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BEHAVIOR OF SPHERICAL PARTICLES AT LOW REYNOLDS NUMBERS IN A FLUCTUATING TRANSLATIONAL FLOW—PRELIMINARY EXPERIMENTS

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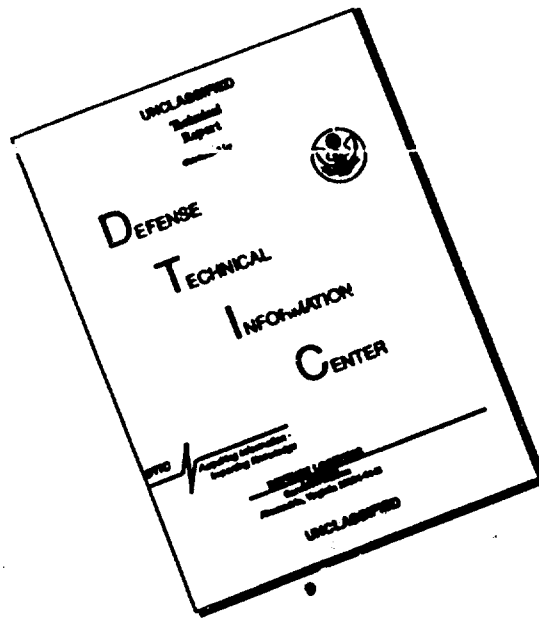
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FOREWORD

This report covers that portion of the work performed under Contract F33615-69-C-1069 from 1 September 1970 through 31 August 1971. The project engineer monitoring this contract was Mr. S. H. Hasinger (LE), Energy Conversion Research Laboratory, Aerospace Research Laboratories. Technical contributions to this work were made by Mr. Murray K. Hill, Dr. Edward E. Zukoski, Dr. W. Duncan Rannie, and the Principal Investigator, Dr. Frank E. Marble. We are particularly indebted to Dr. H. von Ohain, Chief Scientist, A. R. L., and to Mr. S. H. Hasinger, Energy Conversion Research Laboratory, for their many technical discussions which were of singular importance in choosing areas of emphasis.

Another report published under above contract is ARL 72-0018 "An Experimental Investigation of Particle Motion in a Liquid Fluidized Bed" by Charles A. Willus.

ABSTRACT

The behavior of small spheres in non-steady translational flow has been studied experimentally for values of Reynolds number from 0 to 3000. The aim of the work was to improve our quantitative understanding of particle transport in turbulent gaseous media, a process of extreme importance in powerplants and energy transfer mechanisms.

Particles, subjected to strong sinusoidal oscillations parallel to the direction of steady translation, were found to have changes in average drag coefficient depending upon their translational Reynolds number, the frequency and amplitude of the oscillations. When the Reynolds number based on the sphere diameter was less than 200, the symmetric translational oscillation had negligible effect on the average particle drag.

For Reynolds numbers exceeding 300, the effective drag coefficient was significantly increased in a particular frequency range. For example, an increase in drag coefficient of 25 per cent was observed at a Reynolds number of 3000 when the amplitude of the oscillation was 2 per cent of the sphere diameter and the disturbance frequency was approximately the Strouhal frequency. The occurrence of the maximum effect at frequencies between one and two times the Strouhal frequency strongly suggests non-linear interaction between wake vortex shedding and the oscillation in translational motions. Flow visualization studies support this suggestion.

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1. INTRODUCTION

The two outstanding fundamental problems that limit the technological development of dusty gas dynamics are: (1) the near-field interaction between solid particles or between a particle and a solid surface, and (2) the behavior of solid particles in a turbulent medium.

The first of these has been considered in some detail analytically¹ and experimentally², utilizing a fluidized bed in the study of interactions with large volume fractions of solid particles.

The second of these problems, the movement of solid particles in a turbulent flow field, has been considered by some investigators^{3,4} to concern a problem of particle drag coefficient as affected by the stream turbulence level. In the case of micron-sized particles, where the particle size is small compared with the turbulence macroscale, it seems doubtful that this picture is an appropriate one.

On the other hand, other investigators - notably Tchen⁵, Corrsin and Lumley⁶, and Lumley⁷ - consider the migration of particles in a turbulent field possessing certain statistical properties. These efforts come much closer to the problem of technical interest and are limited largely by the turbulence statistics and the general complexity of the calculation. This work does not, however, account for the effects which a dense cloud of particles would have upon the turbulence structure. It is to be expected that, depending upon the particle size, a selected portion of the turbulence spectrum would be altered by particle attenuation similar to the well-known selective attenuation of acoustic fields by particle clouds.

Calculations of the particle transport by a turbulent field have been carried out using essentially a modification to the Stokes law which accounts, in a certain measure, for linear non-steady effects. This is certainly an

understandable assumption since this improves, somewhat, its analytical tractability. However, in many cases of technological interest, the Reynolds number of the particle motion relative to the fluid is large enough so that this relatively simple law is inapplicable. The anomalous, low drag coefficients observed in refs. 3 and 4 at Reynolds numbers between 10 and 10^3 were attributed in ref. 4 to premature transition on the sphere induced by the high level of stream turbulence. It would seem that a more acceptable and profitable approach is to recognize that these small spheres exist in a severely non-steady flow field, in which relative velocities may be reversed by fluctuations, and to understand something of their behavior under these circumstances.

The anomalous behavior of spheres moving freely in fluids is a matter of historical record as well as one of continuing research interest, refs. 8-12, 21. There are obvious changes in the mode of flow in certain ranges of Reynolds number that involve vortex shedding, asymmetric forces, and, in general, significant departures from mean values frequently quoted. In the turbulent transport problem, the situation is further complicated by the strongly non-steady flow to which the particle is subjected.

The current work was stimulated by an effort to understand the measurements of refs. 3, 4, 22, 23 on the basis of the forces acting on a sphere in severely non-steady flow rather than on the basis of turbulent boundary layer transitions on a sphere at Reynolds numbers of the order 10^2 . To accomplish this, it appeared necessary to measure the sphere response to fluctuations in translational velocity which were of the same magnitude as the mean translational velocity. This is a very difficult experimental problem (e. g. ref. 13) in the Reynolds number range of interest.

The technique that has been adopted in the present investigation was suggested by some processes in the chemical industry and its use in refs. 14 and 15 to study problems related to the present one. In our adaptation, we utilize a large, vertical, rigid tube, completely filled with liquid and mounted upon a movable table which oscillates the tube along its axis in a vertical direction. This shaker is capable of providing large amplitude oscillation over a wide frequency range. Spheres, of suitable size and material, are released at the top of the tube and fall, under gravitational influence, through the oscillating liquid. In this manner, because the liquid is effectively incompressible, the experiment duplicates, quite reasonably, the free motions of particles through an oscillating fluid field of large scale and permits drag measurements based upon transit time for a fixed distance. In the preliminary experiments discussed here, frequency was limited to the range 7 to 200 Hz and acceleration to 10 g's.

It is important to point out that the experiment does not duplicate the motion of a particle through a turbulent field, not only because of the single frequency present, but because the length scale of the fluctuating field is effectively infinite. It does, however, suggest modifications to the particle drag law to be utilized in analysis of particle transport in a turbulent field when turbulence scale is larger than sphere diameter.

used successfully by McLaughlin¹⁶ in dropping steel spheres. Since one requirement of the present experiment was to test spheres over a range of densities, the restriction of using only magnetic materials led to the rejection of this mechanism. The idea of holding the sphere at the end of a pipe with a partial vacuum was also tried and rejected. Baird, et al.¹⁴ had reported dropping balls through a 5/8" diameter tube and had not reported any difficulty. Tubes of various diameters were tried, and it was found that a tube of approximately 1.2 ball diameters gave reasonably satisfactory results.

To retrieve the spheres from the test section, a 3½" deep plexi-glass insert was installed at the bottom of the tube. The top was machined with a conical depression and a hole was bored in the center. A 1/4" I. D. by 3" brass bucket was placed in the hole. When the bucket was full of spheres it could be retrieved through the top of the apparatus.

