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GYROTHEODOLITES

By

V.Yu. Torochkov

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TECHNICAL TRANSLATION

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NOTICE

FOREWORD

The development of contemporary instrument making has reached such a level that still another group of instruments has been added to the precision instruments designed for geodesy -- gyroscopic instruments for determining the direction of geographic meridians. These instruments have various names, such as, "meridian definer," "surveying gyroscopic compass," "land gyroscopic compass," and others. In connection with the wide use in recent years of gyroscopic instruments for determining the direction of geographic meridians in geodetic work, a name has been settled for these instruments, gyrotheodolite, as an organic combination of gyroscopic measuring instrument with the theodolite indicating system. Gyroscopic compasses which permit the determination of geographic azimuths on a fixed base are combined under this name in the book. Working features of gyrotheodolites are high accuracy of determination of the direction of geographic meridians in a relatively short time, independently of natural and meteorological conditions and at any time of year and day. In other words, the gyrotheodolite method of finding the direction of north is autonomous and an all-weather method. This feature determines varied practical uses of gyrotheodolite measurements. Figure 1 shows schematically several technical fields in which, at the present time, gyrotheodolites are used. It should be noted that industrial use of gyrotheodolites has occurred only in the last few years; however, the first results show prospects for a wide range of uses for them.

The elaboration of the theoretical basis of gyrotheodolites is inseparably linked with the names of leading Soviet and foreign scientists.

The introduction of these instruments into practical measurement work and their specific constructive design was assisted to a significant degree by the work of the Soviet scientists V. N. Lavrov, D. N. Ogloblin, I. B. Zhitomirskiy, Yu. S. Lukovatyy, V. M. Nazarov, A. I. Makarov, N. N. Voronkov, F. A. Sumishin, and foreigners O. Rellensman, G. R. Shvendener, D. Lyuderer, Yu. Merkel', F. Pustay, F. Khalmosh.

Several main designs underlie the constructive basis of gyrotheodolites. The most exploited has been the pendulum gyrotheodolite, in which the

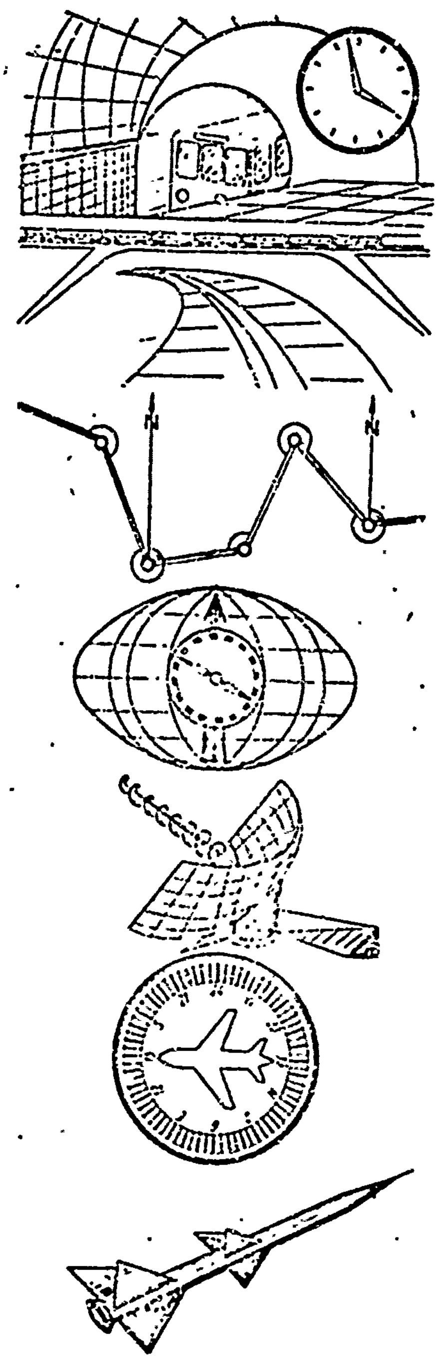

Figure 1. Alternative possible uses of gyrotheodolites

Determination of the azimuths of lines during construction of a subway, in mine shafts, mountain excavations.

Determination of directions during the planning of streets, railroad lines, canals, electric power transmission lines, pipelines, airport runways.

Azimuth control in surveys, azimuthal tying in of aerial photographs, directional control in large-scale mechanical engineering and shipbuilding.

Measurement of magnetic declination, determination of magnetic anomalies, azimuth control of points in geophysical determinations.

Orientation of radio direction finder and television antennas, radio range beacons, optical communication channels.

Control and orientation of navigational aids of airports and aircraft.

Azimuthal control of battle formations of troops, aiming of ballistic missiles.

center of gravity of the gyroscope rotor has been shifted downwards. A lesser development is found in the practical design of two-degrees-of-freedom gyrotheodolites. There are several designs of instruments using a spherical gyroscope, but only laboratory models of gyrostabilized platforms are known, working like the gyrocompass, but of little accuracy. These designs are developed on the basis of the rotary gyroscope. In recent years, a different principle has been used for the purpose of finding the direction of geographical meridians, specifically, instruments with vibrating gyroscope and circular laser configurations. A scheme for development of modern gyrotheodolites is shown in Figure 2.

At the present time, more than fifty types of different gyroscopic instruments for determining the direction of geographical meridians on a fixed base are known. However, accounts of the principles of design and construction of these instruments, as well as the methods for working with them, may be found only in technical descriptions of these instruments, and, partially, in journal articles. The absence of first-hand information on gyrotheodolites in collections, and of methods for working with them, hinders the introduction of this progressive method into geodetic practice.

This attempt to systematize the uncoordinated information on gyrotheodolites and methods of working with them is offered to the reader's attention.

In setting forth the principles of operation and the functional connections of the various gyrotheodolite systems, attention is principally directed to the physical processes taking place in the instrument. Setting forth such material permits geodesist readers to use the book without a preliminary study of special literature. Together with this, the first chapter presents some information on the theory of gyroscopes, which helps in a deeper understanding of the physical phenomena taking place in gyrotheodolites.

Along with widely distributed gyrotheodolites, the book considers those which, at the present time, have not gone beyond the walls of research laboratories. This is necessary so that the reader himself may envision possibile directions for the development of gyrotheodolites.

A considerable part of the book is devoted to measurements with gyrotheodolites, methods of appraising the quality of the work of the instrument, and its errors. Here, too, are considered questions connected with automation of measurement, distant transmission of the results of measurements, and the possibility of constructing gyrotheodolites with aperiodic transitional processes and without introduction of a constant correction. The appendix includes tables in which are collected information on known gyrotheodolites.

The first paragraph of the third chapter of the book was written by B.I. Morozov, at the author's request.

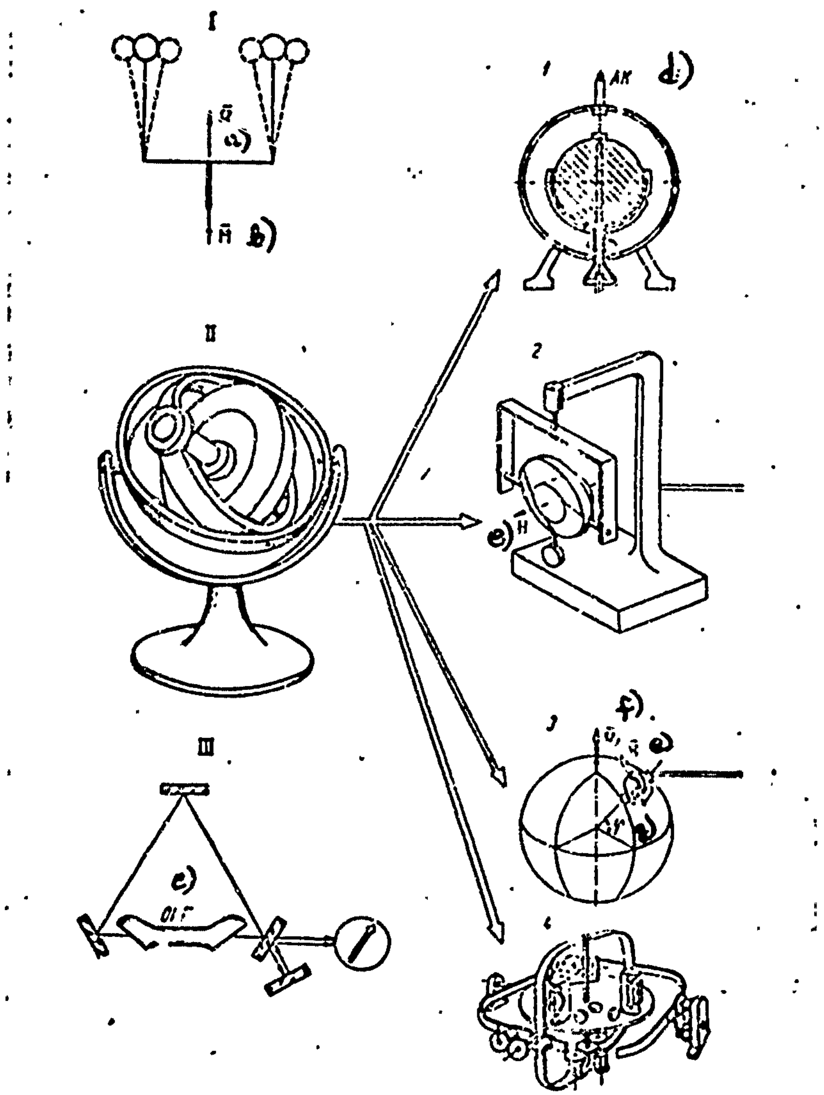

Fig. 2: Diagram of development of gyrotheodolites

Key:

- I. Oscillating gyroscope
- II. Rotary gyroscope
- III. Optical (laser) gyroscope

1. Gyrotheodolite with spherical gyroscope
2. Pendulum gyrothecdolite
3. Two-degrees-of-freedom gyrotheodolite
4. Gyrostabilized platform, operating like a gyroscopic compass

- a. Vector of angular velocity of rotation of the earth
- b. Moment of force vector
- c. Laser
- d. Astronomical compass
- e. Gyroscope kinetic moment vector
- f. Vector of angular velccity of rotation of the earth
- g. Angle turned by gyroscope rotor

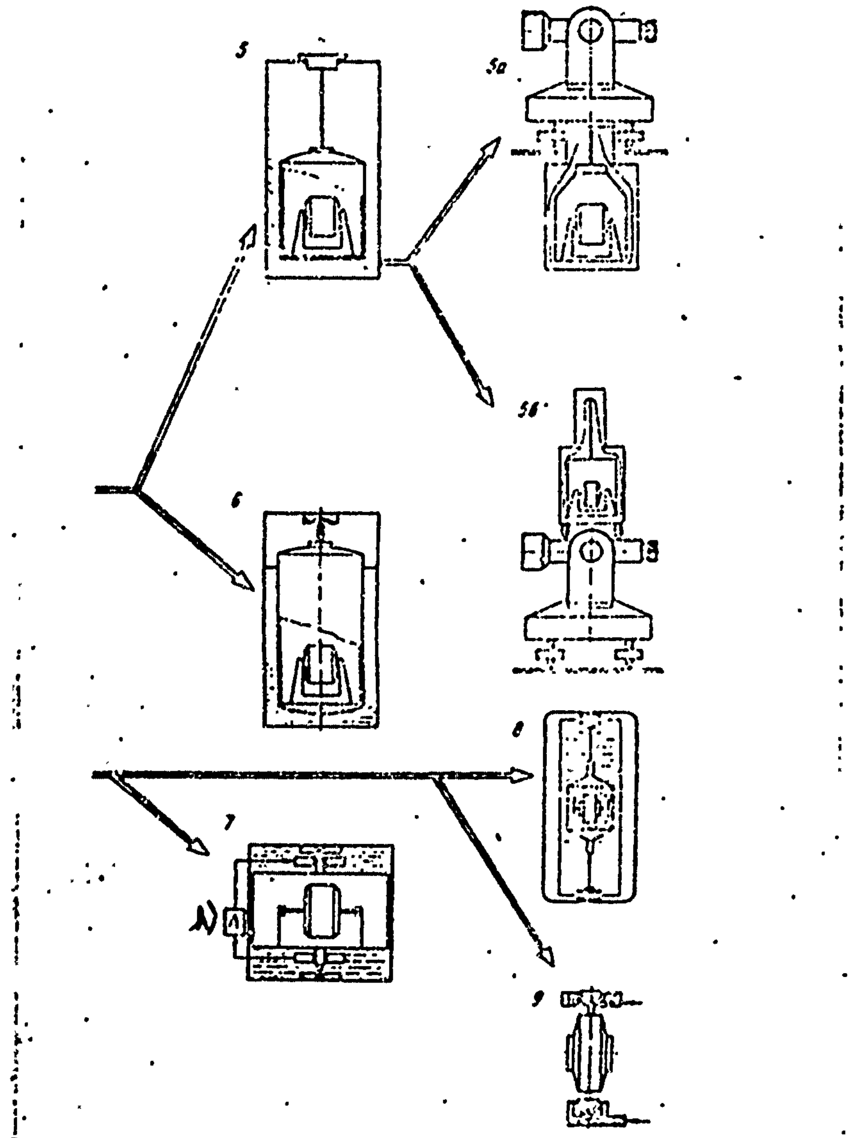

Fig. 2: (continued)

Key:
5. Pendulum gyrotheodolite with torsion suspension
6. Pendulum gyrotheodolite with pivot support
7. Two-degrees-of-freedom gyrotheodolite with core support
8. Two-degrees-of-freedom gyrotheodolite on torsion suspensions
9. Air-supported two-degrees-of-freedom gyrotheodolite

5a. "gyrotheodolite"
5b. "gyroscope attachment"

h. Amplifier

CHAPTER 1

INTRODUCTION TO THE GYROSCOPE

Section 1. Some Information from Mechanics

The movement of a gyrcscopic mechanism is the result of the inte-action of moments of force which act on it, since a gyroscope can have only angular rotation. We consider the basic situation of mechanical rotary movement.

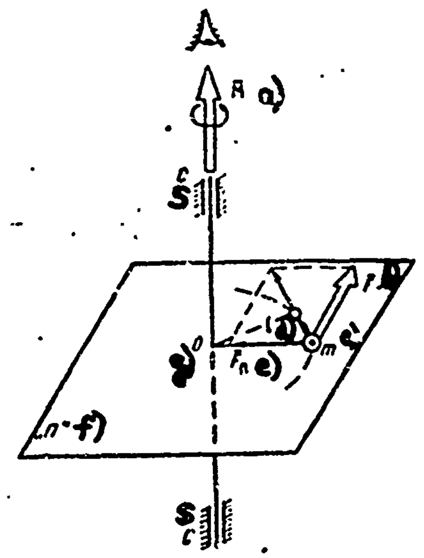

Fig. 3: Moment of force

Key:
- a. Moment of force vector ($\overline{M}$)
- b. Force vector ($\overline{F}$)
- c. Mass (m)
- d. Distance (1)
- e. Centripetal force vector ($\overline{F}_p$)
- f. Plane (P)
- g. Origin (0)

Moment of Force

Let us take an axis SS, to which is attached a point mass m at a distance 1 (Figure 3). We construct plane "P", perpendicular to axis SS in such a manner that a lever connecting mass m with axis SS lies on it. We exert a force $\overline{F}$ on mass m. This force attempts to displace the mass in the same direction as it is directed itself. But the mass is connected to axis SS, which is stopped from linear movement by its bearings. The force with which the bearings hold the axis also acts on mass m. As a result of the geometric summation of force $\overline{F}$ and force F_p we obtain a resultant which turns the lever with mass m. It is not difficult to see that the longer the lever the greater the displacement of the mass by one and the same force $\overline{F}$. On the other hand, the same effect is attained by increase of force while holding the lever constant. Hence the angular movement depends not only on the action of the force but on the distance between the axis of rotation and the place where the force is applied. This relation characterizes the moment of force:

$$M = Fl \qquad (1)$$

where it is assumed that lever l is perpendicular to the line of action of force F (or force vector $\overline{F}$). In order that the direction of movement of the point to which the force is applied may be shown, the moment of force is shown as a vector. The direction of a moment of force vector is defined by the following rule:

The moment of force vector is directed along the axis of rotation in that direction from where the rotation of lever l under the action of force F appears to be counterclockwise.

Angular velocity ω is represented by the increase in angle $\Delta\phi$ in an interval of time Δt:

$$\omega = \frac{\Delta\phi}{\Delta t}. \qquad (2)$$

If the angular velocity is constant we may write that

$$\omega = \frac{\phi}{t}.$$

The angular velocity of all points on a rigid body is the same, but the linear speed V is a variable dependent on the distance of the point under consideration from the axis of rotation

$$V = \omega r. \qquad (3)$$

The angular velocity, like the moment, is shown as a vector, directed along the axis of rotation in that direction from which the rotation of the body appears to be counterclockwise.

Angular acceleration ε appears when the body does not rotate uniformly and we characterize the degree of non-uniformity of this motion:

$$\varepsilon = \frac{\Delta\omega}{\Delta t}. \qquad (4)$$

The angular acceleration vector is located along the spin axis, and its direction is determined by the nature of the change in speed of rotation. If the speed of rotation increases -- the angular acceleration vector is directed in the same direction as the angular velocity vector; if the rotation slows down -- the angular acceleration vector is directed in the opposite direction from the angular velocity vector.

Moment of Inertia of a Body

There is a measure of the inertia of a body for translational motion of a mass which is established by the second law of mechanics:

$$F = ma. \tag{5}$$

Using this law, we consider the movement of a mass fixed to a bar and rigidly connected to the spin axis (Fig. 4). We will consider the bar and the axis to be weightless. Moment M acts directly on the axis, which results in force F acting on the mass, as determined by the relation

$$F = \frac{M}{r}. \tag{6}$$

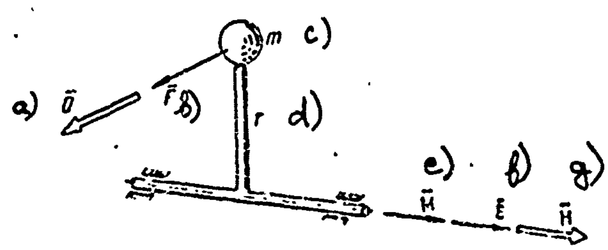

Fig. 4: Moment of inertia

Key:
- a. Moment of inertia vector
- b. Force vector
- c. Mass
- d. Distance
- e. Moment of force vector
- f. Angular acceleration vector
- g. Kinetic moment vector

This force imparts a linear (tangent) speed to mass m

$$a = \frac{F}{m}. \tag{7}$$

We find the angular acceleration. Proceeding from

$$a = \frac{V}{t}, \quad V = \omega r \quad \text{and} \quad \varepsilon = \frac{\omega}{t},$$

we get

$$\varepsilon = \frac{a}{r}. \tag{8}$$

Then substituting Eq. (6) and (7) in Eq. (8), we get

$$\varepsilon = \frac{M}{mr^2}. \tag{9}$$

Comparing expression (7) for linear motion with expression (9) for angular motion, it can be considered that the measure of inertia for rotary motion is the expression

$$I = mr^2, \tag{10}$$

called the moment of inertia of a body relative to its axis of rotation. We cite several examples of the expression for the moment of inertia of some bodies:

For a cylinder of radius R

$$I_ц = 0{,}5mR^2,$$

For a cylindrical ring with outside radius R and inside radius r

$$I_к = 0{,}5m(R^2 - r^2).$$

For a solid ball of radius R

$$I_ш = 0{,}4mR^2.$$

Kinetic Moment

For translational motion, the product

$$m\overline{V} = \overline{Q} \tag{11}$$

is called the quantity of motion. The quantity of motion is represented by a vector, fixed to mass m and coinciding with the direction of the linear speed vector. In rotary motion this vector is at a distance r from the axis of rotation (see Fig. 4). Then, by analogy with the moment of force, we can find the moment of the quantity of motion of the mass relative to its axis of rotation. For this the quantity of motion vector is multiplied by the "lever" -- the shortest distance from the axis of rotation. The product obtained is called the kinetic moment of a body

$$H = Qr. \tag{12}$$

Substituting expressions (11) and (2) in (12), we get

$$\overline{H} = mr^2\overline{\omega}$$

or

$$\overline{H} = I\overline{\omega}. \tag{13}$$

Consequently, the kinetic moment appears as a vector, whose direction coincides with the angular speed vector of its own rotation.

Section 2. Equations for Motion of a Solid Body

The motion of a solid body is defined by three basic elements: an external perturbation (force or moment), its inertia, and that resistance to motion which the body shows to the external perturbation cited. The algebraic sum of these elements is the differential equation of motion of a body.

We consider disk 1, suspended on string 2 in a vessel containing a fluid (Fig. 5, a). We apply moment M_B to the disk (this will turn the disk, twisting the string) and we observe the motion of the disk. The motion of the disk under the action of the moment begins only after overcoming its inertia, owing to which the body (disk) attempts to retain its position of rest. The turning of the disk will twist the string, and the arising of a of a moment of torque -- brakes the motion of the disk. The rotation of the disk will be hindered by the fluid, creating a moment of fluid friction.

The basic law defining the interaction of a solid body with a moment of force is the second law of mechanics

$$M = I\varepsilon$$

or

$$I\varepsilon - M = 0 \tag{14}$$

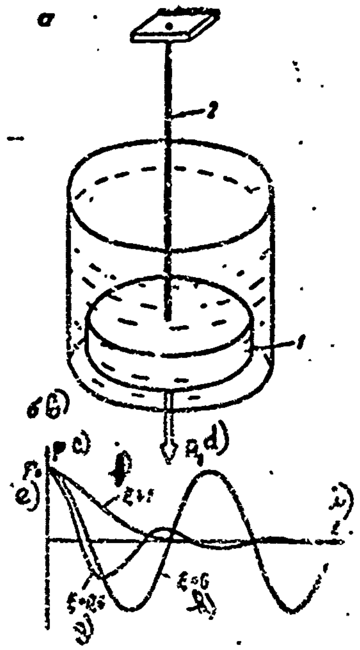

Fig. 5: Solid body on a torsion suspension:

a. Kinematic diagram
b. Graph of the transitional processes

1. Solid body (disk)
2. Torsion member

Key:

c. Deflection of the system
d. Moment of force
e. Initial deflection of the system
f. Coefficient of relative damping $\geq$ 1
g. Coefficient of relative damping = 0.4
h. Coefficient of relative damping = 0
i. Time

In the case being considered, M designates the sum of moments: external M_B, torque M_K and the moment of fluid friction M_M. Since moments M_K and M_M oppose the motion engendered by the action of moment M_B, it is obvious that they are opposite in sign to moment M_B.

We substitute in (14) the moments under consideration:

$$I\varepsilon - (M_{\text{в}} - M_{\kappa} - M_{\kappa c}) = 0. \qquad (15)$$

We take into account that $\varepsilon = \ddot{\varphi}$ (φ -- angle through which the disk turns, $\ddot{\varphi}$ -- second derivative by time of angle φ), $M_{\text{н}} = \kappa\varphi$ (κ -- angular rigidity of the twisted string), $M_{\text{м}} = c\dot{\varphi}$ (c -- damping coefficient).

Substituting the foregoing expressions in (15), we obtain the differential equation of movement of the disk in fluid

$$I\ddot{\varphi} + c\dot{\varphi} + \kappa\varphi = M_{\text{в}}. \qquad (16)$$

The inertial moment, like the external, is present in the differential equation of any body having mass.

Moments of resistance $c\dot{\psi}$ and $\kappa\psi$ are distinguishing features of every system considered and characterize its specific features, the determination of which is a basic aim of the study of one or another phenomenon, including the gyroscopic moment phenomenon.

The left part of Eq. (16) defines the interaction of the internal (proper) moments, when a system, having been removed from its state of rest, is again returned to this state. This movement is written down in a similar equation:

$$I\ddot{\varphi} + c\dot{\varphi} + \kappa\varphi = 0. \qquad (17)$$

Eq. (17) is often written down in dimensionless form:

$$\tau^2\ddot{\varphi} + 2\xi\tau\dot{\varphi} + \varphi = 0. \qquad (18)$$

This equation is often called the oscillating component, and its solution is written in the following form (when $\xi<1$):

$$\varphi = \varphi_0 e^{-\frac{\xi}{\tau}t} \sin\left(\frac{1}{\tau}\sqrt{1-\xi^2}\,t + \delta\right). \qquad (19)$$

when ψ_0 -- initial deflection of the system;

$\tau = \sqrt{\frac{I}{k}}$ -- time constant of the system;

$\xi = \frac{c}{2\sqrt{Ik}}$ -- relative damping coefficient;

δ -- phase angle;

t -- time.

Graphical solutions of Eq. (19) for various values of ξ are shown in Fig. 5b, and are called transition processes, since the process is characterized by a transition of the system from one established state ($\phi=\phi_0$) to another established state ($\phi=0$). The transition process for $\xi \geq 1$ is called aperiodic, for $1>\xi>0$ -- oscillatory damping and for $\xi=0$ -- undamped oscillation. The latter is most characteristic of the oscillation of the kinetic moment vector of pendulum gyrotheodolites about the direction of a geographic meridian. The frequency of oscillation in the transition process will be

for $\xi=0$, $$\omega_c = \frac{1}{\tau}, \tag{20}$$

for $0<\xi<1$ $$\omega_n = \omega_c \sqrt{1-\xi^2}.$$

The angular velocity ω_c is also called the proper frequency of the system; then the period of undamped harmonic oscillation is

$$T = \frac{2\pi}{\omega_c} = 2\pi\tau. \tag{21}$$

The time during which the oscillating system comes to rest is called the time for termination of the transition process, and it is found in equation form as

$$t_m = \frac{(3 \div 4)\tau}{\xi}. \tag{22}$$

Section 3. Gyroscopic Moment

The basis of the structure of a gyroscope is a rapidly rotating fly wheel which can swing around two perpendicular axes (Fig. 6). Rotation about the axis of symmetry of the fly wheel (Z) is called proper rotation; rotation about the other two axes (X and Y) -- the universal suspension axes -- is called gyroscopic precession. Simultaneously, we give the rotor angular velocity $\bar{\omega}$ of proper motion and angular velocity of precession $\bar{\omega}_y$, and we consider elementary mass dm, which will take part in two movements at the same time: with angular velocity $\bar{\omega}$ and angular velocity of precession $\bar{\omega}_y$. In consequence of the proper rotation, mass dm has peripheral linear speed $V_c=\omega R$, the magnitude of which remains constant, since the angular velocity and radius R remain constant. The angular velocity of precession $\bar{\omega}_y$ also imparts to mass dm linear speed $V_{\text{п}}$, that is: when mass dm is located on the positive half-axis Y, the speed $V_{\text{п}}=0$, since the radius (distance from axis of rotation to mass) equals zero; when mass dm is situated on the negative half-axis X (owing to its proper rotation), the speed $V_{\text{п}}=-\omega_y R$; when mass dm is situated on the negative half-axis Y, the speed $V_{\text{п}}=0$, and when located on the positive half-axis X, the speed $V_{\text{п}}=\omega_y R$. Consequently, during the action of mass dm at two constant angular velocities, the directions of whose vectors are mutually perpendicular, mass dm has a variable linear speed $V_{\text{п}}$. The change in speed $V_{\text{п}}$ is characterized by the change in distance between axis of rotation X and mass dm, resulting from the rotation of mass dm with angular velocity ω. In accordance with Eq. (3), it follows from Fig. 4 that speed $V_{\text{п}}$ may be written:

$$V_{\text{п}} = \omega_y R \sin\varphi. \qquad (23)$$

The change in speed of movement of any mass, including mass dm, appears in consequence of the action of a force on this mass. In accordance with the second law of mechanics, for the elementary mass under consideration, the elementary force is defined from the expression:

$$dF = dm \cdot a. \qquad (24)$$

where $a=\dot{V}_{\text{п}}$ -- acceleration of mass dm.

Differentiating expression (23), we find the acceleration

$$\dot{V}_{\text{п}} = R\omega_y\omega\cos\varphi. \qquad (25)$$

where $\omega=\dot{\psi}$.

Substituting (25) in (24) we find

$$dF = dm R\omega_y\omega\cos\varphi. \qquad (24')$$

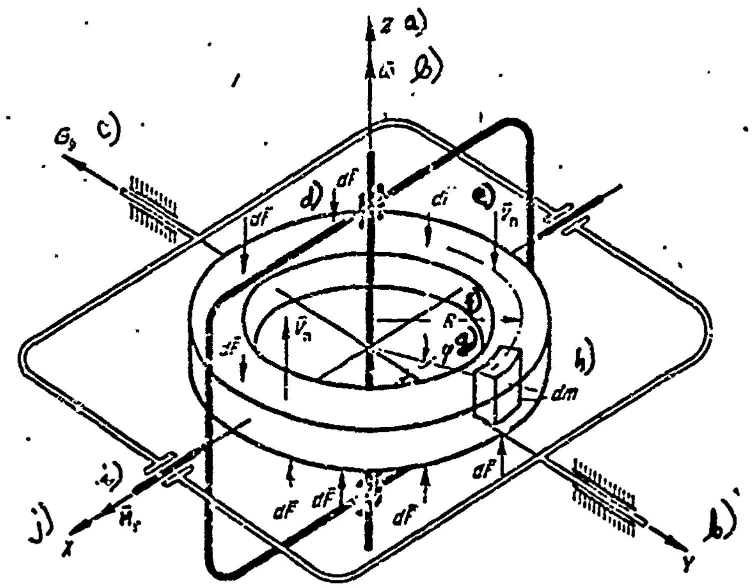

Fig. 6: Rapidly spinning rotor on a universal suspension

Key:

a. Z axis
b. Angular velocity vector
c. Angular velocity of precession vector
d. Force differential
e. Linear speed vector
f. Radius
g. Disk angle of rotation
h. Mass
i. Gyroscopic moment vector
j. X axis
k. Y axis

From (24') and (23) it follows that, between the force applied to mass dm and the speed with which it moves under the action of the force, a displacement in phase equal to $\frac{\pi}{2}$ takes place, the speed "retiring" from the force.

An elementary force applied to mass dm generates, relative to the universal suspension axes X and Y, elementary moments

$$dM_y = 2dFR\sin\varphi,$$
$$dM_x = 2dFR\cos\varphi.$$

Let us imagine that the mass of the rotating fly wheel is distributed uniformly in a thin ring. Then elementary mass dm can be represented in the form

$$dm = \frac{m}{2\pi}d\varphi.$$

Substituting the express. for elementary force (24') and elementary mass in the equations for the elementary moments and then integrating the expressions obtained for one revolution of the rotor, we will have:

$$\left.\begin{array}{l} M_x = mR^2\omega\omega_y \\ M_y = 0 \end{array}\right\}. \qquad (26)$$

The result obtained establishes the correlation between angular velocity (ω_y) and the moment (M_x): if an external moment acts on the gyroscope along axis X, angular velocity of precession ω_y arises along axis Y and, the other way around, if axis Y has an angular velocity of precession ω_y, there arises along axis X a moment of gyroscopic reaction $M_x = M_r$ (M_r is called the gyroscopic moment).

This conclusion appears in consequence of the consideration of the peripheral speed V_n, arising in the body and participating simultaneously in two rotations, the angular velocity vectors of which are perpendicular.

Taking into consideration Eq. (10), (13) and (26), magnitude of the gyroscopic moment may be written in the following form:

$$M_r = H\omega_y. \qquad (26')$$

where H -- kinetic moment of the gyroscope.

The gyroscopic moment appears as a vector and is generally defined by the vector product:

$$\overline{M}_r = \overline{H}\times\overline{\omega}_y. \qquad (26'')$$

Then its magnitude (modulus)

$$M_r = H\omega_y \sin(\widehat{\bar{H}, \bar{\omega}_y'}).$$

Vector $\bar{M}_r$ is directed perpendicular to the plane formed by vectors $\bar{H}$ and $\bar{\omega}_y$, in that direction from which superposition of vector $\bar{H}$ on vector $\bar{\omega}_y$ by the shortest distance appears to be a counterclockwise movement.

The action of an external moment leads to the appearance of an angular velocity of precession

$$\omega_{np} = \frac{M_{nt}}{H}.$$

The vector of this angular velocity is directed perpendicular to the plane formed by vectors $\bar{H}$ and $\bar{M}_{BH}$, and in that direction from which superposition of vector $\bar{H}$ on vector $\bar{M}_{BH}$ by the shortest distance appears to be a counterclockwise movement.

Section 4. A Gyroscope under the Action of External Moments

Let us consider a gyroscope, to the interior gimbal of which is applied external moment M_{BH} (Fig. 7, a). In accordance with the equation for the gyroscopic moment, an angular velocity of precession $\omega_{\Pi p}$ appears on the gyroscope, directed along the axis of the exterior gimbal. The reaction of the gyroscope to this angular velocity of precession will be the gyroscopic moment $\bar{M}_\Gamma$, directed along the axis of the interior gimbal in the opposite direction from moment $\bar{M}_{BH}$.

The angular velocity of precession

$$\omega_{np} = \frac{M_{nt}}{H},$$

then the gyroscopic moment

$$M_r = H\omega_{np}$$

or

$$M_r = M_{nt}.$$

Consequently, the gyroscopic moment is equal in magnitude to the external moment. But, since these moments have opposite directions, the gyroscopic moment fully compensates for the external moment and the interior gimbal proves to be free from its action.

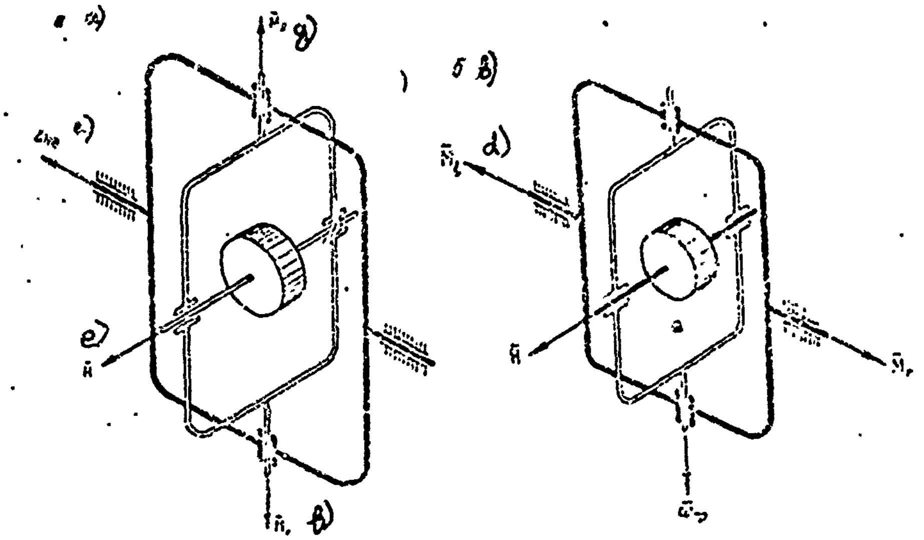

Fig. 7: Action of an external moment on a gyroscope

Key:
- a. Fig. 7a
- b. Fig. 7b
- c. Angular velocity of precession
- d. External moment on the exterior gimbal
- e. Gyroscope kinetic moment vector
- f. Gyroscopic moment
- g. Moment of gyroscopic reaction

A stabilizing property of a gyroscope is manifested by the phenomenon under consideration. The stabilization of the interior gimbal requires an expenditure of energy. This energy is generated in the gyroscope at the expense of the precession of the exterior gimbal. Consequently, stabilization of the interior gimbal is accomplished at the expense of exterior precession.

Under the action of an external moment M_E on the exterior gimbal (see Fig. 7, b), precession of the interior gimbal takes place, as a consequence of which the gyroscopic moment appears, which compensates for the external moment. Consequently, the mechanism by which stabilization is generated is the same as in the preceding case, only this time stabilization of the external gimbal is accomplished at the expense of precession of the interior.

However, this uniformity hides a definite difference in the stabilization of the external moments. During action of an external moment on the axis of the interior gimbal its stabilization will be continued independent of time. Stabilization of the exterior gimbal will be continued only in the course of that time which is necessary for the turning of the interior gimbal through an angle of $\frac{\pi}{2}$. By the turning of the interior gimbal to an angle of $\frac{\pi}{2}$ the axis of the gyroscope rotor coincides with the axis of the exterior gimbal, resulting in the loss of one degree of freedom (by means of superposition of the axes). And along with this -- a loss of stability of the gyroscope. In this case, the gyroscope begins to behave like a solid body; that is, it acquires a rotation in the direction of the external moment applied to the exterior gimbal.

From this consideration it follows that, practically, it is necessary to avoid a longitudinal application of a moment to the exterior gimbal, if steps are not taken before hand to eliminate the coincidence of the rotor axis with the axis of the exterior gimbal.

