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Materiel Test Procedure 3-1-005
U. S. Army Field Artillery Board

U. S. ARMY TEST AND EVALUATION COMMAND

BACKGROUND DOCUMENT

FIELD ARTILLERY STATISTICS

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TABLE OF CONTENTS

GLOSSARY

The definitions and notations in this MTP are listed alphabetically. The Greek symbols used are listed below along with their names. The definition of each Greek symbol is then found alphabetically by name.

α - alpha
β - beta
χ - chi
Δ - delta (capital)
δ - delta
ε - epsilon
γ - gamma
λ - lambda
μ - mu
ω - omega
π - pi
ρ - rho
Σ - sigma (capital)
σ - sigma
τ - tau

$| \; |$ - Absolute value symbols; the enclosed term becomes positive regardless of the original sign.

AR - Maintenance action rate; number of actions per hour.

α - Small Greek letter alpha used to denote the level of significance or the risk of Type I error. (Confidence level = $1-\alpha$.)

A_α - Lower boundary for the one-sided unbiased Type A test.

$A_{1-\alpha}$ - Upper boundary for the one-sided unbiased Type A test.

A_a - Achieved availability.

A_i - Inherent availability.

A_o - Operational availability.

AMT - Active maintenance time.

AP - Aiming point; target.

β - Small Greek letter beta used to denote the risk of a Type II error.

B_L - Lower boundary for the two-sided unbiased Type A test.

B_U - Upper boundary for the two-sided unbiased Type A test.

χ^2 - The square of the small Greek letter chi used to denote the chi-square distribution.

CPE - Circular probable error; the radius of a circle, centered at the mean, in which 50% of the population lies.

CV - Critical value to which a test result is compared in order to make a decision.

CN - Critical number to which the ratio of successive difference method to standard deviation method for computation of parable error is compared to decide whether a trend existed or not.

d.f. - Degree of freedom to which subscripts may be added as necessary; e.g., $d.f._1$ or $d.f._2$; a numerical value dependent upon sample size and the number of estimated parameters.

Δ - Capital Greek letter delta used to denote the deviation of each reading from the mean.

Δ_A - Capital Greek letter delta used to denote the deviation of each reading from the mean for a Type A item.

Δ_B - Capital Greek letter delta used to denote the deviation of each reading from the mean for a Type B item.

δ - Small Greek letter delta used as a subscript to denote the successive differences method for computing PE and standard deviation.

D - Amount of doubt; a defined area which requires continued testing to insure that borderline equipment is adequately tested.

d_m - The distance between a data point and the mean of all data points.

e.d.f. - Effective number of degrees of freedom.

ε - Small Greek letter epsilon used to denote an amount of error either to help determine a realistic sample size or to determine how close a population value is to a sample value at a desired confidence level.

e^{-x} - Exponential reliability; e = 2.71828.

F - The F distribution; the ratio of two variances, each generated from two samples which have normal distributions.

f - Total number of failures.

f_r - Failure rate; number of failures per hour.

f_{rt} - Failure rate; number of failures per hour where continued testing is necessary.

f_s - Total number of system failures.

γ - Small Greek letter gamma used to denote the ratio of the sample standard deviation divided by the required standard deviation.

K - The number of products tested.

λ - Small Greek letter lambda used to denote the population proportion.

λ_o - Small Greek letter lambda with subscript zero used to denote the required proportion found in the Requirements Document or from a comparable item.

ln - Natural logarithm.

LCL - Lower confidence limit.

$\underline{M}$ - Maintainability; the probability that an item will be retained in or restored to a specified condition within a period of time, when the maintenance is performed in accordance with prescribed procedures and resources.

MA - Total number of maintenance actions.

MR - Maintenance ratio; amount of active maintenance time per hour.

M_1 - Mean time between failures (lower confidence limit). NOTE: The parameter may be rounds or miles instead of time.

M_2 - Mean time between failures (upper confidence limit).

MDT - Mean downtime.

$\overline{M}$ - Mean active maintenance time; total maintenance time divided by the number of maintenance actions.

MPI - The mean point of impact; the mean horizontal coordinates for ground bursts.

MTBF - Mean time between failures.

$MTBF_t$ - Mean time between failures where continued testing is necessary.

MTBM - Mean time between maintenance.

MTTR - Mean time to repair.

m - Miss distance; the distance between the aiming point and MPI.

MP - Mission (operational) profile, generally found in the Requirements Document.

μ - Small Greek letter mu used to denote the population mean.

μ_A - Small Greek letter mu used to denote the population mean for a Type A item.

μ_B - Small Greek letter mu used to denote the population mean for a Type B item.

μ_o - Small Greek letter mu with subscript zero used to denote the required mean found in the Requirements Document or from a comparable item.

N - Number of samples; sample size.

N_A - Number of samples for a Type A item.

N_B - Number of samples for a Type B item.

N_t - Sample size required to test the criteria; computed before testing starts.

N_{min} - Used when computing combined system reliability; the sample size for that individual component of a system which is tested fewer times than the other components.

OC - Operating-characteristic curve used to determine required sample size for testing given criteria.

ω - Small Greek letter omega used to denote allowable maintenance action time as prescribed in the Requirements Document.

π - Capital Greek letter pi used to represent the product of items; e.g.,

$$\prod_{i=1}^{N} x_i = (x_1)(x_2)\ (x_N)$$

p - The probability of an event occurring. (It cannot be less than zero or greater than one.)

PE - Probable error to which necessary subscripts are added to denote types of PE; e.g., PE_R (range probable error), PE_D (deflection probable error), or PE_H (height of burst probable error); a deviation from μ such that 50% of the observations may be expected to lie between μ-PE and μ+PE.

PE_A - Probable error for a Type A item to which necessary subscripts are used to denote types of PE_A; e.g., PE_{A_R} (range probable error for a Type A item), PE_{A_D} (deflection probable error for a Type A item), or PE_{A_H} (height of burst probable error for a Type A item).

PE_B - Probable error for a Type B item to which necessary subscripts are added to denote types of PE_B; e.g., PE_{B_R}, PE_{B_D}, or PE_{B_H},

P - Sample Proportion; the ratio of the items possessing a given characteristic divided by the sample size.

P_A - Sample Proportion for a Type A item.

P_B - Sample Proportion for a Type B item.

P_o - The required maximum proportion of defectives; P_o equals λ_o, if λ_o is in terms of defectives or P_o equals the quantity $(1-\lambda_o)$, if λ_o is in terms of successes.

P_U - Upper limit for the proportion of defectives; the difference between P_o and the amount of doubt $(P_U = P_o - D)$.

$\overline{POB}$ - The mean point of burst; the mean coordinates for air bursts.

q - The ratio of the range of the observations to the standard deviation; the studentized range (q) distribution.

R - Reliability; the extent to which a test yields the same results on repeated trials.

ρ - Small Greek letter rho used to denote the population reliability.

ρ_o - Small Greek letter rho with subscript zero used to denote the required reliability prescribed in the Requirements Document.

R_U - Upper limit for the reliability; the sum of ρ_o and the amount of doubt $(R_U = \rho_o + D)$.

R_{PE} - Point estimate reliability; the number of successes divided by the sample size; achieved reliability.

RR - Repair rate.

RT - Repair time which is the result of a failure.

Σ - Capital Greek letter sigma used to denote the sum of items; e.g.,

$$\sum_{i=1}^{N} x_i = x_1+x_2+x_3+ \ldots x_N$$

σ - Small Greek letter sigma used to denote the population standard deviation.

σ_A - Small Greek letter sigma used to denote the population standard deviation for a Type A item.

σ_B - Small Greek letter sigma used to denote the population standard deviation for a Type B item.

σ_E - Small Greek letter sigma used to denote the population standard deviation for eastings.

σ_N - Small Greek letter sigma used to denote the population standard deviation for northings.

σ_d - Small Greek letter sigma used to denote the population standard deviation of the differences between paired readings for a Type A item and a Type B item.

σ_o - Small Greek letter sigma with subscript zero used to denote the required standard deviation prescribed in the Requirements Document.

s - Sample Standard deviation of the sample; a measure of deviation from the mean.

s_A - Sample standard deviation for a Type A item.

s_B - Sample standard deviation for a Type B item.

s_δ - Sample standard deviation computed by the successive differences method.

s_E - Sample standard deviation for eastings.

s_N - Sample standard deviation for northings.

s_d - Sample standard deviation of the differences between paired readings for a Type A item and a Type B item.

s_p - Sample standard deviation of combined items when individual population standard deviations are unknown but assumed equal.

s^2 - Sample variance; standard deviation squared.

s_1^2 - Sample variance with the suspected outlier deleted.

$s^2_{1_\delta}$ - Sample variance with the suspected outlier deleted, computed by the successive differences method.

s^2_K - Average variance of K number of products.

sc - Total number of successes.

t - The variable of the Student t distribution.

τ - Small Greek letter tau used to denote the population probable error.

τ_o - Small Greek letter tau with subscript zero used to denote the required probable error prescribed in the Requirements Document.

TM - Total active maintenance manhours.

T_m - Total number of miles.

T_t - Total number of hours; total time.

UCL - Upper confidence limit.

x - A variable which may be assigned values.

x_A - A variable which may be assigned values relative to a Type A item.

x_B - A variable which may be assigned values relative to a Type B item.

x_d - The difference between two readings.

$\overline{X}$ - Sample mean or sample average.

$\overline{X}_A$ - Sample mean for a Type A item.

$\overline{X}_B$ - Sample mean for a Type B item.

$\overline{X}_d$ - Sample mean for a particular set of differences.

Z - Standard units of measure on a normal curve with a mean of zero and a standard deviation of one.

$<$ - Less than; $a < b$ is read, a is less than b.

$\leq$ - Less than or equal to; $a \leq b$ is read, a is less than or equal to b.

$>$ - Greater than; $a > b$ is read, a is greater than b.

$\geq$ - Greater than or equal to; $a \geq b$ is read, a is greater than or equal to b.

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U. S. ARMY TEST AND EVALUATION COMMAND
BACKGROUND DOCUMENT

FIELD ARTILLERY STATISTICS

1. PURPOSE

This Materiel Test Procedure (MTP) is a guide for the project officer for planning the test and analyzing the test data.

2. SCOPE

a. This MTP encompasses all necessary aspects of statistical procedures for service tests (ST). This MTP does not give the theoretical background for the statistical tests. The scope includes:

(1) Concepts.
(2) Median.
(3) Mean.
(4) Standard deviation.
(5) Proportion.
(6) Accuracy and precision.
(7) Reliability.
(8) Maintenance evaluation.

b. The statistical procedures presented herein are applicable to testing of Field Artillery materiel.

3. BACKGROUND

a. Statistics is an essential tool for evaluating results of tests conducted on newly developed items and for measuring and evaluating the degree of uncertainty associated with the test data. Statistical analysis usually consists of generating a result pertinent to the test item and then comparing that result to a stated requirement prescribed in the Requirements Document. In the absence of stated requirements, the development of statistical results will be of value in comparing the characteristics of a new item to those of a standard item or in determining the characteristics of a new item.

b. Population is the whole class about which conclusions are to be drawn. However, population characteristics can rarely be determined exactly because of the unavailability of all the items in the population, the expense of examining every item of the population or even a large number of items, or the destructive nature of the examination. Consequently, population characteristics must be inferred from an examination of a part of the population -- a randomly selected sample. The statistical approach to examining and predicting the population characteristic will depend upon the size of the randomly selected sample. As the sample size is increased, there is greater confidence in the result being a true representation of the true population characteristic. The project officer will make every effort to utilize the proper sample size.

4. CONCEPTS

4.1 POPULATION AND SAMPLE

A population is any finite or infinite collection of individual things, objects, or events, which is determined by some property that distinguishes between things that do and things that do not belong. In contrast, a sample is defined as a portion of a population. The sample even though a portion of the population plays an important role in predicting the characteristics of the population. Due to the size of the population, the prohibitive costs in testing the population, and, in most cases, the destructive nature of the tests; testing the entire population is impossible and impractical. However, a sample may be tested and the findings from that sample used to predict characteristics of the population. A random sample; i.e., the sample is chosen such that every individual in the population has an equal chance of being chosen, is the best type of sample to test. If separate random samples are drawn, the two samples are independent; i.e., one does not rely on the other. However, in many cases, a true random selection is not feasible; e.g., the prototype. Testing agencies may be furnished only one prototype of an item to test. The results of the test will reflect only on the prototype and not on the production items (population). In order to obtain a random sample and to accurately forecast such characteristics as reliability and availability for the population, random samples of production line items must be subjected to the same tests as the prototype.

4.2 FAILURE

A failure is defined as the inability of an item to perform within previously specified limits. Failures are classified as chargeable or non-chargeable. Non-chargeable failures do not count against the test item. Since a decision to accept or to reject an item can be altered if certain failures are not counted, it is necessary to carefully decide whether a failure is chargeable or non-chargeable. Of course, if failures are ignored, the probability of accepting an unacceptable item is increased. Reference 12a defines chargeable and non-chargeable failures.

4.3 DISTRIBUTION

a. The description of measurements and observations by grouping and classifying is an essential part of statistics. The grouping of data into classes is known as a frequency distribution (or simply a distribution) and consists of essentially choosing the classes into which the data are to be grouped, sorting or tallying the data into the appropriate classes, and counting the number of items in each class. Choosing the classes into which the data are to be grouped involves determining the class width, called a class interval; the number of class intervals needed to contain all the data, normally between six and 15; and the class intervals such that each measurement is contained in one and only one interval. Equal class intervals should be used whenever possible to aid in the ease of grouping and for quick and accurate reading of the data. As an example, Figure 1 has the data grouped and classified. The distribution has seven equal class intervals, each class interval contains 2500 meters, and each range measurement which is recorded to the nearest meter is contained in one and only one class interval.

b. The graphical representation of a distribution is called a histogram and is illustrated at Figure 1. A population distribution, which consist of many small classes, can be pictured as a curve which is approximated by the histogram. The curve is known as the probability density function or the distribution function and always has an area of one. The probability that a random observation will fall in any interval, a to b, is the area under the curve from a to b.

c. A distribution generally is one of two types, continuous or discrete. In the case of a continuous distribution, a random observation may assume any value between and including the minimum and maximum values; however, a discrete distribution allows an observation to assume only certain values. For example, when firing ammunition, a range is obtained; this range reading may be any reading between and including the minimum and maximum ranges for the ammunition. An example of a discrete distribution is obtained by making repeated tests on 8 different charges under similar conditions. In each test, the charge fired may take on only one of the 8 values 1, 2, ..., 8. The binomial distribution is the most important discrete distribution Field Artillery statistics uses (paragraph 4.15.3, page 17).

d. Although frequency distributions present data in a relatively compact form, give a good overall picture, and contain information which is adequate for many purposes, there are some limitations. For instance, the maximum and minimum values are not disclosed; nor is the average value (overall or by class) available.

4.4 MEASURES OF CENTRAL LOCATION

4.4.1 MEAN.

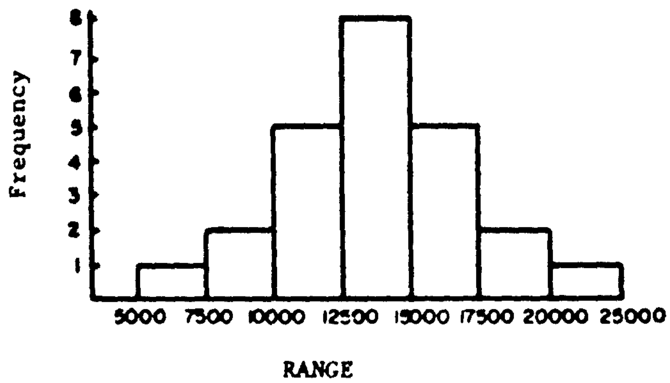
The sample mean, or average ($\bar{X}$), of a number of sample readings (N) is a description of the central location. The mean is determined by summing the values of all of the sample readings and dividing by N. A population mean (μ) can be defined for the whole population. $\bar{X}$ is generally a good estimate of μ. Figure 2 illustrates the mean.

4.4.2 MEDIAN

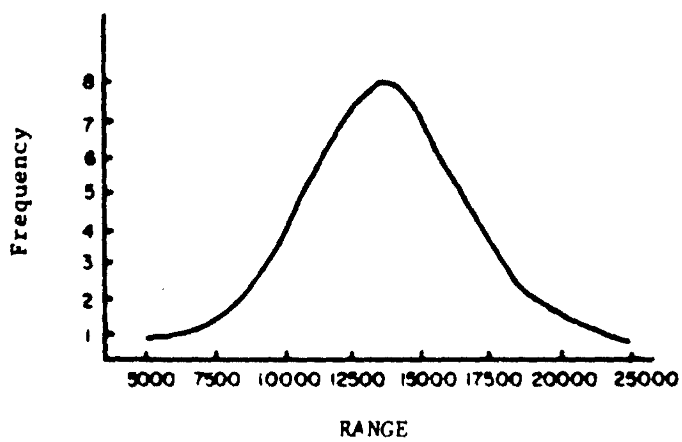
a. The median is the midpoint of the readings when they are arranged in ascending or descending order. The median is the middle reading of an odd number of readings or is the average of the middle two readings of an even number of readings. Thus, 1/2 of the readings are larger than the median, and 1/2 of the readings are smaller than the median.

DISTRIBUTION

RANGE	FREQUENCY	RANGE	FREQUENCY
5000-7499	1	15000-17499	5
7500-9999	2	17500-19999	2
10000-12499	5	20000-22499	1
12500-14999	8		



HISTOGRAM



DISTRIBUTION CURVE

Figure 1

CENTRAL MEASURES OF LOCATION

RANGE		
495	1225	Mean = 1188
745	1235	Median = 1000
750	1238	Mode = 1125 (1000 to 1250)
865	1249	Mode of Raw Data = 1000
950	1450	Midrange = 1755
975	1485	
995	1720	
1000	1950	
1000	2250	
1000		

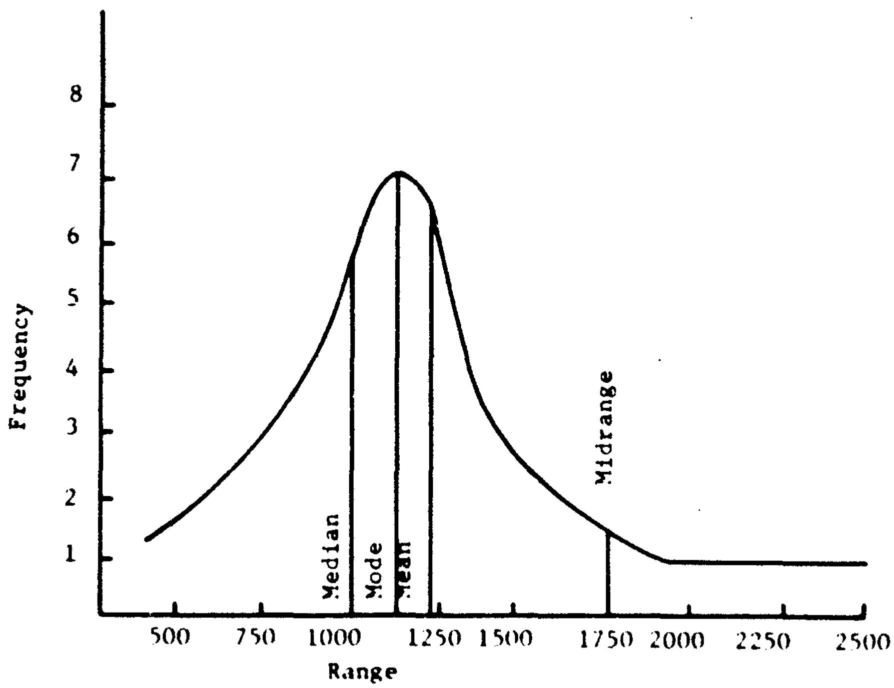


Figure 2

b. The population median and mean are equal for a normal distribution (paragraph 4.15.1, page 15) and the sample median may be used to estimate the population mean; however, the median is not as good an estimate as the sample mean. If the distribution is not normal, the population median and mean may not be equal. Figure 2 illustrates the median.

4.4.3 MODE

a. The mode of a set of raw data is the value which occurs most often; e.g., if the raw data is 10, 8, 12, 8, 9, 10, 8; then 8 occurs three times; 10, two times; and 12 and 9, one time. Therefore, the mode is 8. A mode of high concentration gives a rough but quick measure of central location but is not unique; i.e., several readings may occur the same number of times. Therefore, if more than one high point exists, the mode is not very useful.

b. The mode of a set of data that has been converted into a frequency distribution is the midpoint of the interval which contains the most readings; e.g., the interval from 1 to 5 has one sample; 6 to 10, two samples; 11 to 15, six samples; 16 to 20, two samples; and 21 to 25, zero samples. The mode is 13 (the interval from 11 to 15 which has six samples). Figure 2 illustrates the mode.

4.4.4 MIDRANGE

The midrange is the sum of the smallest and the largest readings divided by two. This is a good measure of the central location for samples of five or fewer, though it is not as good as the mean. Figure 2 illustrates the midrange.

4.5 MEASURES OF DISPERSION

4.5.1 STANDARD DEVIATION

a. The standard deviation (s) is a measure of dispersion from $\bar{X}$. The amount of variation of that dispersion depends on the distances of the readings from the mean. The difference between the mean and each reading $(x-\bar{X})$ represents the deviation from the mean (Δ) and suggests that the average of the deviations might be used as a measure of the variation of the N readings. Since these deviation values are positive and negative and a normal distribution exists, the sum is zero as is the mean of the variation.

b. Since the size of the deviations and not the direction (sign) of the deviation are of interest, the direction (sign) can be ignored. The sum is then positive, and the result is the mean deviation (paragraph 4.5.3, page 7). However, there exists another and better way of eliminating the direction (sign) of the deviations and that is squaring the deviations. A square cannot be negative. The average of the squared deviations is the variance (s^2). The square root of the variance is the standard deviation (s). Originally s was computed in the following manner:

(1) Square the difference between the mean and reading i.e., $(x-\overline{X})^2$.
(2) Sum the squares; i.e., $\Sigma(x-\overline{X})^2$.
(3) Average the sum by dividing by N; i.e., $\frac{\Sigma(x-\overline{X})^2}{N}$.
(4) Find the square root of the average; i.e., $s = \sqrt{\frac{\Sigma(x-\overline{X})^2}{N}}$.
(The square root is used to compensate for the fact that the deviations were squared.)

c. In recent years there has been a tendency to divide by N-1 rather than by N. The reason for this is that if s^2 is used to estimate a population variance (σ^2), the mean obtained is usually too small and biased if N is the divisor. Therefore, N-1 as a divisor yields a truer estimate of the population variance. Since the population is the item of interest rather than only a few samples, N-1 will be used throughout this MTP in computing s^2 or s; i.e.,

$$s = \sqrt{\frac{\Sigma(x-\overline{X})^2}{N-1}}$$

(see paragraph 7.1, page 64, for computations). The population standard deviation (σ) is a measure of the extent to which a population characteristic varies from one item to another.

NOTE: The standard deviation may also be computed by the following formula:

$$s = \sqrt{\frac{N\Sigma x^2 - (\Sigma x)^2}{N(N-1)}}$$

4.5.2 RANGE

The range is the difference between the smallest and the largest readings in the sample. The range multiplied by the appropriate factor from Table B-1, page 2-1, approximates σ for a small sample ($N \leq 10$) and a normal distribution (paragraph 4.15.1, page 15).

4.5.3 MEAN DEVIATION

The mean deviation of a normal distribution is the mean of the deviations from the mean or median of the N sample members. The deviations from the mean (median) is the absolute value of the mean (median) subtracted from the reading. The mean deviation multiplied by a factor from Table B-2, page 2-2, approximates σ for a small sample ($N \leq 10$) and a normal distribution (see paragraph 4.15.1, page 15).

4.5.4 PROBABLE ERROR (RANGE, DEFLECTION, AND HEIGHT OF BURST)

The probable error (PE) is a measure of deviation from μ such that 50% of the observations may be expected to lie between μ-PE and μ+PE. However, certain conditions must exist for the PE to have any meaning. These are independent (random) samples, normal distribution, and large sample size.

PE may be expressed for various parameters, range (PE_R), deflection (PE_D), and height of burst (PE_H). For the population probable error (τ), $\tau = 0.6745\sigma$ and $\sigma = 1.4826\,\tau$. Since a sample is being examined as a representative of the population, PE = 0.6745s and s = 1.4826PE. Firing tables and other data concerning Field Artillery precision contain the appropriate PE's. When testing for precision, end results are often expressed in terms other than PE. This occurs in modern day testing because prototype samples are not random representations of production line items, the normal distribution is not appropriate in many cases, and small sample sizes bias the PE. The more modern standard deviation is in wider use as a measure of dispersion than is the probable error because s is commonly computed for statistical analysis. Due to the freedom to use small or large sample sizes, the wider applications of the standard deviation, and the ease of calculation, statistical tests involving standard deviation comparisons are more widely used than those involving PE comparisons.

4.5.5 CIRCULAR PROBABLE ERROR

The circular probable error (CPE or CEP) is a measure of deviation from μ and defines the radius of the circle which is centered at the mean and in which 50% of the observations are contained. CPE = 1.1774 times the population standard deviation for the easting (σ_E) when σ_E equals the population standard deviation for the northing (σ_N). When $\sigma_E \neq \sigma_N$, the CPE is called the equivalent CPE and equals .5887 ($\sigma_E+\sigma_N$). In terms of a sample, the equivalent CPE = .5887 (s_E+s_N). However, as for the PE, certain conditions must exist for the CPE to have any meaning; these are independent (random samples, a bivariate normal distribution, and a large sample size. Firing tables and other data concerning Field Artillery precision may contain the CPE. When testing for precision end results are often expressed in terms other than CPE. This occurs in modern day testing because prototype samples are not random representations of production line items, the bivariate normal distribution is not appropriate, and small sample sizes bias the CPE. The bivariate normal distribution is a representation of the measure of dispersion for two variables (see paragraph 4.15.2, page 15 and paragraph 9.2.4, page 118).

4.6 RELIABILITY

a. Reliability is the probability of an item functioning adequately for the period of time intended under the operating conditions encountered. Along with the numerical value of the reliability, a fraction or a percent value, the following are necessary:

(1) Define precisely a success or satisfactory performance.
(2) Specify the time base or operating cycles over which such performance is to be sustained; e.g., hours, miles, or rounds. This factor is particularly important since the probability value is based on completing a mission or task. For example, if the probability of a test item operating for 50 hours is 0.65 or 65%, then on the average 65 times out of 100 trials the test item would be functioning after a 50-hour operating period.

(3) Specify the environment or use conditions which will prevail. Typical of these conditions are temperature, humidity, shock, and vibration. Without these various conditions the reliability definition would be relatively meaningless.

b. Due to the various types of test items and the various distributions which apply, reliability may be evaluated by several methods (see paragraph 10, page 118).

4.7 TEST OF A STATISTICAL HYPOTHESIS

The investigator's objective can often be translated into an hypothesis (assumption or claim) concerning the test item. This hypothesis, called the null hypothesis, usually states that the test item does not meet the stated requirements. This explains why it is called the null (not) hypothesis. A decision is made to accept or reject the null hypothesis using the test data from the sample. Failure to reject the null hypothesis does not necessarily mean that the hypothesis is true but merely indicates that the sample is compatible with the kind of population described in the null hypothesis. The same is true if the null hypothesis is rejected; the fact is merely recognized that the sample is not compatible with the kind of population described in the null hypothesis. Associated with the null hypothesis are two types of errors (paragraph 4.8, page 10), and a significance level (paragraph 4.9, page 10). In general, to test a null hypothesis and construct statistical decision criteria, the following outline is used:

a. Formulate the null hypothesis so that it states that the test item does not meet the stated requirements. The null hypothesis is a numeric expression; e.g., $\overline{X} > 25$.

b. Formulate an alternative hypothesis so that the rejection of the null hypothesis is equivalent to the acceptance of the alternative hypothesis. The alternative hypothesis is also a numeric expression e.g., $\overline{X} \geq 25$.

c. Specify the probability to be risked as a Type I error. If possible, desired, or necessary, also make some specifications about the probability of a Type II error for a given alternate value of the parameter concerned.

d. Use the appropriate statistical theory (e.g., paragraphs 6.2, page 36, and 6.3, page 45) to test the null hypothesis.

NOTE: In some cases when the null hypothesis has been rejected, a reserve judgment decision will be made instead of accepting the alternative hypothesis; e.g., insufficient sampling to produce conclusive results.

4.8 TYPES OF ERROR

4.8.1 TYPE I ERROR

The Type I error is rejection of the null hypothesis when it is true. The risk of Type I error is the level of significance (α). It is the more important of the two error types, since rejecting an item when in fact it is good is better economically than accepting an item when in fact it is bad. The value of α is arbitrary but will sometimes be found in the Requirements Document. In the event the significance level or confidence level (confidence level = 1 - significance level) is not specified in the Requirements Document, α = .10 or confidence level = .90 will be used.

4.8.2 TYPE II ERROR

The Type II error is the acceptance of the null hypothesis when it is false. The risk of a Type II error is denoted by β. The value of β is not as restricted as that of α. In the event α and β are highly restricted, the sample size must be very large to reach an accept or reject decision. When β is not specified in the Requirements Document, .20 will be used.

4.9 LEVEL OF SIGNIFICANCE.

a. The risk of making a Type I error (α) equals the level of significance of the test. The null hypothesis serves as an origin or base. From the null hypothesis the test criterion may be a two-sided test (two-tail test) or a one-sided test (one-tail test). The two-sided test involves an area at each extreme of the distribution curve (note Figure 3A); e.g., if α = .05 or 5%, then the shaded areas in Figure 3A are each equal to 2.5% of the total area under the curve. The one-sided test is only concerned with the area under the curve at one extreme (note Figure 3B); e.g., if α = .05 or 5%, then the shaded area in Figure 3B is equal to 5% of the total area under the curve. When the stated requirement is in the shaded area, the null hypothesis is accepted which means that the item is not acceptable.

b. In general, a test is said to be one-sided or two-sided (one-tailed or two-tailed) depending on whether α is concentrated at one end of the curve (left or right) or is divided into two areas with the areas situated at opposite ends of the curve (see Figure 3).

4.10 CONFIDENCE INTERVAL, LIMITS, AND LEVEL

a. When estimating a population measure, such as μ, by a sample measure, such as $\overline{X}$, μ has a value somewhere near $\overline{X}$. How near μ is to $\overline{X}$ is determined by an interval constructed about $\overline{X}$; and, at a specified confidence level, μ lies in this interval. This interval is called the confidence interval. The interval between the shaded areas of Figure 3A is an example of a confidence interval (see paragraph 6.1.2.1, page 27).

b. The end points of the confidence interval are called confidence limits. Thus, there exist an upper confidence limit (UCL) and a lower confidence limit (LCL). The LCL and UCL are shown in Figure 3A. In

ONE-SIDED AND TWO-SIDED AREAS

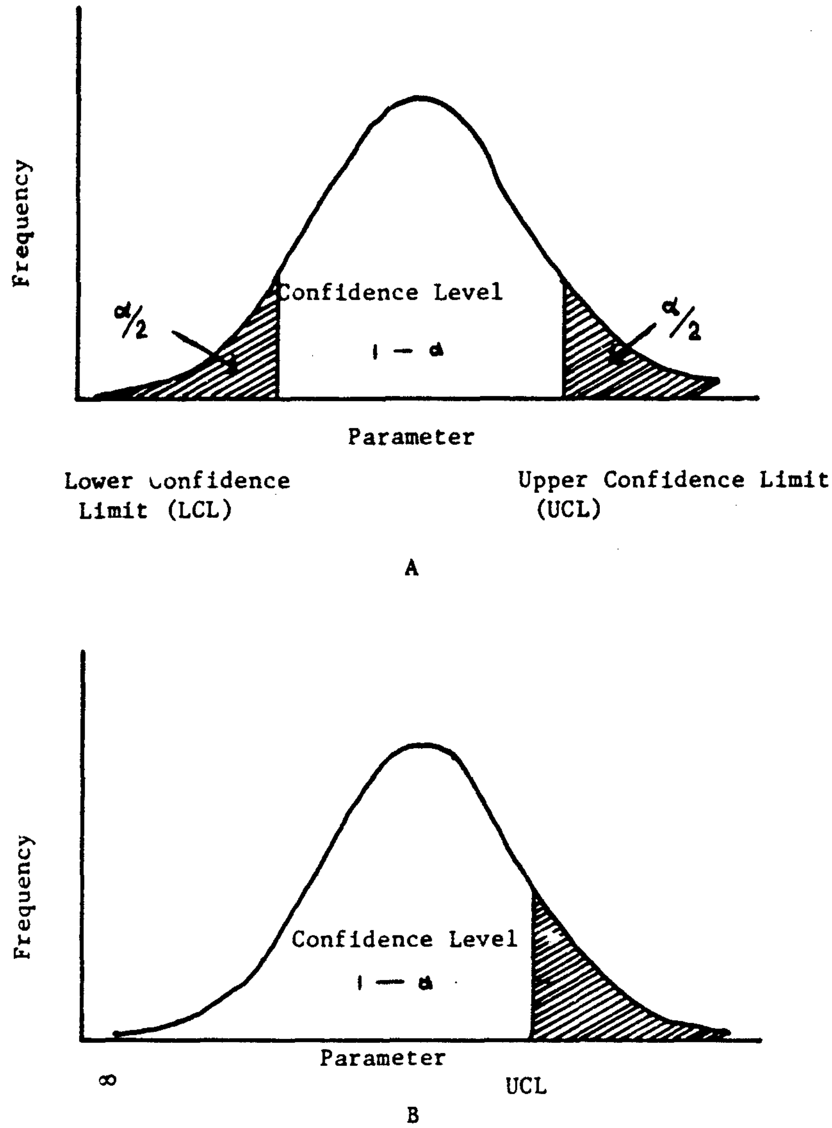


Figure 3

the case of the one-sided test, only one confidence limit is used for testing purposes. Figure 3B is an example where only the UCL is used.

c. The confidence level is 1-α. In both Figures 3A and 3B the confidence levels are the unshaded areas, and the area that α represents is either concentrated at one end of the curve as in the one-sided test or is divided into two sections as in the two-sided test. In the event the same area at one extreme of the distribution curve is considered for the one-sided test and the two-sided test, the confidence levels will be different. For example, to test whether a test item differs from a standard (two-sided test) or whether a test item is less than a standard (one-sided test), a value for α must be chosen. When the one-sided test is used, α is the area at one extreme; and the confidence level is $1-\alpha$ (.95 for $\alpha = .05$). If the same area at the right extreme appears at the left extreme, the result is a two-sided test with the confidence level being one minus the area at both extremes (.90 for .05 area at each extreme). Figure 4 illustrates this difference.

4.11 SIGNIFICANT DIFFERENCE

a. One of the most frequent uses of statistics is in testing for differences. Comparisons are conducted with the appropriate statistical test applied to the results of the test to determine whether there is sufficient justification in concluding that there is a difference either between the test item and the stated requirements or between the test item and a standard item. The test item may be evaluated in such terms as the mean ($\bar{X}$), proportion (P), standard deviation (s), or probable error (PE) while the respective requirements are in terms of the required mean (μ_0), required proportion (λ_0), required standard deviation (σ_0), or required probable error (τ_0). The test item and standard item are evaluated with respect to the same term, e.g., their means. Ordinarily, the statistical test applied to the results observed on a sample will point the way to a decision between a pair of alternatives. For some tests, the two alternative decisions will be formally stated as follows:

(1) The population mean, or any other parameter, of test item A is greater than that of standard item B.
(2) There is no reason to believe that the population mean, or any other parameter, of test item A is greater than that of standard item B.

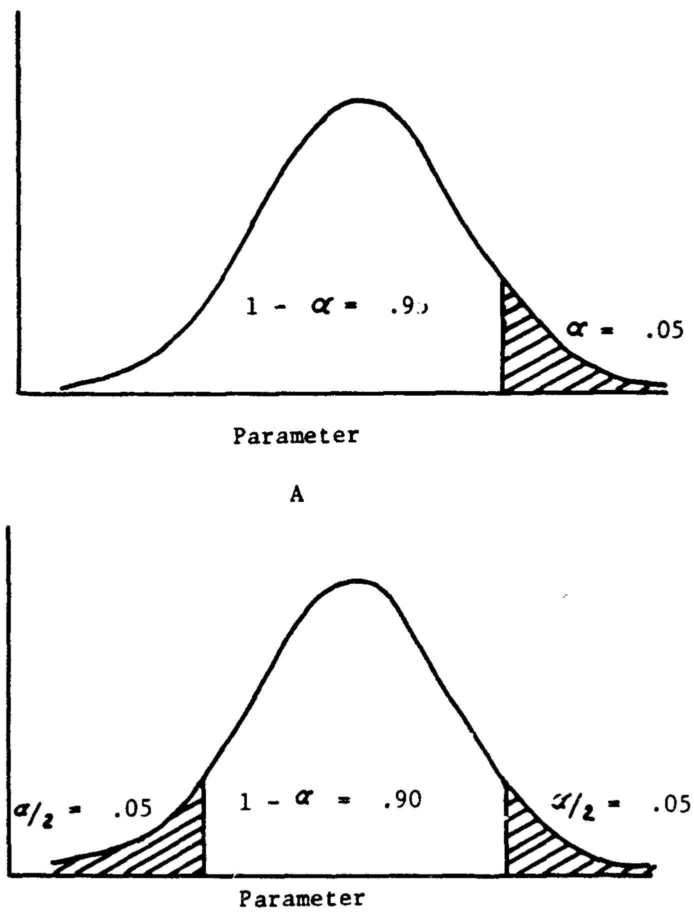
b. In other cases, the formal statement of the two alternative decisions will be:

(1) The population mean, or any other parameter, of test item A is less than that of standard item B.
(2) There is no reason to believe that the population mean, or any other parameter, of test item A is less than that of standard item B.

c. The problem is just how large the difference must be in order to conclude that the two items differ or that the observed difference is "statistically significant"? A difference may be statistically significant and yet be unimportant for all practical purposes. However, the size of the difference is dependent upon several factors:

(1) The amount of variability in the items of each type (test item and standard item).
(2) The number of items of each type.
(3) The amount of risk allowed in stating that a difference exists when there is none (Type I error).

ONE-SIDED AND TWO-SIDED COMPARISON



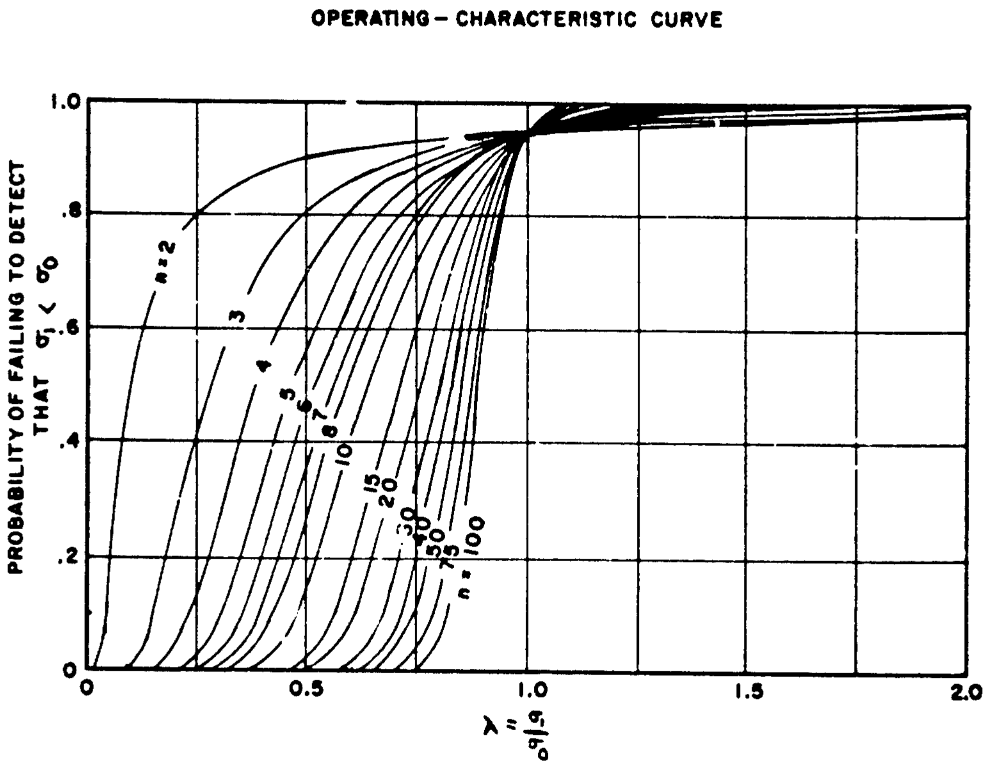
B
Figure 4

4.12 DEGREES OF FREEDOM

The degrees of freedom (d.f.) is a numerical value usually generated by the sample size minus the number of estimated parameters. This procedure may vary depending upon the parameters involved and the distribution and test being applied. If the d.f. is needed, the process for obtaining it will be supplied as a part of the statistical test procedure.

4.13 OPERATING-CHARACTERISTIC CURVE

a. An operating-characteristic (OC) curve is used to determine one of four values given three of them; e.g., the Type II error can be determined if N, α, and λ are known (see Figure 5). An example of an OC curve is illustrated at Figure 5.



Operating characteristics of the one-sided χ^2 test to determine whether the standard deviation σ_1 of a new product is less than the standard deviation σ_0 of a standard (α = .05).

Adapted with permission from Annals of Mathematical Statistics, Vol. 17, No. 2, June 1946, from article entitled "Operating Characteristics for the Common Statistical Tests of Significance" by C. D. Ferris, F. E. Grubbs, and C. L. Weaver.

Figure 5

b. Since tables are easier to read than OC curves, the OC curves, retaining all of their inherent qualities, have been transferred to tables for use in this MTP.

4.14 AMOUNT OF ERROR WHEN DETERMINING SAMPLE SIZE

a. The error size (ε) is a critical factor in determining sample sizes. The next paragraph contains suggestions which will assist the project officer in compromising between excessive sampling and loss of confidence in results obtained.

b. In many cases the Requirements Document will specify a permissible error. If an error is not specified, the project officer must use his judgment. An error of one percent of the required mean or the standard item mean has been used in some of the illustrated cases (see paragraph 6.1.3, page 33, and paragraph 6.2.3, page 43). This is considered appropriate since the timing and recording of the data for the tests may be easily controlled. However when the test item has a large standard deviation, an error as great as five percent may be acceptable in order to keep sample sizes reasonable.

4.15 PARTICULAR DISTRIBUTIONS

4.15.1 NORMAL DISTRIBUTION

a. The normal distribution is by far the most important continuous distribution (see pages 2 to 4). Due to the laws of chance repeated measurements of the same physical quantity occur with such a dispersion that a pattern (distribution) is evident and can be closely approximated by a certain kind of continuous distribution, referred to as the "normal curve of errors." The graph of a normal distribution is a bell-shaped curve that extends indefinitely in both directions (see Figure 6A).

b. The mean is at the peak of the distribution, and the standard deviation determines the spread of the distribution. The physical area from a to b under two normal distributions may not be equal (see Figure 6B). Since construction of separate tables of normal curve areas for each conceivable pair of values for μ and σ is impractical, areas are tabulated only for the so-called standard normal distribution which has a mean of zero and a standard deviation of one. The conversion of a normal distribution to a standard normal distribution is accomplished by using the equation $Z = \frac{x-\mu}{\sigma}$ (see Figure 7A). With the conversion to standard units, Table B-3, page 2-3, may be used. The entries in this table are the areas under the standard normal distribution between the mean ($Z = 0$) and $Z = .01, \ldots, 3.09$. The negative values of Z (areas to the left of the mean) are not needed by virtue of the symmetry of a normal curve about its mean; e.g., the area between $Z = -1.33$ and $Z = 0$ is the same as the area between $Z = 0$ and $Z = 1.33$, which is 0.4082. In the event the percentage of area under the curve to the left of a given value of Z is desired, Table B-3, page 2-3 and this value of Z are used to determine the percent from the mean. If Z is positive, the percentage of the area to the left of Z equals .50 plus the value obtained from Table B-3; e.g., if $Z = .92$ the percent of area is $.50 + .3212$ which is .8212 or 82.12% of the area. If Z is negative, the percentage of the area to the left of Z equals .50 minus the value obtained from Table B-3; e.g., if $Z = -.92$, the percent of area is $.50 - .3212$ which is .1788 or 17.88% of the area.

c. The percentage of area between two Z values can be determined by obtaining the areas for the Z values from Table B-3, page 2-3, and either subtracting the smaller area from the larger area if both Z values are on the same side of the mean or adding the areas if the Z values are on opposite sides of the mean (see Figure 7B).

4.15.2 BIVARIATE NORMAL DISTRIBUTION

a. A bivariate normal distribution is a popula[illegible] in which each member is dependent on two variables (values); e.g., eas[illegible]ng and northing.

NORMAL DISTRIBUTION CURVE

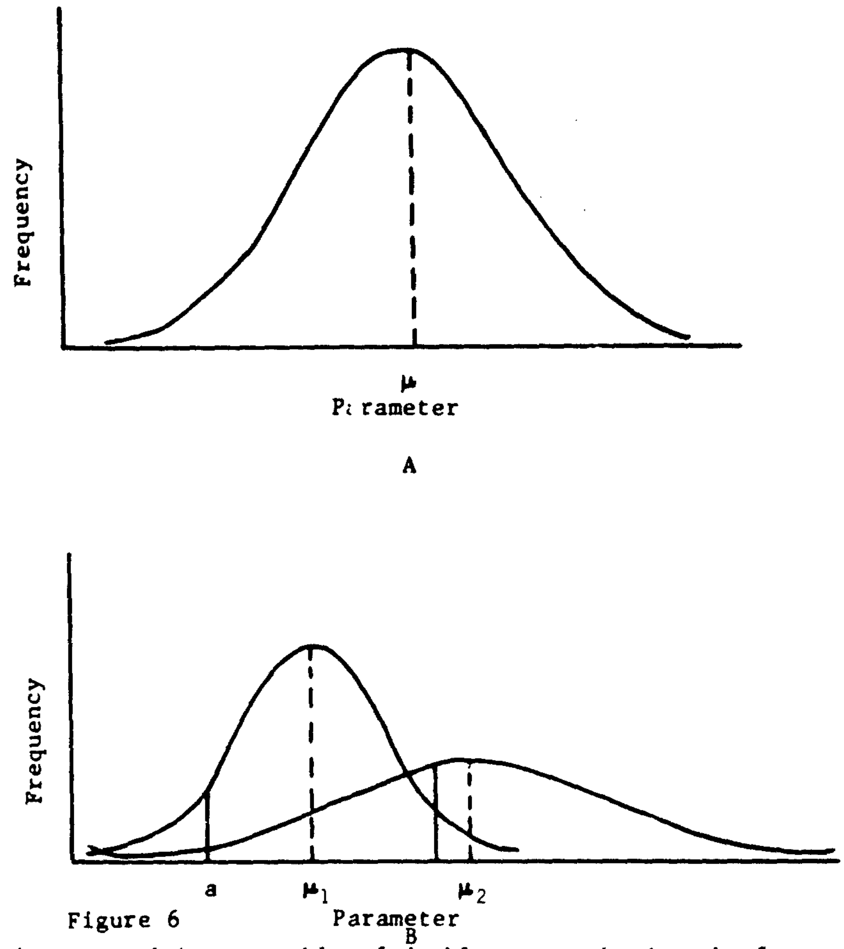


Figure 6

The data may be grouped into a table of double entry showing the frequencies of pairs of values lying within given class intervals. Each row in such a table gives the frequency distribution of the first variable for the members of the population in which the second variable lies within the limits stated on the left of the row. A similar statement can be made about the columns. A grouped frequency distribution of the type in Tables A-1a and A-1b, page 1-1 may be termed a bivariate frequency distribution.

b. The shape of the bivariate normal population is a normal distribution in three dimensions, rising to its greatest height at the center and fading away to tangency (see Figure 8). Some properties of the bivariate normal distribution are:

COMPUTING AREA UNDER NORMAL CURVE

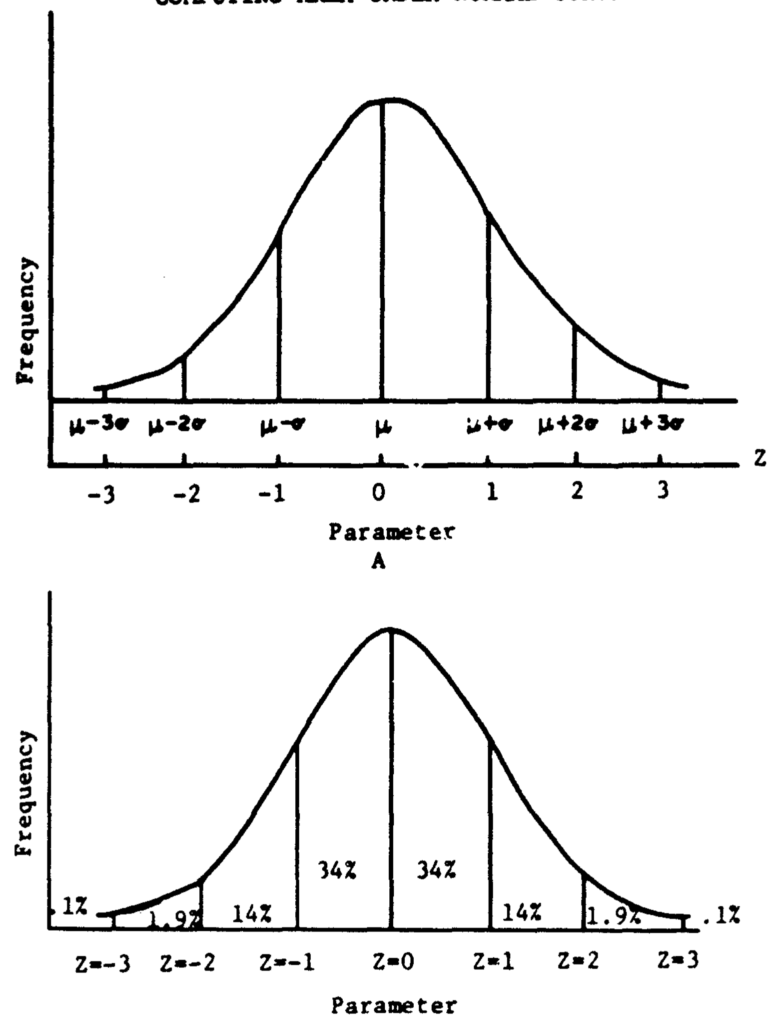


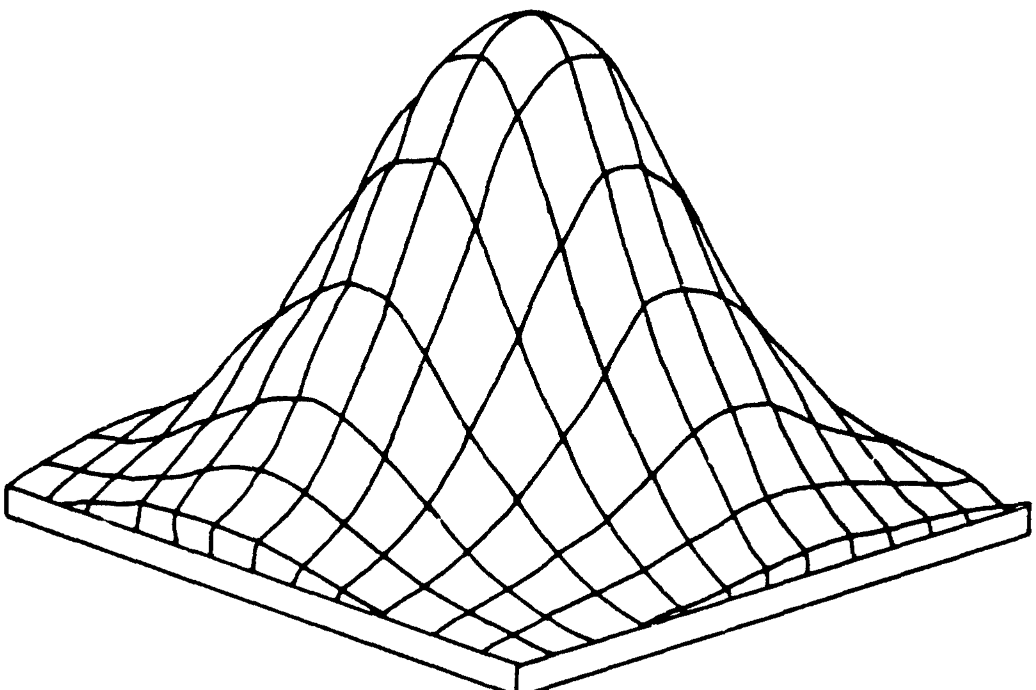
Figure 7

(1) Each section perpendicular to each axis can be transformed into a normal distribution. This means that the data from each column and each row are samples from a normal distribution.
(2) All of the transformed distributions perpendicular to each axis have the same population standard deviation, and all of the population means lie on a straight line.
(3) The distribution is dependent on the standard deviations, the means, and the covariance (amount of dependency) of the two variables.

4.15.3 BINOMIAL DISTRIBUTION

a. Problems arise as to the number of successes or failures in N trials. To handle problems of this type, a special probability function,

BIVARIATE NORMAL DISTRIBUTION CURVE



The ideal symmetrical bivariate
Figure 8

the binomial distribution, is needed. This distribution applies only when the probability of a success or failure remains constant from trial to trial and the trials are independent. Table B-18, page 2-74, contains tables for 0 to 100 failures. Table B-18 is used to establish reliability, confidence level, and sample size for a given number of failures. Paragraph 10.1, page 119, explains the use of Table B-18 and gives various applications of the binomial distribution.

b. If the probability of an event occurring is p, the mean of the binomial distribution is Np and the standard deviation is N(p) (1-p). As N increases, the binomial distribution tends to the normal.

4.15.4 POISSON DISTRIBUTION

a. The Poisson distribution was developed for studying rare events where N is large and the mean (Np) is much less than 15. Under such conditions the binomial distribution remains noticeably skew, and the normal approximation is unsatisfactory. The Poisson distribution is a limiting form of the binomial distribution such that as N tends to infinity while p tends to zero, $\mu = Np$ is constant.

b. If a series of independent distributions are each Poisson distributions with means μ_1, μ_2, ..., the sum follows a Poisson distribution with mean equal to $\mu_1 + \mu_2 + \ldots$

c. The Poisson distribution plays an important role in the inspection and quality control of manufactured goods. It is used to ascertain that the proportion of defective items in a large lot is small.

d. The distribution is dependent on the mean (μ), which equals the variance (σ^2).

4.15.5 EXPONENTIAL DISTRIBUTION

a. For systems that are renewed by repair or maintenance, a failure rate is employed. This rate is thus given as age-dependent. The exponential distribution e^{-x} may characterize the lifetime to failure for one or more of the following reasons:

(1) The principal cause of failure is a chance effect from the environment.
(2) A large serial system; i.e., one which fails when any part fails, will have an exponential lifetime to failure if the failures are independent and if repair times are negligibly short.
(3) There may be many independent external possible causes of failure that tend simultaneously and continuously to threaten the system.

b. The combined effect of (1), (2), and (3) can be summarized as follows: if an operating system has a very large number of components and if the components are sufficiently independent; then the failure of one component is an independent binary process. In this case there are only two possible outcomes for all observations of the component; i.e., success or failure. Each observation selects one outcome at random, and the observations are independent. Consequently, the average operating time until failure of each component causes the probability of the system operating to decrease rapidly and exponentially as operating time increases (see Figure 9).

4.15.6 STUDENT t DISTRIBUTION

The Student t distribution approximates the normal distribution and is symmetrical about the mean. For large samples the standardized mean is zero, and the standard deviation is one. The distinction is obvious only for samples of less than 30. With samples of less than 30, there is a slightly higher probability of values falling into the two tails. Figure 10 illustrates the t distribution and its relationship to the normal distribution.

EXPONENTIAL RELIABILITY CURVE

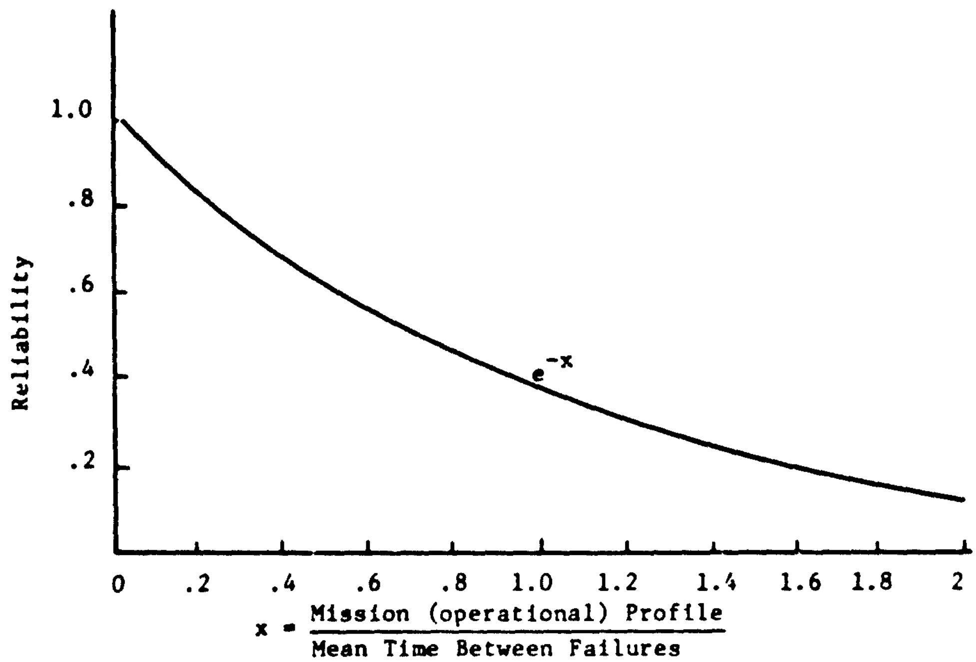


Figure 9

4.15.7 F DISTRIBUTION

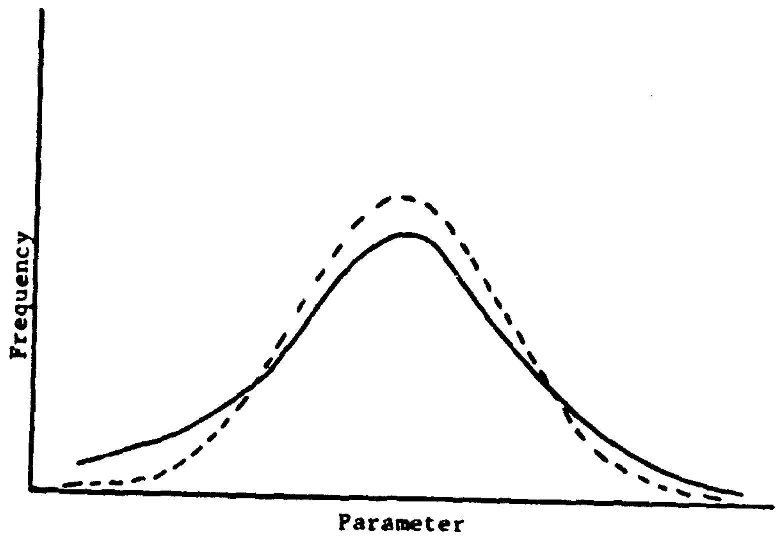
a. If two sample variances are generated from two samples which have normal distributions, the ratio of the two variances (called a variance ratio) forms a sampling distribution called the F Distribution. The distribution is dependent on the respective degrees of freedom, N_1-1 and N_2-1. Figure 11 illustrates the F distribution curve.

b. The F distribution is very helpful in determining the equality of two population standard deviations (see paragraph 7.3, page 74, for method and example).

4.15.8 CHI-SQUARE DISTRIBUTION

a. For many tests σ is needed but is unknown. Although s is by far the most popular estimate of the standard deviation of a population, it is not the only estimate; and confidence intervals for σ based on s are

STUDENT t DISTRIBUTION CURVE



— Student t-distribution curve

– – – Normal distribution curve

Figure 10

F DISTRIBUTION CURVE

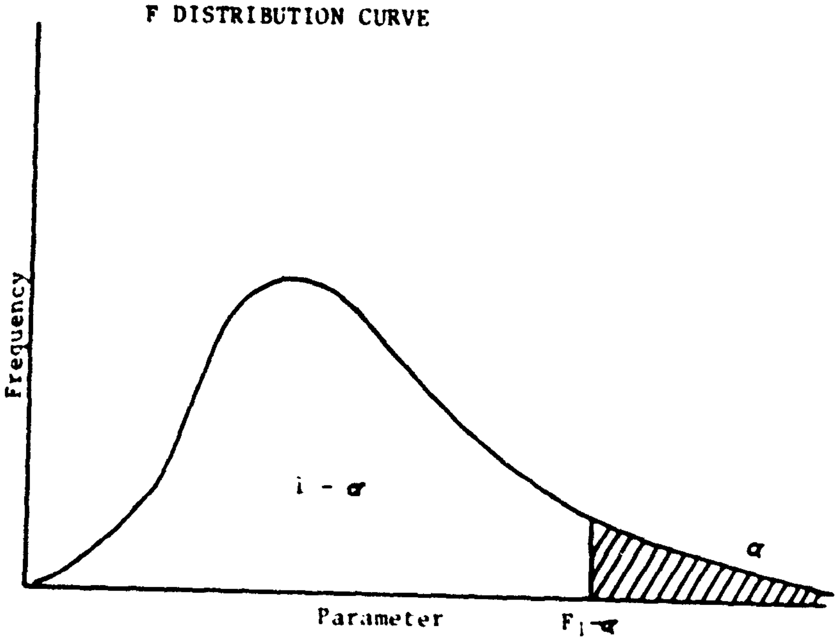


Figure 11

often stated with a given confidence level. The theory on which these confidence intervals are based assumes that the population from which the sample is obtained has roughly the shape of a normal distribution and is called the chi-square (χ^2) distribution. An example of a chi-square distribution is shown in Figure 12A; in contrast to the normal and t distribution, its domain is restricted to the nonnegative real numbers.

b. The χ^2 distribution is also different from those previously discussesd in that the area under the curve is summed from the χ^2 point to the right. The value for $\chi^2_{1-\alpha}$ represents an area of α under the curve (right-hand tail, see Figure 12A), while χ^2_α represents an area of $1-\alpha$ to the right under the curve (see Figure 12B). Due to the shape of the χ^2 curve the point values of $\chi^2_{\alpha/2}$ and $\chi^2_{1-\alpha/2}$ will be different even though the significance levels are equal (see Figure 12C). This distinction is important due to the fact the distribution is not symmetrical; thus, a table containing values corresponding to areas in either tail of the distribution is necessary. Thus, with a confidence level of $1-\alpha$,

$$\frac{(N-1)s^2}{\chi^2_{\alpha/2}} < \sigma < \frac{(N-1)s^2}{\chi^2_{1-\alpha/2}}$$

As the sample size decreases, the interval for σ becomes wider. Therefore, in most tests applying the chi-square distribution, a normal sample size is needed ($N \geq 30$).

4.16 ROUND OFF PROCEDURES

a. Since all measuring equipment has limited accuracy, the measurements are also of limited accuracy and thus consist of numbers which have been rounded off; e.g., if an instrument is accurate to tenths of minutes and a time measurement is 12.2 minutes, the time may actually have been any value between 12.15 and 12.25 minutes.

b. When test data are used to compute test item characteristics, such as the mean and standard deviation, the results must be consistent with the original data; i.e., the mean weight of a group of projectiles cannot be more accurate than the individual weights used to compute the mean. The following are some basic rules concerning significant figures and the rounding of data:

(1) Significant figures (significant digits) are the digits of a number that begin with the first digit on the extreme left that is not a zero and that end with the last digit on the right that is not a zero or that is a zero which is considered accurate. For example:

(a) 12304 has five significant digits.
(b) 1.0200 has five significant digits.

(When a number ends with a zero which is on the right of the decimal point, the zero is significant.)

CHI-SQUARE DISTRIBUTION CURVE

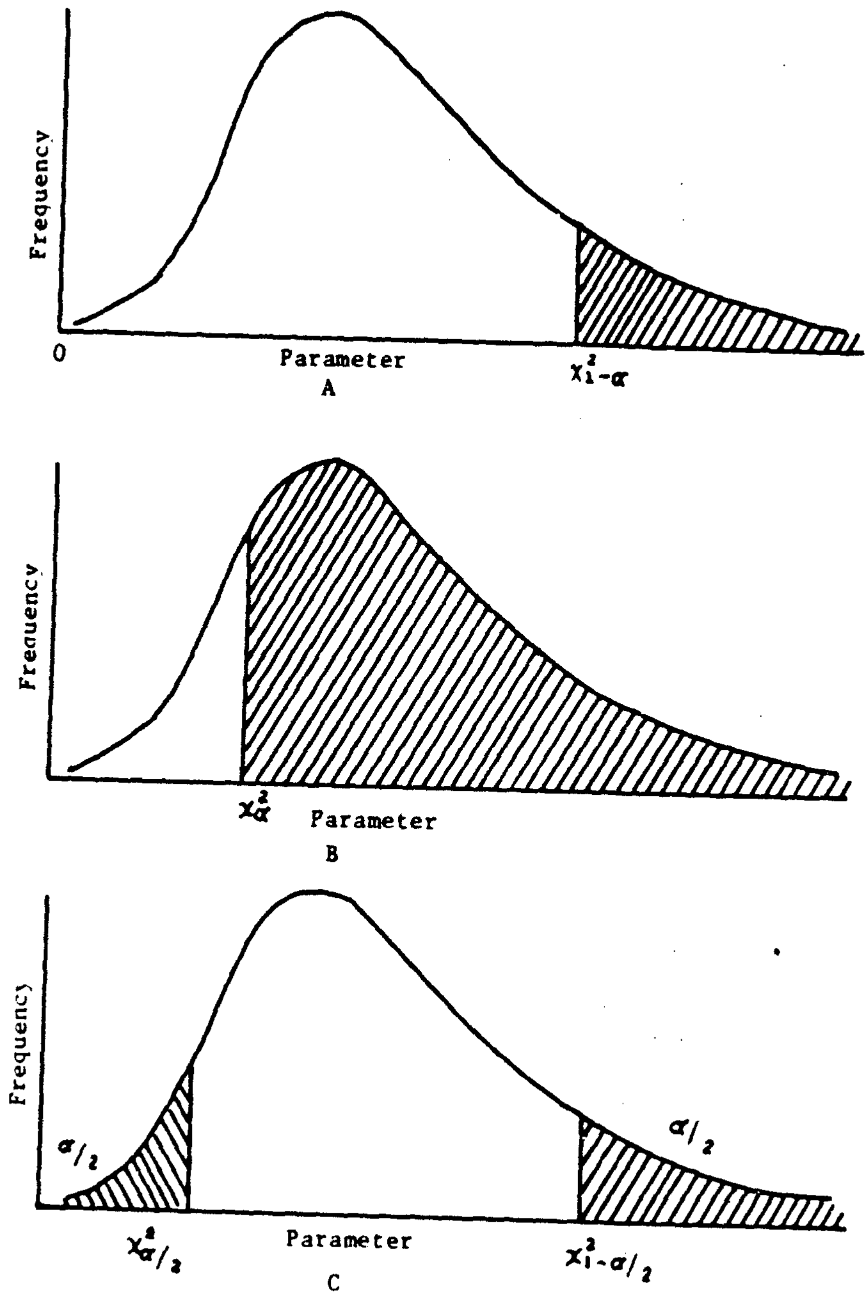


Figure 12

(c) .0003 has one significant digit.
(d) 5200 has two, three, or four significant digits depending on whether the instrument used to obtain this measurement is accurate to hundreds, tens, or units, respectively.
(e) 100.0 has four significant figures.

(2) The result of a series of arithmetic operations must be rounded off to an accuracy consistent with the least accurate measurement in the original data. The generally recommended procedure is to carry at least two extra significant digits throughout the computations before rounding off the final result. (If a calculator is used for the computations will depend on the capacity of the calculator.)

(3) Some numbers are considered totally accurate due to the fact that they are not the result of a measurement, and thus they do not limit the number of significant digits in the final result; e.g., degrees of freedom (d.f.), required or desired significance levels, and requirements such as μ_0 and σ_0. Values taken from tables are accurate only to the number of digits given in the table.

(4) Generally, rounding off of numbers will be performed in accordance with the standard Field Artillery round off rules; however, special procedures must be used in the following cases:

(a) When the sample size which must be tested in order to prove an hypothesis is calculated, the answer must be a whole number since a fraction of a sample cannot be tested. Furthermore, if 36.2 samples are needed, then 36 samples are not enough. Therefore, calculated sample sizes must be rounded off to the next larger whole number.
(b) When one-sided or two-sided confidence limits are calculated, the answer must usually be rounded off. However, since the unrounded limits (UCL and LCL) define an interval for a specified confidence level, care must be used to insure that the desired confidence level is not decreased when the limits are rounded off. Since the confidence level increases as the interval increases, the UCL must always be rounded up; and the LCL must always be rounded down.

5. MEDIAN

5.1 OBJECTIVE

To determine the midpoint of the readings such that half of the readings are above and half are below the median.

5.2 DATA REQUIRED

A list of sample readings.

5.3 PROCEDURE

a. N is odd.

(1) List the readings in descending or ascending order.
(2) Use the middle reading for the median.

b. N is even

(1) List the readings in descending or ascending order.
(2) Use the average of the two middle readings for the median.

5.4 EXAMPLE

a. Case I.

Given:
N = 5

Procedure:	Example:
(1) List the readings in order.	(1) 15 13.5 12.7 12 11.9
(2) Use the $\frac{N+1}{2}$ reading for the median.	(2) $\frac{N+1}{2} = \frac{5+1}{2}$ $= 3$ The median is the 3rd reading. The median = 12.7

b. Case II

Given:
N = 6

Procedure:	Example:
(1) List the readings in order	(1) 250 245 230 228 225 224.6

(2) Use the $\frac{N}{2}$ and $\frac{N}{2}$ + readings to compute the median which is the average of the two. The median =

$$= \frac{(\frac{N}{2}\text{ reading}) + (\frac{N}{2} + 1\text{ reading})}{2}$$

(2) $\frac{N}{2} = 3$

$\frac{N}{2} + 1 = 4$

Use the 3rd and 4th readings to compute:

The median $= \frac{230+228}{2}$

$= \frac{458}{2}$

$= 229$

5.5 ANALYSIS

The median equals the mean if the population is normally distributed; otherwise, it is only another measure of central location, which denotes the midpoint of the total dispersions.

6. MEAN

6.1 ESTIMATE OF THE POPULATION MEAN (μ).

6.1.1 BEST SINGLE ESTIMATE OF μ.

6.1.1.1 OBJECTIVE

To determine the best point estimate of the population mean for a normal distribution.

6.1.1.2 DATA REQUIRED

A list of sample readings; e.g., the time required for prepare for action under daylight conditions.

6.1.1.3 PROCEDURE

a. Sum the list of data for the parameter.

b. Divide the sum by the number of readings recorded to obtain the mean of the parameter.

6.1.1.4 EXAMPLE

Given:
Sample data at Table A-2a, page 1-2.

Procedure:

a. Sum the parameter.

Example:

a. Sum = 1037.0 min

b. Compute:

$\overline{X}$ = Sum/no. of readings

NOTE: Mean may be expressed as MTBF, MMBF, MTTR.

b. $\overline{X}$ = 1037.0/12

= 86.417 min.

= 86.4 min.

6.1.1.5 ANALYSIS

The sample mean, or average, is a value which is typical or representative of a set of data. The mean is the most commonly used measure of central location.

6.1.2 CONFIDENCE INTERVAL ESTIMATES

6.1.2.1 TWO-SIDED INTERVAL WITH σ UNKNOWN

6.1.2.1.1 OBJECTIVE

To determine a two-sided confidence interval which is expected to bracket μ at the desired confidence level when σ is unknown.

6.1.2.1.2 DATA REQUIRED

A list of sample readings.

6.1.2.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}$ (see paragraph 6.1.1.3, page 26).

c. Compute s (see paragraph 7.1.1.3, page 64).

d. Use Table B-5, to obtain $t_{1-\alpha/2}$ for N-1 d.f.

e. Compute ε as follows:

(1) Multiply s by step d.
(2) Divide step (1) by the square root of N.

f. Add ε to $\overline{X}$ to obtain the UCL, and subtract ε from $\overline{X}$ to obtain the LCL.

g. Conclude that μ is equal to or between the UCL and LCL at the desired confidence level.

6.1.2.1.4 EXAMPLE

Given:
Sample data at Table A-2a, page 1-2.

Procedure:

a. Choose the confidence level $(1-\alpha)$.

b. Compute $\bar{X}$.

c. Compute s.

d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha/2}$ for N-1 d.f.

e. Compute:

$$\varepsilon = \frac{t_{1-\alpha/2}(s)}{\sqrt{N}}$$

f. Compute:

$$UCL = \bar{X} + \varepsilon$$
$$LCL = \bar{X} - \varepsilon$$

g. Conclude that $\mu \leq UCL$ and $\mu \geq LCL$ (the upper and lower confidence limit values) at a $100(1-\alpha)\%$ confidence level.

Example:

a. $\alpha = .05$
$1-\alpha = .95$
$1-\alpha/2 = .975$

b. $\bar{X} = 1037.0/12$
$= 86.417$ min.
$= 86.4$ min.

See paragraph 6.1.1.4, page 26, for computations.

c. $s = \sqrt{\frac{51.9563}{12-1}}$

$= \sqrt{4.7233}$

$= 2.173$ min.

$= 2.2$ min.

See paragraph 7.1.1.4, page 65 for computations.

d. $t_{.975}$ for 11 d.f. $= 2.201$

e. $\varepsilon = \frac{(2.201)\ (2.173)}{\sqrt{12}}$

$= \frac{4.783}{3.464}$

$= 1.381$

f. $UCL = 86.417 + 1.381$
$= 87.798$ min.
$= 87.8$ min.
$LCL = 86.417 - 1.381$
$= 85.036$ min.
$= 85.0$ min.

g. Conclude that $\mu \leq 87.8$ and $\mu \geq 85.0$ at a 95% confidence level.

6.1.2.1.5 ANALYSIS

The two-sided interval surrounds μ such that $\mu \leq$ UCL and $\mu \geq$ LCL at a $100(1-\alpha)\%$ confidence level. Due to σ being unknown, the confidence interval will be as large as possible. Application of the Student t test, which is designed for small sample size, also contributes to a larger confidence interval than a normal test for a particular ε.

6.1.2.2 ONE-SIDED INTERVAL WITH σ UNKNOWN

6.1.2.2.1 OBJECTIVE

To determine a one-sided confidence interval such that μ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level when σ is unknown.

6.1.2.2.2 DATA REQUIRED

A list of sample readings.

6.1.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}$ (see paragraph 6.1.1.3, page 26).

c. Compute s (see paragraph 7.1.1.3, page 64).

d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for N-1 d.f.

e. Compute ε as follows:

(1) Multiply s by step d.
(2) Divide step (1) by square root of N.

f. Add ε to $\overline{X}$ to obtain the UCL (or subtract ε from $\overline{X}$ to obtain the LCL).

g. Conclude that μ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level.

6.1.2.2.4 EXAMPLE

Given:
Sample data at Table A-2a, page 1-2.

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .01$ $1-\alpha = .99$

b. Compute $\overline{X}$.

b. $\overline{X}$ = 86.417 min.
= 86.4 min.

c. Compute s.

c. s = 2.173 min.
= 2.2 min.
See paragraph 6.1.2.1.4, page 27, for computations.

d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for N-1 d.f.

d. $t_{.99}$ for 11 d.f. = 2.718

e. Compute:

$$\varepsilon = \frac{t_{1-\alpha}(s)}{\sqrt{N}}$$

e.
$$\varepsilon = \frac{(2.718)(2.173)}{\sqrt{12}} = \frac{5.906}{3.464} = 1.705$$

f. Compute:

UCL = $\overline{X} + \varepsilon$
(or LCL = $\overline{X} - \varepsilon$

f. UCL = 86.417 + 1.705
= 88.122
= 88.2 min.
(or LCL = 86.417 - 1.705
= 84.712
= 84.7 min.)

g. Conclude that $\mu \le$ UCL (or $\mu \ge$ LCL) at a 100(1-α)% confidence level.

g. Conclude that $\mu \le$ 88.2 min. (or $\mu \ge$ 84.7 min.) at a 99% confidence level.

6.1.2.2.5 ANALYSIS

The one-sided interval surrounds μ such that $\mu \le$ UCL (or $\mu \ge$ LCL) at a 100(1-α)% confidence level. Since α is concentrated at one end of the curve, the value $t_{1-\alpha}$ is used instead of the value of $t_{1-\alpha/2}$.

6.1.2.3 TWO-SIDED INTERVAL WITH σ KNOWN

6.1.2.3.1 OBJECTIVE

To determine a two-sided confidence interval which is expected to bracket μ at the desired confidence level when σ is known.

6.1.2.3.2 DATA REQUIRED

A list of sample readings and σ, which is known from a standard item, history, or a Requirements Document.

6.1.2.3.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}$ (see paragraph 6.1.1.3, page 26).

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

d. Compute ε as follows:

(1) Multiply σ by step c.
(2) Divide step (1) by the square root of N.

e. Add ε to $\overline{X}$ to obtain the UCL, and subtract ε from $\overline{X}$ to obtain the LCL.

f. Conclude that μ is equal to or between the UCL and LCL at the desired confidence level.

6.1.2.3.4 EXAMPLE

Given:
Sample data at Table A-2a, Page 1-2.
σ = 2.0 min.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. $\alpha = .05$ $1-\alpha = .95$ $1-\alpha/2 = .975$
b. Compute $\overline{X}$.	b. $\overline{X} = 86.417$ min. $= 86.4$ min. See paragraph 6.1.1.4, page 26, for computations.
c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.	c. $Z_{.975} = 1.960$
d. Compute: $\varepsilon = \frac{Z_{1-\alpha/2}(\sigma)}{\sqrt{N}}$	d. $\varepsilon = \frac{(1.96)(2.0)}{\sqrt{12}}$ $= \frac{3.920}{3.464}$ $= 1.132$
e. Compute: $UCL = \overline{X} + \varepsilon$ $LCL = \overline{X} - \varepsilon$	e. UCL = 86.417 + 1.132 = 87.549 = 87.6 min. LCL = 86.417 - 1.132 = 85.285 = 85.2 min.

f. Conclude that $\mu \leq$ UCL and $\mu \geq$ LCL at a $100(1-\alpha)\%$ confidence level.

f. Conclude that $\mu \leq 87.6$ min and $\mu \geq 85.2$ min at a 95% confidence level.

6.1.2.3.5 ANALYSIS

The two-sided interval surrounds μ such that $\mu \leq$ UCL and $\mu \geq$ LCL at a $100(1-\alpha)\%$ confidence level. When the value of σ is known, this procedure will be used in preference to that in paragraph 6.1.2.1, page 27, because it will, in most cases, lead to a narrower confidence interval relative to the known σ.

6.1.2.4 ONE-SIDED INTERVAL WITH σ KNOWN

6.1.2.4.1 OBJECTIVE

To determine a one-sided confidence interval such that μ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level when σ is known.

6.1.2.4.2 DATA REQUIRED

A list of sample readings and σ, which is known from a standard item, history, or Requirements Document.

6.1.2.4.3 PROCEDURE

a. Chodse the desired confidence level.

b. Compte $\overline{X}$ (see paragraph 6.1.1.3, page 26).

c. Use Table B-4, page 2-4, to obtain $z_{1-\alpha}$.

d. Compute ε as follows:

(1) Multiply σ by step c.
(2) Divide step (1) by the square root of N.

e. Add ε to $\overline{X}$ to obtain the UCL (or subtract ε from $\overline{X}$ to obtain the LCL).

f. Conclude the μ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level.

6.1.2.4.4 EXAMPLE

Given:
Sample data at Table A-2a, page 1-2.
$\sigma = 2.0$ min.

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .01$ $1-\alpha = .99$
b. Compute $\overline{X}$	b. $\overline{X} = 86.417$ $= 86.4$ min. See paragraph 6.1.1.4, page 26, for computations.
c. Use Table B-4, page 2-4 to obtain $Z_{1-\alpha}$.	c. $Z_{.99} = 2.326$
d. Compute: $\varepsilon = \frac{Z_{1-\alpha}(\sigma)}{\sqrt{N}}$	d. $\varepsilon = \frac{(2.326)(2.0)}{\sqrt{12}} = \frac{4.652}{3.464} = 1.343$
e. Compute: $UCL = \overline{X} + \varepsilon$ (or $LCL = \overline{X} - \varepsilon$)	e. UCL = 86.417 + 1.343 = 87.760 = 87.8 min. (or LCL = 86.417 - 1.343 = 85.074 - 85.0 min.)
f. Conclude that $\mu \le$ UCL (or $\mu \ge$ LCL) at a $100(1-\alpha)\%$ confidence level.	f. Conclude that $\mu \le 87.8$ min (or $\mu \ge 85.0$ min.) at a 99% confidence level.

6.1.2.4.5 ANALYSIS

The one-sided interval surrounds μ such that $\mu \le$ UCL (or $\mu \ge$ LCL) at a $100(1-\alpha)\%$ confidence level. Since α is concentrated at one end of the curve, the value of $Z_{1-\alpha}$ is used instead of the value of $Z_{1-\alpha/2}$. When σ is known, this procedure will be used in preference to that in paragraph 6.1.2.2, page 29.

6.1.3 SAMPLE SIZE REQUIRED TO ESTIMATE THE POPULATION MEAN

6.1.3.1 SAMPLE SIZE REQUIRED TO ESTIMATE μ WITH σ UNKNOWN

6.1.3.1.1 OBJECTIVE

To determine the sample size (N_t) required in order to state that μ is equal to or between $\overline{X} + \varepsilon$ and $\overline{X} - \varepsilon$ at the desired confidence level when σ is unknown.

6.1.3.1.2 DATA REQUIRED

The s and the d.f. from a previously tested sample.

6.1.3.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Choose the allowable amount of error.

c. Use Table B-5, page 2-5, to obtain $t_{1-\alpha/2}$ for N-1 d.f.

d. Compute N_t as follows:

(1) Square step c.
(2) Square s.
(3) Square ε.
(4) Multiply step (1) by step (2).
(5) Divide step (4) by step (3).
(6) Round step (5) to the next larger whole number.

e. Conclude that N_t samples are required in order to state that μ is equal to or between $\overline{X} + \varepsilon$ and $\overline{X} - \varepsilon$ at the desired confidence level.

6.1.3.1.4 EXAMPLE

Given:
s = 2.2 min.
N = 12

Procedure:	Example:
a. Choose the confidence level (1-α).	a. $\alpha = .05$ $1-\alpha/2 = .975$
b. Choose ε.	b. $\varepsilon = .8$ min.
c. Use Table B-5, page 2-5 to obtain $t_{1-\alpha/2}$ for N-1 d.f.	c. $t_{.975}$ for 11 d.f. = 2.201
d. Compute: $N_t = \frac{(t_{1-\alpha/2})^2 (s)^2}{\varepsilon^2}$	d. $N_t = \frac{(2.201)^2(2.2)^2}{(.8)^2}$ $= \frac{(4.844)(4.84)}{(.64)}$ $= \frac{23.445}{64}$ $= 36.632$ $= 37$
e. Conclude that N_t samples are required in order to state that $\mu \le \overline{X} + \varepsilon$ and $\mu \ge \overline{X} - \varepsilon$ at a 100(1-α)% confidence level.	e. If 37 samples are tested and $\overline{X}$ computed, conclude that $\mu \le \overline{X} + .8$ min. and $\mu \ge \overline{X} - .8$ min. at a 95% confidence level.

6.1.3.1.5 ANALYSIS

If N_t samples are tested and $\overline{X}$ is computed, conclude that $\mu \leq \overline{X} + \varepsilon$ and $\mu \geq \overline{X} - \varepsilon$ at a 100(1-α)% confidence level.

6.1.3.2 SAMPLE SIZE REQUIRED TO ESTIMATE μ WITH σ KNOWN

6.1.3.2.1 OBJECTIVE

To determine the N_t required in order to state that μ is equal to or between $\overline{X} + \varepsilon$ and $\overline{X} - \varepsilon$ at the desired confidence level when σ is known.

6.1.3.2.2 DATA REQUIRED

σ, which is known from a standard item, history, or Requirements Document.

6.1.3.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Choose the allowable amount of error.

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

d. Compute N_t as follows:

(1) Square step c.
(2) Square σ.
(3) Square ε.
(4) Multiply step (1) by step (2).
(5) Divide step (4) by step (3).
(6) Round step (5) to the next larger whole number.

e. Conclude that N_t samples are required in order to state that μ is equal to or between $\overline{X} + \varepsilon$ and $\overline{X} - \varepsilon$ at the desired confidence level.

6.1.3.2.4 EXAMPLE

Given:
σ = 2.0 min.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. α = .05 1-α = .95 1-α/2 = .975
b. Choose ε.	b. ε = .8 min.

c. Use Table B-4, page 2-4 to obtain $Z_{-\alpha/2}$.

c. $Z_{.975} = 1.960$

d. Compute:

$$N_t = \frac{(Z_{-\alpha/2})^2(\sigma)^2}{\varepsilon^2}$$

d.
$$N_t = \frac{(1.96)^2(2.0)^2}{(.8)^2} = \frac{(3.842)(4.00)}{.64} = \frac{15.37}{.64} = 24.02 = 25$$

e. Conclude that N_t samples are required in order to state that $\mu < \bar{X} + \varepsilon$ and $\mu > \bar{X} - \varepsilon$ at a $100(1-\alpha)\%$ confidence level.

e. If 25 samples are tested and $\bar{X}$ computed, conclude that $\mu < \bar{X} + .8$ min. and $\mu > \bar{X} - .8$ min. at a 95% confidence level.

6.1.3.2.5 ANALYSIS

If N_t samples are tested and $\bar{X}$ is computed conclude that $\mu < \bar{X} + \varepsilon$ and μ $\bar{X} - \varepsilon$ at a $100(1-\alpha)\%$ confidence level.

6.2 COMPARING AN OBSERVED MEAN ($\bar{X}$) TO A REQUIREMENT (μ_o)

a. An observed mean is generated from a sample and is representative of μ. This value of $\bar{X}$ is then compared to a stated requirement (μ_0). However, looking at the values of $\bar{X}$ and μ_0 to decide whether μ is greater than μ_0 or μ is less than μ_0 at a confidence level is insufficient. Since the decision pertains to the population, statistical tests must be applied to $\bar{X}$ to determine whether μ is greater than μ_0 or μ is less than μ_0.

b. There exist two possibilities for the relationship of $\bar{X}$ to However, for each possibility there are two approaches; i.e., σ may be known or unknown; and the appropriate test must be chosen on that basis. Following are the assumptions and the circumstances for each possible relationship:

(1) $\bar{X}$ greater than μ_0.

(a) The null hypothesis is greater than μ_0.
(b) The alternative hypothesis is there is no reason to believe μ is greater than μ_0.
(c) The use of this test is appropriate when μ_0 is a maximum value for μ to satisfy. In the event that μ must not be greater than μ_0, this test would be appropriate.

(2) $\overline{X}$ less than μ_o.

(a) The null hypothesis is μ is less than μ_o.
(b) The alternative hypothesis is there is no reason to believe μ is less than μ_o.
(c) The use of this test is appropriate when μ_o is a minimum value for μ to satisfy. In the event that μ must meet or exceed μ_o, this test would be appropriate.

6.2.1 $\overline{X}$ GREATER THAN μ_o

6.2.1.1 $\overline{X}$ GREATER THAN μ_o WITH σ UNKNOWN

6.2.1.1.1 OBJECTIVE

To determine whether μ is greater than μ_o at the desired confidence level when the value of σ is unknown.

6.2.1.1.2 DATA REQUIRED

A list of sample readings.

6.2.1.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}$ (see paragraph 6.1.1.3, page 26).

c. Compute s (see paragraph 7.1.1.3, page 64).

d. Use Table B-5, page 2-5, to obtain $t_{\Gamma\alpha}$ for N-1 d.f.

e. Compute ε as follows:

(1) Multiply s by step d.
(2) Divide step (1) by the square root of N.

f. Subtract ε from $\overline{X}$ to obtain the LCL which is the lower bound for μ at the desired confidence level. The confidence interval for μ is from the LCL to $+\infty$.

g. If μ_o is less than the LCL, decide that μ is greater than μ_o; otherwise, there is no reason to believe μ is greater than μ_o at the desired confidence level.

6.2.1.1.4 EXAMPLE

Given:
μ_o = 85.0 min.
Sample data at Table A-2a, page 1-2.

Procedure:

a. Choose the confidence level $(1-\alpha)$.

b. Compute $\overline{X}$.

c. Compute s.

d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for N-1 d.f.

e. Compute:

$$\varepsilon = \frac{t_{1-\alpha}(s)}{\sqrt{N}}$$

f. Compute:

$$LCL = \overline{X} - \varepsilon$$

g. If $\mu_0 < LCL$, decide that $\mu > \mu_0$; otherwise, there is no reason to believe $\mu > \mu_0$ at a $100(1-\alpha)\%$ confidence level.

Example:

a. $\alpha = .05$
$1-\alpha = .95$

b. $\overline{X} = 86.417$
$= 86.4$ min.
See paragraph 6.1.2.1.4 b, page 28, for computations.

c. $s = 2.173$
$= 2.2$ min.
See paragraph 6.1.2.1.4 c, page 28, for computations.

d. $t_{.95}$ for 11 d.f. $= 1.796$

e. $\varepsilon = \frac{(1.796)(2.173)}{\sqrt{12}}$
$= \frac{3.903}{3.464}$
$= 1.127$

f. $LCL = 86.417 - 1.127$
$= 85.290$
$= 85.2$ min.

g. Since $85.0 < 85.2$, decide that $\mu > 85.0$ min. at a 95% confidence level.

6.2.1.1.5 ANALYSIS

If $\mu_0 < LCL$, the null hypothesis that $\mu > \mu_0$ is accepted; otherwise. there is no reason to believe $\mu > \mu_0$ at a $100(1-\alpha)\%$ confidence level when T is unknown. The $100\ (1-\alpha)\%$ confidence interval for μ is from the LCL to $+\infty$.

6.2.1.2 $\overline{X}$ GREATER THAN μ_0 WITH σ KNOWN

6.2.1.2.1 OBJECTIVE

To determine whether μ is greater than μ_0 at the desired confidence level when σ is known.

6.2.1.2.2 DATA REQUIRED

A list of sample readings and σ, which is known from a standard item, history, or Requirements Document.

6.2.1.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}$ (see paragraph 6.1.1.3, page 26).

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

d. Compute ε as follows:

(1) Multiply σ by step c.
(2) Divide step (1) by the square root of N.

e. Subtract ε from $\overline{X}$ to obtain the LCL which is the lower bound for μ at the desired confidence level. The confidence interval for μ is from the LCL to $+\infty$.

f. If μ_0 is less than the LCL, decide that μ is greater than μ_0; otherwise ther is no reason to believe μ is greater than μ_0 at the desired confidence level.

6.2.1.2.4 EXAMPLE

Given:
σ = 1.4 min.
μ_0 = 83.0 min.
Sample data at Table A-2a, page 1-2.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. $\alpha = .10$ $1-\alpha = .90$
b. Compute $\overline{X}$.	b. $\overline{X} = 86.417$ $= 86.4$ min. See paragraph 6.1.1.4, page 26, for computations.
c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.	c. $Z_{.90} = 1.282$
d. Compute: $\varepsilon = \frac{Z_{1-\alpha}(\sigma)}{\sqrt{N}}$	d. $\varepsilon = \frac{(1.282)(1.4)}{\sqrt{12}} = \frac{1.7948}{3.464} = .518$

e. Compute:

$LCL = \bar{X} - \varepsilon$

e. $LCL = 86.417 - .518$
$= 85.899$
$= 85.8$ min.

f. If $\mu_o < LCL$ decide that $\mu > \mu_o$; otherwise, there is no reason to believe $\mu > \mu_o$ at a $100(1-\alpha)\%$ confidence level.

f. Since $83.0 < 85.8$ decide that $\mu > 83.0$ min. at a 90% confidence level.

6.2.1.2.5 ANALYSIS

If $\mu_o < LCL$, the null hypothesis that $\mu > \mu_o$ is accepted; otherwise, there is no reason to believe $\mu > \mu_o$ at a $100(1-\alpha)\%$ confidence level when T is unknown. The $100\ (1-\alpha\ )\%$ confidence interval for μ is LCL to $+ \infty$

6.2.2 $\bar{X}$ LESS THAN μ_o.

6.2.2.1 $\bar{X}$ LESS THAN μ_o WITH σ UNKNOWN

6.2.2.1.1 OBJECTIVE

To determine whether μ is less than μ_o at the desired confidence level when σ is unknown.

6.2.2.1.2 DATA REQUIRED

A list of sample readings.

6.2.2.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\bar{X}$ (see paragraph 6.1.1.3, page 26).

c. Compute s (see paragraph 7.1.1.3, page 64).

d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for N-1 d.f.

e. Compute ε as follows:

(1) Multiply s by step d.
(2) Divide step (1) by the square root of N.

f. Add ε to $\bar{X}$ to obtain the UCL which is the upper bound for μ at the desired confidence level. The confidence interval for μ is from $-\infty$ to the UCL.

g. If μ_o is greater than the UCL, decide that μ is less than μ_o; otherwise, there is no reason to believe μ is less than μ_o at the desired confidence level.

6.2.2.1.4 EXAMPLE

Given:
$\mu_0 = 87.0$ min.
Sample data at Table A-2a, page 1-2.

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .10$ $1-\alpha = .90$
b. Compute $\overline{X}$.	b. $\overline{X} = 86.417$ $= 86.4$ min. See paragraph 6.1.2.1.4 b, page 27, for computations.
c. Compute s.	c. $s = 2.173$ $= 2.2$ min. See paragraph 6.1.2.1.4 c, page 27, for computations.
d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for N-1 d.f.	d. $t_{.90}$ for 11 d.f. $= 1.363$
e. Compute: $\varepsilon = \frac{t_{1-\alpha}(s)}{\sqrt{N}}$	e. $\varepsilon = \frac{(1.363)(2.173)}{\sqrt{12}}$ $= \frac{2.962}{3.464}$ $= .855$
f. Compute: $UCL = \overline{X} + \varepsilon$	f. $UCL = 86.417 + .855$ $= 87.272$ $= 87.3$ min.
g. If $\mu_0 > UCL$, decide that $\mu < \mu_0$; otherwise, there is no reason to believe $\mu < \mu_0$ at a $100(1-\alpha)\%$ confidence level.	g. Since $87.0 \not> 87.3$, decide that there is no reason to believe that $\mu < 87.0$ min, at a 90% confidence level.

6.2.2.1.5 ANAYLSIS

If $\mu_0 > UCL$, the null hypothesis that $\mu < \mu_0$ is accepted, otherwise, there is no reason to believe $\mu < \mu_0$ at a $100(1-\alpha)\%$ confidence level. The $100(1-\alpha)\%$ confidence interval for μ is from $-\infty$ to UCL.

6.2.2.2 $\overline{X}$ LESS THAN μ_0 WITH σ KNOWN

6.2.2.2.1 OBJECTIVE

To determine whether μ is less than μ_0 at the desired confidence level when σ is known.

6.2.2.2.2 DATA REQUIRED

A list of sample readings and σ, which is known from a standard item, history, or Requirements Document.

6.2.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}$ (see paragraph 6.1.1.3, page 26).

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

d. Compute ε as follows:

(1) Multiply σ by step c.
(2) Divide step (1) by the square root of N.

e. Add ε to $\overline{X}$ to obtain the UCL which is the upper bound for μ at the desired confidence level. The confidence interval for μ is from $-\infty$ to UCL.

f. If μ_0 is greater than the UCL, decide that μ is less than μ_0; otherwise, there is no reason to believe μ is less than μ_0 at the desired confidence level.

6.2.2.2.4 EXAMPLE

Given:
σ = 2.2 min.
μ_0 = 88.0 min.
Sample data at Table A-2a, page 1-2.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. α = .10 $1-\alpha$ = .90
b. Compute $\overline{X}$.	b. $\overline{X}$ = 86.417 = 86.4 min. See paragraph 6.1.1.4, page 26, for computations.
c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.	c. $Z_{.90}$ = 1.282
d. Compute: $\varepsilon = \frac{Z_{1-\alpha}(\sigma)}{\sqrt{N}}$	d. $\varepsilon = \frac{(1.282)(2.2)}{\sqrt{12}} = \frac{2.820}{3.464} = .814$

e. Compute:

$UCL = \overline{X} + \varepsilon$

e. UCL = 86.417 + .814
= 87.231
= 87.3 min.

f. If $\mu_0 > UCL$, decide that $\mu < \mu_0$; otherwise, there is no reason to believe $\mu < \mu_0$ at a $100(1-\alpha)\%$ confidence level.

f. Since 88.0 > 87.3, decide that $\mu < 88.0$ min. at a 90% confidence level.

6.2.2.2.5 ANALYSIS

If $\mu_0 > UCL$, the null hypothesis that $\mu < \mu_0$ is accepted; otherwise there is no reason to believe $\mu<\mu_0$ at a desired confidence level. The $100\ (1-\alpha)\%$ confidence interval for μ is from $-\infty$ to UCL.

6.2.3 DETERMINATION OF SAMPLE SIZE

6.2.3.1 OBJECTIVE

To determine the N_t required to determine whether μ is equal to or greater than $\mu_0 + \varepsilon$ (or equal to or less than $\mu_0 - \varepsilon$) at the desired confidence level when:

a. σ is known.

b. σ is unknown.

6.2.3.2 DATA REQUIRED

a. σ, which is known from a standard item, history, or Requirements Document.

b. An approximation of the value that σ will assume.

6.2.3.3 PROCEDURE

a. Choose α and β, the probabilities of making Type I and Type II errors respectively.

b. Choose the allowable amount of error.

c. Compute d^2, an intermediate value, as follows:

(1) Divide ε by σ.
(2) Square step (1).

d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

e. If σ is known, compute N_t as follows:

(1) Add $Z_{1-\alpha}$ to $Z_{1-\beta}$.
(2) Square step (1).

(3) Divide step (2) by step c and round to the next larger whole number.

f. If σ is unknown, add 3 to step e for $\alpha = .01$, 2 for $\alpha = .05$, or 1 for $\alpha = .10$.

g. Conclude that N_t samples are required to determine whether μ is equal to or greater than $\mu_o + \varepsilon$ (or equal to or less than $\mu_o - \varepsilon$) at the desired confidence level.

6.2.3.4 EXAMPLE

Given:
$\sigma = .12$

Procedure:	Example:
a. Choose α and β.	a. $\alpha = .01$ $1-\alpha = .99$ $\beta = .20$ $1-\beta = .80$
b. Choose ε.	b. $\varepsilon = .05$
c. Compute: $d^2 = (\varepsilon/\sigma)^2$	c. $d^2 = (.05/.12)^2$ $= (.4167)^2$ $= .1736$
d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.	d. $Z_{.99} = 2.326$ $Z_{.80} = .84$
e. When σ is known, compute: $N_t = \frac{(Z_{1-\alpha}+Z_{1-\beta})^2}{d^2}$	e. $N_t = \frac{(2.326+.84)^2}{.1736}$ $= \frac{(3.166)^2}{.1736}$ $= \frac{10.024}{.1736}$ $= 57.74$ $= 58$
f. When σ is unknown and a value is assumed, compute: (1) $\alpha = .01$ $N_t = \frac{(Z_{1-\alpha}+Z_{1-\beta})^2}{d^2} + 3$ (2) $\alpha = .05$ $N_t = \frac{(Z_{1-\alpha}+Z_{1-\beta})^2}{d^2} + 2$	f. Since $\alpha = .01$ $N_t = \frac{(2.326+.84)^2}{.1736} + 3$ $= 61$

(3) $\alpha = .10$

$$N_t = \frac{(Z_{1-\alpha}+Z_{1-\beta})^2}{d^2} + 1$$

g. Conclude that N_t samples are required to determine whether $\mu \geq \mu_0 + \varepsilon$ (or $\mu \leq \mu_0 - \varepsilon$) at a $100(1-\alpha)\%$ confidence level.

g. Conclude that 58 samples, for σ known and equal to .12 (or 61 samples for σ assumed equal to .12), must be tested in order to determine whether $\mu \geq \mu_0 + .05$ (or $\mu \leq \mu_0 - .05$) at a 99% confidence level.
NOTE: If σ is really less than .12, N_t is more than adequate.

6.2.3.5 ANALYSIS

If σ is overestimated, the consequences are twofold: first, N_t is overestimated; second, by employing a N_t that is larger than necessary, the actual value of β will be somewhat less than intended at the same confidence level, a consequence which is never undesirable. On the other hand if σ is underestimated, N_t is underestimated. Therefore, N_t must be recomputed and additional items must be tested if possible. β will be larger than intended at the same confidence level. Thus, overestimating σ is safer than underestimating σ.

6.3 COMPARING TWO OBSERVED MEANS

a. An observed mean is generated from a sample and is representative of μ. This value of $\overline{X}$ is then required to meet a standard item $\overline{X}$ which is representative of the standard items population. Looking at the values of the means ($\overline{X}_A$ and $\overline{X}_B$) to decide whether μ_A is greater than μ_B or μ_A is less than μ_B at a confidence level is insufficient. Since the decision pertains to the populations, statistical tests must be applied $\overline{X}_A$ and $\overline{X}_B$ to determine whether μA is greater than μB or μA is less than B. The statistical tests use the sample means as estimates of the population means.

b. Type A generally represents the test item and Type B the standard item when testing the hypothesis that μ_A is greater than μ_B. However, to prove that the average performance of the test item is less than that of the standard item, Type A must represent the standard item so that the hypothesis, μ_A is greater than μ_B, can be tested.

c. When the null hypothesis is μ_A is greater than μ_B, the alternative hypothesis is there is no reason to believe that μ_A is greater than μ_B.

d. There are four different procedures available to test the null hypothesis. Following are the conditions which dictate the appropriate test:

(1) The variabilities of A and B are unknown but assumed equal ($\sigma_A = \sigma_B$). This test also applies when N_A = NB even though $\sigma A \neq \sigma B$ (see paragraph 6.3.11 page 46.

(2) The variabilities of A and B are unknown but assumed unequal ($\sigma_A \neq \sigma_B$) for unequal sample sizes (see paragraph 6.3.1.2, page 49).

(3) The variabilities of A and B are known from previous experience; thus, σ_A may or may not equal σ_B (see paragraph 6.3.1.3, page 51).

(4) The observations are paired; i.e., individual Type A and Type B items are tested alternately such that the items in each pair are tested under the same condition. Obviously, $N_A = N_B$ (see paragraph 6.3.1.4, page 53).

NOTE: The procedure in subparagraph (1) is also valid for paired observations since $N_A = N_B$; however, the procedure in subparagraph (4) is only valid for paired observations.

6.3.1 $\overline{X}_A$ GREATER THAN $\overline{X}_B$

6.3.1.1 σ_A AND σ_B UNKNOWN BUT ASSUMED EQUAL

6.3.1.1.1 OBJECTIVE

To determine whether μ_A is greater than μ_B at the desired confidence level when the population standard deviations of A and B are unknown but σ_A is assumed equal to σ_B.

6.3.1.1.2 DATA REQUIRED

A list of sample readings.

6.3.1.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}_A$ and $\overline{X}_B$ (see paragraph 6.1.1.3, page 26).

c. Compute $\Sigma\Delta_A^2$ and $\Sigma\Delta_B^2$ as follows:

(1) Compute the deviation from the mean for each reading ($\Delta_A = X_A - \overline{X}_A$ and $\Delta_B = X_B - \overline{X}_B$)

(2) Square each deviation (Δ_A^2 and Δ_B^2).

(3) Sum the squared deviations for each of the two items ($\Sigma\Delta_A^2$ and $\Sigma\Delta_B^2$).

d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for (N_A+N_B-2) d.f.

e. Compute the combined standard deviation (s_p) of the items as follows:

(1) Add $\Sigma\Delta_A^2$ to $\Sigma\Delta_B^2$.

(2) Add N_A TO N_B.

(3) Subtract 2 from step (2).

(4) Divide step (1) by step (3).

(5) Find the square root of step (4).

f. Compute ε as follows:

(1) Add N_A to N_B.

(2) Multiply N_A by N_B.

(3) Divide step (1) by step (2).

(4) Multiply step e by the square root of step (3).

(5) Multiply step d by step (4).

g. Subtract ε from $\overline{X}_A$ to obtain the LCL.

h. If $\overline{X}_B$ is less than the LCL, decide that μ_A is greater than μ_B; otherwise, there is no reason to believe μ_A is greater than μ_B at the desired confidence level.

6.3.1.1.4 EXAMPLE

Given:
Sample data at Table A-2b, page 1-3, and Table A-2c, page 1-4.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. α = .05 1-α = .95
b. Compute: $\overline{X}_A$ $\overline{X}_B$	b. $\overline{X}_A$ = 5401.40 = 5401 meters $\overline{X}_B$ = 5372.25 = 5372 meters See paragraph 6.1.1.4, page 26.
c. Compute: $\Sigma\Delta_A^2$ $\Sigma\Delta_B^2$	c. $\Sigma\Delta_A^2$ = 3,552.80 $\Sigma\Delta_B^2$ = 2,899.70 See paragraph 7.1.1.4, page 65.

d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for (N_A+N_B-2) d.f.

d. $N_A+N_1-2 = 20+20-2$
$= 38$

$t_{.95}$ for 38 d.f. = 1.687

NOTE: If the necessary d.f. is not in Table B-5, page 2-5, interpolate to find the value of $t_{1-\alpha}$.

10 d.f. [8 d.f. [$t_{.95}$ for 30 d.f. = 1.697
$t_{.95}$ for 38 d.f. =] x
$t_{.95}$ for 40 d.f. = 1.684] .013

$$\frac{8}{10} = \frac{x}{.013}$$

$(10)\ x = (.013)(8)$
$x = (.013)(8)/10$
$= .104/10$
$= .010$

Since the t value decreases for increasing d.f., subtract .010 from 1.697. Thus, $t_{.95}$ for 38 d.f. = 1.687

e. Compute:

$$s_p = \sqrt{\frac{\Sigma\Delta_A^2+\Sigma\Delta_B^2}{N_A+N_B-2}}$$

e.

$$s_p = \sqrt{\frac{3{,}552.80+2899.70}{38}}$$
$= 13.03$ meters

f. Compute:

$$\varepsilon = t_{1-\alpha}(s_p)\sqrt{\frac{N_A+N_B}{N_A N_B}}$$

f. $\varepsilon = (1.586\ (13.03)\ \sqrt{40/400}$
$= (1.687)(13.03)\ \sqrt{.1}$
$= (21.37\ (.3162)$
$= 6.947$

g. Compute:

$LCL = \overline{X}_A - \varepsilon$

g. LCL = 5401.40-6.95
= 5394.45
= 5394 meters

h. If $\overline{X}_B <$ LCL, decide that $\mu_A > \mu_B$; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a $100(1-\alpha)\%$ confidence level.

h. Since 5372 < 5394, decide that $\mu_A >$ 5372 meters at a 95% confidence level.

6.3.1.1.5 ANALYSIS

If $\overline{X}_B$ < LCL, the null hypothesis that $\mu_A > \mu_B$ is accepted; otherwise, there is no reason to believe $\mu_A > \mu_B$ at the $100(1-\alpha)\%$ confidence level when σ_A and σ_B are unknown and σ_A is assumed equal to σ_B.

6.3.1.2 σ_A AND σ_B UNKNOWN BUT ASSUMED UNEQUAL

6.3.1.2.1 OBJECTIVE

To determine whether μ_A is greater than μ_B when the population standard deviations of A and B are unknown but are assumed unequal.

6.3.1.2.2 DATA REQUIRED

A list of sample readings.

6.3.1.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}_A$, s_A^2, $\overline{X}_B$, and s_B^2 (see paragraphs 6.1.1.3, page 26, and 7.1.1.3, page 64).

c. Compute V_A AND V_B, intermediate values, by dividing s_A^2 by N_A and s_B^2 by N_B respectively.

d. Compute the effective number of d.f. (e.d.f.) as follows:

(1) Add V_A to V_B.
(2) Square step (1).
(3) Square V_A.
(4) Square V_B.
(5) Divide step (3) by the sum of N_A+1.
(6) Divide step (4) by the sum of N_B+1.
(7) Add step (5) to step (6).
(8) Divide step (2) by step (7).
(9) Subtract 2 from step (8).
(10) Round step (9).

e. Use Table B-5, page B-5, to obtain t[illegible] for the e.d.f.

f. Compute ϵ as follows:

(1) Add V_A to V_B.
(2) Multiply step e by the square root of step (1).

g. Subtract ϵ from $\overline{X}_A$ to obtain the LCL.

h. If $\overline{X}_B$ is less than the LCL, decide that μ_A is greater than μ_B; otherwise, there is no reason to believe μ_A is greater than μ_B at the desired confidence level.

6.3.1.2.4 EXAMPLE

Given:
Sample data at Table A-2b, page 1-3 and Table A-2d, page 1-5.

Procedure:

a. Choose the confidence level $(1-\alpha)$.

b. Compute:

$\overline{X}_A$

s_A^2

$\overline{X}_B$

s_B^2

c. Compute:

$V_A = s_A^2/N_A$

$V_B = s_B^2/N_B$

d. Compute:

$$\text{e.d.f.} = \frac{(V_A + V_B)^2}{[V_A^2/(N_A+1)]+[V_B^2/(N_B+1)]} - 2$$

Example:

a. $\alpha = .05$
$1-\alpha = .95$

b. $\overline{X}_A = 5401.40$
$= 5401$ meters
$s_A^2 = 186.99$
$= 187$
$\overline{X}_B = 5378.30$
$= 5378$ meters
$s_B^2 = 165.48$
$= 165$

See paragraphs 6.1.1.4, page 26, and 7.1.1.4, page 65.

c. $V_A = 186.99/20$
$= 9.3495$
$V_B = 165.48/10$
$= 16.548$

d.

$$\text{e.d.f.} = \frac{(9.35+16.55)^2}{[(9.35)^2/21]+[(16.55)^2/11)]} - 2$$

$$= \frac{(25.90)^2}{(87.42/21)+(273.9/11)} - 2$$

$$= \frac{670.8}{4.16 + 24.90} - 2$$

$$= \frac{670.8}{29.06} - 2$$

$= 23.08 - 2$

$= 21.08$

$= 22$

e. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for e.d.f.

e. $t_{.95}$ for 22 d.f. = 1.717

f. Compute:

$$\varepsilon = t_{1-\alpha} \sqrt{V_A + V_B}$$

f. $\varepsilon = (1.717) \sqrt{9.35 + 16.55}$
$= (1.717) \sqrt{25.90}$
$= (1.717)\ (5.089)$
$= 8.738$

g. Compute:

$LCL = \overline{X}_A - \varepsilon$

g. LCL = 5401.40 - 8.74
= 5392.66
= 5392 meters

h. If $\overline{X}_B$ < LCL, decide that $\mu_A > \mu_B$; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a 100(1-α)% confidence level.

h. Since 5378 < 5392, decide that μ_A > 5378 meters at a 95% confidence level.

6.3.1.2.5 ANALYSIS

If $\overline{X}_B$ < LCL, the null hypothesis that $\mu_A > \mu_B$ is accepted; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a 100(1-α)% confidence level when σ_A and σ_B are unknown and σ_A assumed equal to σ_B.

6.3.1.3 σ_A AND σ_B KNOWN FROM PREVIOUS EXPERIENCE

6.3.1.3.1 OBJECTIVE

To determine whether μ_A is greater than μ_B when σ_A and σ_B are known from previous experience.

6.3.1.3.2 DATA REQUIRED

A list of sample readings and σ_A and σ_B, which are known from previous testing.

6.3.1.3.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

c. Compute $\overline{X}_A$ and $\overline{X}_B$ (see paragraph 6.1.1.3, page 26).

d. Compute ε as follows:

(1) Square σ_A.

(2) Square σ_B.

(3) Divide step (1) by N_A.

(4) Divide step (2) by N_B.

