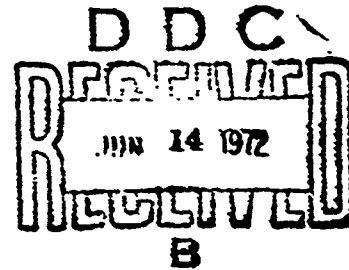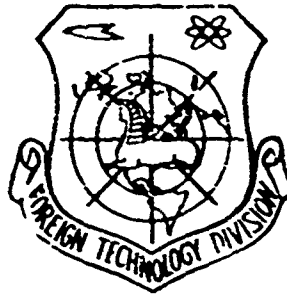FTD-HC-23-1839-71

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VORTEX FORMATION IN CHANNEL BETWEEN BLOCKS
OF FRICTION THRUST BEARINGS

by

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13. ABSTRACT

It is shown that the hydrodynamic phenomena occurring in the
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approximately analyzed, providing a basis for clarification
of the heat calculations and conditions providing for absence
of vacuum zones. The problem of vortex formation in the
channel is analyzed considering the influence of lubrication
carried from beneath one block to beneath the other. Convo-
lutions of the flow with turbulent motion are considered.
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VORTEX FORMATION IN CHANNEL BETWEEN BLOCKS OF FRICTION THRUST BEARINGS

The load capacity of friction thrust bearings is not only determined by phenomena that occur in the lubricant film, but also depends largely on what happens in the bearing as a whole. This is explained basically by three circumstances. First, heat transfer through the blocks and collar and thermal processes in the film that is transported through the channel between the blocks from beneath one block to the other have a considerable influence on the temperature layer of the film. Since the viscosity depends on temperature, the noted phenomena have a direct effect on the magnitude of the hydrodynamic reaction of the lubricant wedge. Second, the transfer of heat to the oil flowing in the body of the bearing is accompanied by uneven heating of the blocks, which causes warping, distortion of the geometry of the lubricating film and in great measure leads to reduction of its load capacity. Finally, vacuum zones may form within the bearing under known conditions, which, as shown experimentally [6], has a negative effect on working capacity. Analysis of the block channel is obviously very important in investigation of the hydrodynamics and heat exchange in the body of a bearing, since oil flows directly from it into the oil wedge. Experimental studies [2, 6] and also tests on determination of the optimal filling coefficient [7] show that processes in the block channel actually have a considerable influence on bearing operation and, in particular, on its thermal regime [2]. Here, so far as can be judged according to [6], the main feature of oil motion in the channel is the formation of longitudinal vortexes. The latter are created as a result of rotation of the volumes of oil between the blocks under the influence of forces of friction from the bearing collar.

An attempt is made below to carry out an approximate analysis of the hydrodynamic phenomena that occur in the block channel, which, in turn, can serve as the basis for refining thermal calculations and explaining conditions that en are the absence of vacuum zones.

The problem is laid out schematically as follows. The channel is r-placed by a recess of rectangular cross section (Figure 1), the open siue of which is bounded by a moving plate. The distance between the plate and the walls of the groove is h and the depth of the groove is b. It is also assumed that there is no motion of the lubricant along axis z. Analysis of twisting of the flow with consideration of longitudinal flow along the channel is described in the second part of this work.
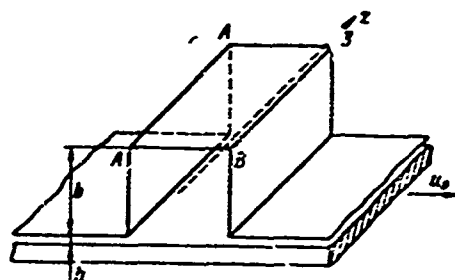

Figure 1.

The plane problem of the motion in the slot was solved [4, 11] for h = 0 and [5] for h $\neq$ 0. In these works the complete system of Navier-Stokes equations is examined and they are numerically integrated with the aid of computers. Due to the limited memory volume of the computer, however, the solution was found only for a comparatively small number of Reynolds numbers Re = Ub/ν. Meanwhile the problem can be simplified for high Reynolds numbers (for the block channel Re = 10^4-10^5) by dividing the flow into two zones: 1) boundary layer and 2) nonviscous core, in which the forces of friction can be ignored.