Considering the interaction of the gyroscope with external moments, it is advisable to give some attention to the "external" moments themselves. Only for an ideal gyroscope can it be considered that there is an absence of external moments on the suspension axes. In practical designs there is always a cause for the arising of a force located on some "lever" of the axes of rotation of the gyroscope suspension. Consequently, there is always an "external" moment on the suspension axes of a gyroscope: from the force of friction in the bearings, from imbalance, from the action of current supply mechanisms, and others. These moments must be considered, since in many cases they determine the accuracy of operation of gyroscopic equipment. By selection of designs and technology of manufacture, the total harmful moment in contemporary gyrotheodolites has been decreased to a magnitude on the order of 10^{-6}g-cm.

Section 5. Equations for Gyroscopic Motion

The use of a gyroscope as a measuring instrument consists of determining the direction of the spin axis of its rotor compared to a direction connected with the earth. In view of the fact that the spin axis of an ideal gyroscope remains unchanging in its direction compared with the stars, it will appear to an observer standing on the earth that the spin axis of the gyroscope shifts.

To insure the measuring function on earth, a gyroscope is equipped with supplementary mechanisms (adjusting systems). As a result, it is proved that the direction which is found with the help of a gyroscope is the direction of the dynamic equilibrium of the kinetic moment vector, determined from the differential equation of motion.

The gyroscope, in distinction from the system with one degree of freedom considered in Fig. 5, has three degrees of freedom, of which two determine the position of the spin axis of the gyroscope rotor, and the third defines the proper rotation. The first two degrees of freedom, connected with rotation of the gyroscope kinetic moment vector axis about its universal suspension axes, are of the most interest for measuring purposes. This turning of the kinetic moment vector $\bar{H}$ is plotted in Fig. 8 with two angles: angle α defines the turning of the external gimbal, angle β -- the internal gimbal. Angles α and β are measured relative to a fixed coordinate system. Angle ϕ is measured from line <u>on</u> and defines the turning of the gyroscope rotor.

Two coordinate systems are used for working out the gyroscope motion equations: the mobile system XYZ, which rotates together with the gyroscope, and the fixed system $\eta\xi\zeta$, relative to which the gyroscope motion is considered. The gyroscope axes are connected with these coordinate systems in the following manner: the rotor axis and the interior gimbal axis are situated along any two axes of system XYZ, then the third axis is directed so that it forms three-dimensional coordinate axes with the first two; the axis of the exterior gimbal is situated along one of the axes of the fixed system $\eta\xi\zeta$, since the axis of the exterior gimbal is connected across its bearings and casing to a fixed base, relative to which the position of the kinetic moment vector is measured.

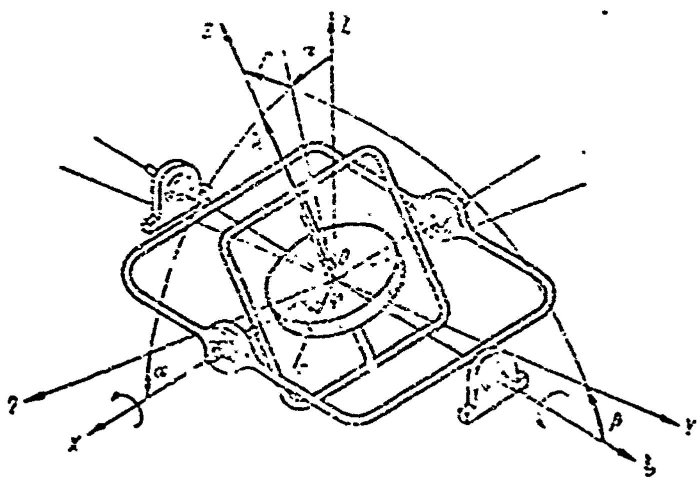

Fig. 8: Angles turned by a gyroscope

At the first instant of time we will consider that the axes of the mobile and fixed coordinate systems coincide (Fig. 9, a), and for this the kinetic moment vector $\bar{H}$ is directed along axis Z, and the exterior axis -- along axis ξ. Now, if an external moment $\bar{M}_x$ acts on the axis of the interior gimbal, the reaction of the gyroscope to this moment will be angular velocity of precession of the exterior gimbal $\bar{\dot{\alpha}}$. But before the exterior gimbal begins to precess, its inertia must be overcome, that is, the initiation of precession $\bar{\dot{\alpha}}$ is opposed by moment of inertia $I_\xi\bar{\ddot{\alpha}}$.

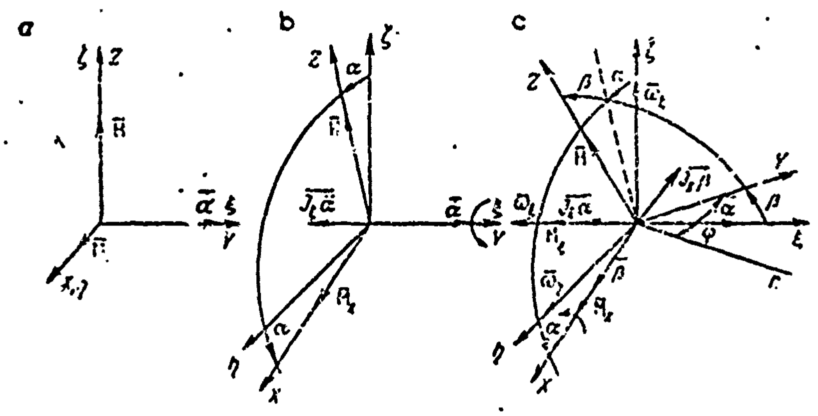

Fig. 9: Diagram of coordinate axes for derivation of the equation for gyroscope motion

In view of the fact that the external moment is applied to the interior gimbal, which moves together with the exterior gimbal, vector $\bar{M}_x$ shifts together with axis X. The precession of the exterior gimbal leads to this, that axis Z of the proper rotation of the gyroscope rotor (vector $\bar{H}$) and the axis of the interior gimbal turn through angle α (Fig. 9, b).

Now let the external moment $\bar{M}_\xi$ be applied to the exterior gimbal (Fig. 9, c). The reaction of the gyroscope to this moment will be precession of the interior gimbal with angular velocity $\dot{\beta}$, with the result that in a short time it turns through angle β. Axis Z of the proper rotation of the rotor and axis Y turn together with the interior gimbal. Before movement of the interior gimbal begins, the inertia must also still be overcome, since there arises on its axis moment of inertia $I_x\ddot{\beta}$.

Let us find the moments applied to the axes of rotation of the gyroscope gimbals.

Exterior gimbal:
External moment "$-M_\xi$";

Moment of inertia "$-I_\xi\ddot{\alpha}$";

Gyroscopic moment "$+H\dot{\beta}\cos\beta$".

Interior gimbal:
External moment "$+M_x$";

Moment of inertia "$-I_x\ddot{\beta}$";

Gyroscopic moment "$-H\dot{\alpha}\cos\beta$".

Summarizing these moments, we obtain the differential equations of motion of the exterior and interior gimbals:

$$\left.\begin{array}{l} -M_\xi - I_\xi\ddot{\alpha} + H\dot{\beta}\cos\beta = 0 \\ M_x - I_x\ddot{\beta} - H\dot{\alpha}\cos\beta = 0 \end{array}\right\}.$$

The equations obtained are the technical equations of the gyroscope. Reversing the signs in these equations and moving the external moments to the right hand side:

$$\left.\begin{array}{l} I_\xi\ddot{\alpha} - H\dot{\beta}\cos\beta = -M_\xi \\ I_x\ddot{\beta} + H\dot{\alpha}\cos\beta = M_x \end{array}\right\}. \qquad (27)$$

Eq. (27) are correct for the situation when the base and its connected coordinate system $\eta\xi\zeta$ is fixed with reference to inertial space. However, in practical problems this condition is not observed. Therefore, let us find out what changes in the equation for gyroscope motion result if the coordinate system $\eta\xi\zeta$ rotates with angular velocity $\underline{\Omega}_0$. Let this velocity have as its projections ω_η, ω_ξ, ω_ζ (see Fig. 9, c). Projecting the vectors of these velocities on the axes of the gyroscope suspension, we find that its base rotates relative to the axis of the exterior gimbal with angular velocity ω_ξ, and relative to the interior -- with angular velocity defined by the sum of $\omega_\eta\cos\alpha - \omega_\zeta\sin\alpha$. In addition, a change in the angular velocity of the proper rotation takes place of a magnitude of

$$\omega_\zeta\cos\alpha\cos\beta + \omega_\xi\sin\beta + \omega_\eta\cos\beta\sin\alpha.$$

In view of the great magnitude of the angular velocity of proper rotation we will disregard its changes. Let us consider further the velocity under consideration in the equation for gyroscope motion, but first we transform Eq. (27):

$$\left.\begin{aligned} \tau_\xi \ddot{\alpha} - \dot{\beta}\cos\beta &= -\omega_{np}(M_\xi) \\ \tau_x \ddot{\beta} + \dot{\alpha}\cos\beta &= \omega_{np}(M_x) \end{aligned}\right\}, \tag{28}$$

where $\frac{H}{{}^xW} = ({}^xW)^{dn}\omega$, $\frac{H}{{}^\xi W} = ({}^\xi W)^{dn}\omega$, $\frac{H}{{}^xJ} = \tau_x$, $\frac{H}{{}^\xi J} = \tau_\xi$

Eq. (28) is analogous to Eq. (27), but it is written, not in the form of moments acting along the suspension axes, but in the form of angular velocities of the suspension axes. Now, considering that the velocities discussed above still exist relative to the suspension axes, the equation for gyroscope motion must be written in the form:

$$\left.\begin{aligned} \tau_\xi \ddot{\alpha} - (\dot{\beta} + \omega_\eta \cos\alpha - \omega_\zeta \sin\alpha)\cos\beta &= -\omega_{np}(M_\xi) \\ \tau_x \ddot{\beta} + (\dot{\alpha} - \omega_\xi)\cos\beta &= \omega_{np}(M_x) \end{aligned}\right\}$$

or

$$\left.\begin{aligned} J_\xi \ddot{\alpha} - H(\dot{\beta} + \omega_\eta \cos\alpha - \omega_\zeta \sin\alpha)\cos\beta &= -M_\xi \\ J_x \ddot{\beta} + H(\dot{\alpha} - \omega_\xi)\cos\beta &= M_x \end{aligned}\right\}. \tag{29}$$

Depending on the function of the gyroscope and the magnitudes of angles α and β, sometimes the possibility of disregarding several items is presented in Eq. (29). In widespread technical situations there are "truncated" equations for gyroscope mo on:

$$\left.\begin{aligned} H\dot{\beta} &= M_\xi + H\omega_\zeta \sin\alpha - H\omega_\eta \cos\alpha \\ H\dot{\alpha} &= M_x + H\omega_\xi \end{aligned}\right\}. \tag{30}$$

Or without taking into account the angular velocity of rotation of the base

$$\left.\begin{aligned} H\dot{\beta} &= M_\xi \\ H\dot{\alpha} &= M_x \end{aligned}\right\}. \tag{30}$$

It is not difficult to see that Eq. (30) are the gyroscopic moment equations, analogous to Eq. (26') or (26").

Section 6. Definitions

Considering the essence of gyroscopic phenomena, let us develop some properties of gyroscopes.

A gyroscope possesses the ability to maintain a given direction of the rotor axis in space (relative to the stars). It resists the action of any force which attempts to move the rotor from the plane of its rotation; this resistance is a measure of the stability of the gyroscope or of its gyroscopic inertia.

The action of an external force on a gyroscope leads to a change in position of the rotor, with the vector of angular velocity of displacement of the rotor perpendicular to the moment of external force vector.

The deviation of the axis of proper rotation of the gyroscope rotor compared to the theoretical position is called gyroscopic drift, which is caused mainly by imbalance in a gyroscope part, friction and the inertia of the universal suspension.

In order to give the gyroscope the ability of maintaining a given direction relative to earth, it is equipped with an adjusting mechanism which establishes a controlling moment of force on the universal suspension axes. In aligning the spin axis of the rotor with the axis of rotation of the external gimbal, the gyroscope loses stability and begins to rotate in the direction of the applied moment.

A gyroscope in which one of the axes of rotation of the universal suspension is fixed, is called two-degrees-of-freedom, and it possesses the property of being able to align the axis of proper rotation with the vector of angular velocity of rotation of the base. A gyroscope is called free, if no external moment whatever acts on it.

The axis of proper rotation of the gyroscope rotor is called the principal axis of the gyroscope or simply the gyroscope spin axis.

CHAPTER II

GYROTHEODOLITES

Section 1. The Principle of Determining a Geographic Meridian by Use of a Gyroscope

The use of a gyroscope for determining the direction of geographic meridians is based on using the properties of the gyroscope and the daily rotation of the earth. The angular velocity Ω of rotation of the earth may be distributed into two components: the horizontal ω_{Γ} and vertical ω_{t} (Fig. 10). The horizontal component ω_{Γ} lies in a horizontal plane and indicates the direction of the geographic meridian of the site. The vertical component ω_{B} is directed along the vertical of the site; relative to space, it rotates in an east-west plane. If we construct this plane by some method, the normal to it will indicate the direction of the geographic meridian. Specifically, this plane may be constructed with the help of a gyroscope. Let us suppose that in the first instant the spin axis of a free gyroscope rotor coincides with the direction of the vertical of the site. In the following instant, as a result of the daily rotation, the vertical of the site relative to space turns to the west. The vertical of the site and the spin axis of the gyroscope rotor intersect at the center of the earth and define the east-west plane in this manner. Since the gyroscope spin axis remains fixed all the time, the angle between it and the vertical of the site will continually increase; after six hours it will be equal to $\frac{\pi}{2}$, which leads to the coincidence of the planes of the gimbals and loss of stability by the gyroscope. In order for this not to happen, the principal axis of the gyroscope is made to move after the vertical of the site.

The movement of the gyroscope spin axis following the vertical takes place at some angle of delay, so that by this angle the east-west plane (two intersecting lines: the gyroscope spin axis and the vertical of the site) is formed. The execution of such a movement of the gyroscope spin axis is accomplished with the help of an adjustment system. From Fig. 10 we can see that the magnitude of this angular velocity must be equal to the angular velocity of the horizontal component of the earth's rotation. Gyrotheodolites with a spherical gyroscope (see Fig. 2) are constructed by the principle being considered.

The result of the interaction of the horizontal component of the angular velocity of rotation of the earth with the two-degrees-of-freedom gyroscope also permits a determination of the direction of a geographic meridian. In this case the spin axis of the gyroscope rotor is located in the horizontal plane and the axis of rotation of the gyroscope gimbal establishes the vertical. The interaction of the kinetic moment vector with the angular velocity $\overline{\omega}_{\Gamma}$ generates a gyroscopic moment, forcing vector $\overline{H}$ to be set in the direction of vector ω_{Γ} and, consequently, to indicate the direction of a geographic meridian at a given point in an area (two-degrees-of-freedom gyrotheodolite -- see Fig. 2).

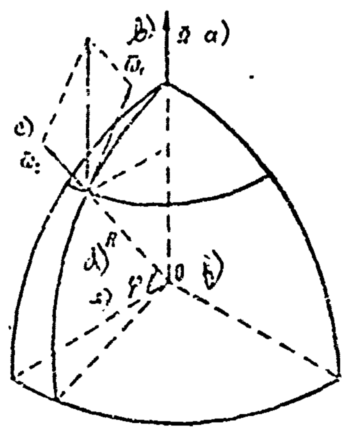

Fig. 10: Components of angular velocity of rotation of the earth

Key:
a. Vector of angular velocity of rotation of the earth
b. Horizontal component of earth's angular velocity of rotation
c. Vertical component of earth's angular velocity of rotation
d. Radius of the earth
e. Angle through which the gyroscope rotor is turned
f. Center of the earth

The examples considered do not exhaust the possibilities of constructing gyrotheodolite schemes, but they illustrate two basic principles: first, the use of the properties of vertical of a site and a three-degrees-of-freedom gyroscope and second, the use of the reaction of a two-degree-of-freedom gyroscope on angular velocity ($\overline{\omega}_r$ or $\overline{\Omega}$).

There are various modifications to gyrotheodolite schemes which use one of the principles indicated. In recent years, to gyroscopic methods for determining the direction of geographic meridians has been added the method of measuring the angular velocity of rotation of the earth with the help of laser instruments. They use the method of determining the velocity of rotation of the base by the difference in motion of two opposing light beams.

Section 2. Pendulum gyrotheodolites

The greatest progress has been achieved with gyrotheodolites based on a three-degrees-of-freedom gyroscope with a horizontally placed spin axis of the rotor and a center of gravity displaced downward relative to a fixed point. The displacement of the center of gravity leads to the origin of a physical pendulum in the gyroscope, the principal property of which is tracking the vertical of the site. Thus, in the pendulum gyrotheodolite use is made of the properties of the free three-degrees-of freedom gyroscope and the verticals of a site (of the horizontal plane). Let us examine the principles of operation of the pendulum gyrotheodolite.

At some point on the earth's surface let us set a gyroscope in such a manner that the kinetic moment vector of the gyroscope lies in a horizontal plane and is directed to the east (Fig. 11, position I). In this position, the line of action of the force of gravity (vector $\bar{G}$) lies along the axis of rotation of the exterior gimbal, and in relation to the axis of rotation of the interior gimbal, the moment of the force of gravity does not arise. After some time the angular position of the earth relative to the gyroscope changes. Specifically, the plane of the horizon turns relative to vector $\bar{H}$ through angle β_1 (position II). In consequence of such a turning of the vertical of the site, the line of action of the force of gravity vector $\bar{G}$ will no longer go through the axis of rotation of the exterior gimbal. This leads to the appearance of a moment of the force of gravity $\bar{M}_G$, applied to the axis of rotation of the interior gimbal and in our direction. The reaction of the gyroscope to the moment generated will be precession $\bar{\alpha}$ of the exterior gimbal of the gyroscope. The direction of the precession is such that the end of vector $\bar{H}$ moves to the south (towards us) and after some time coincides with the direction of the geographical meridian (position III). However, the spin axis of the gyroscope cannot establish itself in this position, since the end of vector $\bar{H}$ is still elevated a little above the plane of the horizon and, consequently, is the moment of the force of gravity $\bar{M}_G$, which forces the spin axis of the gyroscope to move farther to the west (position IV). The rotation of the earth always leads to the result that, relative to space, the east end of the horizontal sinks and the west -- rises. Therefore, to the extent that the end of vector $\bar{H}$ turns to the west, elevated above the plane of the horizon, the plane of the horizon itself will approach the spin axis of the gyroscope. In a few moments vector $\bar{H}$ finds itself located in the horizontal plane (position V), and at that time the force of gravity moment $\bar{M}_G$ vanishes and the precession of the exterior gimbal $\bar{\alpha}$ ceases ($\bar{\alpha}=0$). But the earth continues to rotate, and in the next moment the end of vector $\bar{H}$ is now below the plane of the horizon (position VI). This gives rise anew to the appearance of the force of gravity moment $\bar{M}_G$. But the direction of vector $\bar{M}_G$ will be opposite to that direction which was the elevated position of the gyroscope spin axis above the plane of the horizon (position II). In consequence of the change in direction of vector $\bar{M}_G$, the direction of precession $\bar{\alpha}$ of the exterior gimbal changes: the gyroscope spin axis will move from the west to the north, going past the position of the geographical meridian and stopping in a position analogous to position I. Thus, the

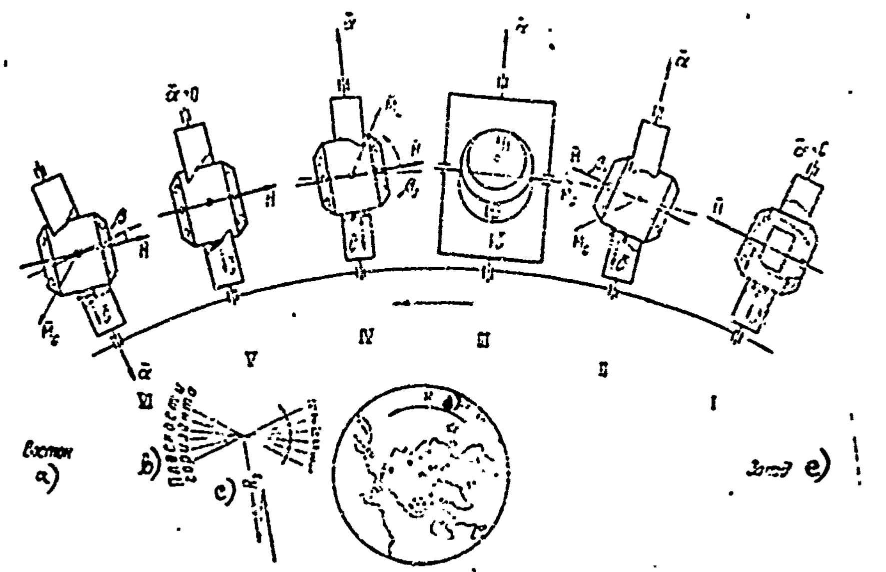

Fig. 11: Pendulum gyrocompass on the earth

Key:
- a. East
- b. Plane of the horizon
- c. Radius of the earth
- d. Angular velocity of earth rotation
- e. West

gyroscope spin axis performs an oscillation relative to the plane of the horizon and the plane of the geographic meridian at a given point in an area. Consequently, the pendulum moment generated because of the property of a free gyroscope of holding constant the position of vector $\bar{H}$ in space is a correcting moment, which insures the movement of the gyroscope axis around the direction of the meridian and the plane of earth's horizon.

Let us form an equation for the movement of a pendulum gyrotheodolite. For this, the coordinate system $\eta\xi\zeta$ is bound fast to the earth, as a consequence of which it will rotate relative to space together with the earth. Let us direct axis $O\zeta$ along the vertical of the site, $O\eta$ -- to the south in the horizontal plane, and axis $O\xi$ -- to the east along the horizontal plane (Fig. 12). The mobile coordinate system XYZ has a common origin with the fixed system. The gyroscope axes are connected with these coordinate systems in the following manner: the axis of the exterior gimbal is located vertically along axis $O\zeta$; the axis of the interior gimbal

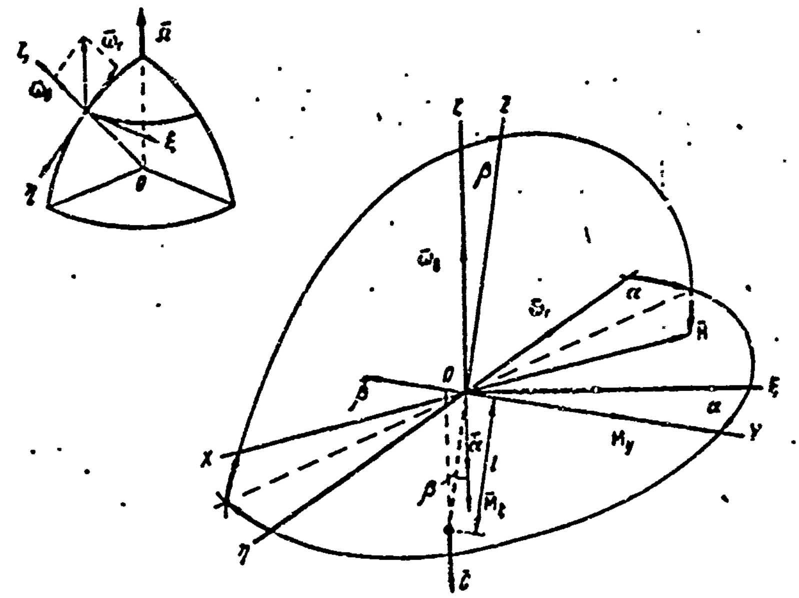

Fig. 12: Coordinate axes for a pendulum gyrotheodolite

coincides with axis OY, the spin axis of the rotor is directed along axis OX in the negative direction (to the north), and axis OZ is directed in such a way that, with axis OY and OX, it forms a rectangular coordinate system with axes XYZ. The center of gravity of the gyroscope is displaced along axis OZ.

To establish the nature of the movement of the gyrotheodolite, we apply an external moment successively to the interior and exterior gimbals of its universal suspension and we find the moments -- inertial, gyroscopic and adjusting -- aroused by the displacement of the center of gravity. As a result, we find that the equations for motion of a pendulum theodolite will be the expressions:

$$\left.\begin{aligned} I_\zeta\ddot{\alpha} + H\omega_r \sin\alpha\cos\beta + H\dot{\beta}\cos\beta - M_\zeta = 0 \\ I_y\ddot{\beta} - H\dot{\alpha}\cos\beta + H\omega_n\cos\beta + M_G + M_y = 0 \end{aligned}\right\}. \qquad (31)$$

The designations in these equations: I_ζ, I_y -- moments of inertia of the gyroscope relative to the axes of the exterior and interior gimbals respectively; α, β -- the angles turned by the exterior and interior gimbals, ω_{Γ}, ω_{B} -- horizontal and vertical components of the angular velocity of rotation of the earth; M_{ζ}, M_{y} -- external moments on the axes of a gyroscope suspension. The force of gravity moment M_G, as follows from the coordinate axis scheme, may be represented in the following form:

$$M_G = Gl \sin\beta,$$

where G -- the suspended weight forming the pendulum;
l -- the arm by which the weight is attached.

The movement of the gyroscope axes proceeds sufficiently slowly that the angular accelerations $\ddot{\alpha}$ and $\ddot{\beta}$ are insignificant, and the inertial moments in comparison with the gyroscopic may be disregarded. The external moments M_{ζ} and M_y also are small, since this situation is basic to the design of the gyrotheodolite. Then, excluding the external moments from consideration and counting angles α and β as trifles, for

$$\cos\alpha = \cos\beta = 1, \quad \sin\beta = \beta, \quad \sin\alpha = \alpha,$$

we obtain Eq. (31) in the following form

$$\left.\begin{aligned} H\omega_{\Gamma}\alpha + H\dot{\beta} = 0 \\ -H\dot{\alpha} + H\omega_{B} + Gl\beta = 0 \end{aligned}\right\}. \qquad (32)$$

If we assume $\alpha=0$ in the first equation, then the angular velocity $\dot{\beta}=0$, and the angle turned by the interior gimbal β equals a constant. Consequently, if the spin axis of the gyroscope rotor is in the plane of the meridian, the interior gimbal will not precess, and the pendulum moment insures movement of the gyroscope together with the earth. Deflection of the gyroscope spin axis from the plane of the meridian causes a movement of the interior gimbal and, together with it, change in the pendulum moment, and signifies a precession of the exterior gimbal, bringing the spin axis of the gyroscope to the plane of the meridian. Consequently, the pendulum moment M_G always insures movement of the gyroscope together with the earth. But retention of vector $\bar{H}$ in the plane of the geographic meridian during this movement is accomplished by the gyroscopic moment $H\omega_{\Gamma}\alpha$, which is called the directing moment M_H of the gyrotheodolite. From (31) we have:

$$M_H = H\Omega\cos\varphi\sin\alpha \approx H\Omega\alpha\cos\varphi. \qquad (33)$$

From the first equation of system (32) we find the expression for angle α, and from the second -- for angle β:

$$\alpha = -\frac{\dot{\beta}}{\omega_r}, \tag{34}$$

$$\beta = \frac{H\dot{\alpha} - H\omega_s}{Gl}. \tag{35}$$

Differentiating the expressions obtained by time, and assuming that H = constant, G = constant, l = constant, ω_r = constant, ω_B = constant:

$$\dot{\alpha} = -\frac{\ddot{\beta}}{\omega_r},$$

$$\dot{\beta} = \frac{H\ddot{\alpha}}{Gl}.$$

Let us substitute these expressions in (32) and, after simple transformations, we will have two independent equations:

$$\ddot{\alpha} + s^2\alpha = 0, \tag{36}$$

$$\ddot{\beta} + s^2\beta = -\omega_s\omega_r. \tag{37}$$

In these equations

$$s^2 = \frac{\omega_r Gl}{H} = \frac{\Omega Gl \cos\varphi}{H}. \tag{38}$$

Let us find the solution of the differential equations for motion of a gyrotheodolite, assuming that, at the first instant of time, the gyroscope spin axis is in the horizontal plane and is directed exactly to the north, that is, $\alpha(0)=0$ and $\beta(0)=0$, but that here the gyroscope moves together with earth relative to space, giving $\dot{\alpha}(0)=\omega_B$. Then, from the general solution for the angle turned by the interior gimbal

$$\alpha = C_1 \cos st + C_2 \sin st$$

and taking account of the specified initial conditions, we obtain $C_1=0$, $C_2=\frac{\omega_B}{s}$, or

$$\alpha = \frac{\omega_s}{s} \sin st. \tag{39}$$

In order to find the solution for the equation which defines the angle turned by the interior gimbal β, we differentiate expression (39) and substitute it in (35). Then, we obtain

$$\beta = \beta_0 (\cos st - 1), \tag{40}$$

where

$$\beta_0 = -\frac{H\omega_B}{Gl}. \tag{41}$$

It follows from the relations found for angles α and β that the gyroscope spin axis accomplishes undamped oscillation, both in the horizontal plane (oscillation relative to the direction of the geographic meridian) and in the vertical (movement of the interior gimbal). If the oscillation of the gyroscope spin axis in the horizontal plane takes place about the zero value of angle α (angle $\alpha=0$, then, when the gyroscope spin axis is directed to the north -- see the coordinate axis diagram), then in the vertical plane the gyroscope spin axis oscillates relative to a position which is elevated above the plane of the horizon by angle β_0. This inclination of the gyroscope is necessary to insure uninterrupted movement of its exterior gimbal in space following the plane of the geographic meridian of earth with angular velocity ω_B. If this inclination did not exist, the exterior gimbal would oscillate about the plane displaced with relation to the plane of the meridian, that is, the oscillation would be asymmetric about the north direction. Actually, the plane of the meridian at a given point on the earth's surface rotates in space with angular velocity $\omega_B=\Omega\sin\varphi$. In order to cause precession of the exterior gimbal with such a velocity, a constant moment must be applied to the interior gimbal. In a pendulum gyrotheodolite this moment is generated automatically by the inclination of the interior gimbal. The trajectory of the movement of the spin axis of a pendulum gyrotheodolite is shown in Fig. 13. The angular velocity and the characteristic frequency of oscillation of the interior and exterior gimbals are identical and, in accordance with (20), are equal to

$$\omega_c = s = \sqrt{\frac{\Omega Gl\cos\varphi}{H}} \quad \text{(radians/sec)}, \tag{42}$$

whence we get the period of oscillation

$$T = 2\pi\sqrt{\frac{H}{\Omega Gl\cos\varphi}} \quad \text{(sec)}. \tag{43}$$

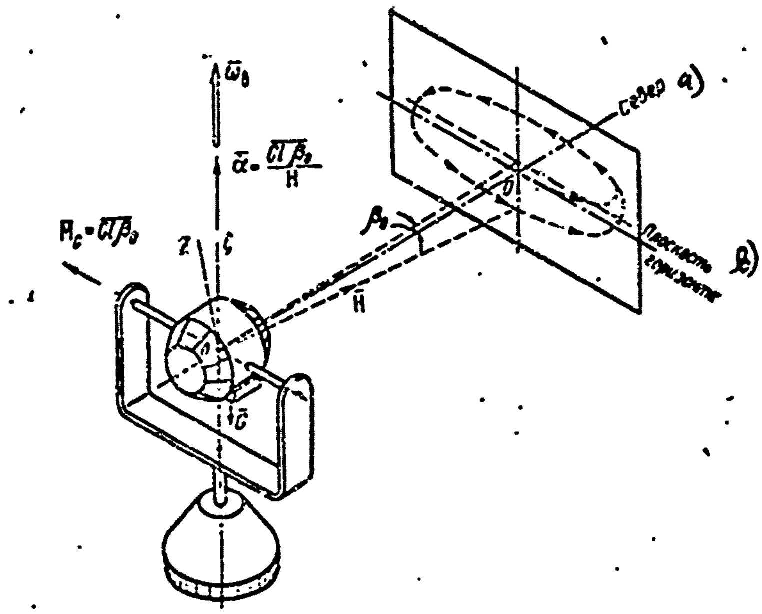

Fig. 13: Trajectory of the movement of the gyrotheodolite on a pictured plane

Key: a. North
b. Plane of the horizon

In modern gyrotheodolites the period of oscillation lies within the limits of 3 to 12 minutes.

The movement of the gyroscope spin axis under consideration is correct for ideal operating conditions. In practical gyrotheodolites deleterious moments are always introduced, which, on the one hand, lead to damping of the oscillations of the gyroscope axes (that is, damping is introduced), and on the other hand -- generates asymmetric oscillations of the gyroscope spin axis, that is, displace the equilibrium position of the oscillatory process. These appear as direct errors in determining the direction of the geographical meridian. Therefore, on the one hand, extremely high design and technological requirements are placed on gyrotheodolites, and on the other hand -- such methods are adapted to accomplishing precisely the measurements which exclude the greatest number of errors from the results of measurement.

At the present time, many different pendulum gyrotheodolites have been developed and adapted to practice. By their structural features they may be separated into two large groups, depending on the form of suspension of the sensitive element (gyromotor): gyrotheodolites with fluid suspension and gyrotheodolites with torsion suspension.

The typical diagram of the first of the gyrotheodolites named showing construction, is represented in Fig. 14. The sensitive element (ЧЭ) is the structure represented as a hollow, hermetically sealed cylinder 5, in the lower part of which is fixed gyromotor 7. The sensitive element floats in a liquid which fills reservoir 8 and has an insignificant buoyancy (5-20g). The ЧЭ is centered relative to the reservoir by means of pivot 1, fastened to the cover of the casing, and ruby (or agate) bearing 2, which is pressed against the pivot under the action of the lifting force of the liquid in accordance with Archimedes' law. The point of suspension of the sensitive element is point D, at which is located the center of gravity of the volume of liquid displaced by cylinder 5. Relative to this suspension point D, the center of gravity of the ЧЭ is displaced downward and is located at point C. The distance between points D and C is equal to approximately 100-120 mm. Owing to such locations of the center of volume of the liquid displaced and the center of gravity, a pendulum moment is generated, and the geometrical axis of the sensitive element attempts to maintain a vertical position, which leads to a gyroscopic effect.

The supply of electric energy to the gyroscope rotor is accomplished without mechanical contact with the ЧЭ; feed to the gyromotor is accomplished through the liquid. Consequently, the liquid, in addition to its function of support, must fulfill the role of conductor of electrical energy. A suitable mixture of alcohol, water and borax or water, borax and formalin as a preservative substance may be used as such a liquid. Electrical energy is transmitted from the casing of the gyrotheodolite to the gyromotor through the liquid by electrodes 6 and 9.

The locking mechanism is located on the top cover of the ЧЭ casing. Counterclockwise rotation of gear 18 is transmitted through screw 19 and pushrod 17 to lever 16, which pulls down cone 15, bringing the ЧЭ up against the cushioning ring 10. In non-working status the ЧЭ presses against ring 10 and, for cone 15 to exert continual pressure during transportation, the locking device is equipped with a spring latch which falls between the gear teeth. The electrical connection is blocked with the locking mechanism; the electrical cable connection cannot be disconnected from the ЧЭ if the ЧЭ is not locked beforehand. This is done so that the instrument is always locked before being removed from the tripod.

For excluding errors in operation of the gyrotheodolite under consideration, the center of gravity of the ЧЭ must lie on its axis of symmetry. This is accomplished by the use of two weights -- ballast 11 and balancing 12.

