AD 742610

FSTC-HT-23-387-72

ARMY MATERIEL COMMAND

U.S. ARMY
FOREIGN SCIENCE AND TECHNOLOGY CENTER

NUMERICAL SOLUTION OF EQUATIONS IN THE TWO PARAMETER THEORY OF THE BOUNDARY LAYER

BY

Ye. F. Ozerova and L. M. Simuni

COUNTRY: USSR

This document is a rendition of the original foreign text without any analytical or editorial comment.

15

UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Foreign Science and Technology Center US Army Materiel Command Department of the Army	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE
NUMERICAL SOLUTION OF EQUATIONS IN THE TWO PARAMETER THEORY OF THE BOUNDARY LAYER.

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Translation

5. AUTHOR(S) (First name, middle initial, last name)
Ye. F. Ozerova, L. M. Simuni

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
19 April, 1972	11	N/A

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S) FSTC-HT-23- 387-72
b. PROJECT NO c. T702301 2301	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
d. Requester SMUFA Frankford Arsenal	ACSI K 0971

10. DISTRIBUTION STATEMENT
Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	US Army Foreign Science and Technology Center

13. ABSTRACT

Numerical solution of problems of the parametric boundary layer theory in a two-parameter approximation. The partial differential boundary layer equation is solved by a finite difference method using an implicit difference scheme, and the results obtained serve as the initial data. The derivatives in the original equation are replaced by finite difference equations, and the resulting system is solved by a brute force method. A table lists the numerical results obtained by computer.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

UNCLASSIFIED
Security Classification

UNCLASSIFIED
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
NUMERIC SOLUTION BOUNDARY LAYER PROBLEM BOUNDARY LAYER THEORY BOUNDARY LAYER EQUATION COUNTRY CODE: UR SUBJECT CODE: 20, 12						

UNCLASSIFIED
Security Classification

TECHNICAL TRANSLATION

FSTC-HT-23- 387-72

ENGLISH TITLE: NUMERICAL SOLUTION OF EQUATIONS IN THE TWO PARAMETER THEORY OF THE BOUNDARY LAYER.

FOREIGN TITLE: CHISLENNOYE RESHENIYE URAVNENIY DVUKH PARAMETRICHESKOY TEORII POGRANICHNOGO SLOYA

AUTHOR: Ye. F. Ozerova, L. M. Simuni

SOURCE: Not Applicable

Translated for FSTC by ACSI

NOTICE

NUMERICAL SOLUTION OF EQUATIONS IN THE TWO-PARAMETER THEORY OF THE BOUNDARY LAYER

by

Ye. V. Ozerova and L. M. Simuni

1. Formulation of the problem

In solving problems in the parametric theory of the boundary layer [1,2] in the two-parameter approximation, we arrive at the equation

$$\frac{\partial^3\Phi}{\partial\xi^3} + \frac{F+2f_1}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\xi^2} + \frac{f_1}{B^2}\left[1-\left(\frac{\partial\Phi}{\partial\xi}\right)^2\right] =$$
$$= \frac{Ff_1+f_2}{B^2}\left(\frac{\partial\Phi}{\partial\xi}\frac{\partial^2\Phi}{\partial f_1\partial\xi} - \frac{\partial\Phi}{\partial f_1}\frac{\partial^2\Phi}{\partial\xi^2}\right) +$$
$$+ \frac{(f_1+2F)f_2}{B^2}\left(\frac{\partial\Phi}{\partial\xi}\frac{\partial^2\Phi}{\partial f_2\partial\xi} - \frac{\partial\Phi}{\partial f_2}\frac{\partial^2\Phi}{\partial\xi^2}\right), \qquad (1)$$

where

$$F = 2\left\{B\frac{\partial^2\Phi}{\partial\xi^2}\Big|_{\xi=0} - \left[2+\frac{1}{B}\int_0^\infty\left(1-\frac{\partial\Phi}{\partial\xi}\right)d\xi\right]f_1\right\}$$

with the boundary conditions

$$\Phi = \frac{\partial\Phi}{\partial\xi} = 0 \text{ for } \xi=0, \quad \frac{\partial\Phi}{\partial\xi}\to 1 \text{ as } \xi\to\infty$$

