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Volume I

IMPLEMENTATION STUDIES FOR A RELIABILITY-BASED STATIC STRENGTH CRITERIA SYSTEM

Volume I, Evaluation

M. C. Campion, L. C. Hanson, D. S. Morcock, et al.
Lockheed-Georgia Company

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FOREWORD

This report was prepared by Lockheed-Georgia Company, Marietta, Georgia, for the Design Criteria Branch of the Structures Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Air Force Contract No. F33(615)-71-C-1129, Project No. 1367, "Structural Design Criteria," Task No. 136714, "Airframe Structural Design Adequacy."

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Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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LIST OF SYMBOLS

AMSTR	Intended mean strength of the structural design
DF	Design factor = FS(1 + MS)
dx, dX	Interval width
DSNLD	Factored load used for sizing the structure
δP_f	Probability of failure when strength is in the interval $x \pm \frac{1}{2} dx$
F_{bru}, F_{bry}	Ultimate and yield strengths in bearing
F_{cy}	Yield strength in compression
FS	Design factor of safety
F_{su}	Ultimate strength in shear
F_{tu}, F_{ty}	Ultimate and yield strengths in tension
GWT	Design gross weight
MS	Design margin of safety
n_z, N_z	Normal load factor
p	Probability of a value in the interval $x \pm \frac{1}{2} dx$
p_{s_M}, $p(\bar{x}_i)$	Probability that mean strength is in the interval $x \pm \frac{1}{2} dx$
p_{x_s}, $p_s(x)$	Probability that strength is in the interval $x \pm \frac{1}{2} dx$
P	Probability of value less than (or greater than) X
P_F	Probability of failure
P_L	Probability that load equals or exceeds X
P/PU	Test strength as fraction of intended ultimate strength
R	Reliability = $1 - P_F$
s	Standard deviation

$\bar{S}$	Indicated mean strength of the fleet
S_{ALL}	Design allowable strength (number of standard deviations below the mean)
SUMA, SUMB	Fractions of total allotted to A and B families of double-family distribution
TF	Test factor (applied to UNFLD)
UNFLD	Unfactored design load used as basis for sizing the structure
v	Coefficient of variation = S/mean
v_A, v_B	Coefficients of variation of A and B families of double-family distribution
v_T	Resultant coefficient of variation of double-family distribution
W	Aircraft weight
WS	Wing station
x, X	General variable
$\bar{x}_A$, $\bar{x}_B$	Means of the A and B families of a double-family distribution
x_i	Particular value of the variable
$\bar{x}_i$	Particular value of the probable mean
$\bar{x}_T$	Resultant mean of double-family distribution
X_T	Test result
y, Y	Gumbel transform of the probability (P) of a value less than X

SECTION I
INTRODUCTION

Many attempts have been made to achieve the realization of techniques for applying reliability methods to the definition of structural strength. The most comprehensive of these was prepared by Innes Bouton and others and is described in AFFDL-TR-67-107. The three volumes of that report discussed previous methods and derived proposed methods covering both time-independent (static) and time-dependent (fatigue) strength. The full range of interactions with non-structural, operational, executive and contractual areas was discussed.

The study described in the present report was aimed at reviewing the proposed method for applying probabilistic techniques to the assessment of static strength reliability. This review was to identify the data requirements of the proposed method, the necessary changes to specifications and design handbooks, the interfaces with non-structural design areas and the steps to be taken during implementation of the method.

SECTION II

SUMMARY

A clear understanding of the various operations incorporated into the proposed static strength reliability analysis of AFFDL-TR-67-107 is necessary to its successful implementation. Section III provides a simple worked example which illustrates each step in turn using, first, dummy data and then realistic data. The categories of required data are defined.

Sections IV through IX discuss each category in turn, by means of studies of data pertinent to the C-141A cargo transport aircraft. Section X then summarizes the findings in the form of a trial application of the method to the wing of the C-141A.

Sections XI and XII discuss, respectively, the updating of the data to reflect the state of knowledge at each stage during the design and operational life of a vehicle, and the form in which the required data might be standardized.

Specific steps required to achieve the short-term and long-term implementation of the method are described in Section XIII, and the necessary changes to existing MIL-A specifications and AFSC Design Handbooks are summarized in Section XIV. Section XV contains the conclusions and recommendations resulting from the study.

Five appendices follow the main text. Appendix I outlines a technique for the use of bi-modal (double-family) statistical distributions; the Gumbel distribution of extremes is employed as an example, but the method is valid for a range of statistical distributions. Appendix II contains the basic equations of the computer program used in the study; this uses double-family Gumbel distributions, a constant calculation interval, and employs Bayes' theorem to incorporate the effects of test results, but is otherwise similar to the original program; many of the intermediate results are, however, printed. Appendix III describes the program, its input requirements and operation.

Appendix IV contains sample runs made with the program, and Appendix V shows the analysis of load and strength data using double-family representations.

SECTION III

EXAMPLE OF PROPOSED ANALYSIS SYSTEM

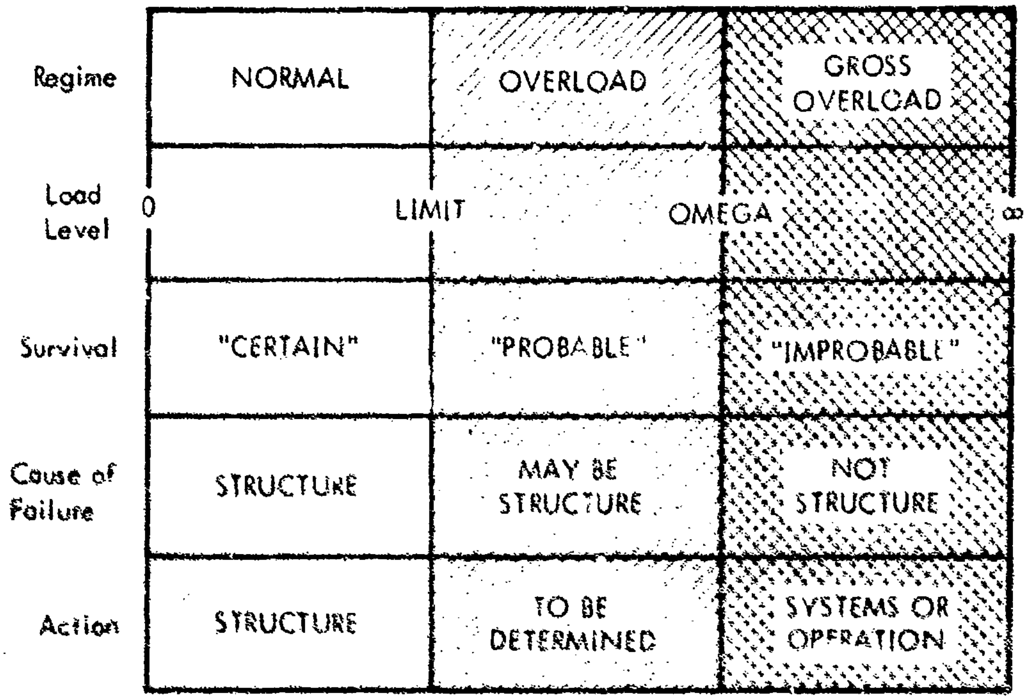
3.1 Introduction

Reference 1 discusses in detail the underlying philosophy of a reliability-based system of structural design criteria. Reference 2 summarizes the essential ingredients in brief fashion. Both documents are based on certain assumptions, some consciously recognized, but some unconsciously incorporated in the analytical procedures. Certain basic decisions must be made at intervals throughout the application of the proposed system, and many of those who would be responsible for the decisions will probably not be fully conversant with the mathematical processes involved.

The purpose of this Section is to illustrate,as far as possible, the physical meanings of the various steps in the process. The data required for each step will be identified, and its use demonstrated.

3.2 Computer Program

The computer program used for these examples was a modified version of that in reference 1, since many of the intermediate stages, which are necessary to an understanding of the implications,are not made visible in that program. The modified program, which is described in Appendix II, differs from the original in several respects. The statistical functions used are based on Gumbel's first asymptotic theory of extremes, rather than on a choice of normal, log-normal or Weibull distributions. Furthermore, the skewness of the loads spectrum is assumed positive (the tail extending towards higher loads), but the skewnessof the strength distribution is assumed negative (the tail extending towards lower strengths). Use is also made of double-family distributions (see Appendix I) to enable recognition of measured samples exhibiting such characteristics.



Regime	NORMAL	OVERLOAD	GROSS OVERLOAD
Load Level	0	LIMIT	OMEGA ∞
Survival	"CERTAIN"	"PROBABLE"	"IMPROBABLE"
Cause of Failure	STRUCTURE	MAY BE STRUCTURE	NOT STRUCTURE
Action	STRUCTURE	TO BE DETERMINED	SYSTEMS OR OPERATION

FIGURE 1 OPERATIONAL REGIMES

Provision is made for the resultant strength of the structure to be represented by a basic material strength distribution on which may be superimposed a second distribution attributable to the variations caused by fabrication processes.

Two interpretations of test results are incorporated in the modified program. The first is the same as that in the original program of reference 1, and recognizes the consequences of survival of the test load. In the case of multiple tests, this need not be interpreted as N tests surviving the lowest load carried. The second, which has been added, permits recognition of the implications of failures at different known loads. Bayesian techniques are employed to perform the modifications to the probable strength distribution, as recommended in reference 3.

3.3 Data Used

To provide greater clarity of the steps involved, the data used have greater dispersion that could normally be expected in practice. Hence, the numerical values must not be regarded as realistic. A realistic application is described in sub-section III-6.

3.4 The Two Design Conditions

a. The philosophy of the proposed system incorporates a number of interesting features, and the relationships between these must be fully understood if the application is to be realized. The operational regime of the aircraft is divided into the three areas shown in figure 1 (see reference 2).

b. At all conditions up to those possible within the specified limits of normal operation the probability of structural failure should be negligible; the desired reliability must be very close to unity.

c. At conditions above these, but only up to some "overload" level considered to be feasible, the structure should have a progressively diminishing chance of survival. In the proposed system this is represented by evaluation of the calculation of the risk of failure at a chosen design overload level defined as the omega condition.

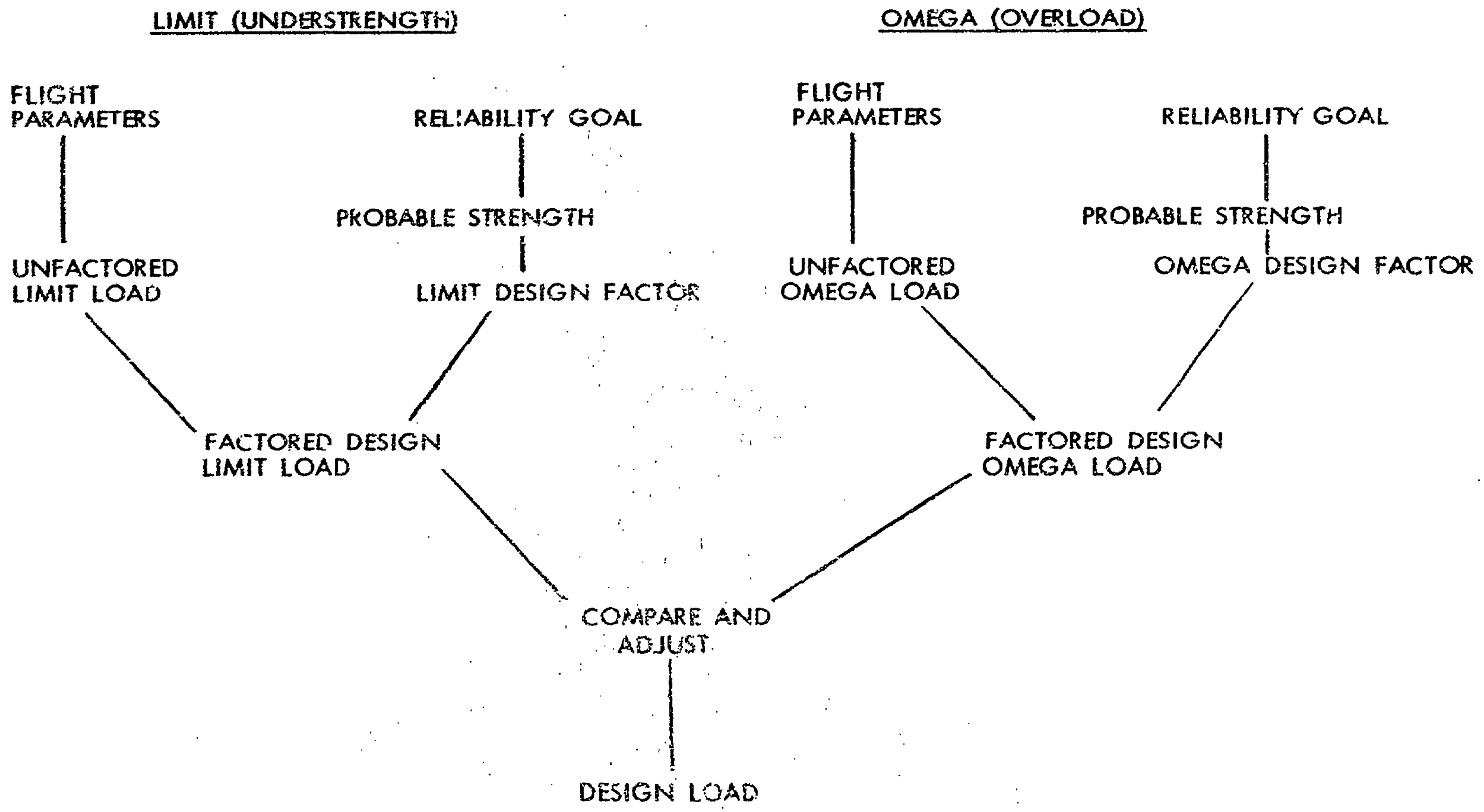


FIGURE 2 - DESIGN FLOW

d. At load levels beyond the omega condition, no structural performance is evaluated. Failure is regarded as inevitable and the responsibility for failure is assigned entirely to operational or sub-system management.

e. The structural design process must therefore start with the recognition of two simultaneous design conditions. Figure 2 shows the various steps involved. The critical load system occurring within the normal operational limits is evaluated to determine the unfactored design limit load. From a knowledge of the strength distribution appropriate to limit conditions (load interaction, temperature, pressure, etc.), a design factor can be selected which will enable the desired reliability goal to be attained; this should recognize the probability of discrepancies between the intended and actual strength levels (see sub-sections III-5 (v) and III-7 (iv) and Section VII). This is applied to the unfactored design limit load to give the factored design limit load. A corresponding sequence of calculations will result in a value for the factored design omega load.

f. These two factored load levels must then be compared. If they are equal, then a structure designed to the common load will meet both reliability requirements without penalty. The two other situations are more likely:

   (1) if the factored limit load is less than the factored omega load, then either the omega condition should be reduced in level or reliability, or the limit condition can be raised in level or reliability without penalty

   (2) if the factored omega condition is less than the factored limit condition, then a greater overload capacity can be provided (load or reliability), or the limit condition penalizes a design with the chosen overload capacity.

g. The test procedure becomes a means of disclosing the probability that the actual strength distribution differs from that intended. The test results will change the predicted reliability levels in a manner depending on the number of independent tests and on the test load. Predictions or assumptions may be included in the choice of design factor if so desired.

3.5 Worked Example

a. The first step requires the selection of the unfactored design load (UNFLD). This may be based either on the normal operational regime (limit load), or on the overload regime (omega load) as discussed in sub-section III-4; the difficulties of a meaningful definition are described in Section IV and V, but for the present example it is sufficient to assume that a limit value of 100 units has been selected, as shown in figure 3. This unfactored design load is used as a basis for defining the initial sizing of the structure.

b. The second step matches the factored design load (DSNLD) (a design safety factor, FS, on limit load may be incorporated if so desired, together with a design margin of safety, MS), where

$$\text{DSNLD} = \text{UNFLD} \times \text{FS} \times (1 + \text{MS}) \qquad \text{-(1)}$$

to some specified strength level defined as a number of standard deviations (S_{ALL}) below the intended mean strength. Conventionally, this will be implicit in the design allowable strengths, but must be specifically recognized in these statistical terms. The intended mean strength, AMSTR, is therefore known, since

$$\text{DSNLD} = \text{AMSTR}\,(1 - S_{ALL}) \qquad \text{-(2)}$$

Figure 4 shows this step in graphical terms. The assumed strength variation was assumed to be a double-family distribution containing a sub-family of weaker specimens.

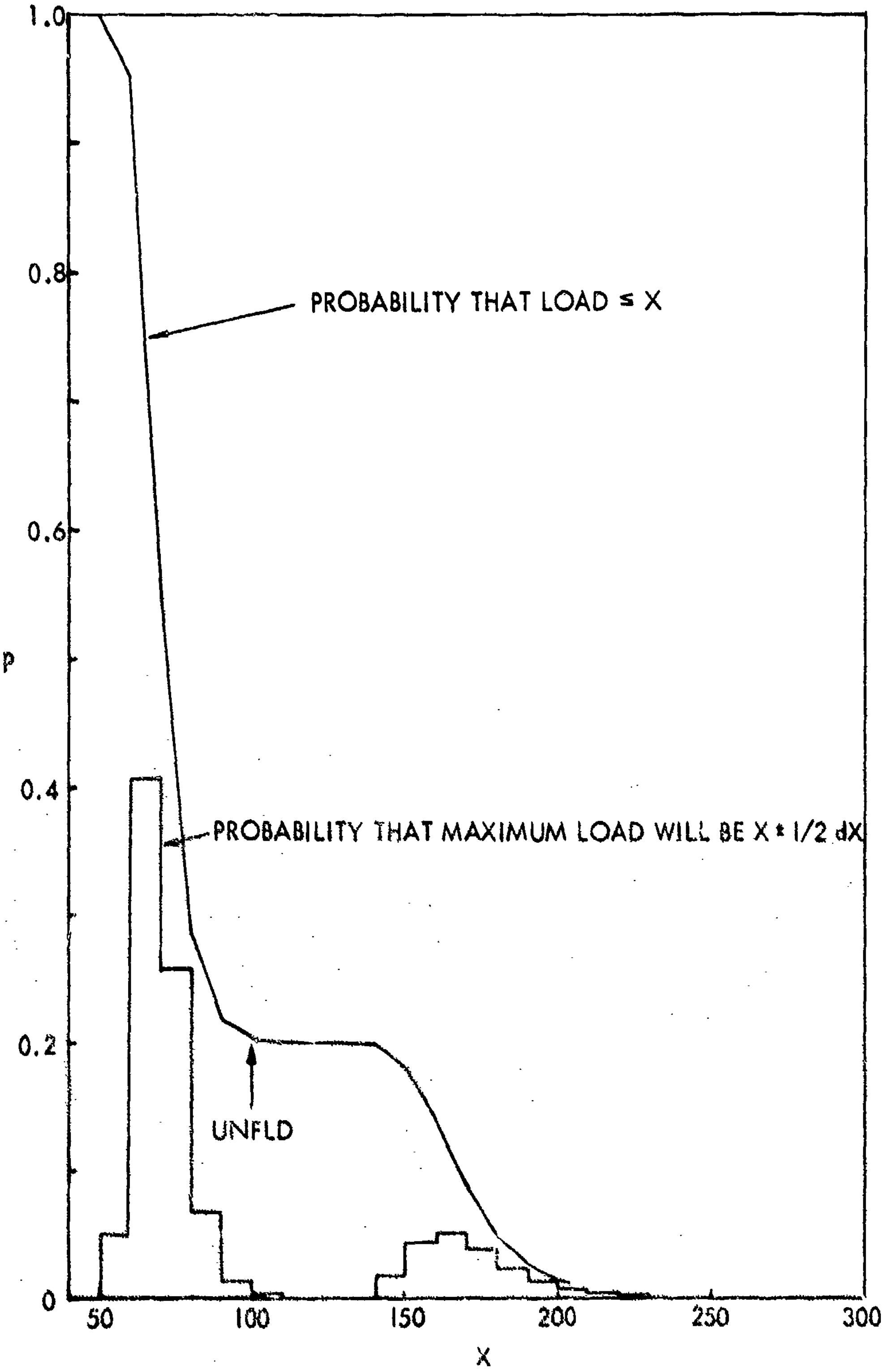


FIGURE 3 LOAD SPECTRUM FOR FIRST EXAMPLE

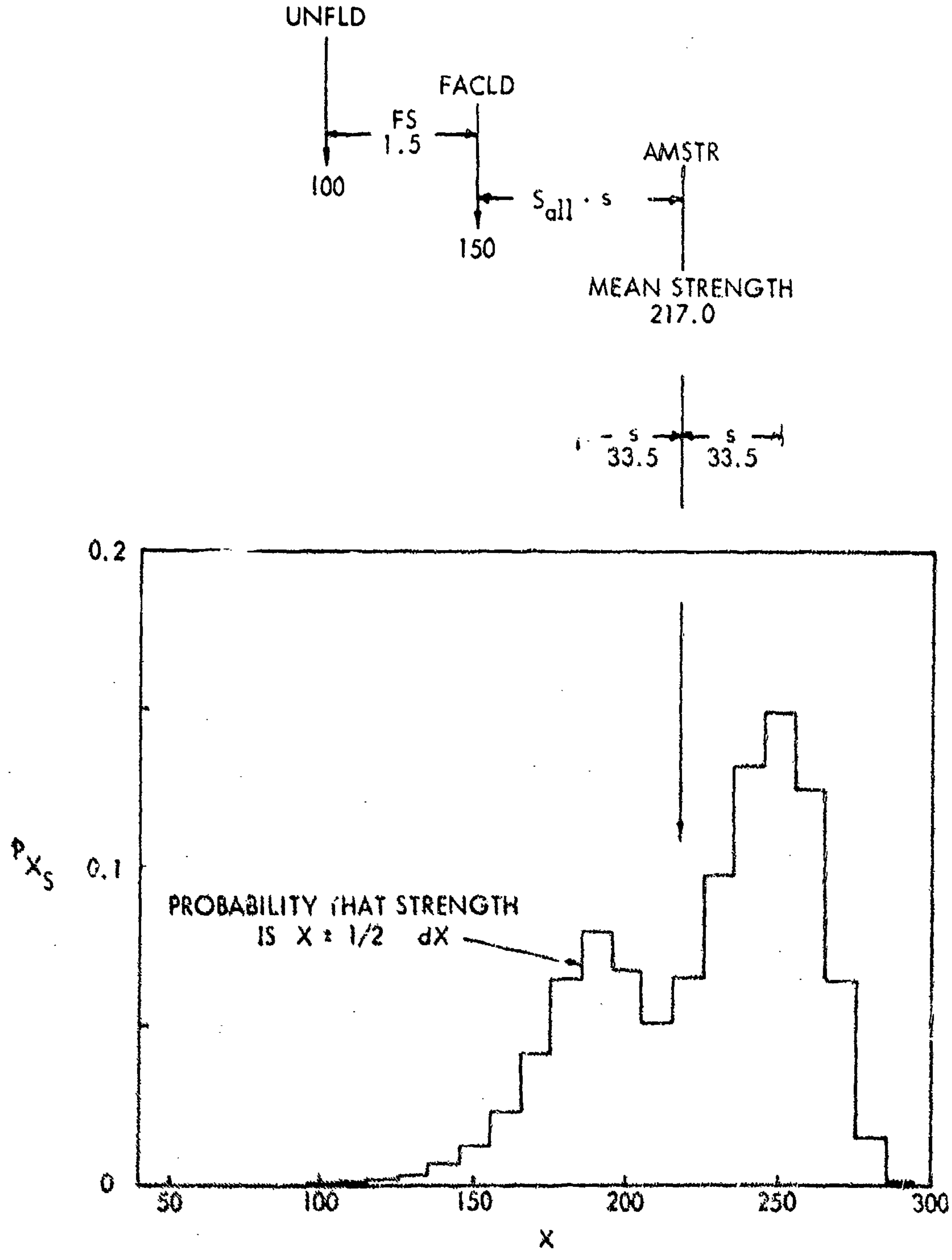


FIGURE 4 INTENDED MEAN STRENGTH (FIRST EXAMPLE)

c. So far, the procedure is virtually indistinguishable from that in use in the present deterministic criteria systems, but from this point onward the added consequences of a probabilistic system begin to emerge.

d. The intended reliability can now be evaluated, based on the premise that the actual mean strength of the whole production run of the particular structural item under consideration will actually be AMSTR. This implies not only that the loads and strength variations are correct, but also that there are no discrepancies of any kind in the design, the analysis, the material, the fabrication or the assembly of the structure. If this assumption of "no error" is made, then the probability of failure, if the strength is X (± 1/2dx), is given approximately by the product of the probability that the strength is in that band multiplied by the probability that the load exceeds X (i.e. failure occurs if the load exceeds the strength). This can be expressed as

$$\delta P_F(X) = p_S(X) \cdot P_L(X) \qquad \text{-(3)}$$

where $\delta P_F(X)$ is the contribution to the total probability of failure,

$p_S(X)$ is the probability that the strength is X (± 1/2dx)

$P_L(X)$ is the probability that the load exceeds X

Summing the incremental values of $\delta P_F(X)$ gives the total probability of failure

$$P_F(X) = \sum_{x=0}^{X} \delta P_F(X) \qquad \text{-(4)}$$

Figure 5 shows the two stages graphically.

e. The next step represents a major change between the conventional and probabilistic processes, namely the quantitative assessment of the probability of a discrepancy between the intended mean strength of the fleet and the achieved strength. This point is discussed at length in reference 1, but since it must be fully recognized, it is briefly outlined here, and described again in Section VII.

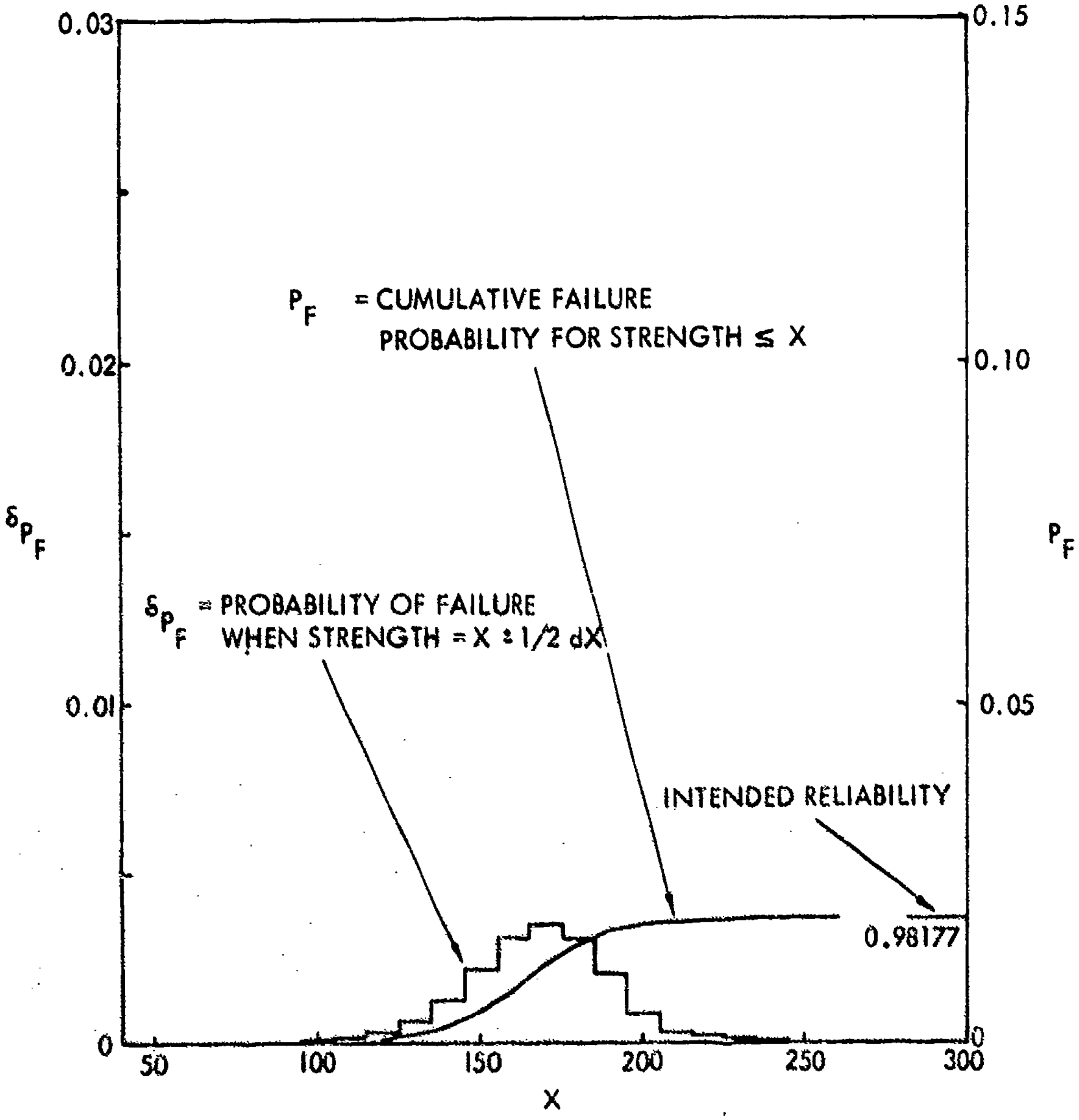


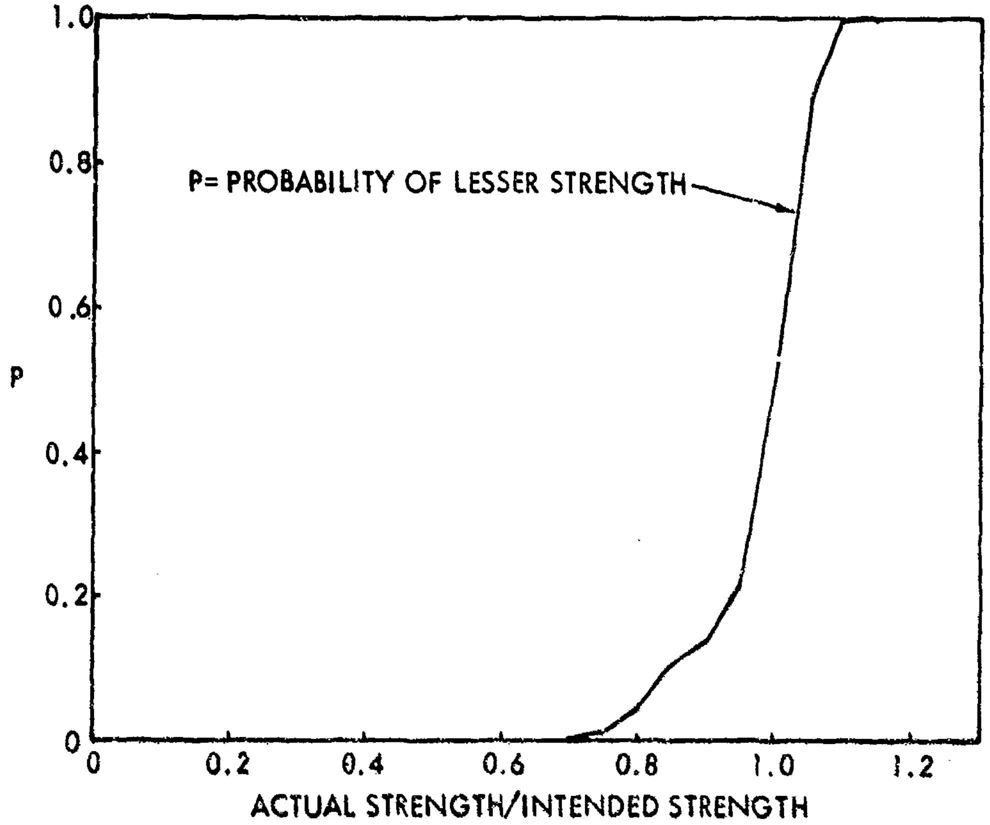
FIGURE 5 INTENDED RELIABILITY (FIRST EXAMPLE)

It is commonly accepted that analytical methods alone are insufficient to guarantee the strength of a structure, particularly where design and manufacturing processes are advancing more rapidly than the supporting analytical tools. Arithmetical errors, either major or minor, are encountered in practice, as are deliberate processes of underdesign to save weight. The net effect is reflected in accumulated test failure experience.

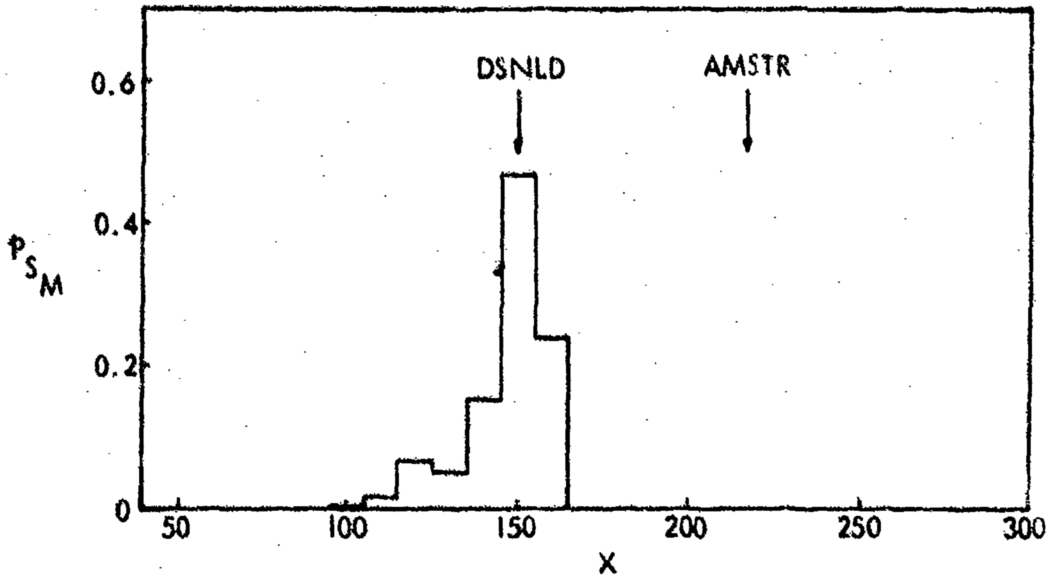
An interesting numerical observation described in reference 1 may be reiterated for emphasis. Suppose the design allowable strength to correspond to the 99 per cent probability of survival; then only one test article in 100 should be expected to fail at load levels lower than the fully factored design load, a situation which is not confirmed by actual test experience. Even if the mean strength (as determined by small-scale tests) is used as the allowable strength, then no more than one half of the static ultimate tests should result in failure.

Objective consideration of real-life static test performance leads to the inescapable conclusion that the achieved mean strength of a design may be less than the intended mean strength because of discrepancies in design, material, fabrication or assembly.

f. The choice of the specific error function to be used is discussed in Section VII. For this illustrative example, a double-family distribution was assumed for the ratio of probable actual mean strength to intended mean strength. Figures 6(a) and 6(b) show the assumed distribution; the sub-family with its mean at 1.0 can be regarded as covering tolerances in reading design data from curves, in "round-off" errors and other similar practices; the other sub-family has its mean at 0.8 and can be considered to represent discrepancies due to arithmetic errors, to faulty quality-control of material, poor assembly and so on. Ten per cent of the total population is assigned to this second sub-family.



(a) error function



(b) Probable Mean Strength Distribution

FIGURE 6 PROBABLE ERROR (FIRST EXAMPLE)

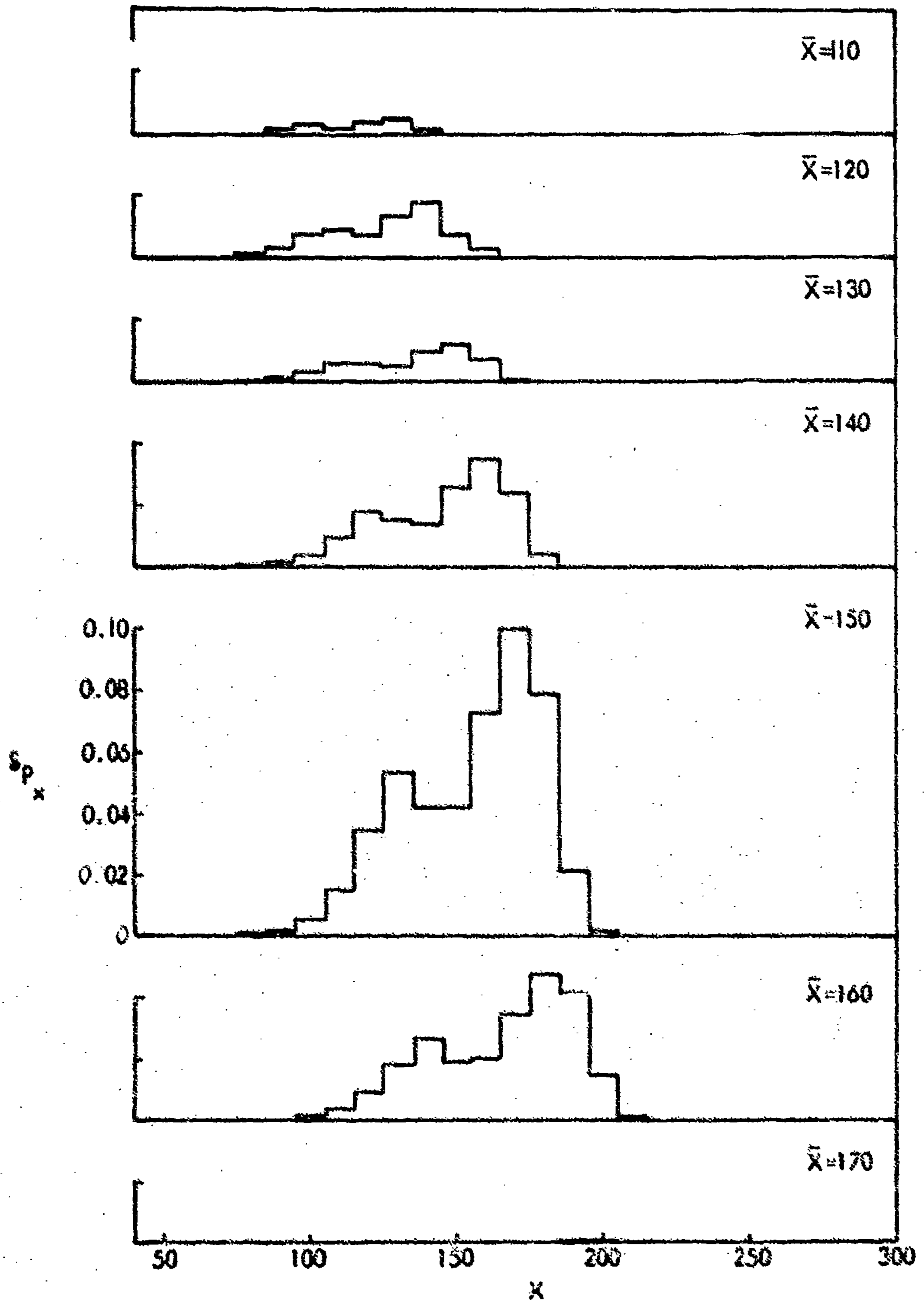
g. The assumed distribution of mean strengths is then combined with the assumed strength variation to produce a distribution of probable individual strengths. This is achieved by taking each mean strength level, $\bar{x}_i$, in turn, and assuming a sub-group containing $p(x_i)$ of the total with a distribution scaled from the basic strength distribution; this results in a series of contributions to the probability $p_s(x_i, \bar{x}_i)$ that the strength is x_i when the mean is x_i, as shown in figure 7(a). Summing for each x_i gives the total probability of each strength level, as shown in figure 7(b).

h. The failure risk and the reliability can now be revised to recognize the assumed probable discrepancies, but before the incorporation of knowledge from any tests. Figure 8 shows the two stages involved, which are identical to those described in paragraph (iv) above.

i. The next step, the incorporation of test results, requires a different interpretation of the purposes of static testing from that commonly held. The conventional view is that if the test article survives the designated load, then the design is proved, but this has no validity in a probabilistic context. The essence of probabilism (reference 4) is that a discrepancy remains, however slim that chance may be. This is due to the possibility that the test article may be from the stronger end of the distribution; the reliability estimate must recognize the existence of the weakest member of the fleet.

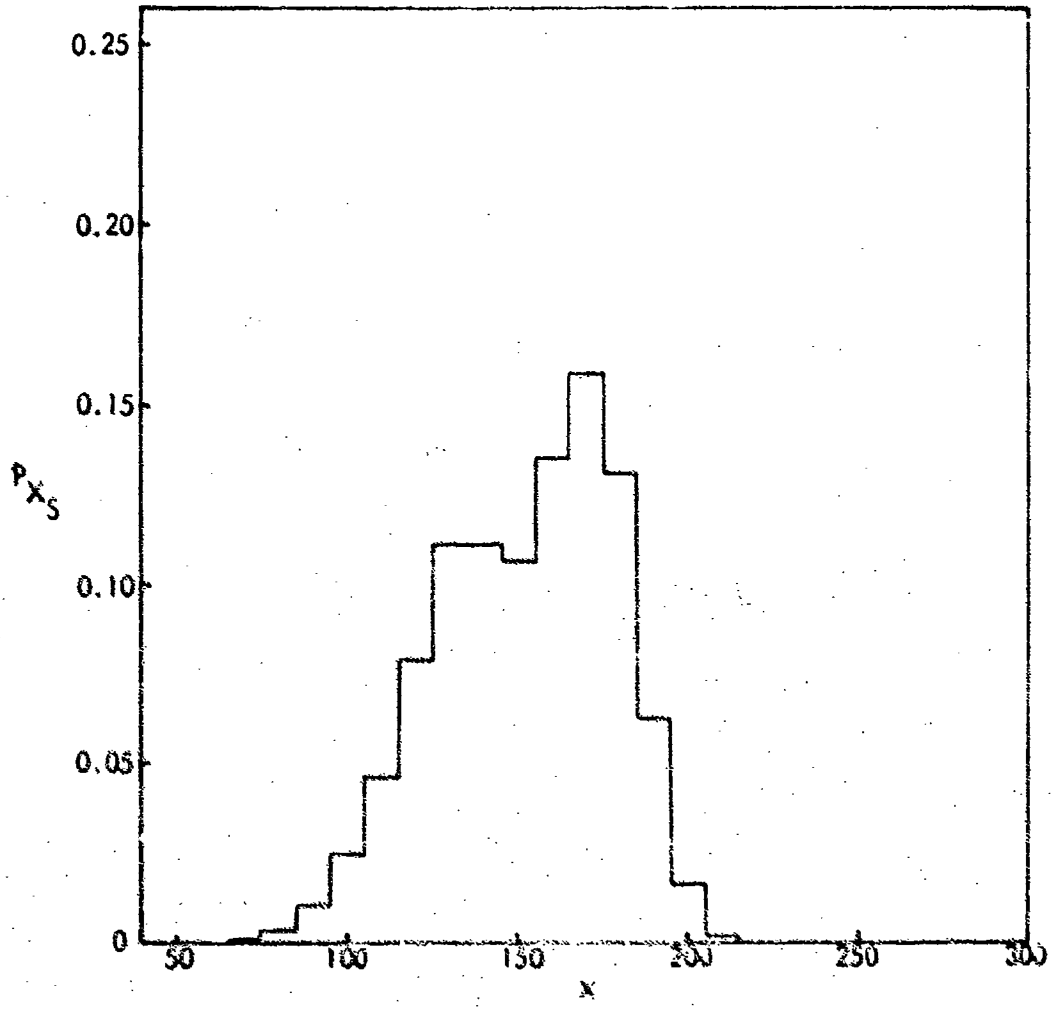
Hence the intention of the test requires re-interpretation, and as stated in reference 1, becomes the means of disclosing whether there are discrepancies in the design, fabrication or assembly processes which result in the actual strength levels being different from those intended.

j. The mathematical application of Bayes' theorem to this specific problem is well-understood; reference 5 is one example of the available literature. Briefly, the reasoning is as follows, for the case of "survival" tests:



(a) Incremental distributions

FIGURE 7 INDIVIDUAL STRENGTH DISTRIBUTION (FIRST EXAMPLE)



(b) Resultant distribution

FIGURE 7 CONCLUDED

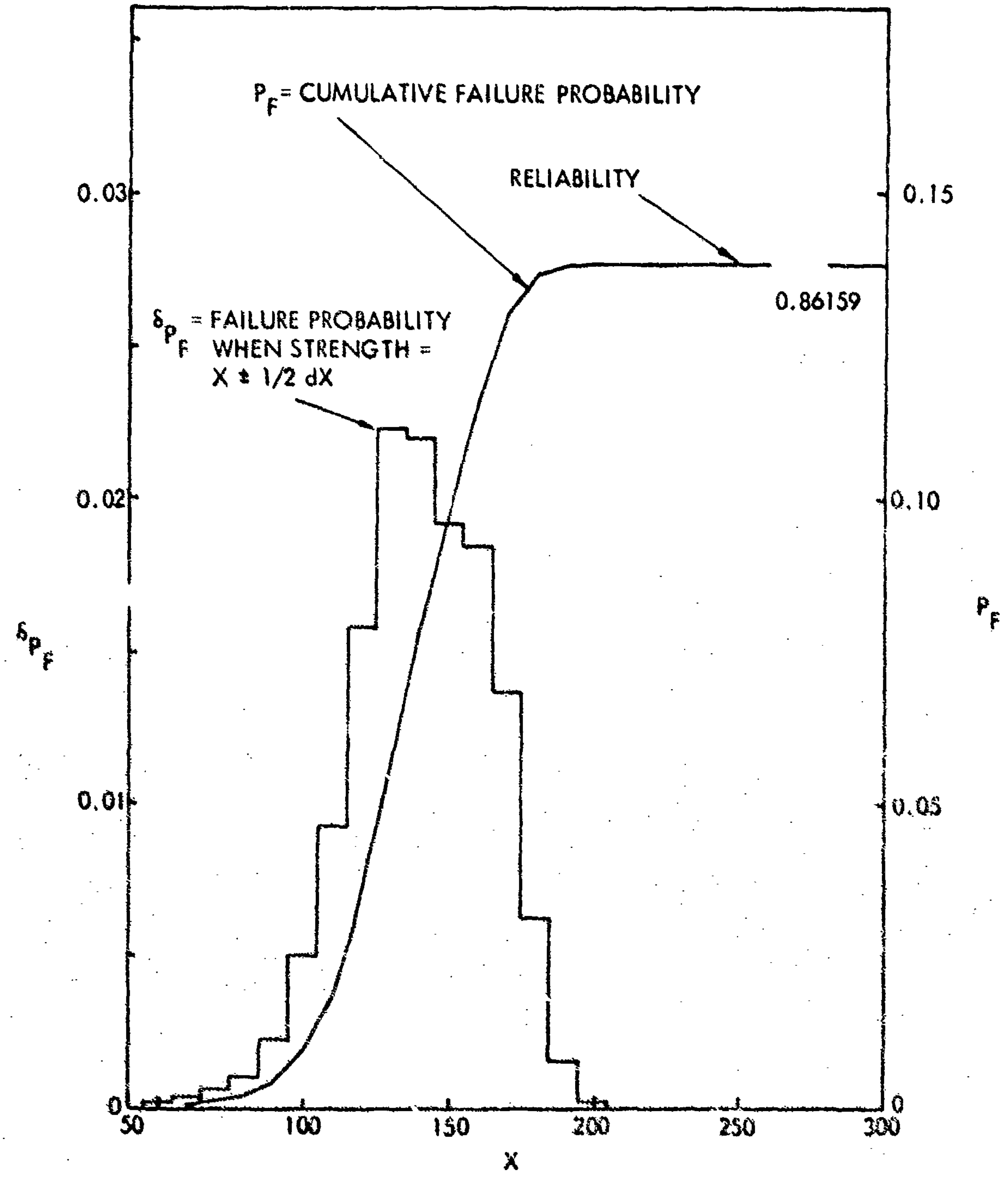


FIGURE 8 RELIABILITY WITH PROBABLE ERROR (FIRST EXAMPLE)

let $P(x_i;\bar{x}_i)$ be the probability* that the strength exceeds x_i; when the mean strength has the value $\bar{x}_i$; and

let $p(\bar{x}_i)$ be the probability that the mean strength is $\bar{x}_i(\pm 1/2dx)$

Initially, an assumed distribution of $\bar{x}_i$ is used as a prior distribution $p(\bar{x}_i)$. Now let one test be performed to a load X_{T_1} and let the specimen survive this test. The posterior distribution of mean strengths is then given by

$$p^1(\bar{x}_i;X_{T_1}) = \frac{P(X_{T_1};\bar{x}_i) \cdot p(\bar{x}_i)}{\sum\left|P(X_{T_1};\bar{x}_i) \cdot p(\bar{x}_i)\right|} \qquad \text{-(5)}$$

where the summation is performed for the whole range of $\bar{x}_i$ required to ensure that $\sum\left\{p(\bar{x}_i)\right\}$ equals unity, and the denominator represents a normalizing factor which retains the total posterior probability of $\bar{x}_i$ as unity.

The effect of equation (5) is therefore to update the assumed distribution of mean strengths as a result of knowledge gained from the test, this knowledge being that the strength of the specimen was greater than X_{T_1}.

If several tests are made successively, the posterior distribution from the first test becomes the prior distribution for the second test, and so on.

Figure 9 shows the revised distributions of probable mean strength which are derived from one and two tests to survive a load of 150. These revised mean strength distributions lead in turn to updated distributions of probable individual strength (figure 10 shows the effects).

k. The failure probabilities and reliabilities are then re-evaluated to give values appropriate to the new state of knowledge (see figure 11).