In consequence of the operation of the gyromotor, a local magnetic field is generated which, interacting with the magnetic field of earth and other external magnetic fields, causes supplementary external

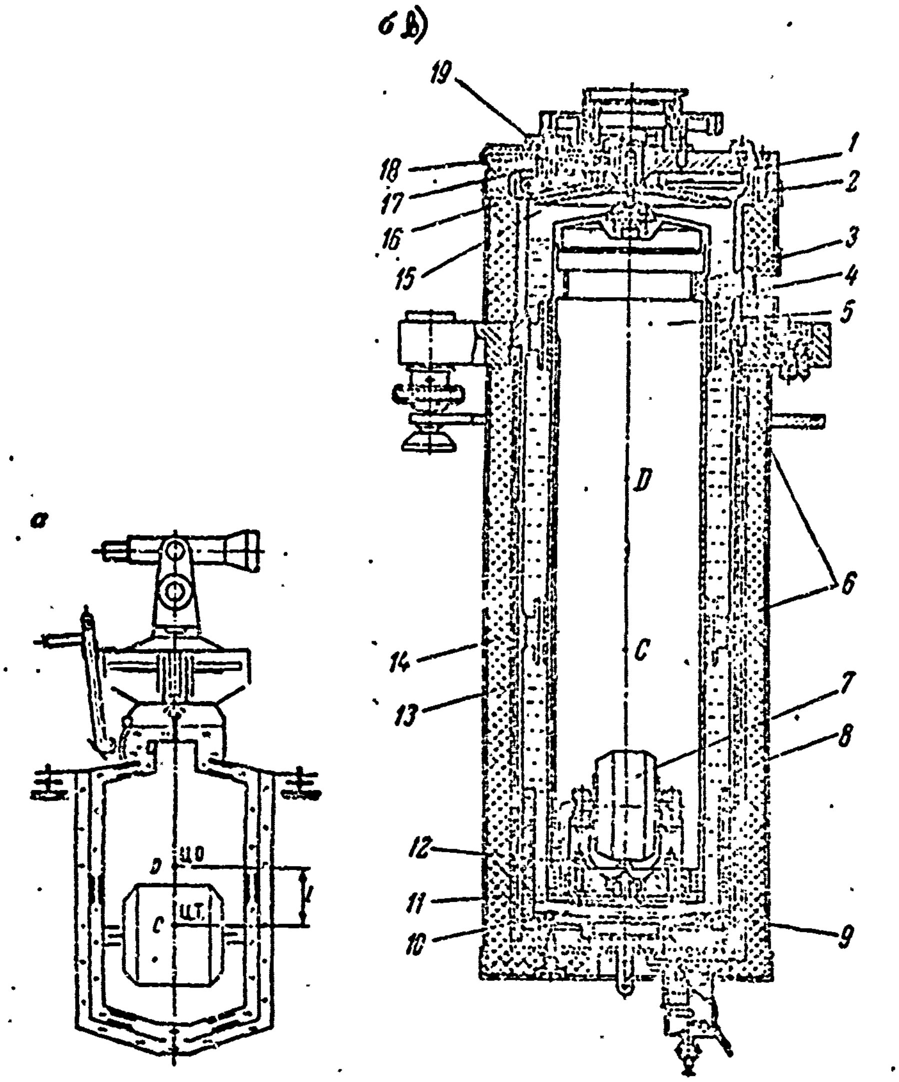

Fig. 14: Pendulum gyrotheodolite centered on a pivot in a supporting liquid

Key:
a. Kinematic diagram
b. Construction of the gyroscope assembly

1. Pivot
2. Bearing
3. ЧЭ mirror
4. Window
5. Sensitive element (ЧЭ)
6. Electrodes
7. Gyromotor
8. Liquid reservoir
9. Elcctrodes
10. Cushioning ring
11. Ballast weight
12. Balancing weight
13. Magnetic shield
14. Insulation
15. Cone
16. Lever
17. Pushrod
18. Gear
19. Screw

moments of force, introducing errors in the operation of the gyrotheodolite. Therefore, the ЧЭ must be isolated from external magnetic fields. This purpose is served by magnetic shield 13, composed of several thin sheets of magnetically permeable material -- permalloy (a nickel-iron alloy). Calculations show that a magnetic field with a flux density of 0.1 gauss is capable of displacing the equilibrium position of the gyrotheodolite ЧЭ oscillations with a kinetic moment $H=(4-5)\ 10^3$ g-cm-sec to an angle of up to 5". From this it follows that the magnetic screen must have the capability of attenuating the internal magnetic field by a hundred times, for a natural anomaly which reaches only two gauss.

Insulating layer 14 is placed between the magnetic shield and the protective external housing.

Mirror 3, observed through window 4, serves to follow the movement of the ЧЭ.

In gyrotheodolites similar to those under consideration, an adjustment in the buoyancy is usually accomplished so that the pressure between pivot 1 and bearing 2 changes negligibly during a change of temperature. To do this such a composition of liquid and ЧЭ volume is selected that the change in volume is compensated for by a change in the specific gravity of the liquid displaced. For example, when the temperature rises the volume of all liquids and, consequently, the volume of liquid displaced by the ЧЭ will be increased, but while this happens the specific gravity of this liquid will be decreased. To accomplish the adjustment, it is necessary to select temperature parameters of the liquid and the volume of the sensitive element in such a way that

$$V_{t_0} d_{t_0} = V_{t_1} d_{t_1},$$

where V_{t_0}, d_{t_0} -- volume of liquid displaced and its specific gravity at a temperature t_0° C;

V_t, d_{t_1} -- volume and specific gravity at t_1° C.

For the purpose of decreasing an effect of the liquid, the frequently appearing statistical moment ("adhesive" effect), the outer surface of the ЧЭ is polished.

Among the deficiencies of gyrotheodolites similar to those under discussion, the foremost has to do with instability of the moments of friction between the pivot and the bearing and the viscosity of the fluid, which lead to a decrease in the acccuracy and stability of operation of the instrument.

At the present time gyrotheodolites with torsion suspensions have a basic difference. Structurally, these gyroscopic systems are realized in two versions: in the form of an organic combination of a gyroscopic part with azimuth disk (gyrotheodolites) and in the form called a gyroscopic attachment, installed on the top or the bottom of the theodolite.

The principal scheme of suspension of a gyromotor on a thin, elastic tape (torsion) is portrayed in Fig. 15. If it is considered that, in the initial instant, vector $\overline{H}$ is situated in the horizontal plane, then at the following instant in time (similar to position II in Fig. 11) the plane of the horizon is deflected from vector $\overline{H}$ and pendulum moment $\overline{M}_G$ arises, the vector of which will be directed perpendicular to the plane of Fig. 15. The interaction of the gyroscope with this pendulum moment leads to the turning of vector $\overline{H}$ in the direction of the vector of moment $\overline{M}_G$ and, consequently, the twisting of the torsion member. As of the result of this, torsion moment $\overline{M}_3$ arises, which is proportional to its angle of twist. This moment opposes movement of the gyroscope axis. After a short time the positions of vector $\overline{H}$ and the plane of the horizon change relative to each other (see, for example, position VI, Fig. 11), from which the pendulum moment begins in a different direction and changes the direction of movement of the gyroscope axis. During this the torsion member untwists at first and, then, begins to twist in the opposite direction. In this manner, the twisting of the torsion member always generates an opposing moment. The reaction of the gyroscope to this moment will be precession of the interior gimbal (rising above vector $\overline{H}$ in the case of Fig. 15). Change of the angular position of vector $\overline{H}$ relative to the plane of the horizon from the action of the twist moment of the torsion member leads to a change in the equilibrium position of oscillation of the gyroscope axis relative to the geographic meridian. From this it turns out that the use of a torsion suspension of a gyromotor would worsen the conditions of operation of the gyroscope as a gyrocompass. However, if the angle of twisting of the torsion member is followed without interruption and, following it, the place where it is fastened revolves, then the moment of twisting of the torsion member will not act on the gyroscope. In addition, high stability of the elastic characteristic of the torsion member generates, through small angles, strongly symmetric moments on the gyroscope, which have a negligible effect on the accuracy of operations.

Since the gyromotor is connected to the casing (base), only through the torsion member, other, mainly chance, perturbing factors (liquid friction, friction between the pivot and the bearing -- Fig. 14) are absent.

Elimination of the moment of twisting of the torsion member introduces into the measuring process the continuous operation of tracking of the gyroscope and the synchronous turning of the place where the torsion member is fastened (built in). The position of the fastening site in which the moment of twisting of the torsion member equals zero does not remain constant in the process of operating the instrument. Therefore, during measurements with such gyrotheodolites, it is necessary to determine this position so that, during oscillation of the gyroscope spin axis around the direction of the meridian, this position of the fastening site coincides with the mark on the gyroscope. The tracking of the gyroscope and

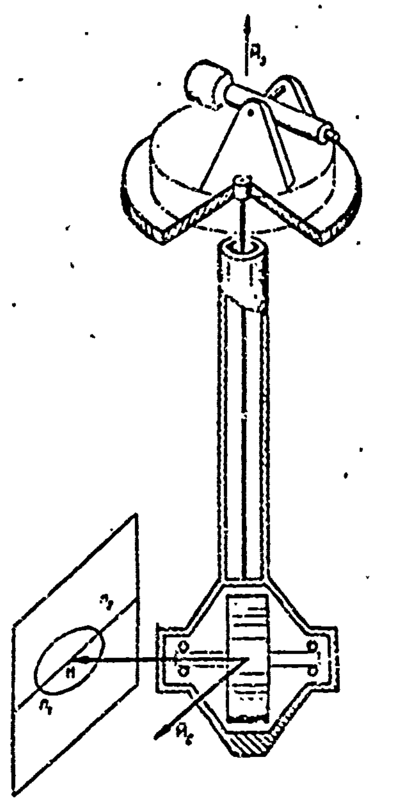

Fig. 15: Gyroscope on a torsion suspension

turning of the torsion member are accomplished both by hand (the KT-1 and Gi-B1 gyrotheodolites) and automatically (MT-1, Gi-B2, IMK-604).

Introduction of automatic tracking eliminates irregularities in motion, variability in the residual angle of twisting and, also, frees the observer from the tiresome operation of tracking the torsion member and its twisting. Due to the accuracy of automatic tracking of the twisting angle of the torsion member, the 4"-6" error in determining the direction of the geographical meridian is reduced by 20-25%, compared with tracking by hand [45]. Accuracy in tracking of 4"-6" means that only within these limits will there be irregularity in twisting of the torsion member which generates a definite perturbation of the gyroscope. For a torsion member with rectangular section, the twisting moment M_3 is determined by the equation

$$M_3 = \frac{EI_c}{l}\gamma.$$

where $E=8\cdot10^5$ kg/cm^2 -- twisting modulus of the torsion member material;

$I_c=\frac{bh^3}{3}$ -- moment of inertia of the section;

b -- width of the tape;
h -- thickness of the tape;
l -- lenth of the tape;
γ -- angle turned by the torsion member.

For a torsion member with the measurements: b=0.04cm, h=0.005cm, l=13cm and angle $\gamma=5''$, the twisting moment $M_3=1\cdot10^{-6}$g-cm. The absolute value of this moment is small, but the directing moment of the gyrotheodolite is smallish. In accordance with Eq. (33) for latitude $\varphi=60°$ and the gyroscope with kinetic moment H=4000g-cm-sec, when the angle of deviation of vector H from the plane of the geographical meridian $\alpha=10''$, the directing moment $M_H=5.5\cdot10^{-6}$g-cm. From a comparison of moments $\bar{M}_H$ and $\bar{M}_3$, the inference follows of an expedient increase in the accuracy of the tracking system for the twisting of the torsion member (decrease in angle γ). An increase in the accuracy of this tracking decreases the angle within whose limits the angular position of the torsion member is arbitrary.

From among examples of gyrotheodolite construction produced by the Hungarian optical factory (MOM), let us examine the basic principles of a constructive solution for gyrotheodolites with torsion suspension. A diagram of the Gi-B1 gyrotheodolite, which is the basic instrument for the Gi-B2 gyrotheodolite, is represented in Fig. 16. The gyrotheodolite is composed of two main parts: azimuth disk (theodolite) and gyroscopic, which are combined in a single instrument and transported as such. However, its construction permits replacement of the gyroscopic unit in case it is necessary.

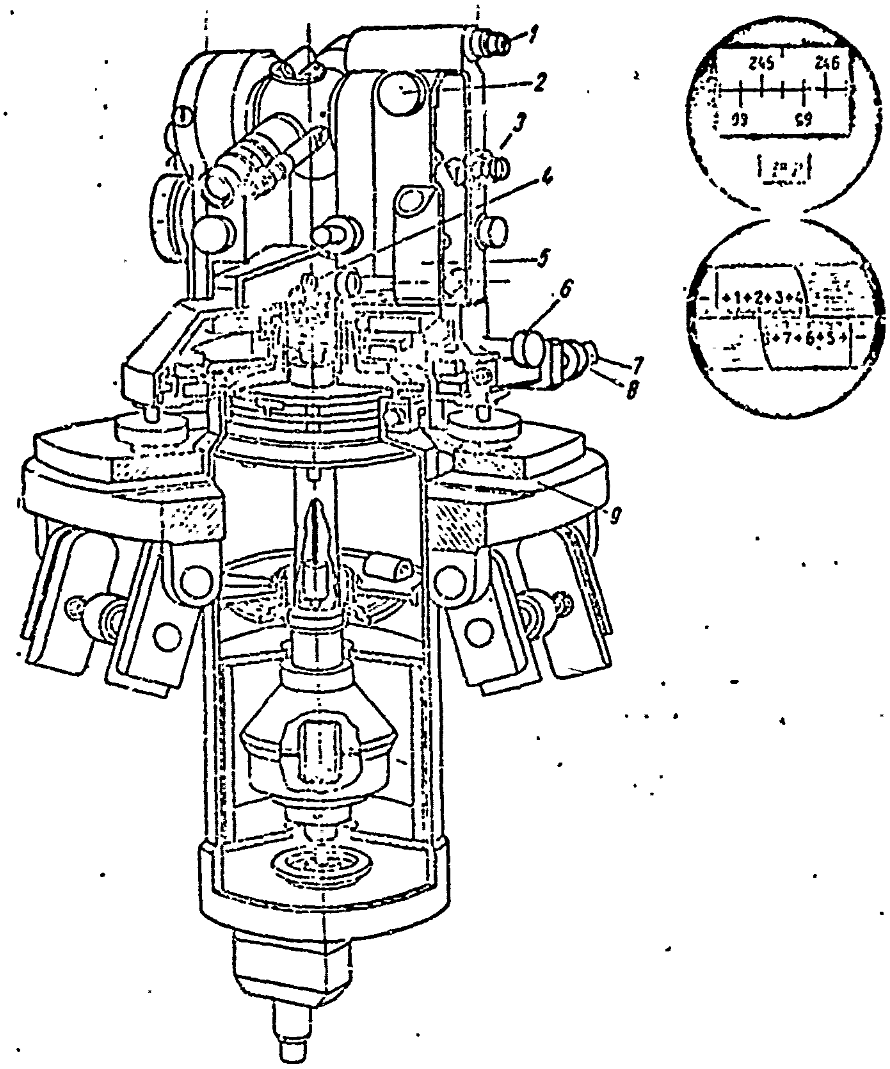

Fig. 16: Kinematic diagram of the Gi-B1 gyrotheodolite:

1. Optical microscope eyepiece
2. Knob
3. Automatic collimator eyepiece
4. Sensitive element mirror
5. Illuminating lamp housing
6. Alidade anchor screw
7. Coarse aiming reducing gear knob
8. Fine aiming reducing gear knob
9. Torsion member zero position regulating screw

The azimuth disk section is executed on the base of an MOM model Te-Bl second-measuring theodolite, into which changes necessary for operation with the gyroscope unit have been introduced: in the right hand column, the optical telescope has been replaced by the automatic collimator for tracking the gyroscope oscillations; a supplementary eyepiece elbow is located on it for reading the micrometer; the vertical axis system is hollow, and the sensitive element mirror is situated inside it.

Observation of the sensitive element by mirror 4 is conducted through eyepiece 3 of the automatic collimator. The upper and lower parts of a scale move in opposite directions in its field of view.

The distance between two thickened marks is equal to double the angle through which the gyroscope turns relative to the optical axis of the automatic collimator. The value of the smallest division on the scale of the Gi-Bl is equal to 30". The source of light for the automatic collimator scale is an electric light bulb placed in housing 5. Determination of the position of the point where the torsion member is fastened, about which the twisting moment M_3 is near zero (the zero torsion point), is done by the automatic collimator scale. According to the measurements obtained, the position of the zero torsion point may be changed by turning regulator screw 9, the head of which has divisions two times larger than the divisions on the automatic collimator scale.

During operation of the gyromotor, tracking of the ЧЭ mirror is accomplished by turning the alidade, with anchor screw 7 fastened down, aiming, by means of rotation of coarse reducing worm gear knob 7 and fine aiming reducing gear knob 8, while continually keeping the thickened marks of the automatic collimator scale lined up. During the approach to the reversing point (the point of change of direction of movement), coincidence of the marks is accomplished particularly thoroughly, and, at the moment the moving image of the automatic collimator scale stops, a reading is taken from the horizontal circle by use of knob 2 on eyepiece 1 of the supplementary indicating micrometer.

The gyroscopic part, or gyroscope unit, is produced as a complete independent construction (Fig. 17). Gyroscope motor rotor 14 is located in vacuum chamber 13, which is suspended on torsion member 11 from lifting plate 4. This plate rests on three pins 5 on rotating ring 6. The rotating ring has lug 8, on which the turning of upper torsion member fastening assemply 3 is accomplished with the help of micrometer screw 7, for correcting the position of its zero point. Rotation of the torsion member after the gyroscope during its oscillation about the direction of the meridian is accomplished by rotation of the lifting plate. Tubular bar 9 of the gyroscope motor, into which cuts are made for a bracket fastening the upper end of the torsion member, bears mirror 1 and electrical conductor 2. The oscillation of the gyroscope axis is observed through the mirror with the help of the automatic collimation system. The gyroscope motor is a three phase electric motor; therefore, there must be three isolated electrical circuits for its operation. Two silver spiral springs 2 are used for transmitting the two outer

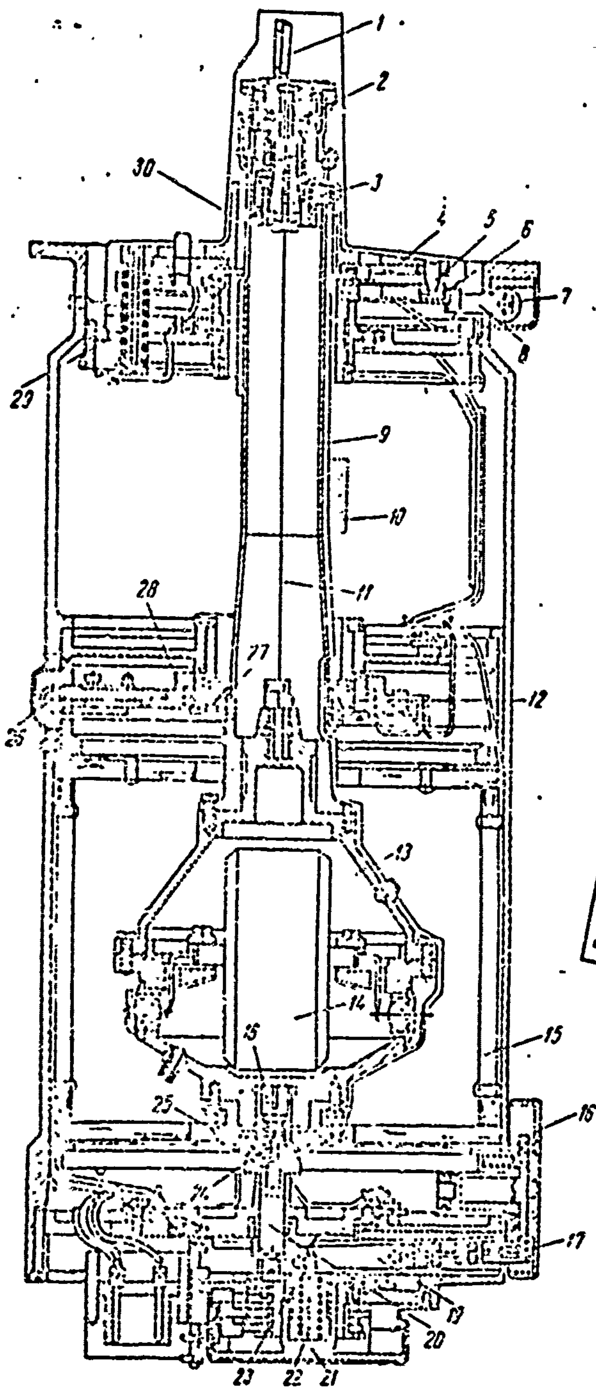

Fig. 17· Gi-B1 pendulum gyro-theodolite gyroscope assembly

Key:
1. Sensitive element mirror
2. Electric current supply
3. Upper torsion element fastening assembly
4. Lifting plate
5. Support pins
6. Rotating ring
7. Micrometer screw
8. Rotating ring lug
9. Hollow bar
10. Position mirror for Gi-B2 torsion element twist tracking system
11. Torsion element
12. Conical insert
13. Gyroscope chamber
14. Gyroscope motor
15. Magnetic protective screen
16. Base locking mechanism knob
17. Pin
18. Plug
19. Lever
20. Locking pin
21. Auxiliary locking mechanism knob
22. Spring
23. Pushrod
24. Mushroom
25. Current collector
26. Worm gear
27. Contact ring
28. Conical ring
29. Spring
30. Guide

phases, and the third phase is transmitted across the torsion member. The spiral spring conductors have mutually opposing moment characteristics, with the result that the sum of the twisting moments acting on the gyroscope is negligible. The conductors mentioned serve only to maintain the rotation of the rotor. Starting and stopping of the gyroscope motor is accomplished across different contact groups. At the bottom of the gyroscope chamber are situated two current collectors 25 which, while the gyroscope is in locked position (see below) come into contact with current carrying lamellae, introduced along spherical segment-mushroom 24 of the locking mechanism. The third phase completes the circuit on the body of the gyroscope chamber. Current is supplied by these conductors during starting-up of the gyroscope motor. Transmission of braking current (in positive stopping) to the gyroscope motor is accomplished across the cone shaped half-ring contacts 28 and contact ring 27. Stopping is accomplished by transmission of a constant current to the stator winding of the gyroscope motor.

The gyroscope unit is equipped with turning mechanisms, serving in the locked status for setting the reflecting surface of mirror 1 of the ЧЭ perpendicular to the optical axis of the automatic collimator. This position is determined visually during alignment of the central marks of the automatic collimator. Turning of the gyroscope assembly is accomplished by use of a pair of worm gears 26, rotating thrust ring 28 with conical insert 12, which carries along the gyroscope assembly tightened to it in locked condition.

The locking mechanism is intended for securing the gyroscope unit in non-operating (non-measuring) condition (including while being transported) when stopping and starting. For locking the gyroscope assembly, knob 16, which has a cam which moves pin 17 and which itself forces in lever 19, is rotated. This lever, pushing against rod 20 of the locking mechanism, which ends in mushroom 24, displaces the latter upwards, putting stress on rubber membrane stretched over plug 18, raises the gyroscope assembly upwards and presses conical ring 27 tightly against conical half-ring 12 of the turning mechanism. In this position the torsion ribbon is freed from the weight of the gyroscope chamber and, consequently, the lifting plate is also freed from it. The latter is squeezed upward by strong springs, on account of which the torsion element is stretched with a force of about 600g. The stressed state of the torsion element in the period between start-ups insures a high stability of its mechanical characteristics, owing to maintainance of stability of the internal structure of the material.

One more supplementary locking mechanism is provided in the structure of the gyroscope unit to increase the reliability of locking. By rotation of knob 21, spring 22 is compressed and, in addition, lever 19 is tightened through pushrod 23. A safety cutout is located in the locking mechanism knob which locks the power supply cut-off in locked condition.

To decrease the deleterious effect of external magnetic fields, the gyrotheodolite has special magnetic screen 15, which is cylindrical in shape, is made of 18 sheets of permalloy of a thickness of about 0.25mm and possesses a great magnetic permeability. The magnetic screen is electrically insulated from the remaining parts of the gyroscope assembly.

In addition to the construction of the Gi-B1 gyroscope assembly under consideration, provisionally indicated on the cylindrical column is mirror 10, used for twisting of the torsion element[1] in the Gi-B2 gyrotheodolite.

The torsion element is fastened to the gyroscope chamber in such a manner that the axis of the gyroscope rotor is perpendicular to the plane of the torsion ribbon. In this position, the torsion element will show the least resistance to the gyroscope during oscillations of its vector $\bar{H}$ relative to the plane of the horizon. The torsion ribbon has a rectangular section of approximately 0.04×0.005cm and a length of 13cm. The distance from the point where the lower end of the torsion element is fastened to the center of gravity of the gyroscope is 7.2cm, the speed of rotation of the rotor is about 21,000 rpm and kinetic moment H≈4000g-cm-sec.

Subsequent development of the gyrotheodolite under consideration is model Gi-B2, in which a photoelectric tracking system is used, insuring automatic turning of the torsion element linkage points after the gyroscope. The kinematic and structural diagram of this tracking system is shown in Fig. 18. Light from source 12 is shaped into a parallel beam by condenser 11 and goes further through prisms 10 and 4 and falls on mirror 3 (mirror 10 in Fig. 17), installed on sensitive element 8. Reflecting off mirror 3 and, further, off prisms 4 and 10, the light falls on photocell 5, the signal from which drives motor 7 through amplifier 6. A split photoresistor is used as a photoreceptor, in which the magnitude of the output current depends on the magnitude of the light beam falling on it. The sides of the photoresistor (5a and 5b) are included in an electrical bridge. The light beam (diffusion circle KP), in the absence of angular deviation, shines on both halves of the photoresistor uniformly, and the electric current across the bridge is equal to zero. As the gyroscope turns relative to the housing to angle α_{Γ}, there is a displacement of the light spot, and illumination on one half of the photoresistor increases and on the other, decreases. As a result of this, a current appears across the bridge, the magnitude of which is proportional to angle α_{Γ} and the polarity of which is determined by the direction of displacement of the light spot (sign of angle α_{Γ}). However, the magnitude of the current across the bridge will be proportional to angle α_{Γ} only when the light spot shines simultaneously on both halves of the photoresistor. When the light spot illuminates only one of the halves of the photoresistor, the magnitude of the light beam will not change; therefore, the current across the bridge will not change (the relation between the

[1] [Fig. 17, Key Item 10, indicates that mirror 10 is used for tracking the twisting of the torsion element.]

angular position of the light spot and the current across the bridge is shown in the graph of Fig. 18).

The current across the bridge is the input signal to the amplifier, where a constant current is amplified and transformed into alternating current with a frequency of ~400hz, since the turning of the housing behind the torsion element is accomplished by a two-phase induction motor with a DID-0.5 hollow rotor operating at an increased frequency.

In the Gi-B2 gyrotheodolite, the position of the zero point is adjusted by turning the upper fastening point of the torsion element. This displacement is accomplished by use of worm gear 9 of the tracking system, but in this case supporting housing 14 is secured to exterior housing 15 by use of lever 16. After release of lever 16, the tracking housing turns to the former position (while switching on the photoelectric system) with relation to mirror 3, but the zero point of the torsion element proves to be displaced during this by the magnitude of change of the zero point. Therefore, when tracking the gyroscope position, the zero angle of turning will approximately match zero twisting of the torsion element.

The presence of an automatic system for twisting the torsion member frees the observer from the necessity of continuously observing the movement of the sensitive element. Therefore, in construction, the Gi-B2 gyroscope unit is not connected mechanically with the theodolite alidade. This permits accomplishment of geodetic tying-in of observed points during the period of movement of the gyroscope from one reversing point to another, while the gyroscope motor is starting up and stopping.

The characteristics discussed are the principal differences of the Gi-B2 from the Gi-B1 gyrotheodolite. But, simultaneously with these, the construction of the magnetic screen in the Gi-B2 gyrotheodolite has been improved, several changes in construction of the locking mechanism have been made and the power supply unit has been modernized.

We assign the Gi-B1 and Gi-B2 instruments as the basis for discussion of other gyrotheodolites, and we will indicate distinguishing characteristics relative to them of the remaining pendulum instruments for determining the direction of a geographical meridian.

MT1 gyrotheodolite (USSR) -- pendulum, with torsion suspension of the sensitive element in air, with photoelectric system for tracking twisting of the torsion element; kinetic moment H=10,000 g-cm-sec and H=23,000 g-cm-sec [15].

LMK-604 gyrotheodolite ("Lear-Electronic" Company, Munich) -- pendulum, with torsion suspension in air, twisting of the torsion member is accomplished by use of an electromechanical tracking system with induction sensor of the angle which serves to measure the angular position of the gyroscope [50].

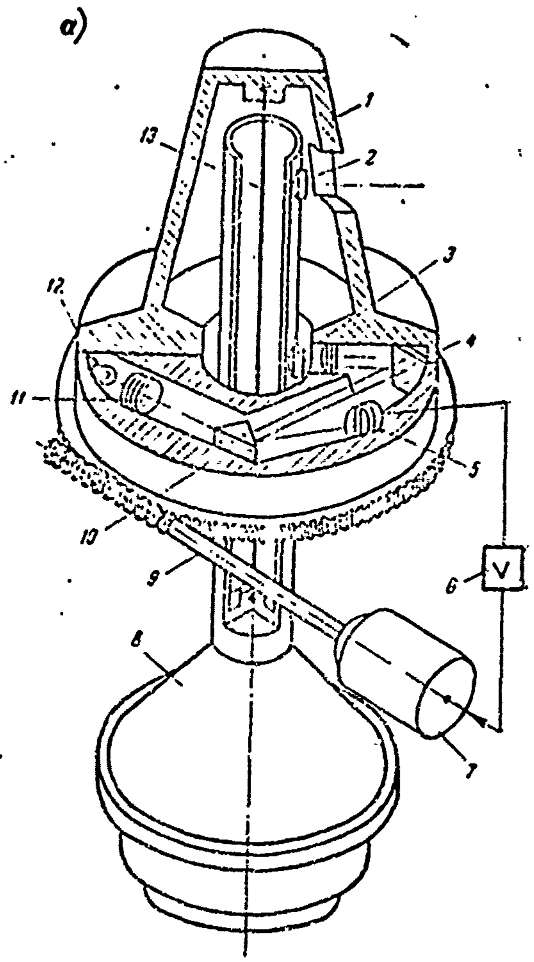

Fig. 18: Diagram of Gi-B2 gyrotheodolite

Key:

a. Kinematic diagram of the system for tracking twisting of the torsion member

1. Tracking housing
2. Sensitive element mirror for observing reversing points
3. Tracking system mirror
4. Prism
5. Photocell
6. Amplifier
7. Motor
8. Sensitive element
9. Worm gear
10. Prism
11. Condensor
12. Light source
13. Torsion element

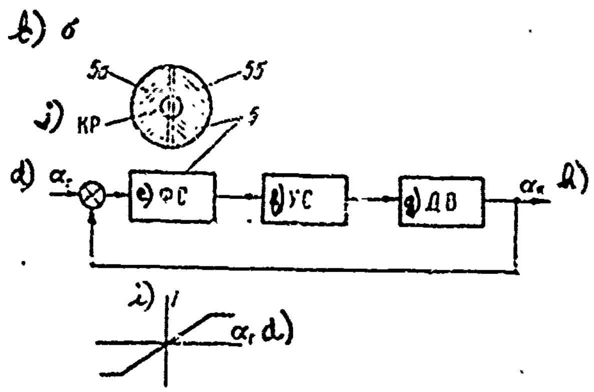

Fig. 18: Continued

Key:

b. Tracking system structural scheme
d. Angle turned by the gyroscope
e. Photoresistor
f. Amplifier
g. Motor
h. Angle turned by the housing
i. Current across the bridge
j. Diffusion circle

c)

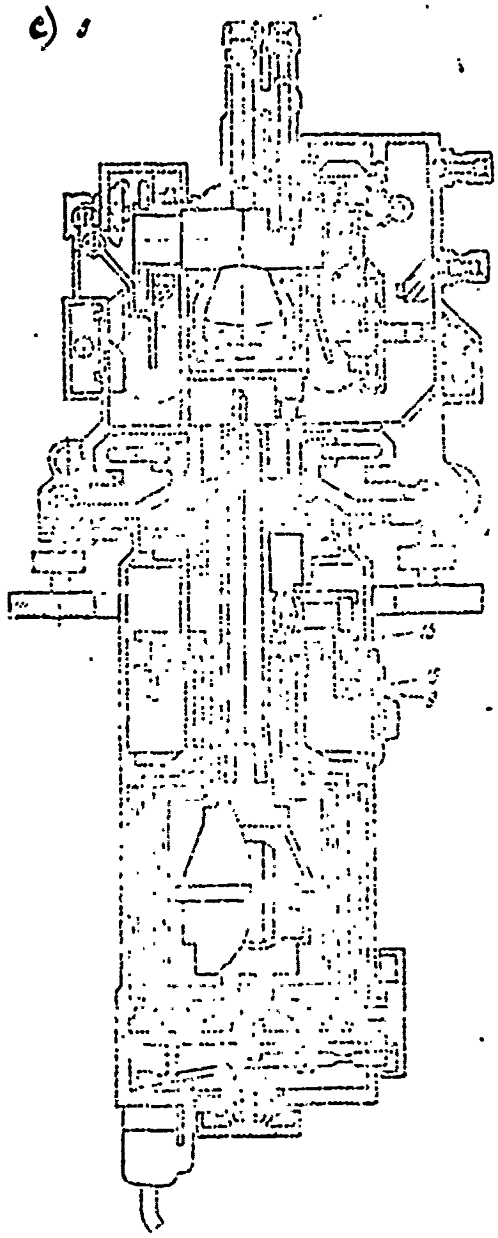

Fig. 18: Continued

Key: c. Instrument cross-section

9. Worm gear
14. Support housing
15. Exterior housing
16. Lever

"Gitar" gyrotheodolite ("GYTAR" -- Gyro Theodolite Azimuth Reference, USA) -- pendulum, with torsion suspension in air, with system for tracking twisting of the torsion element [6]. Probably, subsequent development of this instrument in the military engineer-topographical laboratories of the USA will be a gyrotheodolite with electrical damping of the oscillation, as a result of which its axis will be fixed by the direction of the geographic meridian (accuracy ~1', measurement time 12 min).

MRK -2 gyrotheodolite (Meridianrichtungskreisel, German Peoples Republic) -- pendulum, with torsion suspension in air, with tracking of the torsion arm (Fig. 19). Mercury conductor (liquid); the mercury ... are shut off when the instrument is locked.

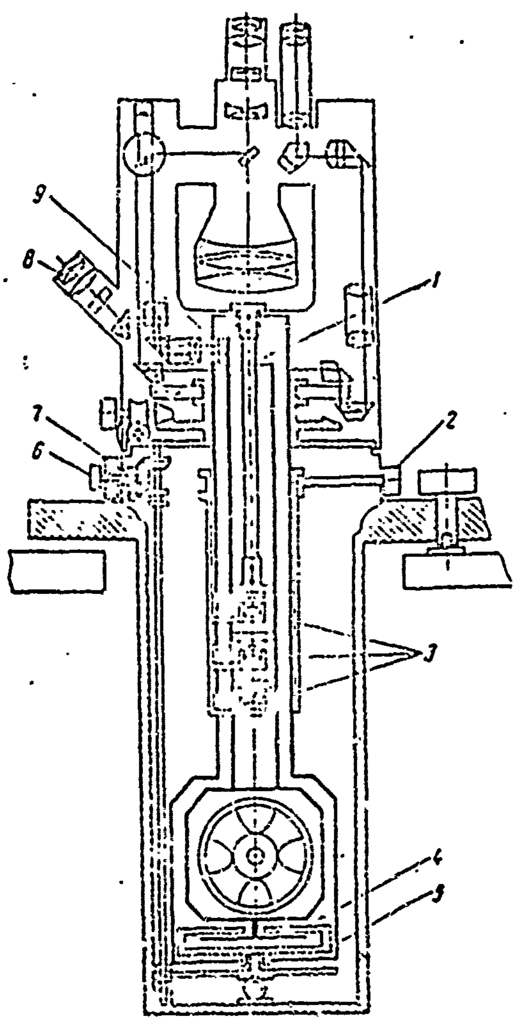

Fig. 19: MRK -2 gyrotheodolite

Key:
1. Torsion element
2. Current supply lever with eccentric
3. Mercury conductors
4. Damping rotor
5. Damping housing
6. Gyroscope locking lever
7. Eccentric
8. Automatic collimating system eyepiece
9. Sensitive element mirror

The present instrument has a relatively small period of oscillation (2-2.5 min); it is advertised by the manufacturing plant as a method of measuring the equilibrium position of sensitive elements oscillations without synchronous turning of the fastening point of the upper end of the torsion element. For this, it is necessary to have a small (within the field of vision of the automatic collimator system) amplitude of oscillation of the sensitive element, for which a special mechanism to regulate the amplitude of sensitive element oscillation is provided in the MRK-2 [28]. If the liquid level in damper bath 5 is such that rotor 4 of the damper does not touch it, the sensitive element oscillations will be practically undamped (see Fig. 19). But if rotor 4 is immersed in the liquid, the oscillation will be damped. Regulating the degree of immersion of rotor 4 in the liquid by vertical shifting of bath 5 by use of eccentric 7 can regulate the amplitude of sensitive element oscillations.

KT-1A gyrotheodolite (German Federal Republic) -- pendulum, with torsion suspension in air, permitting orientation without determining the instrumental correction of the apparatus. For this, gyroscope motor casing 6 is fastened to axle 9, which is perpendicular to the spin axis of rotor 10 (Fig. 20). Mirror 13 is fastened on this axle 9 of the gyroscope motor housing. It is used for observation of oscillations of sensitive element 11, suspended on torsion element 4 (the plane of the torsion ribbon is perpendicular to the gyroscope rotor spin axis, which is shown in Fig. 20 in a position perpendicular to the plane of the sketch).