In this statement for the case h = 0, as it applies to the diagram in Figure 2 (wall 1 is fixed and wall 2 rotates around axis O), the problem was examined in [10, 12-14]. Here, in view of the symmetry of flow it was assumed that the pressure gradient along the boundary is equal to zero and the velocity on the periphery U^0 = const.
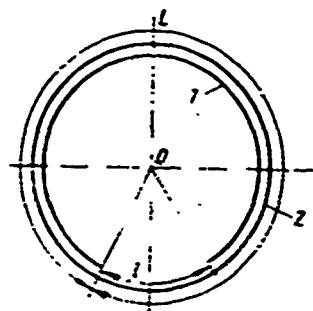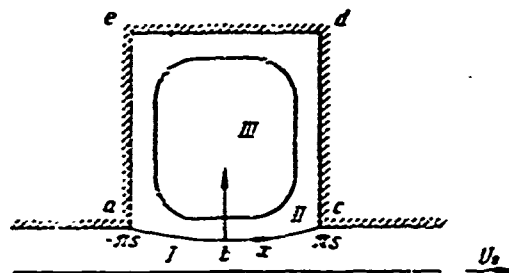

Figure 2.

Figure 3.

On the basis of analysis of the equations of laminar asymptotic boundary layer, written in the Prandtl-Mises form, the authors of [10, 12, 11] showed that the mean square velocity in the boundary layer along each line is the same and equal to

$$u_m^2 = \frac{1}{L} \int_0^L u^2 dx = U^{02} = \text{const.} \qquad (1)$$

Hence for velocity U^0 is found the expression

$$U^* = U_0 \sqrt{s}, \quad s = l/L, \qquad (2)$$

where U_0 is the velocity of wall 2. It was also shown [10] that in the general case of plane flow with closed streamlines the vortex rot v in the nonviscous core has a constant value.

The application of this approach to cavities of rectangular form poses known difficulties in connection with the fact that the pressure through the length of the boundary layer is constant and, moreover, vortex formation is possible in angles A and B of the grooves. Angular vortexes were actually discovered both as a result of numerical solution of the complete Navier-Stokes equation system [4, 5, 11] and experimentally [12]. Meanwhile experimental investigation, carried out for a groove of square cross section for Re = 10^5 (the pressure here was laminar) revealed that the pressure gradient along the perimeter of the groove is small and the velocity along the periphery of the core is approximately constant and equal to $U^0 = 0.5U_0$ [12]. This result can be explained by the fact that the formation of angular vortexes leads to separation of the streamlines from the walls, with the result that their form approaches circular and the flow in the nonviscous core becomes close to the flow according to the diagram in Figure 2. As regards the numerical value of velocity along the periphery of the vortex $U^0 = 0.5U_0$, it can be found from (1) for L = 4l,

i.e., schematic representation of the flow in the form of a flow core and unbroken laminar boundary layer yields suitable results for grooves of square cross section. Apparently the pressure gradient along the stream-lines can be regarded equal to zero only for square grooves. In grooves of rectangular cross section the difference of the streamlines in the core from circles will be more pronounced, leading to variability of velocity and pressure along the periphery of the core. Meanwhile in the practical sense the first case is of greatest importance, since the form of cross section of the block channel in most bearings is close to square. From now on, therefore, the pressure gradient will be assumed equal to zero.

Examined below, in the specified statement, is the problem of vortex formation in the channel with consideration of the effect of the lubricant, transferred from beneath one block to beneath the other (h ≠ 0). At the end of the article are presented certain considerations of twisting of the flow in the turbulent flow regime.

We will examine the motion of the lubricant according to the diagram in Figure 3.

We will separate the region filled with oil into three zones: nonviscous core III, boundary layer II, oil stream I, forced from beneath the preceding block. We will assume that there is a separating streamline, which separates zones I and II. From the physical standpoint this assumption is quite natural. It is also substantiated by the results of numerical solution [5]. Since the thickness of the layer in zone I is small, the motion of the lubricant in region I, as in region II, can be described by the boundary layer equations. The latter, under the conditions of the examined problem, are conveniently taken in the Prandtl-Mises form [3]. Then, assuming $\partial p/\partial x = 0$, we obtain

$$\frac{1}{\nu}\frac{\partial u_*^2}{\partial x_*} = u_* \frac{\partial^2 u_*^2}{\partial \psi_*^2}.$$

Here x_* is a coordinate on the streamline, ψ_* is the stream function ($u_* = \partial\psi_*/\partial y_*$, $v_* = -\partial\psi_*/\partial x_*$), u_*, v_* are the longitudinal and lateral velocity components, respectively, y_* is the lateral coordinate.