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*The semi-colon denotes that P refers to the distribution of x_i for a given value of $\bar{x}_i$

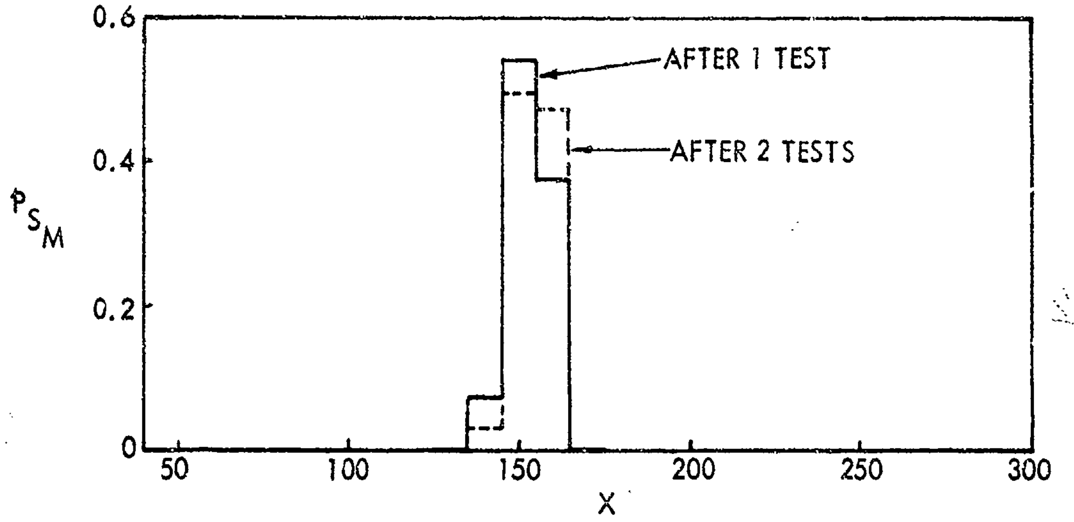


FIGURE 9 MEAN STRENGTH AFTER SURVIVAL TESTS

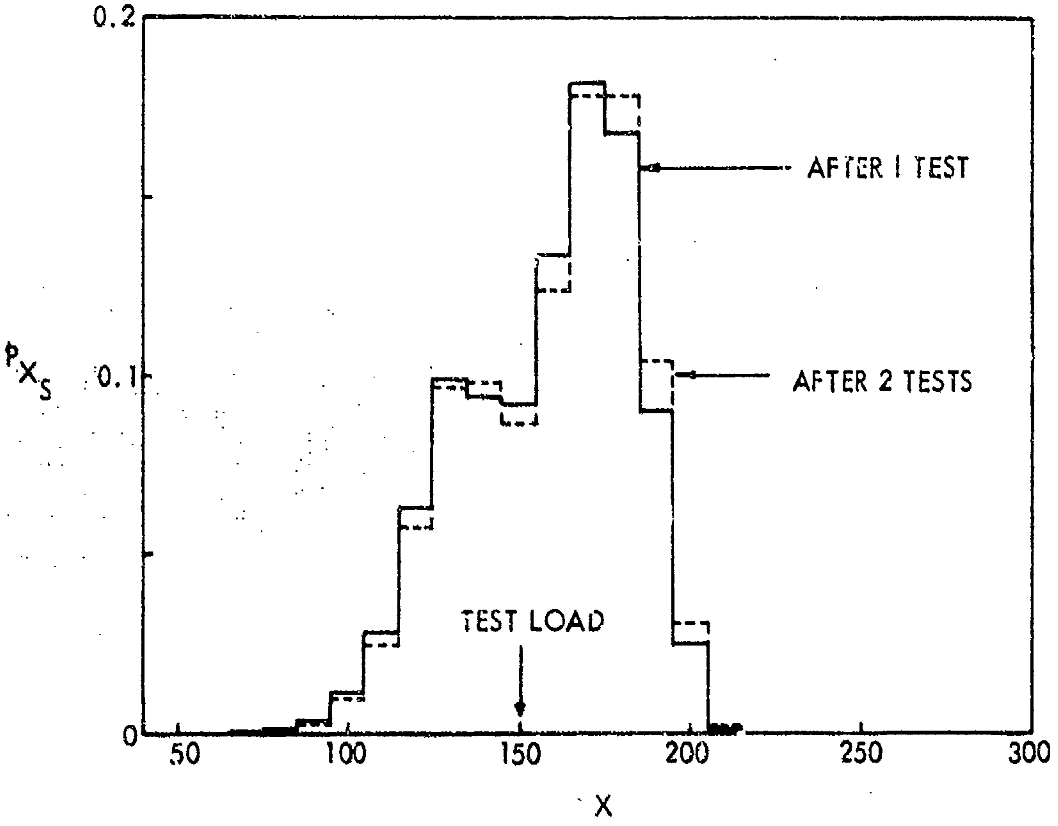


FIGURE 10 INDIVIDUAL STRENGTH DISTRIBUTION AFTER SURVIVAL TESTS

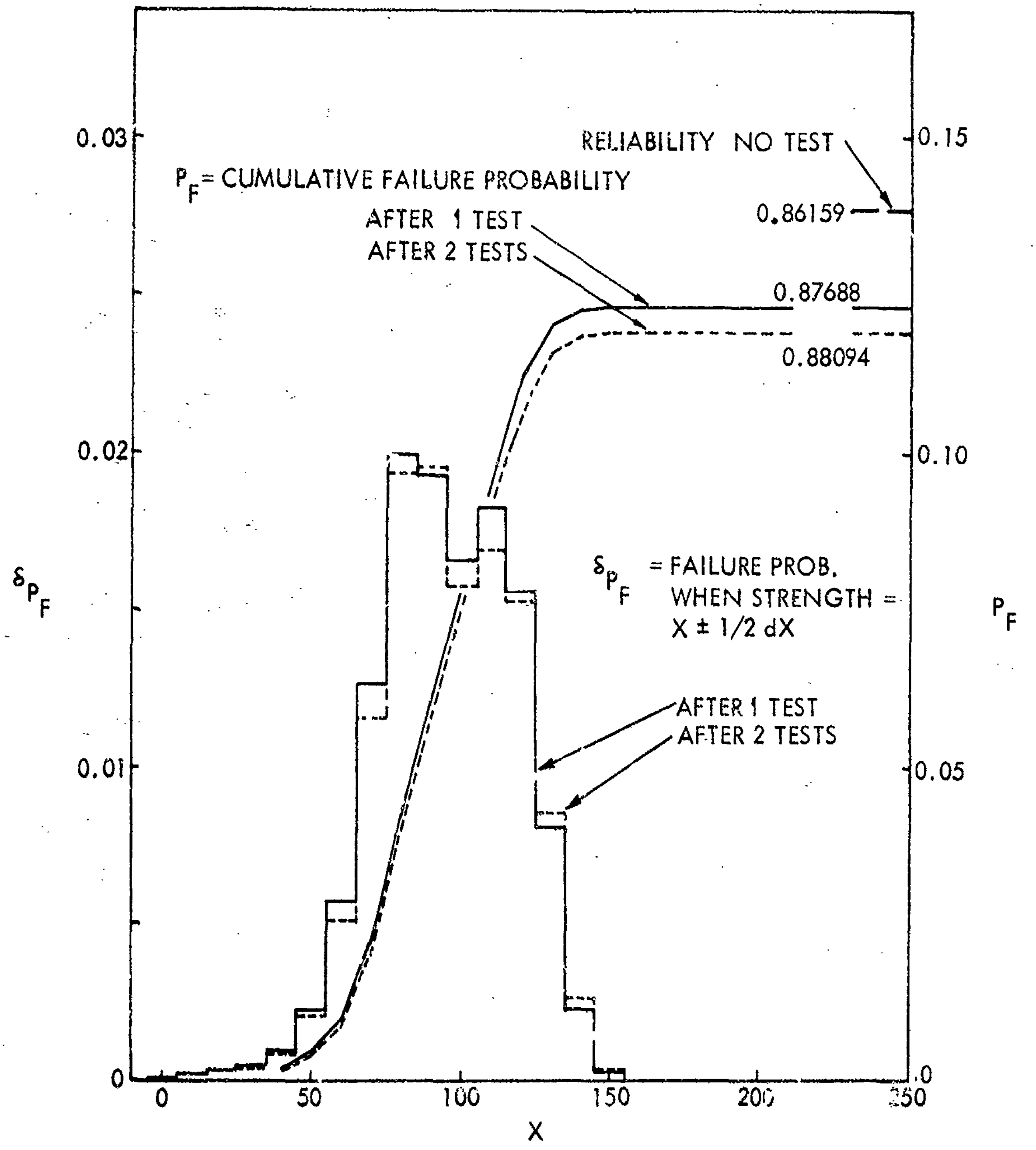


FIGURE 11 RELIABILITY AFTER SURVIVAL TESTS

l. An alternative test interpretation will exist when the failing load is known.

let $p(x_i; \bar{x}_i)$ be the probability that the strength is x_i ($\pm$ 1/2dx) when the mean strength is $\bar{x}_i$ ($\pm$ 1/2dx)

let $p(\bar{x}_i)$ be unchanged from the previous definition

The posterior distribution of $\bar{x}_i$ is then

$$p^{\prime}(\bar{x}_i; X_{T_i}) = \frac{p(X_{T_i}; \bar{x}_i) \cdot p(\bar{x}_i)}{\sum\left\{p(X_{T_i}; \bar{x}_i) \cdot p(\bar{x}_i)\right\}} \qquad -(6)$$

and is used as before to yield the updated distribution of individual strength (see figures 12 and 13 for the example of two test failures at 150).

m. The revised failure probabilities and reliabilities can be computed from the updated strength distributions to reflect the known fact that the strength of each test specimen was 150. Figure 14 shows the results graphically.

n. In general, tests will not lead to the same result, and the methods described above remain valid if the X_T values are changed from test to test. The order in which the values occur is immaterial the same final results being obtained, for example, for a test to 150 followed by a test to 180 and for a test to 180 followed by a test to 150. The intermediate estimates after the first test will differ. The difference in interpretation between survival tests and failure tests is discussed in Section IX.

o. The results of the computations in the different steps of the analysis are summarized in Table I. Comments illustrating the interpretation of the values are:

(1) the process of matching the (factored) design load to an allowable strength set at two standard deviations below the mean implies that the intended mean strength of the fleet is 217.0. The basic strength distribution (double-tamily) has a standard deviation of 217.0 x 0.154 = 33.4 which explains the large difference in values. Practical data would reduce this substantially (see Section III-

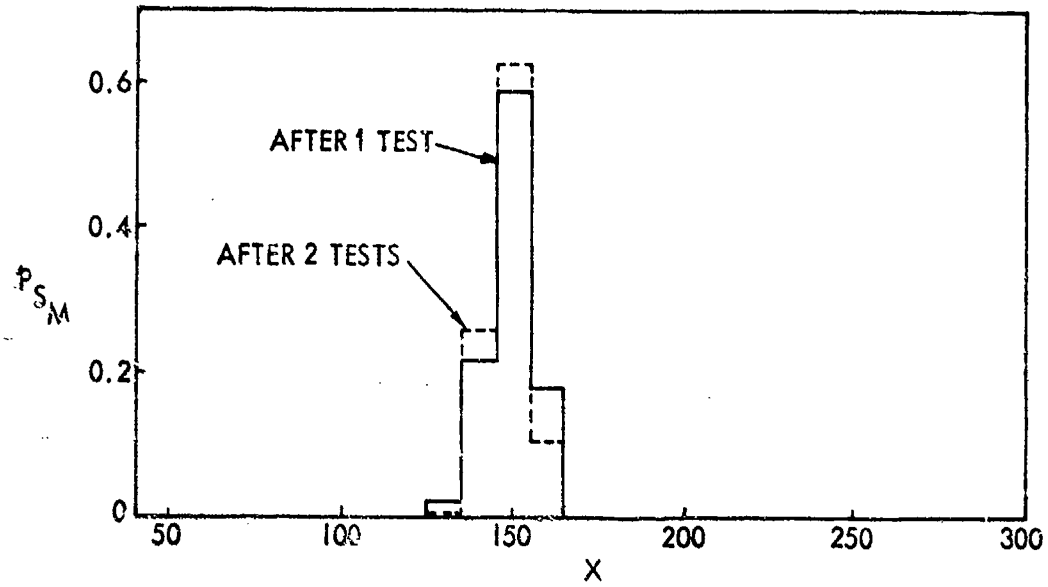


FIGURE 12   MEAN STRENGTH AFTER TEST FAILURES

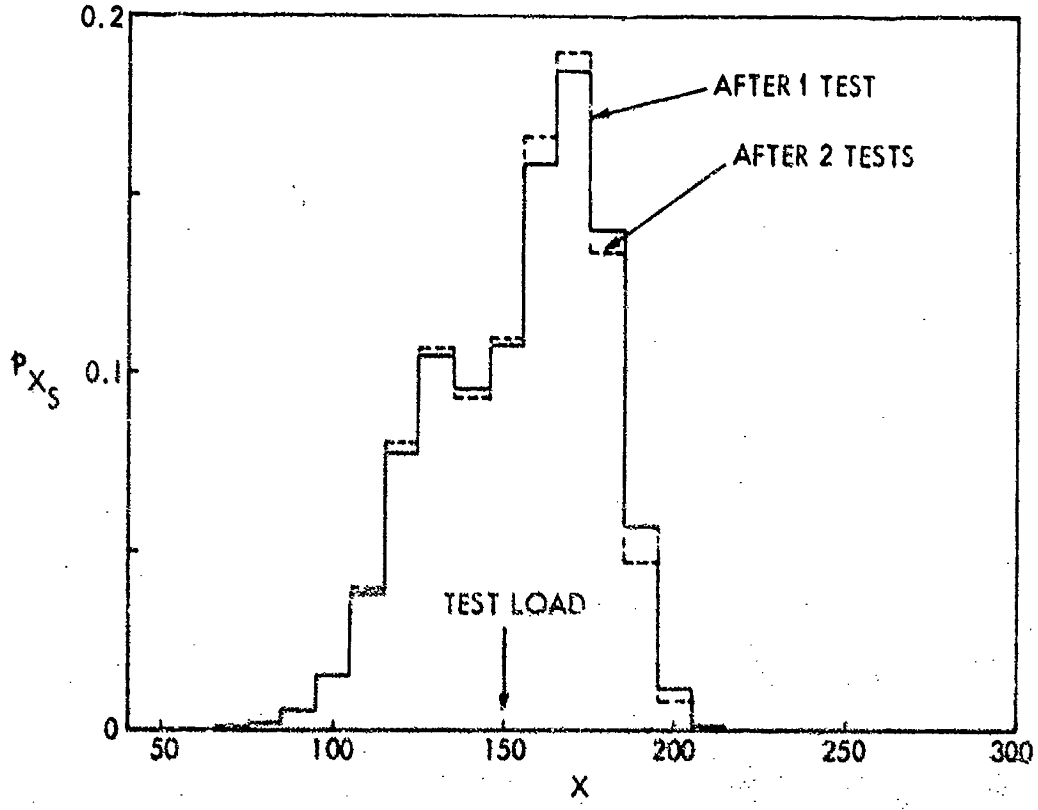


FIGURE 13 INDIVIDUAL STRENGTH DISTRIBUTION AFTER TEST FAILURES

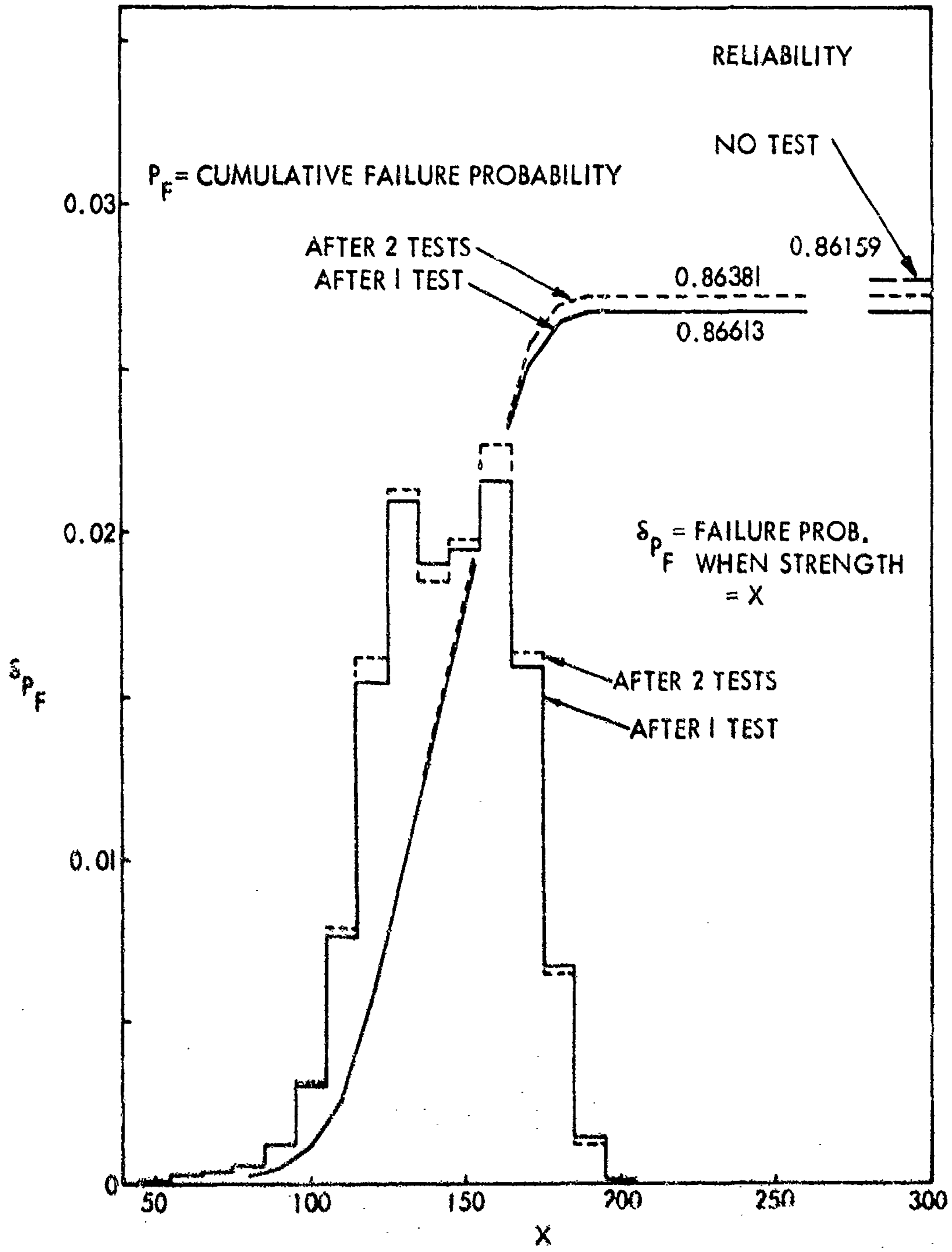


FIGURE 14 RELIABILITY AFTER TEST FAILURES

TABLE I - SUMMARY OF RESULTS OF EXAMPLE

CONDITION	MEAN STRENGTH	COEFF. OF VARIATION	TOTAL* RISK	TOTAL REL.
No error, no test	217.0	0.154	0.01823	0.98177
Prob. error, no test	152.0	0.170	0.13841	0.86159
Survival tests:				
1st to 150	158.0	0.155	0.12312	0.87688
2nd to 150	159.4	0.154	0.11906	0.88094
Failure tests:				
1st to 150	154.2	0.156	0.13387	0.86613
2nd to 150	153.4	0.155	0.13619	0.86381
1st to 150	154.2	0.156	0.13387	0.86613
2nd to 180	163.3	0.151	0.10844	0.89156
1st to 180	164.0	0.151	0.10667	0.89333
2nd to 150	163.3	0.151	0.10844	0.89156

UNFLD = 100, FS = 1·5, MS =0, DSNLD = 150, SALL = 2·0

The intended failure risk is 0.01823, the intended reliability being 0.982.

(2) Recognition of the probable existence of discrepancies reduces the predicted actual mean strength to 152.0 and, at the same time, increases the coefficient of variation to 0.170, resulting in a standard deviation of 25.8. The predicted failure risk increases seventy six times with a corresponding decrease in reliability to 0.862.

(3) After one test surviving 150, the probable mean strength of the fleet reverts upward to 158.0 with the coefficient of variation dropping well back to 0.155. The Bayesian update uses the test result to indicate a smaller error than was assumed, and revises the reliability to a slightly better value of 0.877.

(4) The second test has less influence, resulting in an improvement to 0.881.

(5) A test failure at 150 tends to affirm the assumed error definition implying a fleet mean strength of 154.2; the revised reliability (0.866) is only a little better than that corresponding to the "no test" situation.

(6) The second test failure at 150 confirms the error assumption, and lowers the fleet mean strength further (to 153.4), the reliability dropping very slightly to 0.864.

(7) If the second test failure is at 180, the results of the first test (failure at 150) are raised by a significant amount. The fleet mean strength improves to 163.3, the reliability moving to 0.892. However, the values are still well below the intended ("no error") values, which emphasizes the fact that testing to load levels in the neighborhood of the factored design load do not prove the absence of discrepancies between the intended and actual strength variation among the total population.

(8) It is seen from Table I that reversing the order of the two failure tests leads to the same final values. The intermediate values, after the first test to 180, are compatible with the achievement of this test level.

3.6 Example with Realistic Data

a. An example based on realistic data for the C-141 Cargo Transport follows. The procedure is as described in the previous sub-section but more reliance can be placed on the absolute values of the results. The assumptions made are as follows:

(1) The loads distribution was based on a single-family Gumbel distribution of the maximum load occurring per aircraft lifetime; integration from right to left yields the necessary probability that a load less than or equal to x will occur: design limit load was set at 100 with a design factor of 1.5.

(2) The basic strength distribution was assumed to be of Gumbel form with coefficient of variation of 0.06.

(3) The design allowable used for sizing the structure was taken to be 2.326 standard deviations below the mean (99 per cent exceedence).

(4) The assumed error function was based on retrospective analysis of C-141 wing test data (component and static test); this is discussed further in Section VII.

(5) Testing was assumed to consist of two separate tests, each surviving 150 (i.e. the test factor was equated to the design factor of 1.5 in the conventional manner).

b. Figure 15 shows the load distribution and the intended strength distributions. Because of the wide numerical ranges, logarithmic plots have been chosen throughout. Figure 16 gives the corresponding failure probability distribution and reliability. It will be noted that the (low) failure risk is due almost entirely to the few very weak specimens which are certain to incur loads exceeding their strength, and that there is little risk of the high loads causing failure. This emphasizes the interpretation in reference 1 of "under-strength protection".

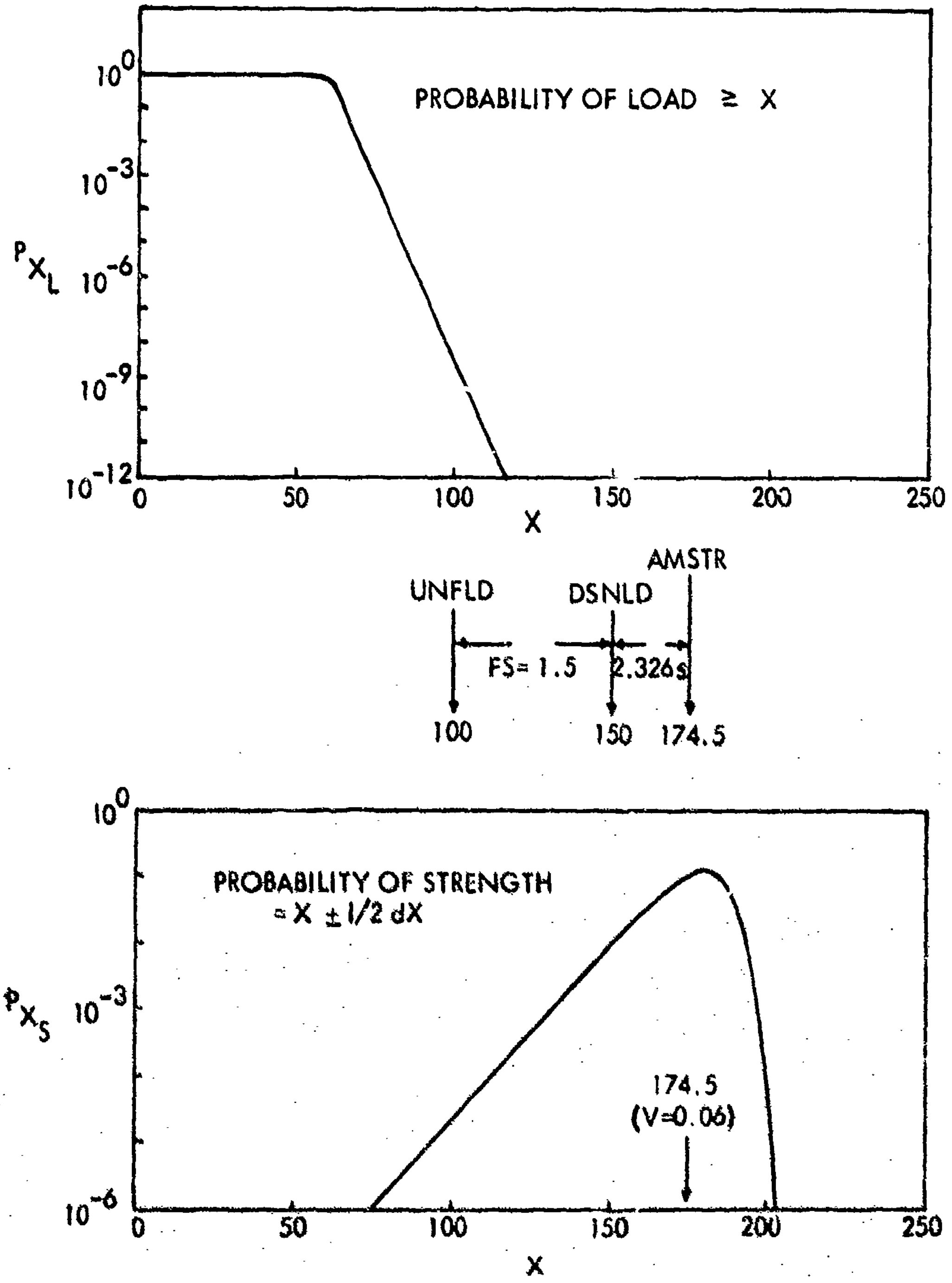


FIGURE 15 LOAD SPECTRUM AND INTENDED STRENGTH (SECOND EXAMPLE)

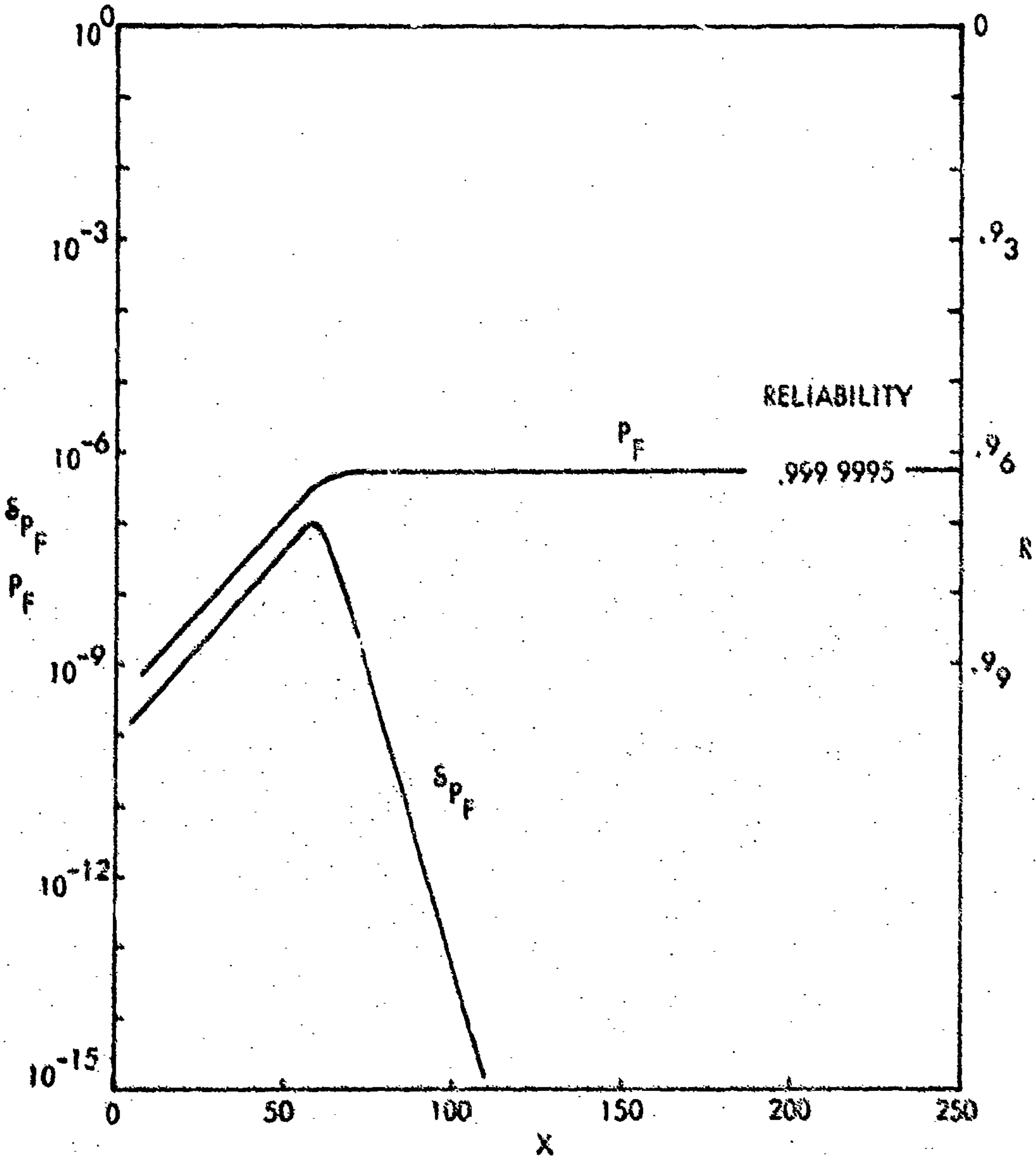


FIGURE 16 INTENDED RELIABILITY (SECOND EXAMPLE)

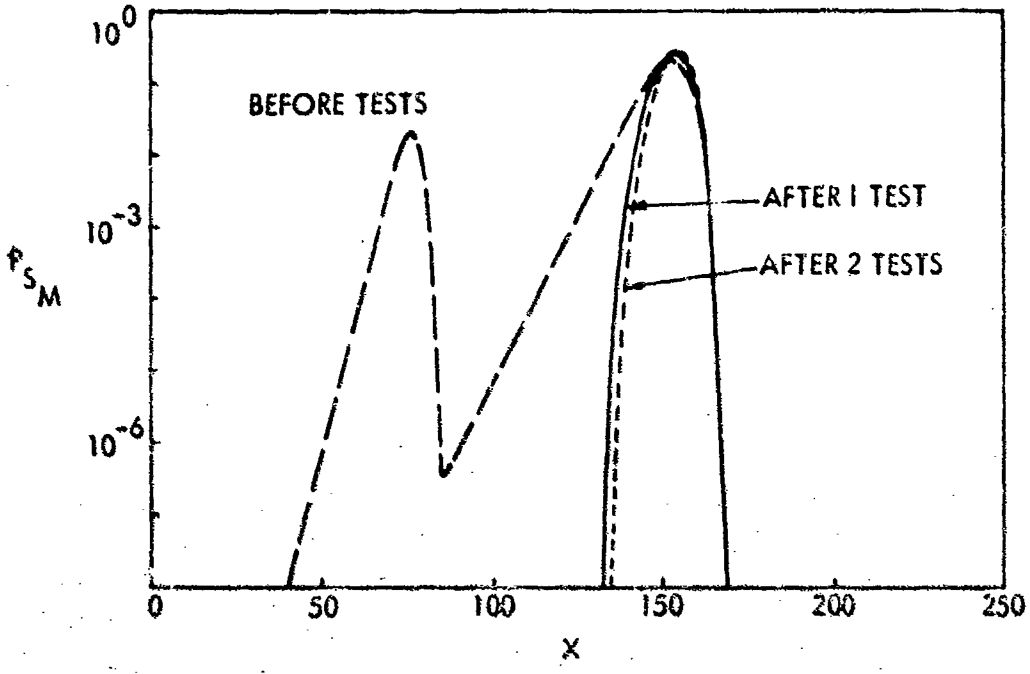
c. Figures 17(a) and 17(b) shows the effect of the assumed error function on the probable strength, exaggerated by the logarithmic scale. The same figures show the updated distributions following the tests. It is important to note that the assumed error function implies a low probability of surviving the test; the fact that the test was survived thus effectively denies the existence of the weaker sub-family of strength and amounts to a self-compensating process. In practice, this will tend to alleviate penalties which might exist due to over-conservative assumptions; conversely, premature failure will correct the strength distribution by implying a greater probability of a discrepancy. Techniques such as the use of Bayes' theorem may prove to be the key to the effective use of the proposed system of reference 1.

d. Figure 18 illustrates the variation of the failure distribution and reliability as the test data is accumulated.

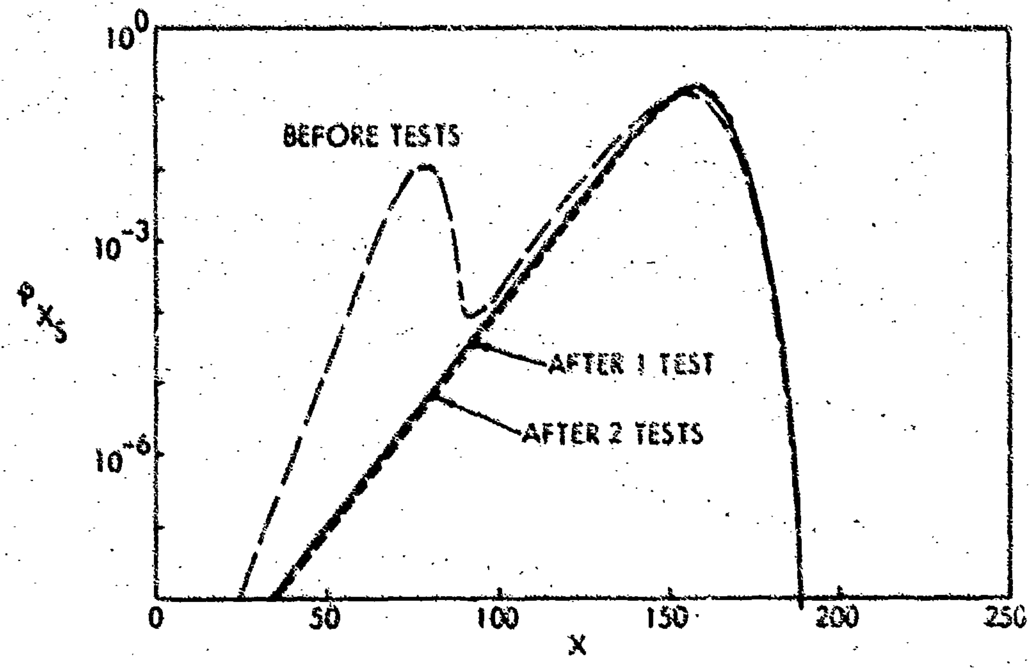
e. The C-141 example is summarized in Table II, and leads to the following comments:

(1) matching the factored design load (150) to an allowable strength at 2.326 standard deviation below the mean implies that the intended mean strength of the fleet is at 174.5 (i.e. at 1.745 times the unfactored load). The probability of surviving one test to 150 is found to be 0.972, which implies that only one specimen in 36 should fail to carry the 150 per cent test load. The intended reliability is almost one.

(2) the assumed error function reduces the probable mean strength of the fleet to 146.3 and doubles the coefficient of variation. The probability of surviving a test to 150 drops to 0.50, so that if the error assumptions are correct, one specimen in two should fail below 150 per cent load. The predicted reliability reduces to 0.9986.



(a) Mean strength distribution



(b) Individual Strength Distribution

FIGURE 17 STRENGTH DISTRIBUTION, BEFORE AND AFTER TESTS (SECOND EXAMPLE)

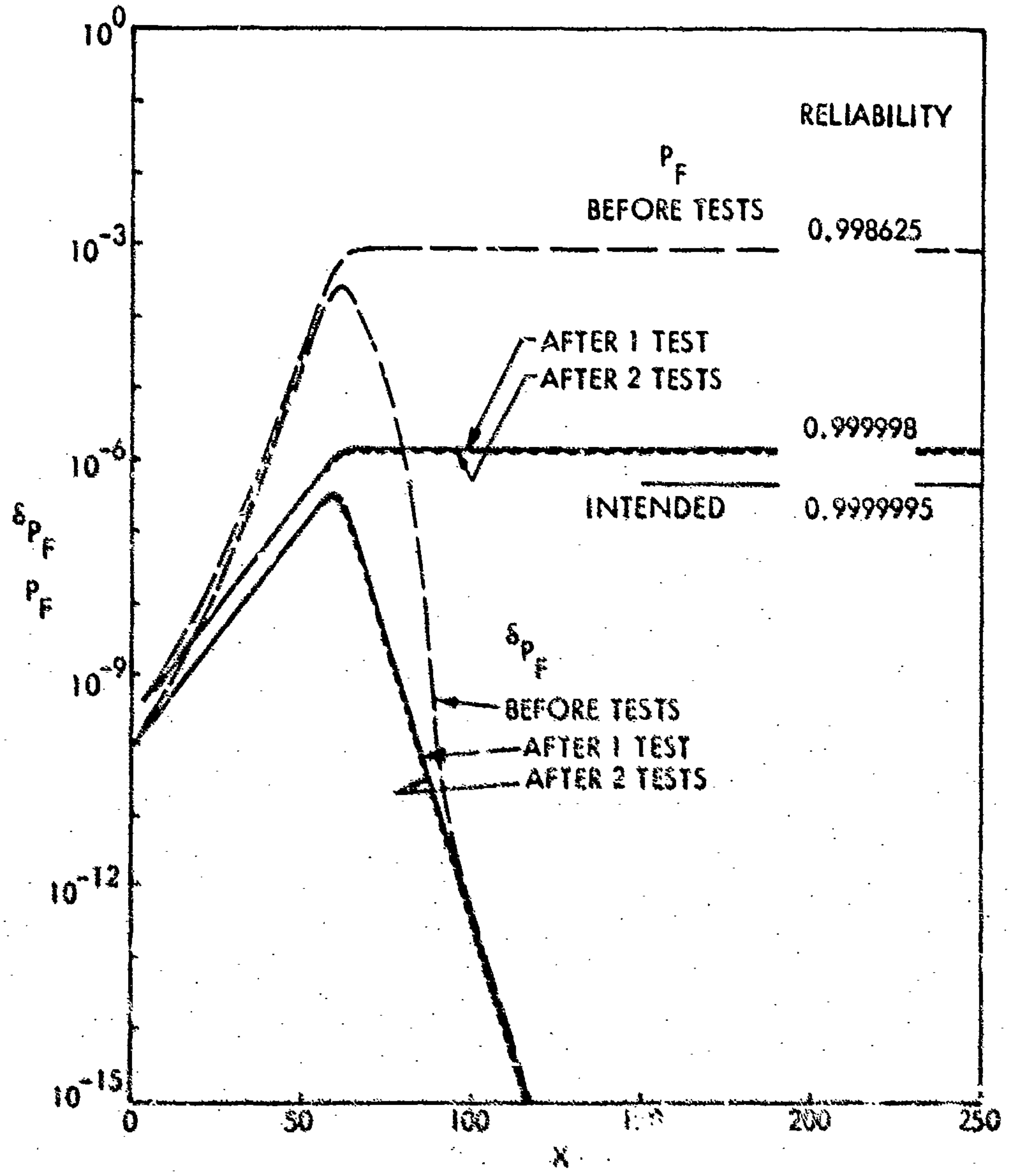


FIGURE 18 RELIABILITY BEFORE AND AFTER TESTS (SECOND EXAMPLE)

TABLE II - SUMMARY OF C-141 EXAMPLE

CONDITION	MEAN STRENGTH	COEFF. OF VARIATION	TOTAL RISK	TOTAL REL.
No error, no test	174.5	0.060	0.0000005	0.999999
Probable error, no test	146.3	0.133	0.001374	0.998625
After one test surviving 150	152.5	0.066	0.0000018	0.999998
After two tests surviving 150	153.5	0.065	0.0000017	0.999998

UNFLD = 100, FS = 1·5, MS = 0, DSNLD = 150, S_{ALL} = 2.326

(3) survival of the first test indicates that the likely errors are less extreme than assumed. The update process raises the predicted mean strength of the fleet to 152·5, reducing the coefficient of variation almost to the intended value. The reliability (0.999 998) is also restored almost to the original value.

(4) the second test has virtually no effect on the reliability.

3.7 Data Categories

a. This sub-section identifies the data requirements of the proposed method in general terms. Each category is discussed separately in later Sections of this report.

b. Load data:

The philosophy of reference 1 (summarized in a clearer manner in reference 2) considers the operational experience of a fleet of aircraft to be divided into three areas, separated by boundaries defined as "limit condition" and "omega condition" respectively (see sub-section III-4). Load levels up to the maximum which is likely to occur in normal usage must not result in failure due to unduly weak strength; in different words, the probable risk of failure of the weakest likely member of the fleet must be acceptably remote. A corollary of this is the necessity that the operator must be able to apply the limit definition in order to achieve the desired reliability.

Now the kernel of the reliability prediction is the comparison of the probability of a certain strength and the probability of a greater load. It is therefore essential that load and strength must be expressible in terms of the same quantity.

When only a single parameter is involved (as in the cases discussed in references 1 and 2), no real problem arises. In most realistic conditions, combinations of parameters will be necessary for both load (load factor, weight, speed, etc.) and strength (bending, torsion, pressure, etc.), and the choice of basic parameters is less obvious. These points are discussed in later sections.

When multiple parameters occur, it is not possible to select a single limit (or omega) load level with a probability which can be directly related to a reliability level. Nevertheless, a single value is necessary for the initial sizing of the structure.

The basic data required therefore consists of:

(1) design unfactored load levels (based on normal operational or limit conditions and based on desired overload, or omega, conditions)

(2) design factors (and design margins of safety) to be used in conjunction with the unfactored loads in order to determine the structural configuration

(3) declared load levels (limit and omega) at which the chosen reliability goals are to be met

(4) probability distributions of the limit and omega loads, which may be quite separate since the parameter being overloaded may not be the primary parameter.

Sections IV and V explores these features in greater detail.

c. Strength Data:

A means is required for establishing the probable variation of strength relative to the mean strength, and this definition must be in terms of the single principal parameter used to define the load. It will generally be necessary, therefore, to perform separate analyses at constant values of each secondary parameter. The resultant strength of a real structure will involve not only the properties of the basic material, but also the variability introduced by fabrication and assembly processes. A design allowable level (a number of standard deviations below the mean) is required for establishing the initial sizes of the structural members. The basic statistical properties of the resultant strength distribution (i.e. the coefficients of variation of the sub-families, the relative locations of the means of the sub-families and the relative proportions of the population assigned to the sub-families) are assumed constant as the predicted mean strength of the system is updated. Section VI discusses the nature of these items.

d. Error Function:

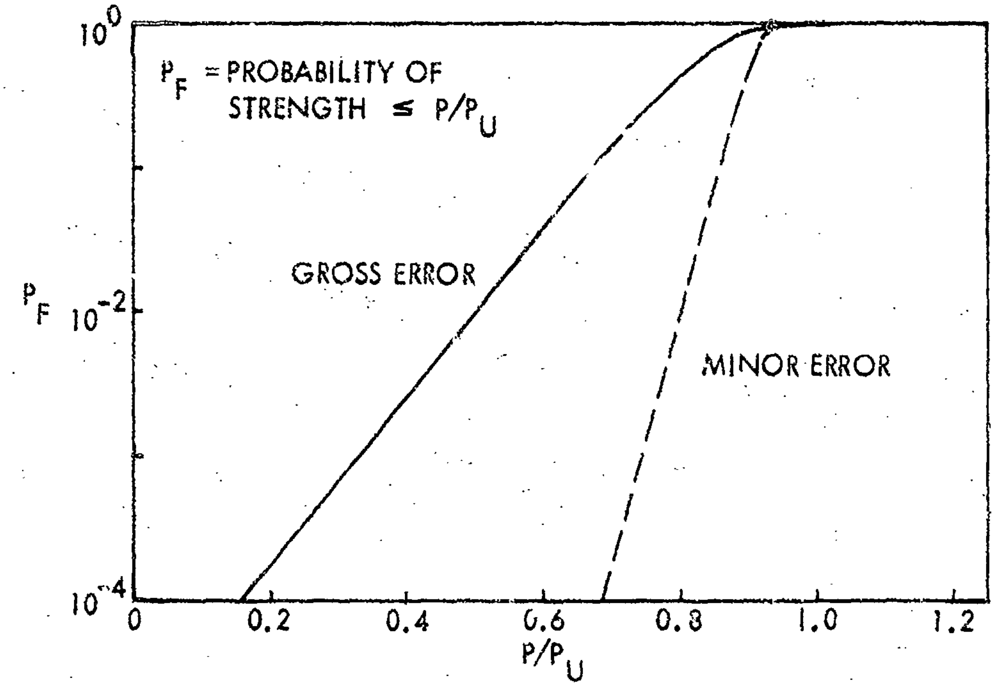
In order to employ the automatic update feature introduced by Bayesian methods, the predicted actual strength is required to be a function of two variables. These are taken to be the basic strength distribution relative to the mean strength, and the probable distribution of mean strength. The "no error" condition can be analyzed independently as described in sub-section III-5, but cannot be assumed as the prior distribution of Bayes' theorem since equations (5) or (6) do not result in any change when only one value of x_i exists. Hence, some assumed distribution of mean strength is required, however narrow this may be.

In practice, there will be few instances where the design and construction methods are so well established that the choice of an undisclosed error can be truly claimed to be negligible. The choice of error function can initially be arbitrary, or may be based on an individual company's experience of its own procedures. It is important that the interaction between the original error function and the updating by test results is appreciated; a gross error function implies little chance of surviving a high test load, and if the test load is survived, it will result in a drastic improvement of the predicted strengths. Conversely, an optimistic error function implies near certainty of passing the test; if the test fails, a drastic reduction in the predicted strength will result. The whole process tends to be self-compensating. Figures 19(a), 19(b), 19(c) and 19(d) illustrate this tendency.

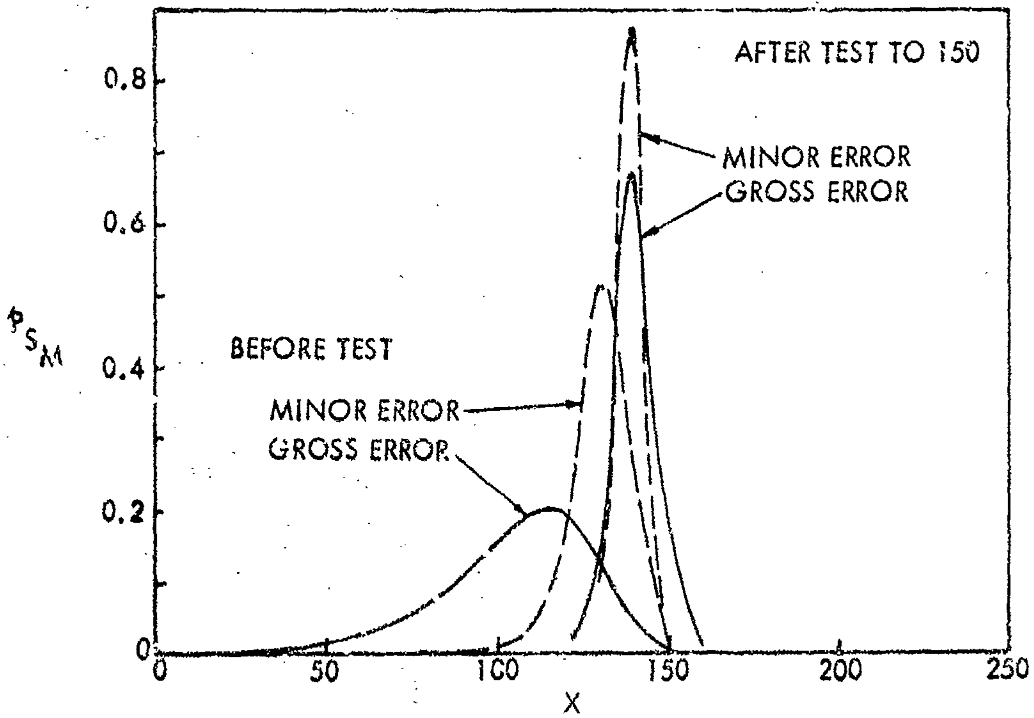
Section VII describes the practical assessment of suitable functions from test experience.

e. Reliability Goal:

This subject is addressed in Section VIII and the only comments necessary at this stage are that no obvious rationale has been detected for the values to be used. Even if the remainder of the system is probabilistic in nature, the chosen reliability levels will probably retain a deterministic character.

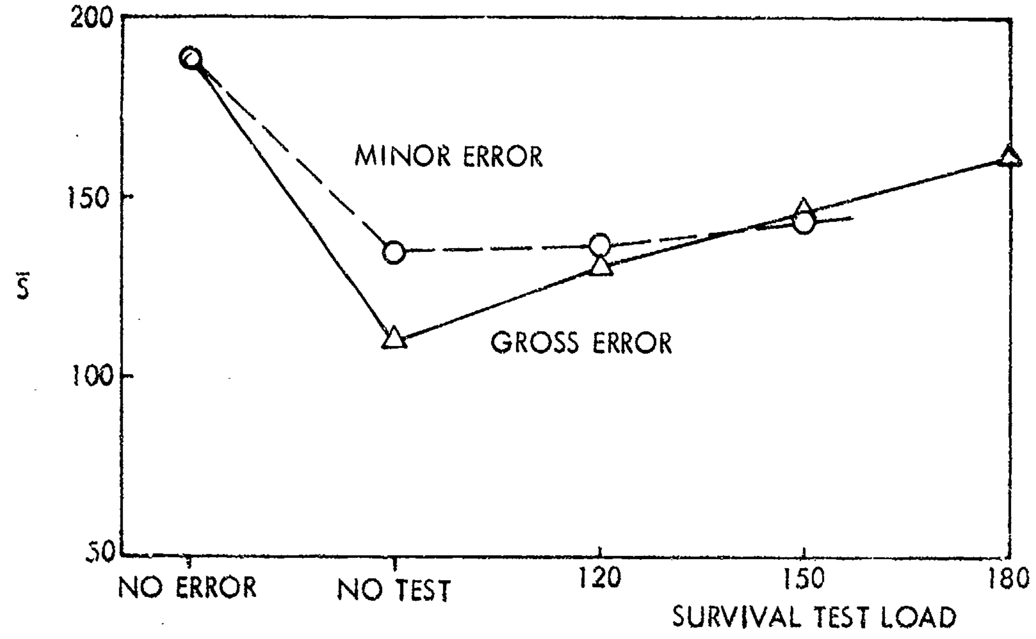


(a) Error functions

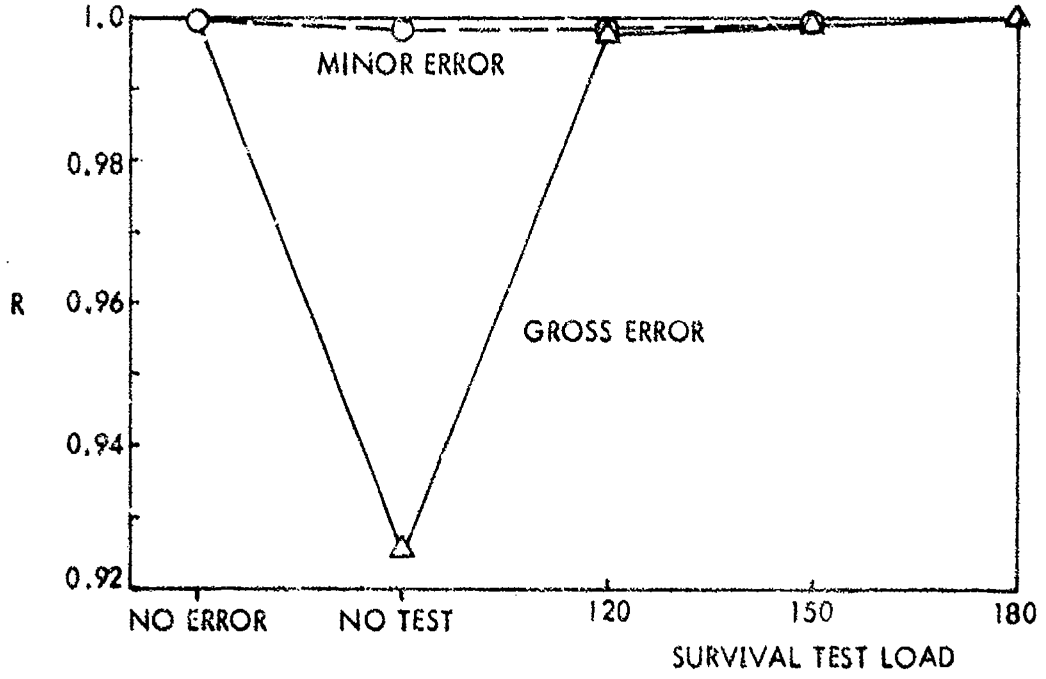


(b) Mean strength distributions

FIGURE 19 EFFECTS OF GROSS AND MINOR ERRORS



(c) Resultant fleet mean strength



(d) Reliability

FIGURE 19 CONCLUDED

f. Test Factors:

Once the essential nature of the static strength tests is accepted as the means of disclosing discrepancies between the intended strength distribution and the probable actual strength distribution, and not as a means of "proving compliance" with a design obligation, then the logic by which test factors can be selected can be developed.