After carrying out the measurement in the first position of the rotor (pictured in Fig. 20), the gyroscope motor stands still, and lever 8 is moved to the lowest position by handle 7. Axle 9 of the gyroscope motor housing is rotated through 180° together with lever 8, as a result of which the "north" end of the axle proves to be directed to the south, and the "south" -- to the north. The sensitive element mirror rotates together with the rotation of the gyroscope motor housing. Fixing the rotation of the gyroscope motor strictly on 180° is accomplished by use of conical holes and pins.

For determining the direction of the geographic meridian in the second position of the rotor, the goniometer part, together with the locked sensitive element is rotated through 180°, after which the usual measuring processes are carried out.

The final result of measurement is the average of the determinations obtained in the two rotor positions.

The scheme of constructive solution produced permits elimination of a number of instrumental errors, such as: eccentricity of the goniometer part, nonconcordance of the axes of the optical systems and gyroscope motor axes, the effect of inclination of the tripod, invariable composition of internal magnetic fields and others.

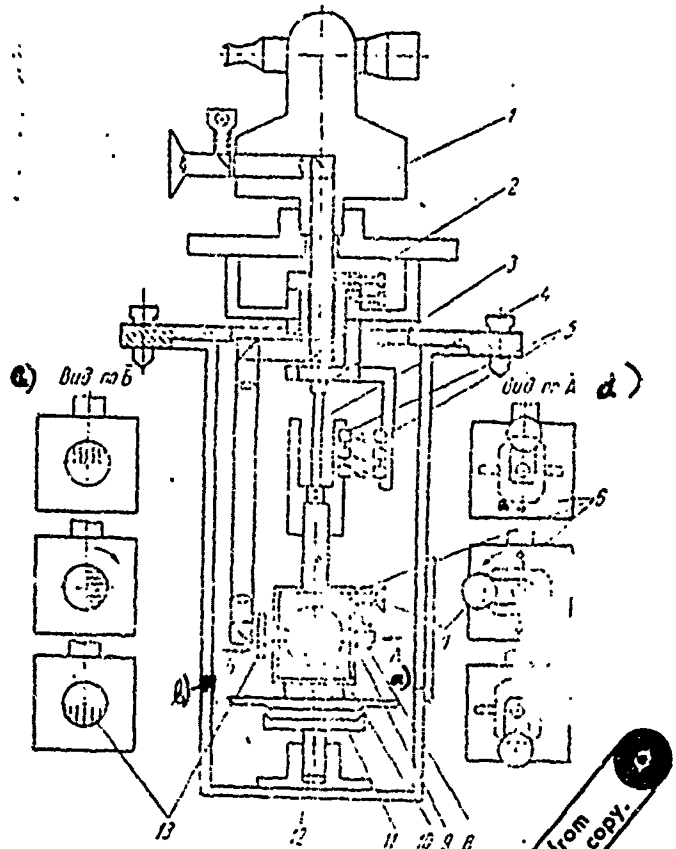

Fig. 20: The KT1-A gyrotheodolite

Key:
a. A
b. B
c. View from B
d. View from A

1. Theodolite
2. Plug
3. Upper torsion element fasteni ..
4. Torsion element
5. Electric current supply
6. Gyroscope unit
7. Locking handle
8. Lever
9. Gyroscope motor housing axle
10. Gyroscope rotor
11. Sensitive element
12. Lock
13. Sensitive element mirror

The error in determining the direction of the geographical meridian is on the order of 15" for a duration of measurement of 2-2.5 hr; the gyroscope moment $H=8.5\cdot 10^3$g-cm-sec.

MW-10 gyrotheodolite (German Federal Republic) -- pendulum, with torsion suspension of the sensitive element in liquid, has explosion-proof housing[1], and a liquid power supply mechanism, analogous to the power supply in Fig. 14. Suspension of the sensitive element in a liquid permits isolation from explosive parts of the surrounding environment, in which electrical sparks may arise. On the other hand, suspension in liquid permits the torsion element to have a small cross-section and, consequently, a small rigidity.

Attempting to decrease the size and weight of the apparatus while simultaneously shortening the time necessary for making a measurement, led to the creation of the so-called gyroscope attachment, in which the gyroscopic measuring part is removable and is set on the theodolite telescope. At the present time four groups of gyroscope attachments ("gyroadapters") are known: gyroscope attachment models Gi-C and Gi-D of the MOM works (Hungarian Peoples Republic), model TK gyroscope attachment (Theodolitkreisel) of the "Fennel'" Company, gyroscope attachments of the "Vil'd" Company works (GAK-1, ARK-1) and the MVSh3 and MVT2 "gyroadapters" (developments of the All-Union Scientific Research Institute of Mine Surveying, USSR). The MVSh3 and MVT2 gyroscope adapters [15] have such a structural composition that the gyroscopic part is located under the optical theodolite and is connected to a carrier. The MVSh3 sensitive element has a cylindrical form, suspended in liquid, and centered on a pin (like the sensitive element pictured in Fig. 14). The sensitive element in the MVT2 gyroscope adapter has a torsion suspension. The MVT2 gyroscope adapter is equipped with a photoelectric system for tracking twisting of the torsion element. Both gyroscope adapters indicated have explosion-proof configurations.

The MOM works gyroscope attachment differs principally from the model TK and GAK gyroscope attachments in that with them observation of the oscillations of the sensitive element and geodetic tying-in are carried out by use of one and the same telescope. In the model TK and model GAK gyroscope attachments these optical channels are separate. On the one hand, this permits observation to be conducted independently of movement of the ЧЭ, and on the other hand -- increases the number of parts in the construction, with the help of which movement of the gyroscope spin axis is transmitted by an optical system, reproducing the equilibrium position of the gyroscope spin axis on the terrain, which can introduce additional errors into the measuring process.

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[1] The instrument can operate in such a configuration in an explosive gas environment, for example, in fire damp surroundings, when electrical sparks (in power supplies, gyroscope motors and others) may cause an explosion.

The Gi-C2 gyroscope attachment, which is a pendulum gyrotheodolite with torsion suspension in air, is depicted in Fig. 21. Observation of the sensitive element oscillations is carried out by use of the theodolite telescope, on which there are three posts 12 with nuts 13 for fastening the gyroscope attachment housing. An optical connection of the gyroscope attachment and theodolite is created by use of base prism 8 for observation of the ЧЭ oscillations. Prism 8 is made of glass with a low coefficient of thermal expansion. During observations of objects on the terrain, the base prism is moved aside from the theodolite telescope objective. Construction of the gyroscope attachment ЧЭ is similar to the construction of the MOM works gyrotheodolite discussed earlier.

Electric power supply from connection 11 to the gyroscope attachment is transmitted through current carrying terminals installed on posts 12. In locked position, the gyroscope motor receives power from current feed receptacle 1 through split hemispherical current feeds 2 (phases I and III) and through the body (phase II). In unlocked position (after starting the gyroscope motor), electric energy is transmitted to the gyroscope motor through current feed ribbon 3 and torsion element 4. In locked position, the torsion element is in a stressed condition, created by springs 5 (Fig. 21a).

Flat parallel plate 7 is set in the bottom of the ЧЭ for observation of azimuthal oscillations. A light beam from the light source goes through measuring grid 15 and adjusting grid 16 and, further, through the flat parallel plate, the collimator objective and the base prism to the theodolite telescope. Using the property of the flat parallel plate, there is a parallel displacement of the outgoing beam relative to the entering one. The magnitude of this displacement depends on the angle at which the incoming beam falls; the greater the angle of inclination of the beam from normal to the entrance surface of the plate, the greater the displacement of the exiting beam. Considering that the angle at which the beam falls will change during azimuthal oscillations of the ЧЭ, on the exit edge of the plate this angle will correspond to the linear displacement of the measuring grid. In this case, the focal length of the optical system is such that the measuring grid will be observed in the telescope in a form which is analogous to the form of the grid of the Gi-31 automatic collimator. Superposing of the thick marks of the measuring grid during rotation of the theodolite alidade determines the position of the reversing points.

In locked status, the flat parallel plate is removed from the path of the optical system beam, the image of the measuring grid is blurred and the image of the regulating grid will be observed clearly in the field of view of the telescope. Using the image of this grid, the optical axis of the collimator is set in a position parallel to the optical axis of the telescope, assuming that there are no errors in the base prism. This setting is made by turning the gyroscope attachment housing by use of regulating screw 9. Systematic errors (corrections) connected with different placements of the gyroscope attachment on the theodolite and with different thermal deformations of the gyroscope attachment and

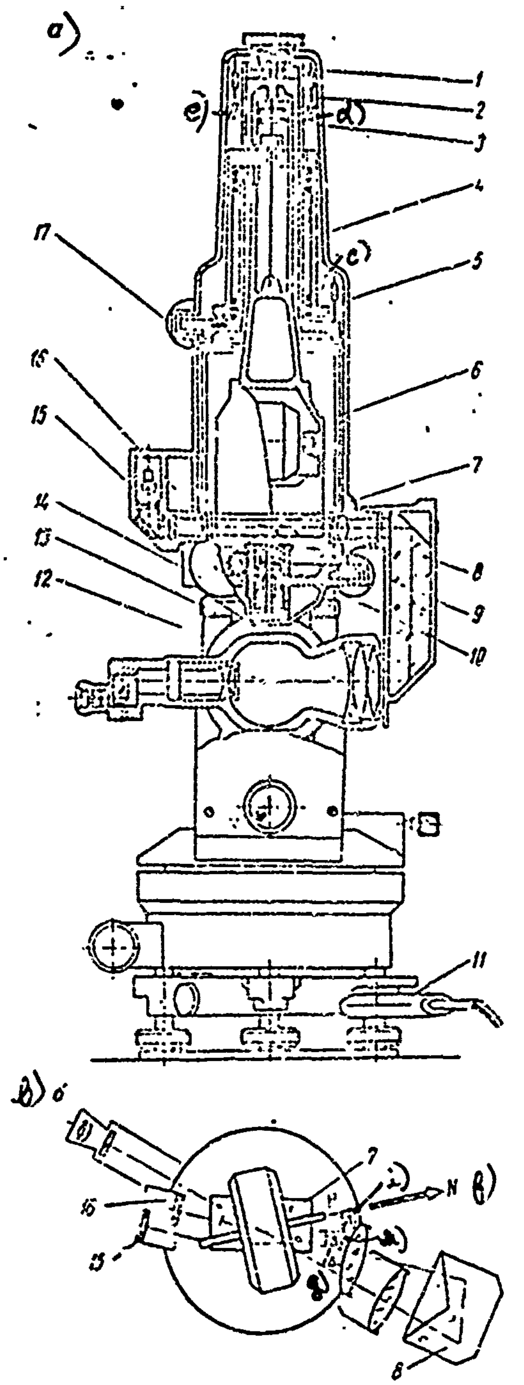

Fig. 21: Model Gi-C2 gyroscope attachment

Key:

a. Gyroscope attachment
b. Scheme for production of the constant correction
c. Current feed phase I
d. Current feed phase III
e. Current feed phase II
f. North
g. Divergence of beams from parallel
h. Angle based on non-coincidence of measuring and adjusting grids
i. Angle between the normal to the flat parallel plate and kinetic moment vector

1. Current feed receptacle
2. Split hemispheric current lead
3. Current lead ribbons
4. Torsion element
5. Springs
6. Magnetic protective screen
7. Flat parallel plate
8. Base prism
9. Regulating screw
10. Lock rod
11. Plug connection
12. Posts (3 ea)
13. Nuts
14. Micrometer screw knob
15. Measuring grid
16. Adjusting grid
17. [Not identified]

theodolite are eliminated from the results of measuring by introduction of this adjustment. The gyroscope attachment housing regulator for making the optical axes of the telescope parallel and the collimator are back-coupled, locking two independent structures (gyroscope attachment and theodolite) into a single whole.

The Hungarian MOM optical works produces the Gi-C1, Gi-C2, Gi-D1 and Gi-D2 gyroscope attachments. The Gi-C1 model is distinguished from the Gi-C2 by the telescope optical system (the latest model has a dividing prism). The Gi-C model gyroscope attachments are supplied with 34 v current with a frequency of 410 hz; Gi-D model gyroscope attachments are supplied with current of a frequency of 300 hz, which shortens the period of oscillation by 20-30% and, consequently, the time required for determining the direction of a geographical meridian, but together with this decreases the accuracy.

Two designs of the Vil'd Company gyroscope attachments are shown in Fig. 22. The gyroscopic parts of both instruments are identical; the difference concerns the theodolite part: in one case (GAK-1), the gyroscope attachment and theodolite are produced as independent structures, and in the second ARK-1), the gyroscopic part and the theodolite are united in a common structure. From the point of view taken above of division of gyroscopic instruments into gyrotheodolites and gyroscope attachments, the ARK-1 belongs more to the first by grouping the basic parts together in construction but lets it be considered a gyroscope attachment at the same time.

The GAK-1 gyroscope motor casing 9 has damping plate 5 below, and at the top tubular bar 12, on the upper part of which torsion element 14 is fastened. Optical system 11 and gyroscope mark 10 are located in the bar. Upper clamp 15 of the torsion element can be turned by use of screw 16 for adjustment of the torsion element zero point position. The gyroscope unit is locked by tightening pins 7 of the damping plate to lugs 8 by rotation of screw rod 4. Whether the gyroscope is in locked or unlocked position can be determined through the inspection window. Fastening of the whole gyroscope attachment to the theodolite is accomplished by cover nuts 3. Starting and stopping of the gyromotor is carried out in locked position. After unlocking, the gyroscope oscillates about the equilibrium position. In view of the fact that the method of determining the equilibrium position recommended by the company consists in observation of reversing points without twisting the torsion element, the amplitude of oscillations must be small (within the limits of the field of view of the scale). Regulation of the amplitude of oscillations of the gyroscope is accomplished by friction of spring leaf 6 of the locking mechanism on the damping plate pins. After establishment of the required amplitude of oscillations, the spring leaf is dropped down by the locking mechanism bar.

The gyroscope oscillations are observed through telescope 1, which has scale 2 in the field of view, on which the gyroscope mark is projected.

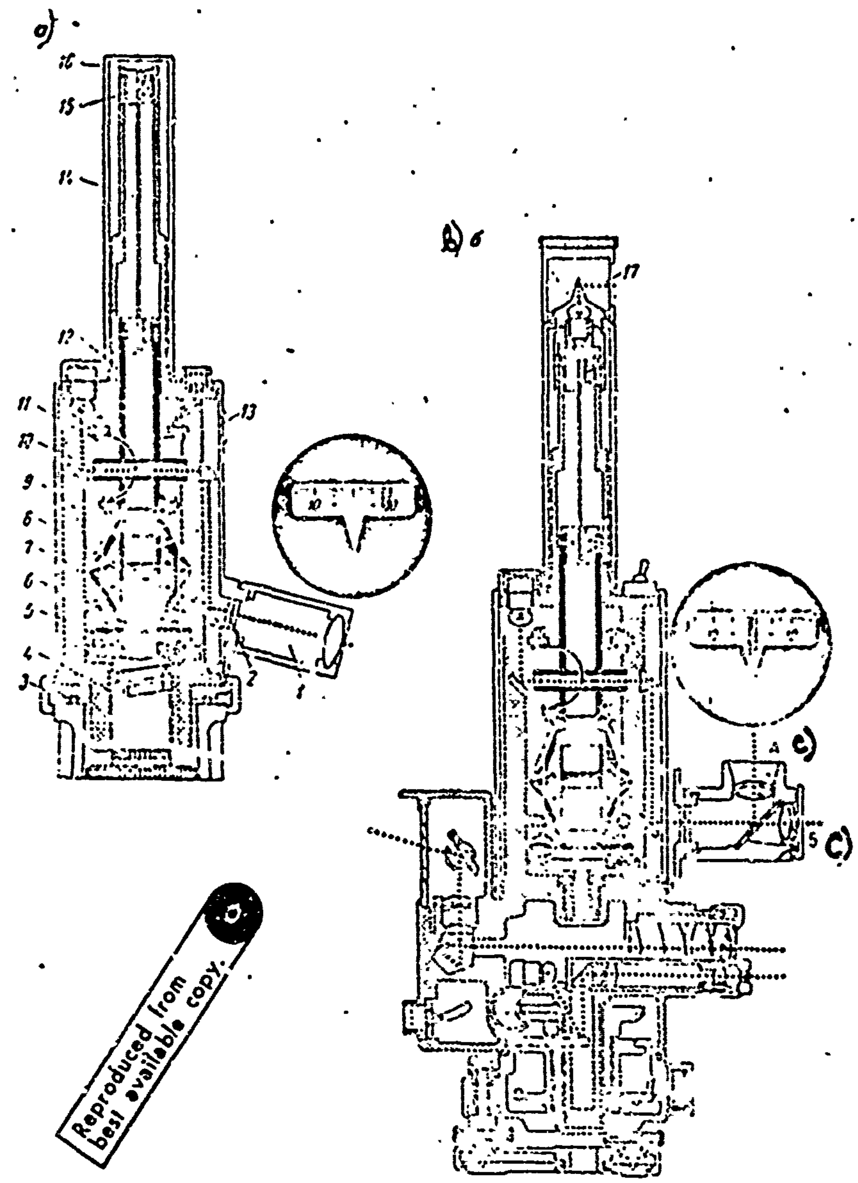

Fig. 22: Vil'd Company gyroscope attachments

Key:

a. GAK-1 gyroscope attachment
b. ARK-1 gyroscope attachment
c. Eye-pieces A and 3 for simultaneous observation of the gyroscope marks by two operators

1. Telescope
2. Scale

Fig. 22: Continued

Key:
3. Cover nut
4. Locking rod
5. Damping plate
6. Spring
7. Pin
8. Lug
9. Gyroscope motor
10. Gyroscope mark
11. Gyroscope optical system
12. Tubular bar
13. Window
14. Torsion element
15. Upper torsion element fastener
16. Regulating screws
17. Luminous mark

The ARK-1 gyroscope unit differs hardly at all from the GAK-1. The ARK-1 instrument has a modified locking rod drive, has luminous mark 17 located on the upper part for sighting on the instrument from a different point and observation of the gyroscope unit oscillation can be carried out simultaneously by two observers through eye-pieces A and B.

The model TK gyroscope attachments of the Fennel Company has a model GAK-1 scheme; the TK-3 gyroscope attachment is produced, in addition to the usual variant, in explosion-proof configuration.

The designs of gyroscopic instruments for determining the direction of geographical meridians on a fixed base considered above are the most widely distributed. Careful consideration of designers' decisions clearly show their principal aim: attainment of accuracy in a given size and weight of instrument. For this two principal routes must be considered: first, increasing accuracy of operation (accuracy of ascertainment) of the gyroscopic part itself and, second, increasing the accuracy of finding positions of the gyroscope spin axes (or equilibrium position of oscillations) and transfer of the positions found to the terrain.

In connection with this, it is advisable to consider the so-called internal and external accuracy.

For internal accuracy of a gyrotheodolite, we will understand the deviation of a single measurement from the average value obtained from a given series of measurements. With the Gi-B1 gyrotheodolite, observation of 24-30 reversing points are made in one continuous operation in the course of 2.5-3 hr. Groups of four points are formed from the reversing point readings, by which the direction of the geographical meridian is determined. Approximately 1500 readings were processed in all, giving a mean direction of north, relative to which the errors of individual measurements were distributed in the following manner [45]:

$$\Delta_r = \pm(1 \div 3)'' - 30\%$$
$$\Delta_r = \pm(3 \div 6)'' - 60\%$$
$$\Delta_r = \pm(6 \div 9)'' - 10\%.$$

It follows from the results adduced that the bulk of the measurements lie within the limits of ±6". At the same time, a measurement error obtained under actual conditions, that is, at different temperatures of the surroundings, after moving, under varying durations of gyroscope motor operations and so forth, and calculation of the relative direction of the meridian, taken from astronomical observations, is about ±15". Such a discrepancy is caused by a number of reasons, but the principal one of them is instability of design elements "located" between the measuring axis of the gyroscope and the optical axis of the theodolite telescope, by use of which the direction determined with the gyroscope is transferred to the terrain. The angular error between the gyroscope axes is called the instrumental correction (the "constant" of the instrument, "constant" correction and others). Let us consider, using the example of the design of the Gi-C2 (see Fig. 21), the scheme of the development of these instrumental corrections, assuming that the gyroscope spin axis lies exactly in the plane of the meridian. Then the difference between the direction of north obtained with the gyroscope and that previously known (astronomically) at a given point on the terrain will introduce a systematic error, the instrumental correction, which depends on the angle of deviation between the incoming and outgoing light beams in base prism 8, measured in the horizontal plane (deviation of the light beams from parallelism) -- Δ_1, from failure of the optical axes to coincide during sighting on the measuring 15 and regulating 16 grids, which leads to angle Δ_2, and from the angle between the normal to flat parallel plate 7 and the rotor spin axis (more accurately, the kinetic moment vector) -- Δ_3.

The instrumental correction is determined in this manner

$$\Delta = \Delta_1 + \Delta_2 + \Delta_3.$$

The principal causes of changes in correction Δ are thermal deformation and material aging. Research shows that the stability of measurement is 20-30% poorer under field conditions than in the laboratory or in measurements underground (in mine shafts) [45]. In the model Gi-B gyrotheodolite, where the number of component elements is greater than in the Gi-C, the deviation in the correction during a change in temperature of surroundings from -10°C to +25°C may reach 30-40" in individual measurements with a mean deviation of ±10" [37]. The correction does not remain constant in the course of a single start-up, since, in the time of observation (30min), the condition of temperature equilibrium will still not be reached in various parts of the gyrotheodolite.

One example of increasing the stability of the instrumental correction is shortening the optical beam path and the number of elements of construction. A diagram of the construction of a gyrotheodolite with a torsion suspension is given in Fig. 23, in which the automatic collimation system for tracking the gyroscope unit is absent [46]. The role of these systems is fulfilled by telescope 1 of the theodolite. Mirror 8 of sensitive element 6 is fixed at an angle to the vertical axis, which permits observation of movement of the sensitive element when the telescope is inclined around horizontal axis 2. Post 5, connecting the sensitive element mirror with the gyroscope unit, is made of glass with a small coefficient of thermal expansion. In addition, the glass, having poor thermal conductivity, facilitates localization of the effect of temperature and, consequently, thermal deformation of measuring parts of the instrument in the period of a start-up.

All possible measures to insure the greatest mechanical and thermal stability of the mutual locations of instrument elements are provided for in actual construction of gyrotheodolites. But, together with this, development of methods of measurement which can materially increase the accuracy of determinations has great significance. This is of particular concern to engineering and geodetic work, which is carried out in a relative coordinate system, where the principal condition for use of measuring instruments is stability of their results for specified measurements (in a specified period of time). In this sense, one may be guided by the internal accuracy of a gyrotheodolite (gyroscope attachment), which, as many investigations show, lies within the limits of a few angular seconds.

From considerations introduced above, of instruments for determining the direction of a geographical meridian, it may be found that, in recent years, simultaneously with finding methods of increasing the accuracy of operation, definite attention has been given to such questions as:

- -- decreasing the time necessary for one determination,
- -- increasing the reliability of measurement results,
- -- eliminating calculating operations from the measuring process,
- -- automation of the measuring process,
- -- development of individual methods applicable to the instrument model and the kind of work being done.

Section 3. Two-degrees-of-freedom Gyrotheodolites

The basis of construction of two-degrees-of-freedom gyrotheodolites lies in gyroscopes which have only two degrees of freedom. Loss of one degree of freedom in a three-degrees-of-freedom gyroscope gives it the ability to establish vector $\bar{H}$ along the direction of the vector of the angular velocity with which its housing rotates. If the gyroscope housing is fixed to the earth, the angular velocity of the base is the horizontal component of the angular velocity of rotation of the earth, and vector $\bar{H}$, being fixed along the direction of the vector of this velocity, will indicate the direction of north.

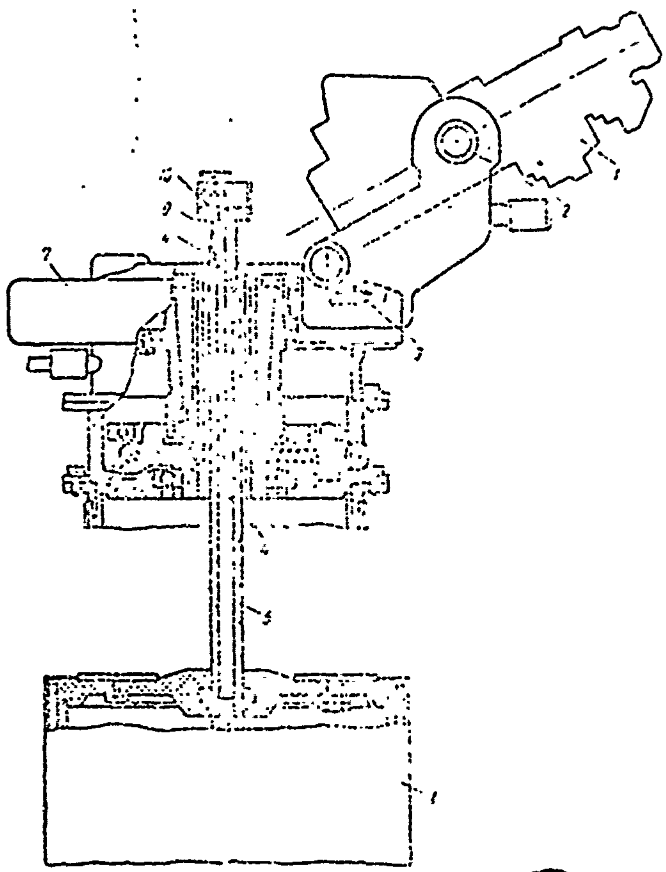

Fig. 23: Construction of a torsion suspension

Key:
1. Automatic collimator telescope
2. Horizontal axis
3. Limb
4. Lower fastening of the torsion element
5. Glass (quartz) tubular post
6. Sensitive element
7. Housing
8. Sensitive element mirror
9. Torsion element
10. Upper fastening of the torsion element

Let us take some horizontal plane "Γ", in which we locate the spin axis of the rotor of a two-degrees-of-freedom gyroscope (Fig. 24). The vector of the horizontal component $\bar{\omega}_{\Gamma}$ of the angular velocity $\bar{\Omega}$ of rotation of the earth lies in this plane, and the vector of the vertical component ω_{B} of the angular velocity of rotation of earth is directed perpendicular to plane "Γ". The kinetic moment vector $\bar{H}$ of the gyroscope under consideration, interacting with the angular velocities ω_{Γ} and ω_{B}, creates two gyroscopic moments, one of which is directed along the axis of rotation of the gimbal (vertically up), and the second, perpendicular to the kinetic moment vector in the horizontal plane. Under the action of the first gyroscopic moment, the gyroscope rotor spin axis will attempt to turn and line itself up with horizontal component $\bar{\omega}_{\Gamma}$ of the angular velocity of rotation of the earth. The second gyroscopic moment is cancelled by a reaction in the bearings of the gyroscope suspension gimbals. The spin axis of the gyroscope establishes itself in the direction of north in this manner. This method of determining geographical meridians is simpler than that used in pendulum gyrotheodolites, and it permits the direction of the geographic meridian to be physically reproduced in the instrument. However, putting the principle of the two-degrees-of-freedom gyrotheodolite into practice encounters serious difficulties, for the frictional moment in the bearings, the power supply mechanism moment and others must be less than the gyroscopic moment to insure fixing the gyroscope rotor spin axis in the plane of the meridian. To obtain an accuracy of operation which is satisfactory from a geodetic point of view, the total of deleterious moments on the suspension axes of such a gyroscope must be not greater than $2\cdot10^{-6}$g-cm. Attaining such a small moment is a difficult assignment; therefore, keeping the principal scheme given in Fig 24 in the apparatus, such methods of measuring can be used as show the least effect of deleterious moments on the suspension axes. The basic contents of this method consist in finding that position of the gyroscope rotor spin axis in which the gyroscopic moment

$$M_{\Gamma} = H\omega_{\Gamma} \text{[illegible]} \qquad (44)$$

attains the maximum value. This happens when the gyroscope axis is located in the east-west plane (for an instrument located on the equator, with kinetic moment $H=10^{3}$g-cm-sec, $M_{\Gamma max}=7\cdot10^{-2}$g-cm). Finding the gyroscope axis position in which $M_{\Gamma}=M_{\Gamma max}$ thereby determines the position of the east-west plane, the normal to which indicates the direction of north at a given point on the ground. The English "PIM" gyrotheodolite (precision meridian indicator PIM) is built by this scheme. Exterior casing 3, within which is located housing 4 of gyroscope motor 5, is connected with vertical axis 2 of theodolite 1 (Fig. 25). The gyroscope motor housing is hermetically sealed and fill with helium at close to atmospheric pressure. The kinetic moment of the rotor $H=2\cdot10^{3}$g-cm-sec, the revolution rate is 24,000 rpm, and rotor diameter is 50 mm. The space between the gyroscope motor housing and the exterior case (0.2 mm) is filled with a liquid which provides buoyancy to the gyroscope motor and generates a damping moment.

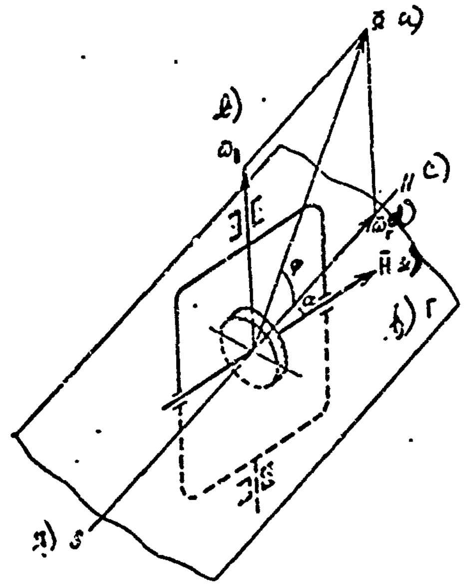

Fig. 24: Diagram of a two-degrees-of-freedom gyroscope

Key:
- a. Vector of earth's angular velocity of rotation
- b. Vertical component of "a"
- c. North
- d. Horizontal component of "a"
- e. Kinetic moment vector
- f. Plane Γ
- g. South

The gyroscope motor housing can rotate inside the exterior case around the vertical axis. The gyroscope motor housing suspension is on pin bearings. Pins 6 are made from tungsten carbide and insets 8 of agate or ruby. Transmission of electrical energy is accomplished across silver band conductors 0.17 mm wide and 0.012 mm thick. Position of the housing relative to the exterior case is determined by use of angle sensor 7. An external moment may be applied to the gyroscope motor housing axis of rotation by using moment pick-up 10. The angle and moment pick-ups are connected together across amplifier mechanism 9 in the tracking system scheme. The mechanism being discussed is covered on the outside by case 11, which has thermal and magnetic shielding. Limb 12 is connected to the theodolite. The interior volume of case 3 has a thermostat and the temperature inside the case is held at 71±0.1°C. As long as this temperature is not reached, a blocking mechanism operates, which precludes making measurements. This blocking system is excited by the parameters of the

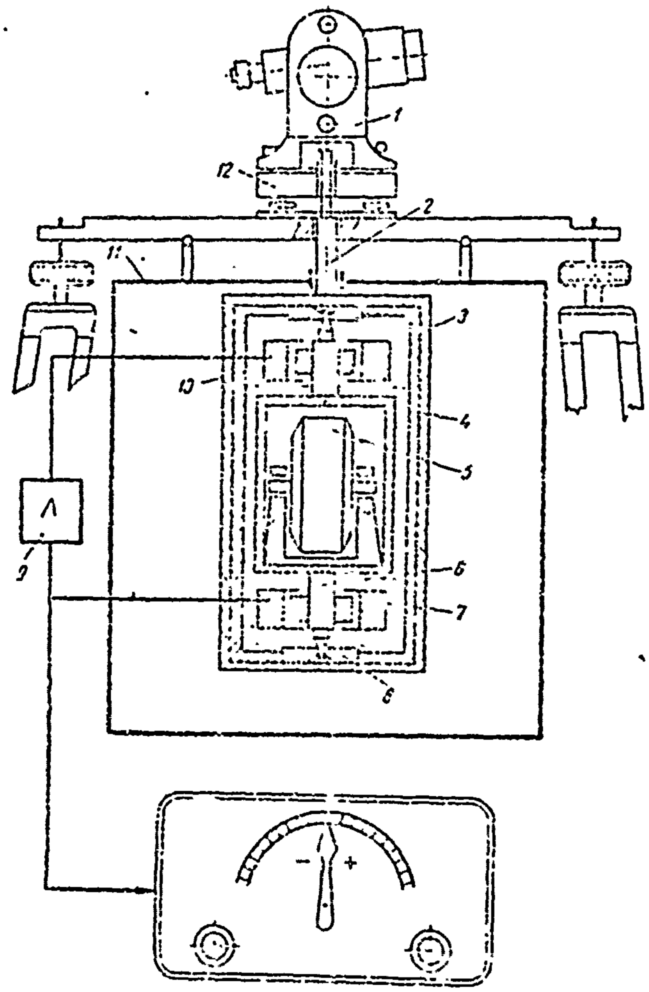

Fig. 25: PIM gyrotheodolite diagram

Key:
1. Theodolite
2. Vertical axis
3. Exterior housing
4. Gyroscope motor housing
5. Gyroscope motor
6. Pins
7. Angle sensor
8. Bearing (bushing)
9. Amplifier
10. Moment sensor
11. Protective housing
12. Limb

measuring scheme, which are designed for quite definite coefficients of damping, depending on the viscosity of the liquid in a given instrument. Therefore, obtaining stable results of measurement requires stability in temperature and, consequently, in viscosity of the liquid. It takes from 5 to 10 min to warm up the instrument, depending on the temperature of the outside air; the instrument is designed to operate in the temperature range between -31°C and +55°C [56].

After the instrument has warmed up and the speed of rotation of the gyroscope rotor becomes constant, the measuring process begins. The gyroscope rotor attempts to turn so that its axis coincides with the vector of the horizontal component of the angular velocity of rotation of the earth. This turning leads to the appearance of a signal from angle sensor 7, which is amplified and sent to moment pick-up 10. The moment, acting on axis 6, does not allow the gyroscope to rotate and simultaneously shows its position relative to the meridian, since the magnitude of this moment will be variable: when the rotor axis is located in the plane of the geographical meridian, the moment will be equal to zero, and when located in the east-west plane, the moment will be at a maximum. When the gyroscope axis is directed along the north-south line and when along the east-west line can be established by observing the electrical measuring apparatus shown, which is included in the chain of formation of the directing moment.

The movement of the suspension axis of the gyroscope under consideration is written in equation form

$$I\ddot{\beta} + c\dot{\beta} + k\beta = M_{\Gamma}, \quad (45)$$

where I -- moment of inertia of housing 4 relative to axis 6;
c -- damping coefficient;
k -- amplification coefficient of amplifier 9;
M_{Γ} -- gyroscopic moment, see (44);
β -- angle turned by the gyroscope relative to housing 3.

In order to insure the optimal nature of damping of the gyroscope axis oscillations, moment kβ should be variable, for which the amplification coefficient k becomes a variable. Proceding from (45), we can obtain:

[illegible] (46)

In the steady state (after damping of the oscillations)

[illegible] (47)

From the relations set forth it follows that the magnitude of the coefficient of amplification k must be found by solving the differential equation for the gyroscope. There is an analog computer which solves Eq. (46) in the "PIM" control assembly for this. As applied to the "PIM" gyrotheodolite, the angle and moment sensors provide, in the steady state, measurement of angle β within the limits of a few angular seconds.

The measuring process with the "PIM" gyrotheodolite consists in determining the direction in which the magnitude of the moment on sensor 10 will be maximum, by means of rotation of the theodolite together with the gyroscope about the vertical axis. After the unknown position for the gyroscope is found, all transition processes in the system must be allowed to come to completion, which takes 2-3 min. Then a reading is made by the theodolite limb, and a correction is subtracted from the limb reading, using the measuring apparatus in the control assembly. This correction facilitates finding the maximum gyroscopic moment. In measuring with the electrical measuring apparatus, two readings during clockwise rotation of the theodolite limb and two readings during counterclockwise rotation usually are made. For a given gyrotheodolite, as for pendulum instruments, there is an instrumental correction; the error in determining the direction of a geographical meridian with the "PIM" is 10-15"; convergence of the results of measurement at one setting is 2-3"; the weight of the gyroscope unit with the theodolite and tripod is 16.4 kg [56]. Examples of entries during work with the "PIM" gyrotheodolite are given in Table 1.