The order of the solution of the problem is as follows. We are given an expression for the velocity on segment abc, separating the streamlines. Let this expression contain n unknown parameters. Then, after determining the velocities in zones I and II, we calculate the force of friction on line abc approaching it from zones I and II, respectively. Having now broken line abc down into n segments and requiring that the forces of friction on each of them, determined by the above-stated method, are equal to each other, we find n equations for determination of n unknown parameters. Further, we restrict ourselves to the simplest case of the uniparametric problem. Then all that remains is to calculate the summary force of friction on the streamline abc.

To find the approximate solution of the problem, following [12, 14], we will linearize equation (1), averaging the coefficient in front of $\partial^2 u_*^2/\partial\psi_*^2$.

In addition we will proceed to the dimensionless variables and introduce the Reynolds number Re, assuming (L is the dimensional length of contour abcde)

$$x = \frac{2\pi x_*}{L}, \quad y = \frac{2\pi y_*}{L}, \quad u = \frac{u_*}{U_*}, \quad v = \frac{v_*}{U_*}, \quad \mathrm{Re} = \frac{U_*L}{2\pi\nu}. \tag{3}$$

Then we obtain approximately for zones I and II

$$\frac{\partial u^2}{\partial x} = \varkappa_1^2 \frac{\partial^2 u^2}{\partial \psi^2}, \quad \frac{\partial u^2}{\partial x} = \varkappa_2^2 \frac{\partial^2 u^2}{\partial \psi^2}, \quad \psi = \frac{\psi_*}{U_*L}, \quad \varkappa_1^2 = \frac{u_{1m}}{4\pi^2 R}, \quad \varkappa_2^2 = \frac{u_{2m}}{4\pi^2 \mathrm{Re}}, \tag{4}$$

4

where u_{1m} and u_{2m} are the average values of the dimensionless velocity in zones I and II.

Satisfying the condition of periodicity in terms of x in zone II, we seek u^2 in the form of the real part of the series

$$u^2 = \text{вещ. ч.} \sum_{-\infty}^{\infty} a_k(\psi)\exp(ikx).$$

Substituting this into the second of equations (4), we obtain for $a_k'(\psi)$ the equation system

$$\frac{c\,a_k}{c\psi^2} - \frac{ik}{\varkappa_2^2}a_k = 0.$$

Viewing the boundary layer as asymptotic [10, 12-14], we find that the velocity and therefore functions $a_k(\psi)$ should be bounded at $\psi \to \infty$. Solving the equations for $a_k(\psi)$ with consideration of this circumstance, we find

$$u^2 = a_{00} + 2\,\text{вещ. ч.}\sum_{k=}^{\infty} a_k \exp[-\sqrt{k}(1+i)\eta_2 + ikx], \quad \eta_2 = \frac{\psi}{\varkappa_2\sqrt{2}} \tag{5}$$

The coefficients a_{k0} and $a_{(i)}$ are determined as a function of the form of the function $u(x)$ on the separating streamline abc. We will consider that $\psi = 0$ on line abc and assume

$$u^2 = \Phi\cos\frac{x}{2s} \quad (\psi = 0, \quad -\pi s < x < \pi s). \tag{6}$$

Then, assuming that velocity $u = 0$ on idea and expanding $u^2(x, \psi = 0)$ into a Fourier series on the segment $-\pi < x < \pi$, we obtain

$$a_{00} = s\Phi_\pi; \quad a_k = -\frac{\Phi_\pi}{4s} \frac{\cos k\pi s}{1 - (k \cdot 2s)^2}, \quad \Phi_\pi = \frac{2}{\pi}\Phi. \tag{7}$$

The summary force of friction on the separating streamline will be (with an accuracy to a constant cofactor)

$$T = -\frac{1}{2}\int_{-\pi s}^{\pi s}\frac{\partial u^2}{\partial \psi}\Big|_{\psi=0}dx. \tag{8}$$

