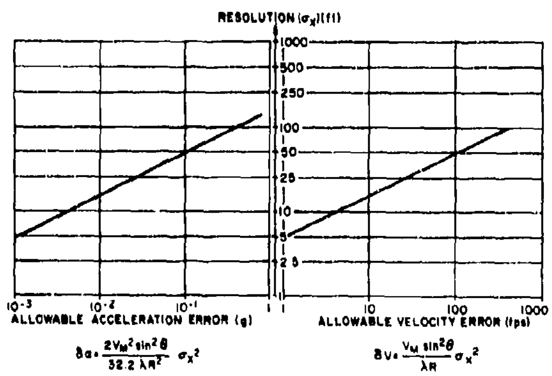

Figure 1-18. Coherent Resolution - Motion Compensation Requirements V_m = 1000 fps, θ = 30°, R = 60,000 ft.

The number of resolution cells required in the range dimension is determined by the missile distance ($V_m T_i$) flown during the sampling period (T_i) divided by the resolution cell as modified by the non-orthogonality of the resolution cell to the missile track ($\sigma_x/\cos\theta$). In the radar's azimuth direction the number of resolution cells required is the swath width desired (S) modified by the non-orthogonality of the azimuth direction to the missile track divided by the azimuth resolution ($S/\sigma_x \cos\theta$). The total number of words then is

$$W = \frac{2V_m T_i S}{\sigma_x^2}$$

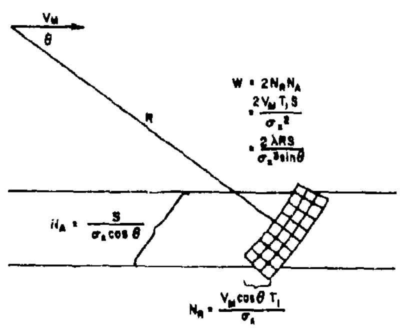

Figure 1-19. Plan Geometry for Search Map

This can be rewritten in a more enlightening form by substituting for T_i thus

$$W = \frac{\lambda RS}{\sigma_x^3 \sin \theta}$$

This equation is plotted on Figure 1-20 using the spatial resolution (σ_x) as the independent variable.

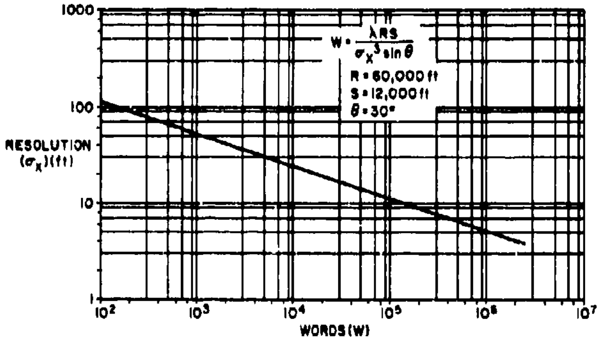

Figure 1-20. Processing Memory Requirements

SECTION II

The previous Section treated the detection of targets in various forms of clutter background. This section will address the attack of such targets. Here again the effects of unwanted returns and their minimization will be of primary interest, while only peripheral concern will be given to the more conventional radar related miss distance contributor.

The homing phase of attack in all terminal guidance problems is similar in that motion of the vehicle with respect to the target must be determined and controlled. Most particularly, the components of the relative motion perpendicular to the target line of sight must be driven to zero at impact to achieve the high level of accuracy required of tactical missiles.

To highlight the special aspects of such applications, the problem is examined in terms of the most conventional implementation method, a tracking approach.

In a tracking terminal guidance system, the radar beam center (boresight) is accurately "fastened" to the target for portions of or throughout the homing phase, thereby providing a means of determining the required guidance parameters. These parameters are missile to target range, range rate, angle and angle rate. To assure precision of these measurements it is, of course, mandatory that the radar line-of-sight remain very accurately fixed on the desired target despite the existence of unwanted reflectors in the radar's field of view. This, in turn, is the fundamental limit of the ground attack problem and is the main subject of the following discussion.

The tracking problem starts where detection left off. Detection required that the target be spatially or contrast distinguishable from other reflectors located within the same beam and range gate width. This was shown to be possible by the utilization of a relative velocity discrimination approach. It was further shown that the desired spatial resolution was obtained by "viewing" the target at some squint angle or at an angle with respect to the velocity vector. Since tracking requires continued "visibility" of target during homing, it follows that the squint angle or "head angle" (angle between target line-of-sight and missile velocity vector) must be retained. On the other hand, impact on a non-moving target requires closure of this angle, hence the need for special guidance considerations. These are indicated in Figure 2-1. At the initiation of terminal guidance, the target is shown to be at some squint angle θ. At that time the spatial resolution of the target area (in azimuth) is