and the auxiliary conditions

$$F \geqslant 0\ ; \quad \frac{\partial^2\Phi}{\partial\xi^2} \geqslant 0.$$

Let us introduce the substitution

$$u = \frac{\partial \varphi}{\partial \xi}; \quad v = -\frac{\partial \varphi}{\partial f_1}; \quad w = -\frac{\partial \varphi}{\partial f_2},$$

then equation (1) reduces to the system

$$\left.\begin{aligned} &\frac{\partial^2 u}{\partial \xi^2} + \frac{F + 2f_1}{2B^2}\varphi\frac{\partial u}{\partial \xi} + \frac{f_1}{B^2}(1-u^2) = \frac{Ff_1 + f_2}{B^2}\left(u\frac{\partial u}{\partial f_1} + \right. \\ &\quad \left. + v\frac{\partial u}{\partial \xi}\right) + \frac{(f_1 + 2F)f_2}{B^2}\left(u\frac{\partial u}{\partial f_2} + w\frac{\partial u}{\partial \xi}\right); \\ &\frac{\partial v}{\partial \xi} + \frac{\partial u}{\partial f_1} = 0; \\ &\frac{\partial w}{\partial \xi} + \frac{\partial u}{\partial f_2} = 0, \end{aligned}\right\} \qquad (2)$$

where

$$F = 2\left\{B\left.\frac{\partial u}{\partial \xi}\right|_{\xi=0} - \left[2 + \frac{1}{B}\int_0^\infty (1-u)\,d\xi\right]f_1\right\}$$

with the boundary conditions $u = 0,\ v = 0,\ w = 0$ for $\xi = 0$;

$$u \to 1 \text{ as } \xi \to \infty.$$

The point $f_1 = 0$, $f_2 = 0$ is a singular point of system (2) and in it the unknown functions must satisfy the one-dimensional equation

$$\frac{\partial^2 u}{\partial \xi^2} + \frac{F}{2B^2}\varphi\frac{\partial u}{\partial \xi} = 0, \qquad (3)$$

where

$$F = 2B\left.\frac{\partial u}{\partial \xi}\right|_{\xi=0}; \quad \varphi = \int_0^\xi u\,d\xi$$

with the boundary conditions

$$u = 0 \text{ for } \xi = 0;\ u \to 1 \text{ as } \xi \to \infty.$$

Assuming in system (2) $f_2 = 0$, we get the system

$$\left.\begin{aligned} &\frac{Ff_1}{B^2}u\frac{\partial u}{\partial f_1} + \frac{Ff_1}{B^2}v\frac{\partial u}{\partial \xi} = \frac{\partial^2 u}{\partial \xi^2} + \frac{F + 2f_1}{2B^2}\varphi\frac{\partial u}{\partial \xi} + \frac{f_1}{B^2}(1-u^2); \\ &\frac{\partial v}{\partial \xi} + \frac{\partial u}{\partial f_1} = 0, \end{aligned}\right\} \qquad (4)$$

where

$$F = 2\left\{B\left.\frac{\partial u}{\partial \xi}\right|_{\xi=0} - \left[2 + \frac{1}{B}\int_0^\infty (1-u)\,d\xi\right]f_1\right\}; \quad \varphi = \int_0^\xi u\,d\xi$$

with the boundary conditions

$$u = v = 0 \text{ for } \xi = 0; \quad u \to 1 \text{ as } \xi \to \infty.$$

4. Solution of the problem

The problem is solved by the finite differences method.

1. Replacing the derivatives in equation (3) with finite-difference relations, we get the system

$$\left.\begin{aligned} &\frac{u_{\kappa+1}^n - 2u_\kappa^n + u_{\kappa-1}^n}{(\Delta\xi)^2} + \frac{F^{n-1}}{2B^2}\varphi_\kappa^{n-1}\frac{u_{\kappa+1}^n - u_{\kappa-1}^n}{2\Delta\xi} = 0; \\ &F^{n-1} = 2B\,\frac{-3u_0^{n-1} + 4u_1^{n-1} - u_2^{n-1}}{2\Delta\xi}. \end{aligned}\right\} \quad (5)$$

with the boundary condition $u_0 = 0$, $u_K = 1$.