The timing mechanism, as shown in figure 2, comprised two light beams falling on two photocells connected to an electronic timing circuit. As spheres in the Reynolds number range considered tended to move in spiral or zigzag patterns with amplitude greater than a diameter, it was not possible to have the sphere intercept the complete light beam. Rather, a broad beam of small thickness (1½" by 1/16") had to be used. It was then necessary to detect a small change in the light intensity when the sphere passed through the beam. Because of this, a light source powered by direct current was used, because alternating current produced an oscillation in intensity of the same magnitude as the signal. The light sources used were commercial flashlights (pk 4 bulbs) powered by 9 V batteries through rheostats so as to make the intensity adjustable. The light then passed through two 1½" by 1/16" slits eight inches apart

to produce a thin, collimated beam. The beam, in passing through the liquid, was focused by the curvature of the water-glass, glass-air interfaces on to the 1/4" diameter face of the photocell. The electrical circuit, shown in figure 3, consisted of a two-stage preamplifier, a single-stage amplifier, and a monostable multivibrator. The signal from the photocell is amplified to approximately a 1-volt pulse, and the 100 - 150 K resistor is adjusted so that this signal (but not the noise) will trigger the multivibrator. The multivibrator in turn puts out a 4.5-volt square wave with rise time less than 1 millisecond. This pulse is sufficient to trigger a Hewlett-Packard model 5262-A time interval unit. The signal from the second photocell, of course, stops the timer.

A Ling model A-175 electrodynamic shaker was used in these experiments. The shaker is capable of producing motions corresponding to input signals from a signal generator. Only sinusoidal motions were used in these experiments. The frequency range of the shaker is 5 to 4000 cps. At low frequencies the table is limited to one inch peak-to-peak displacement: at high frequency it is limited to 1500 lbs. of force output. In this case, the weight of the table and the apparatus is 75 lbs., limiting the accelerations to 20 g's above 20 Hz. An Endevco model 2242 accelerometer mounted on the shaker table monitors the acceleration of the apparatus and is included in the shaker feedback loop. The frequency of the oscillation is measured with a Hewlett-Packard model 521 DR electronic counter. In the preliminary experiments reported here, the maximum acceleration has been limited to 10 g's and to frequencies between 7 and 200 Hz.

The spheres used in these preliminary experiments were Hartford-Universal grade 200 type 440-C stainless steel, grade 200 type 2017 alu-

minum, and tungsten carbide. The sphericity (permissible difference between largest and smallest measured diameter) is given as .0002". Spheres used in these experiments ranged from 3/32" diameter to 3/16" diameter. In addition, several experiments were carried out with brass cones with a total included angle of 70°. Distilled water and mixtures of water and glycerin were used as the test fluids.

As a critical check on the operation of the drop mechanism and timer, steady-state drag measurements were made for a wide range of Reynolds numbers and were compared with values summarized in ref. 21. Because our data fell within the scatter band of these previous results, it was felt that the apparatus was operating satisfactorily.

Flow visualization studies are being carried out in a rectangular and open channel about 20' long, 18" wide and 18" deep. Visualization is achieved by introducing dye into the wake, and visual and photographic studies (16 mm motion pictures) are being made of the flow. Velocity oscillations with amplitudes of 10 - 20 per cent of the mean velocity are imposed on the mean flow by exciting the fundamental longitudinal mode of shallow water waves. One- to four-inch diameter spheres are held on a 1/40" diameter sting which extends downstream from the rear stagnation point.

3. PROCEDURE

To determine the average velocity of a falling sphere, time elapsed as the spheres fell between the two light beams, described earlier, was measured and the average speed taken as the ratio of length between the beams and the travel time. This very simple measurement is complicated by the fact that for Reynolds numbers above 300, spheres do not fall in a straight line, but actually in a helical and/or a zigzag pattern, e. g. refs. 17 and 18. Consequently, the velocity as determined above is not the true velocity but the vertical component. Fortunately, this complication is not serious in the present work for two reasons.

First, in this experiment we are most interested in the ratio of the velocity in oscillating flow to that in steady flow. So long as the nature of the flow is unchanged, e. g. if the sphere still falls in a spiral in an oscillatory as well as a non-oscillating flow, then the ratio of the vertical components of velocity will be meaningful. Visual observation of sphere trajectories indicates that for the conditions considered here, no changes in gross motion occurred when the tube was oscillated.

Second, these visual observations showed that the wavelength of the spiral or zigzag trajectories were of order of 8 - 12 inches or greater and had an amplitude less than one inch for the sphere trajectories used in making the drag calculations. Hence, the difference between vertical velocity component and true velocity magnitudes is less than 3 per cent for the trajectories studied here.

At least ten measurements in still fluid and five in the oscillating fluid cases were taken so that effects of complex trajectories were averaged. In order to assure that there was no interference from the preceding sphere, about 10 - 15 seconds' gap was left between releases. This

would correspond to each sphere being roughly 2000 diameters behind the preceding one.

The first measuring station was located far enough below the sphere drop point to ensure that the spheres fell more than 40 diameters. On the basis of tests carried out with an initial length of 25 and 50 diameters and previous work reported in the literature¹⁸, this length appears to be more than enough to allow the spheres to reach their terminal velocities at all Reynolds numbers tested. A measuring length of 6 inches was sufficient to allow accurate measurement of the length itself and the resulting periods of fall. Errors in velocity measurements were estimated to be no more than ± 1 per cent, which was also approximately the scatter between individual measurements.

Temperature drift in the test fluids was no more than 1°C and average values were used to obtain fluid density and viscosity values based on handbook data or falling ball viscosimeter tests.

The expected uncertainties in the determination of the dimensionless parameters introduced in the next section were less than 2 per cent.

4. RESULTS

From the numerous detailed experimental observations of sphere workers, e.g. ref. 8, it has become clear that the flow phenomena may be divided into three regimes which can be characterized by the Reynolds number based upon sphere diameter. For low Reynolds number regime, $0 < Re < 10$, the flow remains attached to the sphere and a wake exists only in the sense of Oseen flow, that is, there exists some asymmetry between upstream and downstream flow fields. In the intermediate regime, $10 < Re < 300$, there exists a stable recirculation region or wake attached to the sphere, and the wake is generally stable and time independent. Toward the extreme upper range of this regime the wake exhibits fluctuations and shows a distinct tendency to become unstable. In the high range, for $Re > 300$, vorticity is shed in a more or less regular manner: in the Reynolds number range from 300 up to at least 800, discrete vortices are shed in a variety of complex but very regular patterns, some of which are described in ref. 8. Above 800, discrete shedding continues but becomes less regular.

In carrying out experiments to investigate the effects on sphere drag of imposing a regular oscillation on the fluid, a range of frequencies must be selected for study. For Reynolds numbers greater than 300, there are good reasons (for example, the natural resonance of sphere-wake system which produces regular shedding, and previous work with circular cylinders) to expect a resonance near the vortex shedding frequency. Goldberg and Florsheim¹⁹ have developed an empirical formula for the Strouhal number as a function of the Reynolds number for the range $300 < Re < 600$. From this relation, the vortex shedding frequency f_{ST} can be expressed as:

$$f_{ST} = 0.387 \left(1 - \frac{270}{Re} \right) \frac{u}{d}, \quad (1)$$

where u and d are respectively the translational velocity and diameter of the sphere, and $Re = ud/\nu$ is the sphere Reynolds number. In addition, for Reynolds numbers between 2,000 and 10,000, values of shedding frequency of about $0.4 u/d$ are reported in ref. 20. These results suggest that equation (1) may give values of shedding frequency which are accurate to within 5 or 10 per cent for the whole range $300 \leq Re \leq 10,000$.