$$\sigma_x = \frac{\lambda R}{2V \sin\theta \; T_I} .$$

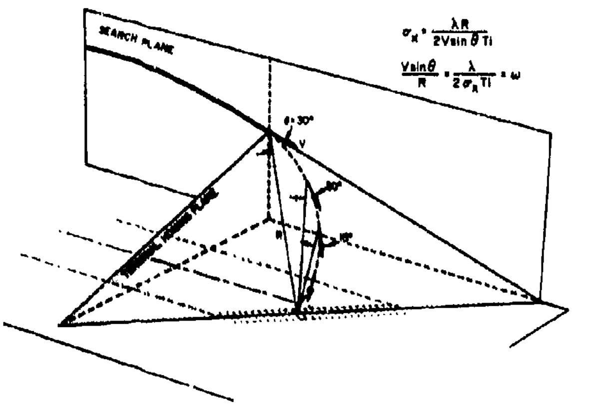

Figure 2-1. Homing Trajectory

Discrimination requires that this level of spatial resolution be maintained, while target impact (without overshoot) requires that θ be reduced to zero at or before R equals zero. A slight re-ordering of terms in the spatial resolution equation yields the desired answer. Consider the same expression in the form

$$\frac{V \sin\theta}{R} = \frac{\lambda}{2\sigma_x T_i} = \omega .$$

This form indicates that constant spatial resolution is maintained indeed, if the cross line of sight component of velocity is reduced linearly with range, or equivalently, if the line of sight rotation (ω) is maintained at a constant and deterministic value. The resulting trajectory is that of a bead on a string moving constant angular velocity, as shown in Figure 2-1 and is an arc of a circle for a vehicle with constant velocity. The resulting attack plane guidance law is the conventional proportional navigation law modified by a bias

$$\left[\eta = N\left(\dot{\psi} - \frac{V \sin\theta}{R}\right)\right].$$

Note also that this guidance law modification is required only in the projected horizontal plane (assuming largely horizontal scatterer extent) while elevation guidance remains conventional. A further point of interest is that this "curved path" approach should theoretically be continued at least until the desired spatial resolution element fully fills the physical beam ($R = \sigma_x/\beta$). However, this later requirement is modified in practice by system lag considerations.

The foregoing are general considerations; now how are these implemented? Consider the problem illustrated in Figure 2-2. The radar beam is shown to illuminate a substantial surface area which quite naturally can be expected to contain a number of surface objects, yet the beam must be accurately centered on the desired target rather than the centroid of radar returns. To achieve this objective, all returns but those originating from the desired resolution cell must be "gated out". Towards this end consider Figure 2-3. This figure indicates the ground imagery which had been available prior to target acquisition, it was mapped with the radar beam in a known angular relationship with the vehicle's coordinate axis. The target, by virtue of a designation process, was identified to be located at a specific

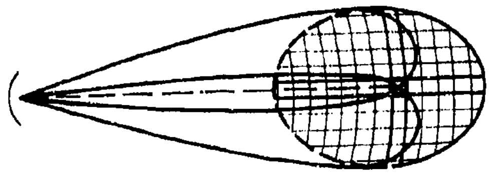

Figure 2-2. Target Tracking

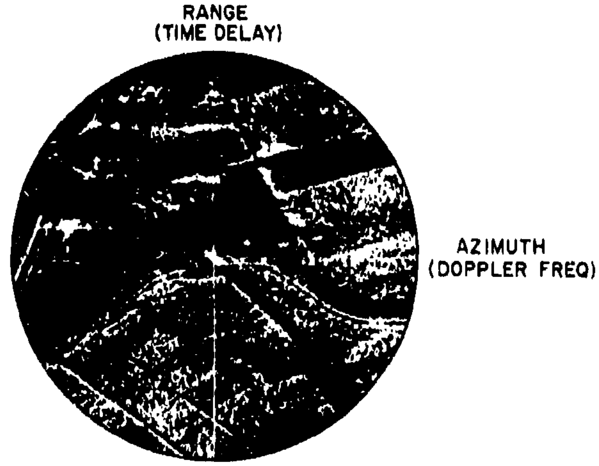

Figure 2-3. Radar Imagery

range and doppler address on this imagery. Thus, three basic gating parameters (after appropriate updating for change in position, speed, etc.) can be inserted into the system to define the target. The system must now center the beam on the returns emanating from the stipulated angle, range, and doppler coordinates.

Towards this end, the beam is steered so as to initially maximize the return coming from this cell. Subsequently, the system continues to maximize the return in this narrow band but permits the values of doppler, range and angle to change as the missile changes its relative displacement with respect to the target. Three discrimination loops are used in the process.

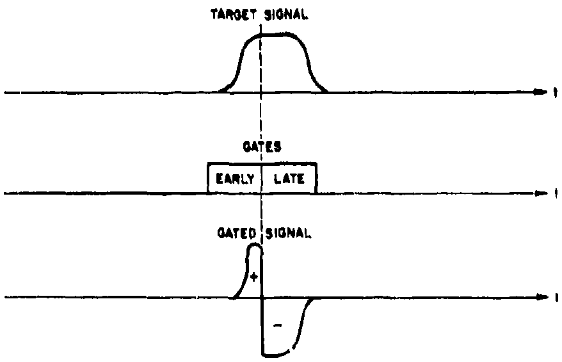

Figure 2-4. Split Gate Range Tracking

The first is a range tracking loop; this could be a split range gate as shown in Figure 2-4. Two contiguous gates are used, an early gate and a late gate. The received target radar pulse is sampled by each gate and the difference in energy outputs between the gates result in an error signal to reposition the gates with the sign of the error signal indicating the direction the gates are to be repositioned. The second, the doppler tracking loop could operate in an analogous fashion. Here two fixed bandpass filters could be used as opposed to the range gates. The target designation is used to set the output frequency of a voltage controlled oscillator which is mixed with the target's doppler such that the resultant frequency lies in the range of the contiguous filters. An error signal is generated from the difference energy between the filters and it is used to correct the oscillator.