Since equation (3) is nonlinear, it is solved by the method of iterations, where the superscript n in system (5) denotes the iteration number. System (5) is reduced to the form

$$a_\kappa^n u_{\kappa-1}^n - 2b_\kappa^n u_\kappa^n + c_\kappa^n u_{\kappa+1}^n = g_\kappa^n, \quad (6)$$

where

$$a_\kappa^n = 4B^2 - F^{n-1}\varphi_\kappa^{n-1}\Delta\xi; \quad b_\kappa^n = 4B^2;$$

$$c_\kappa^n = 4B^2 + F^{n-1}\varphi_\kappa^{n-1}\Delta\xi; \quad g_\kappa^n = 0.$$

System (6) is solved by the dispersion method [3] with different $\Delta\xi$ and different K such that a further increase in K and a decrease in $\Delta\xi$ with desired accuracy is not reflected in the results. Finally K = 40, $\Delta\xi$ = 0.15 was chosen for the results to agree in the third decimal place.

2. Replacing the derivatives in system (4) with finite-difference relations, we get the system

$$\left(\frac{F^{n-1} f_1}{B^2} u^n\right)_{i+\frac{1}{2},\kappa} \frac{u^n_{i+1,\kappa} - u^n_{i,\kappa}}{\Delta f_1} + \left(\frac{F^{n-1} f_1}{B^2} v^n\right)_{i+\frac{1}{2},\kappa} \times$$

$$\times \frac{s_2 (u_{i+1,\kappa+1} - u_{i+1,\kappa-1}) + (1 - s_2)(u_{i,\kappa+1} - u_{i,\kappa-1})}{2\Delta\xi} =$$

$$= \frac{s_1 (u^n_{i+1,\kappa+1} + u^n_{i+1,\kappa-1} - 2u^n_{i+1,\kappa}) + (1 - s_1)(u^n_{i,\kappa+1} + u^n_{i,\kappa-1} - 2u^n_{i,\kappa})}{(\Delta\xi)^2} +$$

$$+ \left(\frac{F^{n-1} + 2f_1}{2B^2} \varphi^{n-1}\right)_{i+\frac{1}{2},\kappa} \times$$

$$\times \frac{s_2 (u^n_{i+1,\kappa+1} - u^n_{i+1,\kappa-1}) + (1 - s_2)(u^n_{i,\kappa+1} - u^n_{i,\kappa-1})}{2\Delta\xi} +$$

$$+ \left\{\frac{f_1}{B^2} [1 - (u^n)^2]\right\}_{i+\frac{1}{2},\kappa};$$

$$v^n_{i+1,\kappa+1} = v^n_{i+1,\kappa} - \frac{\Delta\xi}{2} \frac{1}{\Delta f_1} (u^n_{i+1,\kappa+1} + u^n_{i+1,\kappa} - u^n_{i,\kappa+1} - u^n_{i,\kappa});$$

$$F^{n-1} = 2\left\{B \frac{\partial u}{\partial \xi}\Big|_{\xi=0} - \left[2 + \frac{1}{B}\int_0^\infty (1 - u^{n-1})\, d\xi\right] f_1\right\} \qquad (7)$$

with the boundary conditions

$$u_0 = v_0 = 0, \quad u_K = 1.$$

where

$$\frac{\partial u^{n-1}}{\partial \xi}\Big|_{\xi=0} = \frac{-3u_0^{n-1} + 4u_1^{n-1} - u_2^{n-1}}{2\Delta\xi}.$$

The value of the function at the halfway point equals the arithmetic mean of its values in the neighboring points.

To solve system (7), we take the solution of system (5) as the initial values of the functions u, Φ, . System (7) reduces to the form

$$a^n_{i,\kappa} u^n_{i+1,\kappa-1} - 2b^n_{i,\kappa} u_{i+1,\kappa} + c^n_{i,\kappa} u^n_{i+1,\kappa+1} = g^n_{i,\kappa}, \qquad (8)$$

where

$$a^n_{i,\kappa} = -s_1 - \frac{s_2 \Delta\xi}{4B^2}\left[(F + 2f_1)\Phi\right]^{n-1}_{i+\frac{1}{2},\kappa} + \frac{s_2\Delta\xi}{2B^2}\left[F^{n-1}f_1 v^n\right]_{i+\frac{1}{2},\kappa};$$

$$2b^n_{i,\kappa} = 2s_1 + \frac{(\Delta\xi)^2}{B^2}\left(\frac{F^{n-1}f_1}{\Delta f_1}u^n\right)_{i+\frac{1}{2},\kappa};$$

