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A PROBLEM IN OPTIMAL REPLACEMENT

by

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The authors formulate the optimum replacement problem as a concave integer programming problem, and develop an algorithm to solve the formulated problem.

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Formulation of the Problem

We shall consider a system consisting of n elements. Assume that the probability of failure in a definite time period is known for each element and equals q_i . In addition to the basic element, there are z_i spare elements (z_i is the number of replacements). It is assumed that the spare elements function in parallel with the basic element, or that they can be instantaneously connected with the system when the main element fails.

A particular element of the system is considered to have failed during the one-time failure of the basic element and all spare elements. Then the probability of failure during a definite time period equals, for the i th element, $q_i^{1+z_i}$ where the number of spares is z_i .

Suppose that the failure of any of the n elements leads to the failure of the complete system. Then the probability that the system will not fail during a definite time period equals

$$\phi = \prod_{i=1}^n (1 - q_i^{x_i}), \quad (1)$$

where $x_i = 1 + z_i$.

The quantity ϕ can be considered as a quantitative indicator of the degree of reliability of the system. From here on, for brevity, we shall call this quantity the reliability of the system.

Every i th element of the system is characterized by definite dimensions: weight, cost, etc., which are expressed in terms of the coefficient a_{ij} (j th characteristic of the i th element). Assume that admissible dimensions, weight

and other similar characteristics of the system b_j , are given. Then the optimal replacement problem can be formulated as follows. To determine

$$\max \Phi = \prod_{i=1}^n (1 - q_i^{x_i}) \quad (2)$$

with the conditions

$$\sum_{i=1}^n a_{ij} x_i \leq b_j, \quad (j = 1, 2, \dots, m); \quad (3)$$

where $x_i \geq 1$ are integers. (4)

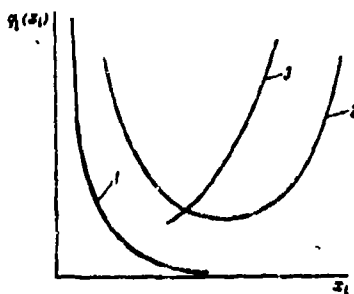
In the context of the problem, x_i must be integers, since these represent the number of elements. Here the optimality criterion Φ is a concave function of the variables x_i . Thus the optimal replacement program is formulated as a concave integer programming problem. At the present time, direct methods for solving problems of this class do not exist.

Problem (2) through (4) can be replaced by the problem of finding

$$\min (-\ln \Phi) = \min \sum_{i=1}^n -\ln(1 - q_i^{x_i}) = \min \sum_{i=1}^n \phi_i(x_i)$$

subject to restrictions (3) and (4).

Here $\phi_i(x_i) = -\ln(1 - q_i^{x_i})$ is a convex function (see the figure, curve 1). The objective function for this problem has the property that it is a sum of convex functions, each of which depends only on a single variable. A procedure for reducing this problem to a linear programming problem is described in the literature.¹ The optimal solution for the latter problem is an integer solution.



¹ D.B. Yudin and Ye.G. Goldstein, "Problems and Methods of Linear Programming," Sovetskoye Radio, Moscow, 1961.

However, a practical application of this method is difficult because of the high dimensionality of the corresponding linear programming problem. The same remark also applies to dynamic programming methods, the application of which to problems (2) through (4) has been studied.¹

An important special case of the optimal replacement problem is the case when the dimensions of the system are restricted only to one parameter, for example, weight or volume. Then the number of restrictions of type (3) equals one ($m = 1$).

In this case, the problem can be solved efficiently by the following method.

We will consider the problem:

$$\min \sum_{i=1}^n \phi_i(x_i) \min FX. \quad (5)$$

with

$$\sum_{i=1}^n a_i x_i \leq b, \quad (6)$$

where $x_i \geq x_{i0}$ can assume only discrete values

$$x_{i0} + kh, \quad (k = 1, 2, \dots). \quad (7)$$

Here $\phi_i(x_i)$ are convex, monotonically decreasing functions of a single variable; $h > 0$ is the discrete step size, $b > 0$ and $a_i > 0$ ($i = 1, \dots, n$). For the all-integer case, $x_{i0} = 1$, and $h = 1$. Generally speaking, the discrete step size can be different for various variables: $h = h(i)$.

This case is easily reduced to the case $h = \text{const}$ by introducing new variables of the type $x'_i = kx_i$ and is the case considered here.