These tracking loops can continuously drive a slaved range gate and doppler filter which is centered over the target. The next step is to drive the antenna in angle to maximize the return through these filters.

Angle tracking can be implemented in a number of ways. For purposes of illustration phase comparison monopulse will be discussed. This type of angle tracking is analogous to the interferometer devices used by radio astronomers thus it is sometimes referred to as interferometer radar tracking. Figure 2-5 represents the technique in one plane.

The antenna aperture is divided into two parts (per axis) where the distance between antenna 1 and the target is R_1. The distance between antenna 2 and the target is R_2 where $R_2 = R_1 + d\sin\theta$. The phase difference, $\Delta\phi$, as received at the two antennas is the difference in distances times $(2\pi/\lambda)$ to convert into phase or $2\pi/\lambda\ d\sin\theta$ where λ is the radiation wavelength. Thus for small angles $(\sin\theta \approx \theta)$ $\Delta\phi \approx (2\pi d/\lambda)\ \theta$ and the phase difference is proportional to the desired angle. This phase difference signal can be measured by a phase detector, whose output can then either be used as a direct reading for guidance or it can be used as an error signal to drive the antenna system to a null position.

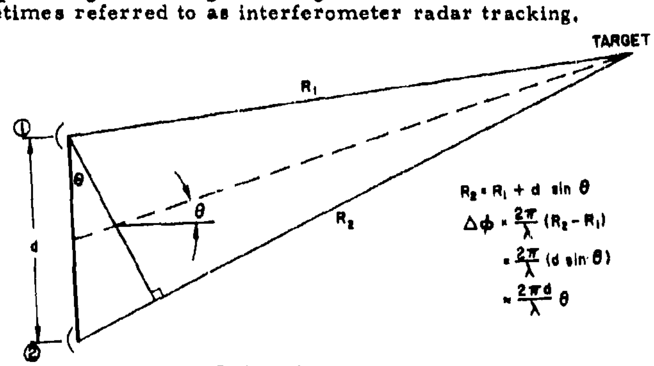

Figure 2-5. Phase Monopulse Angle Determination

If angular data is required in two planes a second orthogonal set of measuring axes is required to determine the orthogonal angle. In practice the four antennas are a single antenna structure of four phase centers as shown in Figure 2-6.

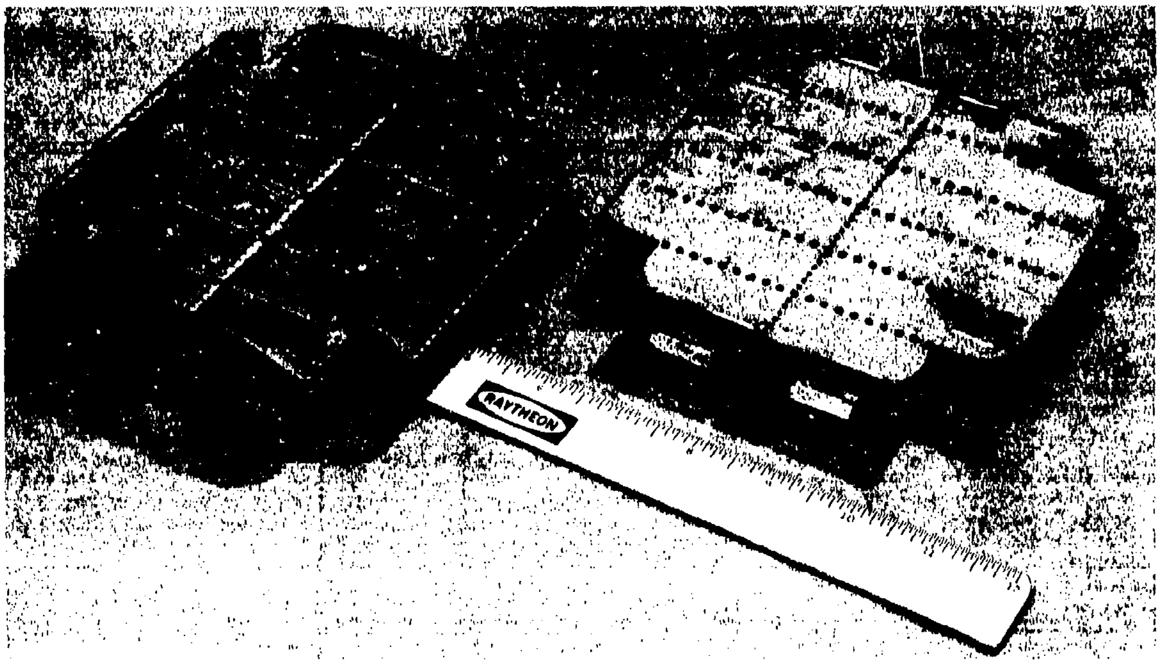

Figure 2-6. Monopulse Antenna Hardware

At this juncture, it is appropriate to provide a broader view of the variety of guidance information-yielding schemes currently under investigation. These are largely still in the early stages of development and thus not suited to extensive discussion at this time. However, some appreciation of the power of the pulse-doppler approach can be gained by even a superficial treatment of these. They fall into two basic categories, both use the multiplicity of scatterers contained in the radar's field of view to improve the accuracy of guidance information with respect to the targets. The first category recognizes that returns from non-target scatterers (assuming a fixed target domain) can be spatially related to the target by virtue of doppler and range differences. Hence, the apparent target return can be greatly improved by permitting the monopulse system not only to balance the target's return but also to balance the normalized returns from equi-angularly displaced secondary scatterers. Possible methods of implementation depend on tactical circumstances but it is evident that the "aim point" contrast can be significantly improved in this manner.