(5) Add step (3) to step (4).

(6) Multiply step b by the square root of step (5).

e. Subtract ε from $\bar{X}_A$ to obtain the LCL.

f. If $\bar{X}_B$ is less than the LCL, decide that μ_A is greater than μ_B; otherwise, there is no reason to believe μ_A is greater than μ_B at the desired confidence level.

6.3.1.3.4 EXAMPLE

Given:
Sample data at Table A-2b, page 1-3, and Table A-2c, page 1-4.

$\sigma_A = 14.0$ meters

$\sigma_B = 12.0$ meters

Procedure:	Example:
a. Choose the confidence level (1-α).	a. $\alpha = .05$ $1-\alpha = .95$
b. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.	b. $Z_{.95} = 1.645$
c. Compute: $\bar{X}_A$ $\bar{X}_B$	c. $\bar{X}_A = 5401.40$ $= 5401$ meters $\bar{X}_B = 5372.25$ $= 5372$ meters See paragraph 6.3.1.1.4, page 47.
d. Compute: $\varepsilon = Z_{1-\alpha}\sqrt{\frac{\sigma_A^2}{N_A}+\frac{\sigma_B^2}{N_B}}$	d. $\varepsilon = (1.645)\sqrt{[(14.0)^2/20]+[(12.0)^2/20]}$ $= (1.645)\sqrt{(196.0/20)+(144.0/20)}$ $= (1.645)\sqrt{9.80 + 7.20}$ $= (1.645)\sqrt{17.00}$ $= (1.645)\ (4.12)$ $= 6.78$
e. Compute: $LCL = \bar{X} - \varepsilon$	e. $LCL = 5401.40 - 6.78$ $= 5394.62$ $= 5394$ meters

f. If $\overline{X}_B$ < LCL, decide that $\mu_A > \mu_B$; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a 100(1-α)% confidence level.

f. Since 5372 < 5394, decide that μ_A > 5372 meters at a 95% confidence level.

6.3.1.3.5 ANALYSIS

If $\overline{X}_B$ < LCL, the null hypothesis that $\mu_A > \mu_B$ is accepted; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a 100(1-α)% confidence level when σ_A and σ_B are known from previous testing.

6.3.1.4 PAIRED OBSERVATIONS

6.3.1.4.1 OBJECTIVE

To determine whether μ_A is greater than μ_B when the observations are paired (see subparagraph 6.3 d (4), page 45.

6.3.1.4.2 DATA REQUIRED

A list of paired sample readings.

6.3.1.4.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute the difference between the reading for Type A and the reading for Type B ($x_d = x_A - x_B$) for each pair of observations.

c. Compute the mean of the differences ($\overline{X}_d$), (see paragraph 6.1.1.3, page 26).

d. Compute the standard deviation of the differences (s_d), (see paragraph 7.1.1.3, page 64).

e. Use Table B-5, to obtain $t_{1-\alpha}$ for N-1 d.f.

f. Compute ε as follows:

(1) Divide step d by the square root of N.
(2) Multiply step e by step (1).

g. If $\overline{X}_d$ is greater than ε, decide that μ_A is greater than μ_B; otherwise, there is no reason to believe μ_A is greater than μ_B at the desired confidence level.

6.3.1.4.4 EXAMPLE

Given:
Sample data at Table A-2e, page 1-6.

Procedure:

a. Choose the confidence level $(1-\alpha)$.

b. Compute x_d for each pair of readings.

$$x_d = x_A - x_B$$

c. Compute $\overline{X}_d$.

d. Compute s_d.

e. Use Table B-5, page 2-5 to obtain $t_{1-\alpha}$ for N-1 d.f.

f. Compute:

$$\varepsilon = t_{1-\alpha} \frac{s_d}{\sqrt{N}}$$

g. If $\overline{X}_d > \varepsilon$, decide that $\mu_A > \mu_B$; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a $100(1-\alpha)\%$ confidence level.

Example:

a. $\alpha = .05$
$1-\alpha = .95$

b. (1) $x_d = 146 - 141 = 5$

(2) $x_d = 141 - 143 = -2$

See Table A-2e, page 1-6 for complete list.

c. $X_d = -0.10 = -0.1$ amp. hr.
See paragraph 6.1.1.4, page 26.

d. $s_d = 2.81 = 2.8$ amp. hr.
See paragraph 7.1.1.4, page 65.

e. $t_{.95}$ for 9 d.f. $= 1.833$

f. $\varepsilon = (1.833)(2.81)/\sqrt{10}$
$= 5.15/3.16$
$= 1.63$
$= 1.6$

g. Since $-0.1 \not> 1.6$, decide that there is no reason to believe $\mu_A > \mu_B$ at a 95% confidence level.

6.3.1.4.5 ANALYSIS

If $\overline{X}_d > \varepsilon$, the null hypothesis that $\mu_A > \mu_B$ is accepted; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a $100(1-\alpha)\%$ confidence level when the observations are paired.

6.3.1.5 DETERMINATION OF SAMPLE SIZE

6.3.1.5.1 OBJECTIVE

To determine the N_t required to determine whether μ_A is equal to or greater than $\mu_B + \varepsilon$ (or equal to or less than $\mu_B - \varepsilon$) at the desired confidence level when:

a. Case I. The variabilities of A and B are unknown but assumed equal.

b. Case II. The variabilities of A and B are unknown but assumed equal.

c. Case III. The variabilities of A and B are known from previous experience.

d. Case IV. The observations are paired (see subparagraph 6.3 d (4), page 45).

6.3.1.5.2 DATA REQUIRED

a. Case I. An approximation of the value that σ ($\sigma_A = \sigma_B$) will assume.

b. Case II. An approximation of the values that σ_A and σ_B will assume.

c. Case III. The values of σ_A and σ_B which are known from previous experience.

d. Case IV. An approximation of the population standard deviation of the differences ($\sigma_d \approx |\tilde{\sigma}_A - \tilde{\sigma}_B|$ where $\tilde{\sigma}_A$ and $\tilde{\sigma}_B$ are approximations) for the pairs concerned.

6.3.1.5.3 PROCEDURE

a. Case I.

(1) Choose α and β, the probabilities of making Type I and Type II errors respectively.
(2) Choose the allowable amount of error.
(3) Compute d^2, an intermediate value, as follows:

(a) Square σ.
(b) Multiply step (a) by 2.
(c) Square ε.
(d) Divide step (c) by step (b).

(4) Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.
(5) Compute N_t ($N_t = N_A = N_B$) as follows:

(a) Add $Z_{1-\alpha}$ to $Z_{1-\beta}$.
(b) Square step (a).
(c) Divide step (b) by step (3).
(d) If $\alpha = .01$, add 2 to step (c) and round up; and if $\alpha = .05$, add 1 to step (c) and round up.

(6) Conclude that N_t samples of each item are required to determine whether μ_A is equal to or greater than $\mu_B + \varepsilon$ (or equal to or less than $\mu_B - \varepsilon$) at the desired confidence level.

b. Case II.

(1) Choose α and β, the probabilities of making Type I and Type II errors respectively.

(2) Choose the allowable amount of error.
(3) Compute d^2, an intermediate value, as follows:

(a) Square σ_A.
(b) Square σ_B.
(c) If $N_A = N_B$, add step (a) to step (b).
(d) If N_A is a multiple of N_B; i.e., $N_A = C(N_B)$, multiply step (b) by C and add the product to step (a).
(e) Square ε.
(f) Divide step (e) by the value from step (c) if $N_A = N_B$ or by the value from step (d) if $N_A = C(N_B)$.

(4) Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.
(5) Compute N_t ($N_t = N_A = N_B$) as follows:

(a) Add $Z_{1-\alpha}$ to $Z_{1-\beta}$.
(b) Square step (a).
(c) Divide step (b) by step (3) and round up.

(6) Conclude that N_t samples of each item are required to determine whether μ_A is equal to or greater than $\mu_B + \varepsilon$ (or equal to or less than $\mu_B - \varepsilon$) at the desired confidence level.

c. Case III.

Same as Case II.

d. Case IV.

Same as paragraph 6.2.3.3, page 43

NOTE: σ in paragraph 6.2.3.3 represents σ_d.

6.3.1.5.4 EXAMPLE

a. Case I.

Given:
$\sigma = 2.6$

Procedure:	Example:
(1) Choose α and β.	(1) $\alpha = .05$ $1-\alpha = .95$ $\beta = .20$ $1-\beta = .80$
(2) Choose ε.	(2) $\varepsilon = 1.05$

(3) Compute:

$d^2 = \varepsilon^2/2\sigma^2$

(3) $d^2 = 1.05^2/2(2.6)^2$

$= 1.1025/2(6.76)$

$= 1.1025/13.52$

$= .08155$

(4) Use Table B-4, page 2-4 to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

(4) $Z_{.95} = 1.645$

$Z_{.80} = .84$

(5) Compute:

(a) For $\alpha = .01$,

$$N_t = \frac{(Z_{1-\alpha} + Z_{1-\beta})^2}{d^2} + 2$$

(b) For $\alpha = .05$,

$$N_t = \frac{(Z_{1-\alpha} + Z_{1-\beta})^2}{d^2} + 1$$

Since $\alpha = .05$,

$$N_t = \frac{(1.645 + .84)^2}{.08155} + 1$$

$$= \frac{(2.485)^2}{.08155} + 1$$

$$= \frac{6.175}{.08155} + 1$$

$= 75.72 + 1$

$= 76.72$

$= 77$

(6) Conclude that N_t samples of each item are required to determine whether $\mu_A \geq \mu_B + \varepsilon$ (or $\mu_A \leq \mu_B - \varepsilon$) at a $100(1-\alpha)\%$ confidence level.

(6) Conclude that 77 samples of each item for σ assumed and equal to 2.6, must be tested in order to determine whether $\mu_A \geq \mu_B + 1.05$ (or $\mu_A \leq \mu_B - 1.05$) at a 95% confidence level.

b. Case II.

Given
$\sigma_A = 2.2$

$\sigma_B = 3.0$

$N_A = N_B$

Procedure:

(1) Choose α and β.

Example:

(1) $\alpha = .05$

$1-\alpha = .95$

$\beta = .20$

$1-\beta = .80$

(2) Choose ε.

(2) $\varepsilon = .75$

(3) (a) If $N_A = N_B$, compute:

$$d^2 = \frac{\varepsilon^2}{\sigma_A^2 + \sigma_B^2}$$

(3) Since $N_A = N_B$ is assumed,

$$d^2 = \frac{.75^2}{(2.2)^2+(3.0)^2}$$

$$= \frac{.5625}{4.84 + 9.0}$$

$$= \frac{.5625}{13.84}$$

$$= .0406$$

(b) If $N_A = C(N_B)$, compute:

$$d^2 = \frac{\varepsilon^2}{\sigma_A^2 + C(\sigma_B^2)}$$

(4) Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

(4) $Z_{.95} = 1.645$

$Z_{.80} = .84$

(5) Compute:

$$N_t = \frac{(Z_{1-\alpha} + Z_{1-\beta})^2}{d^2}$$

(5) $$N_t = \frac{(1.645 + .84)^2}{.0406}$$

$$= \frac{(2.485)^2}{.0406}$$

$$= \frac{6.175}{.0406}$$

$$= 152.10$$

$$= 153$$

(6) Conclude that N_t samples of each item are required to determine whether $\mu_A \geq \mu_B + \varepsilon$ (or $\mu_A \leq \mu_B - \varepsilon$) at a $100(1-\alpha)\%$ confidence level.

(6) Conclude that 153 samples of each item, for σ_A assumed and equal to 2.2 and σ_B assumed and equal to 3.0, must be tested in order to determine whether $\mu_A \geq \mu_B + .75$ (or $\mu_A \leq \mu_B - .75$) at a 95% confidence level.

c. Case III.

Same as Case II.

d. Case IV.

Same as paragraph 6.2.3.4, page 44.

NOTE: σ in paragraph 6.2.3.4 represents σ_d.

6.3.1.5.5 ANALYSIS

If σ is overestimated, the consequences are twofold; first, N_t is overestimated; second, by employing a N_t that is larger than necessary, the actual value of β will be somewhat less than intended at the same confidence level, a consequence which is never undesirable. On the other hand if σ is underestimated, N_t is underestimated. Therefore, N_t must be recomputed and additional items must be tested if possible. β will be larger than intended at the same confidence level. Thus, overestimating σ is safer than underestimating σ.

6.3.2 COMPARING THE MEANS OF SEVERAL PRODUCTS

6.3.2.1 OBJECTIVE

To determine whether the means of several products differ.

6.3.2.2 DATA REQUIRED

A list of sample readings.

6.3.2.3 PROCEDURE

a. Case I. $N_1 = N_2 = N_3 = \ldots = N_K = N$

(1) Choose the desired confidence level.
(2) Compute s^2 for each product (see paragraph 7.1.1.3, page 64).
(3) Compute $d.f._1$ and $d.f._2$ as follows:

(a) Set $d.f._1$ equal to K.
(b) Sum the N's and subtract step (a) to obtain $d.f._2$.

(4) Compute the average variance (s_K^2) as follows:

(a) Sum s^2's.
(b) Divide step (a) by the number of products (K).

(5) Use Table B-6, page 2-6, to obtain $q_{1-\alpha}$ for ($d.f._1$, $d.f._2$) d.f.
(6) Compute ε as follows:

(a) Multiply step (5) by the square root of step (4).
(b) Divide step (a) by the square root of N.

(7) If the absolute difference between any two sample means is greater than ε, decide that those means differ; otherwise, there is no reason to believe the means differ at the desired confidence level.

b. Case II. The N's are unequal.

(1) Choose the desired confidence level.
(2) Compute s^2 for each product (see paragraph 7.1.1.3, page 64).
(3) Compute d.f.$_1$ and d.f.$_2$ as follows:

(a) Set d.f.$_1$ equal to K.
(b) Sum the N's and subtract step (a) to obtain d.f.$_2$.

(4) Compute s_K^2 as follows:

(a) Subtract one from each N.
(b) Multiply step (a) by the s^2 of the particular sample.
(c) Sum the products generated by step (b).
(d) Divide step (c) by step (3) (b).

(5) Use Table B-6, page 2-6, to obtain $q_{1-\alpha}$ for (d.f.$_1$, d.f.$_2$) d.f.
(6) Compute H, an intermediate value, as follows:

(a) Take the reciprocal of each N (1/N).
(b) Sum the reciprocals generated by step (a).
(c) Divide K by step (b).

(7) Compute ε as follows:

(a) Multiply step (5) by the square root of step (4).
(b) Divide step (a) by the square root of step (6).

(8) If the absolute difference between any two sample means is greater than ε, decide that those means differ; otherwise, there is no reason to believe the means differ at the desired confidence level.

6.3.2.4 EXAMPLE

a. Case I.

Given:

$N_1 = N_2 = N_3 = N_4 = N$

Sample data at Table A-2f, page 1-7.

Procedure:	Example:
(1) Choose the confidence level (1-α).	(1) $\alpha = .10$ $1-\alpha = .90$
(2) Compute: $s_1^2, s_2^2, s_3^2, s_4^2$	(2) $s_1^2 = 406.00$ $s_2^2 = 574.80$ $s_3^2 = 636.80$ $s_4^2 = 159.30$ See paragraph 7.1.1.4, page 65 for example computations.

(3) Compute:

$d.f._1 = K$

$d.f._2 = (\Sigma N) - K$

(3) $d.f._1 = 4$

$d.f._2 = (5+5+5+5)-4$

$= 20 - 4$

$= 16$

(4) Compute:

$$s_K^2 = \frac{\Sigma s^2}{K}$$

(4)

$$s_K^2 = \frac{406.00+574.80+636.80+159.30}{4}$$

$$= \frac{1776.90}{4}$$

$$= 444.22$$

(5) Use Table B-6, page 2-6, to obtain $q_{1-\alpha}$ for $(d.f._1, d.f._2)$ d.f.

(5) $q_{.90}$ for (4,16) d.f. = 3.52

(6) Compute:

$$\varepsilon = \frac{q_{1-\alpha}\sqrt{s_K^2}}{\sqrt{N}}$$

(6) $\varepsilon = (3.52)\ \sqrt{444.22}/\sqrt{5}$

$= (3.52)(21.07)/2.236$

$= 74.17/2.236$

$= 33.17$

(7) If the absolute difference between any two sample means is greater than ε, decide that those means differ; otherwise, there is no reason to believe the means differ at a $100(1-\alpha)\%$ confidence level.

(7) 1 and 3

Is $|534.00-562.60| > 33.2?$

$28.6 \not> 34$

Since $29 \not> 34$, decide that the means of Types 1 and 3 do not differ at a 90% confidence level.

NOTE: Check the difference between the smallest and largest means first. If that difference is not greater than ε, conclude that none of the remaining differences will be larger than ε. However, if the difference is greater than ε, then continue checking the remaining 5 differences.

b. Case II.

Given:

$N_1 \neq N_2 \neq N_3 \neq N_4$

Sample data at Table A-23, page 1-8.

Procedure:

(1) Choose the confidence level $(1-\alpha)$.

(2) Compute:

$s_1^2, s_2^2, s_3^2, s_4^2$

(3) Compute:

$d.f._1 = K$

$d.f._2 = (\Sigma N) - K$

(4) Compute:

$$s_K^2 = \frac{(N_1-1)s_1^2+(N_2-1)s_2^2+(N_3-1)s_3^2+(N_4-1)S_4^2}{d.f._2}$$

(5) Use Table B-6, page 2-6, to obtain $q_{1-\alpha}$ for $(d.f._1, d.f._2)$ d.f.

(6) Compute:

$$H = \frac{K}{\frac{1}{N_1}+\frac{1}{N_2}+\frac{1}{N_3}+\frac{1}{N_4}}$$

Example:

(1) $\alpha = .10$

$1-\alpha = .90$

(2) $s_1^2 = 4912.90$

$s_2^2 = 310.70$

$s_3^2 = 212.50$

$s_4^2 = 1190.50$

See paragraph 7.1.1.4, page 65 for example.

(3) $d.f._1 = 4$

$d.f._2 = (7+5+9+7)-4$

$= 28-4$

$= 24$

(4)

$$s_K^2 = \frac{(6)(4912.90)+(4)(310.70)+(8)(212.50)+(6)(1190.50)}{24}$$

$$= \frac{29477.4+1242.8+1700.0+7143.0}{24}$$

$$= \frac{39,563.2}{24}$$

$$= 1648.47$$

(5) $q_{.90}$ for $(4,24) = 3.42$

(6)

$$H = \frac{4}{\frac{1}{7}+\frac{1}{5}+\frac{1}{9}+\frac{1}{7}}$$

$$= \frac{4}{.143+.200+.111+.143}$$

$$= \frac{4}{.597}$$

$$= 6.70$$

(7) Compute:

$$\varepsilon = \frac{q_{1-\alpha}\sqrt{s_K^2}}{\sqrt{H}}$$

(7)

$$\varepsilon = (3.42)\sqrt{1648.47}\ /\sqrt{6.70}$$
$$= (3.42)(40.60)/2.59$$
$$= 138.85/2.59$$
$$= 53.64$$

(8) If the absolute difference between any two sample means is greater than ε, decide that those means differ; otherwise, there is no reason to believe the means differ at a 100(1-α)% confidence level.

(8) (a) 2 and 3

Is $|5011.20-5584.67| > 54?$

573 > 54

Since 573 > 54, decide that the means of Types 2 and 3 differ at a 90% confidence level.

NOTE: Since the difference between the smallest and largest mean produces a difference decision, repeat step (8) for the next largest difference.

(b) 3 and 4

Is $|5584.67-5147.86| > 54?$

436.81 > 54?

437 > 54

Since 437 > 54, decide that the means of Types 3 and 4 differ at a 90% confidence level.

(c) 1 and 3

Is $|5222.29-5584.67| > 54?$

362.38 > 54?

362 > 54

Since 362 > 54, decide that the means of Types 1 and 3 differ at a 90% confidence level.

(d) 1 and 2

Is $|5222.29-5011.20| > 54?$

211.09 > 54?

211 > 54

Since 211 > 54, decide that the means of Types 1 and 2 differ at a 90% confidence level.

(e) 2 and 4

Is $|5011.20-5147.86| > 54?$

> 54?

137 > 54

Since 137 > 54, decide that the means of Types 2 and 4 differ at a 90% confidence level.

(f) 1 and 4

Is $|5222.29-5147.86| > 54?$

74.43 > 54?

74 > 54

Since 74 > 54, decide that the means of Type 1 and 4 differ at a 90% confidence level.

6.3.2.5 ANALYSIS

The population means of several products may be compared by computing the absolute difference between any two sample means and comparing this value to a computed ε. The decision is relative only to the two products compared. Therefore, this test only reveals the relationship between the means of two items at a desired confidence level and does not necessarily reveal a difference between one mean and all of the remaining means.

7. STANDARD DEVIATION

7.1 ESTIMATE OF THE POPULATION STANDARD DEVIATION (s).

7.1.1 BEST SINGLE ESTIMATE of s.

7.1.1.1 OBJECTIVE

To determine the best point estimate of the population standard deviation for a normal distribution.

7.1.1.2 DATA REQUIRED

A list of sample readings.

7.1.1.3 PROCEDURE

a. Compute $\bar{X}$ (see paragraph 6.1.1.3, page 26).

b. Find the deviation of each reading from the mean by subtracting the mean from each reading; i.e., $\Delta = x - \bar{X}$.

c. Square each deviation; i.e., Δ^2.

d. Sum the squared deviations; i.e., $\Sigma\Delta^2$.

e. Compute s as follows:

(1) Divide step d by N-1.
(2) Find the square root of step (1).

7.1.1.4 EXAMPLE

Given:

Sample data at Table A-3a, page 1-9.

Procedure:	Example:
a. Compute $\bar{X}$.	a. $\bar{X} = 1094/10$ $= 109.40$ $= 109$ min. See paragraph 6.1.1.4, page 26.
b. Compute: $\Delta = X - \bar{X}$	b. (1) $\Delta = 100.00 - 109.40$ $= -9.40$ (2) $\Delta = 125.00 - 109.40$ $= 15.60$ See Table A-3a, page 1-9, for complete list.
c. Square each Δ.	c. (1) $\Delta^2 = (-9.40)^2$ $= 88.36$ (2) $\Delta^2 = (15.60)^2$ $= 243.36$ See Table A-3a, page 1-9 for complete list.
d. Sum the Δ^2.	$\Sigma\Delta^2 = 810.4$
e. Compute: $s = \sqrt{\frac{\Sigma\Delta^2}{N-1}}$	e. $s = \sqrt{\frac{810.40}{10-1}}$ $= \sqrt{\frac{810.40}{9}}$ $= \sqrt{90.04}$ $= 9.49$ $= 9$ min.

7.1.1.5 ANALYSIS

The standard deviation is a unit measure of dispersion around the mean. In the case of the normal distribution, 68% of the area under the curve is between $\bar{X} + s$ and $\bar{X} - s$ with μ centered at $\bar{X}$ or, in terms of the population, between $\mu + \sigma$ and $\mu - \sigma$ (see Figure 13).

7.1.2 CONFIDENCE INTERVAL ESTIMATES

7.1.2.1 TWO-SIDED INTERVAL

7.1.2.1.1 OBJECTIVE

To determine a two-sided confidence interval which is expected to bracket σ at the desired confidence level.

7.1.2.1.2 DATA REQUIRED

A list of sample readings.

AREA UNDER THE NORMAL CURVE

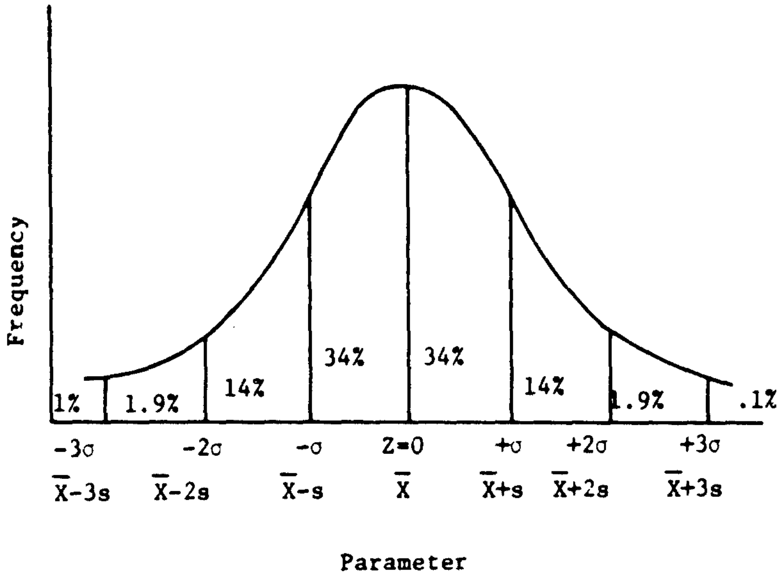


Figure 13

7.1.2.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute s (see paragraph 7.1.1.3, page 64).

c. Use Table B-9, page 2-35, to obtain B_U (upper bound) and B_L (lower bound) for N-1 d.f.

d. Multiply s by B_U to obtain the UCL and multiply s by B_L to obtain the LCL.

e. Conclude that σ is equal to or between the UCL and LCL at the desired confidence level.

7.1.2.1.4 EXAMPLE

Given:

Sample data at Table A-3a, page 1-9.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. $\alpha = .05$ $1-\alpha = .95$
b. Compute s.	b. $s = 9.49$ $= 9$ min. See paragraph 7.1.1.4, page 65.

c. Use Table B-9, page 2-35, to obtain B_U and B_L for α and N-1 d.f.

c. For α = .05 and 9 d.f.,
B_U = 1.746
B_L = .6657

d. Compute:

UCL = (B_U) s

LCL = (B_L) s

d. UCL = (1.746)(9.49)
= 16.57
= 17 min.
LCL = (.6657)(9.49)
= 6.32
= 6 min.

e. Conclude that σ ≤ UCL and σ ≥ LCL at a 100(1-α)% confidence level.

e. Conclude that σ ≤ 17 min. and σ ≥ 6 min. at a 95% confidence level.

7.1.2.1.5 ANALYSIS

The two-sided interval surrounds σ such that σ ≤ UCL and σ ≥ LCL at a 100(1-α)% confidence level.

7.1.2.2 ONE-SIDED INTERVAL

7.1.2.2.1 OBJECTIVE

To determine a one-sided confidence interval such that σ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level.

7.1.2.2.2 DATA REQUIRED

A list of sample readings.

7.1.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute s (see paragraph 7.1.1.3, page 64).

c. Use Table B-10, page 2-37, to obtain $A_{1-\alpha}$ (or A_α) for N-1 d.f.

d. Multiply $A_{1-\alpha}$ by s to obtain the UCL (or multiply A_α by s to obtain the LCL).

e. Conclude that σ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level.

7.1.2.2.4 EXAMPLE

Given:

Sample data at Table A-3a, page 1-9.

Procedure:

a. Choose the confidence level (1-α).

b. Compute s.

Example:

a. α = .05
1-α = .95

b. s = 9.49
= 9 min.
See paragraph 7.1.1.4, page 65.

c. Use Table B-10, page 2-37, to obtain $A_{1-\alpha}$ (or A_α) for N-1 d.f.	c. For 9 d.f., $A_{.95} = 1.645$ (or $A_{.05} = .7293$)
d. Compute: $UCL = A_{1-\alpha}\ s$ or $LCL = A_\alpha\ S$	d. $UCL = (1.645)(9.49) = 15.61 = 16$ min. (or $LCL = (.7293)(9.49) = 6.92 = 6$ min.)
e. Conclude that $\sigma \leq UCL$ (or $\sigma \geq LCL$) at a $100(1-\alpha)\%$ confidence level.	e. Conclude that $\sigma \leq 16$ min. (or $\sigma \geq 6$ min.) at a 95% confidence level.

7.1.2.2.5 ANALYSIS

The one-sided interval surrounds σ such that $\sigma \leq UCL$ (or $\mu \geq LCL$) at a $100(1-\alpha)\%$ confidence level.

7.1.3 SAMPLE SIZE REQUIRED TO ESTIMATE THE POPULATION STANDARD DEVIATION

7.1.3.1 OBJECTIVE

To determine the N_t required in order to state that σ lies within a specified percentage of its true value at the desired confidence level.

7.1.3.2 DATA REQUIRED

None.

7.1.3.3 PROCEDURE

a. Choose the desired confidence level.

b. Choose the allowable percentage of error.

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

d. Compute N_t as follows:

(1) Square step c.
(2) Square step b.
(3) Multiply step (2) by 2.
(4) Divide step (1) by step (3) and round to the next larger whole number.

e. Conclude that N_t samples are required in order to state that σ lies within an allowable percentage of error ot its true value at the desired confidence level.

7.1.3.4 EXAMPLE

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .05$ $1-\alpha = .95$ $1-\alpha/2 = .975$

b. Choose the percentage of error.	b. Percentage of error = .10
c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.	c. $Z_{.975} = 1.96$
d. Compute: $$N_t = \frac{Z_{1-\alpha/2}^{2}}{2(\text{percentage of error})^2}$$	d. $$N_t = \frac{(1.96)^2}{2(.10)^2} = \frac{3.84}{2(.01)} = \frac{3.84}{.02} = 192$$
e. Conclude that N_t samples are required in order to state that σ lies within an allowable percentage of its true value at a $100(1-\alpha)\%$ confidence level.	e. Conclude that 192 samples are required in order to state that σ lies within 10% of its true value at a 95% confidence level.

7.1.3.5 ANALYSIS

N_t samples are required in order to state that σ lies within a certain percentage of its true value at a $100(1-\alpha)\%$ confidence level.

7.2 COMPARING AN OBSERVED STANDARD DEVIATION (s) TO A REQUIREMENT (σ_0)

a. An observed standard deviation is generated from a sample and is representative of σ. This value of s is then compared to a stated requirement (σ_0). However, looking at the values of s and σ_0 to decide whether σ is greater than σ_0 or σ is less than σ_0 at a confidence level is insufficient. Since the decision pertains to the population, statistical tests must be applied to s to determine whether σ is greater than σ_0 or σ is less than σ_0.

b. There exist two possibilities for the relationship of s to σ_0. Following are the assumptions and the circumstances for each possible relationship:

(1) s greater than σ_0.

(a) The null hypothesis is σ is greater than σ_0.
(b) The alternative hypothesis is there is no reason to believe σ is greater than σ_0.
(c) The use of this test is appropriate when σ_0 is a maximum value for σ to satisfy. In the event that σ must not be greater than σ_0, this test would be appropriate.

(2) s less than σ_0.

(a) The null hypothesis is σ is less than σ_0.
(b) The alternative hypothesis is there is no reason to believe that σ is less than σ_0.

(c) The use of this test is appropriate when σ_0 is a minimum value for σ to satisfy. In the event that σ must meet or exceed σ_0, this test would be appropriate.

7.2.1 <u>s GREATER THAN σ_0.</u>

7.2.1.1 <u>OBJECTIVE</u>

To determine whether σ is greater than σ_0 at the desired confidence level.

7.2.1.2 <u>DATA REQUIRED</u>

A list of sample readings.

7.2.1.3 <u>PROCEDURE</u>

a. Choose the desired confidence level.

b. Use Table B-10, page 2-37, to obtain A_α for N-1 d.f.

c. Compute s (see paragraph 7.1.1.3, page 64).

d. Multiply step c by step b to obtain the LCL. The confidence interval for σ is from the LCL to $+\infty$.

e. If σ_0 is less than the LCL, decide that σ is greater than σ_0; otherwise, there is no reason to believe σ is greater than σ_0 at the desired confidence level.

7.2.1.4 <u>EXAMPLE</u>

Given:

σ_0 = 7.0 min.
Sample data at Table A-3a, page 1-9.

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. α = .05 $1-\alpha$ = .95
b. Use Table B-10, page 2-37, to obtain A_α for N-1 d.f.	b. For 9 d.f. $A_{.05}$ = .7293
c. Compute s.	c. s = 9.49 = 9 min. See paragraph 7.1.1.4, page 65.
d. Compute: LCL = A_α (s)	d. LCL = (.7293)(9.49) = 6.921 = 6 min.
e. [illegible] LCL, decide that [illegible]; otherwise, there is no reason to believe [illegible] at a 100(1-α)% confidence level.	e. Since 7.0 [illegible], decide that there is no reason to believe [illegible] 7.0 min. at a 95% confidence level.

7.2.1.5 ANALYSIS

If σ_0 < LCL, the null hypothesis that $\sigma > \sigma_0$ is accepted; otherwise, there is no reason to believe $\sigma > \sigma_0$ at a 100(1-α)% confidence level. The 100(1-α)% confidence interval for σ is from the LCL to + ∞.

7.2.2 s LESS THAN σ_0

7.2.2.1 OBJECTIVE

To determine whether σ is less than σ_0 at the desired confidence level.

7.2.2.2 DATA REQUIRED

A list of sample readings.

7.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-10, page 2-37, to obtain $A_{1-\alpha}$ for N-1 d.f.

c. Compute s (see paragraph 7.1.1.3, page 64).

d. Multiply step c by step b to obtain the UCL. The confidence interval for σ is from - ∞ to the UCL.

e. If σ_0 is greater than the UCL, decide that σ is less than σ_0; otherwise, there is no reason to believe σ is less than σ_0 at the desired confidence level.

7.2.2.4 EXAMPLE

Given:

σ_0 = 12.0 min.
Sample data at Table A-3a, page 1-9.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. α = .05 1-α = .95
b. Use Table B-10, page 2-37, to obtain $A_{1-\alpha}$ for N-1 d.f.	b. For 9 d.f., $A_{.95}$ = 1.645
c. Compute s.	c. s = 9.49 = 9 min. See paragraph 7.1.1.4, page 65.
d. Compute: UCL = $A_{1-\alpha}$(s)	d. UCL = (1.645)(9.49) = 15.611 = 16 min.
e. If σ_0 > UCL, decide that $\sigma < \sigma_0$; otherwise, there is no reason to believe $\sigma < \sigma_0$ at a 100(1-α)% confidence level.	e. Since 12.0 ≯ 16, decide that there is no reason to believe that σ < 12.0 min. at a 95% confidence level.

7.2.2.5 ANALYSIS

If $\sigma_0 > UCL$, the null hypothesis that $\sigma < \sigma_0$ is accepted; otherwise, there is no reason to believe $\sigma < \sigma_0$ at a $100(1-\alpha)\%$ confidence level.

7.2.3 DETERMINATION OF SAMPLE SIZE

7.2.3.1 OBJECTIVE

To determine the N_t required to determine whether σ is greater than $\gamma\ \sigma_0$ (or less than $\gamma\ \sigma_0$) at the desired confidence level.

7.2.3.2 DATA REQUIRED

None.

7.2.3.3 PROCEDURE

a. Choose α and β, the probabilities of making Type I and Type II errors respectively.

b. Estimate s based on experience or a comparable item.

c. Divide s by σ_0 to obtain γ, an intermediate value.

d. Use Table B-11, page 2-38, to obtain N_t which corresponds to γ and the chosen values of α and β. If one of these values is not contained in the table, continue with step e.

e. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

f. Compute N_t as follows:

(1) Multiply $Z_{1-\beta}$ by step c.
(2) Add step (1) to $Z_{1-\alpha}$.
(3) Divide step (2) by:

(a) $\gamma-1$, if s is greater than σ_0.
(b) $1-\gamma$, if s is less than σ_0.

(4) Square step (3).
(5) Multiply step (4) by 1/2.
(6) Add 1 to step (5) and round to the next larger whole number.

g. Conclude that N_t samples are required to determine whether σ is greater than $\gamma\ \sigma_0$ (or is less than $\gamma\ \sigma_0$) at the desired confidence level.

NOTE: When $\gamma > 1$, then s is greater than σ_0; and the null hypothesis is that $\sigma > \gamma\ \sigma_0$. When $\gamma < 1$, then s is less than σ_0; and the null hypothesis is that $\sigma < \gamma\ \sigma_0$.

7.2.3.4 EXAMPLE

Given:

$\sigma_0 = 7.3$

Procedure:

a. Choose α and β.

b. Estimate s.

c. Compute:

$$\gamma = \frac{s}{\sigma_o}$$

d. Use Table B-11, page 2-38, to obtain N_t which corresponds to γ and chosen values of α and β. If one of these values is not contained in Table B-11, page 2-38, continue with step e.

e. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

f. (1) If $s > \sigma_o$ ($\gamma > 1$), compute:

$$N_t = 1+(1/2)\left(\frac{Z_{1-\alpha}+\gamma(Z_{1-\beta})}{\gamma-1}\right)^2$$

(2) If $s < \sigma_o$ ($\gamma < 1$), compute:

$$N_t = 1+(1/2)\left(\frac{Z_{1-\alpha}+\gamma(Z_{1-\beta})}{1-\gamma}\right)^2$$

g. Conclude that N_t samples are required to determine whether $\sigma > \gamma\ \sigma_o$ (or $\sigma < \gamma\ \sigma_o$) at a $100(1-\alpha)\%$ confidence level.

Example:

a. $\alpha = .05$
$1-\alpha = .95$
$\beta = .20$
$1-\beta = .80$

b. $s = 9.5$

c.

$$\gamma = \frac{9.5}{7.3} = 1.3$$

d. Since $\beta = .20$ is not contained in Table B-11, page 2-38, continue with step e.

e. $Z_{.95} = 1.645$

$Z_{.80} = .840$

f. Since 9.5 > 7.3 (1.3 > 1)

$$N_t = 1+(1/2)\left(\frac{1.645+(1.3)(.804)}{1.3-1}\right)^2$$
$$= 1+(1/2)\left(\frac{1.645+1.092}{.3}\right)^2$$
$$= 1+(1/2)\left(\frac{2.737}{.3}\right)^2$$
$$= 1+(1/2)(9.123)^2$$
$$= 1+(1/2)(83.229)$$
$$= 1+41.614$$
$$= 42.614$$
$$= 43$$

g. Conclude that 43 samples must be tested in order to determine whether $\sigma > 1.3\ \sigma_o$ at a 95% confidence level.

7.2.3.5 ANALYSIS

a. Initial N_t.

At specified significance levels of α and β, N_t samples are required to determine whether $\sigma > \gamma\ \sigma_o$ (or $\sigma < \gamma\ \sigma_o$). As γ approaches 1, a very large sample size is required.

b. Adequacy of N_t.

(1) s greater than σ_o.

After the initial N_t samples have been tested, s must be computed and compared to the initial estimated s. If the computed s is greater than the initial s, the initial N_t is adequate; however, if the computed s is less than the initial s, the initial N_t is inadequate. If N_t is inadequate, N_t must be recomputed using the computed s in place of the initial s; and additional samples must be tested if possible.

(2) s less than σ_o.

After the initial N_t samples have been tested, s must be computed and compared to the initial estimated s. If the computed s is less than the initial s, the initial N_t is adequate; however, if the computed s is greater than the initial s, the initial N_t is inadequate. If N_t is inadequate, N_t must be recomputed using the computed s in place of the initial s; and additional samples must be tested if possible.

7.3 COMPARING TWO OBSERVED STANDARD DEVIATIONS

a. An observed standard deviation is generated from a sample and is representative of σ. This value of s is then required to meet a standard item s which is representative of the standard items population. Looking at the values of the standard deviations (s_A and s_B) to decide whether σ_A is greater than σ_B or σ_A is less than σ_B at a confidence level is insufficient. Since the decision pertains to the populations, statistical tests must be applied to s_A and s_B to determine whether σ_A is greater than σ_B or σ_A is less than σ_B. The statistical tests use the sample standard deviations as estimated of the population standard deviations.

b. Type A generally represents the test item and Type B the standard item when testing the hypothesis that σ_A is greater than σ_B. However, to prove that the average performance of the test item is less than that of the standard item, Type A must represent the standard item so that the hypothesis, σ_A is greater than σ_B, can be tested.

c. When the null hypothesis is σ_A is greater than σ_B, the alternative hypothesis is there is no reason to believe that σ_A is greater than σ_B.

d. The use of this test is appropriate when σ_B is a maximum value for σ_A to satisfy. In the event σ_A must not be greater than σ_B, this test would be appropriate.

7.3.1 s_A GREATER THAN s_B

7.3.1.1 OBJECTIVE

To determine whether σ_A is greater than σ_B at the desired confidence level.

7.3.1.2 DATA REQUIRED

A list of sample readings.

7.3.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $d.f._1$ and $d.f._2$ as follows:

(1) Subtract 1 from N_A to obtain $d.f._1$.
(2) Subtract 1 from N_B to obtain $d.f._2$.

c. Use Table B-8, page 2-18, to obtain $F_{1-\alpha}$, which is the UCL, for $(d.f._1, d.f._2)$ d.f.

d. Compute s_A^2 and s_B^2 (see paragraph 7.1.1.3, page 64).

e. Divide s_A^2 by s_B^2 to obtain the computed value of F.

f. If F is greater than the UCL, decide that σ_A is greater than σ_B; otherwise, there is no reason to believe σ_A is greater than σ_B at the desired confidence level.

7.3.1.4 EXAMPLE

Given:
Sample data at Tables A-3a, page 1-9, and A-3b, page 1-10

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .05$ $1-\alpha = .95$
b. Compute: $d.f._1 = N_A - 1$ $d.f._2 = N_B - 1$	b. $d.f._1 = 10 - 1$ $= 9$ $d.f._2 = 12 - 1$ $= 11$
c. Use Table B-8, page 2-18, to obtain $F_{1-\alpha}$ for $(d.f._1, d.f._2)$ d.f. $UCL = F_{1-\alpha}$	c. $F_{.95}$ for (9,11) d.f. = 2.90 UCL = 2.90
d. Compute s_A^2 and s_B^2	d. $s_A^2 = 810.4/9$ $= 90.04$ $= 90$ min $s_B^2 = 151/11$ $= 13.73$ $= 14$ min.

e. Compute:

$$F = \frac{s_A^2}{s_B^2}$$

e.

$$F = \frac{90.04}{13.73}$$

$$= 6.558$$

$$= 6.56$$

f. If $F > UCL$, decide that $\sigma_A > \sigma_B$; otherwise, there is no reason to believe $\sigma_A > \sigma_B$ at a $100(1-\alpha)\%$ confidence level.

f. Since $6.56 > 2.90$, decide that σ_A is greater than σ_B at a 95% confidence level.

7.3.1.5 ANALYSIS

If $F > UC'$, the null hypothesis that $\sigma_A > \sigma_B$ is accepted; otherwise, there is no reason to believe $\sigma_A > \sigma_B$ at a $100(1-\alpha)\%$ confidence level.

7.3.2 DETERMINATION OF SAMPLE SIZE

7.3.2.1 OBJECTIVE

To determine the N_t required to determine whether σ_A is greater than $\gamma\ \sigma_B$ (or less than $\gamma\ \sigma_B$) at the desired confidence level.

7.3.2.2 DATA REQUIRED

None.

7.3.2.3 PROCEDURE

a. Choose α and β, the probabilities of making Type I and Type II errors respectively.

b. Estimate s_A and s_B based on experience or comparable items.

c. Divide s_A by s_B to obtain γ, an intermediate value.

d. Use Table B-12, page 2-41, to obtain N_t which corresponds to γ and the chosen value of α and β. If one of these values is not contained in the table, continue with step e.

e. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

f. Compute N_t as follows:

(1) Add $Z_{1-\alpha}$ and $Z_{1-\beta}$.
(2) Divide step (1) by the natural logarithm of step c ($\ln \gamma$).
(3) Square step (2).
(4) Add 2 to step (3) and round up.

g. Conclude that N_t samples are required to determine whether σ_A is greater than $\gamma\ \sigma_B$ (or is less than $\gamma\ \sigma_B$) at the desired confidence level.

NOTE: When $\gamma > 1$, then s_A is greater than s_B; and the null hypothesis is that $\sigma_A > \sigma_B$. When $\gamma < 1$, then s_A is less than s_B; and the null hypothesis is that $\sigma_A < \gamma\, \sigma_B$.

7.3.2.4 EXAMPLE

Procedure:	Example:
a. Choose α and β.	a. $\alpha = .05$ $1-\alpha = .95$ $\beta = .20$ $1-\beta = .80$
b. Estimate s_A and s_B.	b. $s_A = 6.0$ $s_B = 4.8$
c. Compute: $\gamma = {s_A}/{s_B}$	c. $\gamma = \frac{6.0}{4.8}$ $= 1.250$
d. Use Table B-12, page 2-41, to obtain N_t which corresponds to γ and the chosen values of α and β. If one of these values is not contained in the table, continue with step e.	d. Since $\gamma = 1.250$ is not contained in Table B-12, page 2-41, continue with step e.
e. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.	e. $Z_{.95} = 1.645$ $Z_{.80} = .840$
f. Compute: $N_t = 2 + \left(\frac{Z_{1-\alpha}+Z_{1-\beta}}{\ln(\gamma)}\right)^2$	f. $N_t = 2 + \left(\frac{1.645+.840}{\ln 1.25}\right)^2$ $= 2 + \left(\frac{2.485}{.2231}\right)^2$ $= 2 + (11.14)^2$ $= 2 + 124.1$ $= 126.1$ $= 127$
g. Conclude that N_t samples are required to determine whether $\sigma_A > \gamma\sigma_B$ (or $\sigma_A < \gamma\, \sigma_B$) at a $100(1-\alpha)\%$ confidence level.	g. Conclude that 127 samples of eacn item must be tested in order to determine whether $\sigma_A > 1.25\, \sigma_B$ at a 95% confidence level.

7.3.2.5 ANALYSIS

a. Initial N_t.

At specified significant levels of α and β, N_t samples are required to determine whether $\sigma_A > \gamma\ \sigma_B$ (or $\sigma_A < \gamma\ \sigma_B$). As γ approaches 1, a very large sample size is required.

b. Adequacy of N_t.

(1) s_A greater than s_B.

After the initial N_t samples have been tested, s_A and s_B must be computed. Their ratio (s_A/s_B) must then be computed and compared to the initial ratio determined for the initial N_t. If the computed ratio is greater than the initial ratio, the initial N_t is adequate; however, if the computed ratio is less than the initial ratio, the initial N_t is inadequate. If N_t is inadequate, N_t must be recomputed using the computed ratio in place of the initial ratio; and additional samples must be tested if possible.

(2) s_A less than s_B.

After the initial N_t samples have been tested, s_A and s_B must be computed. Their ratio (s_A/s_B) must then be computed and compared to the initial ratio determined for the initial N_t. If the computed ratio is less than the initial ratio, the initial N_t is adequate; however, if the computed ratio is greater than the initial ratio, the initial N_t is inadequate. If N_t is inadequate, N_t must be recomputed using the computed ratio in place of the initial ratio; and additional samples must be tested if possible.

8. PROPORTION

For some kinds of tests there may be no way to obtain actual measurements. An item may be subjected to a test when the result of that particular test can be expressed only in terms of a pre-established classification of possible results. The simplest kind of classification, and the one most widely used, consists of just two mutually exclusive categories; e.g., success and failure or perfect and defective. The ratio generated, the number of items having the characteristic divided by N, is known as a proportion (P) or a success-attempt ratio. In all examples P is computed relative to failures (f); however, other variables, such as successes, may be substituted.

8.1 ESTIMATE OF THE POPULATION PROPORTION (P)

8.1.1 BEST SINGLE ESTIMATE of P

8.1.1.1 OBJECTIVE

To determine the best point estimate of the population proportion (P).

8.1.1.2 DATA REQUIRED

N and the number of elements possessing the given characteristic.

8.1.1.3 PROCEDURE

a. Divide the number of sample items which have the characteristic by the total number of items in the sample.

b. Conclude that P is the best estimate of the proportion of population of items which will have the given characteristics.

8.1.1.4 EXAMPLE

Given:

N = 10

f = 4

Procedure:	Example:
a. Compute: P = characteristic/N	a. P = f/N = 4/10 = .4
b. Conclude that P is the best estimate of the proportion of population items which will have the given characteristic.	b. Conclude that .4 is the best estimate of λ, the fraction of population items that will fail.

8.1.1.5 ANALYSIS

The best single estimate of λ is the observed proportion of items having this characteristic in a random sample from the population; i.e., the number of sample items which have the characteristic divided by the total number of items in the sample.

8.1.2 CONFIDENCE INTERVAL ESTIMATES

8.1.2.1 TWO-SIDED INTERVAL FOR $N \leq 30$

8.1.2.1.1 OBJECTIVE

To determine a two-sided confidence interval which is expected to bracket λ at the desired confidence level when N is equal to or less than 30.

8.1.2.1.2 DATA REQUIRED

N and the number of elements possessing the given characteristic.

8.1.2.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-13, page 2-43, to obtain the UCL and LCL which correspond to N and the number of elements possessing the given characteristic at the desired confidence level.

c. Conclude that λ is equal to or between the UCL and LCL at the desired confidence level.

8.1.2.1.4 EXAMPLE

Given:

N - 10 ($N \leq 30$)

f = 4

Procedure:	Example:
a. Choose the confidence level ($1-\alpha$).	a. $\alpha = .05$ $1-\alpha = .95$
b. Use Table B-13, page 2-43, to obtain the UCL and LCL which correspond to N and the number of elements possessing the given characteristic at a $100(1-\alpha)\%$ confidence level.	b. For N = 10, f = 4, and $1-\alpha = .95$, UCL = .733 = .8 LCL = .150 = .1
c. Conclude that $\lambda \leq$ UCL and $\lambda \geq$ LCL at a $100(1-\alpha)\%$ confidence level.	c. Conclude that $\lambda \leq .8$ and $\lambda \geq .1$ at a 95% confidence level.

8.1.2.1.5 ANALYSIS

The two-sided interval surrounds λ such that $\lambda \leq$ UCL and $\lambda \geq$ LCL at a $100(1-\alpha)\%$ confidence level.

8.1.2.2 TWO-SIDED INTERVAL FOR $N > 30$

8.1.2.2.1 OBJECTIVE

To determine a two-sided confidence interval which is expected to bracket λ at the desired confidence level when N is greater than 30.

8.1.2.2.2 DATA REQUIRED

N and the number of elements possessing the given characteristic.

8.1.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

c. Compute P (see paragraph 8.1.1.3, page 79).

d. Compute the UCL and LCL as follows:

(1) Multiply P by the quantity (1-P).
(2) Divide step (1) by N.

(3) Find the square root of step (2).
(4) Multiply step b by step (3).
(5) Add step (4) to P to determine the UCL and subtract step (4) from P to determine the LCL.

e. Conclude that λ is equal to or between the UCL and LCL at the desired confidence level.

8.1.2.2.4 EXAMPLE

Given:

$N = 150$ $(N > 30)$
$f = 60$

Procedure:

a. Choose the confidence level (1-α

b. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

c. Compute:

P = characteristic/N

d. Compute:

$$UCL = P+Z_{1-\alpha/2}\sqrt{\frac{P(1-P)}{N}}$$

$$LCI = P-Z_{1-\alpha/2}\sqrt{\frac{P(1-P)}{N}}$$

Example:

a. $\alpha = .10$
$1-\alpha = .90$
$1-\alpha/2 = .95$

b. $Z_{.95} = 1.645$

c. $P = 60/150$
$= .40$

d.

$$UCL = .40+1.645\sqrt{\frac{.4(1-.4)}{150}}$$
$$= .40+1.645\sqrt{\frac{.4(.6)}{150}}$$
$$= .40+1.645\sqrt{\frac{.24}{150}}$$
$$= .40+1.645\sqrt{.0016}$$
$$= .40+1.645(.04)$$
$$= .40+.07$$
$$= .47$$

$$LCL = .40-1.645\sqrt{\frac{.4(1 - .4)}{150}}$$
$$= .40-1.645(.04)$$
$$= .40-.07$$
$$= .33$$

e. Conclude that $\lambda \leq$ UCL and $\lambda \geq$ LCL at a 100(1-α)% confidence level.

e. Conclude that $\lambda \leq .47$ and $\lambda \geq .33$ at a 90% confidence level.

8.1.2.2.5 ANALYSIS

The two-sided interval surrounds λ such that $\lambda \leq$ UCL and $\lambda \geq$ LCL at a 100(1-α)% confidence level.

8.1.2.3 ONE-SIDE INTERVAL FOR $N \leq 30$

8.1.2.3.1 OBJECTIVE

To determine a one-sided confidence interval such that λ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level when N is equal to or less than 30.

8.1.2.3.2 DATA REQUIRED

N and the number of elements possessing the given characteristic.

8.1.2.3.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-14, page 2-50, to obtain the UCL (or the LCL) which corresponds to N and the number of elements possessing the given characteristic at the desired confidence level.

c. Conclude that λ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level.

8.1.2.3.4 EXAMPLE

Given:

$N = 10$ ($N \leq 30$)

$f = 4$

Procedure:

a. Choose the confidence level (1-α).

b. Use Table B-14, page 2-50, to obtain the UCL (or the LCL) which corresponds to N and the number of elements possessing the given characteristic at a 100(1-α)% confidence level.

c. Conclude that $\lambda \leq$ UCL (or $\lambda \geq$ LCL) at a 100(1-α)% confidence level.

Example:

a. $\alpha = .05$
$1-\alpha = .95$

b. For $N = 10$, $f = 4$, and $1 - \alpha = .95$.

UCL = .696

(or LCL = 1 - .850

= .150

c. Conclude that $\lambda \leq .696$ (or $\lambda \geq .150$) at a 95% confidence level.

8.1.2.3.5 ANALYSIS

The one-sided interval surrounds λ such that $\lambda \leq$ UCL (or $\lambda \geq$ LCL) at a 100(1-α)% confidence level.

8.1.2.4 ONE-SIDED INTERVAL FOR N > 30

8.1.2.4.1 OBJECTIVE

To determine a one-sided confidence interval such that λ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level when N is greater than 30.

8.1.2.4.2 DATA REQUIRED

N and the number of elements possessing the given characteristic.

8.1.2.4.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

c. Compute P (see paragraph 8.1.1.3, page 79).

d. Compute the UCL (or LCL) as follows:

(1) Multiply P by the quantity (1-P).
(2) Divide step (1) by N.
(3) Find the square root of step (2).
(4) Multiply step b by step (3).
(5) Add step (4) to P to obtain the UCL (or subtract step (4) from P to obtain the LCL).

e. Conclude that λ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level.

8.1.2.4.4 EXAMPLE

Given:

N = 150 (N > 30)
f = 60

Procedure:	Example:
a. Choose the confidence level (1-α).	a. α = .10 1-α = .90
b. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.	b. $Z_{.90}$ = 1.282
c. Compute:	c. P = f/N
P = characteristic/N	= 60/150 = .40

d. Compute:

$$UCL = P + Z_{1-\alpha}\sqrt{\frac{P(1-P)}{N}}$$

$$\left(\text{or } LCL = P - Z_{1-\alpha}\sqrt{\frac{P(1-P)}{N}}\right)$$

e. Conclude that $\lambda \le UCL$ (or $\lambda \ge LCL$) at a $100(1-\alpha)\%$ confidence level.

d.
$$UCL = .40 + 1.282\sqrt{\frac{.4(1-.4)}{150}}$$
$$= .40 + 1.282\sqrt{\frac{.4(.6)}{150}}$$
$$= .40 + 1.282\sqrt{\frac{.24}{150}}$$
$$= .40 + 1.282\sqrt{.0016}$$
$$= .40 + 1.282(.04)$$
$$= .40 + .05$$
$$= .45$$
$$(\text{or } LCL = .40 - .05$$
$$= .35)$$

e. Conclude that $\lambda \le .45$ (or $\lambda \ge .35$) at a 90% confidence level.

8.1.2.4.5 ANALYSIS

The one-sided interval surrounds λ such that $\lambda \le UCL$ (or $\lambda \ge LCL$) at a $100(1-\alpha)\%$ confidence level.

8.1.3 SAMPLE SIZE REQUIRED TO ESTIMATE THE POPULATION PROPORTION

8.1.3.1 SAMPLE SIZE WITH A SPECIFIED LIMIT OF ERROR IN BOTH DIRECTIONS

8.1.3.1.1 OBJECTIVE

To determine the N_t required in order to state that λ is equal to or between P + [illegible] and P − [illegible] at the desired confidence level.

8.1.3.1.2 DATA REQUIRED

None.

8.1.3.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Choose the allowable amount of error.

c. Choose a value for P in the following manner:

(1) If no prior information is available and if λ is believed to be in the neighborhood of 0.5, use P = 0.5. The largest sample size will be required when P = 0.5, and the purpose of the rules is to be as conservative as possible.

(2) If λ can safely be assumed to be less than o.5, let P be the largest reasonable guess for λ.

(3) If λ can safely be assumed to be breater than 0.5, let P be the smallest reasonable guess for λ.

d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

e. Compute N_t as follows:

(1) Square step d.
(2) Multiply P by the quantity (1-P).
(3) Multiply step (1) by step (2).
(4) Square ε.
(5) Divide step (3) by step (4).
(6) Round the result of step (5) up to the next whole number.

f. Conclude that N_t samples are required in order to state that λ is equal to or between $P + \varepsilon$ and $P - \varepsilon$ at the desired confidence level.

8.1.3.1.4 <u>EXAMPLE</u>

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .10$ $1-\alpha/2 = .95$
b. Choose ε.	b. $\varepsilon = .10$
c. Choose P.	c. $P = 0.5$
d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.	d. $Z_{.95} = 1.645$
e. Compute: $N_t = \frac{(Z_{1-\alpha/2})^2 (P)(1-P)}{\varepsilon^2}$	e. $N_t = \frac{(1.645)^2(.5)(1-.5)}{(.10)^2} = \frac{(2.706)(.5)(.5)}{.01} = \frac{(2.706)(.25)}{.01} = \frac{.6765}{.01} = 67.65 = 68$
f. Conclude that N_t samples are required in order to state that $\lambda \leq P + \varepsilon$ and $\lambda \geq P - \varepsilon$ at a $100(1-\alpha)\%$ confidence level.	f. If 68 samples are tested and P computed, conclude that $\lambda \leq P + .10$ and $\lambda \geq P - .10$ at a 90% confidence level.

8.1.3.1.5 ANALYSIS

If N_t samples are tested and P is computed, conclude that $\lambda \leq P + \varepsilon$ and $\lambda \geq P - \varepsilon$ at a 100(1-α)% confidence level.

8.1.3.2 SAMPLE SIZE WITH A SPECIFIED LIMIT OF ERROR IN ONLY ONE DIRECTION

8.1.3.2.1 OBJECTIVE

To determine the N_t required in order to state that λ is equal to or less than $P + \varepsilon$ (or equal to or greater than $P - \varepsilon$) at the desired confidence level.

8.1.3.2.2 DATA REQUIRED

None.

8.1.3.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Choose the allowable amount of error.

c. Choose the value of P in the following manner:

(1) If no prior information is available and if λ is believed to be in the neighborhood of 0.5, use P = 0.5. The largest sample size will be required when P = 0.5, and the purpose of the rules is to be as conservative as possible.
(2) If λ can safely be assumed to be less than 0.5, let P be the largest reasonable guess for λ.
(3) If λ can safely be assumed to be greater than 0.5, let P be the smallest reasonable guess for λ.

d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

e. Compute N_t as follows:

(1) Square step d.
(2) Multiply P by the quantity (1-P).
(3) Multiply step (1) by step (2).
(4) Square ε.
(5) Divide step (3) by step (4).
(6) Round the result of step (5) up to the next whole number.

f. Conclude that N_t samples are required in order to state that λ is equal to or less than $P + \varepsilon$ (or equal to or greater than $P - \varepsilon$) at the desired confidence level.

8.1.3.2.4 EXAMPLE

Procedure:	Example:
a. Choose the confidence level 1-α).	a. α = .10 1-α = .90

b. Choose ε.

b. $\varepsilon = .10$

c. Choose P.

c. $P = 0.5$

d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

d. $Z_{.90} = 1.282$

e. Compute:

$$N_t = \frac{(Z_{1-\alpha})^2(P)(1-P)}{\varepsilon^2}$$

e.

$$N_t = \frac{(1.282)^2(0.5)(1-0.5)}{(.10)^2}$$

$$= \frac{(1.644)(0.5)(0.5)}{.01}$$

$$= \frac{(1.644)(.25)}{.01}$$

$$= \frac{.4110}{.01}$$

$$= 41.10$$

$$= 42$$

f. Conclude that N_t samples are required in order to state that $\lambda \leq P + \varepsilon$ or $(\lambda \geq P - \varepsilon)$ at a $100(1-\alpha)\%$ confidence level.

f. If 42 samples are tested and P computed, conclude that $\lambda \leq P + .10$ at a 90% confidence level.

8.1.3.2.5 ANALYSIS

If N_t samples are tested and P is computed, $\lambda \leq P + \varepsilon$ (or $\lambda \geq P - \varepsilon$) at a 100)1-$\alpha$)% confidence level.

8.2 COMPARING AN OBSERVED PROPORTION (P) TO A REQUIREMENT (λ_o)

a. An observed proportion is generated from a sample and is representative of λ. This value of P is then compared to a stated requirement (λ_o). However, looking at the values of P and λ_o to decide whether λ is greater than λ_o or λ is less than λ_o at a confidence level is insufficient. Since the decision pertains to the population, statistical tests must be applied to P to determine whether λ is greater than λ_o or λ is less than λ_o.

b. There exist two possibilities for the relationship of P to λ_o. Following are the assumptions and the circumstances for each possible relationship:

(1) P greater than λ_o.

(a) The null hypothesis is λ is greater than λ_o.
(b) The alternative hypothesis is there is no reason to believe λ is greater than λ_o.
(c) The use of this test is appropriate when λ_o is a maximum value for λ to satisfy. In the event that λ must not be greater than λ_o, this test would be appropriate.

(2) P is less than λ_0.

(a) The null hypothesis is λ is less than λ_0.
(b) The alternative hypothesis is there is no reason to believe λ is less than λ_0.
(c) The use of this test is appropriate when λ_0 is a minimum value for λ to satisfy. In the event that λ must meet or exceed λ_0, this test would be appropriate.

8.2.1 P GREATER THAN λ_0

8.2.1.1 SMALL SAMPLE SIZE

8.2.1.1.1 OBJECTIVE

To determine whether λ is greater than λ_0 at the desired confidence level when N is equal to or less than 30.

8.2.1.1.2 DATA REQUIRED

Success-failure data.

8.2.1.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-14, page 2-50, to obtain the LCL which corresponds to N and the number of elements possessing the given characteristic at the desired confidence level.

c. If λ_0 is less than the LCL, decide that λ is greater than λ_0; otherwise, there is no reason to believe λ is greater than λ_0 at the desired confidence level.

8.2.1.1.4 EXAMPLE

Given:

$N = 20$ $(N \leq 30)$
$f = 3$
$\lambda_0 = .100$

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .05$ $1-\alpha = .95$
b. Use Table B-14, page 2-50, to obtain the LCL which corresponds to N and the number of elements possessing the given characteristic at a $100(1-\alpha)\%$ confidence level.	b. For $1-\alpha = .95$, $N = 20$, and $N-f = 17$, the tabled value is .958. This must be subtracted from 1; hence, $LCL = 1 - .958$ $= .042$

c. If λ_0 < LCL, decide that $\lambda > \lambda_0$; otherwise, there is no reason to believe $\lambda > \lambda_0$ at a 100(1-α)% confidence level.

c. Since .100 $\nless$ .042, decide that there is no reason to believe λ > .100 at a 95% confidence level.

8.2.1.1.5 ANALYSIS

If λ_0 < LCL, the null hypothesis that $\lambda > \lambda_0$ is accepted; otherwise, there is nc reason to believe $\lambda > \lambda_0$ at a 100(1-α)% confidence level.

8.2.1.2 LARGE SAMPLE SIZE

8.2.1.2.1 OBJECTIVE

To determine whether λ is greater than λ_0 at the desired confidence level when N is greater than 30.

8.2.1.2.2 DATA REQUIRED

Success-failure data.

8.2.1.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute Z as follows:

(1) Multiply λ_0 by N.
(2) Subtract step (1) from the number of items having the given characteristic.
(3) Add .5 to step (2).
(4) Multiply the quantity (1-λ_0) by step (1).
(5) Divide step (3) by the square root of step (4).

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$, which is the UCL.

d. If Z is greater than the UCL, decide that λ is greater than λ_0; otherwise, there is no reason to believe λ is greater than λ_0 at the desired confidence level.

8.2.1.2.4 EXAMPLE

Given:

N = 100 (N > 30)
f = 7
λ_0 = .06

Procedure:

a. Choose the confidence level (1-α).

Example:

a. α = .10
1-α = .90

b. Compute:

$$Z = \frac{f - N\lambda_0 + .5}{\sqrt{N\lambda_0(1-\lambda_0)}}$$

b.

$$Z = \frac{7-100(.06)+.5}{\sqrt{100(.06)(-.06)}}$$

$$= \frac{7-6+.50}{\sqrt{6\ (.94)}}$$

$$= \frac{1.50}{\sqrt{5.64}}$$

$$= \frac{1.50}{2.375}$$

$$= .633$$

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

$UCL = Z_{1-\alpha}$

c. $Z_{.90} = 1.282$

UCL = 1.282

d. If $Z > UCL$, decide that $\lambda > \lambda_0$; otherwise, there is no reason to believe $\lambda > \lambda_0$ at a $100(1-\alpha)\%$ confidence level.

d. Since $.633 \not> 1.282$, decide that there is no reason to believe $\lambda > .06$ at a 90% confidence level.

8.2.1.2.5 ANALYSIS

If $Z > UCL$, the null hypothesis that $\lambda > \lambda_0$ is accepted; otherwise, there is no reason to believe $\lambda > \lambda_0$ at a $100(1-\alpha)\%$ confidence level.

8.2.2 P LESS THAN λ_0

8.2.2.1 SMALL SAMPLE SIZE

8.2.2.1.1 OBJECTIVE

To determine whether λ is less than λ_0 at the desired confidence level when N is equal to or less than 30.

8.2.2.1.2 DATA REQUIRED

Success-failure data.

8.2.2.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-14, page 2-50, to obtain the UCL which corresponds to N and the number of elements possessing the given characteristic at the desired confidence level.

c. If λ_0 is greater than the UCL, decide that λ is less than λ_0; otherwise, there is no reason to believe λ is less than λ_0 at the desired confidence level.

8.2.2.1.4 EXAMPLE

Given:

$N = 20\ (N \ngtr 30)$
$f = 3$
$\lambda_0 = .200$

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .05$ $1-\alpha = .95$
b. Use Table B-14, page 2-50, to obtain the UCL which corresponds to N and the number of elements possessing the given characteristics at a $100(1-\alpha)\%$ confidence level.	b. For $1-\alpha = .95$, $N = 20$, and $f = 3$, $UCL = .344$
c. If $\lambda_0 > UCL$, decide that $\lambda < \lambda_0$; otherwise, there is no reason to believe $\lambda < \lambda_0$ at a $100(1-\alpha)\%$ confidence level.	c. Since $.200 \not> .344$, decide that there is no reason to believe $\lambda < .200$ at the 95% confidence level.

8.2.2.1.5 ANALYSIS

If $\lambda_0 > UCL$, the null hypothesis that $\lambda < \lambda_0$ is accepted; otherwise, there is no reason to believe $\lambda < \lambda_0$ at a $100(1-\alpha)\%$ confidence level.

8.2.2.2 LARGE SAMPLE SIZE

8.2.2.2.1 OBJECTIVE

To determine whether λ is less than λ_0 at the desired confidence level when N is greater than 30.

8.2.2.2.2 DATA REQUIRED

Success-failure data.

8.2.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute Z as follows:

(1) Multiply λ_0 by N.
(2) Subtract step (1) from the number of items having the given characteristic.
(3) Subtract .5 from step (2).
(4) Multiply the quantity $(1-\lambda_0)$ by step (1).
(5) Divide step (3) by the square root of step (4).

c. Use Table B-4, page 2-4, to obtain Z_α, which is the LCL.

d. If Z is less than the LCL, decide that λ is less than λ_0; otherwise, there is no reason to believe λ is less than λ_0 at the desired confidence level.

8.2.2.2.4 EXAMPLE

Given:

$N = 100$ $(N > 30)$
$f = 7$
$\lambda_0 = .08$

Procedure:

a. Choose the confidence level $(1-\alpha)$.

b. Compute:

$$Z = \frac{f - N\lambda_0 - .5}{\sqrt{N\lambda_0(1-\lambda_0)}}$$

c. Use Table B-4, page 2-4, to obtain Z_α.

$LCL = Z_\alpha$

d. If $Z < LCL$, decide that $\lambda < \lambda_0$; otherwise, there is no reason to believe $\lambda < \lambda_0$ at a $100(1-\alpha)\%$ confidence level.

Example:

a. $\alpha = .10$
$1-\alpha = .90$

b.

$$Z = \frac{7-100(.08)-.5}{\sqrt{100(.08)(1-.08)}} = \frac{7-8-.5}{\sqrt{8(.92)}} = \frac{-1.5}{\sqrt{7.36}} = \frac{-1.5}{2.71} = -.554$$

c. $Z_{.10} = -1.282$

$LCL = -1.282$

d. Since $-.554 \not< -1.282$, decide that there is no reason to believe $\lambda < .08$ at a 90% confidence level.

8.2.2.2.5 ANALYSIS

If $Z < LCL$, the null hypothesis that $\lambda < \lambda_0$ is accepted; otherwise, there is no reason to believe $\lambda < \lambda_0$ at a $100(1-\alpha)\%$ confidence level.

8.2.3 DETERMINATION OF SAMPLE SIZE

8.2.3.1 OBJECTIVE

To determine the N_t required to determine whether λ is equal to or greater than $\lambda_0 + \varepsilon$ (or equal to or less than $\lambda - \varepsilon$) at the desired confidence level.

8.2.3.2 DATA REQUIRED

λ which is known from a standard item, history, or Requirements Document.

8.2.3.3 PROCEDURE

a. Choose α and β, the probabilities of making Type I and Type II errors respectively.

b. Choose the allowable amount of error.

c. Estimate the test item proportion by adding ε to λ.

d. Use Table B-15, page 2-54, to obtain θ_1, which corresponds to P, and θ_o, which corresponds to λ.

e. Compute d^2, an intermediate value, as follows:

(1) Subtract θ_o from θ_1.
(2) Square Step (1).

f. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

g. Compute N_t as follows:

(1) Add $Z_{1-\alpha}$ to $Z_{1-\beta}$.
(2) Square step (1).
(3) Divide step (2) by step e.
(4) Round step (3) to the next larger whole number.

h. Conclude that N_t samples are required to determine whether λ is equal to or greater than $\lambda_o + \varepsilon$ (or equal to or less than $\lambda_o - \varepsilon$) at the desired confidence level.

8.2.3.4 EXAMPLE

Given:

$\lambda = .41$

Procedure:	Example:
a. Choose α and β.	a. $\alpha = .05$ $1-\alpha = .95$ $\beta = .20$ $1-\beta = .80$
b. Choose ε.	$\varepsilon = .23$
c. Estimate P as follows: $P = \lambda + \varepsilon$	c. $P = .41 + .23$ $= .64$
d. Use Table B-15, page 2-54, to obtain θ_1, which corresponds to P, and θ_o, which corresponds to λ.	d. For $P = .64$, $\theta_1 = 1.85$ For $\lambda = .41$, $\theta_o = 1.39$
e. Compute: $d^2 = (\theta_1 - \theta_o)^2$	e. $d^2 = (1.85-1.39)^2$ $= (.46)^2$ $= .2116$
f. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.	f. $Z_{.95} = 1.645$ $Z_{.80} = .840$

g. Compute:

$$N_t = \frac{(Z_{1-\alpha}+Z_{1-\beta})^2}{d^2}$$

g.

$$N_t = \frac{(1.645+.840)^2}{.2116}$$

$$= \frac{(2.485)^2}{.2116}$$

$$= \frac{6.1752}{.2116}$$

$$= 29.18$$

$$= 30$$

h. Conclude that N_t samples are required to determine whether $\lambda \geq \lambda_o + \varepsilon$ (or $\lambda \leq \lambda_o - \varepsilon$) at a $100(1-\alpha)\%$ confidence level.

h. Conclude that 30 samples for λ known and equal to .41 must be tested in order to determine whether $\lambda \geq \lambda_o + .23$ at a 95% confidence level.

8.2.3.5 ANALYSIS

N_t samples are required to determine whether $\lambda \geq \lambda_o + \varepsilon$ (or $\lambda \leq \lambda_o - \varepsilon$) at a $100(1-\alpha)\%$ confidence level.

8.3 COMPARING TWO OBSERVED PROPORTIONS

a. An observed proportion is generated from a sample and is representative of λ. This value of P is then required to meet a standard item P which is representative of the standard item's population. Looking at the values of the proportions (P_A and P_B) to decide whether λ_A is greater than λ_B or λ_A is less than λ_B at a confidence level is insufficient. Since the decision pertains to the populations, statistical tests must be applied to determine whether λ_A is greater than λ_B or λ_A is less than λ_B. The statistical tests use the sample proportions as estimates of the population proportions.

b. Type A generally represents the test item and Type B the standard item when testing the hypothesis that λ_A is greater than λ_B. However, to prove that the average performance of the test item is less than that of the standard item, Type A must represent the standard item so that the hypothesis, λ_A is greater than λ_B, can be tested.

c. When the null hypothesis is λ_A is greater than λ_B, the alternative hypothesis is there is no reason to believe that λ_A is greater than λ_B.

d. The use of this test is appropriate when λ_B is a maximum value for λ_A to satisfy.

8.3.1 P_A GREATER THAN P_B

8.3.1.1 SMALL SAMPLE SIZE

8.3.1.1.1 OBJECTIVE

To determine whether λ_A is greater than λ_B at the desired confidence level when neither N_A nor N_B is greater than 20.

8.3.1.1.2 DATA REQUIRED

Success-failure data.

8.3.1.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Arrange the data as in Table A-4a, Part I, page 1-11.

c. Focus on the class of interest and compute the following intermediate values:

(1) h_A, the ratio of the class of interest to the sample size for Type A; i.e., $h_A = I_A/N_A$ or $h_A = II_A/N_A$.
(2) h_B, the ratio of the class of interest to the sample size for Type B; i.e., $h_B = I_B/N_B$ or $h_B = II_B/N_B$.

d. If h_A is greater than h_B, continue with step e; however, if h_A is not greater than h_B, decide that the data give no reason to believe that λ_A is greater than λ_B at the desired confidence level.

e. Arrange the data so that the results of the larger sample are in the first row (see Table A-4a, Part II, page 1-11.

f. Compute the following intermediate values:

(1) h_1, the ratio of class I to the sample size for the item having the larger sample size; i.e., $h_1 = I_1/N_1$.
(2) h_2, the ratio of Class I to the sample size for the item having the smaller sample size; i.e., $h_2 = I_2/N_2$.
(3) g_1, the ratio of class II to the sample size for the item having the larger sample size, i.e., $g_1 = II_1/N_1$.
(4) g_2, the ratio of class II to the sample size for the item having the smaller sample size; i.e., $g_2 = II_2/N_2$.

g. Focus attention on that class (I or II) which produces a proportion for the larger sample which is larger than or equal to the respective proportion for the smaller sample. Depending on the class chosen, let I_1 (or II_1) equal a_1, an intermdeiate value, and I_2 (or II_2) equal a_2, an intermediate value.

h. Use Table B-16, page 2-55, to obtain a tabled a_2 which corresponds to the two sample sizes and a_1 at the desired confidence level.

i. If a_2 from step g is less than or equal to the table a_2, decide that λ_A is greater than λ_B with regard to the class of interest; otherwise, there is no reason to believe λ_A is greater than λ_B at the desired confidence level.

8.3.1.1.4 EXAMPLE

Given:
Sample data at Table A-4a, Part I, page 1-11.

Procedure:

a. Choose the confidence level $(1-\alpha)$.

b. Arrange the data.

c. Focus on the class of interest and compute one of the following:

(1) Class I.

$h_A = I_A/N_A$

$h_B = I_B/N_B$

(2) Class II

$h_A = II_A/N_A$

$h_B = II_B/N_B$

d. If $h_A > h_B$, continue with step e. If $h_A < h_B$, decide that the data give no reason to believe that λ_A is greater than λ_B with respect to the class of interest at a $100(1-\alpha)\%$ confidence level.

e. Arrange the data so that the results of the larger sample are in the first row.

f. Compute:

$h_1 = I_1/N_1$

$h_2 = I_2/N_2$

$g_1 = II_1/N_1$

$g_2 = II_2/N_2$

Example:

a. $\alpha = .05$
$1-\alpha = .95$

b. See Table A-4a, Part I, page 1-11.

c. Focus on class II.

$h_A = 2/6$
$= .333$
$= .3$

$h_B = 2/10$
$= .200$
$= .2$

d. Since $.3 > .2$, continue with step e.

e. See Table A-4a, Part II, page 1-11.

f. $h_1 = 8/10$
$= .800$
$= .8$

$h_2 = 4/6$
$= .667$
$= .7$

$g_1 = 2/10$
$= .200$
$= .2$

$g_2 = 2/6$
$= .333$
$= .3$

g. (1) If $h_1 \geq h_2$, focus attention on class I with

$a_1 = I_1$

$a_2 = I_2$

(2) If $g_1 \geq g_2$, focus attention on class II with

$a_1 = II_1$

$a_2 = II_2$

g. Since .8 > .7, focus attention on class I.

$a_1 = 8$

$a_2 = 4$

h. Use Table B-16, page 2-55, to obtain a tabled a_2 which corresponds to N_1, N_2, and a_1 at a $100(1-\alpha)\%$ confidence level.

NOTE: Since this is a one-sided test, use the α which is not in parentheses.

h. For $N_1 = 10$, $N_2 = 6$, $a_1 = 8$, and $\alpha = .05$, the tabled $a_2 = 1$.

i. If $a_2 \leq$ the table value of a_2 from step h, decide that $\lambda_A > \lambda_B$ with respect to the original class of interest; otherwise, there is no reason to believe $\lambda_A > \lambda_B$ with respect to the original class of interest at a $100(1-\alpha)\%$ confidence level.

i. Since $4 < 1$, decide that there is no reason to believe $\lambda_A > \lambda_B$ with respect to the number of failures at a 95% confidence level.

8.3.1.1.5 ANALYSIS

If $a_2 \leq$ table value of a_2, the null hypothesis that $\lambda_A > \lambda_B$ is accepted; otherwise, there is no reason to believe $\lambda_A > \lambda_B$ at a $100(1-\alpha)\%$ confidence level. In the event that the confidence level desired is not within the scope of Table B-16, page 2-55, the test for the large sample size must be applied. The results will not be as accurate but will still be useful. In the event that a_1 or a_2 or both are missing for the given sample sizes and confidence level in Table B-16, page 2-55, conclude that the sample sizes are considered insufficient for accepting or rejecting the null hypothesis.

8.3.1.2 LARGE SAMPLE SIZE

8.3.1.2.1 OBJECTIVE

To determine whether λ_A is greater than λ_B at the desired confidence level when either N_A or N_B is greater than 20.

8.3.1.2.2 DATA REQUIRED

Success-failure data.

8.3.1.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-7, page 2-12, to obtain χ^2_α for 1 d.f.

c. Add N_A to N_B to obtain T_N, an intermediate value.

d. Compute AB, an intermediate value, as follows:

(1) Multiply I_A by II_B.
(2) Multiply I_B by II_A.
(3) Subtract step (2) from step (1) and take the absolute value of the difference (disregard the sign).

e. Compute J, an intermediate value, as follows:

(1) Add I_A to I_B to obtain T_I, an intermediate value.
(2) Add II_A to II_B to obtain T_{II}, an intermediate value.
(3) Multiply N_A, N_B, T_I, and T_{II} together.

f. Compute χ^2 as follows:

(1) Divide step c by 2.
(2) Subtract step (1) from step d.
(3) Square step (2).
(4) Multiply step (3) by step c.
(5) Divide step (4) by step e.

g. Focus on the class of interest and compute the following intermediate values:

(1) h_A, the ratio of the class of interest to the sample size for Type A; i.e., $h_A = I_A/N_A$ or h_A N^2
(2) h_B, the ratio of the class of interest to the sample size for Type B; i.e., $h_B = I_B/N_B$ or $h_B = II_B/N_B$.

h. If χ^2 is greater than or equal to χ^2_α for 1 d.f. and h_A is larger than h_B, decide that λ_A is greater than λ_B with regard to the class of interest; otherwise, there is no reason to believe λ_A is greater than λ_B at the desired confidence level.

8.3.1.2.4 EXAMPLE

Given:

Sample data at Table A-4b, page 1-11.

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .10$ $1-\alpha = .90$
b. Use Table B-7, page 2-12, to obtain χ^2_α for 1 d.f.	b. $\chi^2_{.20}$ for 1 d.f. = 1.64
c. Compute: $T_N = N_A+N_B$	c. $T_N = 216 + 216$ $= 432$

d. Compute:

$$AB = \left| I_A \; II_B - I_B \; II_A \right|$$

d.

$$AB = \left|(181)(56)-(160)(35)\right|$$
$$= \left|10,136 - 5,600\right|$$
$$= 4,536$$

e. Compute:

$$J = (N_A)\ (T_I)\ (T_{II})\ (N_B)$$

e. $J = (216)(341)(91)(216)$

$= (73,656)(91)(216)$

$= 1,447,782,336$

f. Compute:

$$\chi^2 = \frac{TN(AB - \frac{T_N}{2})^2}{J}$$

f.

$$\chi^2 = \frac{432(4536-216)^2}{1,447,782,336}$$
$$= \frac{432(4,320)^2}{1,447,782,336}$$
$$= \frac{432(18,662,400)}{1,447,782,336}$$
$$= \frac{8,062,156,800}{1,447,782,336}$$
$$= 5.5686$$
$$= 5.57$$

NOTE: The formula for χ^2 has been broken down for simplicity and the complete formula is

$$\chi^2 = \frac{(N_A+N_B)\left(\left|I_A II_B - I_B II_A\right| - N_A+N_B\right)^2}{(N_A)(I_A+I_B)(II_A+II_B)N_B}$$

g. Focus on the class of interest and compute one of the following:

(1) Class I

$h_A = I_A/N_A$

$h_B = I_B/N_B$

(2) Class II

$h_A = II_A/N_A$

$h_B = II_B/N_B$

g. Focus on class I.

$h_A = 181/216$

$= .83796$

$= .838$

$h_B = 160/216$

$= .74074$

$= .741$

h. If $\chi^2 \geq \chi^2_\alpha$ for 1 d.f. and $h_A > h_B$, decide that $\lambda A > \lambda B$ with regard to the class of interest; otherwise, there is no reason to believe $\lambda_A > \lambda_B$ at a 100(1-α)% confidence level.

h. Since 5.57 ≥ 1.64 and .838 > .741, decide that the proportion of hits for $\lambda_A > \lambda_B$ at a 90% confidence level.

8.3.1.2.5 ANALYSIS

If $\chi^2 \geq \chi^2_{2\alpha}$ for 1 d.f. and $h_A > h_B$, the null hypothesis that $\lambda_A > \lambda_B$ is accepted; otherwise, there is no reason to believe $\lambda_A > \lambda_B$ at a $100(1-\alpha)\%$ confidence level. The sample size for P_A or P_B must exceed 20. If the confidence level desired is unavailable for P_A and P_B less than 20, the chi-square test will be used to test $\lambda_A > \lambda_B$.

8.3.2 DETERMINATION OF SAMPLE SIZE

8.3.2.1 OBJECTIVE

To determine the N_t (N_t=N_A=N_B) required to determine whether λA. is equal to or greater than $\lambda_B + \varepsilon$ (or equal to or less than $\lambda_B - \varepsilon$) at the desired confidence level.

8.3.2.2 DATA REQUIRED

None.

8.3.2.3 PROCEDURE

a. Choose α and β, the probabilities of making Type I and Type II errors respectively.

b. Choose the allowable amount of error.

c. Estimate one of the proportions, either P_A or P_B. Make this estimate as close to 0.5 as is reasonable.

d. Compute the other proportion as follows:

(1) If P_A is estimated, subtract step b from P_A to obtain P_B.
(2) If P_B is estimated, add step b to P_B to obtain P_A.

e. Use Table B-15, page 2-54, to obtain θ_A, which corresponds to P_A, and θ_B, which corresponds to P_B.

f. Compute d^2, an intermediate value, as follows:

(1) Subtract θ_B from θ_A.
(2) Square step (1).

g. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

h. Compute n, an intermediate value, as follows:

(1) Add $Z_{1-\alpha}$ to $Z_{1-\beta}$.
(2) Square step (1).
(3) Divide step (2) by step f.
(4) Round step (3) up to the next whole number.

i. Multiply step h by 2 to obtain N_t.

j. Conclude that N_t samples are required to determine whether λ_A is equal to or greater than $\lambda_B + \varepsilon$ (or equal to or less than $\lambda_B - \varepsilon$) at the desired confidence level.

8.3.2.4 EXAMPLE

Procedure:	Example:
a. Choose α and β.	a. $\alpha = .05$ $1-\alpha = .95$ $\beta = .20$ $1-\beta = .80$
b. Choose δ.	b. $\delta = .20$
c. Estimate P_A or P_B.	c. $P_A = .70$
d. (1) If P_A is estimated, compute: $P_B = P_A - \delta$ (2) If P_B is estimated, compute: $P_A = P_B + \delta$	d. Since P_A is estimated, $P_B = .70 - .20$ $= .50$
e. Use Table B-15, page 2-54, to obtain θ_A, which corresponds to P_A, and θ_B, which corresponds to P_B.	e. For $P_A = .70$, $\theta_A = 1.98$ For $P_B = .50$, $\theta_B = 1.57$
f. Compute: $d^2 = (\theta_A-\theta_B)^2$	f. $d^2 = (1.98-1.57)^2$ $= (.41)^2$ $= .1681$
g. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.	g. $Z_{.95} = 1.645$ $Z_{.80} = .840$
h. Compute: $n = \frac{(Z_{1-\alpha}+Z_{1-\beta})^2}{d^2}$	h. $n = \frac{(1.645+.840)^2}{.1681}$ $= \frac{(2.485)^2}{.1681}$ $= \frac{6.175}{.1681}$ $= 36.73$ $= 37$
i. $N_t = 2n$	i. $N_t = 2\ (37)$ $= 74$

j. Conclude that N_t samples are required to determine whether $\lambda_A \geq \lambda_B + \varepsilon$ (or $\lambda_A \leq \lambda_B - \varepsilon$) at a $100(1-\alpha)\%$ confidence level.

j. Conclude that 74 samples of each item must be tested to determine whether $\lambda_A \geq \lambda_B + .20$ at a 95% confidence level.

8.3.2.5 ANALYSIS

a. Initial N_t.

N_t samples are required to determine whether $\lambda_A \geq \lambda_B + \varepsilon$ (or $\lambda_A \leq \lambda_B - \varepsilon$) at a $100(1-\alpha)\%$ confidence level. Unfortunately, the sample size required depends on the unknown population values of the two proportions involved. Very often the experimenter has some idea of the magnitude of (or an upper bound for) one of these values and then must specify the size of the difference which the experiment is designed to detect. The largest sample size is required when the true proportions are in the neighborhood of 0.5. Thus, a careful examination must be made in order to estimate the proportion accurately rather than arbitrarily using a value close to .5 so that the sample size can be kept at a minimum.

b. Adequacy of N_t.

After the initial N_t has been tested, P_A and P_B must be computed. N_t is then recomputed using the computed proportions in place of the estimated proportions to determine whether the initial N_t was adequate.

9. ACCURACY AND PRECISION

9.1 ACCURACY

9.1.1 OBJECTIVE

To determine the accuracy of a test item.

9.1.2 DATA REQUIRED

The aiming point (AP) or target, the coordinates of the points of impact or points of burst, the set time, and the achieved time.

9.1.3 PROCEDURE

a. Case I: Cannon.

(1) Compute the mean point of impact (MPI) for ground bursts as follows:

(a) Compute the mean of the eastings ($\overline{EAST}$).
(b) Compute the mean of the northings ($\overline{NORTH}$).

(2) Compute the mean point of burst ($\overline{POB}$) and the mean time as follows:

(a) Compute $\overline{EAST}$.
(b) Compute $\overline{NORTH}$.

(c) Compute the mean of the heights ($\overline{HEIGHT}$).
(d) Compute the mean time.

(3) List the MPI as the mean easting and mean northing ($\overline{EAST}$, $\overline{NORTH}$) and the $\overline{POB}$ as the mean easting, mean northing, and mean height ($\overline{EAST}$, $\overline{NORTH}$, $\overline{HEIGHT}$).

(4) Compute the miss distance (m) for the MPI as follows:

(a) Subtract the $\overline{EAST}$ from the AP easting ($EAST_{AP}$).
(b) Square step (a).
(c) Subtract the $\overline{NORTH}$ from the AP northing ($NORTH_{AP}$).
(d) Square step (c).
(e) Add step (b) to step (d) and find the square root.

(5) Compute m and the miss time for the $\overline{POB}$ as follows:

(a) Subtract the $\overline{EAST}$ from the $EAST_{AP}$.
(b) Square step (a).
(c) Subtract the $\overline{NORTH}$ from the $NORTH_{AP}$.
(d) Square step (c).
(e) Subtract the $\overline{HEIGHT}$ from the AP height ($HEIGHT_{AP}$).
(f) Square step (e).
(g) Add step (b) to step (d).
(h) Add step (g) to step (f) and find the square root to obtain the m.
(i) Subtract the set time from the mean time to obtain the miss time.

b. Case II: Missile systems (limited sample).

(1) Plot each point of impact or point of burst relative to its AP and determine the distance over or short and the distance right or left.
(2) Compute the mean AP using all of the AP coordinates in a given range band.
(3) Plot the points of impact or points of burst relative to the mean AP, using the distances from step (1).
(4) Compute the MPI or $\overline{POB}$ and mean time for the points relative to the mean AP.
(5) Compute m for MPI the same as for a cannon.
(6) Compute m and the miss time for the $\overline{POB}$ the same as for a cannon.

9.1.4 EXAMPLE

a. Case I: Cannon.

Given:
AP: (2784,3501)

Sample data at Table A-1a, page 1-1.

Procedure:	Example:
(1) Compute the following for the MPI:	(1) (a) $\overline{EAST}$ = 2565.67 = 2566
(a) $\overline{EAST}$	

(b) $\overline{NORTH}$

(b) $\overline{NORTH} = 3256.47$
$= 3256$

(2) Compute the following for the $\overline{POB}$:

(a) $\overline{EAST}$

(b) $\overline{NORTH}$

(c) $\overline{HEIGHT}$

(d) Mean time

(2)

(3) List the following:

(a) MPI: $(\overline{EAST}, \overline{NORTH})$

(b) $\overline{POB}$: $(\overline{EAST}, \overline{NORTH}, \overline{HEIGHT})$

(3) MPI: (2566,3256)

(4) For the MPI, compute:

$$m = \sqrt{\qquad\qquad}$$

(4)

$$m = \sqrt{(2784-2565.67)^2+(3501-3256.47)^2}$$
$$= \sqrt{(218.33)^2+(244.53)^2}$$
$$= \sqrt{47668+59795}$$
$$= \sqrt{107,463}$$
$$= 327.82$$
$$= 328$$

(5) For the $\overline{POB}$, compute:

(5)

(a) $m = \sqrt{(EAST_{AP}-\overline{EAST})^2+(NORTH_{AP}-\overline{NORTH})^2+(HEIGHT_{AP}-\overline{HEIGHT})^2}$

(b) miss time = mean time - set time.

b. Case II. Missile systems (limited sample).

Given:

Sample data at Table A-5a, page 1-12.

Procedure:

(1) Plot each point relative to its AP.

(2) Compute the mean AP.

(3) Plot each point relative to the mean AP

Example:

(1) (a) (2350,3100)

(b) (1649,2031)

See Table A-5a, page 1-12 for complete list.

(2) $\overline{EAST}_{AP} = 21548/10$
$= 2155$
$\overline{NORTH}_{AP} = 22091/10$
$= 2209$

(3) (a) (2005,2304)
(b) (2267, 2415)
See Table A-5a, page 1-12, for complete list.

(4) Compute:

(a) MPI.

(b) $\overline{POB}$ and mean time.

(4) MPI: (2148,2274)

$\overline{EAST} = 21482/10$

$= 2148.20$

$= 2148$

$\overline{NORTH} = 22743/10$

$= 2274.30$

$= 2274$

(5) Compute for the MPI:

$$m = \sqrt{(\overline{EAST}_{AP}-\overline{EAST})^2+(\overline{NORTH}_{AP}-\overline{NORTH})^2}$$

(5)

$$m = \sqrt{(2154.80-2148.20)^2+(2209.10-2274.30)^2}$$

$$= \sqrt{(6.60)^2+(65.20)^2}$$

$$= \sqrt{43.56+4251.04}$$

$$= 65.53$$

$$= 66$$

(6) For the $\overline{POB}$, compute:

(6)

(a) $m = \sqrt{(\overline{EAST}_{AP}-\overline{EAST})^2+(\overline{NORTH})_{AP}-\overline{NORTH})^2+(\overline{HEIGHT}_{AP}-\overline{HEIGHT})^2}$

(b) miss time = mean time - set time

9.1.5 ANALYSIS

a. The miss distance is the distance that the MPI or the $\overline{POB}$ missed the AP and describes the accuracy of the test item. The smaller the miss distance, the better the accuracy of the test item. The miss distance must be compared to the stated requirement to determine whether the requirement was met.

b. Due to sampling techniques used for missiles, an average AP must be determined within a range band. The miss distance is the distance that the MPI or $\overline{POB}$ (relative to the average AP) missed the average AP. The miss distance must be compared to the stated requirement to determine whether the requirement was met. Unless the sample size is at least six, conclusions for accuracy cannot be drawn with any reasonable level of confidence.

9.2 PRECISION

9.2.1 PROBABLE ERROR COMPUTATION

9.2.1.1 STANDARD DEVIATION METHOD

9.2.1.1.1 OBJECTIVE

To obtain the system PE and each subsystem PE using the standard deviation method.

9.2.1.1.2 DATA REQUIRED

A list of sample readings.

9.2.1.1.3 PROCEDURE

a. Compute s, (see paragraph 7.1.1.3, page 64).

b. Multiply step a by .6745 to obtain PE.

9.2.1.1.4 EXAMPLE

Given:

Sample data at Table A-5b, page 1-13.

Procedure:

a. Compute:

$$s = \sqrt{\frac{\Sigma\Delta^2}{N-1}}$$

b. $PE = 0.6745(s)$.

Example:

a.

$$s = \sqrt{\frac{38,650.00}{16-1}}$$

$$= \sqrt{\frac{38,650.00}{15}}$$

$$= \sqrt{2,576.67}$$

$$= 50.76$$

$$= 51$$

See paragraph 7.1.1.4, page 65, for computations.

b. $PE = 0.6745(50.76)$

$= 34.24$

$= 34$

9.2.1.1.5 ANALYSIS

The PE is a measure of deviation from μ such that 50% of the observations may be expected to lie between μ - PE and μ + PE. This method is the best estimate of the population PE(τ) unless a trend exists which can be attributed to a non-system condition, such as weather, in which case use of the successive differences method is the best approach. A test comparing the two methods of computing PE can be made to determine whether a trend did exist but was not evident (see paragraph 9.2.1.3, page 108, for details).

9.2.1.2 SUCCESSIVE DIFFERENCES METHOD

9.2.1.2.1 OBJECTIVE

To determine the system PE and each subsystem PE using the successive differences method when there is a suspected trend.

9.2.1.2.2 DATA REQUIRED

A list of sample readings.

9.2.1.2.3 PROCEDURE

a. Compute the differences ($x_d = x_i - x_{i+1}$) between consecutive readings.

b. Square each difference.

c. Sum the squares.

d. Compute s_δ as follows:

(1) Divide step C by the quantity (n-1).
(2) Divide step (1) by the quantity 2.
(3) Find the square root of step (2).

e. Multiply step d by .6745 to obtain the PE.

9.2.1.2.4 EXAMPLE

Given:

Sample data at Table A-5c, page 1-14 (same as data at Table A-5b, page 1-13).

Procedure:

a. Compute the differences between consecutive readings:

$$x_d = x_i - x_{i+1}$$

Example:

a. (1) Difference between 1 and 2:

$$x_d = 1248-1100$$
$$= 148$$

(2) Difference between 2 and 3:

$$x_d = 1100-1260$$
$$= -160$$

See Table A-5c, page 1-14, for complete list.

b. Square each x_d.

b. (1) $x_d^2 = (148)^2$
$= 21{,}904$

(2) $x_d^2 = (-160)^2$
$= 25{,}600$

See Table A-5c, page 1-14, for complete list.

c. Sum the x_d^2.

c. $\Sigma x_d^2 = 85{,}020$

d. Compute:

$$s_\delta = \sqrt{\frac{\Sigma x_d^2}{2(N-1)}}$$

d. $$s_\delta = \sqrt{\frac{(85{,}020)}{2(16-1)}}$$
$$= \sqrt{\frac{(85{,}020)}{30}}$$
$$= \sqrt{2{,}834}$$
$$= 53.23$$
$$= 53$$

e. Compute:

$PE = .6745(s_\delta)$

e. $PE = .6745(53.23)$
$= 35.90$
$= 36$

9.2.1.2.5 ANALYSIS:

The PE is a measure of deviation from such that 50% of the observations may be expected to lie between μ-PE and μ+PE. If a trend which can be attributed to a nor-system condition, such as weather, is suspected then this method will yield the best estimate of τ. A test comparing the two methods of computing PE can be made to determine whether a trend existed but was not evident (see paragraph 9.2.1.3, for details).

9.2.1.3 TREND ANALYSIS

9.2.1.3.1 OBJECTIVE

To determine whether a trend exists and whether the standard deviation method or the successive differences method yields the best estimate of τ.

9.2.1.3.2 DATA REQUIRED

s^2 and s_δ^2.

9.2.1.3.3 PROCEDURE

a. Choose the desired confidence level.

b. Divide s_δ^2 by s^2.

c. Use Table B-23, page 2-133, to obtain the critical number (CN) for N samples at the desired confidence level.

d. If s_δ^2/s^2 is less than CN, decide that a trend exists and that the successive differences method yields the best estimate of τ; otherwise, a trend does not exist at the desired confidence level and the standard deviation method yields the best estimate of τ.

9.2.1.3.4 EXAMPLE

Given:

s^2 = 2,576.67 (see paragraph 9.2.1.1.4, page 106).
s_δ^2 = 2,834.00 (see paragraph 9.2.1.2.4, page 107).

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .05$ $1-\alpha = .95$
b. Compute: $\frac{s_\delta^2}{s^2}$	b. $\frac{s_\delta^2}{s^2} = \frac{2{,}834.00}{2{,}576.67} = 1.0999$
c. Use Table B-23, page 2-133, to obtain CN for N samples at a $100(1-\alpha)\%$ confidence level.	c. For N = 16 and $1-\alpha = .95$, CN = .6136
d. If $s_\delta^2/s^2 <$ CN, decide that a trend exists at the desired confidence level and that the successive differences method yields the best estimate of τ; otherwise, conclude that a trend does not exist and the standard deviation method yields the best estimate of τ.	d. Since $1.0999 \nless .6136$, decide that a trend did not exist at a 95% confidence level and that the standard deviation method is the best estimate of τ. Thus, PE = 34.

9.2.1.3.5 ANALYSIS

a. The PE is a measure of deviation from μ such that 50% of the observation may be expected to lie between μ-PE and μ+PE. Elements, such as tube warming and wear, weapon seating, or non-random met changes, within a data point will have a progressive effect on the magnitude of the data collected. Thus, a trend test is important for detecting gradual increases or decreases in some selected parameters within each point. Range, height, deflection, time of flight, muzzle velocity and projectile weight are parameters which may be examined for a trend.

b. If a trend exists at a $100(1-\alpha)\%$ confidence level, the probable error computed by the successive differences method will be the best estimate of τ; otherwise, the probable error computed by the standard deviation method will be the best estimate of τ.

9.2.1.4 OUTLIERS

9.2.1.4.1 OBJECTIVE

To identify any outliers which may be present.

9.2.1.4.2 DATA REQUIRED

A list of sample readings.

9.2.1.4.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute s^2 for all readings.

c. Isolate the reading which deviates most from the mean as a suspected outlier.

d. Compute s_1^2 with the suspected outlier deleted.

(1) Compute the mean of the readings with the suspected outlier deleted.
(2) Compute the differences between each reading and the mean.
(3) Square each difference.
(4) Sum the squares.
(5) Divide step (4) by the quantity (N-2).

e. Divide s_1^2 by s^2.

f. Use the "First Outlier CV" column of Table B-17, page 2-71, to obtain the critical value (CV) for N samples at the desired confidence level.

g. If s_1^2/s^2 is less than the CV, decide that the reading is an outlier; otherwise, there is no reason to believe that the reading is an outlier at the desired confidence level.

h. If the reading is an outlier, exclude it from the data and proceed to step i. If the reading is not an outlier, return the reading to the set of data, and no further examination of the data is required.

i. With the outlier deleted, isolate the reading which deviates most from the mean. If this suspected outlier is on the same side of the mean as the outlier, proceed to step j. If both readings are not on the same side of the mean, conclude that it is invalid to eliminate both as outliers. Therefore the suspected outlier is retained with no further examination of the data required.

j. Compute the standard deviation with both the outlier and suspected outlier removed (s_2^2).

k. Divide s_2^2 by s^2.

l. Use the "Second Outlier CV" column of Table B-17, page 2-71, to obtain the CV for N samples at the desired confidence level.

m. If s_1^2/s^2 is less than the CV, decide that the reading (suspected outlier) is an outlier at the desired confidence level; otherwise, there is no reason to believe that the reading is an outlier.

n. If the reading is an outlier, exclude it from the data. If it is not an outlier, return the reading to the set of data. In either event, no further examination of the data is required.

9.2.1.4.4 <u>EXAMPLE</u>

Given:

Sample data at Table A-5b, page 1-13 (or Table A-5c, page 1-14).

Procedure:	Example:
a. Choose the confidence level (1-α).	a. $\alpha = .05$ $1-\alpha = .95$
b. Compute: $s^2 = \frac{\Sigma\Delta^2}{N-1}$	b. $s^2 = 2{,}576.67$ See paragraph 9.2.1.1.4, page 106, for computation.
c. Isolate suspected outlier.	c. Isolate reading number 2, 1100 meters.
d. Compute: $s_1^2 = \frac{\Sigma\Delta^2}{N-2}$ (Standard deviation with suspected outlier deleted)	d. $s_1^2 = 858.93$
e. Compute: $\frac{s_1^2}{s^2}$	e. $\frac{s_1^2}{s^2} = \frac{858.93}{2576.67}$ $= .3333$
f. Use the "First Outlier CV," column of Table B-17, page 2-71, to obtain CV for N samples at a 100(1-α)% confidence level.	f. For N = 16 and 1-α = .95, CV = .6166
g. If $s_1^2/s^2 < CV$, decide that the reading is an outlier; otherwise, there is no reason to believe that the reading is an outlier at a 100(1-α)% confidence level.	g. Since .3333 < .6166, decide that 1100 is an outlier at a 95% confidence level.

h. If the reading is an outlier, exclude it from the data and proceed to step i. If the reading is not an outlier, return the reading to the set of data, and no further examination of the data is required.

h. Since reading number 2 is an outlier, reading number 12, 1325 meters, is the next suspected outlier to isolate.

i. With the outlier deleted, isolate the reading which deviates most from the mean. If this suspected outlier is on the same side of the mean as the outlier, proceed to step j. If both readings are not on the same side of the mean, conclude that it is invalid to eliminate both as outliers. Therefore, the suspected outlier is retained with no further examination of the data required.

i. Since reading number 12 is on a different side of the mean than reading number 2, conclude that reading number 12 is not an outlier. Return the reading to the set of data. Conclude that reading number 2 is the only outlier in the set of data. No further analysis of the data is required.

j. Compute s_2^2.

k. Compute:

$$\frac{s_2^2}{s^2}$$

l. Use the "Second Outlier CV" column of Table B-17, page 2-71, to obtain the CV for N samples at a 100(1-α)% confidence level.

m. If $s_2^2/s^2 < CV$, decide that the reading is an outlier at a 100(1-α)% confidence level; otherwise, there is no reason to believe that the reading is an outlier.

n. If the reading is an outlier, exclude it from the data. If it is not an outlier, return the reading to the set of data. In either event, no further examination of the data is required.

9.2.1.4.5 ANALYSIS

The dispersion of the data with the suspected outlier included is compared to the dispersion with the outlier removed. If this ratio falls below a certain value (Table B-17, page 2-71), the reading is deleted as an outlier at the desired confidence level. This particular method for isolating outliers is used due to small samples and the fact that only one or two observations will possibly be outliers with any confidence.

9.2.2 COMPARING PROBABLE ERRORS (PE's)

As stated in paragraph 4.5.4, page 7, the PE is a measure of deviation from μ such that 50% of the observations may be expected to lie between μ - PE and μ + PE. Since the PE is a function of the standard deviation (PE = $.6745s_\delta$ or PE .6745 s), the same tests used for the comparison of standard deviations will be used to compare PE's for a significant difference.

9.2.2.1 COMPARING AN OBSERVED PE TO A REQUIREMENT

a. An observed PE is generated from a sample and is representative of τ. This value of PE is then compared to a stated requirement (τ_0). However, looking at the values of PE and the requirement to decide whether τ is greater than τ_0 or τ is less than τ_0 at a confidence level is insufficient. Since the decision pertains to the population, statistical tests must be applied to PE to determine whether τ is greater than τ_0 or τ is less than τ_0.

b. There exist two possibilities for the relationship of PE to τ_0. Following are the assumptions and the circumstances for each possible relationship:

(1) PE greater than τ_0.

(a) The null hypothesis is τ is greater than τ_0.
(b) The alternative hypothesis is there is no reason to believe τ is greater than τ_0.
(c) The use of this test is appropriate when τ_0 is a maximum value for τ to satisfy. In the event that τ must not be greater than τ_0, this test would be appropriate.

(2) PE less than τ_0.

(a) The null hypothesis is τ is less than τ_0.
(b) The alternative hypothesis is there is no reason to believe that τ is less than τ_0.
(c) The use of this test is appropriate when τ_0 is a minimum value for τ to satisfy. In the event that τ must meet or exceed τ_0, this test would be appropriate.

c. In order to test the above hypotheses when given the values of PE and τ_0, s and σ_0 must be computed; and the appropriate test as described in paragraphs 7.2.1 through 7.2. 2, page 70 through 71 must be performed. The values of s and σ_0 are determined by multiplying PE and τ_0 each by 1.4826. Since the PE is a multiple of s, the conclusions drawn concerning standard deviations will also hold true for probable errors; e.g., if the null hypothesis that σ is less than σ_0 is accepted at a $100(1-\alpha)\%$ confidence level, then the null hypothesis that τ is less than τ_0 can also be accepted at the same confidence level.

9.2.2.2 COMPARING TWO OBSERVED PE's

a. An observed probable error is generated from a sample and is representative of τ. This value of PE is then required to meet a standard item PE which is representative of the standard items population. Looking at the values of the probable errors (PE_A and PE_B) to decide whether τ_A is greater than τ_B or τ_A is less than τ_B at a confidence level is insufficient. Since the decision pertains to the populations, statistical tests must be applied to P_A and P_B to determine whether τ_A is greater than τ_B or τ_A is less than τ_B. The statistical tests use the sample PE's as estimates of the population PE's.

b. Type A generally represents the test item and Type B, the standard item when testing the hypothesis that τ_A is greater than τ_B. However, to prove that the PE of the test item is less than that of the standard item, Type A must represent the standard item so that the hypothesis, τ_A is greater than τ_B, can be tested.

c. When the null hypothesis is τ_A is greater than τ_B, the alternative hypothesis is there is no reason to believe that τ_A is greater than τ_B.

d. This test is appropriate when τ_B is a maximum value for τ_A to satisfy.

e. In order to test the above hypothesis when given the values of PE_A and PE_B, s_A and s_B must be computed; and the appropriate test as described in paragraphs 7.3.1 and 7.3.2, pages 74 and 76, must be performed. The values of s_A and s_B are determined by multiplying PE_A and PE_B each by 1.4826. Since the PE is a multiple of s, conclusions drawn concerning standard deviations will also hold true for probable errors; e.g., if the null hypothesis that σ_A is greater than σ_B is accepted at a $100(1-\alpha)\%$ confidence level then the null hypothesis that τ_A is greater than τ_B can also be accepted at the same confidence level.

9.2.2.3 DETERMINATION OF SAMPLE SIZE

a. The determination of N_t is necessary to assure that there is a sufficient sample upon which to base a decision to accept or reject a null hypothesis at a specified confidence level.

b. The values of s and σ_0 are determined by multiplying the PE and τ_0 each by 1.4826. N_t is determined by following the appropriate procedure as described in paragraph 7.2.3, page 72.

c. The values of s_A and s_B are determined by multiplying PE_A and PE_B each by 1.4826. N_t is determined by following the appropriate procedure as described in paragraph 7.3.2, page 76.

9.2.3 CIRCULAR PROBABLE ERROR

9.2.3.1 COMPUTATION

9.2.3.1.1 OBJECTIVE

To determine the radius of a circle such that 50% of the population lie within the circle.

9.2.3.1.2 DATE REQUIRED

List of sample eastings and corresponding northings.

9.2.3.1.3 PROCEDURE

a. Compute s for the eastings (s_E), (see paragraph 7.1.1.3, page 64).

b. Compute s for the northings (s_N), (see paragraph 7.1.1.3, page 64).

c. Compute the CPE as follows:

(1) If s_E equals s_N, multiply s_E by 1.1774 to obtain the CPE.

(2) If s_E is not equal to s_N, compute the equivalent CPE as follows:

(a) Add s_E to s_N.

(b) Multiply step (1) by .5887.

9.2.3.1.4 EXAMPLE

Given:

Sample data at Table A-5e, page 1-17.

Procedure:

a. Compute s_E:

$$s_E = \sqrt{\frac{\Sigma(\text{East}-\overline{\text{EAST}})^2}{N-1}}$$

b. Compute s_N:

$$s_N = \sqrt{\frac{\Sigma(\text{North}-\overline{\text{NORTH}})^2}{N-1}}$$

Example:

a. $s_E = \sqrt{\frac{1,650,542}{15-1}}$

$= \sqrt{1,650,542/14}$

$= \sqrt{117,895.9}$

$= 343.36$

$= 343$

b. $s_N = \sqrt{\frac{3,389,046}{15-1}}$

$= \frac{3,389,046}{14}$

$= 242,074.7$

$= 492.01$

$= 492$

c. Compute one of the following:

(1) If $s_E = s_N$ compute:
$CPE = 1.1774\ s_E$

(2) If $s_E \neq s_N$ compute
Equivalent CPE = $.5887\ (s_E+s_N)$

c. Since 343.36 ≠ 492.01,
Equivalent CPE
= .5887 (343.36+492.01)
= .5887 (835.37)
= 491.78
= 492

9.2.3.1.5 ANALYSIS

The CPE is the radius of a circle within which 1/2 or 50% of the population lies. The following is a list of multiples of the CPE and the percentages of the population which lie within the respective circles for a circular normal distribution:

a. 2(CPE) contains 93.75% of the population.

b. 3(CEP) contains 99.81% of the population.

c. 3.5(CPE) contains 99.99% of the population.

9.2.3.2 OUTLIERS

9.2.3.2.1 OBJECTIVE

To identify an outliers which may be present.

9.2.3.2.2 DATA REQUIRED

A list of sample eastings and corresponding northings.

9.2.3.2.3 PROCEDURE

a. Compute the CPE for all of the readings.

b. Compute the distance from the mean (d_m) for each set of coordinates using data from step a.

c. Isolate each suspected outlier beginning with the largest distance from the mean.

d. Recompute the CPE, with the suspected outlier deleted, as follows:

(1) Compute s_E (and s_N) as follows:

(a) Compute the mean of the remaining eastings (northings).
(b) Compute the deviation of each remaining reading from the mean.
(c) Square each deviation.
(d) Sum the squared deviations.
(e) Since N_1 is the sample size with the suspected outlier deleted, divide step (d) by the quantity (N_1-1).
(f) Find the square root of step (e).

(2) Add s_E to s_N.

(3) Multiply step (2) by .5887.

e. Compute the d_m between the suspected outlier and the mean of the remaining readings (suspected outlier deleted).

f. If d_m is greater than 3.5(CPE), decide that the reading is an outlier; otherwise, there is no reason to believe the reading is an outlier.

g. If the reading is an outlier, exclude it from the data and repeat step c (with $N = N_1$) through step f. If the reading is not an outlier, return the reading to the set of data; and no further examination of the data is required.