The fundamental aim is the prediction of the risk of failure within a specified range of load levels, and the selection of a target test level which will indicate that the chosen risk (the complement of the reliability) will be met. If a lower test strength is achieved, further studies can be initiated to evaluate the trade-off between load probability and reliability. The test factors can therefore be selected from a knowledge of the pre-test data and cost-optimized with respect to the probability of destroying the specimens, the number of specimens and the level of loading. Section IX gives further details.

One other feature emerges from the example described earlier; a test failure at, say 150, can also be regarded as a test surviving 149. However, the probabilistic differences may not be negligible for the failure implies no probability that the specimen has a strength exceeding 150, whereas the survival does include the probability of greater strengths. This anomaly is pursued in Section IX.

g. The final data requirement is simply the recognition that at any given stage in the design, test and operational life of the aircraft, the appropriate data should reflect the current stage of knowledge. Progressive updating of all parameters is necessary to the full assessment of the reliability of the fleet.

SECTION IV
CHOICE OF INITIAL DESIGN LOADS

4.1 Introduction

At the outset of the design of a new aircraft, little definitive information will be available to define the probabilities of the loads or the strength levels. The maximum use must be made of approximations to permit the preliminary design iterations to proceed; the structural configuration, materials and methods of fabrication will usually be varied during this stage. It is necessary for a deterministic definition of the design loads to be clearly defined as a means to the sizing of the structure; this item in the design chain cannot be treated on a probability basis within the procedures currently in general use, and any change to introduce such a basis would be a cause of disruption.

This section examines these related problems as they would occur during the design of a cargo transport aircraft (C-141 data was used), but with the implied advantage of prior knowledge of the probable utilization (in practice, this could frequently be obtained from accumulated data on an existing aircraft of similar type).

4.2 Available Statistics

Appropriate data which can be used in the application of statistical methods to determination of design loads appear to exist in quantity only for the following parameters:

a. Symmetrical maneuver load factors
b. Gust intensities
c. Landing sinking speeds

The information which is available concerning these parameters in many cases is probably inadequate to establish probability levels appropriate to Omega load levels without extreme extrapolation. Also, it is quite obvious that loading conditions cannot be defined with these parameters alone.

However, where at least one significant parameter of a loading condition can be defined adequately through statistics it appears that the statistical approach can be used. This can be done in the following manner:

a. Select appropriate statistical data concerning a significant parameter and extrapolate the data as necessary (using extreme value techniques, for example).

b. Select other significant statistics from mission profile information, also extrapolating to necessary extremes.

c. Combine the above statistics using joint probability techniques to select conditions appropriate to the designated structural reliability goals. (See Section VIII.)

d. Select other parameters necessary to completely define loading conditions from the basic requirements of the MIL-A-8860 Series.

4.3 Design Limit Conditions

In the context of the new procedure, limit loads represent those which may be attained in normal operations within normal operational envelopes and Omega loads are those which result from exceeding normal limitations due to an unusual occurrence. Therefore, in selecting limit conditions, normal operational limitations should be used, such as:

a. Speeds not exceeding V_H

b. Center of gravity limits not including a design tolerance

c. Weights not exceeding maximum gross weight

d. Payloads not exceeding placarded limits

e. Etc.

4.4 Design Omega (Overload) Conditions

However, in the selection of Omega load cases, statistically defined parameters, mission profile extrapolated parameters, and MIL-A-8860 Series parameters should have no individual limits except those set by reasonability. For example, weights exceeding maximum design gross weight should be considered if statistics or extrapolated mission profile data indicate such, speeds up to V_L and possibly beyond should be used if statistics are available for verification, payloads exceeding limit payloads should be included, and center of gravity limits should include at least the Military Specification tolerance.

4.5 Example Using C-141 Data

a. As an example of how such a procedure might be applied using a minimum of statistical data the following C-141 landing loads analysis is offered. Table III shows the C-141 landing weight occurrences for one design lifetime of 12,000 landings as derived from the C-141 design mission profiles as shown in Reference 7.

Applying extreme value theory to these statistic results in the cumulative occurrences of landing weight shown in Figure 20. Extreme value theory applied to landing sinking speed data is shown in Figure 21. Two sets of sinking speed data are shown, one from MIL-A-008866A and the other from Figure 7.5 of Reference 8, for aircraft weighing over 150,000 pounds.

b. Figure 22 results from applying the joint probability of the landing weight statistics and the sinking speed statistics to obtain the combinations of landing weight and sinking speed to be considered. A probability of occurrence of once per 12,000 landings was used for limit conditions and a probability of 10^{-3} per 12,000 landings for Omega conditions.

The sinking speed data from MIL-A-008866A appears to be quite high as compared to that from Reference 8. Since the source of the MIL-A-008866A data is unknown and the reference 8 data is known to be statistically based, the Reference 8 data will be used in the subsequent analysis.

c. Limit combinations of landing weight and sinking speed are chosen along the 1.0 probability line and analyzed in accordance with MIL-A-008862A requirements. The only requirements of MIL-A-008862A which have been replaced are the landing weight sinking speed combinations. However, since limit conditions represent normal operations, the maximum gross weight which should be considered is the landplane landing design gross weight of 257,500 pounds.

TABLE III

C-141 LANDING WEIGHT OCCURRENCES

WEIGHT (LBS)	OCCURRENCES	CUMULATIVE OCCURRENCES
142,650	34	12,000
143,090	17	11,966
145,067	17	11,949
147,060	80	11,932
149,355	17	11,852
166,040	138	11,835
174,530	857	11,697
180,074	672	10,840
197,105	8200	10,168
206,150	1968	1,968

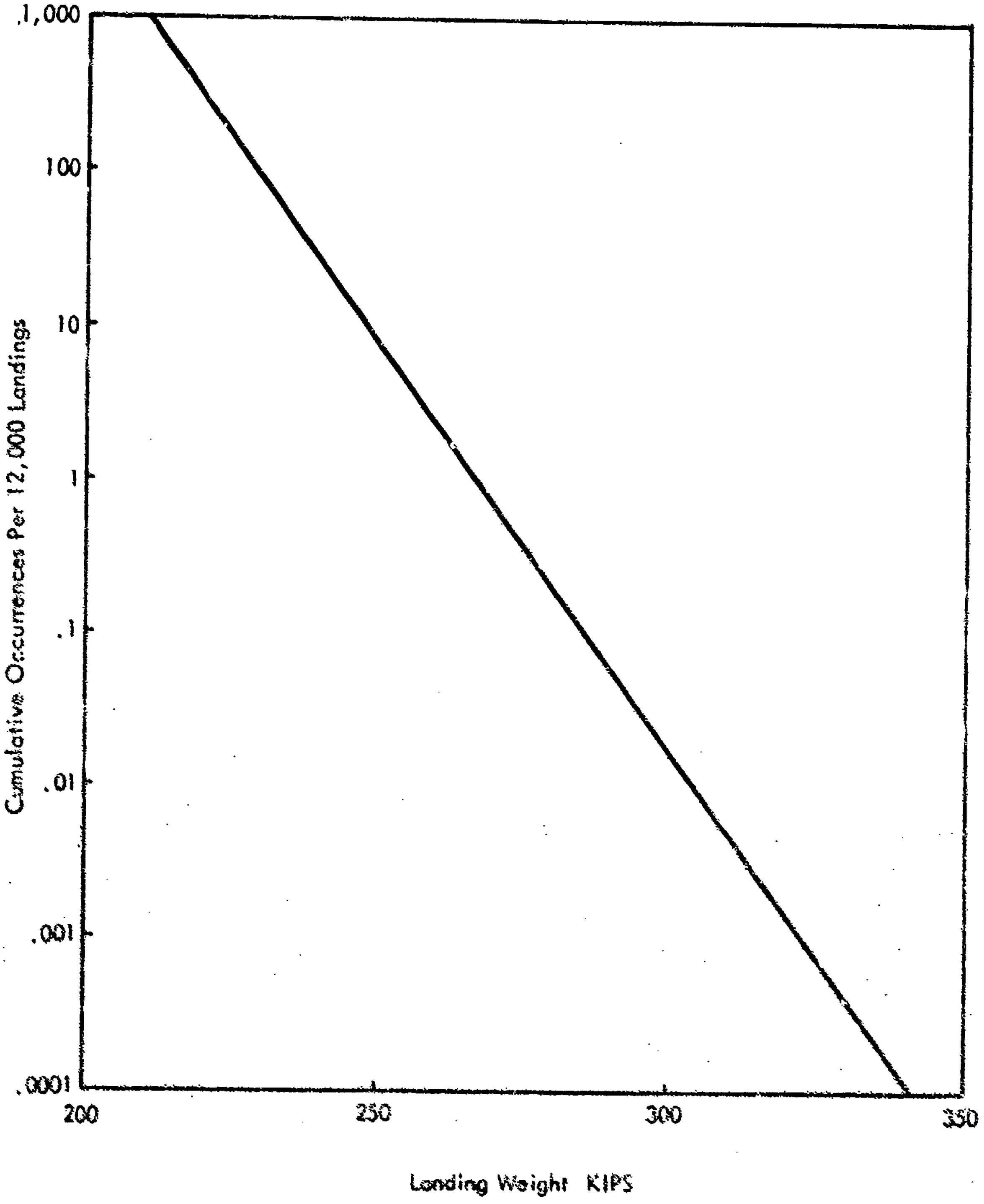


FIGURE 20 C-141 LANDING WEIGHTS

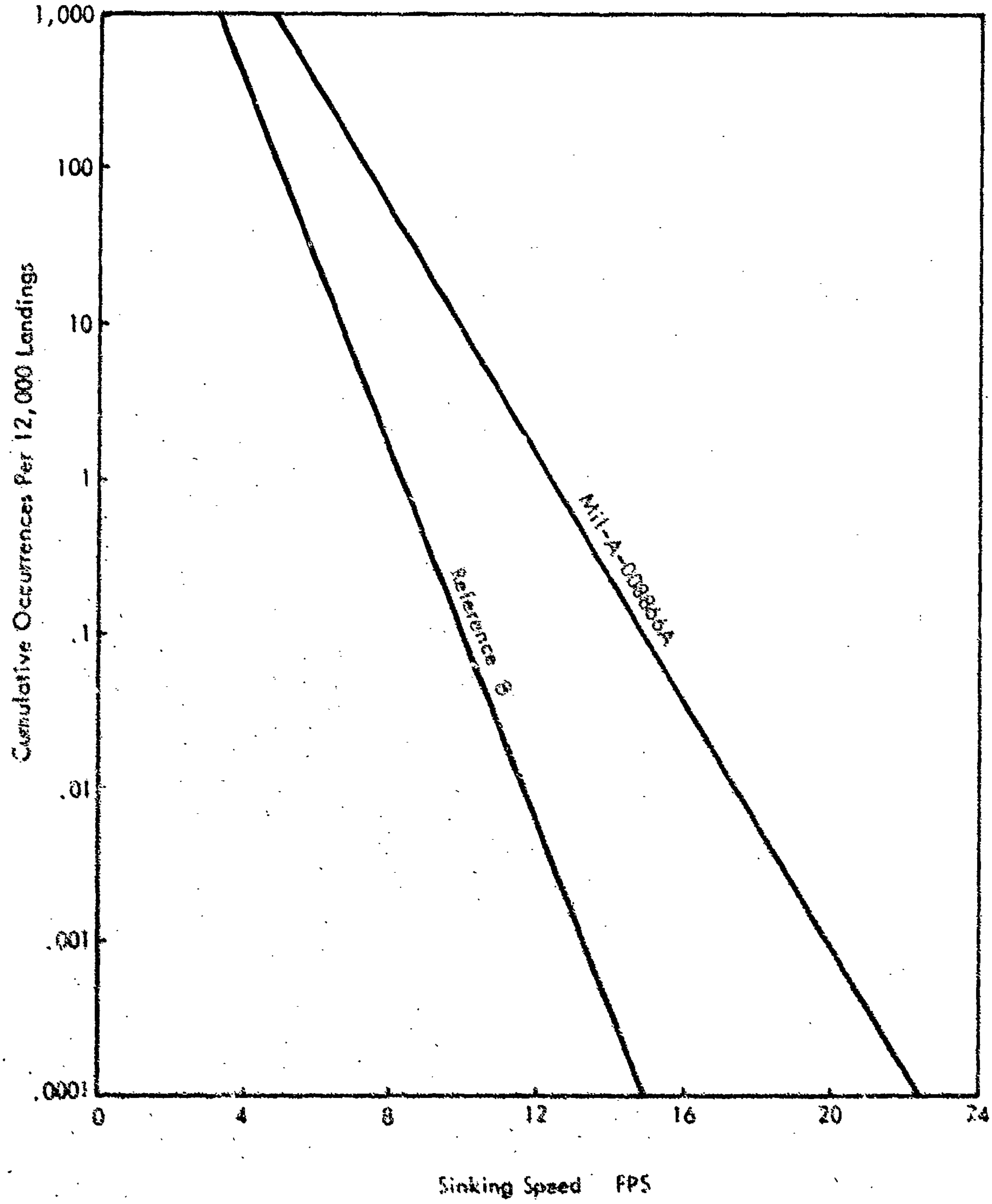


FIGURE 21 LANDING SINK SPEED STATISTICS

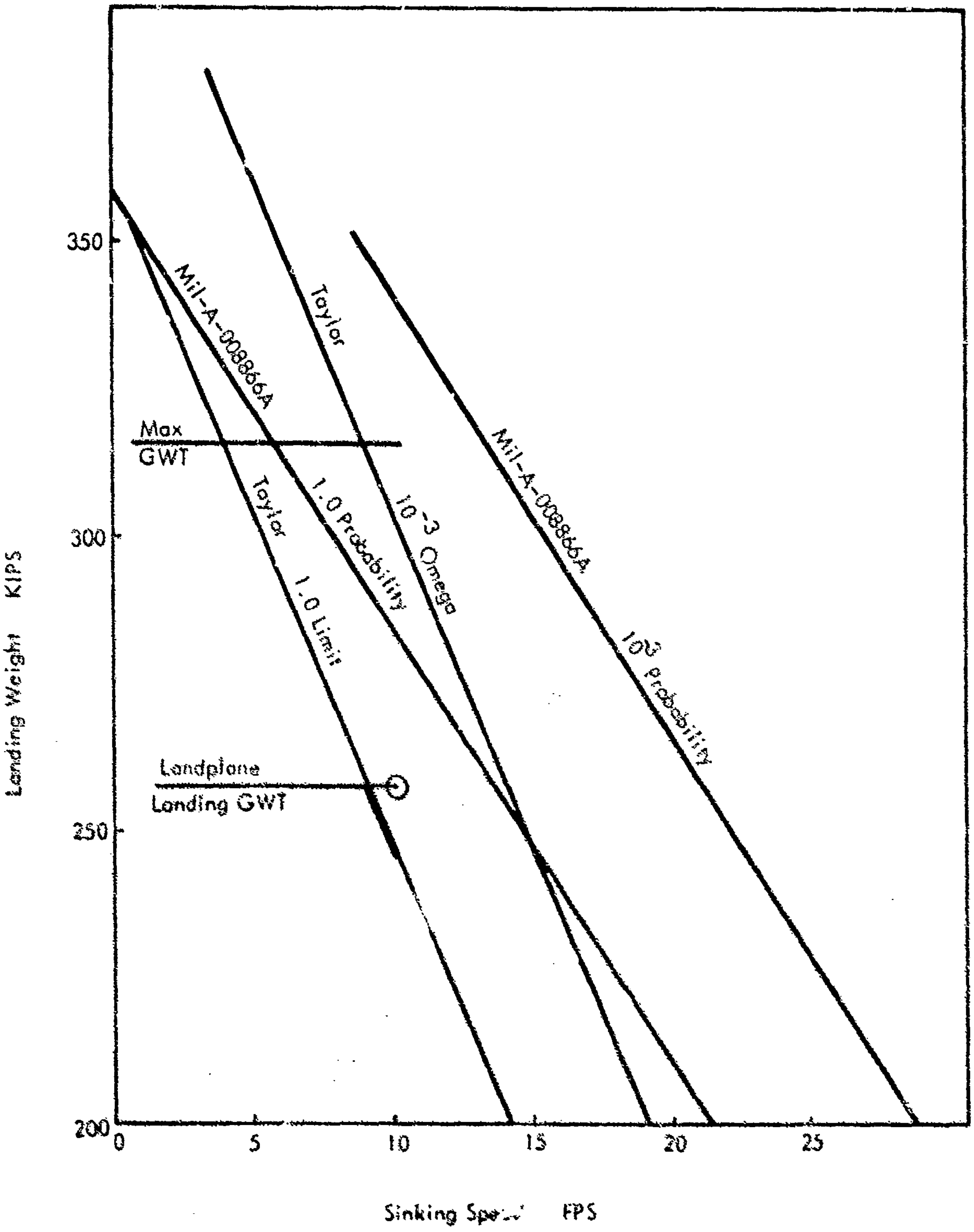


FIGURE 22 LANDING WEIGHT - SINK SPEED PROBABILITIES

Also, a decision must be made as to what the sinking speed restriction is for normal operations. If this is selected as ten feet per second, then the range of limit conditions to be investigated is very small and is as shown by the heavy part of the limit line in Figure 22. This region of investigation is also very close to the actual C-141 design point of 10 feet per second at 257,500 pounds.

d. Omega combinations of landing weight and sinking speed are chosen along the 10^{-3} probability line and analyzed in accordance with MIL-A-008862A requirements. However, the energies involved in using these data are extremely large and lead one to question the validity of the data, particularly when the sinking speed statistics used did not include any data at higher sinking speeds than seven feet per second. It appears that, even for Omega conditions, rational limits must be set on extrapolation of statistics in order to result in a reasonable structural design. A possible rational cutoff of sinking speed for the Omega case might be the reserve energy absorption value of 125 percent of limit sinking speed given in MIL-A-008862A. This would still make the Omega case the designing case in terms of energy requirements.

e. Another example of an approach to selecting design load conditions using a minimum of statistical information follows. This example deals with the selection of positive symmetrical maneuver conditions for the C-141 using payload statistics derived from C-141 usage data and maneuver load factor statistics from MIL-A-008866A.

Table IV reproduces the positive maneuver load factor spectra for $C_{TRANSPORT}$ aircraft from Table VII of MIL-A-008866A. The values shown are in terms of cumulative occurrences per 1000 flight hours by mission segment. Table V shows the percentage of time the C-141 spends in each of the mission segments based on actual usage data.

TABLE IV

MANEUVER LOAD FACTOR SPECTRA $C_{TRANSPORT}$

REFERENCE TABLE VII MIL-A-008866A (USAF)

N_Z	LOGISTICS			TRAINING			REFUEL
	ASCENT	CRUISE	DESCENT	ASCENT	CRUISE	DESCENT	
1.2	11,000	825	13,000	60,000	45,000	35,000	8,000
1.4	380	30	435	5,600	4,000	3,500	850
1.6	25	3	28	500	350	800	110
1.8	4.5	0.7	5	70	35	250	20
2.0	1.8			15	5	90	2.5
2.2				4	1	35	
2.4				2		11	
2.6				1		4.5	
2.8						1.5	

TABLE V

C-141 USAGE DATA (FLIGHT HOUR BASIS)

LOGISTICS 84.3%			TRAINING 15.7%		
CLIMB	CRUISE	DESCENT	CLIMB	CRUISE	DESCENT
13.6%	81.1%	5.3%	18.2%	49.6%	32.2%

Extrapolating the maneuver load factor data of Table IV and applying the percentage utilizations of Table V results in the maneuver load factor exceedances of Figure 23 for one C-141 lifetime of 30,000 flight hours.

f. C-141 usage data also provides payload utilization information which is summarized in Figure 24. The data points are shown and extrapolation is used to determine possible extremes of payload. Note that payloads are extrapolated beyond the design limit payload. Truncation of payloads at 120% in this case is arbitrary. However, in actual cases, reasonable upper limits can probably be established through cargo density-available volume relationships or other means.

Maneuver load factor - payload joint probabilities are shown in Figure 25 as derived from the data of Figures 23 and 24. In accordance with the recommendations of Section VIII, a 10^{-3} probability of occurrence per aircraft lifetime is used for Omega conditions and a 1.0 probability of occurrence per aircraft lifetime is used for limit conditions.

g. In order to facilitate the selection of design loading conditions from these data, real payload and gross weight values are introduced and the product of maneuver load factor and gross weight is plotted against payload as shown in Figure 26 for limit conditions and in Figure 27 for Omega conditions. Lacking further statistics, it is assumed that the load factor - payload combinations can occur with any given fuel quantity present.

h. In Figure 26 the range of limit conditions to be investigated is shown and since limit conditions represent normal operations, the envelope is cut off by a design payload limitation and a maximum takeoff gross weight limitation. To complete the loads analysis, the symmetrical maneuver analysis requirements of MIL-A-008861A are to be applied using limitations on center of gravity limits, speeds, etc. established by normal operational placard. Note that the maximum $N_Z W$ of some 874,000 in Figure 26 is of the same order

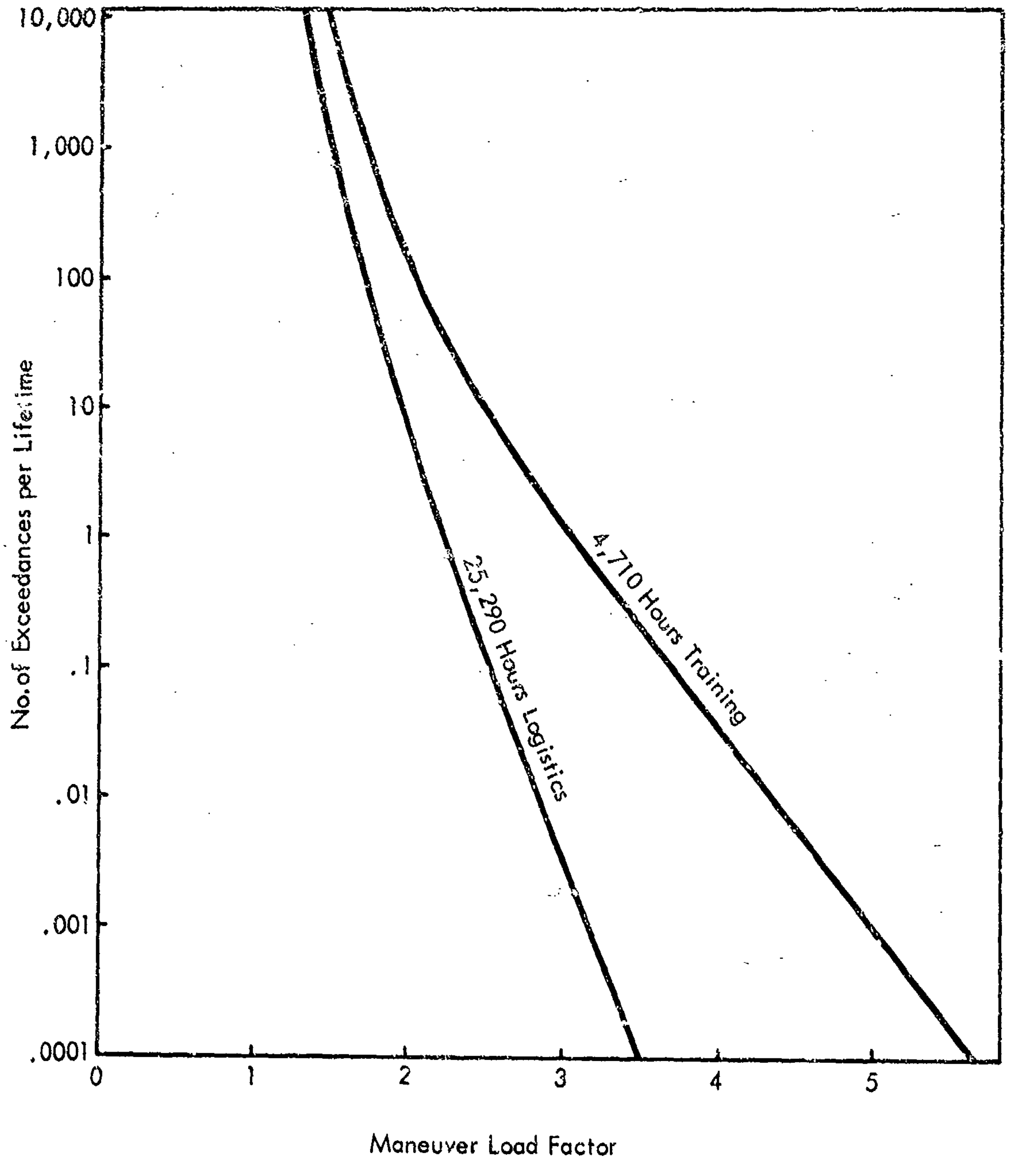


FIGURE 23 C-141 MANEUVER LOAD FACTOR PROBABILITY

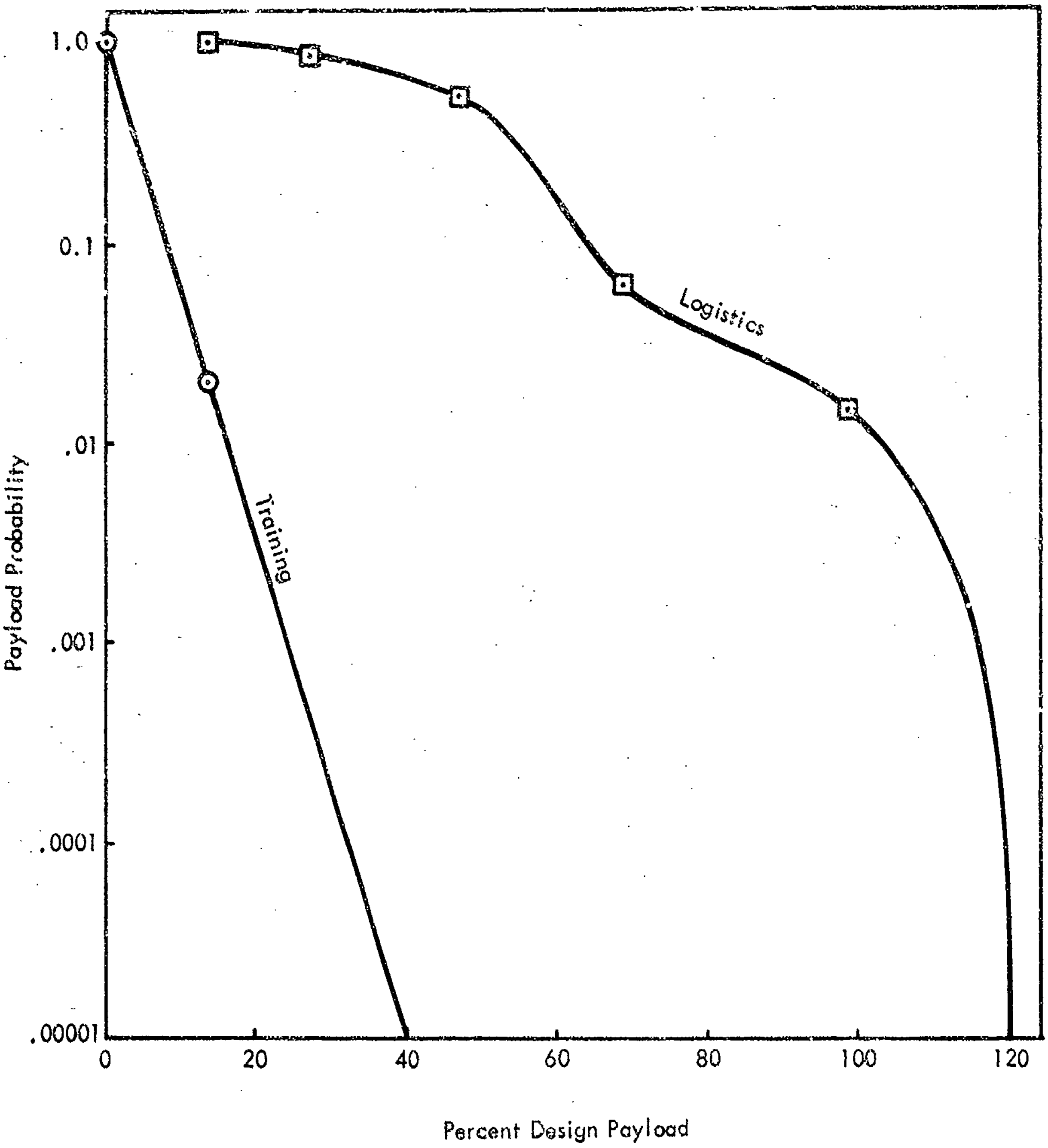


FIGURE 24 C-141 PAYLOAD PROBABILITY

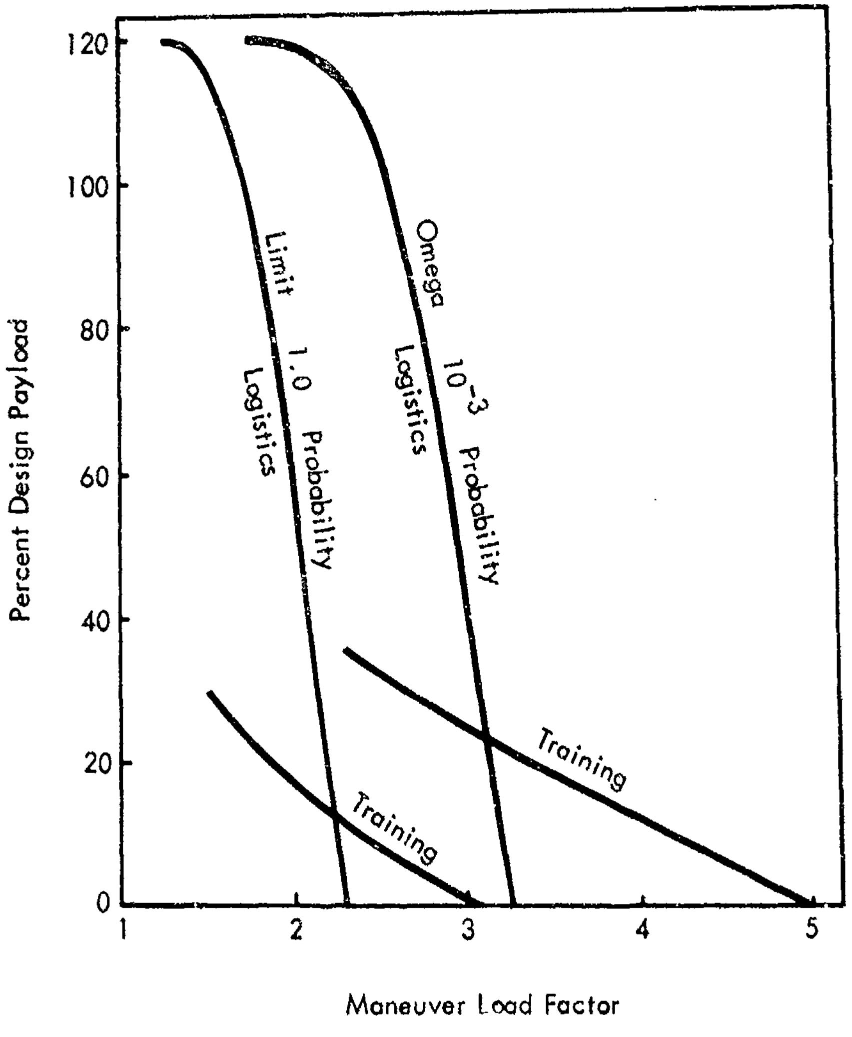


FIGURE 25 C-141 LOAD FACTOR - PAYLOAD PROBABILITIES

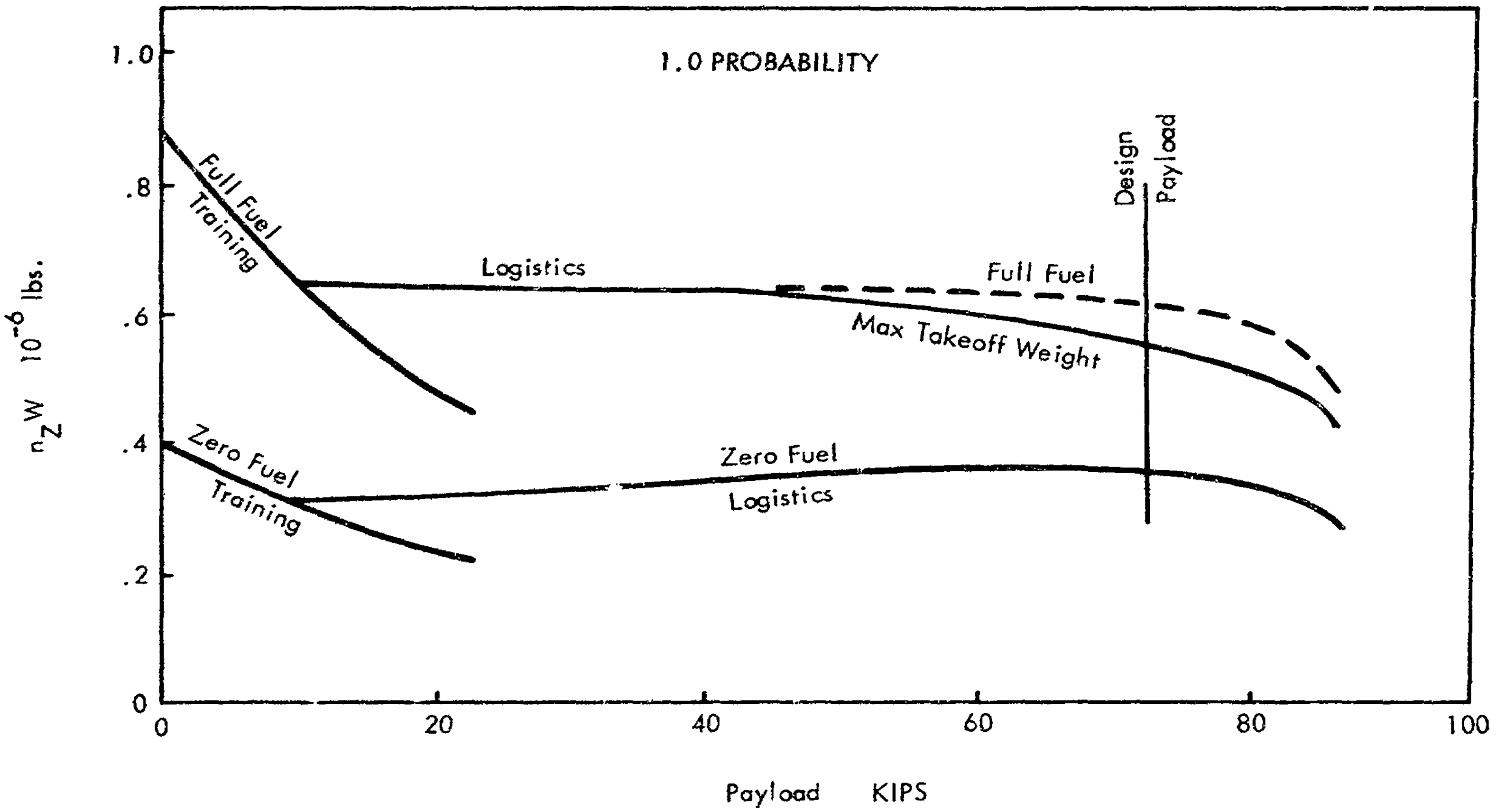


FIGURE 26 C-141 LIMIT LOAD CONDITIONS

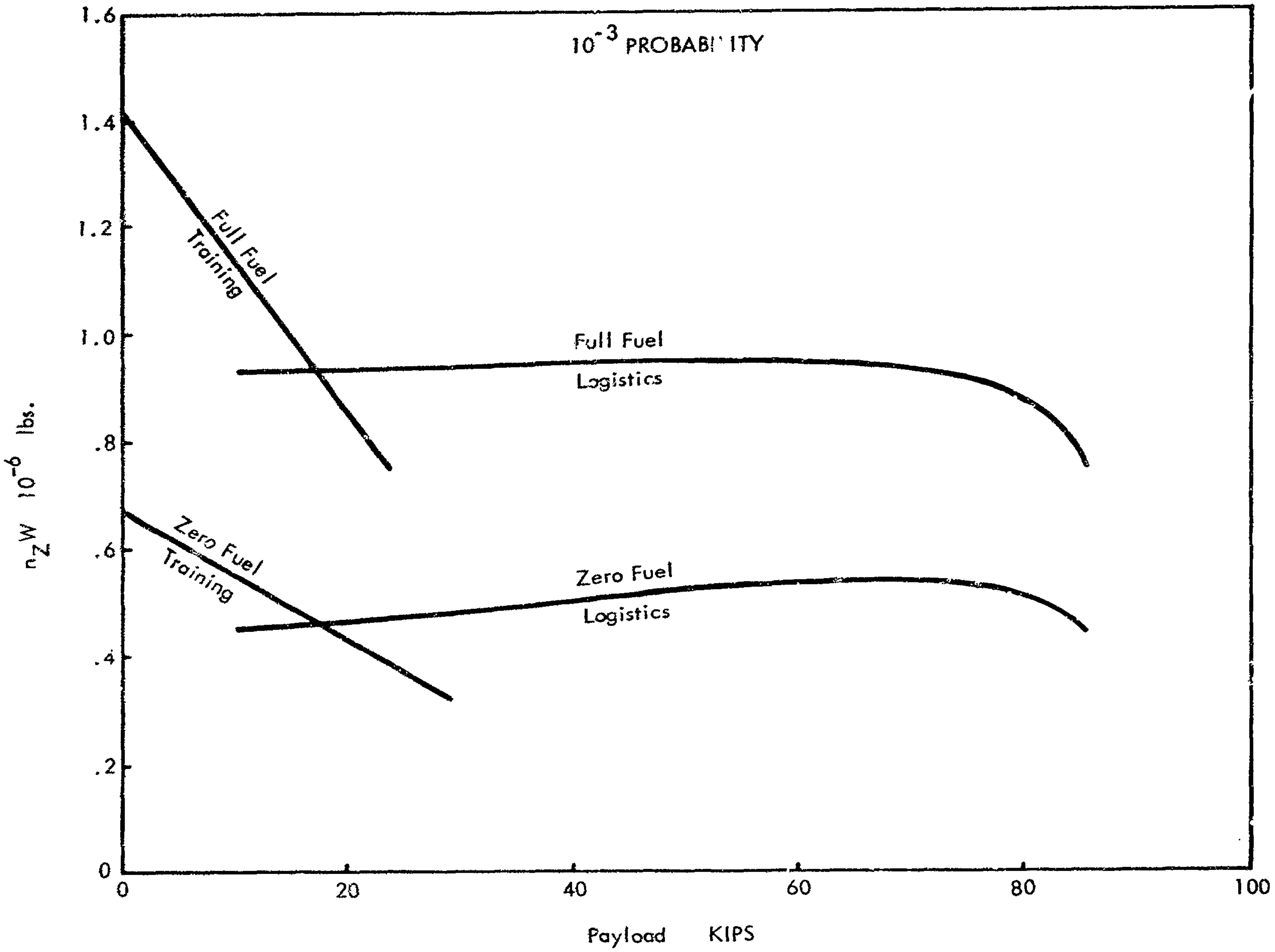


FIGURE 27 C-141 OMEGA LOAD CONDITIONS

as the C-141 design limit $n_Z W$ of approximately 800,000. The relative criticality of the two levels is not known, however, since they occur at entirely different combinations of fuel weight, payload, and maneuver load factor.

i. In Figure 27 the range of Omega conditions to be investigated is shown. With the lack of further statistics, these combinations are analyzed using the symmetrical maneuver requirements of MIL-A-008861A. Note that the maximum $n_Z W$ of approximately 1,400,000 in Figure 27 is of the same order as the C-141 ultimate $n_Z W$ of approximately 1,200,000. Again, the relative criticality of the two levels is not known.

SECTION V
CHOICE OF DESIGN LOADS

5.1 Introduction

a. Reference 1 has shown that a purely probabilistic determination of design load requirements is not acceptable for the design of flight vehicles. Rather, the probabilistic loads descriptions must be used to obtain discrete deterministic limit and omega design conditions. The loads for the design conditions are then utilized for stress design analysis just as if the loads had been calculated using the present deterministic design criteria, except that the factor of safety is that required for a given structural reliability instead of an arbitrary value such as 1.5. Thus, once the deterministic design conditions are obtained, continuity is maintained with the present design procedures.

b. There are three main problem areas involved in the determination of the design conditions and loads for a structural reliability analysis. The three problem areas are as follows:

1) The determination of loads spectra which adequately reflect the utilization of the flight vehicle and the extreme maximum loads.
2) The representation of loads by a single parameter which is compatible with strength.
3) The selection of limit and omega conditions and loads in a multi-parameter, multi-load source environment.

c. It is the intent of this section to determine the data required and available, solutions to the three problem areas and to recommend procedures for determining the design loads while maintaining continuity with the present design procedures.

5.2 Data Required for the Determination of Loads Spectra

a. In order to determine design load requirements on a structural reliability basis, separate limit and omega load spectra must be calculated. When only one or two parameters determine the external loads, such as longitudinal load factor for a rocket, load spectra can be easily calculated. However, for most aircraft the structural loads are a function of several parameters which are often dependent upon each other. Therefore, the probability of occurrence of each of the parameters cannot be separately determined and then combined to get the joint probability of occurrence.

b. Two separate influences can be postulated whose combination is essential to the proper definition of probabilistic load spectra.

1) Probability of Configuration: The combinations of parameters such as gross weight, weight distribution, height, speed, and aerodynamic configuration.

2) Probability of Load Source: Several further parameters are involved in the determination of structural load levels for each load source (e.g. gust, symmetric maneuver, etc.). In the case of wing steady symmetric maneuver loads, for example, the principal parameter is vertical load factor. The probability of load occurrence must be obtained for each of the load sources.

c. The probability of configuration can be determined either by the analysis of assumed mission profiles or, in the case of operational aircraft, by the analysis of aircraft usage data. The mission profiles are based upon the operator's intended or actual usage of the aircraft. In the past, mission profiles have been generated primarily for fatigue analysis. As a result, only average flight conditions within the operational limitations (no omega conditions) were considered. For example, the C-141 logistics design mission profiles consider only standard handbook climb, cruise, and descent speed-altitude schedules. Such profiles are acceptable for fatigue analysis where primary concern is average loading conditions, but not for a statistical determination of extreme loading conditions, as required for the proposed structural design criteria. Figure 28 demonstrates the scatter in the speed-altitude statistics for the C-141 medium range logistics mission data as obtained from the

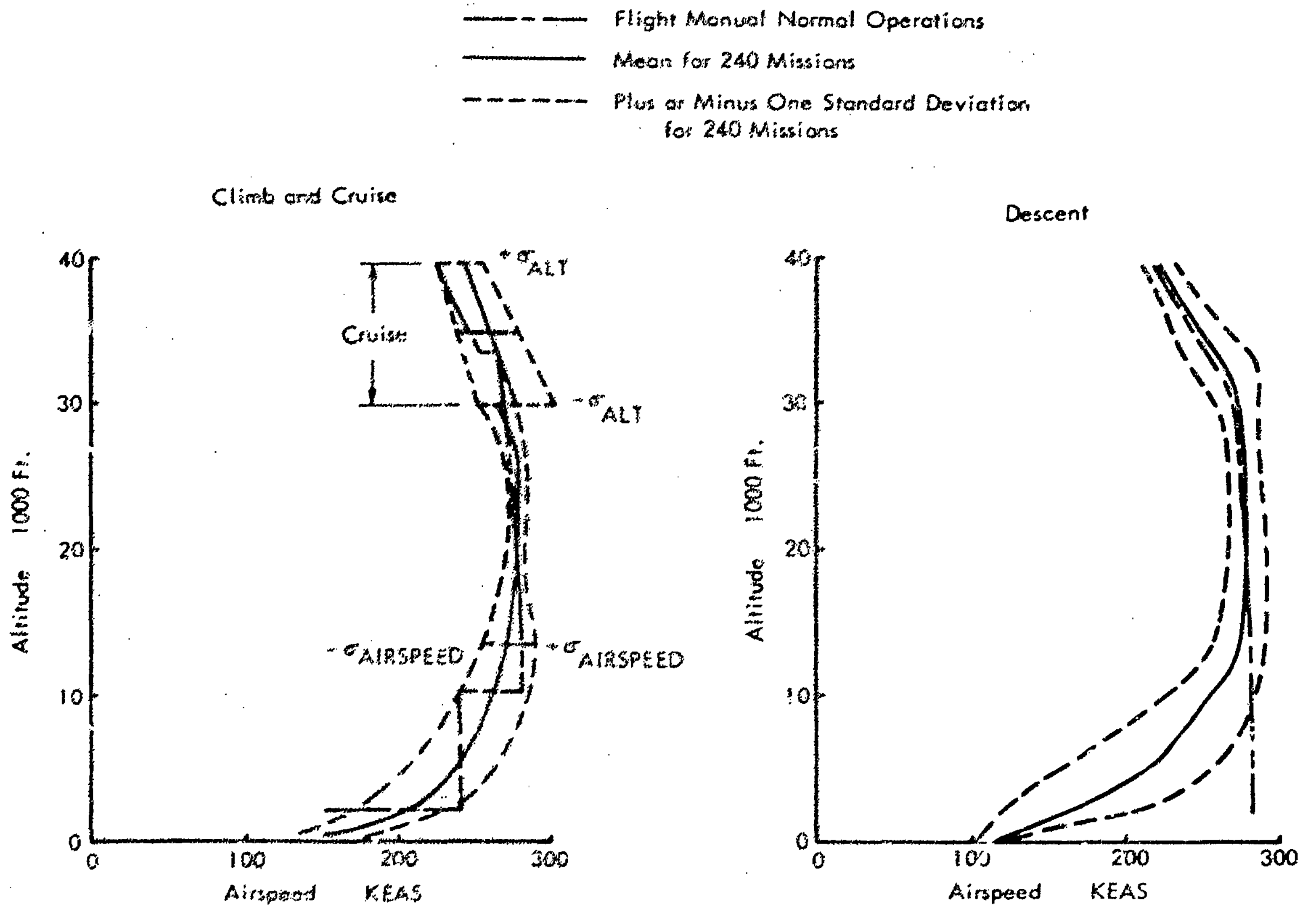


FIGURE 28 SPEED-ALTITUDE STATISTICS FOR MEDIUM RANGE LOGISTICS MISSIONS
C-141

analysis of V-G-H data. The statistical scatter of flight and mass parameters can be accounted for in the mission profiles by one of two methods.

1) Develop a large number of mission profiles which encompass the significant ranges of the parametric statistical scatter.
2) Develop a limited number of mission profiles but bias the parameters such that extreme loads due to the actual parametric scatter are included. For example, since increased airspeed causes increased gust loads, rather than use the mean airspeed for a given altitude, the airspeed should be biased above the mean.

In order to account for such statistical scatter in the mission profile parameters, data must be obtained either from similar operational aircraft or from computer simulated analyses.