The other school of thought essentially dispenses with the angular information yielded by tracking. This approach recognizes that while the target doppler represents only a velocity scalar, other reflectors can be used to derive other velocity vector components thus establishing a sort of velocity-space frame of reference. Possible schemes to extract guidance information from such a field of points vary and as such are subject to various limitations. Just to be a little more specific, one of these approaches is now somewhat further developed. As shown in Figure 2-7 the target's doppler is a specific component of the vehicle's velocity; namely, $V \cos \theta$. It can also be inferred from the diagram that, for a diving vehicle some scatterers (assuming that these are illuminated) will be exposed to the vehicle's full velocity, hence return a doppler of V. These later scatterers then appear at maximum doppler. If both values are measured (the accuracy of measurement is extremely precise due to the high level of coherence), the ratio of

$$\frac{f_{d_{target}}}{f_{d_{max}}} = \frac{V \cos \theta}{V} = \cos \theta$$

can be determined. Now, knowing both θ and V, the cross component of velocity or the component of velocity leading to a target miss namely $V \sin \theta$ can be computed.

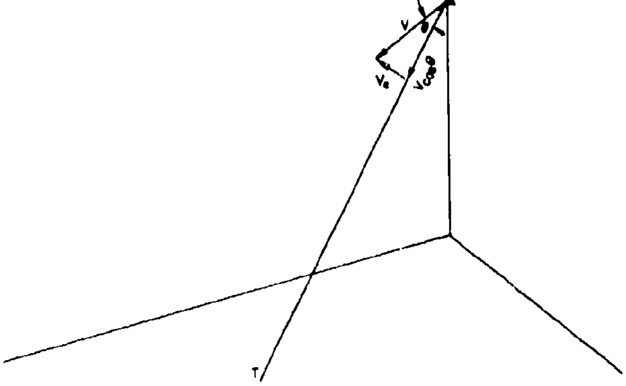

Figure 2-7. Terminal Geometry

Next, the inertial orientation of this "undesired" velocity component can be established on the basis of secondary methods and finally it can be inserted into the vehicle's midcourse system. The net effect of these steps is to "velocity update" the vehicle's on-board inertial sensors with respect to the target line of sight. It should be evident that these inertial components can be subsequently used to steer the missile to this target line of sight subject only to inertial acceleration biases. The key to this approach is the extremely accurate scaler velocity measurements inherent to this sort of doppler measurement.

The foregoing was intended only to broaden the audience's insight into the general utility of the approach and in the process, to establish the close relationship between the radar and inertial measurements required for narrow band sensors.

Returning to the more conventional method of guidance, angle tracking, it is now time to consider some of its limits. Usually one of the system objectives is to use the narrowest tracking gate possible commensurate with the target of interest (usable tracking bandwidths may also be limited by implementation constraints). The target bandwidth is limited in its narrowness by both target spatial and motion considerations. The spatial aspects of the problem have been previously discussed; the motion considerations are caused by apparent acceleration or differential velocity along the radar-target line-of-sight. Thus, for example, if a target moves normal to the radar beam, a line-of-sight acceleration occurs and hence a time increasing (or decreasing) target doppler (or chirp). Another example is a target consisting of an ensemble of scatters which have rotational motion about some metacenter. Here the target has an instantaneous bandwidth defined by the relative velocity of the target's component scatterers relative to the radar. Incidentally the component scatterers relative velocity effect is also responsible for the instantaneous bandwidth of extended flexible scattering surfaces such as rain clutter as was shown in Figure 1-12.

To this point the doppler tracking filter has been treated as a perfectly matched filter to the target bandwidth. Previously the requirement for motion compensation and practical limits thereof and the spectrum widening of moving targets were discussed. The essence of this then is that the exact shape of the received waveform is not known perfectly, and even if it were, no assurance exists that a perfect matched filter could be constructed to handle the target spectrum of interest. Therefore the tracking filter will in practice necessarily have to be some compromise. If the filter is too narrow it will lose target signal, if too wide the increased bandwidth results in increased noise and the inclusion of unwanted background energy thus reducing both the S/N and S/C ratios. In addition to these effects some more subtle effects in angle tracking are caused due to our imperfect filters. For example, consider a filter whose width at its half power represents the radar's spatial resolution in azimuth. (Figure 2-8). A competing radar reflector having the same amplitude as the target and spaced 1/2 filter width away from the target (3 db width) will bias the monopulse angle sensing circuit 1/4 the spatial resolution away from the target. The effect is to bias the aimpoint of the angle tracking circuits thus it can be called target wander. If the target and/or near target points scintillate in amplitude the problem is further aggravated as more or less of the target and/or background energy fall within the doppler tracking loop creating noise. At least two possibilities to handle this aimpoint problem exist. The first is to continue to redesignate the target during the terminal trajectory. This is possible since the homing trajectory allows continuous mapping of the target area. Second, it is possible to smooth the aimpoint noise since the radar sensing rate is usually high compared to the missile's requirement for guidance data.