9.2.3.2.4 EXAMPLE

Given:

Sample data at Table A-5e, page 1-17 and Table A-5f, page 1-18.

Procedure:	Example:
a. Compute the CPE for all of the readings.	a. CPE = 491.78 = 492 See Table A-5e, page 1-17.
b. For each set of coordinates, compute: $d_m = \sqrt{\Delta E^2 + \Delta N^2}$	b. (1) $d_m = \sqrt{10,914+30,276}$ $= \sqrt{41,190}$ = 202.95 (2) $d_m = \sqrt{341+35,344}$ $= \sqrt{35,685}$ = 188.90 See Table A-5e, page 1-17 for complete list.
c. Isolate the suspected outlier.	c. Isolate reading number 3.
d. (1) Compute s_E for the remaining eastings.	d. (1) $s_E = \sqrt{1,469,036/(14-1)}$ $= \sqrt{113,002.8}$ = 333.16 = 333
(2) Compute s_N for the remaining northings.	(2) $s_N = \sqrt{1,505,179/(14-1)}$ $= \sqrt{115,783.0}$ = 340.27 = 340
(3) Compute: $CPE = .5887(s_E+s_N)$	(3) CPE = .5887(333.16+340.27) = .5887(673.43) = 396.45 = 396

e. For the suspected outlier, compute:

$$d_m \approx \sqrt{\Delta E^2 + \Delta N^2}$$

e. For reading 3,

$$d_m \approx \sqrt{195,302+2,018,417}$$
$$\approx \sqrt{2,213,719}$$
$$= 1487.86$$
$$= 1488$$

f. If $d_m > 3.5$(CPE), decide that the reading is an outlier; otherwise, there is no reason to believe that the reading is an outlier.

f. Since 1488 > 1190, decide that the reading is an outlier.

g. (1) If the reading is an outlier, exclude it from the data and repeat step c (with $N=N_1$) through step f.

(2) If the reading is not an outlier, return the reading to the set of data; and no further examination of the data is required.

g. (1) Since the reading number 3 is an outlier, exclude if from the data and repeat step c (with N=14) through step f.

(2) If reading number 6 (next suspected outlier) is not an outlier, return it to the set of data; and no further examination of the data is required.

9.2.3.2.5 ANALYSIS

a. The distance between the suspected outlier and the mean of the remaining readings must be greater than 3.5(CPE) for the reading to be an outlier.

b. The easiest approach to identifying outliers is to use a computer. The formulae and comparisons are the same as the manual method just outlined; however, each coordinate is checked for the possibility of being an outlier.

9.2.4 BIVARIATE NORMAL DISTRIBUTION

At this time the applications of the bivariate distribution are not fully developed. Therefore, no use is made of it in this MTP. The bivariate distribution is mentioned because a discussion and demonstration problem will be added when correct procedures for its use are developed.

10. RELIABILITY

a. Statements; such as, "the minimum system reliability is 90% with a confidence level of at least 95%, infer that on the average the test item will function successfully in 90 cases out of 100 and that 95 times out of 100 the 90% figure will be achieved or exceeded. If 46 samples tested with 1 failure occurring, Table B-18, page 2-74, shows that the R is at least 90%. The confidence level is 95%, since for one failure the 90% "Reliability" row and the 95% "Confidence Level" column intersect at N = 46.

Therefore, a confidence level of 95% indicates that if 100 groups, each containing 46 samples, were tested then on the average five of these groups would have more than one failure and 95 of these groups would have one or zero failures.

b. That high requirements place limitations on acceptability is intuitively evident. Stringent limitations require sufficient sampling to provide an objective view of the test item. However, in the interest of economy, testing must be accomplished with a minimum number of samples. This may be accomplished by decreasing the desired reliability (confidence level) while holding the confidence level (desired reliability fixed. Therefore, serious consideration must be given to sample size, the related R, and the desired confidence level.

10.1 SUCCESS FAILURE

10.1.1 DETERMINATION OF RELIABILITY

10.1.1.1 OBJECTIVE

To determine the population reliability (ρ) of the test item at the desired confidence level. The required reliability (ρ_0) and the confidence level are usually directed by a higher authority or a Requirements Document.

10.1.1.2 DATA REQUIRED

The number of failures (f) and N for a success-failure type test.

10.1.1.3 PROCEDURE

a. Case I:

(1) Use Table B-18, page 2-74, to obtain the intersection of the "Reliability" row and the "Confidence Level" column for the number of failures which occurred (see page 2-125 for 75% confidence level).

(2) If N is equal to or larger than the intersection value, decide that ρ is equal to or greater than ρ_0 (testing may cease); otherwise, there is no reason to believe ρ is equal to or greater than ρ_0 at the desired confidence level (testing may cease with a reject decision or testing must continue with a decision being made at a later date).

b. Case II: Reliability confidence limits.

(1) Compute the two-sided UCL and LCL as follows:

(a) Choose the desired confidence level.

(b) Perform the following calculations to obtain the UCL:

1. Compute d.f.$_2$ as follows:

a. Multiply the number of success (sc) by 2.

b. Add 2 to step a.

2. Compute d.f.$_2$ as follows:

   a. Multiply sc by 2.
   b. Multiply N by 2.
   c. Subtract step a from step b.

3. Use Table B-8, page 2-18, to obtain $F_{1-\alpha/2}$ for (d.f.$_1$, d.f.$_2$) d.f.
4. Compute the following:

   a. Add 1 to sc.
   b. Subtract sc from N.
   c. Divide step a by step b.
   d. Multiply step c by step 3.
   e. Add 1 to step d.
   f. Divide 1 by step e.
   g. Subtract step f from 1.

(c) Perform the following calculations to obtain the LCL:

1. Compute d.f.$_1$ as follows:

   a. Multiply f by 2.
   b. Add 2 to step a.

2. Compute d.f.$_2$ as follows:

   a. Multiply f by 2.
   b. Multiply N by 2.
   c. Subtract step a from step b.

3. Use Table B-8, page 2-18, to obtain $F_{1-\alpha/2}$ for (d.f.$_1$, d.f.$_2$) d.f.
4. Compute the following:

   a. Add 1 to f.
   b. Subtract f from N.
   c. Divide step a by step b.
   d. Multiply step c by step 3.
   e. Add 1 to step d.
   f. Divide 1 by step e.

(d) Conclude that ρ is equal to or between the UCL and LCL at the desired confidence level.

(2) Compute the one-sided UCL as follows:

(a) Choose the desired confidence level.
(b) Compute d.f.$_1$ as follows:

   1. Multiply sc by 2.
   2. Add 2 to step 1.

(c) Compute d.f.$_2$ as follows:

   1. Multiply sc by 2.

2. Multiply N by 2.
3. Subtract step 1 from step 2.

(d) Use Table B-8, page 2-18, to obtain $F_{1-\alpha}$ for ($d.f._1$, $d.f._2$) d.f.

(e) Perform the following calculations:

1. Add 1 to sc.
2. Subtract sc from N.
3. Divide step 1 by step 2.
4. Multiply step 3 by step (d).
5. Add 1 to step 4.
6. Divide 1 by step 5.
7. Subtract step 6 from 1 to obtain the UCL.

(f) Conclude that ρ is equal to or less than the UCL at the desired confidence level.

(3) Compute the one-sided LCL as follows:

(a) Choose the desired confidence level.

(b) Compute $d.f._1$ as follows:

1. Multiply f by 2.
2. Add 2 to step 1.

(c) Compute $d.f._2$ as follows:

1. Multiply f by 2.
2. Multiply N by 2.
3. Subtract step 1 from step 2.

(d) Use Table B-8, page 2-18, to obtain $F_{1-\alpha}$ for ($d.f._1$, $d.f._2$) d.f.

(e) Perform the following calculations:

1. Add 1 to f.
2. Subtract f from N.
3. Divide step 1 by step 2.
4. Multiply step 3 by step (d).
5. Add 1 to step 4.
6. Divide 1 by step 5 to obtain the LCL.

(f) Conclude that ρ is equal to or greater than the LCL at the desired confidence level.

NOTE: The LCL is usually referred to as the R of the test item at the desired confidence level and it is the reliability which is compared to the requirements.

10.1.1.4 EXAMPLE

a. Case I:

Given:

$\rho_o = .90$

$1-\alpha = .90$

$N = 52$

$f = 5$

Procedure:

(1) Use Table B-18, page 2-74, to obtain the intersection of the "Reliability" row and the "Confidence Level" column for the number of failures which occurred.

(2) If N is equal to or larger than the intersection value, decide that $\rho \geq \rho_0$; otherwise, there is no reason to believe $\rho \geq \rho_0$ at a $100(1-\alpha)\%$ confidence level.

NOTE: To determine the achieved reliability at the desired confidence level, continue with Case II.

Example:

(1) For $f = 5$, $\rho_0 = .90$, and $1-\alpha = .90$, the intersection is $N = 91$.

(2) Since $91 > 52$, decide that there is no reason to believe that $\rho \geq .90$ at a 90% confidence level.

b. Case II: Reliability confidence limits.

Given:

$N = 52$

$f = 5$

Procedure:

(1) Compute the two-sided LCL and UCL as follows:

(a) Choose the confidence level $(1-\alpha)$.

(b) Compute:

1. $d.f._1 = 2(sc)+2$
2. $d.f._2 = 2N-2(sc)$

Example:

(1)

(a) $\alpha = .05$
$1-\alpha = .95$
$1-\alpha/2 = .975$

(b) 1. $d.f._1 = 2(47) + 2 = 96$

2. $d.f._2 = 2(52) - 2(47) = 10$

3 Use Table B-8, page 2-18, to obtain $F_{1-\alpha/2}$ for $(d.f._1, d.f._2)$ d.f.

4. $UCL = 1 - \frac{1}{1+\left(\frac{sc+1}{N-sc}\right)F_{1-\alpha/2}}$

(c) Compute:

1. $d.f._1 = 2f + 2$

2. $d.f._2 = 2N - 2f$

3. Use Table B-8, page 2-18, to obtain $F_{1-\alpha/2}$ for $(d.f._1, d.f._2)$ d.f.

4. $LCL = \frac{1}{1+\left(\frac{f+1}{N-f}\right)F_{1-\alpha/2}}$

3. $F_{.975}$ for (96,10)d.f. approximates closely

$F_{.975}$ for (100,10)d.f.

$F_{.975}$ for (100,10)d.f.= 3.18

4. $UCL = 1 - \frac{1}{1+\left(\frac{47+1}{52-47}\right)F_{.975}}$

$= 1 - \frac{1}{1+(9.600)\ (3.18)}$

$= 1 - \frac{1}{1+30.53}$

$= 1 - \frac{1}{31.53}$

$= 1 - .0317$

$= .9683$

$= .97$

(c) 1. $d.f._1 = 2(5) + 2$

$= 12$

2. $d.f._2 = 2(52)-2(5)$

$= 94$

3. $F_{.975}$ for (12,94)d.f. approximates closely

$F_{.975}$ for (12,90)d.f.= 2.09

4. $LCL = \frac{1}{1+\left(\frac{5+1}{52-5}\right)F_{.975}}$

$= \frac{1}{1+(.1277)\ (2.09)}$

$= \frac{1}{1+.2668}$

$= \frac{1}{1.2668}$

$= .7894$

$= .78$

(d) Conclude that $\rho \leq$ UCL and $\rho \geq$ LCL at a $100(1-\alpha)\%$ confidence level.

(d) Conclude that $\rho \leq .97$ and $\rho \geq .78$ at a 95% conficence level.

(2) Compute the one-sided UCL for ρ as follows:

(2)

(a) Choose the confidence level $(1-\alpha)$.

(a) $\alpha = .05$
$1-\alpha = .95$

(b) Compute:

$$d.f._1 = 2(sc) + 2$$

(b) $d.f._1 = 2(47) + 2$
$= 94 + 2$
$= 96$

(c) Compute:

$$d.f._2 = 2(N)-2(sc)$$

(c) $d.f._2 = 2(52)-2(47)$
$= 104-94$
$= 10$

(d) Use Table B-8, page 2-18, to obtain $F_{1-\alpha}$ for $(d.f._1, d.f._2)$ d.f.

(d) $F_{.95}$ for (96,10)d.f. approximates closely

$F_{.95}$ for (100,10)d.f.

$F_{.95}$ for (100,10)d.f.= 2.6

(e) Compute:

$$UCL = 1 - \frac{1}{1+\left(\frac{sc+1}{N-sc}\right)F_{1-\alpha}}$$

(e)

$$UCL = 1 - \frac{1}{1+\left(\frac{47+1}{52-47}\right)F_{.95}}$$

$$= 1 - \frac{1}{1+\left(\frac{48}{5}\right)(2.6)}$$

$$= 1 - \frac{1}{1+(9.600)(2.6)}$$

$$= 1 - \frac{1}{1+24.96}$$

$$= 1 - \frac{1}{25.96}$$

$$= 1 - .0385$$

$$= .9615$$

$$= .97$$

(f) Conclude that $\rho \leq$ UCL at a $100(1-\alpha)\%$ confidence level.

(f) Conclude that $\rho \leq .97$ at a 95% confidence level.

(3) Compute the one-sided LCL for ρ as follows:

(3)

(a) Choose the confidence level $(1-\alpha)$.

(a) $\alpha = .05$

$1-\alpha = .95$

(b) Compute:

$d.f._1 = 2(f)+2$

(b) $d.f._1 = 2(5)+2$

$= 12$

(c) Compute:

$d.f._2 = 2(N) - 2(f)$

(c) $d.f._2 = 2(52)-2(5)$

$= 104-10$

$= 94$

(d) Use Table B-8, page 2-18, to obtain $F_{1-\alpha}$ for $(d.f._1, d.f._2)$ d.f.

(d) $F_{.95}$ for (12,94)d.f. approximates closely

$F_{.95}$ for (12,90)d.f.

$F_{.95}$ for (12,90)d.f. = 1.86

(e) Compute:

$$LCL = \frac{1}{1+\left(\frac{f+1}{N-f}\right)F_{1-\alpha}}$$

(e)

$$LCL = \frac{1}{1+\left(\frac{5+1}{52-5}\right)F_{.95}}$$

$$= \frac{1}{1+\left(\frac{6}{47}\right)(1.86)}$$

$$= \frac{1}{1+(.1277)(1.86)}$$

$$= \frac{1}{1+.2374}$$

$$= \frac{1}{1.2374}$$

$= .8081$

$= .80$

(f) Conclude that $\rho \geq$ LCL at a $100(1-\alpha)$% confidence level.

(f) Conclude that $\rho \geq .80$ at a 95% confidence level.

NOTE: .80 is referred to as the reliability of the test item at a 95% confidence level.

10.1.1.5 ANALYSIS

a. Case I:

If $N \geq$ the intersection value (Table B-18, page 2-74) the null hypothesis that $\rho \geq \rho_0$ is accepted; otherwise, there is no reason to believe $\rho \geq \rho_0$ at a 100 1-α% confidence level.

b. Case II:

(1) The two-sided interval surrounds ρ such that $\rho \leq$ UCL and $\rho \geq$ LCL at a 100(1-α)% confidence level.
(2) The one-sided interval surrounds ρ such that $\rho \leq$ UCL at a 100(1-α)% confidence level.
(3) The one-sided interval surrounds ρ such that $\rho \geq$ LCL at a 100 (1-α)% confidence level.

10.1.2 DETERMINATION OF SAMPLE SIZE

10.1.2.1 OBJECTIVE

a. To determine the absolute minimum N_t required to establish ρ_0 at the desired confidence level.

b. To determine the minimum N_t required to establish ρ_0 at the desired confidence level when the average number of failures is known from previous testing or a comparable item.

10.1.2.2 DATA REQUIRED

a. None.

b. The average number of failures known from a standard item, history, or Requirements Document.

10.1.2.3 PROCEDURE

a. Case I: Determination of an absolute minimum N_t.

(1) Use Table B-18, page 2-74, to obtain the intersection of the "Reliability" row and the "Confidence Level" column for zero failures.
(2) Conclude that the intersection value is the absolute minimum N_t since zero failures constitutes the ideal situation.

b. Case II: Determination of N_t.

(1) Use Table B-18, page 2-74, to obtain the intersection of the "Reliability" row and the "Confidence Level" column for the average number of failures known from a standard item, history, or Requirements Document.

(2) Conclude that the intersection value is the minimum N_t. Generally the test item must be as good as previous test results from a standard item. Note that in most cases this N_t will be larger than the absolute minimum N_t generated in Case I.

10.1.2.4 EXAMPLE

a. Case I: Determination of an absolute minimum N_t.

Given:

$\rho_o = .95$

$1-\alpha = .90$

$f = 0$

Procedure:

(1) Use Table B-18, page 2-74, to obtain the intersection of the "Reliability" row and the "Confidence Level" column for zero failures.

(2) Conclude that the intersection value is the absolute minimum N_t since zero failures constitures the ideal situation.

Example:

(1) For $f = 0$, $\rho_o = .95$, and $1-\alpha = .90$,

$N_t = 45$

(2) For zero failures, conclude that 45 samples are required to achieve $\rho = .95$ at a confidence level of 90%.

b. Case II: Determination of N_t

Given:

$\rho_o = .95$

$1-\alpha = .90$

Average number of failures for the standard item = 6

Procedure:

(1) Use Table B-18, page 2-74, to obtain the intersection of the "Reliability" row and the "Confidence Level" column for the average number of failures.

(2) Conclude that the intersection value is the minimum N_t. Generally the test item must be as good as previous test results from a standard item.

NOTE: In most cases this N_t will be larger than the absolute minimum N_t generated in Case I.

Example:

(1) For $f - 6$, $\rho_o = .95$, and $1-\alpha = .90$,

$N_t = 209$.

(2) For no more than six failures, conclude that 209 samples are required to achieve $\rho = .95$ at a 90% confidence level.

NOTE: $209 > 45$

10.1.2.5 ANALYSIS

a. Initial N_t.

At a specified confidence level, reliability, and number of failures, N_t samples are required to determine whether $\rho \geq \rho_0$. Zero failures will generate the absolute minimum N_t.

b. Adequacy of N_t.

After the initial N_t samples have been tested, R must be computed at the desired confidence level for the number of failures that occurred. If the computed R is equal to or greater than ρ_0, the initial N_t is adequate; however, if the computed R is less than ρ_0, the initial N_t is inadequate. If N_t is inadequate, N_t must be recomputed using the number of failures which have occurred, ρ_0, and the desired confidence level; and additional must be tested if possible or a reject decision made.

10.1.3 SEQUENTIAL ANALYSIS FOR SUCCESS-FAILURE

a. When testing an hypothesis using the sequential method, the project officer is able to make one of the following three decisions at any stage of testing:

(1) Accept the hypothesis.
(2) Reject the hypothesis.
(3) Continue the experiment by collecting additional data.

b. Usually a ρ_0 of .95 with a high degree of assurance is required. In order to achieve assurance of such a high ρ_0, the project officer would have to conduct excessive testing; e.g., many thousands of rounds. This may be impractical; however, using the following statistical approach, the project officer will achieve the predetermined confidence level for reaching the accept decision.

c. If certain criteria are set up graphically, a decision can be made to accept, reject, or continue testing the test item after each sample is tested. This graph uses three areas to represent the decisions to accept, reject, or continue testing the test item. The accept region is below a boundary line determined by the subtraction of the maximum proportion of defectives (P_0) and the confidence levels for rejection and acceptance. The continue testing area is above the accept boundary line and below the reject boundary line. The size of this area, which is an area of doubt for the test item, is determined by the project officer (see paragraph 4.14, page 14). The area of doubt is designed for a test item which may be good but has gotten off to a slow start. In this case, a longer period of time will be required to satisfy the doubts concerning acceptability of the test item. The reject boundary line is determined by P_0 and the confidence levels for rejection and acceptance. The area above this boundary line is the area of rejection. A graph of this type is illustrated by Figure 14. The number of samples are plotted on the horizontal axis with each increment representing one sample. The number of failures are plotted on the vertical axis with each increment representing one failure.

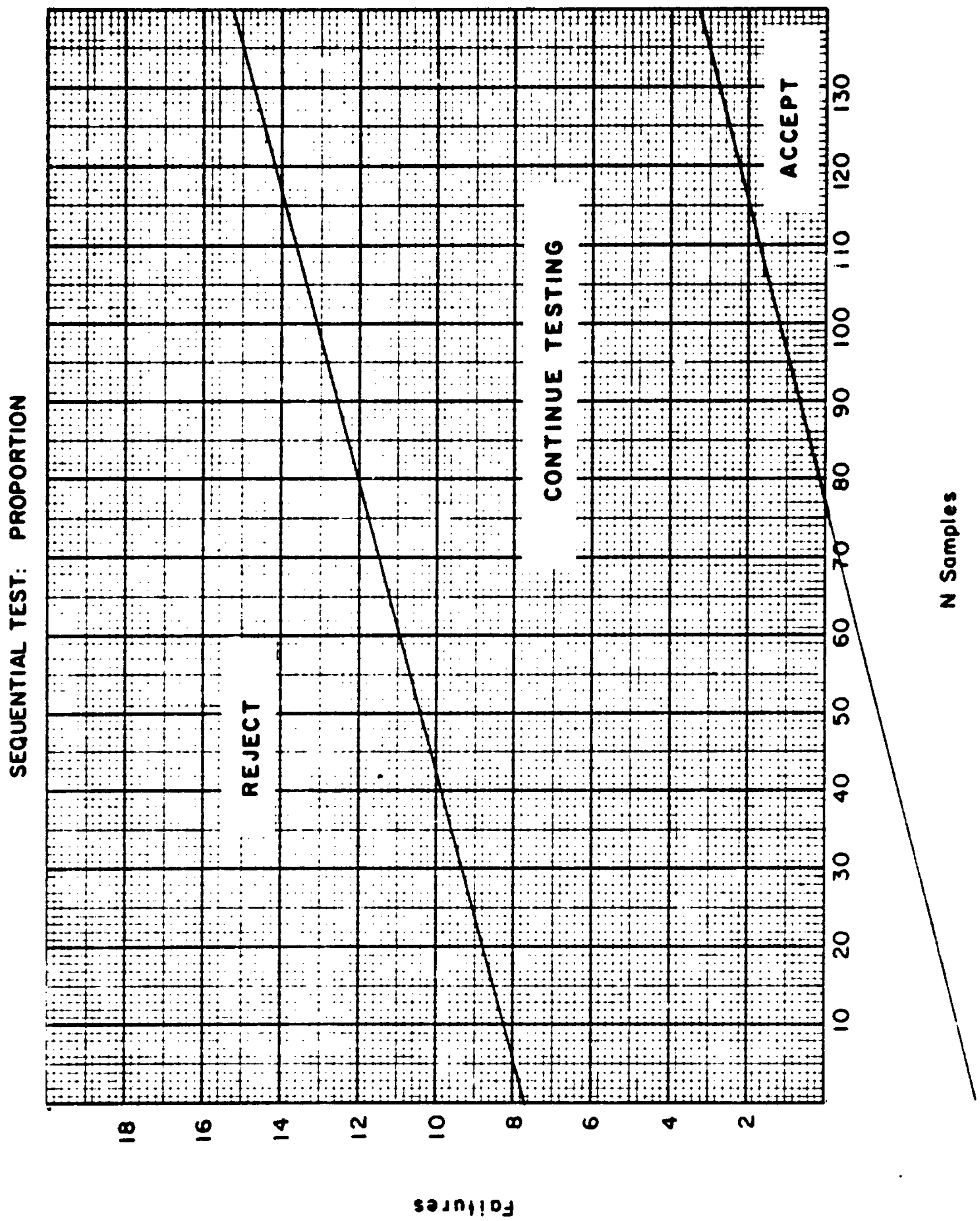

SEQUENTIAL TEST: PROPORTION
REJECT
CONTINUE TESTING
ACCEPT
N Samples
10 20 30 40 50 60 70 80 90 100 110 120 130
Failures
18 16 14 12 10 8 6 4 2

Figure 14

d. The construction of the two boundary lines is described in the procedure paragraph below.

10.1.3.1 OBJECTIVE

To determine whether the proportion of defective test items is equal to or less than P_o at the desired confidence level.

10.1.3.2 DATA REQUIRED

N and f.

10.1.3.3 PROCEDURE

a. Construct the boundary lines as follows:

(1) Choose α and β, the probabilities of making Type I and Type II errors respectively.
(2) Choose the amount of doubt, the proportion of defectives allowable for continued testing.
(3) Use Table B-19, page 2-127, to obtain a and b for α and β.
(4) Subtract D from P_o to obtain the upper limit for the proportion of defectives (P_U).

NOTE: P_o equals λ_o, if λ_o is in terms of defectives. P_o equals the quantity $(1-\lambda_o)$, if λ_o is in terms of successes.

(5) Compute U, an intermediate value, as follows:

(a) Divide P_o by step (4).
(b) Subtract step (4) from 1.
(c) Subtract P_o from 1.
(d) Divide step (b) by step (c).
(e) Multiply step (a) by step (d).
(f) Find the natural logarithm of step (e).

(6) Compute V, an intermediate value, as follows:

(a) Subtract step (4) from 1.
(b) Subtract P_o from 1.
(c) Divide step (a) by step (b).
(d) Find the natural logarithm of step (c).
(e) Divide step (d) by step (5).

(7) Determine the accept boundary line as follows:

(a) Divide the value b in step (3) by step (5).
(b) Multiply step (6) by N.
(c) Add step (a) to step (b) to determine the maximum allowable f for accepting the test item ($f_{ACCEPT} = \frac{b}{U} + V(N)$).
(d) Choose two values for N and substitute them into the above equation to determine two points on the accept boundary line.

NOTE: Use N=0 and N= some large value; such as 50, 100, or 150.

(e) Draw the accept boundary line using the two points determined from step (d).

(8) Determine the reject boundary line as follows:

(a) Divide the value a in step (3) by step (5).
(b) Multiply step (6) by N.
(c) Add step (a) to step (b) to determine the minimum allowable f for rejecting the test item ($f_{REJECT} = \frac{a}{U} + V(N)$).
(d) Choose two values for N and substitute them into the above equation to determine two points on the reject boundary line.
(e) Draw the reject boundary line using the two points determined from step (d).

(9) If the two lines are not parallel, check the computations and plotted points.

b. Plot the sample data on the sequential graph as follows:

(1) Plot the cumulative sample size and f after each sample.
(2) After plotting each point, decide whether to accept, reject, or continue testing the test item.

NOTE: An accept decision may be made before another failure occurs in the event that the sample size increases sufficiently after the last failure to cross the accept boundary line.

10.1.3.4 EXAMPLE

a. Construct the boundary lines as follows:

Given:

$P_o = .07$ (7 failures out of 100; reliability of 93%)

Procedure:	Example:
(1) Choose α and β.	(1) $\alpha = .05$ $1-\alpha = .95$ $\beta = .20$ $1-\beta = .80$
(2) Choose D.	(2) $D = .02$
(3) Use Table B-19, page 2-127, to obtain a and b for α and β.	(3) $a = 2.773$ $b = -1.558$
(4) Compute: $P_U = P_o - D$	(4) $P_U = .07 - .02 = .05$
(5) Compute: $U = \ln \left({}^{P_o}/_{P_U} \right) \left(\frac{1-P_U}{1-P_o} \right)$	(5) $U = \ln \left(\frac{.07}{.05} \right) \left(\frac{1-.05}{1-.07} \right)$ $= \ln (1.4000) \left(\frac{.95}{.93} \right)$ $= \ln (1.4000)(1.0215)$ $= \ln 1.4301$ $= .35775$

(6) Compute:

$$V = \frac{\ln\left(\frac{1-P_U}{1-P_O}\right)}{U}$$

(6) $$V = \frac{\ln\left(\frac{1-P_U}{1-P_O}\right)}{.35775}$$

$$= \frac{\ln(1.0215)}{.35775}$$

$$= \frac{.021282}{.35775}$$

$$= .059488$$

(7) Compute:

$$f_{ACCEPT} = \frac{b}{U} + V(N)$$

(7) $$f_{ACCEPT} = \frac{-1.558}{.35775} + .059488(N)$$

$$= -4,355+.059488(N)$$

When N = 0, f_{ACCEPT} = -4.355

When N = 100, f_{ACCEPT} = 1.594

Plot the points

(0,-4.355) and (100,1.594)
to determine the accept boundary line.

(8) Compute:

$$f_{REJECT} = \frac{a}{U} + V(N)$$

(8) $$f_{REJECT} = \frac{2.773}{.35775} + .059488(N)$$

$$= 7.751+.059488(N)$$

When N = 0, f_{REJECT} = 7.751

When N = 100, f_{REJECT} = 13.700

Plot the points
(0,7.75) and (100,13.700)
to determine the reject boundary line.

(9) If the two lines are not parallel, check the computations and plotted points.

b. Plot the sample data on the sequential graph as follows:

Given:

Requirements and boundary lines from step a.
Sample data at Table A-6a, page 1-19.

Procedure:

(1) Plot the cumulative sample size and failure after each sample,
(N_i, f_i).

Example:

(1) (a) (30,1)

(b) (75,2)

See Table A-6a, page 1-19, for complete list.

(2) After plotting each point, decide to accept, reject, or continue testing the test item.

(2) For failures 1 through 3, decide to continue testing. At failure number 4 decide to accept the test item. A decision to accept the test item could have been made when N was 134 and f was 3 since the accept boundary was crossed (see Table A-6b, page 1-20).

NOTE: From Table B-18, page 2-74, when $f=3, \rho_0=.95$, and $1-\alpha=.95$, the intersection value is 153; thus, fewer samples (N=134) are needed using the sequential method.

10.1.3.5 ANALYSIS

a. The sequential method generally minimizes testing time and N due to the fact that a decision to accept or reject is made as soon as possible after the first failure. Since all failures are not necessarily chargeable failures, decisions will be altered if certain failures are not counted. If the project officer ignores a failure, the probability of accepting an unacceptable item is increased. Therefore, the project officer must carefully decide what constitutes a failure (see paragraph 4.2, page 2).

b. Due to the advantages just discussed, the sequential method should be used whenever possible (see subparagraph 10.2c, page 134).

10.2 RELIABILITY RELATIVE TO CONTINUOUS TESTING

a. When measuring R for the continuous testing situation, the failure rate is assumed to approach the exponential distribution (see paragraph 4.15.2, page 15). In this case there are three measures of R that are of interest to the project officer. These are:

(1) The determination of mean time, miles, or rounds between failures and the limits for the mean at a desired confidence level (see paragraph 10.2.1, page 134).
(2) The determination of a computed R (see paragraph 10.2.2, page 145).
(3) The determination of the R based on ρ_0 and the desired confidence level (see paragraph 10.2.3, page 147).

b. The first two determinations are simple and straightforward but are biased by limitations on N. The third, which is the only sequential analysis method, is a truer representation of the population.

c. Sequential analysis is superior to nonsequential analysis whenever the data become available serially and the cost of the data (in terms of time, labor, or material) is approximately proportional to the amount of data. Nonsequential analysis is superior whenever the amount of data is fixed or the cost of the data is largely overhead, hence more or less independent of the amount of data. Superiority consists of minimizing the set of quantities N, α, and β. Sequential and nonsequential tests differ in the constraints under which this set is minimized. Nonsequential tests treat N as fixed and are designed so that either risk α or risk β is minimized when the other is fixed. Sequential tests treat N as a variable and are designed so that for fixed risks, α and β, the expected (average) number of trials required to reach a decision is minimized. If for a nonsequential test N is made large enough so that, with α fixed, β will not exceed a predetermined amount, this value of N will exceed (frequently by as much as 100 percent) the N required for a sequential test for the same α and β. Thus, when N is readily subject to variation, sequential tests are superior; when N is not readily varied, nonsequential tests are superior.

d. Examples of the solution for each determination are in the following paragraphs. In all examples mean time between failures (MTBF) is used. Other means, such as mean miles between failures (MMBF) or mean rounds between failures (MRBF), may be used when applicable.

10.2.1 MEANS AND LIMITS

10.2.1.1 MEANS

10.2.1.1.1 OBJECTIVE

To determine the mean time between failures.

10.2.1.1.2 DATA REQUIRED

A list of sample readings; e.g., operating time (primary parameter) and failures (secondary parameter).

10.2.1.1.3 PROCEDURE

a. Sum the primary parameter.

b. Sum the secondary parameter.

c. Divide step a by step b.

10.2.1.1.4 EXAMPLE

Given:

Sample data at Table A-6c, page 1-21.

Procedure:	Example:
a. Sum the primary parameter; e.g., total time (T_t) or total miles (T_m).	a. T_t = 3752 hours

b. Sum the secondary parameter; e.g., total failures (f).	b. f = 12
c. Compute: $MTBF = \frac{Tt}{f}$	c. MTBF = 3752/12 = 312.56 = 313 hours

NOTE: In the event a test is time terminated and zero failures occurred, a point estimate of the MTBF cannot be determined but a LCL may be computed (see paragraph 10.2.1.3).

10.2.1.1.5 ANALYSIS

The sample mean, or average, is a value which is typical or representative of a set of data. The mean is the most commonly used measure of central location.

10.2.1.2 LIMITS USING THE STUDENT t DISTRIBUTION

10.2.1.2.1 OBJECTIVE

To determine the two-sided and one-sided limits for the MTBF using the t distribution.

10.2.1.2.2 DATA REQUIRED

A list of sample readings; e.g., operating time (primary parameter) and failures (secondary parameter).

10.2.1.2.3 PROCEDURE

a. Case I: UCL and LCL (two-sided limits), also referred to as M_2 and M_1.

(1) Choose the desired confidence level.
(2) Use Table B-5, page 2-5, to obtain $t_{1-\alpha/2}$ for f-1 d.f.
(3) Compute the MTBF (see paragraph 10.2.1.1.3, page 134).
(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.
(5) Compute s (see paragraph 7.1.1.4, page 65).

NOTE: The sample size is the number of failures.

(6) Compute ε as follows:

(a) Multiply step (2) by step (5).
(b) Divide step (a) by the square root of f.

(7) Add step (6) to step (3) to obtain the UCL and subtract step (6) from step (3) to obtain the LCL.
(8) Conclude that the population MTBF is equal to or less than the UCL and equal to or greater than the LCL at the desired confidence level.

b. Case II: UCL (one-sided limit), also referred to as M_2.

(1) Choose the desired confidence level.
(2) Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for f-1 d.f.
(3) Compute the MTBF (see paragraph 10.2.1.1.3, page 134).
(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.
(5) Compute s (see paragraph 7.1.1.3, page 64).
(6) Compute ε as follows:

(a) Multiply step (2) by step (5).
(b) Divide step (a) by the square root of f.

(7) Add step (6) to step (3) to obtain the UCL.
(8) Conclude that the population MTBF is equal to or less than the UCL at the desired confidence level.

c. Case III: LCL (one-sided limit), also referred to as M_1.

(1) Choose the desired confidence level.
(2) Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for f-1 d.f.
(3) Compute the MTBF (see paragraph 10.2.1.1.3, page 134).
(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.
(5) Compute s (see paragraph 7.1.1.3, page 64).

NOTE: The sample size is the number of failures.

(6) Compute ε as follows:

(a) Multiply step (2) by step (5).
(b) Divide step (a) by the square root of f.

(7) Subtract step (6) from step (3) to obtain the LCL.
(8) Conclude that the population MTBF is equal to or greater than the LCL at the desired confidence level.

10.2.1.2.4 EXAMPLE

a. Case I: UCL and LCL (two-sided limits), also referred to as M_2 and M_1.

Given:

Sample data at Table A-6d, page 1-22.

Procedure:	Example:
(1) Choose the confidence level ($1-\alpha$).	(1) $\alpha = .10$ $1-\alpha = .90$ $1-\alpha/2 = .95$
(2) Use Table B-5, page 2-5, to obtain $t_{1-\alpha/2}$ for (f-1) d.f.	(2) $f-1 = 5$ $t_{.95}$ for 5 d.f. = 2.015

Procedure	Example
(3) Compute the MTBF.	(3) MTBF = 207 hours See paragraph 10.2.1.1.4, page 134.
(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.	(4) (a) Time to failure 1 = 200 hours (b) Time between failures 2 and 1 = 410-200 = 210 hours
(5) Compute: $s = \sqrt{\frac{\Sigma\Delta^2}{f-1}}$	(5) $s = \sqrt{\frac{1411}{6-1}}$ = 16.80 = 17 hours See paragraph 7.1.1.4, page 65.
(6) Compute: $\varepsilon = \frac{t_{1-\alpha/2}\,(s)}{\sqrt{f}}$	(6) $\varepsilon = \frac{2.015(16.80)}{\sqrt{6}}$ = 33.85/2.449 = 13.82
(7) Compute: UCL = MTBF + ε LCL = MTBF − ε	(7) UCL = 206.83 + 13.82 = 220.65 = 221 hours LCL = 206.83 − 13.82 = 193.01 = 193 hours
(8) Conclude that the population MTBF $\leq$ UCL and the population MTBF $\geq$ LCL at a $100(1-\alpha)\%$ confidence level.	(8) Conclude that the population MTBF $\leq$ 221 hours and the population MTBF $\geq$ 193 hours at a 90% confidence level.

b. Case II: UCL (one-sided limit), also referred to as M_2.

Given:

Sample data at Table A-6d, page 1-22.

Procedure:	Example:
(1) Choose the confidence level $(1-\alpha)$.	(1) $\alpha = .10$ $1-\alpha = .90$
(2) Use Table B-5, page 2-5. to obtain $t_{1-\alpha}$ for (f-1) d.f.	(2) f-1 = 5 $t_{.90}$ for 5 d.f. = 1.476
(3) Compute the MTBF.	(3) MTBF = 1241/6 = 207 hours See paragraph 10.2.1.1.4, page 134.

(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.

(4) (a) Time to failure 1 = 200 hours.
(b) Time between failures 2 and 1 = 410-200 = 210 hours

See Table A-6d, page 1-22 for complete list.

(5) Compute:

$$s = \sqrt{\frac{\Sigma\Delta^2}{f-1}}$$

(5) $s = \sqrt{\frac{1410.6}{6-1}}$

$= \sqrt{282.1}$

$= 16.80$

$= 17$ hours

See paragraph 7.1.1.4, page 65.

(6) Compute:

$$\varepsilon = \frac{t_{1-\alpha}(s)}{\sqrt{f}}$$

(6) $\varepsilon = \frac{(1.476)(16.80)}{\sqrt{6}}$

$= \frac{24.80}{2.449}$

$= 10.13$

(7) Compute:

UCL = MTBF + ε

(7) UCL = 207 + 10.13
= 217.13
= 218 hours

(8) Conclude that the population MTBF $\leq$ UCL at a $100(1-\alpha)\%$ confidence level.

(8) Conclude that the population MTBF $\leq$ 218 hours at a 90% confidence level.

c. Case III: LCL (one-sided limit), also referred to as M_1.

Given:

Sample data at Table A-6d, page 1-22.

Procedure:

(1) Choose the confidence level $(1-\alpha)$.

(2) Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for $(f-1)$ d.f.

(3) Compute the MTBF.

(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.

Example:

(1) $\alpha = .10$
$1-\alpha = .90$

(2) $f-1 = 5$
$t_{.90}$ for 5 d.f. = 1.476

(3) MTBF = 207 hours.
See paragraph 10.2.1.1.4, page 134.

(4) (a) Time to failure 1 = 200 hours.
(b) Time between failures 2 and 1
= 410-200
= 210 hours

(5) Compute:

$$s = \sqrt{\frac{\Sigma\Delta^2}{f-1}}$$

(5) $s = 16.80$
$= 17$ hours
See paragraph 7.1.1.4, page 65.

(6) Compute:

$$\varepsilon = \frac{t_{1-\alpha}(s)}{\sqrt{f}}$$

(6) $$\varepsilon = \frac{(1.476)(16.80)}{\sqrt{6}}$$
$= 10.13$

(7) Compute:

LCL = MTBF − ε

(7) LCL = 207 − 10.13
= 196.87
= 196 hours

(8) Conclude that the population MTBF ≥ LCL at a 100(1−α)% confidence level.

(8) Conclude that population MTBF ≥ 196 hours at a 90% confidence level.

10.2.1.2.5 ANALYSIS

a. The two-sided interval surrounds the population MTBF such that the population MTBF ≤ UCL and the population MTBF ≥ LCL at a 100(1−α)% confidence level.

b. The one-sided interval surrounds the population MTBF such that the population MTBF ≤ UCL at a 100(1−α)% confidence level.

c. The one-sided interval surrounds the population MTBF such that the population MTBF ≥ LCL at a 100(1−α)% confidence level. M_1 (the LCL) is generally considered the MTBF of the population since the population MTBF will be at least M_1 at a 100(1−α)% confidence level. If comparing M_1 to the required MTBF produces an accept decision for the test item, then on the average the test item will function as required at a 100(1−α)% confidence level.

d. The method used to compute M_1 and M_2 uses s to estimate σ. If the time between two failures is close to the MTBF, s will be small; and M_1 will be close to the MTBF. However, if the times between failures are erratic (close to the MTBF in some cases and far from the MTBF in other cases), s will be large; and the interval between the M_1 and the MTBF will increase. Since this method uses the student t distribution, f should be less than or equal to 30.

NOTE: The application of the student t assumes that the MTBF's are approximately normally distributed.

10.2.1.3 LIMITS USING THE χ^2 DISTRIBUTION

10.2.1.3.1 OBJECTIVE

To determine the two-sided and one-sided limits for the MTBF using the χ^2 distribution.

10.2.1.3.2 DATA REQUIRED

A list of sample readings; e.g., operating time (primary parameter) and failures (secondary parameter).

10.2.1.3.3 PROCEDURE

a. Case I: UCL and LCL (two-sided limits), also referred to as M_2 and M_1.

(1) Choose the desired confidence level.
(2) Use Table B-20, page 2-128, to obtain $LF_{1-\alpha/2}$ for:

(a) f + 1 d.f., if a time terminated test.
(b) f d.f., if a failure terminated test.

(3) Use Table B-21, page 2-129, to obtain the $UF_{1-\alpha/2}$ for f d.f., for both the time and failure terminated test.
(4) If the test is a time terminated test, compute the following:

(a) Multiply step (2)(a) by T_t.
(b) Divide step (a) by the quantity (f+1) to obtain the LCL.
(c) Multiply step (3) by T_t.
(d) Divide step (c) by the value of f to obtain the UCL.

(5) If the test is a failure terminated test, compute the following:

(a) Multiply step (2) (b) by T_t.
(b) Divide step (a) by f to obtain the LCL.
(c) Multiply step (3) by T_t.
(d) Divide step (c) by f to obtain the LCL.

NOTE: To maintain accuracy, the six decimal number found in Table B-20, page 2-128, or B-21, page 2-129,must be used.

(6) Conclude that the population MTBF is equal to or between the UCL and LCL at the desired confidence level.

b. Case II. UCL (one-sided limit), also referred to as M_2.

(1) Choose the desired confidence level.
(2) Use Table B-21, page 2-129, to obtain the $UF_{1-\alpha}$ for f d.f., for both the time and failure terminated test.

(a) Multiply step (2) by T_t.
(b) Divide step (a) by f.
(3) Compute the UCL as follows:
NOTE: To maintain accuracy, the six decimal number found in Table B-21, page 2-129, must be used.

(3) Conclude that the population MTBF is equal to or less than the UCL at the desired confidence level.

c. Case III. LCL (one-sided), also referred to as M_1.

(1) Choose the desired confidence level.
(2) Use Table B-20, page 2-128, to obtain $LF_{1-\alpha}$ for:

(a) f+1 d.f., if a time terminated test.
(b) f d.f., if a failure terminated test.

(3) If a time terminated test, compute the LCL as follows:

(a) Multiply step (2)(a) by T_t.
(b) Divide step (a) by the quantity (f+1).

(4) If a failure terminated test compute the LCL as follows:

(a) Multiply step (2)(b) by T_t.
(b) Divide step (a) by f.

NOTE: To maintain accuracy, the six decimal number found in Table B-20, page 2-128, must be used.

(5) Conclude that the population MTBF is equal to or greater than the LCL at the desired confidence level.

10.2.1.3.4 EXAMPLE

a. Case I: UCL and LCL (two-sided limits), also referred to as M_2 and M_1.

Given:
Sample data at Table A-6c, page 1-21.

Procedure:	Example:
(1) Choose the confidence level (1-α).	(1) $\alpha = .05$ $1-\alpha = .95$
(2) Use Table B-20, page 2-128, to obtain $LF_{1-\alpha/2}$ for: (a) f+1 d.f., if a time terminated test. (b) f d.f., if a failure terminated test.	(2) $LF_{.975}$ for 13 d.f. = .620525
(3) Use Table B-21, page 2-129, to obtain $UF_{1-\alpha/2}$ for f d.f.	(3) $UF_{.975}$ for 12 d.f. = 1.935484
(4) Compute for a time terminated test: $LCL = \frac{(LF_{1-\alpha/2})(T_t)}{f+1}$ $UCL = \frac{(UF_{1-\alpha/2})(T_t)}{f}$	(4) Since the test is time terminated, $LCL = \frac{(.6205926)(3752)}{(12+1)}$ $= 179.095926$ $= 179$ hours $UCL = \frac{(1.935484)(3752)}{12}$ $= 605.161330$ $= 606$

(5) Compute for a failure terminated test:

$$LCL = \frac{(LF_{1-\alpha/2})(T_t)}{f}$$

$$UCL = \frac{(UF_{1-\alpha/2})(T_t)}{f}$$

(6) Conclude that the population MTBF ≤ UCL and the population MTBF ≥ LCL at a 100(1-α)% confidence level.

(6) Conclude that the population MTBF ≤ hours and the population MTBF ≥ 179 hours at a 95% confidence level.

b. Case II. UCL (one-sided limit), also referred to as M_2.

Given:

Sample data at Table A-6c, page 1-21.

Procedure:

(1) Choose the confidence level (1-α).

(2) Use Table B-21, page 2-129, to obtain the $UF_{1-\alpha}$ for f d.f., for both a time and failure terminated test.

(3) Compute

$$UCL = \frac{(UF_{1-\alpha})\ (T_t)}{f}$$

(4) Conclude that the population MTBF ≤ UCL at a 100(1-α)% confidence level.

Example:

(1) α = .05
1-α = .95

(2) $UF_{.95}$ for 12 d.f. = 1.739130

(3)

$$UF_{.95} = \frac{(1.739130)(3752)}{12}$$

$$= 543.767980$$

$$= 543$$

(4) Conclude that the population MTBF ≤ 543 hours at a 95% confidence level.

c. Case III: LCL (one-sided limit), also referred to as M_1.

Given:

Sample data at Table A-6c, page 1-21.

Procedure:

(1) Choose the confidence level $(1-\alpha)$.
(2) Use Table B-20, page 2-128, to obtain $LF_{1-\alpha}$ for:

(a) f+1 d.f., if time terminated.
(b) f d.f., if failure terminated.

(3) Compute for a time terminated test:

$$LCL = \frac{(LF_{1-\alpha})(T_t)}{(f+1)}$$

(4) Compute for a failure terminated test:

$$LCL = \frac{(LF_{1-\alpha})(T_t)}{f}$$

(a) Conclude that the population MTBF $\geq$ LCL at a $100(1-\alpha)\%$ confidence level.

Example:

(1) $\alpha = .05$
$1-\alpha = .95$
(2) $LF_{.95}$ for 13 d.f. = .668380

(3) Since the test is time terminated,

$$LCL = \frac{(.668380)(3752)}{12+1}$$
$$= 192.907836$$
$$= 192 \text{ hours}$$

(5) Conclude that the population MTBF $\geq$ 192 hours at a 95% confidence level.

NOTE: Although the confidence level is numerically the same for all three cases, M_1 and M_2 different values (see Figure 15).

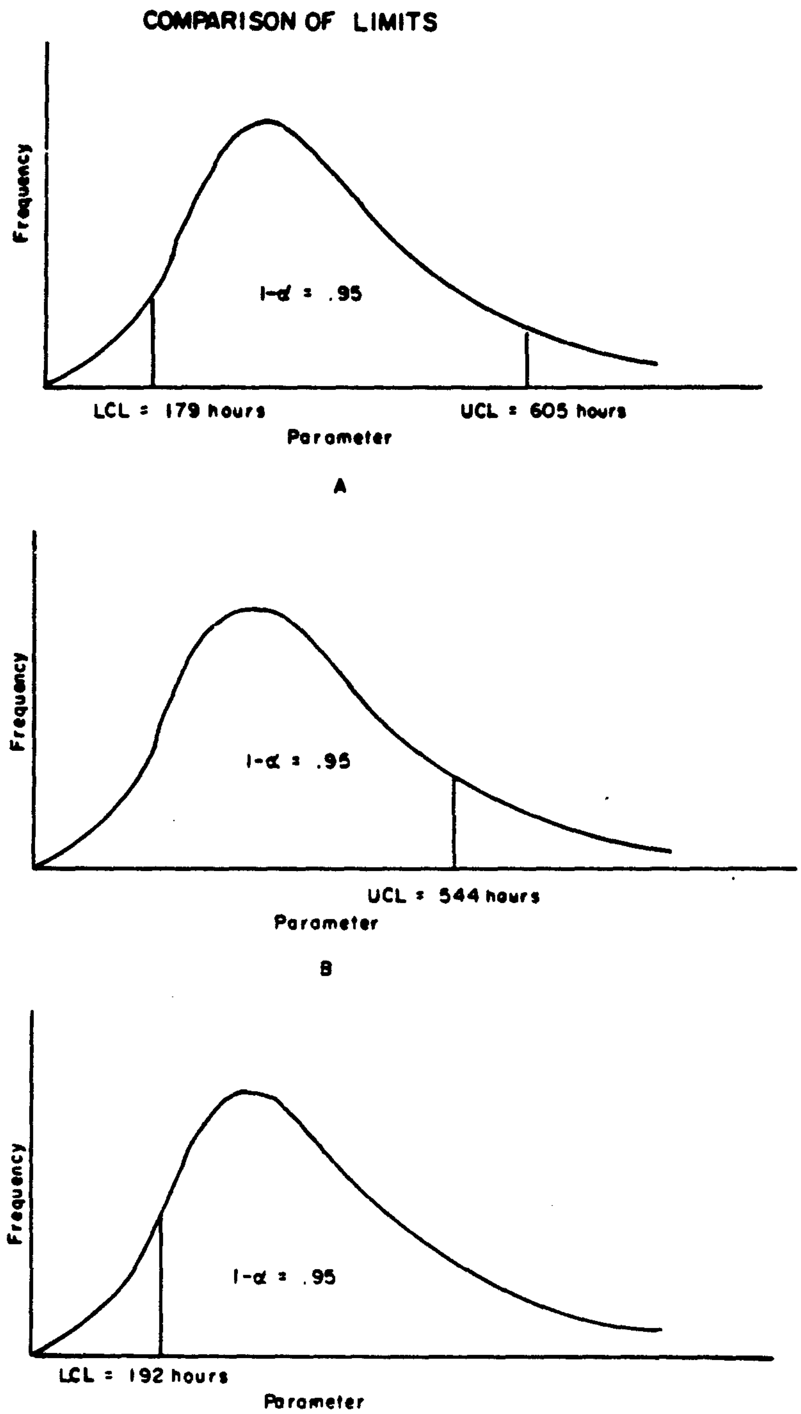
10.2.1.3.5 ANALYSIS

a. The two-sided interval surrounds the population MTBF such that the population MTBF $\leq$ UCL and the population MTBF $\geq$ LCL at a $100(1-\alpha)\%$ confidence level.

b. The one-sided interval surrounds the population MTBF such that the population MTBF $\leq$ UCL at a $100(1-\alpha)\%$ confidence level.

c. The one-sided interval surrounds the population MTBF such that the population MTBF $\geq$ LCL at a $100(1-\alpha)\%$ confidence level. M_1 (the LCL) is generally considered the MTBF of the population since the population MTBF will be at least M_1 at a $100(1-\alpha)\%$ confidence level. If comparing M_1 to the required MTBF produces an accept decision for the test item, then on the average the test item will function as required at a $100(1-\alpha)\%$ confidence level.

d. The method used to compute M_1 and M_2 is dependent upon the type of test conducted; i.e., time terminated or failure terminated. The time terminated test produces a more conservative estimate for the LCL of the population MTBF since a safety factor of one is added to the number of failures which occurred.


COMPARISON OF LIMITS
Frequency
1-α = .95
LCL = 179 hours
UCL = 605 hours
Parameter
A
Frequency
1-α = .95
UCL = 544 hours
Parameter
B
Frequency
1-α = .95
LCL = 192 hours
Parameter

C

Figure 15

10.2.2 APPLICATION OF THE EXPONENTIAL DISTRIBUTION

10.2.2.1 OBJECTIVE

To determine the reliability for those items which demonstrate an exponential lifetime to failure.

10.2.2.2 DATA REQUIRED

The mission (operational) profile (MP), T_t, and f.

10.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute the MTBF (see paragraph 10.2.1.3, page 134).

c. Use Table B-20, page 2-128, to obtain the $LF_{1-\alpha}$ for:

(1) f+1 d.f., if a time terminated test.
(2) F d.f., if a failure terminated test

d. Compute the LCL as follows:

(1) For a time terminated test, multiply step c by T and divide by the quantity (f+1).
(2) For a failure terminated test, multiply step b by step c.

NOTE: To maintain accuracy, the six decimal number found in Table B-20, page 2-128.

e. Compute R as follows:

(1) Divide MP by step d.
(2) Use Table B-22, page 2-130, to obtain e raised to the negative power of step (1).

f. Conclude that ρ is equal to or greater than R at the desired confidence level.

g. If R is equal to or greater than ρ_0, decide that ρ is equal to or greater than ρ_0; otherwise, there is no reason to believe ρ is equal to or greater than ρ_0 at the desired confidence level.

10.2.2.4 EXAMPLE.

Given:

$\rho_0 = .75$

Sample data at Table A-6c, page 1-21.

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .05$ $1-\alpha = .95$
b. Compute: $MTBF = \frac{T_t}{f}$	b. $MTBF = \frac{3752}{12}$ $= 312.67$ $= 313$
c. Use Table B-20, Page 2-128, to obtain $LF_{1-\alpha}$ for: (1) f=1 d.f., if a time terminated test. (2) f d.f., if a failure terminated test.	c. LF for 13 d.f. = .668380
d. Compute: (1) For a time terminated test: $LCL = \frac{(LF_{1-\alpha})(T_t)}{}$ (2) For a failure terminated test: $LCL = LF_{-} (MTBF)$	d. Since the example is a time terminated test, $LCL = \frac{(.668380)\ (3752)}{12 + 1}$ $= \frac{2507.761760}{13}$ $= 192$ hours
e. Compute: $R = e^{-\frac{MP}{LCL}}$	e. $R = e^{\frac{-48}{192.90}}$ $= e^{-.249}$ $= .7796$
f. Conclude that $\rho \geq R$ at a 100 $(1-\alpha)$% confidence level.	f. Conclude that $\rho \geq .77$ at a 95% confidence level.
g. If $R \geq \rho o$, decide that $\rho \geq \rho o$; otherwise, there is no reason to believe $\rho \geq \rho o$ at a 100 $(1-\alpha)$% confidence level.	g. Since $.77 \geq 75$, decide that $\rho \geq .75$ at a 95% confidence level.

10.2.2.5 ANALYSIS

The reliability at a $100(1-\alpha)\%$ confidence level is computed using the LCL of the MTBF. If $R \geq \rho_0$, the null hypothesis that $\rho \geq \rho_0$ is accepted; otherwise, there is no reason to believe $\rho \geq \rho_0$ at a $100(1-a)\%$ confidence level.

10.2.3 SEQUENTIAL ANALYSIS

a. When testing an hypothesis using the sequential method, the project officer is able to make one of the following three decisions at any stage of testing:

(1) Accept the hypothesis.
(2) Reject the hypothesis.
(3) Continue the experiment by collecting additional data.

b. Usually a ρ_0 of .95 with a high degree of assurance is required. In order to achieve assurance of such a high ρ_0, the project officer would have to conduct excessive testing; e.g., many thousands of miles, hours, or rounds. This may be impractical; however, using the following statistical approach, the project officer will achieve the predetermined confidence level for reaching the accept decision.

c. If certain criteria are set up graphically, a decision can be made to accept, reject, or continue testing the test item at any time. This graph uses three areas to represent the decisions to accept, reject, or continue testing the test item. The accept region is above a boundary line determined by the addition of an amount of doubt (D) to ρ_0 and the confidence levels for rejection and acceptance. The continue testing area is below the accept boundary line and above the reject boundary line. The size of this area, which is an area of doubt for the test item, is determined by the project officer (see paragraph 4.14, page 14). The area of doubt is designed for a test item which may be good but has gotten off to a slow start. In this case, a longer period of time will be required to satisfy the doubts concerning acceptability of the test item. The reject boundary line is determined by ρ_0 and the confidence levels for rejection and acceptance. The area below this boundary line is the area of rejection. A graph of this type is illustrated at Figure 16. Failures are plotted on the horizontal axis with each increment representing one failure and hours are plotted on the vertical axis.

d. The construction of the two boundary lines is described in the procedure paragraph below.

10.2.3.1 OBJECTIVE

To determine whether ρ is equal to or greater than ρ_0 at the desired confidence level.

10.2.3.2 DATA REQUIRED

The MP and T_t.

10.2.3.3 PROCEDURE

a. Construct the boundary lines as follows:

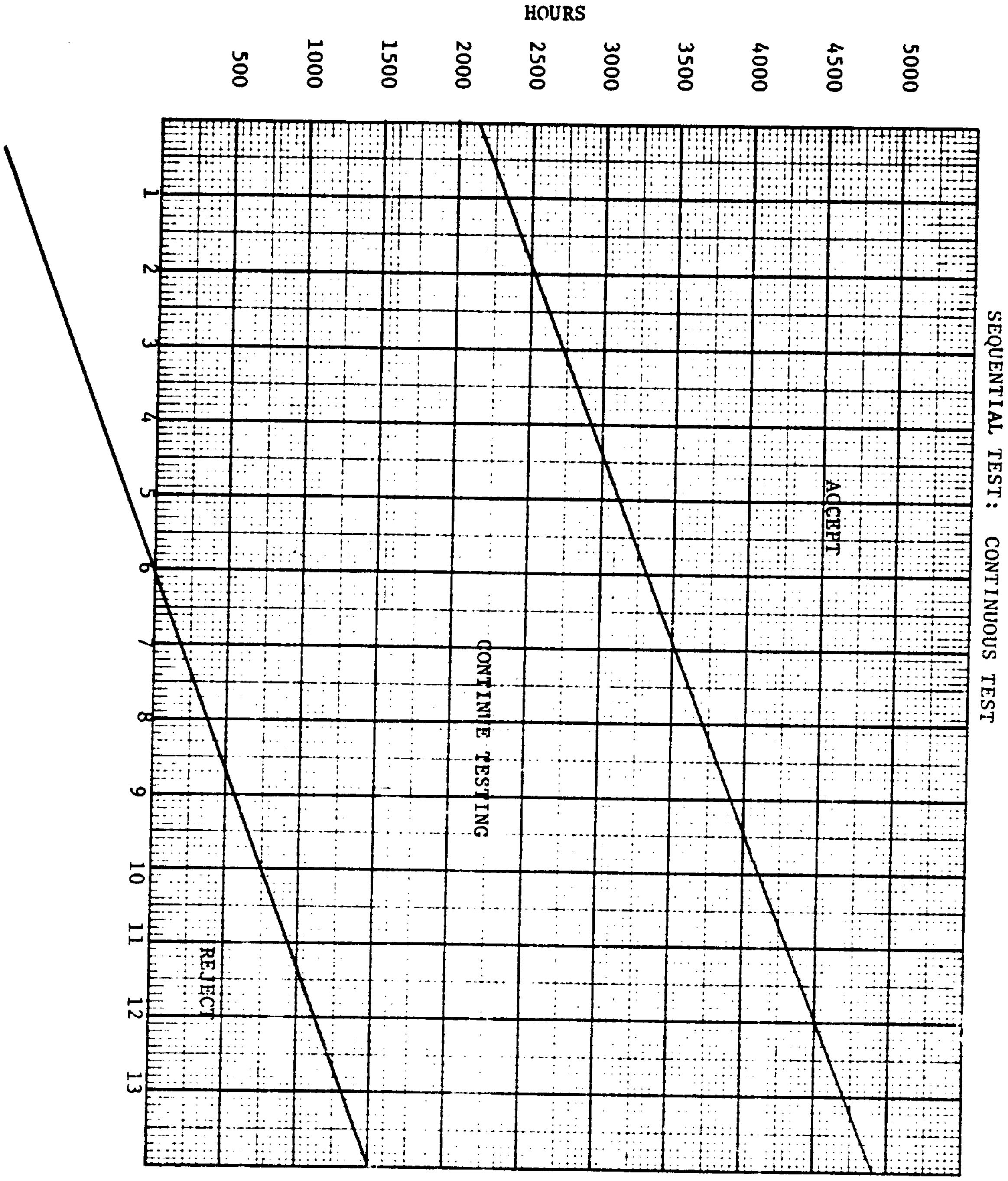

SEQUENTIAL TEST: CONTINUOUS TEST
HOURS
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
1
2
3
4
5
6
7
8
9
10
11
12
13
ACCEPT
CONTINUE TESTING
REJECT

Figure 16

(1) Choose α and β, the probabilities of making Type I and Type II errors respectively.
(2) Determine the upper limit (R_U), which is the sum of ρ_o and the amount of doubt ($R_U = \rho_o + D$).
(3) Use Table B-19, page 2-127, to obtain a and b for α and β.
(4) Compute the required MTBF as follows:

(a) Find the natural logarithm of step (2).
(b) Divide the negative of the MP by step (a).

(5) Compute the mean time between failures for continued testing ($MTBF_t$) as follows:

(a) Find the natural logarithm of ρ_o.
(b) Divide the negative of the MP by step (a).

(6) Divide 1 by step (4) to obtain the failure rate (f_r).
(7) Divide 1 by step (5) to obtain the failure rate to continue testing (f_{rt}).
(8) Subtract step (6) from step (7) to obtain U, an intermediate value.
(9) Compute V, an intermediate value, as follows:

(a) Divide step (4) by step (5).
(b) Find the natural logarithm of step (a).
(c) Divide step (b) by step (8).

(10) Determine the accept boundary line as follows:

(a) Divide the value a in step (3) by step (8).
(b) Multiply step (9) by f.
(c) Add step (a) to step (b) to determine the minimum hours to test in order to make an accept decision ($T_{ACCEPT} = \frac{a}{U} + V(f)$).
(d) Choose two values for f and substitute them into the above equation to determine two points on the accept boundary line.

NOTE: Use f = 0 and f = some large value; such as, 4, 6, or 10.

(e) Draw the accept boundary line using the two points determined from step (d).

(11) Determine the reject boundary line as follows:

(a) Divide the value b in step (3) by step (8).
(b) Multiply step (9) by f.
(c) Add step (a) to step (b) to determine the maximum hours to test in order to make a reject decision ($T_{REJECT} = \frac{b}{U} + V(f)$).
(d) Choose two values for f and substitute them into the above equation to determine two points on the reject boundary line.

(e) Draw the reject boundary line using the two points determined from step (d).

(12) If the two lines are not parallel, check the computations and plotted points.

b. Plot the sample data on the sequential graph as follows:

(1) Plot the cumulative operating hours at appropriate interval.

(2) After plotting each point, decide whether to accept, reject, or continue testing the test item.

NOTE: An accept decision may be made before another failure occurs in the event that the number of operating hours increases sufficiently after the last failure to cross the accept boundary line.

10.2.3.4 EXAMPLE

a. Construct the boundary lines as follows:

Given:

$\rho_o = .75$
MP = 50 hours

Procedure:	Example:
(1) Choose α and β.	(1) $\alpha = .05$ $1-\alpha = .95$ $\beta = .20$ $1-\beta = .80$
(2) $R_U = \rho_o + D$	(2) $R_U = .75 + .05$ $= .80$
(3) Use Table B-19, page 2-127, to obtain a and b for α and β.	(3) a = 2.773 b = -1.558
(4) Compute: $MTBF = \frac{-MP}{\ell n \ Ru}$	(4) $MTBF = \frac{-50.000}{\ell n \ .800} = \frac{-50.000}{-0.22314} = 224.07$
(5) Compute: $MTBF_t = \frac{-MP}{\ln \rho_o}$	(5) $MTBF_t = \frac{-50.000}{\ln .750} = \frac{-50.000}{-0.28768} = 173.80$

(6) Compute:

$$f_r = \frac{1}{MTBF}$$

(6)

$$f_r = \frac{1}{224.07} = .0044629$$

(7) Compute:

$$f_{rt} = \frac{1}{MTBF_t}$$

(7)

$$f_{rt} = \frac{1}{173.80} = .0057536$$

(8) Compute:

$$U = f_{rt} - f_r$$

(8) $U = .0057536 - .0044629 = .0012908$

(9) Compute:

$$V = \frac{\ln\left(\frac{MTBF}{MTBF_t}\right)}{U}$$

(9)

$$V = \frac{\ln\left(\frac{224.07}{173.80}\right)}{.0012908} = \frac{\ln 1.29}{.0012908} = \frac{.25404}{.0012908} = 196.81$$

(10) Compute:

$$T_{ACCEPT} = \frac{a}{U} + V(f)$$

(10)

$$T_{ACCEPT} = \frac{2.773}{.0012908} + 196.81(f) = 2148.2+196.81(f)$$

When $f = 0$, $T_{ACCEPT} = 2150$

When $f = 7$, $T_{ACCEPT} = 3530$

Plot the points (0,2150) and (7,3530) to determine the accept boundary line.

(11) Compute:

$$T_{REJECT} = \frac{b}{U} + V(f)$$

(11)

$$T_{REJECT} = \frac{-1.558}{.0012908} + 196.81(f) = -1207+196.81(f)$$

When $f = 0$, $T_{REJECT} = -1207$

When $f = 7$, $T_{REJECT} = 170$

Plot the points (0,-1207) and (7,170) to determine the reject boundary line.

(12) If the two lines are not parallel, check the computations and plotted points

(12)

b. Plot the sample data on the sequential graph as follows:

Given:

Requirements and boundary lines from step a.
Sample data at Table A-6e, page 1-22.

Procedure:	Example:
(1) Plot the cumulative operating hours at appropriate intervals (f_i, T_i).	(1) (a) (1,175) (b) (2,490) See Table A-6e, page 1-22 for complete list.
(2) After plotting each point, decide to accept, reject, or continue testing the test item.	(2) For failures 1 through 4, decide to continue testing. Decide to accept the test item when T = 3137 hours and f = 5 since the accept boundary line is crossed. See Table A-6f, page 1-23.

10.2.3.5 ANALYSIS

a. The sequential method generally minimizes testing time and N due to the fact that a decision to accept or reject is made as soon as possible after the first failure. Since all failures are not necessarily chargeable failures, decisions will be altered if certain failures are not counted. If the project officer ignores a failure, the probability of accepting an unacceptable item is increased. Therefore, the project officer must carefully decide what constitutes a failure (see paragraph 4.2, page 2).

b. Due to the advantages just discussed, the sequential method should be used whenever possible (see paragraph 10.2c, page 134).

10.3 COMBINED RELIABILITY

If a number of components of a system are connected in such a way that the failure of any one component causes a failure of the system, then these components are considered to be functionally in series. The reliability of such a system can be determined by the following method.

10.3.1 OBJECTIVE

a. Case I: To determine the reliability of a system based on the individual reliabilities of its components.

b. Case II: To determine the reliability of an individual component of a system.

10.3.2 DATA REQUIRED

a. Case I: N and f for each component.

b. Case II: N and f for the component tested.

10.3.3 PROCEDURE

a. Case I: Reliability of independent serial systems.

(1) Choose the desired confidence level.
(2) Compute the point estimate reliability (R_{PE}) as follows:

(a) Subtract f from N for each component.
(b) Divide step (a) by N for each respective component.
(c) Multiply the results of step (b) by each other.

(3) Compute the system failures (f_s) as follows:

(a) Subtract step (2) from 1.
(b) Multiply step (a) by the minimum N of the components.

(4) Co_ute the LCL using Case II of paragraph 10.1.1.4, page 122.

NOTE: When using f_s to determine d.f.$_1$ and d.f.$_2$, round off the results.

(5) Conclude that the ρ for the system is the LCL at the desired confidence level.

b. Case II: Reliability of a component.

See Case III of paragraph 10.2.2.3, page 144.

10.3.4 EXAMPLE

a. Case I: Reliability of independent serial systems.

Given:
Sample data at Table A-6g, page 1-24.

Procedure:	Example:
(1) Choose the confidence level (1-α).	(1) $\alpha = .10$ $1-\alpha = .90$
(2) Compute: $R_{PE} = \prod \frac{N_i - f_i}{N_i}$	(2) $R_{PE} = \left(\frac{90-2}{90}\right)\left(\frac{90-4}{90}\right)\left(\frac{45-1}{45}\right)\left(\frac{45-3}{45}\right)$ $= (.9778)(.9556)(.9778)(.9333)$ $= (.9344)(.9126)$ $= .8527$
(3) Compute: $f_s = N_{min}(1-R_{PE})$	(3) $f_s = 45(1-.8527)$ $= 45(.1473)$ $= 6.628$

(4) Compute:

$d.f._1 = 2(f_s)+2$

$d.f._2 = 2(N_{min})-2(f_s)$

$$LCL = \frac{1}{1+\left(\frac{f_s+1}{N_{min}-f_s}\right) F_{1-\alpha}}$$

See paragraph 10.1.1.4, Case II, page 122, for details.

NOTE: N is the minimum N of the components and f is f_s.

(4) $d.f._1 = 2(6.628)+2$
$= 13.256+2$
$= 15.256$
$= 15$

$d.f._2 = 2(45)-2(6.628)$
$= 90-13.256$
$= 76.744$
$= 76$

NOTE: Use 15 and 70 in F tables.

$$LCL = \frac{1}{1+\left(\frac{6.628+1}{45-6.628}\right) F_{.90}}$$

$$= \frac{1}{1+\left(\frac{7.628}{38.37}\right)(1.58)}$$

$$= \frac{1}{1+(.1988)(1.58)}$$

$$= \frac{1}{1.3141}$$

$= .7610$
$= .76$

(5) Conclude that the ρ for the system is the LCL at a $100(1-\alpha)\%$ confidence level.

(5) Conclude that the ρ for the system is .76 at a 90% confidence level.

b. Case II: Reliability of a component. See Case II (3) of paragraph 10.1.1.4, page 122.

10.3.5 ANALYSIS

a. Case I. The point estimate (achieved) reliability of an independent serial system is determined by multiplying together the point estimate reliability of the components. The number of system failures is determined by multiplying the minimum sample size of the components by the quantity $(1-R_{PE})$. The R of the system is then determined as a LCL (see paragraph 10.1.1.4, Case II, page 122). The project officer will compare this R to ρ_0 to determine whether $\rho \geq \rho_0$ at a $100(1-\alpha)\%$ confidence level.

b. Case II. R at a $100(1-\alpha)\%$ confidence level is computed as a LCL. The project officer will compare this R to ρ_0 to determine whether $\rho \geq \rho_0$ at a $100(1-\alpha)\%$ confidence level.

11. MAINTENANCE EVALUATION

11.1 MAINTENANCE RATIO

11.1.1 OBJECTIVE

To determine the maintenance ratio (MR) for the test item.

11.1.2 DATA REQUIRED

Records of active maintenance manhours and T_t.

11.1.3 PROCEDURE

a. Sum the active maintenance manhours to obtain the total maintenance manhours (TM).

b. Sum the hours of operation to obtain T_t.

c. Divide TM by T_t.

11.1.4 EXAMPLE

Given:
Sample data at Table A-7b, page 1-26.

Procedure:	Example:
a. Compute: $TM = \Sigma$ active maintenance manhours.	a. TM = 9.25 manhours
b. Compute: $T_t = \Sigma$ operating time.	b. T_t = 109.75 hours
c. Compute: $MR = \frac{TM}{T_t}$	c. $MR = \frac{9.25}{109.75}$ = .084282 = .0842 manhours per hour

11.1.5 ANALYSIS

The MR indicates the amount of active maintenance manhours required per operating hour for the test item.

11.2 MAINTAINABILITY

11.2.1 OBJECTIVE

To determine the maintainability ($\underline{M}$).

11.2.2 DATA REQUIRED

a. Active maintenance time (AMT), the number of maintenance actions (MA), the required maintenance action time (ω).

b. Time to repair (RT), ω, and f.

11.2.3 PROCEDURE

a. Case I: Maintainability, based on all MA's.

(1) Sum the AMT's (ΣAMT).
(2) Divide step (1) by MA to obtain the mean active maintenance time ($\bar{M}$).

(3) Determine the maintenance action rate (AR) by dividing 1 by step (2).

(4) Compute M as follows:

(a) Multiply step (3) by ω.
(b) Raise the exponential (e) to the negative power of step (a) (see Table B-22, page 2-130).
(c) Subtract step (b) from 1.

(5) Conclude that the M is the probability of completing an MA of the population within prescribed limits based on the sample.

b. Case II: Maintainability, based only on failures.

(1) Compute Y, an intermediate value, as follows:

(a) If f is equal to or less than 3, compute:

1. Use Table B-22, page 2-130 to obtain e raised to the negative power of f.
2. Subtract step 1 from 1.

(b) If f is greater than 3, set Y equal to 1.

(2) Sum the repair time (ΣRT).
(3) Divide step (a) by f to obtain the mean time to repair (MTTR).
(4) Divide 1 by step (3) to obtain the repair rate (RR).
(5) Compute U, an intermediate value, as follows:

(a) Multiply step (4) by ω.
(b) Use Table B-22, page 2-130, to obtain e raised to the negative power of step (a).
(c) Subtract step (b) from 1.

(6) Multiply step (1) by step (5) to obtain M.
(7) Conclude that the M is the probability of completing a failure within prescribed limits based on the sample.

11.2.4 EXAMPLE

a. Case I: Maintainability, based on all MA's.
Given:
ω = .5 hour
MA = 22
Sample data at Table A-7a, page 1-25.

Procedure:	Example:
(1) Compute: ΣAMT	(1) ΣAMT = 16.9 hours
(2) Compute: $\overline{M} = \frac{\Sigma AMT}{MA}$	(2) $\overline{M} = \frac{16.9}{22}$ = .7682 = .77 hour per action.

(3) Compute:

$$AR = \frac{1}{\overline{M}}$$

(3) $AR = \frac{1}{.77045}$
= 1.3017
= 1.30 actions per hr.

(4) Compute:

$$\underline{M} = 1-e^{-(AR)(\omega)}$$

Use Table B-22, page 2-130.

(4)

$$\underline{M} = 1-e^{-(1.3017)(.5)}$$
$= 1-e^{-.651}$
$= 1-.5215$
$= .4785$
$= .48$

(5) Conclude that the $\underline{M}$ is the probability of completing an MA within prescribed limits based on the sample.

(5) Conclude that .48 is the probability of completing an MA in .5 hour or less based on the sample.

b. Case II: Maintainability, based only on failures
Given:
ω = .5 hours
f = 5
Sample data at Table A-7a, page 1-25.

Procedure:

(1) Compute:

(a) If $f \leq 3$, compute:

$$Y = 1-e^{-f}$$

(b) If $f > 3$, compute:

Y = 1

(2) Compute:
ΣRT

(3) Compute:

$$MTTR = \frac{\Sigma RT}{f}$$

(4) Compute:

$$RR = \frac{1}{MTTR}$$

(5) Compute:

$$U = 1-e^{-(RR)(\omega)}$$

Use Table B-22, page 2-130.

Example:

(1) Since 5 > 3,
Y = 1

(2) ΣRT = 4.8 hours

(3) $MTTR = \frac{4.8}{5}$
= .960 hr. per failure

(4) $RR = \frac{1}{.960}$
= 1.04 failures per hr. of repair

(5)

$$U = 1-e^{-(1.04)(.5)}$$
$= 1-e^{-.520}$
$= 1-.5945$
$= .4055$
$= .41$

(6) Compute:

$\underline{M} = Y(U)$

(7) Conclude that the $\underline{M}$ is the probability of completing a failure within prescribed limits based on the sample.

(6) $\underline{M} = (1)\ (.41)$

$= .41$

(7) Conclude that .41 is the probability of completing a failure in .5 hour or less based on the sample.

11.2.5 ANALYSIS

Maintainability is a characteristic of design and installation which is expressed as the probability than an item will be retained in or restored to a specified condition within a given period of time, when the maintenance is performed in accordance with prescribed procedures and resources. The maintainability increases exponentially with time for a given maintenance action rate. The greater the time available to perform a MA, the greater will be the probability of successfully performing the maintenance action.

11.3 AVAILABILITY

Availability is a measure of the degree to which an item is in the operable and committable state when the mission is called for at an unknown (random) point in time. Availability actually consists of two components: maintainability and reliability. Poor reliability can be offset by correspondingly improved maintainability. For test purposes availability is broken down into three types which are discussed in the following paragraphs.

11.3.1 INHERENT AVAILABILITY

11.3.1.1 OBJECTIVE

To determine the inherent availability (A_i) of the test item as an estimate of the population availability.

11.3.1.2 DATA REQUIRED

T_t, f, and RT's.

11.3.1.3 PROCEDURE

a. Compute MTBF (see paragraph 10.2.1.1.3, page 134).

b. Compute the mean time to repair (MTTR) as follows:

(1) Sum the RT's (ΣRT).
(2) Divide step (1) by f.

c. Compute A_i as follows:

(1) Add step a to step b.
(2) Divide step a by step (1).

d. Conclude that the inherent availability of the sample is $100(A_i)$%.

11.3.1.4 EXAMPLE

Given:

Sample data at Table A-7b, page 1-25.

Procedure:

a. Compute:

$$MTBF = \frac{T_t}{f}$$

b. Compute:

$$MTTR = \frac{\Sigma RT}{f}$$

c. Compute:

$$A_i = \frac{MTBF}{MTBF+MTTR}$$

d. Conclude that the inherent availability of the sample is $100(A_i)\%$.

Example:

a. $MTBF = 109.8/3$
$= 36.600$ hours

b. $MTTR = 4.8/3$
$= 1.600$ hours per failure

c. $A_i = \frac{36.600}{36.600+1.600}$
$= \frac{36.600}{38.200}$
$= .95811$
$= .958$

d. Conclude that the inherent availability of the sample is 95.8%.

11.3.1.5 ANALYSIS

A_i is the probability that a system or equipment, when used under stated conditions without consideration for any scheduled or preventive maintenance in an ideal support environment; i.e., when all tools, parts, manpower, and manuals are available, will operate satisfactorily at any given time. A_i excludes ready time, preventive maintenance downtime , supply downtime, and waiting or administrative downtime. A_i is a prediction of the population inherent availability.

11.3.2 ACHIEVED AVAILABILITY

11.3.2.1 OBJECTIVE

To determine the achieved availability (A_a) of the test item.

11.3.2.2 DATA REQUIRED

T_t, MA, and AMT.

11.3.2.3 PROCEDURE

a. Divide T_t by MA to obtain the mean time between maintenance (MTBM)

b. Compute $\overline{M}$ as follows:

(1) Sum the AMT's.
(2) Divide step (1) by MA.

c. Compute A_a as follows:

(1) Add step a and step b.
(2) Divide step a by step (1).

d. Conclude that the achieved availability of the sample is $100(A_a)\%$.

11.3.2.4 EXAMPLE

Given:
Sample data at Table A-7b, page 1-26.

Procedure:

a. Compute:

$$MTBM = \frac{T_t}{MA}$$

b. Compute:

$$\overline{M} = \frac{\Sigma AMT}{MA}$$

c. Compute:

$$A_a = \frac{MTBM}{MTBM+\overline{M}}$$

d. Conclude that the achieved availability of the sample is $100(A_a)\%$.

Example:

a. MTBM = 109.8/7
= 15.686
= 15.7 hr. per MA

b. $\overline{M}$ = 6.8/7
= .971
= .97 Active maintenance time per MA

c. $$A_a = \frac{15.686}{15.686+.971} = \frac{15.686}{16.657} = .941706 = .94$$

d. Conclude that the achieved availability of the sample is 94%.

11.3.2.5 ANALYSIS

A_a is the probability that a system or equipment, when used under stated conditions in an ideal support environment, will operate satisfactorily at any given time. A_a is the sample's achieved availability and excludes supply downtime and waiting or administrative downtime.

11.3.3 OPERATIONAL AVAILABILITY

11.3.3.1 OBJECTIVE

To determine the operational availability (A_o) of the test item.

11.3.3.2 DATA REQUIRED

T_t, MA, AMT, and delay time (supply and administrative downtime).

11.3.3.3 PROCEDURE

a. Compute MTBM (see paragraph 11.3.2.3, page 158).

b. Sum the AMT's and the delay time.

c. Divide step b by MA to obtain the mean downtime (MDT).

d. Compute A_o as follows:

(1) Add step a and step c.
(2) Divide step a by step (1).

e. Conclude that the operational availability of the sample in a test support environment is $100(A_o)\%$.

11.3.3.4 EXAMPLE

Given:
Sample data at Table A-7b, page 1-26.

Procedure:	Example:
a. Compute: $MTBM = \frac{T_t}{MA}$	a. $MTBM = 109.8/7$ $= 15.686$ $= 15.7$ hrs. per MA
b. Compute: ΣAMT Σ delay time	b. $\Sigma AMT = 6.8$ Σ delay time $= 8.8$
c. Compute: $MDT = \frac{\Sigma AMT + \Sigma \text{ delay time}}{MA}$	c. $MDT = \frac{6.8 + 8.8}{7}$ $= \frac{15.6}{7}$ $= 2.228$ $= 2.23$ hrs. per down
d. Compute: $A_o = \frac{MTBM}{MTBM+MDT}$	d. $A_o = \frac{15.686}{15.686+2.228}$ $= \frac{15.686}{17.914}$ $= .875628$ $= .876$
e. Conclude that the operational availability of the sample in a test support environment is $100(A_o)\%$.	e. Conclude that the operational availability of the sample in a test support environment is 87%.

11.3.3.5 ANALYSIS

A_o is the probability that a system or equipment, when used under stated conditions in a real support environment, will operate satisfactorily at any given time. A_o includes ready time, maintenance downtime, preventive maintenance downtime, supply downtime, and waiting or administrative downtime.

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INCLOSURE I

TABLE A-1a

BIVARIATE NORMAL DISTRIBUTION RAW DATA

READING NUMBER	EASTING	NORTHING
1	2500	3218
2	2601	3305
3	2575	3279
4	2581	3221
5	2560	3250
6	2590	3261
7	2565	3249
8	2575	3250
9	2560	3239
10	2560	3251
11	2576	3270
12	2553	3251
13	2550	3280
14	2570	3245
15	2549	3278

TABLE A-1b

BIVARIATE NORMAL DISTRIBUTION GROUPED DATA

NORTH \ EAST	2500-2519	2520-2539	2540-2559	2560-2579	2580-2599	2600-2619	TOTAL
3300-3319						1	1
3280-2399							0
3260-3279			2	2	1		5
3240-3259			1	4	1		6
3220-3239				1	1		2
3200-3219	1						1
TOTAL	1	0	3	7	3	1	

TABLE A-2a

MEAN

TEST:

Prepare for action under daylight condition.

TIME (minutes)	Δ	Δ^2
89.3	2.883	8.312
90.4	3.983	15.864
86.0	-.417	.174
83.6	-2.817	7.936
84.4	-2.017	4.068
86.1	-.317	.100
86.0	-.417	.174
88.0	1.583	2.506
86.7	.283	.080
87.4	.983	.966
86.1	-.317	.100
83.0	-3.417	11.676

$N = 12$

$\overline{X} = 86.417$

$= 86.4$ min.

$\Sigma\Delta^2 = 51.9567$

$s^2 = 4.723$

$s = 2.173$

$= 2.2$ min.

TABLE A-2b

COMPARING TWO MEANS (TYPE A: Test Item)

ROUND NUMBER	RANGE (meters)	Δ_A	Δ_A^2
1	5440	38.60	1489.96
2	5379	-22.40	501.76
3	5402	.60	.36
4	5400	-1.40	1.96
5	5400	-1.40	1.96
6	5397	-4.40	19.36
7	5383	-18.40	338.56
8	5400	-1.40	1.96
9	5405	3.60	12.96
10	5395	-6.40	40.96
11	5397	-4.40	19.36
12	5390	-11.40	129.96
13	5402	.60	.36
14	5389	-12.40	153.76
15	5406	4.60	21.16
16	5420	18.60	345.96
17	5423	21.60	466.56
18	5401	-0.40	.16
19	5400	-1.40	1.96
20	5399	-2.40	5.76

$\bar{X}_A$ = 5401.40

= 5401 meters

Σ Range_A = 108,028

$\Sigma\ \Delta_A^2$ = 3552.80

s_A^2 = 186.99

s_A = 13.67

= 14 meters

TABLE A-2c

COMPARING TWO MEANS (TYPE B: Standard Item)

ROUND NUMBER	RANGE (meters)	Δ_B	Δ_B^2
1	5380	7.75	60.06
2	5374	1.75	3.06
3	5374	1.75	3.06
4	5390	17.75	315.06
5	5351	-21.25	451.56
6	5348	-24.25	588.06
7	5350	-22.25	495.06
8	5370	-2.25	5.06
9	5374	1.75	3.06
10	5390	17.75	315.06
11	5381	8.75	76.56
12	5374	1.75	3.06
13	5380	7.75	60.06
14	5375	2.75	7.56
15	5390	17.75	315.06
16	5370	-2.25	5.06
17	5359	-13.25	175.56
18	5370	-2.25	5.06
19	5370	-2.25	5.06
20	5375	2.75	7.56

$\overline{X}_B$ = 5372 meters

Σ $Range_B$ = 107,445

$\Sigma\ \Delta_B^2$ = 2899.70

s_B^2 = 152.62

s_B = 12.35

= 12 meters

TABLE A-2d

COMPARING TWO MEANS WITH VARIABILITY ASSUMED UNEQUAL

(Type B: Standard Item)

ROUND NUMBER	RANGE (meters)	Δ_B	Δ_B^2
1	5345	33.30	1108.89
2	5387	9.30	86.49
3	5385	7.30	53.29
4	5374	-3.70	13.69
5	5385	7.30	53.29
6	5388	10.30	106.09
7	5375	-2.70	7.29
8	5385	7.30	53.29
9	5379	1.30	1.69
10	5380	2.30	5.29

$\bar{X}_B = 5378.30$

$= 5378$ meters

$\Sigma \text{ Range}_B = 53{,}783$

$\Sigma\Delta_B^2 = 1489.30$

$s_B^2 = 165.48$

$s_B = 12.86$

$= 13$ meters

TABLE A-2e

COMPARING MEANS OF PAIRED OBSERVATIONS

CAPACITY OF BATTERIES
(ampere hours)

BATTERY A	BATTERY B	$x_d = x_A - x_B$	Δ	Δ^2
146.0	141.0	5.0	5.10	26.01
141.5	143.5	-2.0	-1.90	3.61
135.2	139.2	-4.0	-3.90	15.21
142.1	139.1	3.0	3.10	9.61
140.3	140.3	0.0	.10	.01
143.3	141.3	2.0	2.10	4.41
138.0	138.0	0.0	.10	.01
137.0	140.0	-3.0	-2.90	8.41
142.0	142.0	0.0	.10	.01
136.9	138.9	-2.0	-1.90	3.61

$N = 10$

$\Sigma\ x_d = -1$

$\overline{X}_d = .10$

$\Sigma\ \Delta^2 = 70.90$

$s_d^2 = 7.88$

$s_d = 2.81$

$= 3$

TABLE A-2f

COMPARING MEANS OF SEVERAL PRODUCTS

The following data is related to the life span of a resistor (in hours).

RESISTOR

	TYPE 1	TYPE 2	TYPE 3	TYPE 4
	518	502	554	555
	560	574	598	567
	538	528	579	550
	510	534	538	535
	544	538	544	540
Σx_i =	2670 hours	2682 hours	2813 hours	2747 hours
N_i =	5	5	5	5
$\bar{X}_i$ =	534.00	536.40	562.60	549.40
=	534	536	563	549
s_i^2 =	406.00	574.80	636.8	159.30
=	406 hours	575 hours	636 hours	159 hours

TABLE A-2g

COMPARING MEANS OF SEVERAL PRODUCTS

The following data is related to the range of a particular ammunition fired from various guns (meters).

	TYPE 1	TYPE 2	TYPE 3	TYPE 4
	5,120	5,000	5,581	5,130
	5,300	5,010	5,590	5,150
	5,285	5,032	5,580	5,205
	5,291	4,989	5,595	5,100
	5,202	5,025	5,598	5,125
	5,170		5,589	5,175
	5,188		5,580	5,150
			5,598	
			5,551	
Σx_i =	36,556 meters	25.056 meters	50,262 meters	36.035 meters
N_i =	7	5	9	7
$\bar{X}_i$ =	5,222.29	5,011.20	5584.67	5,147.86
=	5,222	5,011	5585	5,148
s_i^2 =	4,912.90	310.7	212.50	1190.48
=	4,913 meters	311 meters	212 meters	1190 meters

TABLE A-3a

STANDARD DEVIATION (TYPE A: Test Item)

READING (minutes)	$\Delta(\Delta_A)$	$\Delta^2(\Delta^2_A)$
100	-9.40	88.36
125	15.60	243.36
98	-11.40	129.96
100	-9.40	88.36
112	2.60	6.76
115	5.60	31.36
120	10.60	112.36
110	.60	.36
100	-9.40	88.36
114	4.60	21.16

For Type A item

$N = 10$	$N = 10$
$\bar{X} = 109.40$	$\bar{X}_A = 109.40$
$= 109$ min.	$= 109$ min.
$\Sigma\Delta^2 = 810.40$	$\Sigma\Delta^2_A = 810.40$
$s^2 = 90.04$	$s^2_A = 90.04$
$s = 9.49$	$s_A = 9.49$
$= 9$ min.	$= 9$ min.

TABLE A-3b

STANDARD DEVIATION (TYPE B: Standard Item)

READING (minutes)	Δ_B	Δ_B^2
86	-2.50	6.25
84	-4.50	20.25
93	4.50	20.25
85	-3.50	12.25
91	2.50	6.25
84	-4.50	20.25
90	1.50	2.25
92	3.50	12.25
85	-3.50	12.25
94	5.50	30.25
91	2.50	6.25
87	-1.50	2.25

$N = 12$

$\bar{X}_B = 88.5$

$= 88$ min.

$\Sigma\Delta_B^2 = 151.00$

$s_B^2 = 13.73$

$s_B = 3.71$

$= 4$ min.

TABLE A-4a

PART I

FUSE TYPE	SUCCESS (Class I)	FAILURE (Class II)	TOTAL
TYPE A	$I_A = 4$	$II_A = 2$	$N_A = 6$
TYPE B	$I_B = 8$	$II_B = 2$	$N_B = 10$
TOTAL	12	4	16

PART II

FUSE TYPE	SUCCESS (Class I)	FAILURE (Class II)	TOTAL
Larger Sample (Type B)	$I_1 = 8$	$II_1 = 2$	$N_1 = 10$
Smaller Sample (Type A)	$I_2 = 4$	$II_2 = 2$	$N_2 = 6$
TOTAL	12	4	16

TABLE A-4b

Given Characteristic: Proportion of Hits

FUZE TYPE	SUCCESS (Class I)	FAILURE (Class II)	TOTAL
Type A	$I_A = 181$	$II_A = 35$	$N_A = 216$
Type B	$I_B = 160$	$II_B = 56$	$N_B = 216$
TOTAL	$T_I = 341$	$T_{II} = 91$	$T_N = 432$

TABLE A-5a

AP	POINTS OF IMPACT	DIFFERENCE R(+) O(+) L(-) S(-)	POINTS OF IMPACT AROUND T E MEAN AP
(2500,3005)	(2350,3100)	(-150,95	(2005,2304)
(1537,1825)	(1649,2031)	(112,206)	(2267,2415)
(2041,2800)	(2175,2520)	(134,-280)	(2289,1929)
(3000,1945)	(2793,2275)	(-207,330)	(1948,2539)
(1874,1700)	(1954,1439)	(80,-261)	(2235,1948)
(1500,2734)	(1748,3088)	(248,354)	(2403,2563)
(2273,1679)	(2345,2310)	(72,631)	(2227,2840)
(1725,2600)	(1539,2415)	(-186,-185)	(1969,2024)
(2758,1503)	(2833,1100)	(75,-403)	(2230,1806)
(2340,2300)	(2094,2466)	(-246,166)	(1909,2375)

Mean AP: (2155,2209)

TABLE A-5b

PE: STANDARD DEVIATION

READING NUMBER	READING (meters)	Δ	Δ^2
1	1248	-10.00	100.0
2	1100	-158.00	24,964.0
3	1260	2.00	4.0
4	1300	42.00	1,764.0
5	1260	2.00	4.0
6	1234	-24.00	576.0
7	1287	29.00	841.0
8	1275	17.00	289.0
9	1290	32.00	1,024.0
10	1280	22.00	484.0
11	1225	-33.00	1,089.0
12	1325	67.00	4,489.0
13	1223	-35.00	1,225.0
14	1299	41.00	1,681.0
15	1268	10.00	100.0
16	1254	-4.00	16.0

$N = 16$

$\overline{X} = 1258.00$

$= 1258$ meters

$\Sigma\Delta^2 = 38,650.0$

$s^2 = 2,576.67$

$s = 50.76$

$= 51$ meters

TABLE A-5c

PE SUCCESSIVE DIFFERENCES

READING NUMBER	READING (meters)	d	d^2
1	1248		
2	1100	148	21,904
3	1260	-160	25,600
4	1300	-40	1,600
5	1260	40	1,600
6	1234	26	676
7	1287	-53	2,809
8	1275	12	144
9	1290	-15	225
10	1280	10	100
11	1225	55	3,025
12	1325	-100	10,000
13	1223	102	10,404
14	1299	-76	5,776
15	1268	31	961
16	1254	14	196

$N = 16$

$\Sigma d^2 = 85{,}020$

$s_d^2 = 5{,}668.00$

$s_d = 75.29$

$= 75$

TABLE A-5d

OUTLIER FOR PE

READING NUMBER	READING (meters)	Δ	Δ^2
1	1248	20.53	421
2	1100	Isolate as an outlier	
3	1260	-8.53	73
4	1300	31.47	990
5	1260	-8.53	73
6	1234	-34.53	1192
7	1287	18.47	341
8	1275	6.47	42
9	1290	21.47	461
10	1280	11.47	132
11	1225	-43.53	1895
12	1325	56.47	3189
13	1223	-45.53	2073
14	1299	30.47	928
15	1268	-.53	0
16	1254	-14.53	211

$N_1 = 15$

$\overline{X}_1 = 1268.53$

$= 1269$ meters

$\Sigma\Delta_1^2 = 12021$

$s_1^2 = 858.64$

$= 859$ meters

TABLE A-5e

CIRCULAR PROBABLE ERROR (CPE)

READING NUMBER	EASTING COORDINATE	ΔE	ΔE^2
1	48270	-104.47	10,914
2	48356	-18.47	341
3	47962	-412.47	170,132
4	48001	-373.47	139,480
5	48512	137.53	18,915
6	47570	-804.47	647,172
7	48830	455.53	207,508
8	48781	406.53	165,267
9	48329	-45.47	2,068
10	48659	284.53	80,957
11	48238	-136.47	18,642
12	48762	387.53	149,404
13	48325	-49.47	2,447
14	48515	140.53	19,749
15	48507	132.53	17,564

$N = 15$

$\overline{EAST} = 48374.47$

$= 48374$

$\Sigma \Delta E^2 = 1,650,542$

$s_E^2 = 117,895.9$

$s_E = 343.36$

$= 343$

TABLE A-5e

CPE continued

READING NUMBER	NORTHING COORDINATE	ΔN	ΔN^2	d_m
1	46530	-174.00	30,276	203
2	46516	-188.00	35.344	189
3	45378	-1326.00	1,758,276	1389
4	45971	733.00	537,289	822
5	46831	127.00	16,129	188
6	46972	268.00	71,824	847
7	47015	311.00	96,721	552
8	46505	-199.00	39,601	453
9	47230	526.00	276,676	528
10	46993	289.00	83,521	406
11	47020	316.00	99,856	344
12	47044	340.00	115,600	516
13	46845	141.00	19,881	149
14	46570	-134.00	17,956	194
15	47140	436.00	190,096	456

N = 15

$\overline{\text{NORTH}}$ = 46704.00
= 46704

$\Sigma \Delta N^2$ = 3,389,046

s_N^2 = 242,074.7

s_N = 492.01
= 492

TABLE A-5f

OUTLIERS FOR CPE

SUSPECTED OUTLIER	E	Δ	N	Δ	d_m
3	47962	441.93	45378	1420.71	1488.2

READING NUMBER	E	ΔE	ΔE^2
1	48270	-133.93	17,937
2	48356	-47.93	2,297
4	48001	-402.93	162,353
5	48512	108.07	11,679
6	47570	-833.93	695,439
7	48830	426.07	181,536
8	48781	377.07	142,182
9	48329	-74.93	5,615
10	48659	255.07	65,061
11	48238	-165.93	27,533
12	48762	358.07	128,214
13	48325	-78.97	6,230
14	48515	111.07	12,337
15	48507	103.07	10,623

$N_1 = 14$

$\overline{EAST} = 48403.93$

$= 48404$

$\Sigma \Delta E^2 = 1,469,036$

$s_E^2 = 113,002.8$

$s_E = 336.16$

$= 336$

TABLE A-5f continued

READING NUMBER	N	ΔN	$ΔN^2$
1	46530	-268.71	72,205
2	46516	-282.71	79,925
4	45971	-827.71	685,104
5	46831	32.29	1,043
6	46972	173.29	30,029
7	47015	216.29	46,781
8	46505	-293.71	86,266
9	47230	431.29	186,011
10	46993	194.29	37,749
11	47020	221.29	48,969
12	47044	245.29	60,167
13	46845	46.29	2,143
14	46570	-228.71	52,308
15	47140	341.29	116,479

$N_1 = 14$

$\overline{NORTH} = 46798.71$

$= 46799$

$\Sigma \Delta N^2 = 1,505,179$

$s_N^2 = 115,783.0$

$s_N = 340.27$

$= 340$

TABLE A-6a

SEQUENTIAL TESTING: SUCCESS - FAILURE

FAILURE	SAMPLES TESTED	COORDINATES
1	30	(30,1)
2	75	(75,2)
3	110	(110,3)
4	160	(160,4)

TABLE A-6b

SEQUENTIAL TEST: PROPORTION

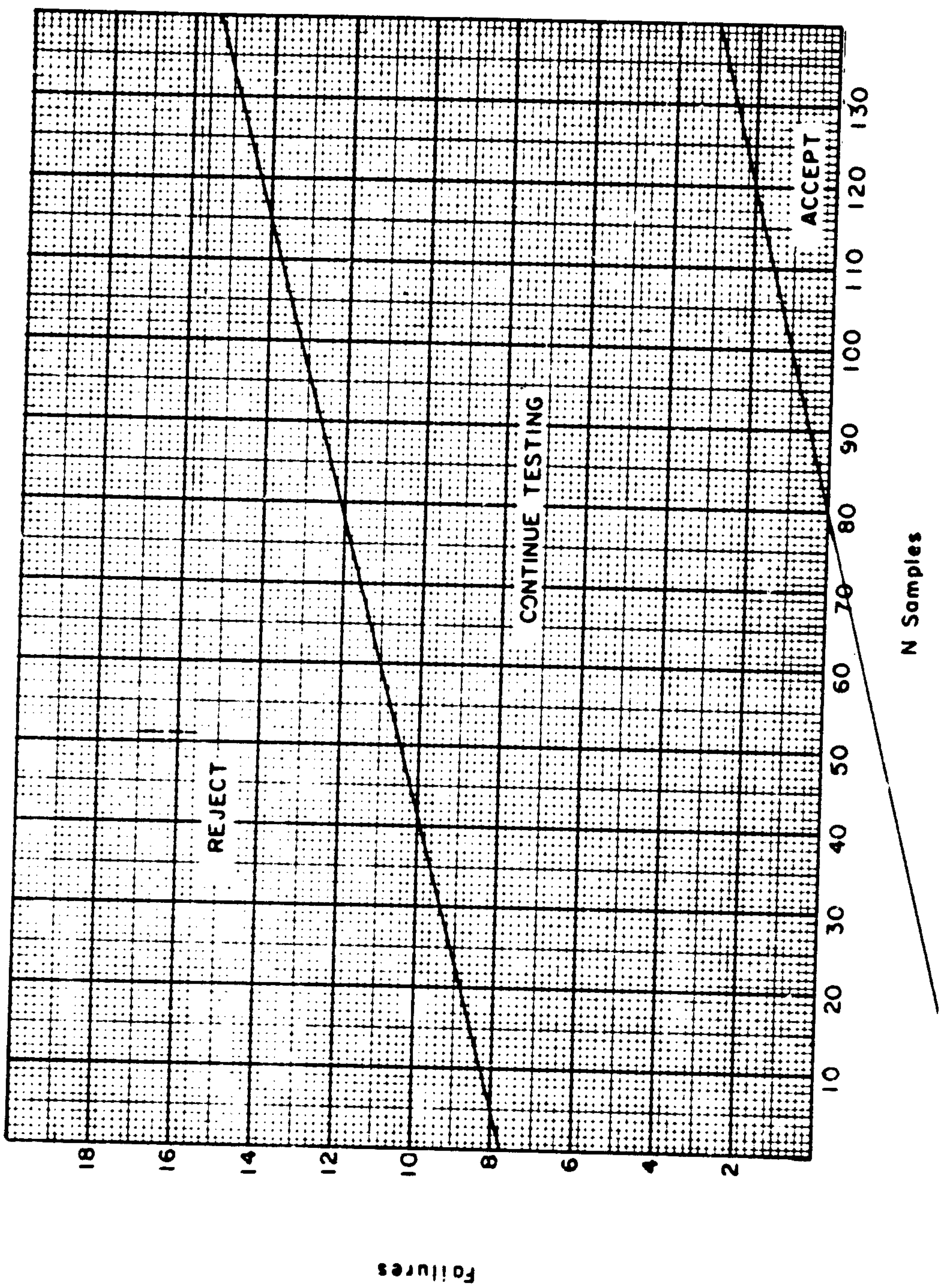


TABLE A-6c

FAILURES	OPERATING HOURS
1	360
2	275
3	320
4	311
5	285
6	290
7	318
8	314
9	340
10	298
11	300
12	310
	31*
	3752

* Since no failure corresponds to the 31 under Operating Hours, this is a time terminated test.

TABLE A-6d

FAILURES	T_t (hours)	HOURS BETWEEN FAILURES	Δ	Δ^2
1	200	200	-6.83	46.6
2	410	210	3.17	10.0
3	595	185	-21.83	476.5
4	816	221	14.17	200.8
5	1046	230	23.17	536.8
6	1241	195	-11.83	139.9

$F = 6$

$MTBF = 206.83$

$= 207$ hours

$\Sigma\Delta^2 = 1410.6$

$s^2 = 282.1$

$s = 16.80$

$= 17$ hours

TABLE A-6e

SEQUENTIAL TESTING: CONTINUOUS TEST

FAILURE NUMBER	OPERATING TIME (hours)	COORDINATE
1	175	(1,175)
2	490	(2,490)
3	985	(3,985)
4	1500	(4,1500)
5	2495	(5,2495)
6	3290	(6,3290)
7	4075	(7,4075)

TABLE A-6f

SEQUENTIAL TEST: CONTINUOUS TEST

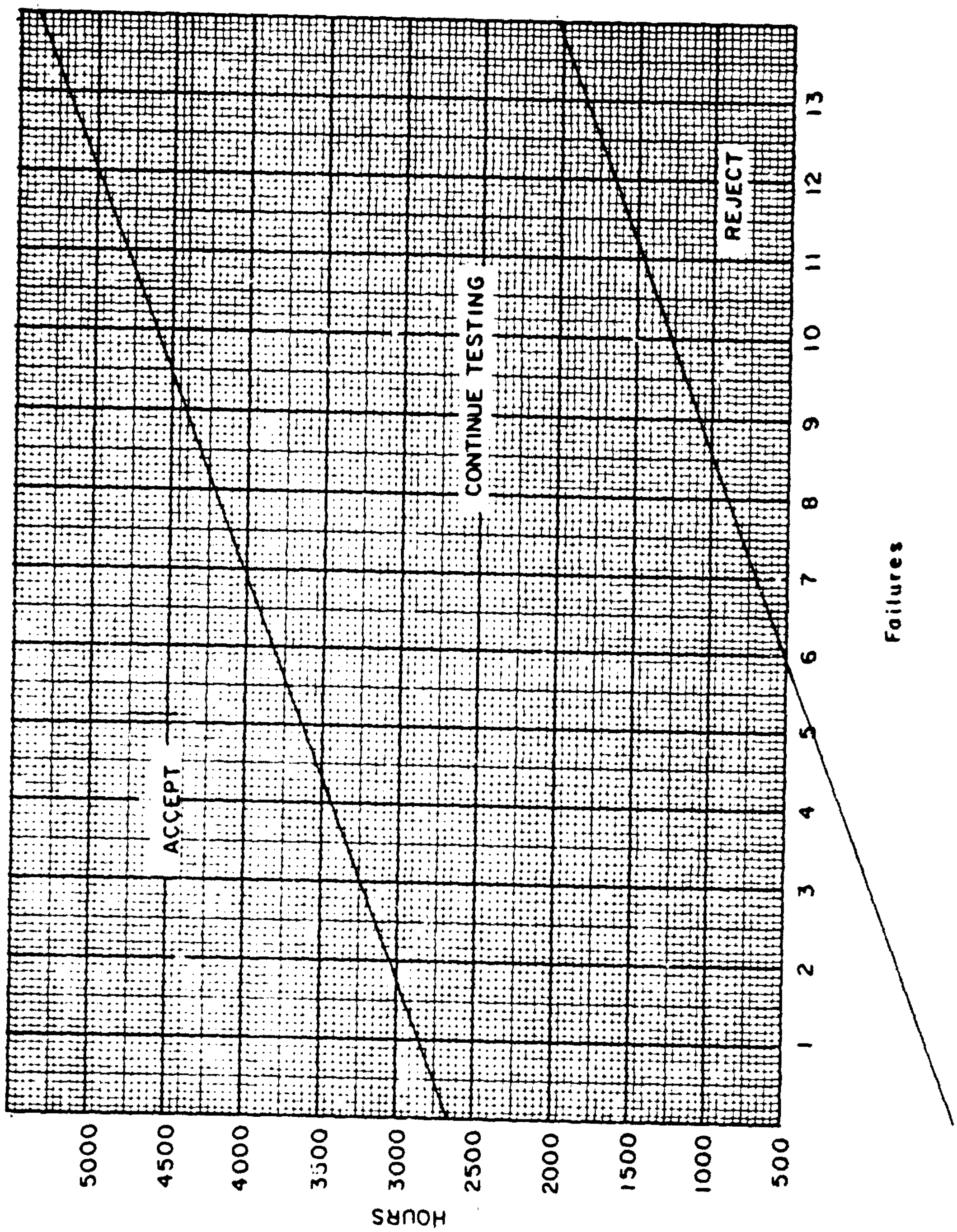


TABLE A-6g

COMBINED RELIABILITY

COMPONENT NUMBER	SAMPLE SIZE	FAILURES
1	90	2
2	90	4
3	45	1
4	45	3

TABLE A-7a

MAINTAINABILITY

MAINTENANCE ACTION	MAINTENANCE ACTION TIME
1	.6
2 Failure	1.2
3	.4
4	.9
5	2.3
6 Failure	2.0
7	1.4
8	.7
9	.8
10 Failure	.6
11	.1
12 Failure	.5
13	.4
14	.4
15	1.0
16	1.1
17 Failure	.5
18	.3
19	.3
20	.4
21	.6
22	.4

TABLE A-7b

DATE	3 MAR	4 MAR	5 MAR	6 MAR	7 MAR
1. Operating time	22.0	23.0	21.3	23.0	20.5
2. Active Maintenance*					
a. Time	1.50	.5	2.0	.5	2.3
b. Manhours	2.0	1.0	3.0	1.0	2.3
3. Number of MA	2	1	2	1	1
4. Failures	1	0	1	0	1
5. Time to Repair Failures	1.5	0.0	2.0	0.0	1.3
6. Delay time	0.0	0.0	7.0	0.0	1.8

Remarks: Date

5 Mar: Delay time - driver sick.
7 Mar: Delay time - supply delay.

*Includes preventive and corrective maintenance action.

INCLOSURE II

INCLOSURE II
STATISTICAL TABLES

TABLE B-1

RANGE

Range to estimate σ : k · Range

SAMPLE	k
2	.886
3	.591
4	.486
5	.430
6	.395
7	.370
8	.351
9	.337
10	.325

TABLE B-2

MEAN DEVIATION

Mean Deviation estimate of σ : c · mean deviation

SAMPLE SIZE	c
2	.886
3	.591
4	.377
5	.302
6	.237
7	.203
8	.172
9	.153
10	.135

TABLE B-3

NORMAL DISTRIBUTION

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4732	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4960	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Source: From THEORY AND PROBLEMS OF STATISTICS by Murray R. Spiegel.

TABLE B-4

CUMULATIVE NORMAL DISTRIBUTION - VALUES OF z_p

Values of z_p corresponding to P for the normal curve.
z is the standard normal variable

P	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.00	-	-2.33	-2.05	-1.88	-1.75	-1.64	-1.55	-1.48	-1.41	-1.34
.10	-1.28	-1.23	-1.18	-1.13	-1.08	-1.04	-0.99	-0.95	-0.92	-0.88
.20	-0.84	-0.81	-0.77	-0.74	-0.71	-0.67	-0.64	-0.61	-0.58	-0.55
.30	-0.52	-0.50	-0.47	-0.44	-0.41	-0.39	-0.36	-0.33	-0.31	-0.28
.40	-0.25	-0.23	-0.20	-0.18	-0.15	-0.13	-0.10	-0.08	-0.05	-0.03
.50	0.00	0.03	0.05	0.08	0.10	0.13	0.15	0.18	0.20	0.23
.60	0.25	0.28	0.31	0.33	0.36	0.39	0.41	0.44	0.47	0.50
.70	0.52	0.55	0.58	0.61	0.64	0.67	0.71	0.74	0.77	0.81
.80	0.84	0.88	0.92	0.95	0.99	1.04	1.08	1.13	1.18	1.23
.90	1.28	1.34	1.41	1.48	1.55	1.64	1.75	1.88	2.05	2.33

Special Values

P	.001	.005	.010	.025	.050	.100
z_p	-3.090	-2.576	-2.326	-1.960	-1.645	-1.282

P	.999	.995	.990	.975	.950	.900
z_p	3.090	2.576	2.326	1.960	1.645	1.282

TABLE B-5

PERCENTILES OF THE t DISTRIBUTION

df	$t_{.60}$	$t_{.70}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.875}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$
1	.325	.727	1.000	1.376	1.963	2.414	2.078	6.314	12.706	31.821	63.657
2	.289	.617	.816	1.061	1.386	1.604	1.886	2.920	4.303	6.965	9.925
3	.277	.584	.765	.978	1.250	1.423	1.638	2.353	3.182	4.541	5.841
4	.271	.569	.741	.941	1.190	1.344	1.533	[illegible]32	2.776	3.747	4.604
5	.267	.559	.727	.920	1.156	1.301	1.476	[illegible]5	2.571	3.365	4.032
6	.265	.553	.718	.906	1.134	1.273	1.440	1.943	2.447	3.143	3.707
7	.263	.549	.711	.896	1.119	1.254	1.415	1.895	2.365	2.998	3.499
8	.262	.546	.706	.889	1.108	1.240	1.397	1.860	2.306	2.896	3.355
9	.261	.543	.703	.883	1.100	1.230	1.383	1.833	2.262	2.821	3.250
10	.260	.542	.700	.879	1.093	1.221	1.372	1.812	2.228	2.764	3.1[illegible]
11	.260	.540	.697	.876	1.088	1.214	1.363	1.796	2.201	2.718	3.106
12	.259	.539	.695	.873	1.083	1.209	1.356	1.782	2.179	2.681	3.055
13	.259	.538	.694	.870	1.079	1.204	1.350	1.771	2.160	2.650	3.012
14	.258	.537	.692	.868	1.076	1.200	1.345	1.761	2.145	2.624	2.977
15	.258	.536	.691	.866	1.074	1.197	1.341	1.753	2.131	2.602	2.947
16	.258	.535	.690	.865	1.071	1.194	1.337	1.746	2.120	2.583	2.921
17	.257	.534	.689	.863	1.069	1.191	1.333	1.740	2.110	2.567	2.898
18	.257	.534	.688	.862	1.067	1.189	1.330	1.734	2.101	2.552	2.878
19	.257	.533	.688	.861	1.066	1.187	1.328	1.729	2.093	2.539	2.861
20	.257	.533	.687	.860	1.064	1.185	1.325	1.725	2.086	2.528	2.845
21	.257	.532	.686	.859	1.063	1.183	1.323	1.721	2.080	2.518	2.831
22	.256	.532	.686	.858	1.061	1.182	1.321	1.717	2.074	2.508	2.819
23	.256	.532	.685	.858	1.060	1.180	1.319	1.714	2.069	2.500	2.807
24	.256	.531	.685	.857	1.059	1.179	1.318	1.711	2.064	2.492	2.797
25	.256	.531	.684	.856	1.058	1.178	1.316	1.708	2.060	2.485	2.787
26	.256	.531	.684	.856	1.058	1.177	1.315	1.706	2.056	2.479	2.779
27	.256	.531	.684	.855	1.057	1.176	1.314	1.703	2.052	2.473	2.771
28	.256	.530	.683	.855	1.056	1.175	1.313	1.701	2.048	2.467	2.763
29	.256	.530	.683	.854	1.055	1.174	1.311	1.699	2.045	2.462	2.756
30	.256	.530	.683	.854	1.055	1.173	1.310	1.697	2.042	2.457	2.750
40	.255	.529	.681	.851	1.050	1.167	1.303	1.684	2.021	2.423	2.704
60	.254	.527	.679	.848	1.046	1.162	1.296	1.671	2.000	2.390	2.660
120	.254	.526	.677	.845	1.041	1.156	1.289	1.658	1.980	2.358	2.617
**	.253	.524	.674	.842	1.036	1.150	1.282	1.645	1.960	2.326	2.576

Linear interpolation may be used to obtain the t value for the d.f. value which is not in the table; however, the following formula is a more accurate method of obtaining the t value

$$t_{1-\alpha,d.f.} = t_{1-\alpha,d.f._u} + \left(\frac{1/d.f. - 1/d.f._u}{1/d.f._L - 1/d.f._u} \right) \left(t_{1-\alpha,d.f._L} - t_{1-\alpha,d.f._u} \right)$$

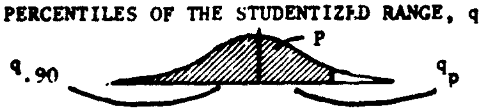
where d.f. = degrees of freedom not in table.