Omega conditions must also be included in the mission profiles. Such conditions may be the result of intentional violation of the operating restrictions such as exceeding maximum cargo weight, or the result of the failure of such items as automatic controls, engines, etc. The selection of the omega conditions is based upon the analysis of operational data for similar aircraft and/or computer simulated analyses which include probabilistic systems failures

It is not recommended that separate sets of mission profiles be developed for fatigue and static strength structural reliability analyses. Rather, one set of mission profiles should be established which adequately meet the requirements for both analyses.

d. Probability of Load Source

For a given load source, such as steady vertical maneuver, the probability of occurrence of the principal parameters must be determined. For a conventional aircraft, the probabilities of occurrence for such parameters can be determined from either

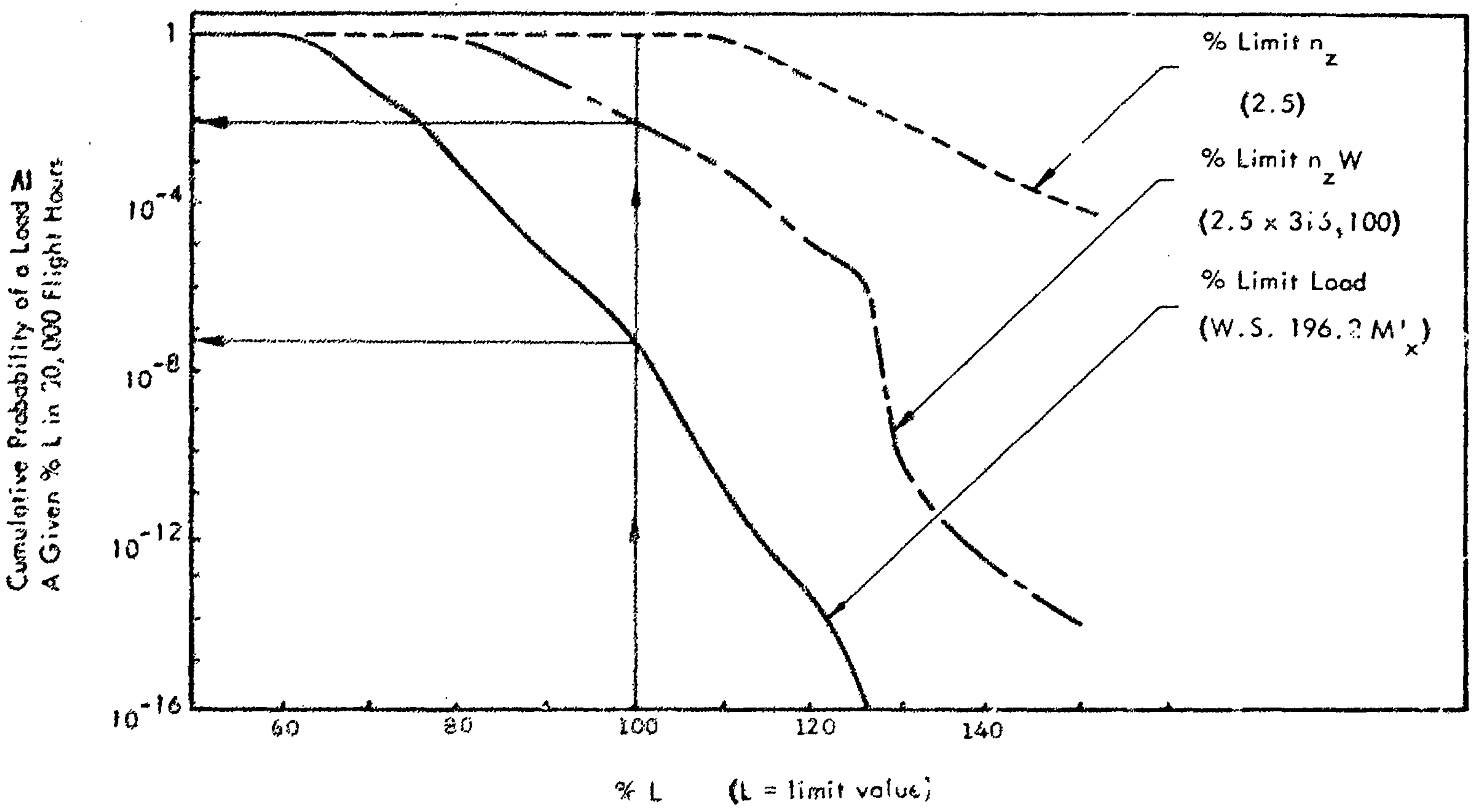


FIGURE 29 VERTICAL MANEUVER FOR 30,000 HOUR DESIGN LIFETIME

C-141

aircraft V-G-H/G-L-S[*] data for similar aircraft or from Reference 9. For instance, the probability of occurrence of vertical maneuver [illegible] factor is delineated in MIL-A-8866 for various types of aircraft. However, there are some parameters, such as aileron input during rolling maneuvers, which require measurements not only of [illegible]al deflection (magnitude), but also a time history so that deflection rates, and deflection duration can be ascertained. As a result, much of the data needed to determine probabilistic loads spectra are either not available at all or not available in sufficient detail. Also, in the case of a radically new design such data would not be available (with the possible exceptions of atmospheric turbulence and runway roughness data) until either V-G-H/G-L-S data for the aircraft has been collected and analyzed, or obtained from computer simulation analyses.

e. In order to evaluate the significance of various parameters for strength vertical maneuver, a statistical analysis was performed for the C-141 cargo transport. As discussed previously, steady vertical maneuver is just one of the many load sources which contribute to the total probability of load occurrence. Figure 29 presents the results of the steady vertical analysis for the 30,000 flight hour design lifetime. Three different parameters were selected to represent per cent limit load as follows:

1) Vertical Load Factor (N_Z) - This parameter reflects only the mission usage of the aircraft.
2) Vertical Load Factor - Gross Weight Product ($N_Z \times W$) - This parameter reflects both mission and gross weight utilization.
3) Actual Wing Bending Moment (M'_x) - This parameter reflects mission, gross weight, weight distribution, center of gravity, airspeed, altitude, and aerodynamic configuration utilization.

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[*] G-L-S = Ground Load Survey

TABLE VI

SUMMARY OF PRESENT STATISTICAL LOADS ANALYSIS CAPABILITIES

LOAD SOURCE	DATA AVAILABILITY	RECOMMENDED ANALYSIS METHODS	SIGNIFICANT ANALYSIS PROBLEMS
Atmospheric Turbulence	Large amounts of data are available (References 9, 10, and 13)	Power Spectral Density (References 9, 10, 11 and 13)	Load Component Phasing
Taxi, Takeoff, and Runout	Data is available (Reference 12)	Power Spectral Density (Reference 9 and 12)	Load Component Phasing
Landing Impact	Large amounts of sink rate data are available for conventional aircraft, but quite limited for STOL aircraft. (Reference 9	Dynamic Flexible	Some load phasing complications
Steady Vertical Maneuver	Large amounts of vertical load factor data are available for conventional aircraft (Reference 9)	Quasi-Static Flexible	None
Abrupt Vertical Maneuver Rudder Induced Yaw Aileron Roll	In general, more data is required. In particular, compatible time histories of the pertinent control surface displacements, rotational accelerations, and load factors are required.	Dynamic overall aircraft Quasi-static or dynamic flexible loads analysis	None
Engine Failure Gust Upset Automatic Controls and Other System Failures	Statistical data can be determined for the probability of occurrence of such events. Pilot corrective actions and subsequent aircraft response can be determined by computer simulation for individual aircraft.	Same as above	None
Towing, Pivoting, Turning and Other Ground Operations	Some data is available, but may be best analyzed by arbitrary criteria Reference 9).	Arbitrary	None

Grossly different probabilities of loads are obtained based upon the choice of parameter. As tabulated below, the probability of 100% limit load occurrence decreases significantly as more of the utilization parameters are included in the analysis.

Parameter:	N_Z	N_ZW	M'_X
Cumulative Probability of Limit Load	10^0	10^{-2}	10^{-7}

f. Statistical loads analysis must therefore properly account for the utilization of all parameters which significantly affect the loads. Overall load parameters such as N_Z or N_ZW can lead to overly conservative design load requirements.

5.3 Data Available and Methods of Determining Load Spectra

a. Table VI presents a summary of data available, recommended analysis methods and significant analysis problem areas. Even where it is indicated that large amounts of data are available, more data would be useful, as the available data is primarily within the operating restrictions. Therefore, the data must often be extrapolated to obtain data in the omega operational regime. Such extrapolation can be accomplished by fitting the available statistical data with a probabilistic distribution such as Gumbel's extreme-value distribution.

b. Of the loads sources considered in Table VI, there are only two, atmospheric turbulence and taxi, takeoff and runout operations, for which power-spectral rather than discrete loads analyses are recommended. Power-spectral methods are recommended for the analysis of atmospheric turbulence in lieu of a discrete gust approach. Reference 10 contains an evaluation of power spectral gust analysis. The response of an aircraft to atmospheric turbulence and the resultant structural loads depends not only upon the gust velocity and wavelength at a given instant, but also upon the immediately preceding turbulence. Clear air, thunderstorm, and low level turbulence predominately display the characteristics of continuous turbulence with some severe discrete gusts. The turbulence is random in nature with varying gust velocities and wavelengths, which supports the continuous turbulence model of the atmosphere used for power-spectral gust analyses as opposed to a discrete gust model. Also, in a discrete gust analysis, the elastic mode effects are highly sensitive to gust wavelength. The predominant practice for discrete gust analyses is to assume a gust wavelength which is a given multiple of the wing mean aerodynamic chord length. However, in order to be realistic, the discrete gust analysis would have to account for the joint probability of gust wavelength and velocity. Such data are not available and are not likely to become available.

c. It has long been recognized that atmospheric turbulence is three dimensional with spatial distributions. Recent analyses have indicated that maximum structural loads may be obtained from combined vertical and lateral gust velocities. Power-spectral gust analysis can be extended to include the response of the aircraft to three dimensional spatial dependent turbulence. Reference 11 has developed a feasible approach to such analysis. Further work along these lines is being continued at the Lockheed-Georgia Company under contract with the Air Force Flight Dynamics Laboratory.

Therefore, it is concluded that the power-spectral gust analysis will yield design loads which most adequately reflect both the actual atmospheric turbulence and the elastic mode effects.

d. Power-spectral analysis of atmospheric turbulence does have one significant problem which involves the determination of load component phasing. For a large, dynamically responding, flexible aircraft, the loads are not in phase. For example, wing root bending may maximize at a quite different time than wing root torsion or shear. It is necessary to determine load phase relationships in order to obtain discrete load conditions for the structural reliability and design stress analyses.

e. Power-spectral analysis is also recommended for the analysis of taxi, takeoff and runout, for essentially the same reasons as for atmospheric turbulence. Power-spectral analysis for such ground operations is analogous to that for atmospheric turbulence except runway and taxiway roughness is the source of the power-spectral density rather than atmospheric turbulence. References 9 and 12 contain criteria and methods for the power-spectral analysis of taxi conditions.

f. As noted in Table VI sufficient data is not available for a purely probabilistic determination of all loads spectra. This is true particularly for abrupt control input conditions such as abrupt vertical maneuver, aileron roll or rudder kick. In such instances either assumptions can be made in order to obtain the loads spectra, or loads spectra will not be determined, thus forcing the selection of limit and omega load conditions on an engineering judgment basis.

5.4 Recognition of Aircraft Limitations

a. It has often been suggested, as in reference 13, that there is no practical limitation of maneuver load factor capability for modern high speed transports. This is true at high airspeeds coupled with low mach numbers, but at the airspeed - Mach Number combinations at which the aircraft is predominantly operated, there can be definite limitations. Figure 30 presents the 2.5g symmetrical maneuver stall speeds for the C-141 transport at two different gross weights. Approximately 65% of an average C-141 lifetime is spent in cruise during logistic missions. The flight manual cruise-climb schedule is also shown on Figure 30 as a function of gross weight. It is noted that for the two gross weights shown the cruise equivalent airspeed is well below the corresponding 2.5g stall speeds. In fact the maximum obtainable load factor obtainable for the cruise-climb schedule is approximately 1.6 for both gross weights. Therefore, it is concluded that it is not realistic to take limited measured load factor spectra and extrapolate to some extreme load factor without accounting for aircraft limitations such as aerodynamic stall or control limits.

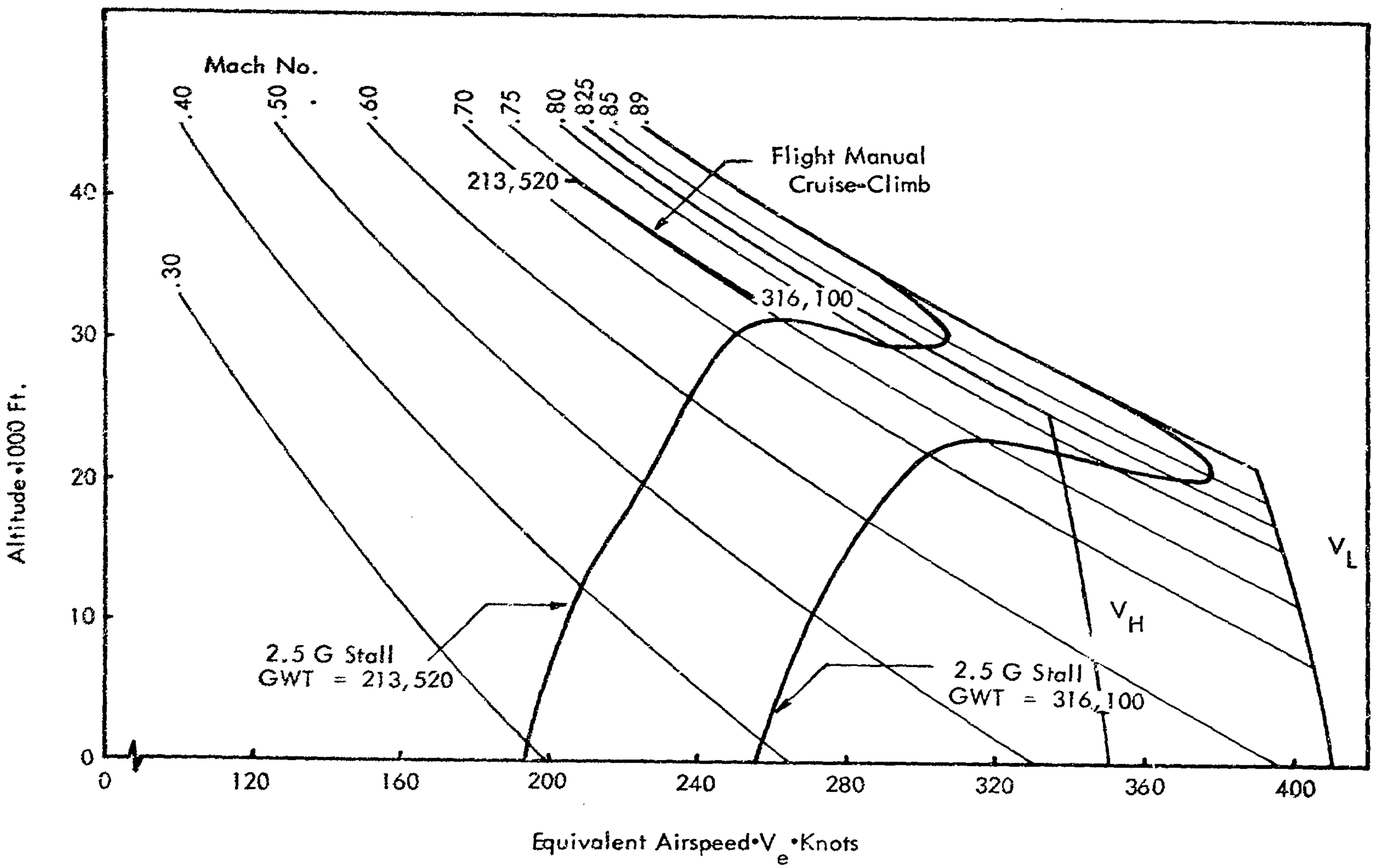


FIGURE 30 C-141 2.5 G SYMMETRICAL MANEUVER STALL DATA

b. Figure 31 shows the effects of considering aerodynamic stall limitations on a C-141 wing vertical bending moment spectrum. At 100% of limit vertical bending moment, the truncation of the load factor spectrum at stall lift coefficients causes an approximate three decade decrease in the probability of exceedance.

It is therefore concluded that statistical analyses which neglect aircraft limitations can be overly conservative.

5.5 Loads Representation Compatible with Strength

a. One of the basic requirements of the proposed structural design criteria is that the loads must be expressed by a single parameter which is compatible with strength so that the loads and strength probability distributions can be integrated to obtain the structural reliability. This is a difficult problem, as the strength requirements for a given flight condition are determined by

1) six component external loading
2) internal loading due to such sources as pressurization or thermal effects
3) possible strength degradation at extreme temperatures.

b. The most accurate way to account for the six component external loading together with internal loading due to pressurization, etc., would be to perform a stress spectral analysis. Strength degradation due to elevated temperatures could be accounted for by grouping similar strength degradation conditions and performing individual probability of failure calculations, the sum of which must be equal to the desired probability of failure. Also, since stress spectral analyses are performed for particular structural locations, such as a joint at a given wing station, an error function which is applicable to joint structure in particular could be used rather than a general error function. However, stress spectral analyses are not feasible during initial design stages. Therefore, an alternative approach must be taken in order to initially define the strength requirements. Obviously some concessions must be made in the extent that the loads representation is compatible with the strength.

c. Under present design procedures, design loads are often obtained by developing load envelopes at selected component stations. The conditions which define the loads envelopes are then the design load conditions. After stress analysis of the design load conditions, strength envelopes are obtained by expanding the load envelopes to zero margin of safety. Figure 32 presents typical envelopes for a C-141 wing station. If vertical bending moment and torsion are taken to be the significant load components, an approximation of the per cent strength for a given load condition can be obtained by ratioing the magnitude of the bending-torsion vector to the magnitude of the envelope vector having the same direction. A similar procedure can be adopted for the proposed structural design criteria, except that the envelopes are defined by the statistical limit and omega load conditions. The load spectra for limit and omega conditions are then defined as a per cent of the appropriate loads envelopes. Pressurization and thermal stress effects either result in increased or decreased external loads capability at a given component station. The loads spectra could be adjusted to approximate such effects by factoring the loads spectra for each individual flight condition. For instance, if pressurization for flight at a given altitude were to decrease bending moment capability by 10% at a particular load station, the load spectra for flight at that altitude should have the load magnitudes multiplied by 1.11 (1/.9). Strength degradation due to extreme temperatures can also be handled by factoring the loads spectra for such conditions if the strength coefficient of variation, (S) does not vary significantly. If S does vary significantly, then loads spectra for separate strength degradation regimes can be calculated and separate probability of failure analyses performed with the total probability of failure divided amongst the strength degradation regimes.

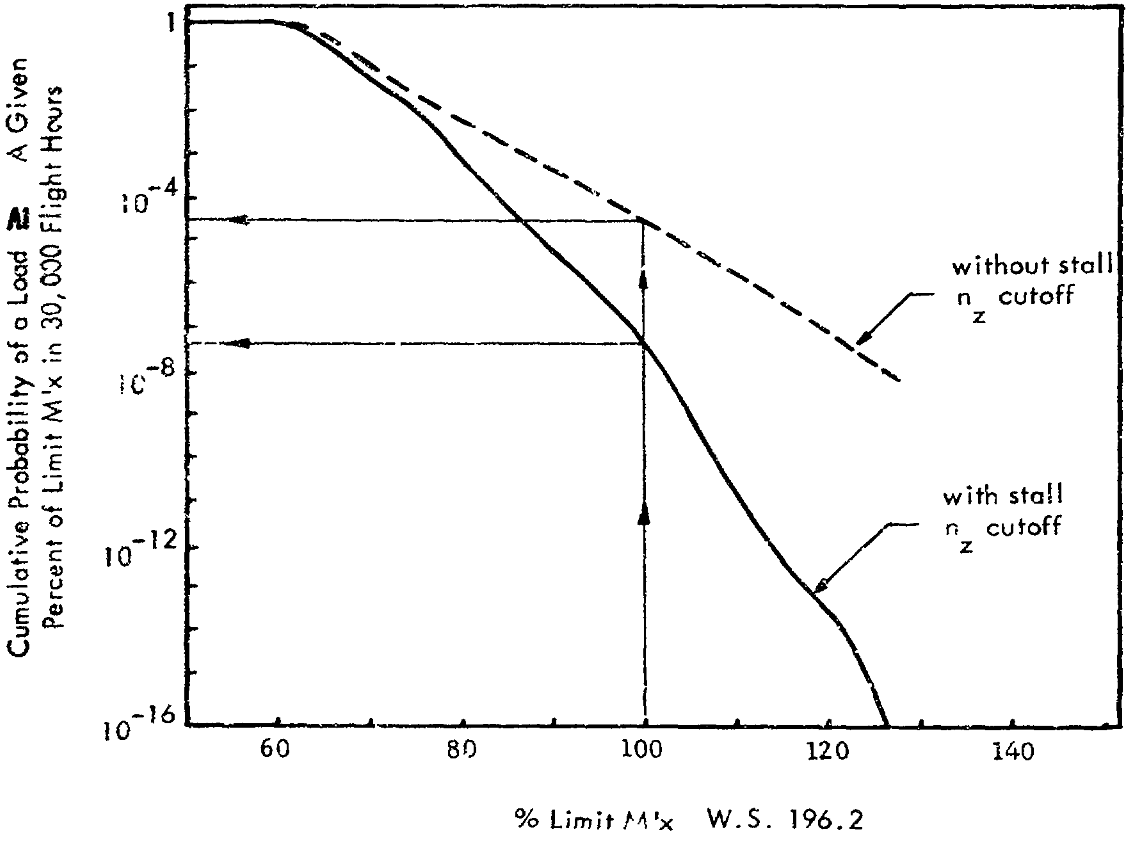


FIGURE 31 EFFECT OF STALL CUTOFF ON MANEUVER LOAD SPECTRUM

C-141

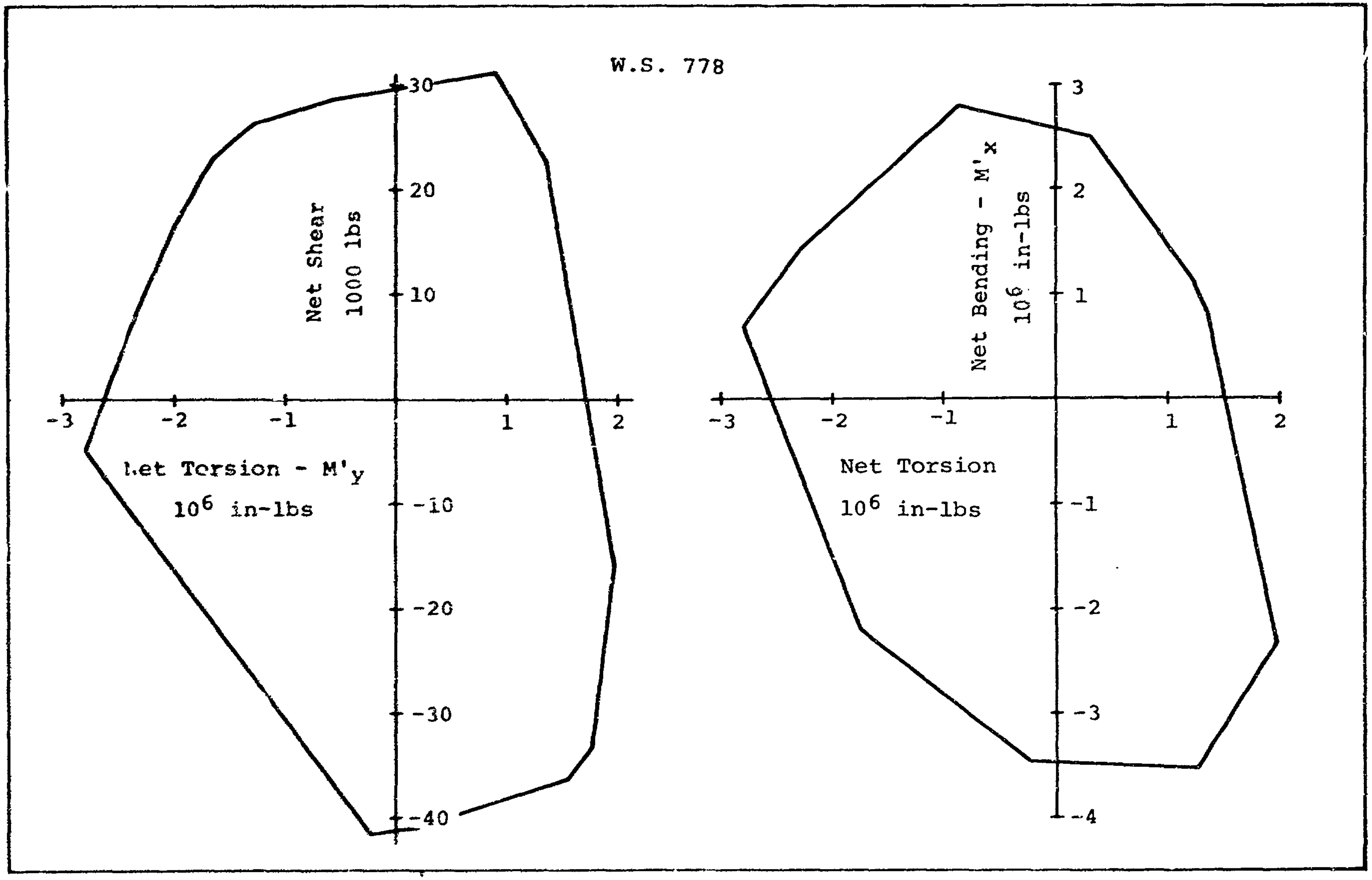


FIGURE 32 EXAMPLES OF C-141 PARTIAL LIMIT STRENGTH ENVELOPES

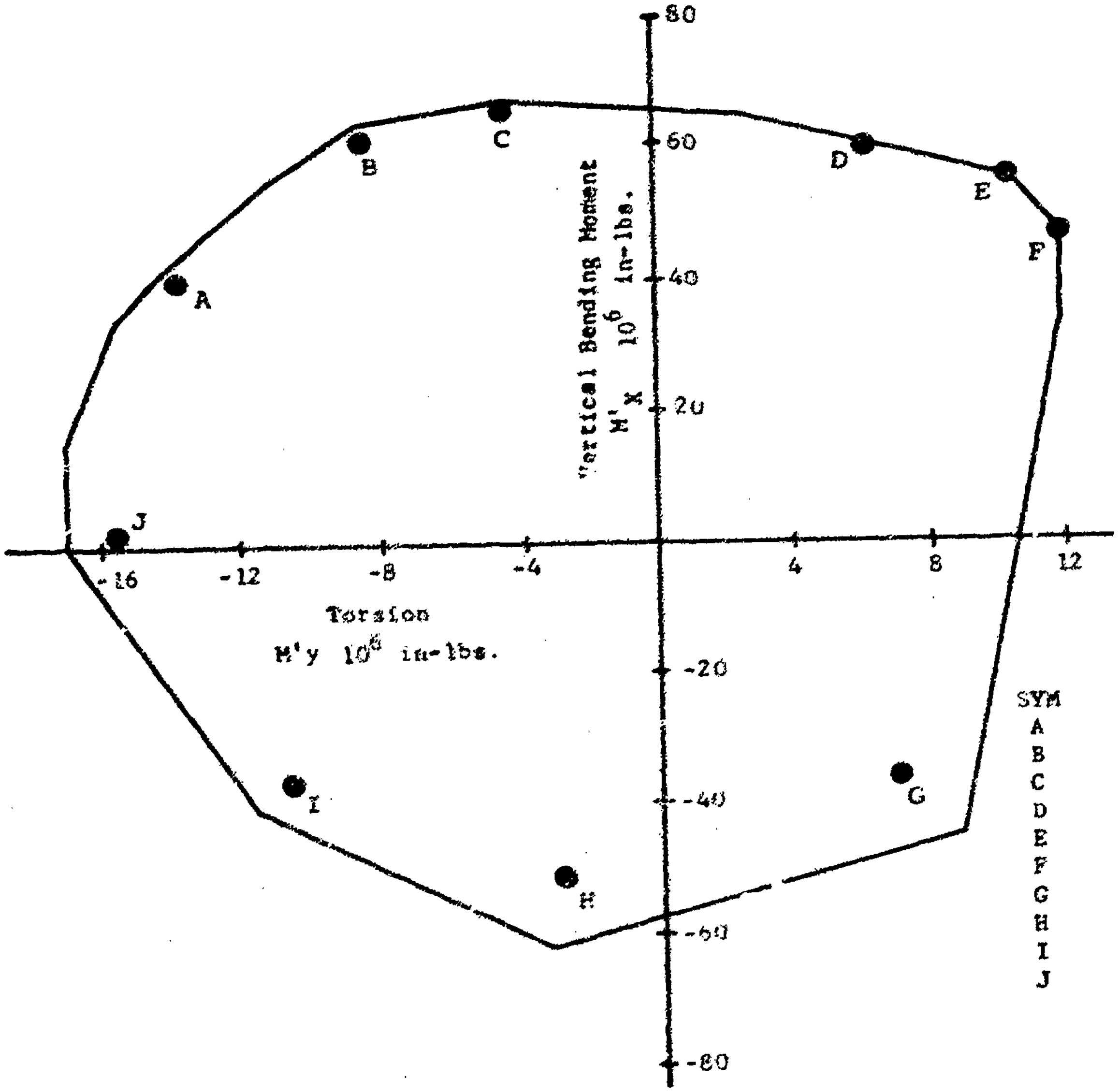
d. It is recognized that such approximations may involve considerable error. However, for the initial design analysis such approximations must be made in order to make the loads representation compatible with that for strength. More sophisticated methods such as stress spectral analysis can be used for subsequent or update analyses.

5.6 Selection of Limit and Omega Conditions and Loads

a. As previously discussed, deterministic limit and omega conditions must be selected in order to maintain continuity with present design procedures. In general, several limit and omega conditions will be necessary in order to adequately design the structure. Figure 33 presents a vertical bending moment - torsion partial limit strength envelope together with the original design load requirements for a C-141 inboard wing station. Positive maneuver alone causes six different design conditions with widely varying vertical bending moment-torsion combinations. Other C-141 wing stations have different design conditions. For example, the vertical bending moment requirements for the outboard wing are primarily caused by aileron roll conditions rather than vertical maneuver. In addition, different major structural components have different loading conditions which cause maximum loads. For instance, lateral gust may cause significant vertical stabilizer and fuselage aftbody loads, but have negligible effect on wing loads.

b. Following the determination of the aircraft utilization data required, data available, and the methods to be used in the determination of the limit and omega load spectra, the following paragraphs outline the procedures used to determine the limit and omega load conditions.

c. The normal operational limits for the parameters which are user controlled must be determined in order to differentiate between limit and omega operational conditions. These limits should be based upon statistics where possible. For example, maximum allowable vertical maneuver load factor can be presented as a



Wing Station 135

SYM	DESIGN CONDITION	
A	2.0g Steady Man.	Takeoff Flaps
B	2.5g Steady Man.	Clean
C	2.5g Steady Man.	Clean
D	2.5g Steady Man.	Clean
E	2.5g Steady Man.	Spoilers Ext.
F	2.5g Abrupt Man.	Spoilers Ext.
G	0.0g Abrupt Man.	Clean
H	2.0g Taxi	Clean
I	-1.0g Abrupt Man.	Clean
J	-50 FPS Gust	Takeoff Flaps

FIGURE 33 C-141 WING PARTIAL LIMIT STRENGTH VERTICAL BENDING - TORSION ENVELOPE

function of cargo weight or any other significant load parameter on a joint probability of exceedance basis, such as is demonstrated in Section IV. As also discussed in Section IV, when lacking applicable statistics, the limit airspeed can be taken as V_H, the maximum level flight speed, while the omega airspeed can be taken as V_L, dive speed. Maximum limit fuel-cargo combinations can be obtained from fuel tanks full with the intended fuel density and limit gross weight considerations while those for omega can be obtained from fuel tanks full with increased fuel density, and omega gross weight limitations. The center of gravity limits for limit conditions can be those without adverse tolerances, while those for omega conditions are expanded to include at least the Military Specification tolerances.

The limits on user controlled parameters define the normal, overload, and gross overload operational regimes for each of the parameters. As such, the limits must be presented in a form which can be readily adhered to by the user. For instance, maximum maneuver load factor should not be a function of several parameters such that the allowable load factor would be constantly varying during a flight. Figure 34 presents examples of limits for user controlled parameters. It must be noted that the combinations of the user controlled parameters to be used for design have not been determined yet. Such combinations are obtained by the determination of limit and omega design conditions.

d. Load control stations must be selected for each major structural component in order to obtain a minimum feasible number of structural locations for consideration in the design loads investigation. As previously discussed, significant combinations of load components are then selected in order to obtain load envelopes which are the most compatible with the strength requirements.

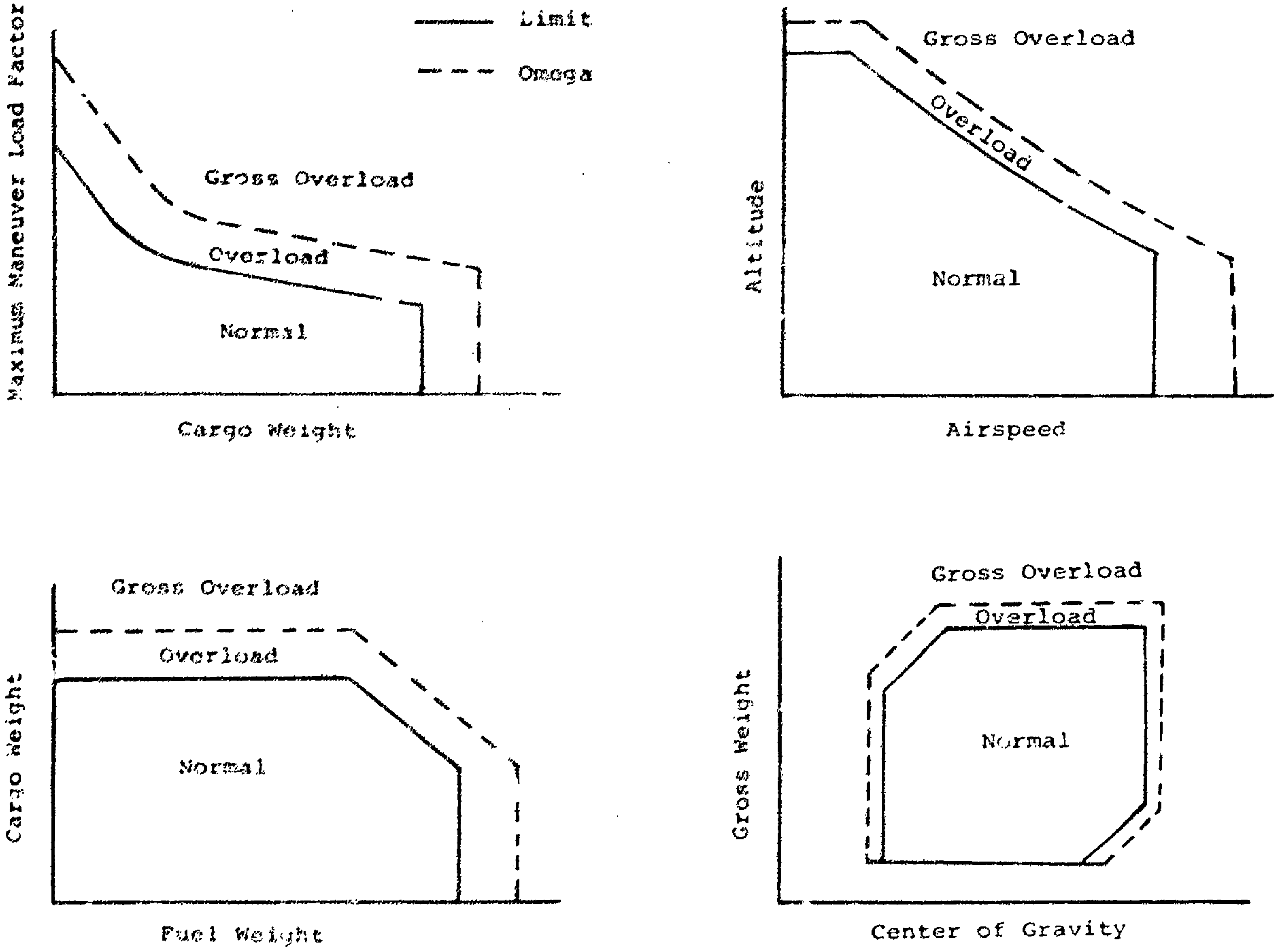


FIGURE 34 EXAMPLES OF LIMITS ON USER-CONTROLLED PARAMETERS

If a stress spectral analysis is conducted, the load components are converted directly to stress at individual locations. As such, envelopes are not needed.

e. There are several loads sources for which the loads are determined mainly by user controlled parameters. These loads sources are: directional, lateral, and vertical maneuvers, and manual landing impact. The maximum obtainable loads for such loads sources are mainly determined by the limits on the user controlled parameters. Such maximum loads may occur for combinations of parameters having a remote probability of occurrence yet they still represent operations within the defined limits of normal operation.

There are two ways in which the load envelopes can be determined for such loads sources. The first approach is purely statistical in that the loads envelopes are determined on a probability of occurrence basis. That is, the maximum loads would be those that occur for the limit and omega probabilities of occurrence. Such an envelope is shown in Figure 35.

However there is a drawback associated with this approach for multi-parameter environments. The limit and omega envelopes define the normal, overload, and gross overload regimes. As such, the individual user must know what combinations of user controlled parameters will result in loads which are within either the normal or overload regimes. If the envelopes are defined on a probability basis, then it would still be possible to obtain loads in the overload or gross overload regimes for combinations of user controlled parameters while each of the parameters is within the previously prescribed limits for normal operation.

It would also be possible to obtain loads in the gross overload regime for combinations of parameters while each of the parameters is within the previously prescribed limits for overload operation. Granted, the probability of occurrence of such combinations of parameters may be remote, nevertheless, in order to maintain

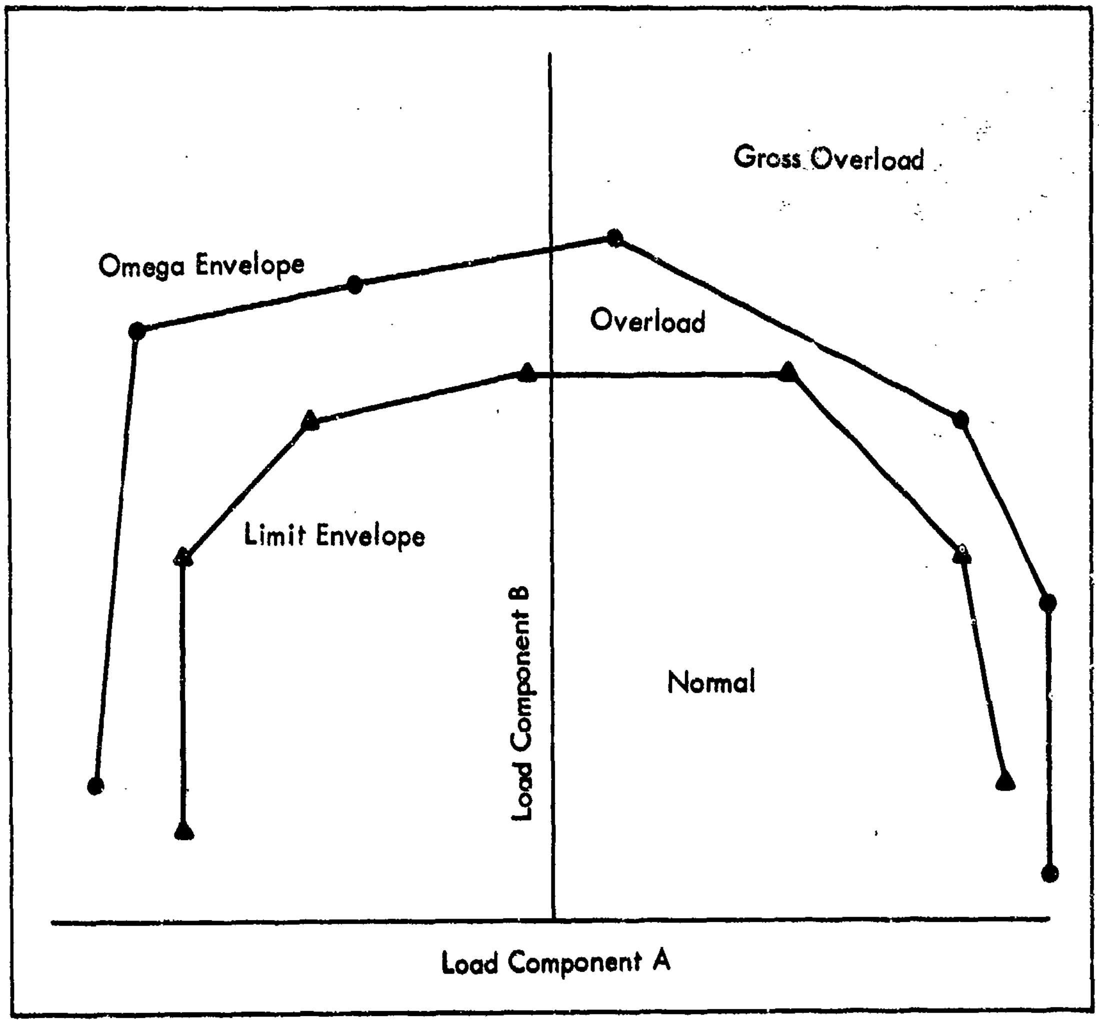


FIGURE 35 EXAMPLE OF LOAD ENVELOPES

confidence in the structural reliability design analysis, the user must know what combinations of user controlled parameters will maintain the flight vehicle loads within either the normal or at least the overload regimes.

In a multi-parameter environment, the determination of the limit and omega conditions on a purely probabilistic basis will not adequately define such combinations of user controlled parameters. Therefore, it is suggested that at least initially the limit and omega conditions should be those which cause the maximum loads for all combinations of user controlled parameters when each of the parameters is within the respective limit or omega restrictions. As a result, it would be impossible to have loads in the overload or gross overload regime if the loads are caused entirely by user controlled parameters and if each of the parameters is within the prescribed limits of normal operation. It would also be impossible to obtain loads in the gross overload regime if each of the parameters is within the prescribed overload limits. Thus, the user can confidently operate the flight vehicle anywhere within the prescribed limits and still remain within the desired load regimes.

Such an approach not only maintains continuity with present design procedures, but also provides additional confidence in the structural reliability analysis.

As previously discussed, separate envelopes must be developed for flight regimes where such effects as thermal stress or pressurization significantly affect the external load capability of the structure.

f. There are several loads sources which are beyond the control of the user. Such load sources include atmospheric turbulence, and landing impact for automatic landings. Severe atmospheric turbulence can be avoided to some extent, but such avoidance techniques are reflected in the models of atmospheric turbulence which are based on measured data. As a result, other than adhering to the intended utilization of the flight vehicle and maintaining the user controlled parameters within the prescribed limits, the user has relatively no control over the probability of occurrence of such loads sources. Therefore, the limit and omega conditions for such loads sources can be established on a purely probabilistic basis. Limit and omega load envelopes for the selected loads control station are then developed such that the probability of exceedence of the respective envelopes [illegible] that for limit or omega conditions as discussed in Section VII. The limit and omega conditions for each of the individual load sources are then those conditions which define the corners of the load envelopes.

As previously discussed separate loads envelopes must be developed for flight regimes where such effects as thermal stress or pressurization significantly affect the external load capability of the structure.

g. As previously discussed, the loads spectra must be represented in terms of a single parameter which is compatible with strength. The loads spectra are therefore represented as percentages of the pertinent envelopes. Where sufficient data is not available to develop load spectra, load spectra must either be assumed or a given load level designated which has a probability of occurrence of one.

If a stress spectral analysis is performed, the loads are expressed directly in terms of stress at a given structural location, which is the ideal situation.

h. As previously discussed, there are several loads sources, such as vertical maneuver, gust, and landing impact. Each of the loads sources has an independent probability of occurrence. Therefore, the loads spectra for each loads source are independent. For independent probability distributions, the total probability of occurrence is expressed by the following law, as stated in Reference 14.

$$P_r\left(\sum_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P_r(A_i) - \sum_{j>i} P_r(A_iA_j) + \sum_{k>j>i} P_r(A_iA_jA_k)$$

$$- \sum_{l>k>j>i} P_r(A_iA_jA_kA_l) + \cdots \qquad (7)$$

Where: $P_r\left(\sum_{i=1}^{n} A_i\right)$ is the total probability that a given load will occur for at least one of n loads sources.

$P_r(A_i)$ is the probability that the load will occur for the ith load source, A_i

$P_r(A_iA_j)$ is the joint probability that the load will occur for either the ith or jth load source, A_i or A_j.

and so on.

For three loads sources the law is:

$$P(A_1+A_2+A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1A_2) - P(A_1A_3)$$
$$-P(A_2A_3)+P\ (A_1A_2A_3) \qquad (8)$$

It should be noted that although atmospheric turbulence is independent of other load sources, such as landing impact, the various components of atmospheric turbulence, such as positive and negative vertical and lateral gusts, are not. This is due to the isotropic nature of atmospheric turbulence. Therefore, if the structural reliability of a given structure is .999 for positive vertical gust, and the same for negative vertical gust, the total structural reliability is .999, <u>not</u> .998.

i. There are two reasons for initially not using the preceding law.

1) Discrete deterministic design conditions must be obtained. In order to obtain such design conditions, the individual load sources must be analyzed independently.

2) There are some load sources for which load probability spectra may not be developed.

j. Therefore, the procedure to be used is as follows:

1) Perform a separate reliability analysis for each of the loads sources and obtain the factored design load requirements for each loads source at a given structural location.
2) Merge the individual load source design requirements to obtain the overall requirements at the given structural location.
3) Use the law of total probability for all of the load spectra available, and obtain the total reliability for the overall design requirements from (2).
4) If necessary, apply a factor to the overall design load requirements in order to obtain the desired total reliability at a given structural location.
5) The structural reliability for each individual loads source can then be determined based upon the overall design loads requirements.

The preceding methods therefore account for the relative distribution of the total structural reliability between the individual loads sources and also allow the selection of deterministic limit and omega design conditions, together with the required design load levels.

SECTION VI
CHOICE OF ALLOWABLE STRENGTH

6.1 Introduction

a. It would seem at first that a probabilistic criteria system only requires knowledge of the mean strength and a measure of the dispersion (the standard deviation, for example). Two factors dispel this illusion; the first is the necessity for some simple definition which can be used by the designers to assess the required sizes of the structural members, and the second is the restraint imposed by the need to associate load and strength distributions on a common scale.

b. Present practice uses particular values in the observed statistical distribution as design allowables. In terms of local linear stress, these are adequate. Difficulties arise as soon as the realistic load systems are invoked. The combinations of bending moment, shear, torsion, end load and transverse pressure which exist within a structure such as a wing make the selection of the allowable load less than clear. Other sections of this report describe the definition of the load system in a form which permits the probability of failure to be assessed (failure being the association of a load with a lower strength, or of a strength with a higher load). This section describes some of the features which constitute the description of the allowable load for a structure.

c. Present design evaluation processes frequently require the assessment of the permissible value of one load parameter, and this is generally performed for discrete values of other parameters. For example, the permissible normal pressure on a certain wing panel might be assessed at specific levels of vertical load factor, or even at specific combinations of vertical load factor, gross weight, Mach number and altitude. The solution in deterministic terms is arduous and inexact; if the statistical distribution of the permissible pressure is required, then the problem expands by several magnitudes.

d. The most practical approach, within the present state of knowledge, appears to be the relatively crude one of determining the allowable statistical properties of one parameter at a time, assuming conservative and constant values of the other parameters. This will generally lead to over-estimates of the risk, a result which is at least conservative.

f. The remainder of this section discusses the current methods of defining allowable strength, together with areas where further work would permit the derivation of at least part of the data required by a probabilistic system of design criteria.

6.2 Material Basic Properties

a. The choice of materials in the initial design is based on structural integrity, cost, weight, ease of fabrication and maintenance requirements. Trade-off is also considered for each of these factors to determine the optimum material for each aircraft component. The material selection process begins with analysis of the problem, which results in detailed specification of the material requirement. The material requirements are derived from the study of the attributes, functions and performance of the product being developed as well as the environment in which it will operate. The successful functioning of a product is heavily dependent upon the materials. The functional requirements are directly dependent upon the desired attributes of the product or upon the function the product is designed to perform.

The design strength can be related to that of the material and the geometric configuration The material strength is designated as an allowable strength. This strength can be either tension, compression, shear or bearing depending on the load. The geometric configuration defines the strength of a component and its load carrying capability prior to failure. Such strengths as column, panel buckling and crippling strength or instability, fall in this category.

b. The material allowable strength data are established by standard well-established tests. The Mil-HDBK-5A Guidelines define the types and number of tests and the number of heats or production lots needed to generate the required test data. These data will include sufficient specimens for a statistical analysis to be performed to determine the data scatter and distribution. Statistical values have a notation of "A" value which represent a 99% exceedence with 95% confidence, "B" value a 90% exceedence with 95% confidence or "S" value a guarantee of minimum by the producer and normally included in the procurement specification - "S" value does not have any statistical significance. These properties normally pertain only to the yield and ultimate tensile stresses of the material. Other material properties or allowables such as shear, bearing and compression yield are derived using limited amounts of test data and a presumed relationship as a ratio of the A and B values of Ftu and Fty. Fatigue properties vital in design are not within the scope of this study.

c. The environmental effect on the design strength is also accounted for in component design where environment exists as a real factor. The influence of temperature on material strength allowables is usually expressed as a factor to reduce the room temperature material strength allowables depending on the severity and duration. Creep and thermal instability are also material properties to be taken into consideration where severe thermal environment is a design condition of the vehicle. Materials in contact in a humid environment should be chosen to avoid galvanic corrosion.

6.3 Effect of Processing and Fabrication on Material Properties

a. The processing operation will almost always have some effect on the material functional or service performance properties. The material allowable documents (Mil-HDBKs, Spec., etc.) present allowables for material as processed by the producer in sheet, plate or extrusion form and with subsequent heat treatment imposed on these materials. It should be recognized, however, that the users' final configuration of the material bears little resemblance to initial materials on which the allowable strength parameters were based. A point of controversy which has always existed and has not yet been clearly resolved is the problem of heat treatment by the vendor or producer (T6 temper) as compared to heat treatment by user or fabricator (T62 temper). These two conditions are known to give different allowable properties yet the designer does not differentiate between the two tempers due to lack of knowledge in the initial design as to the severity of fabrication requirements. Another problem of significant impact on material properties that has been ignored is the degradation of properties of extrusions due to stretch forming in the O or W conditions. The stretch forming severely strains the material which results in surface crystallization when the material is solution treated and aged with an appreciable reduction of allowables. Precautionary notes are currently included in material allowables documents which point to this degradation. However, since the degradation depends on the percent stretch and thickness, this knowledge is indefinite in the initial design and usually ignored in the strength design of the part. Processing and manufacturing techniques such as chem-milling, grinding, anodizing, machining, shot peening, etc. are usually not considered in the static material properties used.

6.4 Design Strength Related to Utilization

a. The strength design is not confined to material properties per se; the configuration and/or geometry of the part controls the load carrying capacity. The geometry of a part designed to carry column load limits the compressive stress of the material as related to compressive yield strength of the basic material. A number of design allowable curves are usually generated for each material to relate strength to utilization. The buckling or crippling strength of panels or stiffeners, skin buckling, column curves, torsion and bending moduli of rupture, lug efficiency curves etc. are geometry-dependent. Typical test data for elastic moduli (tension, compressive and tangent) are used in the equations defining the design allowable with no consideration for statistical variation of the material properties.

b. Another parameter that has significant affect on the strength is assembly technique. This includes fabrication processes, such as welding, bonding, riveting and bolting. Each of these processes is unique in the method of load transfer and application. The most widely used method of assembly is by mechanical fasteners. Each fastener system will have its own effect on the overall strength of the fabricated structure due to such variables as type of installation; tightness of the fit, manner of loading distribution, shear and bearing strength, joint yield and deformation and the relative stiffness of the fastener and sheet. Many of these effects are offset in design by increasing the thickness of the member in the region of the connection, but this in turn introduces eccentric loading paths and uneven stress distributions. Appendix V includes the results of analyses of sample groups of riveted and bolted joints.

c. The design allowable values for mechanical and welded joints are established by experimental means in accordance with Mil-HDBK-5 Guidelines (reference 15). The design joint strengths are computed from experimental data by taking the average of test values in the bearing and shear bearing areas and dividing by a factor such as 1.15. The shear strength of fasteners is computed using the cross sectional area of the fastener and the specified fastener material shear strength. The strength of welded joints is also established experimentally; however, the strength is compared to basic or parent material data and a reduction factor is imposed to compensate for the degradation, if any, due to the welding process. Similar approaches are utilized in arriving at design data for chemical-and diffusion-bonded joints.

d. Other variables that can contribute to some extent to the differences in material properties are testing technique, test machine and instrumentation used, and interpretation of the data.