$$c^n_{i,\kappa} = s_1 + \frac{s_2\Delta\xi}{4B^2}\left[(F + 2f_1)\Phi\right]^{n-1}_{i+\frac{1}{2},\kappa} - \frac{s_2\Delta\xi}{2B^2}\left[F^{n-1}f_1 v^n\right]_{i+\frac{1}{2},\kappa};$$

$$\begin{aligned} g^n_{i,\kappa} = -\Big[(1 - s_1) + \frac{(1 - s_2)\Delta\xi}{4B^2}\left[(F + 2f_1)\Phi\right]^{n-1}_{i+\frac{1}{2},\kappa} - \\ - \frac{(1 - s_2)\Delta\xi}{2B^2}\left(F^{n-1}f_1 v^n\right)_{i+\frac{1}{2},\kappa}\Big] u^n_{i,\kappa+1} + \\ + \left[2(1 - s_1) - \frac{(\Delta\xi)^2}{B^2}\left(\frac{F^{n-1}f_1}{\Delta f_1}u^n\right)_{i+\frac{1}{2},\kappa}\right] u^n_{i,\kappa} - \\ - \Big[(1 - s_1) - \frac{(1 - s_2)\Delta\xi}{4B^2}\left[(F + 2f_1)\Phi\right]^{n-1}_{i+\frac{1}{2},\kappa} + \\ + \frac{(1 - s_2)}{2B^2}\left(F^{n-1}f_1 v^n\right)_{i+\frac{1}{2},\kappa}\Big] u^n_{i,\kappa-1} - \\ - \frac{(\Delta\xi)^2}{B^2}\left\{f_1\left[1 - (u^n)^2\right]\right\}_{i+\frac{1}{2},\kappa} \quad (0 < s_1 \leq 1, \quad 0 \leq s_2 \leq 1). \end{aligned}$$

System (3) is solved by the dispersion method. The solution is arrived at to the right from the point $f_1 = 0$ to the point at which $F^{n-1} = 0$, and to the left up to the point where $\left.\frac{\partial u}{\partial \xi}\right|_{\xi=0} = 0$.

By choosing the values of the parameters s_1 and s_2, we can get different difference schemes. For a given accuracy the most economical scheme, that is, the one that can be used with the largest Δf_1 values, proved to be the scheme which is obtained when $s_1 = s_2 = 0.5$. Automatic selection of the pitch was employed in selecting the Δf_1 values ensuring the given accuracy. Calculation of the coefficients at the halfway points was made iteratively until the results agreed for a given pitch with the given accuracy.

3. Replacing the derivatives in system (2) with finite-difference relations, we get the system

$$
\begin{aligned}
&\frac{s_1\left(u^n_{i+1,\kappa+1,j+1}-2u^n_{i+1,\kappa,j+1}+u^n_{i+1,\kappa-1,j+1}\right)+(1-s_1)\left(u^n_{i,\kappa+1,j+1}-2u^n_{i,\kappa,j+1}+u^n_{i,\kappa-1,j+1}\right)}{(\Delta\xi)^2}+\\
&+\left(\frac{F^{n-1}+2f_1}{2B^2}\,\varphi^{n-1}\right)_{i+\frac{1}{2},\kappa,j+1}\times\\
&\times\frac{s_2\left(u^n_{i+1,\kappa+1,j+1}-u^n_{i+1,\kappa-1,j+1}\right)+(1-s_2)\left(u^n_{i,\kappa+1,j+1}-u^n_{i,\kappa-1,j+1}\right)}{2\Delta\xi}+\\
&+\left[\frac{f_1}{B^2}\left[1-(u^n)^2\right]\right]_{i+\frac{1}{2},\kappa,j+1}=\\
&=\left(\frac{F^{n-1}f_1+f_2}{B^2}\,u^n\right)_{i+\frac{1}{2},\kappa,j+1}\frac{u^n_{i+1,\kappa,j+1}-u^n_{i,\kappa,j+1}}{\Delta f_1}+\\
&+\left(\frac{F^{n-1}f_1+f_2}{B^2}\,v^n\right)_{i+\frac{1}{2},\kappa,j+1}\times\\
&\times\frac{s_2\left(u^n_{i+1,\kappa+1,j+1}-u^n_{i+1,\kappa-1,j+1}\right)+(1-s_2)\left(u^n_{i,\kappa+1,j+1}-u^n_{i,\kappa-1,j+1}\right)}{2\Delta\xi}+\\
&+s_4\left(\frac{(f_1-2F^{n-1})f_2}{B^2}\right)_{i+1,\kappa,j}\left(\frac{u^n_{i+1,\kappa,j+1}-u^n_{i+1,\kappa,j}}{\Delta f_2}u^n_{i+1,\kappa,j}+\right.\\
&\left.+\frac{u^n_{i+1,\kappa+1,j}-u^n_{i+1,\kappa-1,j}}{2\Delta\xi}\,w^n_{i+1,\kappa,j+1}\right)+\\
&+s_3\left(\frac{(f_1+2F^{n-1})f_2}{B^2}\right)_{i,\kappa,j}\left(u^n_{i,\kappa,j}\frac{u^n_{i,\kappa,j+1}-u^n_{i,\kappa,j}}{\Delta f_2}+\right.\\
&\left.+w^n_{i,\kappa,j+1}\frac{u^n_{i,\kappa+1,j}-u^n_{i,\kappa-1,j}}{2\Delta\xi}\right);\\
&v^n_{i+1,\kappa+1,j+1}=v^n_{i+1,\kappa,j+1}-\frac{\Delta\xi}{2\Delta f_1}\left(u^n_{i+1,\kappa,j+1}-u^n_{i,\kappa,j+1}+\right.\\
&\left.+u^n_{i+1,\kappa+1,j+1}-u^n_{i,\kappa+1,j+1}\right);\\
&w^n_{i+1,\kappa+1,j+1}=w^n_{i+1,\kappa,j+1}-\frac{\Delta\xi}{2\Delta f_2}\left(u^n_{i+1,\kappa,j+1}-u^n_{i+1,\kappa,j}+\right.\\
&\left.+u^n_{i+1,\kappa+1,j+1}-u^n_{i+1,\kappa+1,j}\right)
\end{aligned}
\qquad (2)
$$