Because it was expected that the Strouhal frequency would be a critical frequency for the phenomena being investigated, and because equation (1) was the best estimate available, values of f_{ST} calculated from this equation were used to normalize the frequency of oscillation in data presentations. The ratio fd/u would have been a satisfactory parameter, but in order to emphasize the Strouhal frequency,

$$\frac{f}{f_{ST}} = \left(\frac{fd}{u} \right) \left(\frac{1}{0.387(1 - 270/Re)} \right)$$

was used instead. No deep significance is attributed to equation (1) except that it was the best estimate for f_{ST} available to us.

At lower frequencies, i. e., $Re < 300$, no such natural frequency appears to exist. In this range, it was decided to concentrate our investigation in the frequency range about $fd/u \approx 1$, where the period of oscillation is close to the time required for the sphere to fall one diameter.

In the Stokes-Oseen regime of Reynolds number, one is able to give (see ref. 6) a reasonably valid analytical description of the instantaneous forces acting on a sphere under non-steady conditions. The application of this equation to a simple sinusoidal oscillation leads to a prediction of no net change in the average sphere drag or the average falling velocity. This follows, in general, from the linearity of the drag law. Our experi-

ments yielded, as should be expected, an accurate confirmation of this result, even when the ratio of fluid amplitude to sphere diameter A/d was larger than 5. The frequencies for these tests were in the range $\frac{1}{2} \leq fd/u \leq 1.5$.

Tests in the intermediate Reynolds number range showed that at large amplitude-to-sphere ratios, A/d as large as 1.0, a small velocity decrease could be produced. For example, at $Re = 200$, $A/d = 1.4$, and $fu/d \approx 1$, the terminal velocity decreased about 2 per cent. This change is of the same order of magnitude as that calculated from a quasi-steady drag model due to the non-linearity of the drag law at Reynolds numbers of 200. This result is surprising because at this Reynolds number the sphere wake is well formed, and for the values of A/d used the particles were moving into their own wakes a significant part of a diameter.

Even when $A/d \approx 1$, only a small change in particle terminal velocities could be produced for oscillation frequencies in the range $\frac{1}{2} \leq fd/u \leq 2$ when the Reynolds number was less than 200. That is, when no natural shedding phenomena were present, no appreciable decrease was produced in the terminal velocity.

For Reynolds numbers greater than 300, a definite and systematic effect on the average drag coefficient was observed for small values of A/d . In discussing the results, it is convenient to introduce a number of dimensionless parameters. The dimensionless parameters selected to characterize the average terminal falling velocity u of a sphere of density ρ and diameter d which is moving through a fluid characterized by its density ρ_f and kinematic viscosity ν when the fluid is moving with a sinusoidal oscillation characterized by its amplitude A and frequency f cycles per seconds or ω radians per second, are:

fractional velocity change	$\frac{-\Delta u}{u} = (u_0 - u)/u$
Reynolds number	$Re = ud/\nu$
amplitude ratio	A/d
density ratio	ρ/ρ_f
frequency ratio	f/f_{ST} or w/w_{ST}

Here, u_0 is the fall velocity with no oscillation and is a function of Re and ρ/ρ_f , u is the time-averaged fall velocity with oscillation present, and f_{ST} is the shedding frequency whose value is assumed to depend on the Reynolds number, d and u as shown above by equation (1). These dimensionless groups form a complete set if the characterizations of spheres, fluid, and oscillation are complete. Alternate parameters, such as a velocity fluctuation ratio Aw/u or acceleration ratio Aw^2/g , were rejected because the actual amplitude of particle motion A_p (which is closely related to A) relative to the diameter and the frequency relative to the Strouhal frequency rather than the acceleration or velocity fluctuation ratios were felt to be of more importance.

Because the average terminal speed of the spheres changed as oscillation amplitude and frequency were changed, it was at times convenient to substitute for Re and f_{ST} two slightly different parameters, Re_0 and f_{ST0} which are the values of Reynolds number and shedding frequency based on u_0 , the terminal velocity of a sphere falling through a non-oscillating fluid.

It is important to note that A is the amplitude of fluid motion and not that of the particle motion relative to the fluid, A_p . Values of A_p/A , which depend on all the dimensionless parameters, are estimated to be between 0.5 and 1.0 for the present experiments at high Reynolds numbers.

For the high Reynolds number regime and for frequencies far below the natural shedding frequency f_{ST} , no appreciable changes in terminal velocity were observed. The few per cent changes observed here were of the same size as those changes predicted from a quasi-steady model.

However, when oscillations with frequencies near f_{ST} were used, systematic changes in terminal velocity were observed which were sensitive functions of all the parameters. Most of the experiments were carried out in one of two ways. First, spheres of a fixed type were dropped into a given fluid and the frequency was changed while the oscillation amplitude was held fixed. Thus, Re_o , A/d , ρ/ρ_f were held constant while f/f_{ST} was varied. Second, the amplitude of the oscillation was changed while fluid, sphere, and frequency were held fixed. Because terminal velocity changed (up to 15 per cent) during these tests, the Reynolds number also changed. These changes have been ignored in reporting the data, and Re_o rather than Re is typically used to characterize particle Reynolds number.

Response curves for a wide range of parameters are shown in figures 4, 5, and 6. In these figures, the fractional velocity change $(u_o - u)/u \equiv -\Delta u/u$ is plotted against f/f_{ST} for fixed values of the amplitude ratio A/d and other parameters. Several curves given in figure 4 show the response out to frequencies of $3.5 f_{ST}$. As the frequency is increased, the response to the oscillations begins around $1/4 f_{ST}$, increases to a peak near f_{ST} , and remains roughly constant out to about $2 f_{ST}$. The response then decreases rapidly and becomes less than a few percent around $3.5 f_{ST}$. The data show that for large amplitude oscillations a definite peak exists around f_{ST} and they suggest a weak peak may exist

near $2f_{ST}$. However, this second peak is too weak to be defined unambiguously by the present data. The data presented in figures 5 and 6 show similar trends and illustrate the dependence on the other three parameters.

Although large variations in the magnitudes of velocity changes and in the shape of the response curves do occur as the parameters are changed, the description given above is generally typical of that obtained for all values of A/d and ρ/ρ_f as long as the Reynolds number was above the 300 to 500 range. That is, as long as natural vortex shedding exists in a steady flow, this resonance-like phenomenon, with the maximum response centered around $1.5f_{ST}$, is present when a sphere falls through an oscillating fluid.

Visualization studies confirm that vortex shedding plays an important part in the response of the sphere to the oscillating flow. Preliminary studies show that a discrete mass of fluid is detached from the sphere wake during each cycle of the imposed oscillation when the oscillation frequency is between $1/2$ and $3/2$ of the Strouhal frequency, as estimated by equation (1). As the amplitude is increased, the volume of fluid shed appears to increase.

An increase in oscillation amplitude was found to cause an increase in the general level of the fractional velocity change and to change slightly the shape of the response curve for $f > f_{ST}$. The data of figures 4, 5, and 6 illustrate these features. For $1000 \leq Re \leq 3000$, the fractional velocity change increases roughly as $(A/d)^{2/3}$ when f/f_{ST} is held fixed and when $f/f_{ST} \leq 1.0$.

To supplement data of the type given in these figures, a number of tests were carried out in which the frequency was held fixed at, say, $0.6f_{ST0}$, and the amplitude was changed. Here, f_{ST0} , the Strouhal

frequency with no applied oscillations, was used rather than f_{ST} because it was more convenient. In addition, it was found that the response increased linearly with A/d when the experiment was carried out with fixed values of f/f_{ST0} and $f < 0.6f_{ST0}$. Data obtained in this manner are presented in figures 7 and 8, and they show that the fractional velocity change increases with Reynolds number and density ratio.

In the range for which the fractional velocity change increases linearly with amplitude, the dependence on Reynolds number is most easily analyzed by examining the plot of $(-\Delta u)/(A/d)$ versus Re shown in figure 9. This amounts to examining the slopes of the curves shown in figures 7 and 8 for $f/f_{ST0} = 0.614$. The sensitivity to oscillations increases markedly for $Re > 500$ and continues to increase throughout the range of Reynolds numbers investigated.