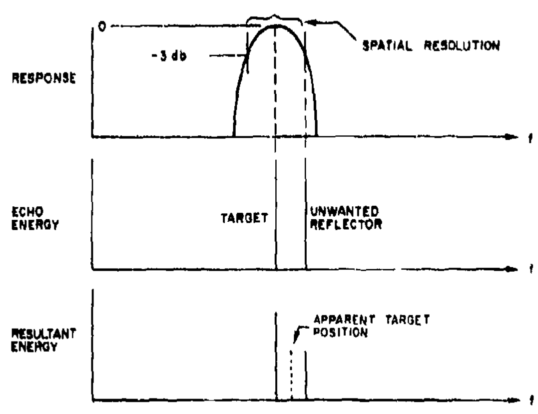

Figure 2-8. Aimpoint Biasing

Both target fade and radar glint or phase scintillation are caused by the coherent phase addition of radar returns from the radar reflecting surfaces in the target area of interest. Radar glint or scintillation can be thought of as a phase front distortion. Since our radar angle measurement using either a phase or amplitude type comparison system is in reality only measurement of the phase front, this results in angle noise; i.e., the random motion of the apparent position of the target about its physical center as seen by the radar. Target fade is an amplitude effect in which the amplitude of a complex target due to its multiplicity of reflecting surfaces varies as a function of the phase addition of complex returns.

The classic experiment to demonstrate the effects of angle noise uses two radar reflectors as shown on Figure 2-9. The phase front of the two reflector targets can be determined by finding the phase difference at the radar as a function of the angle ψ. The approach is to determine the difference in the round trip range, radar to reflectors to radar, and convert to phase.

$$\Delta R = 2 L \sin \psi$$

$$\Delta \phi = \frac{2\pi}{\lambda} \Delta R = \frac{4\pi}{\lambda} L \sin \psi$$

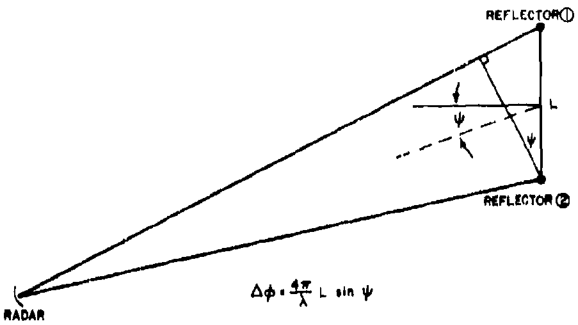

Figure 2-9. Two Reflector Target Glint

This phase difference represents a phase front distortion or an angle error to the angle tracking circuits. Highly complex targets such as aircraft have been modeled by Dean Howard (Ref. 1) as producing a Gaussian distributed angle glint of zero mean where the $3\pm$ sigma limit is the length of the target; i.e., $\sigma = L/6$. Experiments have shown that the standard deviation of angle glint ranges from 0.1 to 0.3 of the target length. These experiments imply that by taking several independent samples of the distorted phase front one can estimate the true target position. One problem then is to provide some change in the geometry of the situation such that several independent samples can be measured. This can be done by changing the transmitted frequency or wavelength, λ, as a function of time. A radar that does this is called a frequency agile radar. From the equation $\Delta\phi = (4\pi/\lambda) L \sin\psi$ note that this technique has maximum efficacy when the reflectors are separated in range, i.e., $\psi = 90^o$, and is insensitive to separations in azimuth. Another method of doing this is by imposing a velocity to the radar as shown in Figure 2-10. The velocity (if the velocity is not directed towards the middle of the apparent separation of the reflectors) changes the ψ angle as a function of time thus $\dot{\Delta\phi} = (4\pi/\lambda) L \cos\psi\dot{\psi}$ where $\dot{\psi}$ is the turning rate about the target. This can be rewritten using the missile velocity and range to the target as

$$\dot{\Delta\phi} = \frac{4\pi}{\lambda} L \cos\psi \frac{V \sin\theta}{R}$$

Recalling the trajectory requirements for constant target contrast (resolution) it was necessary that $V \sin\theta/R$, the LOS rate, ω, be a constant. This trajectory which is necessary for target contrast, speeds the glint rate much in the same manner as frequency agility. Figure 2-11 shows the power spectral density of glint for both a non-turning trajectory and one with a constant turning rate of 0.03 radians/second. These runs were made against the same target thus the total energy of the two spectras are the same.