6.5 Design Strength Scatter Assessment

a. It is evident from the preceding discussion that the strength of a component is dependent on the accurate assessment of the variables involved. The interaction and contribution of the different parameters such as fabrication, assembly, environment, etc., will have to be incorporated into the basic material data in the component design to attain a realistic estimate of the reliability and probability of failure.

b. Basic material strength and scatter are functions of the inherent characteristics of the alloy as produced by the manufacturer, and the quality control measures for acceptability. Generally, the scatter of strength of the material used in aircraft parts has been truncated. This truncation is an adjustment of material property data distribution to compensate for the censoring effect of an imposed specification. The effect of censoring is of major concern in attempting to predict the occurrence of extreme values of deviation. If an assumption is made that the specification is 100% effective, and there is no probability that a value in the procured product will be lower that the specification minimum, then this would solve most of the material problems with respect to minimum values and reliability of strength predictions. This rarely happens. In reality the distribution of strength is probably somewhere between a complete or uncensored distribution and a truncated distribution. Therefore, the minimum strength values used in design and analysis are conservative and the use of statistical material strength data should recognize the non-Gaussian distribution of the truncated data. This can be accomplished by plotting material test data to determine the mean and a pseudo-standard deviation as shown in the examples using C-141 data. (See Appendix V).

c. The material strength in the above discussion is confined to yield and ultimate strength (Fty and Ftu) which are governed by specification. The other material design values are derived as ratios of Fty and Ftu using paired tests to establish the ratios, and are usually limited to ten pairs of tests each, for compression yield, shear ultimate, and bearing yield and ultimate (F_{cy}, F_{su}, F_{bry} and F_{bru}).

d. The material strength data distribution for the purpose of deriving allowables is assumed to be either normally distributed or skewed (Reference 15). An evaluation of test data on three metallic and one non-metallic material indicate that a double family type of distribution fits the test data closer than a single family normal distribution curve. The second family lies in the tail extending toward the lower strength, the region where the "A" & "B" values of material strength is determined. The double family distribution curves for an aluminum, titanium and steel alloys are presented in Appendix V to illustrate a means of recognition of measured samples exhibiting such characteristics. The double family distribution make the statistical significance of the customary A & B allowable strength values other than values at 2.36 & 1.282 standard deviations from the mean; alternatively, they can be interpreted at these locations, but having properties other than 99 per cent and 90 per cent exceedence.

e. Design strength scatter of members that are influenced by geometry and loads (other than tensile) is difficult to define. Members such as columns which depend on geometry, material thickness tolerance, and compression stress-strain relationships will have a wide scatter variation. Limited numbers of tests are normally performed to define a design curve for a material and related geometry with no attention paid to scatter, mean or standard deviation. The final strength is verified by testing a typical component with the assumption that the behavior of the component will represent those of the structure.

f. The effect of manufacturing and fabrication processes on the basic design strength is seldom considered in initial design. Only when a problem arises or where past experience has indicated a degradation to exist, is an adjustment in the material allowables applied. The scatter in the material properties and the definition of the mean and variance is not considered. An assessment of the strength distribution of the basic material prior and after processing, to validate the strength allowable used, is generally overlooked.

g. As indicated in this discussion, the initial design contains a series of factors and conditions which affect the overall strength, scatter and reliability, that are difficult to evaluate and incorporate in the basic design. For implementation of the proposed statistical design method these factors and their effect have to be accounted for whether individually or collectively, through initial component test and empirical derived error functions typical for certain types of structure.

It is felt that certain parameters such as material allowable strength and joint design strength are basic and should be well defined and established in the initial design. Design detail, fabrication and process methods should be considered based on similar design of earlier aircraft. However, prior to the final design release typical component and sufficient material strength test should be run to verify the initial design and establish a backlog of strength data for each process for future analysis and usage.

SECTION VII

CHOICE OF ERROR FUNCTION

7.1 Introduction

An error function is required to describe design strength variations. Many mathematical expressions for the error function can be formulated. The purpose of this section is to outline different error functions available to define the variation of actual strength from the intended strength and to present the C-141 wing test data available to aid in the choice of the error function distribution.

7.2 Basis for Error Function Definition

a. The initial design contains many factors affecting its overall strength and scatter which are difficult to evaluate. For implementation of the proposed statistical design method, these factors can be grouped together through component tests into empirically derived "error functions" typical for certain types of structure. At the beginning of a design, data obtained for similar designs on earlier aircraft can be used as a basis for sizing the members and for reliability estimates. Prior to design release, element tests and limited component tests will have been run to confirm the strength of the design. (These tests can be evaluated in conjunction with the earlier tests to provide a broader data base for mean strength and scatter estimates.)

b. It is felt that the basic material strength should be considered a separate entity from the "error function." The material strength and scatter are basic to the design, and are not entirely under the control of the aircraft manufacturer; whereas, design details, fabrication methods, and test detail effects may be updated through later redesigns or retests, and are more under the manufacturer's control.

7.3 Possible Definitions

a. The choice of error function definition has a significant effect on the (pre-test) calculated reliability. A number of expressions can be used to describe the error function. Six curves using four different definitions are shown in Figure 36; these are plotted output from the computer program described in Appendix III. All curves are based on an intended design strength of 100 percent and do not account for variation in material strength.

b. Curve 1 represents the error function defined by Bouton in Reference 1, based on data collected by Jablecki (Reference 16). The "standard Jablecki" curve is characterized by a reliability of 0.99 at one-third of the intended design strength.

Curve 2 is a Jablecki distribution which results in 0.999 reliability at one-third the intended design strength and corresponds to the "ten times better" curve used in Reference 1.

Curve 3 represents the type of error function suggested by Freudenthal (Reference 17) with values based on a reliability of 0.98 at 80 percent.

Curve 4 is a Gumbel distribution which matches Curve 3 at 80 percent and 100%; these two curves show the effect of using the two different error functions for the same two points input into the computer program.

Curve 5 is a "worse" Gumbel distribution which illustrates the effect of changing the point at which a reliability of 0.98 is demonstrated, from 80 percent to 50 percent of the intended design strength.

Curve 6 is an example of the double family Gumbel distribution mentioned in Section III.5(vi). This type of distribution is used throughout this report to approximate both the exhibited material scatter and the exhibited C-141 wing strength scatter.

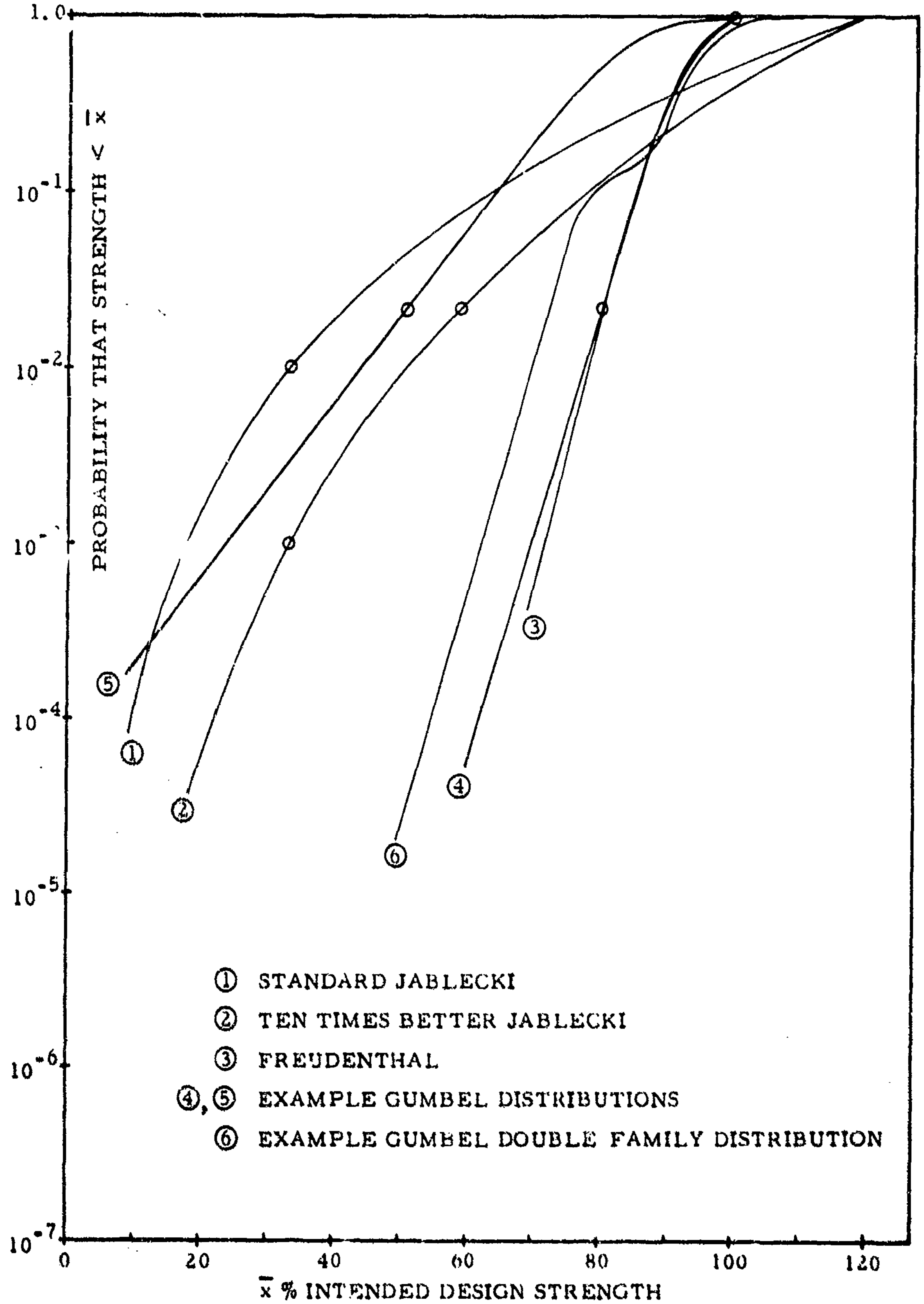


FIGURE 36 ERROR FUNCTION DEFINITIONS

TABLE VII

C-141 WING COMPONENT STRENGTH TESTS

Tested Components	Reference	Type Test	Number of Test Specimens	Test Failure Loads % Ultimate	Remarks
Center-to-inner wing beam cap and panel joints	18	Tension Compression	4 2	96.3 110.0 117.0 176.7 90.3 94.3	Four different beam cap joints - one panel joint
Inner-outer wing joint	19	Tension	3	90.0 123.0 126.0	Specimens 2 & 3 redesigned configuration of 1
Rib diagonals	20	Compression	5	52.2 83.5 89.0 105.4 136.5	No two specimens same configuration
Center wing panel	21	Compression	2	98.0 104.7	One specimen had access cutout
Outer wing panel	22	Compression	2	91.0 99.0	

c. From the above, it is seen that the distribution can be described in several ways and the definition employed in a particular application must be chosen to fit data appropriate to that application. The reliability estimate will be affected by the choice of distribution equation.

7.4 C-141 Wing Strength Scatter

a. To determine the amount of data available pertaining to the strength scatter of a specific aircraft structure, the C-141 wing is used as a typical example. During the development stage of the C-141, component static tests of selected parts of the wing were conducted. The complete wing was then static tested during the full-scale test program.

b. Component static tests of the C-141 wing structure were conducted in 1962-1964 to determine either the optimum configuration or ultimate strength of selected parts of the wing. Table VII summarizes those test results which are used herein. During review of the tests, the following characteristics were noted:

(1) Most of the test specimens were the same scale as the actual aircraft structure.

(2) The number of specimens per test group ranged from two to six; the maximum number of specimens of the same configuration was two.

(3) Nearly all specimens were uniaxially loaded.

(4) Loading jig effects invalidated some of the test results.

(5) The design strength of many specimens was not reported. Because of the time interval lag from 1964 to 1971, backup data (stress analyses not formally reported) were not available for any tests. Therefore, determination of the design strength, where not reported, was not attempted. Four test groups (not listed in Table VII) were found to be in this category.

TABLE VIII

C-141 FULL-SCALE WING STRENGTH TESTS

Test Condition	Critical Structure	Principal Loads			% Ultimate Reached
		Bending		Torsion	
		Up	Down		
2.0g taxi	Inner wing		x		100
Abrupt Maneuver	Inner wing		x		100
Negative accelerated roll	Outer wing		x		100
2.5g maneuver	Center & inner wings	x			80, 95
2.0g flap maneuver	Rear beam	x		x	100
SSCBM transport	Inner wing	x			100
2.0g roll maneuver	Outer wing	x		x	100
Transient gust	Outer wing	x		x	100
Negative checked roll	Outer wing	x		x	100
Wing jacking	Jacking points	x			100
Pylon tests (6)	Pylon support str.				100

c. Sixteen wing/pylon tests were conducted on the C-141 full-scale static test airplane to confirm the design ultimate strength capability of the structure. These tests are listed in Table VIII; detailed test procedures and results are presented in Reference 18. During one test, impending failure of some rib diagonals was detected at 80% ultimate design load; the test was discontinued at that point, the diagonals were redesigned, and the structure tested to the scheduled 95% ultimate design load. All other tests were successfully completed to 100% ultimate design load. Therefore, the structure was evidenced to equal or exceed the design ultimate strength, but the extent of overstrength is not known.

d. Obviously, the available data are insufficient to define the strength scatter of a particular part of the wing structure. However, for the overall wing, the combined data can be used to indicate the probability of failure. The exhibited probability of failure of the original design is shown in Figure 37; the observed data include the component tests of original design configurations and the one static test which was discontinued at 80% ultimate design load. The fitted curve is a double family Gumbel distribution; the values of the shaping parameters for this curve are shown on the plot. The Standard Jablecki and Freudenthal error function definitions (Curves 1 and 3, respectively, of Figure 36) are superimposed for comparison.

If the full-scale tests are assumed to represent the ultimate strength of the wing and used to modify the probability of failure, the observed data and corresponding fitted Gumbel distribution shown in Figure 38 result. All of the low strength structures detected during component tests, however, are still included in the observed data. To update the strength variation, the tests of these obsolete configurations are deleted and the component tests of the corresponding redesign configurations substituted.

The observed data and fitted curve of Figure 39 result; this can be considered to represent the achieved strength of the final configuration of the wing, as best can be defined using the available data.

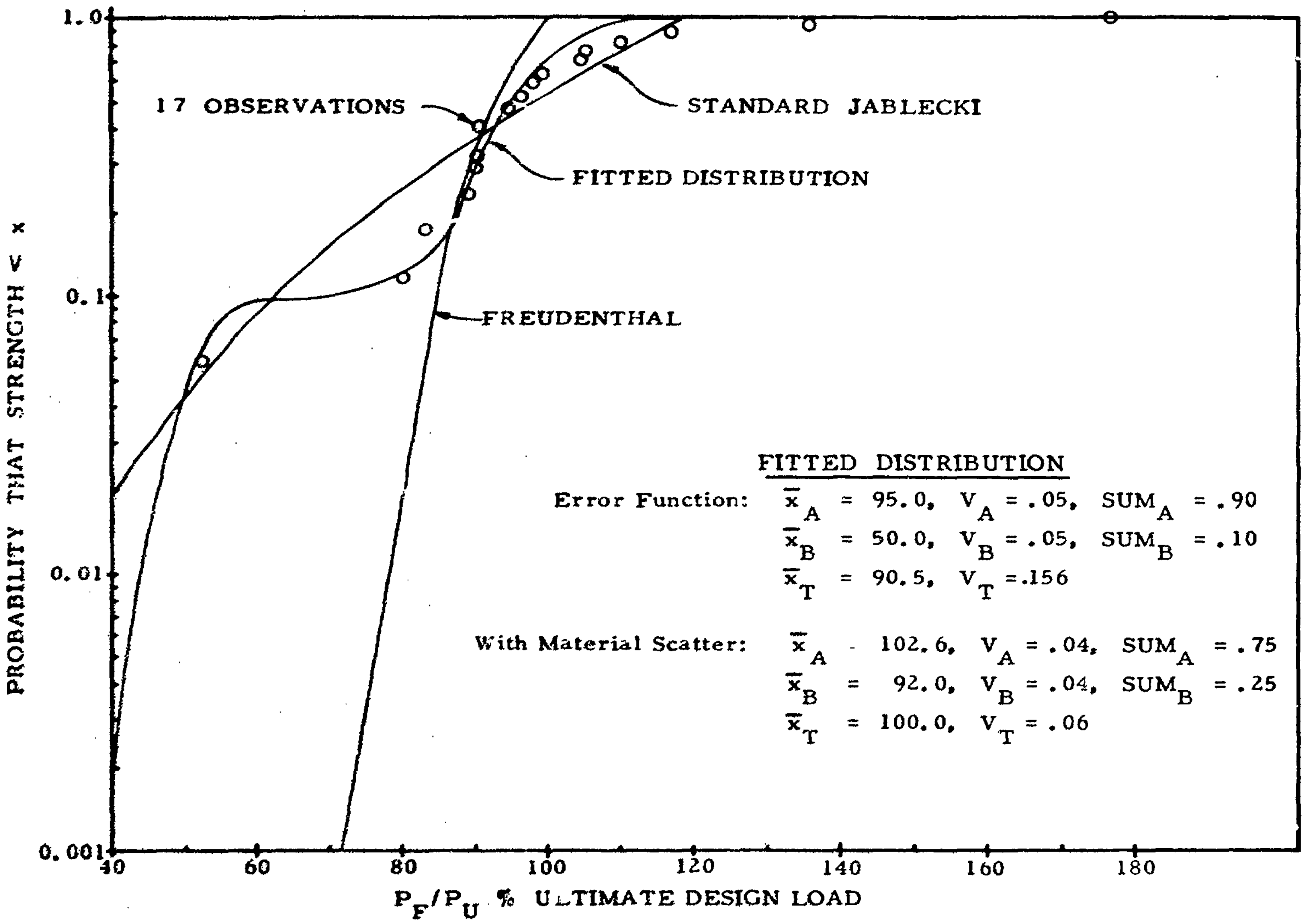


FIGURE 37 C-141 ACHIEVED STRENGTH - ORIGINAL CONFIGURATION

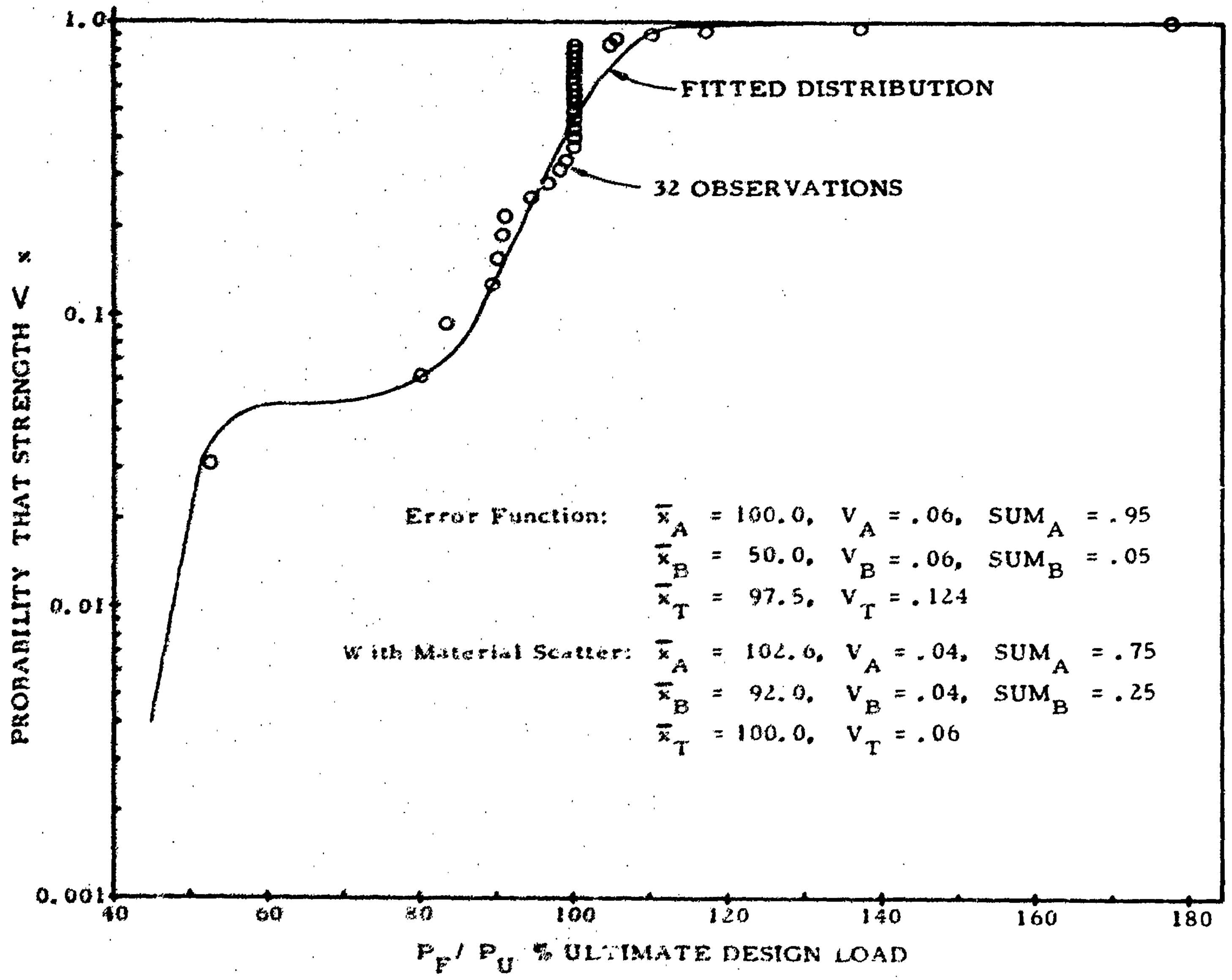


FIGURE 38 C-141 ACHIEVED STRENGTH - ORIGINAL AND STATIC TEST

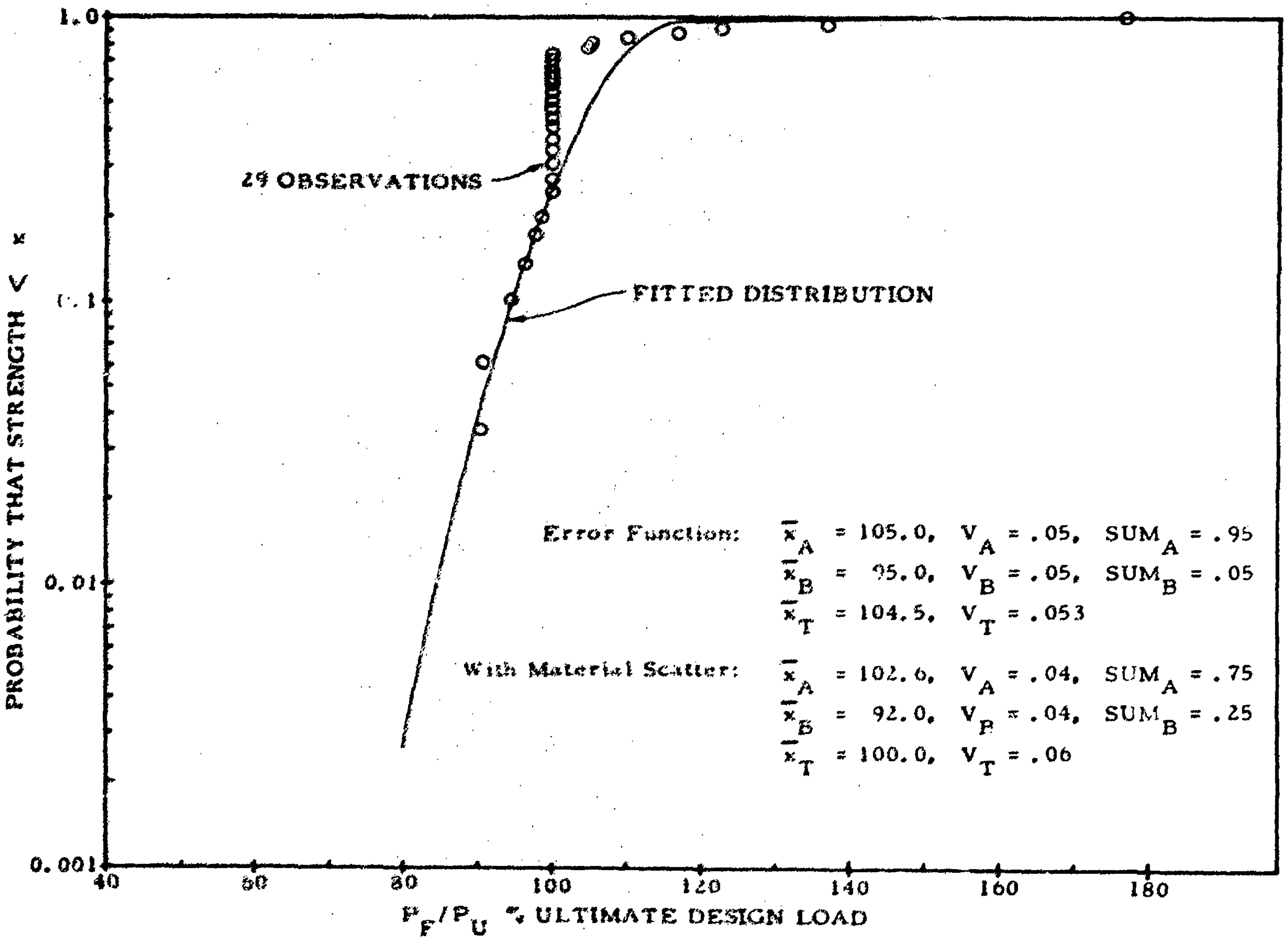


FIGURE 39 C-141 ACHIEVED STRENGTH - FINAL CONFIGURATION

7.5 Error Function for Initial Design

a. At the time of the initial design, the error function can be estimated from the mass of background data provided by Jablecki, Freudenthal, and/or such data as are shown herein for the C-141 original design configuration. An appropriate expression for this error function can be formulated. The C-141 tests, using a Gumbel double-family distribution, after consideration for material scatter, shows an error function as indicated in Figure 37.

b. The error function for initial reliability estimates can be used to predict reliability prior to completion of tests and the resulting redesigns. This error function can be used in conjunction with pre-test-completion flight restrictions to evaluate the reliability of the aircraft with the limited flight loads resulting from these restrictions. It can also be used to predict the probability of survival of static test loads, as discussed in Section IX. This may permit tradeoff decisions between design factors, test factors, and reliability predictions which can minimize overall test cost.

c. During the early design phases, several error functions can be used which are based on varying amounts of data. One function, based on pre-test (other aircraft) data, is described above. Another error function can be estimated based on assumed test results to provide a reliability prediction for the final configuration. An intermediate error function based on component tests can also be estimated. These error functions and the resulting reliability predictions can aid in decisions regarding the number of component and full-scale tests to be performed, probabilities of survival of the tests, number of specimens, and acceptability of the final configuration.

d. Obviously, updating of the initial error function definitions is necessary as data become available, to provide a proper base for later decisions and predictions.

SECTION VIII

CHOICE OF STRUCTURAL RELIABILITY GOAL

8.1 Introduction

The selection of structural reliability goals for flight vehicles is a formidable task when one considers that the relationship between the chosen goal and the final computed structural reliability is unknown. The chosen goal may be used as a means of selecting levels of limit and omega loads but the computed structural reliability depends also on the shape of the load spectra, the strength scatter of materials and structural components, and the chosen error function.

8.2 A Proposed Approach

a. Reference 24 recommends space vehicle structural reliability levels and associated probabilities of exceeding limit and ultimate (omega) conditions which are reproduced herein as Table IX. This table is also shown as Table I in Reference 1 where it is implied to be applied to aircraft. In fact, elsewhere in Reference 1 typical structural reliability goals of 0.99 for fighter aircraft, 0.9999 for liaison aircraft, and 0.999999 for transports are mentioned apparently in accord with the respective high risk, standard risk, and low risk vehicle columns of Table IX.

b. In the presentation of these data, the actual relationship between probabilities of exceeding limit or omega conditions and structural reliability goals is not expounded upon except for the assumption that the probability of exceeding the omega condition is the complement of the structural reliability. That is; S.R. = 1.0 - Omega condition probability.

c. Figure 40 shows the probability of Exceeding Limit Condition and Probability of Exceeding Omega Condition values of Table IX plotted as a function of (ref. 1) Table I Structural Reliability Goals. The validity of the limit condition trend of Figure 40 is very questionable. In particular, it does not follow that the structural reliability is zero merely because a limit load is exceeded once per aircraft lifetime.

TABLE IX

REFERENCE 1 STRUCTURAL RELIABILITY OBJECTIVES

Class	Standard Vehicles	Low Risk Vehicles	High Risk Vehicles
Structural Reliability Goal	0.9999	0.999999	0.99
Probability of Exceeding Limit Condition	0.01	0.001	0.1
Probability of Exceeding Omega Condition	0.0001	0.000001	0.01
Conditional Limit Reliability	0.999999	0.99999999	0.9999
Conditional Omega Reliability	0.99	0.99	0.99

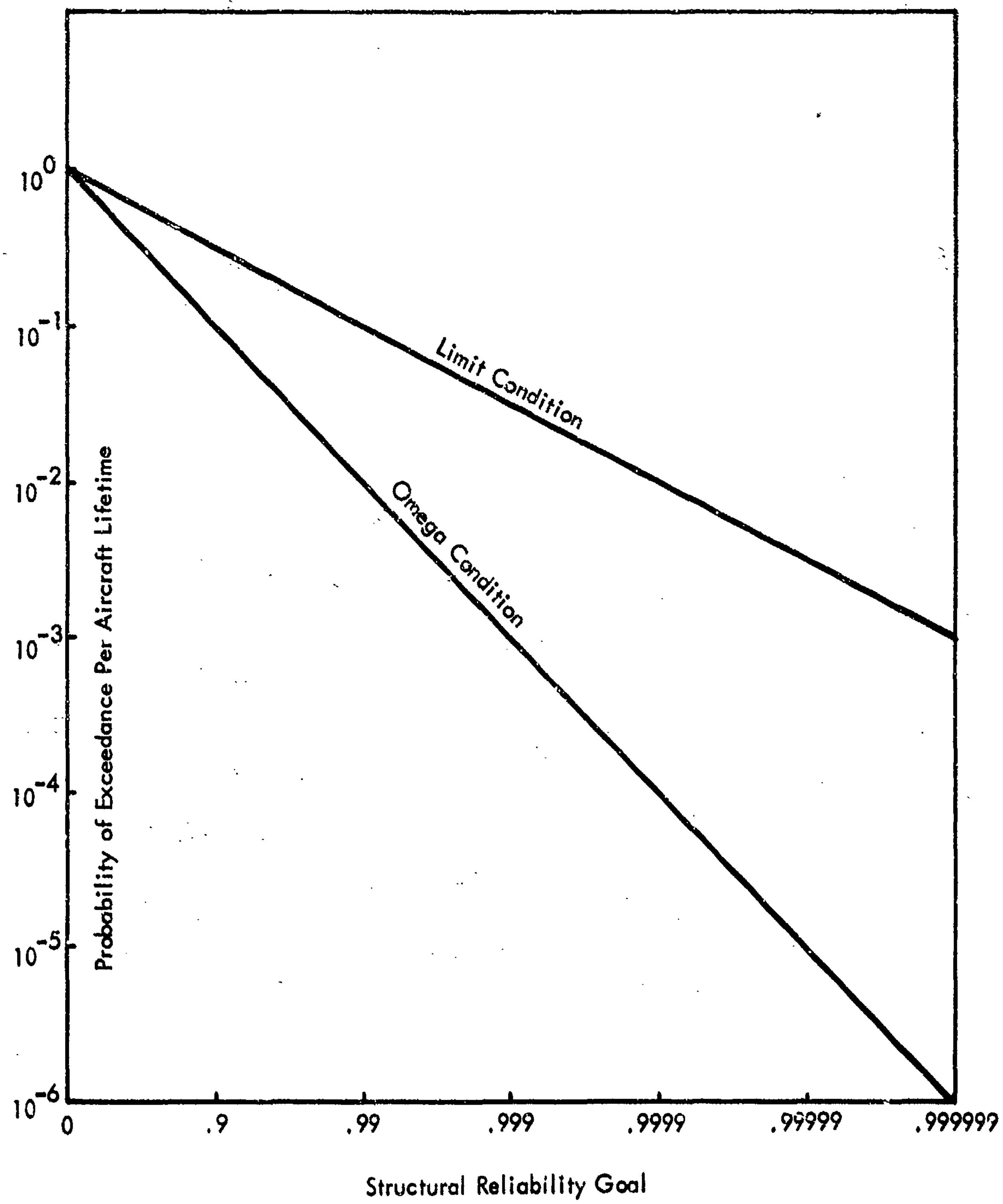


FIGURE 40 REFERENCE 1 STRUCTURAL RELIABILITY GOALS

8.3 Existing Data

a. Reference 13 presents mission analysis results for gust exceedences in terms of frequency of exceedence per average flight hour of limit strength and ultimate strength of various components of the Electra (188), Constellation (749), and 720B aircraft. These data are reproduced in Table X. Assuming that these are all 30,000 hour lifetime aircraft, the number of exceedences per aircraft lifetime are also shown as well as the implied structural reliability for each condition assuming that structural reliability is the complement of number of exceedences of the ultimate strength.

b. Examining the structural reliability values derived from Table X, it is quite obvious that the Constellation (749) tail is not critical for vertical gusts and that the values for Electra (188) aft body and 749 tail occurrences of ultimate strength due to lateral gust are unbelievably high. There is evidence that the lateral gust statistics used in these analyses were excessively conservative which would explain the derivation of such low structural reliability values for supposedly successful aircraft.

Table XI derived from Reference 6 shows data similar to that of Table X of overall computed failure rates due to gust.

c. Using only the structural reliability values of Table XI and those of Table X which appear to be rationally derived, it may be seen that the structural reliability of these aircraft on the basis of gust condition lies between 0.999 and 0.9999 in almost all cases.

If aircraft such as these have been operating satisfactorily at such implied structural reliability levels, then the 0.999999 typical structural reliability goal suggested in Reference 1 for transports seems unduly severe.

TABLE X

REFERENCE 13 MISSION ANALYSIS RESULTS

Component and Airplane	Frequency of Exceedence, N(y), Per Average Flight Hour		Exceedence Per 30,000 Hour Life		Structural Reliability
	Limit Strength	Ultimate Strength	Limit	Ultimate	
Wing					
188	2.1×10^{-5}	1.4×10^{-8}	.63	.00042	.99958
749	1.8×10^{-5}	4.2×10^{-9}	.54	.000126	.999874
720B	1.1×10^{-5}	-	.33	-	-
Body & Tail-Vert. Gust					
188 (Forebody)	6.0×10^{-6}	1.0×10^{-9}	.18	.00003	.99997
749 (Tail)	4.5×10^{-9}	1.7×10^{-14}	.000135	$.51 \times 10^{-9}$	.9999999995
720B (Aftbody)	1.0×10^{-9}	-	.00003	-	-
Body & Tail-Lat. Gust					
188 (Aftbody)	6.0×10^{-5}	5.0×10^{-7}	1.8	.015	.985
749 (Tail)	2.5×10^{-4}	5.0×10^{-6}	7.5	.15	.85
720B, Yaw Damper Off (Tail)	4.0×10^{-6}	-	.12	-	-
720B, Yaw Damper On (Body)	1.2×10^{-8}	-	.00036	-	-

TABLE XI

OVERALL GUST FAILURE RATES

Aircraft	Exceedences of Ultimate Load Per 30,000 Hour Life	Structural Reliability
Electra (188)	.00084	.99916
Constellation (749)	.00025	.99975
B-52	.0007	.9993
C-5A (indicated)	.0005	.9995

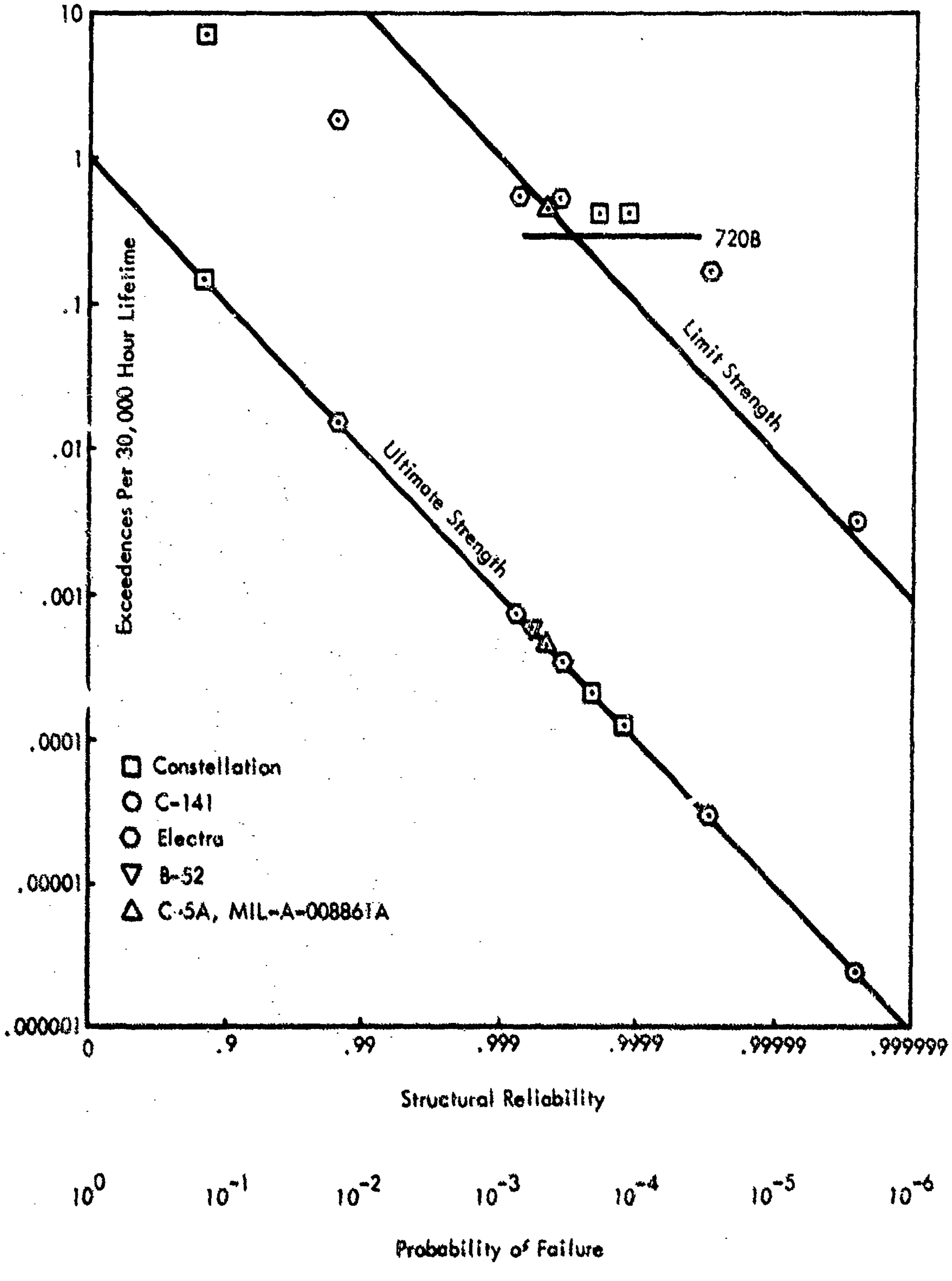


FIGURE 41 AIRCRAFT GUST ANALYSIS RESULTS

d. In Figure 41, the values of Tables X and XI are plotted to establish trends of limit condition occurrence versus structural reliability goal. In addition, C-141 limit and ultimate vertical gust levels are shown with structural reliability levels derived using the methods of Reference 1 with a coefficient of variation in strength of 0.05. An extremely high structural reliability is indicated for the C-141 data since it is not a gust critical aircraft.

e. Note that in Figure 41 the structural reliability levels are derived as the complement of the number of exceedences of ultimate load per aircraft lifetime. Limit load exceedences are placed at the structural reliability levels derived from the ultimate exceedences. The 720B limit data is shown as a horizontal line since there was no corresponding ultimate data available from which to establish a structural reliability level.

f. The mission analysis portion of the continuous turbulence analysis criteria developed by Reference 13 and adopted in the U.S. SST criteria and in Reference 23 calls for a limit load exceedence not more often that 2.0×10^{-5} times per flight hour. Reference 23 states that the probability of survival to this gust encounter should be equal to or greater than 0.9995. Using this as the structural reliability goal, the point is placed on Figure 41 using a lifetime of 30,000 hours for which there are 0.6 occurrences. The same structural reliability on the ultimate load line corresponds to the C-5A ultimate lateral gust design case.

Based on these statistics it appears that, in order to maintain a level of safety comparable to present transports, a structural reliability goal of 0.999 to 0.9999 is applicable rather than the goal of 0.999999 implied by Reference 1.

g. It is recommended that a structural reliability goal of 0.999 be used for military transports since they are primarily cargo carriers and can be somewhat more risky than commercial transports from which most of the available statistics were derived.

In Figure 41, a line which represents fairly well the limit strength exceedence statistics lies three orders of magnitude higher than the ultimate (or omega) line. If it is thought to be feasible to establish occurrences of limit conditions versus structural reliability goal, this line should be a better representation of such levels than the line established in Reference 24 and shown in Figure 40 of this document.

In present transport design practice, a limit load is thought of as a load level which occurs approximately once per aircraft lifetime. For the recommended structural reliability goal of 0.999, the proposed line does in fact allow exactly one occurrence of a given limit load per aircraft lifetime.

8.4 Other Reliability-Based Criteria

a. Reference 25 established the following probability of occurrence concepts for use in the design of the Concorde supersonic transport:

1. Frequent. Occurring more than 10^{-3} per hour of flight.
2. Reasonably Probable. Of the order 10^{-3} to 10^{-5} per hour of flight. These terms are collectively known as recurrent and are expected to occur from time to time during the operation of each particular airplane of a type.
3. Remote. Of the order of 10^{-5} to 10^{-7} per hour of flight. Not likely to occur often during the operation of an airplane type but may happen a few times during the total operational life of the type.
4. Extremely Remote. Not expected to occur more than 10^{-7} per hour of flight. Unlikely to occur during the total operational life of all airplanes of the type but, nevertheless, has to be considered as being possible.
5. Extremely Improbable. So Extremely Remote that it can be stated with confidence that it should not occur.

For a 30,000 hour aircraft the occurrences per lifetime for these levels are:

Frequent	More than 30
Reasonably Probable	0.3 to 30
Remote	0.003 to 0.3
Extremely Remote	Less than 0.003

b. Examining the data of Figure 41 using this nomenclature it may be seen that, for the recommended structural reliability levels, limit conditions fall in the Reasonably Probable category which is where they should be. Ultimate (omega) conditions are in the Extremely Remote range which is also the proper placement. The minimum structural reliability necessary to place Ultimate (Omega) conditions in the Extremely Remote category is approximately 0.997.

8.5 Fighter Data

a. Table XII shows F-100 limit wing bending moment occurrences and computed structural reliability levels derived from Reference 1 for a coefficient of variation in strength of 0.08.

Surprisingly high structural reliability levels were derived in the referenced analysis when it is considered that the implied recommended fighter structural reliability is 0.99.

b. In Figure 42 the F-100 data is superimposed on the limit and ultimate (omega) variation with structural reliability previously shown in Figure 41. Note that the proposed limit condition line correlates fairly well with the F-100 limit data. Also shown is the Reference 23 mission analysis continuous turbulence limit condition point of 2×10^{-5} occurrences per hour applied to a 4,000 hour fighter which results in 0.08 occurrences per lifetime. Even though it is unlikely that a fighter would be gust critical, it appears that the 2×10^{-5} occurrences per hour is not appropriate. The value was derived in Reference 13 based strictly on commercial transport data and application of the same value to other aircraft types may not be valid.

8.6 Suggested Goals

Based on the preceding analysis, the structural reliability goals and corresponding exceedences of limit and ultimate (omega) conditions of Table XIII are recommended for analytical applications of the new method.

TABLE XII

F-100 WING STRUCTURAL RELIABILITY

Item	Occurrences Per Lifetime	Structural Reliability		
		Normal	Lognormal	Weibull
Wing Root Limit Bending Moment	70	.992	.99925	.9983
Wing Root Limit Bending Moment (Revised)	0.2	.99958	.99973	.9989
Wing Mid-Span Limit Bending Moment	0.08	.999974	.999978	.9999

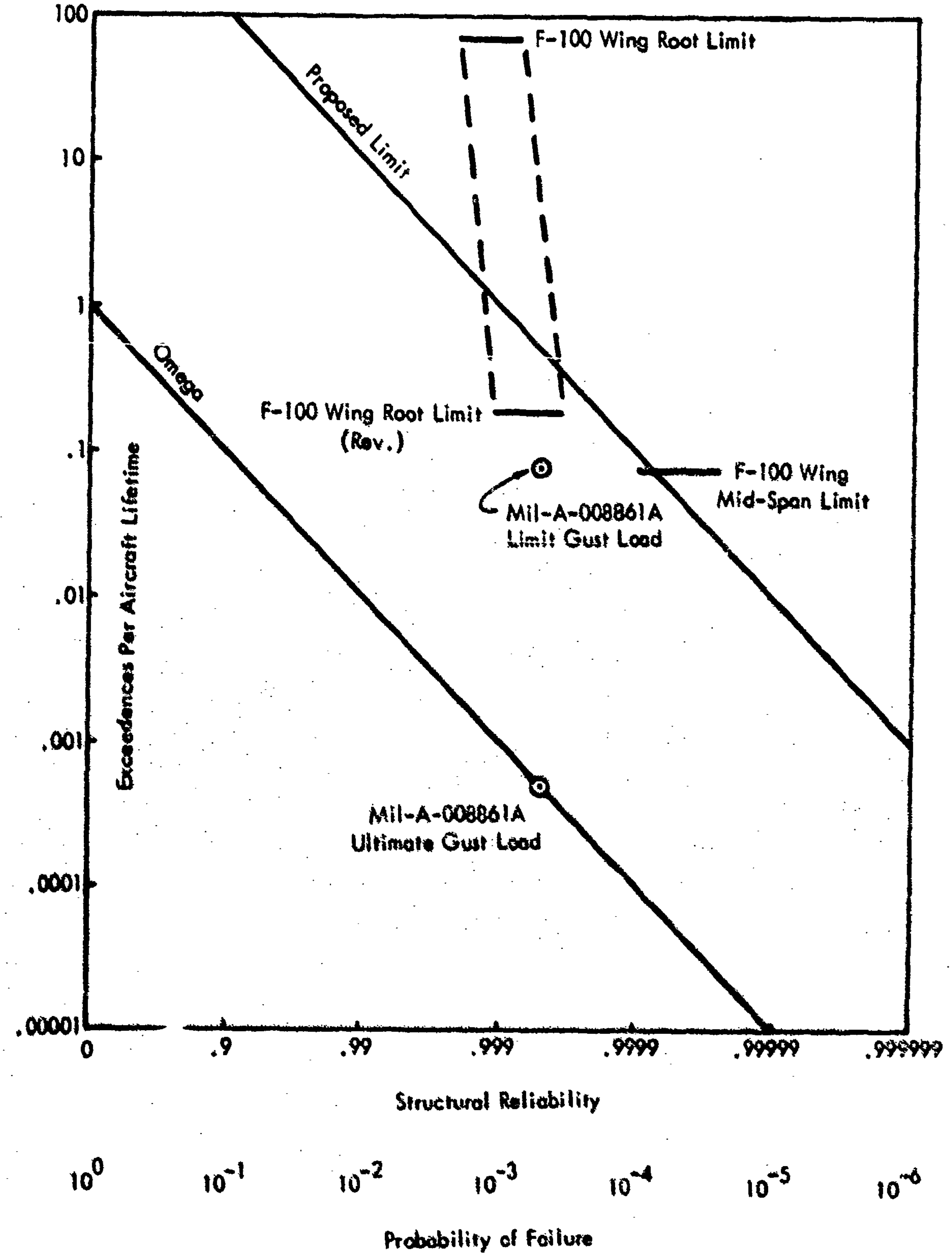


FIGURE 42 FIGHTER DESIGN LOAD EXCEEDENCES

TABLE XIII

STRUCTURAL RELIABILITY OBJECTIVES

Aircraft Types	High Risk	Standard	Low Risk
	A, F, TF	O, T, U, B_I, B_{II}, C	Commercial Transports
Structural Reliability Goal	0.99	0.999	0.9999
No. Exceedences of Limit Condition Per Aircraft Lifetime	10	1	0.1
Probability of Exceeding Omega Condition in Aircraft Lifetime	0.01	0.001	0.0001

SECTION IX

CHOICE OF DESIGN AND TEST FACTOR

9.1 Required Factors

The basic chain of events in the reliability calculations has been described in Section III. Figure 43 summarizes this chain for reference. The chosen unfactored load (limit or omega) is multiplied by a design factor (FS) and increased by a design margin of safety (MS) to give a factored design load (DSNLD). This is matched to an allowable strength, defined as S_{all} standard deviations below the intended mean strength, AMSTR, which is therefore determined. The intended strength distribution is modified by the error function so as to give probable strength distribution. Tests, made to load levels defined by UNFLD * TEST FACTOR, then yield updated probable strength distributions which in turn lead to failure probabilities and reliabilities.

9.2 Design Factors

a. It is apparent that the intended (no error) strength level of the structure is related to the unfactored load by three factors, FS, MS, and S_{all}, which all achieve a similar effect. They provide a margin to cover the likely presence of resultant discrepancies between the intended minimum strength and the actual strength of the weakest aircraft in the fleet.

For convenience, the ensuing discussion assumes the design margin of safety to be zero. The logic is easily modified to incorporate non-zero values where appropriate.

b. The value of S_{all} establishes the design allowables currently in use, but whereas current methods require no other data, the probabilistic system also requires the standard deviation (and distribution function) to be known.