The dependence on the Reynolds number of the peak values of the fractional velocity change curves (obtained for a fixed value of A/d) is similar to that shown on figure 9, but values for any given A/d are considerably larger. For example, when $A/d = .02$, $\Delta u/u = 0.12$ at $Re = 3000$ and $\rho/\rho_f = 7.8$.

The response also increases with density ratio. This increase is marked between $\rho/\rho_f = 2.83$ and 7.83 (see figure 9), and is almost absent between 7.83 and 14 (see figures 5 and 9). This result holds for $1000 \leq Re \leq 3000$.

The dependence of fractional velocity change on the density ratio can only enter the problem through the dependence of A_p/A on density ratio. The results obtained here appear to show that the dependence of particle amplitude on fluid amplitude is insensitive to density ratio when

$\rho/\rho_f > 8$ and consequently that the results obtained for steel and tungsten carbide balls in water may be directly applicable to the prediction of motion of dense spheres in air.

In order to examine whether terminal velocity changes discussed above were due to movement of the separation point, one series of tests was carried out with 70° (total included cone angle) brass cones. The test Reynolds number was about 4000 and the frequency used was 60 per cent of the value of f_{STo} calculated from equations given in ref. 19. The fractional velocity change ratio was almost identical to that obtained with spheres at similar conditions. Because the separation point for the cone is fixed, it is evident that movement of this point is not the phenomenon of interest here.

Although the preceding paragraphs have dealt with velocity changes, these can be translated easily into drag coefficient changes by use of the relationship

$$\frac{C_D - C_{Do}}{C_{Do}} = \left(\frac{u_o}{u}\right)^2 - 1 ,$$

which follows from the usual definition of the drag coefficient with the speed taken as the average speed of the sphere. For small values of $(-\Delta u/u)$ one gets

$$\frac{\Delta C_D}{C_{Do}} \equiv \frac{C_D - C_{Do}}{C_{Do}} \approx \left(\frac{-2\Delta u}{u}\right) .$$

The drag changes found during these experiments ranged up to about 25 per cent. For a Reynolds number of 3000 and a frequency near the Strouhal frequency, this drag rise was achieved with A/d values of 0.02 for steel spheres.

5. DISCUSSION

Analysis of a quasi-steady drag model in which the instantaneous drag coefficient is based on the instantaneous velocity has been carried out. This work shows that even in the purely quadratic drag law region, drag or velocity changes of only a few per cent would be expected. In addition, the resonance-like peak observed near f_{ST} would not be predicted by such an approach.

These preliminary data suggest several conclusions. First, it is clear that a non-linear resonance phenomenon connected with the normal vortex shedding mechanism and involving vortex shedding is responsible for the observed terminal velocity decrease and drag increase. It is interesting to note that this mechanism appears to be operative far beyond the Reynolds number range where regular vortex shedding has been observed visually. (Experience with vortex shedding from cylinders leads one to expect that some regular shedding component would exist for spheres in this regime even though it has not been observed visually.)

If the frequency corresponding to the maximum value of the fractional velocity change or the beginning of the plateau region is taken as the Strouhal frequency, then the present data show that a direct use of the empirical equation of Goldberg and Florsheim¹⁹ is justified for Reynolds numbers in the range 1000 - 3000.

Second, fractional changes in terminal velocity as large as 12 per cent could be obtained at high Reynolds numbers (3000) and high density ratios (7.83) with amplitude of fluid motion of only 2 per cent of the sphere diameter. The corresponding drag increase of more than 25 per cent is an order of magnitude greater than that calculated on the basis of a quasi-steady model. The effect becomes smaller as the Reynolds num-

ber decreases and appears to be negligible at Reynolds numbers below 300 even when the fluid amplitude is equal to the sphere diameter.

Third, for particle-to-fluid density ratios greater than 8, the value of A_p/A appears to be close enough to one that the influence of density ratio is negligible.

Fourth, at high frequencies and large amplitude-to-diameter ratios, values of $A\omega/u$ as large as 10 - 25 per cent were obtained. In all cases, the imposition of such oscillations produced drag increases; this result is in contrast to that reported in refs. 3, 4 where drag decreases were found when particles moved through turbulent streams.

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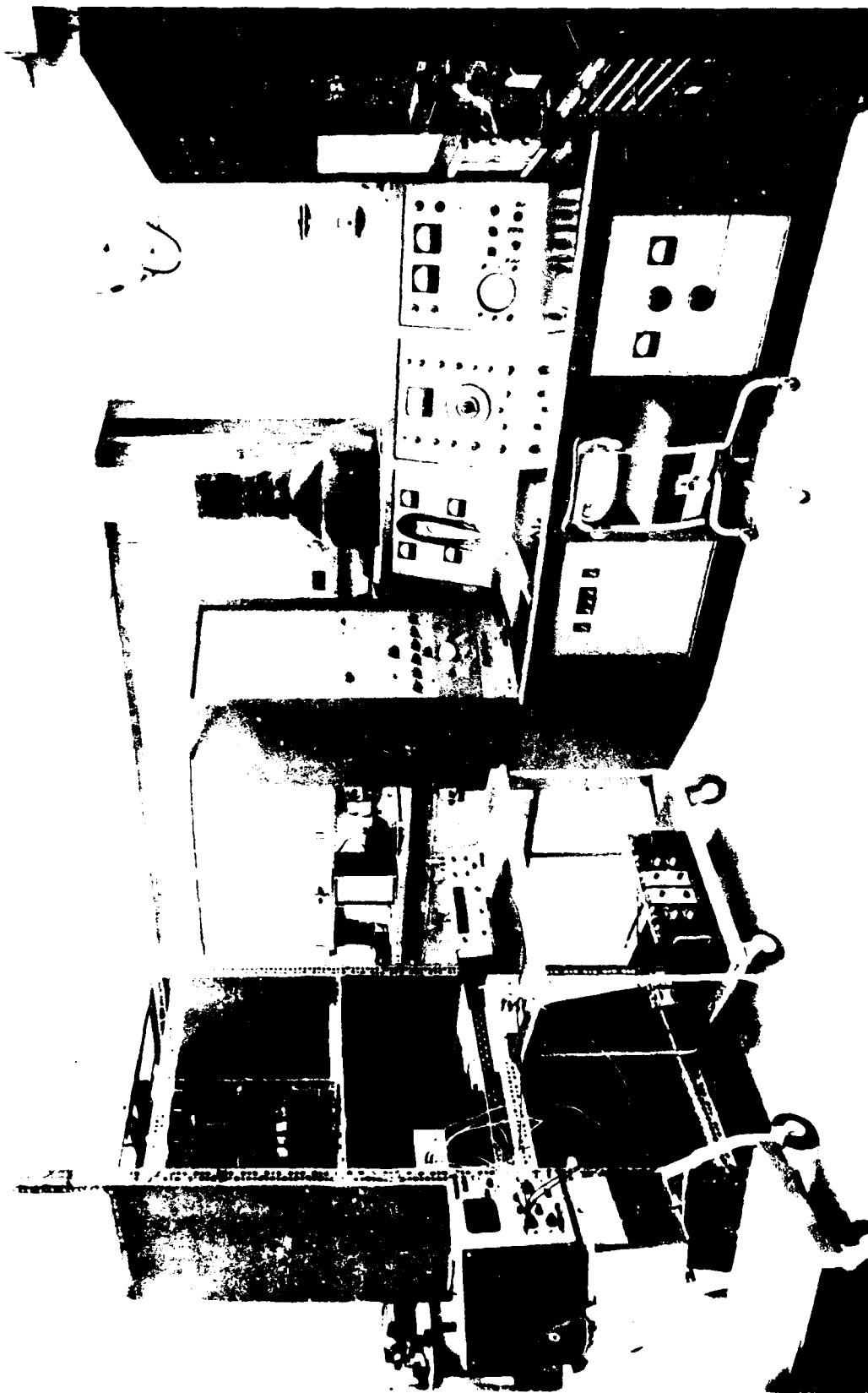


Fig. 1. View of the experimental apparatus.