$d.f._u$ = upper value, degree of freedom just larger than d.f.

$d.f._L$ = lower value, degree of freedom just smaller than d.f.

TABLE B-6

PERCENTILES OF THE STUDENTIZED RANGE, q



d.f.1 / d.f.2	2	3	4	5	6	7	8	9	10
1	8.93	13.44	16.36	18.49	20.15	21.51	22.64	23.62	24.48
2	4.13	5.73	6.77	7.54	8.14	8.63	9.05	9.41	9.72
3	3.33	4.47	5.20	5.74	6.16	6.51	6.81	7.06	7.29
4	3.01	3.98	4.59	5.03	5.39	5.68	5.93	6.14	6.33
5	2.85	3.72	4.26	4.66	4.98	5.24	5.46	5.65	5.82
6	2.75	3.56	4.07	4.44	4.73	4.97	5.17	5.34	5.50
7	2.68	3.45	3.93	4.28	4.55	4.78	4.97	5.14	5.28
8	2.63	3.37	3.83	4.17	4.43	3.65	4.83	4.99	5.13
9	2.59	3.32	3.76	4.08	4.34	4.54	4.72	4.87	5.01
10	2.56	3.27	3.70	4.02	4.26	4.47	4.64	4.78	4.91
11	2.54	3.23	3.66	3.96	4.20	4.40	4.57	4.71	4.84
12	2.52	3.20	3.62	3.92	4.6	4.35	4.51	4.65	4.78
13	2.50	3.18	3.59	3.88	4.12	4.30	4.46	4.60	4.72
14	2.49	3.16	3.56	3.85	4.08	4.27	4.42	4.56	4.68
15	2.48	3.14	3.54	3.83	4.05	4.23	4.39	4.52	4.64
16	2.47	3.12	3.52	3.80	4.03	4.21	4.36	4.49	4.61
17	2.46	3.11	3.50	3.78	4.00	4.18	4.33	4.46	4.58
18	2.45	3.10	3.49	3.77	3.98	4.16	4.31	4.44	4.55
19	2.45	3.09	3.47	3.75	3.97	4.14	4.29	4.42	4.53
20	2.44	3.08	3.46	3.74	3.95	4.12	4.27	4.40	4.51
24	2.42	3.05	3.42	3.69	3.90	4.07	4.21	4.34	4.44
30	2.40	3.02	3.39	3.65	3.85	4.02	4.16	4.28	4.38
40	2.38	2.99	3.35	3.60	3.80	3.96	4.10	4.21	4.32
60	2.36	2.96	3.31	3.56	3.75	3.91	4.04	4.16	4.25
120	2.34	2.93	3.28	3.52	3.71	3.86	3.99	4.10	4.19
∞	2.33	2.90	3.24	3.48	3.66	3.81	3.93	4.04	4.13

TABLE B-6 continued

PERCENTILES OF THE STUDENTIZED RANGE, q

$q_{.90}$

d.f.₁ \ d.f.₂	11	12	13	14	15	16	17	18	19	20
1	25.24	25.92	26.54	27.10	27.62	28.10	28.54	28.96	29.35	29.71
2	10.01	10.26	10.49	10.70	10.89	11.07	11.24	11.39	11.54	11.68
3	7.49	7.67	7.83	7.98	8.12	8.25	8.37	8.48	8.58	8.68
4	6.49	6.65	6.78	6.91	7.02	7.13	7.23	7.33	7.41	7.50
5	5.97	6.10	6.22	6.34	6.44	6.54	6.63	6.71	6.79	6.86
6	5.64	5.76	5.87	5.98	6.07	6.16	6.25	6.32	6.40	6.47
7	5.41	5.53	5.64	5.74	5.83	5.91	5.99	6.06	6.13	6.19
8	5.25	5.36	5.46	5.56	5.64	5.72	5.80	5.87	5.93	6.00
9	5.13	5.23	5.33	5.42	5.51	5.58	5.66	5.72	5.79	5.85
10	5.03	5.13	5.23	5.32	5.40	5.47	5.54	5.61	5.67	5.73
11	4.95	5.05	5.15	5.23	5.31	5.38	5.45	5.51	5.57	5.63
12	4.89	4.99	5.08	5.16	5.24	5.31	5.37	5.44	5.49	5.55
13	4.83	4.93	5.02	5.10	5.18	5.25	5.31	5.37	5.43	5.48
14	4.79	4.88	4.97	5.05	5.12	5.19	5.26	5.32	5.37	5.43
15	4.75	4.84	4.93	5.01	5.08	5.15	5.21	5.27	5.32	5.38
16	4.71	4.81	4.89	4.97	5.04	5.11	5.17	5.23	5.28	5.33
17	4.68	4.77	4.86	4.93	5.01	5.07	5.13	5.19	5.24	5.30
18	4.65	4.75	4.83	4.90	4.98	5.04	5.10	5.16	5.21	5.26
19	4.63	4.72	4.80	4.88	4.95	5.01	5.07	5.13	5.18	5.23
20	4.61	4.70	4.78	4.85	4.92	4.99	5.05	5.10	5.16	5.20
24	4.54	4.63	4.71	4.78	4.85	4.91	4.97	5.02	5.07	5.12
30	4.47	4.56	4.64	4.71	4.77	4.83	4.89	4.94	4.99	5.03
40	4.41	4.49	4.56	4.63	4.69	4.75	4.81	4.86	4.90	4.95
60	4.34	4.42	4.49	4.56	4.62	4.67	4.73	4.78	4.82	4.86
120	4.28	4.35	4.42	4.48	4.54	4.60	4.65	4.69	4.74	4.78
∞	4.21	4.28	4.35	4.41	4.47	4.52	4.57	4.61	4.65	4.69

TABLE B-6 continued

PERCENTILES OF THE STUDENTIZED RANGE, q

$q_{.95}$

d.f.₁ / d.f.₂	2	3	4	5	6	7	8	9	10
1	17.97	26.98	32.82	37.08	40.41	43.12	45.40	47.36	49.07
2	6.08	8.33	9.80	10.88	11.74	12.44	13.03	13.54	13.99
3	4.50	5.91	6.82	7.50	8.04	8.48	8.85	9.18	9.46
4	3.93	5.04	5.76	6.29	6.71	7.05	7.35	7.60	7.83
5	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99
6	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49
7	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16
8	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92
9	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74
10	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60
11	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49
12	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39
13	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32
14	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25
15	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20
16	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15
17	2.98	3.63	4.02	4.30	4.52	4.70	4.86	4.99	5.11
18	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07
19	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04
20	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01
24	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92
30	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82
40	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73
60	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65
120	2.80	3.36	3.68	3.92	4.10	4.24	4.36	4.47	4.56
∞	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47

TABLE B-6 continued

PERCENTILES OF THE STUDENTIZED RANGE, q
q .95

d.f.$_1$ / d.f.$_2$	11	12	13	14	15	16	17	18	19	20
1	50.59	51.96	53.20	54.33	55.36	56.32	57.22	58.04	58.83	59.56
2	14.39	14.75	15.08	15.38	15.65	15.91	16.14	16.37	16.57	16.77
3	9.72	9.95	10.15	10.35	10.52	10.69	10.84	10.98	11.11	11.24
4	8.03	8.21	8.37	8.52	8.66	8.79	8.91	9.03	9.13	9.23
5	7.17	7.32	7.47	7.60	7.72	7.83	7.93	8.03	8.12	8.21
6	6.65	6.79	6.92	7.03	7.14	7.24	7.34	7.43	7.51	7.59
7	6.30	6.43	6.55	6.66	6.76	6.85	6.94	7.02	7.10	7.17
8	6.05	6.18	6.29	6.39	6.48	6.57	6.65	6.73	6.80	6.87
9	5.87	5.98	6.09	6.19	6.28	6.36	6.44	6.51	6.58	6.64
10	5.72	5.83	5.93	6.03	6.11	6.19	6.27	6.34	6.40	6.47
11	5.61	5.71	5.81	5.90	5.98	6.06	6.13	6.20	6.27	6.33
12	5.51	5.61	5.71	5.80	5.88	5.95	6.02	6.09	6.15	6.21
13	5.43	5.53	5.63	5.71	5.79	5.86	5.93	5.99	6.05	6.11
14	5.36	5.46	5.55	5.64	5.71	5.79	5.85	5.91	5.97	6.03
15	5.31	5.40	5.49	5.57	5.65	5.72	5.78	5.85	5.90	5.96
16	5.26	5.35	5.44	5.52	5.59	5.66	5.73	5.79	5.84	5.90
17	5.21	5.31	5.39	5.47	5.54	5.61	5.67	5.73	5.79	5.84
18	5.17	5.27	5.35	5.43	5.50	5.57	5.63	5.69	5.74	5.79
19	5.14	5.23	5.31	5.39	5.46	5.53	5.59	5.65	5.70	5.75
20	5.11	5.20	5.28	5.36	5.43	5.49	5.55	5.61	5.66	5.71
24	5.01	5.10	5.18	5.25	5.32	5.38	5.44	5.49	5.55	5.59
30	4.92	5.00	5.08	5.15	5.21	5.27	5.33	5.38	5.43	5.47
40	4.82	4.90	4.98	5.04	5.11	5.16	5.22	5.27	5.31	5.36
60	4.73	4.81	4.88	4.94	5.00	5.06	5.11	5.15	5.20	5.24
120	4.64	4.71	4.78	4.84	4.90	4.95	5.00	5.04	5.09	5.13
∞	4.55	4.62	4.68	4.74	4.80	4.85	4.89	4.93	4.97	5.01

TABLE B-6 continued

PERCENTILES OF THE STUDENTIZED RANGE, q

q.99

d.f.1 / d.f.2	2	3	4	5	6	7	8	9	10
1	90.03	135.0	164.3	185.6	202.2	215.8	227.2	237.0	245.6
2	14.04	19.02	22.29	24.72	26.63	28.20	29.53	30.68	31.69
3	8.26	10.62	12.17	13.33	14.24	15.00	15.64	16.20	16.69
4	6.51	8.12	9.17	9.96	10.58	11.10	11.55	11.93	12.27
5	5.70	6.98	7.80	8.42	8.91	9.32	9.67	9.97	10.24
6	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10
7	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37
8	4.75	5.64	6.20	6.62	6.96	7.24	7.47	7.68	7.86
9	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.33	7.49
10	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21
11	4.39	5.15	5.62	5.97	6.25	6.48	6.67	6.84	6.99
12	4.32	5.05	5.50	5.84	6.10	6.32	6.51	6.67	6.81
13	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67
14	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54
15	4.17	4.84	5.25	5.56	5.80	5.99	6.16	6.31	6.44
16	4.13	4.79	5.19	5.49	5.72	5.92	6.08	6.22	6.35
17	4.10	4.74	5.14	5.43	5.66	5.85	6.01	6.15	6.27
18	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20
19	4.05	4.67	5.05	5.33	5.55	5.73	5.89	6.02	6.14
20	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09
24	3.96	4.55	4.91	5.17	5.37	5.54	5.69	5.81	5.92
30	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76
40	3.82	4.37	4.70	4.93	5.11	5.26	5.39	5.50	5.60
60	3.76	4.28	4.59	4.82	4.99	5.13	5.25	5.36	5.45
120	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30
∞	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16

TABLE B-6 continued

PERCENTILES OF THE STUDENTIZED RANGE, q

$q_{.99}$

d.f.1 / d.f.2	11	12	13	14	15	16	17	18	19	20
1	253.2	260.0	266.2	271.8	277.0	281.8	286.3	290.4	294.3	298.0
2	32.59	33.40	34.13	34.81	35.43	36.00	36.53	37.03	37.50	37.95
3	17.13	17.53	17.89	18.22	18.52	18.81	19.07	19.32	19.55	19.77
4	12.57	12.84	13.09	13.32	13.53	13.73	13.91	14.08	14.24	14.40
5	10.48	10.70	10.89	11.08	11.24	11.40	11.55	11.68	11.81	11.93
6	9.30	9.48	9.65	9.81	9.95	10.08	10.21	10.32	10.43	10.54
7	8.55	8.71	8.86	9.00	9.12	9.24	9.35	9.46	9.55	9.65
8	8.03	8.18	8.31	8.44	8.55	8.66	8.76	8.85	8.94	9.03
9	7.65	7.78	7.91	8.03	8.13	8.23	8.33	8.41	8.49	8.57
10	7.36	7.49	7.60	7.71	7.81	7.91	7.99	8.08	8.15	8.23
11	7.13	7.25	7.36	7.46	7.56	7.65	7.73	7.81	7.88	7.95
12	6.94	7.06	7.17	7.26	7.36	7.44	7.52	7.59	7.66	7.73
13	6.79	6.90	7.01	7.10	7.19	7.27	7.35	7.42	7.48	7.55
14	6.66	6.77	6.87	6.96	7.05	7.13	7.20	7.27	7.33	7.39
15	6.55	6.66	6.76	6.84	6.93	7.00	7.07	7.14	7.20	7.26
16	6.46	6.56	6.66	6.74	6.82	6.90	6.97	7.03	7.09	7.15
17	6.38	6.48	6.57	6.66	6.73	6.81	6.87	6.94	7.00	7.05
18	6.31	6.41	6.50	6.58	6.65	6.73	6.79	6.85	6.91	6.97
19	6.25	6.34	6.43	6.51	6.58	6.65	6.72	6.78	6.84	6.89
20	6.19	6.28	6.37	6.45	6.52	6.59	6.65	6.71	6.77	6.82
24	6.02	6.11	6.19	6.26	6.33	6.39	6.45	6.51	6.56	6.61
30	5.85	5.93	6.01	6.08	6.14	6.20	6.26	6.31	6.36	6.41
40	5.69	5.76	5.83	5.90	5.96	6.02	6.07	6.12	6.16	6.21
60	5.53	5.60	5.67	5.73	5.78	5.84	5.89	5.93	5.97	6.01
120	5.37	5.44	5.50	5.56	5.61	5.66	5.71	5.75	5.79	5.83
∞	5.23	5.29	5.35	5.40	5.45	5.49	5.54	5.57	5.61	5.65

TABLE B-7

PERCENTILES OF THE χ^2 DISTRIBUTION

d.f. \ 1-α	.9995	.999	.995	.99	.975	.95	.90	.80	.75	.70	.60	.50
1	$.0^6 393$	$.0^5 157$	$.0^4 393$	$.0^3 157$	$.0^3 982$	$.0^2 393$	.0158	.0642	.102	.148	.275	.455
2	$.0^2 100$	$.0^2 20$	.0100	.0201	.0506	.103	.211	.446	.575	.713	1.02	1.39
3	.0153	.0243	.0717	.115	.216	.352	.584	1.00	1.21	1.42	1.87	2.37
4	.0639	.0908	.207	.297	.484	.711	1.06	1.65	1.92	2.19	2.75	3.36
5	.158	.210	.412	.554	.831	1.15	1.61	2.34	2.67	3.00	3.66	4.35
6	.299	.381	.676	.872	1.24	1.64	2.20	3.07	3.45	3.83	4.57	5.35
7	.485	.598	.989	1.24	1.69	2.17	2.83	3.82	4.25	4.67	5.49	6.35
8	.710	.857	1.34	1.65	2.18	2.73	3.49	4.59	5.07	5.53	6.42	7.34
9	.972	1.15	1.73	2.09	2.70	3.33	4.17	5.38	5.90	6.39	7.36	8.34
10	1.26	1.48	2.16	2.56	3.25	3.94	4.87	6.18	6.74	7.27	8.30	9.34
11	1.59	1.83	2.60	3.05	3.82	4.57	5.58	6.99	7.58	8.15	9.24	10.3
12	1.93	2.21	3.07	3.57	4.40	5.23	6.30	7.81	8.44	9.03	10.2	11.3
13	2.31	2.62	3.57	4.11	5.01	5.89	7.04	8.63	9.30	9.93	11.1	12.3
14	2.70	3.04	4.07	4.66	5.63	6.57	7.79	9.47	10.2	10.8	12.1	13.3
15	3.11	3.48	4.60	5.23	6.26	7.26	8.55	10.2	11.0	11.7	13.0	14.3
16	3.54	3.94	5.14	5.81	6.91	7.96	9.31	11.2	11.9	12.6	14.0	15.3
17	3.98	4.42	5.70	6.41	7.56	8.67	10.1	12.0	12.8	13.5	14.9	16.3
18	4.44	4.90	6.26	7.01	8.23	9.39	10.9	12.9	13.7	14.4	15.9	17.3
19	4.91	5.41	6.84	7.63	8.91	10.1	11.7	13.7	14.6	15.4	16.9	18.3
20	5.40	5.92	7.43	8.26	9.59	10.9	12.4	14.6	15.5	16.3	17.8	19.3
21	5.90	6.45	8.03	8.90	10.3	11.6	13.2	15.4	16.3	17.2	18.8	20.3
22	6.40	6.98	8.64	9.54	11.0	12.3	14.0	16.3	17.2	18.1	19.7	21.3
23	6.92	7.53	9.26	10.2	11.7	13.1	14.8	17.2	18.1	19.0	20.7	22.3
24	7.45	8.08	9.89	10.9	12.4	13.8	15.7	18.1	19.0	19.9	21.7	23.3
25	7.99	8.65	10.5	11.5	13.1	14.6	16.5	18.9	19.9	20.9	22.6	24.3
26	8.54	9.22	11.2	12.2	13.8	15.4	17.3	19.8	20.8	21.8	23.6	25.3
27	9.09	9.80	11.8	12.9	14.6	16.2	18.1	20.7	21.7	22.7	24.5	26.3
28	9.66	10.4	12.5	13.6	15.3	16.9	18.9	21.6	22.7	23.6	25.5	27.3
29	10.2	11.0	13.1	14.3	16.0	17.7	19.8	22.5	23.6	24.6	26.5	28.3
30	10.8	11.6	13.8	15.0	16.8	18.5	20.6	23.4	24.5	25.5	27.4	29.3
31	11.4	12.2	14.5	15.7	17.5	19.3	21.4	24.3	25.4	26.4	28.4	30.3
32	12.0	12.8	15.1	16.4	18.3	20.1	22.3	25.1	26.4	27.4	29.4	31.3
33	12.6	13.4	15.8	17.1	19.0	20.9	23.1	26.0	27.3	28.3	30.3	32.3
34	13.2	14.1	16.5	17.8	19.8	21.7	24.0	26.9	28.2	29.2	31.3	33.3
35	13.8	14.7	17.2	18.5	20.6	22.5	24.8	27.8	29.1	30.2	32.3	34.3
36	14.4	15.3	17.9	19.2	21.3	23.3	25.6	28.7	30.0	31.1	33.3	35.3
37	15.0	16.0	18.6	20.0	22.1	24.1	26.5	29.6	30.9	32.1	34.2	36.3
38	15.6	16.6	19.3	20.7	22.9	24.9	27.3	30.5	31.9	33.0	35.2	37.3
39	16.3	17.3	20.0	21.4	23.7	25.7	28.2	31.4	32.8	33.9	36.2	38.3
40	16.9	17.9	20.7	22.2	24.4	26.5	29.1	32.3	33.7	34.9	37.1	39.3

NOTE: $.0^4 393$ means .0000393.

TABLE B-7 continued

PERCENTILES OF THE χ^2 DISTRIBUTION

d.f. \ 1-α	.40	.30	0.25	.20	.10	.05	.025	.01	.005	.001	.0005
1	.708	1.07	1.32	1.64	2.71	3.84	5.02	6.63	7.88	10.8	12.1
2	1.83	2.41	2.77	3.22	4.61	5.99	7.38	9.21	10.6	13.8	15.2
3	2.95	3.67	4.11	4.64	6.25	7.81	9.35	11.3	12.8	16.3	17.7
4	4.04	4.88	5.39	5.99	7.78	9.49	11.1	13.3	14.9	18.5	20.0
5	5.13	6.06	6.63	7.29	9.24	11.1	12.8	15.1	16.7	20.5	22.1
6	6.21	7.23	7.84	8.56	10.6	12.6	14.4	16.8	18.5	22.5	24.1
7	7.28	8.38	9.04	9.80	12.0	14.1	16.0	18.5	20.3	24.3	26.0
8	8.35	9.52	10.2	11.0	13.4	15.5	17.5	20.1	22.0	26.1	27.9
9	9.41	10.7	11.4	12.2	14.7	16.9	19.0	21.7	23.6	27.9	29.7
10	10.5	11.8	12.5	13.4	16.0	18.3	20.5	23.2	25.2	29.6	31.4
11	11.5	12.9	13.7	14.6	17.3	19.7	21.9	24.7	26.8	31.3	33.1
12	12.6	14.0	14.8	15.8	18.5	21.0	23.3	26.2	28.3	32.9	34.8
13	13.6	15.1	16.0	17.0	19.8	22.4	24.7	27.7	29.8	34.5	36.5
14	14.7	16.2	17.1	18.2	21.1	23.7	26.1	29.1	31.3	36.1	38.1
15	15.7	17.3	18.2	19.3	22.3	25.0	27.5	30.6	32.8	37.7	39.7
16	16.8	18.4	19.4	20.5	23.5	26.3	28.8	32.0	34.3	39.3	41.3
17	17.8	19.5	20.5	21.6	24.8	27.6	30.2	33.4	35.7	40.8	42.9
18	18.9	20.6	21.6	22.8	26.0	28.9	31.5	34.8	37.2	42.3	44.4
19	19.9	21.7	22.7	23.9	27.2	30.1	32.9	36.2	38.6	43.8	46.0
20	21.0	22.8	23.8	25.0	28.4	31.4	34.2	37.6	40.0	45.3	47.5
21	22.0	23.9	24.9	26.2	29.6	32.7	35.5	38.9	41.4	46.8	49.0
22	23.0	24.9	26.0	27.3	30.8	33.9	36.8	40.3	42.8	48.3	50.5
23	24.1	26.0	27.1	28.4	32.0	35.2	38.1	41.6	44.2	49.7	52.0
24	25.1	27.1	28.2	29.6	33.2	36.4	39.4	43.0	45.6	51.2	53.5
25	26.1	28.2	29.3	30.7	34.4	37.7	40.6	44.3	46.9	52.6	54.9
26	27.2	29.2	30.4	31.8	35.6	38.9	41.9	45.6	48.3	54.1	56.4
27	28.2	30.3	31.5	32.9	36.7	40.1	43.2	47.0	49.6	55.5	57.9
28	29.2	31.4	32.6	34.0	37.9	41.3	44.5	48.3	51.0	56.9	59.3
29	30.3	32.5	33.7	35.1	39.1	42.6	45.7	49.6	52.3	58.3	60.7
30	31.3	33.5	34.8	36.3	40.3	43.8	47.0	50.9	53.7	59.7	62.2
31	32.3	34.6	35.9	37.4	41.4	45.0	48.2	52.2	55.0	61.1	63.6
32	33.4	35.7	37.0	38.5	42.6	46.2	49.5	53.5	56.3	62.5	65.0
33	34.4	36.7	38.1	39.6	43.7	47.4	50.7	54.8	57.6	63.9	66.4
34	35.4	37.8	39.2	40.7	44.9	48.6	52.0	56.1	59.0	65.2	67.8
35	36.5	38.9	40.2	41.8	46.1	49.8	53.2	57.3	60.3	66.6	69.2
36	37.5	39.9	41.3	42.9	47.2	51.0	54.4	58.6	61.6	68.0	70.6
37	38.5	41.0	42.4	44.0	48.4	52.2	55.7	59.9	62.9	69.3	72.0
38	39.6	42.0	43.5	45.1	49.5	53.4	56.9	61.2	64.2	70.7	73.4
39	40.6	43.1	44.6	46.2	50.7	54.6	58.1	62.4	65.5	72.1	74.7
40	41.6	44.2	45.6	47.3	51.8	55.8	59.3	63.7	66.8	73.4	76.1

TABLE B-7 continued

PERCENTILES OF THE χ^2 DISTRIBUTION

d.f. \ 1-α	.9995	.999	.995	.99	.975	.95	.90	.80	.75	.70	.60	.50
41	17.5	18.6	21.4	22.9	25.2	27.3	29.9	33.3	34.6	35.8	38.1	40.3
42	18.2	19.2	22.1	23.7	26.0	28.1	30.8	34.2	35.6	36.8	39.1	41.3
43	18.8	19.9	22.9	24.4	26.8	29.0	31.6	35.1	36.5	37.7	40.0	42.3
44	19.5	20.6	23.6	25.1	27.6	29.8	32.5	36.0	37.4	38.6	41.0	43.3
45	20.1	21.3	24.3	25.9	28.4	30.6	33.4	36.9	38.4	39.6	42.0	44.3
46	20.8	21.9	25.0	26.7	29.2	31.4	34.2	37.8	39.3	40.3	43.0	45.3
47	21.5	22.6	25.8	27.4	30.0	32.3	35.1	38.7	40.2	41.5	43.9	46.3
48	22.1	23.3	26.5	28.2	30.8	33.1	35.9	39.6	41.1	42.4	44.9	47.3
49	22.8	24.0	27.2	28.9	31.6	33.9	36.8	40.5	42.1	43.4	45.9	48.3
50	23.5	24.7	28.0	29.7	32.4	34.8	37.7	41.4	43.0	44.3	46.9	49.3
51	24.1	25.4	28.7	30.5	33.2	35.6	38.6	42.4	43.9	45.3	47.8	50.3
52	24.8	26.1	29.5	31.2	34.0	36.4	39.4	43.3	44.9	46.2	48.8	51.3
53	25.5	26.8	30.2	32.0	34.8	37.3	40.3	44.2	45.8	47.2	49.8	52.3
54	26.2	27.5	31.0	32.8	35.6	38.1	41.2	45.1	46.7	48.1	50.8	53.3
55	26.9	28.2	31.7	33.6	36.4	[illegible]	42.1	46.0	47.7	49.1	51.7	54.3
56	27.6	28.9	32.5	34.3	37.2	39.8	42.9	47.0	48.6	50.0	52.7	55.3
57	28.2	29.6	33.2	35.1	38.0	40.6	43.8	47.9	49.6	51.0	53.7	56.3
58	28.9	30.3	34.0	35.9	38.8	41.5	44.7	48.8	50.5	51.9	54.7	57.3
59	29.6	31.0	34.8	36.7	39.7	42.3	45.6	49.7	51.4	52.9	55.6	58.3
60	30.3	31.7	35.5	37.5	40.5	43.2	46.5	50.6	52.4	53.8	56.6	59.3
61	31.0	32.5	36.3	38.3	41.3	44.0	47.3	51.6	53.3	54.8	57.6	60.3
62	31.7	33.2	37.1	39.1	42.1	44.9	48.2	52.5	54.2	55.7	58.6	61.3
63	32.5	33.9	37.8	39.9	43.0	45.7	49.1	53.4	55.2	56.7	59.6	62.3
64	33.2	34.6	38.6	40.6	43.8	46.6	50.0	54.3	56.1	57.6	60.5	63.3
65	33.9	35.4	39.4	41.4	44.6	47.4	50.9	55.3	57.1	58.6	61.5	64.3
66	34.6	36.1	40.2	42.2	45.4	48.3	51.8	56.2	58.0	59.5	62.5	65.3
67	35.3	36.8	40.9	43.0	46.3	49.2	52.7	57.1	58.9	60.5	63.5	66.3
68	36.0	37.6	41.7	43.8	47.1	50.0	53.5	58.0	59.9	61.4	64.4	67.3
69	36.7	38.3	42.5	44.6	47.9	50.9	54.4	59.0	60.8	62.4	65.4	68.3
70	37.5	39.0	43.3	45.4	48.8	51.9	55.3	59.9	61.8	63.3	66.4	69.3
71	38.2	39.8	44.1	46.2	49.6	52.6	56.2	60.8	62.7	64.3	67.4	70.3
72	38.9	40.5	44.8	47.1	50.4	53.5	57.1	61.8	63.7	65.3	68.4	71.3
73	39.6	41.3	45.6	47.9	51.3	54.3	58.0	62.7	64.6	66.2	69.2	72.3
74	40.4	42.0	46.4	48.7	52.1	55.2	58.9	63.6	65.6	67.2	70.3	73.3
75	41.1	42.8	47.2	49.5	52.9	56.1	59.8	64.5	66.5	68.1	71.3	74.3
76	41.8	43.5	48.0	50.3	53.3	56.9	60.7	65.5	67.4	69.1	72.3	75.3
77	42.6	44.3	48.8	51.1	54.6	57.8	61.6	66.4	68.4	70.0	73.2	76.3
78	43.3	45.0	49.6	51.9	55.5	58.7	62.5	67.3	69.3	71.0	74.2	77.3
79	44.1	45.8	50.4	52.7	56.3	59.5	63.4	68.3	70.3	72.0	75.2	78.3
80	44.8	46.5	51.2	53.5	57.2	60.4	64.3	69.2	71.2	72.9	76.2	79.3

TABLE B-7 continued

PERCENTILES OR THE χ^2 DISTRIBUTION

d.f. \ 1-α	.40	.30	.025	.20	.10	.05	.025	.01	.005	.001	.0005
41	42.7	45.2	46.7	48.4	52.9	56.9	60.6	65.0	68.1	74.7	77.5
42	43.7	46.3	47.8	49.5	54.1	58.1	61.8	66.2	69.3	76.1	78.8
43	44.7	47.3	48.9	50.5	55.2	59.3	63.0	67.5	70.6	77.4	80.2
44	45.7	48.4	49.9	51.6	56.4	60.5	64.2	68.7	71.9	78.7	81.5
45	46.8	49.5	51.0	52.7	57.5	61.7	65.4	70.0	73.2	80.1	82.9
46	47.8	50.5	52.1	53.8	58.6	62.8	66.6	71.2	74.4	81.4	84.2
47	48.8	51.6	53.1	54.9	59.8	64.0	67.8	72.4	75.7	82.7	85.6
48	49.9	52.6	54.2	56.0	60.9	65.2	69.0	73.7	77.0	84.0	86.9
49	50.9	53.7	55.3	57.1	62.0	66.3	70.2	74.9	78.2	85.4	88.2
50	51.9	54.7	56.3	58.2	63.2	67.5	71.4	76.2	79.5	86.7	89.6
51	52.9	55.8	57.4	59.2	64.3	68.7	72.6	77.4	80.7	88.0	90.9
52	53.9	56.8	58.5	60.3	65.4	69.8	73.8	78.6	82.0	89.3	92.2
53	55.0	57.9	59.5	61.4	66.5	71.0	75.0	79.8	83.3	90.6	93.5
54	56.0	58.9	60.6	62.5	67.7	72.2	76.2	81.1	84.5	91.9	94.8
55	57.0	60.0	61.7	63.6	68.8	73.3	77.4	82.3	85.7	93.2	96.2
56	58.0	61.0	62.7	64.7	69.9	74.5	78.6	83.5	87.0	94.5	97.5
57	59.1	62.1	63.8	65.7	71.0	75.6	79.8	84.7	88.2	95.8	98.8
58	60.1	63.1	64.9	66.8	72.2	76.8	80.9	86.0	89.5	97.0	100.1
59	61.1	64.2	65.9	67.9	73.3	77.9	82.1	87.2	90.7	98.3	101.4
60	62.1	65.2	67.0	69.0	74.4	79.1	83.3	88.4	92.0	99.6	102.7
61	63.2	66.3	68.0	70.0	75.5	80.2	84.5	89.6	93.2	100.9	104.0
62	64.2	67.3	69.1	71.1	76.6	81.4	85.7	90.8	94.4	102.2	105.3
63	65.2	68.4	70.2	72.2	77.7	82.5	86.8	92.0	95.6	103.4	106.6
64	66.2	69.4	71.2	73.3	78.9	83.7	88.0	93.2	96.9	104.7	107.9
65	67.2	70.5	72.3	74.4	80.0	84.8	89.2	94.4	98.1	106.0	109.2
66	68.3	71.5	73.3	75.4	81.1	86.0	90.3	95.6	99.3	107.3	110.5
67	69.3	72.6	74.4	76.5	82.2	87.1	91.5	96.8	100.6	108.5	111.7
68	70.3	73.6	75.5	77.6	83.3	88.3	92.7	98.0	101.8	109.8	113.0
69	71.3	74.6	76.5	78.6	84.4	89.4	93.9	99.2	103.0	111.1	114.3
70	72.4	75.7	77.6	79.7	85.5	90.5	95.0	100.4	104.2	112.3	115.6
71	73.4	76.7	78.6	80.8	86.6	91.7	96.2	101.6	105.4	113.6	116.9
72	74.4	77.8	79.7	81.9	87.7	92.8	97.4	102.8	106.6	114.8	118.1
73	75.4	78.8	80.7	82.9	88.8	93.9	98.5	104.0	107.9	116.1	119.4
74	76.4	79.9	81.8	84.0	90.0	95.1	99.7	105.2	109.1	117.3	120.7
75	77.5	80.9	82.9	85.1	91.1	96.2	100.8	106.4	110.3	118.6	121.9
76	78.5	82.0	83.9	96.1	92.2	97.4	102.0	107.6	111.5	119.9	123.2
77	79.5	83.0	85.0	87.2	93.3	98.5	103.2	108.8	112.7	121.1	124.5
78	80.5	84.0	86.0	88.3	94.4	99.6	104.3	110.0	113.9	122.3	125.7
79	81.5	85.1	87.1	89.3	95.5	100.7	105.5	111.1	115.1	123.6	127.0
80	82.6	86.1	88.1	90.4	96.6	101.9	106.6	112.3	116.3	124.8	128.3

TABLE B-7 continued

PERCENTILES OF THE χ^2 DISTRIBUTION

d.f. \ 1-α	.9995	.999	.995	.99	.975	.95	.90	.80	.75	.70	.60	.50
81	45.5	47.3	52.0	54.4	58.0	61.3	65.2	70.1	72.2	73.9	77.2	80.3
82	46.3	48.0	52.8	55.2	58.8	62.1	66.1	71.1	73.1	74.8	78.1	81.3
83	47.0	48.8	53.6	56.0	59.7	63.0	67.0	72.0	74.1	75.8	79.1	82.3
84	47.8	49.6	54.4	56.8	60.5	63.9	67.9	72.9	75.0	76.8	80.1	83.3
85	48.5	50.3	55.2	57.6	61.4	64.7	68.8	73.9	76.0	77.7	81.1	84.3
86	49.3	51.1	56.0	58.5	62.2	65.6	69.7	74.8	76.9	78.7	82.1	85.3
87	50.0	51.9	56.8	59.3	63.1	66.5	70.6	75.7	77.9	79.6	83.0	86.3
88	50.8	52.6	57.6	60.1	63.9	67.4	71.5	76.7	78.8	80.6	84.0	87.3
89	51.5	53.4	58.4	60.9	64.8	68.2	72.4	77.6	79.8	81.6	85.0	88.3
90	52.3	54.2	59.2	61.8	65.6	69.1	73.3	78.6	80.7	82.5	86.0	89.3
91	53.0	54.9	60.0	62.6	66.5	70.0	74.2	79.5	81.7	83.5	87.0	90.3
92	53.8	55.7	60.8	63.4	67.4	70.9	75.1	80.4	82.6	84.4	88.0	91.3
93	54.5	56.5	61.6	64.2	68.2	71.8	76.0	81.4	83.6	85.4	88.9	92.3
94	55.3	57.2	62.4	65.1	69.1	72.6	76.9	82.3	84.5	86.4	89.9	93.3
95	56.1	58.0	63.2	65.9	69.9	73.5	77.8	83.2	85.5	87.3	90.9	94.3
96	56.8	58.8	64.1	66.7	70.8	74.4	78.7	84.2	86.4	88.3	91.9	95.3
97	57.6	59.6	64.9	67.6	71.6	75.3	79.6	85.1	87.4	89.2	92.9	96.3
98	58.4	60.4	65.7	68.4	72.5	76.2	80.5	86.1	88.3	90.2	93.8	97.3
99	59.1	61.1	66.5	69.2	73.4	77.0	81.4	87.0	89.3	91.2	94.8	98.3
100	59.9	61.6	67.3	70.1	74.2	77.9	82.4	87.9	90.2	92.1	95.8	99.3

TABLE B-7 continued

PERCENTILES OF THE χ^2 DISTRIBUTION

d.f. \ 1-α	.40	.30	.25	.20	.10	.05	.025	.01	.005	.001	.0005
81	83.6	87.2	89.2	91.5	97.7	103.0	107.8	113.5	117.5	126.1	129.5
82	84.6	38.2	90.2	92.5	98.9	104.1	108.9	114.7	118.7	127.3	130.8
83	85.6	89.2	91.3	93.6	99.9	105.3	110.1	115.9	119.9	128.6	132.0
84	86.6	90.3	92.3	94.7	101.0	106.4	111.2	117.1	121.1	129.8	133.3
85	87.7	91.3	93.4	95.7	102.1	107.5	112.4	118.2	122.3	131.0	134.5
86	88.7	92.4	94.4	96.8	103.2	108.6	113.5	119.4	123.5	132.3	135.8
87	89.7	93.4	95.5	97.9	104.3	109.8	114.7	120.6	124.7	133.5	137.0
88	90.7	94.4	96.5	98.9	105.4	110.9	115.8	121.8	125.9	134.7	138.3
89	91.7	95.5	97.6	100.0	106.5	112.0	117.0	122.9	127.1	136.0	139.5
90	92.8	96.5	98.6	101.1	107.6	113.1	118.1	124.1	128.3	137.2	140.8
91	93.8	97.6	99.7	102.1	108.7	114.3	119.3	125.3	129.5	138.4	142.0
92	94.8	98.6	100.7	103.2	109.8	115.4	120.4	126.5	130.7	139.7	143.3
93	95.9	99.6	101.8	104.2	110.9	116.5	121.6	127.6	131.9	140.9	144.5
94	96.8	100.7	102.8	105.3	111.9	117.6	122.7	128.8	133.1	142.1	145.8
95	97.9	101.7	103.9	106.4	113.0	118.8	123.9	130.0	134.2	143.3	147.0
96	98.9	102.8	104.9	107.4	114.1	119.9	125.0	131.1	135.4	144.6	148.2
97	99.9	103.8	106.0	108.5	115.2	121.0	126.1	132.3	136.6	145.8	149.5
98	100.9	104.8	107.0	109.5	116.3	122.1	127.3	133.5	137.8	147.0	150.7
99	101.9	105.9	108.1	110.6	117.4	123.2	128.4	134.6	139.0	148.2	151.9
100	102.9	106.9	109.1	111.7	118.5	124.3	129.6	135.8	140.2	129.4	153.2

For larger degrees of freedom:

$$\chi^2_{1-\alpha} = \frac{1}{2}\left(Z_\alpha + \sqrt{2(d.f. -1)}\right)^2$$

approximately, where d.f. = degrees of freedom and Z_α is given in Table B-4.

TABLE 8-8*

PERCENTILES OF THE F DISTRIBUTION

$F_{.50}$

d.f.1 / d.f.2	1	2	3	4	5	6	7	8	9	10	12	15
1	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03	2.04	2.07	2.09
2	.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33	1.34	1.36	1.38
3	.585	.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17	1.18	1.20	1.21
4	.549	.828	1941	1.00	1.04	1.06	1.08	1.09	1.10	1.11	1.13	1.14
5	.528	.799	.907	.965	1.00	1.02	1.04	1.05	1.06	1.07	1.09	1.10
6	.515	.780	.886	.942	.977	1.00	1.02	1.03	1.04	1.05	1.06	1.07
7	.506	.767	.871	.926	.960	.983	1.00	1.01	1.02	1.03	1.04	1.05
8	.499	.757	.860	.915	.948	.971	.988	1.00	1.01	1.02	1.03	1.04
9	.494	.749	.852	.906	.939	.962	.978	.990	1.00	1.01	1.02	1.03
10	.490	.743	.845	.899	.932	.954	.971	.983	.992	1.000	1.010	1.021
11	.486	.739	.840	.893	.926	.948	.964	.977	.986	.994	1.004	1.015
12	.484	.735	.835	.888	.921	.943	.959	.972	.981	.989	1.000	1.010
13	.482	.733	.832	.884	.917	.939	.958	.969	.978	.985	.995	1.006
14	.480	.730	.829	.881	.914	.936	.955	.966	.974	.981	.992	1.003
15	.478	.726	.826	.878	.911	.933	.948	.960	.970	.977	.989	1.000
16	.477	.724	.824	.876	.908	.930	.945	.959	.969	.975	.986	.997
17	.476	.723	.822	.874	.906	.928	.942	.958	.966	.973	.984	.994
18	.474	.722	.820	.872	.904	.926	.940	.956	.964	.971	.981	.992
19	.473	.720	.818	.870	.902	.924	.939	.953	.962	.969	.980	.990
20	.472	.718	.816	.868	.900	.922	.938	.950	.959	.966	.977	.989
21	.471	.717	.815	.866	.899	.921	.935	.949	.958	.966	.976	.987
22	.470	.716	.814	.865	.897	.919	.934	.948	.957	.965	.975	.986
23	.469	.715	.813	.864	.895	.918	.933	.946	.956	.963	.974	.984
24	.469	.714	.812	.863	.895	.917	.932	.944	.953	.961	.972	.983
25	.469	.713	.811	.862	.894	.916	.931	.943	.952	.961	.971	.982
26	.468	.712	.810	.861	.893	.915	.930	.942	.951	.960	.970	.981
27	.468	.711	.809	.860	.892	.914	.929	.941	.950	.959	.970	.980
28	.467	.710	.809	.859	.891	.914	.928	.940	.950	.958	.969	.979
29	.466	.709	.808	.859	.891	.913	.927	.940	.949	.958	.968	.978
30	.466	.709	.807	.858	.890	.912	.927	.939	.948	.955	.966	.978
40	.463	.705	.802	.854	.885	.907	.922	.934	.943	.950	.961	.972
50	.462	.703	.800	.851	.883	.904	.918	.931	.940	.946	.959	.969
60	.461	.701	.798	.849	.880	.901	.917	.928	.937	.945	.956	.967
70	-	-	-	.848	.879	.900	.915	.927	.936	.945	.955	.965
80	-	-	-	.847	.878	.899	.914	.926	.935	.944	.954	.964
90	-	-	-	.946	.877	.898	.913	.925	.934	.943	.953	.963
100	-	-	-	.845	.876	.897	.912	.924	.933	.942	.952	.962
120	.458	.697	.793	.844	.875	.896	.912	.923	.932	.939	.950	.961
500	.455	.693	.789	.839	.870	.891	.907	.919	.928	.937	.947	.958

*See Note on page 2-34.

TABLE 2-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.50}$

$d.f._2$ \ $d.f._1$	1	2	3	4	5	6	7	8	9	10	12	15
1	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.41	9.49
2	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.41
3	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.46
4	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
5	1.69	1.85	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
6	1.62	1.76	1.79	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.76
7	1.57	1.70	1.72	1.71	1.71	1.71	1.70	1.70	1.69	1.69	1.68	1.68
8	1.54	1.66	1.66	1.66	1.66	1.65	1.64	1.64	1.64	1.63	1.62	1.62
9	1.51	1.62	1.63	1.62	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.57
10	1.49	1.60	1.59	1.59	1.59	1.58	1.57	1.56	1.56	1.551	1.542	1.532
11	1.47	1.58	1.57	1.56	1.56	1.55	1.54	1.53	1.53	1.523	1.513	1.503
12	1.46	1.56	1.55	1.54	1.54	1.53	1.52	1.51	1.51	1.500	1.490	1.479
13	1.45	1.55	1.54	1.53	1.52	1.51	1.50	1.50	1.49	1.481	1.470	1.458
14	1.44	1.53	1.53	1.52	1.50	1.49	1.48	1.47	1.47	1.464	1.453	1.441
15	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.430	1.438	1.426
16	1.43	1.52	1.51	1.51	1.48	1.47	1.47	1.45	1.45	1.438	1.426	1.413
17	1.42	1.51	1.50	1.50	1.47	1.46	1.46	1.45	1.44	1.427	1.415	1.401
18	1.41	1.51	1.50	1.49	1.46	1.46	1.45	1.44	1.43	1.417	1.405	1.391
19	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.409	1.396	1.382
20	1.40	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.401	1.388	1.373
21	1.40	1.49	1.48	1.46	1.45	1.43	1.43	1.42	1.40	1.394	1.381	1.366
22	1.40	1.48	1.47	1.46	1.44	1.43	1.42	1.41	1.39	1.388	1.374	1.359
23	1.39	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.383	1.368	1.353
24	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.377	1.363	1.348
25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.373	1.358	1.342
26	1.39	1.47	1.45	1.43	1.43	1.41	1.40	1.39	1.38	1.368	1.354	1.338
27	1.38	1.46	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.364	1.349	1.333
28	1.38	1.46	1.44	1.42	1.42	1.40	1.39	1.38	1.37	1.360	1.345	1.329
29	1.38	1.45	1.44	1.42	1.41	1.39	1.39	1.37	1.36	1.357	1.342	1.325
30	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.354	1.338	1.322
40	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.330	1.314	1.296
50	1.35	1.43	1.42	1.39	1.38	1.36	1.35	1.34	1.33	1.316	1.299	1.281
60	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.306	1.289	1.270
70	1.35	1.41	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.300	1.282	1.263
80	1.35	1.41	1.40	1.38	1.37	1.34	1.32	1.31	1.30	1.295	1.277	1.257
90	1.34	1.40	1.40	1.37	1.36	1.34	1.32	1.31	1.30	1.291	1.273	1.253
100	1.34	1.40	1.39	1.37	1.36	1.33	1.31	1.30	1.29	1.288	1.270	1.250
120	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.283	1.265	1.244
500	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.266	1.246	1.225

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.75}$

d.f.₂ \ d.f.₁	1	2	3	4	5	6	7	8	9	10	12	15
1	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.41	9.49
2	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.41
3	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.46
4	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08
5	1.69	1.85	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89
6	1.62	1.76	1.79	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.76
7	1.57	1.70	1.72	1.71	1.71	1.71	1.70	1.70	1.69	1.69	1.68	1.68
8	1.54	1.66	1.66	1.66	1.66	1.65	1.64	1.64	1.64	1.63	1.62	1.62
9	1.51	1.62	1.63	1.62	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.57
10	1.49	1.60	1.59	1.59	1.59	1.58	1.57	1.56	1.56	1.551	1.542	1.532
11	1.47	1.53	1.57	1.56	1.56	1.55	1.54	1.53	1.53	1.523	1.513	1.503
12	1.46	1.56	1.55	1.54	1.54	1.53	1.52	1.51	1.51	1.500	1.490	1.479
13	1.45	1.55	1.54	1.53	1.52	1.51	1.50	1.50	1.49	1.481	1.470	1.458
14	1.44	1.53	1.53	1.52	1.50	1.49	1.48	1.47	1.47	1.464	1.453	1.441
15	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.450	1.438	1.426
16	1.43	1.52	1.51	1.51	1.48	1.47	1.47	1.45	1.45	1.438	1.426	1.413
17	1.42	1.51	1.50	1.50	1.47	1.46	1.46	1.45	1.44	1.427	1.415	1.401
18	1.41	1.51	1.50	1.49	1.46	1.46	1.45	1.44	1.43	1.417	1.405	1.391
19	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.409	1.396	1.382
20	1.40	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.401	1.388	1.373
21	1.40	1.49	1.48	1.46	1.45	1.43	1.43	1.42	1.40	1.394	1.381	1.366
22	1.40	1.48	1.47	1.46	1.44	1.43	1.42	1.41	1.39	1.388	1.374	1.359
23	1.39	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.383	1.368	1.353
24	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.377	1.363	1.348
25	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.373	1.358	1.342
26	1.39	1.47	1.45	1.43	1.43	1.41	1.40	1.39	1.38	1.368	1.354	1.338
27	1.38	1.46	1.45	1.43	1.42	1.40	1.39	1.38	1.37	1.364	1.349	1.333
28	1.38	1.46	1.44	1.42	1.42	1.40	1.39	1.38	1.37	1.360	1.345	1.329
29	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.357	1.342	1.325
30	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.354	1.338	1.322
40	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.330	1.314	1.296
50	1.35	1.43	1.42	1.39	1.38	1.36	1.35	1.34	1.33	1.316	1.299	1.281
60	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.306	1.289	1.270
70	1.35	1.41	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.300	1.282	1.263
80	1.35	1.41	1.40	1.38	1.37	1.34	1.32	1.31	1.30	1.295	1.277	1.257
90	1.34	1.40	1.40	1.37	1.36	1.34	1.32	1.31	1.30	1.291	1.273	1.253
100	1.34	1.40	1.39	1.37	1.36	1.33	1.31	1.30	1.29	1.288	1.270	1.250
120	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.283	1.265	1.244
500	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.266	1.246	1.225

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.75}$

d.f.₂ \ d.f.₁	20	25	30	40	50	60	70	80	90	100	120	500
1	9.58	9.63	9.67	9.71	9.74	9.76	9.76	9.77	9.78	9.78	9.80	9.85
2	3.43	3.43	3.44	3.45	3.45	3.46	3.46	3.46	3.47	3.47	3.47	3.48
3	2.46	2.46	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47	2.47
4	2.09	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08
5	1.88	1.88	1.88	1.88	1.88	1.87	1.87	1.87	1.87	1.87	1.87	1.87
6	1.76	1.75	1.75	1.75	1.75	1.74	1.74	1.74	1.74	1.74	1.74	1.74
7	1.67	1.67	1.66	1.66	1.66	1.65	1.65	1.65	1.65	1.65	1.65	1.65
8	1.61	1.60	1.60	1.59	1.59	1.59	1.59	1.58	1.58	1.58	1.58	1.58
9	1.56	1.56	1.55	1.55	1.54	1.54	1.54	1.53	1.53	1.53	1.53	1.53
10	1.522	1.516	1.512	1.506	1.502	1.500	1.498	1.497	1.496	1.495	1.493	1.488
11	1.492	1.485	1.480	1.474	1.470	1.467	1.465	1.463	1.462	1.461	1.460	1.454
12	1.467	1.459	1.454	1.447	1.443	1.440	1.438	1.436	1.435	1.434	1.432	1.454
13	1.446	1.438	1.432	1.425	1.420	1.417	1.414	1.413	1.411	1.410	1.408	1.402
14	1.428	1.419	1.413	1.405	1.400	1.397	1.394	1.393	1.391	1.390	1.388	1.381
15	1.412	1.403	1.397	1.389	1.383	1.380	1.377	1.375	1.374	1.372	1.370	1.363
16	1.398	1.389	1.382	1.374	1.368	1.365	1.362	1.360	1.358	1.357	1.355	1.347
17	1.386	1.377	1.370	1.361	1.355	1.351	1.348	1.346	1.344	1.343	1.341	1.333
18	1.376	1.366	1.359	1.349	1.343	1.339	1.336	1.334	1.332	1.331	1.329	1.320
19	1.366	1.356	1.349	1.339	1.333	1.329	1.326	1.323	1.321	1.320	1.318	1.308
20	1.357	1.347	1.340	1.330	1.323	1.319	1.316	1.313	1.311	1.310	1.307	1.298
21	1.350	1.339	1.331	1.321	1.315	1.310	1.307	1.304	1.302	1.301	1.298	1.289
22	1.343	1.332	1.324	1.313	1.307	1.302	1.299	1.296	1.294	1.293	1.290	1.280
23	1.336	1.325	1.317	1.306	1.300	1.295	1.291	1.289	1.287	1.285	1.282	1.272
24	1.330	1.319	1.311	1.300	1.293	1.288	1.285	1.282	1.280	1.278	1.275	1.265
25	1.325	1.313	1.305	1.294	1.287	1.282	1.278	1.275	1.273	1.271	1.269	1.258
26	1.320	1.308	1.300	1.288	1.281	1.276	1.272	1.270	1.267	1.265	1.263	1.251
27	1.315	1.303	1.295	1.283	1.276	1.271	1.267	1.264	1.262	1.260	1.257	1.245
28	1.311	1.299	1.290	1.279	1.271	1.266	1.262	1.259	1.257	1.255	1.252	1.240
29	1.307	1.295	1.286	1.274	1.266	1.261	1.257	1.254	1.252	1.250	1.247	1.235
30	1.303	1.291	1.282	1.270	1.262	1.257	1.253	1.250	1.247	1.245	1.242	1.230
40	1.276	1.262	1.252	1.239	1.230	1.224	1.220	1.216	1.213	1.211	1.208	1.193
50	1.259	1.245	1.235	1.220	1.211	1.204	1.199	1.196	1.192	1.190	1.186	1.170
60	1.248	1.233	1.223	1.208	1.198	1.191	1.185	1.181	1.178	1.175	1.171	1.153
70	1.240	1.225	1.214	1.198	1.188	1.181	1.175	1.171	1.167	1.164	1.160	1.141
80	1.235	1.219	1.207	1.191	1.181	1.173	1.167	1.163	1.159	1.156	1.152	1.131
90	1.230	1.214	1.202	1.186	1.175	1.167	1.161	1.157	1.153	1.150	1.145	1.124
100	1.226	1.210	1.198	1.182	1.170	1.162	1.156	1.152	1.148	1.144	1.139	1.117
120	1.221	1.204	1.192	1.175	1.163	1.155	1.149	1.144	1.140	1.136	1.131	1.107
500	1.199	1.181	1.168	1.149	1.136	1.126	1.118	1.112	1.107	1.103	1.096	1.062

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.80}$

d.f.₂ \ d.f.₁	2	3	4	5	6	7	8	9	10	12	15
2	3.75	3.26	3.31	3.36	3.41	3.45	3.46	3.50	3.52	3.55	3.58
3	3.60	2.92	2.87	2.87	2.88	2.89	2.90	2.90	2.91	2.92	2.92
4	3.24	2.55	2.48	2.46	2.45	2.45	2.45	2.45	2.44	2.44	2.44
5	3.03	2.35	2.26	2.23	2.21	2.20	2.19	2.19	2.19	2.18	2.17
6	2.90	2.22	2.12	2.08	2.06	2.05	2.04	2.03	2.03	2.02	2.01
7	2.80	2.13	2.03	1.98	1.96	1.94	1.93	1.92	1.92	1.90	1.89
8	2.73	2.06	1.96	1.91	1.89	1.87	1.86	1.85	1.84	1.82	1.81
9	2.68	2.02	1.91	1.86	1.83	1.81	1.80	1.79	1.78	1.76	1.75
10	2.64	1.98	1.87	1.82	1.79	1.77	1.75	1.74	1.73	1.72	1.72
11	2.60	1.95	1.83	1.79	1.76	1.73	1.72	1.70	1.69	1.69	1.69
12	2.58	1.92	1.81	1.76	1.73	1.71	1.69	1.68	1.66	1.65	1.65
13	2.55	1.90	1.78	1.73	1.70	1.68	1.66	1.64	1.63	1.62	1.60
14	2.53	1.88	1.76	1.71	1.69	1.66	1.64	1.62	1.61	1.59	1.57
15	2.52	1.87	1.75	1.70	1.68	1.64	1.62	1.61	1.60	1.58	1.56
16	2.50	1.85	1.74	1.68	1.65	1.62	1.60	1.59	1.58	1.56	1.54
17	2.49	1.84	1.72	1.67	1.63	1.61	1.59	1.58	1.56	1.54	1.52
18	2.48	1.83	1.71	1.66	1.62	1.60	1.58	1.56	1.55	1.53	1.51
19	2.47	1.82	1.70	1.65	1.61	1.59	1.57	1.55	1.54	1.52	1.50
20	2.46	1.82	1.70	1.64	1.61	1.58	1.56	1.55	1.53	1.51	1.49
21	2.45	1.81	1.69	1.63	1.60	1.57	1.55	1.53	1.52	1.50	1.48
22	2.44	1.80	1.68	1.62	1.59	1.56	1.54	1.53	1.51	1.49	1.47
23	2.44	1.79	1.67	1.62	1.58	1.56	1.53	1.52	1.50	1.48	1.46
24	2.43	1.79	1.67	1.61	1.58	1.55	1.53	1.52	1.50	1.48	1.46
25	2.43	1.78	1.66	1.61	1.57	1.54	1.52	1.51	1.49	1.47	1.44
26	2.42	1.78	1.66	1.60	1.56	1.54	1.52	1.50	1.49	1.46	1.44
27	2.42	1.77	1.65	1.60	1.56	1.53	1.51	1.49	1.48	1.46	1.43
28	2.41	1.77	1.65	1.59	1.56	1.53	1.51	1.49	1.48	1.45	1.43
29	2.41	1.77	1.65	1.59	1.55	1.53	1.50	1.49	1.47	1.45	1.42
30	2.41	1.77	1.64	1.59	1.55	1.52	1.50	1.48	1.47	1.45	1.42
40	2.38	1.74	1.62	1.56	1.53	1.50	1.47	1.46	1.44	1.42	1.39
50	2.36	1.73	1.60	1.54	1.50	1.47	1.45	1.43	1.42	1.39	1.36
60	2.36	1.72	1.60	1.54	1.50	1.47	1.44	1.43	1.41	1.38	1.36
70	2.34	1.71	1.59	1.53	1.49	1.46	1.43	1.41	1.40	1.37	1.34
80	2.34	1.70	1.58	1.52	1.48	1.45	1.43	1.41	1.39	1.36	1.33
90	2.33	1.70	1.58	1.52	1.48	1.45	1.42	1.40	1.39	1.36	1.33
100	2.33	1.70	1.57	1.51	1.47	1.44	1.42	1.40	1.38	1.35	1.33
120	2.33	1.70	1.57	1.51	1.47	1.44	1.42	1.40	1.38	1.35	1.32
500	2.31	1.68	1.56	1.49	1.45	1.42	1.40	1.37	1.36	1.33	1.30

TABLE 2-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.80}$

d.f.₂ \ d.f.₁	20	25	30	40	50	60	70	80	90	100	120	500
2	3.60	3.61	3.63	3.65	3.66	3.66	3.66	3.66	3.67	3.68	3.68	3.69
3	2.93	2.93	2.94	2.94	2.95	2.95	2.95	2.95	2.95	2.95	2.96	2.96
4	2.44	2.44	2.44	2.44	2.44	2.44	2.44	2.43	2.43	2.43	2.43	2.43
5	2.17	2.16	2.16	2.16	2.16	2.16	2.15	2.15	2.15	2.15	2.15	2.15
6	2.00	1.99	1.99	1.98	1.98	1.98	1.97	1.97	1.97	1.97	1.97	1.97
7	1.88	1.87	1.87	1.86	1.86	1.86	1.85	1.85	1.85	1.85	1.85	1.84
8	1.80	1.78	1.78	1.77	1.77	1.77	1.76	1.76	1.76	1.76	1.76	1.75
9	1.73	1.72	1.72	1.71	1.70	1.70	1.69	1.69	1.69	1.69	1.69	1.68
10	1.682	1.671	1.664	1.654	1.648	1.644	1.641	1.639	1.637	1.636	1.634	1.626
11	1.642	1.630	1.622	1.612	1.605	1.601	1.598	1.595	1.593	1.592	1.590	1.581
12	1.608	1.596	1.588	1.577	1.570	1.565	1.562	1.559	1.557	1.555	1.553	1.543
13	1.580	1.568	1.559	1.547	1.540	1.535	1.531	1.528	1.526	1.525	1.522	1.512
14	1.557	1.543	1.534	1.522	1.514	1.509	1.505	1.502	1.500	1.498	1.495	1.485
15	1.536	1.522	1.513	1.500	1.492	1.487	1.483	1.480	1.477	1.475	1.472	1.461
16	1.518	1.504	1.494	1.481	1.473	1.467	1.463	1.460	1.457	1.455	1.452	1.440
17	1.503	1.488	1.477	1.464	1.455	1.450	1.445	1.442	1.439	1.437	1.434	1.422
18	1.489	1.474	1.463	1.449	1.440	1.434	1.430	1.426	1.424	1.422	1.418	1.406
19	1.476	1.461	1.450	1.436	1.427	1.420	1.416	1.412	1.410	1.407	1.404	1.391
20	1.465	1.449	1.438	1.423	1.414	1.408	1.403	1.400	1.397	1.395	1.391	1.377
21	1.455	1.439	1.427	1.413	1.403	1.397	1.392	1.388	1.385	1.383	1.379	1.365
22	1.446	1.429	1.418	1.403	1.393	1.386	1.381	1.378	1.375	1.372	1.369	1.354
23	1.437	1.421	1.409	1.393	1.384	1.377	1.372	1.368	1.365	1.362	1.359	1.344
24	1.430	1.413	1.401	1.385	1.375	1.368	1.363	1.359	1.356	1.353	1.350	1.334
25	1.423	1.406	1.393	1.377	1.367	1.360	1.355	1.351	1.348	1.345	1.341	1.326
26	1.416	1.399	1.387	1.370	1.360	1.353	1.347	1.343	1.340	1.337	1.333	1.318
27	1.410	1.393	1.380	1.364	1.353	1.346	1.340	1.336	1.333	1.330	1.326	1.310
28	1.405	1.387	1.374	1.357	1.343	1.339	1.334	1.330	1.326	1.324	1.319	1.303
29	1.400	1.382	1.369	1.352	1.341	1.333	1.328	1.323	1.320	1.317	1.313	1.296
30	1.395	1.377	1.364	1.346	1.335	1.328	1.322	1.318	1.314	1.311	1.307	1.290
40	1.360	1.340	1.326	1.307	1.295	1.286	1.280	1.275	1.271	1.268	1.263	1.24[illegible]
50	1.339	1.318	1.303	1.283	1.270	1.261	1.254	1.249	1.245	1.241	1.236	1.214
60	1.324	1.303	1.288	1.267	1.253	1.244	1.236	1.231	1.226	1.222	1.217	1.193
70	1.314	1.293	1.277	1.255	1.241	1.231	1.223	1.218	1.213	1.209	1.203	1.177
80	1.307	1.285	1.269	1.246	1.232	1.221	1.214	1.207	1.203	1.198	1.192	1.165
90	1.301	1.278	1.262	1.240	1.225	1.214	1.206	1.199	1.194	1.190	1.184	1.156
100	1.296	1.273	1.257	1.234	1.219	1.208	1.200	1.193	1.188	1.183	1.177	1.148
120	1.289	1.266	1.249	1.225	1.210	1.199	1.190	1.183	1.178	1.173	1.166	1.135
500	1.262	1.237	1.219	1.193	1.175	1.162	1.152	1.143	1.137	1.131	1.122	1.078

TABLES B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.85}$

d.f.₂ \ d.f.₁	2	3	4	5	6	7	8	9	10	12	15
2	5.52	4.84	4.91	5.00	5.06	5.12	5.16	5.20	5.22	5.26	5.31
3	4.72	3.83	3.76	3.75	3.75	3.76	3.77	3.77	3.77	3.78	3.79
4	4.10	3.21	3.10	3.06	3.04	3.03	3.03	3.02	3.02	3.01	3.00
5	3.75	2.88	2.75	2.70	2.67	2.65	2.64	2.63	2.62	2.61	2.60
6	3.54	2.67	2.54	2.49	2.45	2.43	2.41	2.40	2.39	2.37	2.35
7	3.39	2.55	2.41	2.34	2.30	2.28	2.26	2.24	2.23	2.21	2.19
8	3.29	2.46	2.31	2.24	2.20	2.17	2.15	2.13	2.12	2.10	2.08
9	3.21	2.39	2.24	2.17	2.12	2.09	2.07	2.05	2.03	2.01	1.99
10	3.15	2.33	2.18	2.11	2.06	2.03	2.01	1.99	1.971	1.946	1.921
11	3.10	2.29	2.14	2.06	2.02	1.98	1.96	1.94	1.919	1.894	1.866
12	3.06	2.26	2.10	2.02	1.98	1.94	1.92	1.89	1.877	1.851	1.824
13	3.03	2.23	2.07	1.99	1.94	1.91	1.88	1.86	1.842	1.814	1.788
14	3.00	2.20	2.04	1.97	1.92	1.88	1.85	1.83	1.813	1.786	1.755
15	2.98	2.18	2.02	1.94	1.89	1.86	1.83	1.81	1.787	1.759	1.729
16	2.96	2.16	2.00	1.92	1.87	1.84	1.81	1.78	1.776	1.759	1.705
17	2.94	2.15	1.99	1.91	1.86	1.82	1.79	1.77	1.747	1.717	1.685
18	2.93	2.13	1.97	1.89	1.84	1.80	1.77	1.75	1.729	1.700	1.668
19	2.91	2.12	1.96	1.88	1.83	1.79	1.76	1.73	1.716	1.685	1.652
20	2.90	2.11	1.95	1.87	1.81	1.78	1.75	1.72	1.702	1.670	1.637
21	2.89	2.10	1.94	1.86	1.80	1.76	1.73	1.71	1.690	1.658	1.625
22	2.88	2.09	1.93	1.85	1.79	1.75	1.72	1.70	1.678	1.647	1.612
23	2.87	2.08	1.92	1.84	1.78	1.74	1.71	1.69	1.668	1.637	1.603
24	2.86	2.08	1.91	1.83	1.78	1.74	1.71	1.68	1.660	1.627	1.593
25	2.85	2.07	1.90	1.82	1.77	1.73	1.70	1.67	1.652	1.619	1.584
26	2.85	2.06	1.90	1.81	1.76	1.72	1.69	1.67	1.643	1.611	1.576
27	2.84	2.06	1.89	1.81	1.75	1.71	1.68	1.66	1.637	1.603	1.568
28	2.83	2.05	1.89	1.80	1.75	1.71	1.68	1.65	1.630	1.596	1.562
29	2.83	2.05	1.88	1.80	1.74	1.70	1.67	1.65	1.624	1.590	1.555
30	2.82	2.04	1.88	1.79	1.74	1.70	1.67	1.64	1.619	1.585	1.549
40	2.79	2.01	1.84	1.76	1.70	1.66	1.63	1.60	1.577	1.543	1.505
50	2.76	1.99	1.82	1.74	1.68	1.64	1.60	1.58	1.554	1.518	1.479
60	2.75	1.98	1.81	1.72	1.67	1.62	1.59	1.56	1.538	1.502	1.462
70	2.74	1.97	1.80	1.71	1.65	1.61	1.58	1.55	1.526	1.490	1.450
80	2.73	1.96	1.79	1.71	1.65	1.60	1.57	1.54	1.518	1.481	1.440
90	2.72	1.95	1.79	1.70	1.64	1.60	1.56	1.54	1.511	1.474	1.433
100	2.72	1.95	1.78	1.70	1.64	1.59	1.56	1.53	1.506	1.469	1.427
120	2.72	1.94	1.78	1.69	1.63	1.59	1.55	1.52	1.499	1.460	1.419
500	2.67	1.92	1.75	1.66	1.60	1.56	1.52	1.49	1.469	1.430	1.386

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.85}$

d.f.₂ \ d.f.₁	20	25	30	40	50	60	70	80	90	100	120	500
2	5.35	5.38	5.39	5.41	5.42	5.44	5.44	5.45	5.45	5.45	5.46	5.47
3	3.80	3.80	3.80	3.81	3.81	3.81	3.81	3.81	3.81	3.81	3.82	3.82
4	3.00	2.99	2.99	2.99	2.99	2.98	2.98	2.98	2.98	2.98	2.98	2.98
5	2.58	2.58	2.57	2.56	2.56	2.55	2.55	2.55	2.55	2.55	2.54	2.54
6	2.33	2.32	2.32	2.31	2.30	2.30	2.30	2.29	2.29	2.29	2.29	2.28
7	2.17	2.16	2.15	2.14	2.13	2.13	2.12	2.12	2.12	2.11	2.11	2.10
8	2.05	2.04	2.03	2.01	2.01	2.00	2.00	2.00	1.99	1.99	1.99	1.98
9	1.96	1.95	1.94	1.92	1.91	1.91	1.90	1.90	1.90	1.90	1.89	1.88
10	1.894	1.877	1.866	1.851	1.842	1.836	1.331	1.827	1.825	1.823	1.820	1.807
11	1.838	1.822	1.809	1.793	1.784	1.777	1.771	1.768	1.766	1.764	1.761	1.747
12	1.793	1.775	1.762	1.745	1.736	1.729	1.724	1.721	1.717	1.714	1.710	1.697
13	1.755	1.736	1.724	1.707	1.695	1.688	1.683	1.680	1.676	1.673	1.570	1.655
14	1.724	1.704	1.690	1.675	1.661	1.653	1.648	1.645	1.642	1.638	1.635	1.619
15	1.697	1.676	1.661	1.643	1.632	1.624	1.619	1.614	1.611	1.608	1.604	1.588
16	1.673	1.652	1.637	1.619	1.606	1.598	1.592	1.588	1.584	1.582	1.577	1.560
17	1.652	1.630	1.616	1.596	1.584	1.576	1.569	1.565	1.562	1.558	1.554	1.537
18	1.633	1.611	1.596	1.576	1.563	1.555	1.549	1.544	1.540	1.537	1.532	1.515
19	1.617	1.595	1.579	1.558	1.546	1.537	1.531	1.526	1.521	1.518	1.514	1.496
20	1.603	1.579	1.563	1.543	1.529	1.520	1.514	1.509	1.505	1.502	1.497	1.478
21	1.588	1.566	1.549	1.528	1.515	1.506	1.499	1.494	1.490	1.487	1.482	1.462
22	1.577	1.554	1.537	1.515	1.502	1.493	1.485	1.481	1.476	1.474	1.468	1.447
23	1.566	1.541	1.525	1.503	1.490	1.479	1.474	1.468	1.463	1.460	1.454	1.434
24	1.555	1.532	1.514	1.493	1.478	1.469	1.462	1.456	1.452	1.449	1.443	1.423
25	1.546	1.521	1.505	1.482	1.468	1.459	1.452	1.446	1.441	1.437	1.433	1.411
26	1.538	1.512	1.496	1.474	1.459	1.449	1.441	1.436	1.431	1.427	1.423	1.400
27	1.529	1.505	1.487	1.465	1.450	1.440	1.433	1.427	1.421	1.419	1.413	1.390
28	1.523	1.497	1.479	1.456	1.441	1.431	1.424	1.419	1.413	1.410	1.404	1.382
29	1.515	1.490	1.472	1.449	1.434	1.424	1.416	1.410	1.406	1.402	1.396	1.373
30	1.509	1.484	1.466	1.441	1.427	1.416	1.409	1.403	1.397	1.395	1.388	1.366
40	1.463	1.437	1.417	1.392	1.375	1.363	1.355	1.348	1.343	1.339	1.332	1.306
50	1.436	1.407	1.388	1.360	1.343	1.331	1.321	1.315	1.308	1.304	1.296	1.268
60	1.419	1.389	1.368	1.340	1.321	1.308	1.299	1.291	1.285	1.280	1.273	1.242
70	1.404	1.375	1.353	1.325	1.306	1.293	1.282	1.275	1.268	1.263	1.256	1.222
80	1.395	1.364	1.343	1.313	1.294	1.280	1.269	1.262	1.256	1.249	1.242	1.206
90	1.388	1.357	1.335	1.304	1.285	1.271	1.259	1.252	1.244	1.239	1.231	1.194
100	1.381	1.351	1.328	1.298	1.277	1.263	1.252	1.243	1.237	1.231	1.222	1.184
120	1.373	1.341	1.319	1.286	1.266	1.251	1.239	1.231	1.223	1.217	1.209	1.168
500	1.337	1.304	1.280	1.244	1.221	1.204	1.191	1.180	1.172	1.165	1.153	1.097

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.90}$

$d.f._2$ \ $d.f._1$	1	2	3	4	5	6	7	8	9	10	12	15
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.50	2.50	2.46
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66
50	2.82	2.42	2.21	2.07	1.99	1.91	1.85	1.80	1.76	1.73	1.68	1.62
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60
70	-	-	-	2.08	1.96	1.87	1.81	1.76	1.72	1.69	1.64	1.58
80	-	-	-	2.07	1.95	1.86	1.80	1.75	1.71	1.68	1.63	1.56
90	-	-	-	2.06	1.94	1.85	1.79	1.74	1.70	1.67	1.62	1.56
100	-	-	-	2.06	1.93	1.85	1.79	1.74	1.70	1.66	1.61	1.55
120	2.75	2.35	2.13	2.05	1.92	1.84	1.78	1.73	1.69	1.65	1.60	1.54
500	2.71	2.30	2.08	2.01	1.89	1.74	1.84	1.69	1.65	1.61	1.56	1.50

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.90}$

d.f.₂ \ d.f.₁	20	25	30	40	50	60	70	80	90	100	120	500
1	61.74	62.00	62.26	62.53	62.66	62.79	-	-	-	-	63.06	63.33
2	9.44	9.45	9.46	9.47	9.47	9.47	-	-	-	-	9.48	9.49
3	5.18	5.18	5.17	5.16	5.15	5.15	-	-	-	-	5.14	5.13
4	3.84	3.83	3.82	3.80	3.79	3.79	-	-	-	-	3.78	3.76
5	3.21	3.19	3.17	3.16	3.15	3.14	-	-	-	-	3.12	3.10
6	2.84	2.82	2.80	2.78	2.77	2.76	-	-	-	-	2.74	2.72
7	2.59	2.58	2.56	2.54	2.53	2.51	-	-	-	-	2.74	2.72
8	2.42	2.40	2.38	2.36	2.35	2.34	-	-	-	-	2.49	2.47
9	2.30	2.28	2.25	2.23	2.22	2.21	-	-	-	-	2.32	2.29
10	2.20	2.18	2.16	2.13	2.12	2.11	2.11	2.10	2.10	2.09	2.08	2.07
11	2.12	2.10	2.08	2.05	2.04	2.03	2.02	2.02	2.01	2.01	2.00	1.98
12	2.06	2.04	2.01	1.99	1.97	1.96	1.95	1.95	1.94	1.94	1.93	1.91
13	2.01	1.98	1.96	1.93	1.91	1.90	1.90	1.89	1.89	1.88	1.88	1.86
14	1.96	1.94	1.91	1.89	1.87	1.86	1.85	1.64	1.84	1.83	1.83	1.81
15	1.92	1.90	1.87	1.85	1.83	1.82	1.81	1.80	1.80	1.79	1.79	1.76
16	1.89	1.87	1.84	1.81	1.79	1.78	1.77	1.76	1.76	1.76	1.75	1.73
17	1.86	1.84	1.81	1.78	1.76	1.75	[illegible]	1.73	1.73	1.72	1.72	1.69
18	1.84	1.81	1.78	1.75	1.73	1.72	1.71	1.70	1.70	1.70	1.69	1.66
19	1.81	1.79	1.76	1.73	1.71	1.70	1.69	1.68	1.67	1.67	1.67	1.64
20	1.79	1.77	1.74	1.71	1.69	1.68	1.66	1.66	1.65	1.65	1.64	1.61
21	1.78	1.75	1.72	1.69	1.67	1.66	1.64	1.64	1.63	1.63	1.62	1.59
22	1.76	1.73	1.70	1.67	1.65	1.64	1.63	1.62	1.61	1.61	1.60	1.57
23	1.74	1.72	1.69	1.66	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.56
24	1.73	1.70	1.67	1.64	1.62	1.61	1.59	1.59	1.58	1.58	1.57	1.54
25	1.72	1.69	1.66	1.63	1.60	1.59	1.58	1.57	1.57	1.56	1.56	1.52
26	1.71	1.68	1.65	1.61	1.59	1.58	1.57	1.56	1.55	1.55	1.54	1.51
27	1.70	1.67	1.64	1.60	1.58	1.57	1.55	1.55	1.54	1.54	1.53	1.50
28	1.69	1.66	1.63	1.59	1.57	1.56	1.54	1.53	1.53	1.52	1.52	1.49
29	1.68	1.65	1.62	1.58	1.56	1.55	1.53	1.52	1.52	1.51	1.51	1.47
30	1.67	1.64	1.61	1.57	1.55	1.54	1.52	1.51	1.51	1.50	1.50	1.46
40	1.61	1.57	1.54	1.51	1.48	1.47	1.45	1.44	1.43	1.43	1.42	1.38
50	1.56	1.53	[illegible]	1.52	1.44	1.42	1.41	1.40	1.39	1.38	1.37	1.34
60	1.54	1.51	1.44	1.44	1.41	1.40	1.38	1.37	1.36	1.35	1.35	1.30
70	1.52	1.45	1.41	1.48	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.32
80	1.51	1.47	1.44	[illegible]	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.26
90	1.50	1.46	1.43	1.46	1.36	1.34	1.33	1.32	1.31	1.30	1.29	1.24
100	1.49	1.45	1.42	[illegible]	1.35	1.33	1.32	1.31	1.30	1.29	1.29	1.23
120	1.48	1.44	1.40	1.37	1.34	1.32	1.30	1.29	1.28	1.27	1.26	1.21
500	1.43	1.39	1.36	1.31	1.28	1.26	1.24	1.23	1.21	1.20	1.19	1.12

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.95}$

d.f.₂ \ d.f.₁	1	2	3	4	5	6	7	8	9	10	12	15
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92
50	4.04	3.19	2.80	2.61	2.42	2.30	2.20	2.13	2.07	2.02	1.95	1.87
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84
70	-	-	-	2.55	2.37	2.24	2.15	2.08	2.02	1.97	1.89	1.81
80	-	-	-	2.54	2.35	2.23	2.13	2.06	2.00	1.95	1.87	1.79
90	-	-	-	2.53	2.34	2.21	2.12	2.05	1.99	1.94	1.86	1.78
100	-	-	-	2.52	2.33	2.20	2.11	2.04	1.98	1.93	1.85	1.76
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75
500	3.84	3.00	2.60	2.44	2.26	2.13	2.04	1.96	1.90	1.85	1.77	1.68

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.95}$

d.f.₂ \ d.f.₁	20	25	30	40	50	60	70	80	90	100	120	500
1	248.0	249.1	250.1	251.1	251.6	252.2	-	-	-	-	253.3	254.3
2	19.45	19.45	19.46	19.47	19.48	19.48	-	-	-	-	19.49	19.50
3	8.66	8.64	8.62	8.59	8.58	8.57	-	-	-	-	8.55	8.53
4	5.80	5.77	5.75	5.72	5.71	5.69	-	-	-	-	5.66	5.63
5	4.56	4.53	4.50	4.46	4.45	4.43	-	-	-	-	4.40	4.36
6	3.87	3.84	3.81	3.77	3.76	3.74	-	-	-	-	3.70	3.67
7	3.44	3.41	3.38	3.34	3.32	3.30	-	-	-	-	3.27	3.23
8	3.15	3.12	3.08	3.04	3.03	3.01	-	-	-	-	2.97	2.93
9	2.94	2.90	2.86	2.83	2.82	2.79	-	-	-	-	2.75	2.71
10	2.77	2.74	2.70	2.66	2.65	2.62	2.62	2.61	2.61	2.60	2.58	2.56
11	2.65	2.61	2.57	2.53	2.51	2.49	2.49	2.48	2.47	2.47	2.45	2.43
12	2.54	2.51	2.47	2.43	2.41	2.38	2.38	2.37	2.36	2.36	2.34	2.32
13	2.46	2.42	2.38	2.34	2.32	2.30	2.29	2.28	2.27	2.27	2.25	2.22
14	2.39	2.35	2.31	2.27	2.24	2.22	2.21	2.20	2.20	2.19	2.18	2.15
15	2.33	2.29	2.25	2.20	2.18	2.16	2.15	2.14	2.13	2.13	2.11	2.08
16	2.28	2.24	2.19	2.15	2.12	2.11	2.09	2.08	2.08	2.07	2.06	2.02
17	2.23	2.19	2.15	2.10	2.08	2.06	2.04	2.03	2.03	2.02	2.01	1.97
18	2.19	2.15	2.11	2.06	2.03	2.02	2.00	1.99	1.98	1.98	1.97	1.93
19	2.16	2.11	2.07	2.03	2.00	1.98	1.96	1.95	1.95	1.94	1.93	1.89
20	2.12	2.08	2.04	1.99	1.96	1.95	1.93	1.92	1.91	1.90	1.90	1.86
21	2.10	2.05	2.01	1.96	1.93	1.92	1.90	1.89	1.88	1.87	1.87	1.82
22	2.07	2.03	1.98	1.94	1.91	1.89	1.87	1.86	1.85	1.85	1.84	1.79
23	2.05	2.01	1.96	1.91	1.88	1.86	1.85	1.84	1.83	1.82	1.81	1.77
24	2.03	1.98	1.94	1.89	1.86	1.84	1.82	1.81	1.80	1.80	1.79	1.74
25	2.01	1.96	1.92	1.87	1.84	1.82	1.80	1.79	1.78	1.78	1.77	1.72
26	1.99	1.95	1.90	1.85	1.82	1.80	1.78	1.77	1.76	1.76	1.75	1.70
27	1.97	1.93	1.88	1.84	1.80	1.79	1.77	1.75	1.75	1.74	1.73	1.68
28	1.96	1.91	1.87	1.82	1.79	1.77	1.75	1.74	1.73	1.72	1.71	1.67
29	1.94	1.90	1.85	1.81	1.77	1.75	1.73	1.72	1.71	1.71	1.70	1.65
30	1.93	1.89	1.84	1.79	1.76	1.74	1.72	1.71	1.70	1.69	1.68	1.63
40	1.84	1.79	1.74	1.69	1.66	1.64	1.62	1.60	1.59	1.59	1.58	1.52
50	1.78	1.72	1.68	1.63	1.59	1.57	1.55	1.54	1.53	1.52	1.51	1.45
60	1.75	1.70	1.65	1.59	1.55	[illegible]	1.51	1.50	1.49	1.48	1.47	1.40
79	1.72	1.66	1.62	1.56	1.52	1.50	1.48	1.47	1.46	1.45	1.43	1.37
80	1.70	1.64	1.60	1.54	1.50	1.48	1.46	1.44	1.43	1.42	1.41	1.34
90	1.68	1.62	1.58	1.52	1.49	1.46	1.44	1.43	1.41	1.40	1.39	1.32
100	1.67	1.61	1.57	1.51	1.47	1.45	1.43	1.41	1.40	1.39	1.37	1.30
120	1.66	1.61	1.55	1.50	1.45	1.43	1.40	1.39	1.37	1.36	1.35	1.28
500	1.59	1.52	1.48	1.41	1.37	1.34	1.32	1.30	1.28	1.27	1.25	1.16

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.975}$

d.f.₂ \ d.f.₁	1	2	3	4	5	6	7	8	9	10	12	15
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18
50	5.35	3.99	3.40	3.10	2.85	2.68	2.56	2.46	2.38	2.31	2.21	2.10
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06
70	-	-	-	3.02	2.77	2.60	2.48	2.38	2.30	2.24	2.13	2.02
80	-	-	-	3.00	2.75	2.53	2.45	2.36	2.28	2.21	2.11	2.00
90	-	-	-	2.98	2.73	2.56	2.44	2.34	2.26	2.19	2.09	1.98
100	-	-	-	2.97	2.72	2.55	2.42	2.32	2.24	2.18	2.07	1.96
120	-	-	-	2.94	2.69	2.53	2.40	2.30	2.22	2.16	2.05	1.94
500	5.02	3.69	3.12	2.86	2.61	2.44	2.32	2.22	2.14	2.07	1.97	1.86

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.975}$

d.f.₂ \ d.f.₁	20	25	30	40	50	60	70	80	90	100	120	500
1	993.1	997.2	1001.	1006.	1008.	1010.	-	-	-	-	1014.	1018.
2	39.45	39.46	39.46	39.47	39.48	39.40	-	-	-	-	39.49	39.50
3	14.17	14.12	14.08	14.04	13.00	13.95	-	-	-	-	13.95	13.90
4	8.56	8.51	8.46	8.41	8.36	8.31	-	-	-	-	8.31	8.26
5	6.33	6.28	6.23	6.18	6.13	6.07	-	-	-	-	6.01	6.02
6	5.17	5.12	5.07	5.01	4.96	4.90	-	-	-	-	4.90	4.85
7	4.47	4.42	4.36	4.31	4.26	4.20	-	-	-	-	4.20	4.14
8	4.00	3.95	3.89	3.84	3.79	3.73	-	-	-	-	3.73	3.67
9	3.67	3.61	3.56	3.51	3.45	3.39	-	-	-	-	3.39	3.33
10	3.42	3.37	3.31	3.26	3.24	3.20	3.21	3.19	3.18	3.18	3.14	3.12
11	3.23	3.17	3.12	3.06	3.05	3.00	3.00	2.99	2.98	2.97	2.94	2.92
12	3.07	3.02	2.96	2.91	2.88	2.85	2.84	2.83	2.82	2.81	2.79	2.75
13	2.95	2.89	2.84	2.78	2.75	2.72	2.71	2.70	2.69	2.68	2.66	2.62
14	2.84	2.79	2.73	2.67	2.64	2.61	2.60	2.59	2.58	2.57	2.55	2.51
15	2.76	2.70	2.64	2.59	2.55	2.52	2.51	2.50	2.49	2.48	2.46	2.42
16	2.68	2.63	2.57	2.51	2.47	2.45	2.43	2.42	2.41	2.40	2.38	2.34
17	2.62	2.56	2.50	2.44	2.41	2.38	2.36	2.35	2.34	2.33	2.32	2.27
18	2.56	2.50	2.44	2.38	2.35	2.32	2.30	2.29	2.28	2.27	2.26	2.21
19	2.51	2.45	2.39	2.33	2.29	2.27	2.25	2.24	2.23	2.22	2.20	2.15
20	2.46	2.41	2.35	2.29	2.25	2.22	2.20	2.19	2.18	2.17	2.16	2.10
21	2.42	2.37	2.31	2.25	2.21	2.18	2.16	2.15	2.14	2.13	2.11	2.06
22	2.39	2.33	2.27	2.21	2.17	2.14	2.12	2.11	2.10	2.09	2.08	2.02
23	2.36	2.30	2.24	2.18	2.13	2.11	2.09	2.07	2.06	2.05	2.04	1.98
24	2.33	2.27	2.21	2.15	2.10	2.08	2.06	2.04	2.03	2.02	2.01	1.95
25	2.30	2.24	2.18	2.12	2.08	2.05	2.03	2.01	2.00	1.99	1.98	1.92
26	2.28	2.22	2.16	2.09	2.05	2.03	2.00	1.99	1.98	1.97	1.95	1.89
27	2.25	2.19	2.13	2.07	2.03	2.00	1.98	1.96	1.95	1.94	1.93	1.87
28	2.23	2.17	2.11	2.05	2.00	1.98	1.96	1.94	1.93	1.92	1.91	1.85
29	2.21	2.15	2.09	2.03	1.98	1.96	1.94	1.92	1.91	1.90	1.89	1.82
30	2.20	2.14	2.07	2.01	1.96	1.94	1.92	1.90	1.89	1.88	1.87	1.80
40	2.07	2.01	1.94	1.88	1.83	1.80	1.78	1.76	1.75	1.74	1.72	1.66
50	1.99	1.92	1.86	1.79	1.75	1.72	1.70	1.68	1.67	1.66	1.63	1.57
60	1.94	1.88	1.81	1.74	1.69	1.66	1.64	1.62	1.61	1.59	1.58	1.50
70	1.91	1.83	1.77	1.70	1.66	1.62	1.60	1.58	1.57	1.55	1.53	1.46
80	1.88	1.80	1.75	1.67	1.63	1.59	1.57	1.55	1.54	1.52	1.50	1.43
90	1.86	1.78	1.73	1.65	1.60	1.57	1.55	1.53	1.51	1.50	1.48	1.40
100	1.84	1.77	1.71	1.64	1.59	1.55	1.53	1.51	1.49	1.48	1.46	1.38
120	1.82	1.75	1.69	1.61	1.56	1.52	1.50	1.48	1.46	1.45	1.43	1.34
500	1.73	1.65	1.59	1.51	1.46	1.42	1.39	1.37	1.35	1.33	1.31	1.19

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

$F_{.99}$

d.f.₂ \ d.f.₁	1	2	3	4	5	6	7	8	9	10	12	15
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.92
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52
50	7.20	5.08	4.37	3.76	3.42	3.19	3.02	2.89	2.78	2.69	2.56	2.41
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35
70	-	-	-	3.64	3.30	3.07	2.90	2.77	2.67	2.58	2.44	2.30
80	-	-	-	3.61	3.27	3.04	2.87	2.74	2.63	2.55	2.41	2.27
90	-	-	-	3.58	3.24	3.01	2.84	2.71	2.61	2.52	2.38	2.24
100	-	-	-	3.55	3.22	2.99	2.82	2.69	2.59	2.50	2.36	2.22
120	6.85	4.79	3.95	3.52	3.19	2.96	2.79	2.66	2.56	2.47	2.33	2.19
500	6.63	4.61	3.78	3.40	3.07	2.84	2.67	2.54	2.44	2.35	2.22	2.07

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

F .99

d.f.2 \ d.f.1	20	25	30	40	50	60	70	80	90	100	120	500
1	6209	6235	6261	6287	6300	6313	-	-	-	-	6339	6366
2	99.45	99.46	99.47	99.47	99.48	99.48	-	-	-	-	99.49	99.50
3	26.69	26.60	26.50	26.41	26.37	26.32	-	-	-	-	26.22	26.13
4	14.02	13.93	13.84	13.75	13.70	13.65	-	-	-	-	13.56	13.46
5	9.55	9.47	9.38	9.29	9.25	9.20	-	-	-	-	9.11	9.02
6	7.40	7.31	7.23	7.14	7.10	7.06	-	-	-	-	6.97	6.88
7	6.16	6.07	5.99	5.91	5.87	5.82	-	-	-	-	5.74	5.65
8	5.36	5.28	5.20	5.12	5.08	5.03	-	-	-	-	4.95	4.86
9	4.81	4.73	4.65	4.57	4.53	4.48	-	-	-	-	4.40	4.31
10	4.41	4.33	4.25	4.17	4.16	4.08	4.10	4.08	4.07	4.06	4.00	3.97
11	4.10	4.02	3.94	3.86	3.84	3.78	3.78	3.77	3.75	3.74	3.69	3.66
12	3.86	3.78	3.70	3.62	3.59	3.54	3.54	3.52	3.50	3.49	3.45	3.41
13	3.66	3.59	3.51	3.43	3.39	3.34	3.33	3.32	3.30	3.29	3.25	3.21
14	3.51	3.43	3.35	3.27	3.23	3.18	3.17	3.15	3.14	3.13	3.09	3.04
15	3.37	3.29	3.21	3.13	3.09	3.05	3.03	3.02	3.00	2.99	2.96	2.90
16	3.26	3.18	3.10	3.02	2.98	2.93	2.92	2.90	2.88	2.87	2.84	2.79
17	3.16	3.08	3.00	2.92	2.88	2.83	2.82	2.80	2.78	2.77	2.75	2.69
18	3.08	3.00	2.92	2.84	2.79	2.75	2.73	2.71	2.70	2.69	2.66	2.60
19	3.00	2.92	2.84	2.76	2.71	2.67	2.65	2.64	2.62	2.61	2.58	2.52
20	2.94	2.86	2.78	2.69	2.65	2.61	2.59	2.57	2.55	2.54	2.52	2.45
21	2.88	2.80	2.72	2.64	2.59	2.55	2.53	2.51	2.49	2.48	2.46	2.39
22	2.83	2.75	2.67	2.58	2.53	2.50	2.47	2.45	2.44	2.43	2.40	2.33
23	2.78	2.70	2.62	2.54	2.49	2.45	2.42	2.40	2.39	2.38	2.35	2.28
24	2.74	2.66	2.58	2.49	2.44	2.40	2.38	2.36	2.34	2.33	2.31	2.24
25	2.70	2.62	2.54	2.45	2.40	2.36	2.34	2.32	2.30	2.29	2.27	2.20
26	2.66	2.59	2.50	2.42	2.37	2.33	2.30	2.28	2.27	2.25	2.23	2.16
27	2.63	2.55	2.47	2.38	2.33	2.29	2.27	2.25	2.23	2.22	2.20	2.12
28	2.60	2.52	2.44	2.35	2.30	2.26	2.24	2.23	2.20	2.19	2.17	2.09
29	2.57	2.49	2.41	2.33	2.27	2.23	2.21	2.19	2.17	2.16	2.14	2.06
30	2.55	2.47	2.39	2.30	2.25	2.21	2.18	2.16	2.15	2.13	2.11	2.03
40	2.37	2.27	2.20	2.11	2.06	2.02	1.99	1.97	1.95	1.94	1.92	1.83
50	2.26	2.16	2.10	2.01	1.95	1.91	1.88	1.86	1.84	1.83	1.80	1.71
60	2.20	2.10	2.03	1.94	1.88	1.84	1.81	1.78	1.76	1.75	1.73	1.63
70	2.15	2.05	1.98	1.89	1.83	1.78	1.75	1.73	1.71	1.70	1.67	1.57
80	2.11	2.01	1.94	1.85	1.79	1.75	1.71	1.69	1.67	1.65	1.63	1.53
90	2.08	1.99	1.92	1.82	1.76	1.72	1.68	1.66	1.64	1.62	1.60	1.49
100	2.06	1.97	1.85	1.80	1.74	1.69	1.66	1.63	1.61	1.60	1.57	1.47
120	2.03	1.93	1.86	1.76	1.70	1.66	1.62	1.60	1.58	1.56	1.53	1.42
500	1.91	1.81	1.74	1.63	1.57	1.52	1.49	1.45	1.43	1.41	1.38	1.23

NOTE: The tables for $F_{.80}$ and $F_{.85}$ were generated using the formula $F \approx e^{2w}$ where:

$$\lambda = \frac{Z_\alpha^2 - 3}{6}$$

$$h = 2\left(\frac{1}{d.f._2 - 1} + \frac{1}{d.f._1 - 1}\right)^{-1}$$

$$w = \frac{-Z_\alpha(h+\lambda)}{h} - \left(\frac{1}{d.f._1 - 1} - \frac{1}{d.f._2 - 1}\right)\left(\lambda + \frac{5}{6} - \frac{2}{3h}\right)$$

The approximation is accurate enough for practical uses when $d.f._1$ and $d.f._2 \geq 10$. However, the formula has been used for $d.f._1 < 10$ and $d.f._2 < 10$ for $F_{.80}$ and $F_{.85}$ because no tables were available.

Values other than the ones found in the standard F tables were also supplied using the above formula where possible and if not possible, dashes were left, e.g., $F_{.90}, (80, 2) = -$. In the event of a dash occurring, use the smaller d.f. which appears in the table for computation purposes; e.g., use $F_{.90}, (60, 2) = 9.47$ for $F_{.90}, (80, 2)$.

TABLE B-9

FACTORS FOR COMPUTING TWO-SIDED CONFIDENCE LIMITS FOR σ

Degrees of Freedom ν	α = .05		α = .01		α = .001	
	B_U	B_L	B_U	B_L	B_U	B_L
1	17.79	.3576	86.31	.2969	844.4	.2480
2	4.859	.4581	10.70	.3879	33.29	.3291
3	3.183	.5178	5.449	.4453	11.65	.3824
4	2.567	.5590	3.892	.4865	6.938	.4218
5	2.248	.5899	3.175	.5182	5.085	.4529
6	2.052	.6143	2.764	.5437	4.128	.4784
7	1.918	.6344	2.495	.5650	3.551	.5000
8	1.820	.6513	2.311	.5830	3.167	.5186
9	1.746	.6657	2.173	.5987	2.896	.5348
10	1.686	.6784	2.065	.6125	2.689	.5492
11	1.638	.6896	1.980	.6248	2.530	.5621
12	1.598	.6995	1.909	.6358	2.402	.5730
13	1.564	.7084	1.851	.6458	2.298	.5845
14	1.534	.7166	1.801	.6549	2.210	.5942
15	1.509	.7240	1.758	.6632	2.136	.6032
16	1.486	.7308	1.722	.6710	2.073	.6116
17	1.466	.7372	1.688	.6781	2.017	.6193
18	1.448	.7430	1.658	.6848	1.968	.6266
19	1.432	.7484	1.632	.6909	1.925	.6333
20	1.417	.7535	1.609	.6968	1.886	.6397
21	1.404	.7582	1.587	.7022	1.851	.6457
22	1.391	.7627	1.568	.7074	1.820	.6514
23	1.380	.7669	1.550	.7122	1.791	.6568
24	1.370	.7709	1.533	.7169	1.765	.6619
25	1.360	.7747	1.518	.7212	1.741	.6668
26	1.351	.7783	1.504	.7253	1.719	.6713
27	1.343	.7817	1.491	.7293	1.698	.6758
28	1.335	.7849	1.479	.7331	1.679	.6800
29	1.327	.7880	1.467	.7367	1.661	.6841
30	1.321	.7909	1.457	.7401	1.645	.6880
31	1.314	.7937	1.447	.7434	1.629	.6917
32	1.308	.7964	1.437	.7467	1.615	.6953
33	1.302	.7990	1.428	.7497	1.601	.6987
34	1.296	.8015	1.420	.7526	1.588	.7020
35	1.291	.8039	1.412	.7554	1.576	.7052
36	1.286	.8062	1.404	.7582	1.564	.7083
37	1.281	.8085	1.397	.7608	1.553	.7113
38	1.277	.8106	1.390	.7633	1.543	.7141
39	1.272	.8126	1.383	.7658	1.533	.7169
40	1.268	.8146	1.377	.7681	1.523	.7197
41	1.264	.8166	1.371	.7705	1.515	.7223
42	1.260	.8184	1.365	.7727	1.506	.7248
43	1.257	.8202	1.360	.7748	1.498	.7273
44	1.253	.8220	1.355	.7769	1.490	.7279
45	1.249	.8237	1.349	.7789	1.482	.7320
46	1.246	.8253	1.345	.7809	1.475	.7342
47	1.243	.8269	1.340	.7828	1.468	.7364
48	1.240	.8285	1.335	.7847	1.462	.7386
49	1.237	.8300	1.331	.7864	1.455	.7407
50	1.234	.8314	1.327	.7882	1.449	.7427

Adapted with permission from Biometrika, Vol. 47, (1960), from article entitled "Table for Making Inferences About the Variance of a Normal Distribution" by D. V. Lindley, D.A. East, and P.A. Hamilton

TABLE 8-9 continued

FACTORS FOR COMPUTING TWO-SIDED CONFIDENCE LIMITS FOR σ

Degrees of Freedom d.f.	α = .05		α = .01		α = .001	
	a_U	a_L	a_U	a_L	a_U	a_L
51	1.232	.8329	1.323	.7899	1.443	.7446
52	1.229	.8343	1.319	.7916	1.437	.7466
53	1.226	.8356	1.315	.7932	1.432	.7485
54	1.224	.8370	1.311	.7949	1.426	.7503
55	1.221	.8383	1.308	.7964	1.421	.7521
56	1.219	.8395	1.304	.7979	1.416	.7539
57	1.217	.8408	1.301	.7994	1.411	.7556
58	1.214	.8420	1.298	.8008	1.406	.7573
59	1.212	.8431	1.295	.8022	1.402	.7589
60	1.210	.8443	1.292	.8036	1.397	.7605
61	1.208	.8454	1.289	.8050	1.393	.7621
62	1.206	.8465	1.286	.8063	1.389	.7636
63	1.204	.8475	1.283	.8076	1.385	.7651
64	1.202	.8486	1.280	.8088	1.381	.7666
65	1.200	.8496	1.277	.8101	1.377	.7680
66	1.199	.8506	1.275	.8113	1.374	.7694
67	1.197	.8516	1.272	.8125	1.370	.7708
68	1.195	.8525	1.270	.8137	1.366	.7722
69	1.194	.8535	1.268	.8148	1.363	.7735
70	1.192	.8544	1.265	.8159	1.360	.7749
71	1.190	.8553	1.263	.8170	1.356	.7761
72	1.189	.8562	1.261	.8181	1.353	.7774
73	1.187	.8571	1.259	.8191	1.350	.7787
74	1.186	.8580	1.257	.8202	1.347	.7799
75	1.184	.8588	1.255	.8212	1.344	.7811
76	1.183	.8596	1.253	.8222	1.341	.7822
77	1.182	.8604	1.251	.8232	1.338	.7834
78	1.181	.8612	1.249	.8242	1.336	.7845
79	1.179	.8620	1.247	.8252	1.333	.7856
80	1.178	.8627	1.245	.8261	1.330	.7868
81	1.176	.8635	1.243	.8270	1.328	.7878
82	1.176	.8642	1.241	.8279	1.325	.7889
83	1.174	.8650	1.239	.8288	1.323	.7899
84	1.173	.8657	1.238	.8297	1.320	.7909
85	1.172	.8664	1.236	.8305	1.318	.7920
86	1.171	.8671	1.235	.8314	1.316	.7930
87	1.170	.8678	1.233	.8322	1.313	.7939
88	1.168	.8684	1.231	.8331	1.311	.7949
89	1.167	.8691	1.230	.8338	1.309	.7959
90	1.166	.8697	1.228	.8346	1.307	.7968
91	1.165	.8704	1.227	.8354	1.305	.7977
92	1.164	.8710	1.225	.8362	1.303	.7987
93	1.163	.8716	1.224	.8370	1.301	.7996
94	1.162	.8722	1.222	.8377	1.298	.8004
95	1.161	.8729	1.221	.8385	1.297	.8013
96	1.160	.8734	1.219	.8392	1.295	.8022
97	1.159	.8741	1.218	.8399	1.293	.8031
98	1.158	.8746	1.217	.8406	1.291	.8039
99	1.158	.8752	1.216	.8413	1.290	.8047
100	1.157	.8757	1.214	.8420	1.288	.8055

TABLE B-10

FACTORS FOR COMPUTING ONE-SIDED CONFIDENCE LIMITS FOR σ

Degrees of Freedom d.f.	$A_{.05}$	$A_{.95}$	$A_{.025}$	$A_{.975}$	$A_{.01}$	$A_{.99}$	$A_{.005}$	$A_{.995}$
1	.5103	15.947	.4461	31.910	.3882	79.786	.3562	159.576
2	.5778	4.415	.5207	6.285	.4660	9.975	.4344	14.124
3	.6196	2.920	.5665	3.729	.5142	5.111	.4834	6.467
4	.6493	2.372	.5992	2.84	.5489	3.669	.5188	4.396
5	.6721	2.089	.6242	2.453	.5757	3.003	.5464	3.485
6	.6903	1.915	.6444	2.202	.5974	2.623	.5688	2.980
7	.7054	1.797	.6612	2.035	.6155	2.377	.5875	2.660
8	.7183	1.711	.6754	1.916	.6310	2.204	.6037	2.439
9	.7293	1.645	.6878	1.826	.6445	2.076	.6177	2.278
10	.7391	1.593	.6987	1.755	.6564	1.977	.6301	2.154
11	.7477	1.551	.7084	1.698	.6670	1.898	.6412	2.056
12	.7554	1.515	.7171	1.651	.6765	1.833	.6512	1.976
13	.7624	1.485	.7250	1.611	.6852	1.779	.6603	1.909
14	.7688	1.460	.7321	1.577	.6931	1.733	.6686	1.854
15	.7747	1.437	.7387	1.548	.7004	1.694	.6762	1.806
20	.7979	1.358	.7650	1.44	.7297	1.556	.7071	1.640
25	.8149	1.308	.7843	1.380	.7511	1.473	.7299	1.542
30	.8279	1.274	.7991	1.337	.7678	1.416	.7477	1.475
40	.8470	1.228	.8210	1.279	.7925	1.343	.7740	1.390
50	.8606	1.199	.8367	1.243	.8103	1.297	.7931	1.337
60	.8710	1.179	.8487	1.217	.8239	1.265	.8078	1.299
70	.8793	1.163	.8583	1.198	.8349	1.241	.8196	1.272
80	.8861	1.151	.8662	1.183	.8439	1.222	.8293	1.250
90	.8919	1.141	.8728	1.171	.8515	1.207	.8376	1.233
100	.8968	1.138	.8785	1.161	.8581	1.195	.8446	1.219

For large degrees of freedom, we may use the approximate formula:

$$A_{1-\alpha} = \sqrt{2d.f.} \Big/ \left(Z_\alpha + \sqrt{2(d.f.) - 1}\right)$$

where Z_α is found in Table B-4, page 2-4.