Figure 43 illustrates again the need for a single load value (DSNLD) which can be used in conjunction with design allowables to enable the sizes of the structural members to be established.

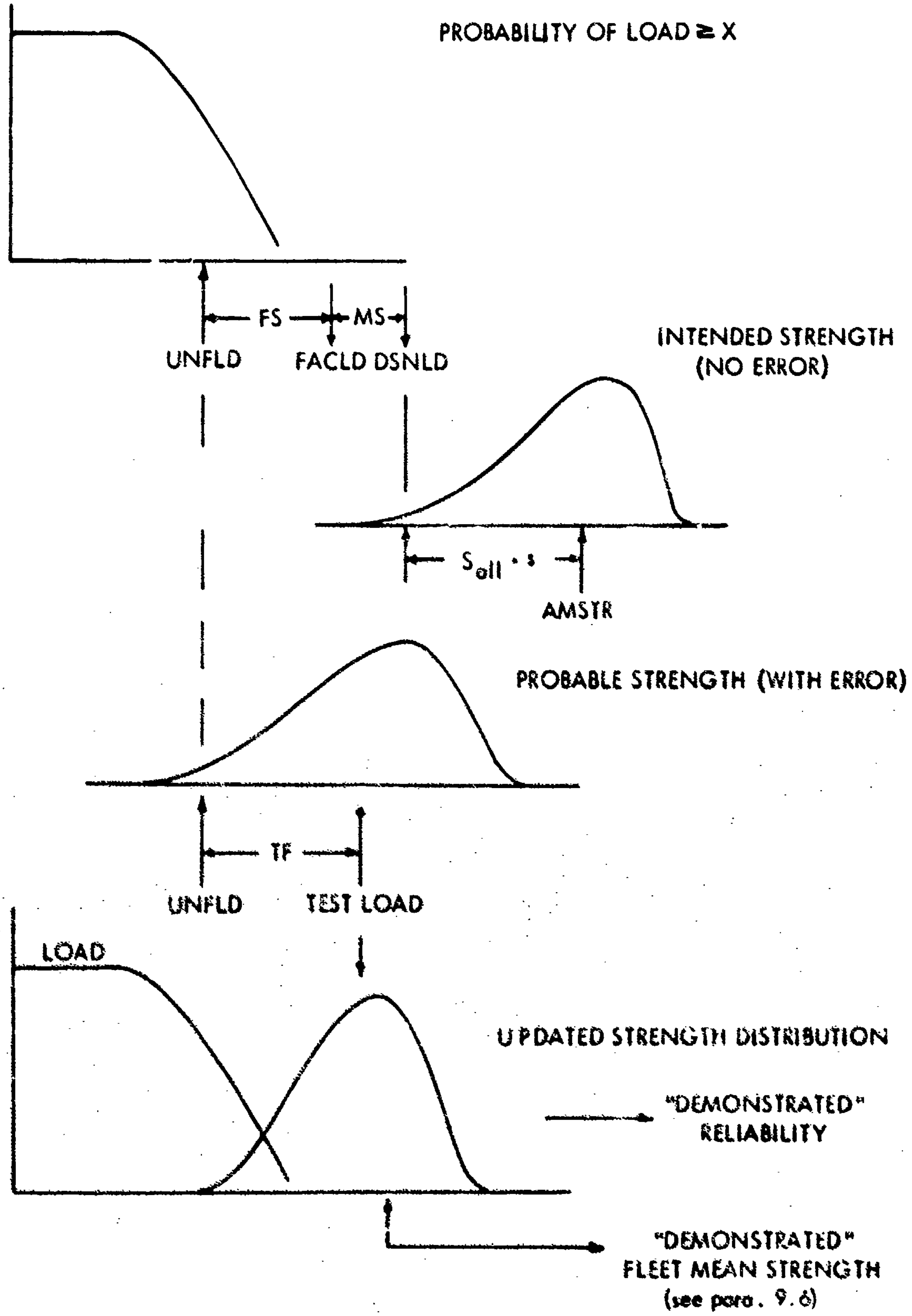


FIGURE 43 SUMMARY OF DESIGN CHAIN

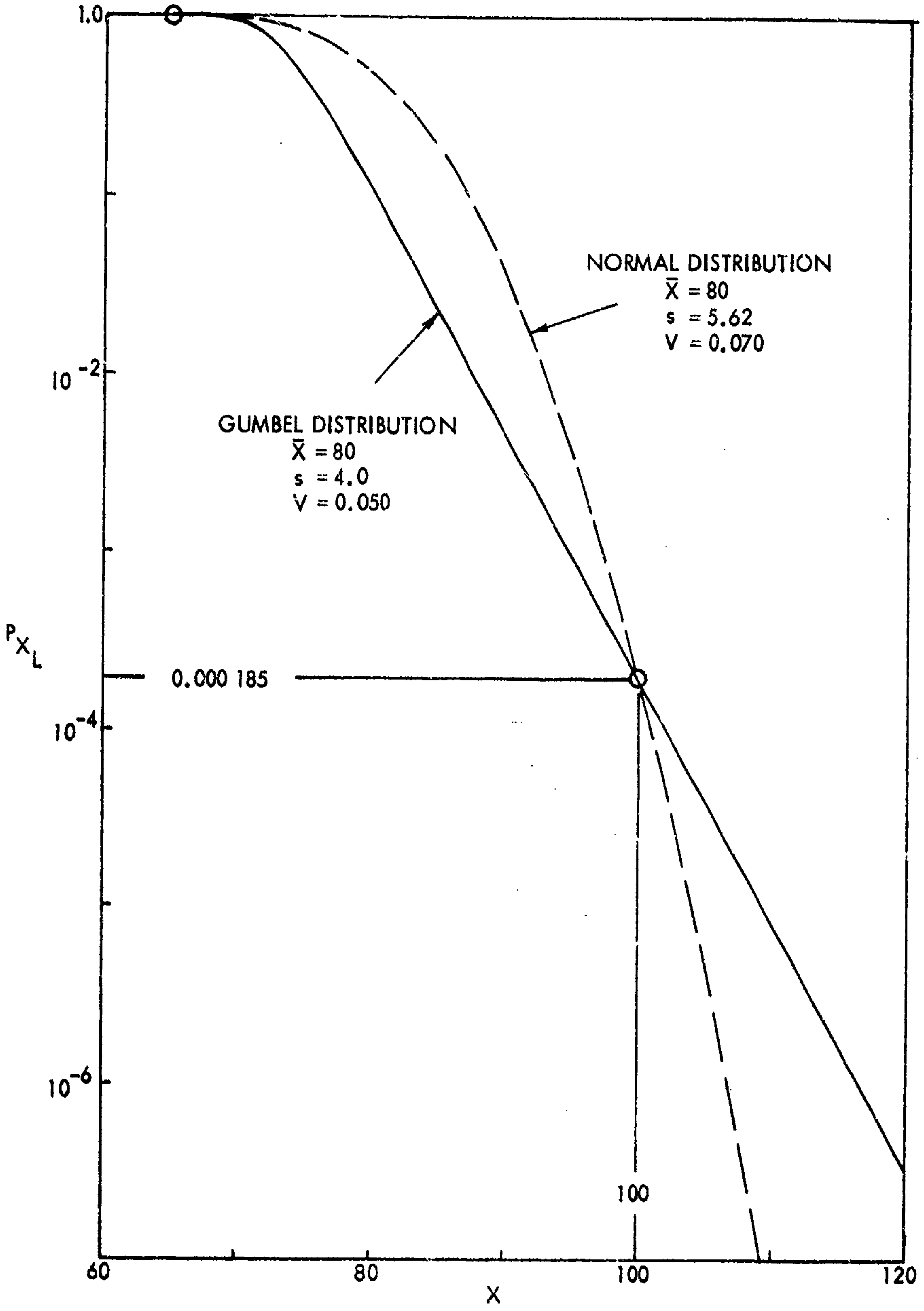


FIGURE 44 TWO DIFFERENT LOAD SPECTRA

c. Reference 1 suggests that relationships can be established between the probability of occurrence of the unfactored load and the desired reliability. This is correct, provided that a number of particular assumptions are made regarding other parameters, namely

1) load spectrum location, and distribution parameters

2) strength distribution shape and distribution parameters

3) error function

4) test load level.

d. The influence of load spectrum shape is illustrated by the following example. Figure 44 shows two load spectra which both reach 65.0 at a probability of one, and both reach 100at a probability of 0.000185. One is a Gumbel distribution and the other a normal distribution. The common value of 100 is chosen as UNFLD. Reliability estimates were made for both load spectra, assuming identical values for all other functions (strength, error and test level).

Figure 45 shows the failure density distributions before testing and after surviving one test to 150. Figure 46 shows the cumulative failure probabilities and the reliabilities. Table XIV summarizes the results. It is seen that whereas the demonstrated fleet mean strengths* do not vary, the failure risks differ widely although they are low. The particular values in the example show the intended failure probabilities (no error, no test) to differ by a factor of 2.5, decreasing to 1.6 when the probable error is added, and increasing to 2.6 after testing. In all cases, the normal load distribution gives the higher risk because of the increased load probabilities between 65 and 100.

e. The attained ("demonstrated") reliability, after the test, is 0.9999992 for the Gumbel loads distribution, and 0.9999979 for the normal distribution. If the concept of reference 1 is adopted, that the reliability is the complement of the probability of the load, a value of 0.9998115 results, which is some

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*See para. 9.6.

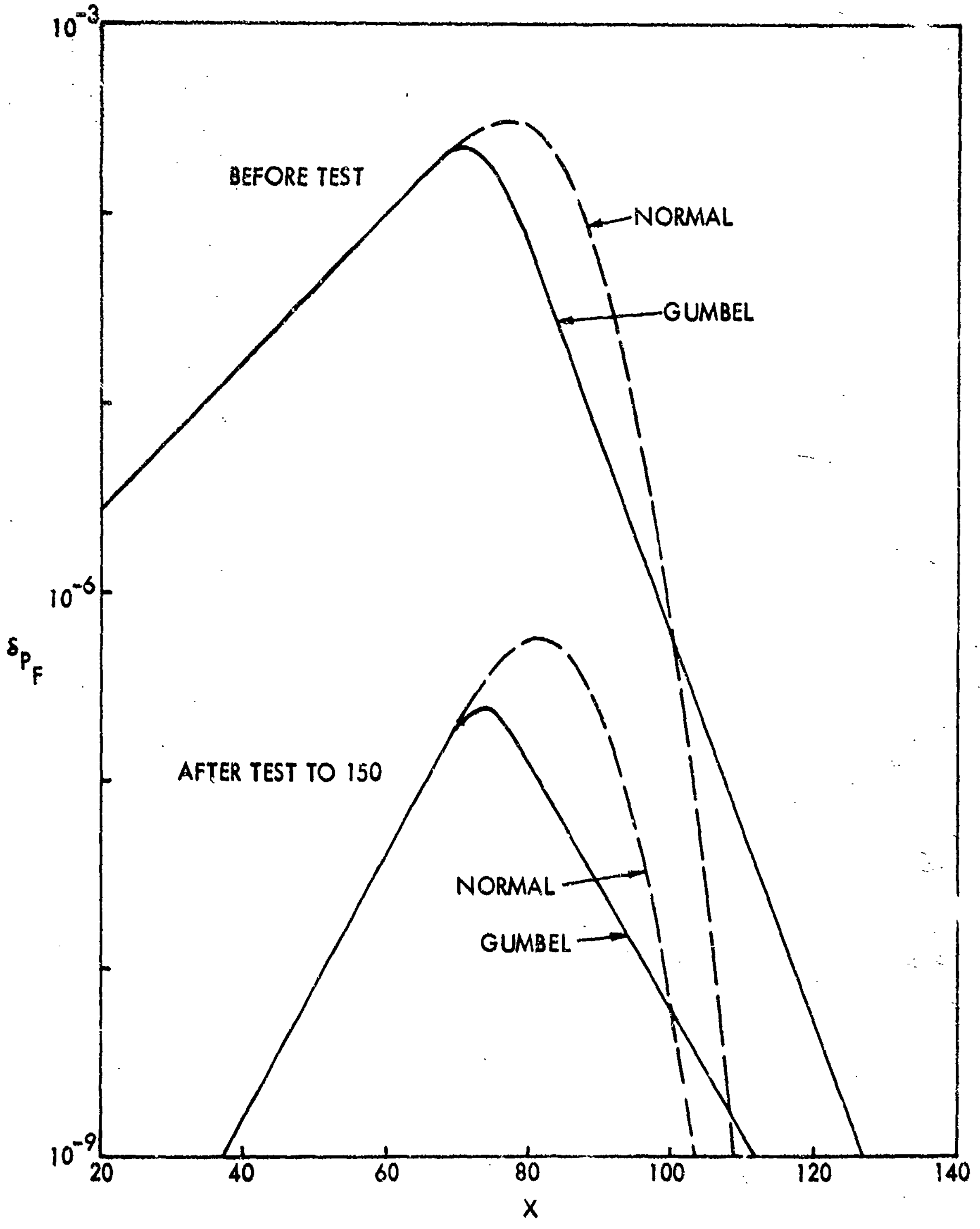

FIGURE 45 FAILURE DENSITY DISTRIBUTIONS FOR TWO DIFFERENT LOAD SPECTRA

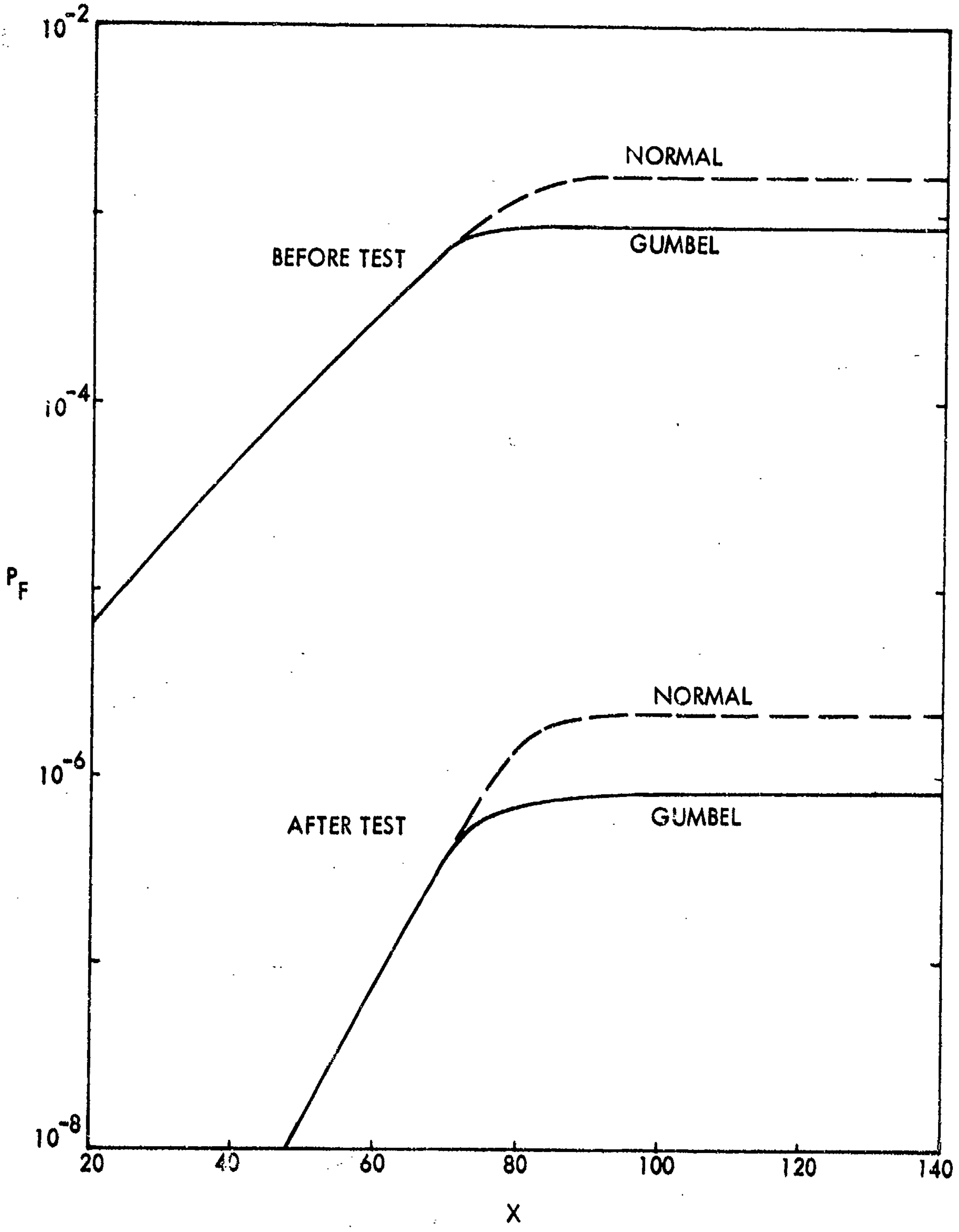

FIGURE 46 RELIABILITIES FOR TWO DIFFERENT LOAD SPECTRA

TABLE XIV

SUMMARY OF RESULTS OF DIFFERENT LOAD SPECTRA

Quantity	Load Distribution	
	Gumbel	Normal
No Error (Intended)		
Failure Probability	0.00000043	0.00000105
Reliability	0.99999957	0.99999895
Fleet Mean Strength*	166.9	166.9
With Error, No Test		
Failure Probability	0.00088822	0.00145386
Reliability	0.99911178	0.99854614
Fleet Mean Strength*	152.5	152.5
With Error, After Test to 150		
Failure Probability	0.00000081	0.00000211
Reliability	0.99999919	0.99999789
Fleet Mean Strength*	161.8	161.8

*See para. 9.6.

two decades lower. It is evident that the reliability will only be the complement of the load probability for certain combinations of all of the other parameters discussed above. This fact complicates the problem associated with the selection of the initial load levels to be used to size the structural members.

f. Studies performed during the preparation of this report have revealed some of the inherent relationships between the various factors. If the application of the proposed criteria system is summarized in the following manner, certain practical procedures can be formulated:

> "to provide structural members whose sizes are determined by matching a factored design load to an allowable strength, and to test to load levels such that, allowing for probable discrepancies between the intended and actual strength levels, the desired reliability is demonstrated."

It will be obvious that the same results will be achievable by designing to a high factored load level, and testing to a moderate load level, and by designing to a modest factored load level with the testing performed to a higher level. It will also be apparent that the probability of sustaining the test load diminishes as the test load level increases. These trends suggest that an optimum combination might exist in terms of total cost (reference 5, for example, discusses this concept), but the formal logic of such a procedure remains undeveloped. Section IX-4 explores the interaction between the factors.

9.3 Test Factors

a. Earlier sections of this report have alluded to the difference between survival tests and failure tests. Reference 1 describes the interpretation of survival tests; the probability that the test specimen has a strength greater than the test load is estimated from the mean strength distribution (including the probable discrepancy). Bayes' theorem is applied to yield an updated mean strength distribution which leads to the updated individual strength distribution and to the reliability.

The knowledge of an actual test failure load is much more difficult to incorporate into a practical analysis, since the probability of an exact value is mathematically indeterminate. It must be replaced by the estimated probability of a value within a certain interval, and this will vary with the width of the interval, a fact which inhibits uniform interpretation.

b. Figure 47 shows the reliability levels computed for different intervals (dx), with all other values unchanged. For test loads of 150 percent of the unfactored design load, a variation of thirteen times is observed in the risk of failure as the interval width changes from 2 to 20. This ambiguity suggests that for practical reasons, rather than for logical reasons, all tests should be interpreted as tests surviving a given load level. If a failure does occur, then a level just below the failure level is regarded as the load survived.

Figure 48 compares the results of tests to various load levels, regarded in the two different ways.

9.4 Combined Factors

a. Adopting the approach described above, a study was made using data pertinent to the C-141 wing root bending moment in the vertical gust cases. The load spectrum used is shown in Figure 49. The strength variations and error function were the same as those described in earlier sections of this report. Using 100 as the unfactored design load, the design factor and test factor were varied through the range 1.0 to 1.8, and the fleet mean strength* and probability of failure were calculated before and after the survival test. For the case of a design factor of 1.5, the failure test condition was evaluated with an interval width of 5.

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*See para. 9.6.

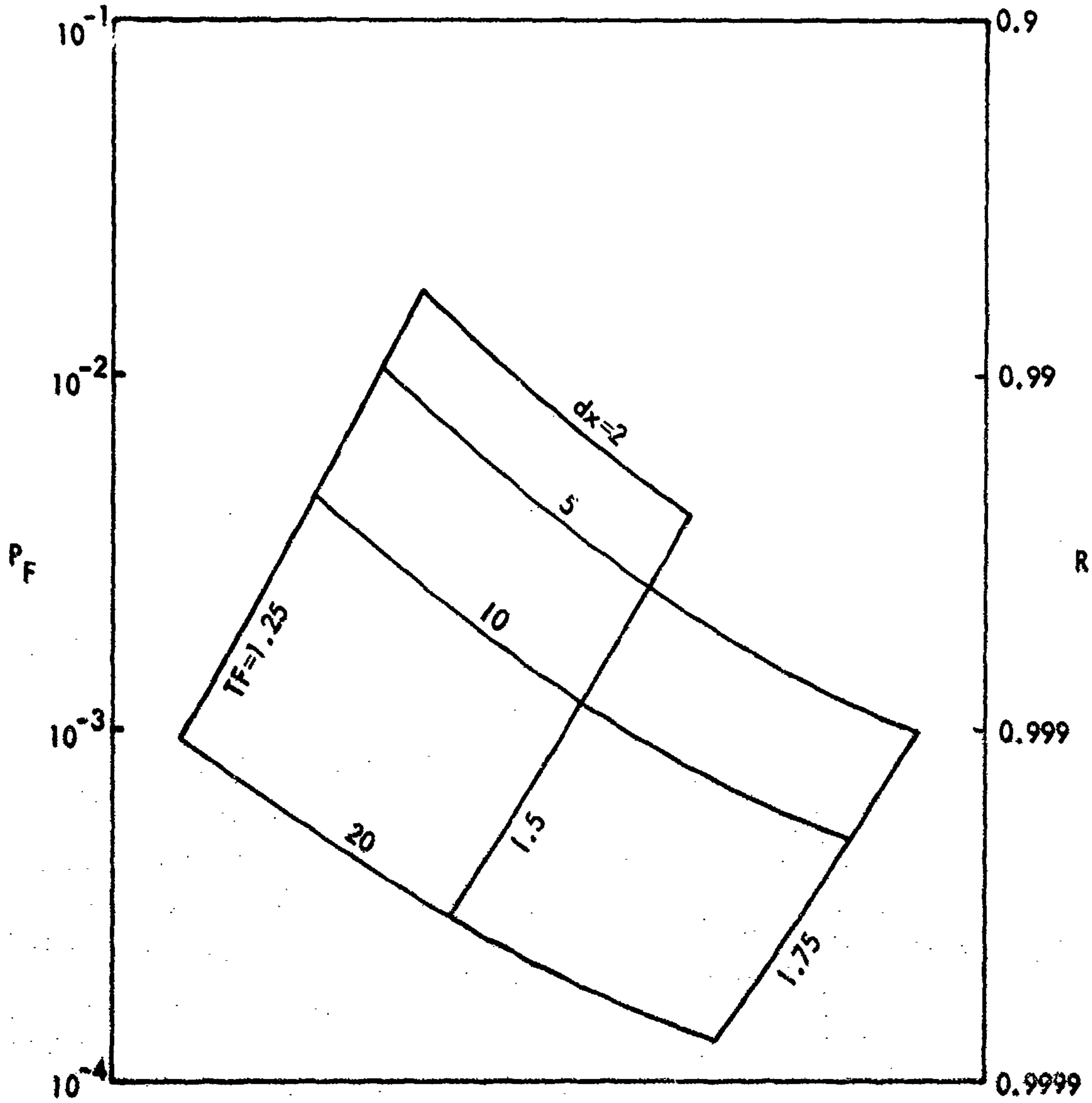

FIGURE 47 RELIABILITIES FOR TEST FAILURE LOADS AND INTERVAL WIDTHS

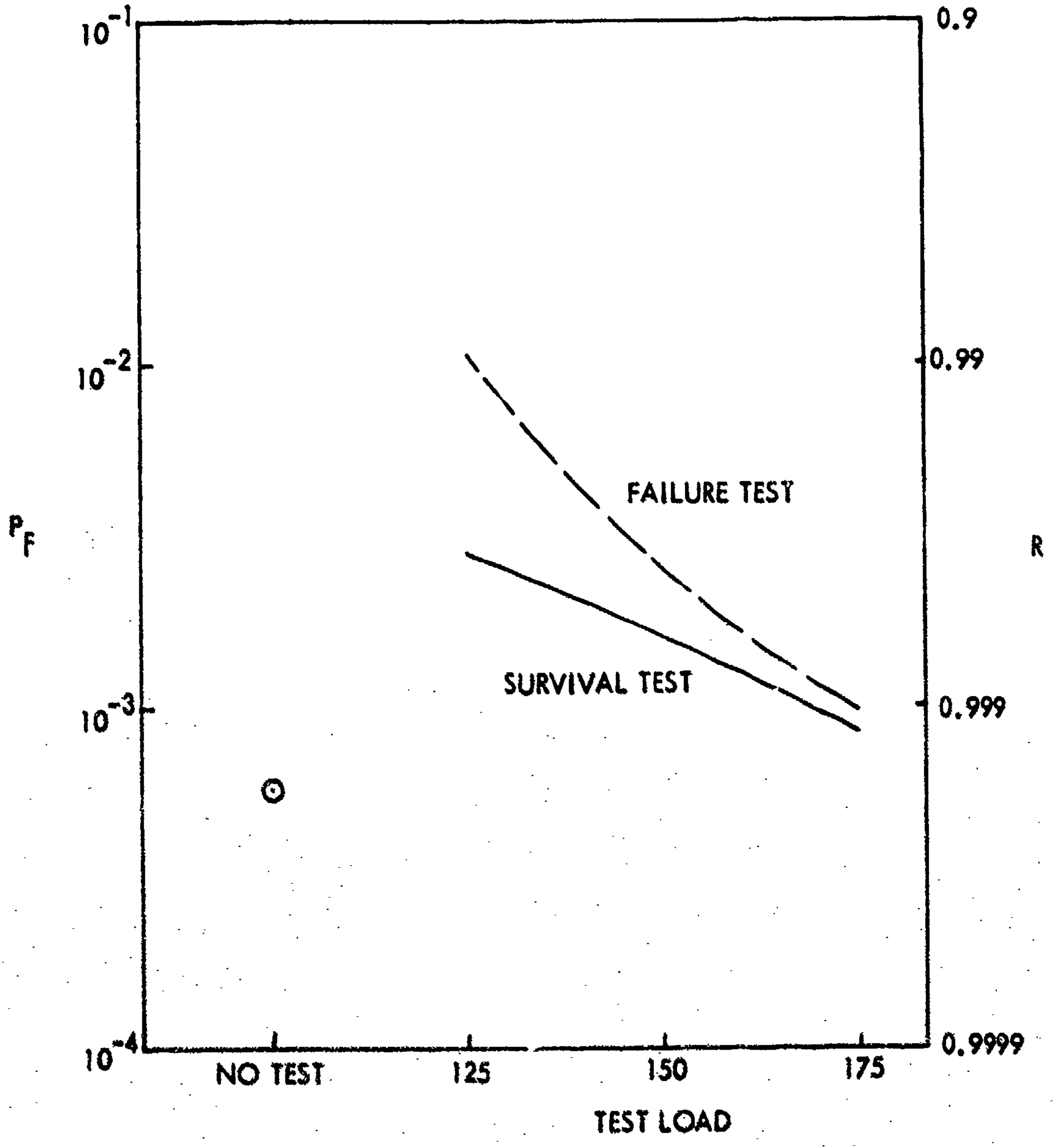

FIGURE 48 SURVIVAL AND FAILURE TESTS

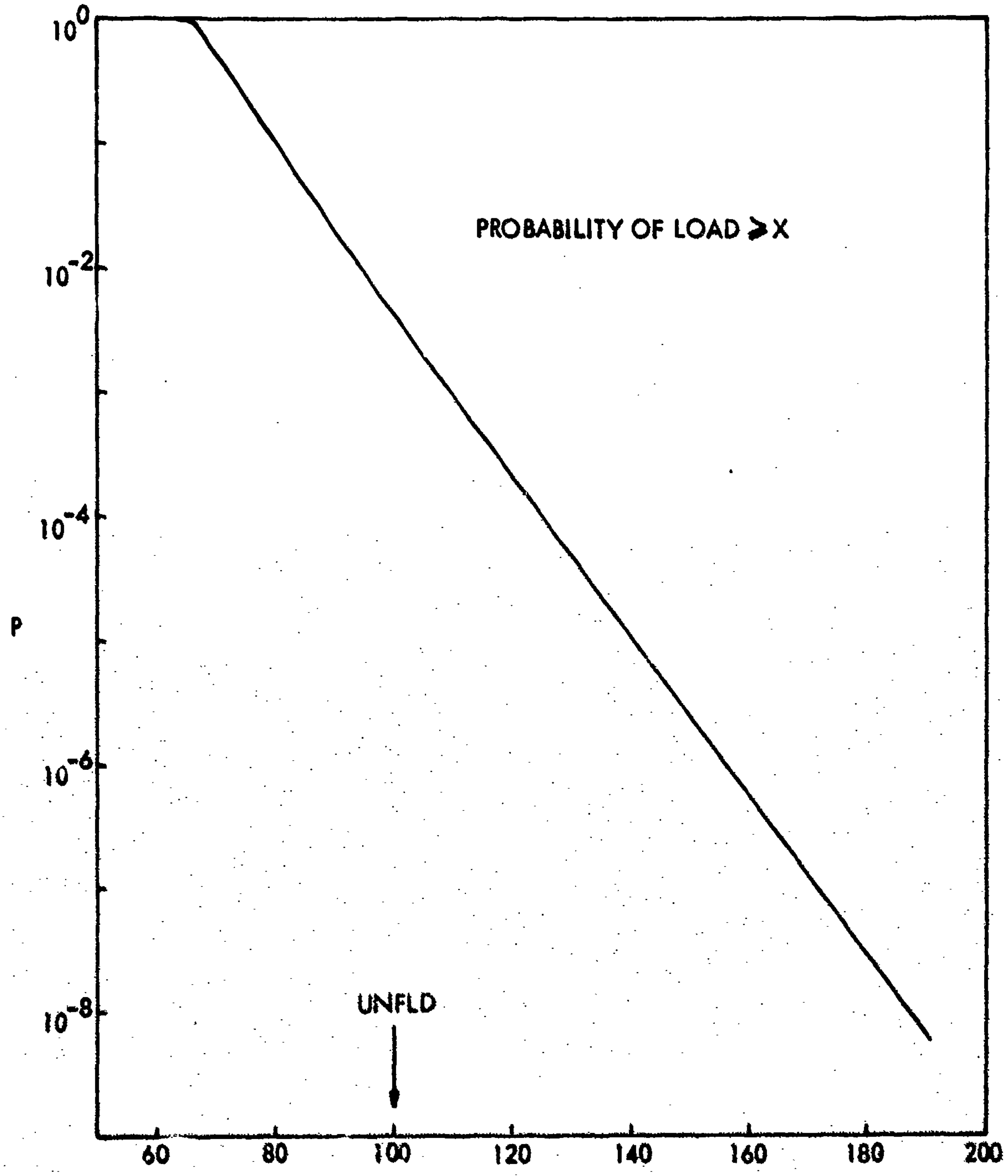

FIGURE 49 C-141 GUST BENDING MOMENT AT WING ROOT

b. Figure 50 shows the reliabilities for the survival tests, together with the intended (no error) values and the probable (no test) values. The actual design process used 1.5 as the design and test factor, the appropriate points being marked A and B. Point B can be regarded as the intended reliability level (0.99999980) and Point A as the value demonstrated by the conventional test to 1.5 times the unfactored design load (reliability = 0.9999983). Figure 51 compares the failure and survival test results for a 1.5 design factor. Figure 52 shows the fleet mean strength* demonstrated by the survival tests, and figure 53 compares the same quantity for survival and failure tests, with the design factor of 1.5.

c. A study of figure 50 reveals some intriguing trends. If the conventional test to 150 percent load is replaced by a test surviving 100 percent load, the demonstrated reliability only drops from 0.9999983 to 0.9999947; figure 52 shows that a test to 150 percent indicates a fleet mean strength* of 165.1, compared with the intended value of 175.5 (no error), but that testing to 100 percent still indicates a fleet mean strength of 162.0. For these particular numbers, the value of a test above the 100 percent level must be questioned, apart from its effect in reducing the probable standard deviation of strength from 8.2 percent to 7.1 percent, as shown in figure 54.

d. Figures 50 and 52 shows the variations for the case when the design and test factors are equal. This assumption can be made in order to simplify the choice of values to be used in a particular case, and will probably be necessary if charts of standard values are to be prepared (see Section XII).

9.5 Non-Destructive Testing

a. The features previously discussed lead to the question whether a series of non-destructive tests ("proof tests") can be used in place of a single test to a higher load level. It is necessary, in this context, to emphasize that each test must be on a separate article, and to point out that an operational

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*See para. 9.6.

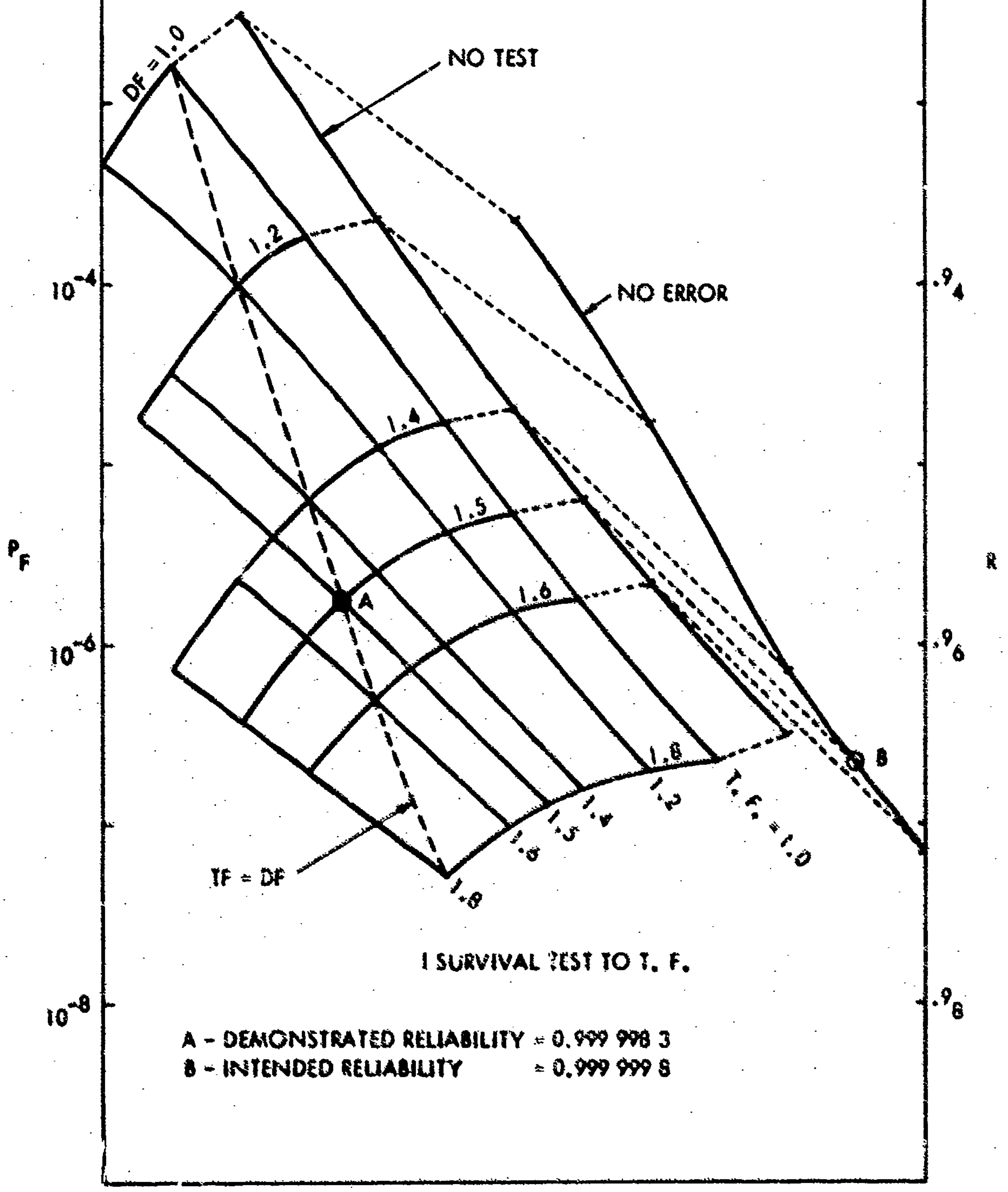


FIGURE 50 DESIGN AND TEST FACTORS FOR GIVEN RELIABILITY LEVELS

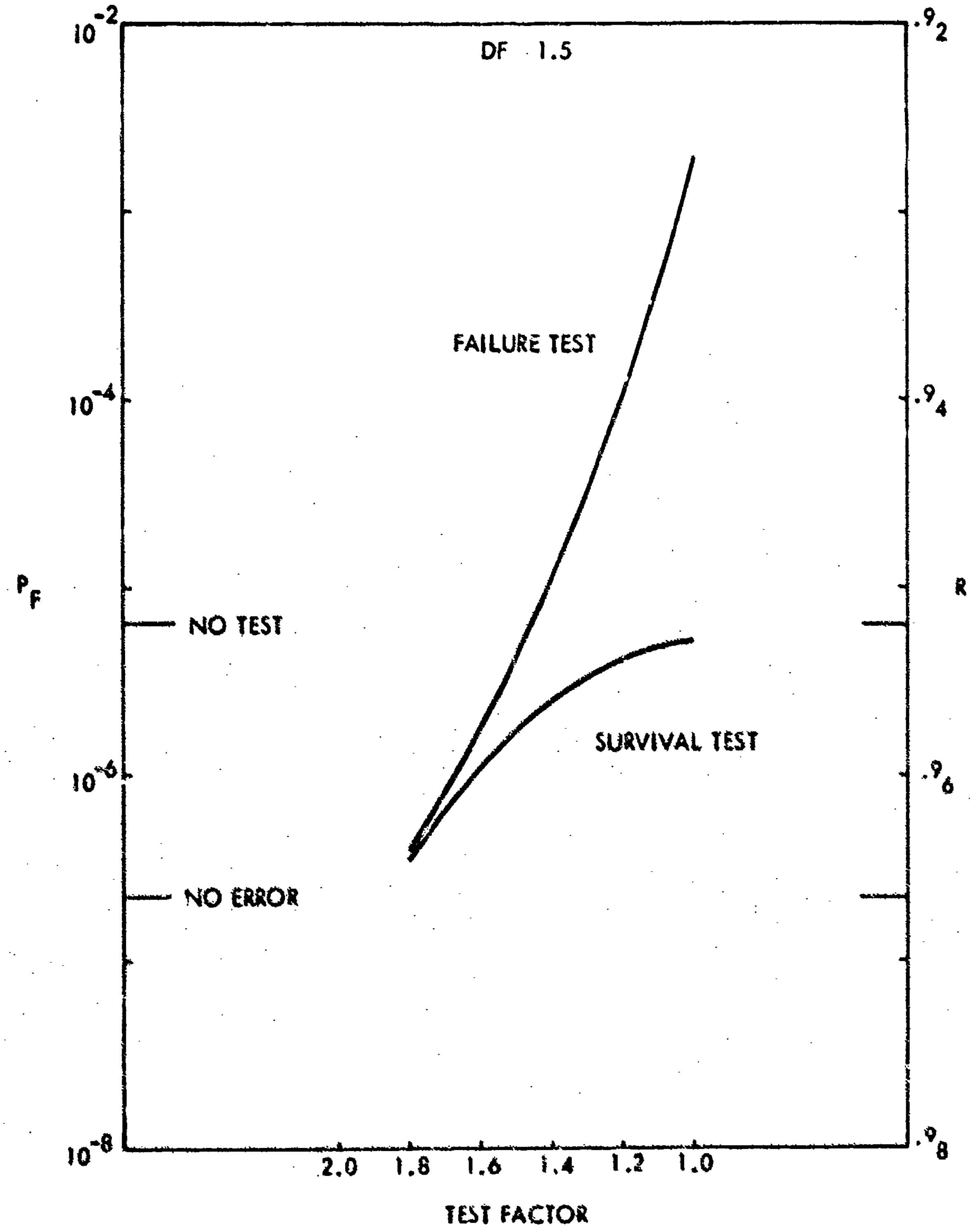


FIGURE 51 FAILURE AND SURVIVAL TEST FACTORS FOR GIVEN RELIABILITY LEVEL

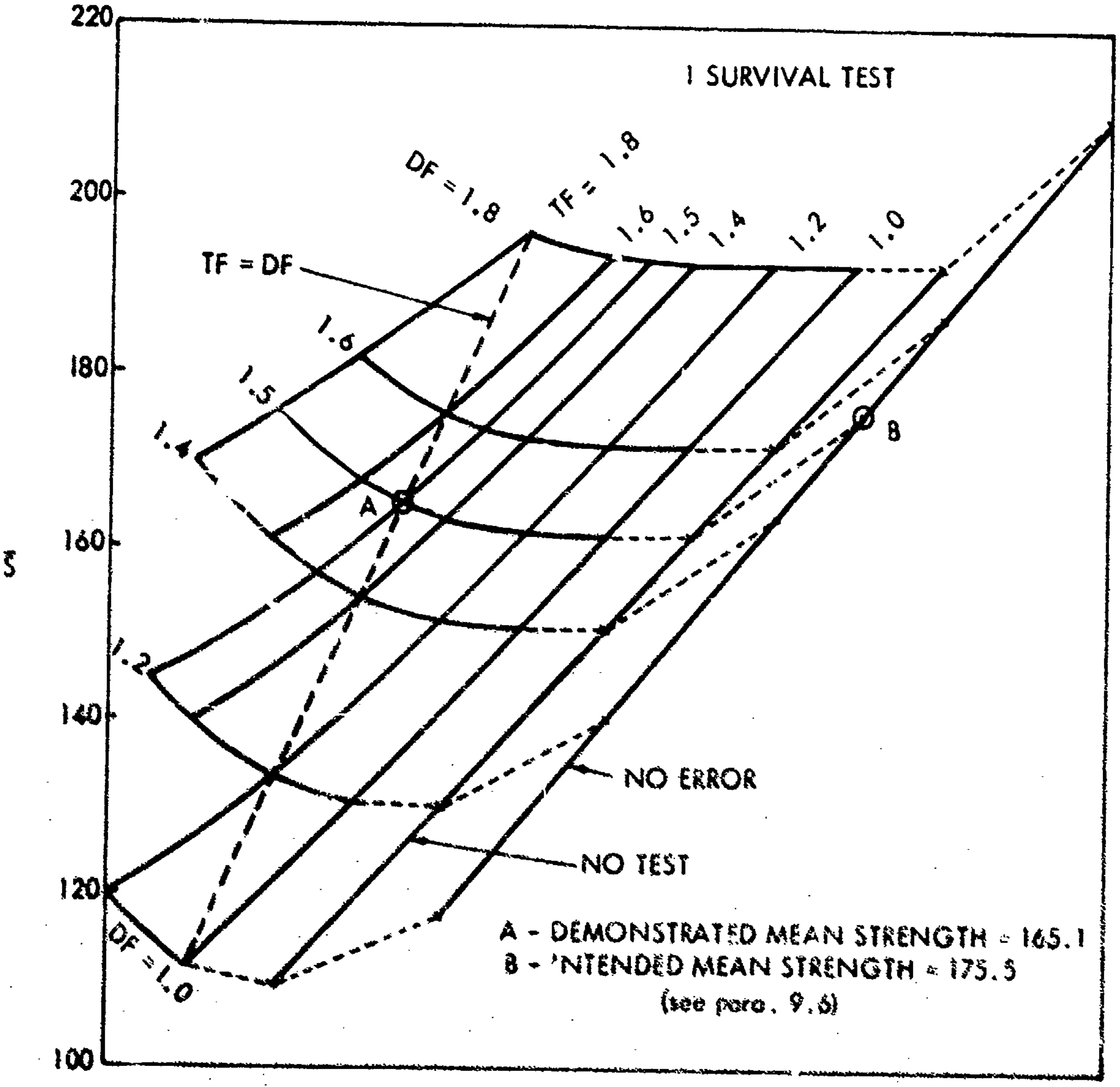


FIGURE 52 FLEET MEAN STRENGTH FOR VARIOUS DESIGN AND TEST FACTORS

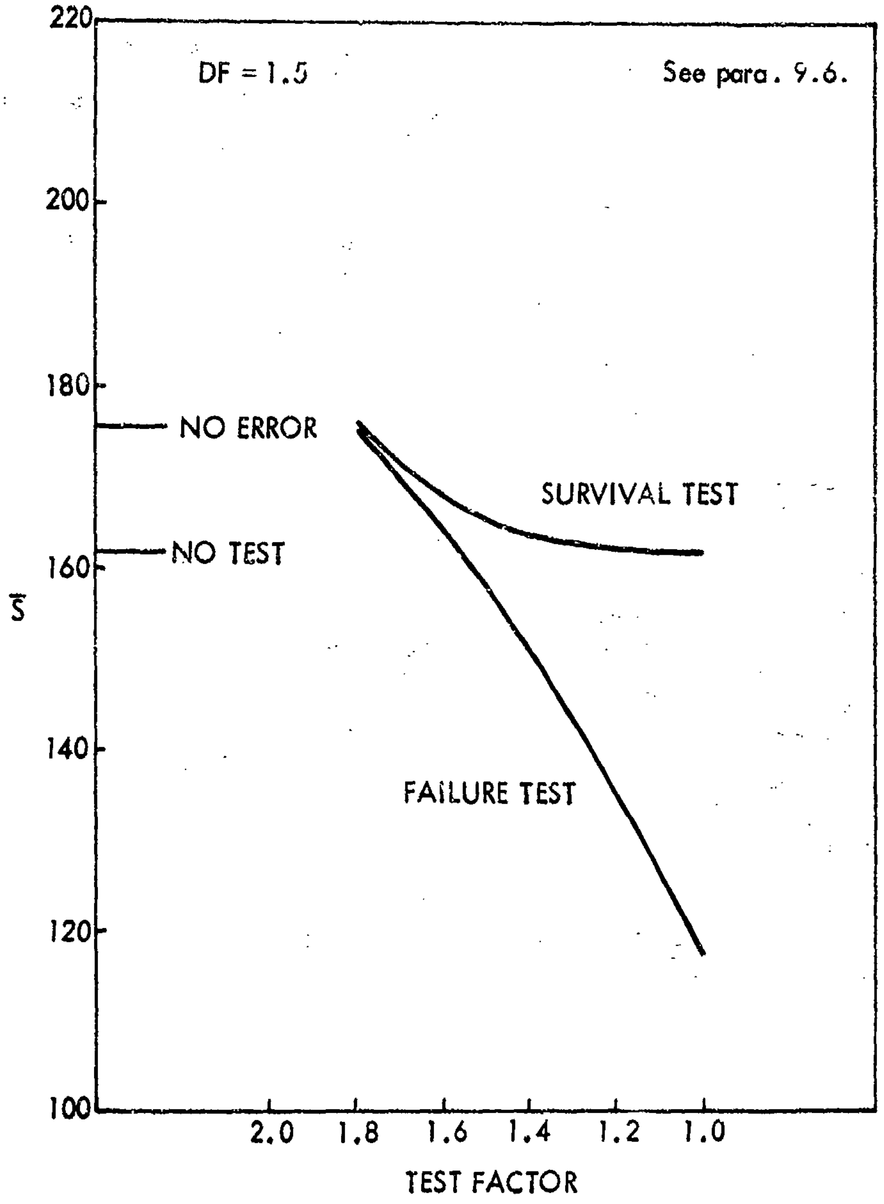
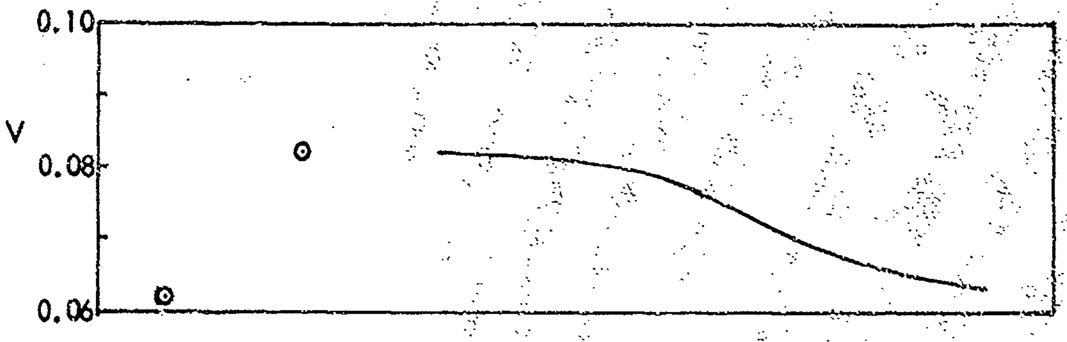
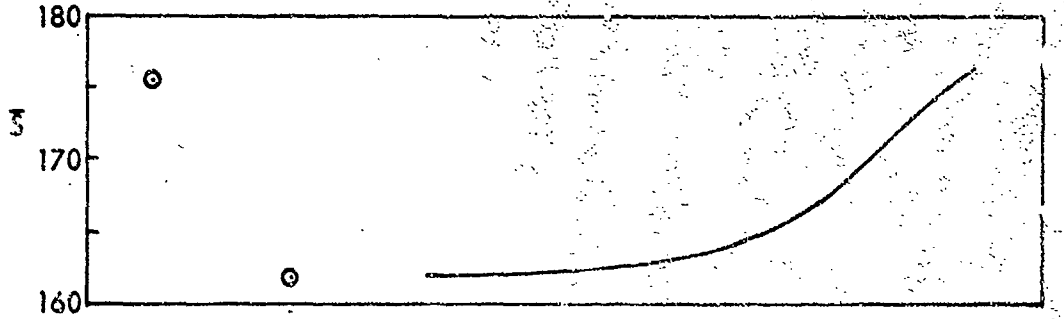


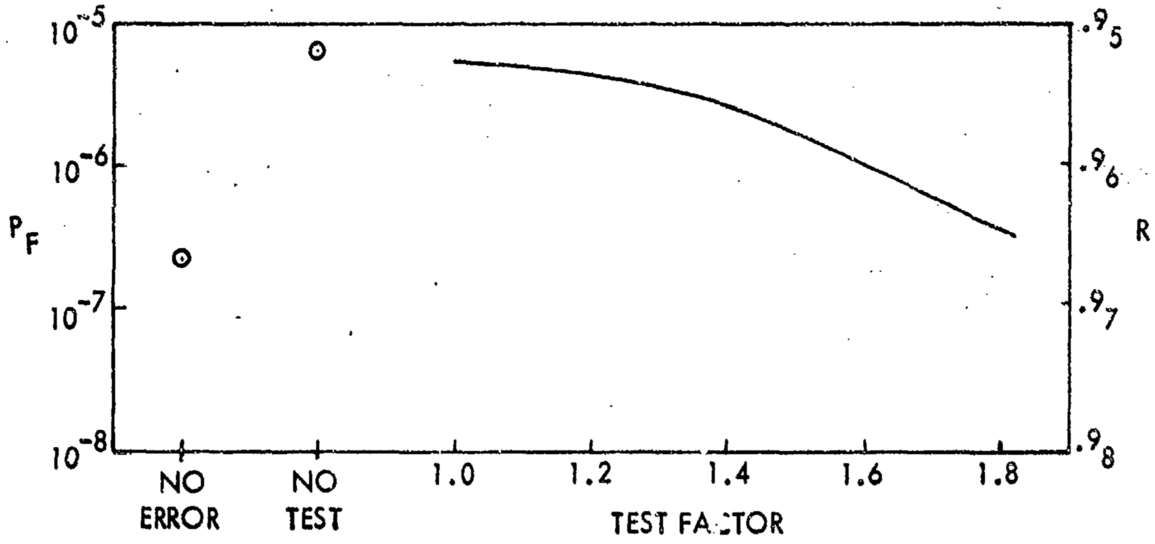
FIGURE 53 FLEET MEAN STRENGTH FOR SURVIVAL AND FAILURE TESTS TO VARIOUS LEVELS



(a) Coefficient of variation



(b) Fleet mean strength (see para. 9.6)



(c) Failure probability and reliability

FIGURE 54 EFFECT OF TEST FACTOR (DF = 1.5) experience has the same influence as a laboratory test; hence the acceptance tests on each aircraft can provide a much greater volume of pertinent data for probability-based criteria than is normally available under the present system.

b. The data used for the example described above was used for a trade-off study between one test to 150 percent of the unfactored load and ten tests to 100 percent of the unfactored load. Figure 55 shows the reliabilities obtained from one test to factored load levels compared with those from a series of tests to 100 percent load. It is seen that for this particular example repeated testing to 100 percent has little influence, and that ten such tests are equivalent to one test to 108 percent load.

c. A further study was then made using a wider range of values. Trade-off rates between different numbers of tests to different test factors are illustrated by Figure 56. It is observed that for the particular data used, one test to 150 percent of the unfactored design load could be replaced by two tests to 147 percent, three to 145 percent, five to 142 percent or ten to 139 percent. The chances of surviving the same series are shown in Figure 57. Before the testing, the probabilities of surviving the same series of tests are 0.77, 0.69, 0.65, 0.59 and 0.46 respectively, so that the best chance of "demonstrating" the reliability occurs with a single test to the highest test load.

d. Suppose now that the first test only survived 143 percent, testing of four futher specimens to this level would now be required to "demonstrate" the same level of reliability as the original aim. But the center plot of Figure 57 shows that, at this stage, the chances of surviving this new test series have dropped to 0.75. Deductions of this kind can be made from plots of this type, to aid in the assessment of the optimum test program based on available data at any time.