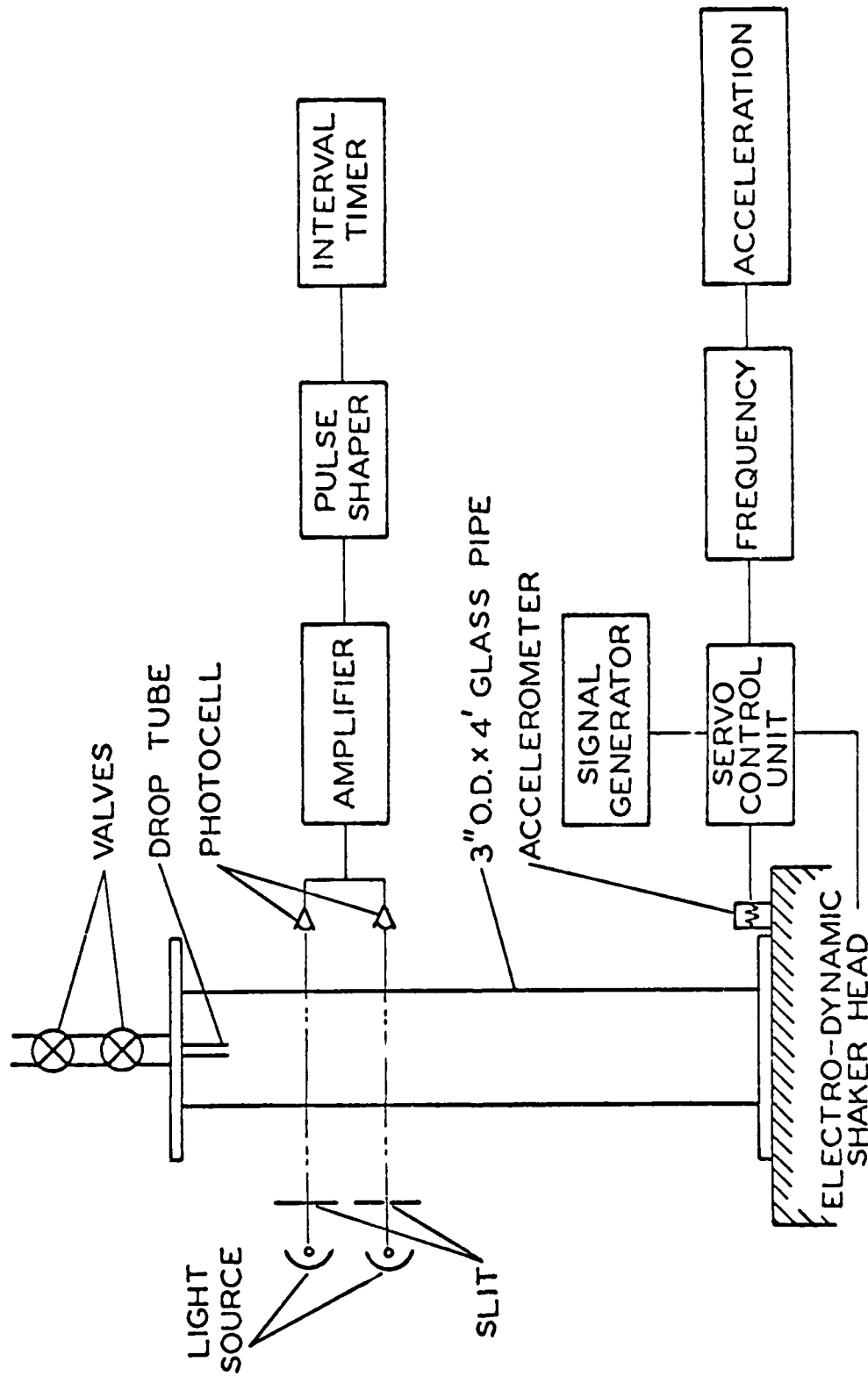


Figure 2. Schematic diagram of the experimental apparatus

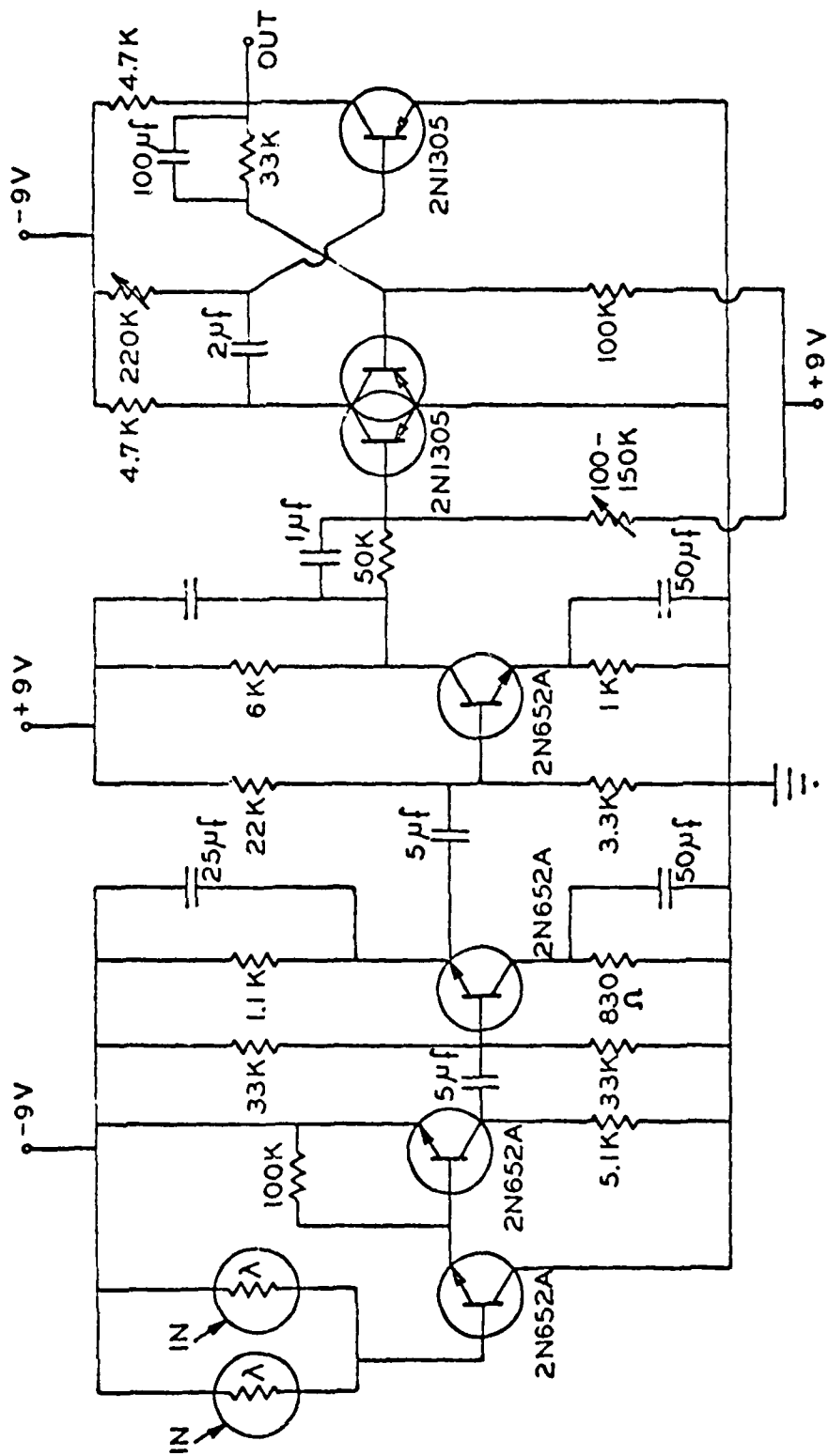


Figure 3. Photoelectric timing circuit

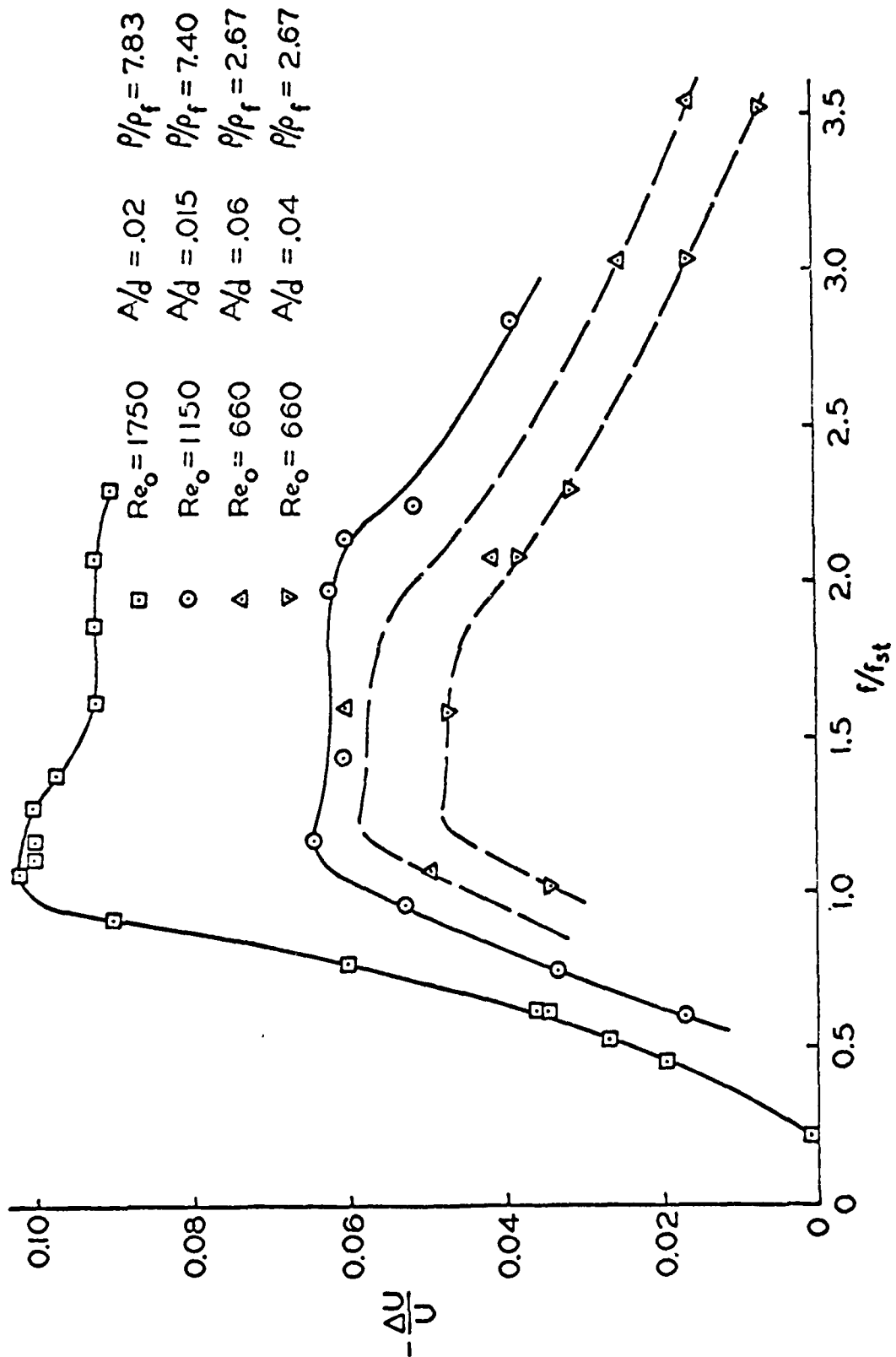