Suppose that for all i , $x_i = x_{i0}$. It is clear that in this case inequality (6) is satisfied, since in the contrary case a solution does not exist. We will determine a decrease in the objective function (5) by increasing each of the unknowns by h : $\phi_i(x_{i0}) - \phi_i(x_{i0} + h) > 0$ ($i = 1, 2, \dots, n$). We remember that ϕ_i are monotonically decreasing functions.

Dividing this quantity by a_i , we obtain δ_i -- the decrease in the objective function per unit weight. In the general case, by weight we mean a

¹ R. Bellman and S. Dreyfus, "Applied Dynamic Programming," Nauka, Moscow, 1965.

property which can be expressed by the coefficient b where the unit weight of each variable x_i equals a_i .

Among all unknowns we find that unknown which insures a maximum decrease in the objective function per unit weight, i.e., we find

$$\max_i \frac{\varphi_i(x_{i0}) - \varphi_i(x_{i0} + h)}{a_i} = \frac{\varphi_k(x_{k0}) + \varphi_k(x_{k0} + h)}{a_k} \quad (8)$$

We increase the value of x_k by h and verify condition (6). If it is satisfied, then for x_k we have a new value which has been increased by h in comparison with the previous value. We again compute the quantities (8), but here, instead of x_{k0} we take the new value $x_{k0} + h$. We notice that the quantity (8) must be computed only for $i = k$, because for all remaining i values it remains unchanged. This considerably shortens the computational effort. We again determine

$$\max_i \frac{\varphi_i(x_i) - \varphi_i(x_i + h)}{a_i} = \frac{\varphi_r(x_r) - \varphi_r(x_r + h)}{a_r}$$

and if condition (6) is satisfied, we increase x_r by h . The computation continues until either the increase of the current x_r by h leads to:
a) attainment of equality in (6) (then the value x_r is replaced by $x_r + h$, and the computation terminates), b) the violation of condition (6).

We then determine x_j , for which the quantity

$$\frac{\varphi_j(x_j) - \varphi_j(x_j + h)}{a_j} = \delta_j$$

assumes a value which is next to

$$\max_i \frac{\varphi_i(x_i) - \varphi_i(x_i + h)}{a_i}$$

If, while increasing this x_j by h condition (6) is satisfied, x_j is replaced by $x_j + h$ and the calculation continues. If among all x_j there is no x_j which can be increased by h without violating condition (6), computation terminates.

It can be shown that the method shown above gives an exact solution to problems (5) through (7) if it terminates in attainment of equality in (6).

If for the solution X^* which was obtained condition (6) has the form of an inequality, then the solution, generally speaking, need not be optimal. In this case let X_{opt} be the optimum solution. Then $F(X^*) \geq F(X_{opt})$.

We will determine a solution x which will be obtained when condition (6) is violated for the first time, and without regard for this, the corresponding x_i

are increased by h and the computation is considered terminated. Clearly then the inequality $F(X^*) \geq F(X_{\text{opt}}) \geq F(\bar{X})$ will hold.

From here we have an estimate for the constructed solution $F(X^*) - F(X_{\text{opt}}) \leq F(X^*) - F(\bar{X})$.

For the optimum replacement problem,

$$\delta_i = \frac{\varphi_i(x_i) - \varphi_i(x_i + h)}{a_i} = \frac{-\ln(1 - q_i^{x_i}) + \ln(1 - q_i^{x_i + h})}{a_i} = \frac{1}{a_i} \ln \frac{1 - q_i^{x_i + h}}{1 - q_i^{x_i}}$$

In addition to problems (5) through (7), the above method is applicable in the following cases.

1. $\delta_i(x_{i0}) > 0$ and $\phi_i(x_i)$ are convex functions which are not necessarily monotonically decreasing (curve 2). In this case, every time after increasing the current x_i by h , one must verify the sign of $\delta_i(x_i + h)$. In addition to the usual condition for terminating the calculation, in this case, each x_i can be increased only until the condition $\delta_i \geq 0$ takes place.

2. $\phi_i(x_i)$ are monotonically increasing convex functions, with $\delta_i(x_{i0}) < 0$ (line 3) and condition (6) has the form

$$\sum_{i=1}^n a_i x_i \geq b. \quad (6')$$

3. $\phi_i(x_i)$ are convex functions and condition (6') holds. In this case, for x_{i0} we can take some quantities which exceed considerably their optimum values and for which $\delta_i(x_{i0}) < 0$, and condition (6') is satisfied. The movement takes place in the direction of decreasing x_i .

4. Problems (5) through (7), in the case of continuous change of variables.

For a sufficiently small step, an optimum solution can be attained with arbitrary, preassigned precision. To reduce the amount of computation, the step size need only be decreased sufficiently near the optimum.