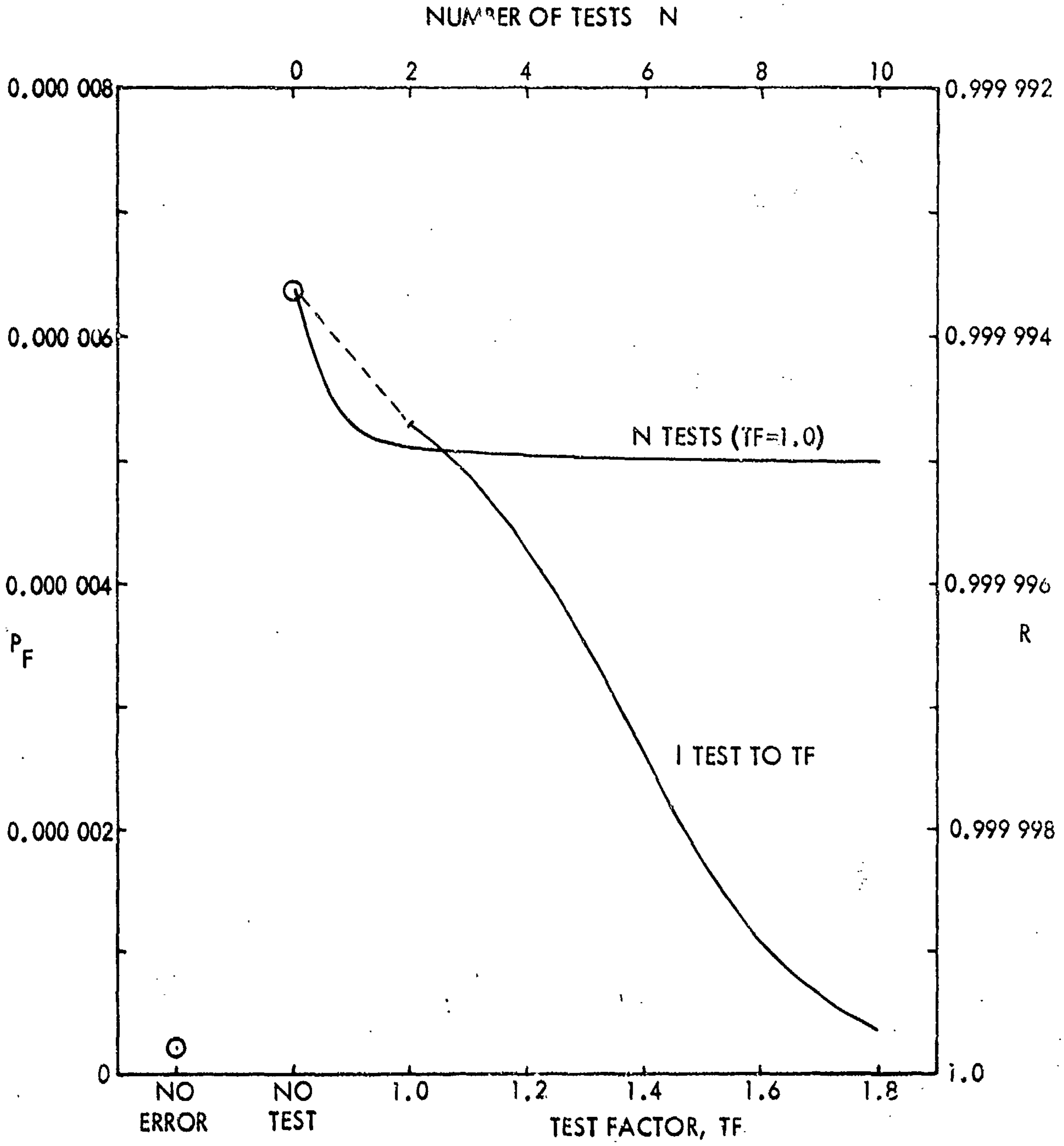


FIGURE 55 EQUIVALENCE OF LIMIT LOAD TESTS FOR C-141

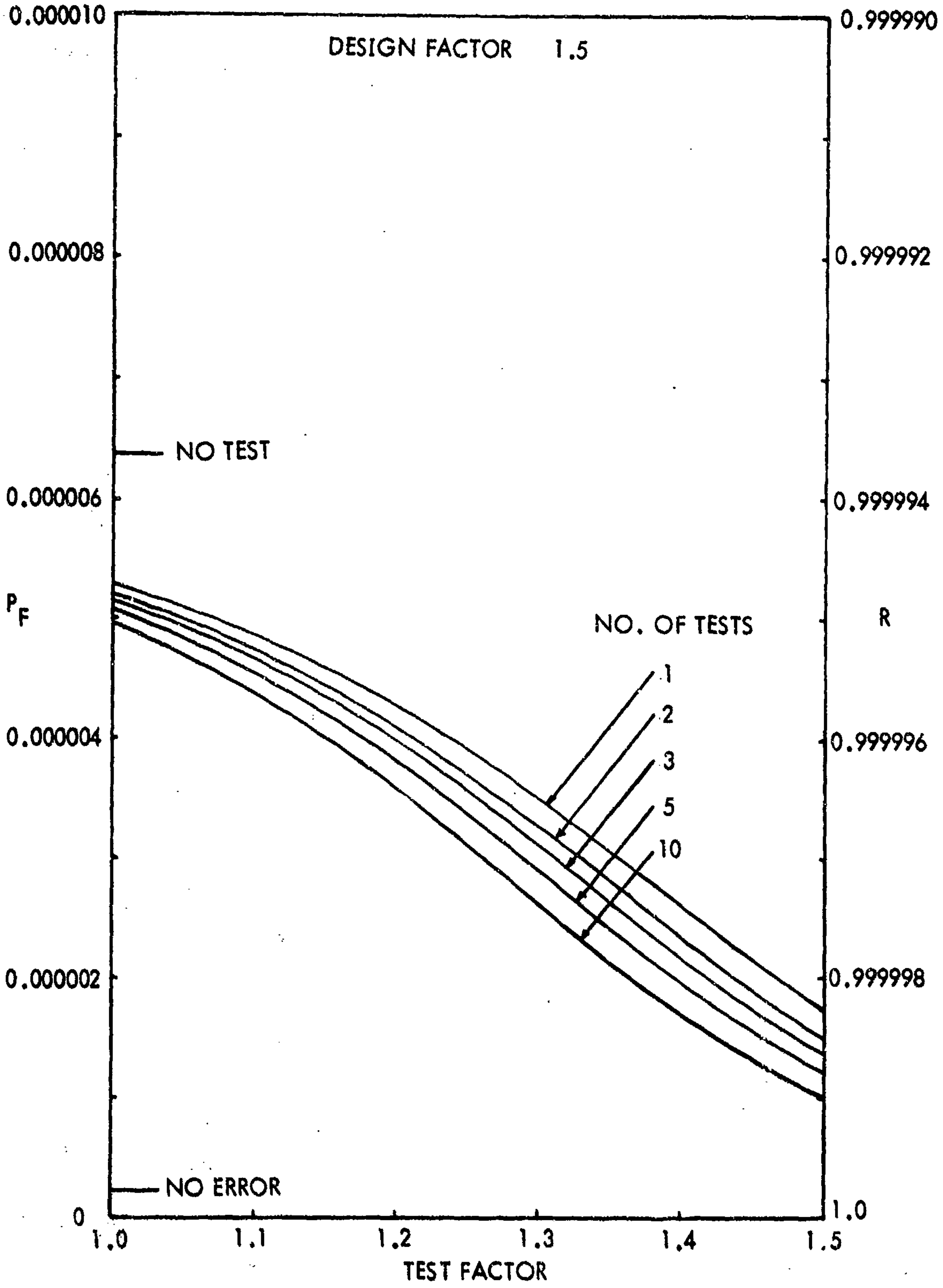


FIGURE 56. EQUIVALENCE OF REPEATED TESTS TO DIFFERENT LEVELS (DESIGN FACTOR 1.5)

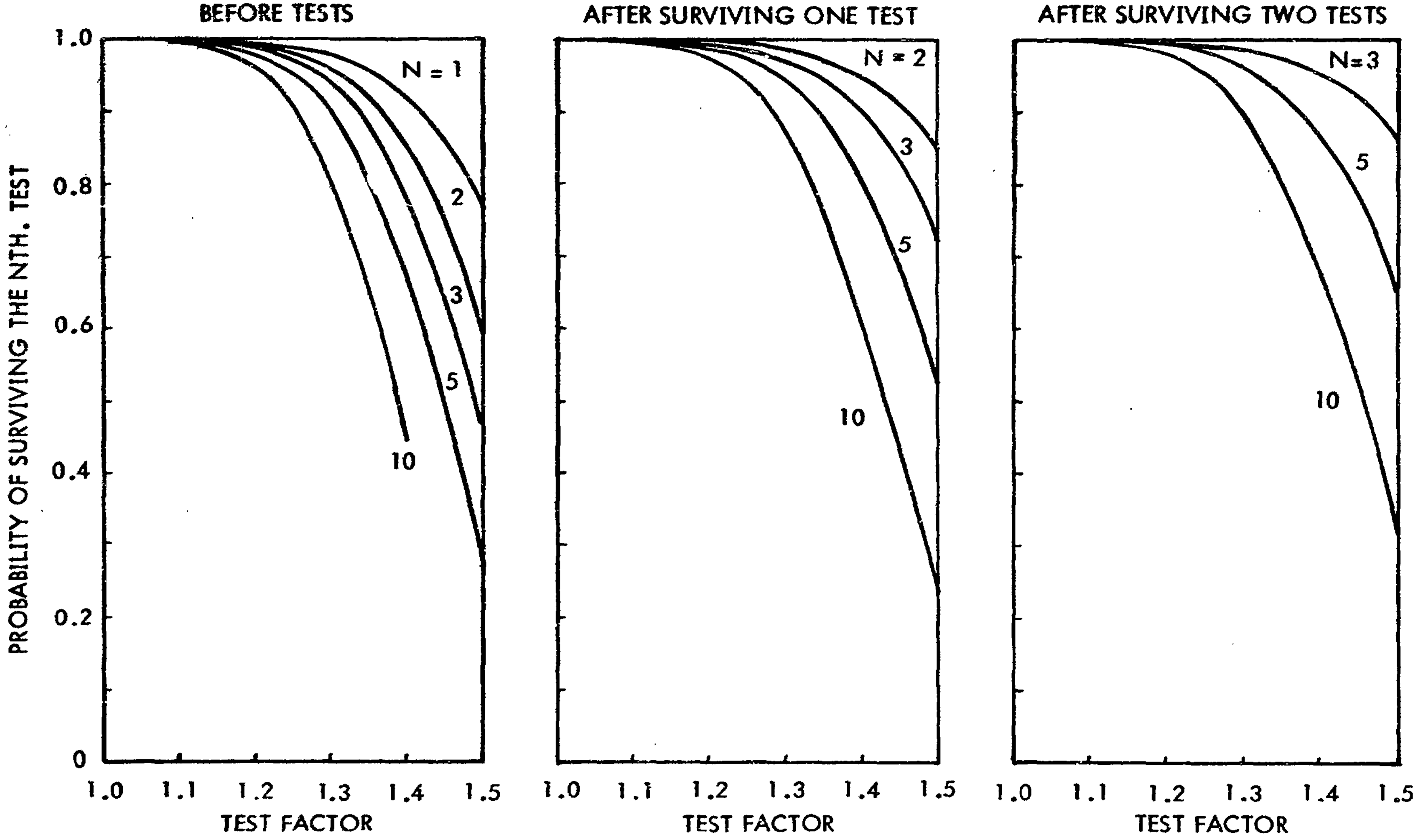


FIGURE 57. PROBABILITY OF SURVIVING REPEATED TESTS TO DIFFERENT LEVELS
(DESIGN FACTOR = 1.5)

9.6 Indicated Mean Strength

a. Some explanation of the meaning of the "demonstrated" or "indicated" mean strength is required. It is assumed in the analysis that the strength scatter about the mean is known, but that the location of the mean is not known. The form of the "error function" provides a means whereby the probable distribution of the actual mean strength may be defined. Interpretation of the test results by the employment of Bayes' theorem takes the form of modifying this assumed distribution of probable mean strengths. It must be emphasized that at no time will the actual value of the mean strength be determinate.

b. At each stage of the analysis (no error, with error but before tests, after tests), the implied probability distribution of mean strength can be derived. With the assumed scatter about the mean, it is then possible to derive the implied total distribution of individual strength (see figure 7). The mean of this resultant distribution is the quantity referred to as the "fleet mean strength." It cannot be regarded as the actual mean strength, but can be taken as an indication of the most likely value of the mean, if all of the other assumptions are valid. The scatter about this mean is equally important and changes from step to step as additional data becomes available (see figure 54(a) for example).

SECTION X
TRIAL APPLICATION TO THE C-141 CARGO TRANSPORT

10.1 Introduction

a. In order to demonstrate the procedures, interfaces, and decisions involved in the structural reliability analysis, a trial application of the techniques and data presented in this report is performed for the C-141 cargo transport aircraft.

b. The C-141 (Figure 58) is a land-based, heavy logistic cargo transport designed to airlift various types of combat support equipment, supplies, personnel, and air-evac patients. The C-141 fleet consists of 281 aircraft which have flown a total of three million flight hours without the loss of a single aircraft due to either understrength structure or overload.

Table XV presents a summary of the C-141 structural criteria.

c. The C-141 fleet is now undergoing the Individual Service Life Monitoring Program (IASLMP), which is a portion of the Aircraft Structural Integrity Program (ASIP). Under IASLMP, the utilization of mission types and such parameters as cargo weight, fuel weight, Mach number, and altitude are recorded for each aircraft. Therefore, the actual utilization of individual aircraft or that for the hypothetical fleet average aircraft can be determined. Under the life history recording program of ASIP, 26,741 hours of velocity-load factor - altitude (VGH) data have been collected. Such data are used to determine maneuver load factor spectra and atmospheric turbulence parameters which are representative of the environment in which the C-141 is flown.

d. Since the C-141 fleet has demonstrated that the structural reliability resulting from the criteria to which it was designed is more than adequate, it is of interest to determine what the structural reliability actually is. Also, it is of interest to determine what the design loads for the C-141 would be using the structural reliability technique presented in this report. It is therefore the intent of this section to perform such analyses using the C-141 utilization and VGH data.

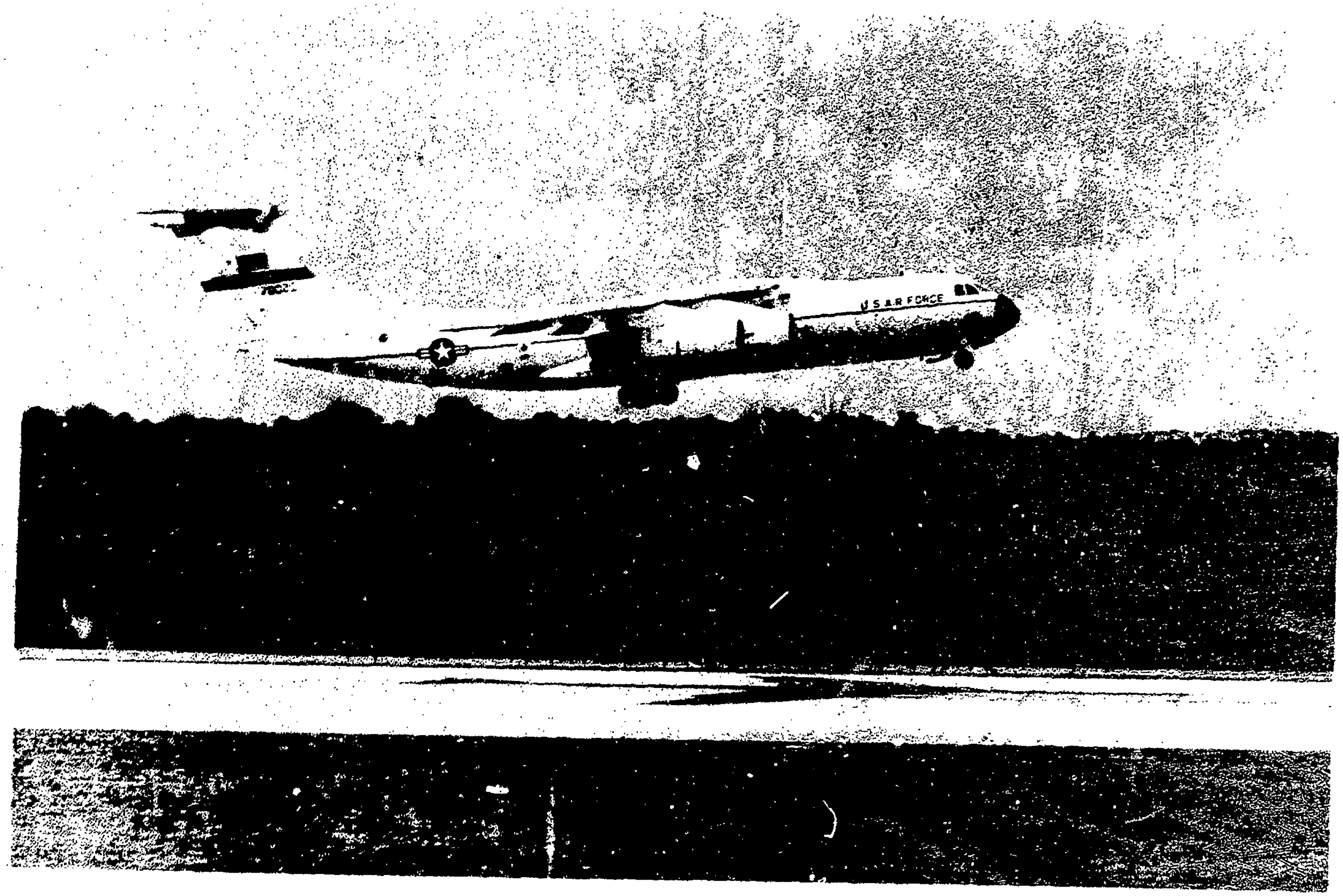


FIGURE 58 C-141A STARLIFTER CARGO TRANSPORT

TABLE XV

C-141 OPERATIONAL CRITERIA

DESIGN WEIGHTS

Condition	Weight (Lb.)
Maximum Flight Gross Weight	316,100 (Original) 323,100 (Updated)
Maximum Cargo Weight	72,131
Maximum Fuel Weight for Flight	151,452
Maximum Zero Fuel Weight	204,670
Maximum Landing Weight (6 ft/sec sink speed)	316,100 (Original) 323,100 (Updated)
Normal Landing Weight (10 ft/sec sink speed)	257,500

DESIGN SPEEDS

Condition	Speed		
Limit	410 KCAS to	21,000 Ft. then	M = 0.89
Maximum Level Flight	350	25,000	0.225
Rough Air Penetration Speed	270	36,800	0.225
Spoiler Placard	350	19,800	0.750
T.O. Flap Placard	200	24,200	0.420
Landing Flap Placard	185	24,700	0.450

DESIGN MANEUVER VERTICAL LOAD FACTORS

Case	Configuration	Load Factor
Positive Symmetrical Maneuver	Clean and Spoiler	2.5
	Flap	2.0

10.2 Calculation of Structural Reliability for C-141

a. A comprehensive calculation of the structural reliability of the C-141 is beyond the scope of this report due to the sheer magnitude of the amount of analysis required. Also, such a comprehensive analysis is not necessary as the structural reliability can be calculated for selected loads sources at one structural location. As such, the results reflect only the structural reliabilities for the structural location and load sources selected, which are sufficient for demonstration purposes.

b. Wing Station 135, which is located about one-third of the distance between the wing root and the inboard pylon, is selected for the structural reliability analysis. This station was used as a loads control station for the inboard wing during the original design loads analysis. As such, the original design load conditions for the inboard wing were determined by performing a loads envelope analysis for selected load components at wing station 135.

Positive vertical maneuver is the source of six different design conditions for positive vertical bending moment - torsion requirements at wing station 135. Therefore, positive vertical maneuver is selected as one of the loads sources for the structural reliability analysis. Positive, discrete gust did not cause any design load conditions for the C-141 wing. However, the gust loads were of significant magnitude. In view of this fact, and the fact that power-spectral gust analysis has been recommended for use in the structural reliability analysis, positive vertical gust is also selected as a loads source for the structural reliability analysis.

c. Approximately 764,000 flight hours of C-141 IASLMP usage data is used to establish the utilization of the fleet average C-141 aircraft. The usage data is broken down by mission type and a grid of 2268 fuel, cargo, Mach number and altitude data block combinations which represent the operational regimes of the C-141. By using all of the significant data block usage data, the statistical scatter inherent in the data is retained, rather than just the mean values. There are thirteen different missions into which the usage data are classified. These thirteen missions can be broken down into three distinct groups; logistics, training, and airdrop. Table XVI gives a summary of the

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TABLE XVI

C-141 IASLMP MISSION UTILIZATION

MISSION TYPE	AVERAGE CARGO WEIGHT (LB.)*	% FLIGHT HOUR UTILIZATION
Logistics	36,660	83.9
Training	4,900	15.7
Airdrop	14,400	0,4

*Design Cargo Weight = 72,131 Lbs.

C-141 mission utilization and average cargo weights. The airdrop missions represent only 0.4% of the total flight hour utilization and therefore will be considered as logistics missions for this analysis. Thus, only two separate types of missions, logistics and training, are required to adequately represent the utilization of the C-141.

d. Maneuver vertical load factor spectra were obtained by reducing the maneuver vertical load factor data for 13,264 flight hours of VGH data. Due to the limited range of the vertical load factor data, the data was fit by extreme-value double family distributions in order to allow extrapolation to larger vertical load factors. The vertical load factor data was reduced on an extreme-value basis by retaining only the maximum vertical load factor for constant time intervals. A ten hour time interval was used for the logistics missions, while a five hour time interval was used for the shorter training missions. The resultant data then determine the probability that a load factor will occur as a maximum during the given time interval, not just the probability that it will occur.

The maneuver load factor spectra vary significantly between mission types and mission segment. Therefore, separate spectra were determined for logistics and training missions cruise and non-cruise segments. The resulting maneuver load factor spectra are presented in Figures 59 and 60 for logistics and training missions respectively.

e. Maneuver vertical bending moment-torsion loads spectra are then calculated using the mission data block utilization, maneuver load factor spectra and mean and incremental maneuver loads data. As discussed in Section V, aircraft limitations such as aerodynamic stall and control limits should be included in a statistical loads analysis. The C-141 maneuver capability is primarily limited by aerodynamic stall rather than control limits. Therefore, the maneuver load spectra for each discrete flight condition is truncated when the stall lift coefficient is obtained.

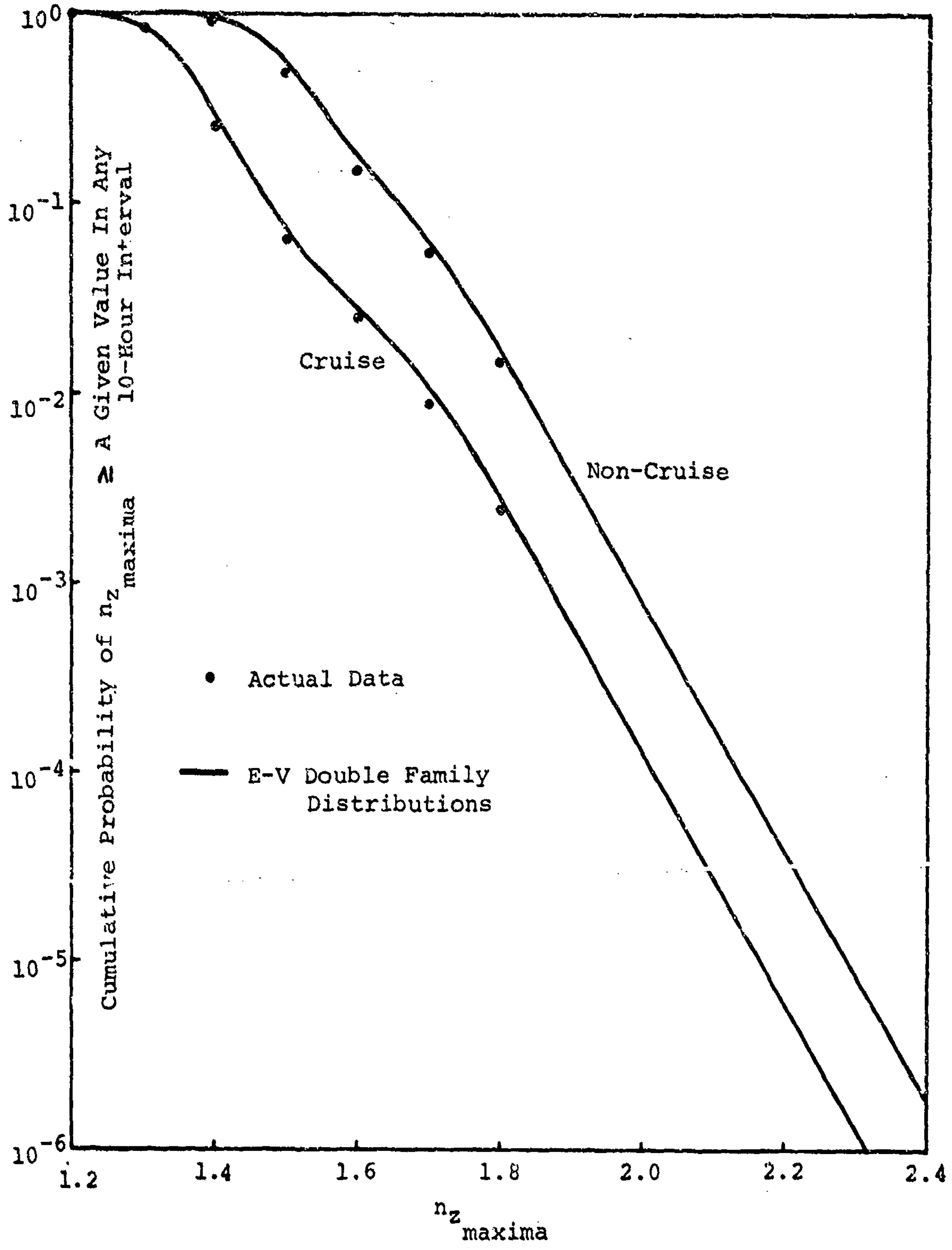


FIGURE 59 C-141 LOGISTICS MISSIONS MANEUVER SPECTRA

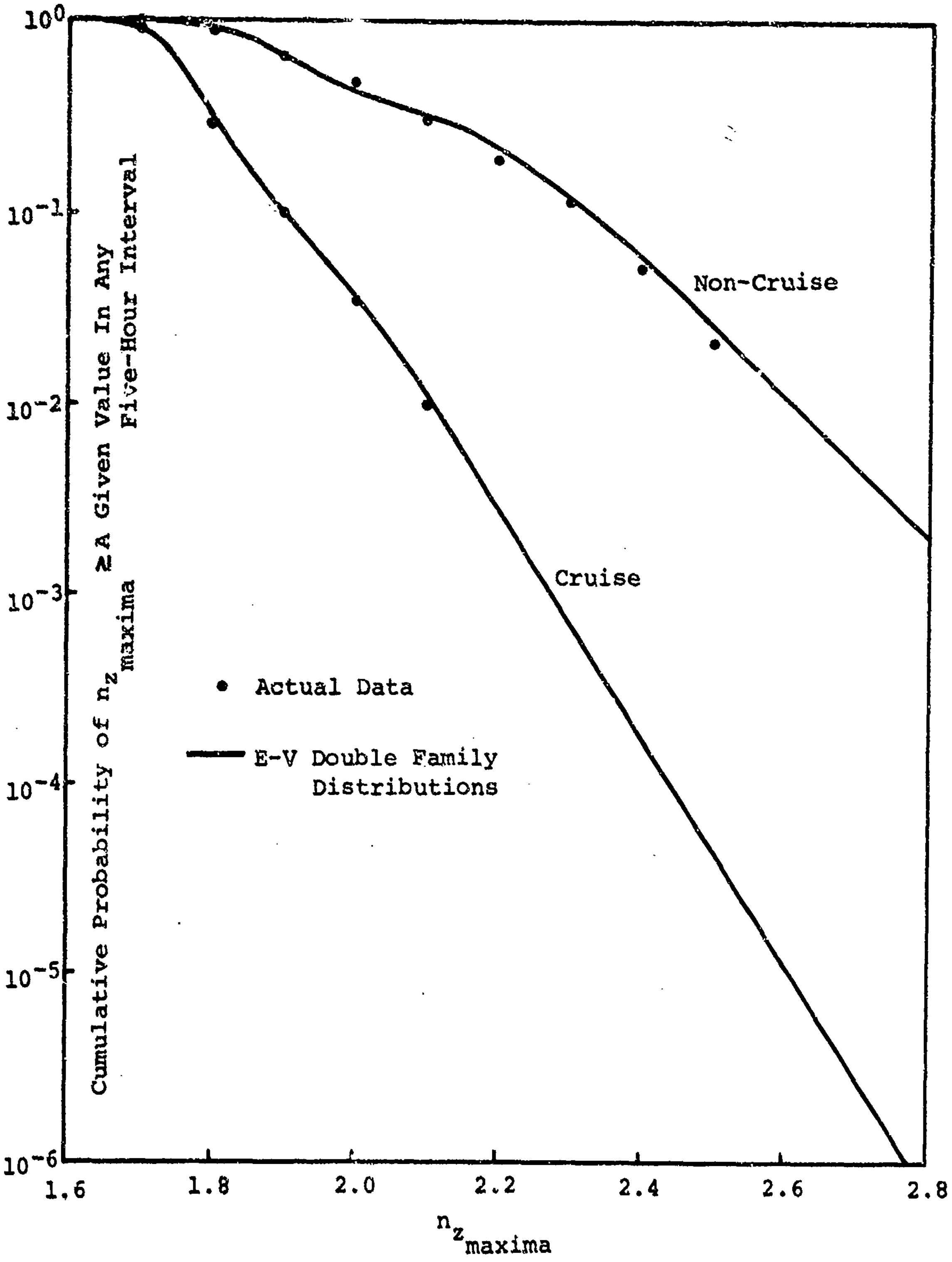


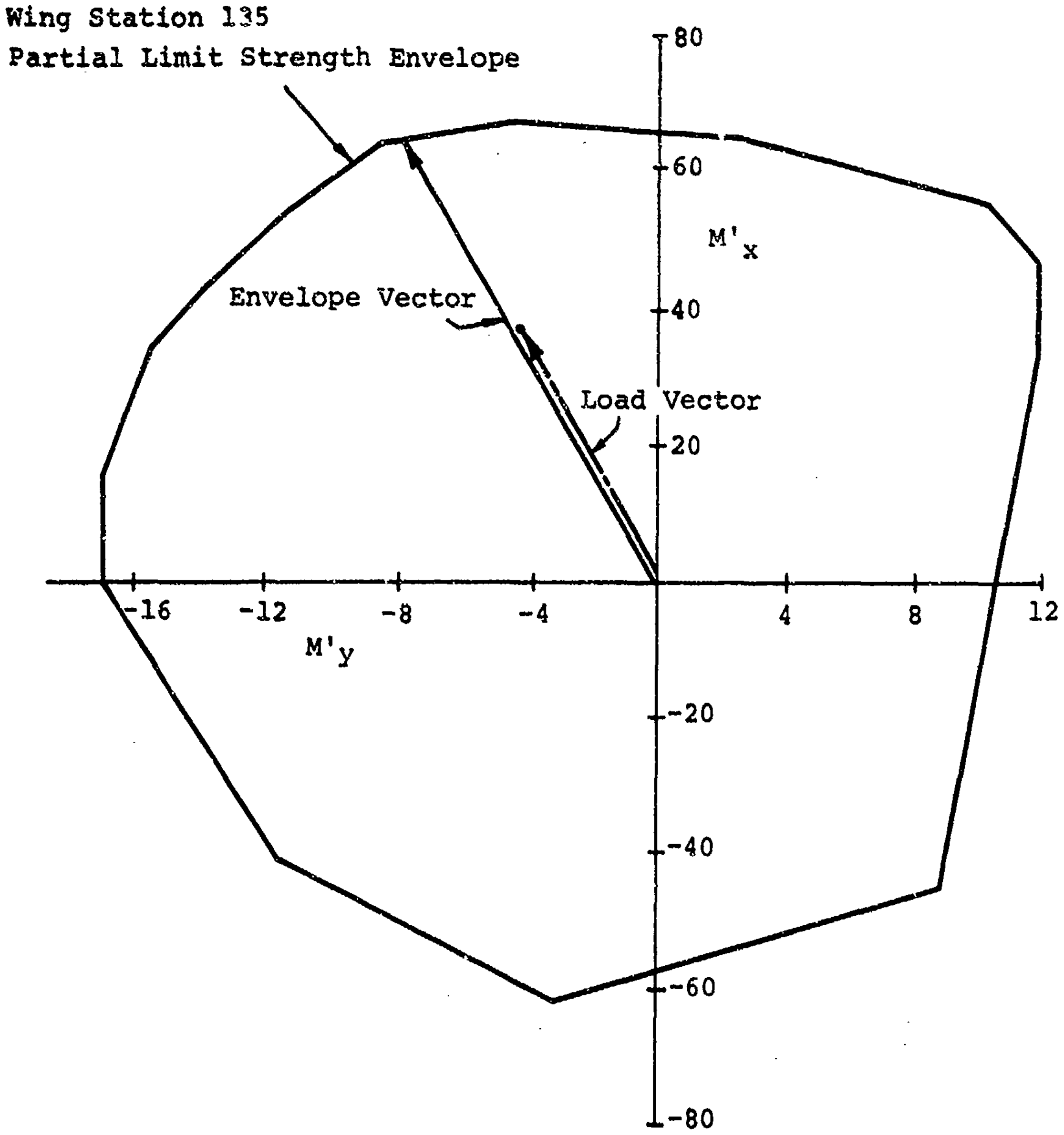
FIGURE 60 C-141 TRAINING MISSIONS MANEUVER SPECTRA

As discussed in Section V, the loads spectra must be presented in a form which is compatible with structural strength. For C-141 vertical maneuver conditions, vertical bending moment and torsion are the most significant wing load components. Therefore, the vertical bending moment-torsion loads spectra are converted to percent of partial limit vertical bending moment-torsion strength as shown by Figure 61. The envelope is called a partial limit strength envelope because the envelope is formed from limit conditions having zero margins of safety due to the combined six components of load. The use of the partial limit strength vertical bending indirect torsion envelope thus places qualifying assumptions on the other four load components, but it is felt to be a satisfactory approximation. Thermodynamic and internal pressurization effects are not significant for the C-141 wing and therefore are not included here. The resulting spectra are presented by Figure 62 for the C-141 30,000 flight hour design lifetime.

f. Using 16,430 hours of C-141 VGH gust vertical load factor data, gust environmental parameters have previously been derived. The procedure involves the generation of generalized peak load factor spectra and curve fitting to obtain the gust environmental parameters.

The aircraft structure is defined by three rigid body modes and 15 symmetrical modes of flexible vibration. The aerodynamic representation includes such effects as variation of the lift curve slope with Mach number, downwash on the horizontal stabilizer and Kussner and Wagner lift growth functions. The von Karman power-spectral equation and a varying scale of turbulence, L, are used to define the R.M.S. load response to a unit R.M.S. gust velocity, A, and the characteristic frequency of response, N_o. The von Karman power-spectral equation is shown below.

$$\frac{\Phi(\Omega, L)}{\sigma_w^2} = \frac{L(1 + 4.781\Omega^2 L^2)}{\pi(1 + 1.793\Omega^2 L^2)^{11/6}} \qquad 9$$



$$\% \text{ Limit Strength} = \frac{\text{Magnitude of Load Vector}}{\text{Magnitude of Envelope Vector}} \times 100$$

FIGURE 61 CONVERSION OF MANEUVER LOAD TO PERCENT LIMIT STRENGTH C-141

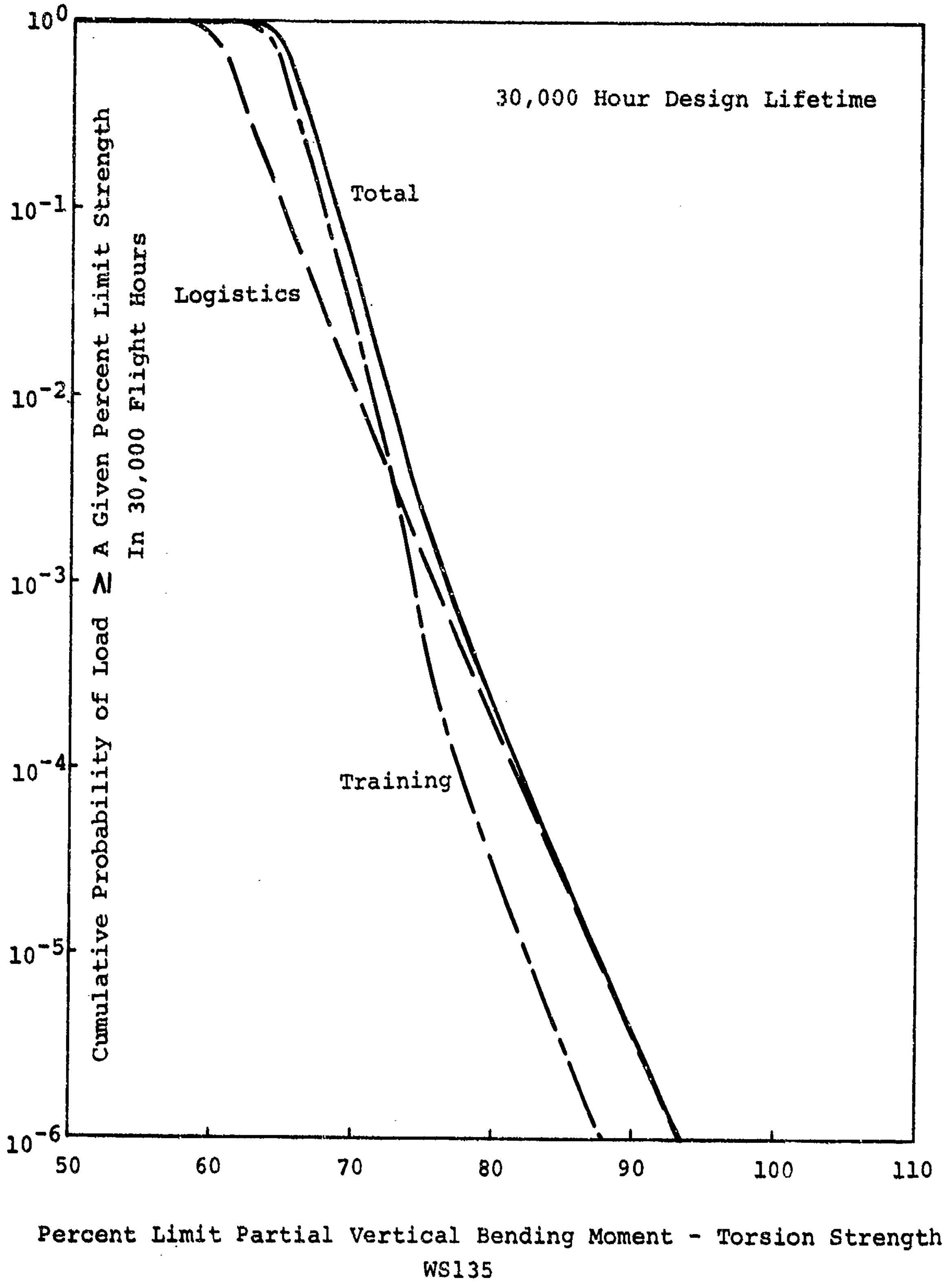


FIGURE 62 C-141 MANEUVER PERCENT PARTIAL LIMIT STRENGTH SPECTRA

Where:	$\Phi(\Omega, L)$	is the power spectral density input function
	σ_w	is the R.M.S. gust velocity (ft/sec)
	L	is the scale of turbulence (ft)
	Ω	is the reduced frequency (rad/ft)

Table XVII presents the scale of turbulence and the other turbulence parameters as derived for the C-141.

g. Peak gust load spectra are then determined for the C-141 utilization by use of the generalized exceedance equation separately for each data block.

$$N_p(Y) = N_{o_y} T \left[P_1 \exp\left(- \frac{Y - \bar{Y}}{b_1 \bar{A}_y} \right) + P_2 \exp\left(- \frac{Y - \bar{Y}}{b_2 \bar{A}_y} \right) \right] \qquad 10$$

Where:

N_p is the cumulative number of occurrences of load greater than or equal to Y

N_{oy} is the characteristic frequency of response for load Y (CPS)

Y is the total load

$\bar{Y}$ is the mean 1.0g flight load

T is the flight time in seconds

$\bar{A}_y$ is the R.M.S. load response to an R.M.S. gust velocity of one ft/sec

P_1, P_2, b_1, and b_2 are as defined by Table XVII

h. The spectra for individual load components are obtained independently by use of the generalized exceedence equation. Therefore, in order to determine a given state of loading, the phasing of the load components must be determined. Reference 13 presents two methods for determining load component phasing; both of which

TABLE XVII

VERTICAL GUST TURBULENCE PARAMETERS

ALTITUDE (1000 FT.)	P_1	P_2	b_1	b_2	L
0-1	.95	.0045	2.8	6.0	500
1-2	.47	.0034	3.1	6.2	1600
2-5	.27	.0021	3.2	6.5	1650
5-10	.13	.001	3.2	7.9	1860
10-20	.057	.0004	3.2	7.9	2250
20-30	.039	.0002	3.2	8.3	3250
30-40	.031	.00013	3.2	8.0	4250
>40	.027	.0001	3.2	7.2	5350

P_1	is the percent of time spent in non-storm turbulence.
P_2	is the percent of time spent in storm turbulence.
b_1	is the composite R.M.S. gust velocity for non-storm turbulence (ft/sec).
b_2	is the composite R.M.S. gust velocity for storm turbulence (ft/sec).
L	is the scale of turbulence (ft)

involve a significant amount of analysis. Such methods are not applied here as it is felt that for the C-141 wing station selected, a satisfactory representation of the gust load spectra in terms of percent limit strength can be obtained by the use of vertical bending moment alone. This is demonstrated by referring to Figure 63 which shows that over a range of torsion from -10 to +7 million in-lbs, the allowable limit bending moment varies from a maximum of 67 to minimum of 60 million in-lbs, a variation of only 10%. Therefore, an approximation of the percent of limit strength for a given bending moment can be obtained by taking the 100% limit strength bending moment as the reduced value of 60 million in-lbs. Techniques such as this would have to be used in preliminary design analyses where such simplifications are a necessity.

The resulting spectra are presented by Figure 64 for the C-141 30,000 hour design lifetime.

i. In order to perform the structural reliability analysis following the determination of the loads spectra, the strength scatter due to fabrication and material variations and an error function must be determined. Section VI presents an extreme-value double family fit of the strength scatter for the primary material used for the C-141 wing, 7075-T6. Section VI also presents an extreme-value double family fit of the strength scatter due to the fabrication of riveted joints. Section VII presents error functions as derived from C-141 static test data. The material and fabrication data from Section VI and the error function for the original C-141 configuration are selected for the structural reliability calculations. These data together with the loads spectra are input to the modified structural reliability program. The results are as follows for one test surviving 150% of limit load (ultimate).