Figure 4. Fractional change in terminal velocity vs. non-dimensional frequency for values of non-dimensional frequency up to 3.5

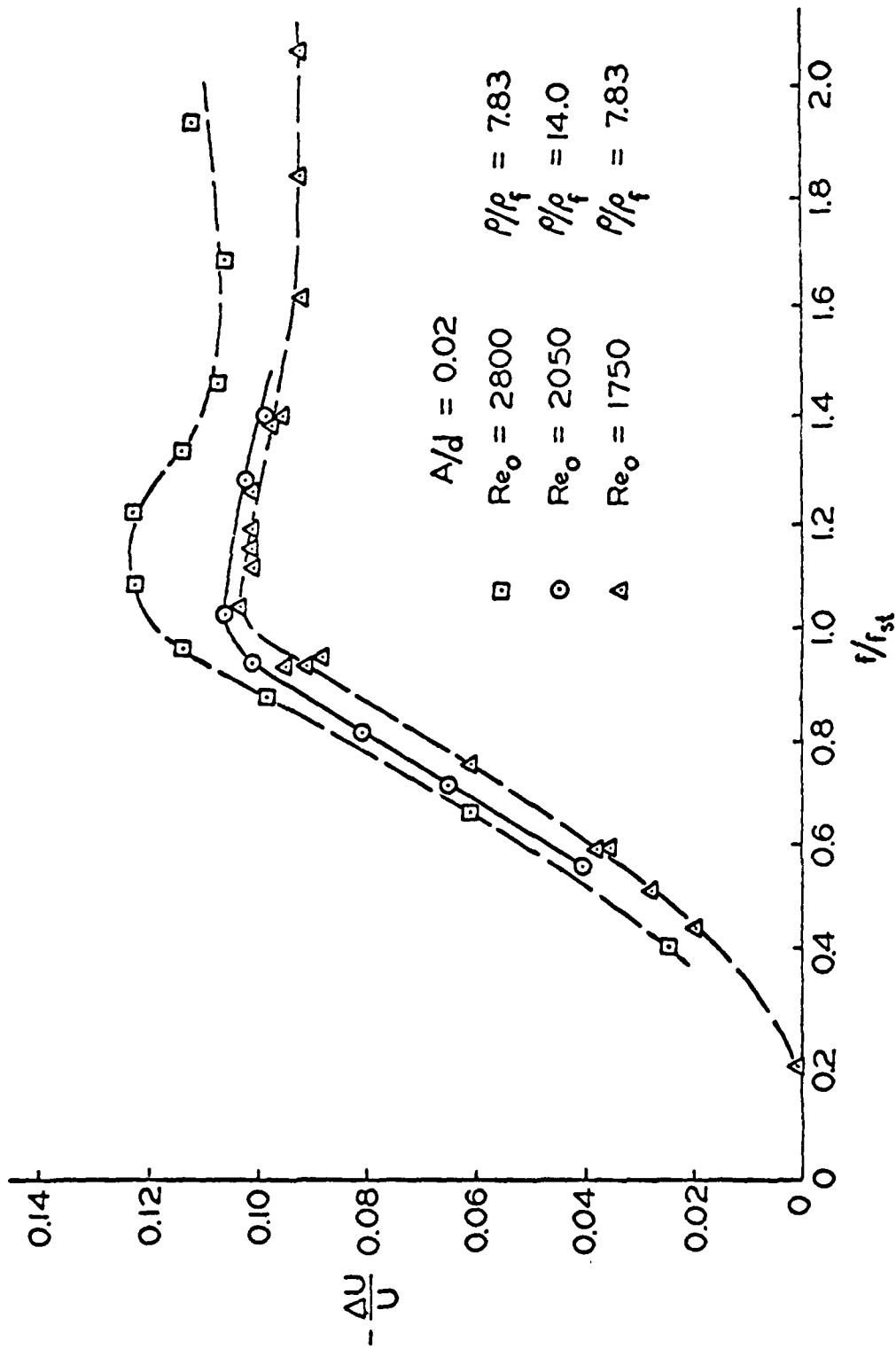


Figure 5. Fractional change in terminal velocity vs. non-dimensional frequency for various Reynolds numbers and two density ratios