Table 1

Instrumental correction Δ=1°17'26"

Measurements	By the limb	By the instrument			
		Clockwise		Counterclockwise	
		On the scale	Angular equivalent	On the scale	Angular equivalent
Aiming west	270° 10' 00"	3 1/4	-03' 15"	2 1/2	-02' 30"
Aiming east	90 00 00	4 3/4	-04 45	7 1/4	-07 15
Total	360 10 00	-17' 45"			
Average	180 05 00	-04' 26"			
North	00 05 00				
Instrumental correction	1 17 26				
Approximate north	358 47 34				
Average correction	-00 04 26				
True north	358 43 08				

In principle, determination of the east-west plane using a two-degrees-of-freedom gyroscope is more advantageous than direct determination of the plane of the meridian, since in this case the deleterious moments on the suspension axes have a weaker effect on the results of measurement. In practice, however, to find the exact position of the gyroscope axes at which the gyroscopic moment is at a maximum is also a sufficiently complicated problem, since the gyroscope is a dynamic system and the angle turned by the gyroscope stays equal to the angle turned by the theodolite only after completion of the transition process. Lack of the possibility of determining the maximum gyroscopic moment directly when turning the gyrotheodolite leads to the necessity for introducing supplementary reading mechanisms and taking no fewer than four readings and the maximum agreement among which, as follows from Table 1, reaches -5'.

Finding the maximum gyroscopic moment, it is necessary to create a maximum restoring moment. The incoming signal to the moment sensor is the angle turned by the gyroscope gimbal axis, and in the ideal case the maximum moment must be at a zero angle turned. In actual conditions, this angle reaches several seconds, which is a direct error of the instrument. All this leads to the principal specific technical difficulties in constructive fulfillment of the idea.

Therefore, construction of gyrotheodolites which determine the line of the meridian directly also is of great interest. The diagram of a two-degrees-of-freedom gyrotheodolite is presented in Fig. 26 where sensitive element 7 is suspended in liquid and its orientation relative to housing 2 is carried out by use of low-moment torsion leaf 9. The sensitive element is a hermetically sealed housing within which rotor 5 rotates. Under the action of the horizontal component of the vector of angular velocity of rotation of the earth a gyroscopic moment appears, which attempts to bring the gyroscope rotor spin axis to the plane of the geographical meridian which crosses the position of the gyrotheodolite. The gyroscope meets a counteraction by the torsion suspension during its movement to the plane of the meridian. To compensate for the twisting moment of the torsion suspension, the angle turned by the gyroscope and subsequent turning of the torsion fastening point is carried out, using automatic collimator system 3, by turning gimbals 8, which are connected with the moving part of the theodolite. At some position of gimbal 8, when the movement of the gyroscope has ceased, the automatic collimator image from prism 4 coincides with the zero mark on the automatic collimator scale. In this position, the angle to which the torsion element is twisted is nearly zero, and the gyroscope spin axis is located close to the geographical meridian. Since the goniometer part of theodolite 1 is connected with gimbals 8, the theodolite indicates the direction of north. Tracking the angle of twisting of the torsion member may be accomplished visually, but this process may be automated. For this, the light beam reflected from prism 4 is transformed into an electric current by use of an optical-electronic system. The current is amplified by amplifier 6 and is transmitted further to adjusting motor 10. The motor rotates gimbal 8 through a reducing gear during such time as the electric signal input to amplifier 6 does not diminish to zero, which will witness to the setting of the telescope optical axis on the meridian [23].

Let us make up the equation for motion of the gyrotheodolite under consideration, for which we will assume that the kinetic moment vector $\bar{H}$ of the gyrotheodolite is in the horizontal plane and inclined from the direction of the meridian by angle $\angle YO\xi = \alpha$ (see Fig. 26, b). Then, under the action of horizontal ω_r and vertical ω_B components of the angular velocity of rotation of the earth, a gyroscopic moment arises, which gives rise to precession of vector $\bar{H}$. This precession counteracts torsion moments $k\alpha$ (around the vertical axis) and $\chi\beta$ (around the horizontal axis), damping moments $c\dot{\beta}$ and $c\dot{\alpha}$ (damping coefficient, for simplicity taken as equal in both channels) and moments of inertia $I_\zeta\ddot{\alpha}$ and $I_x\ddot{\beta}$. In accordance with the diagram of coordinate axes (see Fig. 26, b) the equation for movement of a gyrothecdolite, assuming all angles are insignificant, will be:

$$\left.\begin{aligned} I_\zeta\ddot{\alpha} + c\dot{\alpha} + k\alpha + H\dot{\beta} - H\omega_r(\alpha_0 - \alpha) = 0 \\ I_x\ddot{\beta} + c\dot{\beta} + \chi\beta - H\dot{\alpha} - H\omega_B = 0 \end{aligned}\right\} \quad (48)$$

These equations are true for cases when the angular position of the torsion elements relative to the position of the gyrotheodolite remains fixed.

Taking into account that when carrying out measurements, the torsion element fastening brackets turn together with gimbals 8, the moment from twisting of the torsion element around the vertical axis will be determined precisely with the automatic collimating system (by angle α_k). For the design shown in Fig. 26, c, the automatic collimator permits detection of angular shifts from one angular second up; the total rigidity of the two torsion elements k=0.3 g-cm. Assuming that the translational process of a gyrotheodolite comes to an end, from (48) we find:

$$\left.\begin{aligned} k\alpha_k - H\omega_r(\alpha_0 - \alpha) = 0 \\ \chi\beta - H\omega_B = 0 \end{aligned}\right\} \quad (49)$$

Given the angle of elevation of the gyroscope rotor spin axis above the plane of the horizon, the necessary rigidity of the torsion element χ can be determined from the second equation. The accuracy of operation of the gyrotheodolite may be decided by the first equation.

If the angle turned by the gyroscope spin axis α is equal to its initial position relative to the direction of the meridian (α_0), the gyroscope spin axis will be directed towards the north. Therefore, the difference (α_0-α) determines the accuracy of setting of the gyroscope spin axis relative to the plane of the geographic meridian.

From Eq. (49) we have:

$$\alpha_0 - \alpha = \frac{k\alpha}{H\omega_r}. \qquad (50)$$

For the construction in Fig. 26, c, kinetic moment $H = 1.5 \cdot 10^3$ g-cm-sec, then for latitude $\phi = 60°$

$$\Delta\alpha = \alpha_0 - \alpha \approx 7''.$$

Taking into account that an additional moment from the residual angle of twisting of the torsion elements is another deleterious moment, the magnitude of the error should double.

One of the variants in construction of a two-degrees-of-freedom gyrotheodolite with torsion suspension is presented in Fig. 26, c. The gyroscope motor is installed in hermetic casing 16 of the sensitive element, suspended on torsion elements 13. The tension on the torsion suspension is adjusted with screw 14. Glass 11 is a supporting element of construction which permits attainment of a high coaxial alignment of the upper and lower axes. Power is supplied to the gyromotor across low-moment current leads. Adjusting nuts 15 are intended for balancing the ЧЭ. Photocell 12 controls the turning of the casing by motor 18 through a dynamic amplifier. Valve system 17 with a filter permits compensation for changes in pressure inside the apparatus, which are excited by temperature changes. The TB-1 theodolite is used as the azimuth disk part. The time for completion of the translational process of the automatic turning system of the theodolite to the direction of the geographical meridian is about 3 min.

The two gyrotheodolites under consideration determine the direction of the geographical meridian in the following manner: in the first case, by construction in the horizontal plane of a normal to the east-west plane; in the second -- by finding a line lying in the horizontal plane and directed to the north. In addition to these versions, one more scheme for finding the plane of the geographical meridian can be constructed by use of the two-degrees-of-freedom gyroscopes, when the unknown plane is determined by two lines radiating from one point: vertical lines and lines situated in the plane of the meridian and forming an arbitrary angle with the plane of the horizon. Construction with an instrument of the vertical lines is accomplished physically with a pendulum, and the lines in the plane of the meridian, by use of a two-degrees-of-freedom gyroscope. For this, the gyroscope rotor spin axis is situated vertically, and the axis of rotation of the interior gimbal, horizontally. Then the interaction of vector $\overline{H}$ with the angular velocity of rotation of earth $\overline{\Omega}$ causes a deflection of the gyroscope spin axis from the vertical direction in the

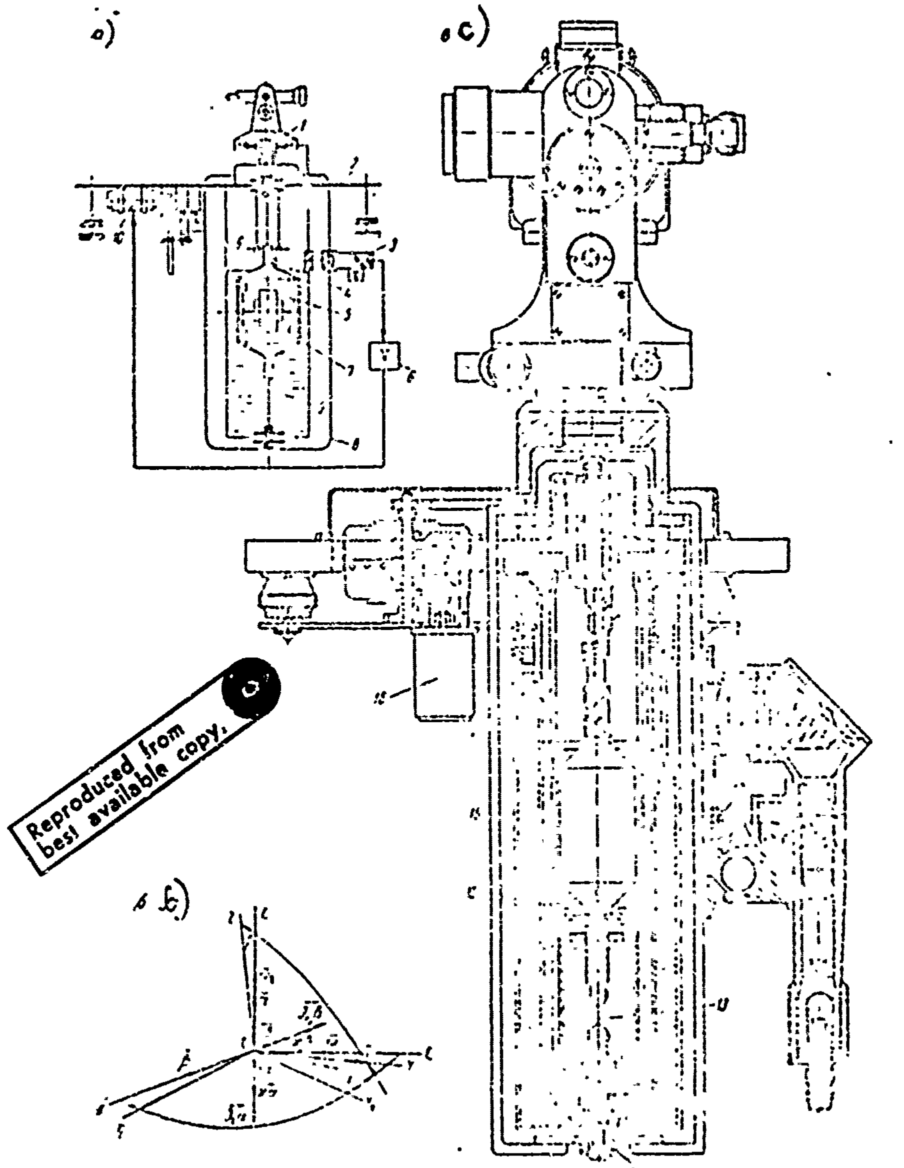

Fig. 26: Gyrotheodolite floating on torsion elements (GPT)

Key:
a. Diagram of principles
b. Coordinate axes
c. Construction

1. Theodolite
2. Housing
3. Optical system
4. Sensitive element prism
5. Gyroscope motor
6. Amplifier
7. Sensitive element
8. Gimbal
9. Torsion element
10. Adjusting motor
11. Glass
12. Photocell

Fig. 26: Continued

Key:
13. Torsion element
14. Adjusting screw
15. Balancing weight
16. Hermetic casing
17. Valves
18. Motor

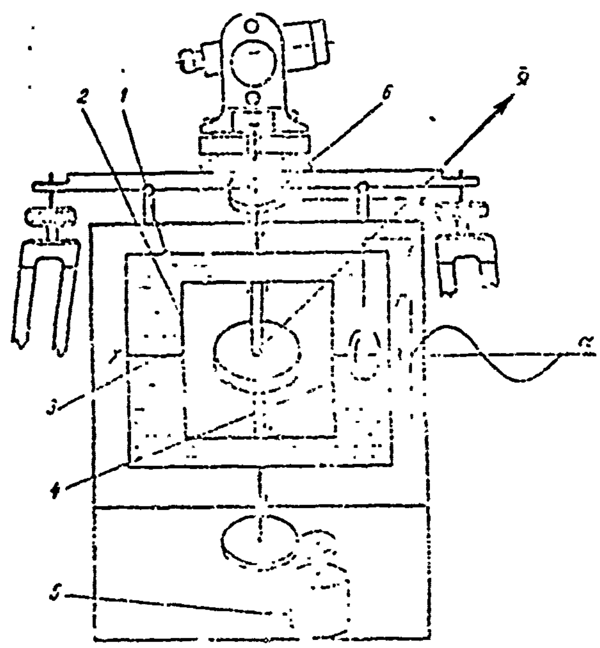

Fig. 27: Gyrotheodolite on two torsion elements with vertical spin axis

Key:
1. Exterior housing
2. Gyroscope motor housing
3. Torsion element
4. Angle sensor
5. Motor
6. Angle sensor

plane of the meridian. Obviously, this inclination will occur only in case the suspension axis of the gyroscope lies in the east-west plane. Since the task of any gyrotheodolite is to find the meridional or east-west plane, then the usual scheme of the two-degrees-of-freedom gyroscope, requiring knowledge of the position these planes, cannot be used in the method under consideration. In the version of constructive implementation of the given method described below, the necessity for previous knowledge of the east-west plane is eliminated.

Let us consider the constructive system of an apparatus (Fig. 27) in which housing 1 is filled with a heavy liquid and gyroscope motor casing 2 is installed inside in a hermetic structure. Orientation of the gyroscope motor housing relative to the casing is accomplished by use of two torsion elements 3, the angle of twisting of which is measured by the angle sensor 4. Housing 1 is set to rotating with a constant speed about the vertical axis by motor 5. The angle turned by the casing around the vertical axis is measured by angle sensor 6.

The kinetic moment vector of the gyroscopic mechanisms under consideration always attempt to coincide with the vector of angular velocity of rotation of the earth, but in practice this matching does not happen, since moments on the part of the torsion elements are proportional to the angle turned by the gyroscope about the horizontal axis. Therefore, the gyroscope will incline only towards vector $\overline{\Omega}$. The magnitude of angle of inclination of the gyroscope β will be determined by the equality of gyroscopic moment M_r and torsion moment M_{tor}:

$$M_{tor} = M_r. \qquad (51)$$

Torsion moment

$$M_{tor} = k\beta, \qquad (52)$$

where k is the rigidity of the two torsion elements.

In view of the facts that the angular velocity vector $\overline{\Omega}$ is always found in the plane of the meridian and that the azimuthal location of the line of the torsion element fastening points (axis XX) changes, the angle of deflection of the gyroscope will also be different. Let us designate the minimum rigidity of the torsion elements (along the line of their fastening points) across k_1, then:

$$k = k_1 + k_0 \sin\alpha, \qquad (53)$$

where k_0 is the value of the rigidity of the torsion elements, changing with the angle turned by the gyroscope;
α is the angle turned by the gyroscope about the vertical axis, calculated from the north-south line.

Taking into account (52) and (53), from (51) we find:

$$\beta=\frac{M_r}{k_1+k_0 \sin \alpha}, \tag{54}$$

where $M_r = H\Omega \cos \varphi$.

Imparting a positive rotation to the gyroscope around the vertical axis, we find that angle β changes periodically: when α=0 the angle turned by the gyroscope around the torsion elements is at a maximum, and when $\alpha=\frac{\pi}{2}$ this angle is quite insignificant. Noting the reading on the theodolite limb at which the angle of inclination of the gyroscope about the horizontal axis reaches a maximum, the same is the determination of the direction of the geographical meridian at a given point on the ground.

Originally, the torsion element with a two-degrees-of-freedom gyroscope was used in a gyrotheodolite, developed by the Italian company Filotecnica Salmoiraghi s. p. A. [55].

Gyroscope rotor 4 with vertical spin axis suspended on torsion element 3, in most cases insures obtaining a gyroscope with three degrees of freedom (Fig. 28). However, besides the axis of proper rotation, the two other degrees of freedom are restricted. Inclination of the gyroscope rotor spin axis from the vertical will cause a deformation of the torsion element and, consequently, the appearance of an opposing moment; in addition, inclination of the gyroscope from the vertical position causes the appearance of a pendulum moment. Therefore, not withstanding the apparent freedom of movement of the gyroscope relative to three axes, it works like a two-degrees-of-freedom gyroscope, that is, it attempts to match its kinetic moment vector with the vector of the external angular velocity. Weight 5, which forms a physical pendulum, is located beside the gyroscope on exactly the same kind of suspension. The gyroscope axis and the pendulum axis form a plane, which may be the plane of the meridian under the conditions described above. The gyroscope axis, shifting from the vertical direction towards the angular velocity vector of earth's rotation, slowly performs damping oscillations around the equilibrium position. These oscillations take place in the plane of the geographical meridian. Consequently, in determining the plane in which the gyroscope spin axis oscillates, we determine the direction of the geographical meridian itself. For finding this plane, the optical line connecting the gyroscope with the pendulum may be used, reproducing the oscillation of the gyroscope in the field of view of telescope 2, which is firmly connected to casing 6 of the gyrotheodolite. The position of the gyroscope in the telescope field of view is defined by a mark, and of the pendulum by a bisector. When the gyroscope motor is not working, owing to turning of the housing of the

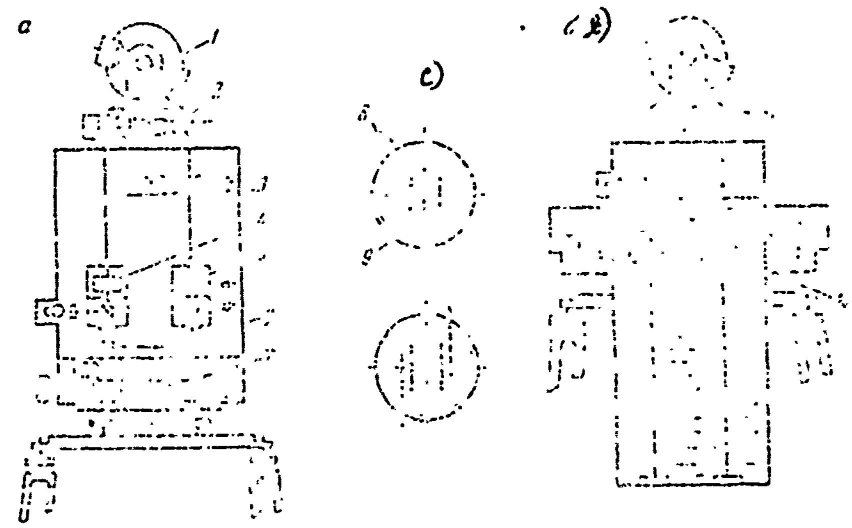

Fig. 28: Gyrotheodolite on one torsion element with vertical rotor spin axis

Key: a. Diagram of gyroscope and physical pendulum
b. Diagram with two gyroscopes
c. Field of view

1. Theodolite	6. Exterior housing
2. Telescope	7. Worm gear
3. Torsion element	8. Pendulum bisector
4. Gyroscope	9. Gyroscope mark
5. Pendulum	10. Second gyroscope

gyrotheodolite around the vertical axis, by use of worm gear 7, gyroscope mark 9 is fixed symmetrical relative to pendulum bisector 8 (Fig. 28, c). After starting up the gyroscope rotor, the gyroscope axis oscillates in the plane of the meridian. If the initial orientation of the gyroscope housing (pendulum bisector) were exactly in the direction of the mid-day line, the gyroscope mark would oscillate along the bisector marks in the telescope field of view. If the initial orientation were approximate, a sideways component in the movement of the mark relative to the bisector would appear. Two extreme positions of the mark in the presence of a sideways component of movement are shown by dotted lines in Fig. 28, c. Turning the gyrotheodolite housing, and together with it the pendulum bisector, achieves a position in which the gyroscope mark oscillates only along the bisector marks. Considering that the optical axis of the telescope is parallel to the bisector marks, the position of the telescope reproduces the direction of the geographical meridian at a given point on the ground in the instrument. Ter[illegible]n objects may also be observed by use of telescope 2. Sighting on [illegible]ome object on the terrain (or special installation), the image of which is projected on the optical axis, the direction of the geographical meridian may be "anchored" on the earth's surface. In addition, sighting theodolite telescope 1 on some object, the direction of the meridian at a given point on the ground is transmitted to the azimuth disk and may be read subsequently from any selected direction.

Fig. 28, a shows only the principal scheme of the gyrotheodolite being discussed. The use of two gyroscopes having opposite directions of rotation of the rotor (Fig. 28, b) is proposed, with the aim of practical realization of an increase in accuracy of operation. The gyroscopes themselves are placed in a heavy liquid, which has a slight positive buoyancy for this use. The static picture is left as before in the field of vision of the telescope. When the gyroscope motors are operating, not only the mark, but the bisector, oscillates. By turning the gyrotheodolite housing, a position is reached in which the transverse movement of the marks is absent.

The magnitude of the displacement of the gyroscope mark relative to the vertical marks has a considerable reflection on the accuracy of determining the plane of the meridian. Disregarding the twisting moments of the torsion element, the equilibrium position of the gyroscope will be determined by the equality

$$M_{\Gamma}=M_{M}, \tag{55}$$

where $M_{\Gamma}=H\Omega\cos\phi$, the gyroscopic moment;
$M_{M}= Pl\sin\beta$, the pendulum restoring moment;
P, the weight of sensitive element 10;
l, length of the torsion element;
β, angle turned by the gyroscope spin axis, calculating from the vertical direction.

The approximate linear displacement of the ЧЭ can be found from the relation

$$S=l\sin\beta. \tag{56}$$

Substituting the relations presented in (55), we obtain:

$$S=\frac{H}{P}\Omega\cos\varphi. \tag{57}$$

In this equation

$$H=J\omega, \tag{58}$$

$$J=mr^2=\frac{P}{g}r^2. \tag{59}$$

Substituting Eq. (58) and (59) in (57), there will be the following relation:

$$S = \frac{P_r}{P} \cdot \frac{r^2}{g} \cdot \omega\Omega \cos\varphi, \qquad (60)$$

where ω -- angular velocity of proper rotation of the rotor;
r -- inertial radius;
m -- gyroscope rotor mass;
P_r -- weight of the gyroscope rotor;
g -- acceleration of gravity.

This equation shows that, for selection of a necessary value of displacement S for given rotor parameters, the relation between the weight of the gyroscope and the weight of the entire ЧЭ can be varied. It is found that the weight of the ЧЭ must be less than the weight of the gyroscope for acceptable gyroscope parameters in practice. To insure this condition, the sensitive element is installed in a liquid, matching the specific gravity of the latter and the volume of the ЧЭ.

Two-degrees-of-freedom gyrotheodolites have considerably less distribution than pendulum gyrotheodolites, but the potential simplicity of automating the measuring process opens wide perspectives to them, especially in investigating azimuthal deformations of heavy engineering works.

Section 4. Gyrotheodolites with spherical gyroscopes (with spherical gyroscope suspension)

Use of the property of free gyroscopes of retaining the direction their axes in space permits determination of the position of the east-west plane. Realization of this method in instruments is possible only by using gyroscopes with spherical suspension. At the present time several models of spherical gyroscope suspensions are known; however, only aerodynamic (gas) suspensions have practical use now. The spherical gyroscope (Fig. 29) consists of two basic parts: rotating cup 1 and rotating sphere 2, slightly elevated above cup 1 by gas pressure, which provides a clearance between the cup and sphere. The cup is rotated by an electric motor and this rotation is transmitted to the sphere by the viscosity of the gas.

At the initial moment we will assume that the axis of rotation of the bowl and axis of rotation of the sphere coincide (the sphere rotates about the axis of cylindrical hole 3). In the next instant, the cup, together with the earth, shifts relative to space and, consequently, relative to the axis of the sphere (gyroscope). Angle α, which lies in the east-west plane, appears between the axis of the sphere and the axis of the cup as a result of this. A moment component, directed perpendicular to the kinetic moment vector $\bar{H}$, appears as a consequence of

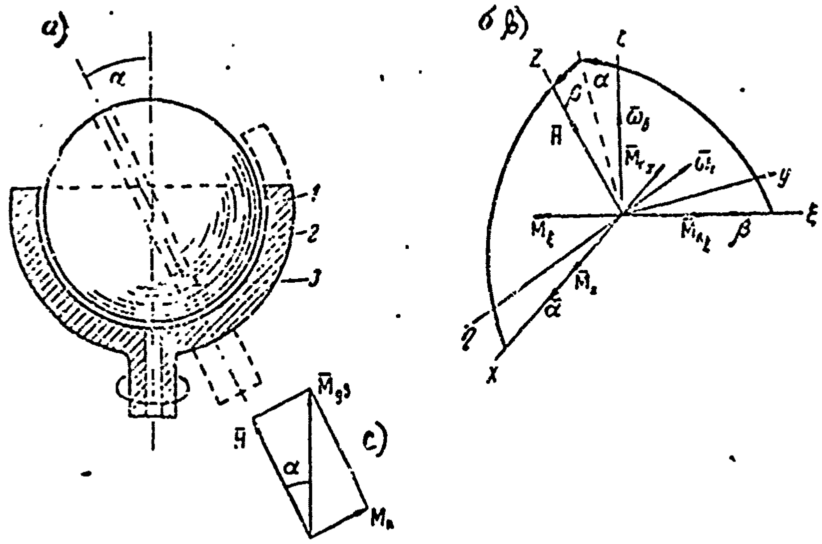

Fig. 29: Diagram of a gyrotheodolite with spherical suspension

Key:
a. Kinematic diagram
b. Coordinate axes
c. Vector diagram

1. Cup
2. Ball
3. Cylindrical hole

this angle. This component is directed in such a way that precession of the sphere axis arises in the east-west plane following the cup axis (around the vertical of the site). Despite the movement of the sphere after the cup, angle α maintains a definite magnitude, necessary for the creation of just this movement. The magnitude of this concordant angle depends on the value of the horizontal component of the angular velocity of rotation of the earth (that is, on the latitude of the place) and on the strength of the aerodynamic interaction of the gas layer between the sphere and cup. After starting the rotation of the sphere (at a given latitude) concordant angle α appears and begins to increase, but with the increase in the concordant angle, the angular velocity of precession of the sphere increases. Only when the speed of precession of the sphere becomes equal to the angular velocity of rotation of the cup, does angle α stop changing. If the strength of the aerodynamic interaction is stable in the apparatus, the magnitude of angle α may be a characteristic of the latitude of the point on the earth's surface at which the spherical gyrotheodolite is located.

Let us form an equation for movement of a spherical gyrotheodolite, for which we assume that two external moments act on the gyroscope: moments M_ξ, directed from east to west, and M_χ, directed initially from north to south (see Fig. 29, b). In a short time the gyroscope axis inclines at angles α and β under the action of these moments, and in consequence of this inclination corrective moments $M_{\kappa\xi}$ and $M_{\kappa\chi}$ appear, attempting to restore vector $\bar{H}$ to the initial position. In accordance with the coordinate axis diagram, equations for movement of a spherical gyroscope, on the assumption that angles α and β are trivial, will have the following form:

$$\left.\begin{aligned} -I_x\ddot{\beta} + H\dot{\alpha} + M_{\kappa\xi} &= M_\xi - H\omega_r \\ I_y\ddot{\alpha} + H\dot{\beta} + M_{\kappa\chi} &= M_\chi \end{aligned}\right\} \quad (61)$$

In these equations the correcting moments

$$\left.\begin{aligned} M_{\kappa\xi} &= k_\text{ш}\alpha \\ M_{\kappa\chi} &= k_\text{ш}\beta \end{aligned}\right\}, \quad (62)$$

where $k_\text{ш}$ is the proportionality coefficient.

If the external moments, which have an insignificant magnitude for the gyroscopes under consideration, are disregarded, and the nutational oscillations are not accounted for, Eo. (61) take the form:

$$H\dot{\alpha} + k_\text{ш}\alpha = H\omega_r \quad (63)$$

$$H\dot{\beta} + k_\text{ш}\beta = 0. \quad (64)$$

The solution of Eq. (63) is written in the following form:

$$\alpha = \frac{H}{k_\text{ш}}\omega_r\left(1 - e^{-\frac{k_\text{ш}}{H}t}\right), \quad (65)$$

from which the fixed value of the angle of inclination of the sphere from the axis of the cup turns out to be

$$\alpha_{\text{уст}} = \frac{H}{k_\text{ш}}\omega_r$$

or

$$\alpha_{\text{уст}} = \frac{H\Omega}{k_\text{ш}}\cos\varphi. \quad (66)$$

In the equations cited [17, 51]

$$k_{ш} = \frac{8\pi R^4 \mu}{3\nu}\lambda, \qquad (67)$$

where R -- radius of the sphere,
μ -- viscosity of the supporting gas;
ν -- clearance between sphere and cup;
λ -- relative angular velocity of the sphere and cup.

Assuming in (66) that H, Ω and $k_{ш}$ are constant, we obtain a relation between the angle of inclination of the sphere axis α_{yCT} and the latitude of the point where the gyrotheodolite is located.

The proportionality coefficient $k_{ш}$ plays a basic role in the operation of a gyroscope with spherical suspension and, as follows from (66), to increase angle α_{yCT} it is necessary to decrease $k_{ш}$. But decreasing coefficient $k_{ш}$ leads to a delay in the translational process, which follows from (65). Therefore, an arbitrary change of $k_{ш}$ is inadmissable, and its magnitude must be chosen from a combined consideration of the dynamic and static characteristics of the instrument. Practically, as follows from [54], $k_{ш}$ has a relatively high value. In order to decrease coefficient $k_{ш}$, the cup has minimum dimensions: the central angle of inclusion is a little larger than π in order to protect the sphere from falling out of the cup when it is inclined (Fig. 30). The so-called inverted scheme of spherical gyroscope (see Fig. 32) is conformed to for the purpose of decreasing coefficient $k_{ш}$. Let us examine the set-up of the gyrotheodolite with a spherical gyroscope [52].

The axis of rotation of cup 8 of the gyrotheodolite (see Fig. 30) is fixed in the vertical position by use of levels 5 and is rotated by electric motor 11. The cup axis of rotation has holes 10, by which air under pressure of ~1 atm is fed to cavity 7, which is closed off by elastic membrane 9. Air is fed from this cavity through jets 12 under sphere 4 and lifts it slightly as a result of which there is a clearance between sphere and cup of ~25 microns. The cup has cylindrical hole 6, owing to which it always rotates around only a single axis, which almost coincides with the axis of this hole. Mirror 15 is located above the cylindrical hole, and is used to observe the movement of the sph..e and to measure angle α_{yCT}. In the apparatus under consideration, d:termination of the east-west plane is carried out automatically by use of a photoelectric tracking system. A two-channel automatic collimator 16 determines the direction and value of inclination of housing 2 of the instrument relative to the sphere and sends a corresponding signal through amplifier 14 to azimuthal turning motor 13. As a result of this turning, telescope 1 will be oriented to the north. The automatic collimator can count off the geographical latitude of the place where the instrument is located simultaneously with the azimuth direction, using electrical measuring apparatus 3.

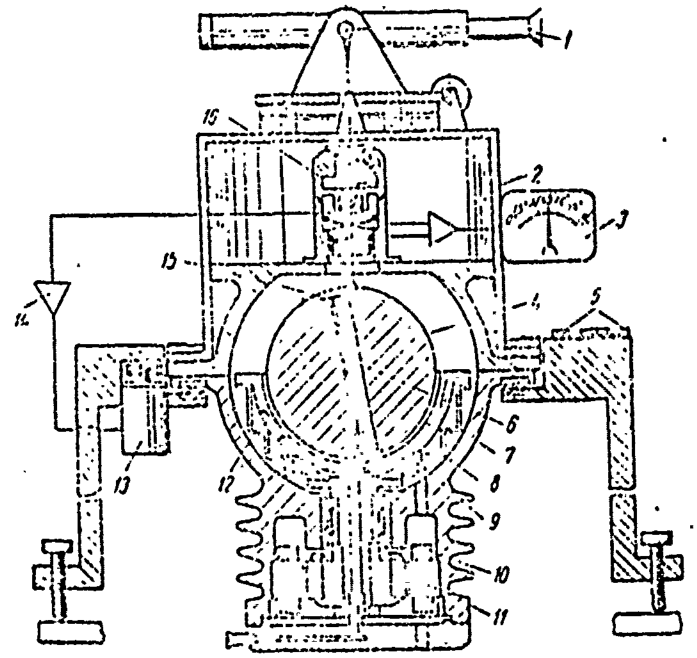

Fig. 30: Spherical gyrotheodolite

Key:
1. Telescope
2. Housing
3. Latitude indicator
4. Gyroscope (sphere)
5. Levels
6. Internal cylindrical hole
7. Air cavity
8. Cup
9. Diaphragm
10. Air channel
11. Gyroscope electric motors
12. Jet
13. Azimuthal turning motor
14. Amplifier
15. Gyroscope mirror
16. Optical sensor

The two-channel automatic collimator is made according to the optical system of D. D. Maksutov. A pencil of light from miniature lamp 2 reflects from the mirror surfaces of lenses 3 and 1 and, further, through the transparent part of dividing pyramidal prism 4, reaches mirror surface 9 of correcting lens 6 (Fig. 31, a). The light pencil reflects from it onto the lower mirror surface of lens 5, from where, going through correcting lens 6, it is formed into a parallel beam and falls on mirror 7, which is fastened to sphere 8. Reflecting from this mirror, the beam falls on smooth side 11 of prism 4 and further on photocell 10. Two cases of the position of the housing of the instrument relative to the

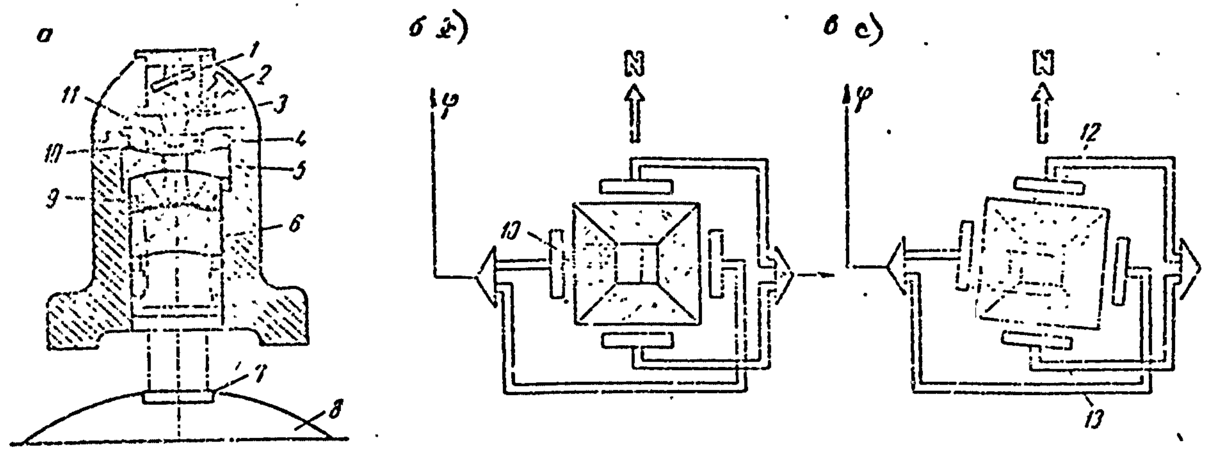

Fig. 31: Spherical gyrotheodolite measuring scheme

Key:

a. Optical system diagram
b. Arrangement of the elements of the apparatus in exact aiming
c. Rotated position of the apparatus

1, 3, 5. Optical system lenses
2. Light source
4. Prism
6. Correcting lens
7. Gyroscope mirror
8. Sphere
9. Mirror surface
10. Western photocell
11. Mirror side of the prism
12. Northern photocell
13. Southern photocell

east-west plane are shown schematically in Fig. 31, b and c. In the precise location of the measuring plane of the instrument in the east-west plane (Fig. 31, b), light reflected from mirror 7 illuminates only one (western) photocell 10, the magnitude of the signal from which is proportional to the geographic latitude. If the measurement plane of the instrument is located at some angle to the east-west plane of earth, light reflected from mirror 7 will fall on the "northern" 12 or on the "southern" 13 photocell. In the case shown in Fig. 31, c, the "southern" photocell is illuminated, and the electric signal from it will be further amplified and then sent to the azimuth motor. This motor turns the instrument housing relative to the sphere and, consequently, the automatic collimator, in such a way that the signal from photodiode 13 will equal zero (analogous to Fig. 31, b). In the variant under consideration, the gyrotheodolite has the following parameters [54]: the top is made of steel; the sphere[1], with a diameter of ~62 mm made of quartz or ceramic, rotates with a speed ~600 sec^{-1}; accelerating time of the sphere is about 1 min; time for completion of the displacement process (establishing angle α_{yCT}), about 7 min; error in determination of the meridian ±24".