Substituting here u^2 from (5) and a_{k0} from (7), we find

$$T = -\frac{\Phi_m}{x_2} f(s), \quad f(s) = \frac{s}{\sqrt{2}} \sum_{k=1}^{\infty} \frac{\sin 2\pi ks}{\sqrt{k}(1 - 4k^2 s^2)}$$

$$f(0,25) = \frac{1}{\sqrt{2}} \left\{ \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{2n+1}[4-(2n+1)^2]} + \frac{\pi}{8\sqrt{2}} \right\} \approx 0,5. \tag{9}$$

We will now examine zone I (the first of equations (4)). Equation (4) must be solved for the conditions

$$u^2 = \frac{\pi}{2} \Phi_m \cos\frac{x}{2s} \quad \text{for} \quad \psi = 0, \; u^2 = 1 \quad \text{for} \quad \psi = -c. \tag{10}$$

Here (-c) is the stream function on the moving plate. If we assume that the oil flow rate through unit width of zone I is $0.5U_0 h$, then (see (1)) $c = 0.5h/L$.

We will seek the solution in the form

$$u^2 = -\frac{\psi}{c} + a_1(\psi)\exp\left(\frac{xi}{2s}\right) + a_{-1}(\psi)\exp\left(-\frac{xi}{2s}\right), \tag{11}$$

where in view of (10) the functions $\alpha_1(\psi)$ and $\alpha_{-1}(\psi)$ should satisfy the conditions

$$a_1(0) = a_{-1}(0) = \frac{\pi}{4}\Phi_m, \quad a_1(-\delta_1) = a_1(-\delta_1) = 0, \quad \delta_1 = \frac{c}{2x_1\sqrt{s}}. \tag{12}$$

Substituting (11) into (4) we find for α_1 and α_{-1} differential equations of the second order. Solving these equations for conditions (12) and then substituting α_1 and α_{-1} into (11) we find

$$u^2 = -\frac{\psi}{c} + \frac{\pi}{2}\Phi_m \text{ вещ. ч. } \left\{ \frac{\text{sh } [(\delta_1 - \eta_1)(1 - i)]}{\text{sh } [\delta(1 - i)]} e^{\frac{xi}{2s}} \right\}, \quad \eta_1 = \frac{\psi}{2x_1\sqrt{s}}. \tag{13}$$

Hence for the force of friction on line abc on approach from zone I we obtain with the aid of (8)

$$T = -\frac{\pi s}{c}[1 - \delta_1\chi(\delta_1)\Phi_m],$$

$$\chi(\delta_1) = \frac{(\text{sh } 2\delta_1 + \sin 2\delta_1)(\text{ch } 2\delta_1 + \cos 2\delta_1)}{\text{sh}^2 2\delta_1 + \sin^2 2\delta_1}\Phi_m.$$

6

Further we will examine the case of small δ_1. Calculations show that when $\delta_1 < 1$ the function $\chi(\delta_1) \approx \delta_1^{-1}$, so that

$$T \approx \frac{\pi s}{c}(1 - \Phi_m). \tag{14}$$

Equating (9) and (14) we obtain the equation

$$1 - \Phi_n = \varphi \Phi_m, \quad \varphi = \frac{c f(s)}{\pi s \chi_2} = \frac{f(s)}{s}\frac{h}{L}\sqrt{\frac{R}{u_{2m}}}. \tag{15}$$

The value u_{2m}^2, on the basis of (1), is assumed to be equal to u_2^2 on the outer boundary of the boundary layer in zone II [12, 14]. We obtain (see (5) and (7))

$$u_{2m}^2 = s\Phi_m. \tag{16}$$

Assuming now (for a square groove) $s = 0.25$, we find from (15) and (16)

$$1 - \Phi_m = \varphi^\circ \Phi_m^{3/4}, \quad \varphi^\circ = 2.83\frac{h\sqrt{R}}{L}. \tag{17}$$