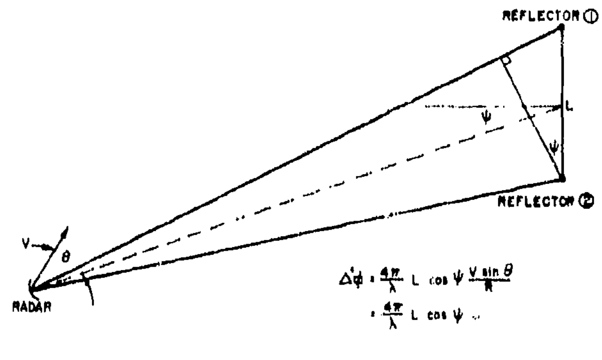

Figure 2-10. Two Reflector Target Glint Pulse-Doppler

Finally this leads to the possibility of guiding the missile through use of a pulse-doppler updated inertial implementation. Such an implementation attempts to capitalize on the long term accuracy of the radar and the short term stability of inertial devices.

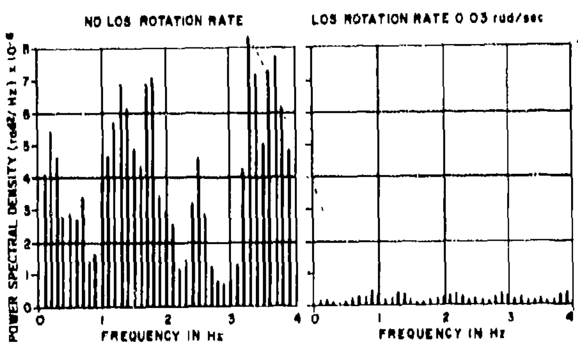

Figure 2-11. Glint Reduction

The inertial system's accelerometer outputs will be utilized to smooth the radar tracking noise. Figure 2-12 is a simplified diagram of such a radar-inertial tracking system.

The tracking radar essentially keeps the target on the boresighted antenna axis while measuring the antenna gimbal rates necessary to do so. In addition, range and range rate to the target are measured. The radar's angle tracking loop as has been discussed suffers from noise due to target scintillation and other effects. We define an antenna coordinate system, A, with the x axis along the antenna axis with y and z axis to be the orthogonal gimbal axes and let ψ and θ be the rotation angles of these coordinates relative to the body frame, B, about the z and y axes, respectively. The radar measurements are $\psi, \dot{\psi}, \theta, \dot{\theta}, |R|, |\dot{R}|$. The velocity of the missile with respect to the target in antenna coordinates is given by:

$$V_x^A = \dot{R}$$

$$V_Y^A = R\dot{\theta}$$

$$V_z^A = R\dot{\psi}$$

and the coordinate transformation T_A^B, between antenna coordinates and body coordinates is given by the angles ψ and θ.

Ref. 1 Merril I Skolnik, "Radar Handbook", Chapter 28 McGraw-Hill Book Co., 1970

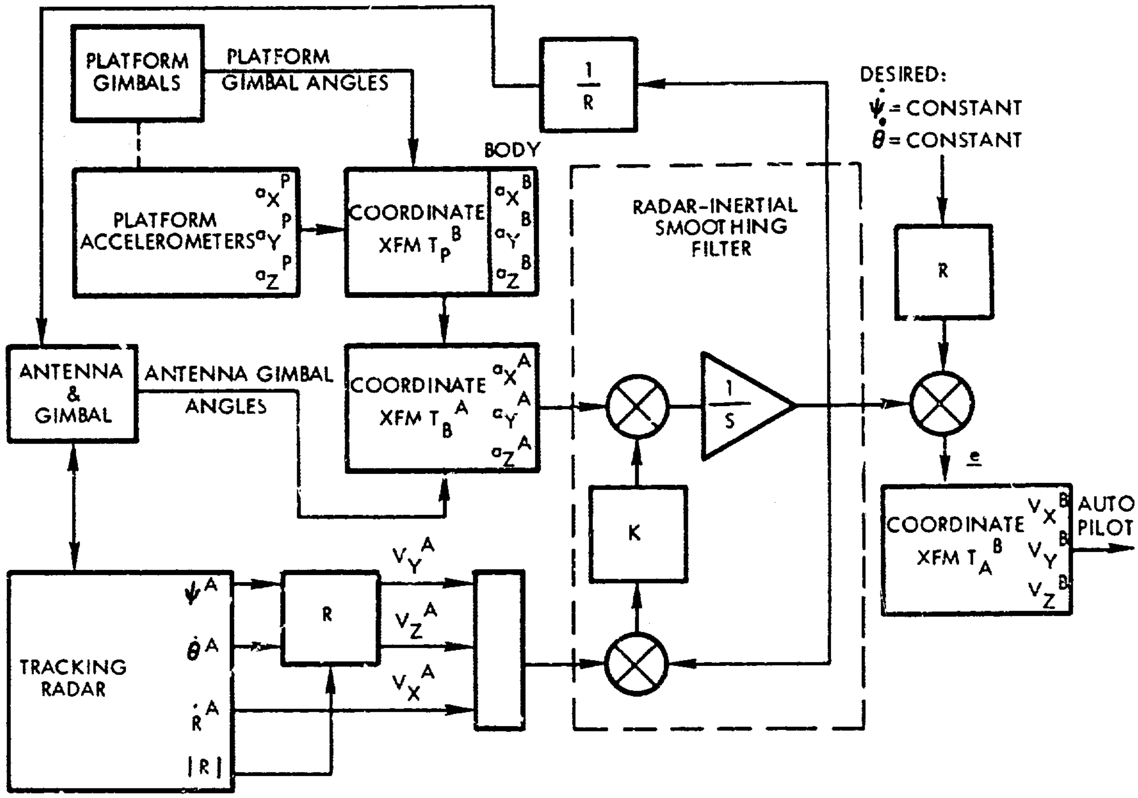

Figure 2-12. Radar-Inertial Tracking

The accelerometer measurements are made in platform coordinates which are related to the body coordinates by the measured platform gimbal angles.