with the boundary conditions

$$u_0=v_0=w_0=0,\quad u_K=1.$$

Here

$$F^{n-1}=2\left\{B\frac{\partial u^{n-1}}{\partial\xi}\Big|_{\xi=0}-\left[2-\frac{1}{B}\int_0^\infty(1-u^{n-1})\,d\xi\right]f_1\right\}.$$

The solution of system (7) was adopted for the initial function values when solving system (9).

System (9) is reduced to the form

$$a^n_{i+1,\kappa,j+1} u^n_{i+1,\kappa-1,j+1} - 2b^n_{i+1,\kappa,j+1} u^n_{i+1,\kappa,j+1} + c^n_{i+1,\kappa,j+1} u^n_{i+1,\kappa+1,j+1} = g^n_{i+1,\kappa,j+1} \qquad (10)$$

where

$$a^n_{i+1,\kappa,j+1} = s_1 - \frac{s_2 \Delta\xi}{4B^2}\left[(F+2f_1)\varphi\right]^{n-1}_{i+\frac{1}{2},\kappa,j+1} + \frac{s_2 \Delta\xi}{2B^2}\left[(F^{n-1}f_1+f_2)v^n\right]_{i+\frac{1}{2},\kappa,j+1};$$

$$2b^n_{i+1,\kappa,j+1} = 2s_1 + \frac{(\Delta\xi)^2}{B^2}\left(\frac{F^{n-1}f_1+f_2}{\Delta f_1}u^n\right)_{i+\frac{1}{2},\kappa,j+1} + \frac{S_4(\Delta\xi)^2}{B^2}\left(\frac{(f_1+2F^{n-1})f_2}{\Delta f_2}u^n\right)_{i+1,\kappa,j};$$

$$c^n_{i+1,\kappa,j+1} = s_1 + \frac{s_2 \Delta\xi}{4B^2}\left[(F+2f_1)\varphi\right]^{n-1}_{i+\frac{1}{2},\kappa,j+1} - \frac{s_2 \Delta\xi}{2B^2}\left[(F^{n-1}f_1+f_2)v^n\right]_{i+\frac{1}{2},\kappa,j+1};$$