TABLE B-11

$s < \sigma_0$ $(PE < \tau_0)$

γ	α = .001 β = .05	α = .001 β = .10	α = .01 β = .05	α = .01 β = .10	α = .05 β = .05	α = .05 β = .10	α = .10 β = .10
1	8	8	5	5	[illegible]	3	3
2	11	10	7	7	5	4	3
.3	15	14	10	9	6	6	4
.4	21	20	14	13	9	8	6
.5	32	29	21	19	14	12	9
.6	53	48	36	31	23	20	15
.7	101	90	69	59	45	37	28
.8	244	213	167	142	111	91	68
.9	1046	902	726	607	490	393	298

TABLE B-11 continued

$s > \sigma_0$ (PE > τ_0)

γ	α = .001 β = .05	α = .001 β = .10	α = .01 β = .05	α = .01 β = .10	α = .05 β = .05	α = .05 β = .10	α = .10 β = .10
1.1	1202	1014	857	699	598	468	364
1.2	322	269	233	188	165	128	101
1.3	153	127	112	90	81	62	50
1.4	92	76	68	55	50	38	31
1.5	63	52	47	38	35	27	22
1.6	47	38	36	28	27	20	17
1.7	37	30	28	22	22	16	14
1.8	30	24	23	18	18	14	12
1.9	25	20	20	15	16	12	10
2.0	22	17	17	13	14	10	9
2.1	19	15	15	12	12	9	8
2.2	17	14	14	11	11	8	7
2.3	15	12	13	10	10	8	7
2.4	14	11	12	9	9	7	6
2.5	13	10	11	8	9	7	6
2.6	12	10	10	8	8	6	6
2.7	11	9	9	7	8	6	5

TABLE B-11 continued

$s > \sigma_0$ (PE > τ_0)

γ	$\alpha = .001$ $\beta = .05$	$\alpha = .001$ $\beta = .10$	$\alpha = .01$ $\beta = .05$	$\alpha = .01$ $\beta = .10$	$\alpha = .05$ $\beta = .05$	$\alpha = .05$ $\beta = .10$	$\alpha = .10$ $\beta = .10$
2.8	11	8	9	7	8	6	5
2.9	10	8	8	7	7	5	5
3.0	10	8	8	6	7	5	5
3.1	9	7	8	6	7	5	5
3.2	9	7	7	6	6	5	4
3.3	8	7	7	6	6	5	4
3.4	8	6	7	5	6	5	4
3.5	8	6	7	5	6	5	4

TABLE B-12

DETERMINATION OF SAMPLE SIZE

(s_A and s_B)

$s_A < s_B$

s_A/s_B	$\alpha = .01$ $\beta = .05$	$\alpha = .01$ $\beta = .10$	$\alpha = .05$ $\beta = .05$	$\alpha = .05$ $\beta = .10$	$\alpha = .10$ $\beta = .10$
.1	5	5	5	4	4
.2	9	8	7	6	5
.3	13	11	10	8	7
.4	21	18	15	13	10
.5	35	30	25	20	16
.6	63	52	44	35	28
.7	126	105	88	70	54
.8	319	264	220	175	135
.9	1423	1175	978	774	595
			$s_A > s_B$		
1.1	1738	1435	1194	946	726
1.2	477	394	328	260	200
1.3	232	192	160	127	98
1.4	142	117	98	78	61
1.5	98	82	68	55	42
1.6	74	61	51	41	32
1.7	59	49	41	33	26
1.8	48	40	34	27	22
1.9	41	34	29	23	18

TABLE B-12 continued

DETERMINATION OF SAMPLE SIZE

(s_A and s_B)

$s_A > s_B$

s_A/s_B	$\alpha = .01$ $\beta = .05$	$\alpha = .01$ $\beta = .10$	$\alpha = .01$ $\beta = .05$	$\alpha = .05$ $\beta = .10$	$\alpha = .10$ $\beta = .10$
2.0	35	30	25	20	16
2.1	31	26	22	18	14
2.2	28	23	20	16	13
2.3	25	21	18	15	12
2.4	23	19	17	14	11
2.5	21	18	15	13	10

TABLE B-13

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

Upper limits are underlined. The observed proportion in a random sample is f/N.

n = 1

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.900	0	.950	0	.990
1	.100	1	.050	1	.010	1

n = 2

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.684	0	.776	0	.900
1	.051	.949	.025+	.975-	.005+	.995-
2	.316	1	.224	1	.100	1

n = 3

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.536	0	.632	0	.785-
1	.035-	.804	.017	.865	.003	.941
2	.196	.965+	.135+	.983	.059	.997
3	.464	1	.368	1	.215+	1

n = 4

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.500	0	.527	0	.684
1	.026	.680	.013	.751	.003	.859
2	.143	.857	.098	.902	.042	.958
3	.320	.974	.249	.987	.141	.997
4	.500	1	.473	1	.316	1

n = 5

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.379	0	.500	0	.602
1	.021	.621	.010	.657	.002	.778
2	.112	.753	.076	.811	.033	.894
3	.247	.888	.189	.924	.106	.967
4	.379	.979	.343	.990	.222	.998
5	.621	1	.500	1	.398	1

n = 6

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.345-	0	.402	0	.536
1	.017	.542	.009	.598	.002	.706
2	.093	.667	.063	.729	.027	.827
3	.201	.799	.153	.847	.085-	.915+
4	.333	.907	.271	.937	.173	.973
5	.458	.983	.402	.991	.294	.998
6	.655+	1	.598	1	.464	1

n = 7

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.316	0	.377	0	.500
1	.015-	.500	.007	.554	.001	.643
2	.079	.684	.053	.659	.023	.764
3	.170	.721	.129	.775-	.071	.858
4	.279	.830	.225+	.871	.142	.929
5	.316	.921	.341	.947	.236	.977
6	.500	.985+	.446	.993	.357	.999
7	.684	1	.623	1	.500	1

n = 8

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.255-	0	.315+	0	.451
1	.013	.418	.006	.500	.001	.590
2	.069	.582	.046	.685-	.020	.707
3	.147	.745+	.111	.711	.061	.802
4	.240	.760	.193	.807	.121	.879
5	.255-	.853-	.289	.889	.198	.939
6	.418	.931	.315+	.954	.293	.980
7	.582	.987	.500	.994	.410	.999
8	.745+	1	.685-	1	.549	1

n = 9

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.232	0	.289	0	.402
1	.012	.391	.006	.443	.001	.598
2	.061	.515+	.041	.558	.017	.656
3	.129	.610	.098	.711	.053	.750
4	.210	.768	.169	.749	.105+	.829
5	.232	.790	.251	.831	.171	.895-
6	.390	.871	.289	.902	.250	.947
7	.485-	.939	.442	.959	.344	.983
8	.609	.988	.557	.994	.402	.999
9	.768	1	.711	1	.598	1

n = 10

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.222	0	.267	0	.376
1	.010	.352	.005+	.397	.001	.512
2	.055-	.500	.037	.603	.016	.624
3	.116	.648	.087	.619	.048	.703
4	.188	.659	.150	.733	.093	.782
5	.222	.778	.222	.778	.150	.850
6	.341	.812	.267	.850	.218	.907
7	.352	.884	.381	.913	.297	.952
8	.500	.945+	.397	.963	.376	.984
9	.648	.990	.603	.995-	.488	.999
10	.778	1	.733	1	.624	1

n = 11

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.197	0	.250	0	.359
1	.010	.315+	.005-	.369	.001	.500
2	.049	.423	.033	.500	.014	.593
3	.105-	.577	.079	.631	.043	.660
4	.169	.685	.135+	.667	.084	.738
5	.197	.698	.200	.750	.134	.806
6	.302	.803	.250	.800	.194	.866
7	.315+	.831	.333	.865-	.262	.916
8	.423	.895+	.369	.921	.340	.957
9	.577	.951	.500	.967	.407	.986
10	.685-	.990	.631	.995+	.500	.999
11	.803	1	.750	1	.641	1

n = 12

f	90% lower	90% upper	95% lower	95% upper	99% lower	99% upper
0	0	.184	0	.236	0	.321
1	.009	.294	.004	.346	.001	.445+
2	.045+	.398	.030	.450	.013	.555-
3	.096	.500	.072	.550	.039	.679
4	.154	.602	.123	.654	.076	.698
5	.184	.706	.181	.706	.121	.765+
6	.271	.729	.236	.764	.175-	.825+
7	.294	.816	.294	.819	.235-	.879
8	.398	.846	.346	.877	.302	.924
9	.500	.904	.450	.929	.321	.961
10	.602	.955-	.550	.970	.445+	.987
11	.706	.991	.654	.996	.555-	.999
12	.816	1	.764	1	.679	1

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TABLE B-13 continued

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

f	90%		95%		99%		f	90%		95%		99%	
n = 13							n = 14						
0	0	.173	0	.225+	0	.302	0	0	.163	0	.207	0	.286
1	.008	.276	.004	.327	.001	.429	1	.007	.261	.004	.312	.001	.392
2	.042	.379	.028	.434	.012	.523	2	.039	.365+	.026	.389	.011	.500
3	.088	.470	.066	.520	.036	.594	3	.081	.422	.061	.500	.033	.608
4	.142	.545-	.113	.587	.069	.698	4	.131	.578	.104	.611	.064	.636
5	.173	.621	.166	.673	.111	.727	5	.163	.594	.153	.629	.102	.714
6	.246	.724	.224	.740	.159	.787	6	.224	.645+	.206	.688	.146	.751
7	.276	.754	.260	.776	.213	.841	7	.261	.739	.207	.793	.195-	.805+
8	.379	.827	.327	.834	.273	.889	8	.355-	.776	.312	.794	.249	.854
9	.455+	.858	.413	.887	.302	.931	9	.406	.837	.371	.847	.286	.898
10	.530	.912	.480	.934	.406	.964	10	.422	.869	.389	.896	.364	.936
11	.621	.958	.566	.972	.477	.988	11	.578	.919	.500	.939	.392	.967
12	.724	.992	.673	.996	.571	.999	12	.635-	.961	.611	.974	.500	.989
13	.827	1	.775-	1	.698	1	13	.739	.993	.688	.996	.608	.999
							14	.837	1	.793	1	.714	1
n = 15							n = 16						
0	0	.154	0	.191	0	.273	0	0	.147	0	.179	0	.264
1	.007	.247	.003	.302	.001	.373	1	.007	.235+	.003	.273	.001	.357
2	.036	.326	.024	.369	.010	.461	2	.034	.305+	.023	.352	.010	.451
3	.076	.400	.057	.448	.031	.539	3	.071	.381	.053	.429	.029	.525-
4	.122	.500	.097	.552	.059	.627	4	.114	.450	.090	.500	.055+	.579
5	.154	.600	.142	.631	.094	.672	5	.147	.550	.132	.571	.088	.643
6	.205+	.674	.191	.668	.135-	.727	6	.189	.619	.178	.648	.125+	.705-
7	.247	.675-	.192	.706	.179	.771	7	.235+	.695-	.179	.727	.166	.739
8	.325+	.753	.294	.808	.229	.821	8	.299	.701	.272	.728	.212	.788
9	.326	.795-	.332	.809	.273	.865+	9	.305+	.765-	.273	.821	.261	.834
10	.400	.846	.369	.858	.328	.906	10	.381	.811	.352	.822	.295+	.875-
11	.500	.878	.448	.903	.373	.941	11	.450	.853	.429	.868	.357	.912
12	.600	.924	.552	.943	.461	.969	12	.550	.886	.500	.910	.421	.945-
13	.674	.964	.631	.976	.539	.990	13	.619	.929	.571	.947	.475+	.971
14	.753	.993	.698	.997	.627	.999	14	.695-	.966	.648	.977	.549	.990
15	.846	1	.809	1	.727	1	15	.765-	.993	.727	.997	.643	.999
							16	.853	1	.821	1	.736	1

TABLE B-13 continued

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

n = 17

f	90%		95%		99%	
0	0	.140	0	.167	0	.243
1	.006	.225+	.003	.254	.001	.346
2	.032	.290	.021	.337	.006	.413
3	.067	.364	.050	.417	.077	.500
4	.107	.432	.085-	.489	.052	.587
5	.140	.500	.124	.544	.082	.620
6	.175+	.568	.166	.594	.117	.662
7	.225+	.636	.167	.663	.155+	.757
8	.277	.710	.253	.746	.197	.758
9	.290	.723	.254	.747	.242	.803
10	.364	.775-	.337	.833	.243	.845
11	.432	.825-	.406	.834	.338	.883
12	.500	.860	.456	.876	.380	.918
13	.568	.893	.511	.915+	.413	.948
14	.636	.933	.583	.950	.500	.973
15	.710	.968	.663	.979	.587	.991
16	.775-	.994	.746	.997	.654	.999
17	.860	1	.833	1	.757	1

n = 18

f	90%		95%		99%	
0	0	.135-	0	.157	0	.228
1	.006	.216	.003	.242	.001	.318
2	.030	.277	.020	.325-	.008	.397
3	.063	.349	.047	.381	.025+	.466
4	.101	.419	.080	.444	.049	.534
5	.135-	.482	.116	.556	.077	.603
6	.163	.536	.156	.619	.110	.682
7	.216	.584	.157	.625+	.145+	.686
8	.257	.651	.236	.675+	.184	.7[illegible]2
9	.277	.723	.242	.758	.226	.774
10	.349	.743	.325-	.764	.228	.815
11	.416	.784	.375-	.843	.314	.855-
12	.464	.837	.381	.844	.318	.890
13	.518	.865+	.444	.884	.397	.923
14	.581	.899	.556	.920	.466	.951
15	.651	.937	.619	.953	.534	.975-
16	.723	.970	.675+	.980	.603	.992
17	.78[illegible]	.994	.758	.997	.682	.999
18	.865+	1	.843	1	.772	1

n = 19

f	90%		95%		99%	
0	0	.130	0	.150	0	.218
1	.006	.209	.003	.232	.001	.305+
2	.028	.265+	.019	.316	.008	.383
3	.059	.337	.044	.365-	.024	.455+
4	.095+	.387	.075+	.426	.046	.515+
5	.130	.440	.110	.500	.073	.564
6	.151	.560	.147	.574	.103	.617
7	.209	.613	.150	.635+	.137	.695-
8	.238	.514	.222	.655+	.173	.707
9	.265+	.663	.232	.688	.212	.782
10	.337	.735-	.312	.768	.218	.788
11	.386	.762	.345-	.778	.293	.827
12	.387	.791	.365-	.850	.305+	.863
13	.440	.849	.426	.853	.383	.897
14	.560	.870	.500	.890	.436	.927
15	.613	.905-	.574	.925-	.485-	.954
16	.663	.941	.635+	.956	.545-	.976
17	.735-	.972	.684	.981	.617	.992
18	.791	.994	.768	.997	.695-	.999
19	.870	1	.850	1	.782	1

n = 20

f	90%		95%		99%	
0	0	.126	0	.143	0	.209
1	.005+	.203	.003	.222	.001	.293
2	.027	.255-	.018	.294	.008	.375-
3	.056	.328	.042	.351	.023	.424
4	.090	.367	.071	.411	.044	.500
5	.126	.422	.104	.467	.069	.576
6	.141	.500	.140	.533	.098	.601
7	.201	.578	.143	.589	.129	.637
8	.221	.633	.209	.649	.163	.707
9	.255-	.642	.222	.706	.200	.726
10	.325	.675+	.293	.707	.209	.791
11	.358	.745+	.294	.778	.274	.800
12	.367	.779	.351	.791	.293	.837
13	.422	.799	.411	.857	.363	.871
14	.500	.859	.467	.860	.399	.902
15	.578	.874	.533	.896	.424	.931
16	.633	.910	.589	.929	.600	.956
17	.672	.944	.649	.958	.576	.977
18	.745+	.973	.706	.982	.625+	.992
19	.797	.995-	.778	.997	.707	.999
20	.874	1	.857	1	.791	1

TABLE B-13 continued

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

n = 21

f	90%		95%		99%	
0	0	.123	0	.137	0	.201
1	.005+	.192	.002	.213	.000	.283
2	.026	.245-	.017	.277	.007	.347
3	.054	.307	.040	.338	.022	.409
4	.086	.353	.068	.398	.041	.466
5	.121	.407	.099	.455+	.065+	.534
6	.130	.458	.132	.506	.092	.591
7	.191	.542	.137	.551	.122	.653
8	.192	.593	.197	.602	.155-	.661
9	.245-	.647	.213	.662	.189	.717
10	.306	.693	.276	.723	.201	.743
11	.307	.694	.277	.724	.257	.799
12	.353	.755+	.338	.787	.283	.811
13	.407	.808	.398	.803	.339	.845+
14	.458	.809	.449	.863	.347	.878
15	.542	.870	.494	.868	.409	.908
16	.593	.879	.545-	.901	.466	.935-
17	.647	.914	.602	.932	.534	.959
18	.693	.946	.662	.960	.591	.978
19	.755+	.974	.723	.983	.653	.993
20	.808	.995	.787	.998	.717	1.000
21	.877	1	.863	1	.799	1

n = 22

f	90%		95%		99%	
0	0	.116	0	.132	0	.194
1	.005-	.182	.002	.205+	.000	.273
2	.024	.236	.016	.264	.007	.334
3	.051	.289	.038	.326	.021	.396
4	.082	.340	.065-	.389	.039	.454
5	.115-	.393	.094	.424	.062	.505-
6	.116	.444	.126	.500	.088	.550
7	.181	.500	.132	.576	.116	.604
8	.182	.556	.187	.582	.147	.666
9	.236	.607	.205+	.617	.179	.682
10	.289	.660	.260	.674	.194	.727
11	.290	.710	.264	.736	.242	.758
12	.340	.711	.326	.740	.273	.806
13	.393	.764	.383	.795-	.318	.821
14	.444	.818	.418	.813	.334	.853
15	.500	.819	.424	.868	.396	.884
16	.556	.884	.500	.874	.450	.912
17	.607	.885+	.576	.906	.495+	.938
18	.660	.918	.611	.935+	.546	.961
19	.711	.949	.674	.962	.604	.979
20	.764	.976	.736	.984	.666	.993
21	.818	.995+	.795-	.998	.727	1.000
	.884	1	.868	1	.806	1

n = 23

f	90%		95%		99%	
0	0	.111	0	.127	0	.187
1	.005-	.174	.002	.198	.000	.265+
2	.023	.228	.016	.255-	.007	.323
3	.049	.274	.037	.317	.020	.386
4	.078	.328	.062	.361	.038	.429
5	.110	.381	.090	.409	.059	.500
6	.111	.431	.120	.457	.084	.571
7	.173	.479	.127	.543	.111	.580
8	.174	.522	.178	.591	.140	.616
9	.228	.569	.198	.639	.171	.677
10	.273	.619	.247	.640	.187	.702
11	.274	.672	.255-	.683	.229	.735-
12	.328	.726	.317	.745+	.265+	.771
13	.381	.727	.360	.753	.298	.813
14	.431	.772	.361	.802	.323	.829
15	.478	.826	.409	.822	.384	.860
16	.512	.827	.457	.873	.420	.889
17	.569	.889	.543	.880	.429	.916
18	.619	.890	.591	.910	.500	.941
19	.672	.922	.639	.938	.571	.962
20	.726	.951	.683	.963	.614	.980
21	.772	.977	.745+	.984	.677	.993
22	.826	.995+	.802	.998	.735-	1.000
23	.889	1	.873	1	.813	1

n = 24

f	90%		95%		99%	
0	0	.105+	0	.122	0	.181
1	.004	.165+	.002	.191	.000	.259
2	.022	.221	.015+	.246	.006	.313
3	.047	.264	.035-	.308	.019	.364
4	.075-	.317	.059	.347	.036	.416
5	.105-	.370	.086	.396	.057	.464
6	.105+	.423	.115-	.443	.080	.536
7	.165-	.448	.122	.500	.106	.584
8	.165+	.552	.169	.557	.133	.636
9	.221	.553	.191	.604	.163	.638
10	.259	.587	.234	.653	.181	.687
11	.264	.630	.246	.661	.216	.720
12	.317	.683	.308	.692	.257	.743
13	.370	.736	.339	.754	.280	.784
14	.413	.741	.347	.766	.313	.819
15	.447	.779	.396	.809	.362	.837
16	.448	.835-	.443	.831	.364	.867
17	.552	.835+	.500	.878	.416	.894
18	.577	.895-	.557	.885+	.464	.920
19	.630	.895+	.604	.914	.536	.943
20	.683	.925+	.653	.941	.584	.964
21	.736	.953	.692	.965+	.636	.981
22	.779	.978	.754	.985-	.687	.994
23	.835-	.996	.809	.998	.741	1.000
24	.895-	1	.878	1	.819	1

TABLE B-13 continued

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

n = 25

f	90%		95%		99%	
0	0	.102	0	.118	0	.175+
1	.004	.159	.002	.185+	.000	.246
2	.021	.214	.014	.238	.006	.305-
3	.045-	.255-	.034	.303	.018	.352
4	.072	.307	.057	.336	.034	.403
5	.101	.362	.082	.384	.054	.451
6	.102	.390	.110	.431	.077	.500
7	.158	.432	.118	.475-	.101	.549
8	.159	.500	.161	.525+	.127	.597
9	.214	.568	.185+	.569	.155+	.648
10	.246	.610	.222	.616	.175+	.658
11	.255-	.611	.238	.664	.205+	.695+
12	.307	.640	.296	.683	.245+	.754
13	.360	.693	.317	.704	.246	.755-
14	.389	.745+	.336	.762	.305-	.795-
15	.390	.754	.384	.778	.342	.825-
16	.432	.786	.431	.815-	.352	.845-
17	.500	.841	.475-	.839	.403	.873
18	.568	.842	.525+	.882	.451	.899
19	.610	.898	.569	.890	.500	.923
20	.638	.899	.616	.918	.549	.946
21	.693	.928	.664	.943	.597	.966
22	.745+	.955+	.697	.966	.648	.982
23	.786	.979	.762	.986	.695+	.994
24	.841	.996	.815-	.998	.754	1.000
25	.898	1	.882	1	.825-	1

n = 26

f	90%		95%		99%	
0	0	.098	0	.114	0	.170
1	.004	.152	.002	.180	.000	.235-
2	.021	.209	.014	.230	.006	.298
3	.043	.247	.032	.283	.017	.342
4	.069	.299	.054	.325+	.033	.393
5	.097	.343	.079	.374	.052	.442
6	.098	.377	.106	.421	.073	.487
7	.151	.419	.114	.465-	.097	.526
8	.152	.460	.154	.506	.122	.562
9	.209	.540	.180	.542	.149	.607
10	.233	.581	.212	.579	.170	.658
11	.247	.623	.230	.626	.195-	.678
12	.299	.657	.282	.675-	.234	.702
13	.342	.658	.283	.717	.235-	.765+
14	.343	.701	.325+	.718	.298	.766
15	.377	.753	.374	.770	.322	.805+
16	.419	.767	.421	.788	.342	.830
17	.460	.791	458	.820	.393	.851
18	.540	.848	[illegible]	.846	.438	.878
19	.581	.849	[illegible]	.886	.474	.903
20	.623	.902	[illegible]	.894	.513	.927
21	.657	.903	.626	[illegible]	.558	.948
22	.701	.931	.675-	[illegible]	.607	.967
23	.753	.957	.717	.968	.658	.983
24	.791	.979	.770	.986	.702	.994
25	.848	.996	.820	.998	.765+	1.000
26	.902	1	.886	1	.830	1

TABLE B-13 continued

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

n = 27

f	90%		95%		99%	
0	0	.093	0	.110	0	.166
1	.004	.146	.002	.175	.000	.225-
2	.020	.204	.013	.223	.006	.297
3	.042	.239	.031	.270	.017	.332
4	.066	.291	.052	.316	.032	.384
5	.093	.327	.076	.364	.050	.419
6	.094	.365+	.101	.415	.070	.461
7	.145+	.407	.110	.437	.093	.539
8	.146	.447	.148	.500	.117	.581
9	.204	.500	.175-	.563	.143	.587
10	.221	.553	.202	.570	.166	.617
11	.239	.593	.223	.598	.185-	.668
12	.291	.635-	.269	.636	.224	.702
13	.326	.673	.270	.684	.225-	.716
14	.327	.674	.316	.730	.284	.775+
15	.365+	.709	.364	.731	.298	.776
16	.407	.761	.402	.777	.332	.815+
17	.447	.779	.430	.798	.383	.834
18	.500	.796	.437	.825+	.413	.857
19	.553	.854	.500	.852	.419	.883
20	.593	.855-	.563	.890	.461	.907
21	.635-	.906	.585+	.899	.539	.930
22	.673	.907	.636	.924	.581	.950
23	.709	.934	.684	.948	.616	.968
24	.761	.958	.730	.969	.668	.983
25	.796	.980	.777	.987	.703	.994
26	.854	.996	.825+	.998	.775+	1.000
27	.907	1	.890	1	.834	1

n = 28

f	90%		95%		99%	
0	0	.090	0	.106	0	.162
1	.004	.140	.002	.170	.000	.218
2	.019	.201	.013	.217	.005+	.273
3	.040	.232	.030	.259	.016	.323
4	.064	.284	.050	.307	.031	.365-
5	.089	.312	.073	.357	.048	.408
6	.090	.355-	.098	.384	.068	.449
7	.139	.396	.106	.424	.089	.500
8	.140	.435+	.142	.463	.112	.551
9	.197	.473	.170	.537	.137	.592
10	.208	.527	.192	.576	.162	.635
11	.232	.565-	.217	.616	.175+	.636
12	.284	.604	.258	.619	.214	.677
13	.310	.645+	.259	.645+	.218	.727
14	.312	.688	.307	.693	.272	.728
15	.355-	.690	.355-	.741	.273	.782
16	.396	.716	.381	.742	.323	.786
17	.435+	.768	.384	.783	.364	.825-
18	.473	.792	.424	.808	.365-	.838
19	.527	.803	.463	.830	.408	.863
20	.565-	.860	.537	.858	.449	.888
21	.604	.861	.576	.894	.500	.911
22	.645+	.910	.616	.902	.551	.932
23	.688	.911	.643	.927	.592	.952
24	.716	.936	.693	.950	.635+	.969
25	.768	.960	.741	.970	.677	.984
26	.799	.981	.783	.987	.727	.995-
27	.860	.996	.830	.998	.782	1.000
28	.910	1	.894	1	.838	1

n = 29

f	90%		95%		99%	
0	0	.087	0	.103	0	.160
1	.004	.135-	.002	.166	.000	.211
2	.018	.190	.012	.211	.005+	.263
3	.039	.225-	.029	.251	.015+	.316
4	.062	.279	.049	.299	.030	.354
5	.086	.303	.070	.340	.046	.397
6	.087	.345-	.094	.374	[illegible]	[illegible]
7	.134	.385+	.103	.413	.086	.477
8	.135-	.425	.136	.451	.108	.523
9	.189	.463	.166	.500	.132	.562
10	.190	.500	.184	.549	.157	.603

n = 30

f	90%		95%		99%	
0	0	.084	0	.100	0	.152
1	.004	.130	.002	.163	.000	.206
2	.018	.183	.012	.205+	.005+	.256
3	.037	.219	.028	.244	.015-	.310
4	.059	.266	.047	.292	.028	.345-
5	.083	.295-	.068	.325-	.045-	.388
6	.084	.336	[illegible]	[illegible]	.062	.420
7	.129	.376	.100	.403	.083	.469
8	.130	.416	.131	.440	.104	.505+
9	.182	.455+	.163	.476	.127	.538
10	.183	.492	.175+	.524	.151	.570

TABLE B-13 continued

CONFIDENCE LIMITS FOR A PROPORTION (TWO SIDED)

n = 29						
f	90%		95%		99%	
11	.225-	.537	.211	.587	.165+	.646
12	.276	.575+	.247	.626	.206	.654
13	.294	.615-	.251	.669	.211	.684
14	.303	.655+	.299	.661	.260	.737
15	.345-	.697	.339	.701	.263	.740
16	.385+	.706	.340	.749	.316	.789
17	.4[illegible]5-	.724	.374	.753	.346	.794
18	.463	.775+	.413	.789	.354	.835-
19	.500	.810	.451	.816	.397	.843
20	.537	.811	.500	.834	.438	.868
21	.575+	.865+	.549	.864	.477	.892
22	.615-	.866	.587	.897	.523	.914
23	.655+	.913	.626	.906	.562	.935-
24	.697	.914	.660	.930	.603	.954
25	.721	.938	.701	.951	.646	.970
26	.775+	.961	.749	.971	.684	.985-
27	.810	.982	.789	.988	.737	.995-
28	.865+	.996	.834	.998	.789	1.000
29	.913	1	.897	1	.840	1

N = 30						
f	90%		95%		99%	
11	.219	.524	.205+	.560	.152	.612
12	.265-	.554	.236	.597	.198	.655+
13	.266	.584	.244	.636	.206	.671
14	.295-	.624	.292	.675+	.249	.692
15	.336	.664	.324	.676	.256	.744
16	.376	.705+	.325-	.708	.308	.751
17	.416	.734	.364	.756	.329	.794
18	.446	.735+	.403	.764	.345-	.802
19	.476	.781	.440	.795-	.388	.848
20	.508	.817	.476	.825-	.430	.849
21	.545-	.818	.524	.837	.462	.873
22	.584	.870	.560	.869	.495-	.896
23	.624	.871	.597	.900	.531	.917
24	.664	.916	.636	.909	.570	.937
25	.705+	.917	.675+	.932	.612	.955+
26	.734	.941	.708	.952	.655+	.972
27	.781	.963	.756	.972	.690	.985+
28	.817	.982	.795-	.988	.744	.995-
29	.870	.996	.837	.998	.794	1.000
30	.916	1	.900	1	.848	1

TABLE B-14

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

If the observed proportion is f/n, enter the table with N and f for an upper one-sided limit. For a lower one-sided limit, enter the table with N and N —— f and subtract the table entry from 1.

f	90%	95%	99%	f	90%	95%	99%	f	90%	95%	99%
	n = 2				n = 3				n = 4		
0	.684	.776	.900	0	.536	.632	.785-	0	.438	.527	.648
1	.949	.975-	.995-	1	.804	.865-	.941	1	.680	.751	.859
				2	.965+	.983	.997	2	.857	.902	.958
								3	.974	.987	.997
	n = 5				n = 6				n = 7		
0	.369	.451	.602	0	.319	.393	.536	0	.280	.348	.482
1	.584	.657	.778	1	.510	.582	.706	1	.453	.521	.643
2	.753	.811	.894	2	.667	.729	.827	2	.596	.659	.764
3	.888	.924	.967	3	.799	.847	.915+	3	.721	.775-	.858
4	.979	.990	.998	4	.907	.937	.973	4	.830	.871	.929
				5	.983	.991	.998	5	.921	.947	.977
								6	.985+	.993	.999
	n = 8				n = 9				n = 10		
0	.250	.312	.438	0	.226	.283	.401	0	.206	.259	.369
1	.406	.471	.590	1	.368	.429	.544	1	.337	.394	.504
2	.538	.600	.707	2	.490	.550	.656	2	.450	.507	.612
3	.655+	.711	.802	3	.599	.655+	.750	3	.552	.607	.703
4	.760	.807	.879	4	.699	.749	.829	4	.646	.696	.782
5	.853	.889	.939	5	.790	.831	.895-	5	.733	.778	.850
6	.931	.954	.980	6	.871	.902	.947	6	.812	.850	.907
7	.987	.994	.999	7	.939	.959	.983	7	.884	.913	.952
				8	.988	.994	.999	8	.945+	.963	.984
								9	.990	.995-	.999
	n = 11				n = 12				n = 13		
0	.189	.238	.342	0	.175-	.221	.319	0	.162	.206	.298
1	.310	.364	.470	1	.287	.339	.440	1	.268	.316	.413
2	.415+	.470	.572	2	.386	.438	.537	2	.360	.410	.506
3	.511	.564	.660	3	.475+	.527	.622	3	.444	.495-	.588
4	.599	.650	.738	4	.559	.609	.698	4	.523	.573	.661
5	.682	.729	.806	5	.638	.685-	.765+	5	.598	.645+	.727
6	.759	.800	.866	6	.712	.755-	.825+	6	.669	.713	.787
7	.831	.865-	.916	7	.781	.819	.879	7	.736	.776	.841
8	.895+	.921	.957	8	.846	.877	.924	8	.799	.834	.889
9	.951	.967	.986	9	.904	.928	.961	9	.858	.887	.931
10	.990	.995+	.999	10	.955-	.970	.987	10	.912	.934	.964
				11	.991	.996	.999	11	.958	.972	.988
								12	.992	.996	.999
	n = 14				n = 15				n = 16		
0	.152	.193	.280	0	.142	.181	.264	0	.134	.171	.250
1	.251	.297	.389	1	.236	.279	.368	1	.222	.264	.349
2	.337	.385+	.478	2	.317	.363	.453	2	.300	.344	.430
3	.417	.466	.557	3	.393	.440	.529	3	.371	.417	.503
4	.492	.540	.627	4	.464	.511	.597	4	.439	.484	.569
5	.563	.610	.692	5	.532	.577	.660	5	.504	.548	.630

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TABLE B-14 continued

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

n = 14 (continued)

f	90%	95%	99%
6	.631	.675-	.751
7	.695+	.736	.805+
8	.757	.794	.854
9	.815-	.847	.898
10	.869	.896	.936
11	.919	.939	.967
12	.961	.974	.989
13	.993	.996	.999

n = 15 (continued)

f	90%	95%	99%
6	.596	.640	.718
7	.658	.700	.771
8	.718	.756	.821
9	.774	.809	.865+
10	.828	.858	.906
11	.878	.903	.941
12	.924	.943	.969
13	.964	.976	.990
14	.993	.997	.999

n = 16 (continued)

f	90%	95%	99%
6	.565+	.609	.687
7	.625-	.667	.739
8	.682	.721	.788
9	.737	.773	.834
10	.790	.822	.875-
11	.839	.868	.912
12	.886	.910	.945-
13	.929	.947	.971
14	.966	.977	.990
15	.993	.997	.999

n = 17

f	90%	95%	99%
0	.127	.162	.237
1	.210	.250	.332
2	.284	.326	.410
3	.352	.396	.480
4	.416	.461	.543
5	.478	.522	.603
6	.537	.580	.658
7	.594	.636	.709
8	.650	.689	.758
9	.703	.740	.803
10	.754	.788	.845
11	.803	.834	.883
12	.849	.876	.918
13	.893	.915+	.948
14	.933	.950	.973
15	.968	.979	.991
16	.994	.997	.999

n = 18

f	90%	95%	99%
0	.120	.153	.226
1	.199	.238	.316
2	.269	.310	.391
3	.334	.377	.458
4	.396	.439	.520
5	.455+	.498	.577
6	.513	.554	.631
7	.567	.608	.681
8	.620	.659	.729
9	.671	.709	.774
10	.721	.756	.816
11	.769	.801	.855-
12	.815-	.844	.890
13	.858	.884	.923
14	.899	.920	.951
15	.937	.953	.975-
16	.970	.980	.992
17	.994	.997	.999

n = 19

f	90%	95%	99%
0	.114	.146	.215+
1	.190	.226	.302
2	.257	.296	.374
3	.319	.359	.439
4	.378	.419	.498
5	.434	.476	.554
6	.489	.530	.606
7	.541	.582	.655+
8	.592	.632	.702
9	.642	.680	.746
10	.690	.726	.788
11	.737	.770	.827
12	.782	.812	.863
13	.825-	.853	.897
14	.866	.890	.927
15	.905-	.925-	.954
16	.941	.956	.976
17	.972	.981	.992
18	.994	.997	.999

n = 20

f	90%	95%	99%
0	.109	.139	.206
1	.181	.216	.289
2	.245-	.283	.358
3	.304	.344	.421
4	.361	.401	.478
5	.415-	.456	.532
6	.467	.508	.583
7	.518	.558	.631
8	.567	.606	.677
9	.615+	.653	.720
10	.662	.698	.761
11	.707	.741	.800
12	.751	.783	.837
13	.793	.823	.871
14	.834	.860	.902
15	.873	.896	.931
16	.910	.929	.956
17	.944	.958	.977
18	.973	.982	.992
19	.995-	.997	.999

n = 21

f	90%	95%	99%
0	.104	.133	.197
1	.173	.207	.277
2	.234	.271	.344
3	.291	.329	.404
4	.345+	.384	.460
5	.397	.437	.512
6	.448	.487	.561
7	.497	.536	.608
8	.544	.583	.653
9	.590	.628	.695+
10	.636	.672	.736
11	.679	.714	.774
12	.722	.755+	.811
13	.764	.794	.845+
14	.804	.832	.878
15	.842	.868	.908
16	.879	.901	.935-
17	.914	.932	.959
18	.946	.960	.978
19	.973	.983	.993
20	.995-	.988	1.000

n = 22

f	90%	95%	99%
0	.099	.127	.189
1	.166	.198	.266
2	.224	.259	.330
3	.279	.316	.389
4	.331	.369	.443
5	.381	.420	.493
6	.430	.468	.541
7	.477	.515+	.587
8	.523	.561	.630
9	.569	.605-	.672
10	.611	.647	.712
11	.654	.689	.750
12	.695+	.729	.786
13	.736	.767	.821
14	.775+	.804	.853
15	.813	.840	.884
16	.850	.874	.912
17	.885+	.906	.938
18	.918	.935+	.961
19	.949	.962	.979
20	.976	.984	.993
21	.995+	.998	1.000

TABLE B-14 continued

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

f	90%	95%	99%	f	90%	95%	99%	f	90%	95%	99%
n = 23				n = 24				n = 25			
0	.095+	.122	.181	0	.091	.117	.175-	0	.088	.113	.168
1	.159	.190	.256	1	.153	.183	.246	1	.147	.176	.237
2	.215+	.249	.318	2	.207	.240	.307	2	.199	.231	.296
3	.268	.304	.374	3	.258	.292	.361	3	.248	.282	.349
4	.318	.355-	.427	4	.306	.342	.412	4	.295-	.330	.398
5	.366	.404	.476	5	.352	.389	.460	5	.340	.375+	.444
6	.413	.451	.522	6	.398	.435-	.505-	6	.383	.420	.488
7	.459	.496	.567	7	.442	.479	.548	7	.426	.462	.531
8	.503	.540	.609	8	.484	.521	.590	8	.467	.504	.571
9	.546	.583	.650	9	.526	.563	.630	9	.508	.544	.610
10	.589	.625-	.689	10	.567	.603	.668	10	.548	.583	.648
11	.630	.665-	.727	11	.608	.642	.705-	11	.587	.621	.684
12	.670	.704	.763	12	.647	.681	.740	12	.625-	.659	.719
13	.710	.742	.797	13	.685+	.718	.774	13	.662	.695-	.752
14	.748	.778	.829	14	.723	.754	.806	14	.699	.730	.784
15	.786	.814	.860	15	.759	.788	.837	15	.735-	.764	.815+
16	.822	.848	.889	16	.795+	.822	.867	16	.770	.798	.845+
17	.857	.880	.916	17	.830	.854	.894	17	.804	.830	.873
18	.890	.910	.941	18	.863	.885+	.920	18	.837	.861	.899
19	.922	.938	.962	19	.895+	.914	.943	19	.869	.890	.923
20	.951	.963	.980	20	.925+	.941	.964	20	.899	.918	.946
21	.977	.984	.993	21	.953	.965+	.981	21	.928	.943	.966
22	.995+	.998	1.000	22	.978	.985-	.994	22	.955+	.966	.982
				23	.996	.998	1.000	23	.979	.986	.994
								24	.996	.998	1.000
n = 26				n = 27				n = 28			
0	.085-	.109	.162	0	.082	.105+	.157	0	.079	.101	.152
1	.142	.170	.229	1	.137	.164	.222	1	.132	.159	.215-
2	.192	.223	.286	2	.185+	.215+	.277	2	.179	.208	.268
3	.239	.272	.337	3	.231	.263	.326	3	.223	.254	.316
4	.284	.318	.385-	4	.275-	.308	.373	4	.265+	.298	.361
5	.328	.363	.430	5	.317	.351	.417	5	.306	.339	.404
6	.370	.405+	.473	6	.358	.392	.458	6	.346	.380	.445-
7	.411	.447	.514	7	.397	.432	.498	7	.385-	.419	.484
8	.451	.487	.554	8	.436	.471	.537	8	.422	.457	.521
9	.491	.526	.592	9	.475-	.509	.574	9	.459	.494	.558
10	.529	.564	.628	10	.512	.547	.610	10	.496	.530	.593
11	.567	.602	.664	11	.549	.583	.645+	11	.532	.565+	.627
12	.604	.638	.698	12	.585-	.618	.679	12	.567	.600	.660
13	.641	.673	.731	13	.620	.653	.711	13	.601	.634	.692
14	.676	.708	.763	14	.655+	.687	.743	14	.635+	.667	.723
15	.711	.742	.794	15	.689	.720	.773	15	.669	.699	.753
16	.746	.774	.823	16	.723	.752	.802	16	.701	.731	.782
17	.779	.806	.851	17	.756	.783	.831	17	.733	.762	.810
18	.812	.837	.878	18	.788	.814	.857	18	.765-	.792	.837
19	.843	.866	.903	19	.819	.843	.883	19	.796	.821	.863
20	.874	.894	.927	20	.849	.871	.907	20	.826	.849	.888
21	.903	.921	.948	21	.879	.899	.930	21	.855+	.876	.911
22	.931	.946	.967	22	.907	.924	.950	22	.883	.902	.932
23	.957	.968	.983	23	.934	.948	.968	23	.911	.927	.952
24	.979	.986	.994	24	.958	.969	.983	24	.936	.950	.969
25	.996	.998	1.000	25	.980	.987	.994	25	.960	.970	.984
				26	.996	.998	1.000	26	.981	.987	.995-
								27	.996	.998	1.000

TABLE B-14 continued

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

f	90%	95%	99%	f	90%	95%	99%
n = 23				n = 30			
0	.076	.098	.147	0	.074	.095+	.142
1	.128	.153	.208	1	.124	.149	.202
2	.173	.202	.260	2	.168	.195+	.252
3	.216	.246	.307	3	.209	.239	.298
4	.257	.288	.350	4	.249	.280	.340
5	.297	.329	.392	5	.287	.319	.381
6	.335-	.368	.432	6	.325-	.357	.420
7	.372	.406	.470	7	.361	.394	.457
8	.409	.443	.507	8	.397	.430	.493
9	.445+	.479	.542	9	.432	.465+	.527
10	.481	.514	.577	10	.466	.499	.561
11	.515+	.549	.610	11	.500	.533	.594
12	.550	.583	.643	12	.533	.566	.626
13	.583	.616	.674	13	.566	.598	.657
14	.616	.648	.705-	14	.599	.630	.687
15	.649	.680	.734	15	.630	.661	.716
16	.681	.711	.763	16	.662	.692	.744
17	.712	.741	.791	17	.692	.721	.772
18	.743	.771	.818	18	.723	.750	.799
19	.774	.800	.843	19	.752	.779	.824
20	.803	.828	.868	20	.782	.807	.849
21	.832	.855-	.892	21	.810	.834	.873
22	.860	.881	.914	22	.838	.860	.896
23	.888	.906	.935-	23	.865+	.885+	.917
24	.914	.930	.954	24	.891	.909	.937
25	.938	.951	.970	25	.917	.932	.955+
26	.961	.971	.985-	26	.941	.953	.972
27	.982	.988	.995-	27	.963	.972	.985+
28	.996	.998	1.000	28	.982	.988	.995-
				29	.996	.998	1.000

TABLE B-15

TABLE OF ARC SINE TRANSFORMATION FOR PROPORTIONS

$\theta = 2 \text{ arc } \sin\sqrt{P}$

P	θ	P	θ	P	θ	P	θ
.00	.00	.25	1.05	.50	1.57	.75	2.09
.01	.20	.26	1.07	.51	1.59	.76	2.12
.02	.28	.27	1.09	.52	1.61	.77	2.14
.03	.35	.28	1.12	.53	1.63	.78	2.17
.04	.40	.29	1.14	.54	1.65	.79	2.19
.05	.45	.30	1.16	.55	1.67	.80	2.21
.06	.49	.31	1.18	.56	1.69	.81	2.24
.07	.54	.32	1.20	.57	1.71	.82	2.27
.08	.57	.33	1.22	.58	1.73	.83	2.29
.09	.61	.34	1.25	.59	1.75	.84	2.32
.10	.64	.35	1.27	.60	1.77	.85	2.35
.11	.68	.36	1.29	.61	1.79	.86	2.37
.12	.71	.37	1.31	.62	1.81	.87	2.40
.13	.74	.38	1.33	.63	1.83	.88	2.43
.14	.77	.39	1.35	.64	1.85	.89	2.47
.15	.80	.40	1.37	.65	1.88	.90	2.50
.16	.82	.41	1.39	.66	1.90	.91	2.53
.17	.85	.42	1.41	.67	1.92	.92	2.57
.18	.88	.43	1.43	.68	1.94	.93	2.61
.19	.90	.44	1.45	.69	1.96	.94	2.65
.20	.93	.45	1.47	.70	1.98	.95	2.69
.21	.95	.46	1.49	.71	2.00	.96	2.74
.22	.98	.47	1.51	.72	2.03	.97	2.79
.23	1.00	.48	1.53	.73	2.05	.98	2.86
.24	1.02	.49	1.55	.74	2.07	.99	2.94
						1.00	3.14

TABLE B-16

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES WITH UNEQUAL SAMPLES

Table B-16 shows (1) for given a_1, n_1, and n_2, the value of a_2, as the whole digit (e.g., for $a_1 = 5$, $n_1 = 5$, $n_2 = 4$, $a_2 = 1$) which is just significant at the probability level quoted in parentheses for a two-sided test and without parentheses for a one-sided test, (2) in small type, for a given n_1, n_2 and $a_1 + a_2$, the exact probability (if there is independence) that a_2 is equal to or less than the integer shown in bold type.

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1 =3, n_2 =3	3	0.50	-	-	-
n =4 n =4	4	0.014	0.014	-	-
3	4	0.029	-	-	-
n_1 =5 n_2 =5	5	1.024	1.024	0.004	0.004
	4	0.024	0.024	-	-
4	5	1.048	0.008	0.008	-
	4	0.040			
3	5	0.018	0.018	-	-
2	5	0.048	-	-	-
n_1 =6 n_2 =6	6	2.030	1.008	1.008	0.001
	5	1.040	0.008	0.008	-
	[illegible]	0.030	-	-	-
5	6	1.015+	1.015+	0.002	0.002
	5	0.013	0.013	-	-
	4	.045+	-	-	-
4	6	1.033	0.005-	0.005-	0.005-
	5	0.024	0.024	-	-
3	6	0.012	0.012	-	-
	5	0.045	-		
2	6	0.036		-	-
n_1 =7 n_2= 7	7	3.035-	2.010+	1.002	1.002
	6	1.015-	1.015-	0.002	0.002
	5	0.010+	0.010+	-	-
	4	0.035-	-	-	-
6	7	2.021	2.021	1.005-	1.005-
	6	1.025+	0.004	0.004	0.004
	5	0.016	0.016	-	-
	4	0.049	-	-	-
5	7	2.045+	1.010+	0.001	0.001
	6	1.045+	0.008	0.008	-
	5	0.027	-	-	-
4	7	1.024	1.024	0.002	0.002
		.015+	.015+	-	-
	5	0.45+	-	-	-
3	7	0.008	0.008	0.008	-
	6	0.033	-	-	-
2	7	.026	-	-	-

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=8 n_2 =8	8	4.038	3.012	2.003	2.003
		2.020	2.020	1.005+	0.001
	6	1.020	1.020	0.003	0.003
	5	0.013	0.013	-	-
	4	0.038	-	-	-
7	8	3.026	2.007	2.007	1.001
	7	2.035-	1.009	1.009	0.001
	6	1.032	0.006	0.006	-
	5	0.019	0.019	-	-
6	8	2.015-	2.015-	1.003	1.003
	7	1.016	1.016	0.002	0.002
	6	0.009	0.009	0.009	-
	5	0.028	-	-	-
5	8	2.035-	1.007	1.007	0.001
	7	1.032	0.005-	0.005-	0.005-
	6	0.016	0.016	-	-
	5	0.044	-	-	-
4	8	1.018	1.018	0.002	0.002
	7	0.010+	0.010+	-	-
	6	0.030	-	-	-
3	8	0.006	0.006	0.006	-
	7	0.024	0.024	-	-
2	8	0.022	0.022	-	-
n_1 =9 n_2 =9	9	5.041	4.015-	3.005-	3.005-
	8	3.025-	3.025-	2.008	1.002
	7	2.028	1.008	1.008	0.001
	6	1.025-	1.025-	0.005-	0.005-
	5	0.015-	0.015-	-	-
	4	0.041	-	-	-
8	9	4.029	3.009	3.009	2.002
	8	3.043	2.013	1.003	1.002
	7	2.044	1.012	0.002	0.002
	6	1.036	0.007	0.007	-
	5	0.020	0.020	0 -	-
7	9	3.019	3.019	2.005-	2.005-
	8	2.024	2.024	1.006	0.001
	7	1.020	1.020	0.003	0.003
	6	0.010+	0.010+	-	-
		.029	-	-	-
6	9	2.044	2.011	1.002	1.002
	8	2.047	1.011	0.002	0.[illegible]
	7	1.[illegible]-	0.006	0.[illegible]	-
	6	0.017	0.017	-	-
	5	0.042	-	-	

Adapted from a table of the same form with probabilities to 4 decimals prepared in the Statistical Engineering Laboratory, National Bureau of Standards, by Anna M. Glinski and John Van Dyke from tables of the Hypergeometric Probability Distribution by Gerald J. Lieberman and Donald B. Owen, Technical Report No. 50 (contract Nonr-225(53) (NR 042-002), applied Mathematics and Statistics Laboratories, Stanford University, Stanford, California.

TABLE B-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
$n_1=9$ $n_2=5$	9	2.027	1.005-	1.005-	1.005-
	8	1.023	1.023	0.003	0.003
	7	0.010+	0.010+	-	-
	6	0.028	-	-	-
4	9	1.014	1.014	0.001	0.001
	8	0.007	0.007	0.007	-
	7	0.021	0.021	-	-
	6	0.049	-	-	-
3	9	1.045+	0.005-	0.005-	.005-
	8	0.018	0.018	-	-
	7	0.045+	-	-	-
2	9	0.018	0.018	-	-
$n_1=10$ $n_2=10$	10	6.043	5.016	4.005+	3.002
	9	4.029	3.010-	3.010-	2.003
	8	3.035-	2.012	1.003	1.003
	7	2.035-	1.010-	1.010-	0.002
	6	1.029	0.005+	0.005+	-
	5	0.016	0.016	-	-
	4	0.043	-	-	-
9	10	5.033	4.011	3.003	3.003
	9	4.050-	3.017	2.005-	2.005-
	8	2.019	2.019	1.004	1.004
	7	1.015-	1.002	0.002	0.002
	6	1.040	0.005	0.008	-
	5	0.022	0.022	-	-
8	10	4.023	4.023	3.007	2.002
	9	3.032	2.009	2.009	1.001
	8	2.031	1.008	1.008	0.001
	7	1.023	1.023	0.004	0.004
	6	0.011	0.011	-	-
	5	0.029	-	-	-
7	10	3.015-	3.015-	2.003	2.003
	9	2.018	2.018	1.004	1.004
	8	1.013	1.013	0.001	0.002
	7	1.036	0.006	0.006	-
	6	0.017	0.017	-	-
	5	0.041	-	-	-
6	10	3.036	2.008	2.008	1.001
	9	2.036	1.008	1.001	0.001
	8	1.024	1.024	0.003	0.003
	7	0.010+	0.010+	-	-
	6	0.026	-	-	-
5	10	2.022	2.022	1.004	1.004
	9	1.017	1.017	0.002	0.002
	8	1.047	0.007	0.007	-
	7	0.019	0.019	-	-
	6	0.042	-	-	-
$n_1=10$ $n_2=4$	10	1.011	1.011	0.001	0.001
	9	1.041	0.005-	0.005-	0.005-
	8	0.015-	0.015-	-	-
	7	0.035-	-	-	-
3	10	1.038	0.003	0.003	0.003
	9	0.014	0.014	-	-
	8	0.035	-	-	-
2	10	0.015+	0.015+	-	-
	9	0.045+	-	-	-
$n_1=11$ $n_2=11$	11	7.045+	6.018	5.006	4.002
	10	5.032	4.012	3.004	3.004
	9	4.040	3.015-	2.004	2.004
	8	3.043	2.015-	1.004	1.004
	7	2.040	1.012	0.002	0.002
	6	1.032	0.006	0.006	-
	5	0.018	0.018	-	-
	4	0.045+	-	-	-
10	11	6.035+	5.012	4.004	4.004
	10	4.021	4.021	3.007	2.002
	9	3.024	3.024	2.007	1.002
	8	2.023	2.023	1.006	0.001
	7	1.017	1.017	0.003	0.003
	6	1.043	0.009	0.009	-
	5	0.023	0.023	-	-
9	11	5.026	4.003	4.003	3.002
	10	4.038	3.012	2.003	2.003
	9	3.040	2.012	1.003	1.003
	8	2.035-	1.009	0.009	0.001
	7	1.025-	1.025-	0.004	0.004
	6	0.012	0.012	-	-
	5	0.030	-	-	-
8	11	4.013	4.018	3.005-	3.005-
	10	3.024	3.024	2.006	1.001
	9	2.023	2.022	1.005-	1.005-
	8	1.015-	1.015-	0.002	0.002
	7	1.037	0.007	0.007	-
	6	0.017	0.017	-	-
	5	0.040			
7	11	4.043	3.011	2.002	2.002
	10	3.047	2.013	1.002	1.002
	9	2.039	1.009	1.004	0.001
	8	1.025-	1.025-	0.004	0.004
	7	0.010+	0.010+	-	-
	6	0.025-	0.025-	-	-
6	11	3.029	2.004	2.006	1.001
	10	2.029	1.005+	1.005+	0.001
	9	1.018	1.018	0.002	0.002

TABLE B-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

		a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
$n_1 = 11$ $n_2 = 6$		8	1.043	0.007	0.007	-
		7	0.017	0.017	-	-
		6	0.037	-	-	-
	5	11	2.018	2.018	1.003	1.003
		10	1.013	1.013	0.001	0.001
		9	1.036	0.005-	0.005-	0.005-
		8	0.013	0.013	-	-
		7	0.029	-	-	-
	4	11	1.009	1.009	1.009	0.001
		10	1.033	0.004	0.004	0.004
		9	0.011	0.011	-	-
		8	0.026	-	-	-
	3	11	1.033	0.003	0.003	0.003
		10	0.011	0.011	-	-
		9	0.027	-	-	-
	2	11	0.013	0.013	-	-
		10	0.038	-	-	-
$n_1 = 12$ $n_2 = 12$		12	8.047	7.019	6.007	5.002
		11	6.034	5.014	4.005-	4.005-
		10	5.045-	..018	3.006	2.002
		9	4.050-	3.020	2.006	1.001
		8	3.050-	2.018	1.005-	1.005-
		7	2.045-	1.014	0.002	0.002
		6	1.034	0.007	0.007	-
		5	0.019	0.019	-	-
		4	0.047	-	-	-
	11	12	7.037	6.014	5.005-	5.005-
		11	5.024	5.024	4.008	3.002
		10	4.029	3.010+	2.003	2.003
		9	3.030	2.009	2.009	1.002
		8	2.026	1.007	1.007	0.001
		7	1.019	1.019	0.003	0.003
		6	1.045-	0.009	0.009	-
		5	0.024	0.024	-	-
	10	12	6.029	5.010-	5.010-	4.003
		11	5.043	4.015+	3.005-	3.005-
		10	4.048	3.017	2.005-	2.005-
		9	3.046	2.015-	1.004	1.004
		8	2.038	1.010+	0.002	0.002
		7	1.026	0.005-	0.005-	0.005-
		6	0.012	0.012	-	-
		5	0.030	-	-	-
	9	12	5.021	5.021	4.006	3.002
		11	4.029	3.009	3.009	2.002
		10	3.029	2.008	2.008	1.002
		9	2.024	2.024	1.006	0.001
		8	1.016	1.016	0.002	0.002
$n_1 = 12$ $n_2 = 9$		7	1.037	0.007	0.007	-
		6	0.017	0.017		-
		5	0.039	-	-	-
	8	12	5.049	4.014	3.004	3.004
		11	3.018	3.018	2.004	2.004
		10	2.015+	2.015+	1.003	1.003
		9	2.040	1.010-	1.010-	0.001
		8	1.025-	1.025-	0.004	0.004
		7	0.010+	0.010+	-	-
		6	0.024	0.024	-	-
	7	12	4.036	3.009	3.009	2.002
		11	3.038	2.010-	2.010-	1.002
		10	2.029	1.006	1.006	0.001
		9	1.017	1.017	0.002	0.002
		8	1.040	0.007	0.007	-
		7	0.016	0.016	-	-
		6	0.034	-	-	-
	6	12	3.025-	3.02^-	2.005-	2.005-
		11	2.022	2.022	1.004	1.004
		10	1.013	1.013	0.002	0.002
		9	1.032	0.005-	0.005-	0.005-
		8	0.011	0.011	-	-
		7	0.025-	0.025-	-	-
		6	0.050-	-	-	-
	5	12	2.015-	2.015-	1.002	1.002
		11	1.010-	1.010-	1.010-	0.001
		10	1.028	0.003	0.003	0.003
		9	0.009	0.009	0.009	-
		8	0.020	0.020	-	-
		7	0.041	-	-	-
	4	12	2.050	1.007	1.007	0.001
		11	1.027	0.003	0.003	0.003
		10	0.008	0.008	0.008	-
		9	0.019	0.019	-	-
		8	0.038	-	-	-
	3	12	1.029	0.002	0.002	0.002
		11	0.009	0.009	0.009	-
		10	0.022	0.022	-	-
		9	0.044	-	-	-
	2	12	0.011	0.011	-	-
		11	0.033	-	-	-
$n_1 = 13$ $n_2 = 13$		13	9.048	8.020	7.007	6.003
		12	7.037	6.015+	5.006	4.002
		11	6.048	5.021	4.008	3.002
		10	4.024	4.024	3.008	2.002
		9	3.024	3.024	2.008	1.002
		8	2.021	2.021	1.006	0.001

TABLE B-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
$n_1=18$ $n_2=13$	7	2.048	1.015+	0.003	0.003
	6	1.037	0.007	0.007	-
	5	0.020	0.020	-	-
	4	0.048	-	-	-
12	13	8.039	7.015-	6.005+	5.002
	12	6.027	5.010-	5.010-	4.003
	11	5.033	4.013	3.004	3.004
	10	4.036	3.013	2.004	2.004
	9	3.034	2.011	1.003	1.003
	8	2.029	1.008	1.008	0.001
	7	1.020	1.020	0.004	0.004
	6	1.046	0.010-	0.010-	-
	5	0.024	0.024	-	-
11	13	7.031	6.011	5.003	5.003
	12	6.048	5.018	4.006	3.002
	11	4.021	4.021	3.007	2.002
	10	3.021	3.021	2.006	1.001
	9	3.050-	2.017	1.004	1.004
	8	2.040	1.011	0.002	0.002
	7	1.027	0.005-	0.005-	0.005-
	6	0.012	0.012	-	-
	5	0.030	-	-	-
10	13	6.024	6.024	5.007	4.002
	12	5.035-	4.012	3.003	3.003
	11	4.037	3.012	2.003	2.003
	10	3.033	2.010+	1.002	1.002
	9	2.026	1.006	1.006	0.001
	8	1.017	1.017	0.003	0.003
	7	1.028	0.007	0.007	-
	6	0.017	0.017	-	-
	5	0.036	-	-	-
9	13	5.017	5.017	4.005-	4.005
	12	4.023	4.023	3.007	2.001
	11	3.022	3.023	2.006	1.001
	10	2.017	2.017	1.004	1.004
	9	2.040	1.010+	0.001	0.001
	8	1.025-	1.02[illegible]-	0.004	0.004
	7	0.010+	0.010+	-	-
	6	0.022	0.022	-	-
	5	[illegible]	-	-	-
8	13	5.042	4.012	3.003	3.003
	12	4.047	3.014	2.003	2.003
	11	3.041	2.011	1.002	1.002
	10	2.029	1.007	1.007	0.001
	9	1.017	1.017	0.002	0.002
	8	1.037	0.006	0.006	-
	7	0.015-	0.015-	-	-
	6	0.022	-	-	-
7	13	4.031	3.007	3.007	2.001
	12	3.031	2.007	2.007	1.001
$n_1=13$ $n_2=7$	11	2.022	1.022	1.004	1.004
	10	1.012	0.012	0.002	0.002
	9	1.029	0.004	0.004	0.004
	8	0.010+	0.010+	-	-
	7	0.022	0.022	-	-
	6	0.044	-	-	-
6	13	3.021	3.021	2.004	2.004
	12	2.017	2.017	1.003	1.003
	11	2.046	1.010-	1.010-	0.001
	10	1.024	1.024	0.003	0.003
	9	1.050-	0.008	0.008	-
	8	0.017	0.017	-	-
	7	0.034	-	-	-
5	13	2.012	2.012	1.002	1.002
	12	2.044	1.008	1.008	0.001
	11	1.022	1.022	0.002	0.022
	10	1.047	0.007	0.007	-
	9	0.015-	0.015-	-	-
	8	0.029	-	-	-
4	13	2.044	1.006	1.006	0.000
	12	1.022	1.022	0.002	0.002
	11	0.006	0.006	0.006	-
	10	0.015-	0.015-	-	-
	9	0.029	-	-	-
3	13	1.025	1.025	0.002	0.002
	12	0.007	0.007	0.007	-
	11	0.018	0.018	-	-
	10	0.036	-	-	-
2	13	0.010-	0.010-	0,010-	-
	12	0.029	-	-	-
$n_1=14$ $n_2=14$	14	10.049	9.020	8.008	7.003
	13	3.038	7.016	6.006	5.002
	12	6.023	6.023	5.009	4.003
	11	5.027	4.011	3.004	3.004
	10	4.026	3.011	2.003	2.003
	9	3.027	2.009	2.009	1.002
	8	2.023	2.023	1.006	0.001
	7	1.016	1.016	0.003	0.003
	6	1.038	1.008	0.008	0 -
	5	0.020	0.020	-	-
	4	0.049	-	-	-
13	14	9.041	8.016	7.006	6.002
	13	7.029	6.011	5.004	5.004
	12	6.037	5.015+	4.005+	3.002
	11	5.041	4.017	3.006	2.001
	10	4.041	3.016	2.005-	2.005-
	9	3.033	2.013	1.002	1.002
	8	2.031	1.009	1.009	0.001

TABLE B-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=14 n_2=13	7	1.021	1.021	0.004	0.004
	6	1.048	0.010+	-	-
	5	0.025-	0.025-	-	-
12	14	8.033	7.012	6.004	6.004
	13	6.021	6.021	5.007	4.001
	12	5.025+	4.009	4.009	3.003
	11	4.026	3.009	3.009	2.002
	10	3.024	3.024	2.007	1.002
	9	2.019	2.019	1.005-	1.005-
	8	2.042	1.012	0.002	0.002
	7	1.028	0.005+	0.005+	-
	6	0.013	0.013	-	-
	5	0.030	-	-	-
11	14	7.026	6.009	6.009	5.003
	13	6.030	5.014	4.004	4.004
	12	5.043	4.015	3.005-	3.005-
	11	4.042	3.015-	2.004	2.004
	10	3.036	2.011	1.003	1.003
	9	2.027	1.007	1.007	0.001
	8	1.017	1.017	0.003	0.003
	7	1.038	0.007	0.007	-
	6	0.017	0.017	-	-
	5	0.038	-	-	-
10	14	6.020	6.020	5.006	4.002
	13	5.028	4.009	4.009	3.002
	12	4.028	3.009	3.009	2.002
	11	3.024	3.024	2.007	2.001
	10	2.018	2.018	1.004	1.004
	9	2.040	1.011	0.002	0.002
	8	1.024	1.024	0.004	0.004
	7	0.010-	0.010-	0.010-	-
	6	0.022	0.022	-	-
	5	0.047	-	-	-
9	14	6.047	5.014	4.004	4.004
	13	4.018	4.018	3.005-	3.005-
	12	3.017	3.017	2.004	2.004
	11	3.042	2.012	1.002	1.002
	10	2.020	1.007	1.007	0.001
	9	1.017	1.017	0.002	0.002
	8	1.036	0.006	0.006	-
	7	0.014	0.014	-	-
	6	0.030	-	-	-
8	14	5.036	4.010-	4.010-	3.002
	12	3.022	2.003	2.003	1.001
	11	2.022	2.022	1.005-	1.005-
	10	2.048	1.012	0.002	0.002
	9	1.026	0.004	0.004	0.004
	8	0.009	0.009	0.009	-
	7	0.020	0.020	-	-
	6	0.040	-	-	-
n_1=14 n_2=7	14	4.026	3.006	3.006	2.001
	13	3.025	2.006	2.006	1.001
	12	2.017	2.017	1.003	1.003
	11	2.041	2.009	1.009	1.001
	10	1.021	1.021	0.003	0.003
	9	1.043	0.007	0.007	-
	8	0.015-	0.015-	-	-
	7	0.030	-	-	-
6	14	3.018	3.018	2.003	2.003
	13	2.014	2.014	1.002	1.002
	12	2.037	1.007	1.007	0.001
	11	1.018	1.018	0.002	0.002
	10	1.038	0.005+	0.005+	-
	9	0.012	0.012	-	-
	8	0.024	0.024	-	-
	7	0.044	-	-	-
5	14	2.010+	2.010+	1.001	1.001
	13	2.037	1.006	1.006	0.001
	12	1.017	1.017	0.002	0.002
	11	1.038	0.005-	0.005-	0.005-
	10	0.011	0.011	-	-
	9	0.022	0.022	-	-
	8	0.040	-	-	-
4	14	2.039	1.005-	1.005-	1.005-
	13	1.019	1.019	0.002	0.002
	12	1.044	0.005-	0.005-	0.005-
	11	0.011	0.011	-	-
	10	0.023	0.023	-	-
	9	0.041	-	-	-
3	14	1.022	1.022	0.001	0.001
	13	0.006	0.006	0.006	-
	12	0.015-	0.015-	-	-
	11	0.029			
2	14	0.008	0.008	0.008	-
	13	0.025	0.025	0 -	-
	12	0.050	-	-	-
n_1=15 n_2=15	15	11.050-	10.021	9.008	8.003
	14	9.040	8.019	7.007	6.003
	13	7.025	6.012+	5.004	5.004
	12	6.030	5.013	4.005-	4.005-
	11	5.033	4.013	3.005-	3.005-
	10	4.033	3.013	2.004	2.004
	8	2.025+	1.007	1.007	0.001
	7	1.018	0.018	0.003	.003
	6	1.040	0.008	0.009	-
	5	0.021	0.021	-	-
	4	0.050-	-	-	-

TABLE 8-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=15 n_2=14	15	10.042	9.017	8.006	7.002
	14	8.031	7.013	6.005-	6.005-
	13	7.041	6.017	5.007	4.002
	12	6.046	5.020	4.007	3.002
	11	5.048	4.020	3.007	2.002
	10	4.046	3.018	2.006	1.001
	9	3.041	2.014	1.004	1.004
	8	2.033	1.009	1.009	0.001
	7	1.022	1.022	0.004	0.004
	6	1.049	0.011		
	5	0.025+	-	-	-
13	15	9.035-	8.013	7.005-	7.005-
	14	7.023	7.023	6.009	5.003
	13	6.029	5.011	4.004	4.004
	12	5.031	4.012	3.004	3.004
	11	4.030	3.011	2.003	2.003
	10	3.026	2.008	2.008	1.002
	9	2.020	2.020	1.005+	0.001
	8	2.043	1.013	0.002	0.002
	7	1.029	0.005+	0.005+	-
	6	0.013	0.013	-	-
	5	0.031	-	-	-
12	15	8.028	7.010-	7.010-	6.003
	14	7.040	6.016	5.006	4.002
	13	6.045	5.019	4.007	3.002
	12	5.049	4.019	3.006	2.002
	11	4.045+	3.017	2.005-	2.005-
	10	3.038	2.012	1.003	1.003
	9	2.028	1.007	1.007	0.001
	8	1.018	1.018	0.003	0.003
	7	1.038	0.007	0.007	-
	6	0.017	0.017	-	-
	5	0.037	-	-	-
11	15	7.022	7.022	6.007	5.002
	14	6.032	5.011	4.003	4.003
	13	5.034	4.012	3.003	3.003
	12	4.032	3.010+	2.003	2.003
	11	3.026	2.008	2.008	1.002
	10	2.019	2.019	1.004	1.004
	9	2.040	1.011	0.002	0.002
	8	1.024	1.024	0.004	0.004
	7	1.049	0.010-	0.010-	-
	6	0.022	0.022	-	-
	5	0.046	-	-	-
10	15	6.017	6.017	5.005-	5.005-
	14	5.023	5.023	4.007	3.002
	13	4.022	4.022	3.007	2.001
	12	3.018	3.018	2.005-	2.005-
	11	3.042	2.013	1.003	1.003
	10	2.029	1.007	0.007	0.001
	9	1.016	1.016	0.002	0.002
	8	1.034	0.006	0.006	-
	7	0.013	0.012	-	-
	6	0.028	-	-	-
9	15	6.042	5.012	4.003	4.003
	14	5.047	4.015-	3.004	3.004

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=15 n_2=9	13	4.042	3.013	2.003	2.003
	12	3.032	2.009	2.009	1.002
	11	2.021	2.021	1.005-	1.005-
	10	2.045-	1.011	0.002	0.002
	9	1.024	1.024	0.004	0.004
	8	1.048	0.009	0.009	-
	7	0.019	0.019	-	-
	6	0.037	-	-	-
8	15	5.032	4.008	4.008	3.002
	14	4.033	3.009	3.009	2.002
	13	3.026	2.006	2.006	1.001
	12	2.017	2.017	1.003	1.003
	11	2.037	1.008	1.008	0.001
	10	1.019	1.019	0.003	0.003
	9	1.038	0.006	0.006	-
	8	0.013	0.013	-	-
	7	0.026	-	-	-
	6	0.050-	-	-	-
7	15	4.023	4.023	3.005-	3.005-
	14	3.022	3.021	2.004	2.004
	13	2.014	2.014	1.002	1.002
	12	2.032	1.007	1.007	0.001
	11	1.015+	1.015+	0.002	0.002
	10	1.032	0.005-	0.005-	0.005-
	9	0.010+	0.010+	-	-
	8	0.020	0.020	-	-
	7	0.038	-	-	-
6	15	3.015+	3.015+	2.003	2.003
	14	2.011	2.011	1.002	1.002
	13	2.031	1.006	1.006	0.001
	12	1.014	1.014	0.002	0.002
	11	1.029	0.004	0.004	0.004
	10	0.009	0.009	0.009	-
	9	0.017	0.017	-	-
	8	0.032	0 -	-	-
5	15	2.009	2.009	2.009	1.001
	14	2.032	1.005-	1.005-	1.005-
	13	1.014	1.014	0.001	0.001
	12	1.031	0.004	0.004	0.004
	11	0.008	0.008	0.008	-
	10	0.016	0.016	-	-
	9	0.030	-	-	-
4	15	2.035+	1.004	1.004	1.004
	14	1.016	1.016	0.001	0.001
	13	1.037	0.004	0.004	0.004
	12	0.009	0.009	0.009	-
	11	0.018	0.018	-	-
	10	0.033	-	-	-
3	15	1.020	1.020	0.001	0.001
	14	0.005-	0.005-	0.005-	0.005-
	13	0.012	0.012	-	-
	12	0.025-	0.025-	-	-
	11	0.043	-	-	-
2	15	0.007	0.007	0.007	-
	14	0.022	0.022	-	-
	13	0.044	-	-	-

TABLE B-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

		Significance Level			
	a_1	0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=16 n_2=16	16	11.022	11.022	10.009	9.003
	15	10.041	9.019	8.008	7.003
	14	8.027	7.012	6.005-	6.005-
	13	7.033	6.015-	5.006	4.002
	12	6.037	5.016	4.006	3.002
	11	5.038	4.016	3.006	2.002
	10	4.037	3.015-	2.005-	2.005-
	9	3.033	2.012	1.003	1.003
	8	2.027	1.008	1.008	0.001
	7	1.019	1.019	0.003	0.003
	6	1.041	0.009	0.009	-
	5	0.022	0.022	-	-
15	16	11.043	10.018	9.007	8.002
	15	9.033	8.014	7.005+	6.002
	14	8.044	7.019	6.008	5.003
	13	6.023	6.023	5.009	4.003
	12	5.024	5.024	4.009	3.003
	11	4.023	4.023	3.008	2.002
	10	4.049	3.020	2.006	1.001
	9	3.043	2.016	1.004	1.004
	8	2.035-	1.010+	0.002	0.002
	7	1.023	1.023	0.004	0.004
	6	0.011	0.011	-	-
	5	0.026	-	-	-
14	16	10.037	9.014	8.005+	7.002
	15	8.025+	7.010-	7.010-	6.003
	14	7.032	6.013	5.005-	5.005-
	13	6.035+	5.014	4.005+	3.001
	12	5.035+	4.014	3.005-	3.005-
	11	4.033	3.012	2.004	2.004
	10	3.028	2.009	2.009	1.002
	9	2.021	2.021	1.006	0.001
	8	2.045-	1.013	0.002	0.002
	7	1.030	0.006	0.006	-
	6	0.013	0.013	-	-
	5	0.031	-	-	-
13	16	9.030	8.011	7.004	7.004
	15	8.047	7.019	6.007	5.002
	14	6.022	6.023	5.008	4.003
	13	5.023	5.023	4.008	3.003
	12	4.022	4.022	3.007	2.002
	11	4.048	3.018	2.005+	1.001
	10	3.039	2.013	1.003	1.003
	9	2.029	1.008	1.008	0.001
	8	1.018	1.018	0.003	0.003
	7	1.038	0.007	0.007	-
	6	0.017	0.017	-	-
	5	0.037	-	-	-

		Significance Level			
	a_1	0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=16 n_2=12	16	8.024	8.024	7.008	6.002
	15	7.036	6.013	5.004	5.004
	14	6.040	5.015-	4.005-	4.005-
	13	5.039	4.014	3.004	3.004
	12	4.034	3.012	2.003	2.003
	11	3.027	2.008	2.008	1.002
	10	2.019	2.019	1.005-	1.005-
	9	2.040	1.011	0.002	0.002
	8	1.024	1.024	0.004	0.004
	7	1.048	0.010-	0.010-	-
	6	0.021	0.021	-	-
	5	0.044	-	-	-
11	16	7.019	7.019	6.006	5.002
	15	6.027	5.009	5.009	4.002
	14	5.027	4.009	4.009	3.002
	13	4.024	4.024	3.008	2.002
	12	3.019	3.019	2.005+	1.001
	11	3.041	2.013	1.003	1.003
	10	2.028	1.007	1.007	0.001
	9	1.016	1.016	0.002	0.002
	8	1.033	0.006	0.006	-
	7	0.013	0.013	-	-
	6	0.027	-	-	-
10	16	7.046	6.014	5.004	5.004
	15	5.018	5.018	4.005+	3.001
	14	4.017	4.017	3.005-	3.005-
	13	4.042	3.014	2.003	2.003
	12	3.032	2.009	2.009	1.002
	11	2.021	2.021	1.005-	1.005-
	10	2.042	1.011	0.002	0.002
	9	1.023	1.023	0.004	0.004
	8	1.045-	0.008	0.008	-
	7	0.017	0.017	-	-
	6	0.035-	-	-	-
9	16	6.037	5.010-	5.010-	4.002
	15	5.040	4.012	3.003	3.002
	14	4.034	3.010-	3.010-	2.002
	13	3.025+	2.007	2.007	1.001
	12	2.016	2.016	1.003	1.003
	11	2.033	1.006	1.008	0.001
	10	1.017	1.017	0.002	0.002
	9	1.034	0.006	0.006	-
	8	0.013	0.012	-	-
	7	0.024	0.024	-	-
	6	0.045+	-	-	-

TABLE B-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=16 n_2=8	16	5.028	4.007	4.007	3.001
	15	4.028	3.007	3.007	2.001
	14	3.021	3.021	2.005-	2.005-
	13	3.047	2.013	1.002	1.002
	12	2.028	1.006	1.006	0.001
	11	1.014	1.014	0.002	0.002
	10	1.027	0.004	0.004	0.004
	9	0.009	0.009	0.009	-
	8	0.017	0.017	-	-
	7	0.033			
7	16	4.020	4.020	3.004	3.004
	15	3.017	3.017	2.003	2.003
	14	3.045+	2.011	1.002	1.002
	13	2.026	1.005-	1.005-	1.005-
	12	1.012	1.012	0.001	0.001
	11	1.024	1.024	0.003	0.003
	10	1.045-	0.007	0.007	-
	9	0.014	0.014	-	-
	8	0.026	-	-	-
	7	0.047	-	-	-
6	16	3.013	3.013	2.002	2.002
	15	3.046	2.009	2.009	1.001
	14	2.025+	1.004	1.004	1.004
	13	1.011	1.011	0.001	0.001
	12	1.023	1.023	0.003	0.003
	11	1.043	0.006	0.006	-
	10	0.012	0.012	-	-
	9	0.023	0.023	-	-
	8	0.040	-	-	-
5	16	3.048	3.008	2.008	1.001
	15	2.029	1.004	1.004	1.004
	14	1.011	1.011	0.001	0.001
	13	1.025+	0.003	0.003	0.003
	12	1.047	0.006	0.006	-
	11	0.012	0.012	-	-
	10	0.023	0.022	-	-
	9	0.0[illegible]	-	-	-
4	16	2.0[illegible]	1.004	1.004	1.004
	15	1.01[illegible]	1.01[illegible]	0.001	0.001
	14	1.0[illegible]	0.003	0.003	0.003
	13	0.007	0.007	0.007	-
	12	0.014	0.014	-	-
	11	0.026	-	-	-
	10	0.043	-	-	-

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=16 n_2=3	16	1.018	1.018	0.001	0.001
	15	0.004	0.004	0.004	0.004
	14	0.010+	0.010+	-	-
	13	0.021	0.021	-	-
	12	0.036	-	-	-
2	16	0.007	0.007	0.007	-
	15	0.020	0.020	-	-
	14	0.039	-	-	-
n_1=17 n_2=17	17	12.022	12.022	11.009	10.004
	16	11.043	10.020	9.008	8.003
	15	9.029	8.013	7.005+	6.002
	14	8.035+	7.016	6.007	5.002
	13	7.040	6.018	5.007	4.003
	12	6.042	5.019	4.007	3.002
	11	5.042	4.018	3.007	2.002
	10	4.040	3.016	2.005+	1.001
	9	3.035+	2.013	1.003	1.003
	8	2.029	1.008	1.008	0.001
	7	1.020	1.020	0.004	0.004
	6	1.043	0.009	0.009	-
	5	0.022	0.022	-	-
16	17	12.044	11.018	10.007	9.003
	16	10.035-	9.015-	8.006	7.002
	15	9.046	8.021	7.009	6.003
	14	7.025+	6.011	5.004	5.004
	13	6.027	5.011	4.004	4.004
	12	5.027	4.011	3.004	3.004
	11	4.025+	3.009	3.004	2.00[illegible]
	10	3.022	3.022	2.007	1.00[illegible]
	9	3.046	2.017	1.004	1.004
	8	2.036	1.011	0.00[illegible]	0.00[illegible]
	7	1.024	1.024	0.005-	0.005-
	6	0.011	0.011	-	-
	5	0.0[illegible]	-	-	-
15	17	11.0[illegible]	10.0[illegible]	9.[illegible]	8.[illegible]
	16	9.027	8.0[illegible]	7.[illegible]	7.[illegible]
	15	8.035+	7.0[illegible]-	6.[illegible]	5.[illegible]
	14	7.04[illegible]	6.0[illegible]	5.[illegible]	4.[illegible]
	13	6.041	5.0[illegible]	4.[illegible]	3.[illegible]
	12	5.0[illegible]	4.016	3.[illegible]+	2.[illegible]
	11	4.025+	3.0[illegible]	2.[illegible]	2.[illegible]
	10	3.023	2.0[illegible]-	2.0[illegible]-	1.[illegible]
	9	2.0[illegible]	2.0[illegible]	1.[illegible]	0.[illegible]
	8	2.04[illegible]	1.0[illegible]	0.[illegible]	0.[illegible]
	7	1.0[illegible]	0.0[illegible]	0.[illegible]	-
	6	0.014	0.014	-	-
	5	0.0[illegible]			

TABLE B-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
$n_1=17$ $n_2=14$	17	10.032	9.012	8.004	8.004
	16	8.021	8.021	7.008	6.003
	15	7.026	6.010-	6.010-	5.003
	14	6.028	5.011	4.004	4.004
	13	5.027	4.010-	4.010-	3.003
	12	4.024	4.024	3.008	2.002
	11	4.049	3.012	2.006	1.001
	10	3.040	2.014	1.003	1.003
	9	2.029	1.008	1.008	0.001
	8	1.018	1.018	0.003	0.003
	7	1.038	0.007	0.007	-
	6	0.017	0.017	-	-
	5	0.036	-	-	-
13	17	9.026	8.009	8.009	7.003
	16	8.040	7.015+	6.005+	5.002
	15	7.045+	6.018	5.006	4.002
	14	6.045+	5.018	4.006	3.001
	13	5.042	4.016	3.005+	2.001
	12	4.035+	3.013	2.004	2.004
	11	3.028	2.009	2.009	1.002
	10	2.019	2.019	1.005-	1.005-
	9	2.040	1.011	0.002	0.002
	8	1.024	1.024	0.004	0.004
	7	1.047	0.010-	0.010-	-
	6	0.021	0.021	-	-
	5	0.043	-	-	-
12	17	8.021	8.021	7.007	6.002
	16	7.030	6.011	5.003	5.003
	15	6.033	5.012	4.004	4.004
	14	5.030	4.011	3.003	3.003
	13	4.026	3.008	3.008	2.002
	12	3.020	3.020	2.007	1.001
	11	3.041	2.013	1.003	1.003
	10	2.028	1.007	1.007	0.001
	9	1.016	1.016	0.002	0.002
	8	1.032	0.006	0.006	-
	7	0.012	0.012	-	-
	6	0.026			
11	17	7.016	7.016	6.005-	6.005-
	16	6.[illegible]	6.[illegible]	5.[illegible]	4.[illegible]
	15	5.[illegible]	5.012	4.[illegible]	3.[illegible]
	14	4.[illegible]	4.[illegible]	3.[illegible]	2.[illegible]
$n_1=17$ $n_2=11$	13	4.042	3.014	2.004	2.004
	12	3.031	2.009	2.009	1.002
	11	2.020	2.020	1.005-	1.005-
	10	2.040	1.011	0.001	0.001
	9	1.022	1.022	0.004	0.004
	8	1.042	0.008	0.008	-
	7	0.016	0.016	-	-
	6	0.033	-	-	-
10	17	7.041	6.012	5.003	5.003
	16	6.047	5.015+	4.004	4.004
	15	5.043	4.014	3.004	3.004
	14	4.034	3.010+	2.002	2.002
	13	3.024	3.024	2.007	1.001
	12	3.049	2.015+	1.003	1.003
	11	2.031	1.007	1.007	0.001
	10	1.016	1.016	0.002	0.002
	9	1.031	0.005+	0.005+	-
	8	0.011	0.011	-	-
	7	0.022	0.022	-	-
	6	0.042	-	-	-
9	17	6.032	5.008	5.008	4.002
	16	5.034	4.010-	4.010-	3.002
	15	4.029	3.008	3.008	2.002
	14	3.020	3.020	2.005-	2.005-
	13	3.042	2.012	1.002	1.002
	12	2.025+	1.006	1.006	0.001
	11	2.048	1.012	0.002	0.002
	10	1.024	1.024	0.004	0.004
	9	1.045-	0.008	0.008	-
	8	0.016	0.016	-	-
	7	0.030	-	-	-
8	17	5.024	5.024	4.006	3.001
	16	4.023	4.023	3.006	2.001
	15	3.017	3.017	2.004	2.004
	14	3.039	2.010-	2.010-	1.002
	13	2.022	2.022	1.004	1.004
	12	2.043	1.010-	1.010-	0.001
	11	1.020	1.020	0.003	0.003
	10	1.039	0.006	0.006	-
	9	0.012	0.012	-	-
	8	0.022	0.022	-	-
	7	0.04[illegible]	-	-	-

TABLE B-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

		Significance Level			
		0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=17 n_2=7	17	4.017	4.017	3.003	3.003
	16	3.014	3.014	2.003	2.003
	15	3.038	2.009	2.009	1.001
	14	2.021	2.0.1	1.004	1.004
	13	2.042	1.009	1.009	0.001
	12	1.[illegible]	1.018	0.002	0.002
	11	1.034	0.005-	0.005-	0.005-
	10	0.010-	0.010-	0.010-	-
	9	0.019	0.019	-	-
	8	0.033	-	-	-
6	17	3.011	3.011	2.002	2.002
	16	3.040	2.008	2.008	1.001
	15	2.021	2.021	1.003	1.003
	14	2.045+	1.009	1.009	0.001
	13	1.018	1.018	0.002	0.002
	12	1.035-	0.005-	0.005-	0.005-
	11	0.009	0.009	0.009	-
	10	0.017	0.017	-	-
	9	0.030	-	-	-
	8	0.050-	-	-	-
5	17	3.043	2.006	2.005	1.001
	16	2.024	2.024	1.003	1.003
	15	1.009	1.009	1.009	0.001
	14	1.021	1.021	0.002	0.002
	13	1.039	0.005-	0.005-	0.005-
	12	0.010-	0.010-	0.010-	-
	11	0.018	0.018	-	-
	10	0.0[illegible]			
	9	0.0[illegible]			
	17	2.029	1.003	1.003	1.003
	16	1.012	1.012	0.001	0.001
	15	1.028	0.003	0.003	0.003
	14	0.006	0.006	0.006	-
	13	0.012	0.012	-	-
	12	0.021	0.021	-	-
	11	0.035+	-	-	-
3	17	1.016	1.016	0.001	0.001
	16	1.046	0.004	0.004	0.004
	15	0.009	0.009	0.009	-
	14	0.018	0.019	-	-
	13	0.031	-	-	-
	12	0.049	-	-	-
[illegible]	17	0.006	0.006	0.006	-
	16	0.018	0.018	-	-
	15	0.035+	-	-	-

		Significance Level			
		0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=18 n_2=18	18	13.023	13.023	12.010-	11.004
	17	12.044	11.020	10.009	9.004
	16	10.030	9.014	8.005	7.002
	15	9.039	8.018	7.008	6.003
	14	8.043	7.020	6.009	5.003
	13	7.046	6.022	5.00[illegible]	4.003
	12	6.047	5.022	4.009	3.003
	11	5.046	4.020	3.008	2.002
	10	4.043	3.018	2.006	1.001
	9	3.038	2.014	1.004	1.004
	8	2.030	1.009	1.009	0.001
	7	1.020	1.020	0.004	0.004
	6	1.044	0.010-	0.010-	-
	5	0.023	0.023	-	-
17	18	13.045+	12.019	11.008	10.003
	17	11.036	10.016	9.007	8.002
	16	10.049	9.023	8.010-	7.004
	15	8.028	7.012	6.005-	6.005-
	14	7.030	6.013	5.005+	4.002
	13	6.031	5.013	4.005-	4.005-
	12	5.030	4.012	3.004	3.004
	11	4.028	3.010+	2.003	2.003
	10	3.023	3.023	2.008	1.002
	9	3.047	2.018	1.005-	1.005-
	8	2.037	1.011	0.002	0.002
	7	1.025-	1.025-	0.005-	0.005-
	6	0.011	0.011	-	-
	5	0.026	-	-	-
16	18	12.039	11.016	10.006	9.002
	17	10.029	9.012	8.005-	8.005-
	16	9.028	8.017	7.007	6.002
	15	8.043	7.019	6.008	5.003
	14	7.04[illegible]	6.020	5.008	4.003
	13	6.045+	5.020	4.007	3.002
	12	5.042	4.018	3.006	2.002
	11	4.037	3.015-	2.004	2.004
	10	3.031	2.011	1.003	1.003
	9	2.023	2.023	1.006	0.001
	8	2.046	1.014	0.002	0.002
	7	1.030	0.006	0.006	-
	6	0.014	0.014	-	-
	5	0.031	-	-	-
15	18	11.033	10.013	9.005-	9.005-
	17	9.023	9.023	8.009	7.003
	16	8.029	7.012	6.004	6.004
	15	7.031	6.013	5.005-	5.005-
	14	6.031	5.013	4.004	4.004
	13	5.029	4.011	3.004	3.004

TABLE 8-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=18 n_2=15	12	4.025+	3.009	3.009	2.003
	11	3.020	3.020	2.006	1.001
	10	3.041	2.014	1.004	1.004
	9	2.030	1.008	1.008	0.001
	8	1.018	1.018	0.003	0.003
	7	1.038	0.007	0.007	-
	6	0.017	0.017	-	-
	5	0.036	-	-	-
14	18	10.028	9.010-	9.010-	8.003
	17	9.043	8.017	7.006	6.002
	16	8.050-	7.021	6.008	5.003
	15	6.022	6.022	5.009	4.003
	14	6.045	5.020	4.007	3.002
	13	5.044	4.017	3.006	2.001
	12	4.037	3.013	2.004	2.004
	11	3.028	2.009	2.009	1.002
	10	2.020	2.020	1.005-	1.005-
	9	2.039	1.011	0.002	0.002
	8	1.024	1.024	0.004	0.004
	7	1.047	0.009	0.009	-
	6	0.020	0.020	-	-
	5	0.043	-	-	-
13	18	9.023	9.023	8.008	7.002
	17	8.034	7.012	6.004	6.004
	16	7.037	6.014	5.005-	5.005-
	15	6.036	5.01	4.004	4.004
	14	5.032	4.012	3.004	3.004
	13	4.027	3.009	3.009	2.002
	12	3.020	3.020	2.006	1.001
	11	3.040	2.013	1.003	1.003
	10	2.027	1.007	1.007	0.001
	9	1.015+	1.015+	0.002	0.002
	8	1.031	0.006	0.006	-
	7	0.012	0.012	-	-
	6	0.025+	-	-	-
12	18	8.018	8.018	7.006	6.002
	17	7.026	6.009	6.009	5.003
	16	6.027	5.009	5.009	4.003
	15	5.024	5.024	4.008	3.002
	14	4.020	4.020	3.006	2.001
	13	4.042	3.014	2.004	2.004
	12	3.030	2.009	2.009	1.002
	11	2.010	2.010	1.005-	1.005-

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=18 n_2=12	10	2.038	1.010+	0.001	0.001
	9	1.021	1.021	0.003	0.003
	8	1.040	0.007	0.007	-
	7	0.016	0.016	-	-
	6	0.031	-	-	-
11	18	8.045+	7.014	6.004	5.004
	17	6.018	6.018	5.006	4.001
	16	5.018	5.018	4.005+	3.001
	15	5.043	4.015-	3.004	3.004
	14	4.033	3.011	2.003	2.003
	13	3.023	3.023	2.007	1.001
	12	3.046	2.014	1.003	1.003
	11	2.020	1.007	1.007	0.001
	10	1.015-	1.015-	0.002	0.002
	9	1.029	0.005-	0.005-	0.005-
	8	0.010+	0.010+	-	-
	7	0.020	0.020	-	-
	6	0.039			
10	18	7.037	6.010+	5.003	5.003
	17	6.041	5.013	4.003	4.003
	16	5.036	4.011	3.003	3.003
	15	4.028	3.009	3.009	2.002
	14	3.019	3.019	2.005-	2.005-
	13	3.039	2.011	1.002	1.002
	12	2.023	2.023	1.005+	0.001
	11	2.043	1.011	0.001	0.001
	10	1.022	1.022	0.003	0.003
	9	1.040	0.007	0.007	-
	8	0.014	0.014	-	-
	7	0.027	-	-	-
	6	0.049	-	-	-
9	18	6.029	5.007	5.007	4.002
	17	5.030	4.008	4.008	3.002
	16	4.023	4.023	3.006	2.001
	15	3.016	3.016	2.004	2.004
	14	3.034	2.008	2.008	1.002
	13	2.019	2.014	1.004	1.004
	12	2.037	1.009	1.009	0.001
	11	1.014	1.018	0.002	0.002
	10	1.033	0.005+	0.005+	-
	9	0.010+	0.010+	-	-
	8	0.020	0.020	-	-
	7	0.036	-	-	-

TABLE B-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=18 n_2=8	18	5.022	5.022	4.005-	4.005-
	17	4.020	4.020	3.004	3.004
	16	3.014	3.014	2.003	2.003
	15	3.032	2.008	2.008	1.001
	14	2.017	2.017	1.003	1.003
	13	2.034	1.007	1.007	0.001
	12	1.015+	1.015+	0.002	0.002
	11	1.028	0.004	0.004	0.004
	10	1.049	0.008	0.008	-
	9	0.016	0.016	-	-
	8	0.028	-	-	-
	7	0.048	-	-	-
7	18	4.015+	4.015+	3.003	3.003
	17	3.012	3.012	2.002	2.002
	16	3.032	2.007	2.007	1.001
	15	2.017	2.017	1.003	1.003
	14	2.034	1.007	1.007	0.001
	13	1.014	1.014	0.002	0.002
	12	1.027	0.004	0.004	0.004
	11	1.046	0.007	0.007	-
	10	0.013	0.013	-	-
	9	0.024	0.024	-	-
	8	0.040	-	-	-
6	18	3.010-	3.010-	3.010-	2.001
	17	3.025+	2.006	2.006	1.001
	16	2.018	2.018	1.003	1.003
	15	2.038	1.007	1.007	0.001
	14	1.015-	1.015-	0.002	0.002
	13	1.003	0.003	0.003	0.003
	12	1.048	0.007	0.007	-
	11	0.013	0.013	-	-
	10	0.022	0.022	-	-
	9	0.037	-	-	-
5	18	3.040	2.006	2.006	1.001
	17	2.021	2.021	1.003	1.003
	16	2.048	1.008	1.003	0.001
	15	1.017	1.017	0.002	0.002
	14	1.033	0.004	0.004	0.004
	13	0.007	0.007	0.007	-
	12	0.014	0.014	-	-
	11	0.024	0.024	-	-
	10	0.039	-	-	-
4	18	2.026	1.003	1.003	1.003
	17	1.010-	1.010-	1.010-	0.001
	16	1.024	1.024	0.002	0.002
	15	1.046	0.005-	0.005-	0.005-
	14	0.010-	0.010-	0.010-	-

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
n_1=18 n_2=4	13	0.017	0.017	-	-
	12	0.029	-	-	-
	11	0.045+	-	-	-
3	18	1.014	1.014	0.001	0.001
	17	1.041	0.003	0.003	0.003
	16	0.008	0.008	0.008	-
	15	0.015+	0.015+	-	-
	14	0.026	-	-	-
	13	0.042	-	-	-
2	18	0.005+	0.005+	0.005+	-
	17	0.016	0.016		-
	16	0.032	-	-	-
n_1=19 n_2=19	19	14.023	14.023	13.010-	12.004
	18	13.045-	12.021	11.009	10.004
	17	11.031	10.015-	9.00	8.003
	16	10.039	9.019	8.009	7.003
	15	9.046	8.022	6.004	6.004
	14	8.050-	7.024	5.004	5.004
	13	6.025+	5.011	4.004	4.004
	12	5.024	5.024	3.003	3.003
	11	5.050-	4.022	3.009	2.003
	10	4.046	3.019	2.008	1.002
	9	3.039	2.015-	1.004	1.004
	8	2.031	1.009	1.009	0.002
	7	1.021	1.021	0.004	0.004
	6	1.045-	0.010-	0.010-	-
	5	0.023	0.023	-	-
18	19	14.045	13.020	12.008	11.003
	18	12.037	11.017	10.007	9.003
	17	10.024	10.024	8.004	8.004
	16	9.030	8.014	7.006	6.002
	15	8.033	7.015+	6.006	5.002
	14	7.035+	6.016	5.006	4.002
	13	6.035-	5.015+	4.006	3.002
	12	5.033	4.014	3.005-	3.005-
	11	4.030	3.011	2.004	2.004
	10	3.025-	3.025-	2.008	1.002
	9	3.049	2.019	1.005+	0.001
	8	2.038	1.042	0.002	0.002
	7	1.025+	0.005-	0.005	0.005-
	6	0.012	0.012	-	-
	5	0.027	-	-	-
17	19	13.040	12.016	11.006	10.002
	18	11.030	10.013	9.005+	8.002
	17	10.040	9.018	8.008	7.003
	16	9.047	8.022	7.009	6.003

TABLE B-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
$n_1=19$ $n_2=17$	15	8.050-	7.023	6.010-	5.004
	14	6.023	6.023	5.010-	4.003
	13	6.049	5.022	4.008	3.003
	12	5.045-	4.019	3.007	2.002
	11	4.039	3.015+	2.005	2.005-
	10	3.032	2.011	1.003	1.003
	9	2.024	2.024	1.007	0.001
	8	2.047	1.015-	0.002	0.002
	7	1.031	0.006	0.006	-
	6	0.014	0.014	-	-
	5	0.031	-	-	-
16	19	12.035-	11.013	10.005-	10.005-
	18	10.024	10.024	9.010-	8.004
	17	9.031	8.013	7.005+	6.002
	16	8.035-	7.015+	6.006	5.002
	15	7.036	6.015+	5.006	4.002
	14	6.034	5.014	4.005+	3.002
	13	5.031	4.012	3.004	3.004
	12	4.027	3.010-	3.010-	2.003
	11	3.021	3.021	2.007	1.002
	10	3.042	2.015-	1.004	1.004
	9	2.030	1.009	1.009	0.001
	8	1.019	1.019	0.003	0.003
	7	1.037	0.007	0.007	-
	6	0.017	0.017	-	-
	5	0.036	-	-	-
15	19	11.029	10.011	9.004	9.004
	18	10.046	9.019	8.007	7.002
	17	8.023	8.023	7.009	6.003
	16	7.025-	7.025-	6.010-	5.003
	15	6.024	6.024	5.011	4.003
	14	5.022	5.022	4.008	3.002
	13	5.045+	4.018	3.006	2.002
	12	4.037	3.014	2.004	2.004
	11	3.029	2.009	2.009	1.002
	10	2.020	2.020	1.005+	0.001
	9	2.033	1.011	0.002	0.002
	8	1.023	1.023	0.004	0.004
	7	1.046	0.009	0.009	
	6	0.020	0.020	-	-
	5	0.042	-	-	-
14	19	10.024	10.024	9.008	8.003
	18	9.047	8.014	7.005-	7.005-
	17	8.041	7.017	6.006	5.002
	16	7.042	6.017	5.006	4.002
	15	6.039	5.015+	4.005+	3.001
	14	5.034	4.011	3.004	3.004
	13	4.027	3.009	3.009	2.003
	12	3.020	3.020	2.006	1.001
	11	3.040	2.013	1.003	1.003
	10	2.027	1.007	1.007	0.001
	9	1.015-	1.015-	0.001	0.001
	8	1.031	0.005+	0.005+	-
	7	0.011	0.011	-	-
	6	0.024	0.024	-	-
	5	0.044			

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
$n_1=19$ $n_2=13$	19	9.020	9.020	8.006	7.002
	18	8.029	7.010+	6.003	6.003
	17	7.031	6.011	5.004	5.004
	16	6.029	5.011	4.003	4.003
	15	5.025+	4.009	4.009	3.003
	14	4.020	4.020	3.006	2.002
	13	4.041	3.015-	2.004	2.004
	12	3.029	2.009	2.009	1.002
	11	2.019	2.019	1.005-	1.005-
	10	2.036	1.010-	1.010-	0.001
	9	1.020	1.020	0.003	0.003
	8	1.038	1.007	0.007	-
	7	0.015-	0.015-	-	-
	6	0.030	-	-	-
12	19	9.049	8.016	7.005-	7.005-
	18	7.022	7.022	6.007	5.002
	17	6.022	6.022	5.007	4.002
	16	5.019	5.019	4.006	3.002
	15	5.042	4.015+	3.004	3.004
	14	4.032	3.011	2.003	2.003
	13	3.023	3.023	2.006	1.001
	12	3.043	2.014	1.003	1.003
	11	2.027	1.007	1.007	0.001
	10	2.050-	1.014	0.002	0.002
	9	1.027	0.005-	0.005-	0.005-
	8	1.050-	0.010-	0.010-	-
	7	0.019	0.019	-	-
	6	0.037	-	-	-
11	19	8.041	7.012	6.003	6.003
	18	7.047	6.016	5.004	5.004
	17	6.043	5.015-	4.004	4.004
	16	5.035+	4.012	3.003	3.003
	15	4.027	3.008	3.008	2.002
	14	3.018	3.018	2.005-	2.005-
	13	3.035+	2.010+	1.002	1.002
	12	2.021	2.021	1.005-	1.005-
	11	2.040	1.010+	0.001	0.001
	10	1.020	1.020	0.003	0.003
	9	1.037	0.006	0.006	-
	8	0.013	0.013	-	-
	7	0.025-	0.025-	-	-
	6	0.046			
10	19	7.033	6.009	6.009	5.002
	18	6.036	5.011	4.003	4.003
	17	5.030	4.009	4.009	3.002
	16	4.022	4.022	3.006	2.001
	15	4.047	3.015-	2.004	2.004
	14	3.032	2.008	2.009	1.002
	13	2.017	2.017	1.004	1.004
	12	2.033	1.008	1.008	0.001
	11	1.016	1.016	0.002	0.002
	10	1.029	0.005-	0.005-	0.005-
	9	0.009	0.009	0.009	-
	8	0.018	0.018	-	-
	7	[illegible]	-	-	-

TABLES B-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
$n_1=19$ $n_2=9$	19	6.026	5.006	5.006	4.001
	18	5.026	4.007	4.007	3.001
	17	4.020	4.020	3.005-	3.005-
	16	4.044	3.017	2.003	2.003
	15	3.028	2.007	2.007	1.001
	14	2.015-	2.015-	1.003	1.003
	13	2.029	1.006	1.006	0.001
	12	1.013	1.013	0.002	0.002
	11	1.024	1.024	0.004	0.004
	10	1.042	0.007	0.007	-
	9	0.013	0.013	-	-
	8	0.024	0.024	-	-
	7	0.043	-	-	-
8	19	5.013	5.019	4.004	4.004
	18	4.017	4.017	3.004	3.004
	17	4.044	3.011	2.002	2.002
	16	3.027	2.006	2.006	1.001
	15	2.014	2.014	1.002	1.002
	14	2.027	1.006	1.006	0.001
	13	2.049	1.011	0.001	0.001
	12	1.021	1.021	0.003	0.003
	11	1.038	0.006	0.006	-
	10	0.011	0.011	-	-
	9	0.020	0.020	-	-
	8	0.034	-	-	-
7	19	4.013	4.013	3.002	3.002
	18	4.047	3.010+	3.002	3.002
	17	3.028	2.006	2.006	1.001
	16	2.014	2.014	1.002	1.002
	15	2.028	1.005+	1.005+	1.001
	14	1.011	1.011	0.001	0.001
	13	1.021	1.021	0.003	0.003
	12	1.037	0.005+	0.005+	-
	11	0.010-	0.010-	0.010-	-
	10	0.017	0.017	-	-
	9	0.030	-	-	-
	8	0.048	-	-	-
6	19	4.050	3.009	3.009	2.001
	18	3.031	2.005+	2.005+	1.001
	17	2.015+	2.015+	1.002	1.002
	16	2.032	1.006	1.006	0.000
	15	1.012	1.012	0.001	0.001
	14	1.023	1.023	0.003	0.003
	13	1.039	0.005+	0.005+	-
	12	0.010-	0.010-	0.010-	-
	11	0.017	0.017	-	-
	10	0.028	-	-	-
	9	0.045+	-	-	-
5	19	3.036	2.005-	2.005-	2.005-
	18	2.018	2.018	1.002	1.002
	17	2.047	1.006	1.006	0.000
	16	1.014	1.014	0.001	0.001
	15	1.028	0.003	0.003	0.003
	14	1.047	0.006	0.006	-
	13	0.011	0.011		
$n_1=19$ $n_2=5$	12	0.019	0.019	-	-
	11	0.030	-	-	-
	10	0.047	-	-	-
4	19	2.024	2.024	1.002	1.002
	18	1.009	1.009	1.009	0.001
	17	1.021	1.021	0.002	0.002
	16	1.040	0.004	0.004	0.004
	15	0.008	0.008	0.008	-
	14	0.014	0.014	-	-
	13	0.024	0.024	-	-
	12	0.037	-	-	-
3	19	1.013	1.013	0.001	0.001
	18	1.038	0.003	0.003	0.003
	17	0.006	0.006	0.006	-
	16	0.013	0.013	-	-
	15	0.023	0.023	-	-
	14	0.036	-	-	-
2	19	0.005-	0.005-	0.005-	0.005-
	18	0.014	0.014	-	-
	17	0.029	-	-	-
	16	0.048	-	-	-
$n_1=20$ $n_2=20$	20	15.024	15.024	13.004	13.004
	19	14.045	13.022	12.010-	11.004
	18	12.032	11.015+	10.007	0.003
	17	11.041	10.020	9.009	8.004
	16	10.048	9.024	7.005-	7.005-
	15	8.027	7.012	6.005+	5.002
	14	7.028	6.013	5.005+	4.001
	13	6.028	5.012	4.005-	4.005-
	12	5.017	4.011	3.004	3.004
	11	4.024	4.024	3.009	2.002
	10	4.048	3.020	2.007	1.002
	9	3.041	2.015+	1.004	1.004
	8	2.032	1.010-	1.010-	0.002
	7	1.072	1.022	0.004	0.004
	6	1.046	0.010+	-	-
	5	1.024	0.024	-	-
19	20	15.047	14.020	13.008	12.003
	19	13.033	12.018	11.008	[illegible]
	18	11.026	10.012	9.005-	9.005-
	17	10.032	9.013-	8.006	7.002
	16	9.036	8.017	7.007	6.003
	15	8.038	7.018	6.008	5.003
	14	7.039	6.019	5.007	4.002
	13	6.038	5.017	4.007	3.002
	12	5.035+	4.015+	3.005+	2.002
	11	4.031	3.011	2.004	2.004
	10	3.026	[illegible]	[illegible]	1.001
	9	2.019	2.013	1.005+	0.001
	8	2.044	1.011	0.001	0.001
	7	1.026	0.005+	0.005+	-
	6	0.012	0.012	-	-
	5	0.027	-	-	-

TABLE 8-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
$n_1=20$ $n_2=18$	20	14.041	13.017	12.007	11.003
	19	12.032	11.014	10.006	9.002
	18	11.043	10.020	9.008	9.003
	17	10.050-	9.024	7.004	7.004
	16	8.026	7.011	6.005-	6.005-
	15	7.027	6.012	5.004	5.004
	14	6.026	5.011	4.004	4.004
	13	5.024	5.024	4.009	3.003
	12	5.047	4.020	3.007	2.002
	11	4.041	3.016	2.005+	1.001
	10	3.033	2.012	1.003	1.003
	9	2.024	2.024	1.007	0.001
	8	2.048	1.015-	0.003	0.003
	7	1.031	0.006	0.006	-
	6	0.014	0.014	-	-
	5	0.031	-	-	-
17	20	13.036	12.014	11.005+	10.002
	19	11.026	10.011	9.004	9.004
	18	10.034	9.015-	8.006	7.002
	17	9.038	8.017	7.007	6.003
	16	8.040	7.018	6.007	5.003
	15	7.039	6.017	5.007	4.002
	14	6.037	5.016	4.006	3.002
	13	5.033	4.013	3.005-	3.005-
	12	4.028	3.010+	2.003	2.003
	11	3.022	3.022	2.007	1.002
	10	3.042	2.015+	1.004	1.004
	9	2.031	1.009	1.009	0.001
	8	1.019	1.019	0.003	0.003
	7	1.027	0.008	0.008	-
	6	0.017	0.017	-	-
	5	0.036	-	-	-
16	20	12.031	11.012	10.004	10.004
	19	11.049	10.021	9.008	8.003
	18	9.026	8.011	7.004	7.004
	17	8.028	7.012	6.004	6.004
	16	7.028	6.012	5.004	5.004
	15	6.026	5.011	4.004	4.004
	14	5.023	5.023	4.009	3.003
	13	5.046	4.019	3.007	2.002
	12	4.038	3.014	2.004	2.004
	11	3.029	2.010-	2.010-	1.002
	10	2.020	2.020	1.005+	0.001
	9	2.039	1.011	0.002	0.002
	8	1.023	1.023	0.004	0.004
	7	1.045+	0.009	0.009	-
	6	0.020	0.020	-	-
	5	0.04	-	-	-
15	20	11.026	10.009	10.009	9.003
	19	10.040	9.015	8.006	7.002
	18	9.046	8.013	7.007	6.002
	17	8.047	7.020	6.008	5.003
	16	7.045-	6.013	5.007	4.002
	15	6.040	5.017	4.006	3.002
	14	5.034	4.013	3.004	3.004

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
$n_1=20$ $n_2=15$	13	4.028	3.010-	3.010-	2.003
	12	3.020	3.020	2.006	1.001
	11	3.039	2.013	1.003	1.003
	10	2.026	1.007	1.007	0.001
	9	2.049	1.015-	0.002	0.002
	8	1.029	0.005+	0.005+	-
	7	0.012	0.012	-	-
	6	0.024	0.024	-	-
	5	0.048	-	-	-
14	20	10.022	10.022	9.007	8.002
	19	9.032	8.012	7.004	7.004
	18	8.035+	7.014	6.005-	6.005-
	17	7.035-	6.013	5.005-	5.005-
	16	6.031	5.012	4.004	4.004
	15	5.026	4.009	4.009	3.003
	14	4.020	4.020	3.007	2.002
	13	4.040	3.015-	2.004	2.004
	12	3.029	2.009	2.009	1.002
	11	2.018	2.018	1.005-	1.005-
	10	2.035+	1.010-	1.010-	0.001
	9	1.019	1.019	0.003	0.003
	8	1.037	0.007	0.007	-
	7	0.014	0.014	-	-
	6	0.029	-	-	-
13	20	9.017	9.017	8.005+	7.002
	19	8.025-	8.025-	7.008	6.003
	18	7.026	6.009	6.009	5.003
	17	6.014	6.014	5.008	4.002
	16	5.021	5.020	4.007	3.001
	15	5.041	4.015+	3.005-	3.005-
	14	4.031	3.011	2.002	2.002
	13	3.022	3.022	2.004	1.001
	12	3.041	2.013	1.003	1.003
	11	2.026	1.007	1.007	0.001
	10	2.047	1.013	0.002	0.002
	9	1.026	0.004	0.004	0.004
	8	1.047	0.009	0.009	-
	7	0.018	0.018	-	-
	6	0.035-	-	-	-
12	20	9.044	8.014	7.004	7.004
	19	7.019	7.018	6.005	5.002
	18	6.018	6.018	5.006	4.002
	17	6.043	5.016	4.005-	4.005-
	16	5.034	4.012	3.003	3.001
	15	4.029+	3.028	3.009	2.002
	14	4.049	3.027	2.007-	2.005-
	13	3.043	2.011-	2.011-	1.001
	12	2.027	2.022	1.007-	1.005-
	11	2.036	1.009	1.009	0.001
	10	1.028	1.024	0.003	0.003
	9	1.034	0.006	0.006	-
	8	0.017	0.017	-	-
	7	0.023	0.023	-	-
	6	0.043	-	-	-

TABLE A-16 continued

TABLES FOR TESTING SIGNIFICANCE IN 2 X 2 TABLES

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
$n_1=20$ $n_2=11$	20	8.037	7.010+	6.003	6.003
	19	7.042	6.013	5.004	5.004
	18	6.037	5.012	4.003	4.003
	17	5.029	4.009	4.009	3.002
	16	4.021	4.021	3.006	2.001
	15	4.042	3.014	[illegible]	[illegible]
	14	3.028	2.008	[illegible]	1.001
	13	2.016	2.016	[illegible]	1.003
	12	2.029	1.007	1.007	0.001
	11	1.014	1.014	[illegible]	[illegible]
	10	1.026	0.004	0.004	0.004
	9	1.046	0.008	[illegible]	-
	8	0.016	0.016	-	
	7	0.029	-	-	-
10	20	7.030	6.005	[illegible]	[illegible]
	19	6.031	5.009	[illegible]	[illegible]
	18	5.026	4.007	[illegible]	[illegible]
	17	4.013	4.018	[illegible]	3.005-
	16	4.039	3.012	[illegible]	[illegible]
	15	3.024	3.024	2.006	1.001
	14	3.045+	2.013	[illegible]	1.003
	13	2.025+	1.006	1.006	0.001
	12	2.045-	[illegible]	0.001	0.001
	11	1.021	1.021	[illegible]	[illegible]
	10	1.037	0.006	0.006	-
	9	0.012	[illegible]	-	-
	8	[illegible]	[illegible]	-	-
	7	0.038	-	-	-
9	20	[illegible]	[illegible]	[illegible]	4.001
	19	5.022	[illegible]	[illegible]	3.001
	18	4.016	4.016	[illegible]	3.004
	17	4.037	3.010+	[illegible]	2.002
	16	3.022	[illegible]	[illegible]	1.001
	15	3.043	2.012	[illegible]	[illegible]
	14	2.023	2.023	[illegible]	1.005-
	13	2.041	[illegible]	[illegible]	0.001
	12	1.018	[illegible]	[illegible]	[illegible]
	11	[illegible]	[illegible]	[illegible]	[illegible]
	10	0.009	[illegible]	[illegible]	-
	9	0.017	0.017	-	-
	8	[illegible]	-	-	-
	7	0.050-	-	-	-
8	20	5.017	[illegible]	[illegible]	[illegible]
	19	[illegible]	[illegible]	[illegible]	[illegible]
	18	4.038	[illegible]	[illegible]	[illegible]
	17	[illegible]	[illegible]	[illegible]	[illegible]
	16	3.044	[illegible]	[illegible]	[illegible]
	15	[illegible]	[illegible]	[illegible]	1.004
	14	[illegible]	[illegible]	[illegible]	[illegible]
	13	[illegible]	[illegible]	[illegible]	[illegible]
	12	[illegible]	[illegible]	[illegible]	[illegible]
	11	[illegible]	[illegible]	[illegible]	-
	10	[illegible]	[illegible]	-	-
	9	[illegible]	[illegible]		-
	8	[illegible]	-	-	-

	a_1	Significance Level 0.05 (0.10)	0.025 (0.05)	0.01 (0.02)	0.005 (0.01)
$n_1=20$ $n_2=7$	20	4.012	4.012	3.002	3.002
	19	4.042	3.009	3.009	2.001
	18	3.024	3.024	2.005-	[illegible]
	17	3.050-	2.011	[illegible]	[illegible]
	16	2.023	2.023	1.004	1.004
	15	2.043	1.009	1.009	[illegible]
	14	1.016	1.016	[illegible]	[illegible]
	13	1.029	0.004	0.004	[illegible]
	12	1.048	0.007	0.007	-
	11	0.013	0.013	-	-
	10	0.022	0.022	-	-
	9	0.036	-	-	-
6	20	[illegible]	[illegible]	[illegible]	[illegible]
	19	[illegible]	[illegible]	[illegible]	[illegible]
	18	[illegible]	[illegible]	[illegible]	[illegible]
	17	[illegible]	[illegible]	[illegible]	[illegible]
	16	[illegible]	[illegible]	[illegible]	[illegible]
	15	1.018	1.018	[illegible]	[illegible]
	14	1.032	[illegible]	0.004	[illegible]
	13	0.007	0.007	0.007	-
	12	0.013	0.013	-	-
	11	0.022	0.022	-	-
	10	0.035-	-	-	-
5	20	3.033	2.004	[illegible]	2.004
	19	[illegible]	2.015	[illegible]	[illegible]
	18	2.038	1.005+	[illegible]	[illegible]
	17	1.012	[illegible]	[illegible]	[illegible]
	16	[illegible]	[illegible]	[illegible]	[illegible]
	15	1.046	[illegible]	[illegible]	[illegible]
	14	[illegible]	[illegible]	0.003	-
	13	0.015-	0.015-	-	-
	12	0.024	0.024	-	-
	11	0.038	-	-	-
4	20	[illegible]	[illegible]	[illegible]	[illegible]
	19	[illegible]	1.004	1.008	[illegible]
	18	1.018	1.019	0.001	0.001
	17	1.035+	0.003	[illegible]	[illegible]
	16	0.007	0.007	0.007	-
	15	0.012	[illegible]	-	-
	14	[illegible]	[illegible]	-	-
	13	[illegible]	-	-	-
	12	[illegible]	-	-	-
[illegible]	20	[illegible]	[illegible]	0.001	[illegible]
	19	[illegible]	[illegible]	[illegible]	[illegible]
	18	[illegible]	[illegible]	[illegible]	-
	17	[illegible]	[illegible]	-	-
	16	[illegible]	[illegible]	-	-
	15	[illegible]	-	-	-
	14	[illegible]	-	-	-
[illegible]	20	[illegible]	[illegible]	[illegible]	[illegible]
	19	[illegible]	[illegible]	-	-
	18	[illegible]	-	-	-
	17	[illegible]	-	-	-
[illegible]	[illegible]	[illegible]			

TABLE B-17

OUTLIER

Table of percentage points for $\frac{s_1^2}{s^2}$ and $\frac{s_2^2}{s^2}$

N	(FIRST OUTLIER CV)				(SECOND OUTLIER CV)		
	.99	.975	.95	.90	.99	.95	.90
3	.0002	.0014	.0054	.0218	NA	NA	NA
4	.0150	.0372	.0741	.1463	.0000	.0024	.0093
5	.0589	.1077	.1693	.2645	.0070	.0366	.0752
6	.1160	.1816	.2540	.3533	.0310	.0947	.1535
7	.1736	.2479	.3235	.4204	.0660	.1530	.2219
8	.2273	.3052	.3804	.4725	.1050	.2069	.2792
9	.2755	.3544	.4277	.5145	.1442	.2545	.3272
10	.3185	.3967	.4673	.5491	.1833	.2964	.366[illegible]
11	.3568	.4334	.5012	.5782	.2170	.3333	.4033
12	.3909	.4655	.5304	.6031	.2498	.3662	.4342
13	.4215	.4940	.5560	.6248	.2800	.3954	.4612
14	.4490	.5191	.5785	.6437	.3079	.4217	.4852
15	.4740	.5417	.5987	.6606	.3335	.4454	.5069
16	.4965	.5621	.6166	.6756	.3574	.4671	.5264
17	.5171	.5805	.6329	.6892	.3796	.4871	.5441
18	.5359	.5972	.6476	.7014	.4001	.5049	.5603
19	.5532	.6125	.6610	.7126	.4191	.5216	.5752

TABLE B-17 continued

N	.99	.975	.95	.90	.99	.95	.90
20	.5693	.6267	.6733	.7228	.4369	.5369	.5889
21	.5840	.6396	.6846	.7324	.4536	.5512	.6017
22	.5977	.6516	.6952	.7411	.4692	.5645	.6134
23	.6104	.6628	.7048	.7492	.4838	.5768	.6245
24	.6224	.6732	.7139	.7567	.4976	.5885	.6347
25	.6335	.6829	.7224	.7636	.5105	.5995	.6443
30		.7232	.7569	.7924			
35		.7540	.7827	.8142			
40		.7776	.8035	.8316			
45		.7969	.8203	.8458			
50		.8129	.8342	.8575			
55		.8264	.8462	.8674			
60		.8377	.8560	.8755			
70		.8565	.8724	.8895			
80		.8709	.8850	.9003			
90		.8827	.8955	.9089			
100		.8922	.9038	.9161			
200		.9390	.9451	.9513			
500		.9719	.9746	.9771			
1000		.8946	.9859	.9872			

TABLE B-17 continued

NOTE: For values between large samples, use linear interpolation; i.e., N = 53 and α = .05:

50 = .8342

5   3   x   .0120

53 =

55 = .8462

$$\frac{3}{5} = \frac{x}{.0120}$$

$$5(x) = 3(.0120)$$

$$x = \frac{.036}{5}$$

$$= .0072$$

Therefore, add .0072 to .8342 giving CV = .8414

TABLE B-18 *

RELIABILITY: SUCCESS - FAILURE

Number of failures = 0

Reliability	Confidence Level .99	.95	.90	.85	.80	.70	.60
.99	459	299	230	189	161	120	92
.98	228	149	114	94	80	60	46
.97	.52	99	76	63	53	40	31
.96	113	74	57	47	40	30	23
.95	90	59	45	37	32	24	18
.94	75	49	38	31	27	20	15
.93	64	42	32	27	23	17	13
.92	56	36	28	23	20	15	11
.91	49	32	25	21	18	13	10
.90	44	29	22	19	16	12	9
.89	40	26	20	17	14	11	8
.88	37	24	19	15	13	10	8
.87	34	22	17	14	12	9	7
.86	31	20	16	13	11	8	7
.85	29	19	15	12	10	8	6
.84	27	18	14	11	10	7	6
.83	25	17	13	11	9	7	5
.82	24	16	12	10	9	7	5
.81	22	15	11	10	8	6	5
.80	21	14	11	9	8	6	5

Number of failures = 1

Reliability	Confidence Level .99	.95	.90	.85	.80	.70	.60
.99	662	473	388	337	299	244	202
.98	330	236	194	168	149	122	101
.97	219	157	129	112	99	81	67
.96	164	117	96	84	74	61	51
.95	130	93	77	67	59	49	40
.94	108	78	64	56	49	40	34
.93	92	66	55	47	42	35	29
.92	81	58	48	41	37	30	25
.91	71	51	42	37	33	27	22
.90	64	46	38	33	29	24	20
.89	58	42	34	30	27	22	18
.88	53	38	31	27	24	20	17
.87	49	35	29	25	23	19	16
.86	45	32	27	23	21	17	14
.85	42	30	25	22	19	16	13
.84	39	28	23	20	18	15	13
.83	37	26	22	19	17	14	12
.82	34	25	21	18	16	13	11
.81	33	24	19	17	15	13	11
.80	31	22	18	16	14	12	10

*See page 2-125 for .75 confidence level.