To understand the radar inertial smoothing filter operation, consider the channel shown in Figure 2-13.

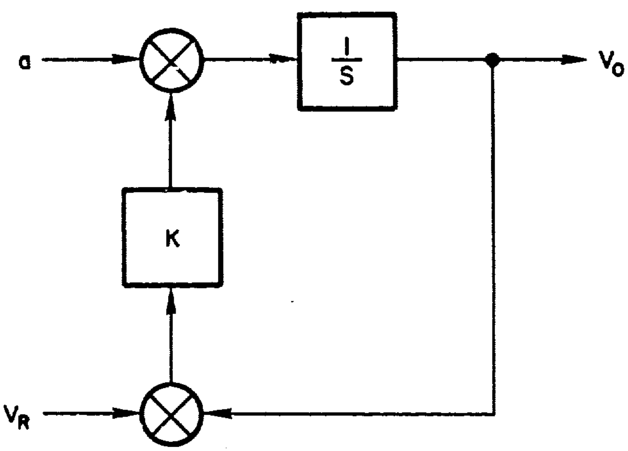

Figure 2-13. Signal Channel

The acceleration, a, is measured by the inertial system and resolved along one of the antenna frame axes, V_R is the radar velocity measurement along that axis, and V_o is the filtered velocity indication along that axis. V_o is given by the expression:

$$V_o = V_R \left(\frac{1}{\left(\frac{1}{K} S + 1\right)}\right) + a \left(\frac{1}{K\left(\frac{1}{K} S + 1\right)}\right)$$

Since a is the time rate of change of velocity V_A as measured by the inertial system accelerometers; we can write a = SV_A. And, the expression for V_o can be rewritten as

$$V_o = V_R \left(\frac{1}{\frac{1}{K} S + 1}\right) + V_A \left(\frac{\frac{1}{K} S}{\frac{1}{K} S + 1}\right)$$

The velocity as measured by the radar can be written: $V_R = V_m + V_T + N$

Where: V_m is the actual missile velocity

V_T is the target velocity

N is the noise of the radar system

Also we can write for the inertial velocity measurement:

$$V_A = V_m + \delta V$$

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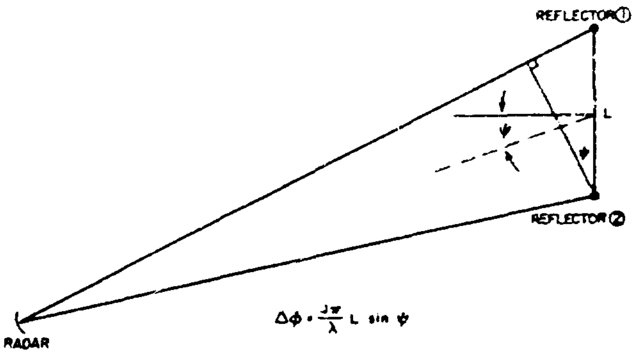

Figure 2-9. Two Reflector Target Glint

This phase difference represents a phase front distortion or an angle error to the angle tracking circuits. Highly complex targets such as aircraft have been modeled by Dean Howard (Ref. 1) as producing a Gaussian distributed angle glint of zero mean where the $3\pm$ sigma limit is the length of the target; i.e., $\sigma = L/6$. Experiments have shown that the standard deviation of angle glint ranges from 0.1 to 0.3 of the target length. These experiments imply that by taking several independent samples of the distorted phase front one can estimate the true target position. One problem then is to provide some change in the geometry of the situation such that several independent samples can be measured. This can be done by changing the transmitted frequency or wavelength, λ, as a function of time. A radar that does this is called a frequency agile radar. From the equation $\Delta\phi = (4\pi/\lambda)\ L \sin\psi$ note that this technique has maximum efficacy when the reflectors are separated in range, i.e., $\psi = 90°$, and is insensitive to separations in azimuth. Another method of doing this is by imposing a velocity to the radar as shown in Figure 2-10. The velocity (if the velocity is not directed towards the middle of the apparent separation of the reflectors) changes the ψ angle as a function of time thus $\dot{\Delta\phi} = (4\pi/\lambda)\ L \cos\psi\dot{\psi}$ where $\dot{\psi}$ is the turning rate about the target. This can be rewritten using the missile velocity and range to the target as