$$\begin{aligned} g^n_{i+1,\kappa,j+1} = &-(1-s_1)\left(u^n_{i,\kappa+1,j+1} - 2u^n_{i,\kappa,j+1} + u^n_{i,\kappa-1,j+1}\right) - \\ &- \frac{(1-s_1)\Delta\xi}{4B^2}\left[(F+2f_1)\varphi\right]^{n-1}_{i+\frac{1}{2},\kappa,j+1}\left(u^n_{i,\kappa+1,j+1} - u^n_{i,\kappa-1,j+1}\right) - \\ &- \frac{(\Delta\xi)^2}{B^2}\left[f_1(1-u^2)\right]^n_{i+\frac{1}{2},\kappa,j+1} - \frac{(\Delta\xi)^2}{B^2}\left(\frac{F^{n-1}f_1+f_2}{\Delta f_1}u^n\right)_{i+\frac{1}{2},\kappa,j+1} u^n_{i,\kappa,j+1} + \\ &+ \frac{(1-s_1)\Delta\xi}{2B^2}\left[(F^{n-1}f_1+f_2)v^n\right]_{i+\frac{1}{2},\kappa,j+1}\left(u^n_{i,\kappa+1,j+1} - u^n_{i,\kappa-1,j+1}\right) + \\ &+ s_3\frac{(\Delta\xi)^2}{B^2}\left[\frac{(f_1+2F)f_2}{\Delta f_2}\right]^{n-1}_{i,\kappa,j}\Big[u^n_{i,\kappa,j}\left(u^n_{i,\kappa,j+1} - u^n_{i,\kappa,j}\right) + \\ &+ w^n_{i,\kappa,j+1}\Delta f_2 \frac{u^n_{i,\kappa+1,j} - u^n_{i,\kappa-1,j}}{2\Delta\xi}\Big] + \\ &+ s_4\frac{(\Delta\xi)^2}{B^2}\left[\frac{(f_1+2F)f_2}{\Delta f_2}\right]^{n-1}_{i+1,\kappa,j}\Big[w^n_{i+1,\kappa,j+1}\frac{\left(u^n_{i+1,\kappa+1,j} - u^n_{i+1,\kappa-1,j}\right)\Delta f_2}{2\Delta\xi} - \\ &- \left(u^n_{i+1,\kappa,j}\right)^2\Big]. \end{aligned}$$

System (10) is solved by the dispersion method. The value of the variable f_1 in which $F_{i,j}f_{1i} + f_{2,j+1} = 0$ was determined in each layer f_{2i} and the f_1 value was adopted as the starting-point for computation at the (j + 1)-th layer. In practical computation, the point where the quantity $F_{i,j}f_{1,i} + f_{2,j+1}$ changes sign was taken as the initial f_1 value. In the first layer, i. e., $j = 1$, $f_1 = 0$ was taken as the initial f_1 value. For the zero layer, the region of integration was found to be bounded by the values $M_2 \leq f_1 \leq M_1$. In those cases when for $f_2 \neq 0$ the bounds $f = 0$ and $\left.\frac{\partial u}{\partial \xi}\right|_{\xi=0} = 0$ did not exceed the region $[M_1, M_2]$, $s_3 = 0$, $s_4 = 1$ was adopted, which yielded a stable scheme. If $f_2 < 0$ and $f_1 < 0$, then the condition $\left.\frac{\partial u}{\partial \xi}\right|_{\xi=0} = 0$ was satisfied for $f_1 \leq M_2$. In this case the region of integration was extended for each layer by one point due to the selection of $s_3 = 1$, $s_4 = 0$ for computation in the layer.

The largest possible Δf_2 value for a given accuracy proved to be $|\Delta f_2| = 0.001$, with which the final calculations were carried out. Some results in the calculations are presented in the table ($\zeta = \left.\frac{\partial u}{\partial \xi}\right|_{\xi=0}$).

All calculations were performed on the BESM-3 computer of the LOTsEMI [expansion unknown] of the USSR Academy of Sciences.

[illegible]lues of the quantities [illegible] and [illegible] in the [illegible]-parameter [illegible]ximation