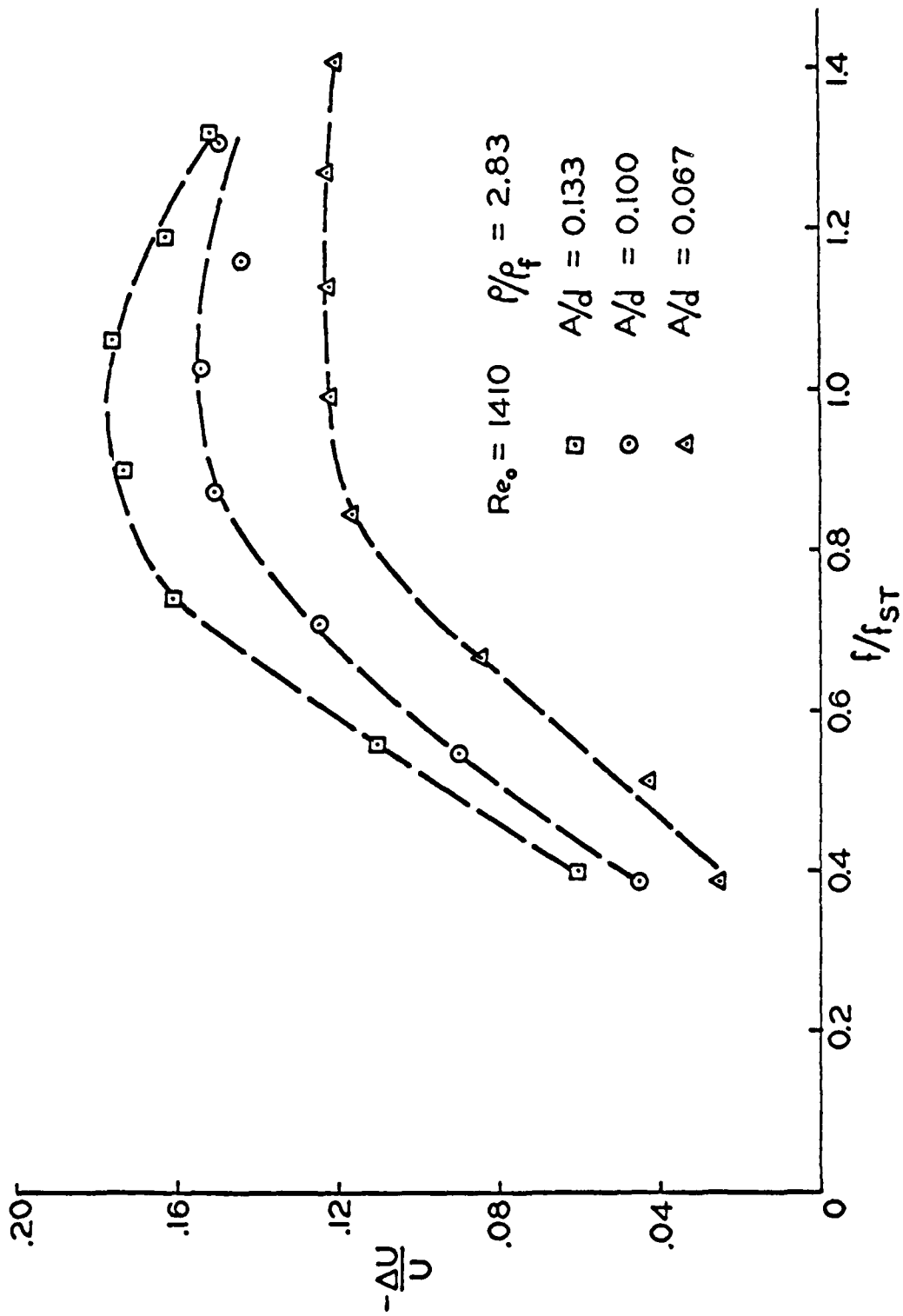


Figure 6. Fractional change in terminal velocity vs. non-dimensional frequency for several amplitudes of particle oscillation

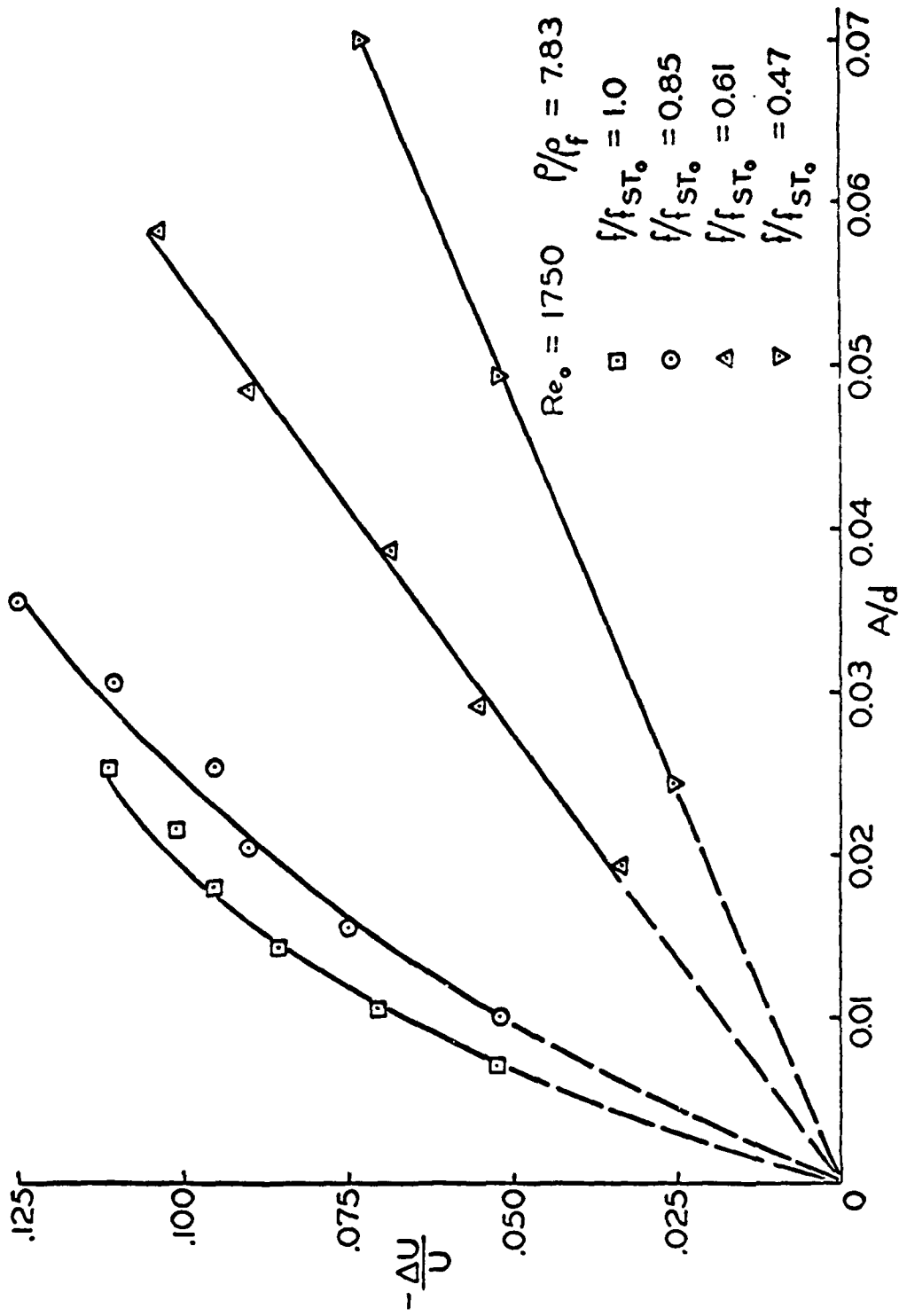


Figure 7. Effect of amplitude ratio on terminal velocity for various ratios of oscillation frequency to Strouhal frequency

