RESEARCH IN NONPARAMETRIC STATISTICS TOPICS WHICH HAVE APPLICATION TO COMMUNICATION THEORY, METEOROLOGY AND CIRCULAR DATA ANALYSIS

Interim (Scientific-Progress) Report
Grant Number AFOSR-71-2009
March 1, 1971 - February 29, 1972

(a) Research Completed.

(i) Some Variants of Chi-square for testing uniformity on the circle. In this paper the problem of testing for uniformity on the basis of a set of observations on the circumference of unit circle was considered. The classical chi-square test was modified to make it invariant under rotations of the circle. Two such modifications were considered and their limiting distributions were obtained.

(ii) Weak convergence of Empirical Distributions Functions of Random Variables subject to Perturbation and Scale Factors. In this paper the weak convergence of empirical distribution functions of random variables subject to random perturbations and random scale factors to a Gaussian process was established. These results were then used to derive the asympotic distributions of test statistics based on spacings which arise in connection with goodness-of-fit problems on the line and the circle. The asymptotic relative efficiencies of such tests were studied.

(iii) On some selection procedures in two-way lay outs. In this paper we consider some robust selection procedures for selecting a subset of t ($< c$) best treatments (with or without regard to orders in a two-factor complete block design involving c (≥ 2) treatments. We also consider the problem of selecting all treatments which are at least as good as the standard one. The procedures considered are based on a class of robust estimates of treatment effects based on rank statistics and form an extension of the existing theory of Puri and Puri (1969) [Annals of Mathematical Statistics, 40, 619-632], and others, to two-way lay outs designs.

(iv) Some Aspects of Nonparametric Inference. This paper deals with some aspects of the theory of univariate nonparametric statistics in a systematic and logically integrated form. It provides the justification and motivation behind the use of nonparametric methods. It surveys some of the most important developments that have taken place in the theory of rank order statistics in recent years. Special attention is given to the asymptotic results in (i) the two sample problem, (ii) one sample problem, (iii) c-sample problem (with ordered as well as unordered alternatives), (iv) two-way layout problem, (v) selection procedures, and (vi) estimation problem. The concept of asymptotic relative efficiency is also discussed.

(v) Bahadur Efficiencies of Some Tests for Uniformity on the Circle. In this paper the asymptotic efficiencies due to Bahadur of several tests that are available for testing uniformity on the circle are investigated. This is done by evaluating the local slopes of the test statistics using large deviation results. These results suggest which test procedure is to be preferred when faced with the problem of testing uniformity on the circle.

(vi) Rank Tests for Some Linear Hypotheses in Paired Comparison Designs. In this paper we extend the theory of rank order tests for paired comparisons to the analysis of covariance problems and derive an alternative class of tests when the model can be described by a smaller number of parameters. The asymptotic Pitman-efficiencies of the proposed tests with respect to corresponding (normal-theory) variance-ratio tests are also studied.

(b) Work in Progress.

(i) Nonparametric procedures are being developed for exchangeable processes and for some other classes of stochastic processes: for example, processes with independent increments, processes whose increments are dependent in some specific manners, etc.

(ii) In most of the statistical theory developed so far in the area of incomplete blocks designs, one usually makes the assumption that the underlying observations within each block are identically distributed. This assumption is known to be unrealistic and unnatural in most situations of practical interest. The result is that the whole theory developed in this direction (parametric as well as nonparametric) is only of very limited scope. Some progress has been made to overcome this basic defect inherent in earlier work in this direction by eliminating the assumption of the identity of the underlying distributions. This enables us to derive the nonparametric (as well as parametric) comprehensive test procedures which are completely robust, which are invulnerable to gross errors, and which provide considerable efficiency improvements over their parametric competitors based on normality assumptions.

(iii) In the area of the analysis of circular data, work is being done to develop techniques which correspond to linear procedures like correlation, regression, and analysis of covariance etc. Work is also being done to develop some two sample and multisample nonparametric techniques for circular populations on the lines of the work of Puri [Ann. Math. Stat. 1964, 102-121].

(iv) Work is also being done in reference to nonparametric procedures in multiple regression models with stochastic as well as nonstochastic predictors. This work deals with nonparametric methods which are applicable to problems in meteorology. Preliminary results have shown that procedures developed are considerably more efficient than the ones considered by Millar [Statistical Prediction by Discriminant Analysis. Meteorological Monographs, vol. 4, 1-54].

(c) Publications (including the papers submitted) under the present grant.

(i) J. S. Rao, "Some variants of chi-square for testing uniformity on the circle." Zeitschrift fur Wahrscheinlichkeitstheorie und verwandte Gebiete. (1972), 22, 33-44.

(ii) J. S. Rao, "Bahadur Efficiencies of some tests for uniformity on the circle." The Annals of Mathematical Statistics (1972), April issue.

(iii) J. S. Rao (and J. Sethuraman), "Weak convergence of empirical distribution functions of random variables subject to perturbations and scale factors," Annals of Mathematical Statistics (submitted).

(iv) Puri, M. L. "On some selection procedures in two-way lay outs." Zeitschrift fur Wahrscheinlichkeitstheorie und verwandte Gebiete (1972) 22, 242-250.

(v) Puri, M. L. "Some Aspects of Nonparametric Inference." Accepted for publication in Review of the International Statistical Institute.

(vi) Puri, M. L. and Sen, P. K. "Rank Tests for Some Linear Hypotheses in Paired Comparison Designs." Sankhyā, Ser. A, (submitted)

(d) Honors Received:

Professor Puri was made the Fellow of the American Statistical Association "for his significant contributions to nonparametric statistical theory and his vigorous efforts to help others develop and disseminate nonparametric theory."

Professor Puri was also made the Fellow of the Institute of Mathematical Statistics "in recognition of his contributions to the development, dissemination, and application of Mathematical Statistics."

(e) Professional Service:

Professor Puri was made the Chairman of the Institute of Mathematical Statistics Committee on Summer Research Institutes.

(f) Meetings Attended (including invited talks)

(i) Mathematischen Forschungsinstitut, Oberwolfach, West Germany. March 20, 1971, Professor M. L. Puri gave one hour invited talk on "Some generalized multivariate one sample problems."

(ii) Sixth Prague Conference on Information Theory, Statistical Decision Functions and Random Processes; (sponsored by Czechoslovak Academy of Sciences) held in Prague, September 19-25, 1971. Professor M. L. Puri gave one hour invited talk on "Statistical Inference in Incomplete Block Designs." He also chaired one session.

(iii) The Fourth Conference in Probability Theory (sponsored by the Academy of the Socialist Republic of Romania) held in Brasov, Romania, September 12-18, 1971. Professor Puri gave one hour invited talk on "Limit Theorems for Rank Statistics." Professor Puri also chaired one session.

(iv) Professor Puri and J. S. Rao attended the 37th session of the International Statistical Institute, held in Washington, D. C. in August 1971.

(v) Professor Puri gave some colloquium talks in the Universities of Montreal, Laval (Quebec, Canada), Purdue, Michigan and Professor J. S. Rao gave colloquium talks at Johns Hopkins University and the Catholic University of America.

(g) Personnel under the present grant.

(i) M. L. Puri (Principal Investigator)
(ii) J. S. Rao (Co-investigator)
(iii) Norman Wykoff (Student-Research Assistant)
(iv) Carl T. Russel (Student-Research Assistant)

Both Norman Wykoff and Carl T. Russel are working on their Ph.D. dissertations under the direction of Professor M. L. Puri.