In a gyrotheodolite with an inverted spherical gyroscope, cup 1 of gyroscope rotor 3 covers rotating sphere 2 (Fig. 32). This sphere and its connected housing 5 are rotated by electric motor 6. The rotating sphere and the cup of the rotor have channels through which air moves to form an air film, providing freedom of rotation of the rotor. The gyrotheodolite has azimuthal turning motor 7, which is directed by a photoelectric tracking system. Light source 9, photocells 8 and the gyroscope rotor, the lateral surfaces of which are made in the form of a many-sided mirror, are parts of this photoelectric azimuth tracking system. Light source 9 and photocells 8 are situated in the meridional plane. The analogous optical-electronic system is fastened in the east-west plane and is used to obtain the geographical latitude by measurement of the value of angle α_{CT}.

A diagram of an azimuthal photoelectric system is presented in Fig. 33, a, and a latitudinal in Fig. 33, b. If the measuring plane of the gyrotheodolite, with which light source 1 and photocells 2 are connected, is oriented strictly to the north, a light beam reflected from rotor 3 will not lead to the appearance of an electrical signal in the measuring circuit (see Fig. 33, a). In this case, the beam from the light source falls on that point of the rotor across which in imagination the axis of apparent drift of the gyroscope rotor passes. If the measurement plane of the gyrotheodolite is situated at an angle to the meridional plane, the point where the light beam falls lies at some distance from the "axis of apparent rotation" of the gyroscope rotor, as a result of which the beam reflected on the photocell is displaced relative to its initial position and causes the appearance of an electrical signal which is proportional to the

1 In operation [54], it has been shown that, in actual construction of the gyrotheodolite, a body of somewhat different form, nearly geoidal, can be used in place of the sphere.

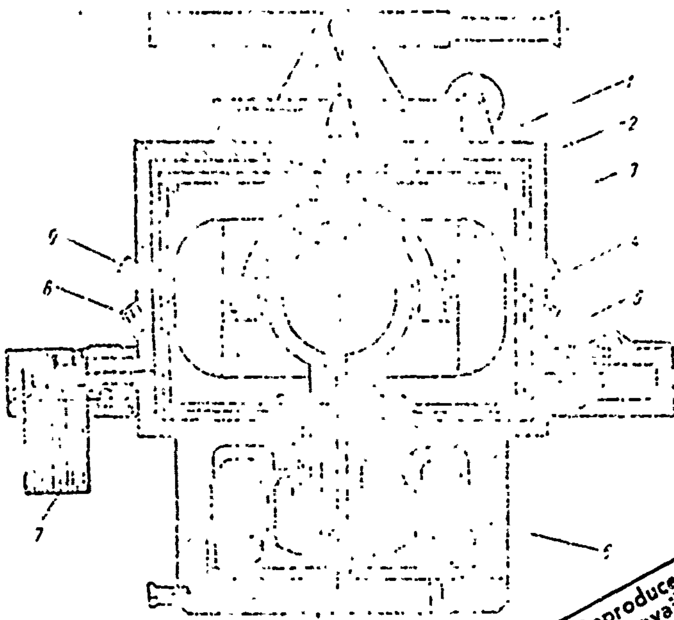

Fig. 32: Gyrotheodolite with inverted spherical gyroscope

Key:
1. Cup
2. Rotating sphere
3. Gyroscope rotor
4. Window
5. Rotating housing
6. Gyroscope motor
7. Azimuth motor
8. Photocell
9. Illuminator

concordant angle of the plane mentioned. This electrical signal reaches the indicator and, further on, through the amplifier, the azimuth motor, which corrects the azimuthal orientation of the gyrotheodolite housing.

For measuring the geographic latitude, a different pair is used, "illuminator 4 and receiver 5," located diametrically opposite from the azimuthal pair, "illuminator 1 and receiver 2" (see Fig. 33, b). When the plane of rotor 3 is located horizontally, illuminator 4 and photocell 5 are set in such a way that there is no electrical signal in the measuring circuit. In operating status, the gyrotheodolite housing, together with the illuminator and photocell, are displaced relative to the gyroscope; this angular displacement causes a shift in the reflected light beam and the appearance of an electrical signal. The magnitude of this signal is proportional to the relative angle of inclination of the gyroscope, which depends in its turn on the latitude of the point where the instrument is located on the earth. The electric signal is transformed into a value of the geographic latitude counted off by indicator 6.

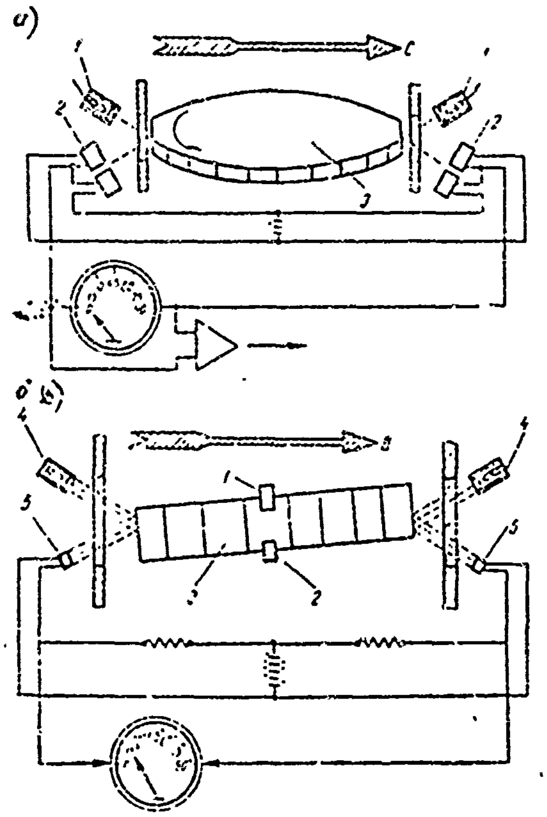

Fig. 33: Measurement scheme of an inverted spherical gyrotheodolite

Key: a. Azimuthal
b. Latitudinal

1. Illuminator
2. Photocell
3. Gyroscope rotor
4. Illuminator
5. Photocell

During operation of a spherical gyrotheodolite on a mobile base, verticalization of the axis of rotation of the gyroscope motor is carried out automatically, and signals characteristic of the azimuth and latitude are used for navigational purposes [51].

Section 5. Gyroscopically Stabilized Platforms Operating like a Gyroscopic Compass (Gyrotheodolite)

Before setting inertial surveying equipment in motion, prior to starting an airplane or other flying machine, the gyroscopic system which is installed in it must be oriented relative to a given direction, of which the most widely used are the direction of the meridian and the direction of the vertical [57].

Let us examine the orientation of the gyroscopic instrument in the direction a geographical meridian. Most widespread are the gyrotheodolite method and the method of tying-in by "remote landmarks", when the inertial surveying equipment or airplane is fixed over a point for which the azimuth to the remote object is known.

Taking into account a substantial increase in accuracy in manufacture of gyroscopic systems, many kinds of gyroscopic systems have appeared in recent years, which are capable of operating, for example, like a gyroscopic compass, a directional gyroscope or a gyromagnetic compass [26]. There is a definite interest in two-mode-of-operation inertial gyroscopically stabilized platforms, which are transferred to a gyrocompass mode of operation before starting (beginning of movement) [58].

Let us consider a tri-axial gyroscopically stabilized platform, on which three two-degrees-of-freedom gyroscopes, with kinetic moments H_ζ, H_η, and H_ξ, and two accelerometers A_η and A_ξ, capable of measuring the inclination relative to axes η and ξ, are installed (Fig. 34). These accelerometers, together with the gyroscopes which have kinetic moments H_η and H_ξ, form a system for bringing the interior platform to a horizontal position. If, for example, for some reason or other the interior platform turns around axis η, this inclination will be measured by accelerometer A_η and a corresponding electrical signal will be sent to moment sensor DM_η. The interaction of this kinetic moment with the moments generated by the indicated sensor DM_η leads to the appearance of precession of the entire platform around axis η, as a result of which the inclination mentioned above is removed. The connection between accelerometer and moment sensor, as between other elements, is accomplished through amplifiers (they are not shown in Fig. 34). Let us observe that, for operation under field conditions in a fixed location, the base accelerometers can be replaced by high-accuracy electrical levels.

In addition to a system for making it horizontal, the platform under consideration is equipped with an unloading system (compensation for external moments on the stabilizing axis) [4, 21].

Let us consider the gyrocompass mode of operation, for which we assume that kinetic moment vector $\overline{H}_\xi$ makes some angle α with the horizontal component vector $\overline{\omega}_r$ of the angular speed of rotation of the earth. Then, vector $\overline{H}_\xi$, in accordance with the gyroscopic moment rule (the rule of N. E. Zhukovskiy), will attempt to match itself with vector $\overline{\omega}_r$ and, after

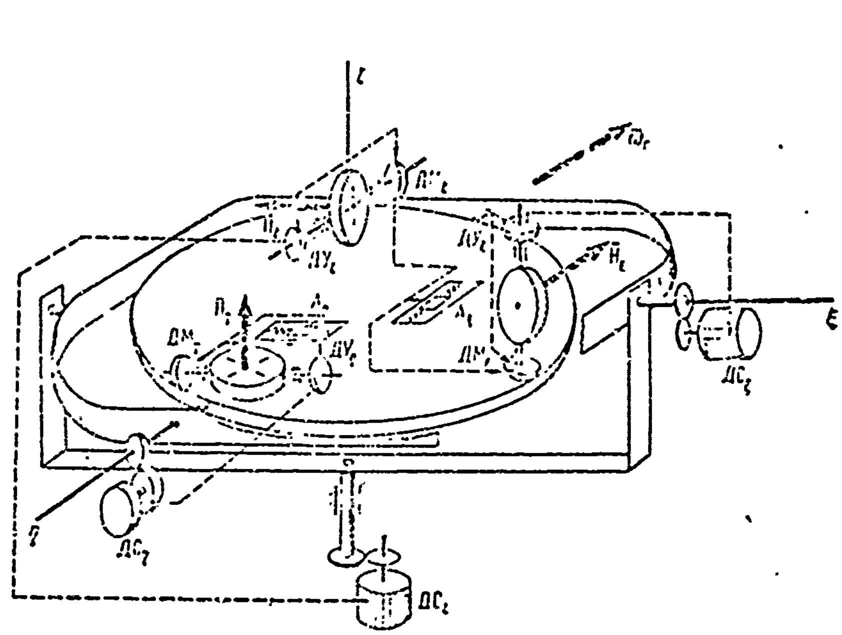

Fig. 34: Gyroscopically stabilized platform

Key: ДМ -- DM
ДС -- DS
ДУ -- DU

a short time, an electrical signal will form on angle sensor DU_ξ, which will be fed to stabilizing motor DS_ξ after amplification. This motor causes the platform to turn around axis ξ, which will be sensed by accelerometer A_ξ. The signal from this accelerometer is transmitted to moment sensor DM_ξ of the system for bringing the platform to the horizontal and to moment sensor M_ξ of the azimuth setting system. Kinetic moment $\bar{H}_\zeta$, interacting with the moment generated by sensor DM_ζ, causes the gyroscopic platform to turn around the vertical axis. This turning will be continued as long as vector $\bar{H}_\xi$ is not parallel to vector ω_Γ, that is, does not indicate the direction of the geographical meridian.

Section 6. Vibrating Gyrotheodolites

Development of means of autonomous determination of the direction of a geographical meridian is taking different routes. The basic aims of this development are to increase the speed of action and accuracy of operation, and to decrease the weight of the instrument and the consumption of electrical energy. Here, known methods of reaching solutions are being improved and new methods are being developed.

If the basis for operation of the usual gyroscopic structures lies in the use of the inertial property of a rapidly rotating mass, a piezoceramic cylinder uses the property of a rapidly vibrating mass as the basis of operation. Such instruments have received the name of vibrating gyroscopes.

Various constructive schemes for creation of a vibrating mass are known [4]. The diagram for a tuning fork model of vibrational gyroscope is given in Fig. 35. Stimulating electromagnet 8 serves to put the blades of tuning fork 7 into continuous vibrational motion. The windings of these electromagnets are supplied with alternating current from amplifier 1, into which is fed a signal from tuning fork blade vibration sensor 6. The tuning fork equipped with such a system of stimulating electromagnets and sensors will vibrate with a natural frequency determined by its physical constants. In this case, vibrations will be undamped, with constant amplitude, since the loss of energy is regularly made up by the stimulating electromagnets.

The action of external angular velocity Ω causes a torsional vibration of stem 5, which is converted into an electrical signal by use of lug 4, moving in the inter-pole gap of torsional vibration sensor 3. The signal received is amplified and equalized with the amplified tuning fork signal. The phase of the signal from torsional vibration sensor 3 is determined by the direction of movement of the lug relative to its poles; the phase of the signal from tuning fork blade vibration sensor 6 is determined by the direction of movement of the tuning fork blades (receding or approaching). As a result of transmission of both signals, phase discriminator 2 forms a direct current signal, the polarity of which determines the direction of the angular velocity vector, and the magnitude of which determines the speed. The "Vibrogiro" instrument for measuring small angular velocities was manufactured on the basis of use of the gyroscopic effect of a vibrating mass (Westinghouse, Pittsburgh, USA) [34]. The "Vibrogiro" is a thin-walled, hollow cylinder of barium-titanium piezoceramic, the height and exterior diameter of which is 13 mm. This cylinder is attached in such a way that its butt-ends remain free and, under the action of an applied voltage, vibrate in a radial direction with a frequency of 100 khz. Under the action of the angular velocity of the base on which the piezoceramic vibrating cylinder is mounted, a piezoelectric signal arises in the latter, the magnitude of which is proportional to the angular velocity. Setting the measuring axis of the cylinder in the horizontal plane and measuring the horizonal component of the angular velocity of rotation of the earth, a position of the cylinder can be found in which its reading axis will indicate the direction of the geographical meridian. The virtues of the "Vibrogiro" are absence of moving parts, small size and weight, insignificant consumption of electric energy and rapid action.

The accuracy of operation of the vibrating gyroscopes still is insufficient for development of instruments based on them for geodetic

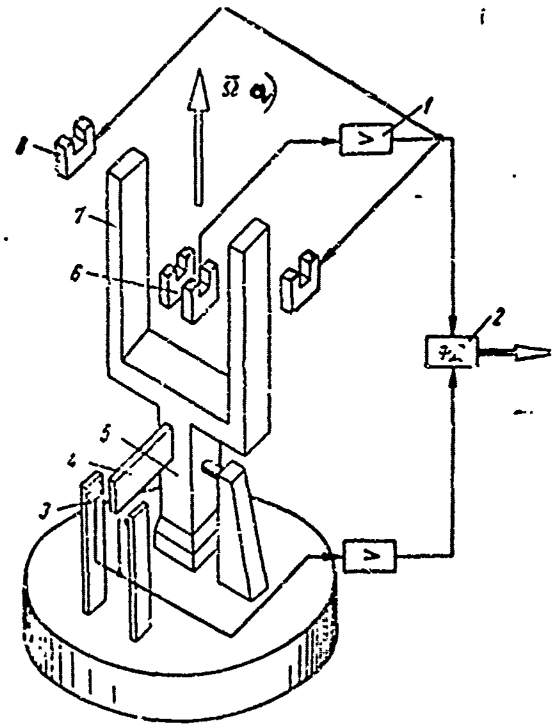

Fig. 35: Kinematic diagram of a vibrating gyroscope

Key:

1. Amplifier
2. Phase discriminator
3. Torsional vibration sensor
4. Lug
5. Stem
6. Vibration sensor
7. Tuning fork blades
8. Stimulating electromagnets

a. Angular velocity vector of earth rotation

purposes. However, their potential arouses a definite interest, especially the absence of the necessity for having bulky and heavy batteries. The angular velocity which can be measured with tuning fork gyroscopes at the present time is $\sim(1-0.5)\cdot10^{-5}$ sec^{-1} [20].

Section 7: Optical Gyrotheodolites

A method of determining the speed of rotation of a base using two light beams moving toward each other, as proposed by Michelson, theoretically permits the detection of very small angular velocities. However, for practical achievement of a mechanism capable of measuring a velocity less than the angular velocity of the earth requires considerable size to get a perceptible difference in the paths of the light beams. For example, Michelson had a large, rectangular area of 60 m^2 and a small one of 33 m^2 (Fig. 36, a). A different method of obtaining the necessary difference in paths of the beams is multiple passages through a loop of small dimensions. But, in this case, there will be a gradual weakening of the light beam by reflection from the mirrors, and it may turn out that, in attaining the necessary path difference, all of the energy of the light beam will be lost.

The use of quantum generators (lasers) permits the regular replenishment of the loss of light, owing to the stimulated emission effect. A circular quantum optical generator can be used as a sensor of the angular velocity of the base. If this mechanism is set on the earth in such a manner that its incoming axis lies in the horizontal plane, then, by turning the generator housing, a position is found in which the angular velocity attains a maximum. In this position, the incoming axis of the laser circuit lies in the plane of the geographic meridian and, consequently, permits determination of the geographical azimuth of the line sought for. Considering that the horizontal component ω_r of the angular velocity of rotation of the earth depends on the geographic latitude, the latter may be determined if the magnitude of ω_r is measured.

On the basis that the circular quantum optical generators, like the two-degrees-of-freedom gyroscopes, measure the angular velocity of the base, they are frequently called optical gyroscopes, optical gyrocompasses and laser gyroscopes.

One of the optical gyroscopes developed [25] consists of a monolithic quartz block 4, in which three channels 10, with a length of 12.5 cm, are drilled at an angle of 120°, forming a light conductor (Fig. 36, b). This triangular light conductor acts as a gas discharge tube, and the three mirrors located at the apexes of the triangle form a circular resonator. The two mirrors 5 and 9 are flat, and the third one is spherical; the mirror reflecting surfaces have a 13-layer dielectric film, so that mirrors 1 and 9 have a covering with a good reflecting capability and mirror 5 is semi-transparent. Prism 7 directs the light signal going through this semi-transparent mirror to photomultiplier 6. The quantum optical generator works on a helium-neon mixture at a pressure of 5 mm Hg, and cathode 3 and two anodes 2, as well as diaphragm 8, which permits a single mode of operation, are found in the quartz block.

If the vibration frequencies of the light beams propagating in opposite directions are identical in a fixed loop, during rotation of the loop, the vibration frequency of one beam increases and that of the other decreases. Comparing these two light beams (using a photocell), the changing beat frequency, which is the difference in vibration frequency of the light beams, can be detected in the photodetector output voltage. It is evident that, the higher the speed of rotation of the circular quantum generator, the higher the beat frequency, measured by the photoelectronic

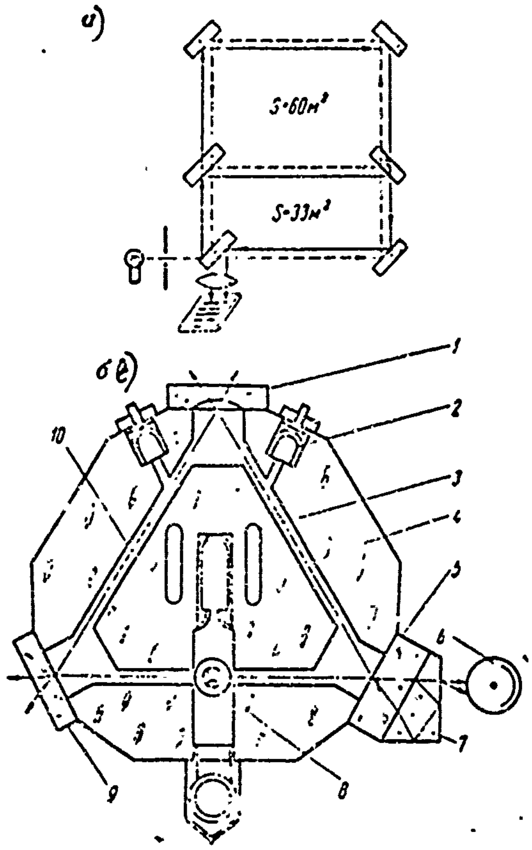

Fig. 36: Diagram of an optical angular velocity sensor

Key:

a. Michelson experiment diagram
b. Laser sensor diagram

1. Spherical mirror
2. Anode
3. Cathode
4. Quartz monoblock
5. Flat mirror
6. Photomultiplier
7. Prism
8. Diaphragm
9. Flat mirror
10. Light channel

multiplier (FEU). Knowing the beat frequency and the loop parameters, the angular velocity of rotation of the base can be determined. The incoming axis of the optical quantum circular generator, along which the vector of measured angular velocity is located, is normal to the plane in which the generator loop (ring) lies. The phenomenon of production of different frequencies by the rotating loop is called the effect of San'yak, who discovered it in 1913.

Known quantum gyroscopes have the following technical characteristics [25]:

sensitivity, $10^{-3\circ}$/hr;
drift, 5 angular sec per day;
time of preparation for operation, 1-2 sec.

CHAPTER III

GYROTHEODOLITE MEASUREMENTS

Section 1. Field of Uses of Gyrotheodolite Measurements

The number of problems being solved with involvement of gyroscopic instruments for determining the direction of the geographical meridian increases in proportion to mastery of the technology of gyrotheodolite measurements, analysis of the measurement properties of gyrotheodolites and increase in their accuracy of operation. A schematic enumeration of several directions where application of gyrotheodolites is possible is given in Fig. 1. Before considering several examples of the use of gyrotheodolites, it should be noted that astronomical azimuths, by which geodetic azimuths are calculated, and further, directional angles, can be measured by use of a gyrotheodolite. The accuracy of operation of gyrotheodolites, cited on the rating plate of the instrument, characterizes the accuracy of determination of the astronomical azimuth. This astronomical azimuth is obtained from two separate measurements: from the measurement of the instrumental correction and measurement of the equilibrium position of ЧЭ oscillations. In those cases when the relative angular orientation of the given lines is required, the "relative" azimuth, defined as the angle between a given line and the equilibrium position of oscillations of the gyrotheodolite sensitive element, can be used. When making measurements under identical meteorological (particularly temperature) conditions and over a short period of time, the error of these measurements will lie within the limits of a few angular seconds [37]. Use of the "relative" azimuth is of great practical interest in carrying out engineering-geodetic work in construction of bridges, dams, erection of large monolithic structures and so forth.

The direct use of an astronomical azimuth is rarely encountered, and serves mainly for initial orientation of space flight apparatus and for tying-in and control of astronavigational apparatus [3].

The use of a geodetic azimuth and directional angle is more widespread at the present time for determination of initial directions in surveying, mine surveying, geophysics and others.

Experience in the use of a wire plumb line for azimuthal tying-in of underground mining shows that the plumb line is not completely still, and, consequently, in underground mining the azimuth cannot be transmitted accurately. The error in transmission of the direction in a mine shaft is, on the average, 3-5'; in addition, twisting (turning) of the geodetic network laid out underground is observed [36, 38]. The use of a gyrotheodolite decreases the time for carrying out work, increases the accuracy of measurements made, insures control and has a significant economic effect. For example, the Gi-B1 gyrotheodolite, costing 9650 rubles, pays for itself in 10-15 measurements, giving an annual saving of 15,000-23,000 rubles [36]. The comparative accuracy of transmission of directions in a mine shaft is characterized by the results given in Table 2 [35].

Table 2

Method of measurement	Depth of shaft (m)			
	300	600	[illegible]	[illegible]
By plumb line	65"	73"	98"	121"
By use of gyrotheodolite	10"	11"	15"	19"

Use of gyrotheodolite measurements in underground surveying decreases the incidental error and, also, decreases the effect of systematic errors. Table 3 shows lateral divergence (in cm) of survey stations at a distance of 1.6 km from the mine shaft [35].

Table 3

Method of measurement	Depth of shaft (m)			
	[illegible]	600	[illegible]	[illegible]
By plumb line	50,2	56,3	75,[illegible]	93,6
By use of gyrotheodolite	7,7	8,5	11,6	11,6

Geodetic work is based on astronomical azimuths obtained from observations at Laplace points. However, the high cost of astronomical observations and their dependence on meterological conditions impose definite limitations on the density of Laplace point locations. It is advisable that construction of a full network based on neighboring Laplace points be carried out by use of a gyrotheodolite, which permits the accumulation of errors in measurement to be excluded.

The most methodical development of gyrotheodolite use is in survey work. Errors of lateral displacement of survey stations arise because of errors in measurement of the angle turned by the theodolite [29]:

$$\Delta S_{\tau} = \pm S_c \frac{m_{\tau}}{\rho''} \sqrt{\frac{n(n+1)(2n+1)}{6}}. \quad (68)$$

The directional angles in gyrotheodolite measurements [21]

$$\Delta S_{r} = \pm S_c \frac{m_{r}}{\rho''} \sqrt{\frac{n+1}{2}}. \quad (69)$$

where S_c -- length of a side;
n -- number of sides in a traverse;
m -- mean error of angle measurement (m_T -- with a theodolite, m_Γ -- with a gyrotheodolite);
ρ" -- 206 265" (number of angular seconds in a radian).

Comparing (69) and (68), we find that

$$m_{\tau} = m_{r} \sqrt{\frac{3}{n(2n+1)}}. \quad (70)$$

In order that the accuracy in systematic determination of points by two survey traverses, by the usual method of laying ar· with the use of a gyrotheodolite, might be the same, Eq. (70) determine the corresponding error in measurement of angles on these traverses. If, for example, the number of stations n=10, the accuracy of measurement of angles with the use of a gyrotheodolite might be ~9 times smaller in comparison with the accuracy of a theodolite; but if the number of stations is 20, the accuracy produced with a gyrotheodolite drops 16 times.

A graph of systematic displacement of survey stations using a theodolite with 2" and 5" accuracy and a gyrotheodolite having an error of 5" is shown in Fig. 37, a. The magnitude of the displacement is plotted on the ordinate and the number of stations (average length of a side 120 m) on the abscissa. Use of a gyrotheodolite for determining directional angles permits transmission of coordinates to great distances from the initial points. Gyrotheodolite measurements substantially decrease the effect of systematic errors (Fig. 37, b). During tying-in of aerial photographs with terrain points by azimuth, the productivity of this work may be increased and, in some cases, the number of special landmarks may be decreased by use of a gyrotheodolite.

Use of a gyrotheodolite in conjunction with light- and radio-distance measuring equipment is especially effective in the measurement of distances.

Combination of a gyroscopic compass with a magnetic needle led to the formation of a geophysical instrument, gyrodeclinator (KD), intended for determinations of magnetic declination, that is, the direction of a magnetic meridian with relation to the direction of a geographic meridian. Error in determination of a geographic meridian is ±15", and of a magnetic meridian

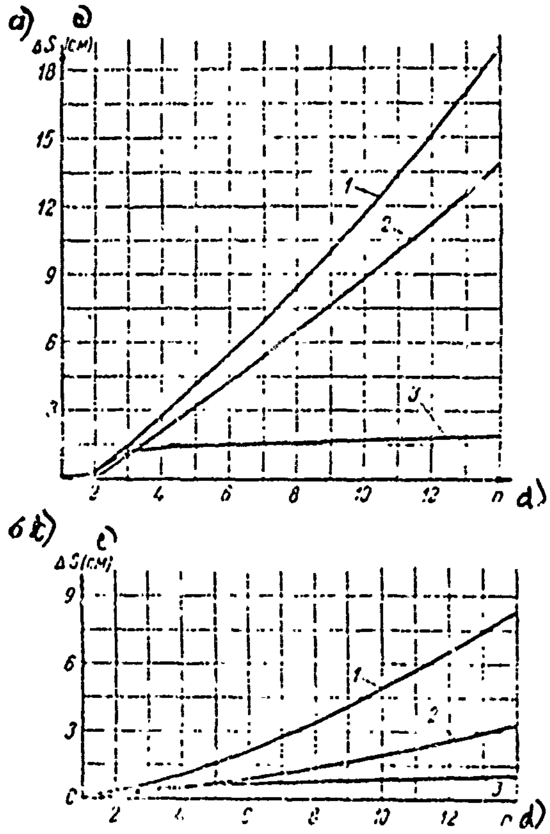

Fig. 37: Graphs of errors in survey station positions

Key:

a. Graph of systematic displacement
b. Graph of the effect of systematic errors
c. ΔS in cm
d. Number of stations

1. Working with a theodolite with a 5" accuracy
2. Working with a theodolite with a 2" accuracy
3. Working with a gyrotheodolite with a 5" accuracy

±20", and the error in determination of the magnetic declination, taking into account changes in the magnetic pole of the earth, is ±40" [48]. The gyrodeclinator permits determination of anomalies in the magnetic field caused by the presence of ore bodies to be determined quickly and independently of the presence of geodetically prepared points. Use of the gyrodeclinator opens new possibilities for the development of iron ore deposits.

In determining azimuthal direction, independence from triangulation networks permits use of the gyrotheodolite in orienting television and radio location station antennae and gyroscopic apparatus for ballistic missiles (Fig. 38). An automatic aiming system has been developed for aiming the "Minuteman" missile for launching from railroad platforms, the setting (azimuth determining) element of which is a gyrotheodolite [8].

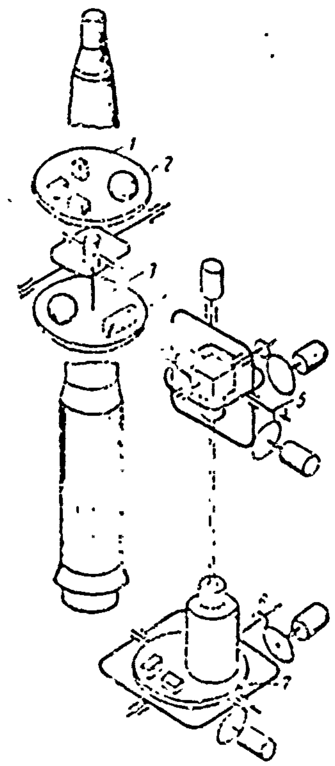

Fig. 38: Aiming scheme of the American "Minuteman" missile

Key:
1. Accelerometer
2. Spherical gyroscope
3. Stabilizing platform
4. Missile control element
5. Upper stabilizing platform
6. Gyrotheodolite
7. Lower stabilizing platform

To bring the gyrotheodolite quickly to the plane of the horizon and to eliminate possible inclination of its housing during the measurement, it is installed on a horizontal stabilizing platform. Transmission of the launch direction of the missile from the gyrotheodolite to the optical angle measuring instrument is accomplished by use of polarizing optical systems. A signal from the optical azimuth disk is transmitted to the inertial platform, which controls the missile launch. A pendulum gyrotheodolite with torsion suspension, which has a sensitive element suspended in liquid (similar to the MW-10) and ЧΘ period of oscillation of 8 min, is used in this aiming system.

Section 2. Gyroscopic Azimuth and Directional Angle

A gyroscope, owing to its basic property, interacts with the angular velocity of the earth (as long as it is fixed on the earth's surface) and determines the direction of the vector of the horizontal component of the angular velocity of the earth relative to space. Therefore, it can be taken into consideration that astronomical azimuth can be determined by use of the gyrotheodolite. In using gyroscopic azimuths in ground measurements, it is necessary to transform this astronomical azimuth to geodetic by the following equation [36]:

$$A = \alpha_0 + \Delta\alpha_1 + \Delta\alpha_2 + \Delta\alpha_3 - \Delta\alpha_4, \qquad (71)$$

where A -- geodetic azimuth on an ellipsoid;
α_0 -- gyroscopic azimuth;
$\Delta\alpha_1$ -- correction for height of the point of the gyrotheodolite stand above the reference ellipsoid;
$\Delta\alpha_2$ -- correction for change from the normal section to a geodetic line;
$\Delta\alpha_3$ -- correction for deviation of the plumb line;
$\Delta\alpha_4$ -- correction for reduction to the ean pole.

Finding the geodetic azimuth on an ellipsoid, we can change to a Gauss-Kreuger projection on a plane and find directional angle Д

$$Д = A + П - Г. \qquad (72)$$

where П -- correction for transfer from an ellipsoid to a plane;
Г -- correction for the Gaussian approach of meridians.

Determination of azimuth by the equations introduced can be accomplished only for an ideal gyroscopic measuring system, in which there is no instrumental error. As was spoken of in Chapter II, in actual construction of gyrotheodolites there is an angular divergence between the gyroscope spin axis and the measuring axis (telescope axis). Therefore, not knowing previously the magnitude of this disagreement, it is impossible to conduct a measurement using a gyrotheodolite. The magnitude of the disagreement is determined experimentally each time by comparing the gyroscopic azimuth $\alpha_Г$ and the azimuth calculated earlier, usually from astronomical observations. This comparison of azimuths, which discloses definite possibilities for simplifying office work resulting from gyrotheodolite measurements, is standardized for gyrotheodolites. Gyrotheodolite measurements are compared, according to the direction on the earth's surface in which they are made. Then, under productive conditions, the astronomical azimuth of the geodetic or relative direction observed can be determined. And what is more, in determining the limits of the linear dimensions of the territory of a given gyrotheodolite operation, standardization of the apparatus may be carried out so that the results of the measurements will turn out directly to be the directional angles.

As a result of standardization and use of known geodetic directions, instrumental correction Δ of the gyrotheodolite is determined (see Fig. 21); its magnitude is found from the following equation:

$$\Delta = Д + N - Л, \tag{73}$$

where N -- [illegible] reading, describing the equilibrium position of oscillation of the ЧЭ;

Л -- Limb reading, describing the geodetic direction at the gyrotheodolite station.

Finding instrumental correction Δ, the directional angle of the unknown direction can be calculated:

$$Д = Л - N + \Delta. \tag{74}$$

Direction to a local feature is obtained by sighting at it through the telescope and subsequently reading the horizontal ring of the theodolite.

Determining the equilibrium position of oscillation of the sensitive element directly by the theodolite limb does not always work out well. It is calculated from readings on the limb which describe only the reversing points. Finding readings N and Л in this way and knowing Δ from standardization, the directional angle is calculated.

Section 3. Determination of the Equilibrium Position of Oscillation of a Gyrotheodolite Sensitive Element

Pendulum gyrotheodolites with torsion suspensions have a practical application at the present time. Therefore, measurement and calculation methods set forth below concern just such gyrotheodolites.

Oscillations of the ЧЭ, with operating and with nonoperating gyroscope motors, have very insignificant damping, so that in practical measurement they can be considered harmonically. These oscillations have a constant period at the moment of measurement and can be considered as functions of the angle and the time. Let us evaluate the effect of damping on a gyrotheodolite.

The principal cause of damping of ЧЭ oscillations is an opposing medium. In the majority of gyrotheodolites this is air, and in some, for example, the MW-10 or АГ gyrotheodolites, it is a liquid. The equation for gyrotheodolite motion (36) was obtained without taking account of damping moments. However, to derive the equation anew, accounting for damping terms, is not necessary, since they will have a form like that in Eq. (18). Then, in accordance with (19), movement of the ЧЭ relative to the equilibrium position must be written in the form

$$\alpha = \alpha_1 e^{-\xi s t} \sin(s\sqrt{1-\xi^2}\,t + \delta), \tag{75}$$

where

$$s = \frac{1}{\tau}. \tag{76}$$

If it is considered that, in this equation, α_1 represents the amplitude of the first reversing point (the point of change of direction of movement) and the initial time reading also is taken at the first reversing point, the amplitude of oscillations at the second reversing point, displaced in time by 0.5 T relative to the first point, is expressed by the equation

$$\alpha_2 = \alpha_1 e^{-\xi s \frac{T}{2}}, \tag{77}$$

where T -- period of oscillation of the ЧЭ.

Taking Eq. (21) into account for the period of oscillation, we write expression (77) in the form

$$\alpha_2 = \alpha_1 e^{-\xi\pi}. \tag{78}$$

The amplitude of oscillation of the ЧЭ may be expressed through readings of the horizontal circle of the theodolite:

$$\left.\begin{array}{l} \alpha_1 = n_1 - N \\ \alpha_2 = N - n_2 \end{array}\right\}. \tag{79}$$

where n_1, n_2 -- readings corresponding to the reversing points;
N -- reading corresponding to the equilibrium position of oscillation of the ЧЭ.