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 2

Reliability	Confidence Level .99	.95	.90	.85	.80	.70	.60
.99	838	628	531	471	427	361	310
.98	418	313	265	235	213	180	155
.97	277	208	176	157	142	120	103
.96	207	156	132	117	106	90	78
.95	165	124	105	94	85	72	62
.94	137	103	88	78	71	60	52
.93	117	88	75	67	60	51	44
.92	102	77	65	58	53	45	39
.91	91	68	58	52	47	40	34
.90	81	61	52	46	42	36	31
.89	74	56	47	42	38	33	28
.88	67	51	43	38	35	30	26
.87	62	47	40	35	32	27	24
.86	57	43	37	33	30	25	22
.85	53	40	34	31	28	24	21
.84	50	38	32	29	26	22	19
.83	47	35	30	27	24	21	18
.82	44	33	28	25	23	20	17
.81	41	31	27	24	22	19	16
.80	39	30	25	23	21	18	15

Number of failures = 3

Reliability	Confidence Level .99	.95	.90	.85	.80	.70	.60
.99	1001	773	667	600	551	476	417
.98	499	386	333	300	275	238	209
.97	332	257	221	199	183	158	139
.96	248	192	166	149	137	119	104
.95	198	153	132	119	110	95	83
.94	164	127	110	99	91	79	69
.93	140	109	94	85	78	68	60
.92	122	95	82	74	68	59	52
.91	109	84	73	66	60	53	46
.90	97	76	65	59	54	47	42
.89	88	69	59	54	49	43	38
.88	81	63	54	49	45	39	35
.87	74	58	50	45	42	36	32
.86	69	53	46	42	39	34	30
.85	64	50	43	39	36	31	28
.84	60	47	40	37	34	29	26
.83	56	44	38	34	32	28	24
.82	53	41	36	32	30	26	23
.81	50	39	34	31	28	25	22
.80	47	37	32	29	27	23	21

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 4

Reliability / Confidence Level

Reliability	.99	.95	.90	.85	.80	.70	.60
.99	1157	913	798	726	671	589	524
.98	577	456	398	362	335	294	262
.97	383	303	265	241	223	196	174
.96	287	227	198	181	167	147	131
.95	229	181	158	144	134	117	105
.94	190	150	132	120	111	98	87
.93	162	129	113	103	95	84	75
.92	142	112	98	90	83	73	65
.91	126	100	87	80	74	65	58
.90	113	89	78	72	66	58	52
.89	102	81	71	65	60	53	47
.88	93	74	65	59	55	49	44
.87	86	68	60	55	51	45	40
.86	79	63	56	51	47	42	37
.85	74	59	52	47	44	39	35
.84	69	55	48	44	41	36	33
.83	65	52	45	42	39	3[illegible]	31
.82	61	49	43	39	36	3[illegible]	29
.81	58	46	40	37	34	31	27
.80	55	44	38	35	33	29	26

Number of failures = 5

Reliability / Confidence Level

Reliability	.99	.95	.90	.85	.80	.70	.60
.99	1307	1049	926	848	790	700	629
.98	652	523	462	423	394	350	314
.97	433	348	308	282	263	233	210
.96	324	261	230	211	197	175	157
.95	259	208	184	169	157	140	126
.94	215	173	153	140	131	116	105
.93	184	148	131	120	112	100	90
.92	160	129	114	105	98	87	78
.91	142	115	101	93	87	77	70
.90	127	103	91	84	78	70	63
.89	116	93	83	76	71	63	57
.88	106	85	76	69	65	58	52
.87	97	79	70	64	60	53	48
.86	90	73	65	59	55	50	45
.85	84	68	60	55	52	46	42
.84	78	63	56	52	48	43	39
.83	73	59	53	49	46	41	37
.82	69	56	50	46	43	38	35
.81	65	53	47	43	41	36	33
.80	62	50	45	41	39	34	31

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 6

Reliability	Confidence Level .99	.95	.90	.85	.80	.70	.60
.99	1453	1182	1051	969	906	811	734
.98	725	590	525	484	453	405	367
.97	482	392	349	322	301	270	245
.96	360	294	262	241	226	202	183
.95	288	234	209	193	180	162	147
.94	239	195	174	160	150	135	122
.93	204	167	149	137	129	115	105
.92	178	146	130	120	112	101	92
.91	158	129	115	106	100	90	81
.90	142	116	104	96	90	81	73
.89	129	105	94	87	81	73	67
.88	118	96	86	79	75	67	61
.87	108	89	79	73	69	62	56
.86	100	82	73	68	64	57	52
.85	93	76	68	63	59	53	49
.84	87	71	64	59	56	50	46
.83	82	67	60	56	52	47	43
.82	77	63	57	52	49	44	41
.81	73	60	54	50	47	42	38
.80	69	57	51	47	44	40	37

Number of failures = 7

Reliability	Confidence Level .99	.95	.90	.85	.80	.70	.60
.99	1596	1312	1175	1088	1022	920	839
.98	796	655	587	543	511	460	419
.97	529	436	390	362	340	306	279
.96	396	326	292	271	255	230	210
.95	316	260	234	216	204	184	168
.94	263	217	194	180	169	153	140
.93	225	185	166	154	145	131	120
.92	196	162	145	135	127	114	105
.91	174	143	129	120	113	102	93
.90	156	129	116	107	101	91	84
.89	141	117	105	98	92	83	76
.88	129	107	96	89	84	76	70
.87	119	98	89	82	78	70	64
.86	110	91	82	76	72	65	60
.85	103	85	77	71	67	61	56
.84	96	79	72	67	63	57	52
.83	90	75	67	63	59	54	49
.82	85	70	63	59	56	51	46
.81	80	66	60	56	53	48	44
.80	76	63	57	53	50	45	42

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 8

Reliability	Confidence Level						
	.99	.95	190	.85	.80	.70	.60
.99	1736	1441	1297	1206	1137	1029	943
.98	866	719	648	602	568	514	471
.97	576	478	431	401	378	343	314
.96	431	358	323	300	283	257	236
.95	344	286	258	240	226	205	188
.94	286	238	215	200	188	171	157
.93	244	203	184	171	161	146	135
.92	213	178	160	149	141	128	118
.91	189	158	142	133	125	114	105
.90	170	142	128	119	113	102	94
.89	154	128	116	108	102	93	86
.88	141	117	106	99	94	85	78
.87	130	108	98	91	86	79	72
.86	120	100	91	85	80	73	67
.85	112	93	85	79	75	68	63
.84	104	87	79	74	70	64	59
.83	98	82	74	69	66	60	55
.82	92	77	70	65	62	57	52
.81	87	73	66	62	59	54	49
.80	83	69	63	59	56	51	47

Number of failures = 9

Reliability	Confidence Level						
	.99	.95	.90	.85	.80	.70	.60
.99	1874	1568	1418	1323	1251	1138	1047
.98	935	782	708	661	625	569	524
.97	662	521	471	440	416	379	349
.96	465	390	353	330	312	284	262
.95	371	311	282	263	249	227	209
.94	309	259	235	219	207	189	174
.93	264	221	201	188	178	162	149
.92	230	193	175	164	155	142	131
.91	204	172	156	146	138	126	116
.90	183	154	140	131	124	113	105
.89	166	140	127	119	113	103	95
.88	152	128	116	109	103	94	87
.87	140	118	107	100	95	87	80
.86	130	109	99	93	88	81	75
.85	121	102	93	87	82	75	70
.84	113	95	87	81	77	70	65
.83	106	89	81	76	72	66	61
.82	100	84	77	72	68	63	58
.81	94	80	73	68	65	59	55
.80	89	76	69	65	61	56	52

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 10

Reliability	Confidence Level .99	.95	.90	.85	.80	.70	.60
.99	2010	1693	1538	1439	1364	1246	1151
.98	1003	845	768	719	681	623	576
.97	667	562	511	479	454	415	384
.96	499	421	383	359	340	311	288
.95	398	336	306	286	272	249	230
.94	331	280	255	238	226	207	192
.93	283	239	218	204	194	177	164
.92	247	209	190	178	169	155	144
.91	219	185	169	158	150	138	128
.90	197	167	152	142	135	124	115
.89	178	151	138	129	123	113	104
.88	163	138	126	118	112	103	96
.87	150	127	116	109	104	95	88
.86	139	118	108	101	96	88	82
.85	130	110	100	94	90	82	76
.84	121	103	94	88	84	77	72
.83	114	97	88	83	79	73	67
.82	107	91	83	78	74	68	64
.81	101	86	79	74	70	65	60
.80	96	82	75	70	67	62	57

Number of failures = 11

Reliability	Confidence Level .99	.95	.90	.85	.80	.70	.60
.99	2144	1818	1658	1555	1476	1354	1255
.98	1070	907	828	777	737	677	627
.97	712	604	551	517	491	451	418
.96	532	452	413	387	368	338	314
.95	425	361	330	310	294	270	251
.94	353	300	274	258	245	225	209
.93	302	257	235	221	210	193	179
.92	264	224	205	193	183	169	157
.91	234	199	182	171	163	150	139
.90	210	174	164	154	146	135	125
.89	190	162	149	140	133	122	114
.88	174	149	136	128	122	112	104
.87	160	137	125	118	112	103	96
.86	149	127	116	109	104	96	89
.85	138	118	108	102	97	90	83
.84	129	111	101	95	91	84	78
.83	121	104	95	90	85	79	74
.82	114	98	90	85	81	74	69
.81	108	93	85	80	76	70	66
.80	102	88	81	76	72	67	62

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 12

Reliability	Confidence Level .99	.95	.90	.85	.80	.70	.60
.99	2277	1941	1776	1670	1588	1461	1359
.98	1136	969	887	834	793	730	679
.97	756	645	590	555	528	487	453
.96	566	483	442	416	396	365	339
.95	451	386	353	332	316	292	271
.94	375	321	294	277	263	243	226
.93	321	274	252	237	226	208	194
.92	280	240	220	207	197	182	170
.91	249	213	195	184	175	162	151
.90	223	191	175	165	157	145	136
.89	202	173	159	150	143	132	123
.88	185	159	146	137	131	121	113
.87	170	146	134	127	121	112	104
.86	158	136	125	117	112	104	97
.85	147	126	116	109	104	97	90
.84	138	118	109	103	98	91	85
.83	129	111	102	96	92	85	80
.82	122	105	96	91	87	80	75
.81	115	99	91	86	82	76	71
.80	109	94	86	82	78	72	68

Number of failures = 13

Reliability	Confidence Level .99	.95	.90	.85	.80	.70	.60
.99	2409	2064	1893	1784	1700	1569	1492
.98	1202	1030	945	891	849	784	731
.97	800	686	629	593	566	522	487
.96	598	513	471	444	424	392	365
.95	478	410	377	355	339	313	292
.94	397	341	313	296	282	261	243
.93	340	292	268	253	242	223	209
.92	297	255	234	221	211	195	182
.91	263	226	208	196	187	174	162
.90	236	203	187	177	169	156	146
.89	214	184	170	160	153	142	133
.88	196	169	155	147	140	130	122
.87	180	156	143	135	129	120	112
.86	167	144	133	126	120	111	104
.85	156	134	124	117	112	104	97
.84	146	126	116	110	105	97	91
.83	137	118	109	103	98	91	86
.82	129	111	103	97	93	86	81
.81	122	105	97	92	88	82	77
.80	115	100	92	87	83	78	73

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 14

Reliability	Confidence Level .99	.95	.90	.85	.80	.70	.60
.99	2539	2185	2010	1898	1811	1676	1565
.98	1267	1091	1004	948	905	837	783
.97	843	726	668	631	603	558	522
.96	631	544	501	473	452	418	391
.95	504	434	400	378	361	334	313
.94	419	361	333	315	300	279	261
.93	358	309	285	269	257	239	223
.92	313	270	249	235	225	209	195
.91	277	240	221	209	200	185	174
.90	249	215	199	188	180	167	156
.89	226	195	180	171	163	151	130
.88	207	179	165	156	149	139	130
.87	190	165	152	144	138	128	120
.86	176	153	141	134	128	119	111
.85	164	142	132	125	119	111	104
.84	154	133	123	117	112	104	98
.83	144	125	116	110	105	98	92
.82	136	118	109	103	99	92	87
.81	128	112	103	98	94	87	82
.80	122	106	98	93	89	83	78

Number of failures = 15

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	2704	2343	2140	2013	1917
.98	1348	1169	1069	1006	958
.97	896	778	711	670	628
.96	670	582	533	502	478
.95	535	465	426	401	382
.94	445	387	354	334	318
.93	380	331	303	286	273
.92	332	289	265	250	238
.91	294	256	235	222	212
.90	264	230	211	199	190
.89	239	209	192	181	173
.88	219	191	176	166	158
.87	201	176	162	153	146
.86	186	163	150	142	135
.85	173	152	140	132	126
.84	162	142	131	124	118
.83	152	134	123	116	111
.82	143	126	116	110	105
.81	135	119	110	104	99
.80	128	113	104	99	94

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 16

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	2832	2463	2256	2126	2028
.98	1412	1229	1126	1062	1013
.97	939	818	750	707	675
.96	702	612	562	530	506
.95	560	489	449	423	404
.94	466	407	373	352	337
.93	398	348	320	302	288
.92	347	304	279	264	252
.91	308	270	248	234	224
.90	276	242	223	211	201
.89	251	220	202	191	183
.88	229	201	185	175	167
.87	211	185	171	161	154
.86	195	172	158	150	143
.85	182	160	147	140	134
.84	170	150	138	131	125
.83	159	141	130	123	118
.82	150	133	122	116	111
.81	142	125	116	110	105
.80	134	119	110	104	100

Number of failures = 17

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	2960	2582	2371	2238	2138
.98	1476	1289	1184	1118	1068
.97	981	858	788	745	711
.96	734	642	590	558	533
.95	586	513	472	446	426
.94	487	427	393	371	355
.93	416	365	336	318	304
.92	363	319	294	278	266
.91	322	283	261	247	236
.90	289	254	234	222	212
.89	262	231	213	201	193
.88	240	211	195	184	177
.87	221	194	179	170	163
.86	204	180	166	158	151
.85	190	168	155	147	141
.84	178	157	145	138	132
.83	167	148	136	129	124
.82	157	139	129	122	117
.81	148	131	122	116	111
.80	141	125	115	110	105

TABLE B-13 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 18

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	3086	2701	2486	2351	2247
.98	1539	1348	1241	1174	1123
.97	1024	897	827	782	748
.96	766	672	616	586	561
.95	611	536	495	463	448
.94	508	446	412	390	373
.93	434	382	352	334	320
.92	379	334	308	292	279
.91	336	296	273	259	248
.90	302	266	246	233	223
.89	273	241	223	212	203
.88	250	221	204	194	186
.87	230	203	188	179	171
.86	213	189	175	166	159
.85	198	176	163	154	148
.84	185	164	152	145	139
.83	174	154	143	136	131
.82	164	146	135	128	123
.81	155	138	128	121	117
.80	147	131	121	115	111

Number of failures = 19

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	3212	2819	2600	2462	2357
.98	1602	1407	1299	1230	1178
.97	1066	937	865	819	784
.96	797	701	648	614	588
.95	636	560	517	491	470
.94	529	466	431	[illegible]	393
.93	452	399	369	[illegible]	[illegible]
.92	395	348	322	306	293
.91	350	309	286	[illegible]	260
.90	314	278	257	[illegible]	234
.89	285	252	233	[illegible]	213
.88	260	231	214	203	[illegible]
.87	240	212	197	187	[illegible]
.86	222	197	183	[illegible]	167
.85	207	183	170	162	155
.84	193	172	159	152	146
.83	181	161	150	143	[illegible]
.82	171	152	141	[illegible]	129
.81	161	144	134	127	122
.80	153	136	127	[illegible]	116

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 20

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	3338	2937	2714	2574	2466
.98	1665	1466	1356	1286	1232
.97	1107	976	903	586	821
.96	828	731	676	642	615
.95	661	584	540	513	492
.94	550	486	450	427	410
.93	470	415	385	366	351
.92	410	363	336	320	307
.91	364	322	299	284	272
.90	327	289	268	255	245
.89	296	263	244	232	223
.88	271	240	223	212	204
.87	249	221	206	196	188
.86	231	205	191	181	174
.85	215	191	178	169	163
.84	201	179	166	158	152
.83	189	168	156	149	143
.82	178	159	148	141	135
.81	168	150	140	133	128
.80	159	142	132	126	122

Number of failures = 21

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	3462	3055	2828	2685	2575
.98	1727	1525	1412	1341	1287
.97	1149	1015	940	893	857
.96	860	760	705	669	642
.95	686	607	563	535	514
.94	570	505	469	445	428
.93	488	432	401	381	366
.92	426	378	351	333	320
.91	377	335	311	296	284
.90	339	301	280	266	256
.89	307	273	254	242	232
.88	281	250	233	221	213
.87	259	230	214	204	196
.86	240	214	199	189	182
.85	223	199	185	177	170
.84	209	186	173	165	159
.83	196	175	163	155	150
.82	185	165	154	147	141
.81	174	156	146	139	134
.80	165	148	138	132	127

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 22

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	3587	3172	2942	2796	2684
.98	1789	1584	1469	1397	1341
.97	1190	1054	978	930	894
.96	890	789	733	697	670
.95	711	630	586	557	535
.94	591	525	487	464	446
.93	505	449	417	397	382
.92	441	392	365	347	334
.91	391	348	324	308	297
.90	351	313	291	277	267
.89	318	284	264	252	242
.88	291	260	242	231	222
.87	268	239	223	213	205
.86	248	222	207	197	190
.85	231	207	193	184	177
.84	216	193	181	172	166
.83	203	182	170	162	156
.82	191	171	160	153	147
.81	181	162	151	145	139
.80	171	154	144	137	132

Number of failures = 23

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	3710	3289	3055	2907	2793
.98	1851	1642	1526	1452	1396
.97	1231	1093	1016	967	930
.96	921	818	761	725	697
.95	735	654	608	579	557
.94	611	544	506	482	464
.93	523	465	433	413	397
.92	456	407	379	361	347
.91	405	361	336	321	309
.90	363	324	302	288	278
.89	330	294	274	262	252
.88	301	269	251	240	231
.87	278	248	232	221	213
.86	257	230	215	205	198
.85	239	214	200	191	184
.84	224	201	188	179	173
.83	210	189	176	168	162
.82	198	178	166	159	153
.81	187	168	157	150	145
.80	177	159	149	143	138

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 24

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	3834	3406	3168	3017	2902
.98	1913	1700	1582	1507	1450
.97	1272	1132	1054	1004	966
.96	952	848	789	752	724
.95	760	677	631	601	579
.94	632	563	525	501	482
.93	540	482	449	429	413
.92	472	421	393	375	361
.91	418	374	349	333	321
.90	376	336	313	299	288
.89	341	305	285	272	262
.88	312	279	261	249	240
.87	287	257	240	230	221
.86	266	238	223	213	205
.85	248	222	208	199	192
.84	232	208	195	186	179
.83	217	195	183	175	169
.82	205	184	172	165	159
.81	194	174	163	156	151
.80	183	165	155	148	143

Number of failures = 25

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	3956	3522	3280	3127	3010
.98	1974	1758	1638	1562	1504
.97	1313	1170	1091	1041	1002
.96	983	876	817	780	751
.95	784	700	753	623	600
.94	652	583	544	519	500
.93	558	499	465	444	428
.92	487	436	407	389	374
.91	432	387	361	345	333
.90	388	347	325	310	299
.89	352	315	295	282	272
.88	322	289	270	258	249
.87	296	266	249	238	230
.86	275	247	231	221	213
.85	256	230	215	206	199
.84	239	215	201	193	186
.83	224	202	189	181	175
.82	212	191	179	171	165
.81	200	180	169	162	156
.80	189	171	160	154	148

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 26

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	4079	3637	3393	3237	3118
.98	2035	1816	1695	1617	1558
.97	1354	1209	1128	1077	1038
.96	1013	905	845	807	778
.95	809	723	676	645	622
.94	673	602	562	537	518
.93	575	515	481	460	444
.92	502	450	421	402	388
.91	445	399	374	357	345
.90	400	359	336	321	310
.89	363	326	305	292	282
.88	332	298	279	267	258
.87	306	275	257	246	238
.86	283	255	239	229	221
.85	264	237	223	213	206
.84	247	222	208	200	193
.83	232	209	196	188	181
.82	218	197	185	177	171
.81	206	186	175	168	162
.80	195	177	166	159	154

Number of failures = 27

Reliability	Confidence Level .99	.95	.90		
.99	4201	3753	3505	3347	3226
.98	2096	1874	1751	1672	1612
.97	1394	1247	1166	1114	1074
.96	1044	934	873	835	805
.95	833	746	698	667	644
.94	693	621	581	556	536
.93	593	532	497	476	459
.92	517	464	435	416	401
.91	459	412	386	369	357
.90	412	370	347	332	321
.89	374	336	315	302	291
.88	342	308	289	276	267
.87	315	284	266	255	246
.86	292	263	247	236	228
.85	272	245	230	220	213
.84	254	229	215	206	200
.83	239	216	202	194	188
.82	225	203	191	183	177
.81	213	192	181	173	168
.80	201	182	172	165	159

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 28

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	4322	3868	3617	3457	3334
.98	2157	1931	1807	1727	1666
.97	1435	1286	1203	1150	1110
.96	1074	963	901	862	832
.95	857	769	720	689	665
.94	713	640	600	574	554
.93	610	548	513	491	474
.92	532	479	449	430	415
.91	472	425	398	382	369
.90	424	382	358	343	331
.89	385	347	325	312	301
.88	352	317	298	285	276
.87	324	292	275	263	254
.86	300	271	255	244	236
.85	280	253	237	228	220
.84	262	237	222	213	206
.83	246	222	209	201	194
.82	232	210	197	189	183
.81	219	198	187	179	173
.80	207	188	177	170	165

Number of failures = 29

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	4443	3983	3729	3566	3442
.98	2217	1989	1862	1782	1720
.97	1475	1324	1240	1187	1146
.96	1104	992	929	889	859
.95	881	792	743	711	687
.94	733	659	618	592	572
.93	627	564	529	507	490
.92	548	493	463	443	428
.91	486	438	411	394	380
.90	436	393	369	354	342
.89	396	357	335	322	311
.88	362	327	307	294	285
.87	333	301	283	272	263
.86	309	279	263	252	244
.85	288	260	245	235	227
.84	269	244	229	220	213
.83	253	229	216	207	200
.82	238	216	203	195	189
.81	225	204	192	185	179
.80	213	194	183	175	170

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 30

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	4564	4098	3840	3676	3549
.98	2278	2046	1918	1836	1774
.97	1515	1362	1277	1223	1182
.96	1134	1020	957	917	886
.95	906	815	765	733	708
.94	753	678	637	610	590
.93	644	581	545	523	505
.92	563	507	477	457	442
.91	499	450	423	406	392
.90	448	405	380	365	353
.89	407	367	345	331	321
.88	372	336	316	304	294
.87	343	310	292	280	271
.86	317	287	271	260	251
.85	296	268	252	242	234
.84	277	251	236	227	220
.83	260	236	222	213	207
.82	245	222	209	201	195
.81	231	210	198	191	185
.80	219	199	188	181	175

Number of failures = 31

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	4685	4213	3952	3785	3657
.98	2338	2104	1974	1891	1827
.97	1555	1400	1315	1260	1217
.96	1164	1049	985	944	913
.95	930	838	787	755	730
.94	773	697	655	628	608
.93	661	597	561	538	520
.92	577	522	490	470	455
.91	512	463	435	418	404
.90	460	416	391	376	364
.89	417	378	356	341	330
.88	382	346	326	313	303
.87	352	319	300	288	279
.86	326	296	278	267	259
.85	304	275	260	249	242
.84	284	258	243	234	226
.83	267	242	229	220	213
.82	251	229	216	207	201
.81	238	216	204	196	190
.80	225	205	194	186	181

TABLE B-18 continued

RELIABILITY: SUCCESS -- FAILURE

Number of failures = 32

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	4805	4327	4063	3894	3764
.98	2398	2161	2029	1946	1881
.97	1595	1439	1352	1296	1253
.96	1194	1077	1013	971	939
.95	954	861	809	776	751
.94	793	716	674	646	625
.93	678	613	577	554	536
.92	592	536	504	484	468
.91	526	476	448	430	416
.90	472	427	403	387	374
.89	428	388	366	351	340
.88	392	355	335	322	312
.87	361	327	309	297	287
.86	334	304	286	275	267
.85	311	283	267	257	249
.84	291	265	250	240	233
.83	274	249	235	226	219
.82	258	235	222	213	207
.81	244	222	210	202	196
.80	231	211	199	192	186

Number of failures = 33

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	4925	4441	4174	4003	3872
.98	2458	2218	2085	2000	1935
.97	1635	1476	1389	1332	1289
.96	1224	1106	1040	998	966
.95	977	884	831	798	772
.94	813	735	692	665	643
.93	695	629	593	569	551
.92	607	550	518	498	482
.91	539	488	460	442	428
.90	484	439	414	397	385
.89	439	398	376	361	350
.88	402	365	344	331	320
.87	370	336	317	305	296
.86	343	312	294	283	274
.85	319	291	274	264	256
.84	299	272	257	247	240
.83	281	256	241	232	225
.82	265	241	228	219	213
.81	250	228	216	208	201
.80	237	216	205	197	191

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 34

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	5044	4555	4285	4112	3979
.98	2517	2275	2140	2054	1988
.97	1675	1514	1425	1369	1325
.96	1254	1134	1068	1026	993
.95	1001	906	854	820	794
.94	833	754	711	683	661
.93	712	646	608	585	566
.92	622	564	532	511	495
.91	552	501	472	454	440
.90	496	450	425	408	396
.89	450	409	386	371	359
.88	412	374	353	340	329
.87	379	345	326	313	304
.86	351	320	302	291	282
.85	327	298	282	271	263
.84	306	279	264	254	246
.83	288	262	248	239	232
.82	271	247	234	225	219
.81	256	234	221	213	207
.80	243	222	210	202	197

Number of failures = 35

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	5164	4669	4395	4221	4086
.98	2577	2331	2196	2109	2042
.97	1715	1552	1462	1405	1360
.96	2184	1163	1096	1053	1020
.95	1025	929	876	842	815
.94	853	773	729	701	679
.93	729	662	624	600	582
.92	637	578	546	525	509
.91	565	513	485	466	452
.90	508	461	436	419	406
.89	461	419	396	381	369
.88	421	384	362	349	338
.87	388	354	334	322	312
.86	360	328	310	298	290
.85	335	306	289	278	270
.84	314	286	271	261	253
.83	295	269	254	245	238
.82	278	254	240	231	225
.81	262	240	227	219	213
.80	249	228	216	208	202

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 36

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	5283	4782	4506	4329	4193
.98	2637	2388	2251	2163	2095
.97	1754	1590	1499	1441	1396
.96	1313	1191	1123	1080	1046
.95	1049	952	898	863	837
.94	872	792	747	719	697
.93	746	678	640	616	597
.92	652	592	559	538	522
.91	578	526	497	478	464
.90	519	473	447	430	417
.89	471	429	406	391	379
.88	431	393	371	358	347
.87	397	362	343	330	320
.86	368	336	318	306	297
.85	343	313	296	285	277
.84	321	293	277	267	260
.83	301	275	261	251	244
.82	284	260	246	237	231
.81	269	246	233	225	218
.80	255	233	221	213	207

Number of failures = 37

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	5402	4896	4616	4438	4300
.98	2696	2445	2306	2217	2149
.97	1794	1628	1536	1477	1432
.96	1343	1219	1151	1107	1073
.95	1073	974	920	885	858
.94	892	811	766	737	715
.93	763	694	656	631	612
.92	667	607	573	552	535
.91	591	538	509	490	475
.90	531	484	458	441	428
.89	482	439	416	400	389
.88	441	402	381	367	356
.87	406	371	351	338	328
.86	377	344	326	314	305
.85	351	321	304	293	284
.84	328	300	284	274	266
.83	308	282	267	258	250
.82	291	266	252	243	236
.81	275	252	239	230	224
.80	261	239	227	219	213

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 38

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	5520	5009	4727	4546	4406
.98	2755	2501	2361	2271	2202
.97	1833	1665	1573	1513	1467
.96	1373	1248	1178	1134	1100
.95	1096	997	942	907	879
.94	912	830	784	755	732
.93	780	710	671	647	627
.92	681	621	587	565	549
.91	605	551	521	502	487
.90	543	495	469	452	438
.89	493	450	426	410	398
.88	451	412	390	376	365
.87	415	379	359	347	337
.86	385	352	333	322	312
.85	359	328	311	300	291
.84	336	307	291	281	273
.83	315	289	274	264	257
.82	297	272	258	249	242
.81	281	258	244	236	229
.80	266	244	232	224	218

Number of failures = 39

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	5638	5122	4837	4654	4513
.98	2814	2558	2416	2325	2255
.97	1873	1703	1609	1549	1503
.96	1402	1276	1206	1161	1126
.95	1120	1019	964	928	901
.94	931	848	802	773	750
.93	797	726	687	662	643
.92	696	635	601	579	562
.91	618	563	533	514	499
.90	555	507	480	462	449
.89	503	460	436	420	408
.88	461	421	399	385	374
.87	424	388	368	355	345
.86	393	360	341	329	320
.85	366	335	318	307	298
.84	343	314	298	288	280
.83	322	295	280	270	263
.82	304	278	264	242	235
.81	287	263	250	242	235
.80	272	250	237	229	223

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 40

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	5757	5235	4947	4762	4620
.98	2873	2614	2471	2379	2309
.97	1912	1741	1646	1585	1538
.96	1432	1304	1233	1188	1153
.95	1143	1042	986	950	922
.94	951	867	821	791	768
.93	814	742	703	677	658
.92	711	649	614	592	575
.91	631	576	546	526	511
.90	567	518	491	473	460
.89	514	470	446	430	418
.88	470	430	408	394	383
.87	433	397	376	363	353
.86	402	368	349	337	328
.85	374	343	325	314	305
.84	350	321	305	294	286
.83	329	302	287	277	269
.82	310	285	270	261	254
.81	293	269	256	247	241
.80	278	256	243	235	228

Number of failures = 41

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	5874	5347	5057	4870	4726
.98	2932	2670	2526	2433	2362
.97	1951	1778	1683	1621	1574
.96	1461	1332	1261	1251	1180
.95	1167	1064	1008	971	943
.94	971	886	839	809	785
.93	830	758	718	693	673
.92	725	663	628	606	588
.91	644	588	558	538	523
.90	578	529	502	484	470
.89	525	480	455	440	427
.88	480	440	417	403	391
.87	442	405	385	371	361
.86	410	376	357	345	33[illegible]
.85	382	350	333	321	313
.84	357	328	312	301	293
.83	336	308	293	283	275
.82	317	291	276	267	260
.81	299	275	262	253	246
.80	284	261	248	240	234

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 42

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	5992	5460	5167	4978	4833
.98	2991	2727	2581	2487	2415
.97	1991	1816	1719	1657	1609
.96	1490	1360	1288	1242	1206
.95	1190	1087	1030	999	964
.94	990	905	857	827	803
.93	847	774	734	708	688
.92	740	677	642	619	602
.91	657	601	570	550	535
.90	590	540	512	495	481
.89	535	490	465	449	437
.88	490	449	476	412	400
.87	451	414	393	380	369
.86	418	384	365	352	343
.85	390	358	340	329	320
.84	365	335	318	308	299
.83	343	315	299	289	282
.82	323	297	283	273	266
.81	305	281	267	259	252
.80	290	267	254	245	239

Number of failures = 43

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	6110	5572	5276	5086	4939
.98	3050	2783	2636	2541	2468
.97	2030	1853	1756	1693	1645
.96	1520	1388	1316	1269	1233
.95	1214	1109	1052	1014	986
.94	1010	923	875	845	821
.93	864	790	750	724	703
.92	755	691	655	633	615
.91	670	613	582	562	546
.90	602	551	523	505	491
.89	546	501	475	459	447
.88	500	458	435	421	409
.87	460	423	401	388	377
.86	427	392	372	360	350
.85	398	365	347	336	327
.84	372	342	325	314	306
.83	349	321	306	296	288
.82	329	303	289	279	272
.81	312	287	273	264	257
.80	295	272	259	251	244

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 44

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	6227	5684	5386	5194	5045
.98	3108	2839	2691	2595	2521
.97	2069	1890	1792	1729	1680
.96	1549	1416	1343	1296	1259
.95	1237	1132	1073	1036	1007
.94	1029	942	894	863	839
.93	881	806	765	739	718
.92	769	705	669	646	628
.91	683	626	594	574	558
.90	613	562	534	516	502
.89	557	511	485	469	456
.88	509	468	444	429	418
.87	469	431	410	396	386
.86	435	400	380	368	358
.85	405	373	355	343	334
.84	379	349	332	321	313
.83	356	328	312	302	294
.82	336	309	295	285	278
.81	318	293	279	270	263
.80	301	278	265	256	250

Number of Failures = 45

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	6344	5796	5495	5302	5152
.98	3167	2895	2745	2649	2574
.97	2108	1928	1829	1765	1715
.96	1578	1444	1370	1323	1286
.95	1260	1154	1095	1057	1028
.94	1049	961	912	881	856
.93	897	822	781	754	734
.92	784	719	683	660	642
.91	696	638	606	586	570
.90	625	574	545	527	513
.89	567	521	495	479	466
.88	519	477	453	438	427
.87	478	440	418	404	394
.86	443	408	388	375	365
.85	413	380	362	350	341
.84	386	356	339	328	319
.83	363	335	319	308	300
.82	342	316	301	291	283
.81	324	299	285	275	268
.80	307	283	270	261	255

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 46

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	6461	5908	5605	5409	5258
.98	3225	2951	2800	2703	2627
.97	2147	1965	1865	1801	1751
.96	1607	1472	1398	1350	1312
.95	12[illegible]4	1176	1117	1079	1049
.94	1068	979	930	898	874
.93	914	838	797	770	749
.92	798	733	696	673	655
.91	708	651	618	598	582
.90	637	585	556	538	523
.89	578	531	505	488	475
.88	529	486	463	447	436
.87	487	448	427	413	402
.86	452	416	396	383	373
.85	421	388	269	357	348
.84	394	363	346	335	326
.83	370	341	325	315	307
.82	349	322	307	297	289
.81	330	304	290	281	274
.80	313	289	275	267	260

Number of failures = 47

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	6578	6020	5714	5517	5364
.98	3284	3007	2855	2756	2680
.97	2185	2002	1901	1836	1786
.96	1636	1500	1425	1376	1339
.95	1307	1199	1139	1100	1070
.94	1087	998	948	916	892
.93	931	854	812	785	764
.92	813	747	710	686	668
.91	721	663	631	610	593
.90	648	596	567	548	534
.89	588	541	515	498	485
.88	538	495	472	456	444
.87	496	457	435	421	410
.86	460	424	404	391	380
.85	428	395	376	364	355
.84	401	370	352	341	333
.83	377	348	331	321	313
.82	355	328	313	303	295
.81	336	310	296	287	280
.80	318	294	281	272	265

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 48

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	6694	6132	5823	5624	5470
.98	3342	3063	2909	2810	2733
.97	2224	2040	1938	1872	1821
.96	1666	1528	1452	1403	1365
.95	1330	1221	1161	1122	1092
.94	1107	1016	966	934	909
.93	947	870	828	800	779
.92	827	761	724	700	681
.91	734	675	643	622	605
.90	660	607	578	559	544
.89	599	551	525	508	495
.88	548	505	481	465	453
.87	505	465	443	429	418
.86	468	432	411	398	388
.85	436	402	384	371	362
.84	408	377	359	348	339
.83	383	354	338	327	319
.82	362	334	319	309	301
.81	342	316	302	292	285
.80	324	300	286	278	271

Number of failures = 49

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	6811	6244	5932	5731	5576
.98	3400	3118	2964	2864	2786
.97	2263	2077	1974	1908	1857
.96	1695	1556	1479	1430	1392
.95	1353	1243	1182	1143	1113
.94	1126	1035	985	952	927
.93	964	886	843	816	794
.92	842	774	737	713	694
.91	747	688	655	633	617
.90	671	618	589	570	555
.89	609	561	535	518	504
.88	558	514	490	474	462
.87	514	474	452	437	426
.86	476	439	419	406	396
.85	444	410	391	378	369
.84	415	384	366	355	346
.83	390	361	344	333	325
.82	368	340	325	315	307
.81	348	322	307	298	291
.80	330	305	292	283	276

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 50

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	6927	6355	6041	5839	5682
.98	3458	3174	3018	2917	2839
.97	2302	2114	2010	1944	1892
.96	1724	1584	1507	1457	1418
.95	1377	1266	1204	1165	1134
.94	1145	1053	1003	970	945
.93	980	902	859	831	809
.92	856	788	751	726	708
.91	760	700	667	645	629
.90	683	629	600	580	566
.89	620	571	545	527	514
.88	567	523	499	483	471
.87	523	482	460	446	434
.86	484	447	427	413	403
.85	451	417	398	386	376
.84	422	391	373	361	352
.83	397	367	351	340	331
.82	374	346	331	321	313
.81	354	328	313	304	296
.80	336	311	297	288	281

Number of failures = 51

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	7043	6467	6150	5946	5788
.98	3516	3230	3072	2971	2892
.97	2340	2151	2047	1979	1927
.96	1753	1611	1534	1484	1445
.95	1400	1288	1226	1186	1155
.94	1165	1072	1021	988	962
.93	997	918	874	846	824
.92	871	802	764	740	721
.91	773	712	679	657	640
.90	694	640	610	591	576
.89	630	581	554	537	523
.88	577	532	508	492	480
.87	531	491	468	454	442
.86	493	455	434	421	411
.85	459	424	405	393	383
.84	430	397	380	368	359
.83	404	374	357	346	338
.82	381	352	337	327	319
.81	360	334	319	309	302
.80	341	316	303	293	286

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 52

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	7159	6578	6259	6053	5893
.98	3574	3285	3127	3025	2945
.97	2379	2188	2083	2015	1962
.96	1781	1639	1561	1510	1471
.95	1423	1310	1248	1207	1176
.94	1184	1090	1039	1006	980
.93	1013	934	890	861	839
.92	885	816	778	753	734
.91	786	725	691	669	652
.90	706	651	621	602	587
.89	641	592	564	547	533
.88	586	542	517	501	488
.87	540	499	477	462	451
.86	501	463	442	429	418
.85	467	432	412	400	390
.84	437	404	386	375	365
.83	410	380	363	352	344
.82	387	359	343	332	325
.81	366	339	324	315	307
.80	347	322	308	299	292

Number of failures = 53

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	7275	6689	6368	6160	5999
.98	3632	3341	3181	3078	2998
.97	2417	2225	2119	2031	1998
.96	1810	1667	1588	1537	1498
.95	1446	1332	1269	1229	1197
.94	1203	1109	1057	1023	997
.93	1030	949	905	877	854
.92	900	830	791	767	747
.91	798	737	703	681	664
.90	717	662	632	612	597
.89	651	602	574	556	543
.88	596	551	526	510	497
.87	549	508	485	470	459
.86	509	471	450	436	426
.85	474	439	420	407	397
.84	444	411	393	381	372
.83	417	387	370	359	350
.82	393	365	349	338	330
.81	372	345	330	320	313
.80	353	327	313	304	297

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 54

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	7390	6800	6476	6267	6105
.98	3690	3396	3236	3132	3051
.97	2456	2262	2155	2086	2033
.96	1839	1695	1615	1564	1524
.95	1469	1354	1291	1250	1219
.94	1222	1127	1075	1041	1015
.93	1046	965	921	892	870
.92	914	844	805	780	760
.91	811	749	715	693	676
.90	729	674	643	623	608
.89	662	612	584	566	552
.88	605	560	535	519	506
.87	558	516	493	478	467
.86	517	479	458	444	433
.85	482	447	427	414	404
.84	451	418	400	388	379
.83	424	393	376	365	356
.82	400	371	355	344	336
.81	378	351	336	326	318
.80	359	333	319	309	302

Number of failures = 55

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	7506	6911	6585	6374	6210
.98	3747	3452	3290	3185	3104
.97	2494	2299	2191	2122	2068
.96	1868	1722	1642	1591	1550
.95	1492	1376	1313	1272	1240
.94	1242	1146	1093	1059	1033
.93	1063	981	936	907	885
.92	928	858	818	793	774
.91	824	761	727	705	687
.90	740	685	654	634	618
.89	672	622	594	576	562
.88	615	569	544	527	515
.87	567	525	502	487	475
.86	525	487	465	451	441
.85	490	454	434	421	411
.84	458	425	407	395	385
.83	431	400	382	371	362
.82	406	377	361	350	342
.81	384	357	341	332	324
.80	364	338	324	315	307

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 56

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	7621	7022	6693	6481	6316
.98	3805	3507	3344	3238	3156
.97	2533	2336	2228	2158	2103
.96	1897	1750	1669	1617	1527
.95	1515	1399	1334	1293	1261
.94	1261	1164	1111	1077	1050
.93	1079	997	952	922	900
.92	943	871	832	807	787
.91	837	774	739	716	699
.90	752	696	665	644	629
.89	682	632	604	585	571
.88	625	578	553	536	523
.87	576	533	510	495	483
.86	534	495	473	459	448
.85	497	461	441	428	418
.84	465	432	413	401	392
.83	437	406	389	377	369
.82	412	383	367	356	348
.81	390	362	347	337	329
.80	370	344	329	320	313

Number of failures = 57

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	7736	7133	6802	6588	6422
.98	3862	3563	3398	3292	3209
.97	2571	2373	2264	2193	2138
.96	1926	1778	1696	1644	1603
.95	1538	1421	1356	1314	1282
.94	1280	1183	1129	1095	1068
.93	1095	1013	967	938	915
.92	957	885	846	820	800
.91	849	786	751	728	711
.90	763	707	675	655	639
.89	693	642	613	595	581
.88	634	588	562	545	532
.87	584	542	518	503	491
.86	542	503	481	467	456
.85	505	469	448	435	425
.84	473	439	420	408	398
.83	444	413	395	384	375
.82	419	389	373	362	354
.81	396	368	353	343	335
.80	376	349	335	325	318

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 58

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	7851	7243	6910	6694	6527
.98	3920	3618	3452	3345	3262
.97	2609	2410	2300	2229	2174
.96	1954	1805	1723	1671	1629
.95	1561	1443	1378	1336	1303
.94	1299	1201	1147	1112	1085
.93	1112	1028	983	953	930
.92	971	899	859	833	813
.91	862	798	763	740	722
.90	775	718	686	666	650
.89	703	652	623	605	590
.88	644	597	571	554	541
.87	593	550	527	511	499
.86	550	510	488	474	463
.85	512	476	456	442	432
.84	480	446	427	414	405
.83	451	419	401	390	381
.82	425	395	379	368	360
.81	402	374	358	348	340
.80	381	555	340	331	323

Number of failures = 59

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	7966	7354	7018	6801	6632
.98	3977	3673	3506	3399	3315
.97	2648	2446	2336	2264	2209
.96	1983	1833	1751	1697	1656
.95	1584	1465	1399	1357	1324
.94	1318	1219	1165	1130	1103
.93	1128	1044	998	968	945
.92	986	913	873	847	826
.91	875	810	775	752	734
.90	786	729	697	676	660
.89	714	662	633	614	600
.88	653	606	580	563	550
.87	602	559	535	519	507
.86	558	518	496	482	471
.85	520	483	463	449	439
.84	487	453	433	421	411
.83	457	425	408	396	387
.82	431	401	385	374	365
.81	408	380	364	354	346
.80	387	360	346	336	328

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 60

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	8081	7464	7126	6908	6738
.98	4035	3729	3561	3452	3367
.97	2686	2483	2372	2300	2244
.96	2012	1861	1778	1724	1682
.95	1607	1487	1421	1378	1345
.94	1337	1238	1183	1148	1120
.93	1144	1060	1013	983	960
.92	1000	926	886	860	839
.91	887	823	787	764	746
.90	798	740	708	687	671
.89	724	672	643	624	610
.88	663	615	589	572	559
.87	611	567	543	527	515
.86	566	526	504	489	478
.85	528	491	470	457	446
.84	494	459	440	428	418
.83	464	432	414	402	393
.82	438	407	391	380	371
.81	414	386	370	359	351
.80	393	366	351	341	334

Number of failures = 61

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	8196	7575	7235	7014	6843
.98	4092	3784	3615	3505	3420
.97	2724	2520	2408	2335	2279
.96	2040	1888	1805	1750	1708
.95	1630	1509	1443	1400	1366
.94	1356	1256	1201	1166	1138
.93	1161	1076	1029	998	975
.92	1014	940	899	873	853
.91	900	835	799	776	757
.90	809	751	718	698	681
.89	734	682	653	634	619
.88	672	624	598	581	567
.87	619	576	551	536	523
.86	574	534	512	497	486
.85	535	498	477	464	453
.84	501	466	447	434	425
.83	471	438	420	409	399
.82	444	414	397	386	377
.81	420	391	375	365	357
.80	398	371	356	347	339

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 62

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	8311	7685	7343	7121	6949
.98	4149	3839	3669	3558	3473
.97	2762	2557	2444	2371	2314
.96	2069	1916	1832	1777	1735
.95	1653	1531	1464	1421	1387
.94	1375	1275	1219	1183	1155
.93	1177	1091	1044	1014	990
.92	1028	954	913	886	866
.91	913	847	811	787	769
.90	820	762	729	708	692
.89	745	692	662	643	629
.88	682	633	607	589	576
.87	628	584	560	544	531
.86	582	542	519	505	493
.85	543	505	484	471	460
.84	508	473	454	441	431
.83	477	445	427	415	406
.82	450	420	403	391	383
.81	426	397	381	371	363
.80	404	377	362	352	344

Number of failures = 63

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	8425	7795	7451	7227	7054
.98	4207	3894	3723	3612	3525
.97	2800	2593	2480	2406	2349
.96	2097	1943	1859	1804	1761
.95	1676	1553	1486	1442	1408
.94	1394	1293	1237	1201	1173
.93	1193	1107	1060	1029	1005
.92	1043	968	926	900	879
.91	925	859	823	799	781
.90	832	773	740	719	702
.89	755	702	672	653	638
.88	691	643	616	598	585
.87	637	592	568	552	540
.86	591	550	527	512	501
.85	550	512	491	478	467
.84	515	480	460	448	438
.83	484	451	433	421	412
.82	457	426	409	397	389
.81	432	403	387	376	368
.80	410	382	367	357	349

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 64

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	8539	7906	7559	7334	7159
.98	4264	3949	3777	3665	3578
.97	2839	2630	2516	2442	2384
.96	2126	1971	1885	1830	1787
.95	1698	1575	1507	1463	1429
.94	1413	1311	1255	1219	1190
.93	1210	1123	1075	1044	1020
.92	1057	981	940	913	892
.91	938	872	835	811	792
.90	843	784	751	729	713
.89	765	712	682	663	648
.88	701	652	625	607	593
.87	646	601	576	560	548
.86	599	557	535	520	508
.85	558	520	499	485	474
.84	522	487	467	454	444
.83	491	458	439	427	418
.82	463	432	414	403	394
.81	438	409	392	382	374
.80	415	388	372	362	355

Number of failures = 65

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	8654	8016	7666	7440	7264
.98	4321	4004	3830	3718	3630
.97	2877	2667	2552	2477	2419
.96	2154	1998	1912	1857	1814
.95	1721	1597	1485	1485	1450
.94	1432	1330	1273	1236	1208
.93	1226	1138	1090	1059	1035
.92	1071	995	953	926	905
.91	951	884	847	823	804
.90	854	795	762	740	723
.89	776	722	692	672	657
.88	710	661	634	616	602
.87	654	609	584	568	556
.86	607	565	542	527	516
.85	565	527	506	492	481
.84	529	494	474	461	451
.83	497	464	446	433	424
.82	469	438	420	409	400
.81	444	414	398	387	379
.80	421	393	378	368	360

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 66

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	8768	8126	7774	7547	7369
.98	4378	4059	3884	3771	3683
.97	2915	2703	2588	2513	2454
.96	2183	2026	1939	1883	1840
.95	1744	1619	1550	1506	1471
.94	1451	1348	1291	1254	1225
.93	1242	1154	1106	1074	1050
.92	1085	1009	967	939	918
.91	963	896	859	835	816
.90	866	806	772	751	734
.89	786	732	702	682	667
.88	719	670	643	625	611
.87	663	618	593	576	564
.86	615	573	550	535	523
.85	573	534	513	499	488
.84	536	500	480	467	457
.83	504	470	452	440	430
.82	475	444	426	415	406
.81	450	420	404	393	385
.80	427	399	383	373	365

Number of failures = 67

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	8882	8236	7882	7653	7474
.98	4435	4114	3938	3824	3736
.97	2953	2740	2624	2548	2489
.96	2211	2053	1966	1910	1866
.95	1767	1641	1572	1527	1492
.94	1470	1366	1309	1272	1243
.93	1258	1170	1121	1089	1065
.92	1100	1023	980	953	931
.91	976	908	871	846	827
.90	877	816	783	761	744
.89	796	741	711	692	676
.88	729	679	652	634	620
.87	672	626	601	585	572
.86	623	581	558	542	531
.85	581	542	520	506	495
.84	543	507	487	474	464
.83	511	477	458	446	436
.82	482	450	432	421	412
.81	456	426	409	398	390
.80	432	404	388	378	370

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 68

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	8996	8346	7990	7759	7580
.98	4492	4169	3992	3877	3788
.97	2990	2777	2659	2583	2524
.96	2240	2081	1993	1936	1892
.95	1789	1663	1593	1548	1513
.94	1489	1384	1327	1289	1260
.93	1275	1185	1136	1105	1080
.92	1114	1036	994	966	944
.91	989	920	883	858	839
.90	888	827	794	772	755
.89	807	751	721	701	686
.88	738	688	660	642	628
.87	681	635	609	593	580
.86	631	589	565	550	538
.85	588	549	527	513	502
.84	551	514	494	481	470
.83	517	483	464	452	443
.82	488	456	438	427	418
.81	462	432	415	404	396
.80	438	410	394	384	376

Number of failures = 69

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	9110	8455	8097	7865	7685
.98	4549	4224	4046	3930	3841
.97	3028	2813	2695	2619	2559
.96	2268	2108	2020	1973	1919
.95	1812	1685	1615	1569	1534
.94	1508	1403	1345	1307	1278
.93	1291	1201	1152	1120	1095
.92	1128	1050	1007	979	958
.91	1001	932	895	870	851
.90	900	838	804	782	765
.89	817	761	731	711	695
.88	748	697	669	651	637
.87	689	643	617	601	588
.86	639	596	573	558	546
.85	596	556	534	520	509
.84	558	521	501	487	477
.83	524	490	471	458	449
.82	494	462	444	433	424
.81	467	437	420	410	401
.80	443	415	399	389	381

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 70

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	9223	8565	8205	7971	7790
.98	4606	4279	4100	3984	3893
.97	3066	2850	2731	2654	2594
.96	2297	2135	2047	1990	1945
.95	1835	1707	1636	1591	1555
.94	1527	1421	1363	1325	1295
.93	1307	1217	1167	1135	1110
.92	1142	1064	1020	992	971
.91	1014	945	906	892	862
.90	911	849	815	793	776
.89	827	771	741	721	705
.88	757	706	678	660	646
.87	698	651	626	609	596
.86	647	604	581	565	553
.85	603	563	541	527	516
.84	565	528	507	494	484
.83	531	496	477	465	455
.82	500	468	450	438	429
.81	473	443	426	415	407
.80	449	420	404	394	386

Number of failures = 71

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	9337	8675	8313	8078	7895
.98	4662	4333	4153	4037	3946
.97	3104	2386	2767	2690	2629
.96	2325	2163	2074	2016	1971
.95	1857	1729	1658	1612	1576
.94	1546	1439	1381	1342	1313
.93	1323	1232	1182	1150	1125
.92	1156	1077	1034	1006	984
.91	1026	957	918	893	874
.90	922	860	826	804	786
.89	837	781	750	730	714
.88	767	715	687	669	655
.87	707	660	634	617	604
.86	655	612	588	573	561
.85	611	571	549	534	523
.84	572	534	514	500	490
.83	537	503	483	471	461
.82	507	474	456	444	435
.81	479	449	432	421	412
.80	455	426	410	399	391

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 72

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	9451	8784	8420	8184	7999
.98	4719	4388	4207	4090	3998
.97	3142	2923	2803	2725	2664
.96	2353	2190	2101	2043	1997
.95	1880	1750	1679	1633	1597
.94	1565	1457	1398	1360	1330
.93	1339	1248	1198	1165	1140
.92	1170	1091	1047	1019	997
.91	1039	969	930	905	886
.90	934	871	837	814	797
.89	848	791	760	740	724
.88	776	725	696	678	663
.87	715	668	642	625	612
.86	663	620	596	580	568
.85	618	578	556	541	530
.84	579	541	521	507	497
.83	544	509	490	477	467
.82	513	480	462	450	441
.81	485	454	437	426	418
.80	460	431	415	405	396

Number of failures = 73

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	9564	8894	8527	8290	8104
.98	4776	4443	4261	4143	4050
.97	3180	2959	2839	2760	2699
.96	2382	2217	2127	2069	2023
.95	1903	1772	1701	1654	1618
.94	1584	1476	1416	1378	1348
.93	1355	1264	1213	1180	1155
.92	1184	1105	1061	1032	1010
.91	1051	981	942	917	897
.90	945	882	847	825	807
.89	858	801	770	749	733
.88	785	734	705	687	672
.87	724	677	650	633	620
.86	671	628	603	588	576
.85	626	585	563	548	537
.84	586	548	527	514	503
.83	550	515	496	483	473
.82	519	486	468	456	447
.81	491	460	443	432	423
.80	466	437	421	410	402

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 74

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	9677	9003	8635	8396	8209
.98	4832	4498	4314	4196	4103
.97	3217	2996	2874	2795	2734
.96	2410	2245	2154	2095	2050
.95	1925	1794	1722	1675	1639
.94	1602	1494	1434	1395	1365
.93	1372	1279	1228	1195	1170
.92	1199	1118	1074	1045	1023
.91	1064	993	954	929	909
.90	956	893	858	835	818
.89	868	811	779	759	743
.88	795	743	714	695	681
.87	733	685	659	641	628
.86	679	635	611	595	583
.85	633	592	570	555	544
.84	593	555	534	520	510
.83	557	522	502	489	479
.82	525	492	474	462	453
.81	497	466	449	437	429
.80	471	442	426	415	407

Number of failures = 75

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	9791	9113	8742	8502	8314
.98	4889	4552	4368	4248	4155
.97	3255	3032	2910	2831	2769
.96	2438	2272	2181	2122	2076
.95	1948	1816	1744	1697	1660
.94	1621	1512	1452	1413	1383
.93	1388	1295	1244	1211	1185
.92	1213	1132	1087	1059	1036
.91	1076	1005	966	940	921
.90	968	904	869	846	828
.89	878	821	789	769	753
.88	804	752	723	704	689
.87	741	693	667	650	636
.86	687	643	619	603	590
.85	641	600	577	562	551
.84	600	562	541	527	516
.83	564	528	508	496	486
.82	532	498	480	468	458
.81	503	472	454	443	434
.80	477	448	431	420	412

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 76

Reliability	Confidence Level .99	.95	.90	85	.80
.99	9904	9222	8849	8607	8419
.98	4946	4607	4422	4301	4208
.97	3293	3069	2946	2866	2804
.96	2466	2299	2208	2148	2102
.95	1971	1838	1765	1718	1681
.94	1640	1530	1470	1431	1400
.93	1404	1310	1259	1226	1200
.92	1227	1145	1101	1072	1049
.91	1089	1017	978	952	932
.90	979	915	879	856	839
.89	889	831	799	778	762
.88	813	761	732	713	698
.87	750	702	675	658	644
.86	695	651	626	610	598
.85	648	607	584	569	558
.84	607	568	547	533	523
.83	570	534	515	502	492
.82	538	504	486	474	464
.81	509	477	460	448	440
.80	483	453	436	426	417

Number of failures = 77

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	10017	9331	8957	8713	8524
.98	5002	4662	4475	4354	4260
.97	3330	3105	2982	2901	2839
.96	2495	2327	2235	2175	2128
.95	1993	1860	1786	1739	1702
.94	1659	1548	1488	1448	1418
.93	1420	1326	1274	1241	1215
.92	1241	1159	1114	1085	1062
.91	1102	1029	990	964	944
.90	990	926	890	867	849
.89	899	841	809	788	772
.88	823	770	741	722	707
.87	758	710	683	666	652
.86	703	659	634	618	605
.85	656	614	591	576	565
.84	614	575	554	540	529
.83	577	541	521	508	498
.82	544	510	492	479	470
.81	515	483	465	454	445
.80	488	458	442	431	423

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 78

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	10130	9441	9064	8819	8628
.98	5059	4716	4529	4407	4312
.97	3368	3141	3017	2937	2874
.96	2523	2354	2261	2201	2154
.95	2016	1881	1808	1760	1723
.94	1678	1566	1506	1466	1435
.93	1436	1341	1290	1256	1229
.92	1255	1173	1128	1098	1075
.91	1114	1041	1002	976	955
.90	1001	936	901	878	859
.89	909	851	818	797	781
.88	832	779	750	731	716
.87	767	718	691	674	660
.86	711	666	642	625	613
.85	663	621	598	583	572
.84	621	582	561	547	536
.83	583	547	527	514	504
.82	550	516	498	485	476
.81	521	489	471	460	451
.80	494	464	447	436	428

Number of failures = 79

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	10243	9550	9171	8925	8733
.98	5115	4771	4582	4460	4365
.97	3406	3178	3053	2972	2909
.96	2551	2381	2288	2228	2181
.95	2038	1903	1829	1781	1744
.94	1696	1585	1523	1484	1452
.93	1452	1357	1305	1271	1244
.92	1269	1186	1141	1111	1088
.91	1126	1054	1013	987	967
.90	1013	947	911	888	870
.89	919	860	828	807	791
.88	842	788	759	739	724
.87	776	727	700	682	668
.86	719	674	649	633	620
.85	671	629	606	590	579
.84	628	589	567	553	542
.83	590	554	533	520	510
.82	556	522	503	491	482
.81	526	494	477	465	456
.80	499	469	452	442	433

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 80

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	10356	9659	9278	9031	8838
.98	5171	4825	4636	4513	4417
.97	3443	3214	3089	3007	2943
.96	2579	2408	2315	2254	2207
.95	2061	1925	1851	1802	1765
.94	1715	1603	1541	1501	1470
.93	1468	1373	1320	1286	1259
.92	1283	1200	1154	1125	1101
.91	1139	1066	1025	999	979
.90	1024	958	922	899	880
.89	929	870	838	817	800
.88	851	797	767	748	733
.87	784	735	708	690	676
.86	727	682	657	641	628
.85	678	636	613	597	586
.84	635	596	574	560	549
.83	597	560	540	527	516
.82	563	528	509	497	487
.81	532	500	482	471	461
.80	505	475	458	447	438

Number of failures = 81

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	10469	9768	9385	9136	8942
.98	5228	4880	4690	4566	4469
.97	3481	3250	3124	3042	2978
.96	2607	2436	2342	2281	2233
.95	2083	1947	1872	1823	1785
.94	1734	1621	1559	1519	1487
.93	1484	1388	1335	1301	1274
.92	1297	1214	1168	1138	1115
.91	1151	1078	1037	1011	990
.90	1035	969	933	909	891
.89	940	880	847	826	809
.88	860	806	776	757	742
.87	793	743	716	698	684
.86	735	690	664	648	635
.85	685	643	620	604	593
.84	642	602	581	566	555
.83	603	566	546	533	522
.82	569	534	515	503	493
.81	538	506	488	476	467
.80	511	480	463	452	443

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 82

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	10581	9877	9492	9242	9047
.98	5284	4934	4743	4619	4522
.97	3518	3287	3160	3078	3013
.96	2636	2463	2368	2307	2259
.95	2106	1969	1893	1845	1806
.94	1753	1639	1577	1536	1505
.93	1500	1404	1351	1316	1289
.92	1311	1227	1181	1151	1128
.91	1164	1090	1049	1023	1002
.90	1046	980	944	920	901
.89	950	890	857	836	819
.88	870	815	785	766	750
.87	802	752	724	706	692
.86	743	697	672	656	643
.85	693	650	627	612	600
.84	649	609	587	573	562
.83	610	573	552	539	529
.82	575	540	521	509	499
.81	544	511	493	482	472
.80	516	485	468	457	449

Number of failures = 83

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	10694	9986	9599	9348	9152
.98	5340	4989	4796	4671	4574
.97	3556	3323	3196	3113	3048
.96	2664	2490	2395	2333	2285
.95	2128	1990	1915	1866	1827
.94	1771	1657	1595	1554	1522
.93	1516	1419	1366	1331	1304
.92	1325	1241	1194	1164	1141
.91	1176	1102	1061	1034	1013
.90	1057	991	954	930	912
.89	960	900	867	845	828
.88	879	824	794	774	759
.87	810	760	732	714	700
.86	751	705	680	663	650
.85	700	658	634	619	606
.84	656	616	594	580	568
.83	616	579	559	545	535
.82	581	546	527	515	505
.81	550	517	499	487	478
.80	522	491	474	463	454

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 84

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	10807	10095	9706	9453	9256
.98	5397	5043	4850	4724	4626
.97	3593	3359	3231	3148	3083
.96	2692	2517	2422	2360	2311
.95	2151	2012	1936	1887	1848
.94	1790	1675	1612	1571	1540
.93	1532	1435	1381	1346	1319
.92	1339	1254	1208	1177	1154
.91	1189	1114	1073	1046	1025
.90	1069	1002	965	941	922
.89	970	910	877	855	838
.88	888	833	803	783	768
.87	819	768	741	723	708
.86	759	713	687	671	658
.85	708	665	641	626	613
.84	663	623	601	586	575
.83	623	585	565	551	541
.82	587	552	533	520	511
.81	556	523	505	493	483
.80	527	496	479	468	459

Number of failures = 85

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	10919	10204	9813	9559	9361
.98	5453	5098	4903	4777	4678
.97	3631	3395	3267	3183	3118
.96	2720	2544	2448	2386	2337
.95	2173	2034	1958	1908	1869
.94	1809	1693	1630	1589	1557
.93	1548	1450	1396	1361	1334
.92	1353	1268	1221	1191	1167
.91	1201	1126	1085	1058	1037
.90	1080	1012	975	951	933
.89	980	920	886	864	847
.88	898	842	812	792	776
.87	827	777	749	731	716
.86	767	721	695	678	665
.85	715	672	648	633	620
.84	670	629	607	593	581
.83	629	592	571	558	547
.82	594	558	539	526	516
.81	562	529	510	498	489
.80	533	502	484	473	464

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 86

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	11032	10312	9920	9665	9465
.98	5509	5152	4957	4830	4731
.97	3668	3432	3302	3218	3153
.96	2748	2572	2475	2412	2362
.95	2196	2056	1979	1929	1890
.94	1827	1711	1648	1607	1574
.93	1564	1466	1412	1376	1349
.92	1367	1281	1234	1204	1180
.91	1214	1138	1096	1069	1048
.90	1091	1023	986	962	943
.89	991	929	896	874	857
.88	907	851	821	801	785
.87	836	785	757	739	724
.86	775	728	702	686	672
.85	723	679	655	640	627
.84	677	636	614	599	588
.83	636	598	577	564	553
.82	600	564	545	532	522
.81	568	534	516	504	494
.80	538	507	490	478	469

Number of failures = 87

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	11144	10421	10027	9770	9570
.98	5565	5206	5010	4883	4783
.97	3706	3468	3338	3253	3187
.96	2776	2599	2502	2439	2390
.95	2218	2077	2000	1950	1911
.94	1846	1730	1666	1624	1592
.93	1580	1481	1427	1391	1364
.92	1381	1295	1248	1217	1193
.91	1226	1150	1108	1081	1060
.90	1102	1034	997	972	953
.89	1001	939	906	884	866
.88	916	860	830	810	794
.87	845	793	765	747	732
.86	783	736	710	693	680
.85	730	686	662	647	634
.84	684	643	620	606	594
.83	643	605	584	570	559
.82	606	570	551	538	528
.81	573	540	521	509	500
.80	544	512	495	484	475

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 88

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	11256	10530	10134	9876	9674
.98	5621	5261	5064	4935	4835
.97	3743	3504	3374	3289	3222
.96	2804	2626	2529	2465	2416
.95	2240	2099	2022	1971	1932
.94	1865	1748	1684	1642	1609
.93	1596	1497	1442	1406	1379
.92	1395	1308	1261	1230	1206
.91	1239	1162	1120	1093	1071
.90	1113	1045	1007	983	969
.89	1011	949	915	893	876
.88	925	869	838	818	803
.87	853	802	773	755	740
.86	791	744	718	701	687
.85	738	694	669	654	641
.84	691	650	627	612	601
.83	649	611	590	576	565
.82	612	576	557	544	534
.81	579	546	527	515	505
.80	550	518	500	489	480

Number of failures = 89

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	11368	10639	10240	9981	9779
.98	5677	5315	5117	4988	4887
.97	3780	3540	3409	3324	3257
.96	2832	2653	2555	2491	2442
.95	2263	2121	2043	1992	1953
.94	1883	1766	1701	1659	1626
.93	1612	1512	1457	1422	1394
.92	1409	1322	1274	1243	1219
.91	1251	1174	1132	1104	1083
.90	1124	1056	1018	994	974
.89	1021	959	925	903	885
.88	935	878	847	827	811
.87	862	810	782	763	749
.86	799	752	725	708	695
.85	745	701	676	661	648
.84	697	656	634	619	607
.83	656	617	596	582	571
.82	618	582	563	550	539
.81	585	551	533	520	511
.80	555	523	506	494	485

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 90

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	11481	10747	10347	10086	9883
.98	5733	5369	5170	5041	4940
.97	3818	3576	3445	3359	3292
.96	2860	2680	2582	2518	2468
.95	2285	2142	2064	2013	1973
.94	1902	1784	1719	1677	1644
.93	1628	1528	1472	1437	1408
.92	1423	1336	1288	1256	1232
.91	1263	1186	1144	1116	1095
.90	1136	1067	1029	1004	985
.89	1031	969	935	912	895
.88	944	887	856	836	820
.87	870	818	790	771	757
.86	807	759	733	716	702
.85	752	708	684	668	655
.84	704	663	640	626	614
.83	662	624	602	588	578
.82	625	588	568	555	545
.81	591	557	538	526	516
.80	561	529	511	499	490

Number of failures = 91

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	11593	10856	10454	10192	9987
.98	5789	5423	5224	5093	4992
.97	3855	3613	3480	3394	3326
.96	2888	2707	2608	2544	2494
.95	2308	2164	2085	2034	1994
.94	1921	1802	1737	1694	1661
.93	1644	1543	1488	1452	1423
.92	1437	1349	1301	1269	1245
.91	1276	1198	1156	1128	1106
.90	1147	1077	1039	1015	995
.89	1041	979	944	922	904
.88	953	896	865	845	829
.87	879	827	798	779	765
.86	815	767	740	723	710
.85	760	715	691	675	662
.84	711	670	647	632	620
.83	669	630	609	595	584
.82	631	594	574	561	551
.81	597	563	544	531	522
.80	566	534	516	505	495

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 92

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	11705	10964	10560	10297	10092
.98	5845	5478	5277	5146	5044
.97	3892	3649	3516	3429	3361
.96	2916	2734	2635	2570	2520
.95	2330	2186	2107	2055	2015
.94	1939	1820	1755	1712	1679
.93	1660	1559	1503	1467	1438
.92	1451	1363	1314	1283	1258
.91	1288	1210	1167	1140	1118
.90	1158	1088	1050	1025	1006
.89	1051	988	954	931	914
.88	963	905	874	853	837
.87	887	835	806	787	773
.86	823	775	748	731	717
.85	767	722	698	682	669
.84	718	677	654	639	627
.83	675	636	615	601	590
.82	637	600	580	567	557
.81	603	568	549	537	527
.80	572	539	521	510	501

Number of failures = 93

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	11817	11073	10667	10403	10196
.98	5901	5532	5330	5199	5096
.97	3930	3685	3551	3464	3396
.96	2944	2761	2662	2597	2546
.95	2352	2207	2128	2076	2036
.94	1958	1838	1772	1729	1696
.93	1676	1574	1518	1482	1453
.92	1465	1376	1327	1296	1271
.91	1301	1222	1179	1151	1129
.90	1169	1099	1061	1036	1016
.89	1061	998	964	941	923
.88	972	914	883	862	846
.87	896	843	814	795	781
.86	831	782	756	738	724
.85	775	730	705	689	676
.84	725	683	660	645	633
.83	682	643	621	607	596
.82	643	606	586	573	563
.81	608	574	555	542	533
.80	577	545	527	515	506

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 94

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	11928	11181	10773	10508	10300
.98	5957	5586	5383	5251	5148
.97	3967	3721	3587	3499	3431
.96	2972	2788	2688	2623	2572
.95	2375	2229	2149	2097	2057
.94	1976	1856	1790	1747	1713
.93	1692	1589	1533	1497	1468
.92	1479	1390	1341	1309	1284
.91	1313	1234	1191	1163	1141
.90	1180	1110	1071	1046	1026
.89	1072	1008	973	950	933
.88	981	923	892	871	855
.87	905	852	822	803	789
.86	839	790	763	746	732
.85	782	737	712	696	683
.84	732	690	667	652	640
.83	688	649	627	613	602
.82	649	612	592	579	568
.81	614	580	560	548	538
.80	583	550	532	520	511

Number of failures = 95

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	12040	11289	10880	10613	10405
.98	6013	5640	5437	5304	5200
.97	4004	3757	3622	3534	3466
.96	3000	2816	2715	2649	2598
.95	2397	2251	2171	2118	2078
.94	1995	1874	1808	1765	1731
.93	1708	1605	1548	1512	1483
.92	1493	1403	1354	1322	1297
.91	1325	1246	1203	1175	1152
.90	1191	1121	1082	1057	1037
.89	1082	1018	983	960	942
.88	990	932	900	880	863
.87	913	860	831	811	797
.86	847	798	771	753	739
.85	789	744	719	703	690
.84	739	697	674	648	646
.83	695	655	633	619	608
.82	655	618	598	585	574
.81	620	585	566	553	544
.80	588	556	537	526	516

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 96

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	12152	11398	10986	10718	10509
.98	6069	5694	5490	5357	5252
.97	4041	3793	3658	3569	3500
.96	3028	2843	2742	2676	2624
.95	2419	2272	2192	2139	2099
.94	2014	1892	1825	1782	1748
.93	1724	1620	1564	1527	1498
.92	1507	1417	1367	1335	1310
.91	1338	1258	1215	1186	1164
.90	1202	1131	1092	1067	1047
.89	1092	1028	993	970	952
.88	1000	941	909	888	872
.87	922	868	839	820	805
.86	855	805	778	761	747
.85	797	751	726	710	697
.84	746	704	680	665	653
.83	701	662	640	625	614
.82	661	624	604	590	580
.81	626	591	572	559	549
.80	594	561	543	531	521

Number of failures = 97

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	12264	11506	11093	10824	10613
.98	6125	5748	5543	5409	5305
.97	4079	3829	3693	3604	3535
.96	3055	2870	2768	2702	2650
.95	2441	2294	2213	2161	2119
.94	2032	1910	1843	1800	1765
.93	1740	1636	1579	1542	1513
.92	1521	1430	1381	1348	1323
.91	1350	1270	1226	1198	1176
.90	1214	1142	1103	1078	1058
.89	1102	1038	1002	979	961
.88	1009	950	918	897	881
.87	930	876	847	828	813
.86	863	813	786	768	754
.85	804	758	733	717	704
.84	753	710	687	671	659
.83	708	668	646	632	620
.82	668	630	610	596	586
.81	632	597	577	564	555
.80	599	566	548	536	527

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 98

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	12375	11614	11199	10929	10717
.98	6181	5803	5596	5462	5357
.97	4116	3865	3729	3639	3570
.96	3083	2897	2795	2728	2676
.95	2464	2315	2234	2182	2140
.94	2051	1928	1861	1817	1783
.93	1756	1651	1594	1557	1528
.92	1534	1444	1394	1361	1336
.91	1362	1282	1238	1210	1187
.90	1225	1153	1114	1088	1068
.89	1112	1047	1012	989	971
.88	1018	959	927	906	889
.87	939	885	855	836	821
.86	871	821	793	776	762
.85	812	765	740	724	711
.84	760	717	693	678	666
.83	714	674	652	638	626
.82	674	636	616	602	591
.81	638	602	583	570	560
.80	605	572	553	541	532

Number of failures = 99

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	12487	11722	11306	11034	10822
.98	6236	5857	5649	5514	5409
.97	4153	3901	3764	3674	3604
.96	3111	2924	2821	2755	2702
.95	2486	2337	2256	2203	2161
.94	2069	1946	1879	1835	1800
.93	1772	1667	1609	1572	1542
.92	1548	1457	1407	1375	1349
.91	1375	1294	1250	1221	1199
.90	1236	1164	1124	1099	1078
.89	1122	1057	1021	998	980
.88	1027	968	936	915	898
.87	947	893	863	844	829
.86	878	829	801	783	769
.85	819	773	747	731	717
.84	767	724	700	685	672
.83	721	681	658	644	633
.82	680	642	621	608	597
.81	643	608	588	576	565
.80	610	577	559	546	537

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Number of failures = 100

Reliability	Confidence Level .99	.95	.90	.85	.80
.99	12598	11831	11412	11139	10926
.98	6292	5911	5703	5567	5461
.97	4190	3937	3799	3710	3639
.96	3139	2951	2848	2781	2728
.95	2508	2359	2277	2224	2182
.94	2088	1964	1896	1852	1817
.93	1788	1682	1624	1587	1557
.92	1562	1471	1420	1388	1362
.91	1387	1306	1262	1233	1210
.90	1247	1175	1135	1109	1089
.89	1132	1067	1031	1008	989
.88	1037	977	945	923	907
.87	956	901	871	852	837
.86	886	836	809	791	776
.85	826	780	754	737	724
.84	774	730	707	691	679
.83	727	687	665	650	639
.82	686	648	627	614	603
.81	649	614	594	581	571
.80	616	582	564	552	542

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Confidence Level = .75

Failures

Reliability	0	1	2	3	4	5	6	7	8	9	10
99	138	269	392	510	627	742	855	968	1079	1190	1301
98	69	134	196	255	313	370	427	483	539	595	650
97	46	89	130	170	209	247	284	322	359	396	433
96	34	67	98	127	156	185	213	241	269	297	324
95	28	53	78	102	125	148	170	193	215	237	259
94	23	45	65	85	104	123	142	161	179	198	216
93	20	38	56	72	89	105	121	137	153	169	185
92	17	33	49	63	78	92	106	120	134	148	162
91	15	30	43	56	69	82	94	107	119	131	144
90	14	27	39	51	62	73	85	96	107	118	129
89	12	24	35	46	56	67	77	87	97	107	117
88	11	22	32	42	52	61	71	80	89	98	107
87	10	20	30	39	48	56	65	74	82	91	99
86	10	19	28	36	44	52	60	68	76	84	92
85	9	18	26	33	41	49	56	64	71	78	86
84	8	16	24	31	39	46	53	60	67	73	80
83	8	15	23	29	36	43	50	56	63	69	76
82	7	15	21	28	34	40	47	53	59	65	71
81	7	14	20	26	32	38	44	50	56	62	67
80	7	13	19	25	31	36	42	48	53	59	64

Reliability	11	12	13	14	15	16	17	18	19	20
99	1411	1521	1630	1739	1847	1956	2064	2172	2279	2387
98	750	760	814	869	923	977	1031	1085	1139	1193
97	470	506	543	579	615	651	687	723	759	795
96	352	379	407	434	461	488	515	542	569	596
95	281	303	325	347	368	390	412	433	455	476
94	234	252	271	289	307	325	343	361	379	397
93	201	216	232	247	263	278	294	309	324	340
92	175	189	203	216	230	243	257	270	284	297
91	156	168	180	192	204	216	228	240	252	264
90	140	151	162	173	184	194	205	216	227	237
89	127	137	147	157	167	177	186	196	206	216
88	117	126	135	144	153	162	171	180	189	198
87	108	116	124	133	141	149	157	166	174	182
86	100	108	115	123	131	138	146	154	161	169
85	93	100	108	116	122	129	136	143	151	158
84	87	94	101	107	114	121	128	134	141	148
83	82	88	95	101	107	114	120	126	133	139
82	77	83	89	95	101	107	113	119	125	131
81	73	79	85	90	96	102	107	113	119	124
80	69	75	80	85	91	96	102	107	113	118

TABLE B-18 continued

RELIABILITY: SUCCESS - FAILURE

Confidence Level = .75

Reliability	Failures 21	22	23	24	25	26	27	28	29	30
99	2494	2601	2708	2815	2922	3028	3135	3241	3347	3453
98	1246	1300	1353	1407	1460	1513	1567	1620	1673	1726
97	830	866	902	937	973	1008	1044	1079	1115	1150
96	622	649	676	703	729	756	782	809	836	862
95	498	519	540	562	583	604	626	647	668	689
94	414	432	450	468	486	503	521	539	556	574
93	355	370	386	401	416	431	446	462	477	492
92	310	324	337	350	364	377	390	404	417	430
91	276	288	300	311	323	335	347	359	370	382
90	248	259	269	280	291	301	312	323	333	344
89	225	235	245	254	264	274	283	293	303	312
88	206	215	224	233	242	251	260	268	277	286
87	190	199	207	215	223	231	240	248	256	264
86	177	184	192	200	207	215	222	230	237	245
85	165	172	179	186	193	200	207	214	221	228
84	154	161	168	174	181	188	194	201	207	214
83	145	152	158	164	170	177	183	189	195	201
82	137	143	149	155	161	167	172	178	184	190
81	130	135	141	147	152	158	163	169	174	180
80	123	129	134	139	144	150	155	160	166	171

Reliability	31	32	33	34	35	36	37	38	39	40
99	3559	3665	3771	3877	3983	4088	4194	4299	4404	4510
98	1779	1832	1885	1938	1990	2043	2096	2149	2201	2254
97	1185	1221	1256	1291	1326	1361	1397	1432	1467	1502
96	888	915	941	968	994	1021	1047	1073	1100	1126
95	710	732	753	774	795	816	837	838	879	900
94	592	609	627	645	662	680	697	715	732	750
93	507	522	537	552	567	582	597	612	627	642
92	443	457	470	483	496	509	522	536	549	562
91	394	406	417	429	441	452	464	476	488	499
90	354	365	375	386	396	407	418	428	439	449
89	322	331	341	351	360	370	379	389	398	408
88	295	304	312	321	330	339	348	356	365	374
87	272	280	288	296	305	313	321	329	337	345
86	252	260	268	275	283	290	298	305	313	320
85	236	243	250	257	264	271	278	285	292	299
84	221	227	234	240	247	254	260	267	273	280
83	208	214	220	226	232	239	245	251	257	263
82	196	202	208	213	219	225	231	237	243	248
81	186	191	197	202	208	213	219	224	230	235
80	176	181	187	192	197	202	208	213	218	223

TABLE B-19

SEQUENTIAL TEST

$a = \ln \left(\frac{1-\beta}{\alpha}\right)$ and $b = \ln \left(\frac{\beta}{1-\alpha}\right)$

The upper number in each cell represents a, the lower number, b.

β \ α	.001	.01	.025	.05	.10	.15	.20	.25
.001	6.907 -6.907	4.604 -6.898	3.688 -6.882	2.995 -6.882	2.302 -6.802	1.896 -6.745	1.608 -6.685	1.385 -6.620
.01	6.898 -4.604	4.595 -4.595	3.679 -4.580	2.986 -4.554	2.293 -4.500	1.887 -4.443	1.599 -4.382	1.376 -4.317
.05	6.856 -2.995	4.554 -2.986	3.638 -2.970	2.944 -2.944	2.251 -2.890	1.846 -2.833	1.558 -2.773	1.335 -2.708
.10	6.802 -2.302	4.500 -2.292	3.583 -2.277	2.890 -2.251	2.197 -2.197	1.792 -2.14	1.504 -2.079	1.281 -2.015
.20	6.685 -1.608	4.382 -1.599	3.466 -1.584	2.773 -1.558	2.079 -1.504	1.674 -1.447	1.386 -1.386	1.163 -1.322
.25	6.620 -1.385	4.317 -1.376	3.401 -1.361	2.708 -1.335	2.015 -1.281	1.609 -1.224	1.322 -1.163	1.099 -1.099
.30	6.551 -1.203	4.248 -1.194	3.332 -1.179	2.639 -1.153	1.946 -1.099	1.540 -1.041	1.253 -.981	1.030 -.916
.40	6.397 -.915	4.094 -.906	3.178 -.891	2.485 -.865	1.792 -.811	1.386 -.754	1.099 -.693	.875 -.629

TABLE B-20

FACTORS FOR DETERMINING LOWER CONFIDENCE LIMIT
FOR THE EXPONENTIAL MEAN LIFE

$LF_{1-\alpha}$

d.f.	.75	.80	.90	.95	.975	.99
1	.722022	.621118	.433839	.333890	.271003	.217155
2	.742115	.667780	.514139	.421496	.360360	.300752
3	.765306	.700935	.566038	.476190	.416667	.357143
4	.784314	.727273	.597015	.516129	.457143	.398010
5	.800000	.746269	.625000	.546448	.487805	.431034
6	.810811	.759494	.648649	.571429	.515021	.458015
7	.818713	.769231	.663507	.590717	.536398	.481010
8	.824742	.780488	.680851	.608365	.555556	.500000
9	.833333	.789474	.692308	.622837	.571429	.517241
10	.840336	.800000	.704225	.636943	.584795	.531915
11	.846154	.805861	.714286	.648968	.597826	.545906
12	.851064	.810811	.722892	.659341	.609137	.558140
13	.855263	.817610	.730337	.668380	.620525	.570175
14	.858896	.823529	.738786	.677966	.629213	.579710
15	.862069	.826446	.744417	.684932	.638298	.589391
16	.864865	.831169	.751174	.692641	.646465	.598131
17	.867347	.835381	.757238	.699588	.653846	.606061
18	.871671	.839161	.762712	.705882	.661765	.614334
19	.873563	.842572	.767677	.711610	.667838	.620915
20	.877193	.845666	.772201	.716846	.674536	.627943
21	.878661	.848485	.776340	.722892	.679612	.634441
22	.881764	.852713	.780142	.727273	.685358	.640466
23	.882917	.885019	.784983	.732484	.690691	.646067
24	.885609	.857143	.788177	.736196	.695652	.651289
25	.888099	.859107	.791139	.740741	.700280	.656168
26	.888889	.862355	.795107	.744986	.704607	.661578
27	.891089	.860000	.797637	.747922	.708661	.665845
28	.893142	.865533	.801144	.751678	.712468	.670659
29	.893683	.868263	.803324	.755208	.716934	.674419
30	.895522	.869565	.806452	.758534	.720288	.678733

Multiply factors of this table by estimated mean time between failures for lower confidence limits.

$$\text{For } f > 30: \quad \text{lower factor} = \frac{2f}{\chi^2_{\alpha,\ 2f}}$$

TABLE B-21

FACTORS FOR DETERMINING UPPER CONFIDENCE LIMIT
FOR THE EXPONENTIAL MEAN LIFE

$UF_{1-\alpha}$

d.f.	.75	.80	.90	.95	.975	.99
1	3.478261	4.484305	9.478673	19.417476	39.525692	99.502488
2	2.083333	2.424242	3.773585	5.625879	8.264463	13.468013
3	1.739130	1.954397	2.727273	3.658537	4.838710	6.880734
4	1.577909	1.742919	2.292264	2.930403	3.669725	4.848485
5	1.483680	1.618123	2.053388	2.538071	3.076923	3.906250
6	1.421801	1.536492	1.904762	2.294455	2.727273	3.361344
7	1.372549	1.478353	1.797176	2.130898	2.486679	3.004292
8	1.344538	1.428571	1.718582	2.010050	2.315485	2.753873
9	1.313869	1.395349	1.651376	1.916933	2.187120	2.567760
10	1.290323	1.369863	1.612903	1.834862	2.085506	2.421308
11	1.279070	1.349693	1.571429	1.788618	2.000000	2.306080
12	1.263158	1.325967	1.528662	1.379130	1.935484	2.201835
13	1.250000	1.313131	1.502890	1.688312	1.884058	2.131148
14	1.233480	1.296296	1.481481	1.656805	1.830065	2.058824
15	1.224490	1.282051	1.456311	1.621622	1.785714	2.000000
16	1.212121	1.274900	1.434978	1.592040	1.748634	1.951220
17	1.205674	1.263940	1.416667	1.566820	1.717172	1.910112
18	1.200000	1.254355	1.406250	1.545064	1.690141	1.875000
19	1.191225	1.245902	1.391941	1.526104	1.679389	1.835749
20	1.186944	1.238390	1.374570	1.509434	1.639344	1.801802
21	1.179775	1.228070	1.363636	1.494662	1.615385	1.772152
22	1.176471	1.222222	1.353846	1.476510	1.594203	1.752988
23	1.170483	1.216931	1.345029	1.464968	1.575342	1.722846
24	1.167883	1.212121	1.337047	1.450151	1.558442	1.702128
25	1.162791	1.207729	1.326260	1.436782	1.543210	1.683502
26	1.158129	1.200924	1.319797	1.428571	1.529412	1.666667
27	1.156317	1.197339	1.310680	1.417323	1.516854	1.646341
28	1.152263	1.191489	1.305361	1.407035	1.505376	1.632653
29	1.148515	1.188524	1.297539	1.397590	1.494845	1.615599
30	1.145038	1.185771	1.290323	1.388889	1.481481	1.600000

Multiply factors of this table by estimated mean time between failures for the upper confidence limit.

$$\text{For } f > 30: \quad \text{upper factor} = \frac{2f}{\chi^2_{1-\alpha,2f}}$$

TABLE B-22

EXPONENTIAL FUNCTION: e^{-x}

x	0	1	2	3	4	5	6	7	8	9
.00	1.0000	.9990	.9980	.9970	.9960	.9950	.9940	.9930	.9920	.9910
.01	.9900	.9890	.9980	.9870	.9860	.9851	.9841	.9831	.9821	.9811
.02	.9802	.9792	.9782	.9773	.9763	.9753	.9743	.9734	.9724	.9714
.03	.9704	.9695	.9685	.9675	.9665	.9656	.9646	.9637	.9627	.9618
.04	.9608	.9598	.9589	.9579	.9570	.9560	.9550	.9541	.9531	.9522
.05	.9512	.9503	.9493	.9484	.9474	.9465	.9455	.9446	.9436	.9427
.06	.9418	.9408	.9399	.9389	.9380	.9371	.9361	.9352	.9343	.9333
.07	.9324	.9315	.9305	.9296	.9287	.9277	.9268	.9259	.9250	.9240
.08	.9231	.9222	.9213	.9204	.9194	.9185	.9176	.9167	.9158	.9148
.09	.9139	.9130	.9121	.9112	.9103	.9094	.9085	.9076	.9066	.9057
.10	.9048	.9039	.9030	.9021	.9012	.9003	.8994	.8985	.8976	.8967
.11	.8958	.8949	.8940	.8932	.8923	.8914	.8905	.8896	.8887	.8878
.12	.8869	.8860	.8851	.8843	.8834	.8825	.8816	.8807	.8799	.8790
.13	.8781	.8772	.8763	.8755	.8746	.8737	.8728	.8720	.8711	.8702
.14	.8694	.8685	.8676	.8668	.8659	.8650	.8642	.8633	.8624	.8616
.15	.8607	.8598	.8590	.8581	.8573	.8564	.8556	.8547	.8538	.8530
.16	.8521	.8513	.8504	.8496	.8487	.8479	.8470	.8462	.8454	.8445
.17	.8437	.8428	.8420	.8411	.8403	.8395	.8386	.8378	.8369	.8361
.18	.8353	.8344	.8336	.8328	.8319	.8311	.8303	.8294	.8286	.8278
.19	.8270	.8261	.8253	.8245	.8237	.8228	.8220	.8212	.8204	.8195
.20	.8187	.8179	.8171	.8163	.8155	.8146	.8138	.8130	.8122	.8114
.21	.8106	.8098	.8090	.8082	.8073	.8065	.8057	.8049	.8041	.8033
.22	.8025	.8017	.8009	.8001	.7993	.7985	.7977	.7969	.7961	.7953
.23	.7945	.7937	.7929	.7922	.7914	.7906	.7898	.7890	.7882	.7874
.24	.7866	.7858	.7851	.7843	.7835	.7827	.7819	.7811	.7804	.7796
.25	.7788	.7780	.7772	.7765	.7757	.7749	.7741	.7734	.7726	.7718
.26	.7711	.7703	.7695	.7687	.7680	.7672	.7664	.7657	.7649	.7641
.27	.7634	.7626	.7619	.7611	.7603	.7596	.7588	.7581	.7573	.7565
.28	.7558	.7550	.7543	.7535	.7528	.7520	.7513	.7505	.7498	.7490
.29	.7483	.7475	.7468	.7460	.7453	.7445	.7438	.7430	.7423	.7416
.30	.7408	.7401	.7393	.7386	.7379	.7371	.7364	.7357	.7349	.7342
.31	.7334	.7327	.7320	.7312	.7305	.7298	.7291	.7283	.7276	.7269
.32	.7261	.7254	.7247	.7240	.7233	.7225	.7218	.7211	.7204	.7196
.33	.7189	.7182	.7175	.7168	.7161	.7153	.7146	.7139	.7132	.7125
.34	.7118	.7111	.7103	.7096	.7096	.7089	.7082	.7075	.7068	.7054
.35	.7047	.7040	.7033	.7026	.7019	.7012	.7005	.6998	.6991	.6983
.36	.6977	.6970	.6963	.6956	.6949	.6942	.6935	.6928	.6921	.6914
.37	.6907	.6900	.6994	.6887	.6880	.6873	.6866	.6859	.6852	.6845
.38	.6839	.6832	.6825	.6818	.6811	.6805	.6798	.6791	.6784	.6777
.39	.6771	.6764	.6757	.6750	.6744	.6737	.6730	.6723	.6717	.6710

TABLE B-22 continued

EXPONENTIAL FUNCTION: e^{-x}

x	0	1	2	3	4	5	6	7	8	9
.40	.6703	.6697	.6690	.6683	.6676	.6670	.6663	.6656	.6650	.6643
.41	.6637	.6630	.6623	.6617	.6610	.6603	.6597	.6590	.6584	.6577
.42	.6570	.6564	.6557	.6551	.6544	.6538	.6531	.6525	.6518	.6512
.43	.6505	.6499	.6492	.6486	.6479	.6473	.6466	.6460	.6453	.6447
.44	.6440	.6434	.6427	.6421	.6415	.6408	.6402	.6395	.6389	.6383
.45	.6376	.6370	.6364	.6357	.6351	.6344	.6338	.6332	.6325	.6319
.46	.6313	.6307	.6300	.6294	.6288	.6281	.6275	.6269	.6263	.6256
.47	.6250	.6244	.6238	.6231	.6225	.6219	.6213	.6206	.6200	.6194
.48	.6188	.6182	.6175	.6169	.6163	.6157	.6151	.6145	.6139	.6132
.49	.6126	.6120	.6114	.6108	.6102	.6096	.6090	.6084	.6077	.6071
.50	.6065	.6059	.6053	.6047	.6041	.6035	.6029	.6023	.6017	.6011
.51	.6005	.5999	.5993	.5987	.5981	.5975	.5969	.5963	.5957	.5951
.52	.5945	.5939	.5933	.5927	.5921	.5916	.5910	.5904	.5898	.5892
.53	.5886	.5880	.5874	.5868	.5863	.5857	.5851	.5845	.5839	.5833
.54	.5827	.5822	.5816	.5810	.5804	.5798	.5793	.5787	.5781	.5775
.55	.5769	.5764	.5758	.5752	.5746	.5741	.5735	.5729	.5724	.5718
.56	.5712	.5706	.5701	.5695	.5689	.5684	.5678	.5672	.5667	.5661
.57	.5655	.5650	.5644	.5638	.5633	.5627	.5621	.5616	.5610	.5606
.58	.5599	.5593	.5588	.5582	.5577	.5571	.5565	.5560	.5554	.5549
.59	.5543	.5538	.5532	.5527	.5521	.5516	.5510	.5505	.5499	.5494
.60	.5488	.5483	.5477	.5472	.5466	.5461	.5455	.5450	.5444	.5439
.61	.5434	.5428	.5423	.5417	.5412	.5406	.5401	.5396	.5390	.5385
.62	.5379	.5374	.5369	.5363	.5358	.5353	.5347	.5342	.5337	.5331
.63	.5326	.5321	.5315	.5310	.5305	.5299	.5294	.5289	.5283	.5278
.64	.5273	.5268	.5262	.5257	.5252	.5247	.5241	.5236	.5231	.5226
.65	.5220	.5215	.5210	.5205	.5200	.5194	.5189	.5184	.5179	.5174
.66	.5169	.5163	.5158	.5153	.5148	.5143	.5138	.5132	.5127	.5122
.67	.5117	.5112	.5107	.5102	.5097	.5092	.5086	.5081	.5076	.5071
.68	.5066	.5061	.5056	.5051	.5046	.5041	.5036	.5031	.5026	.5021
.69	.5016	.5011	.5006	.5001	.4996	.4991	.4986	.4981	.4976	.4971
.70	.4966	.4961	.4956	.4951	.4946	.4941	.4936	.4931	.4926	.4921
.71	.4916	.4912	.4907	.4902	.4897	.4892	.4887	.4882	.4877	.4872
.72	.4868	.4863	.4858	.4853	.4848	.4843	.4838	.4834	.4829	.4824
.73	.4819	.4814	.4809	.4805	.4800	.4795	.4790	.4785	.4781	.4776
.74	.4771	.4766	.4762	.4757	.4752	.4747	.4743	.4738	.4733	.4728
.75	.4724	.4719	.4714	.4710	.4705	.4700	.4695	.4691	.4686	.4681
.76	.4677	.4672	.4667	.4663	.4658	.4653	.4649	.4644	.4639	.4635
.77	.4630	.4626	.4621	.4616	.4612	.4607	.4602	.4599	.4593	.4589
.78	.4584	.4579	.4575	.4570	.4566	.4561	.4557	.4552	.4548	.4543
.79	.4538	.4534	.4529	.4525	.4520	.4516	.4511	.4507	.4502	.4498

TABLE B-22 continued

EXPONENTIAL FUNCTION: e^{-x}

x	0	1	2	3	4	5	6	7	8	9
.80	.4493	.4489	.4484	.4480	.4475	.4471	.4466	.4462	.4457	.4453
.81	.4449	.4444	.4440	.4435	.4431	.4426	.4422	.4418	.4413	.4409
.82	.4404	.4300	.4396	.4391	.4387	.4382	.4378	.4374	.4369	.4365
.83	.4360	.4356	.4352	.4347	.4343	.4339	.4334	.4330	.4326	.4321
.84	.4317	.4313	.4308	.4304	.4300	.4296	.4291	.4287	.4283	.4278
.85	.4274	.4270	.4266	.4261	.4257	.4253	.4249	.4244	.4240	.4236
.86	.4232	.4227	.4223	.4219	.4215	.4211	.4206	.4202	.4198	.4194
.87	.4190	.4185	.4181	.4177	.4173	.4167	.4164	.4160	.4156	.4152
.88	.4148	.4144	.4140	.4135	.4131	.4127	.4123	.4119	.4115	.4111
.89	.4107	.4102	.4098	.4094	.4090	.4086	.4082	.4078	.4074	.4070
.90	.4066	.4062	.4058	.4054	.4049	.4045	.4041	.4037	.4033	.4029
.91	.4025	.4021	.4017	.4013	.4009	.4005	.4001	.3997	.3993	.3989
.92	.3985	.3981	.3977	.3973	.3969	.3965	.3961	.3957	.3953	.3949
.93	.3946	.3942	.3938	.3934	.3930	.3926	.3922	.3918	.3914	.3910
.94	.3906	.3902	.3898	.3894	.3891	.3887	.3883	.3879	.3875	.3871
.95	.3867	.3864	.3860	.3856	.3852	.3848	.3844	.3840	.3837	.3833
.96	.3829	.3825	.3821	.3817	.3814	.3810	.3806	.3802	.3798	.3795
.97	.3791	.3787	.3783	.3779	.3776	.3772	.3768	.3764	.3761	.3757
.98	.3753	.3749	.3746	.3742	.3738	.3734	.3731	.3727	.3723	.3719
.99	.3716	.3712	.3708	.3705	.3701	.3697	.3694	.3690	.3686	.3682

x	1	2	3	4	5	6	7	8	9
	.3679	.1353	.0498	.0183	.0067	.0025	.0009	.0003	.0001

NOTE: To obtain values for e^{-x} in which x is greater than one and not a whole number, multiply the whole number value of e by the fractional value of e.

Example:

$$
\begin{aligned}
e^{-1.213} &= (e^{-1})(e^{-.213}) \\
&= (.3679)(.8082) \\
&= .29733678 \\
&= .297
\end{aligned}
$$

TABLE B-23

TREND TEST

TABLE OF PEPCENTAGE POINTS FOR $\frac{S_{\delta}^2}{S^2}$

N	.99	.95
4	.3128	.3902
5	.2690	.4102
6	.2808	.4452
7	.3070	.4680
8	.3314	.4912
9	.3544	.5122
10	.3759	.5311
11	.3957	.5483
12	.4140	.5638
13	.4309	.5779
14	.4466	.5908
15	.4611	.6027
16	.4746	.6136
17	.4872	.6237
18	.4989	.6330
19	.5100	.6417
20	.5203	.6498
21	.5300	.6574
22	.5392	.6645

TABLE B-23 continued

N	.99	.95
23	.5479	.6712
24	.5561	.6776
25	.5639	.6836
26	.5713	.6893
27	.5784	.6947
28	.5851	.6997
29	.5915	.7045
30	.5976	.7091
31	.6034	.7135
32	.6089	.7177
33	.6142	.7217
34	.6193	.7256
35	.6242	.7294
36	.6290	.7330
37	.6337	.7365
38	.6382	.7399
39	.6425	.7432
40	.6467	.7463
41	.6508	.7493
42	.6548	.7522
43	.6586	.7550
44	.6623	.7577
45	.6659	.7603

TABLE B-23 continued

N	.99	.95
46	.6693	.7628
47	.6726	.7652
48	.6757	.7675
49	.6787	.7697
50	.6816	.7718

SUPPLEMENTARY

INFORMATION

AD-741811

U. S. ARMY TEST AND EVALUATION COMMAND
Aberdeen Proving Ground, Maryland 21005

MTP 3-1-005
AD 741811
CHANGE 1

10 June 1974

FIELD ARTILLERY STATISTICS

MTP 3-1-005, 1 March 1972, is changed as follows:

1. Remove pages and insert pages as indicated below.

Remove pages--	Insert pages--
ix and x	ix and x
7 and 8	7 and 8
9 and 10	9 and 10
15 and 16	15 and 16
21 and 22	21 and 22
25 and 26	25 and 26
35 and 36	35 and 36
37 and 38	37 and 38
39 and 40	39 and 40
43 and 44	43 and 44
45 and 46	45 and 46
51 and 52	51 and 52
53 and 54	53 and 54
55 and 56	55 and 56
57 and 58	57 and 58
63 and 64	63 and 64
65 and 66	65 and 66
67 and 68	67 and 68
69 and 70	69 and 70
71 and 72	71 and 72
77 and 78	77 and 78
79 and 80	79 and 80
83 and 84	83 and 84
85 and 86	85 and 86
87 and 88	87 and 88
89 and 90	89 and 90
91 and 92	91 and 92
93 and 94	93 and 94
95 and 96	95 and 96
97 and 98	97 and 98
99 and 100	99 and 100
103 and 104	103 and 104
105 and 106	105 and 106
107 and 108	107 and 108
113 and 114	113 and 114
115 and 116	115 and 116
119 and 120	119 and 120
125 and 126	125 and 126

MTP 3-1-005
AD 741811
CHANGE 1

10 June 1974

Remove pages--	Insert pages--
127 and 128	127 and 128
129 and 130	129 and 130
131a and 132	131a and 132
133 and 134	133 and 134
135 and 136	135 and 136
137 and 138	137 and 138
139 and 140	139 and 140
141 and 142	141 and 142
143 and 144	143 and 144
145 and 146	145 and 146
151 and 152	151 and 152
153 and 154	153 and 154
155 and 156	155 and 156
157 and 158	157 and 158
159 and 160	159 and 160
161 and 162	161 and 162
1-1 and 1-2	1-1 and 1-2
1-13 and 1-14	1-13 and 1-14
1-19 and 1-20	1-19 and 1-20
1-23 and 1-24	1-23 and 1-24
2-17 and 2-18	2-17 and 2-18
2-33 and 2-34	2-33 and 2-34
2-35 and 2-36	2-35 and 2-36
2-49 and 2-50	2-49 and 2-50
2-53 and 2-54	2-53 and 2-54
2-129 and 2-130	2-129 and 2-130

2. A vertical line in the left margin indicates the changed portion of the revised page.

3. Attach this sheet to the front of the reference copy for information.

$\underline{M}$ - Maintainability; the probability that an item will be retained in or restored to a specified condition within a period of time, when the maintenance is performed in accordance with prescribed procedures and resources.

MA - Total number of maintenance actions.

MR - Maintenance ratio; amount of active maintenance time per hour.

M_1 - Mean time between failures (lower confidence limit).
NOTE: The parameter may be rounds or miles instead of time.

M_2 - Mean time between failures (upper confidence limit).

MDT - Mean downtime.

$\bar{M}$ - Mean active maintenance time; total maintenance time divided by the number of maintenance actions.

MPI - The mean point of impact; the mean horizontal coordinates for ground bursts.

MTBF - Mean time between failures.

$MTBF_t$ - Mean time between failures where continued testing is necessary.

MTBM - Mean time between maintenance.

MTTR - Mean time to repair.

m - Miss distance; the distance between the aiming point and MPI.

MP - Mission (operational) profile, generally found in the Requirements Document.

μ - Small Greek letter mu used to denote the population mean.

μ_A - Small Greek letter mu used to denote the population mean for a Type A item.

μ_B - Small Greek letter mu used to denote the population mean for a Type B item.

μ_o - Small Greek letter mu with subscript zero used to denote the required mean found in the Requirements Document or from a comparable item.

N - Number of samples; sample size.

N_A - Number of samples for a Type A item.

N_B - Number of samples for a Type B item.

N_t - Sample size required to test the criteria; computed before testing starts.

N_{min} - Used when computing combined system reliability; the sample size for that individual component of a system which is tested fewer times than the other components.

OC - Operating-characteristic curve used to determine required sample size for testing given criteria.

ω - Small Greek letter omega used to denote allowable maintenance action time as prescribed in the Requirements Document.

π - Capital Greek letter pi used to represent the product of items; e.g.,

$$\prod_{i=1}^{N} X_i = (X_1)(X_2) \cdot \cdot \cdot (X_N)$$

p - The probability of an event occurring. (It cannot be less than zero or greater than one.)

PE - Probable error to which necessary subscripts are added to denote types of PE; e.g., PE_R (range probable error), PE_D (deflection probable error), or PE_H (height of burst probable error); a deviation from μ such that 50% of the observations may be expected to lie between μ-PE and μ+PE.

PE_A - Probable error for a Type A item to which necessary subscripts are used to denote types of PE_A; e.g., PE_{A_R} (range probable error for a Type A item), PE_{A_D} (deflection probable error for a Type A item), or PE_{A_H} (height of burst probable error for a Type A item).

PE_B - Probable error for a Type B item to which necessary subscripts are added to denote types of PE_B; e.g., PE_{B_R}, PE_{B_D}, or PE_{B_H}.

P - Sample Proportion; the ratio of the items possessing a given characteristic divided by the sample size.

P_A - Sample Proportion for a Type A item.

P_B - Sample Proportion for a Type B item.

P_o - The required maximum proportion of defectives; P_o equals λ_o, if λ_o is in terms of defectives or P_o equals the quantity $(1-\lambda_o)$, if λ_o is in terms of successes.

P_U - Upper limit for the proportion of defectives; the difference between P_o and the amount of doubt $(P_U = P_o - D)$.

$\overline{POB}$ - The mean point of burst; the mean coordinates for air bursts.

q - The ratio of the range of the observations to the standard deviation; the studentized range (q) distribution.

R - Reliability; the extent to which a test yields the same results on repeated trials.

ρ - Small Greek letter rho used to denote the population reliability.

ρ_o - Small Greek letter rho with subscript zero used to denote the required reliability prescribed in the Requirements Document.

R_U - Upper limit for the reliability; the sum of ρ_o and the amount of doubt $(R_U = \rho_o + D)$.

(1) Square the difference between the mean and reading i.e., $(x-\bar{X})^2$.
(2) Sum the squares; i.e., $\Sigma(x-\bar{X})^2$.
(3) Average the sum by dividing by N; i.e., $\frac{\Sigma(x-\bar{X})^2}{N}$.
(4) Find the square root of the average; i.e., $s = \sqrt{\frac{\Sigma(x-\bar{X})^2}{N}}$.
(The square root is used to compensate for the fact that the deviations were squared.)

c. In recent years there has been a tendency to divide by N-1 rather than by N. The reason for this is that if s^2 is used to estimate a population variance (σ^2), the mean obtained is usually too small and biased if N is the divisor. Therefore, N-1 as a divisor yields a truer estimate of the population variance. Since the population is the item of interest rather than only a few samples, N-1 will be used throughout this MTP in computing s^2 or s; i.e.,

$$s = \sqrt{\frac{\Sigma(x-\bar{X})^2}{N-1}}$$

(see paragraph 7.1, page 64, for computations). The population standard deviation (σ) is a measure of the extent to which a population characteristic varies from one item to another.

NOTE: The standard deviation may also be computed by the following formula:

$$s = \sqrt{\frac{N\Sigma x^2 - (\Sigma x)^2}{N(N-1)}}$$

4.5.2 RANGE

The range is the difference between the smallest and the largest readings in the sample. The range multiplied by the appropriate factor from Table B-1, page 2-1, approximates σ for a small sample ($N \leq 10$) and a normal distribution (paragraph 4.15.1, page 15).

4.5.3 MEAN DEVIATION

The mean deviation of a normal distribution is the mean of the deviations from the mean or median of the N sample members. The deviations from the mean (median) is the absolute value of the mean (median) subtracted from the reading. The mean deviation multiplied by a factor from Table B-2, page 2-2, approximates σ for a small sample ($N \leq 10$) and a normal distribution (see paragraph 4.15.1, page 15).

4.5.4 PROBABLE ERROR (RANGE, DEFLECTION, AND HEIGHT OF BURST)

The probable error (PE) is a measure of deviation from μ such that 50% of the observations may be expected to lie between μ-PE and μ+PE. However, certain conditions must exist for the PE to have any meaning. These are independent (random) samples, normal distribution, and large sample size.

PE may be expressed for various parameters, range (PE_R), deflection (PE_D), and height of burst (PE_H). For the population probable error (τ), $\tau = 0.6745\sigma$ and $\sigma = 1.4826\tau$. Since a sample is being examined as a representative of the population, PE = 0.6745s and s = 1.4826PE. Firing tables and other data concerning Field Artillery precision contain the appropriate PE's. When testing for precision, end results are often expressed in terms other than PE. This occurs in modern day testing because prototype samples are not random representations of production line items, the normal distribution is not appropriate in many cases, and small sample sizes bias the PE. The more modern standard deviation is in wider use as a measure of dispersion than is the probable error because s is commonly computed for statistical analysis. Due to the freedom to use small or large sample sizes, the wider applications of the standard deviation, and the ease of calculation, statistical tests involving standard deviation comparisons are more widely used than those involving PE comparisons.