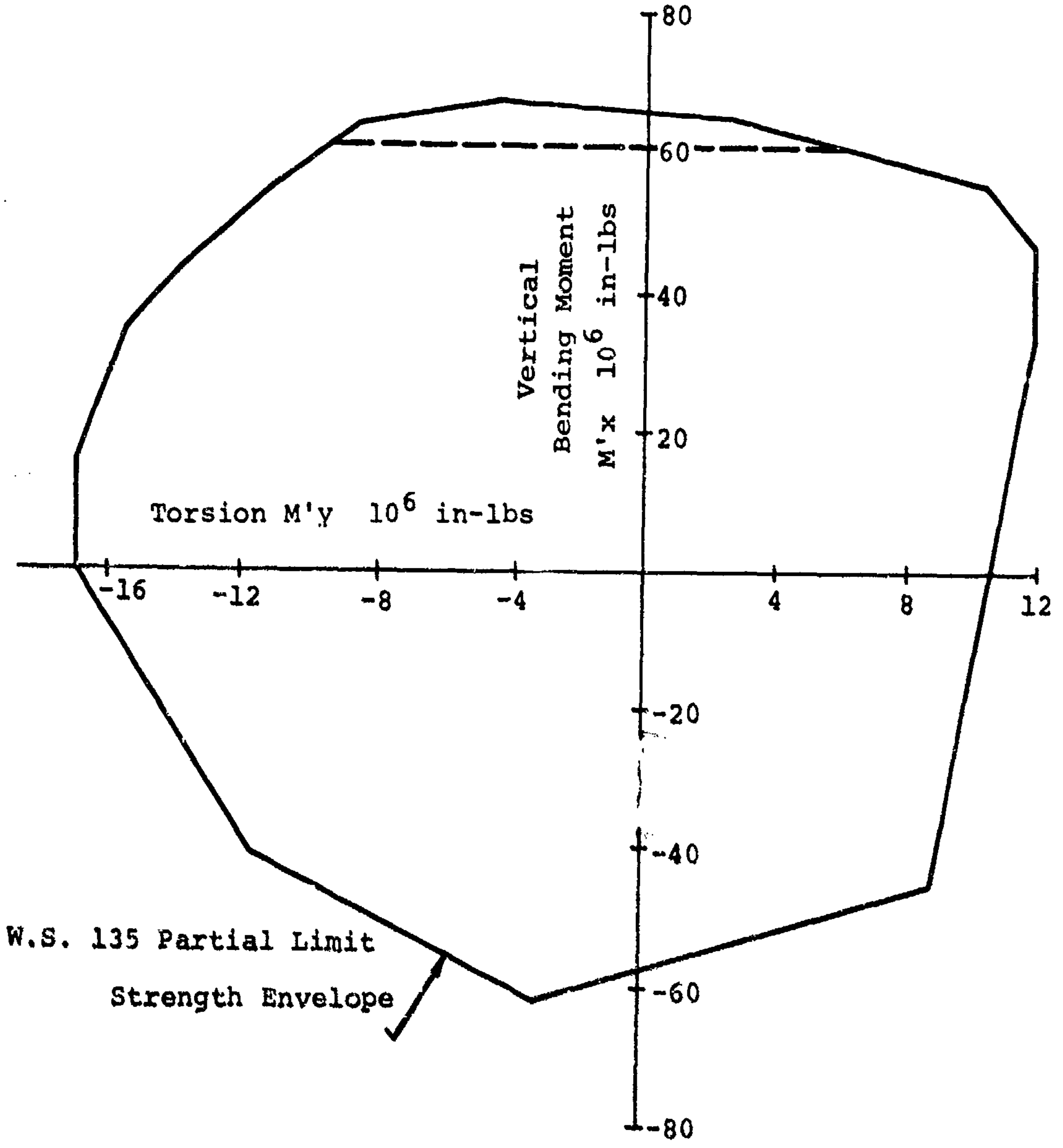


FIGURE 63 VARIATION OF LIMIT VERTICAL BENDING MOMENT WITH TORSION

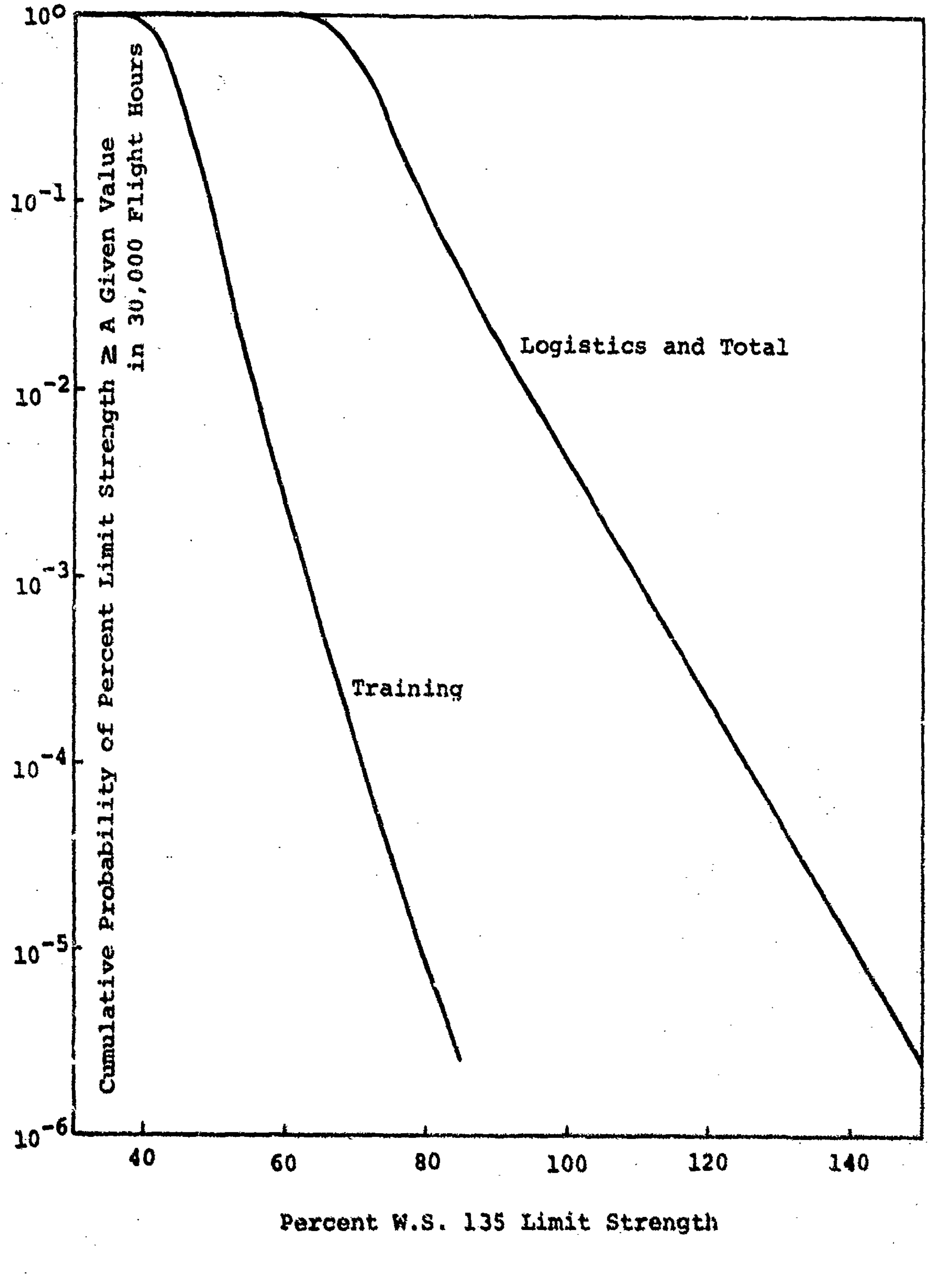


FIGURE 64 C-141 POWER SPECTRAL GUST PERCENT LIMIT STRENGTH SPECTRA

LOADS SOURCE	W.S. 135 STRUCTURAL RELIABILITY
Positive Vertical Maneuver	.999999993
Positive Vertical Gust	.999994

j. As recommended in Section VIII, the structural reliability goal for cargo transport aircraft is .999. Therefore, for the wing station and loads sources considered, the C-141 has structural reliability far in excess of the recommended goal.

k. The comparison of the relative structural reliabilities between the two loads sources is significant. The original C-141 design loads analysis for 2.5g maneuver and discrete gust showed that the aircraft was not gust critical as the maximum gust wing loads did not exceed 80% of the maximum maneuver loads. However, on a statistical basis, just the opposite is true as the wing has a much lower reliability for gust than for maneuver. The merit of the structural reliability analysis is therefore evident as it identifies the strength requirements for individual load sources based upon a common structural reliability goal.

10.3 C-141 Wing Load Requirements for Structural Reliability Goal

a. Since it has been demonstrated that for two loads sources, positive vertical maneuver and positive vertical gust, the structural reliability for a selected C-141 wing station is far in excess of the structural reliability goal, it is of interest to determine what the design load requirements would be in order to just obtain the structural reliability goal.

b. Determination of Limits for User Controlled Parameters

1) As discussed in Section V, limits on the user controlled parameters must be defined such that the areas of normal, overload, and gross overload operation can easily be determined by the user. Positive vertical maneuver is a loads source for which the resultant loads are completely determined by user controlled parameters. As such the limits for the user controlled parameters which are pertinent to positive vertical maneuver will be determined.

2) Figure 65 presents maneuver load factor cumulative occurrence spectra for logistics and training missions representing a 30,000 flight hour design lifetime. The original design vertical maneuver load factor of 2.5 is shown to have a probability of exceedance of approximately 10^{-4} during logistics missions and is equaled or exceeded twelve times during training missions.

At a given vertical maneuver load factor, cargo weight, as shown by Figure 66, has the most significant effect on wing loads. Figure 67 presents cumulative probability spectra for cargo weight utilization during logistics and training missions. The spectrum for logistics missions has been conservatively extrapolated to 120% of the design cargo weight of 72,131 lbs.

3) Since cargo weight and vertical load factor are the most significant parameters for C-141 maneuver wing loads, the magnitude of the maximum wing loads for the three operational regimes can be effectively defined by limiting the maneuver vertical load factor for any given cargo weight. Figure 68 presents curves of vertical load factor versus cargo weight for the limit and omega probabilities of exceedance as recommended in Section VIII. The increase in slope at the lower cargo weights is due to the training missions which have large maneuver vertical load factor, but low cargo weights. The C-141 usage data indicates that an omega cargo weight of 86,500 lbs., 120% of the design maximum cargo, provides sufficient margin for cargo overload.

4) Figure 69 presents the limit and omega cargo-fuel envelopes. The limit envelope is defined by the design cargo weight of 72,131 lbs., the updated maximum flight gross weight of 323,100 lbs., and the design maximum flight fuel weight of 151, 452 lbs. As previously discussed, the maximum cargo weight for omega operations is taken to be 86,500 lbs. The C-141 usage data indicates that an omega gross-weight of 343,100 lbs. provides sufficient margin for omega operations. The C-141 usage data has revealed no instances where the design maximum fuel weight was exceeded. Therefore, no separate omega fuel weight is considered.

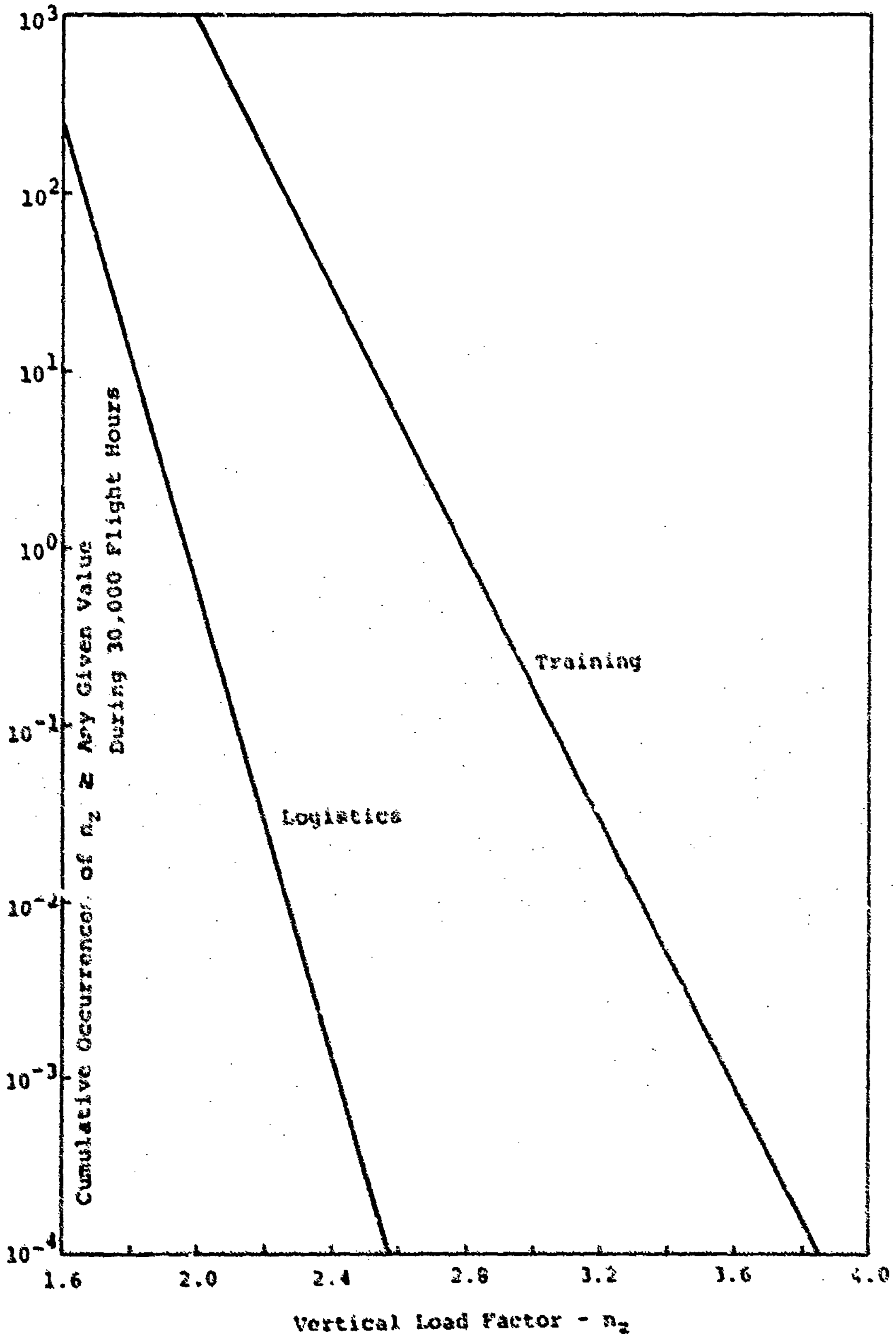


FIGURE 65 C-141 POSITIVE VERTICAL MANEUVER LOAD FACTOR SPECTRA

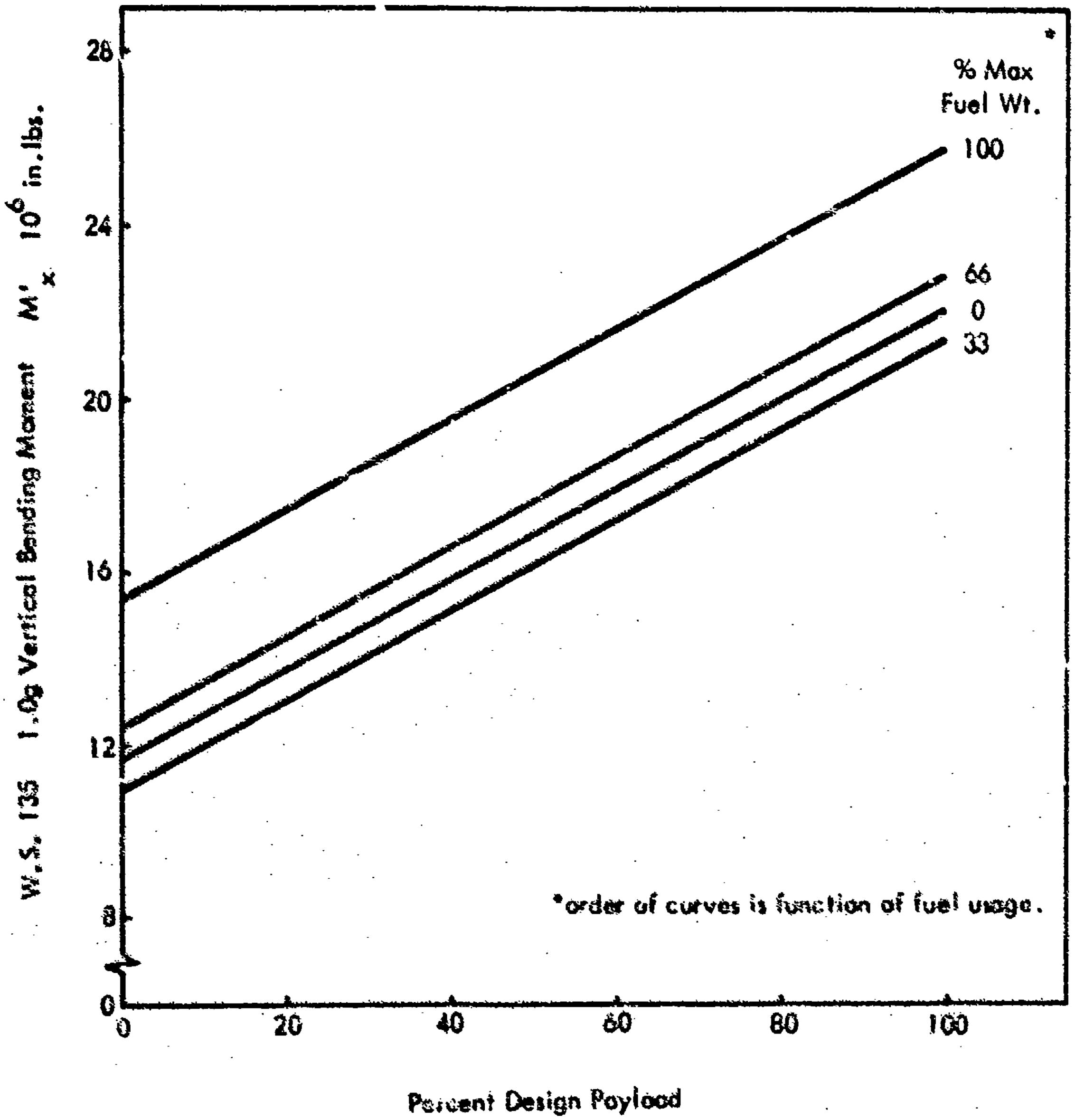


FIGURE 66 TYPICAL EFFECTS OF FUEL AND CARGO ON WING LOADS C-141

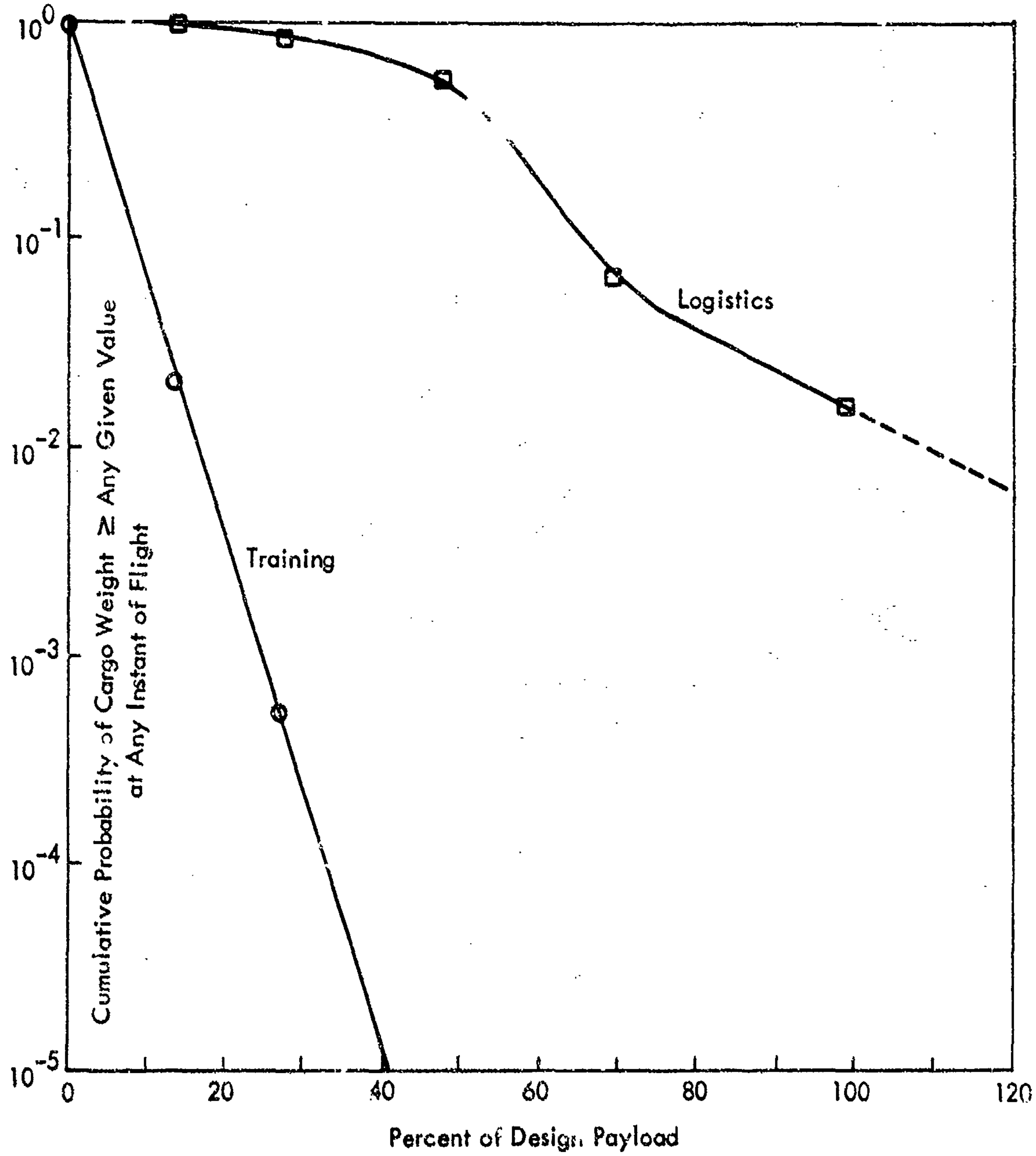


FIGURE 67 C-141 PAYLOAD PROBABILITY

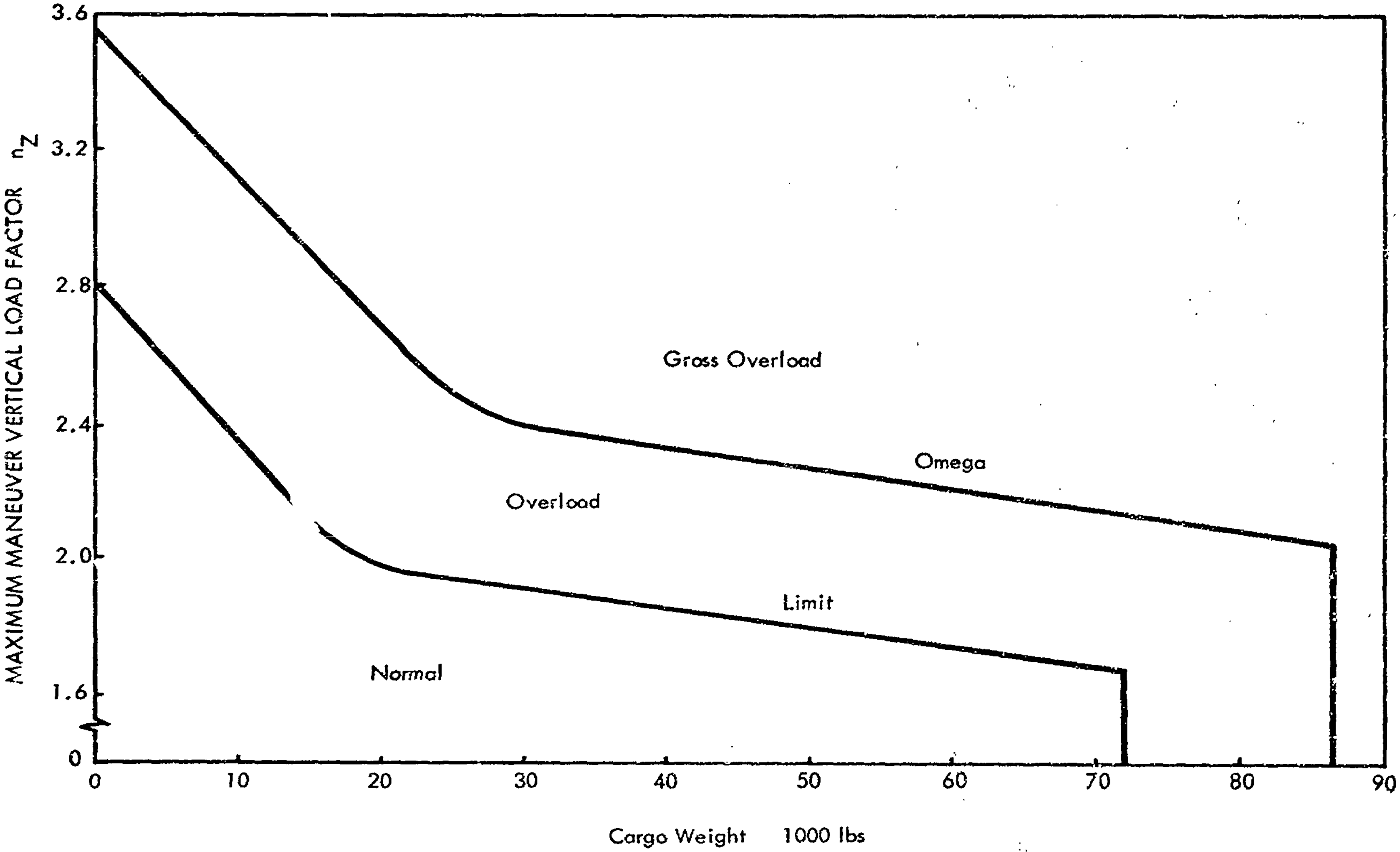


FIGURE 68 LIMIT AND OMEGA MANEUVER VERTICAL LOAD FACTOR vs CARGO WEIGHT
C-141

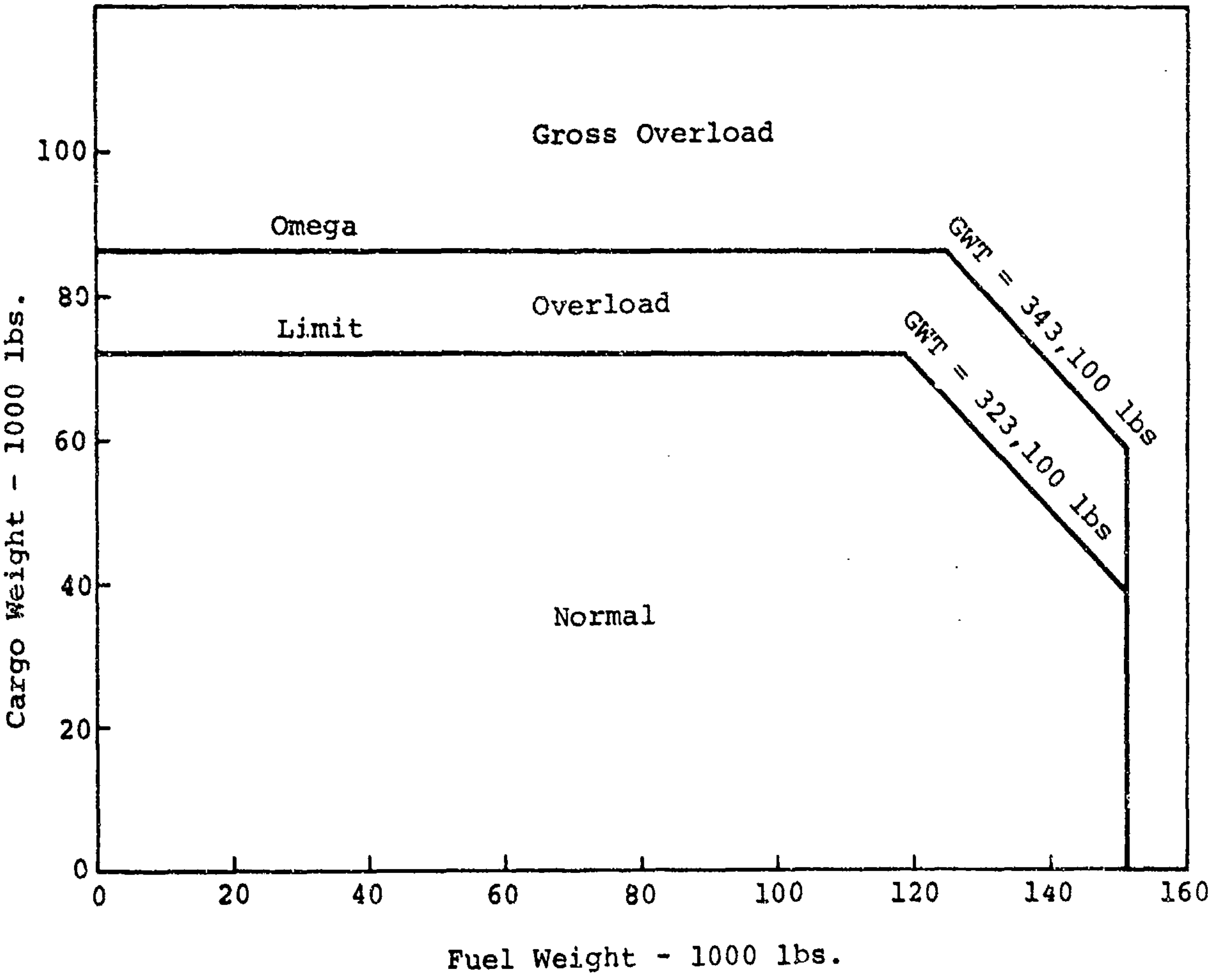


FIGURE 69 C-141 LIMIT AND OMEGA CARGO - FUEL ENVELOPES

5) Figure 70 presents the limit and omega airspeed-altitude combinations for the clean configuration. The C-141 usage data indicates that the maximum level flight speed, V_H, is a sufficient definition of the limit airspeed-altitude combination, while dive speed, V_L, provides sufficient margin for overload operations. The C-141 usage data does not indicate any exceedences of the spoiler and flap placards. Therefore, the limit airspeed-altitude combinations for the spoiler and flap configurations are taken as their respective placard values, and no separate omega values are defined.

6) No exceedences of the design center of gravity envelope have been recorded by the C-141 usage data. Center of gravity has only secondary effects on the C-141 wing loads. Hence, the design center of gravity envelope is taken as the limit envelope, and no omega envelope is defined.

c. Determination of Maneuver Limit and Omega Conditions

1) As discussed in Section V, maneuver limit and omega conditions can be determined by two different methods. The two methods are repeated here for emphasis. The first method is purely probabilistic in that limit and omega conditions are those which produce the highest load levels for the respective limit and omega probabilities of occurrence. Such an approach works well in a single parameter load environment, however in a multiple parameter environment such an approach does not adequately define all combinations of the user controlled parameters which are within the limit and omega conditions.

2) The second method is deterministic in that the limit and omega conditions are defined as those which provide the maximum loads for any combination of user controlled parameters within the respective limit and omega values. Such an approach has the additional advantage of being consistent with present design procedures. It must be noted that the usage of the aircraft is not neglected for the second method, as the limits on the user controlled parameters are determined based upon the usage data. Also, the design factors which are applied to the loads

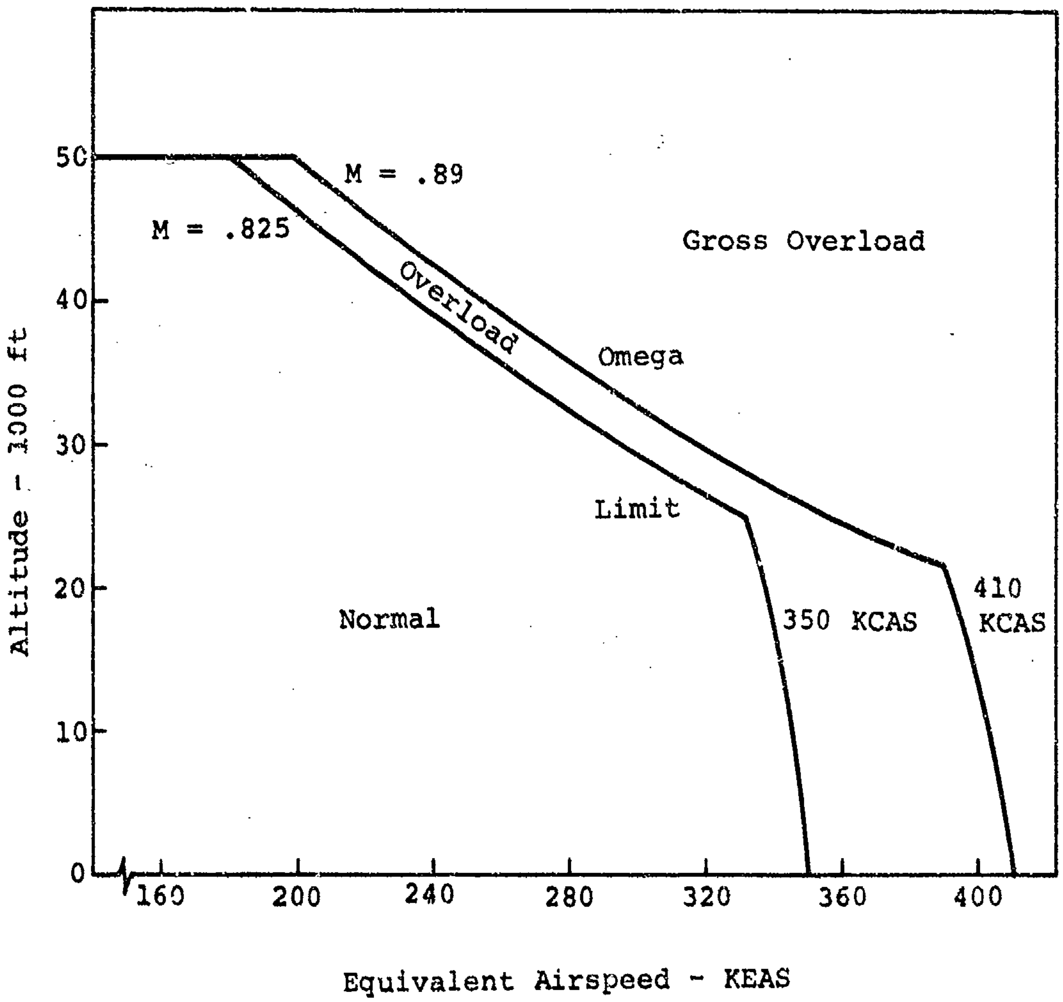


FIGURE 70 C-141 AIRSPEED ALTITUDE ENVELOPES FOR CLEAN CONFIGURATION

for the limit and omega conditions are based upon the loads probability spectra.

3) The deterministic method of obtaining the limit and omega maneuver conditions is therefore used here. The limit and omega conditions for wing station 135 are determined by developing load trend data for the various parameters, and then performing a vertical bending moment torsion envelope analysis. The limit and omega conditions are then those conditions which define the loads envelopes. The resulting load envelopes are presented by Figure 71 and the limit and omega conditions are as shown in Table XVIII.

The limit and omega maneuver loads are presented by Figure 72 as percentages of the respective limit and omega load envelopes.

d. Determination of Positive Gust Limit and Omega Conditions

1) As discussed in Section V, positive gust is a probabilistic loads source which is not directly controlled by the user. That is, the user controlled parameters do not determine the maximum gust loads that can be obtained. Rather, maximum gust loads can be determined only on a probability of occurrence basis.

Using the power-spectral gust equations, methods and turbulence parameters, as previously presented in this section, limit and omega conditions are determined for the C-141 usage data. The selection of the limit and omega conditions is based upon vertical bending moment only. The limit condition is that which provides the largest vertical bending moment, for the limit probability of occurrence of once per lifetime, while that for omega corresponds to the omega probability of exceedance of 10^{-3} times per lifetime. The limit and omega gust load spectra are presented by Figure 73, and the limit and omega conditions are as follows:

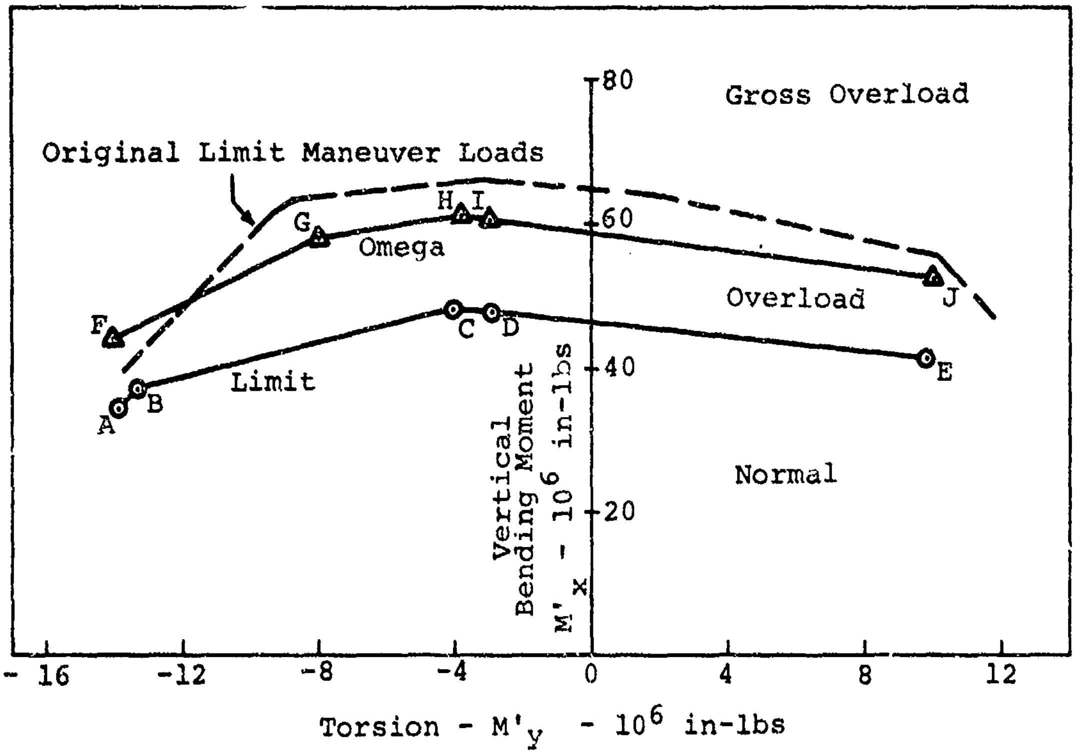


FIGURE 71 C-141 MANEUVER LIMIT AND OMEGA LOADS AND CONDITIONS W.S. 135

TABLE XVIII
MANEUVER CONDITIONS

(a) Limit

CONDITION	G.W. (LB.)	CARGO (LBS.)	FUEL (LB.)	MACH NO.	VELOCITY (KEAS)	CONFIGURATION	THRUST	N_Z
A	323,100	72,181	118,480	.303	200	T.O. Flaps	Flight Idle	1.63
B	283,941	0	151,452	.303	200	T.O. Flaps	Flight Idle	2.8
C	283,941	0	151,452	.825	333	Clean	Flight Idle	2.8
D	283,941	0	151,452	.825	333	Clean	Takeoff	2.8
E	283,941	0	151,452	.363	238	Spoilers	Takeoff	2.8

(b) Omega

CONDITION	G.W. (LB.)	CARGO (LBS.)	FUEL (LB.)	MACH NO.	VELOCITY (KEAS)	CONFIGURATION	THRUST	N_Z
F	343,100	86,500	124,111	.303	200	T.O. Flaps	Flight Idle	2.05
G	343,100	86,500	124,111	.825	378	Clean	Flight Idle	2.05
H	343,100	86,500	124,111	.825	296	Clean	Flight Idle	2.05
I	343,100	86,500	124,111	.825	296	Clean	Takeoff	2.05
J	283,941	0	151,452	.3	254	Spoilers	Takeoff	3.54

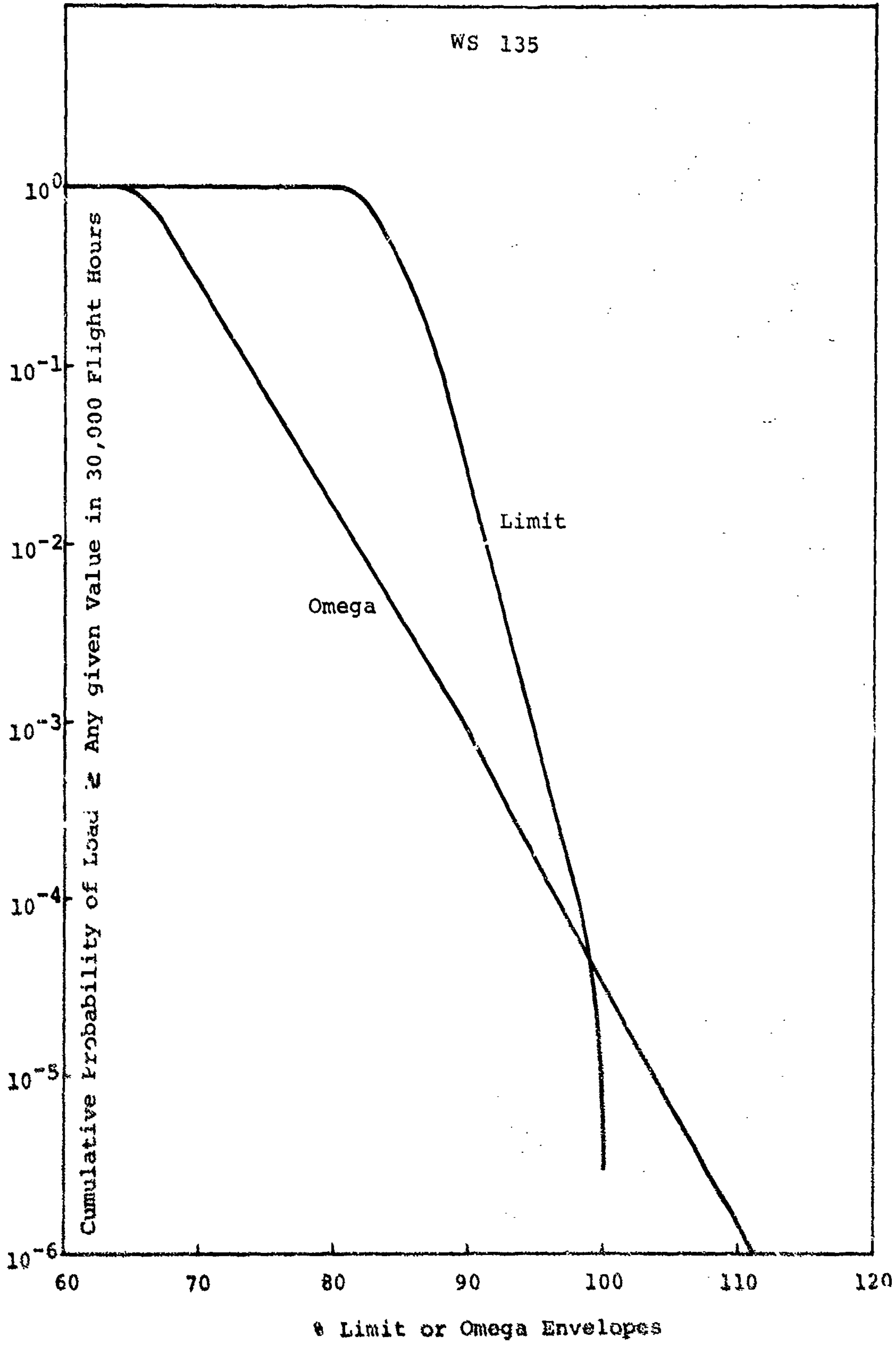
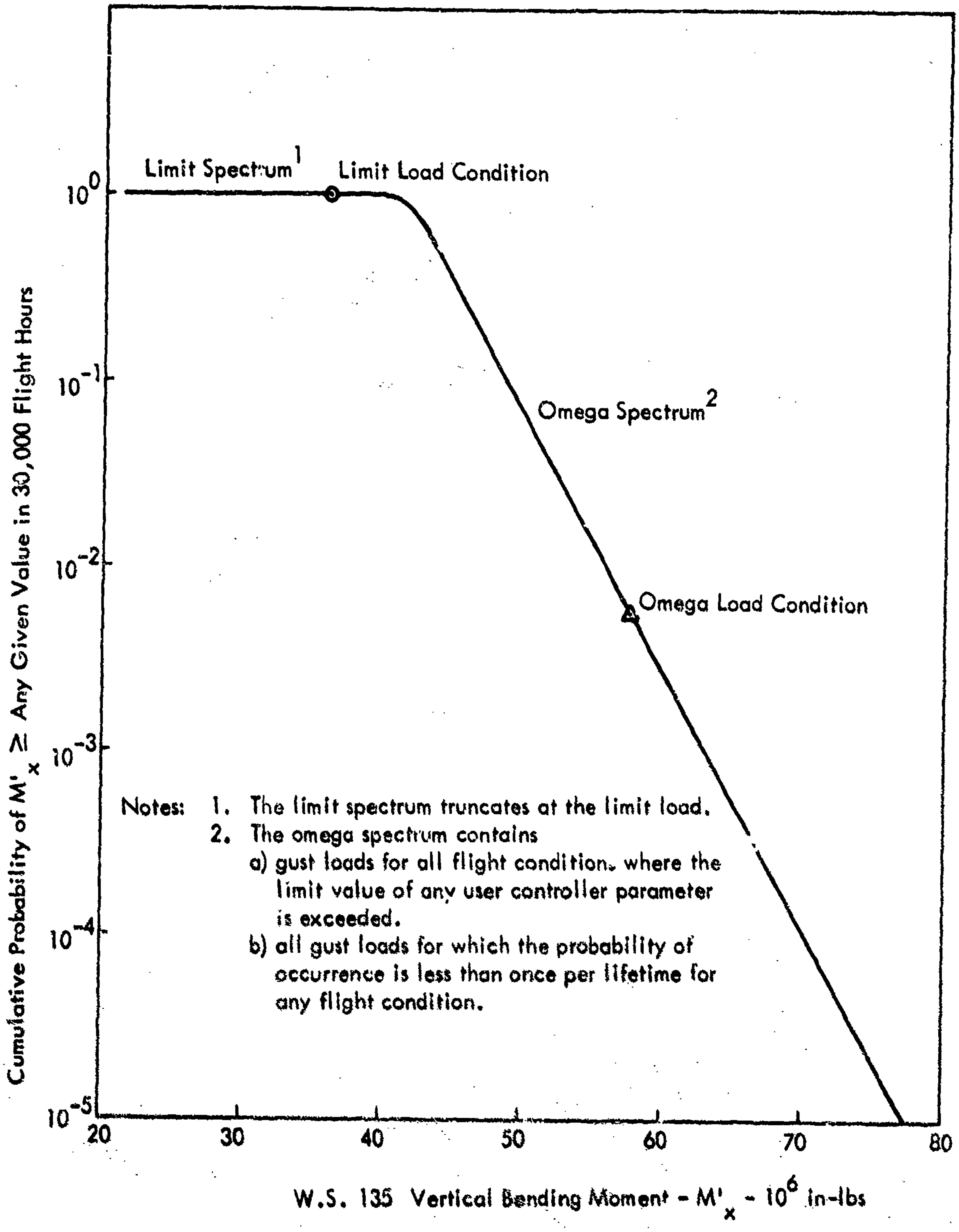


FIGURE 72 C-141 POSITIVE VERTICAL MANEUVER LIMIT AND OMEGA LOAD SPECTRA



Notes:
1. The limit spectrum truncates at the limit load.
2. The omega spectrum contains
   - a) gust loads for all flight conditions where the limit value of any user controller parameter is exceeded.
   - b) all gust loads for which the probability of occurrence is less than once per lifetime for any flight condition.

FIGURE 73 C-141 POSITIVE VERTICAL GUST LIMIT AND OMEGA LOAD SPECTRA

CONDITION	W.S 135 MX (10^6 in-lb)	G.W. (lb)	CARGO (lb)	FUEL (lb)	MACH	VEL. (KEAS)	R.M.S. GUST VEL. (ft/sec)
Limit	36	229,989	60,000	37,500	.49		28.4
Omega	57.6	229,989	60,000	37,500	.755	283	68.2

2) The maximum wing station 135 vertical bending moment for the C-141 original discrete gust design loads analysis is 57.3 million in-lb (limit).

e. Determination of Design Factor and Design Loads

1) The structural reliability analysis is performed separately for maneuver and gust using the limit and omega loads spectra, the C-141 material and fabrication strength scatter, and the C-141 original configuration with static test error function. The design factors required to obtain the structural reliability goals of .99999 for limit and .999 for omega are determined. Assuming one static test is survived with the test factor equal to the design factor, the following design factors are obtained:

LOADS SOURCE	COND.	DESIGN FACTOR
Positive Vertical Maneuver	Limit	1.29
Positive Vertical Maneuver	Omega	1.0
Positive Vertical Gust	Limit	1.51
Positive Vertical Gust	Omega	1.11

2) The factored design loads requirements for both loads sources are presented by Figure 74. The original ultimate design load requirements resulting from both loads sources are also shown for comparison. The omega gust vertical bending moment requirement slightly exceeds both of the limit and omega maneuver requirements. Also, the omega gust requirement exceeds the limit gust requirement by approximately 20%. However, the role is reversed for maneuver as the limit requirements slightly exceed those for omega. Thus, overload operations for maneuver are not as significant as those for gust.

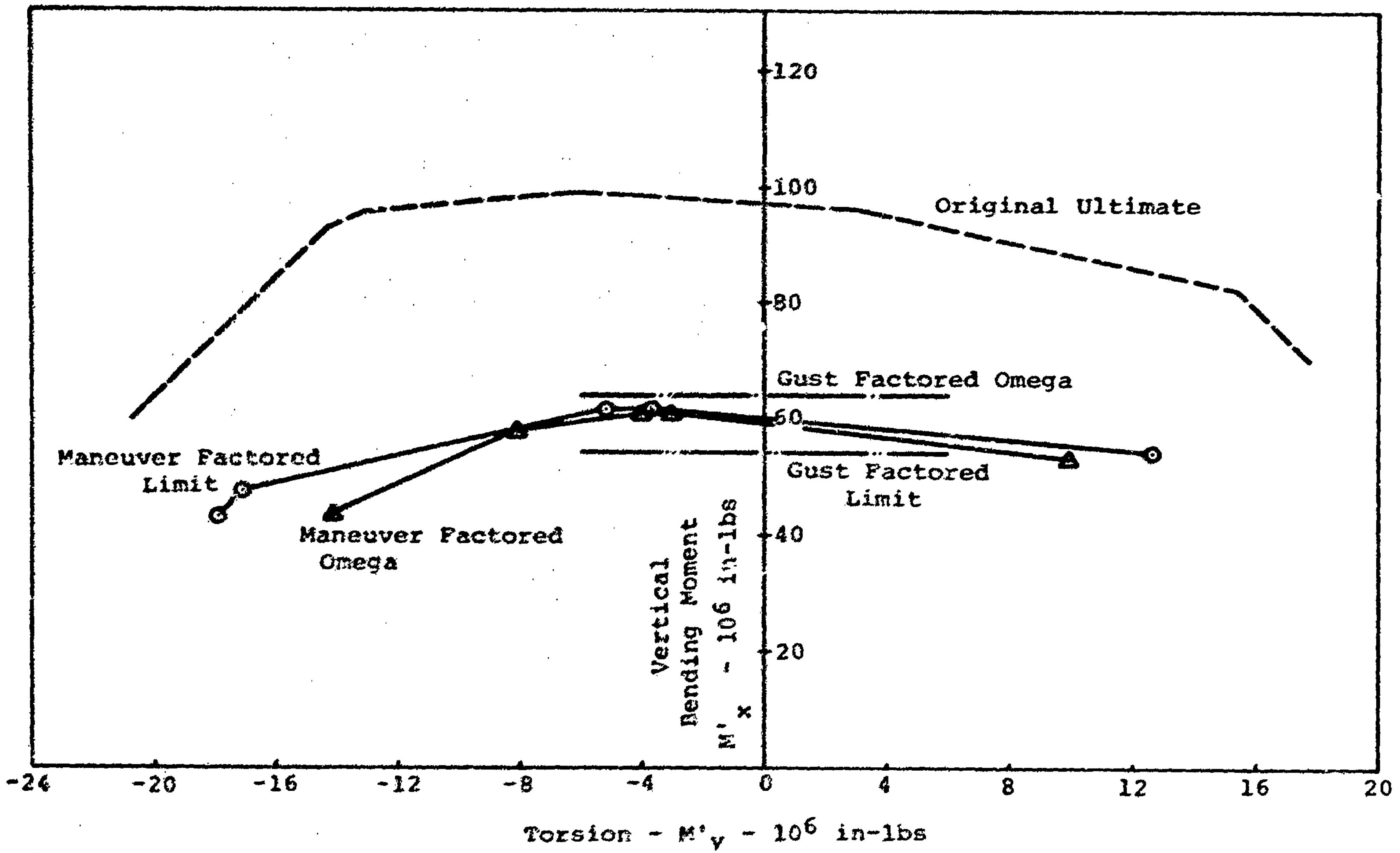


FIGURE 74 C-141 FACTORED LIMIT AND OMEGA DESIGN LOADS
W. S. 135

The structural reliability load requirements are significantly less than the original ultimate load requirements. On a vertical bending moment only basis, a reduction of 35% is indicated at wing station 135 for the two loads sources considered. Since wing station 135 is a loads control station for the inboard wing, the results are applicable to other inboard wing stations.

3) The results of this exercise should not be interpreted to mean that the structural reliability load requirements will always be less than those resulting from the present deterministic methods. Rather, the results are dependent upon the utilization and strength scatter for the individual aircraft being considered.

10.4 Fatigue Endurance Considerations

Only static strength structural reliability has been considered in this analysis. However, in order to adequately define the design load requirements, fatigue and fail-safe requirements must also be included.

The results of the C-141 static strength structural reliability analysis showed that the positive vertical bending moment requirements for the inboard wing are 35% less than those for the original deterministic requirements. However, if the design loads were decreased by 35%, and if the same detail design were used, the stress to load ratio would increase by 35%. Such an increase in stress to load ratio would cause an approximately 90% reduction in fatigue endurance for the C-141 wing root lower surface. Such a reduction is not acceptable. Therefore, the design loads cannot [illegible] determined only by static strength, as fatigue and failsafe considerations may dictate higher load requirements.

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