Taking (79) into account, from expression (78) we find the ratio of two adjacent amplitudes:

$$\frac{\alpha_2}{\alpha_1} = \frac{N - n_2}{n_1 - N} = e^{-\xi\pi}. \tag{80}$$

From expression (80) we find:

$$N - n_2 = (n_1 - N) e^{-\xi\pi}. \tag{81}$$

After reorganization, we obtain from (81) the expression for determining equilibrium position

$$N = n_1 + (n_2 - n_1) \frac{1}{1 + e^{-\xi\pi}}. \tag{82}$$

The factor $\frac{1}{1 + e^{-\xi\pi}}$ accounts for attenuation (damping) of oscillations and, for every model, it has a practically constant magnitude. This factor changes negligibly from specimen to specimen of a given type of gyrotheodolite. For example, for the Gi-B1 gyrotheodolite

$$\frac{1}{1 + e^{-\xi\pi}} \approx 0{,}5003,$$

from which

$$e^{-\xi\pi} = 0{,}9988, \tag{83}$$

which allows us to find the magnitude of the relative damping coefficient

$$\xi \approx 0{,}0003. \tag{84}$$

It should be noted that the magnitude ξ does not remain constant in a gyrotheodolite start-up, but depends on the amplitude of the 43 oscillation. This is explained by the fact that the damping under discussion is excited by an aerodynamic resistance, which depends on the square of the linear speed of the moving body. An increase in amplitude leads to increase in this linear speed and, consequently, to an increase in damping.

The negligible magnitude of the relative damping coefficient generally permits neglect of the theoretical existence of the attenuating factor, since errors connected with locating the reversing point, sighting on local features and others, are greater than the influence of errors connected with ignoring attenuation. Therefore, for example, Eq. (82) can be presented in the form

$$N = n_1 + 0{,}5(n_2 + n_1)$$

or

$$\dot{N} = \frac{n_1 + n_2}{2}. \quad (85)$$

But, in order for this equation to be used in measurements, Eq. (84), which indirectly characterizes the exactness of operation of the gyrotheodolite, must be approximately adhered to. It has been established by experience in operations with pendulum gyrotheodolites with torsion suspensions in air that, for an instrument in good repair, the relation is

$$\frac{N - n_2}{N - n_1} \geqslant 0{,}9950. \quad (86)$$

If the damping appears to be greater than indicated, but the instrument is in good repair, attenuation should be accounted for in the design equations.

The main bases for constructing a scheme for finding the equilibrium position of the oscillation under discussion follows from the constancy of the period and amplitude of oscillation of the sensitive element of a gyrotheodolite: by measuring the angle of inclination, by measuring the time duration and by simultaneously measuring the angle and time.

In order to determine the parameters of the sensitive element oscillation, it is necessary to be able to observe its movement. After unlocking the gyroscope, the sensitive element mark is observed in the automatic collimator telescope, and the effort is made to keep the moving gyroscope mark continually in the center of the automatic collimator scale by rotation of the free alidade. As the sensitive element approaches the reversing point (the point at which the direction of movement changes), the speed of movement of the mark decreases, and at the point itself, it remains motionless for a short instant. At this instant, a reading is taken from the horizontal circle of the theodolite. After this, the alidade moves in the opposite direction to the second reversing point and a reading is taken again from the horizontal circle.

Location of the axis of symmetry of these oscillations is mainly for locating an earth meridian, but the direct equilibrium position permits carrying out a preliminary orientation of the gyrotheodolite.

Obtaining high accuracy in determining a geographical meridian places definite demands on the initial installation of the gyrotheodolite. More accurate preliminary orientation establishes smaller rotation moments during oscillation of the sensitive elements and facilitates the work of the observer. Several methods of preliminary orientation are known: by magnetic surveying compass and directly by gyrotheodolite. In the latter case, orientation may be carried out by observing the amplitude of oscillation (by reversing points) or by knowing the period of oscillation of the ЧЭ.

Preliminary orientation by two reversing points consists in calculating the average values of the readings of these two points and setting the telescope in a position corresponding to this value.

Calculation is carried out by the equation:

$$N = \frac{1}{2}(n_1 + n_2).$$

The accuracy of determining north by these methods is: for gyrotheodolites ±45-60" in a time of 10-12 min, for gyroscope attachment, ±2-3' in 6-9 min.

Preliminary orientation by a known period of oscillation consists in finding one reversing point and fixing the position of the alidade (the sensitive element mark) after a time of 0.25 T (T -- period of oscillation of the sensitive element). A quarter period (0.25 T) -- this is the time which is necessary for the gyroscope to move from the reversing point to the equilibrium position of the oscillation. Owing to the slow speed of movement of the sensitive element near the reversing position it is difficult to determine exactly the moment in time when this speed reaches zero. Therefore, in orienting by this method, the measurement of time begins a little ahead of the reversing point (Fig. 39, a). For this, when a significant decrease in the speed of movement of the ЧЭ is detected, the alidade is fastened, and, at the instant the mark crosses the center of the automatic collimator scale, a stop watch is started (point A in Fig. 39, b). At the moment when the mark crosses the center of the scale again on the return path (point B), time τ is determined by the stop watch and tracking of the sensitive element is continued for a length of time equal to $\left(\frac{1}{4}T + \frac{1}{2}\tau\right)$. At the end of this segment of time, tracking is stopped and the position of the telescope obtained in this manner is nearly the direction of north. The period of oscillation of the sensitive element of a gyrotheodolite depends on the latitude at the point where it stands ϕ; therefore, in a change of latitude, a correction ΔT must be introduced into the value of period T, which is defined from the following equation [53]:

$$\Delta T = T_0 \frac{\operatorname{tg} \varphi_0}{\sqrt{\cos \varphi_0}} \Delta\varphi, \tag{87}$$

then

$$0{,}25T = 0{,}25\,(T_0 + \Delta T). \tag{88}$$

where T_0 -- period of oscillations of the sensitive element at the average latitude of the region of the gyrotheodolite measurements;

$\Delta\phi$ -- latitude increment.

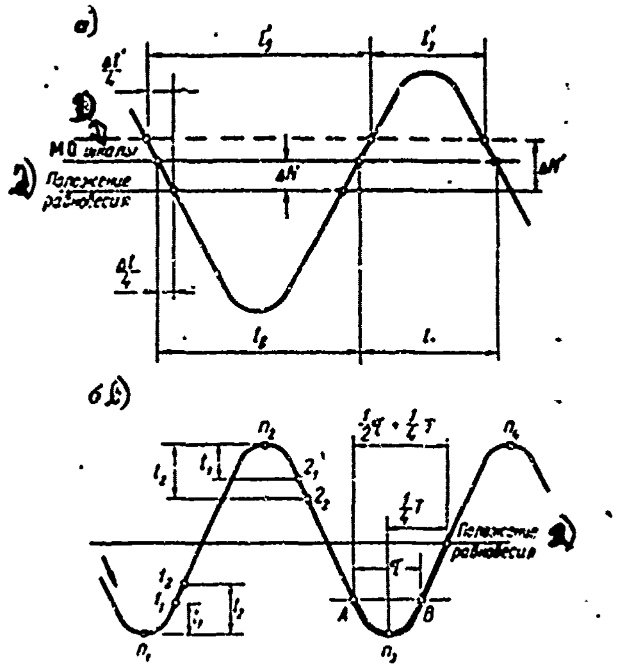

Fig. 39: Determination of the equilibrium position of sensitive element oscillation

Key:
1. Scale
2. Equilibrium position

The accuracy of this method depends on the amplitude of the oscillation. An approximation is given for the GAK-1 gyroscope attachment, presented in the following Table 4.

Table 4

Amplitude of oscillation	Mean error
3	±2'
10'	±7'
30'	±20'

Therefore, a special mechanism for regulating the amplitude of oscillation is frequently provided in a gyroscope attachment.

The preliminary orientation method under consideration has approximately the same accuracy as the two-reversing-point method, but for this method the time necessary to carry out an orientation is decreased by 30-40%.

Methods for accurate orientation assume that the minimum possible number of errors in determining the equilibrium position of the ЧЭ are obtained from the gyrotheodolite. This takes an increase in time necessary for such measurements. Along with this, definite effort has been noticed in recent years in further development of gyrotheodolites, directed towards shortening the measurement times without decreasing accuracy. Solution of these problems will be attained both through constructive improvements in apparatus and in development of methods of measurement.

The reversing point method has achieved the most widespread use for determining the equilibrium position of ЧЭ oscillations. It consists in finding the reversing point by hand or automatic (in the MT1 and Gi-B2 gyrotheodolites) tracking of the sensitive element with the alidade. Investigations conducted by many authors show that four reversing points is the optimum number for determining the equilibrium position of the sensitive element. Using four reversing points, the equilibrium position is found from the following relation (see Fig. 39, a):

$$N_0 = \frac{1}{2}\left(N_0^{I} + N_0^{II}\right), \tag{89}$$

where

$$N_0^{I} = \frac{1}{2}\left(\frac{n_1 + n_2}{2} + \frac{n_2 + n_3}{2}\right), \tag{90}$$

$$N_0^{II} = \frac{1}{2}\left(\frac{n_2 + n_3}{2} + \frac{n_3 + n_4}{2}\right). \tag{91}$$

Some more accurate definition of results of measurement is attained by repeated independent determinations of the equilibrium position of the sensitive element at the same observation point. Together with this, results of investigations [37] show that the values N_0^I and N_0^{II} are different from one another by not more than 2-3''. Considering that every value of N_0^I and N_0^{II} is determined from three reversing points, for the purpose of shortening the observation time, the possibility is presented of finding equilibrium position of the sensitive element not by four, but by three, reversing points. This gives a shortening of time of approximately 20%, but it decreases the accuracy by 5-10%. Further shortening of the observation time is achieved by determining the sensitive element equilibrium position by two reversing points. In this case accuracy of determination of equilibrium point is decreased by 15-20% by comparison with determination by four points. The principal deficiency in the method appears to be the absence of checks on the values obtained. Such a possibility appears if the equilibrium position of the sensitive element is determined by two reversing points with check readings, located symmetrically in time relative to the reversing point (see Fig. 39, b). Introduction of these check readings insures the reliability of measurements and brings the accuracy of determination of the sensitive element equilibrium position close to the accuracy of the three point method. Check readings are taken a short time after the first reversing point (for example, t_1 and t_2). Here, the time is selected arbitrarily. The stop watch is stopped a little past the second reversing point and, at moments of time equal to t_1 and t_2, secondary symmetric readings are taken on the limb (2_1 and 2_2). The point of symmetry in time is obtained with an error of ±(0.3-0.5) sec, which provides a determination of the equilibrium position of sensitive element oscillation with a mean error of ±(5-8)". While taking readings at points 1_1, 1_2,..., 2_1, 2_2 the sensitive element is not tracked, which gives rise to the appearance of a certain twisting moment on the part of the torsion element and, consequently, some distortion of the movement of the sensitive element. Therefore, it is advisable that this method be used with gyrotheodolites which have systems for tracking the twisting of the torsion element (MT-1, IMK-604). It should be noted that symmetrical check points are advisable in the three-reversing-point method, owing to which the reliability of the results of measurements are increased. The use of check points permits determination of the direction of a geographical meridian with an error of about ±20-25" in 9-10 min.

It is known that tracking the sensitive element during its oscillations is a process which takes a lot of work. Therefore, in those cases where the preliminary orientation is carried out with an error of 5-10 angular minutes, the so-called amplitude method may be used, in which successive reversing points are observed by use of a fixed automatic collimator telescope [53]. For this the amplitude of oscillations of the sensitive element of no more than 20-30' should be sought for. With such an amplitude, the influence of twisting of the torsion element on the equilibrium position of the sensitive element may be considered linear-dependent on the angle turned by the sensitive element.

The torsion moment is determined by the equation:

$$M_з = \frac{EI_c}{l}\gamma$$

or

[illegible]

Taking these moments into account, the equations for gyrotheodolite motion (32) take the form

$$\left.\begin{aligned} H\omega_\Gamma\alpha + H\dot{\beta} &= M_z \\ -H\dot{\alpha} + H\omega_z + G\beta &= 0 \end{aligned}\right\}, \qquad (32')$$

from which, after division of the variables, we obtain the equation for azimuthal movement of the ЧЭ:

$$\frac{H}{G\omega_\Gamma}\ddot{\alpha} + \alpha = \frac{k}{H\omega_\Gamma}\gamma. \qquad (92)$$

Solution of this equation shows that oscillations of the gyrotheodolite ЧЭ will take place about the position defined by angle α_Γ:

$$\alpha_\Gamma = \frac{k}{H\omega_\Gamma}\gamma. \qquad (93)$$

Consequently, the twisting moment of the torsion element shifts the equilibrium position of the ЧЭ oscillations. The magnitude of this displacement depends on the latitude of the point where the instrument is standing, since the horizontal component of the angular velocity of rotation of the earth enters into (93).

Determining the equilibrium position of a gyrotheodolite sensitive element oscillation relative to the zero mark of the automatic collimator scale when the gyroscope motor is not operating, and then comparing this position with equilibrium position n_Γ with a working gyroscope motor, the angle twisted by the torsion element is found, and then the correction of the ЧЭ equilibrium position is determined:

$$\Delta N = \frac{k}{H\omega_\Gamma}(n_\Gamma - n_0). \qquad (94)$$

Then the final equilibrium position is found by the following equation

$$N = \frac{n_1 + n_2}{2} + \frac{k}{H\omega_\Gamma}\left(\frac{n_1 + n_2}{2} - \frac{n_1' + n_2'}{2}\right). \qquad (95)$$

The ratio of rigidity k of the torsion ribbon to the gyroscopic moment $H\omega_\Gamma$ is approximately $2.5\cdot10^{-5}$ in middle latitudes for a model Gi-B gyrotheodolite.

The amplitude method is used mainly for determining the zero point of torsion suspensions, but it may be used for the direction of geographic meridians under the conditions described above.

The "passing method" is a further development of the amplitude method. Using the method, the equilibrium position of the sensitive element is determined by means of measuring the segment of time between instants when the sensitive element mark passes across a specified mark on the automatic collimator telescope scale when the alidade is in a fixed position (Fig. 39, b). A condition for use of this method is a small amplitude of sensitive element oscillations. Let us assume that the mark moves from west to east and at the instant when it crosses the zero mark of the automatic collimator telescope scale, the stop watch is started. The mark begins to move in the opposite direction past the reversing point and, at the moment when it again crosses the zero mark, the stop watch is stopped. At this same moment, using a model C-II-1b two-hand thirty-second stop watch, a second count is begun and the second segment of time is measured analogously, only now, when the mark is on the west side relative to the zero mark of the scale. Time segments t_B and t_3 are obtained as a result of the measurements. If a segment of time necessary for the mark to move from the zero mark to the equilibrium position is designated across τ, then:

$$t_B - t_3 = \Delta t = 4\tau. \qquad (96)$$

Taking into account the linearity of the dependence between the angle and the time interval near the equilibrium position, this equilibrium position can be determined relative to the zero mark of the scale. Movement of the sensitive element is written down in the equation

$$\alpha = \alpha_0 \sin st.$$

Near the equilibrium position, when angle (st) is small, it can be written:

$$\alpha = \alpha_0 st, \qquad (97)$$

where α_0 -- amplitude of oscillation;
s -- natural frequency of oscillation, see (38).

Transferring from angle α_0 to a reading on the automatic collimator scale and taking into account the influence of twisting of the torsion element, the ЧЭ equilibrium position will be determined by the equation

$$N = \eta \Delta t + \Delta N_\tau. \qquad (98)$$

A feature of the method under discussion is the possibility of obtaining a very large number of determinations of N, for which any pair of marks located symmetrically with respect to the zero mark can be used.

In this case: $N' = q \Delta t'$

will be distinguished from N by the magnitude of displacement of the selected mark relative to the zero mark.

A comparison of the approximate values of accuracy obtained and the time necessary for one determination of a gyrotheodolite sensitive element equilibrium position by different methods is given in Table 5.

Table 5

Orientation	Method of orientation	Model Gi-B1 gyrotheodolite		Model Gi-C1 gyroscope attachment	
		Mean error	Observation time (in min)	Mean error	Observation time (in min)
Preliminary	By known period	50"	8	3'	4
	By two reversing points . .	50"	11	2'	7
Precise	By four reversing points .	20"	20	25"	14
	By three reversing points .	23"	15	25"	11
	By three reversing points with marking of symmetric check points	23"	15	—	—
	By two reversing points with marking of symmetric check points	25"	10	—	—
	Amplitude method with observation of four reversing points	20"	20	25"	12
	Passing method with measurement of four time intervals	20"	20	25"	12
	Passing method with measurement of two intervals and marking of supplementary intervals by symmetric marks of the scale .	25"	14	—	—

Section 4. Features of Gyroscopic Orientation

At present all possible measures to decrease the influence of harmful moments are taken in the construction of various gyrotheodolites. However, as has been shown above, in order to obtain satisfactory accuracy in gyrotheodolite operations, it is necessary that the level of harmful moments not exceed ~10^{-6}g-cm. There is little likelihood of achieving such small moments by direct methods. However, the error in measurement practical with existing instruments corresponds to an influence of harmful moments of approximately the magnitude indicated. But this is only a formal comparison. Actually, such a correspondence is achieved by methods of making and subsequently processing gyrotheodolite measurements. It follows from this that its own technology of measuring processes must be developed for every type of gyrotheodolite. However, for groups of instruments with identical principal schemes, but of differing constructions, there are a number of common operations whose observance is necessary for obtaining the required accuracy. These general recommendations for pendulum gyrotheodolites with torsion suspensions in air are considered below.

The principal influence on accuracy of determination of the direction of geographical meridians is the stability of the instrumental correction Δ. The magnitude of this correction depends on many factors, the mutual effect of which generates complicated correlative connections. Specifically, the stability of operation of the generators supplying the gyroscope motor, temperature of the surroundings, stability of mechanical characteristics of the torsion element (from start-up to start-up and in the process of start-up), change in the magnetic field, mechanical rigidity of parts of the gyrotheodolite structure (preservation of the relative positions during jolting while being transported) and others have an effect on the correction. For example, a change in the incoming current of 1.5 v gives rise to a change in the instrumental correction of ~14" in the MRK-2 [44]. A given instrument can be qualitatively evaluated by observing the stability of the correction under various conditions. Many publications have been devoted to investigations of the stability of the instrumental correction, part of them, characteristically, only for a given model, but in some more general correlations are established. Results of prolonged observations of the instrumental correction of the model Gi-B gyrotheodolites shows [36, 45] that:

1) accuracy of determination of the correction lies within the limits of ±10" (accuracy of determination of the equilibrium position ~±9", accuracy of the standardized azimuth ~±3");

2) deviation of the correction from the average value obeys the law of normal distribution;

3) the mean value of the deviation of the correction from the average value lies within the limits of ±15".

These statistical values take on the role of criteria for evaluation of the quality of an instrument: i the result of a check test discloses a deviation of the instrumental correction from its rating plate value of greater than ±15", such a gyrotheodolite must be subjected to an additional, many-sided examination. Only highly accurate geodetic networks should be used to determine instrumental corrections. It is advisable for the measuring process itself to be carried out at different times of the day in order to cover various meterological conditions After each start-up of the instrument, the azimuthal orientation, horizontal setting and other things should be changed. The number of start-ups should be 9; the equilibrium position is determined by four reversing points on every start-up.

The main point of these modifications is, in measuring the instrumental correction, to cover as wide a scope as possible of the conditions under which the gyrotheodolite will be used.

A separate question arises periodically of checking of instrumental corrections. A gyrotheodolite should be standardized over 1-2 months under relatively uniform surrounding conditions (especially air temperature); when surrounding conditions change and when the instrument is transported over a considerable distance, the instrumental correction should be determined before beginning work and after it is completed. In this regard, there is a significant comparison of the results of a series of 2000 measurements with the Gi-B1 gyrotheodolite, carried out under experimental mine-drifting and surface conditions [37]. The mean square error of determination of the instrumental correction under laboratory conditions was ±9.0", and the mean error in determination of the azimuth was ±9.5". The picture changes significantly in the field: the error in the correction is ±9.5", but the error in determining azimuth is ±15.5". The difference of 6" is accounted for principally by change in surrounding conditions during the period of measurement on the terrain as well as by inaccuracies in the geodetic network, by which the error in calculated azimuth was determined.

Considering the accuracy of determination of a direction using a gyrotheodolite, the stability of the gyrotheodolite parameters must be watched continuously in the process of a given start-up, as well as from start-up to start-up. The period of free (when the gyromotor is not operating) oscillation of the ЧЭ should be measured with a stop watch during every gyrotheodolite measurement. The period of free oscillation of the sensitive element defines the stability of mechanical characteristics of the oscillating system of the gyroscope suspension and is a sufficiently accurate definition of its working condition. Changes in this period under different surrounding conditions must be insignificant. For example, the period of free oscillation for the Gi-B1 T_{CB}=90 sec, and the permissible change in it ~4-5 sec.

The basic advantage of the torsion suspension over other ЧЭ suspensions is the small harmful moment and the possibility of calculating the residual twisting moment. The twisting moment of the torsion element does not influence the ЧЭ equilibrium position during operation of the gyroscope motor only when the equilibrium position of free oscillations coincides with the equilibrium position of oscillation of the gyroscope unit during the entire start-up. Practical realization of this condition encounters serious difficulties and, therefore, the correction in equilibrium position of the sensitive element is determined by taking into account the linear dependence of the twisting moment of the torsion element on its angle turned. For this, it is necessary to determine the angle turned by the torsion element relative to the equilibrium position of the gyroscope unit, which is often called the zero point or the point of zero torsion (MNT), as well as to know the coefficient of passing from the MNT to the correction in equilibrium position (for MOM gyrotheodolites, this coefficient is "C," the table of which is given in Appendix 1). Use of the table mentioned is not recommended for highly accurate determinations of azimuth. Coefficient "C" should be determined immediately before measuring. Two methods may be used for this: a) carrying out the start-up with the aforementioned determination of displacement of the MNT and b) comparison of the periods of oscillation of the ЧЭ while tracking with the alidade and without tracking (with the alidade fixed). The instrument is oriented by the first method by a previously found or known direction of a geographical meridian. Then the gyroscope motor is started, the ЧЭ is unlocked, during which an amplitude of oscillation of no more than 40' is attained, since the grid image of the automatic collimator will leave the field of view of the automatic collimator at greater amplitudes. Leaving the alidade part of the instrument fixed, the equilibrium position of the ЧЭ is determined by reversing points, observed by the automatic collimator scale. Then, not stopping the gyroscope motor and not locking, the ЧЭ is carefully turned relative to the initial setting to some constant angle, for example +10', and the equilibrium position of ЧЭ oscillation is determined. The gyroscope is then turned to a -10' angle relative to the same initial position and the equilibrium position is determined again by reversing points. If the torsion element twisting moment does not influence the equilibrium position of the ЧЭ, the position found in the first and second turns would differ by the magnitude of the angular displacement, that is, the angle of ±10'. The difference between the angular position of the axis of symmetry of oscillation of the ЧЭ and the angle of ±10' indicates the effect of twisting of the torsion member and permits the amount of it to be determined.

In the absence of an effect of twisting of the torsion elements

$$\left.\begin{aligned} N_{+\psi} &= N_0 + \psi \\ N_{-\psi} &= N_0 - \psi \end{aligned}\right\}, \qquad (99)$$

where N_0, $N_{+\psi}$, $N_{-\psi}$ -- readings on the horizontal ring corresponding to equilibrium positions in initial and slewed positions of the instrument,

ψ -- supplementary angle turned on the scale points.

The influence of twisting of the torsion element leads to the necessity for introducing a compensating item (correction) for maintaining the equality of (99)

$$\left.\begin{aligned} N_{+\psi} + C\varphi_{+} &= N_0 + \psi \\ N_{-\psi} + C\varphi_{-} &= N_0 - \psi \end{aligned}\right\}, \qquad (100)$$

where ψ -- angle of torsion element twisting (on the scale points):

$$\left.\begin{array}{l}\varphi_{+} = \psi - N_{+\psi} \\ \varphi_{-} = \psi - N_{-\psi}\end{array}\right\}. \qquad (101)$$

Subtracting the second equation of system (100) from the first equation of this system and taking (101) into account, after reorganizing, we obtain:

$$C = 1 - \frac{2\psi}{N_{+\psi} - N_{-\psi}}. \qquad (102)$$

The method for determining coefficient C based on comparison of the periods of oscillation of the ЧЭ obtained from tracking the movement of the gyroscope unit with the alidade part and without tracking, follows from comparison of differential equations (36) and (92) for movement of a gyrotheodolite. In the first instance, the oscillation period is determined by Eq. (43) and in the second it has the form

$$T_{kp} = 2\pi\sqrt{\frac{H}{[illegible]}}. \qquad (103)$$

The relation between the period from (43) and the period from (100) leads to the following expression

$$\frac{T^2}{T_{kp}^2} = 1 + [illegible]. \qquad (104)$$

The second term in the right side of Eq. (104) defines the difference in movement of the gyroscope relative to the equilibrium position, that is, determines the magnitude of coefficient C discussed earlier. Then the magnitude of C may be determined by the equation from [42]:

$$C = \frac{T^2}{T_{kp}^2} - 1. \qquad (105)$$

In practice, both methods of determining the coefficient are equivalent, but the principal limitation of the second method arises from the amplitude of oscillation -- it must be small. This limitation is imposed in the first method only by the field of view of the automatic collimator, since the amplitude of oscillation is taken into account to a known extent in Eq. (102).

In order for the influence of twisting of the torsion element to be insignificant, there should be a restriction of the torsion element zero point, which, for example, in the Gi-B1, is the angle between the optical axis of the automatic collimator and the normal to the ЧЭ mirror in its position when the moment of the torsion band equals zero. To determine this position in practice doesn't work out well, and then the equilibrium position of free oscillations is determined, assuming that the rest position and equilibrium positions coincide. The correction for the twisting of the torsion element is introduced by use of the average value of the MNT, which is obtained before beginning and after completing measurements, or MNT measured only after the start-up is completed. In the first case, it is assumed that MNT changes linearly as a result of temperature deformation during the time of the start-up, but, in the second case, temperature stability of the MNT is assumed.

The question of when to determine the MNT is important for highly accurate measurements, and there must be a supplementary examination for every type of instrument. This is connected to the effect of the temperature deformation of an instrument on the MNT. A graph of change in the ЧЭ oscillation equilibrium position as a function of the number n of reversing points (Fig. 40, a) and a graph of displacement of the center of gravity of the Gi-B1 oscillating system in the axial direction (Fig. 40, b), reduced to displacement of the axis of symmetry of oscillations [45] are shown in Fig. 40. From a comparison of the graphs and the times of measurement with instruments, it follows that the whole measuring process is completed before the instrument reaches thermal equilibrium. From the point of view of accurate calculation of the MNT, all measurements with the Gi-B gyrotheodolites should be made over 30-40 min of its operation when the parts of its structure reach temperature stability.

A probability of temperature deformation can explain the periodicity of changes in the average values of a pair of successive readings of the reversing points of the GAK-1 gyroscope attachment [59] by the results of long start-ups (up to 45 reversing points). An idealized graph of the periodic changes of the equilibrium position of oscillations of the gyroscope attachment ЧЭ is represented in Fig. 41, a, and graphs of actual measurements in Fig. 41, b. The period of such changes lies within the limits of 40-100 min and the amplitude does not exceed 30" on the average (60" only in one case). The oscillations shown in Fig. 41, b (quasi-harmonic oscillations of the gyroscope unit equilibrium position) do not arise directly from gyroscope theory, and their cause should be sought in structural features of the apparatus.

The features of gyroscopic orientation considered above directly influence the accuracy of determination of azimuths and, therefore, in one or another form, are accounted for in the final results of measurements.

There is still another series of factors which may change the end results of measurements to one degree or another. When accuracy of measurement is defined in angular seconds, it is impossible to ignore even small

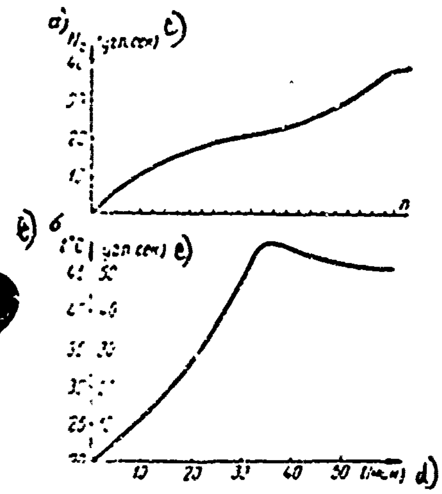

Fig. 40: Graphs of changes in characteristics of a gyrotheodolite due to temperature

Key:
a. Displacement of the equilibrium position
b. Displacement of the center of gravity
c. Angular seconds
d. Minutes

influenes- -- the surroundings. Thus, owing to heating of one side of the trip- ne sun's rays, the additional error may reach a magnitude of 5' ...ations of the tripod under the action of wind gusts are passed o. t. ...torsion element and distort the movement of the sensitive element. ...areless unlocking of the gyroscope unit may change the position of the torsion element zero point by 30-60". Change in amplitude of oscillation of the ...O from 10° to 40° may lead to an additional error in determining azimuth of up to 5".

Everything set forth above shows that provision of high accuracy in determining directions of orientation is possible only by strict execution of all instructions for gyroscope orientation, the specific character of which depends both on the type of gyrotheodolite and on the kind of gyrotheodolite work. Every gyrotheodolite start-up is subject to continuing control by the magnitude of the free and gyroscopic oscillation periods, by the magnitude of damping, by the value of the torsion element zero point and by discrepancies in particular values of the equilibrium position (N_0^I and N_0^{II}). Such control permits replacement of both faults in operation of the instrument and gross errors by observers.

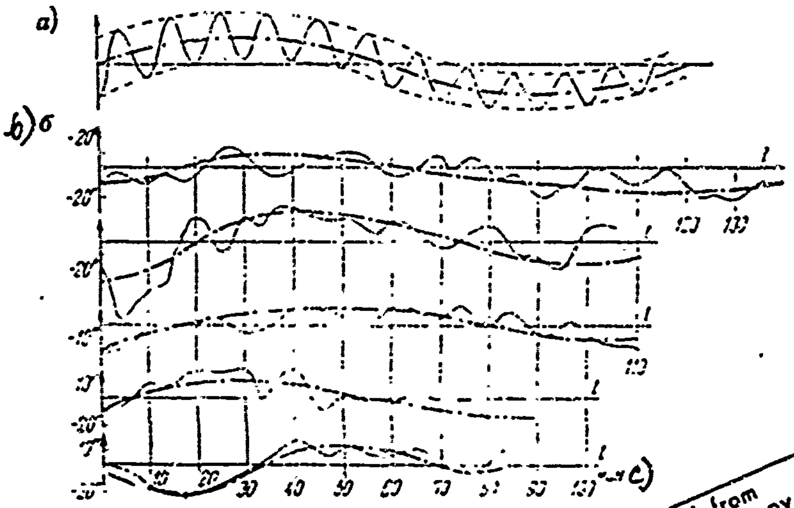

Fig. 41: Quasi-harmonic oscillations of the equilibrium position

Key: a. Ideal
b. Measured
c. Minutes

The accuracy of the gyroscopic azimuth is defined by the sum total of errors, consisting mainly of errors in determination of equilibrium position σ_p, errors in changes of the torsion element zero point σ_T and errors in determining and changing the instrumental correction σ_Δ:

$$\sigma_A = \sqrt{\sigma_p^2 + \sigma_T^2 + \sigma_\Delta^2}. \qquad (106)$$

Section 5. Damping Gyrotheodolite Sensitive Element Oscillations

Gyrotheodolite measurements require unremitting attention of the operator in the course of the entire measuring process, which results in his becoming tired quickly. Trivial external vibrations of the tripod, nonuniform movement of the alidade part when tracking the sensitive element and other hindrances may lead to distortion of the results of measurement. Reversing point readings obtained as a result of the operator's strenuous work must then be processed in an appropriate manner with the corrections resulting from calculations carried out, and only after this can the directional angle be transferred to the terrain.

These conditions of work with gyrotheodolites can be simplified if the ЧЭ oscillation equilibrium position is not determined by reversing points, but by waiting for the state when the oscillations of the gyroscope spin axis calm down. We find from Eq. (75) and taking into account Eq. (22) and (84) that the Gi-B1 gyrotheodolite rotor spin axis will be situated close by the equilibrium position only after 15 days. Obviously, such a period for quieting down of the ЧЭ oscillations is not acceptable for practical use. In order for the time for the ЧЭ oscillations to settle down to be short, the coefficient of relative damping ξ must be within the limits 0.7-0.9. With these values of the relative damping coefficient, the ЧЭ of the Gi-B1 gyrotheodolite will be settled down near its equilibrium position after a time of 6-9 min. During the time that the ЧЭ oscillations are settling down, the observer has the opportunity of carrying out other work, for example, calculation of the correction for transition from gyroscopic azimuth to directional angle, orientation in the locality and others.

Consequently, a special damping mechanism must be introduced into the instrument for rapid settling down of the gyrotheodolite sensitive element oscillations. The use of damping mechanisms in gyroscopic instruments for determining direction of a geographical meridian has been known practically from the moment of appearance of these instruments themselves -- nautical gyrocompasses [14].

However, use of damping mechanisms in gyrotheodolites has a fundamental distinction from damping in nautical gyrocompasses. The kinetic moment of a gyrotheodolite is several orders of magnitude less than that of a nautical gyrocompass, and the accuracy of operation is considerably higher. These features produce an exceptionally strict dependence of the damping moment only on the speed of movement of the sensitive element. The damping mechanism does not have to maintain a constant moment or a moment which is dependent on the angle turned, since these moments will displace the sensitive element oscillation equilibrium position and, consequently, introduce an additional error. As follows from the foregoing, the sum total of deleterious moments on the part of the damper must be less than 10^{-6} g-cm.

Attenuation of the ЧЭ oscillation can be obtained by combination of the gyrotheodolite "interior" gimbal with the damper (see Section 1, Chapter II), by combining the damper with its "exterior" gimbal, or by simultaneous damping by both gimbals. We designate the damping moment of the exterior gimbal axis $c_1\dot{\alpha}$, and of the interior axis $c_2\dot{\beta}$. Then, by analogy with Eq. (32), we obtain the equations for motion of a damped gyrotheodolite:

$$\left.\begin{aligned} H\omega_r\alpha + c_1\dot{\alpha} + H\dot{\beta} = 0 \\ -H\dot{\alpha} + Cl\beta + c_2\dot{\beta} + H\omega_v = 0 \end{aligned}\right\}. \qquad (107)$$

We take advantage of the operator method [21] for further analysis of these equations, and we write Eq. (107) for zero initial conditions in operator form:

$$\left.\begin{array}{l} H\omega_r\alpha + c_1 p\alpha + Hp\beta = 0 \\ -Hp\alpha + Gl\beta + c_2 p\beta + H\omega_a = 0 \end{array}\right\}. \qquad (108)$$

In Eq. (108), in distinction from Eq. (107), angles α and β are Laplace transforms.