$$\dot{\Delta\phi} = \frac{4\pi}{\lambda}\ L \cos\psi\ \frac{V \sin\theta}{R}$$

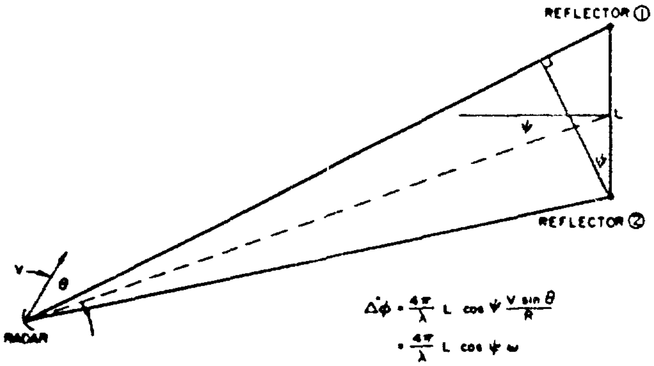

Figure 2-10. Two Reflector Target Glint Pulse-Doppler

Recalling the trajectory requirements for constant target contrast (resolution) it was necessary that $V \sin\theta/R$, the LOS rate, ω, be a constant. This trajectory which is necessary for target contrast, speeds the glint rate much in the same manner as frequency agility. Figure 2-11 shows the power spectral density of glint for both a non-turning trajectory and one with a constant turning rate of 0.03 radians/second. These runs were made against the same target thus the total energy of the two spectras are the same.

Finally this leads to the possibility of guiding the missile through use of a pulse-doppler updated inertial implementation. Such an implementation attempts to capitalize on the long term accuracy of the radar and the short term stability of inertial devices.

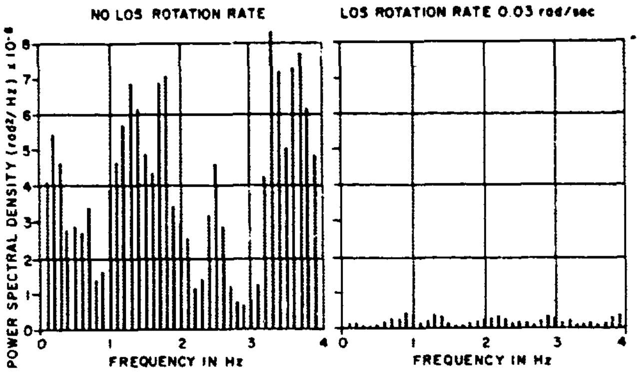

Figure 2-11. Glint Reduction

The inertial system's accelerometer outputs will be utilized to smooth the radar tracking noise. Figure 2-12 is a simplified diagram of such a radar-inertial tracking system.

The tracking radar essentially keeps the target on the boresighted antenna axis while measuring the antenna gimbal rates necessary to do so. In addition, range and range rate to the target are measured. The radar's angle tracking loop as has been discussed suffers from noise due to target scintillation and other effects. We define an antenna coordinate system, A, with the x axis along the antenna axis with y and z axis to be the orthogonal gimbal axes and let ψ and θ be the rotation angles of these coordinates relative to the body frame, B, about the z and y axes, respectively. The radar measurements are $\psi, \dot{\psi}, \theta, \dot{\theta}, |R|, |\dot{R}|$. The velocity of the missile with respect to the target in antenna coordinates is given by:

$$V_x^A = \dot{R}$$

$$V_Y^A = R\dot{\theta}$$

$$V_z^A = R\dot{\psi}$$

and the coordinate transformation T_A^B, between antenna coordinates and body coordinates is given by the angles ψ and θ.

Ref. 1 Merril I Skolnik, "Radar Handbook", Chapter 28 McGraw-Hill Book Co., 1970

Where δV is the inertial velocity error. Combining these two expressions with the expression for V_o yields:

$$V_o = V_m \left(\frac{1}{\frac{1}{K} S + 1} + \frac{\frac{1}{K} S}{\frac{1}{K} S + 1} \right) + \left(V_T + N \right) \left(\frac{1}{\frac{1}{K} S + 1} \right) + \delta V \left(\frac{\frac{1}{K} S}{\frac{1}{K} S + 1} \right)$$

which reduces to

$$V_o = V_m + \left(V_T + N \right) \left(\frac{1}{\frac{1}{K} S + 1} \right) + \delta V \left(\frac{\frac{1}{K} S}{\frac{1}{K} S + 1} \right)$$

From this expression it can be seen that the missile's inertial velocity which is measured by both the radar system and the inertial system is passed directly through the filter. The radar system noise and the radar target motion measurement are passed through a low-pass filter. The inertial measurement noise is passed through a lead lag or high-pass filter.

In this way the long term or low frequency stability of the radar system and the short term or high frequency stability of the inertial system are taken advantage of while the low frequency drift errors of the platform and the high frequency noise of the radar are suppressed.

The constant K is determined by the drift characteristics of the platform and the radar tracker noise characteristic.

If an accurate model of the radar sensor noise and the platform driving noise characteristics are known, V_o could be an optimum estimate of the true velocity generated by a Kalman filter. In this case K would be time varying and would be generated by the Kalman filter based on conditions existing at a particular time.

In conclusion then, we have noted that the attack of small targets in complex backgrounds requires rather massive and time consuming sampling of the target-dynamic radar signature. The sampling intervals associated with the problem can be significantly longer than that desirable for flight control; hence, one needs to marry the radar sensor and the inertial sensors to derive an optimum hybrid system, rather than a separate determination of each. In short for optimum design one must use vehicle motion, phase sensing by the radar sensor, and the short term motion sensing by the inertial sensors analogous to the considerations usually reserved for optimum filtering techniques of other systems.