4.5.5 CIRCULAR PROBABLE ERROR

The circular probable error (CPE or CEP) is a measure of deviation from μ and defines the radius of the circle which is centered at the mean and in which 50% of the observations are contained. CPE = 1.1774 times the population standard deviation for the easting (σ_E) when σ_E equals the population standard deviation for the northing (σ_N). When $\sigma_E \neq \sigma_N$, the CPE is called the equivalent CPE and equals .5887 ($\sigma_E+\sigma_N$). In terms of a sample, the equivalent CPE = .5887 (s_E+s_N). However, as for the PE, certain conditions must exist for the CPE to have any meaning; these are independent (random samples, a bivariate normal distribution, and a large sample size. Firing tables and other data concerning Field Artillery precision may contain the CPE. When testing for precision end results are often expressed in terms other than CPE. This occurs in modern day testing because prototype samples are not random representations of production line items, the bivariate normal distribution is not appropriate, and small sample sizes bias the CPE. The bivariate normal distribution is a representation of the measure of dispersion for two variables (see paragraph 4.15.2, page 15 and paragraph 9.2.4, page 118).

4.6 RELIABILITY

a. Reliability is the probability of an item functioning adequately for the period of time intended under the operating conditions encountered. Along with the numerical value of the reliability, a fraction or a percent value, the following are necessary:

(1) Define precisely a success or satisfactory performance.
(2) Specify the time base or operating cycles over which such performance is to be sustained; e.g., hours, miles, or rounds. This factor is particularly important since the probability value is based on completing a mission or task. For example, if the probability of a test item operating for 50 hours is 0.65 or 65%, then on the average 65 times out of 100 trials the test item would be functioning after a 50-hour operating period.

(3) Specify the environment or use conditions which will prevail. Typical of these conditions are temperature, humidity, shock, and vibration. Without these various conditions the reliability definition would be relatively meaningless.

b. Due to the various types of test items and the various distributions which apply, reliability may be evaluated by several methods (see paragraph 10, page 118).

4.7 TEST OF A STATISTICAL HYPOTHESIS

The investigator's objective can often be translated into an hypothesis (assumption or claim) concerning the test item. This hypothesis, called the null hypothesis, usually states that the test item does not meet the stated requirements. This explains why it is called the null (not) hypothesis. A decision is made to accept or reject the null hypothesis using the test data from the sample. Failure to reject the null hypothesis does not necessarily mean that the hypothesis is true but merely indicates that the sample is compatible with the kind of population described in the null hypothesis. The same is true if the null hypothesis is rejected; the fact is merely recognized that the sample is not compatible with the kind of population described in the null hypothesis. Associated with the null hypothesis are two types of errors (paragraph 4.8, page 10), and a significance level (paragraph 4.9, page 10). In general, to test a null hypothesis and construct statistical decision criteria, the following outline is used:

a. Formulate the null hypothesis so that it states that the test item does not meet the stated requirements. The null hypothesis is a numeric expression; e.g., $\overline{X} > 25$.

b. Formulate an alternative hypothesis so that the rejection of the null hypothesis is equivalent to the acceptance of the alternative hypothesis. The alternative hypothesis is also a numeric expression e.g., $\overline{X} \leq 25$.

c. Specify the probability to be risked as a Type I error. If possible, desired, or necessary, also make some specifications about the probability of a Type II error for a given alternate value of the parameter concerned.

d. Use the appropriate statistical theory (e.g., paragraphs 6.2, page 36, and 6.3, page 45) to test the null hypothesis.

NOTE: In some cases when the null hypothesis has been rejected, a reserve judgment decision will be made instead of accepting the alternative hypothesis; e.g., insufficient sampling to produce conclusive results.

4.8 TYPES OF ERROR

4.8.1 TYPE I ERROR

The Type I error is rejection of the null hypothesis when it is true. The risk of Type I error is the level of significance (α). It is the more important of the two error types, since rejecting an item when in fact it is good is better economically than accepting an item when in fact it is bad. The value of α is arbitrary but will sometimes be found in the Requirements Document. In the event the significance level or confidence level (confidence level = 1 - significance level) is not specified in the Requirements Document, α = .10 or confidence level = .90 will be used.

4.8.2 TYPE II ERROR

The Type II error is the acceptance of the null hypothesis when it is false. The risk of a Type II error is denoted by β. The value of β is not as restricted as that of α. In the event α and β are highly restricted, the sample size must be very large to reach an accept or reject decision. When β is not specified in the Requirements Document, .20 will be used.

4.9 LEVEL OF SIGNIFICANCE.

a. The risk of making a Type I error (α) equals the level of significance of the test. The null hypothesis serves as an origin or base. From the null hypothesis the test criterion may be a two-sided test (two-tail test) or a one-sided test (one-tail test). The two-sided test involves an area at each extreme of the distribution curve (note Figure 3A); e.g., if α = .05 or 5%, then the shaded areas in Figure 3A are each equal to 2.5% of the total area under the curve. The one-sided test is only concerned with the area under the curve at one extreme (note Figure 3B); e.g., if α = .05 or 5%, then the shaded area in Figure 3B is equal to 5% of the total area under the curve. When the stated requirement is in the shaded area, the null hypothesis is accepted which means that the item is not acceptable.

b. In general, a test is said to be one-sided or two-sided (one-tailed or two-tailed) depending on whether α is concentrated at one end of the curve (left or right) or is divided into two areas with the areas situated at opposite ends of the curve (see Figure 3).

4.10 CONFIDENCE INTERVAL, LIMITS, AND LEVEL

a. When estimating a population measure, such as μ, by a sample measure, such as $\bar{X}$, μ has a value somewhere near $\bar{X}$. How near μ is to $\bar{X}$ is determined by an interval constructed about $\bar{X}$; and, at a specified confidence level, μ lies in this interval. This interval is called the confidence interval. The interval between the shaded areas of Figure 3A is an example of a confidence interval (see paragraph 6.1.2.1, page 27).

b. The end points of the confidence interval are called confidence limits. Thus, there exist an upper confidence limit (UCL) and a lower confidence limit (LCL). The LCL and UCL are shown in Figure 3A. In standard item mean has been used in some of the illustrated cases (see paragraph 6.1.3, page 33, and paragraph 6.2.3, page 43). This is considered appropriate since the timing and recording of the data for the tests may be easily controlled. However when the test item has a large standard deviation, an error as great as five percent may be acceptable in order to keep sample sizes reasonable.

4.15 PARTICULAR DISTRIBUTIONS

4.15.1 NORMAL DISTRIBUTION

a. The normal distribution is by far the most important continuous distribution (see pages 2 to 4). Due to the laws of chance repeated measurements of the same physical quantity occur with such a dispersion that a pattern (distribution) is evident and can be closely approximated by a certain kind of continuous distribution, referred to as the "normal curve of errors." The graph of a normal distribution is a bell-shaped curve that extends indefinitely in both directions (see Figure 6A).

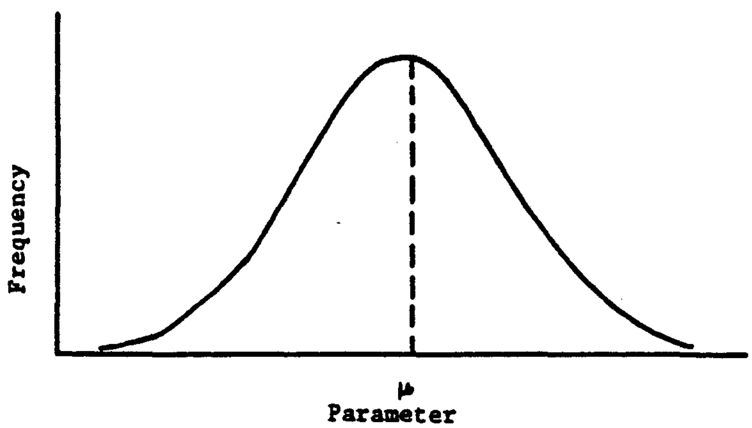
b. The mean is at the peak of the distribution, and the standard deviation determines the spread of the distribution. The physical area from a to b under two normal distributions may not be equal (see Figure 6B). Since construction of separate tables of normal curve areas for each conceivable pair of values for μ and σ is impractical, areas are tabulated only for the so-called standard normal distribution which has a mean of zero and a standard deviation of one. The conversion of a normal distribution to a standard normal distribution is accomplished by using the equation $Z = \frac{x-\mu}{\sigma}$ (see Figure 7A). With the conversion to standard units, Table B-3, page 2-3, may be used. The entries in this table are the areas under the standard normal distribution between the mean (Z = 0) and Z = .01, ..., 3.09. The negative values of Z (areas to the left of the mean) are not needed by virtue of the symmetry of a normal curve about its mean; e.g., the area between Z = -1.33 and Z = 0 is the same as the area between Z = 0 and Z = 1.33, which is 0.4082. In the event the percentage of area under the curve to the left of a given value of Z is desired, Table B-3, page 2-3 and this value of Z are used to determine the percent from the mean. If Z is positive, the percentage of the area to the left of Z equals .50 plus the value obtained from Table B-3; e.g., if Z = .92 the percent of area is .50 + .3212 which is .8212 or 82.12% of the area. If Z is negative, the percentage of the area to the left of Z equals .50 minus the value obtained from Table B-3; e.g., if Z = -.92, the percent of area is .50 - .3212 which is .1788 or 17.88% of the area.

c. The percentage of area between two Z values can be determined by obtaining the areas for the Z values from Table B-3, page 2-3, and either subtracting the smaller area from the larger area if both Z values are on the same side of the mean or adding the areas if the Z values are on opposite sides of the mean (see Figure 7B).

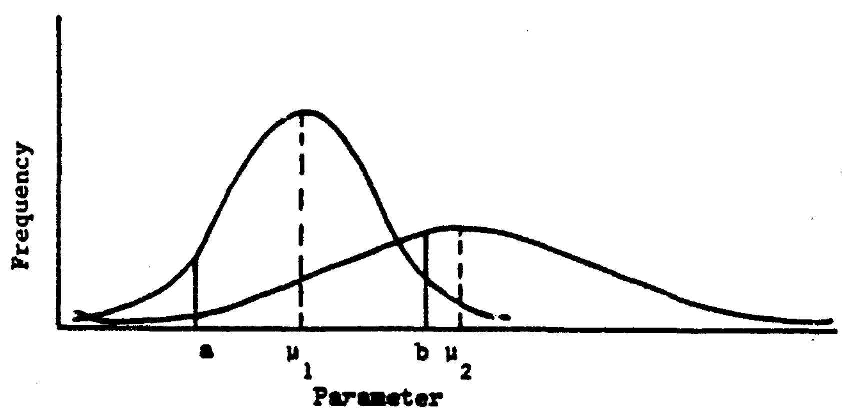
4.15.2 BIVARIATE NORMAL DISTRIBUTION

a. A bivariate normal distribution is a population in which each member is dependent on two variables (values); e.g., easting and northing.

NORMAL DISTRIBUTION CURVE



A



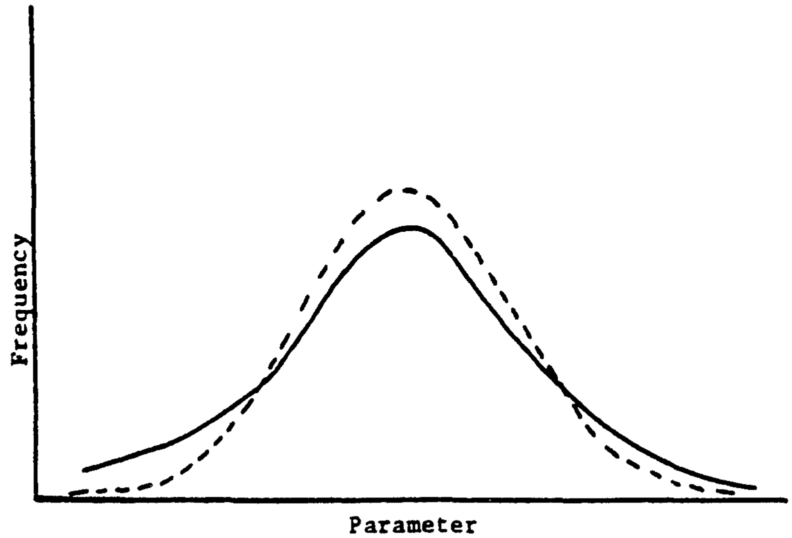
B

Figure 6

The data may be grouped into a table of double entry showing the frequencies of pairs of values lying within given class intervals. Each row in such a table gives the frequency distribution of the first variable for the members of the population in which the second variable lies within the limits stated on the left of the row. A similar statement can be made about the columns. A grouped frequency distribution of the type in Tables A-1a and A-1b, page 1-1 may be termed a bivariate frequency distribution.

b. The shape of the bivariate normal population is a normal distribution in three dimensions, rising to its greatest height at the center and fading away to tangency (see Figure 8). Some properties of the bivariate normal distribution are:

STUDENT t DISTRIBUTION CURVE



— Student t-distribution curve

--- Normal distribution curve

Figure 10

F DISTRIBUTION CURVE

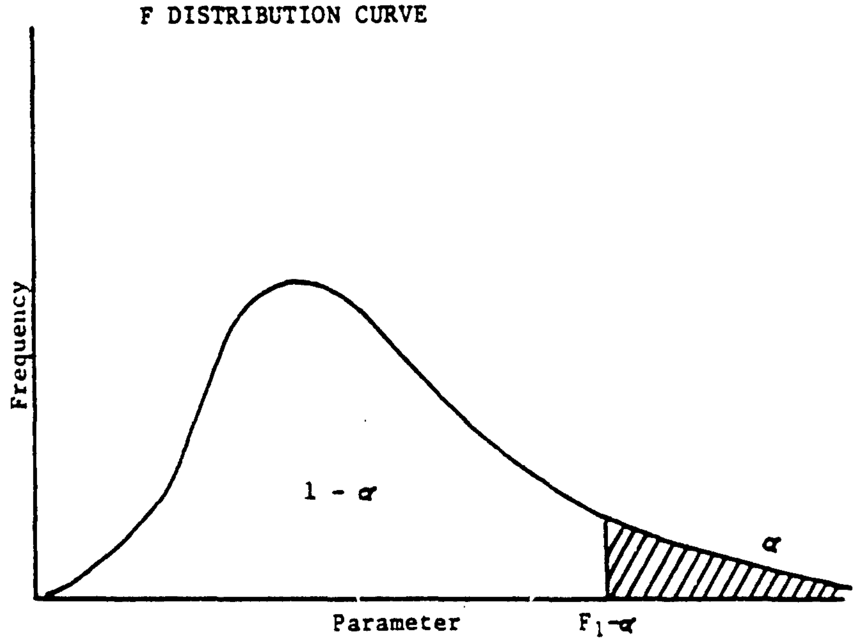


Figure 11

often stated with a given confidence level. The theory on which these confidence intervals are based assumes that the population from which the sample is obtained has roughly the shape of a normal distribution and is called the chi-square (χ^2) distribution. An example of a chi-square distribution is shown in Figure 12A; in contrast to the normal and t distribution, its domain is restricted to the nonnegative real numbers.

b. The χ^2 distribution is also different from those previously discussesd in that the area under the curve is summed from the χ^2 point to the right. The value for $\chi^2_{1-\alpha}$ represents an area of α under the curve (right-hand tail, see Figure 12A), while χ^2_α represents an area of $1-\alpha$ to the right under the curve (see Figure 12B). Due to the shape of the χ^2 curve the point values of $\chi^2{}_{\alpha/2}$ and $\chi^2_{1-\alpha/2}$ will be different even though the significance levels are equal (see Figure 12C). This distinction is important due to the fact the distribution is not symmetrical; thus, a table containing values corresponding to areas in either tail of the distribution is necessary. Thus, with a confidence level of $1-\alpha$,

$$\frac{(N-1)s^2}{\chi^2_{\alpha/2}} < \sigma < \frac{(N-1)s^2}{\chi^2_{1-\alpha/2}}$$

As the sample size decreases, the interval for σ becomes wider. Therefore, in most tests applying the chi-square distribution, a normal sample size is needed ($N \geq 30$).

4.16 ROUND OFF PROCEDURES

a. Since all measuring equipment has limited accuracy, the measurements are also of limited accuracy and thus consist of numbers which have been rounded off; e.g., if an instrument is accurate to tenths of minutes and a time measurement is 12.2 minutes, the time may actually have been any value between 12.15 and 12.25 minutes.

b. When test data are used to compute test item characteristics, such as the mean and standard deviation, the results must be consistent with the original data; i.e., the mean weight of a group of projectiles cannot be more accurate than the individual weights used to compute the mean. The following are some basic rules concerning significant figures and the rounding of data:

(1) Significant figures (significant digits) are the digits of a number that begin with the first digit on the extreme left that is not a zero (if there are any nonzero digits to the left of the decimal point), or with the first digit (zero or nonzero) to the right of the decimal point, and that end with the last digit on the right that is not a zero or that is a zero which is considered accurate. For example:

(a) 12304 has five significant digits.
(b) 1.0200 has five significant digits.

(when a number ends with a zero which is on the right of the decimal point, the zero is significant.)

5.2 DATA REQUIRED

A list of sample readings.

5.3 PROCEDURE

a. N is odd.

(1) List the readings in descending or ascending order.
(2) Use the middle reading for the median.

b. N is even

(1) List the readings in descending or ascending order.
(2) Use the average of the two middle readings for the median.

5.4 EXAMPLE

a. Case I.

Given:
N = 5

Procedure:	Example:
(1) List the readings in order.	(1) 15 13.5 12.7 12 11.9
(2) Use the $\frac{N+1}{2}$ reading for the median.	(2) $\frac{N+1}{2} = \frac{5+1}{2}$ $= 3$ The median is the 3rd reading. The median = 12.7

b. Case II

Given:
N = 6

Procedure:	Example:
(1) List the readings in order	(1) 250 245 230 228 225 224.6

(2) Use the $\frac{N}{2}$ and $\frac{N}{2}$ +1 readings to compute the median which is the average of the two. The median =

$$\frac{(\frac{N}{2} \text{ reading}) + (\frac{N}{2} + 1 \text{ reading})}{2}$$

(2) $\frac{N}{2} = 3$

$\frac{N}{2} + 1 = 4$

Use the 3rd and 4th readings to compute:

$$\text{The median} = \frac{230+228}{2} = \frac{458}{2} = 229$$

5.5 ANALYSIS

The median equals the mean if the population is normally distributed; otherwise, it is only another measure of central location, which denotes the midpoint of the total dispersions.

6. MEAN

6.1 ESTIMATE OF THE POPULATION MEAN (μ).

6.1.1 BEST SINGLE ESTIMATE OF μ.

6.1.1.1 OBJECTIVE

To determine the best point estimate of the population mean for a normal distribution.

6.1.1.2 DATA REQUIRED

A list of sample readings; e.g., the time required for prepare for action under daylight conditions.

6.1.1.3 PROCEDURE

a. Sum the list of data for the parameter.

b. Divide the sum by the number of readings recorded to obtain the mean of the parameter.

6.1.1.4 EXAMPLE

Given:
Sample data at Table A-2a, page 1-2.

Procedure:

a. Sum the parameter.

Example:

a. Sum = 1037.0 min

6.1.3.1.5 ANALYSIS

If N_t samples are tested and $\bar{X}$ is computed, conclude that $\mu \leq \bar{X} + \varepsilon$ and $\mu \geq \bar{X} - \varepsilon$ at a 100(1-α)% confidence level.

6.1.3.2 SAMPLE SIZE REQUIRED TO ESTIMATE μ WITH σ KNOWN

6.1.3.2.1 OBJECTIVE

To determine the N_t required in order to state that μ is equal to or between $\bar{X} + \varepsilon$ and $\bar{X} - \varepsilon$ at the desired confidence level when σ is known.

6.1.3.2.2 DATA REQUIRED

σ, which is known from a standard item, history, or Requirements Document.

6.1.3.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Choose the allowable amount of error.

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

d. Compute N_t as follows:

(1) Square step c.
(2) Square σ.
(3) Square ε.
(4) Multiply step (1) by step (2).
(5) Divide step (4) by step (3).
(6) Round step (5) to the next larger whole number.

e. Conclude that N_t samples are required in order to state that μ is equal to or between $\bar{X} + \varepsilon$ and $\bar{X} - \varepsilon$ at the desired confidence level.

6.1.3.2.4 EXAMPLE

Given:
$\sigma = 2.0$ min.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. $\alpha = .05$ $1-\alpha = .95$ $1-\alpha/2 = .975$
b. Choose ε.	b. $\varepsilon = .8$ min.

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

c. $Z_{.975} = 1.960$

d. Compute:

$$N_t = \frac{(Z_{1-\alpha/2})^2(\sigma)^2}{\varepsilon^2}$$

d. $N_t = \frac{(1.96)^2(2.0)^2}{(.8)^2}$

$= \frac{(3.842)(4.00)}{.64}$

$= \frac{15.37}{.64}$

$= 24.02$

$= 25$

e. Conclude that N_t samples are required in order to state that $\mu < \overline{X} + \varepsilon$ and $\mu > \overline{X} - \varepsilon$ at a 100(1-α)% confidence level.

e. If 25 samples are tested and $\overline{X}$ computed, conclude that $\mu < \overline{X} + .8$ min. and $\mu > \overline{X} - .8$ min. at a 95% confidence level.

6.1.3.2.5 ANALYSIS

If N_t samples are tested and $\overline{X}$ is computed, conclude that $\mu < \overline{X} + \varepsilon$ and $\mu \geq \overline{X} - \varepsilon$ at a 100 (1-α)% confidence level.

6.2 COMPARING AN OBSERVED MEAN ($\overline{X}$) TO A REQUIREMENT (μ_o)

a. An observed mean is generated from a sample and is representative of μ. This value of $\overline{X}$ is then compared to a stated requirement (μ_0). However, looking at the values of $\overline{X}$ and μ_o to decide whether μ is greater than μ_o or μ is less than μ_o at a confidence level is insufficient. Since the decision pertains to the population, statistical tests must be applied to $\overline{X}$ to determine whether μ is greater than μ_o or μ is less than μ_o.

b. There exist two possibilities for the relationship of $\overline{X}$ to μ_o. However, for each possibility there are two approaches; i.e., σ may be known or unknown; and the appropriate test must be chosen on that basis. Following are the assumptions and the circumstances for each possible relationship:

(1) $\overline{X}$ greater than μ_o

(a) The null hypothesis is μ is greater than μ_o.
(b) The alternative hypothesis is there is no reason to believe μ is greater than μ_o.
(c) The use of this test is appropriate when μ_o is a maximum value for μ to satisfy. In the event that μ must not be greater than μ_o, this test would be appropriate.

(2) $\overline{X}$ less than μ_o.

(a) The null hypothesis is μ is less than μ_o.
(b) The alternative hypothesis is there is no reason to believe μ is less than μ_o.
(c) The use of this test is appropriate when μ_o is a minimum value for μ to satisfy. In the event that μ must meet or exceed μ_o, this test would be appropriate.

6.2.1 $\overline{X}$ GREATER THAN μ_o

6.2.1.1 $\overline{X}$ GREATER THAN μ_o WITH σ UNKNOWN

6.2.1.1.1 OBJECTIVE

To determine whether μ is greater than μ_o at the desired confidence level when the value of σ is unknown.

6.2.1.1.2 DATA REQUIRED

A list of sample readings.

6.2.1.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}$ (see paragraph 6.1.1.3, page 26).

c. Compute s (see paragraph 7.1.1.3, page 64).

d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for N-1 d.f.

e. Compute ε as follows:

(1) Multiply s by step d.
(2) Divide step (1) by the square root of N.

f. Subtract ε from $\overline{X}$ to obtain the LCL which is the lower bound for μ at the desired confidence level. The confidence interval for μ is from the LCL to $+\infty$.

g. If μ_o is less than the LCL, decide that μ is greater than μ_o; otherwise, there is no reason to believe μ is greater than μ_o at the desired confidence level.

6.2.1.1.4 EXAMPLE

Given:
μ_o = 85.0 min.
Sample data at Table A-2a, page 1-2.

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .05$ $1-\alpha = .95$
b. Compute $\overline{X}$.	b. $\overline{X} = 86.417$ $= 86.4$ min. See paragraph 6.1.2.1.4 b, page 28, for computations.
c. Compute s.	c. $s = 2.173$ $= 2.2$ min. See paragraph 6.1.2.1.4 c, page 28, for computations.
d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for N-1 d.f.	d. $t_{.95}$ for 11 d.f. $= 1.796$
e. Compute: $\varepsilon = \frac{t_{1-\alpha}(s)}{\sqrt{N}}$	e. $\varepsilon = \frac{(1.796)(2.173)}{\sqrt{12}} = \frac{3.903}{3.464} = 1.127$
f. Compute: $LCL = \overline{X} - \varepsilon$	f. $LCL = 86.417 - 1.127 = 85.290 = 85.2$ min.
g. If $\mu_0 < LCL$, decide that $\mu > \mu_0$; otherwise, there is no reason to believe $\mu > \mu_0$ at a $100(1-\alpha)\%$ confidence level.	g. Since $85.0 < 85.2$, decide that $\mu > 85.0$ min. at a 95% confidence level.

6.2.1.1.5 ANALYSIS

If $\mu_0 < LCL$, the null hypothesis that $\mu > \mu_0$ is accepted; otherwise. there is no reason to believe $u > \mu_0$ at a $100(1-\alpha)\%$ confidence level when σ is unknown. The $100\ (1-\alpha)\%$ confidence interval for μ is from the LCL to $+\ \infty$.

6.2.1.2 $\overline{X}$ GREATER THAN μ_0 WITH σ KNOWN

6.2.1.2.1 OBJECTIVE

To determine whether μ is greater than μ_0 at the desired confidence level when σ is known.

6.2.1.2.2 DATA REQUIRED

A list of sample readings and σ, which is known from a standard item, history, or Requirements Document.

6.2.1.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}$ (see paragraph 6.1.1.3, page 26).

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

d. Compute ε as follows:

(1) Multiply σ by step c.
(2) Divide step (1) by the square root of N.

e. Subtract ε from $\overline{X}$ to obtain the LCL which is the lower bound for μ at the desired confidence level. The confidence interval for μ is from the LCL to $+\infty$.

f. If μ_0 is less than the LCL, decide that μ is greater than μ_0; otherwise there is no reason to believe μ is greater than μ_0 at the desired confidence level.

6.2.1.2.4 EXAMPLE

Given:
$\sigma = 1.4$ min.
$\mu_0 = 83.0$ min.
Sample data at Table A-2a, page 1-2.

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .10$ $1-\alpha = .90$
b. Compute $\overline{X}$.	b. $\overline{X} = 86.417$ $= 86.4$ min. See paragraph 6.1.1.4, page 26, for computations.
c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.	c. $Z_{.90} = 1.282$
d. Compute: $\varepsilon = \frac{Z_{1-\alpha}(\sigma)}{\sqrt{N}}$	d. $\varepsilon = \frac{(1.282)(1.4)}{\sqrt{12}}$ $= \frac{1.7948}{3.464}$ $= .518$

e. Compute:

$$LCL = \overline{X} - \varepsilon$$

f. If $\mu_0 <$ LCL decide that $\mu > \mu_0$; otherwise, there is no reason to believe $\mu > \mu_0$ at a $100(1-\alpha)\%$ confidence level.

e. LCL = 86.417 - .518
= 85.899
= 85.8 min.

f. Since 83.0 < 85.8 decide that $\mu >$ 83.0 min. at a 90% confidence level.

6.2.1.2.5 ANALYSIS

If $\mu_0 <$ LCL, the null hypothesis that $\mu > \mu_0$ is accepted; otherwise, there is no reason to believe $\mu > \mu_0$ at a $100(1-\alpha)\%$ confidence level when σ is unknown. The $100\,(1-\alpha)\%$ confidence interval for μ is LCL to $+\infty$

6.2.2 $\overline{X}$ LESS THAN μ_0.

6.2.2.1 $\overline{X}$ LESS THAN μ_0 WITH σ UNKNOWN

6.2.2.1.1 OBJECTIVE

To determine whether μ is less than μ_0 at the desired confidence level when σ is unknown.

6.2.2.1.2 DATA REQUIRED

A list of sample readings.

6.2.2.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}$ (see paragraph 6.1.1.3, page 26).

c. Compute s (see paragraph 7.1.1.3, page 64).

d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for N-1 d.f.

e. Compute ε as follows:

(1) Multiply s by step d.
(2) Divide step (1) by the square root of N.

f. Add ε to $\overline{X}$ to obtain the UCL which is the upper bound for μ at the desired confidence level. The confidence interval for μ is from $-\infty$ to the UCL.

g. If μ_0 is greater than the UCL, decide that μ is less than μ_0; otherwise, there is no reason to believe μ is less than μ_0 at the desired confidence level.

e. Compute:

$UCL = \overline{X} + \varepsilon$

e. $UCL = 86.417 + .814$
$= 87.231$
$= 87.3$ min.

f. If $\mu_0 >$ UCL, decide that $\mu < \mu_0$; otherwise, there is no reason to believe $\mu < \mu_0$ at a 100(1-α)% confidence level.

f. Since 88.0 > 87.3, decide that $\mu <$ 88.0 min. at a 90% confidence level.

6.2.2.2.5 ANALYSIS

If $\mu_0 >$ UCL, the null hypothesis that $\mu < \mu_0$ is accepted; otherwise there is no reason to believe $\mu < \mu_0$ at a desired confidence level. The 100 (1-α)% confidence interval for μ is from $-\infty$ to UCL.

6.2.3 DETERMINATION OF SAMPLE SIZE

6.2.3.1 OBJECTIVE

To determine the N_t required to determine whether μ is equal to or greater than $\mu_0 + \varepsilon$ (or equal to or less than $\mu_0 - \varepsilon$) at the desired confidence level when:

a. σ is known.

b. σ is unknown.

6.2.3.2 DATA REQUIRED

a. σ, which is known from a standard item, history, or Requirements Document.

b. An approximation of the value that σ will assume.

6.2.3.3 PROCEDURE

a. Choose α and β, the probabilities of making Type I and Type II errors respectively.

b. Choose the allowable amount of error.

c. Compute d^2, an intermediate value, as follows:

(1) Divide ε by σ.
(2) Square step (1).

d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

e. If σ is known, compute N_t as follows:

(1) Add $Z_{1-\alpha}$ to $Z_{1-\beta}$.
(2) Square step (1).

(3) Divide step (2) by step c and round to the next larger whole number.

f. If σ is unknown, add 3 to step e for $\alpha = .01$, 2 for $\alpha = .05$, or 1 for $\alpha = .10$.

g. Conclude that N_t samples are required to determine whether μ is equal to or greater than $\mu_o + \varepsilon$ (or equal to or less than $\mu_o - \varepsilon$) at the desired confidence level.

6.2.3.4 EXAMPLE

Given:
$\sigma = .12$

Procedure:	Example:
a. Choose α and β.	a. $\alpha = .01$ $1-\alpha = .99$ $\beta = .20$ $1-\beta = .80$
b. Choose ε.	b. $\varepsilon = .05$
c. Compute: $d^2 = (\varepsilon/\sigma)^2$	c. $d^2 = (.05/.12)^2 = (.4167)^2 = .1736$
d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.	d. $Z_{.99} = 2.326$ $Z_{.80} = .84$
e. When σ is known, compute: $N_t = \frac{(Z_{1-\alpha}+Z_{1-\beta})^2}{d^2}$	e. $N_t = \frac{(2.326+.84)^2}{.1736} = \frac{(3.166)^2}{.1736} = \frac{10.024}{.1736} = 57.74 = 58$
f. When σ is unknown and a value is assumed, compute: (1) $\alpha = .01$ $N_t = \frac{(Z_{1-\alpha}+Z_{1-\beta})^2}{d^2} + 3$ (2) $\alpha = .05$ $N_t = \frac{(Z_{1-\alpha}+Z_{1-\beta})^2}{d^2} + 2$	f. Since $\alpha = .01$ $N_t = \frac{(2.326+.84)^2}{.1736} + 3 = 61$

(3) $\alpha = .10$

$$N_t = \frac{(Z_{1-\alpha}+Z_{1-\beta})^2}{d^2} + 1$$

g. Conclude that N_t samples are required to determine whether $\mu \geq \mu_o + \varepsilon$ (or $\mu \leq \mu_o - \varepsilon$) at a 100(1-$\alpha$)% confidence level.

g. Conclude that 58 samples, for σ known and equal to .12 (or 61 samples for σ assumed equal to .12), must be tested in order to determine whether $\mu \geq \mu_o + .05$ (or $\mu \leq \mu_o - .05$) at a 99% confidence level.
NOTE: If σ is really less than .12, N_t is more than adequate.

6.2.3.5 ANALYSIS

If σ is overestimated, the consequences are twofold: first, N_t is overestimated; second, by employing a N_t that is larger than necessary, the actual value of β will be somewhat less than intended at the same confidence level, a consequence which is never undesirable. On the other hand if σ is underestimated, N_t is underestimated. Therefore, N_t must be recomputed and additional items must be tested if possible. β will be larger than intended at the same confidence level. Thus, overestimating σ is safer than underestimating σ.

6.3 COMPARING TWO OBSERVED MEANS

a. An observed mean is generated from a sample and is representative of μ. This value of $\bar{X}$ is then required to meet a standard item $\bar{X}$ which is representative of the standard item's population. Looking at the values of the means ($\bar{X}_A$ and $\bar{X}_B$) to decide whether μ_A is greater than μ_B or μ_A is less than μ_B at a confidence level is insufficient. Since the decision pertains to the populations, statistical tests must be applied X_A and X_B to determine whether μ_A is greater than μ_B or μ_A is less than μ_B. The statistical tests use the sample means as estimates of the population means.

b. Type A generally represents the test item and Type B the standard item when testing the hypothesis that μ_A is greater than μ_B. However, to prove that the average performance of the test item is less than that of the standard item, Type A must represent the standard item so that the hypothesis, μ_A is greater than μ_B, can be tested.

c. When the null hypothesis is μ_A is greater than μ_B, the alternative hypothesis is there is no reason to believe that μ_A is greater than μ_B.

d. There are four different procedures available to test the null hypothesis. Following are the conditions which dictate the appropriate test:

(1) The variabilities of A and B are unknown but assumed equal ($\sigma_A = \sigma_B$). This test also applies when $N_A = N_B$ even though $\sigma_A \neq \sigma_B$ (see paragraph 6.3.1.1, page 46).
(2) The variabilities of A and B are unknown but assumed unequal ($\sigma_A \neq \sigma_B$) for unequal sample sizes (see paragraph 6.3.1.2, page 49).
(3) The variabilities of A and B are known from previous experience; thus, σ_A may or may not equal σ_B (see paragraph 6.3.1.3, page 51).
(4) The observations are paired; i.e., individual Type A and Type B items are tested alternately such that the items in each pair are tested under the same condition. Obviously, $N_A = N_B$ (see paragraph 6.3.1.4, page 53).

NOTE: The procedure in subparagraph (1) is also valid for paired observations since $N_A = N_B$; however, the procedure in subparagraph (4) is only valid for paired observations.

6.3.1 $\overline{X}_A$ GREATER THAN $\overline{X}_B$

6.3.1.1 σ_A AND σ_B UNKNOWN BUT ASSUMED EQUAL

6.3.1.1.1 OBJECTIVE

To determine whether μ_A is greater than μ_B at the desired confidence level when the population standard deviations of A and B are unknown but σ_A is assumed equal to σ_B.

6.3.1.1.2 DATA REQUIRED

A list of sample readings.

6.3.1.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute $\overline{X}_A$ and $\overline{X}_B$ (see paragraph 6.1.1.3, page 26).

c. Compute $\Sigma\Delta_A^2$ and $\Sigma\Delta_B^2$ as follows:

(1) Compute the deviation from the mean for each reading ($\Delta_A = X_A - \overline{X}_A$ and $\Delta_B = X_B - \overline{X}_B$).
(2) Square each deviation ($\Delta_A{}^2$ and $\Delta_B{}^2$).
(3) Sum the squared deviations for each of the two items ($\Sigma\Delta_A^2$ and $\Sigma\Delta_B^2$).

d. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for (N_A+N_B-2) d.f.

e. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for e.d.f.

f. Compute:

$$\varepsilon = t_{1-\alpha} \sqrt{V_A+V_B}$$

g. Compute:

$$LCL = \overline{X}_A - \varepsilon$$

h. If $\overline{X}_B < LCL$, decide that $\mu_A > \mu_B$; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a 100(1-α)% confidence level.

e. $t_{.95}$ for 22 d.f. = 1.717

f. $\varepsilon = (1.717) \sqrt{9.35 + 16.55}$
$= (1.717) \sqrt{25.90}$
$= (1.717)\ (5.089)$
$= 8.738$

g. LCL = 5401.40 - 8.74
= 5392.66
= 5392 meters

h. Since 5378 < 5392, decide that $\mu_A > \mu_B$ at a 95% confidence level.

6.3.1.2.5 ANALYSIS

If $\overline{X}_B < LCL$, the null hypothesis that $\mu_A > \mu_B$ is accepted; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a 100(1-α)% confidence level when σ_A and σ_B are unknown and σ_A assumed equal to σ_B.

6.3.1.3 σ_A AND σ_B KNOWN FROM PREVIOUS EXPERIENCE

6.3.1.3.1 OBJECTIVE

To determine whether μ_A is greater than μ_B when σ_A and σ_B are known from previous experience.

6.3.1.3.2 DATA REQUIRED

A list of sample readings and σ_A and σ_B, which are known from previous testing.

6.3.1.3.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

c. Compute $\overline{X}_A$ and $\overline{X}_B$ (see paragraph 6.1.1.3, page 26).

d. Compute ε as follows:

(1) Square σ_A.

(2) Square σ_B.

(3) Divide step (1) by N_A.

(4) Divide step (2) by N_B.
(5) Add step (3) to step (4).
(6) Multiply step b by the square root of step (5).

e. Subtract ε from $\bar{X}_A$ to obtain the LCL.

f. If $\bar{X}_B$ is less than the LCL, decide that μ_A is greater than μ_B; otherwise, there is no reason to believe μ_A is greater than μ_B at the desired confidence level.

6.3.1.3.4 EXAMPLE

Given:
Sample data at Table A-2b, page 1-3, and Table A-2c, page 1-4.

$\sigma_A = 14.0$ meters

$\sigma_B = 12.0$ meters

Procedure:	Example:
a. Choose the confidence level (1-α).	a. $\alpha = .05$ $1-\alpha = .95$
b. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.	b. $Z_{.95} = 1.645$
c. Compute: $\bar{X}_A$ $\bar{X}_B$	c. $\bar{X}_A = 5401.40$ $= 5401$ meters $\bar{X}_B = 5372.25$ $= 5372$ meters See paragraph 6.3.1.1.4, page 47.
d. Compute: $\varepsilon = Z_{1-\alpha}\sqrt{\frac{\sigma_A^2}{N_A} + \frac{\sigma_B^2}{N_B}}$	d. $\varepsilon = (1.645)\sqrt{[(14.0)^2/20]+[(12.0)^2/20]}$ $= (1.645)\sqrt{(196.0/20)+(144.0/20)}$ $= (1.645)\sqrt{9.80 + 7.20}$ $= (1.645)\sqrt{17.00}$ $= (1.645)(4.12)$ $= 6.78$
e. Compute: $LCL = \bar{X}_A - \varepsilon$	e. LCL = 5401.40 - 6.78 = 5394.62 = 5394 meters

f. If $\overline{X}_B$ < LCL, decide that $\mu_A > \mu_B$; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a 100(1-α)% confidence level.

f. Since 5372 < 5394, decide that $\mu_A > \mu_B$ at a 95% confidence level.

6.3.1.3.5 ANALYSIS

If $\overline{X}_B$ < LCL, the null hypothesis that $\mu_A > \mu_B$ is accepted; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a 100 (1-α)% confidence level when σ_A and σ_B are known from previous testing.

6.3.1.4 PAIRED OBSERVATIONS

6.3.1.4.1 OBJECTIVE

To determine whether μ_A is greater than μ_B when the observations are paired (see subparagraph 6.3 d (4), page 46.

6.3.1.4.2 DATA REQUIRED

A list of paired sample readings.

6.3.1.4.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute the difference between the reading for Type A and the reading for Type B ($x_d = x_A - x_B$) for each pair of observations.

c. Compute the mean of the differences ($\overline{X}_d$), (see paragraph 6.1.1.3, page 26).

d. Compute the standard deviation of the differences (s_d), (see paragraph 7.1.1.3, page 64).

e. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for N-1 d.f.

f. Compute ε as follows:

(1) Divide step d by the square root of N.
(2) Multiply step e by step (1).

g. If $\overline{X}_d$ is greater than ε, decide that μ_A is greater than μ_B; otherwise, there is no reason to believe μ_A is greater than μ_B at the desired confidence level.

6.3.1.4.4 EXAMPLE

Given:
Sample data at Table A-2e, page 1-6.

Procedure:

a. Choose the confidence level $(1-\alpha)$.

b. Compute x_d for each pair of readings.

$$x_d = x_A - x_B$$

c. Compute $\overline{X}_d$.

d. Compute s_d.

e. Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for N-1 d.f.

f. Compute:

$$\varepsilon = t_{1-\alpha} \frac{s_d}{\sqrt{N}}$$

g. If $\overline{X}_d > \varepsilon$, decide that $\mu_A > \mu_B$; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a $100(1-\alpha)\%$ confidence level.

Example:

a. $\alpha = .05$
$1-\alpha = .95$

b. (1) $x_d = 146.0 - 141.0 = 5.0$

(2) $x_d = 141.5 - 143.5 = -2.0$

See Table A-2e, page 1-6, for complete list.

c. $\overline{X}_d = -0.10 = -0.1$ amp. hr.

See paragraph 6.1.1.4, page 26.

d. $s_d = 2.81 = 2.8$ amp. hr.

See paragraph 7.1.1.4, page 65.

e. $t_{.95}$ for 9 d.f. = 1.833

f. $\varepsilon = (1.833)(2.81)/\sqrt{10} = 5.15/3.16 = 1.63 = 1.6$

g. Since $-0.1 \not> 1.6$, decide that there is no reason to believe $\mu_A > \mu_B$ at a 95% confidence level.

6.3.1.4.5 ANALYSIS

If $\overline{X}_d > \varepsilon$, the null hypothesis that $\mu_A > \mu_B$ is accepted; otherwise, there is no reason to believe $\mu_A > \mu_B$ at a $100(1-\alpha)\%$ confidence level when the observations are paired.

6.3.1.5 DETERMINATION OF SAMPLE SIZE

6.3.1.5.1 OBJECTIVE

To determine the N_t required to determine whether μ_A is equal to or greater than $\mu_B + \varepsilon$ (or equal to or less than $\mu_B - \varepsilon$) at the desired confidence level when:

a. Case I. The variabilities of A and B are unknown but assumed equal.

b. Case II. The variabilities of A and B are unknown but assumed unequal.

c. Case III. The variabilities of A and B are known from previous experience.

d. Case IV. The observations are paired (see subparagraph 6.3 d (4), page 46).

6.3.1.5.2 DATA REQUIRED

a. Case I. An approximation of the value that σ ($\sigma_A = \sigma_B$) will assume.

b. Case II. An approximation of the values that σ_A and σ_B will assume.

c. Case III. The values of σ_A and σ_B which are known from previous experience.

d. Case IV. An approximation of the population standard deviation of the differences ($\sigma_d = |\sigma_A - \sigma_B|$ where σ_A and α_B are approximations) for the pairs concerned.

6.3.1.5.3 PROCEDURE

a. Case I.

(1) Choose α and β, the probabilities of making Type I and Type II errors respectively.
(2) Choose the allowable amount of error.
(3) Compute d^2, an intermediate value, as follows:

(a) Square σ.
(b) Multiply step (a) by 2.
(c) Square ε.
(d) Divide step (c) by step (b).

(4) Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.
(5) Compute N_t ($N_t = N_A = N_B$) as follows:

(a) Add $Z_{1-\alpha}$ to $Z_{1-\beta}$.
(b) Square step (a).
(c) Divide step (b) by step (3).
(d) If $\alpha = .01$, add 2 to step (c) and round up; and if $\alpha = .05$, add 1 to step (c) and round up.

(6) Conclude that N_t samples of each item are required to determine whether μ_A is equal to or greater than $\mu_B + \varepsilon$ (or equal to or less than $\mu_B - \varepsilon$) at the desired confidence level.

b. Case II.

(1) Choose α and β, the probabilities of making Type I and Type II errors respectively.

(2) Choose the allowable amount of error.
(3) Compute d^2, an intermediate value, as follows:

(a) Square σ_A.
(b) Square σ_B.
(c) If $N_A = N_B$, add step (a) to step (b).
(d) If N_A is a multiple of N_B; i.e., $N_A = C(N_B)$, multiply step (b) by C and add the product to step (a).
(e) Square ε.
(f) Divide step (e) by the value from step (c) if $N_A = N_B$ or by the value from step (d) if $N_A = C(N_B)$.

(4) Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.
(5) Compute N_t ($N_t = N_A = N_B$) as follows:

(a) Add $Z_{1-\alpha}$ to $Z_{1-\beta}$.
(b) Square step (a).
(c) Divide step (b) by step (3) and round up.

(6) Conclude that N_t samples of each item are required to determine whether μ_A is equal to or greater than $\mu_B + \varepsilon$ (or equal to or less than $\mu_B - \varepsilon$) at the desired confidence level.

c. Case III.

Same as Case II.

d. Case IV.

Same as paragraph 6.2.3.3, page 43

NOTE: σ in paragraph 6.2.3.3 represents σ_d.

6.3.1.5.4 <u>EXAMPLE</u>

a. Case I.

Given:
$\sigma = 2.6$

Procedure:	Example:
(1) Choose α and β.	(1) $\alpha = .05$ $1-\alpha = .95$ $\beta = .20$ $1-\beta = .80$
(2) Choose ε.	(2) $\varepsilon = 1.05$

(3) Compute:

$d^2 = \varepsilon^2/2\sigma^2$

(3) $d^2 = 1.05^2/2(2.6)^2$

$= 1.1025/2(6.76)$

$= 1.1025/13.52$

$= .08155$

(4) Use Table B-4, page 2-4 to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

(4) $Z_{.95} = 1.645$

$Z_{.80} = .84$

(5) Compute:

(a) For $\alpha = .01$,

$$N_t = \frac{(Z_{1-\alpha} + Z_{1-\beta})^2}{d^2} + 2$$

(b) For $\alpha = .05$,

$$N_t = \frac{(Z_{1-\alpha} + Z_{1-\beta})^2}{d^2} + 1$$

(5) Since $\alpha = .05$,

$$N_t = \frac{(1.645 + .84)^2}{.08155} + 1$$

$$= \frac{(2.485)^2}{.08155} + 1$$

$$= \frac{6.175}{.08155} + 1$$

$= 75.72 + 1$

$= 76.72$

$= 77$

(6) Conclude that N_t samples of each item are required to determine whether $\mu_A \geq \mu_B + \varepsilon$ (or $\mu_A \leq \mu_B - \varepsilon$) at a $100(1-\alpha)\%$ confidence level.

(6) Conclude that 77 samples of each item, for σ assumed and equal to 2.6, must be tested in order to determine whether $\mu_A \geq \mu_B + 1.05$ (or $\mu_A \leq \mu_B - 1.05$) at a 95% confidence level.

b. Case II.

Given

$\sigma_A = 2.2$

$\sigma_B = 3.0$

$N_A = N_B$

Procedure:

(1) Choose α and β.

Example:

(1) $\alpha = .05$

$1-\alpha = .95$

$\beta = .20$

$1-\beta = .80$

(2) Choose ε.

(2) $\varepsilon = .75$

(3) (a) If $N_A = N_B$, compute:

$$d^2 = \frac{\varepsilon^2}{\sigma_A^2 + \sigma_B^2}$$

(3) Since $N_A = N_B$ is assumed,

$$d^2 = \frac{.75^2}{(2.2)^2 + (3.0)^2}$$

$$= \frac{.5625}{4.84 + 9.0}$$

$$= \frac{.5625}{13.84}$$

$$= .0406$$

(b) If $N_A = C(N_B)$, compute:

$$d^2 = \frac{\varepsilon^2}{\sigma_A^2 + C(\sigma_B^2)}$$

(4) Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

(4) $Z_{.95} = 1.645$

$Z_{.80} = .84$

(5) Compute:

$$N_t = \frac{(Z_{1-\alpha} + Z_{1-\beta})^2}{d^2}$$

(5) $$N_t = \frac{(1.645 + .84)^2}{.0406}$$

$$= \frac{(2.485)^2}{.0406}$$

$$= \frac{6.175}{.0406}$$

$$= 152.10$$

$$= 153$$

(6) Conclude that N_t samples of each item are required to determine whether $\mu_A \geq \mu_B + \varepsilon$ (or $\mu_A \leq \mu_B - \varepsilon$) at a $100(1-\alpha)\%$ confidence level.

(6) Conclude that 153 samples of each item, for σ_A assumed and equal to 2.2 and σ_B assumed and equal to 3.0, must be tested in order to determine whether $\mu_A \geq \mu_B + .75$ (or $\mu_A \leq \mu_B - .75$) at a 95% confidence level.

c. Case III.

Same as Case II.

d. Case IV.

Same as paragraph 6.2.3.4, page 44.

NOTE: σ in paragraph 6.2.3.4 represents σ_d.

(7) Compute:

$$\varepsilon = \frac{q_{1-\alpha}\sqrt{s_K^2}}{\sqrt{N}}$$

(8) If the absolute difference between any two sample means is greater than ε, decide that those means differ; otherwise, there is no reason to believe the means differ at a 100(1−α)% confidence level.

(7) $\varepsilon = (3.42)\sqrt{1648.47}/\sqrt{6.70}$
$= (3.42)(40.60)/2.59$
$= 138.85/2.59$
$= 53.64$

(8) (a) 2 and 3

Is $|5011.20-5584.67| > 54$?

573 > 54

Since 573 > 54, decide that the means of Types 2 and 3 differ at a 90% confidence level.

NOTE: Since the difference between the smallest and largest mean produces a difference decision, repeat step (8) for the next largest difference.

(b) 3 and 4

Is $|5584.67-5147.86| > 54$?

436.81 > 54?

437 > 54

Since 437 > 54, decide that the means of Types 3 and 4 differ at a 90% confidence level.

(c) 1 and 3

Is $|5222.29-5584.67| > 54$?

362.38 > 54?

362 > 54

Since 362 > 54, decide that the means of Types 1 and 3 differ at a 90% confidence level.

(d) 1 and 2

Is $|5222.29-5011.20| > 54$?

211.09 > 54?

211 > 54

Since 211 > 54, decide that the means of Types 1 and 2 differ at a 90% confidence level.

(e) 2 and 4

Is $|5011.20-5147.86| > 54?$

$136.66 > 54?$

$137 > 54$

Since 137 > 54, decide that the means of Types 2 and 4 differ at a 90% confidence level.

(f) 1 and 4

Is $|5222.29-5147.86| > 54?$

$74.43 > 54?$

$74 > 54$

Since 74 > 54, decide that the means of Types 1 and 4 differ at a 90% confidence level.

6.3.2.5 ANALYSIS

The population means of several products may be compared by computing the absolute difference between any two sample means and comparing this value to a computed ε. The decision is relative only to the two products compared. Therefore, this test only reveals the relationship between the means of two items at a desired confidence level and does not necessarily reveal a difference between one mean and all of the remaining means.

7. STANDARD DEVIATION

7.1 ESTIMATE OF THE POPULATION STANDARD DEVIATION (s).

7.1.1 BEST SINGLE ESTIMATE of s.

7.1.1.1 OBJECTIVE

To determine the best point estimate of the population standard deviation for a normal distribution.

7.1.1.2 DATA REQUIRED

A list of sample readings.

7.1.1.3 PROCEDURE

a. Compute $\overline{X}$ (see paragraph 6.1.1.3, page 26).

b. Find the deviation of each reading from the mean by subtracting the mean from each reading; i.e., $\Delta = x - \overline{X}$.

c. Square each deviation; i.e., Δ^2.

d. Sum the squared deviations; i.e., $\Sigma\Delta^2$.

e. Compute s as follows:

(1) Divide step d by N-1.
(2) Find the square root of step (1).

7.1.1.4 EXAMPLE

Given:

Sample data at Table A-3a, page 1-9.

Procedure:	Example:
a. Compute $\bar{X}$.	a. $\bar{X}$ = 1094/10 = 109.40 = 109 min. See paragraph 6.1.1.4, page 26.
b. Compute: $\Delta = X - \bar{X}$	b. (1) Δ = 100.00 - 109.40 = -9.40 (2) Δ = 125.00 - 109.40 = 15.60 See Table A-3a, page 1-9, for complete list.
c. Square each Δ.	c. (1) $\Delta^2 = (-9.40)^2$ = 88.36 (2) $\Delta^2 = (15.60)^2$ = 243.36 See Table A-3a, page 1-9, for complete list.
d. Sum the Δ^2.	d. $\Sigma\Delta^2$ = 810.4
e. Compute: $s = \sqrt{\frac{\Sigma\Delta^2}{N-1}}$	e. $s = \sqrt{\frac{810.40}{10-1}} = \sqrt{\frac{810.40}{9}} = \sqrt{90.04}$ = 9.49 = 9 min.

7.1.1.5 ANALYSIS

The standard deviation is a unit measure of dispersion around the mean. In the case of the normal distribution, 68% of the area under the curve is between $\bar{X} + s$ and $\bar{X} - s$ with μ centered at $\bar{X}$ or, in terms of the population, between $\mu + \sigma$ and $\mu - \sigma$ (see Figure 13).

7.1.2 CONFIDENCE INTERVAL ESTIMATES

7.1.2.1 TWO-SIDED INTERVAL

7.1.2.1.1 OBJECTIVE

To determine a two-sided confidence interval which is expected to bracket σ at the desired confidence level.

7.1.2.1.2 DATA REQUIRED

A list of sample readings.

AREA UNDER THE NORMAL CURVE

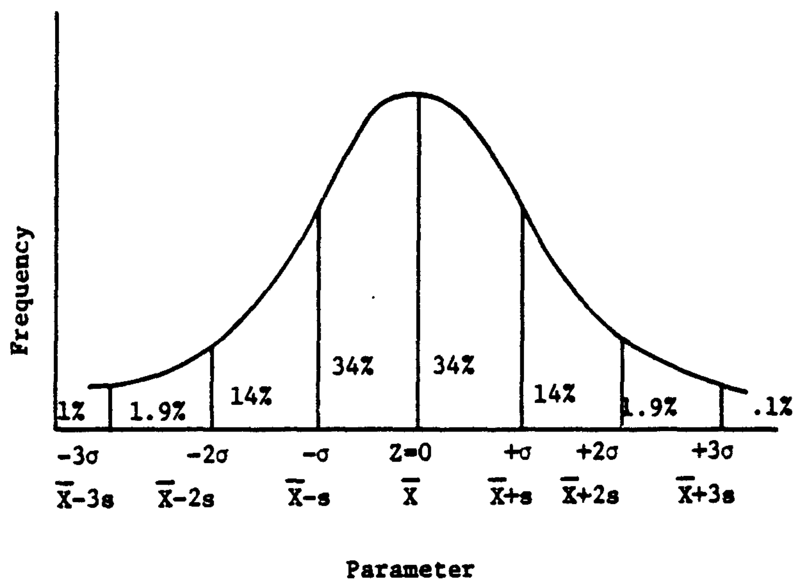


Figure 13

7.1.2.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute s (see paragraph 7.1.1.3, page 64).

c. Use Table B-9, page 2-35, to obtain B_U (upper bound) and B_L (lower bound) for N-1 d.f.

d. Multiply s by B_U to obtain the UCL and multiply s by B_L to obtain the LCL.

e. Conclude that σ is equal to or between the UCL and LCL at the desired confidence level.

7.1.2.1.4 EXAMPLE

Given:

Sample data at Table A-3a, page 1-9.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. α = .05 1-α = .95
b. Compute s.	b. s = 9.49 = 9 min. See paragraph 7.1.1.4, page 65.

c Use Table B-9, page 2-35, to obtain B_U and B_L for α and N-1 d.f.

c. For α = .05 and 9 d.f.,
B_U = 1.746
B_L = .6657

d. Compute:

UCL = (B_U) s

LCL = (B_L) s

d. UCL = (1.746)(9.49)
= 16.57
= 17 min.
LCL = (.6657)(9.49)
= 6.32
= 6 min.

e. Conclude that σ ≤ UCL and σ ≥ LCL at a 100(1-α)% confidence level.

e. Conclude that σ ≤ 17 min. and σ ≥ 6 min. at a 95% confidence level.

7.1.2.1.5 ANALYSIS

The two-sided interval surrounds σ such that σ ≤ UCL and σ ≥ LCL at a 100(1-α)% confidence level.

7.1.2.2 ONE-SIDED INTERVAL

7.1.2.2.1 OBJECTIVE

To determine a one-sided confidence interval such that σ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level.

7.1.2.2.2 DATA REQUIRED

A list of sample readings.

7.1.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute s (see paragraph 7.1.1.3, page 64).

c. Use Table B-10, page 2-37, to obtain $A_{1-\alpha}$ (or A_α) for N-1 d.f.

d. Multiply $A_{1-\alpha}$ by s to obtain the UCL (or multiply A_α by s to obtain the LCL).

e. Conclude that σ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level.

7.1.2.2.4 EXAMPLE

Given:

Sample data at Table A-3a, page 1-9.

Procedure:

a. Choose the confidence level (1-α).

b. Compute s.

Example:

a. α = .05
1-α = .95

b. s = 9.49
= 9 min.
See paragraph 7.1.1.4, page 65.

Procedure	Example
c. Use Table B-10, page 2-37, to obtain $A_{1-\alpha}$ (or A_{α}) for N-1 d.f.	c. For 9 d.f., $A_{.95} = 1.645$ (or $A_{.05} = .7293$)
d. Compute: $UCL = A_{1-\alpha}\, s$ or $LCL = A_{\alpha}\, s$	d. UCL = (1.645)(9.49) = 15.61 = 16 min. (or LCL = (.7293)(9.49) = 6.92 = 6 min.)
e. Conclude that $\sigma \leq$ UCL (or $\sigma \geq$ LCL) at a 100(1-α)% confidence level.	e. Conclude that $\sigma \leq$ 16 min. (or $\sigma \geq$ 6 min.) at a 95% confidence level.

7.1.2.2.5 ANALYSIS

The one-sided interval surrounds σ such that $\sigma \leq$ UCL (or $\sigma \geq$ LCL) at a 100(1-α)% confidence level.

7.1.3 SAMPLE SIZE REQUIRED TO ESTIMATE THE POPULATION STANDARD DEVIATION

7.1.3.1 OBJECTIVE

To determine the N_t required in order to state that σ lies within a specified percentage of its true value at the desired confidence level.

7.1.3.2 DATA REQUIRED

None.

7.1.3.3 PROCEDURE

a. Choose the desired confidence level.

b. Choose the allowable percentage of error.

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

d. Compute N_t as follows:

(1) Square step c.
(2) Square step b.
(3) Multiply step (2) by 2.
(4) Divide step (1) by step (3) and round to the next larger whole number.

e. Conclude that N_t samples are required in order to state that σ lies within an allowable percentage of error ot its true value at the desired confidence level.

7.1.3.4 EXAMPLE

Procedure:	Example:
a. Choose the confidence level (1-α).	a. α = .05 1-α = .95 1-α/2 = .975

b. Choose the percentage of error.

b. Percentage of error = .10

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

c. $Z_{.975} = 1.96$

d. Compute:

$$N_t = \frac{(Z_{1-\alpha/2})^2}{2(\text{percentage of error})^2}$$

d. $$N_t = \frac{(1.96)^2}{2(.10)^2} = \frac{3.84}{2(.01)} = \frac{3.84}{.02} = 192$$

e. Conclude that N_t samples are required in order to state that σ lies within an allowable percentage of its true value at a $100(1-\alpha)\%$ confidence level.

e. Conclude that 192 samples are required in order to state that σ lies within 10% of its true value at a 95% confidence level.

7.1.3.5 ANALYSIS

N_t samples are required in order to state that σ lies within a certain percentage of its true value at a $100(1-\alpha)\%$ confidence level.

7.2 COMPARING AN OBSERVED STANDARD DEVIATION (s) TO A REQUIREMENT (σ_0)

a. An observed standard deviation is generated from a sample and is representative of σ. This value of s is then compared to a stated requirement (σ_0). However, looking at the values of s and σ_0 to decide whether σ is greater than σ_0 or σ is less than σ_0 at a confidence level is insufficient. Since the decision pertains to the population, statistical tests must be applied to s to determine whether σ is greater than σ_0 or σ is less than σ_0.

b. There exist two possibilities for the relationship of s to σ_0. Following are the assumptions and the circumstances for each possible relationship:

(1) s greater than σ_0.

(a) The null hypothesis is σ is greater than σ_0.
(b) The alternative hypothesis is there is no reason to believe σ is greater than σ_0.
(c) The use of this test is appropriate when σ_0 is a **maximum** value for σ to satisfy. In the event that σ must not be greater than σ_0, this test would be appropriate.

(2) s less than σ_0.

(a) The null hypothesis is σ is less than σ_0.
(b) The alternative hypothesis is there is no reason to believe that σ is less than σ_0.

(c) The use of this test is appropriate when σ_0 is a minimum value for σ to satisfy. In the event that σ must meet or exceed σ_0, this test would be appropriate.

7.2.1 s GREATER THAN σ_0.

7.2.1.1 OBJECTIVE

To determine whether σ is greater than σ_0 at the desired confidence level.

7.2.1.2 DATA REQUIRED

A list of sample readings.

7.2.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-10, page 2-37, to obtain A_α for N-1 d.f.

c. Compute s (see paragraph 7.1.1.3, page 64).

d. Multiply step c by step b to obtain the LCL. The confidence interval for σ is from the LCL to $+\infty$.

e. If σ_0 is less than the LCL, decide that σ is greater than σ_0; otherwise, there is no reason to believe σ is greater than σ_0 at the desired confidence level.

7.2.1.4 EXAMPLE

Given:

$\sigma_0 = 7.0$ min.
Sample data at Table A-3a, page 1-9.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. $\alpha = .05$ $1-\alpha = .95$
b. Use Table B-10, page 2-37, to obtain A_α for N-1 d.f.	b. For 9 d.f. $A_{.05} = .7293$
c. Compute s.	c. s = 9.49 = 9 min. See paragraph 7.1.1.4, page 65.
d. Compute: LCL = A_α (s)	d. LCL = (.7293)(9.49) = 6.921 = 6 min.
e. If $\sigma_0 <$ LCL, decide that $\sigma > \sigma_0$, otherwise, there is no reason to believe $\sigma > \sigma_0$ at a 100(1-α)% confidence level.	e. Since $7.0 \nless 6$, decide that there is no reason to believe $\sigma > 7.0$ min. at a 95% confidence level.

7.2.1.5 ANALYSIS

If σ_0 < LCL, the null hypothesis that $\sigma > \sigma_0$ is accepted; otherwise, there is no reason to believe $\sigma > \sigma_0$ at a 100(1-α)% confidence level. The 100(1-α)% confidence interval for σ is from the LCL to + ∞.

7.2.2 s LESS THAN σ_0

7.2.2.1 OBJECTIVE

To determine whether σ is less than σ_0 at the desired confidence level.

7.2.2.2 DATA REQUIRED

A list of sample readings.

7.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-10, page 2-37, to obtain $A_{1-\alpha}$ for N-1 d.f.

c. Compute s (see paragraph 7.1.1.3, page 64).

d. Multiply step c by step b to obtain the UCL. The confidence interval for σ is from - ∞ to the UCL.

e. If σ_0 is greater than the UCL, decide that σ is less than σ_0; otherwise, there is no reason to believe σ is less than σ_0 at the desired confidence level.

7.2.2.4 EXAMPLE

Given:

σ_0 = 12.0 min.
Sample data at Table A-3a, page 1-9.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. α = .05 1-α = .95
b. Use Table B-10, page 2-37, to obtain $A_{1-\alpha}$ for N-1 d.f.	b. For 9 d.f., $A_{.95}$ = 1.645
c. Compute s.	c. s = 9.49 = 9 min. See paragraph 7.1.1.4, page 65.
d. Compute: UCL = $A_{1-\alpha}$(s)	d. UCL = (1.645)(9.49) = 15.611 = 16 min.
e. If σ_0 > UCL, decide that $\sigma < \sigma_0$; otherwise, there is no reason to believe $\sigma < \sigma_0$ at a 100(1-α)% confidence level.	e. Since 12.0 $\not>$ 16, decide that there is no reason to believe that σ < 12.0 min. at a 95% confidence level.

7.2.2.5 ANALYSIS

If σ_0 > UCL, the null hypothesis that $\sigma < \sigma_0$ is accepted; otherwise, there is no reason to believe $\sigma < \sigma_0$ at a 100(1-α)% confidence level.

7.2.3 DETERMINATION OF SAMPLE SIZE

7.2.3.1 OBJECTIVE

To determine the N_t required to determine whether σ is greater than $\gamma\ \sigma_0$ (or less than $\gamma\ \sigma_0$) at the desired confidence level.

7.2.3.2 DATA REQUIRED

None.

7.2.3.3 PROCEDURE

a. Choose α and β, the probabilities of making Type I and Type II errors respectively.

b. Estimate s based on experience or a comparable item.

c. Divide s by σ_0 to obtain γ, an intermediate value.

d. Use Table B-11, page 2-38, to obtain N_t which corresponds to γ and the chosen values of α and β. If one of these values is not contained in the table, continue with step e.

e. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

f. Compute N_t as follows:

(1) Multiply $Z_{1-\beta}$ by step c.
(2) Add step (1) to $Z_{1-\alpha}$.
(3) Divide step (2) by:

(a) γ-1, if s is greater than σ_0.
(b) 1-γ, if s is less than σ_0.

(4) Square step (3).
(5) Multiply step (4) by 1/2.
(6) Add 1 to step (5) and round to the next larger whole number.

g. Conclude that N_t samples are required to determine whether σ is greater than $\gamma\ \sigma_0$ (or is less than $\gamma\ \sigma_0$) at the desired confidence level.

NOTE: When γ > 1, then s is greater than σ_0; and the null hypothesis is that $\sigma > \gamma\ \sigma_0$. When γ < 1, then s is less than σ_0 ; and the null hypothesis is that $\sigma < \gamma\ \sigma_0$.

7.2.3.4 EXAMPLE

Given:

$\sigma_0 = 7.3$

NOTE: When $\gamma > 1$, then s_A is greater than s_B; and the null hypothesis is that $\sigma_A > \gamma\, \sigma_B$. When $\gamma < 1$, then s_A is less than s_B; and the null hypothesis is that $\sigma_A < \gamma\, \sigma_B$.

7.3.2.4 EXAMPLE

Procedure:

a. Choose α and β.

b. Estimate s_A and s_B.

c. Compute:

$$\gamma = s_A / s_B$$

d. Use Table B-12, page 2-41, to obtain N_t which corresponds to γ and the chosen values of α and β. If one of these values is not contained in the table, continue with step e.

e. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

f. Compute:

$$N_t = 2 + \left(\frac{Z_{1-\alpha}+Z_{1-\beta}}{\ln\ (\gamma)}\right)^2$$

g. Conclude that N_t samples are required to determine whether $\sigma_A > \gamma\sigma_B$ (or $\sigma_A < \gamma\, \sigma_B$) at a $100(1-\alpha)\%$ confidence level.

Example:

a. $\alpha = .05$
$1-\alpha = .95$
$\beta = .20$
$1-\beta = .80$

b. $s_A = 6.0$
$s_B = 4.8$

c.
$$\gamma = \frac{6.0}{4.8} = 1.250$$

d. Since $\gamma = 1.250$ is not contained in Table B-12, page 2-41, continue with step e.

e. $Z_{.95} = 1.645$
$Z_{.80} = .840$

f.
$$N_t = 2 + \left(\frac{1.645+.840}{\ln 1.25}\right)^2 = 2 + \left(\frac{2.485}{.2231}\right)^2 = 2 + (11.14)^2 = 2 + 124.1 = 126.1 = 127$$

g. Conclude that 127 samples of each item must be tested in order to determine whether $\sigma_A > 1.25\, \sigma_B$ at a 95% confidence level.

7.3.2.5 ANALYSIS

a. Initial N_t.

At specified significant levels of α and β, N_t samples are required to determine whether $\sigma_A > \gamma\ \sigma_B$ (or $\sigma_A < \gamma\ \sigma_B$). As γ approaches 1, a very large sample size is required.

b. Adequacy of N_t.

(1) s_A greater than s_B.

After the initial N_t samples have been tested, s_A and s_B must be computed. Their ratio (s_A/s_B) must then be computed and compared to the initial ratio determined for the initial N_t. If the computed ratio is greater than the initial ratio, the initial N_t is adequate; however, if the computed ratio is less than the initial ratio, the initial N_t is inadequate. If N_t is inadequate, N_t must be recomputed using the computed ratio in place of the initial ratio; and additional samples must be tested if possible.

(2) s_A less than s_B.

After the initial N_t samples have been tested, s_A and s_B must be computed. Their ratio (s_A/s_B) must then be computed and compared to the initial ratio determined for the initial N_t. If the computed ratio is less than the initial ratio, the initial N_t is adequate; however, if the computed ratio is greater than the initial ratio, the initial N_t is inadequate. If N_t is inadequate, N_t must be recomputed using the computed ratio in place of the initial ratio; and additional samples must be tested if possible.

8. PROPORTION

For some kinds of tests there may be no way to obtain actual measurements. An item may be subjected to a test when the result of that particular test can be expressed only in terms of a pre-established classification of possible results. The simplest kind of classification, and the one most widely used, consists of just two mutually exclusive categories; e.g., success and failure or perfect and defective. The ratio generated, the number of items having the characteristic divided by N, is known as a proportion (P) or a failure-attempt ratio. In all examples P is computed relative to failures (f); however, other variables, such as successes, may be substituted.

8.1 ESTIMATE OF THE POPULATION PROPORTION (P)

8.1.1 BEST SINGLE ESTIMATE of P

8.1.1.1 OBJECTIVE

To determine the best point estimate of the population proportion (λ).

8.1.1.2 DATA REQUIRED

N and the number of elements possessing the given characteristic.

8.1.1.3 PROCEDURE

a. Divide the number of sample items which have the characteristic by the total number of items in the sample.

b. Conclude that P is the best estimate of the proportion of population of items which will have the given characteristics.

8.1.1.4 EXAMPLE

Given:

N = 10

f = 4

Procedure:	Example:
a. Compute: P = characteristic/N	a. P = f/N = 4/10 = .4
b. Conclude that P is the best estimate of the proportion of population items which will have the given characteristic.	b. Conclude that .4 is the best estimate of λ, the fraction of population items that will fail.

8.1.1.5 ANALYSIS

The best single estimate of λ is the observed proportion of items having this characteristic in a random sample from the population; i.e., the number of sample items which have the characteristic divided by the total number of items in the sample.

8.1.2 CONFIDENCE INTERVAL ESTIMATES

8.1.2.1 TWO-SIDED INTERVAL FOR $N \leq 30$

8.1.2.1.1 OBJECTIVE

To determine a two-sided confidence interval which is expected to bracket λ at the desired confidence level when N is equal to or less than 30.

8.1.2.1.2 DATA REQUIRED

N and the number of elements possessing the given characteristic.

8.1.2.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-13, page 2-43, to obtain the UCL and LCL which correspond to N and the number of elements possessing the given characteristic at the desired confidence level.

c. Conclude that λ is equal to or between the UCL and LCL at the desired confidence level.

8.1.2.1.4 EXAMPLE

Given:

$N = 10$ $(N \leq 30)$

$f = 4$

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .05$ $1-\alpha = .95$
b. Use Table B-13, page 2-43, to obtain the UCL and LCL which correspond to N and the number of elements possessing the given characteristic at a $100(1-\alpha)\%$ confidence level.	b. For $N = 10$, $f = 4$, and $1-\alpha = .95$, UCL = .733 LCL = .150
c. Conclude that $\lambda \leq$ UCL and $\lambda \geq$ LCL at a $100(1-\alpha)\%$ confidence level.	c. Conclude that $\lambda \leq .733$ and $\lambda \geq .150$ at a 95% confidence level.

8.1.2.1.5 ANALYSIS

The two-sided interval surrounds λ such that $\lambda \leq$ UCL and $\lambda \geq$ LCL at a $100(1-\alpha)\%$ confidence level.

8.1.2.2 TWO-SIDED INTERVAL FOR $N > 30$

8.1.2.2.1 OBJECTIVE

To determine a two-sided confidence interval which is expected to bracket λ at the desired confidence level when N is greater than 30.

8.1.2.2.2 DATA REQUIRED

N and the number of elements possessing the given characteristic.

8.1.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

c. Compute P (see paragraph 8.1.1.3, page 79).

d. Compute the UCL and LCL as follows:

(1) Multiply P by the quantity (1-P).
(2) Divide step (1) by N.

8.1.2.3.5 ANALYSIS

The one-sided interval surrounds λ such that $\lambda \le$ UCL (or $\lambda \ge$ LCL) at a 100(1-α)% confidence level.

8.1.2.4 ONE-SIDED INTERVAL FOR N > 30

8.1.2.4.1 OBJECTIVE

To determine a one-sided confidence interval such that λ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level when N is greater than 30.

8.1.2.4.2 DATA REQUIRED

N and the number of elements possessing the given characteristic.

8.1.2.4.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

c. Compute P (see paragraph 8.1.1.3, page 79).

d. Compute the UCL (or LCL) as follows:

(1) Multiply P by the quantity (1-P).
(2) Divide step (1) by N.
(3) Find the square root of step (2).
(4) Multiply step b by step (3).
(5) Add step (4) to P to obtain the UCL (or subtract step (4) from P to obtain the LCL).

e. Conclude that λ is equal to or less than the UCL (or equal to or greater than the LCL) at the desired confidence level.

8.1.2.4.4 EXAMPLE

Given:

N = 150 (N > 30)
f = 60

Procedure:	Example:
a. Choose the confidence level (1-α).	a. α = .10 1-α = .90
b. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.	b. $Z_{.90}$ = 1.282
c. Compute:	c. P = f/N
P = characteristic/N	= 60/150 = .40

d. Compute:

$$UCL = P+Z_{1-\alpha}\sqrt{\frac{P(1-P)}{N}}$$

(or $LCL = P-Z_{1-\alpha}\sqrt{\frac{P(1-P)}{N}}$)

e. Conclude that $\lambda \leq$ UCL (or $\lambda \geq$ LCL) at a $100(1-\alpha)\%$ confidence level.

d. $$UCL = .40+1.282\sqrt{\frac{.4(1-.4)}{150}}$$

$$= .40+1.282\sqrt{\frac{.4(.6)}{150}}$$

$$= .40+1.282\sqrt{\frac{.24}{150}}$$

$$= .40+1.282\sqrt{.0016}$$

$$= .40+1.282(.04)$$

$$= .40+.05$$

$$= .45$$

(or $LCL = .40 - .05$
$= .35$)

e. Conclude that $\lambda \leq .45$ (or $\lambda \geq .35$) at a 90% confidence level.

8.1.2.4.5 ANALYSIS

The one-sided interval surrounds λ such that $\lambda \leq$ UCL (or $\lambda \geq$ LCL) at a $100(1-\alpha)\%$ confidence level.

8.1.3 SAMPLE SIZE REQUIRED TO ESTIMATE THE POPULATION PROPORTION

8.1.3.1 SAMPLE SIZE WITH A SPECIFIED LIMIT OF ERROR IN BOTH DIRECTIONS

8.1.3.1.1 OBJECTIVE

To determine the N_t required in order to state that λ is equal to or between $P + \varepsilon$ and $P - \varepsilon$ at the desired confidence level.

8.1.3.1.2 DATA REQUIRED

None.

8.1.3.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Choose the allowable amount of error.

c. Choose a value for P in the following manner:

(1) If no prior information is available and if λ is believed to be in the neighborhood of 0.5, use P = 0.5. The largest sample size will be required when P = 0.5, and the purpose of the rules is to be as conservative as possible.

(2) If λ can safely be assumed to be less than o.5, let P be the largest reasonable guess for λ.
(3) If λ can safely be assumed to be greater than 0.5, let P be the smallest reasonable guess for λ.

d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.

e. Compute N_t as follows:

(1) Square step d.
(2) Multiply P by the quantity (1-P).
(3) Multiply step (1) by step (2).
(4) Square ε.
(5) Divide step (3) by step (4).
(6) Round the result of step (5) up to the next whole number.

f. Conclude that N_t samples are required in order to state that λ is equal to or between P + ε and P - ε at the desired confidence level.

8.1.3.1.4 EXAMPLE

Procedure:	Example:
a. Choose the confidence level (1-α).	a. α = .10 $1-\alpha/2$ = .95
b. Choose ε.	b. ε = .10
c. Choose P.	c. P = 0.5
d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha/2}$.	d. $Z_{.95}$ = 1.645
e. Compute: $N_t = \frac{(Z_{1-\alpha/2})^2(P)(1-P)}{\varepsilon^2}$	e. $N_t = \frac{(1.645)^2(.5)(1-.5)}{(.10)^2} = \frac{(2.706)(.5)(.5)}{.01} = \frac{(2.706)(.25)}{.01} = \frac{.6765}{.01} = 67.65 = 68$
f. Conclude that N_t samples are required in order to state that $\lambda \leq P + \varepsilon$ and $\lambda \geq P - \varepsilon$ at a 100(1-α)% confidence level.	f. If 68 samples are tested and P computed, conclude that $\lambda \leq P + .10$ and $\lambda \geq P - .10$ at a 90% confidence level.

8.1.3.1.5 ANALYSIS

If N_t samples are tested and P is computed, conclude that $\lambda \leq P + \varepsilon$ and $\lambda \geq P - \varepsilon$ at a 100(1-α)% confidence level.

8.1.3.2 SAMPLE SIZE WITH A SPECIFIED LIMIT OF ERROR IN ONLY ONE DIRECTION

8.1.3.2.1 OBJECTIVE

To determine the N_t required in order to state that λ is equal to or less than $P + \varepsilon$ (or equal to or greater than $P - \varepsilon$) at the desired confidence level.

8.1.3.2.2 DATA REQUIRED

None.

8.1.3.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Choose the allowable amount of error.

c. Choose the value of P in the following manner:

(1) If no prior information is available and if λ is believed to be in the neighborhood of 0.5, use P = 0.5. The largest sample size will be required when P = 0.5, and the purpose of the rules is to be as conservative as possible.
(2) If λ can safely be assumed to be less than 0.5, let P be the largest reasonable guess for λ.
(3) If λ can safely be assumed to be greater than 0.5, let P be the smallest reasonable guess for λ.

d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

e. Compute N_t as follows:

(1) Square step d.
(2) Multiply P by the quantity (1-P).
(3) Multiply step (1) by step (2).
(4) Square ε.
(5) Divide step (3) by step (4).
(6) Round the result of step (5) up to the next whole number.

f. Conclude that N_t samples are required in order to state that λ is equal to or less than $P + \varepsilon$ (or equal to or greater than $P - \varepsilon$) at the desired confidence level.

8.1.3.2.4 EXAMPLE

Procedure:	Example:
a. Choose the confidence level 1-α).	a. α = .10 1-α = .90

b. Choose ε.

c. Choose P.

d. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

e. Compute:

$$N_t = \frac{(Z_{1-\alpha})^2 (P)(1-P)}{\varepsilon^2}$$

f. Conclude that N_t samples are required in order to state that $\lambda \le P + \varepsilon$ or $(\lambda \ge P - \varepsilon)$ at a $100(1-\alpha)\%$ confidence level.

b. $\varepsilon = .10$

c. $P = 0.5$

d. $Z_{.90} = 1.282$

e.

$$N_t = \frac{(1.282)^2(0.5)(1-0.5)}{(.10)^2}$$

$$= \frac{(1.644)(0.5)(0.5)}{.01}$$

$$= \frac{(1.644)(.25)}{.01}$$

$$= \frac{.4110}{.01}$$

$$= 41.10$$

$$= 42$$

f. If 42 samples are tested and P computed, conclude that $\lambda \le P + .10$ at a 90% confidence level.

8.1.3.2.5 ANALYSIS

If N_t samples are tested and P is computed, $\lambda \le P + \varepsilon$ (or $\lambda \ge P - \varepsilon$) at a $100(1-\alpha)\%$ confidence level.

8.2 COMPARING AN OBSERVED PROPORTION (P) TO A REQUIREMENT (λ_0)

a. An observed proportion is generated from a sample and is representative of λ. This value of P is then compared to a stated requirement (λ_0). However, looking at the values of P and λ_0 to decide whether λ is greater than λ_0 or λ is less than λ_0 at a confidence level is insufficient. Since the decision pertains to the population, statistical tests must be applied to P to determine whether λ is greater than λ_0 or λ is less than λ_0.

b. There exist two possibilities for the relationship of P to λ_0. Following are the assumptions and the circumstances for each possible relationship:

(1) P greater than λ_0.

(a) The null hypothesis is λ is greater than λ_0.

(b) The alternative hypothesis is there is no reason to believe λ is greater than λ_0.

(c) The use of this test is appropriate when λ_0 is a maximum value for λ to satisfy. In the event that λ must not be greater than λ_0, this test would be appropriate.

(2) P is less than λ_o.

(a) The null hypothesis is λ is less than λ_o.
(b) The alternative hypothesis is there is no reason to believe λ is less than λ_o.
(c) The use of this test is appropriate when λ_o is a minimum value for λ to satisfy. In the event that λ must meet or exceed λ_o, this test would be appropriate.

8.2.1 P GREATER THAN λ_o

8.2.1.1 SMALL SAMPLE SIZE

8.2.1.1.1 OBJECTIVE

To determine whether λ is greater than λ_o at the desired confidence level when N is equal to or less than 30.

8.2.1.1.2 DATA REQUIRED

Success-failure data.

8.2.1.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-14, page 2-50, to obtain the LCL which corresponds to N and the number of elements possessing the given characteristic at the desired confidence level.

c. If λ_o is less than the LCL, decide that λ is greater than λ_o; otherwise, there is no reason to believe λ is greater than λ_o at the desired confidence level.

8.2.1.1.4 EXAMPLE

Given:

$N = 20$ $(N \leq 30)$
$f = 3$
$\lambda_o = .100$

Procedure:	Example:
a. Choose the confidence level (1-α).	a. $\alpha = .05$ $1-\alpha = .95$
b. Use Table B-14, page 2-50, to obtain the LCL which corresponds to N and the number of elements possessing the given characteristic at a 100(1-α)% confidence level.	b. For $1-\alpha = .95$, $N = 20$, and $N-f = 17$, the tabled value is .958. This must be subtracted from 1; hence, $LCL = 1 - .958$ $= .042$

c. If λ_0 < LCL, decide that $\lambda > \lambda_0$; otherwise, there is no reason to believe $\lambda > \lambda_0$ at a 100(1-α)% confidence level.

c. Since .100 ≮ .042, decide that there is no reason to believe λ > .100 at a 95% confidence level.

8.2.1.1.5 ANALYSIS

If λ_0 < LCL, the null hypothesis that $\lambda > \lambda_0$ is accepted; otherwise, there is no reason to believe $\lambda > \lambda_0$ at a 100(1-α)% confidence level.

8.2.1.2 LARGE SAMPLE SIZE

8.2.1.2.1 OBJECTIVE

To determine whether λ is greater than λ_0 at the desired confidence level when N is greater than 30.

8.2.1.2.2 DATA REQUIRED

Success-failure data.

8.2.1.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute Z as follows:

(1) Multiply λ_0 by N.
(2) Subtract step (1) from the number of items having the given characteristic.
(3) Add .5 to step (2).
(4) Multiply the quantity $(1-\lambda_0)$ by step (1).
(5) Divide step (3) by the square root of step (4).

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$, which is the UCL.

d. If Z is greater than the UCL, decide that λ is greater than λ_0; otherwise, there is no reason to believe λ is greater than λ_0 at the desired confidence level.

8.2.1.2.4 EXAMPLE

Given:

N = 100 (N > 30)
f = 7
λ_0 = .06

Procedure:

a. Choose the confidence level (1-α).

Example:

a. α = .10
1-α = .90

b. Compute:

$$Z = \frac{f - N\lambda_o + .5}{\sqrt{N\lambda_o(1-\lambda_o)}}$$

b.

$$Z = \frac{7-100(.06)+.5}{\sqrt{100(.06)(1-.06)}}$$

$$= \frac{7-6+.50}{\sqrt{6\ (.94)}}$$

$$= \frac{1.50}{\sqrt{5.64}}$$

$$= \frac{1.50}{2.375}$$

$$= .633$$

c. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$.

$UCL = Z_{1-\alpha}$

c. $Z_{.90} = 1.282$

UCL = 1.282

d. If Z > UCL, decide that $\lambda > \lambda_o$; otherwise, there is no reason to believe $\lambda > \lambda_o$ at a $100(1-\alpha)\%$ confidence level.

d. Since $.633 \not> 1.282$, decide that there is no reason to believe $\lambda > .06$ at a 90% confidence level.

8.2.1.2.5 ANALYSIS

If Z > UCL, the null hypothesis that $\lambda > \lambda_o$ is accepted; otherwise, there is no reason to believe $\lambda > \lambda_o$ at a $100(1-\alpha)\%$ confidence level.

8.2.2 P LESS THAN λ_o

8.2.2.1 SMALL SAMPLE SIZE

8.2.2.1.1 OBJECTIVE

To determine whether λ is less than λ_o at the desired confidence level when N is equal to or less than 30.

8.2.2.1.2 DATA REQUIRED

Success-failure data.

8.2.2.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-14, page 2-50, to obtain the UCL which corresponds to N and the number of elements possessing the given characteristic at the desired confidence level.

c. If λ_o is greater than the UCL, decide that λ is less than λ_o; otherwise, there is no reason to believe λ is less than λ_o at the desired confidence level.

8.2.2.1.4 EXAMPLE

Given:

$N = 20\ (N \leq 30)$
$f = 3$
$\lambda_o = .200$

Procedure:	Example:
a. Choose the confidence level $(1-\alpha)$.	a. $\alpha = .05$ $1-\alpha = .95$
b. Use Table B-14, page 2-50, to obtain the UCL which corresponds to N and the number of elements possessing the given characteristics at a $100(1-\alpha)\%$ confidence level.	b. For $1-\alpha = .95$, $N = 20$, and $f = 3$, UCL = .344
c. If $\lambda_0 >$ UCL, decide that $\lambda < \lambda_0$; otherwise, there is no reason to believe $\lambda < \lambda_0$ at a $100(1-\alpha)\%$ confidence level.	c. Since $.200 \not> .344$, decide that there is no reason to believe $\lambda < .200$ at the 95% confidence level.

8.2.2.1.5 ANALYSIS

If $\lambda_0 >$ UCL, the null hypothesis that $\lambda < \lambda_0$ is accepted; otherwise, there is no reason to believe $\lambda < \lambda_0$ at a $100(1-\alpha)\%$ confidence level.

8.2.2.2 LARGE SAMPLE SIZE

8.2.2.2.1 OBJECTIVE

To determine whether λ is less than λ_0 at the desired confidence level when N is greater than 30.

8.2.2.2.2 DATA REQUIRED

Success-failure data.

8.2.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute Z as follows:

(1) Multiply λ_0 by N.
(2) Subtract step (1) from the number of items having the given characteristic.
(3) Subtract .5 from step (2).
(4) Multiply the quantity $(1-\lambda_0)$ by step (1).
(5) Divide step (3) by the square root of step (4).

c. Use Table B-4, page 2-4, to obtain Z_α, which is the LCL.

d. If Z is less than the LCL, decide that λ is less than λ_0; otherwise, there is no reason to believe λ is less than λ_0 at the desired confidence level.

8.2.2.2.4 EXAMPLE

Given:

$N = 100$ $(N > 30)$
$f = 7$
$\lambda_0 = .08$

Procedure:

a. Choose the confidence level $(1-\alpha)$.

b. Compute:

$$Z = \frac{f - N\lambda_0 - .5}{\sqrt{N\lambda_0(1-\lambda_0)}}$$

c. Use Table B-4, page 2-4, to obtain Z_α.

$LCL = Z_\alpha$

d. If $Z < LCL$, decide that $\lambda < \lambda_0$; otherwise, there is no reason to believe $\lambda < \lambda_0$ at a $100(1-\alpha)\%$ confidence level.

Example:

a. $\alpha = .10$
$1-\alpha = .90$

b.

$$Z = \frac{7 - 100(.08) - .5}{\sqrt{100(.08)(1-.08)}} = \frac{7-8-.5}{\sqrt{8(.92)}} = \frac{-1.5}{\sqrt{7.36}} = \frac{-1.5}{2.71} = -.554$$

c. $Z_{.10} = -1.282$

$LCL = -1.282$

d. Since $-.554 \not< -1.282$, decide that there is no reason to believe $\lambda < .08$ at a 90% confidence level.

8.2.2.2.5 ANALYSIS

If $Z < LCL$, the null hypothesis that $\lambda < \lambda_0$ is accepted; otherwise, there is no reason to believe $\lambda < \lambda_0$ at a $100(1-\alpha)\%$ confidence level.

8.2.3 DETERMINATION OF SAMPLE SIZE

8.2.3.1 OBJECTIVE

To determine the N_t required to determine whether λ is equal to or greater than $\lambda_0 + \epsilon$ (or equal to or less than $\lambda_0 - \epsilon$) at the desired confidence level.

8.2.3.2 DATA REQUIRED

λ_0 which is known from a standard item, history, or Requirements Document.

8.2.3.3 PROCEDURE

a. Choose α and β, the probabilities of making Type I and Type II errors respectively.

b. Choose the allowable amount of error.

c. Estimate the test item proportion by adding ϵ to λ_0 (or subtracting ϵ from λ_0).

d. Use Table B-15, page 2-54, to obtain θ_1, which corresponds to λ_o, and θ_o, which corresponds to λ.

e. Compute d^2, an intermediate value, as follows:

(1) Subtract θ_o from θ_1.
(2) Square Step (1).

f. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

g. Compute N_t as follows:

(1) Add $Z_{1-\alpha}$ to $Z_{1-\beta}$.
(2) Square step (1).
(3) Divide step (2) by step e.
(4) Round step (3) to the next larger whole number.

h. Conclude that N_t samples are required to determine whether λ is equal to or greater than $\lambda_o + \varepsilon$ (or equal to or less than $\lambda_o - \varepsilon$) at the desired confidence level.

8.2.3.4 EXAMPLE

Given:

$\lambda_o = .41$

Procedure:	Example:
a. Choose α and β.	a. $\alpha = .05$ $1-\alpha = .95$ $\beta = .20$ $1-\beta = .80$
b. Choose ε.	$\varepsilon = .23$
c. Estimate λ as follows: $\lambda = \lambda_o + \varepsilon$	c. $\lambda = .41 + .23$ $= .64$
d. Use Table B-15, page 2-54, to obtain θ_1, which corresponds to λ_o, and θ_o, which corresponds to λ.	d. For $\lambda_o = .41$, $\theta_1 = 1.39$ For $\lambda = .64$, $\theta_o = 1.85$
e. Compute: $d^2 = (\theta_o - \theta_1)^2$	e. $d^2 = (1.85-1.39)^2$ $= (.46)^2$ $= .2116$
f. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.	f. $Z_{.95} = 1.645$ $Z_{.80} = .840$

g. Compute:

$$N_t = \frac{(Z_{1-\alpha}+Z_{1-\beta})^2}{d^2}$$

g.

$$N_t = \frac{(1.645+.840)^2}{.2116}$$

$$= \frac{(2.485)^2}{.2116}$$

$$= \frac{6.1752}{.2116}$$

$$= 29.18$$

$$= 30$$

h. Conclude that N_t samples are required to determine whether $\lambda \geq \lambda_o + \varepsilon$ (or $\lambda \leq \lambda_o - \varepsilon$) at a 100(1-α)% confidence level.

h. Conclude that 30 samples for λ_o known and equal to .41 must be tested in order to determine whether $\lambda \geq \lambda_o + .23$ at a 95% confidence level.

8.2.3.5 ANALYSIS

N_t samples are required to determine whether $\lambda \geq \lambda_o + \varepsilon$ (or $\lambda \leq \lambda_o - \varepsilon$) at a 100(1-α)% confidence level.

8.3 COMPARING TWO OBSERVED PROPORTIONS

a. An observed proportion is generated from a sample and is representative of λ. This value of P is then required to meet a standard item P which is representative of the standard item's population. Looking at the values of the proportions (P_A and P_B) to decide whether λ_A is greater than λ_B or λ_A is less than λ_B at a confidence level is insufficient. Since the decision pertains to the populations, statistical tests must be applied to determine whether λ_A is greater than λ_B or λ_A is less than λ_B. The statistical tests use the sample proportions as estimates of the population proportions.

b. Type A generally represents the test item and Type B the standard item when testing the hypothesis that λ_A is greater than λ_B. However, to prove that the average performance of the test item is less than that of the standard item, Type A must represent the standard item so that the hypothesis, λ_A is greater than λ_B, can be tested.

c. When the null hypothesis is λ_A is greater than λ_B, the alternative hypothesis is there is no reason to believe that λ_A is greater than λ_B.

d. The use of this test is appropriate when λ_B is a maximum value for λ_A to satisfy.

8.3.1 P_A GREATER THAN P_B

8.3.1.1 SMALL SAMPLE SIZE

8.3.1.1.1 OBJECTIVE

To determine whether λ_A is greater than λ_B at the desired confidence level when neither N_A nor N_B is greater than 20.

8.3.1.1.2 DATA REQUIRED

Success-failure data.

8.3.1.1.3 PROCEDURE

a. Choose the desired confidence level.

b. Arrange the data as in Table A-4a, Part I, page 1-11.

c. Focus on the class of interest and compute the following intermediate values:

(1) h_A, the ratio of the class of interest to the sample size for Type A; i.e., $h_A = I_A/N_A$ or $h_A = II_A/N_A$.
(2) h_B, the ratio of the class of interest to the sample size for Type B; i.e., $h_B = I_B/N_B$ or $h_B = II_B/N_B$.

d. If h_A is greater than h_B, continue with step e; however, if h_A is not greater than h_B, decide that the data give no reason to believe that λ_A is greater than λ_B at the desired confidence level.

e. Arrange the data so that the results of the larger sample are in the first row (see Table A-4a, Part II, page 1-11.

f. Compute the following intermediate values:

(1) h_1, the ratio of class I to the sample size for the item having the larger sample size; i.e., $h_1 = I_1/N_1$.
(2) h_2, the ratio of Class I to the sample size for the item having the smaller sample size; i.e., $h_2 = I_2/N_2$.
(3) g_1, the ratio of class II to the sample size for the item having the larger sample size, i.e., $g_1 = II_1/N_1$.
(4) g_2, the ratio of class II to the sample size for the item having the smaller sample size; i.e., $g_2 = II_2/N_2$.

g. Focus attention on that class (I or II) which produces a proportion for the larger sample which is larger than or equal to the respective proportion for the smaller sample. Depending on the class chosen, let I_1 (or II_1) equal a_1, an intermdeiate value, and I_2 (or II_2) equal a_2, an intermediate value.

h. Use Table B-16, page 2-55, to obtain a tabled a_2 which corresponds to the two sample sizes and a_1 at the desired confidence level.

i. If a_2 from step g is less than or equal to the table a_2, decide that λ_A is greater than λ_B with regard to the class of interest; otherwise, there is no reason to believe λ_A is greater than λ_B at the desired confidence level.

8.3.1.1.4 EXAMPLE

Given:
Sample data at Table A-4a, Part I, page 1-11.

Procedure:

a. Choose the confidence level ($1-\alpha$).

b. Arrange the data.

c. Focus on the class of interest and compute one of the following:

(1) Class I.

$h_A = I_A/N_A$

$h_B = I_B/N_B$

(2) Class II

$h_A = II_A/N_A$

$h_B = II_B/N_B$

d. If $h_A > h_B$, continue with step e. If $h_A \leq h_B$, decide that the data give no reason to believe that λ_A is greater than λ_B with respect to the class of interest at a $100(1-\alpha)\%$ confidence level.

e. Arrange the data so that the results of the larger sample are in the first row.

f. Compute:

$h_1 = I_1/N_1$

$h_2 = I_2/N_2$

$g_1 = II_1/N_1$

$g_2 = II_2/N_2$

Example:

a. $\alpha = .05$
$1-\alpha = .95$

b. See Table A-4a, Part I, page 1-11.

c. Focus on class II.

$h_A = 2/6$
$= .333$
$= .3$

$h_B = 2/10$
$= .200$
$= .2$

d. Since $.3 > .2$, continue with step e.

e. See Table A-4a, Part II, page 1-11.

f. $h_1 = 8/10$
$= .800$
$= .8$

$h_2 = 4/6$
$= .667$
$= .7$

$g_1 = 2/10$
$= .200$
$= .2$

$g_2 = 2/6$
$= .333$
$= .3$

g. (1) If $h_1 \geq h_2$, focus attention on class I with

$a_1 = I_1$

$a_2 = I_2$

(2) If $g_1 \geq g_2$, focus attention on class II with

$a_1 = II_1$

$a_2 = II_2$

g. Since .8 > .7, focus attention on class I.

$a_1 = 8$

$a_2 = 4$

h. Use Table B-16, page 2-55, to obtain a tabled a_2 which corresponds to N_1, N_2, and a_1 at a $100(1-\alpha)\%$ confidence level.

NOTE: Since this is a one-sided test, use the α which is not in parentheses.

h. For $N_1 = 10$, $N_2 = 6$,

$a_1 = 8$, and $\alpha = .05$, the tabled $a_2 = 1$.

i. If $a_2 \leq$ the table value of a_2 from step h, decide that $\lambda_A > \lambda_B$ with respect to the original class of interest; otherwise, there is no reason to believe $\lambda_A > \lambda_B$ with respect to the original class of interest at a $100(1-\alpha)\%$ confidence level.

i. Since 4 > 1, decide that there is no reason to believe $\lambda_A > \lambda_B$ with respect to the number of failures at a 95% confidence level.

8.3.1.1.5 ANALYSIS

If $a_2 \leq$ table value of a_2, the null hypothesis that $\lambda_A > \lambda_B$ is accepted; otherwise, there is no reason to believe $\lambda_A > \lambda_B$ at a $100(1-\alpha)\%$ confidence level. In the event that the confidence level desired is not within the scope of Table B-16, page 2-55, the test for the large sample size must be applied. The results will not be as accurate but will still be useful. In the event that a_1 or a_2 or both are missing for the given sample sizes and confidence level in Table B-16, page 2-55, conclude that the sample sizes are considered insufficient for accepting or rejecting the null hypothesis.

8.3.1.2 LARGE SAMPLE SIZE

8.3.1.2.1 OBJECTIVE

To determine whether λ_A is greater than λ_B at the desired confidence level when either N_A or N_B is greater than 20.

8.3.1.2.2 DATA REQUIRED

Success-failure data.

8.3.1.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Use Table B-7, page 2-12, to obtain $\chi^2_{2\alpha}$ for 1 d.f.

c. Add N_A to N_B to obtain T_N, an intermediate value.

d. Compute AB, an intermediate value, as follows:

(1) Multiply I_A by II_B.
(2) Multiply I_B by II_A.
(3) Subtract step (2) from step (1) and take the absolute value of the difference (disregard the sign).

e. Compute J, an intermediate value, as follows:

(1) Add I_A to I_B to obtain T_I, an intermediate value.
(2) Add II_A to II_B to obtain T_{II} , an intermediate value.
(3) Multiply N_A, N_B, T_I, and T_{II} together.

f. Compute χ^2 as follows:

(1) Divide step c by 2.
(2) Subtract step (1) from step d.
(3) Square step (2).
(4) Multiply step (3) by step c.
(5) Divide step (4) by step e.

g. Focus on the class of interest and compute the following intermediate values:

(1) h_A, the ratio of the class of interest to the sample size for Type A; i.e., $h_A = I_A/N_A$ or $h_A = II_A/N_A$.
(2) h_B, the ratio of the class of interest to the sample size for Type B; i.e., $h_B = I_B/N_B$ or $h_B = II_B/N_B$.

h. If χ^2 is greater than or equal to $\chi^2_{2\alpha}$ for 1 d.f. and h_A is larger than h_B , decide that λ_A is greater than λ_B with regard to the class of interest; otherwise, there is no reason to believe λ_A is greater than λ_B at the desired confidence level.

8.3.1.2.4 EXAMPLE

Given:

Sample data at Table A-4b, page 1-11.

Procedure:	Example:
a. Choose the confidence level (1-α).	a. $\alpha = .10$ $1-\alpha = .90$
b. Use Table B-7, page 2-12, to obtain $\chi^2_{2\alpha}$ for 1 d.f.	b. $\chi^2_{.20}$ for 1 d.f. = 1.64
c. Compute: $T_N = N_A+N_B$	c. $T_N = 216 + 216$ $= 432$

d. Compute:

$$AB = |I_A II_B - I_B II_A|$$

d.

$$AB = |(181)(56)-(160)(35)|$$
$$= |10,136 - 5,600|$$
$$= 4,536$$

e. Compute:
$$J = (N_A)\ (T_I)\ (T_{II})\ (N_B)$$

e. $J = (216)(341)(91)(216)$
$= (73,656)(91)(216)$
$= 1,447,782,336$

f. Compute:

$$\chi^2 = \frac{T_N(AB - T_N/2)^2}{J}$$

f.
$$\chi^2 = \frac{432(4536-216)^2}{1,447,782,336}$$
$$= \frac{432(4,320)^2}{1,447,782,336}$$
$$= \frac{432(18,662,400)}{1,447,782,336}$$
$$= \frac{8,062,156,800}{1,447,782,336}$$
$$= 5.5686$$
$$= 5.57$$

NOTE: The formula for χ^2 has been broken down for simplicity and the complete formula is

$$\chi^2 = \frac{(N_A+N_B)\left(\left|I_A II_B - I_B II_A\right| - \frac{N_A+N_B}{2}\right)^2}{(N_A)(I_A+I_B)(II_A+II_B)(N_B)}$$

g. Focus on the class of interest and compute one of the following:

(1) Class I

$h_A = I_A/N_A$

$h_B = I_B/N_B$

(2) Class II

$h_A = II_A/N_A$

$h_B = II_B/N_B$

g. Focus on class I.

$h_A = 181/216$
$= .83796$
$= .838$

$h_B = 160/216$
$= .74074$
$= .741$

h. If $\chi^2 \geq \chi^2_{2\alpha}$ for 1 d.f. and $h_A > h_B$, decide that $\lambda_A > \lambda_B$ with regard to the class of interest; otherwise, there is no reason to believe $\lambda_A > \lambda_B$ at a 100(1-α)% confidence level.

h. Since 5.57 ≥ 1.64 and .838 > .741, decide that the proportion of hits fo- $\lambda_A > \lambda_B$ at a 90% confidence level.

8.3.1.2.5 ANALYSIS

If $\chi^2 \geq \chi^2_{2\alpha}$ for 1 d.f. and $h_A > h_B$, the null hypothesis that $\lambda_A > \lambda_B$ is accepted; otherwise, there is no reason to believe $\lambda_A > \lambda_B$ at a 100(1-α)% confidence level. The sample size for P_A or P_B must exceed 20. If the confidence level desired is unavailable for P_A and P_B less than 20, the chi-square test will be used to test $\lambda_A > \lambda_B$.

8.3.2 DETERMINATION OF SAMPLE SIZE

8.3.2.1 OBJECTIVE

To determine the N_t (N_t=N_A=N_B) required to determine whether λA. is equal to or greater than $\lambda_B + \varepsilon$ (or equal to or less than $\lambda_B - \varepsilon$) at the desired confidence level.

8.3.2.2 DATA REQUIRED

None.

8.3.2.3 PROCEDURE

a. Choose α and β, the probabilities of making Type I and Type II errors respectively.

b. Choose the allowable amount of error.

c. Estimate one of the proportions, either P_A or P_B. Make this estimate as close to 0.5 as is reasonable.

d. Compute the other proportion as follows:

(1) If P_A is estimated, subtract step b from P_A to obtain P_B.
(2) If P_B is estimated, add step b to P_B to obtain P_A.

e. Use Table B-15, page 2-54, to obtain θ_A, which corresponds to P_A, and θ_B, which corresponds to P_B.

f. Compute d^2, an intermediate value, as follows:

(1) Subtract θ_B from θ_A.
(2) Square step (1).

g. Use Table B-4, page 2-4, to obtain $Z_{1-\alpha}$ and $Z_{1-\beta}$.

h. Compute n, an intermediate value, as follows:

(1) Add $Z_{1-\alpha}$ to $Z_{1-\beta}$.
(2) Square step (1).
(3) Divide step (2) by step f.
(4) Round step (3) up to the next whole number.

i. Multiply step h by 2 to obtain N_t.

j. Conclude that N_t samples are required to determine whether λ_A is equal to or greater than $\lambda_B + \varepsilon$ (or equal to or less than $\lambda_B - \varepsilon$) at the desired confidence level.

(c) Compute the mean of the heights ($\overline{HEIGHT}$).
(d) Compute the mean time.

(3) List the MPI as the mean easting and mean northing ($\overline{EAST}$, $\overline{NORTH}$) and the $\overline{POB}$ as the mean easting, mean northing, and mean height ($\overline{EAST}$, $\overline{NORTH}$, $\overline{HEIGHT}$).
(4) Compute the miss distance (m) for the MPI as follows:

(a) Subtract the $\overline{EAST}$ from the AP easting ($EAST_{AP}$).
(b) Square step (a).
(c) Subtract the $\overline{NORTH}$ from the AP northing ($NORTH_{AP}$).
(d) Square step (c).
(e) Add step (b) to step (d) and find the square root.

(5) Compute m and the miss time for the $\overline{POB}$ as follows:

(a) Subtract the $\overline{EAST}$ from the $EAST_{AP}$.
(b) Square step (a).
(c) Subtract the $\overline{NORTH}$ from the $NORTH_{AP}$.
(d) Square step (c).
(e) Subtract the $\overline{HEIGHT}$ from the AP height ($HEIGHT_{AP}$).
(f) Square step (e).
(g) Add step (b) to step (d).
(h) Add step (g) to step (f) and find the square root to obtain the m.
(i) Subtract the set time from the mean time to obtain the miss time.

b. Case II: Missile systems (limited sample).

(1) Plot each point of impact or point of burst relative to its AP and determine the distance over or short and the distance right or left.
(2) Compute the mean AP using all of the AP coordinates in a given range band.
(3) Plot the points of impact or points of burst relative to the mean AP, using the distances from step (1).
(4) Compute the MPI or $\overline{POB}$ and mean time for the points relative to the mean AP.
(5) Compute m for MPI the same as for a cannon.
(6) Compute m and the miss time for the $\overline{POB}$ the same as for a cannon.

9.1.4 EXAMPLE

a. Case I: Cannon.

Given:
AP: (2784,3501)

Sample data at Table A-1a, page 1-1.

Procedure:	Example:
(1) Compute the following for the MPI:	(1) (a) $\overline{EAST}$ = 2565.67 = 2566
(a) $\overline{EAST}$	

Procedure	Example
(b) $\overline{NORTH}$	(b) $\overline{NORTH}$ = 3256.47 = 3256
(2) Compute the following for the $\overline{POB}$:	(2)
(a) $\overline{EAST}$	
(b) $\overline{NORTH}$	
(c) $\overline{HEIGHT}$	
(d) Mean time	
(3) List the following:	(3) MPI: (2566,3256)
(a) MPI: ($\overline{EAST}$, $\overline{NORTH}$)	
(b) $\overline{POB}$: ($\overline{EAST}$, $\overline{NORTH}$, $\overline{HEIGHT}$)	
(4) For the MPI, compute:	(4)
$m = \sqrt{(EAST_{AP}-\overline{EAST})^2+(NORTH_{AP}-\overline{NORTH})^2}$	$m = \sqrt{(2784-2565.67)^2+(3501-3256.47)^2}$ $= \sqrt{(218.33)^2+(244.53)^2}$ $= \sqrt{47668+59795}$ $= \sqrt{107,463}$ $= 327.82$ $= 328$
(5) For the $\overline{POB}$, compute:	(5)

(a) $m = \sqrt{(EAST_{AP}-\overline{EAST})^2+(NORTH_{AP}-\overline{NORTH})^2+(HEIGHT_{AP}-\overline{HEIGHT})^2}$

(b) miss time = mean time - set time.

b. Case II. Missile systems (limited sample).

Given:

Sample data at Table A-5a, page 1-12.

Procedure:	Example:
(1) Plot each point relative to its AP.	(1) (a) (2350,3100) (b) (1649,2031) See Table A-5a, page 1-12 for complete list.
(2) Compute the mean AP.	(2) $\overline{EAST}_{AP}$ = 21548/10 = 2155 $\overline{NORTH}_{AP}$ = 22091/10 = 2209
(3) Plot each point relative to the mean AP	(3) (a) (2005,2304) (b) (2267, 2415) See Table A-5a, page 1-12, for complete list.

(4) Compute:

(a) MPI.

(b) $\overline{POB}$ and mean time.

(4) MPI: (2148,2274)

$\overline{EAST} = 21482/10$

$= 2148.20$

$= 2148$

$\overline{NORTH} = 22743/10$

$= 2274.30$

$= 2274$

(5) Compute for the MPI:

$$m = \sqrt{(\overline{EAST}_{AP}-\overline{EAST})^2+(\overline{NORTH}_{AP}-\overline{NORTH})^2}$$

(5)

$$m = \sqrt{(2154.80-2148.20)^2+(2209.10-2274.30)^2}$$

$$= \sqrt{(6.60)^2+(65.20)^2}$$

$$= \sqrt{43.56+4251.04}$$

$$= 65.53$$

$$= 66$$

(6) For the $\overline{POB}$, compute:

(a) $m = \sqrt{(\overline{EAST}_{AP}-\overline{EAST})^2+(\overline{NORTH}_{AP}-\overline{NORTH})^2+(\overline{HEIGHT}_{AP}-\overline{HEIGHT})^2}$

(b) miss time = mean time - set time

(6)

9.1.5 ANALYSIS

a. The miss distance is the distance that the MPI or the $\overline{POB}$ missed the AP and describes the accuracy of the test item. The smaller the miss distance, the better the accuracy of the test item. The miss distance must be compared to the stated requirement to determine whether the requirement was met.

b. Due to sampling techniques used for missiles, an average AP must be determined within a range band. The miss distance is the distance that the MPI or $\overline{POB}$ (relative to the average AP) missed the average AP. The miss distance must be compared to the stated requirement to determine whether the requirement was met. Unless the sample size is at least six, conclusions for accuracy cannot be drawn with any reasonable level of confidence.

9.2 PRECISION

9.2.1 PROBABLE ERROR COMPUTATION

9.2.1.1 STANDARD DEVIATION METHOD

9.2.1.1.1 OBJECTIVE

To obtain the system PE and each subsystem PE using the standard deviation method.

9.2.1.1.2 DATA REQUIRED

A list of sample readings.

9.2.1.1.3 PROCEDURE

a. Compute s, (see paragraph 7.1.1.3, page 64).

b. Multiply step a by .6745 to obtain PE.

9.2.1.1.4 EXAMPLE

Given:

Sample data at Table A-5b, page 1-13.

Procedure:

a. Compute:

$$s = \sqrt{\frac{\Sigma\Delta^2}{N-1}}$$

b. PE = 0.6745(s).

Example:

a.

$$s = \sqrt{\frac{38,650.00}{16-1}}$$

$$= \sqrt{\frac{38,650.00}{15}}$$

$$= \sqrt{2,576.67}$$

$$= 50.76$$

$$= 51$$

See paragraph 7.1.1.4, page 65, for computations.

b. PE = 0.6745(50.76)

$= 34.24$

$= 34$

9.2.1.1.5 ANALYSIS

The PE is a measure of deviation from μ such that 50% of the observations may be expected to lie between μ - PE and μ + PE. This method is the best estimate of the population PE(τ) unless a trend exists which can be attributed to a non-system condition, such as weather, in which case use of the successive differences method is the best approach. A test comparing the two methods of computing PE can be made to determine whether a trend did exist but was not evident (see paragraph 9.2.1.3, page 108, for details).

9.2.1.2 SUCCESSIVE DIFFERENCES METHOD

9.2.1.2.1 OBJECTIVE

To determine the system PE and each subsystem PE using the successive differences method when there is a suspected trend.

9.2.1.2.2 DATA REQUIRED

A list of sample readings.