$f_2 = 0$			$f_2 = 0{,}01$			$f_2 = 0{,}04$			$f_2 = 0{,}07$		
f_1	F	ζ	f_1	F	ζ	f_1	F	ζ	f_1	F	ζ
0,085	−0,0124	0,3535	0,0825	−0,0010	0,3494	0,08125	−0,0005	0,3430	0,07525	−0,0958	0,3250
0,08	0,0235	0,3511	0,08	0,0091	0,3160	0,08	0,0056	0,3415	0,075	0,0012	0,3234
0,07	0,0706	0,3350	0,075	0,0332	0,3381	0,075	0,0325	0,3314	0,07	0,0274	0,3161
0,06	0,1196	0,3190	0,07	0,0578	0,3302	0,07	0,0586	0,3272	0,06	0,0804	0,3015
0,05	0,1702	0,3031	0,06	0,1080	0,3146	0,06	0,1110	0,3127	0,05	0,1311	[illegible]
0,04	0,2220	0,2870	0,05	0,1597	0,2989	0,05	0,1632	0,2978	0,04	0,1886	0,2715
0,03	0,2752	0,2709	0,04	0,2131	0,2833	0,04	0,2152	0,2822	0,03	0,2439	0,2560
0,02	0,3295	0,2545	0,03	0,2691	0,2680	0,03	0,2674	[illegible]	0,02	0,3000	0,2403
0,01	0,3850	0,2379	0,02	0,3298	0,2541	0,02	0,3211	0,2493	0,01	0,3570	0,2241
0	0,4416	0,2208	0,01	0,4054	0,2408	0,01	0,3761	0,2333	0	0,4149	0,2074
−0,01	0,4998	0,2034	0	0,4727	0,2363	0	0,4332	0,2166	−0,01	0,4738	0,1902
−0,02	0,5591	0,1855	−0,01	0,5160	0,2128	−0,01	0,4912	0,1995	−0,02	0,5339	0,1724
−0,03	0,6197	0,1668	−0,02	0,5732	0,1949	−0,02	0,5503	0,1817	−0,03	0,5951	0,1537
−0,04	0,6817	0,1471	−0,03	0,6311	0,1759	−0,03	0,6109	0,1633	−0,04	0,6578	0,1338
−0,05	0,7454	0,1263	−0,04	0,6910	0,1564	−0,04	0,6732	0,1412	−0,05	0,7221	0,1125
−0,06	0,8110	0,1036	−0,05	0,7528	0,1359	−0,05	0,7374	0,1243	−0,06	0,7885	0,0890
−0,07	0,8792	0,0780	−0,06	0,8166	0,1139	−0,06	0,8037	0,1026	−0,07	0,8578	0,0622
−0,08375	0,9789	0,0325	−0,07	0,8827	0,0899	−0,07	0,8711	0,0774	−0,08	0,9324	0,0295
−0,09	1,0266	−0,0051	−0,08	0,9518	0,0621	−0,08	0,9120	0,0171	−0,085	0,9713	0,0011
—	—	—	−0,085	0,9879	0,00157	−0,085	0,9793	0,0280	−0,08625	0,9812	−0,0048
—	—	—	−0,09	1,025	0,0259	−0,09	1,018	0,0018	—	—	—

[Continuation one]

$f_0 = -0,03$			$f_0 = -0,04$			$f_0 = -0,07$			$f_0 = -0,1$		
f_1	F	ζ	f_1	F	ζ	f_1	F	ζ	f_1	F	ζ
0,08	0,0936	0,3609	0,04	0,2791	0,3119	0,03	0,2985	0,2811	0,03	0,3012	[illegible]
0,07	0,0616	0,3391	0,03	0,2913	0,2793	0,02	0,3663	0,2713	0,02	0,3765	0,2734
0,06	0,1285	0,3225	0,02	0,3574	0,2672	0,01	0,4286	0,2589	0,01	0,4355	0,2619
0,05	0,1778	0,3061	0,01	0,4151	0,2522	0	0,4939	0,2469	0	0,5026	0,2514
0,04	0,2286	0,2698	0	0,4771	0,2386	−0,01	0,5469	0,2294	−0,01	0,5605	0,2334
0,03	0,2820	0,2739	−0,01	0,5290	0,2189	−0,02	0,6038	0,2118	−0,02	0,6196	0,2194
0,02	0,3364	0,2576	−0,02	0,5833	0,2002	−0,03	0,6636	0,1932	−0,03	0,6769	0,2023
0,01	0,3909	0,2407	−0,03	0,6438	0,1813	−0,04	0,7225	0,1738	−0,04	0,7387	[illegible]
0	0,4516	0,2258	−0,04	0,7042	0,1620	−0,05	0,7832	0,1536	−0,05	0,7933	0,1649
−0,01	0,5058	0,2067	−0,05	0,7666	0,1418	−0,06	0,8461	0,1324	−0,06	0,8611	0,1444
−0,02	0,5650	0,1888	−0,06	0,8308	0,1204	−0,07	0,9113	0,1099	−0,07	0,9249	0,1223
−0,03	0,6257	0,1705	−0,07	0,8974	0,0972	−0,08	0,9788	0,0851	−0,08	0,9905	0,0982
−0,04	0,6875	0,1511	−0,08	0,9669	0,0714	−0,09	1,0496	0,0555	−0,09	1,0596	0,0705
−0,05	0,7510	0,1306	−0,09	1,0401	0,0391	−0,095	1,0868	0,0378	−0,095	1,0957	0,0548
−0,06	0,8162	0,1083	−0,0925	1,0503	0,0291	−0,1	1,1254	0,0154	−0,1	1,1331	0,0368
−0,07	0,8839	0,0838	−0,095	1,0783	0,0172	−0,1025	1,1442	−0,001	−0,105	1,1722	0,0143
−0,08	0,9519	0,0551	−0,0975	1,0973	0,0013	—	—	—	−0,1075	1,1908	−0,0024
−0,09	1,0304	0,0137	—	—	—	—	—	—	—	—	—
−0,0925	1,0192	−0,0018	—	—	—	—	—	—	—	—	—