We will assume $h = 20$ mkm $= 20 \cdot 10^{-6}$ m, $U_0 = 50$ m/sec, $\nu = 0.2 \cdot 10^{-4}$ m²/sec, $L = 8 \cdot 10^{-2}$ m (to this value of L corresponds a block channel width of approximately 2 cm). Then $R = 3.2 \cdot 10^{-4}$, $\phi^0 = 0.13$ and equation (15) yields $\phi_m \approx 0.9$[1]. The dimensionless velocity on the periphery of the vortex, in view of (15) is $U^0 = \sqrt{a_{00}} = \sqrt{s\phi_m}$ or for $s = 0.25$ $U^0 = 0.5\sqrt{\phi_m}$. It follows from the above calculations that the lubricating film decreases the velocity on the periphery of the vortex, but this reduction is small when the thickness of the film is small (in the examined case we have instead of $U^0 = 0.5$ $U^0 = 0.5\sqrt{0.9} = 0.47$). This finding means that the flow is close to that obtained if the layer is absent altogether, i.e., if $h = 0$. For $h = 0$, however, the velocity on the line $\psi = 0 - \pi s < x < \pi s$ is constant. Therefore in the examined conditions for velocity on

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[1]Using these results for estimating the value δ_1 from (12) and assuming $u_{m_1} = \sqrt{0.5(1 + \phi_m)} \approx 1$, we obtain

$$\delta_1 = \frac{h}{2L} 2\pi \sqrt{R} \approx 0.14.$$

i.e., $\delta_1 < 1$.

the separating streamline it is advisable to assume $u^2 = \Phi_m = \text{const}$ instead of dependence (6). Finding for this case the solution analogous to the preceding, we again obtain for Φ_m equation (15), but now f(s) is determined according to the equation

$$f(s) = \frac{2}{\pi\sqrt{2}} \sum_{k=1}^{\infty} \frac{\sin^2 k\pi s}{k\sqrt{k}},$$

$$f(0,25) = \frac{1}{\pi\sqrt{2}}\left(1 + \frac{1}{\sqrt{2}}\right)\left(1 - \frac{1}{2\sqrt{2}}\right)\zeta\left(\frac{3}{2}\right) - 0,649,$$

where $\zeta(x) - \zeta$ is the Riemann function, $\zeta(\frac{3}{2}) = 2.61$ [9].

For the above-examined case we have $\phi^0 = 0.17$, $\Phi_m = 0.85$, $u^0 = 0.5\sqrt{0.85} = 0.46$, i.e. u^0 close to the value obtained above.

We will now examine the problem of vortex formation in a groove in the assumption that the motion is turbulent. Here we will assume h = 0. In this case the region occupied by oil is broken down into two parts: the flow core and turbulent boundary layer.

The boundary layer equations will be

$$u_* \frac{\partial u_*}{\partial x_*} + v_* \frac{\partial u_*}{\partial y_*} = \frac{1}{\rho}\frac{\partial \tau_*}{\partial y_*}, \qquad \frac{\partial u_*}{\partial x_*} + \frac{\partial v_*}{\partial y_*} = 0. \tag{18}$$

The force of friction τ_* is given by the dependences [1] .

$$\tau = \mu \frac{\partial u_*}{\partial y_*} \quad \text{for} \quad 0 \leqslant y_* \leqslant \delta_0, \quad \tau = \rho k^2 y_*^2 \left(\frac{\partial u_*}{\partial y_*}\right)^2 \quad \text{for} \quad y_* \geqslant \delta_0, \tag{19}$$

where

$$\frac{\partial u_*}{\partial y_*}\bigg|_{y_*=\delta_1-0} = k_1 \frac{\partial u_*}{\partial x_*}\bigg|_{y_*=\delta_1+0}.$$

Here δ_0 is the thickness of the laminar film, ρ is the density of the lubricant, k and k_1 are constants.

Writing (18) in Prandtl-Mises variables, we obtain

$$\rho \frac{\partial u_*}{\partial x_*} = \frac{\partial \tau_*}{\partial \psi_*}. \tag{20}$$

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Integrating (20) in terms of x_* within the limits from zero to L and considering that on the outer boundary of the boundary layer friction is equal to zero, we will have

$$\int_0^L \tau_* \big|_{v_*=0} \, dx_* = 0.$$

(21)

Equation (21) represents the obvious condition of equilibrium of the boundary layer.