9.2.1.2.3 PROCEDURE

a. Compute the differences ($x_d = x_i - x_{i+1}$) between consecutive readings.

b. Square each difference.

c. Sum the squares.

d. Compute s_δ as follows:

(1) Divide step c by the quantity (N-1).
(2) Divide step (1) by the quantity 2.
(3) Find the square root of step (2).

e. Multiply step d by .6745 to obtain the PE.

9.2.1.2.4 EXAMPLE

Given:

Sample data at Table A-5c, page 1-14 (same as data at Table A-5b, page 1-13).

Procedure:

a. Compute the differences between consecutive readings:

$$x_d = x_i - x_{i+1}$$

Example:

a. (1) Difference between 1 and 2:

$$x_d = 1248-1100 = 148$$

(2) Difference between 2 and 3:

$$x_d = 1100-1260 = -160$$

See Table A-5c, page 1-14, for complete list.

b. Square each x_d.

b. (1) $x_d^2 = (148)^2$
$= 21,904$

(2) $x_d^2 = (-160)^2$
$= 25,600$

See Table A-5c, page 1-14, for complete list.

c. Sum the x_d^2.

c. $\Sigma x_d^2 = 85,020$

d. Compute:

$$s_\delta = \sqrt{\frac{\Sigma x_d^2}{2(N-1)}}$$

d. $$s_\delta = \sqrt{\frac{(85,020)}{2(16-1)}}$$
$$= \sqrt{\frac{(85,020)}{30}}$$
$$= \sqrt{2,834}$$
$$= 53.23$$
$$= 53$$

e. Compute:

$PE = .6745(s_\delta)$

e. $PE = .6745(53.23)$
$= 35.90$
$= 36$

9.2.1.2.5 ANALYSIS:

The PE is a measure of deviation from such that 50% of the observations may be expected to lie between μ-PE and μ+PE. If a trend which can be attributed to a nor-system condition, such as weather, is suspected then this method will yield the best estimate of τ. A test comparing the two methods of computing PE can be made to determine whether a trend existed but was not evident (see paragraph 9.2.1.3, for details).

9.2.1.3 TREND ANALYSIS

9.2.1.3.1 OBJECTIVE

To determine whether a trend exists and whether the standard deviation method or the successive differences method yields the best estimate of τ.

9.2.1.3.2 DATA REQUIRED

s^2 and s_δ^2.

9.2.1.3.3 PROCEDURE

a. Choose the desired confidence level.

b. Divide s_δ^2 by s^2.

c. Use Table B-23, page 2-133, to obtain the critical number (CN) for N samples at the desired confidence level.

9.2.2 COMPARING PROBABLE ERRORS (PE's)

As stated in paragraph 4.5.4, page 7, the PE is a measure of deviation from μ such that 50% of the observations may be expected to lie between μ - PE and μ + PE. Since the PE is a function of the standard deviation (PE = .6745s_δ or PE .6745 s), the same tests used for the comparison of standard deviations will be used to compare PE's for a significant difference.

9.2.2.1 COMPARING AN OBSERVED PE TO A REQUIREMENT

a. An observed PE is generated from a sample and is representative of τ. This value of PE is then compared to a stated requirement (τ_0). However, looking at the values of PE and the requirement to decide whether τ is greater than τ_0 or τ is less than τ_0 at a confidence level is insufficient. Since the decision pertains to the population, statistical tests must be applied to PE to determine whether τ is greater than τ_0 or τ is less than τ_0.

b. There exist two possibilities for the relationship of PE to τ_0. Following are the assumptions and the circumstances for each possible relationship:

(1) PE greater than τ_0.

(a) The null hypothesis is τ is greater than τ_0.
(b) The alternative hypothesis is there is no reason to believe τ is greater than τ_0.
(c) The use of this test is appropriate when τ_0 is a maximum value for τ to satisfy. In the event that τ must not be greater than τ_0, this test would be appropriate.

(2) PE less than τ_0.

(a) The null hypothesis is τ is less than τ_0.
(b) The alternative hypothesis is there is no reason to believe that τ is less than τ_0.
(c) The use of this test is appropriate when τ_0 is a minimum value for τ to satisfy. In the event that τ must meet or exceed τ_0, this test would be appropriate.

c. In order to test the above hypotheses when given the values of PE and τ_0, s and σ_0 must be computed; and the appropriate test as described in paragraphs 7.2.1 through 7.2.2, page 70 through 71 must be performed. The values of s and σ_0 are determined by multiplying PE and τ_0 each by 1.4826. Since the PE is a multiple of s, the conclusions drawn concerning standard deviations will also hold true for probable errors; e.g., if the null hypothesis that σ is less than σ_0 is accepted at a 100(1-α)% confidence level, then the null hypothesis that τ is less than τ_0 can also be accepted at the same confidence level.

9.2.2.2 COMPARING TWO OBSERVED PE's

a. An observed probable error is generated from a sample and is representative of τ. This value of PE is then required to meet a standard item PE which is representative of the standard items population. Looking at the values of the probable errors (PE_A and PE_B) to decide whether τ_A is greater than τ_B or τ_A is less than τ_B at a confidence level is insufficient. Since the decision pertains to the populations, statistical tests must be applied to P_A and P_B to determine whether τ_A is greater than τ_B or τ_A is less than τ_B. The statistical tests use the sample PE's as estimates of the population PE's.

b. Type A generally represents the test item and Type B, the standard item when testing the hypothesis that τ_A is greater than τ_B. However, to prove that the PE of the test item is less than that of the standard item, Type A must represent the standard item so that the hypothesis, τ_A is greater than τ_B, can be tested.

c. When the null hypothesis is τ_A is greater than τ_B, the alternative hypothesis is there is no reason to believe that τ_A is greater than τ_B.

d. This test is appropriate when τ_B is a maximum value for τ_A to satisfy.

e. In order to test the above hypothesis when given the values of PE_A and PE_B, s_A and s_B must be computed; and the appropriate test as described in paragraph 7.3.1, page 74, must be performed.
The values of s_A and s_B are determined by multiplying PE_A and PE_B each by 1.4826. Since the PE is a multiple of s, conclusions drawn concerning standard deviations will also hold true for probable errors; e.g., if the null hypothesis that σA is greater than σB is accepted at a 100(1-α)% confidence level then the null hypothesis that τ_A is greater than τ_B can also be accepted at the same confidence level.

9.2.2.3 DETERMINATION OF SAMPLE SIZE

a. The determination of N_t is necessary to assure that there is a sufficient sample upon which to base a decision to accept or reject a null hypothesis at a specified confidence level.

b. The values of s and σ_0 are determined by multiplying the PE and τ_0 each by 1.4826. N_t is determined by following the appropriate procedure as described in paragraph 7.2.3, page 72.

c. The values of s_A and s_B are determined by multiplying PE_A and PE_B each by 1.4826. N_t is determined by following the appropriate procedure as described in paragraph 7.3.2, page 76.

9.2.3 CIRCULAR PROBABLE ERROR

9.2.3.1 COMPUTATION

9.2.3.1.1 OBJECTIVE

To determine the radius of a circle such that 50% of the population lie within the circle.

9.2.3.1.2 DATE REQUIRED

List of sample eastings and corresponding northings.

9.2.3.1.3 PROCEDURE

a. Compute s for the eastings (s_E), (see paragraph 7.1.1.3, page 64).

b. Compute s for the northings (s_N), (see paragraph 7.1.1.3, page 64).

c. Compute the CPE as follows:

(1) If s_E equals s_N, multiply s_E by 1.1774 to obtain the CPE.

(2) If s_E is not equal to s_N, compute the equivalent CPE as follows:

(a) Add s_E to s_N.
(b) Multiply step (1) by .5887.

9.2.3.1.4 EXAMPLE

Given:

Sample data at Table A-5e, page 1-17.

Procedure:

a. Compute s_E:

$$s_E = \sqrt{\frac{\Sigma(\text{East}-\overline{\text{EAST}})^2}{N-1}}$$

b. Compute s_N:

$$s_N = \sqrt{\frac{\Sigma(\text{North}-\overline{\text{NORTH}})^2}{N-1}}$$

Example:

a. $s_E = \sqrt{\frac{1,650,542}{15-1}}$

$= \sqrt{1,650,542/14}$

$= \sqrt{117,895.9}$

$= 343.36$

$= 343$

b. $s_N = \sqrt{\frac{3,389,046}{15-1}}$

$= \sqrt{\frac{3,389,046}{14}}$

$= \sqrt{242,074.7}$

$= 492.01$

$= 492$

c. Compute one of the following:

(1) If $s_E = s_N$ compute:
CPE = 1.1774 s_E

(2) If $s_E \neq s_N$ compute
Equivalent CPE = .5887 (s_E+s_N)

c. Since 343.36 ≠ 492.01,
Equivalent CPE
= .5887 (343.36+492.01)
= .5887 (835.37)
= 491.78
= 492

9.2.3.1.5 ANALYSIS

The CPE is the radius of a circle within which 1/2 or 50% of the population lies. The following is a list of multiples of the CPE and the percentages of the population which lie within the respective circles for a circular normal distribution:

a. 2(CPE) contains 93.75% of the population.

b. 3(CPE) contains 99.81% of the population.

c. 3.5(CPE) contains 99.99% of the population.

9.2.3.2 OUTLIERS

9.2.3.2.1 OBJECTIVE

To identify an outliers which may be present.

9.2.3.2.2 DATA REQUIRED

A list of sample eastings and corresponding northings.

9.2.3.2.3 PROCEDURE

a. Compute the CPE for all of the readings.

b. Compute the distance from the mean (d_m) for each set of coordinates using data from step a as follows:

$$d_m = \sqrt{(EAST-\overline{EAST})^2+(NORTH-\overline{NORTH})^2} = \sqrt{\Delta E^2+\Delta N^2}$$

c. Isolate each suspected outlier beginning with the largest distance from the mean.

d. Recompute the CPE, with the suspected outlier deleted, as follows:

(1) Compute s_E (and s_N) as follows:

(a) Compute the mean of the remaining eastings (northings).
(b) Compute the deviation of each remaining reading from the mean.
(c) Square each deviation.
(d) Sum the squared deviations.
(e) Since N_1 is the sample size with the suspected outlier deleted, divide step (d) by the quantity (N_1-1).
(f) Find the square root of step (e).

(2) Add s_E to s_N.

Therefore, a confidence level of 95% indicates that if 100 groups, each containing 46 samples, were tested then on the average five of these groups would have more than one failure and 95 of these groups would have one or zero failures.

b. That high requirements place limitations on acceptability is intuitively evident. Stringent limitations require sufficient sampling to provide an objective view of the test item. However, in the interest of economy, testing must be accomplished with a minimum number of samples. This may be accomplished by decreasing the desired reliability (confidence level) while holding the confidence level (desired reliability) fixed. Therefore, serious consideration must be given to sample size, the related R, and the desired confidence level.

10.1 SUCCESS-FAILURE

10.1.1 DETERMINATION OF RELIABILITY

10.1.1.1 OBJECTIVE

To determine the population reliability (ρ) of the test item at the desired confidence level. The required reliability (ρ_0) and the confidence level are usually directed by a higher authority or a Requirements Document.

10.1.1.2 DATA REQUIRED

The number of failures (f) and N for a success-failure type test.

10.1.1.3 PROCEDURE

a. Case I:

(1) Use Table B-18, page 2-74, to obtain the intersection of the "Reliability" row and the "Confidence Level" column for the number of failures which occurred (see page 2-125 for 75% confidence level).

(2) If N is equal to or larger than the intersection value, decide that ρ is equal to or greater than ρ_0 (testing may cease); otherwise, there is no reason to believe ρ is equal to or greater than ρ_0 at the desired confidence level (testing may cease with a reject decision or testing must continue with a decision being made at a later date).

b. Case II: Reliability confidence limits.

(1) Compute the two-sided UCL and LCL as follows:

(a) Choose the desired confidence level.

(b) Perform the following calculations to obtain the UCL:

1. Compute $d.f._1$ as follows:

a. Multiply the number of succes.. (sc) by 2.

b. Add 2 to step a.

2. Compute $d.f._2$ as follows:

a. Multiply sc by 2.
b. Multiply N by 2.
c. Subtract step a from step b.

3. Use Table B-8, page 2-18, to obtain $F_{1-\alpha/2}$ for $(d.f._1, d.f._2)$ d.f.
4. Compute the following:

a. Add 1 to sc.
b. Subtract sc from N.
c. Divide step a by step b.
d. Multiply step c by step 3.
e. Add 1 to step d.
f. Divide 1 by step e.
g. Subtract step f from 1.

(c) Perform the following calculations to obtain the LCL:

1. Compute $d.f._1$ as follows:

a. Multiply f by 2.
b. Add 2 to step a.

2. Compute $d.f._2$ as follows:

a. Multiply f by 2.
b. Multiply N by 2.
c. Subtract step a from step b.

3. Use Table B-8, page 2-18, to obtain $F_{1-\alpha/2}$ for $(d.f._1, d.f._2)$ d.f.
4. Compute the following:

a. Add 1 to f.
b. Subtract f from N.
c. Divide step a by step b.
d. Multiply step c by step 3.
e. Add 1 to step d.
f. Divide 1 by step e.

(d) Conclude that ρ is equal to or between the UCL and LCL at the desired confidence level.

(2) Compute the one-sided UCL as follows:

(a) Choose the desired confidence level.
(b) Compute $d.f._1$ as follows:

1. Multiply sc by 2.
2. Add 2 to step 1.

(c) Compute $d.f._2$ as follows:

1. Multiply sc by 2.

(3) Compute the one-sided LCL for ρ as follows:

(a) Choose the confidence level $(1-\alpha)$.

(b) Compute:

$$d.f._1 = 2(f)+2$$

(c) Compute:

$$d.f._2 = 2(N) - 2(f)$$

(d) Use Table B-8, page 2-18, to obtain $F_{1-\alpha}$ for $(d.f._1, d.f._2)$ d.f.

(e) Compute:

$$LCL = \frac{1}{1+\left(\frac{f+1}{N-f}\right)F_{1-\alpha}}$$

(f) Conclude that $\rho \geq LCL$ at a $100(1-\alpha)\%$ confidence level.

(3)

(a) $\alpha = .05$

$1-\alpha = .95$

(b) $d.f._1 = 2(5)+2$

$= 12$

(c) $d.f._2 = 2(52)-2(5)$

$= 104-10$

$= 94$

(d) $F_{.95}$ for (12,94)d.f. approximates closely

$F_{.95}$ for (12,90)d.f.

$F_{.95}$ for (12,90)d.f. = 1.86

(e)

$$LCL = \frac{1}{1+\left(\frac{5+1}{52-5}\right)F_{.95}}$$

$$= \frac{1}{1+\left(\frac{6}{47}\right)(1.86)}$$

$$= \frac{1}{1+(.1277)(1.86)}$$

$$= \frac{1}{1+.2374}$$

$$= \frac{1}{1.2374}$$

$= .8081$

$= .80$

(f) Conclude that $\rho \geq .80$ at a 95% confidence level.

NOTE: .80 is referred to as the reliability of the test item at a 95% confidence level.

10.1.1.5 ANALYSIS

a. Case I:

If $N \geq$ the intersection value (Table B-18, page 2-74) the null hypothesis that $\rho \geq \rho_0$ is accepted; otherwise, there is no reason to believe $\rho \geq \rho_0$ at a $100(1-\alpha)\%$ confidence level.

b. Case II:

(1) The two-sided interval surrounds ρ such that $\rho \leq$ UCL and $\rho \geq$ LCL at a $100(1-\alpha)\%$ confidence level.
(2) The UCL is determined so that $\rho \leq$ UCL at a $100(1-\alpha)\%$ confidence level.
(3) The LCL is determined so that $\rho \geq$ LCL at a $100(1-\alpha)\%$ confidence level.

10.1.2 DETERMINATION OF SAMPLE SIZE

10.1.2.1 OBJECTIVE

a. To determine the absolute minimum N_t required to establish ρ_0 at the desired confidence level.

b. To determina the minimum N_t required to establish ρ_0 at the desired confidence level when the average number of failures is known from previous testing or a comparable item.

10.1.2.2 DATA REQUIRED

a. None.

b. The average number of failures known from a standard item, history, or Requirements Document.

10.1.2.3 PROCEDURE

a. Case I: Determination of an absolute minimum N_t.

(1) Use Table B-18, page 2-74, to obtain the intersection of the "Reliability" row and the "Confidence Level" column for zero failures.
(2) Conclude that the intersection value is the absolute minimum N_t since zero failures constitutes the ideal situation.

b. Case II: Determination of N_t.

(1) Use Table B-18, page 2-74, to obtain the intersection of the "Reliability" row and the "Confidence Level" column for the average number of failures known from a standard item, history, or Requirements Document.

(2) Conclude that the intersection value is the minimum N_t. Generally the test item must be as good as previous test results from a standard item. Note that in most cases this N_t will be larger than the absolute minimum N_t generated in Case I.

10.1.2.4 EXAMPLE

a. Case I: Determination of an absolute minimum N_t.

Given:

$\rho_o = .95$

$1-\alpha = .90$

$f = 0$

Procedure:	Example:
(1) Use Table B-18, page 2-74, to obtain the intersection of the "Reliability" row and the "Confidence Level" column for zero failures.	(1) For $f = 0$, $\rho_o = .95$, and $1-\alpha = .90$, $N_t = 45$
(2) Conclude that the intersection value is the absolute minimum N_t since zero failures constitutes the ideal situation.	(2) For zero failures, conclude that 45 samples are required to achieve $\rho = .95$ at a confidence level of 90%.

b. Case II: Determination of N_t

Given:

$\rho_o = .95$

$1-\alpha = .90$

Average number of failures for the standard item = 6

Procedure:	Example:
(1) Use Table B-18, page 2-74, to obtain the intersection of the "Reliability" row and the "Confidence Level" column for the average number of failures.	(1) For $f - 6$, $\rho_o = .95$, and $1-\alpha = .90$, $N_t = 209$.
(2) Conclude that the intersection value is the minimum N_t. Generally the test item must be as good as previous test results from a standard item. NOTE: In most cases this N_t will be larger than the absolute minimum N_t generated in Case I.	(2) For no more than six failures, conclude that 209 samples are required to achieve $\rho = .95$ at a 90% confidence level. NOTE: 209 > 45

10.1.2.5 ANALYSIS

a. Initial N_t.

At a specified confidence level, reliability, and number of failures, N_t samples are required to determine whether $\rho \geq \rho_0$. Zero failures will generate the absolute minimum N_t.

b. Adequacy of N_t.

After the initial N_t samples have been tested, R must be computed at the desired confidence level for the number of failures that occurred. If the computed R is equal to or greater than ρ_0, the initial N_t is adequate; however, if the computed R is less than ρ_0, the initial N_t is inadequate. If N_t is inadequate, N_t must be recomputed using the number of failures which have occurred, f, and the desired confidence level; and additional samples must be tested if possible or a reject dicision made.

10.1.3 SEQUENTIAL ANALYSIS FOR SUCCESS-FAILURE

a. When testing an hypothesis using the sequential method, the project officer is able to make one of the following three decisions at any stage of testing:

(1) Accept the hypothesis.
(2) Reject the hypothesis.
(3) Continue the experiment by collecting additional data.

b. Usually a ρ_0 of .95 with a high degree of assurance is required. In order to achieve assurance of such a high ρ_0, the project officer would have to conduct excessive testing; e.g., many thousands of rounds. This may be impractical; however, using the following statistical approach, the project officer will achieve the predetermined confidence level for reaching the accept decision.

c. If certain criteria are set up graphically, a decision can be made to accept, reject, or continue testing the test item after each sample is tested. This graph uses three areas to represent the decisions to accept, reject, or continue testing the test item. The accept region is below a boundary line determined by the subtraction of the maximum proportion of defectives (P_0) and the confidence levels for rejection and acceptance. The continue testing area is above the accept boundary line and below the reject boundary line. The size of this area, which is an area of doubt for the test item, is determined by the project officer (see paragraph 4.14, page 14). The area of doubt is designed for a test item which may be good but has gotten off to a slow start. In this case, a longer period of time will be required to satisfy the doubts concerning acceptability of the test item. The reject boundary line is determined by P_0 and the confidence levels for rejection and acceptance. The area above this boundary line is the area of rejection. A graph of this type is illustrated by Figure 14. The number of samples are plotted on the horizontal axis with each increment representing one sample. The number of failures are plotted on the vertical axis with each increment representing one failure.

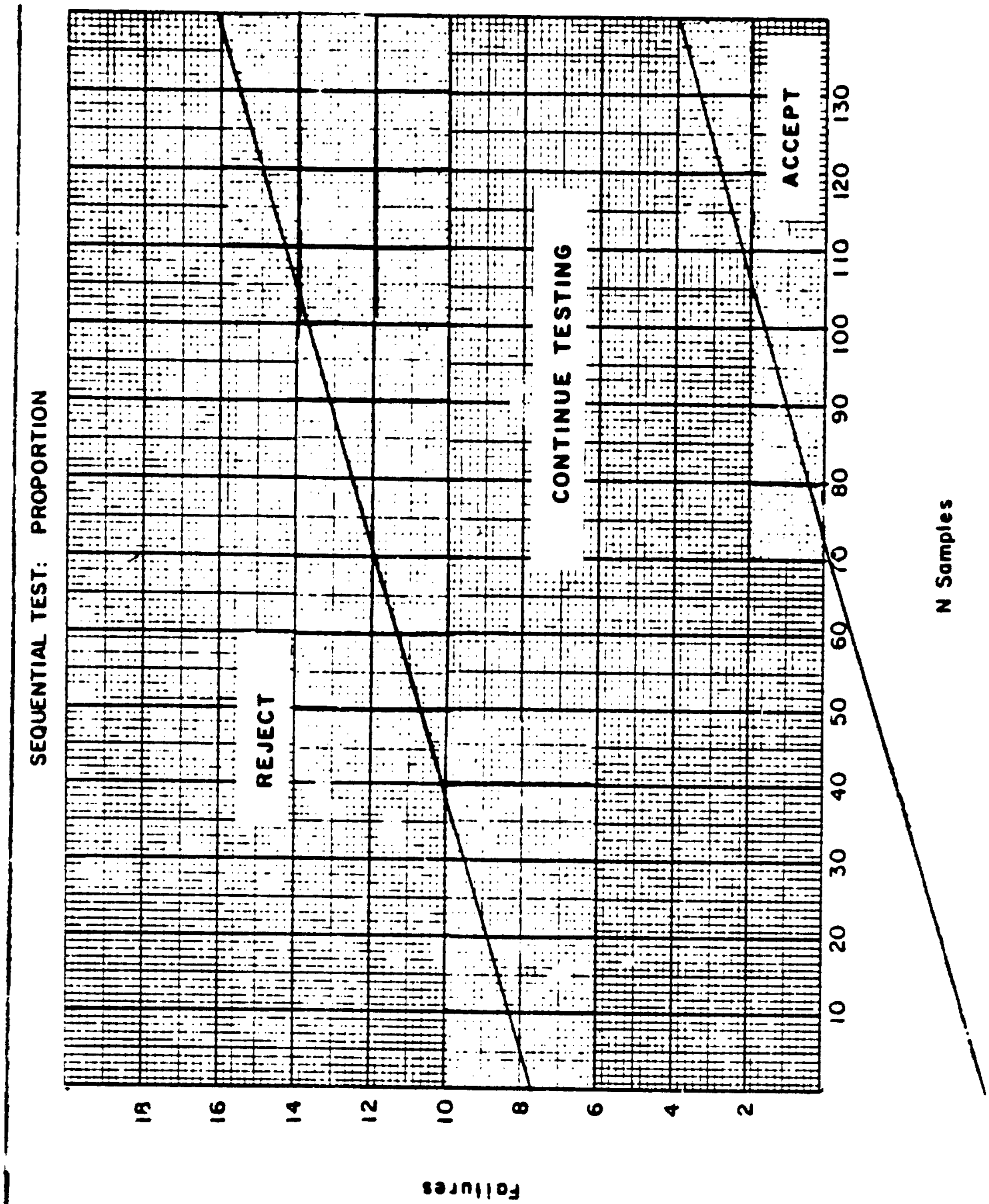

SEQUENTIAL TEST: PROPORTION
REJECT
CONTINUE TESTING
ACCEPT
N Samples
Failures
10 20 30 40 50 60 70 80 90 100 110 120 130
2 4 6 8 10 12 14 16 18

Figure 14

d. The construction of the two boundary lines is described in the procedure paragraph below.

10.1.3.1 OBJECTIVE

To determine whether the proportion of defective test items is equal to or less than P_o at the desired confidence level.

10.1.3.2 DATA REQUIRED

N and f.

10.1.3.3 PROCEDURE

a. Construct the boundary lines as follows:

(1) Choose α and β, the probabilities of making Type I and Type II errors respectively.
(2) Choose the amount of doubt, the proportion of defectives allowable for continued testing.
(3) Use Table B-19, page 2-127, to obtain a and b for α and β.
(4) Subtract D from P_o to obtain the upper limit for the proportion of defectives (P_U).

NOTE: P_o equals λ_o, if λ_o is in terms of defectives. P_o equals the quantity $(1-\lambda_o)$, if λ_o is in terms of successes.

(5) Compute U, an intermediate value, as follows:

(a) Divide P_o by step (4).
(b) Subtract step (4) from 1.
(c) Subtract P_o from 1.
(d) Divide step (b) by step (c).
(e) Multiply step (a) by step (d).
(f) Find the natural logarithm of step (e).

(6) Compute V, an intermediate value, as follows:

(a) Subtract step (4) from 1.
(b) Subtract P_o from 1.
(c) Divide step (a) by step (b).
(d) Find the natural logarithm of step (c).
(e) Divide step (d) by step (5).

(7) Determine the accept boundary line as follows:

(a) Divide the value b in step (3) by step (5).
(b) Multiply step (6) by N.
(c) Add step (a) to step (b) to determine the maximum allowable f for accepting the test item ($f_{ACCEPT} = \frac{b}{U} + V(N)$).
(d) Choose two values for N and substitute them into the above equation to determine two points on the accept boundary line.

10.1.3.4 EXAMPLE

a. Construct the boundary lines as follows:

Given:

$P_o = .07$ (7 failures out of 100; reliability of 93%)

Procedure:	Example:
(1) Choose α and β.	(1) $\alpha = .05$ $1-\alpha = .95$ $\beta = .20$ $1-\beta = .80$
(2) Choose D.	(2) $D = .02$
(3) Use Table B-19, page 2-127, to obtain a and b for α and β.	(3) $a = 2.773$ $b = -1.558$
(4) Compute: $P_U = P_o - D$	(4) $P_U = .07 - .02$ $= .05$
(5) Compute: $U = \ln\left(P_o/P_U\right)\left(\frac{1-P_U}{1-P_o}\right)$	(5) $U = \ln\left(\frac{.07}{.05}\right)\left(\frac{1-.05}{1-.07}\right)$ $= \ln\left(1.4000\right)\left(\frac{.95}{.93}\right)$ $= \ln\ (1.4000)(1.0215)$ $= \ln 1.4301$ $= .35775$

(6) Compute:

$$V = \frac{\ln\left(\frac{1-P_U}{1-P_o}\right)}{U}$$

(6) $V = \frac{\ln\left(\frac{1-.05}{1-.07}\right)}{.35775}$

$= \frac{\ln(1.0215)}{.35775}$

$= \frac{.021282}{.35775}$

$= .059488$

(7) Compute:

$$f_{ACCEPT} = \frac{b}{U} + V(N)$$

(7) $f_{ACCEPT} = \frac{-1.558}{.35775} + .059488(N)$

$= -4.355 + .059488(N)$

When N = 0, $f_{ACCEPT} = -4.355$

When N = 100, $f_{ACCEPT} = 1.594$

Plot the points

(0,-4.355) and (100,1.594)
to determine the accept boundary line.

(8) Compute:

$$f_{REJECT} = \frac{a}{U} + V(N)$$

(8) $f_{REJECT} = \frac{2.773}{.35775} + .059488(N)$

$= 7.751 + .059488(N)$

When N = 0, $f_{REJECT} = 7.751$

When N = 100, $f_{REJECT} = 13.700$

Plot the points
(0,7.75) and (100,13.700)
to determine the reject boundary line.

(9) If the two lines are not parallel, check the computations and plotted points.

b. Plot the sample data on the sequential graph as follows:

Given:

Requirements and boundary lines from step a.
Sample data at Table A-6a, page 1-19.

Procedure:

(1) Plot the cumulative sample size and failure after each sample,
(N_i, f_i).

Example:

(1) (a) (30,1)

(b) (75,2)

See Table A-6a, page 1-19, for complete list.

(2) After plotting each point, decide to accept, reject, or continue testing the test item.

(2) For failures 1 through 3, decide to continue testing. At failure number 4 decide to accept the test item. A decision to accept the test item could have been made when N was 134 and f was 3 since the accept boundary was crossed (see Figure 14, page 129).

NOTE: From Table B-18, page 2-74, when $f=3, \rho_0=.95$, and $1-\alpha=.95$, the intersection value is 153; thus, fewer samples (N=134) are needed using the sequential method.

10.1.3.5 ANALYSIS

a. The sequential method generally minimizes testing time and N due to the fact that a decision to accept or reject is made as soon as possible after the first failure. Since all failures are not necessarily chargeable failures, decisions will be altered if certain failures are not counted. If the project officer ignores a failure, the probability of accepting an unacceptable item is increased. Therefore, the project officer must carefully decide what constitutes a failure (see paragraph 4.2, page 2).

b. Due to the advantages just discussed, the sequential method should be used whenever possible (see subparagraph 10.2c, page 134).

10.2 RELIABILITY RELATIVE TO CONTINUOUS TESTING

a. When measuring R for the continuous testing situation, the failure rate is assumed to approach the exponential distribution (see paragraph 4.15.5, page 19). In this case there are three measures of R that are of interest to the project officer. These are:

(1) The determination of mean time, miles, or rounds between failures and the limits for the mean at a desired confidence level (see paragraph 10.2.1, page 134).
(2) The determination of a computed R (see paragraph 10.2.2, page 145).
(3) The determination of the R based on ρ_0 and the desired confidence level (see paragraph 10.2.3, page 147).

b. The first two determinations are simple and straightforward but are biased by limitations on N. The third, which is the only sequential analysis method, is a truer representation of the population.

c. Sequential analysis is superior to nonsequential analysis whenever the data become available serially and the cost of the data (in terms of time, labor, or material) is approximately proportional to the amount of data. Nonsequential analysis is superior whenever the amount of data is fixed or the cost of the data is largely overhead, hence more or less independent of the amount of data. Superiority consists of minimizing the set of quantities N, α, and β. Sequential and nonsequential tests differ in the constraints under which this set is minimized. Nonsequential tests treat N as fixed and are designed so that either risk α or risk β is minimized when the other is fixed. Sequential tests treat N as a variable and are designed so that for fixed risks, α and β, the expected (average) number of trials required to reach a decision is minimized. If for a nonsequential test N is made large enough so that, with α fixed, β will not exceed a predetermined amount, this value of N will exceed (frequently by as much as 100 percent) the N required for a sequential test for the same α and β. Thus, when N is readily subject to variation, sequential tests are superior; when N is not readily varied, nonsequential tests are superior.

d. Examples of the solution for each determination are in the following paragraphs. In all examples mean time between failures (MTBF) is used. Other means, such as mean miles between failures (MMBF) or mean rounds between failures (MRBF), may be used when applicable.

10.2.1 MEANS AND LIMITS

10.2.1.1 MEANS

10.2.1.1.1 OBJECTIVE

To determine the mean time between failures.

10.2.1.1.2 DATA REQUIRED

A list of sample readings; e.g., operating time (primary parameter) and failures (secondary parameter).

10.2.1.1.3 PROCEDURE

a. Sum the primary parameter.

b. Sum the secondary parameter.

c. Divide step a by step b.

10.2.1.1.4 EXAMPLE

Given:

Sample data at Table A-6c, page 1-21.

Procedure:	Example:
a. Sum the primary parameter; e.g., total time (T_t) or total miles (T_m).	a. T_t = 3752 hours

b. Sum the secondary parameter; e.g., total failures (f).

b. f = 12

c. Compute:

$$MTBF = \frac{Tt}{f}$$

c. MTBF = 3752/12
= 312.56
= 313 hours

NOTE: In the event a test is time terminated and zero failures occurred, a point estimate of the MTBF cannot be determined but a LCL may be computed (see paragraph 10.2.1.3).

10.2.1.1.5 ANALYSIS

The sample mean, or average, is a value which is typical or representative of a set of data. The mean is the most commonly used measure of central location.

10.2.1.2 LIMITS USING THE STUDENT t DISTRIBUTION

10.2.1.2.1 OBJECTIVE

To determine the two-sided and one-sided limits for the MTBF using the t distribution.

10.2.1.2.2 DATA REQUIRED

A list of sample readings; e.g., operating time (primary parameter) and failures (secondary parameter).

10.2.1.2.3 PROCEDURE

a. Case I: UCL and LCL (two-sided limits), also referred to as M_2 and M_1.

(1) Choose the desired confidence level.
(2) Use Table C-5, page 2-5, to obtain $t_{1-\alpha/2}$ for f-1 d.f.
(3) Compute the MTBF (see paragraph 10.2.1.1.3, page 134).
(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.
(5) Compute s (see paragraph 7.1.1.4, page 65).

NOTE: The sample size is the number of failures.

(6) Compute ε as follows:

(a) Multiply step (2) by step (5).
(b) Divide step (a) by the square root of f.

(7) Add step (6) to step (3) to obtain the UCL and subtract step (6) from step (3) to obtain the LCL.
(8) Conclude that the population MTBF is equal to or less than the UCL and equal to or greater than the LCL at the desired confidence level.

b. Case II: UCL (one-sided limit), also referred to as M_2.

(1) Choose the desired confidence level.
(2) Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for f-1 d.f.
(3) Compute the MTBF (see paragraph 10.2.1.1.3, page 134).
(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.
(5) Compute s (see paragraph 7.1.1.3, page 64).
NOTE: The sample size is the number of failures.
(6) Compute ε as follows:

(a) Multiply step (2) by step (5).
(b) Divide step (a) by the square root of f.

(7) Add step (6) to step (3) to obtain the UCL.
(8) Conclude that the population MTBF is equal to or less than the UCL at the desired confidence level.

c. Case III: LCL (one-sided limit), also referred to as M_1.

(1) Choose the desired confidence level.
(2) Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for f-1 d.f.
(3) Compute the MTBF (see paragraph 10.2.1.1.3, page 134).
(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.
(5) Compute s (see paragraph 7.1.1.3, page 64).

NOTE: The sample size is the number of failures.

(6) Compute ε as follows:

(a) Multiply step (2) by step (5).
(b) Divide step (a) by the square root of f.

(7) Subtract step (6) from step (3) to obtain the LCL.
(8) Conclude that the population MTBF is equal to or greater than the LCL at the desired confidence level.

10.2.1.2.4 EXAMPLE

a. Case I: UCL and LCL (two-sided limits), also referred to as M_2 and M_1, respectively.

Given:

Sample data at Table A-6d, page 1-22.

Procedure:	Example:
(1) Choose the confidence level (1-α).	(1) $\alpha = .10$ $1-\alpha = .90$ $1-\alpha/2 = .95$
(2) Use Table B-5, page 2-5, to obtain $t_{1-\alpha/2}$ for (f-1) d.f.	(2) $f-1 = 5$ $t_{.95}$ for 5 d.f. = 2.015

Procedure	Example
(3) Compute the MTBF.	(3) MTBF = 207 hours See paragraph 10.2.1.1.4, page 134.
(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.	(4) (a) Time to failure 1 = 200 hours (b) Time between failures 2 and 1 = 410-200 = 210 hours
(5) Compute: $s = \sqrt{\frac{\Sigma\Delta^2}{f-1}}$	(5) $s = \sqrt{\frac{1411}{6-1}}$ = 16.80 = 17 hours See paragraph 7.1.1.4, page 65.
(6) Compute: $\varepsilon = \frac{t_{1-\alpha/2}\ (s)}{\sqrt{f}}$	(6) $\varepsilon = \frac{2.015(16.80)}{\sqrt{6}}$ = 33.85/2.449 = 13.82
(7) Compute: UCL = MTBF + ε LCL = MTBF − ε	(7) UCL = 206.83 + 13.82 = 220.65 = 221 hours LCL = 206.83 − 13.82 = 193.01 = 193 hours
(8) Conclude that the population MTBF ≤ UCL and the population MTBF ≥ LCL at a 100(1−α)% confidence level.	(8) Conclude that the population MTBF ≤ 221 hours and the population MTBF ≥ 193 hours at ≥ 90% confidence level.

b. Case II: UCL (one-sided limit), also referred to as M_2.

Given:

Sample data at Table A-6d, page 1-22.

Procedure:	Example:
(1) Choose the confidence level (1−α).	(1) α = .10 1−α = .90
(2) Use Table B-5, page 2-5. to obtain $t_{1-\alpha}$ for (f-1) d.f.	(2) f-1 = 5 $t_{.90}$ for 5 d.f. = 1.476
(3) Compute the MTBF.	(3) MTBF = 1241/6 = 207 hours See paragraph 10.2.1.1.4, page 134.

(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.

(4) (a) Time to failure 1 = 200 hours.
(b) Time between failures 2 and 1 = 410-200 = 210 hours

See Table A-6d, page 1-22 for complete list.

(5) Compute:

$$s = \sqrt{\frac{\Sigma\Delta^2}{f-1}}$$

(5) $s = \sqrt{\frac{1410.6}{6-1}}$

$\approx \sqrt{282.1}$

$= 16.80$

$= 17$ hours

See paragraph 7.1.1.4, page 65.

(6) Compute:

$$\varepsilon = \frac{t_{1-\alpha}(s)}{\sqrt{f}}$$

(6) $\varepsilon = \frac{(1.476)(16.80)}{\sqrt{6}}$

$= \frac{24.80}{2.449}$

$= 10.13$

(7) Compute:

UCL = MTBF + ε

(7) UCL = 206.83 + 10.13
= 216.96
= 217 hours

(8) Conclude that the population MTBF $\leq$ UCL at a $100(1-\alpha)\%$ confidence level.

(8) Conclude that the population MTBF $\leq$ 217 hours at a 90% confidence level.

c. Case III: LCL (one-sided limit), also referred to as M_1.

Given:

Sample data at Table A-6d, page 1-22.

Procedure:

(1) Choose the confidence level $(1-\alpha)$.

(2) Use Table B-5, page 2-5, to obtain $t_{1-\alpha}$ for (f-1) d.f.

(3) Compute the MTBF.

(4) Compute the time between failures for each consecutive pair of failures if the data have been recorded as cumulative time.

Example:

(1) $\alpha = .10$
$1-\alpha = .90$

(2) $f-1 = 5$
$t_{.90}$ for 5 d.f. = 1.476

(3) MTBF = 207 hours.

See paragraph 10.2.1.1.4, page 134.

(4) (a) Time to failure 1 = 200 hours.
(b) Time between failures 2 and 1
= 410-200
= 210 hours

(5) Compute:

$$s = \sqrt{\frac{\Sigma\Delta^2}{f-1}}$$

(5) s = 16.80
= 17 hours
See paragraph 7.1.1.4, page 65.

(6) Compute:

$$\varepsilon = \frac{t_{1-\alpha}(s)}{\sqrt{f}}$$

(6) $\varepsilon = \frac{(1.476)(16.80)}{\sqrt{6}}$
= 10.13

(7) Compute:

LCL = MTBF - ε

(7) LCL = 206.83 - 10.13
= 196.70
= 196 hours

(8) Conclude that the population MTBF ≥ LCL at a 100(1-α)% confidence level.

(8) Conclude that population MTBF ≥ 196 hours at a 90% confidence level.

10.2.1.2.5 ANALYSIS

a. The two-sided interval surrounds the population MTBF such that the population MTBF ≤ UCL and the population MTBF ≥ LCL at a 100(1-α)% confidence level.

b. The UCL of the MTBF is determined such that the population MTBF ≤ UCL at a 100(1-α)% confidence level.

c. The LCL of the MTBF is determined such that the population MTBF ≥ LCL at a 100(1-α)% confidence level. M_1 (the LCL) is generally considered the MTBF of the population since the population MTBF will be at least M_1 at a 100(1-α)% confidence level. If comparing M_1 to the required MTBF produces an accept decision for the test item, then on the average, the test item will function as required at a 100(1-α)% confidence level.

d. The method used to compute M_1 and M_2 uses s to estimate σ. If the time between two failures is close to the MTBF, s will be small; and M_1 will be close to the MTBF. However, if the times between failures are erratic (close to the MTBF in some cases and far from the MTBF in other cases), s will be large; and the interval between the M_1 and the MTBF will increase. Since this method uses the student t distribution, f should be less than or equal to 30.

NOTE: The application of the student t assumes that the MTBF's are approximately normally distributed.

10.2.1.3 LIMITS USING THE χ^2 DISTRIBUTION

10.2.1.3.1 OBJECTIVE

To determine the two-sided and one-sided limits for the MTBF using the χ^2 distribution.

10.2.1.3.2 DATA REQUIRED

A list of sample readings; e.g., operating time (primary parameter) and failures (secondary parameter).

10.2.1.3.3 PROCEDURE

a. Case I: UCL and LCL (two-sided limits), also referred to as M_2 and M_1.

(1) Choose the desired confidence level.
(2) Use Table B-20, page 2-128, to obtain $LF_{1-\alpha/2}$ for:

(a) f + 1 d.f., if a time terminated test.
(b) f d.f., if a failure terminated test.

(3) Use Table B-21, page 2-129, to obtain the $UF_{1-\alpha/2}$ for f d.f., for both the time and failure terminated test.
(4) If the test is a time terminated test, compute the following:

(a) Multiply step (2)(a) by T_t.
(b) Divide step (a) by the quantity (f+1) to obtain the LCL.
(c) Multiply step (3) by T_t.
(d) Divide step (c) by the value of f to obtain the UCL.

(5) If the test is a failure terminated test, compute the following:

(a) Multiply step (2) (b) by T_t.
(b) Divide step (a) by f to obtain the LCL.
(c) Multiply step (3) by T_t.
(d) Divide step (c) by f to obtain the UCL.

NOTE: To maintain accuracy, the six decimal number found in Table B-20, page 2-128, or B-21, page 2-129, must be used.

(6) Conclude that the population MTBF is equal to or between the UCL and LCL at the desired confidence level.

b. Case II. UCL (one-sided limit), also referred to as M_2.

(1) Choose the desired confidence level.
(2) Use Table B-21, page 2-129, to obtain the $UF_{1-\alpha}$ for f d.f., for both the time and failure terminated test.

(3) Compute the UCL as follows:
(a) Multiply step (2) by T_t.
(b) Divide step (a) by f.

NOTE: To maintain accuracy, the six decimal number found in Table B-21, page 2-129, must be used.

(4) Conclude that the population MTBF is equal to or less than the UCL at the desired confidence level.

c. Case III. LCL (one-sided), also referred to as M_1.

(1) Choose the desired confidence level.
(2) Use Table B-20, page 2-128, to obtain $LF_{1-\alpha}$ for:

(a) f+1 d.f., if a time terminated test.
(b) f d.f., if a failure terminated test.

(3) If a time terminated test, compute the LCL as follows:

(a) Multiply step (2)(a) by T_t.
(b) Divide step (a) by the quantity (f+1).

(4) If a failure terminated test compute the LCL as follows:

(a) Multiply step (2)(b) by T_t.
(b) Divide step (a) by f.

NOTE: To maintain accuracy, the six decimal number found in Table B-20, page 2-128, must be used.

(5) Conclude that the population MTBF is equal to or greater than the LCL at the desired confidence level.

10.2.1.3.4 EXAMPLE

a. Case I: UCL and LCL (two-sided limits), also referred to as M_2 and M_1.

Given:
Sample data at Table A-6c, page 1-21.

Procedure:	Example:
(1) Choose the confidence level (1-α).	(1) $\alpha = .05$ $1-\alpha = .95$
(2) Use Table B-20, page 2-128, to obtain $LF_{1-\alpha/2}$ for: (a) f+1 d.f., if a time terminated test. (b) f d.f., if a failure terminated test.	(2) $LF_{.975}$ for 13 d.f. = .620525
(3) Use Table B-21, page 2-129, to obtain $UF_{1-\alpha/2}$ for f d.f.	(3) $UF_{.975}$ for 12 d.f. = 1.935484
(4) Compute for a time terminated test: $LCL = \frac{(LF_{1-\alpha/2})(T_t)}{f+1}$ $UCL = \frac{(UF_{1-\alpha/2})(T_t)}{f}$	(4) Since the test is time terminated, $LCL = \frac{(.620525)(3752)}{(12+1)}$ = 179.093062 = 179 hours $UCL = \frac{(1.935484)(3752)}{12}$ = 605.161330 = 606

(5) Compute for a failure terminated test:

$$LCL = \frac{(LF_{1-\alpha/2})(T_t)}{f}$$

$$UCL = \frac{(UF_{1-\alpha/2})(T_t)}{f}$$

(6) Conclude that the population MTBF ≤ UCL and the population MTBF ≥ LCL at a 100(1-α)% confidence level.

(6) Conclude that the population MTBF ≤ 606 hours and the population MTBF ≥ 179 hours at a 95% confidence level.

b. Case II. UCL (one-sided limit), also referred to as M_2.

Given:

Sample data at Table A-6c, page 1-21.

Procedure:

(1) Choose the confidence level (1-α).

(2) Use Table B-21, page 2-129, to obtain the $UF_{1-\alpha}$ for f d.f., for both a time and failure terminated test.

(3) Compute

$$UCL = \frac{(UF_{1-\alpha})\ (T_t)}{f}$$

(4) Conclude that the population MTBF ≤ UCL at a 100(1-α)% confidence level.

Example:

(1) $\alpha = .05$
$1-\alpha = .95$

(2) $UF_{.95}$ for 12 d.f. = 1.739130

(3)

$$UCL = \frac{(1.739130)(3752)}{12}$$

$$= 543.767980$$

$$= 544$$

(4) Conclude that the population MTBF ≤ 544 hours at a 95% confidence level.

c. Case III: LCL (one-sided limit), also referred to as M_1.

Given:

Sample data at Table A-6c, page 1-21.

Procedure:

(1) Choose the confidence level ($1-\alpha$).

(2) Use Table B-20, page 2-128, to obtain $LF_{1-\alpha}$ for:

(a) f+1 d.f., if time terminated.

(b) f d.f., if failure terminated.

(3) Compute for a time terminated test:

$$LCL = \frac{(LF_{1-\alpha})(T_t)}{(f+1)}$$

(4) Compute for a failure terminated test:

$$LCL = \frac{(LF_{1-\alpha})(T_t)}{f}$$

(5) Conclude that the population MTBF $\geq$ LCL at a $100(1-\alpha)$% confidence level.

Example:

(1) $\alpha = .05$
$1-\alpha = .95$

(2) $LF_{.95}$ for 13 d.f. = .668380

(3) Since the test is time terminated,

$$LCL = \frac{(.668380)(3752)}{12+1}$$

$$= 192.907836$$

$$= 192 \text{ hours}$$

(5) Conclude that the population MTBF $\geq$ 192 hours at a 95% confidence level.

NOTE: Although the confidence level is numerically the same for all three cases, M_1 and M_2 take on different values (see Figure 15).

10.? 1.3.5 ANALYSIS

a. The two-sided interval surrounds the population MTBF such that the population MTBF $\leq$ UCL and the population MTBF $\geq$ LCL at a $100(1-\alpha)$% confidence level.

b. The UCL of the MTBF is determined such that the population MTBF $\leq$ UCL at a $100(1-\alpha)$% confidence level.

c. The LCL of the MTBF is determined such that the population MTBF $\geq$ LCL at a $100(1-\alpha)$% confidence level. M_1 (the LCL) is generally considered the MTBF of the population since the population MTBF will be at least M_1 at a $100(1-\alpha)$% confidence level. If comparing M_1 to the required MTBF produces an accept decision for the test item, then on the average, the test item will function as required at a $100(1-\alpha)$% confidence level.

d. The method used to compute M_1 and M_2 is dependent upon the type of test conducted; i.e., time terminated or failure terminated. The time terminated test produces a more conservative estimate for the LCL of the population MTBF since a safety factor of one is added to the number of failures which occurred.

SUPPLEMENTARY

INFORMATION

COMPARISON OF LIMITS

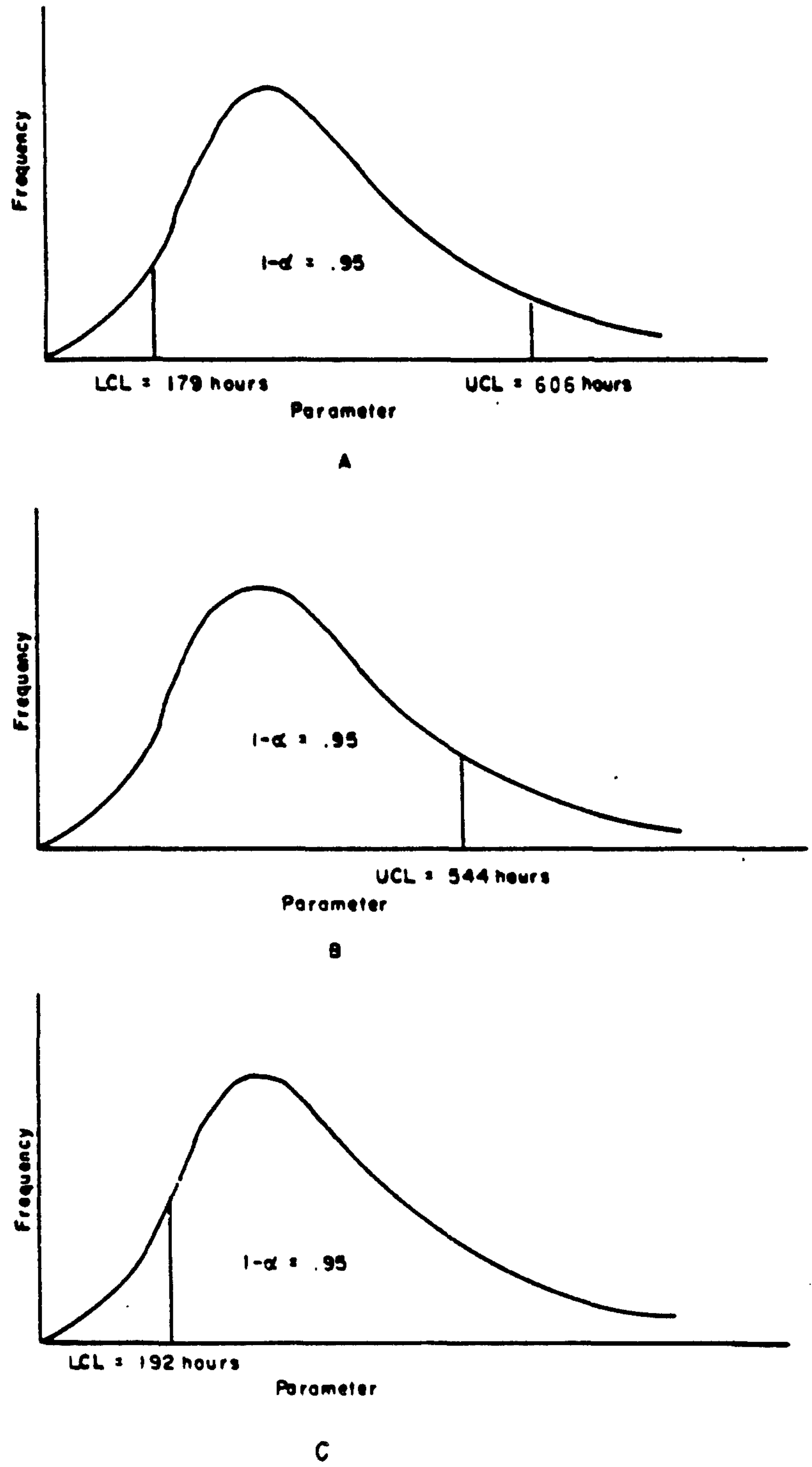


Figure 15

10.2.2 APPLICATION OF THE EXPONENTIAL DISTRIBUTION

10.2.2.1 OBJECTIVE

To determine the reliability for those items which demonstrate an exponential lifetime to failure.

10.2.2.2 DATA REQUIRED

The mission (operational) profile (MP), T_t, and f.

10.2.2.3 PROCEDURE

a. Choose the desired confidence level.

b. Compute the MTBF (see paragraph 10.2.1.1.3, page 134).

c. Use Table B-20, page 2-128, to obtain the $LF_{1-\alpha}$ for:

(1) f+1 d.f., if a time terminated test.
(2) f d.f., if a failure terminated test

d. Compute the LCL as follows:

(1) For a time terminated test, multiply step c by T_t and divide by the quantity (f+1).
(2) For a failure terminated test, multiply step b by step c.

NOTE: To maintain accuracy, the six decimal number found in Table B-20, page 2-128.

e. Compute R as follows:

(1) Divide MP by step d.
(2) Use Table B-22, page 2-130, to obtain e raised to the negative power of step (1).

f. Conclude that ρ is equal to or greater than R at the desired confidence level.

g. If R is equal to or greater than ρ_0, decide that ρ is equal to or greater than ρ_0; otherwise, there is no reason to believe ρ is equal to or greater than ρ_0 at the desired confidence level.

10.2.2.4 EXAMPLE.

Given:

$\rho_0 = .75$

Sample data at Table A-6c, page 1-21.

Procedure:

a. Choose the confidence level $(1-\alpha)$.

b. Compute:

$$MTBF = \frac{T_t}{f}$$

c. Use Table B-20, Page 2-128, to obtain $LF_{1-\alpha}$ for:

(1) f+1 d.f., if a time terminated test.
(2) f d.f., if a failure terminated test.

d. Compute:

(1) For a time terminated test:

$$LCL = \frac{(LF_{1-\alpha})(T_t)}{f+1}$$

(2) For a failure terminated test:

$$LCL = (LF_{1-\alpha})(MTBF)$$

e. Compute:

$$R = e^{-\frac{MP}{LCL}}$$

f. Conclude that $\rho \geq R$ at a $100(1-\alpha)\%$ confidence level.

g. If $R \geq \rho o$, decide that $\rho \geq \rho o$; otherwise, there is no reason to believe $\rho \geq \rho o$ at a $100(1-\alpha)\%$ confidence level.

Example:

a. $\alpha = .05$
$1-\alpha = .95$

b. $MTBF = \frac{3752}{12}$

$= 312.67$

$= 313$

c. $LF_{1-\alpha}$ for 13 d.f. = .668380

d. Since the example is a time terminated test,

$$LCL = \frac{(.668380)(3752)}{12+1}$$

$$= \frac{2507.761760}{13}$$

= 192 hours

e.

$$R = e^{\frac{-48}{192.90}}$$

$$= e^{-.249}$$

$$= .7796$$

f. Conclude that $\rho \geq .77$ at a 95% confidence level.

g. Since $.77 \geq .75$, decide that $\rho \geq .75$, at a 95% confidence level.

(6) Compute:

$$f_r = \frac{1}{MTBF}$$

(6)

$$f_r = \frac{1}{224.07} = .0044629$$

(7) Compute:

$$f_{rt} = \frac{1}{MTBF_t}$$

(7) $$f_{rt} = \frac{1}{173.80} = .0057536$$

(8) Compute:

$$U = f_{rt} - f_r$$

(8) $$U = .0057536 - .0044629 = .0012908$$

(9) Compute:

$$V = \frac{\ln\left(\frac{MTBF}{MTBF_t}\right)}{U}$$

(9)

$$V = \frac{\ln\left(\frac{224.07}{173.80}\right)}{.0012908} = \frac{\ln 1.29}{.0012908} = \frac{.25404}{.0012908} = 196.81$$

(10) Compute:

$$T_{ACCEPT} = \frac{a}{U} + V(f)$$

(10) $$T_{ACCEPT} = \frac{2.773}{.0012908} + 196.81(f) = 2148.2+196.81(f)$$

When f = 0, T_{ACCEPT} = 2148

When f = 7, T_{ACCEPT} = 3530

Plot the points (0,2150) and (7,3530) to determine the accept boundary line.

(11) Compute:

$$T_{REJECT} = \frac{b}{U} + V(f)$$

(11) $$T_{REJECT} = \frac{-1.558}{.0012908} + 196.81(f) = -1207+196.81(f)$$

When f = 0, T_{REJECT} = -1207

When f = 7, T_{REJECT} = 170

Plot the points (0,-1207) and (7,170) to determine the reject boundary line.

(12) If the two lines are not parallel, check the computations and plotted points

(12)

b. Plot the sample data on the sequential graph as follows:

Given:

Requirements and boundary lines from step a.
Sample data at Table A-6e, page 1-22.

Procedure:	Example:
(1) Plot the cumulative operating hours at appropriate intervals (f_i, T_i).	(1) (a) (1,175) (b) (2,490) See Table A-6e, page 1-22 for complete list.
(2) After plotting each point, decide to accept, reject, or continue testing the test item.	(2) For failures 1 through 4, decide to continue testing. Decide to accept the test item when T = 3133 hours and f = 5 since the accept boundary line is crossed. See Figure 16, page 148.

10.2.3.5 ANALYSIS

a. The sequential method generally minimizes testing time and N due to the fact that a decision to accept or reject is made as soon as possible after the first failure. Since all failures are not necessarily chargeable failures, decisions will be altered if certain failures are not counted. If the project officer ignores a failure, the probability of accepting an unacceptable item is increased. Therefore, the project officer must carefully decide what constitutes a failure (see paragraph 4.2, page 2).

b. Due to the advantages just discussed, the sequential method should be used whenever possible (see paragraph 10.2c, page 134).

10.3 COMBINED RELIABILITY

If a number of components of a system are connected in such a way that the failure of any one component causes a failure of the system, then these components are considered to be functionally in series. The reliability of such a system can be determined by the following method.

10.3.1 OBJECTIVE

a. Case I: To determine the reliability of a system based on the individual reliabilities of its components.

b. Case II: To determine the reliability of an individual component of a system.

10.3.2 DATA REQUIRED

a. Case I: N and f for each component.

b. Case II: N and f for the component tested.

10.3.3 PROCEDURE

a. Case I: Reliability of independent serial systems.

(1) Choose the desired confidence level.
(2) Compute the point estimate reliability (R_{PE}) as follows:

(a) Subtract f from N for each component.
(b) Divide step (a) by N for each respective component.
(c) Multiply the results of step (b) by each other.

(3) Compute the system failures (f_s) as follows:

(a) Subtract step (2) from 1.
(b) Multiply step (a) by the minimum N of the components.

(4) Compute the LCL using Case II of paragraph 10.1.1.4, page 122.

NOTE: When using f_s to determine d.f.$_1$ and d.f.$_2$, round off the results.

(5) Conclude that the ρ for the system is the LCL at the desired confidence level.

b. Case II: Reliability of a component.

See Case II(3) of paragraph 10.1.1.4, page 122.

10.3.4 EXAMPLE

a. Case I: Reliability of independent serial systems.

Given:
Sample data at Table A-6g, page 1-24.

Procedure:	Example:
(1) Choose the confidence level (1-α).	(1) $\alpha = .10$ $1-\alpha = .90$
(2) Compute: $R_{PE} = \prod \frac{N_i - f_i}{N_i}$	(2) $R_{PE} = \left(\frac{90-2}{90}\right)\left(\frac{90-4}{90}\right)\left(\frac{45-1}{45}\right)\left(\frac{45-3}{45}\right)$ $= (.9778)(.9556)(.9778)(.9333)$ $= (.9344)(.9126)$ $= .8527$
(3) Compute: $f_s = N_{min}(1-R_{PE})$	(3) $f_s = 45(1-.8527)$ $= 45(.1473)$ $= 6.628$

(4) Compute:

$d.f._1 = 2(f_s)+2$

$d.f._2 = 2(N_{min})-2(f_s)$

$$LCL = \frac{1}{1+\left(\frac{f_s+1}{N_{min}-f_s}\right) F_{1-\alpha}}$$

See paragraph 10.1.1.4, Case II, page 122, for details.
NOTE: N is the minimum N of the components and f is f_s.

(5) Conclude that the ρ for the system is the LCL at a $100(1-\alpha)\%$ confidence level.

(4) $d.f._1 = 2(6.628)+2$
$= 13.256+2$
$= 15.256$
$= 15$

$d.f._2 = 2(45)-2(6.628)$
$= 90-13.256$
$= 76.744$
$= 76$

NOTE: Use 15 and 70 in F tables.

$$LCL = \frac{1}{1+\left(\frac{6.628+1}{45-6.628}\right) F_{.90}}$$

$$= \frac{1}{1+\left(\frac{7.628}{38.37}\right)(1.58)}$$

$$= \frac{1}{1+(.1988)(1.58)}$$

$$= \frac{1}{1.3141}$$

$= .7610$
$= .76$

(5) Conclude that the ρ for the system is .76 at a 90% confidence level.

b. Case II: Reliability of a component.
See Case II (3) of paragraph 10.1.1.4, page 122.

10.3.5 ANALYSIS

a. Case I. The point estimate (achieved) reliability of an independent serial system is determined by multiplying together the point estimate reliability of the components. The number of system failures is determined by multiplying the minimum sample size of the components by the quantity $(1-R_{PE})$. The R of the system is then determined as a LCL (see paragraph 10.1.1.4, Case II, page 122). The project officer will compare this R to ρ_0 to determine whether $\rho \geq \rho_0$ at a $100(1-\alpha)\%$ confidence level.

b. Case II. R at a $100(1-\alpha)\%$ confidence level is computed as a LCL. The project officer will compare this R to ρ_0 to determine whether $\rho \geq \rho_0$ at a $100(1-\alpha)\%$ confidence level.

11. MAINTENANCE EVALUATION

11.1 MAINTENANCE RATIO

11.1.1 OBJECTIVE

To determine the maintenance ratio (MR) for the test item.

11.1.2 DATA REQUIRED

Records of active maintenance manhours and T_t.

11.1.3 PROCEDURE

a. Sum the active maintenance manhours to obtain the total maintenance manhours (TM).

b. Sum the hours of operation to obtain T_t.

c. Divide TM by T_t.

11.1.4 EXAMPLE

Given:
Sample data at Table A-7b, page 1-26.

Procedure:	Example:
a. Compute: $TM = \Sigma$ active maintenance manhours.	a. $TM = 9.25$ manhours
b. Compute: $T_t = \Sigma$ operating time.	b. $T_t = 109.75$ hours
c. Compute: $MR = \frac{TM}{T_t}$	c. $MR = \frac{9.25}{109.75}$ $= .084282$ $= .0842$ manhours per hour

11.1.5 ANALYSIS

The MR indicates the amount of active maintenance manhours required per operating hour for the test item.

11.2 MAINTAINABILITY

11.2.1 OBJECTIVE

To determine the maintainability ($\underline{M}$).

11.2.2 DATA REQUIRED

a. Active maintenance time (AMT), the number of maintenance actions (MA), the allowable maintenance action time (ω).

b. Time to repair (RT), ω, and f.

11.2.3 PROCEDURE

a. Case I: Maintainability, based on all MA's.

(1) Sum the AMT's (ΣAMT).
(2) Divide step (1) bv MA to obtain the mean active maintenance time ($\bar{M}$).

(3) Determine the maintenance action rate (AR) by dividing 1 by step (2).

(4) Compute M as follows:

(a) Multiply step (3) by ω.

(b) Raise the exponential (e) to the negative power of step (a) (see Table B-22, page 2-130).

(c) Subtract step (b) from 1.

(5) Conclude that the M is the probability of completing an MA of the population within prescribed limits based on the sample.

b. Case II: Maintainability, based only on failures.

(1) Compute Y, an intermediate value, as follows:

(a) If f is equal to or less than 3, compute:

1. Use Table B-22, page 2-130 to obtain e raised to the negative power of f.

2. Subtract step 1 from 1.

(b) If f is greater than 3, set Y equal to 1.

(2) Sum the repair time (ΣRT).

(3) Divide step (2) by f to obtain the mean time to repair (MTTR).

(4) Divide 1 by step (3) to obtain the repair rate (RR).

(5) Compute U, an intermediate value, as follows:

(a) Multiply step (4) by ω.

(b) Use Table B-22, page 2-130, to obtain e raised to the negative power of step (a).

(c) Subtract step (b) from 1.

(6) Multiply step (1) by step (5) to obtain M.

(7) Conclude that the M is the probability of completing a failure within prescribed limits based on the sample.

11.2.4 EXAMPLE

a. Case I: Maintainability, based on all MA's.
Given:
ω = .5 hour
MA = 22
Sample data at Table A-7a, page 1-25.

Procedure:	Example:
(1) Compute: ΣAMT	(1) ΣAMT = 16.9 hours
(2) Compute: $\bar{M} = \frac{\Sigma AMT}{MA}$	(2) $\bar{M} = \frac{16.9}{22}$ = .7682 = .77 hour per action.

(3) Compute:

$$AR = \frac{1}{\underline{M}}$$

(4) Compute:

$$\underline{M} = 1 - e^{-(AR)(\omega)}$$

Use Table B-22, page 2-130.

(5) Conclude that the $\underline{M}$ is the probability of completing an MA within prescribed limits based on the sample.

(3) $AR = \frac{1}{.7682}$

$= 1.3148$

$= 1.31$ actions per hr.

(4)

$$\underline{M} = 1 - e^{-(1.3148)(.5)}$$

$= 1 - e^{-.657}$

$= 1 - .5184$

$= .4816$

$= .48$

(5) Conclude that .48 is the probability of completing an MA in .5 hour or less based on the sample.

b. Case II: Maintainability, based only on failures

Given:

$\omega = .5$ hours

$f = 5$

Sample data at Table A-7a, page 1-25.

Procedure:

(1) Compute:

(a) If $f \leq 3$, compute:

$$Y = 1 - e^{-f}$$

(b) If $f > 3$, assume:

$$Y = 1$$

(2) Compute:

ΣRT

(3) Compute:

$$MTTR = \frac{\Sigma RT}{f}$$

(4) Compute:

$$RR = \frac{1}{MTTR}$$

(5) Compute:

$$U = 1 - e^{-(RR)(\omega)}$$

Use Table B-22, page 2-130.

Example:

(1) Since $5 > 3$,

$Y = 1$

(2) $\Sigma RT = 4.8$ hours

(3) $MTTR = \frac{4.8}{5}$

$= .960$ hr. per failure

(4) $RR = \frac{1}{.960}$

$= 1.04$ failures per hr. of repair

(5)

$$U = 1 - e^{-(1.04)(.5)}$$

$= 1 - e^{-.520}$

$= 1 - .5945$

$= .4055$

$= .41$

(6) Compute:

$\underline{M} = Y(U)$

(7) Conclude that the $\underline{M}$ is the probability of completing a failure within prescribed limits based on the sample.

(6) $\underline{M} = (1)\ (.41)$

$= .41$

(7) Conclude that .41 is the probability of completing a failure in .5 hour or less based on the sample.

11.2.5 ANALYSIS

Maintainability is a characteristic of design and installation which is expressed as the probability than an item will be retained in or restored to a specified condition within a given period of time, when the maintenance is performed in accordance with prescribed procedures and resources. The maintainability increases exponentially with time for a given maintenance action rate. The greater the time available to perform a MA, the greater will be the probability of successfully performing the maintenance action.

11.3 AVAILABILITY

Availability is a measure of the degree to which an item is in the operable and committable state when the mission is called for at an unknown (random) point in time. Availability actually consists of two components: maintainability and reliability. Poor reliability can be offset by correspondingly improved maintainability. For test purposes availability is broken down into three types which are discussed in the following paragraphs.

11.3.1 INHERENT AVAILABILITY

11.3.1.1 OBJECTIVE

To determine the inherent availability (A_i) of the test item as an estimate of the population availability.

11.3.1.2 DATA REQUIRED

T_t, f, and RT's.

11.3.1.3 PROCEDURE

a. Compute MTBF (see paragraph 10.2.1.1.3, page 134).

b. Compute the mean time to repair (MTTR) as follows:

(1) Sum the RT's (ΣRT).
(2) Divide step (1) by f.

c. Compute A_i as follows:

(1) Add step a to step b.
(2) Divide step a by step (1).

d. Conclude that the inherent availability of the sample is $100(A_i)\%$.

11.3.1.4 EXAMPLE

Given:

Sample data at Table A-7b, page 1-26.

Procedure:

a. Compute:

$$MTBF = \frac{T_t}{f}$$

b. Compute:

$$MTTR = \frac{\Sigma RT}{f}$$

c. Compute:

$$A_i = \frac{MTBF}{MTBF+MTTR}$$

d. Conclude that the inherent availability of the sample is $100(A_i)\%$.

Example:

a. MTBF = 109.8/3
= 36.600 hours

b. MTTR = 4.8/3
= 1.600 hours per failure

c. $$A_i = \frac{36.600}{36.600+1.600} = \frac{36.600}{38.200} = .95811 = .958$$

d. Conclude that the inherent availability of the sample is 95.8%.

11.3.1.5 ANALYSIS

A_i is the probability that a system or equipment, when used under stated conditions without consideration for any scheduled or preventive maintenance in an ideal support environment; i.e., when all tools, parts, manpower, and manuals are available, will operate satisfactorily at any given time. A_i excludes ready time, preventive maintenance downtime, supply downtime, and waiting or administrative downtime. A_i is a prediction of the population inherent availability.

11.3.2 ACHIEVED AVAILABILITY

11.3.2.1 OBJECTIVE

To determine the achieved availability (A_a) of the test item.

11.3.2.2 DATA REQUIRED

T_t, MA, and AMT.

11.3.2.3 PROCEDURE

a. Divide T_t by MA to obtain the mean time between maintenance (MTBM)

b. Compute $\overline{M}$ as follows:

(1) Sum the AMT's.
(2) Divide step (1) by MA.

c. Compute A_a as follows:

(1) Add step a and step b.
(2) Divide step a by step (1).

d. Conclude that the achieved availability of the sample is $100(A_a)\%$.

11.3.2.4 EXAMPLE

Given:
Sample data at Table A-7b, page 1-26.

Procedure:

a. Compute:

$$MTBM = \frac{T_t}{MA}$$

b. Compute:

$$\overline{M} = \frac{\Sigma AMT}{MA}$$

c. Compute:

$$A_a = \frac{MTBM}{MTBM+\overline{M}}$$

d. Conclude that the achieved availability of the sample is $100(A_a)\%$.

Example:

a. MTBM = 109.8/7
= 15.686
= 15.7 hr. per MA

b. $\overline{M}$ = 6.8/7
= .971
= .97 Active maintenance time per MA

c. $A_a = \frac{15.686}{15.686+.971}$

$= \frac{15.686}{16.657}$

= .941706

= .94

d. Conclude that the achieved availability of the sample is 94%.

11.3.2.5 ANALYSIS

A_a is the probability that a system or equipment, when used under stated conditions in an ideal support environment, will operate satisfactorily at any given time. A_a is the sample's achieved availability and excludes supply downtime and waiting or administrative downtime.

11.3.3 OPERATIONAL AVAILABILITY

11.3.3.1 OBJECTIVE

To determine the operational availability (A_o) of the test item.

11.3.3.2 DATA REQUIRED

T_t, MA, AMT, and delay time (supply and administrative downtime).

11.3.3.3 PROCEDURE

a. Compute MTBM (see paragraph 11.3.2.3, page 159).

b. Sum the AMT's and the delay time.

c. Divide step b by MA to obtain the mean downtime (MDT).

d. Compute A_o as follows:

(1) Add step a and step c.
(2) Divide step a by step (1).

e. Conclude that the operational availability of the sample in a test support environment is $100(A_o)\%$.

11.3.3.4 EXAMPLE

Given:
Sample data at Table A-7b, page 1-26.

Procedure:

a. Compute:

$$MTBM = \frac{T_t}{MA}$$

b. Compute:

ΣAMT

Σ delay time

c. Compute:

$$MDT = \frac{\Sigma AMT + \Sigma \text{ delay time}}{MA}$$

d. Compute:

$$A_o = \frac{MTBM}{MTBM + MDT}$$

e. Conclude that the operational availability of the sample in a test support environment is $100(A_o)\%$.

Example:

a. MTBM = 109.8/7
= 15.686
= 15.7 hrs. per MA

b. $\Sigma AMT = 6.8$

Σ delay time = 8.8

$\Sigma AMT + \Sigma$ delay time = 6.8 + 8.8 = 15.6

c. $MDT = \frac{15.6}{7}$
= 2.228
= 2.23 hrs. per down

d. $A_o = \frac{15.686}{15.686+2.228}$
$= \frac{15.686}{17.914}$
= .875628
= .876

e. Conclude that the operational availability of the sample in a test support environment is 87.6%.

11.3.3.5 ANALYSIS

A_o is the probability that a system or equipment, when used under stated conditions in a real support environment, will operate satisfactorily at any given time. A_o includes ready time, maintenance downtime, preventive maintenance downtime, supply downtime, and waiting or administrative downtime.

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TABLE A-1a

BIVARIATE NORMAL DISTRIBUTION RAW DATA

READING NUMBER	EASTING	NORTHING
1	2500	3218
2	2601	3305
3	2575	3279
4	2581	3221
5	2560	3250
6	2590	3261
7	2565	3249
8	2575	3250
9	2560	3239
10	2580	3251
11	2576	3270
12	2553	3251
13	2550	3280
14	2570	3245
15	2549	3278

TABLE A-1b

BIVARIATE NORMAL DISTRIBUTION GROUPED DATA

NORTH \ EAST	2500-2519	2520-2539	2540-2559	2560-2579	2580-2599	2600-2619	TOTAL
3300-3319						1	1
3280-2399			1				1
3260-3279			1	2	1		4
3240-3259			1	4	1		6
3220-3239				1	1		2
3200-3219	1						1
TOTAL	1	0	3	7	3	1	

TABLE A-2a

MEAN

TEST:

Prepare for action under daylight condition.

TIME (minutes)	Δ	Δ^2
89.3	2.883	8.312
90.4	3.983	15.864
86.0	-.417	.174
83.6	-2.817	7.936
84.4	-2.017	4.068
86.1	-.317	.100
86.0	-.417	.174
88.0	1.583	2.506
86.7	.283	.080
87.4	.983	.966
86.1	-.317	.100
83.0	-3.417	11.676

N = 12

$\overline{X}$ = 86.417

= 86.4 min.

$\Sigma\Delta^2$ = 51.9567

s^2 = 4.723

s = 2.173

= 2.2 min.

TABLE A-5b

PE: STANDARD DEVIATION

READING NUMBER	READING (meters)	Δ	Δ^2
1	1248	-10.00	100.0
2	1100	-158.00	24,964.0
3	1260	2.00	4.0
4	1300	42.00	1,764.0
5	1260	2.00	4.0
6	1234	-24.00	576.0
7	1287	29.00	841.0
8	1275	17.00	289.0
9	1290	32.00	1,024.0
10	1280	22.00	484.0
11	1225	-33.00	1,089.0
12	1325	67.00	4,489.0
13	1223	-35.00	1,225.0
14	1299	41.00	1,681.0
15	1268	10.00	100.0
16	1254	-4.00	16.0

$N = 16$

$\bar{X} = 1258.00$

$= 1258$ meters

$\Sigma\Delta^2 = 38,650.0$

$s^2 = 2,576.67$

$s = 50.76$

$= 51$ meters

TABLE A-5c

PE SUCCESSIVE DIFFERENCES

READING NUMBER	READING (meters)	X_d	X_d^2
1	1248		
2	1100	148	21,904
3	1260	-160	25,600
4	1300	-40	1,600
5	1260	40	1,600
6	1234	26	676
7	1287	-53	2,809
8	1275	12	144
9	1290	-15	225
10	1280	10	100
11	1225	55	3,025
12	1325	-100	10,000
13	1223	102	10,404
14	1299	-76	5,776
15	1268	31	961
16	1254	14	196

$N = 16$

$\Sigma X_d^2 = 85,020$

$s_d^2 = 5,668.00$

$s_d = 75.29$

$= 75$

TABLE A-5f continued

READING NUMBER	N	ΔN	ΔN^2
1	46530	-268.71	72,205
2	46516	-282.71	79,925
4	45971	-827.71	685,104
5	46831	32.29	1,043
6	46972	173.29	30,029
7	47015	216.29	46,781
8	46505	-293.71	86,266
9	47230	431.29	186,011
10	46993	194.29	37,749
11	47020	221.29	48,969
12	47044	245.29	60,167
13	46845	46.29	2,143
14	46570	-228.71	52,308
15	47140	341.29	116,479

$N_1 = 14$

$\overline{NORTH} = 46798.71$

$= 46799$

$\Sigma \Delta N^2 = 1,505,179$

$s_N^2 = 115,783.0$

$s_N = 340.27$

$= 340$

TABLE A-6a

SEQUENTIAL TESTING: SUCCESS - FAILURE

FAILURE	SAMPLES TESTED	COORDINATES
1	30	(30,1)
2	75	(75,2)
3	110	(110,3)
4	160	(160,4)

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TABLE A-6g

COMBINED RELIABILITY

COMPONENT NUMBER	SAMPLE SIZE	FAILURES
1	90	2
2	90	4
3	45	1
4	45	3

TABLE B-7 continued

PERCENTILES OF THE χ^2 DISTRIBUTION

d.f. \ $1-\alpha$	.40	.30	.25	.20	.10	.05	.025	.01	.005	.001	.0005
81	83.6	87.2	89.2	91.5	97.7	103.0	107.8	113.5	117.5	126.1	129.5
82	84.6	88.2	90.2	92.5	98.8	104.1	108.9	114.7	118.7	127.3	130.8
83	85.6	89.2	91.3	93.6	99.9	105.3	110.1	115.9	119.9	128.6	132.0
84	86.6	90.3	92.3	94.7	101.0	106.4	111.2	117.1	121.1	129.8	133.3
85	87.7	91.3	93.4	95.7	102.1	107.5	112.4	118.2	122.3	131.0	134.5
86	88.7	92.4	94.4	96.8	103.2	108.6	113.5	119.4	123.5	132.3	135.8
87	89.7	93.4	95.5	97.9	104.3	109.8	114.7	120.6	124.7	133.5	137.0
88	90.7	94.4	96.5	98.9	105.4	110.9	115.8	121.8	125.9	134.7	138.3
89	91.7	95.5	97.6	100.0	106.5	112.0	117.0	122.9	127.1	136.0	139.5
90	92.8	96.5	98.6	101.1	107.6	113.1	118.1	124.1	128.3	137.2	140.8
91	93.8	97.6	99.7	102.1	108.7	114.3	119.3	125.3	129.5	138.4	142.0
92	94.8	98.6	100.7	103.2	109.8	115.4	120.4	126.5	130.7	139.7	143.3
93	95.8	99.6	101.8	104.2	110.9	116.5	121.6	127.6	131.9	140.9	144.5
94	96.8	100.7	102.8	105.3	111.9	117.6	122.7	128.8	133.1	142.1	145.8
95	97.9	101.7	103.9	106.4	113.0	118.8	123.9	130.0	134.2	143.3	147.0
96	98.9	102.8	104.9	107.4	114.1	119.9	125.0	131.1	135.4	144.6	148.2
97	99.9	103.8	106.0	108.5	115.2	121.0	126.1	132.3	136.6	145.8	149.5
98	100.9	104.8	107.0	109.5	116.3	122.1	127.3	133.5	137.8	147.0	150.7
99	101.9	105.9	108.1	110.6	117.4	123.2	128.4	134.6	139.0	148.2	151.9
100	102.9	106.9	109.1	111.7	118.5	124.3	129.6	135.8	140.2	129.4	153.2

For larger degrees of freedom:

$$\chi^2_{1-\alpha} = \frac{1}{2}\left(Z_\alpha + \sqrt{2(d.f. -1)}\right)^2$$ approximately, where d.f. = degrees of freedom and Z_α is given in Table B-4.

TABLE B-8*

PERCENTILES OF THE F DISTRIBUTION

$F_{.50}$

d.f.₂ \ d.f.₁	1	2	3	4	5	6	7	8	9	10	12	15
1	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03	2.04	2.07	2.09
2	.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33	1.34	1.36	1.38
3	.585	.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17	1.18	1.20	1.21
4	.549	.828	1941	1.00	1.04	1.06	1.08	1.09	1.10	1.11	1.13	1.14
5	.528	.799	.907	.965	1.00	1.02	1.04	1.05	1.06	1.07	1.09	1.10
6	.515	.780	.886	.942	.977	1.00	1.02	1.03	1.04	1.05	1.06	1.07
7	.506	.767	.871	.926	.960	.983	1.00	1.01	1.02	1.03	1.04	1.05
8	.499	.757	.860	.915	.948	.971	.988	1.00	1.01	1.02	1.03	1.04
9	.494	.749	.852	.906	.939	.962	.978	.990	1.00	1.01	1.02	1.03
10	.490	.743	.845	.899	.932	.954	.971	.983	.992	1.000	1.010	1.021
11	.486	.739	.840	.893	.926	.948	.964	.977	.986	.994	1.004	1.015
12	.484	.735	.835	.888	.921	.943	.959	.972	.981	.989	1.000	1.010
13	.482	.733	.832	.884	.917	.939	.958	.969	.978	.985	.995	1.006
14	.480	.730	.829	.881	.914	.936	.955	.966	.974	.981	.992	1.003
15	.478	.726	.826	.878	.911	.933	.948	.960	.970	.977	.989	1.000
16	.477	.724	.824	.876	.908	.930	.945	.959	.969	.975	.986	.997
17	.476	.723	.822	.874	.906	.928	.942	.958	.966	.973	.984	.994
18	.474	.722	.820	.872	.904	.926	.940	.956	.964	.971	.981	.992
19	.473	.720	.818	.870	.902	.924	.939	.953	.962	.969	.980	.990
20	.472	.718	.816	.868	.900	.922	.938	.950	.959	.966	.977	.989
21	.471	.717	.815	.866	.899	.921	.935	.949	.958	.966	.976	.987
22	.470	.716	.814	.865	.897	.919	.934	.948	.957	.965	.975	.986
23	.469	.715	.813	.864	.895	.918	.933	.946	.956	.963	.974	.984
24	.469	.714	.812	.863	.895	.917	.932	.944	.953	.961	.972	.983
25	.469	.713	.811	.862	.894	.916	.931	.943	.952	.961	.971	.982
26	.468	.712	.810	.861	.893	.915	.930	.942	.951	.960	.970	.981
27	.468	.711	.809	.860	.892	.914	.929	.941	.950	.959	.970	.980
28	.467	.710	.809	.859	.891	.914	.928	.940	.950	.958	.969	.979
29	.466	.709	.808	.859	.891	.913	.927	.940	.949	.958	.968	.978
30	.466	.709	.807	.858	890	.912	.927	.939	.948	.955	.966	.978
40	.463	.705	.802	.854	.885	.907	.922	.934	.943	.950	.961	.972
50	.462	.703	.800	.851	.883	.904	.918	.931	.940	.948	.959	.969
60	.461	.701	.798	.849	.880	.901	.917	.928	.937	.945	.956	.967
70	-	-	-	.848	.879	.900	.915	.927	.936	.945	.955	.965
80	-	-	-	.847	.878	.899	.914	.926	.935	.944	.954	.964
90	-	-	-	.846	.877	.898	.913	.925	.934	.943	.953	.963
100	-	-	-	.845	.876	.897	.912	.924	.933	.942	.952	.962
120	.458	.697	.793	.844	.875	.896	.912	.923	.932	.939	.950	.961
500	.455	.693	.789	.839	.870	.891	.907	.919	.928	.937	.947	.958

*See Note on page 2-34.

TABLE B-8 continued

PERCENTILES OF THE F DISTRIBUTION

F .99

d.f.2 \ d.f.1	20	25	30	40	50	60	70	80	90	100	120	500
1	6209	6235	6261	6287	6300	6313	-	-	-	-	6339	6366
2	99.45	99.46	99.47	99.47	99.48	99.48	-	-	-	-	99.49	99.50
3	26.69	26.60	26.50	26.41	26.37	26.32	-	-	-	-	26.22	26.13
4	14.02	13.93	13.84	13.75	13.70	13.65	-	-	-	-	13.56	13.46
5	9.55	9.47	9.38	9.29	9.25	9.20	-	-	-	-	9.11	9.02
6	7.40	7.31	7.23	7.14	7.10	7.06	-	-	-	-	6.97	6.88
7	6.16	6.07	5.99	5.91	5.87	5.82	-	-	-	-	5.74	5.65
8	5.36	5.28	5.20	5.12	5.08	5.03	-	-	-	-	4.95	4.86
9	4.81	4.73	4.65	4.57	4.53	4.48	-	-	-	-	4.40	4.31
10	4.41	4.33	4.25	4.17	4.16	4.08	4.10	4.08	4.07	4.06	4.00	3.97
11	4.10	4.02	3.94	3.86	3.84	3.78	3.78	3.77	3.75	3.74	3.69	3.66
12	3.86	3.78	3.70	3.62	3.59	3.54	3.54	3.52	3.50	3.49	3.45	3.41
13	3.66	3.59	3.51	3.43	3.39	3.34	3.33	3.32	3.30	3.29	3.25	3.21
14	3.51	3.43	3.35	3.27	3.23	3.18	3.17	3.15	3.14	3.13	3.09	3.04
15	3.37	3.29	3.21	3.13	3.09	3.05	3.03	3.02	3.00	2.99	2.96	2.90
16	3.26	3.18	3.10	3.02	2.98	2.93	2.92	2.90	2.88	2.87	2.84	2.79
17	3.16	3.08	3.00	2.92	2.88	2.83	2.82	2.80	2.78	2.77	2.75	2.69
18	3.08	3.00	2.92	2.84	2.79	2.75	2.73	2.71	2.70	2.69	2.66	2.60
19	3.00	2.92	2.84	2.76	2.71	2.67	2.65	2.64	2.62	2.61	2.58	2.52
20	2.94	2.86	2.78	2.69	2.65	2.61	2.59	2.57	2.55	2.54	2.52	2.45
21	2.88	2.80	2.72	2.64	2.59	2.55	2.53	2.51	2.49	2.48	2.46	2.39
22	2.83	2.75	2.67	2.58	2.53	2.50	2.47	2.45	2.44	2.43	2.40	2.33
23	2.78	2.70	2.62	2.54	2.49	2.45	2.42	2.40	2.39	2.38	2.35	2.28
24	2.74	2.66	2.58	2.49	2.44	2.40	2.38	2.36	2.34	2.33	2.31	2.24
25	2.70	2.62	2.54	2.45	2.40	2.36	2.34	2.32	2.30	2.29	2.27	2.20
26	2.66	2.59	2.50	2.42	2.37	2.33	2.30	2.28	2.27	2.25	2.23	2.16
27	2.63	2.55	2.47	2.38	2.33	2.29	2.27	2.25	2.23	2.22	2.20	2.12
28	2.60	2.52	2.44	2.35	2.30	2.26	2.24	2.23	2.20	2.19	2.17	2.09
29	2.57	2.49	2.41	2.33	2.27	2.23	2.21	2.19	2.17	2.16	2.14	2.06
30	2.55	2.47	2.39	2.30	2.25	2.21	2.18	2.16	2.15	2.13	2.11	2.03
40	2.37	2.27	2.20	2.11	2.06	2.02	1.99	1.97	1.95	1.94	1.92	1.83
50	2.26	2.16	2.10	2.01	1.95	1.91	1.88	1.86	1.84	1.83	1.80	1.71
60	2.20	2.10	2.03	1.94	1.88	1.84	1.81	1.78	1.76	1.75	1.73	1.63
70	2.15	2.05	1.98	1.89	1.83	1.78	1.75	1.73	1.71	1.70	1.67	1.57
80	2.11	2.01	1.94	1.85	1.79	1.75	1.71	1.69	1.67	1.65	1.63	1.53
90	2.08	1.99	1.92	1.82	1.76	1.72	1.68	1.66	1.64	1.62	1.60	1.49
100	2.06	1.97	1.85	1.80	1.74	1.69	1.66	1.63	1.61	1.60	1.57	1.47
120	2.03	1.93	1.86	1.76	1.70	1.66	1.62	1.60	1.58	1.56	1.63	1.42
500	1.91	1.81	1.74	1.63	1.57	1.52	1.48	1.45	1.43	1.41	1.38	1.23

NOTE: The tables for $F_{.80}$ and $F_{.85}$ were generated using the formula $F \approx e^{2w}$ where:

$$\lambda = \frac{Z_\alpha^2 - 3}{6}$$

$$h = 2\left(\frac{1}{d.f._2-1} + \frac{1}{d.f._1-1}\right)^{-1}$$

$$w = \frac{-Z_\alpha\sqrt{(h+\lambda)}}{h} - \left(\frac{1}{d.f._1-1} - \frac{1}{d.f._2-1}\right)\left(\lambda + \frac{5}{6} - \frac{2}{3h}\right)$$

The approximation is accurate enough for practical uses when $d.f._1$ and $d.f._2 \geq 10$. However, the formula has been used for $d.f._1 < 10$ and $d.f._2 < 10$ for $F_{.80}$ and $F_{.85}$ because no tables were available.

Values other than the ones found in the standard F tables were also supplied using the above formula where possible and if not possible, dashes were left, e.g., $F_{.90}$, $(80, 2)$ = - . In the event of a dash occurring, use the smaller d.f. which appears in the table for computation purposes; e.g., use $F_{.90}$, $(60, 2) = 9.47$ for $F_{.90}$, $(80, 2)$.

TABLE 2-9

FACTORS FOR COMPUTING TWO-SIDED CONFIDENCE LIMITS FOR σ

Degrees of Freedom d.f.	α = .05		α = .01		α = .001	
	B_U	B_L	B_U	B_L	B_U	B_L
1	17.79	.3576	86.31	.2969	844.4	.2480
2	4.859	.4581	10.70	.3879	33.29	.3291
3	3.183	.5178	5.449	.4453	11.65	.3824
4	2.567	.5590	3.892	.4865	6.938	.4218
5	2.248	.5899	3.175	.5182	5.085	.4529
6	2.052	.6143	2.764	.5437	4.128	.4784
7	1.918	.6344	2.498	.5650	3.551	.5000
8	1.820	.6513	2.311	.5830	3.167	.5186
9	1.746	.6657	2.173	.5987	2.894	.5348
10	1.686	.6784	2.065	.6125	2.689	.5492
11	1.638	.6896	1.980	.6248	2.530	.5621
12	1.598	.6995	1.909	.6358	2.402	.5738
13	1.564	.7084	1.851	.6458	2.298	.5845
14	1.534	.7166	1.801	.6549	2.210	.5942
15	1.509	.7240	1.758	.6632	2.136	.6032
16	1.486	.7308	1.721	.6710	2.073	.6116
17	1.466	.7372	1.688	.6781	2.017	.6193
18	1.448	.7430	1.658	.6848	1.968	.6266
19	1.432	.7484	1.632	.6909	1.925	.6333
20	1.417	.7535	1.609	.6968	1.886	.6397
21	1.404	.7582	1.587	.7022	1.851	.6457
22	1.391	.7627	1.568	.7074	1.820	.6514
23	1.380	.7669	1.550	.7122	1.791	.6568
24	1.370	.7709	1.533	.7169	1.765	.6619
25	1.360	.7747	1.518	.7212	1.741	.6668
26	1.351	.7783	1.504	.7253	1.719	.6713
27	1.343	.7817	1.491	.7293	1.698	.6758
28	1.335	.7849	1.479	.7331	1.679	.6800
29	1.327	.7880	1.467	.7367	1.661	.6841
30	1.321	.7909	1.457	.7401	1.645	.6880
31	1.314	.7937	1.447	.7434	1.629	.6917
32	1.308	.7964	1.437	.7467	1.615	.6953
33	1.302	.7990	1.428	.7497	1.601	.6987
34	1.296	.8015	1.420	.7526	1.588	.7020
35	1.291	.8039	1.412	.7554	1.576	.7052
36	1.286	.8062	1.404	.7582	1.564	.7083
37	1.281	.8085	1.397	.7608	1.553	.7113
38	1.277	.8106	1.390	.7633	1.543	.7141
39	1.272	.8126	1.383	.7658	1.533	.7169
40	1.268	.8146	1.377	.7681	1.523	.7197
41	1.264	.8166	1.371	.7705	1.515	.7223
42	1.260	.8184	1.365	.7727	1.506	.7248
43	1.257	.8202	1.360	.7748	1.498	.7273
44	1.253	.8220	1.355	.7769	1.490	.7279
45	1.249	.8237	1.349	.7789	1.422	.7320
46	1.246	.8253	1.345	.7809	1.475	.7342
47	1.243	.8269	1.340	.7828	1.468	.7364
48	1.240	.8235	1.335	.7847	1.462	.7386
49	1.237	.8300	1.331	.7864	1.455	.7407
50	1.234	.8814	1.327	.7882	1.449	.7427

Adapted with permission from Biometrika, Vol. 47, (1960), from article entitled "Table for Making Infererences About the Variance of a Normal Distribution" by D. V. Lindley, D.A. East, and P.A. Hamilton

TABLE B-9 continued

FACTORS FOR COMPUTING TWO-SIDED CONFIDENCE LIMITS FOR σ

Degrees of Freedom d.f.	α = .05		α = .01		α = .001	
	B_U	B_L	B_U	B_L	B_U	B_L
51	1.232	.8329	1.323	.7899	1.443	.7444
52	1.229	.8343	1.319	.7916	1.437	.7464
53	1.226	.8356	1.315	.7932	1.432	.7485
54	1.224	.8370	1.311	.7949	1.426	.7503
55	1.221	.8383	1.308	.7964	1.421	.7521
56	1.219	.8395	1.304	.7979	1.416	.7539
57	1.217	.8408	1.301	.7994	1.411	.7556
58	1.214	.8420	1.298	.8008	1.406	.7573
59	1.212	.8431	1.295	.8022	1.402	.7589
60	1.210	.8443	1.292	.8036	1.397	.7605
61	1.208	.8454	1.289	.8050	1.393	.7621
62	1.206	.8465	1.286	.8063	1.389	.7636
63	1.204	.8475	1.283	.8076	1.385	.7651
64	1.202	.8486	1.280	.8088	1.381	.7666
65	1.200	.8496	1.277	.8101	1.377	.7680
66	1.199	.8506	1.275	.8113	1.374	.7694
67	1.197	.8516	1.272	.8125	1.370	.7708
68	1.195	.8525	1.270	.8137	1.366	.7722
69	1.194	.8535	1.268	.8148	1.363	.7735
70	1.192	.8544	1.265	.8159	1.360	.7749
71	1.190	.8553	1.263	.8170	1.356	.7761
72	1.189	.8562	1.261	.8181	1.353	.7774
73	1.187	.8571	1.259	.8191	1.350	.7787
74	1.186	.8580	1.257	.8202	1.347	.7799
75	1.184	.8588	1.255	.8212	1.344	.7811
76	1.188	.8596	1.253	.8222	1.341	.7822
77	1.182	.8604	1.251	.8232	1.338	.7834
78	1.181	.8612	1.249	.8242	1.336	.7845
79	1.179	.8620	1.247	.8252	1.333	.7856
80	1.178	.8627	1.245	.8261	1.330	.7868
81	1.176	.8635	1.243	.8270	1.328	.7878
82	1.176	.8642	1.241	.8279	1.325	.7889
83	1.174	.8650	1.239	.8288	1.323	.7899
84	1.173	.8657	1.238	.8297	1.320	.7909
85	1.172	.8664	1.236	.8305	1.318	.7920
86	1.171	.8671	1.235	.8314	1.316	.7930
87	1.170	.8678	1.233	.8322	1.313	.7939
88	1.168	.8684	1.231	.8331	1.311	.7949
89	1.167	.8691	1.230	.8338	1.309	.7959
90	1.166	.8697	1.228	.8346	1.307	.7968
91	1.165	.8704	1.227	.8354	1.305	.7977
92	1.164	.8710	1.225	.8362	1.303	.7987
93	1.163	.8716	1.224	.8370	1.301	.7996
94	1.162	.8722	1.222	.8377	1.298	.8004
95	1.161	.8729	1.221	.8385	1.297	.8013
96	1.160	.8734	1.219	.8392	1.295	.8022
97	1.159	.8741	1.218	.8399	1.293	.8031
98	1.158	.8746	1.217	.8406	1.291	.8039
99	1.158	.8752	1.216	.8413	1.290	.8047
100	1.157	.8757	1.214	.8420	1.288	.8055

TABLE B-13 continued

CONFIDENCE LIMITS FOR A PROPORTION (TWO SIDED)

n = 29						
f	90%		95%		99%	
11	.225-	.537	.211	.587	.165+	.646
12	.276	.575+	.247	.626	.206	.654
13	.294	.615-	.251	.669	.211	.684
14	.303	.655+	.299	.661	.260	.737
15	.345-	.697	.339	.701	.263	.740
16	.385+	.706	.340	.749	.316	.789-
17	.425-	.724	.374	.753	.346	.794
18	.463-	.775+	.413	.789	.354	.835-
19	.500	.810	.451	.816	.397	.843
20	.537	.811	.500	.834	.438	.868
21	.575+	.865+	.549	.864	.477	.892
22	.615-	.866	.587	.897	.523	.914
23	.655+	.913	.626	.906	.562	.935-
24	.697	.914	.660	.930	.603	.954
25	.721	.938	.701	.951	.646	.970
26	.775+	.961	.749	.971	.684	.985-
27	.810	.982	.789	.988	.737	.995-
28	.865+	.996	.834	.998	.789	1.000
29	.913	1	.897	1	.840	1

N = 30						
f	90%		95%		99%	
11	.219	.524	.205+	.560	.152	.612
12	.265-	.554	.236	.597	.198	.655+
13	.266	.584	.244	.636	.206	.671
14	.295-	.624	.292	.675+	.249	.692
15	.336	.664	.324	.676	.256	.744
16	.376	.705+	.325-	.708	.308	.751
17	.416	.734	.364	.756	.329	.794
18	.446	.735+	.403	.764	.345-	.802
19	.476	.781	.440	.795-	.388	.848
20	.508	.817	.476	.825-	.430	.849
21	.545-	.818	.524	.837	.462	.873
22	.584	.870	.560	.869	.495-	.896
23	.624	.871	.597	.900	.531	.917
24	.664	.916	.636	.909	.570	.937
25	.705+	.917	.675+	.932	.612	.955+
26	.734	.941	.708	.952	.655+	.972
27	.781	.963	.756	.972	.690	.985+
28	.817	.982	.795-	.988	.744	.995-
29	.870	.996	.837	.998	.794	1.000
30	.916	1	.900	1	.848	1

TABLE 2-14

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

If the observed proportion is f/n, enter the table with N and f for an upper one-sided limit. For a lower one-sided limit, enter the table with N and N − f and subtract the table entry from 1.

f	90%	95%	99%	f	90%	95%	99%	f	90%	95%	99%
n = 2				n = 3				n = 4			
0	.684	.776	.900	0	.536	.632	.785-	0	.438	.527	.648
1	.949	.975-	.995-	1	.804	.865-	.941	1	.680	.751	.859
				2	.965+	.983	.997	2	.857	.902	.958
								3	.974	.987	.997
n = 5				n = 6				n = 7			
0	.369	.451	.602	0	.319	.393	.536	0	.280	.348	.482
1	.584	.657	.778	1	.510	.582	.706	1	.453	.521	.643
2	.753	.811	.894	2	.667	.729	.827	2	.596	.659	.764
3	.888	.924	.967	3	.799	.847	.915+	3	.721	.775-	.858
4	.979	.990	.998	4	.907	.937	.973	4	.830	.871	.929
				5	.983	.991	.998	5	.921	.947	.977
								6	.985+	.993	.999
n = 8				n = 9				n = 10			
0	.250	.312	.438	0	.226	.283	.401	0	.206	.259	.369
1	.406	.471	.590	1	.368	.429	.544	1	.337	.394	.504
2	.538	.600	.707	2	.490	.550	.656	2	.450	.507	.612
3	.655+	.711	.802	3	.599	.655+	.750	3	.552	.607	.703
4	.760	.807	.879	4	.699	.749	.829	4	.646	.696	.782
5	.853	.889	.939	5	.790	.831	.895-	5	.733	.778	.850
6	.931	.954	.980	6	.871	.902	.947	6	.812	.850	.907
7	.987	.994	.999	7	.939	.959	.983	7	.884	.913	.952
				8	.988	.994	.999	8	.945+	.963	.984
								9	.990	.995-	.999
n = 11				n = 12				n = 13			
0	.189	.238	.342	0	.175-	.221	.319	0	.162	.206	.298
1	.310	.364	.470	1	.287	.339	.440	1	.268	.316	.413
2	.415+	.470	.572	2	.386	.438	.537	2	.360	.410	.506
3	.511	.564	.660	3	.475+	.527	.622	3	.444	.495-	.588
4	.599	.650	.738	4	.559	.609	.698	4	.523	.573	.661
5	.682	.729	.806	5	.638	.685-	.765+	5	.598	.645+	.727
6	.759	.800	.866	6	.712	.755-	.825+	6	.669	.713	.787
7	.831	.865-	.916	7	.781	.819	.879	7	.736	.776	.841
8	.895+	.921	.957	8	.846	.877	.924	8	.799	.834	.889
9	951	.967	.986	9	.904	.928	.961	9	.858	.887	.931
10	.990	.995+	.999	10	.955-	.970	.987	10	.912	.934	.964
				11	.991	.996	.999	11	.958	.972	.988
								12	.992	.996	.999
n = 14				n = 15				n = 16			
0	.152	.193	.280	0	.142	.181	.264	0	.134	.171	.250
1	.251	.297	.389	1	.236	.279	.368	1	.222	.264	.349
2	.837	.385+	.478	2	.317	.363	.453	2	.300	.344	.430
3	.417	.466	.557	3	.393	.440	.529	3	.371	.417	.503
4	.492	.540	.627	4	.464	.511	.597	4	.439	.484	.569
5	.563	.610	.692	5	.532	.577	.660	5	.504	.548	.630

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TABLE B-14 continued

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

f	90%	95%	99%	f	90%	95%	99%
n = 29				n = 30			
0	.076	.098	.147	0	.074	.095+	.142
1	.128	.153	.208	1	.124	.149	.202
2	.173	.202	.260	2	.168	.195+	.252
3	.216	.246	.307	3	.209	.239	.298
4	.257	.288	.350	4	.249	.280	.340
5	.297	.329	.392	5	.287	.319	.381
6	.335-	.368	.432	6	.325-	.357	.420
7	.372	.406	.470	7	.361	.394	.457
8	.409	.443	.507	8	.397	.430	.493
9	.445+	.479	.542	9	.432	.465+	.527
10	.481	.514	.577	10	.466	.499	.561
11	.515+	.549	.610	11	.500	.533	.594
12	.550	.583	.643	12	.533	.566	.626
13	.583	.616	.674	13	.566	.598	.657
14	.616	.648	.705-	14	.599	.630	.687
15	.649	.680	.734	15	.630	.661	.716
16	.681	.711	.763	16	.662	.692	.744
17	.712	.741	.791	17	.692	.721	.772
18	.743	.771	.818	18	.723	.750	.799
19	.774	.800	.843	19	.752	.779	.824
20	.803	.828	.868	20	.782	.807	.849
21	.832	.855-	.892	21	.810	.834	.873
22	.860	.881	.914	22	.838	.860	.896
23	.888	.906	.935-	23	.865+	.885+	.917
24	.914	.930	.954	24	.891	.909	.937
25	.938	.951	.970	25	.917	.932	.955+
26	.961	.971	.985-	26	.941	.953	.972
27	.982	.988	.995-	27	.963	.972	.985+
28	.996	.998	1.000	28	.982	.988	.995-
				29	.996	.998	1.000

TABLE B-15

TABLE OF ARC SINE TRANSFORMATION FOR PROPORTIONS

$$\theta = 2 \text{ arc sin} \sqrt{P}$$

P	θ	P	θ	P	θ	P	θ
.00	.00	.25	1.05	.50	1.57	.75	2.09
.01	.20	.26	1.07	.51	1.59	.76	2.12
.02	.28	.27	1.09	.52	1.61	.77	2.14
.03	.35	.28	1.12	.53	1.63	.78	2.17
.04	.40	.29	1.14	.54	1.65	.79	2.19
.05	.45	.30	1.16	.55	1.67	.80	2.21
.06	.49	.31	1.18	.56	1.69	.81	2.24
.07	.54	.32	1.20	.57	1.71	.82	2.27
.08	.57	.33	1.22	.58	1.73	.83	2.29
.09	.61	.34	1.25	.59	1.75	.84	2.32
.10	.64	.35	1.27	.60	1.77	.85	2.35
.11	.68	.36	1.29	.61	1.79	.86	2.37
.12	.71	.37	1.31	.62	1.81	.87	2.40
.13	.74	.38	1.33	.63	1.83	.88	2.43
.14	.77	.39	1.35	.64	1.85	.89	2.47
.15	.80	.40	1.37	.65	1.88	.90	2.50
.16	.82	.41	1.39	.66	1.90	.91	2.53
.17	.85	.42	1.41	.67	1.92	.92	2.57
.18	.88	.43	1.43	.68	1.94	.93	2.61
.19	.90	.44	1.45	.69	1.96	.94	2.65
.20	.93	.45	1.47	.70	1.98	.95	2.69
.21	.95	.46	1.49	.71	2.00	.96	2.74
.22	.98	.47	1.51	.72	2.03	.97	2.79
.23	1.00	.48	1.53	.73	2.05	.98	2.86
.24	1.02	.49	1.55	.74	2.07	.99	2.94
						1.00	3.14

TABLE B-21

FACTORS FOR DETERMINING UPPER CONFIDENCE LIMIT FOR THE EXPONENTIAL MEAN LIFE

$UF_{1-\alpha}$

d.f.	.75	.80	.90	.95	.975	.99
1	3.478261	4.484305	9.478673	19.417476	39.525692	99.502488
2	2.083333	2.424242	3.773585	5.625879	8.264463	13.468013
3	1.739130	1.954397	2.727273	3.658537	4.838710	6.880734
4	1.577909	1.742919	2.292264	2.930403	3.669725	4.848485
5	1.483680	1.618123	2.053388	2.538071	3.076923	3.906250
6	1.421801	1.536492	1.904762	2.294455	2.727273	3.361344
7	1.372549	1.478353	1.797176	2.130898	2.486679	3.004292
8	1.344538	1.428571	1.718582	2.010050	2.315485	2.753873
9	1.313869	1.395349	1.651376	1.916933	2.187120	2.567760
10	1.290323	1.369863	1.612903	1.834862	2.085506	2.421308
11	1.279070	1.349693	1.571429	1.788618	2.000000	2.306080
12	1.263158	1.325967	1.528662	1.739130	1.935484	2.201835
13	1.250000	1.313131	1.502890	1.688312	1.884058	2.131148
14	1.233480	1.296296	1.481481	1.656805	1.830065	2.058824
15	1.224490	1.282051	1.456311	1.621622	1.785714	2.000000
16	1.212121	1.274900	1.434978	1.592040	1.748634	1.951220
17	1.205674	1.263940	1.416667	1.566820	1.717172	1.910112
18	1.200000	1.254355	1.406250	1.545064	1.690141	1.875000
19	1.191225	1.245902	1.391941	1.526104	1.679389	1.835749
20	1.186944	1.238390	1.374570	1.509434	1.639344	1.801802
21	1.179775	1.228070	1.363636	1.494662	1.615385	1.772152
22	1.176471	1.222222	1.353846	1.476510	1.594203	1.752988
23	1.170483	1.216931	1.345029	1.464968	1.575342	1.722846
24	1.167883	1.212121	1.337047	1.450151	1.558442	1.702128
25	1.162791	1.207729	1.326260	1.436782	1.543210	1.683502
26	1.158129	1.200924	1.319797	1.428571	1.529412	1.666667
27	1.156317	1.197339	1.310680	1.417323	1.516854	1.646341
28	1.152263	1.191489	1.305361	1.407035	1.505376	1.632653
29	1.148515	1.188524	1.297539	1.397590	1.494845	1.615599
30	1.145038	1.185771	1.290323	1.388889	1.481481	1.600000

Multiply factors of this table by estimated mean time between failures for the upper confidence limit.

$$\text{For } f > 30: \text{ upper factor} = \frac{2f}{\chi^2_{1-\alpha,2f}}$$

TABLE B-22

EXPONENTIAL FUNCTION: e^{-x}

x	0	1	2	3	4	5	6	7	8	9
.00	1.0000	.9990	.9980	.9970	.9960	.9950	.9940	.9930	.9920	.9910
.01	.9900	.9890	.9980	.9870	.9860	.9851	.9841	.9831	.9821	.9811
.02	.9802	.9792	.9782	.9773	.9763	.9753	.9743	.9734	.9724	.9714
.03	.9704	.9695	.9685	.9675	.9665	.9656	.9646	.9637	.9627	.9618
.04	.9608	.9598	.9589	.9579	.9570	.9560	.9550	.9541	.9531	.9522
.05	.9512	.9503	.9493	.9484	.9474	.9465	.9455	.9446	.9436	.9427
.06	.9418	.9408	.9399	.9389	.9380	.9371	.9361	.9352	.9343	.9333
.07	.9324	.9315	.9305	.9296	.9287	.9277	.9268	.9259	.9250	.9240
.08	.9231	.9222	.9213	.9204	.9194	.9185	.9176	.9167	.9158	.9148
.09	.9139	.9130	.9121	.9112	.9103	.9094	.9085	.9076	.9066	.9057
.10	.9048	.9039	.9030	.9021	.9012	.9003	.8994	.8985	.8976	.8967
.11	.8958	.8949	.8940	.8932	.8923	.8914	.8905	.8896	.8887	.8878
.12	.8869	.8860	.8851	.8843	.8834	.8825	.8816	.8807	.8799	.8790
.13	.8781	.8772	.8763	.8755	.8746	.8737	.8728	.8720	.8711	.8702
.14	.8694	.8685	.8676	.8668	.8659	.8650	.8642	.8633	.8624	.8616
.15	.8607	.8598	.8590	.8581	.8573	.8564	.8556	.8547	.8538	.8530
.16	.8521	.8513	.8504	.8496	.8487	.8479	.8470	.8462	.8454	.8445
.17	.8437	.8428	.8420	.8411	.8403	.8395	.8386	.8378	.8369	.8361
.18	.8353	.8344	.8336	.8328	.8319	.8311	.8303	.8294	.8286	.8278
.19	.8270	.8261	.8253	.8245	.8237	.8228	.8220	.8212	.8204	.8195
.20	.8187	.8179	.8171	.8163	.8155	.8146	.8138	.8130	.8122	.8114
.21	.8106	.8098	.8090	.8082	.8073	.8065	.8057	.8049	.8041	.8033
.22	.8025	.8017	.8009	.8001	.7993	.7985	.7977	.7969	.7961	.7953
.23	.7945	.7937	.7929	.7922	.7914	.7906	.7898	.7890	.7882	.7874
.24	.7866	.7858	.7851	.7843	.7835	.7827	.7819	.7811	.7804	.7796
.25	.7788	.7780	.7772	.7765	.7757	.7749	.7741	.7734	.7726	.7718
.26	.7711	.7703	.7695	.7687	.7680	.7672	.7664	.7657	.7649	.7641
.27	.7634	.7626	.7619	.7611	.7603	.7596	.7588	.7581	.7573	.7565
.28	.7558	.7550	.7543	.7535	.7528	.7520	.7513	.7505	.7498	.7490
.29	.7483	.7475	.7468	.7460	.7453	.7445	.7438	.7430	.7423	.7416
.30	.7408	.7401	.7393	.7386	.7379	.7371	.7364	.7357	.7349	.7342
.31	.7334	.7327	.7320	.7312	.7305	.7298	.7291	.7283	.7276	.7269
.32	.7261	.7254	.7247	.7240	.7233	.7225	.7218	.7211	.7204	.7196
.33	.7189	.7182	.7175	.7168	.7161	.7153	.7146	.7139	.7132	.7125
.34	.7118	.7111	.7103	.7096	.7096	.7089	.7082	.7075	.7068	.7054
.35	.7047	.7040	.7033	.7026	.7019	.7012	.7005	.6998	.6991	.6983
.36	.6977	.6970	.6963	.6956	.6949	.6942	.6935	.6928	.6921	.6914
.37	.6907	.6900	.6994	.6887	.6880	.6873	.6866	.6859	.6852	.6845
.38	.6839	.6832	.6825	.6818	.6811	.6805	.6798	.6791	.6784	.6777
.39	.6771	.6764	.6757	.6750	.6744	.6737	.6730	.6723	.6717	.6710