Table [Continuation 2?]

$f_0 = -0.13$			$f_0 = -0.15$			$f_0 = -0.16$			$f_0 = -0.22$		
f_1	F	ξ	f_1	F	ξ	f_1	F	ξ	f_1	F	ξ
0,03	0,3039	0,2833	0,03	0,3058	0,2815	0,03	0,3090	0,2851	0,03	0,3112	[illegible]
0,02	0,3745	0,2751	0,02	0,3786	0,2769	0,02	0,3817	0,2783	0,02	0,3844	[illegible]
0,01	0,4109	0,2615	0,01	0,41[illegible]	0,2670	0,01	0,4500	[illegible]	0,01	0,4533	[illegible]
0	0,5099	0,2550	0	0,5157	0,2578	0	0,5203	0,2602	0	0,5242	0,2621
−0,01	0,5685	0,2397	−0,01	0,5752	0,2432	−0,01	0,5809	0,2461	−0,01	0,5857	0,2483
−0,02	0,6290	0,2215	−0,02	0,6366	0,2287	−0,02	0,6431	[illegible]	−0,02	0,6486	0,2333
−0,03	0,6897	0,2086	−0,03	0,6[illegible]	0,2135	−0,03	0,70[illegible]	[illegible]	−0,03	0,7116	0,2213
−0,04	0,7507	0,1907	−0,04	0,7600	0,1974	−0,04	0,7678	0,2023	−0,04	0,7746	[illegible]
−0,05	0,8122	0,1737	−0,05	0,8222	0,1804	−0,05	0,8306	0,1859	−0,05	0,8378	0,19[illegible]
−0,06	0,8745	0,1545	−0,06	0,8851	0,1622	−0,06	0,8939	0,1785	−0,06	0,9016	0,1730
−0,07	0,9360	0,1338	−0,07	0,9490	0,1428	−0,07	0,9582	0,1503	−0,07	0,9661	0,1531
−0,08	1,003	0,1110	−0,08	1,014	0,1215	−0,08	1,023	0,1299	−0,08	1,031	0,1360
−0,09	1,0702	0,0851	−0,09	1,0805	0,0976	−0,09	1,0897	0,1075	−0,09	1,0977	0,1156
−0,095	1,1052	0,0707	−0,095	1,1115	0,0841	−0,095	1,1237	0,0951	−0,095	1,1316	0,1042
−0,1	1,1413	0,0517	−0,1	1,1500	0,0702	−0,1	1,1581	0,0824	−0,1	1,1661	0,0921
−0,105	1,1789	0,0361	−0,105	1,1863	0,0513	−0,105	1,1940	0,0692	−0,105	1,2013	0,0792
−0,11	1,2181	0,0136	−0,11	1,2240	0,0360	−0,11	1,2306	0,0525	−0,11	1,2373	0,0650
−0,1125	1,2369	−0,0025	−0,115	1,2632	0,0131	−0,115	1,2[illegible]	0,0312	−0,115	1,2743	0,0491
—	—	—	−0,1175	1,2823	−0,0027	−0,12	1,308	0,0110	−0,12	1,313	0,0305
—	—	—	—	—	—	−0,1225	1,327	−0,0032	−0,1225	1,332	0,0194
—	—	—	—	—	—	—	—	—	−0,125	1,352	0,0063
—	—	—	—	—	—	—	—	—	−0,12625	1,362	−0,0016