Through τ_{*1} and τ_{*2} we will denote the average forces of friction on the wall on two segments, respectively: $x_* \in (0, l)$ and $x_* \in (l, L-l)$. Then we have from (20) [see also (2)]

$$\tau_{*1} s = \tau_{*2} (1-s).$$

(22)

· If, following Prandtl's example [8], we assume that the force of friction is constant on a line perpendicular to the wall, then, by integrating equations (19) for the segments $x_* \in (0, l)$ and $x_* \in (l, L-l)$, we obtain

$$\tau_{*1} = \frac{\rho U_*^2 (1 - u^\circ)^2}{2} c_1, \qquad \tau_{*2} = \frac{\rho U_*^2 u^{\circ 2}}{2} c_2,$$

(23)

where c_1 and c_2 are the coefficients of resistance, determined by the formulas (δ_1 and δ_2 are the boundary layer thicknesses)

$$c_i = \left(\frac{1}{k} \ln \frac{\delta_i}{\delta_{1i}} + \frac{k_1}{k} \right)^{-2}, \qquad \delta_i = \frac{k_1}{k} \frac{\nu \sqrt{\rho}}{\sqrt{\tau_{*i}}} \qquad (i = 1, 2).$$

Formulas (23) show that the resistance of friction can be determined according to the formulas for tubes if the difference of velocities on the outer boundary of the boundary layer and on the wall is regarded as the characteristic velocity. Using on this basis the interpolation formulas for the coefficients of resistance [1], we obtain

$$c_1 = 0.225 \left[\frac{U_*(1 - u^\circ)\delta_1}{\nu} \right]^{-0.25}, \qquad c_2 = 0.0225 \left[\frac{U_* u^\circ \delta_2}{\nu} \right]^{-0.25}.$$

(24)

We will now consider that on each of the segments the thicknesses of the boundary layers are approximately constant. Then, from (22), (23) and (24) we obtain

$$\left(\frac{1 - u^\circ}{u^\circ} \frac{\delta_1}{\delta_2} \right)^{1/4} \cdot \frac{u^\circ}{1 - u^\circ} = \sqrt{\frac{s}{1-s}}.$$

(25)

The ratio of thicknesses δ_1/δ_2 can be found by assuming that the oil flow rates in films 1 and 2 are equal to each other. Then, assuming for velocity the "one-seventh" wall [1], we have $\delta_1/\delta_2 = 7u^0/(1 + 7u^0)$ and formula (24) yields[1]
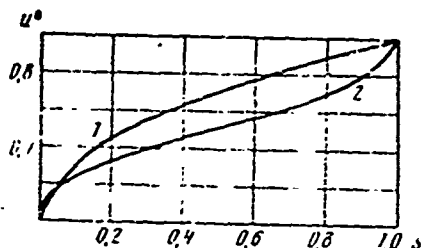

Figure 4.

$$s = \frac{\varphi^2(u^0)}{1 + \varphi^2(u^0)}$$

$$\varphi(u^0) = \left[\frac{7(1 - u^0)}{1 + 7u^0}\right]^{\frac{1}{4}} \frac{u^0}{1 - u^0} \qquad (26)$$

The graph of the function $u^0 = u^0(s)$, plotted according to formula (26), is illustrated in Figure 4 (curve 2). Also shown here is curve 1, plotted according to equation (2), obtained for the case of a laminar boundary layer. Comparison of curves 1 and 2 shows that the intensity of twisting of the flow core in the turbulent regime is less as a whole than in the laminar regime. For $s = 0.25$, in particular, $u^0 = 0.36$.

The twisting of the flow can be judged indirectly on the basis of the pressure drop, which forms through the cross section of the channel as a result of centrifugal forces. It can be determined according to the formula $\Delta p = 0.5\rho v_0^2$, where v_0 is the velocity on the periphery of the twisted core.

The values $(0.2-0.4)U_0$ are given in [6] as the approximate values for v_0. The latter in the laminar regime of motion are less than those that follow from the equations derived above (for channels of square form $v_0 \approx 0.5U_0$ in laminar flow and $v_0 \approx 0.36U_0$ in turbulent flow), which is explained, in particular, by the fact that the twisting of the oil entering the channel develops gradually. Therefore the value $\Delta p = 0.5\rho(u^0 U_0)^2$ should be viewed as the limiting value, established during complete twisting of the flow. Analysis of this problem will be given in the second part of this work.

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