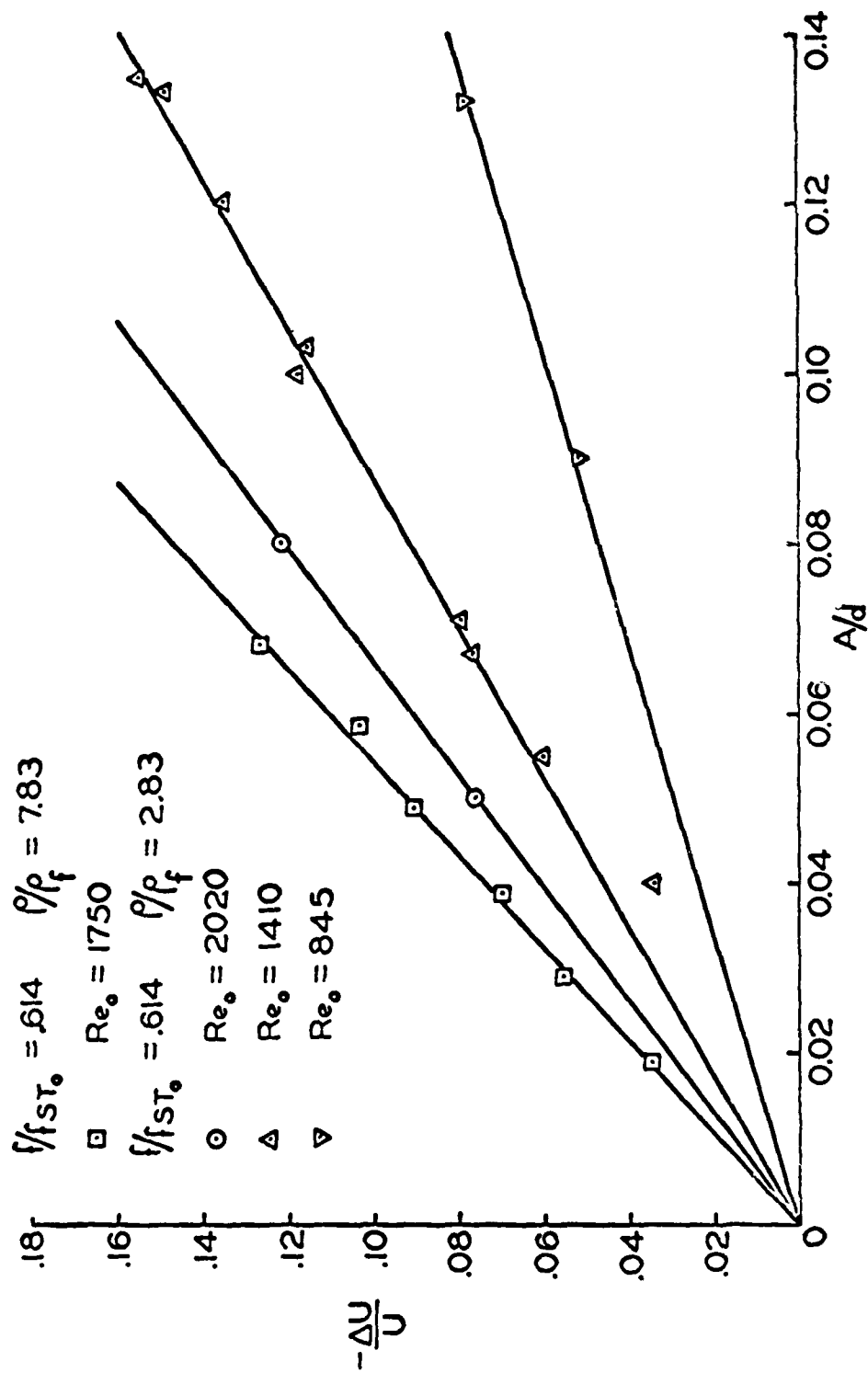


Figure 8. Fractional change in terminal velocity vs. amplitude to diameter ratio for the ratio of oscillation frequency to Strouhal frequency, $f/f_{ST_0} = 0.614$

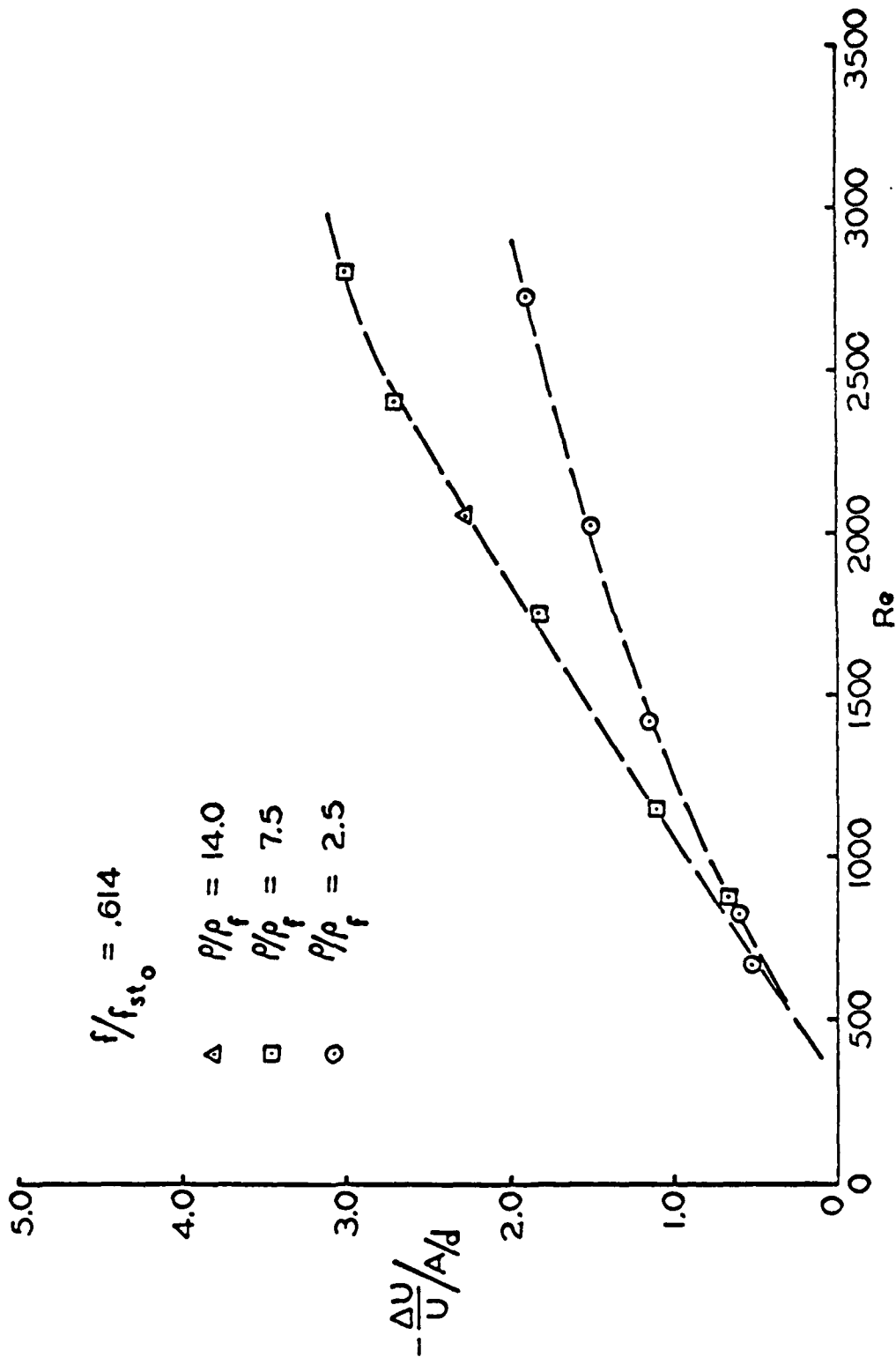


Figure 9. Reynolds number dependence of the slope of the $\Delta u/u$ vs. A/d curves at $f/f_{ST0} = 0.614$