From the first and second equations, respectively, we find:

$$\alpha = -\frac{Hp\beta + H\omega_r\alpha}{c_1 p}, \qquad (109)$$

$$\beta = \frac{Hp\alpha - Gl\beta - H\omega_a}{c_2 p}. \qquad (110)$$

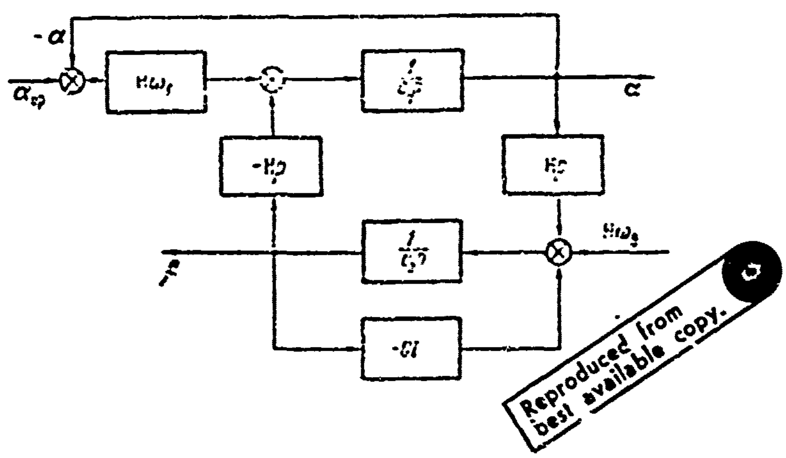

Fig. 42: Structural diagram of a gyrotheodolite with damping

These two equations correspond to the structural diagram given in Fig. 42. Angle α_{op} represents the angle of initial (preliminary) orientation of the gyrotheodolite and the structural diagram indicates the internal interconnections of the gyroscope, as a result of which its axis becomes set in the direction of the geographical meridian. If the preliminary orientation angle is presented on the goniometer part of the instrument in the form of a zero reading, in the ideal case the gyroscope spin axis must turn relative to this reading to angle α_{op}, that is, the gyroscope must measure the value of the preliminary orientation angle. From the structural diagram we present the correlation of gyroscope azimuthal turn angle α with the preliminary orientation angle α_{op}:

$$\alpha = \frac{H\omega_r c_2 p + GlH\omega_r}{(H^2 + c_1c_2)\,p^2 + (c_1Gl + c_2H\omega_r)\,p + H\omega_r Gl}\,\alpha_{op}. \qquad (111)$$

Let us consider two separate cases:
a) only the interior gimbal damps the oscillation (in this case $c_1=0$):

$$\alpha_a = \frac{H\omega_r c_2 p + GlH\omega_r}{H^2p^2 + H\omega_r c_2 p + H\omega_r Gl}\,\alpha_{op}; \qquad (112)$$

b) only the exterior gimbal damps the oscillation (in this case $c_2=0$):

$$\alpha_б = \frac{GlH\omega_r}{H^2p^2 + Glc_1p + H\omega_r Gl}\,\alpha_{op}. \qquad (113)$$

Formula (112) is structurally analogous to formula (111), but formula (113) is simpler than the first two. This simplification has a basic nature, since actual differentiating links ($H\omega_r c_2 p + GlH\omega_r$) are replaced by a simple amplifying link ($GlH\omega_r$). If the preliminary orientation angle is absolutely constant, then substitution of one of the links mentioned in the other does not tell on the nature of the behavior of the gyroscope. However, an insignificant push on the tripod, (for example, from a wind gust) or the base on which the instrument is installed, the differentiating link will be actively emphasized (intensified), in which case it is impossible to speak of the link $GlH\omega_r$. Consequently, damping with the interior gimbal is not advisable in these positions. In addition, there are some technical difficulties in producing an efficient damper structure. One of the versions of damping with the interior gimbal is shown in Fig. 43. Two lower cylinders 3, half filled with liquid, are connected to pendulum weight 5. Upper cylinders 2, which have capillary openings 1, are connected with gyroscope motor housing 4 (by the interior gimbal). When the interior gimbal is inclined the volume between the upper and lower cylinders changes, and air escapes through the capillary openings, as a result of which a force is generated, which creates a damping moment relative to the gyroscope. Experimental work conducted with a similar damping system showed that such a system has negligible damping and is not stable, especially during temperature changes in the surroundings [40].

Further development of damped gyrotheodolites is proceding by way of applying a damping moment to the gyroscope exterior gimbal, that is, by way of direct damping of azimuthal oscillations. This way permits only one part of the damping mechanism to be fastened to the gyroscope, the second to be located on the instrument housing. Since the damping moment must be strictly proportional to the velocity, then, considering that angle α turned by the exterior gimbal is approximately 200 times greater than angle β turned by the interior gimbal, creation of such a moment is simplified in the case of direct damping of azimuthal oscillations.

Eq. (113) may be written down in the form

$$\alpha_б = \frac{1}{\ldots p^2 + \ldots p + 1}\,\alpha_{op}. \qquad (114)$$

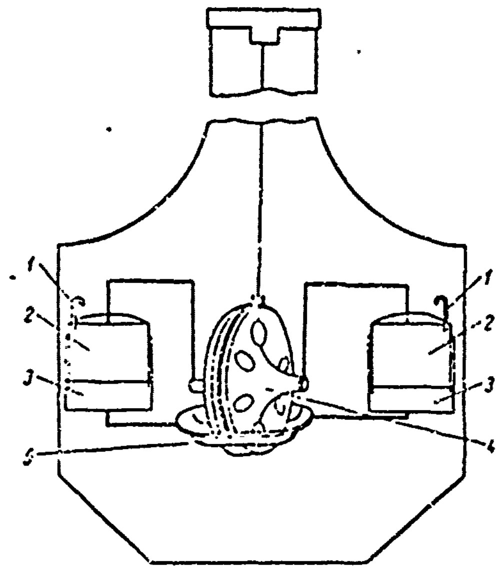

Fig. 43: Damping interior gimbal oscillation

Key:
1. Air jet nozzle
2. Upper cylinder
3. Lower cylinder
4. Interior gimbal (gyroscope motor housing)
5. Exterior gimbal

from which, proceeding from symbols to objects, we obtain a differential equation analogous to (18):

$$T^2\ddot{\alpha}_\beta + 2\xi T\dot{\alpha}_\beta + \alpha_\beta = \alpha_{\beta 0}, \tag{115}$$

where, from expression (38)

$$T^2 = \frac{H}{Gl\omega_r} = \frac{1}{\nu^2},$$

and

$$\xi = \frac{r_1}{2H}\sqrt{\frac{Gl}{H\omega_r}} = \frac{r_1}{2H\omega_r}\nu. \tag{116}$$

We determine the approximate value of damping coefficient c_1 for the type Gi-B gyrotheodolite if we use a magnitude for the relative damping coefficient $\xi=0.5$ (settling down of the oscillation with such a value of ξ will take place over approximately 13 min). From (116), taking (43) into account with a ЧЭ period of oscillations $T \approx 10$ min, we find

$$c_1 \approx 100 H\omega_z.$$

so that for latitude $\phi=60°$ and $H=4000$ g-cm-sec

$$c_1 \approx 15 \text{ Г·см·сек.}$$

In view of the small speed of movement of the gyrotheodolite ЧЭ, obtaining such a value of the damping coefficient is made more difficult by use of a pneumatic mechanism; therefore, liquid or electromagnetic damping is used. Detailed investigations of these forms of damping have shown [31, 40] that liquid damping leads to a significant displacement of the stopping position of the ЧЭ axis relative to the equilibrium position calculated by reversing points. One of the reasons for this displacement is the "sticking effect" (Klebeeffekt) of liquid in the ЧЭ housing, owing to which a moment, in some manner dependent on the angle turned by the sensitive element, begins to act on the gyroscope. This moment, like the moment due to twisting of the torsion element, introduces an error into the ЧЭ settled state.

Practical interest is shown in electromagnetic damping. During rotation of the magnetic field of a solid rotor which possesses good electrical conductivity an induction current arises as a consequence, which interacts with the magnetic field, as a result of which a force arises which opposes the movement of this rotor. This force depends on the design parameters and on the speed of movement of the rotor relative to the poles of the magnet. A damping scheme is shown in Fig. 44, a, in which sensitive element 2 is fastened to torsion element 1 and is joined to copper cylinder 7 through bar 9. Cylinder 7 rotates in an air clearance between permanent magnets 8, which are arranged in a ring. Many layered screen 6 is located above the copper cylinder; observation of the ЧЭ movement is conducted by use of an optical system, consisting of light source 5, mirror 4 and collimator telescope 3 with a scale. Experiments have been conducted with two magnetic systems (Fig. 44, b and c): in the first magnet system the arrangement of magnet poles alternates, and in the second their arrangement is uniform. The value of the inductance in the first system amounts to 3250 gauss, and in the second 4500 gauss. Results of experiments have shown that for both cases the deviation in establishment of the ЧЭ axis relative to the position calculated by reversing points does not exceed ±30"; however, the damping force in the first system is more substantial than in the second. Evidently, this is linked with the fact that in the first magnet system additional induction currents arise which are connected with the changing direction of the magnetic field, while in the second system this field remains in a constant direction. Fig. 45 illustrates the difference in the damping of the ЧЭ oscillations with both magnet systems: curve 1 is drawn for the magnetic system with alternating poles, and curve 2, for a system with uniform arrangement of poles.

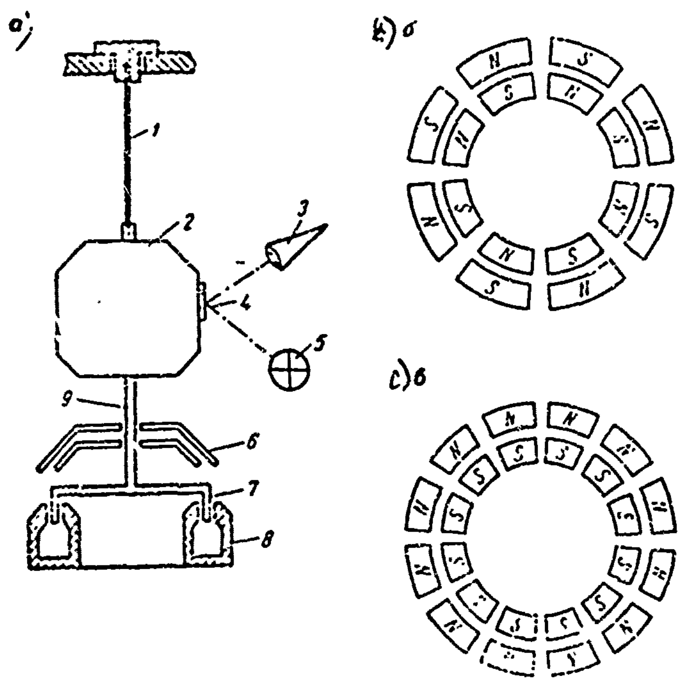

Fig. 44: Damping of azimuthal oscillations

Key:
a. Kinemat'` diagram
b. Magnet ring with alternating poles
c. Magnet ring with uniform pole arrangement

1. Torsion element
2. Sensitive element
3. Telescope
4. Sensitive element mirror
5. Light source
6. Magnetic protective screen
7. Copper cylinder
8. Magnet

Location of the magnet system near the gyroscope unit can't help but effect the equilibrium position of an oscillating system. The influence of the magnetic field of the damper leads to displacement of the zero point of the torsion element. Graphs of the change in zero point of a torsion element for different situations are given in Fig. 46: curve 1 -- magnetic screen absent, curve 2 -- a monolayer magnetic screen is used, and curve 3 -- drawn for the case of use of a three-layer screen. The displacement of the torsion element zero point (MNT) is set forth on the

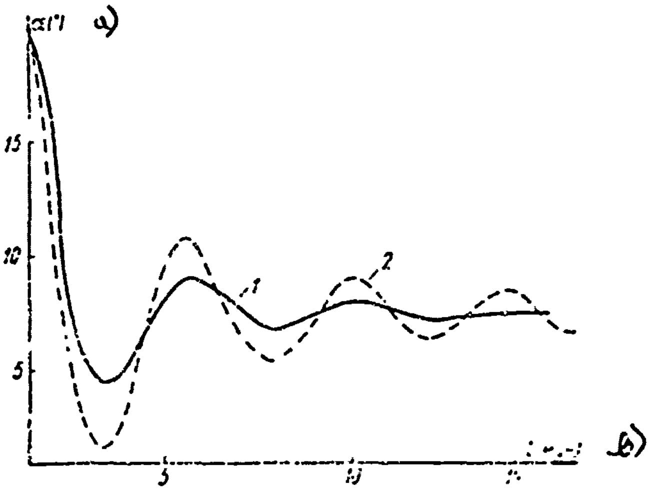

Fig. 45: Graphs of attenuation of azimuthal oscillations of a sensitive element

Key: a. Ordinate, angle turned by sensitive element in degrees
b. Abscissa, time in minutes

1. Graph for magnet system with alternating poles
2. Graph for magnet system with uniform pole arrangement

ordinate, and on the abscissa, the depth L to which the copper cylinder sinks in the air gap between the poles. A negative value of L means that the copper cylinder is located higher than the magnetic ring. Dependence of the displacement of the torsion element zero point on the depth to which the copper cylinder sinks in the interpole clearance is difficult to explain by a basic law of an ideal damping system. It is most likely that all of this is connected with the presence of ferromagnetic impurities in the copper, as well as the transfer of ferromagnetic dust from the body of the magnet to the copper cylinder, remaining after its manufacture.

An experimental model of gyroscope was used with kinetic moment H=2000 g-cm-sec, a magnet system of 6 magnets, a copper cylinder (purity of copper 99.9997%) with a diameter of 120 mm and thickness of 1 mm. The following data were obtained as a result of experiments:

-- time to quiet the ЧЭ oscillations -- 8 min;
-- mean error of the series of measurements -- ±40";
-- mean error of individual measurements -- ±70";
-- total time of measurement (including setting up, starting, stopping) -- 19 min.

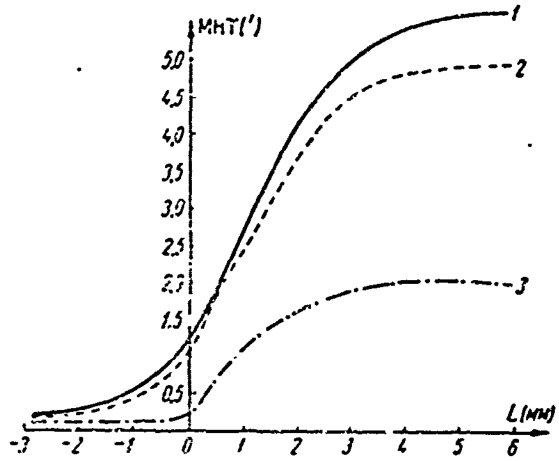

Fig. 46: Graphs of change in zero position of the torsion element

Key: 1. Without magnetic shield
2. One-layer magnetic shield
3. Three-layer magnetic shield

Damping of the oscillations of a gyrotheodolite sensitive element frees the operator from the necessity of conducting continuous observation of the gyroscope. Another feature of a damped gyrotheodolite is the possibility of carrying out the measurements on a mobile base. Conditions for operating a gyrotheodolite on a mobile base are its mounting on an automatic horizontal leveling platform and frequency correlations between linear acceleration of oscillations and the damping frequency of the oscillating system (see Section 2, Chapter I)

ω_H [illegible]

The frequency of a gyroscope system ω_H must be substantially lower than the frequency of change in linear acceleration and azimuthal turning of the base.

In conclusion, we return again to the structural diagram (see Fig. 42) and evaluate the influence of gyroscopic moment $H\omega_B$ on the azimuthal movement of a gyrotheodolite. Using this diagram, we obtain the following relation:

$$\alpha' = \frac{H^2 p^2}{(H^2 + c_1 c_2) p^2 + (c_1 Gl + c_2 H\omega_r) p + H\omega_r Gl} H\omega_B. \qquad (117)$$

The numerator of the fraction is an operator description of a double differentiating link. In view of the fact that gyroscopic moment $H\omega_B$ has a constant value, its product will equal zero and, consequently, angle α' will equal zero. This testifies to the fact that gyroscopic moment $H\omega_B$ does not introduce an additional azimuthal displacement of the gyrotheodolite measuring axis into the scheme under discussion.

Section 6. Automation of Gyrotheodolite Operations

Gyroscopic orientation is connected with the execution of a number of measuring and computing operations, such as orientation in the locality, preliminary orientation of the gyrotheodolite, determination of the equilibrium position, calculation of the geodetic correction, and calculation of the instrumental correction. These operations are distinguished by a great diversity, and automation of several of them is an independent, complex task of modern geodetic instrument making. They are concerned specifically with development of automatic systems for sighting and reading of angles in the horizontal and vertical planes [6]. Therefore, only questions connected with automation of direct measuring operations by the gyroscopic parts of gyrotheodolites are considered below. In the first place, automatic determination of the equilibrium position of ЧЭ oscillations should be considered. One of the versions of solution to these problems is damping of gyroscope oscillations. However, this version is connected with application of moments of external force to the gyroscope and, technically, to attain such a position that stabilized or constant components of the moments are completely absent from the composition of damping moments is a complicated problem. Therefore, damping methods known at present cannot be used in high-accuracy gyrotheodolites, for which the methods usually applied for calculation of equilibrium position using results of observation and measurement of the characteristics of sensitive element oscillation, are abandoned. A principal scheme of a mechanism for automatic calculation of the ЧЭ equilibrium position of the pendulum gyrotheodolite is given in Fig. 47.

The angular position of sensitive element 1, transmitted by use of mirror 2, is determined visually by using automatic collimator telescope 3 and automatically, by using photoelectric telescope 4 [24]. A linear-effect photodiode, which generates an electrical signal proportional to the angle turned by the light beam, is located in the focal plane of telescope 4 for transforming the light energy reflected by mirror 2. This signal exercises control over motor 5, which moves frame 6 and telescope 4, which is mounted on it. The system for moving telescope 4 is constructed in such a manner that, when the light beam reflected from mirror 2 coincides with the optical axis of telescope 4, movement is stopped. The moving part of electrical movement sensor 7, which is shown as a potentiometer in Fig. 47 a, is connected kinematically with telescope 4.

Inductive elements can be used as movement sensors, but a variety of potentiometers, especially with corrections, are quite useful for practical employment. In the latter case, accuracy of operation of the system under consideration is attained by scanning the angular amplitude of the gyrotheodolite sensitive element oscillation over the entire winding of the potentioneter. The output voltage of displacement sensor 7 is fed to in-

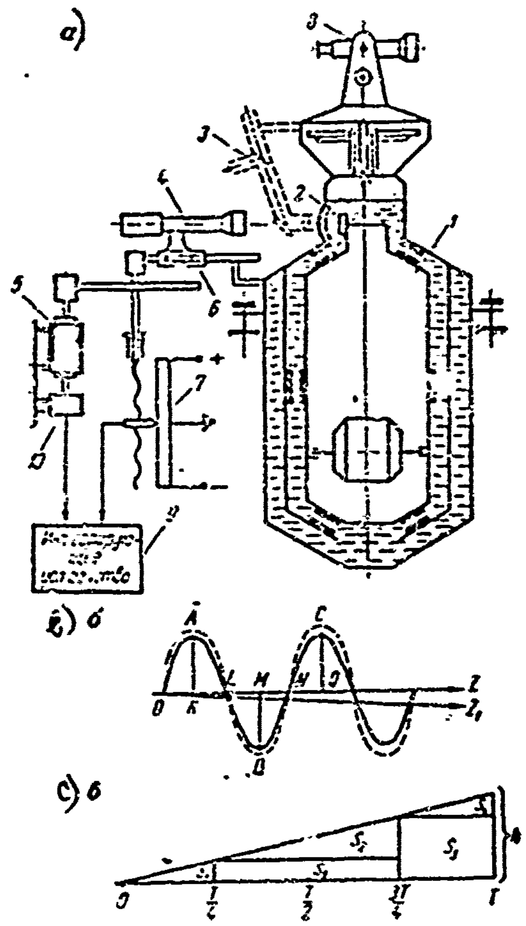

Fig. 47. Scheme for automatic calculation of a gyro-theodolite sensitive element oscillation equilibrium position:

Key: a) kinematic diagram
b) oscillations of a sensitive element
c) correlation of areas
1) sensitive element
2) sensitive element mirror
3) automatic collimator telescope
4) photoelectric automatic collimator telescope
5) motor
6) reducing gear
7) angle sensor
8) theodolite telescope
9) integration mechanism
10) reversing point indicator

tegrating mechanism 9. In view of the fact that the initial setting of theodolite 8 is approximate, it actually is necessary, in operations with the gyrotheodolite, to calculate exactly the error in orientation of theodolite telescope 8; therefore, the slider of potentiometer 7 is set in the position in which its output voltage equals zero. Then, as a result of integrating the voltage from the slider of potentiometer 7, while the gyroscope unit is operating, we will have a certain voltage value, which is proportional to the angular divergence between the 4B oscillation equilibrium position and the direction of the optical axis of theodolite telescope 8, that is, we will have a voltage proportional to the desired angle of divergence.

The dotted line shown in Fig. 47 b is a regular sinusoid, and the solid, a curve obtained from the results of readings with the instrument. If the gyrotheodolite is set up in some direction OZ_1 which does not coincide with the direction of a true medidian, but is close to it and, with the gyroscope motor cut in, only three reversing point readings are taken, the angle to which it is necessary to shift the goniometrical part of previously oriented theodolite 8 can be determined (for simplicity of discussion, we shall conditionally take the instrumental correction $\Delta=0$).

Direction OZ will coincide with the direction of the true meridian only in the event that area LNB is equal to the sum of areas AKL and NCQ (the areas necessary for comparison are determined by integration from readings of the three reversing points A, B, and C). If the equality indicated is not observed, OZ does not coincide with the direction of the true meridian. As long as point O is unchanged (this is the point where the instrument is standing), the difference among these areas:

$$S_{LNB}-(S_{AKL}+S_{NCQ})$$

depends simply on the deviation angle of the directions under consideration. This comparison of areas does not require special proof during undamped sinusoidal oscillations. However, it is easy to show that such a comparison of areas can be made during damped oscillations. The following argument is conducted toward such a conclusion.

Although the decrease in amplitude of oscillations during damping takes place according to the exponential law, the damping taking place in one period may confidently be taken as linear. In such a case, the difference between the area bounded by the undamped sinusoid and the areas bounded by the damped curve has a linear dependence (Fig. 47 c), where T is the period of oscillation of the sensitive element, $h=kT=\xi_s T$ -- see (77).

Let us prove that, if $S_1+S_4+S_5=S_2+S_3$, the damping of oscillations does not affect the difference in areas subject to comparison.

From Fig. 47 c , it follows that:

$$S_1=\frac{kT^2}{32}, \qquad S_1=S_4=\frac{kT^2}{32},$$

$$S_2=S_3=\frac{kT^2}{8}, \qquad S_5=\frac{3kT^2}{16}.$$

Substituting the values found for the areas, we obtain,

$$\frac{kT^2}{32} + \frac{kT^2}{32} + \frac{3kT^2}{16} = \frac{kT^2}{8} + \frac{kT^2}{8}$$

or

$$\frac{kT^2}{4} = \frac{kT^2}{4},$$

which was required to be proven.

In order for the integrator output current obtained to be proportional to the unknown angular correction, the switching on and switching off of the integrator must be accomplished at perfectly defined instants of time. Such instants are the reversing points; for example, the instant of switching on is point A, and the instant of switching off is point C (see Fig. 47 b).

Determination of the moments for turning the integrator on and off, that is, its control, is accomplished by mechanism 10 (see Fig. 47 a), which may be an electrical or a mechanical configuration. There will be a common tachogenerator in the electrical configuration of this mechanism, which will give a zero voltage value at the reversing point instant. The mechanical type is a small-moment, friction clutch with a moving electrical contact, which gives an electrical signal to integrator 9 at the instant motor 5 stops, which corresponds to the reversing point.

The mechanism under discussion for automatic calculation of the gyrotheodolite sensitive element equilibrium position permits automation of the measurement process for finding the direction of a geographical meridian using a gyrotheodolite. The accuracy of the calculating operation is improved here. Thus, photoelectric telescope 4 has a measurement error on the order of 1-2 angular seconds, at the same time as the visual method gives an error of 4-6 angular seconds. Determination of the instant of arrival of a gyrotheodolite at the reversing point in the mechanism under discussion is carried out with an accuracy of 0.7-1 angular seconds, while in the visual method it is several seconds. Use of such a mechanism is particularly advisable for gyrotheodolites with a small period of oscillation, in which it is difficult to insure the required accuracy of reading the reversing point by visual methods.

Some of the methods cited above for determination of the sensitive element oscillation equilibrium position may also be automated. In this regard, the simplest solutions to the problem may be found in methods of automation which use time as a measurement parameter, using a time-impulse reading method.

In finding the equilibrium position and setting it on the limb of the theodolite, it is necessary that this result be transmitted for further processing.

In this case, the best solution may be to provide coding systems, the use of which makes it advisable for all calculations to be carried out by a digital computer, into which the constant correction for a given location of gyrotheodolite can be introduced beforehand. The result obtained from the calculator is a value of the angle to which the telescope must be shifted in order for the angular measurements on the terrain to be the directional angles. If the connection between the gyrotheodolite and the digital computer is not broken off, the latter can check the observers' measurement operations and then print out all angles sighted in cipher form. The computer in inertial surveying equipment [58] solves such a volume of calculations in an exemplary manner.

APPENDIX I

Rating Plate Data for the Gi-B2 Gyrotheodolite

Technical characteristics

A. Gyrotheodolite

Telescope

1. Objective free aperture	45 mm
2. Exit pupil diameter	1.5 mm
3. Magnification of the telescope	30 x
4. Field of view	1°12'
5. Resolution	3.5"
6. Distance measuring coefficient	100
7. Focal length	240 mm
8. Near focusing limit	2 mm

Limbs	Horiz	Vert.
1. Scale diameter	132 mm	68 mm
2. Value of a division	20'	20'
3. Reading microscope magnification	27 x	42 x
4. Value of a micrometer division	1"	1"
5. Micrometer "Ren"	1"	1"
6. Mean square error of a division on the horizontal limb	1"	

Levels

1. Value of a division on the horizontal ring level	16"
2. Dimensions of the horizontal ring level	ϕ 12 x 52 mm
3. Value of a division on the vertical ring level	20"
4. Dimensions of the vertical ring level	ϕ 12 x 52 mm

Gyrotheodolite electrical illumination

1. Automatic collimator illumination	
Lamp type designation	2674
Lamp voltage	12 v
Lamp power	3 watts

2.	Limb illumination	
	Lamp type designation	7563
	Lamp voltage	12 v
	Lamp power	3 watts
3.	Tracking system illumination	
	Lamp type designation	7563
	Lamp voltage	12 v
	Lamp power	3 watts

B. Optical plumbline with circular compass

1.	Optical plumbline depth of field	from 0.5 m to ∞
2.	Optical plumbline telescope magnification	4.25 x
3.	Telescope field of view	6°20'
4.	Accuracy of centering above a point	1 mm
5.	Value of a circular compass division	1°
6.	Precision of compass return	6'
7.	Value of a circular level division	6'/2 mm
8.	Dimensions of circular level	15 x 6 mm
9.	Value of a cylindrical level division	1.5 mm
10.	Dimensions of cylindrical level	10 x 44

C. Electrified illuminated marker

1.	Mean sighting error (with Gi-B2 telescope)	
	By day, from 3 m to 3000 m	±2"
	By night, from 3 m to 6000 m	±1.8"
2.	Lamp type designation	7575
	Lamp voltage	12 v
	Lamp power	5 watts
3.	Greatest eccentricity of the illuminated marker relative to the theodolite vertical axis	0.2 mm
4.	Height of the mark above the theodolite horizontal axis	300 mm
5.	Illuminated marker weight	0.25 kg

D. Semi-conductor triode transformer

1.	Power supply voltage	12 v ±1 v
2.	Power requirements (excluding thermostat and illumination)	
	When starting (2 min)	60 watts
	In operation	13 watts
	When stopping (over 1.8 min)	6 watts

3. Power output

When starting	25 watts
During operation	8 watts

4. Operating voltage output — 3 x 30.5 v ±0.12 v/30 min
5. Operating frequency — 416 hz ± 3.2 x 10^{-5}hz/°C
6. Efficiency factor — 65%
7. Installed thermostat power requirement at surrounding temperature of -40°C — 20 v
8. Energy requirement for one measurement (by hand tracking) — 1.2 ampere hrs
9. Tracking system power requirement — 0.2 ampere hrs
10. Transformer electrical illumination

Lamp type designation	7563	2692	6984
Lamp voltage	12 v	12 v	12 v
Lamp power	3 watts	1.2watts	1.2 watts

E. Weight data

1.	Gyrotheodolite (less sensitive element)	9.5 kg
2.	Sensitive element weight	7 kg
3.	Weight of gyrotheodolite together with case	32.5 kg
4.	Reserve sensitive element with case	12.5 kg
5.	Accessories in chest	16 kg
6.	Tripod weight	7.5 kg
7.	Tripod together with cover	12.5 kg
8.	Three stakes with caps and covers	6 kg
9.	Transformer weight	13.5 kg
10.	Transformer in case	21 kg

Gyrotheodolite Accessories and Spare Parts

No.	Name	Quantity
	Gyrotheodolite in case	
1	Instrument No. 00000	1 each
2	Moisture absorbing unit	2 "
3	Covering made of synthetic material	2 "
4	Screwdriver	1 "
5	Objective cap	1 "
6	Certificate	1 cy
7	Spare gyroscope	3 ea
8	Tripod	1 "
9	Cable	2 "
	Accessories in case	
10	Operating instructions	1 cy
11	Diagram of fundamentals	
12	Assembly diagram	
13	Log book	1 ea
	Upper shelf	
14	Optical centering device No. 000	1 "
15	Electrified illuminated marker	1 "
16	Middle shelf	1 "
17	Thermometer	1 "
18	Screwdriver	2 "
19	Brush	1 "
20	Wrench 12/9	1 "
21	Storage battery cleaning appliance	1 "
22	Rubber lens hood for automatic collimator	1 "
23	Spare compass needles	3 "
24	Blade case	2 "
25	Blades	8 "
26	Pins	2 "
27	Ocular prism	2 "
28	Spare light bulbs	20 "
29	Flannel	1 "
30	Plumb with cord	1 "
31	Lubricator	1 "
32	Pocket flashlight	1 "
33	Hammer with cord	1 "
34	Rubber lens hood for eyepiece	1 "
35	Automatic collimator illuminator housing	1 "
36	Solar lens hood	1 "
37	Daytime illuminating device	1 "

38	Rubber brake membrane	2 ea
39	Tweezers	1 "
40	Flat broaching file	1 "
41	General purpose pincers	1 "
42	Screws	10 "
43	Elastic	10 "
44	Illuminated marker lightbulbs	4 "
45	Lower shelf	
46	Carriage	1 "
47	Plumb device	1 "
48	Transformer in case	1 "
49	Transformer	1 "
50	Storage battery clamps on cable	2 "
51	Transformer lamps	30 "
52	0.7a fuse	10 "
53	5a fuse	10 "

Results of Laboratory and Field Trials
Conducted by the Manufacturing Plant

1. Theodolite part constant (Δ_1) and its changes

No. 000 000	Date	Time	Magnitude of (change) Δ_1
Change in Δ_1 due to shaking	11/18	10 hr 00 min	0"
Change in Δ_1 due to refrigerating to -30°C	11/18	14 hr 00 min	+3"
Magnitude of Δ_1 after an operational sequence consisting of 9 operations	11/27	16 hr 00 min	90°05'52"

2. Sensitive element constant (Δ_2) and its changes

No. 000 000	Date	Time	Magnitude of (change) Δ_2
Change in Δ_2 due to shaking in two operational sequences consisting of 6 operations each	11/18	10 hr 00 min	0"
Change in Δ_2 due to refrigerating to -30°C in operational sequences consisting of 6 and 9 operations	11/18	14 hr 00 min	12"
Magnitude of Δ_2 in an operational sequence consisting of 9 operations	11/27	16 hr 00 min	359°59'12"

3. Gyrotheodolite constant $\Delta = \Delta_1 + \Delta_2$, its changes and mean square error in its determination

No. 000 000	Magnitude (change)	Mean square error Δ	
		Before test	After test
Change in Δ due to shaking in 2 operational sequences consisting of 6 measurements each	0"	±3.3"	±0.0"
Change in Δ due to refrigerating to -30°C in operational sequences consisting of 6 and 9 operations	+2"	±4"	±5.4"
Magnitude of Δ in an operational sequence consisting of 9 operations	89°56'04"		±5.4"

4. Test of zero point slippage (test of free oscillations in the course of 30 min).

No. 000 000	Slippage	Magnitude of slippage (in scale units)
	+	0.4

5. Gyroscope motor heating test (slippage of average values of oscillations, excited by the motor in the course of 2 hrs)

No. 000 000	Slippage	Magnitude of slippage
	±	4.9

6. Errors in angle measurement with the theodolite part

Magnitude of greatest deviation from the mark		Mean square error
±1.7"	-1.9"	±1.7"

Automatic Tracking System Characteristics

1. Tracking error in the case of an optimal magnitude of amplification:
   At -40°C and amplitude of oscillations ±30°, not exceeding ±30",
   At -20°C and amplitude of oscillations ±10°, not exceeding ±15".
2. Reproduction error in automatic tracking system, 10".
3. Sensitivity of photoresistance at t°C+20°C.

	Resistance in the dark	Resistance under illumination at 200 lux
Right	10 megohm	15 kg om[1]
Left	10 megohm	14 kg om[1]

4. Change in the output voltage depending on the angle turned by the sensitive element

α	-3'00"	-1'30"	0'00"	±1'30"	+3'00"
U_{5-14}[1]	+0 v	+2.7 v	0 v	-0 v	-0 v

1 [Terms not known.]

Certificate for Gi-B2 Gyrotheodolite

Theodolite
Theodolite number ....No. 000 000
Theodolite correction constant........ Δ_1=90°05'52"
Gyroscope assembly
Sensitive element number .... No. 000 000
Sensitive element correction
constant .... Δ_2=359°50'12"
Gyrotheodolite correction
constant (Fig. 48)................... $\Delta = \Delta_1 + \Delta_2$=90°05'52"+359°50'12"=$\Delta$ =89°56'04"
Free oscillation period .... T=1 min 18 sec

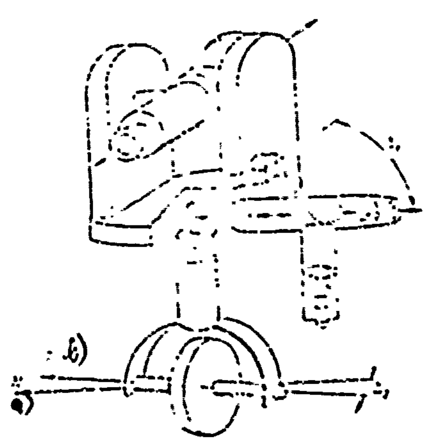

Fig. 48: Diagram representing the instrumental correction of a gyrotheodolite
Key: a) north b) gyroscope moment vector

Coefficient "C" (in angular sec)

(always negative)

a) Географическая широта	b) Период свободных колебаний (сек)										
	64	65	66	67	68	69	70	71	72	73	74
30°	5,95	5,77	5,60	5,43	5,27	5,12	4,98	4,81	4,70	4,58	4,45
35	6,29	6,10	5,92	5,74	5,58	5,41	5,26	5,11	4,97	4,84	4,71
40	6,73	6,52	6,33	6,14	5,96	5,79	5,63	5,47	5,32	[illegible]	[illegible]
45	7,29	7,07	6,86	6,65	6,46	6,27	6,10	5,92	5,76	[illegible]	[illegible]
50	8,02	7,78	7,51	7,32	7,11	6,90	6,70	6,52	6,31	6,17	[illegible]
55	8,99	8,71	8,45	8,20	7,96	7,73	7,51	7,30	7,10	6,91	6,72
60	10,31	10,00	9,70	9,41	9,13	8,87	8,62	8,38	8,15	7,93	7,71
65	12,20	11,83	11,47	11,13	10,81	10,50	10,20	9,91	9,61	9,38	[illegible]
70	15,07	14,61	14,17	13,75	13,35	12,97	12,60	12,25	11,91	11,59	11,28

Key: a) Geographic latitude
b) Period of free oscillations (sec)

a) Географическая широта	b) Период свободных колебаний (сек)										
	75	76	77	78	79	80	81	82	83	84	85
30°	4,31	4,22	4,11	4,01	3,91	3,81	3,72	3,63	3,54	3,45	[illegible]
35	4,58	[illegible]	4,35	4,21	4,13	4,03	3,93	3,83	3,74	[illegible]	[illegible]
40	[illegible]	4,77	4,65	4,53	4,42	4,31	4,20	4,10	[illegible]	3,91	3,82
45	5,31	5,17	[illegible]	4,91	[illegible]	4,67	4,55	[illegible]	[illegible]	4,23	[illegible]
50	[illegible]	[illegible]	[illegible]	5,30	5,26	5,13	5,01	4,89	[illegible]	[illegible]	[illegible]
55	6,55	6,37	6,21	6,05	[illegible]	5,75	5,61	5,48	[illegible]	5,22	[illegible]
60	[illegible]	[illegible]	7,12	6,91	6,77	6,60	6,41	6,28	6,13	[illegible]	[illegible]
65	[illegible]	8,65	8,43	8,21	8,01	7,81	7,62	7,43	7,25	7,08	6,92
70	10,98	10,59	10,41	10,15	9,89	9,65	9,41	9,18	8,96	8,75	[illegible]

Key: [same as above]

APPENDIX 2

Table of Gyrotheodolites

Explanation of the table:

1. Instrument , code number of which is enclosed in a rectangular frame, has explosion-proof configuration.
2. Characteristics denoted by an asterisk symbol (*) are of an orienting nature.
3. The accuracy indicated in the table corresponds to the certificate data (factual accuracy, obtained in several investigations and published, may be a little higher).

Pendulum gyrotheodolites with supporting liquids

Technical characteristics	М-3	МW-2В	МW-3	МУГ-3 [1]	МW-1а
Date of manufacture	1951	1951		1954	1959
Country	[2] СССР	[3] ФРГ	[3] ФРГ	[2] СССР	[3] ФРГ
Accuracy (angular sec)	60	60		90	10
Period of oscillations ($\phi \approx 60°$) (min)	30	30		30	
Weight data on complete sets (kg)	500	640		250	
Form of sensitive element	Gyroscopic sphere				
Method of centering	Electromagnetic				
Dimensions of the torsion element (mm)					
Kinetic moment (g-cm sec)$\cdot 10^3$	115	115		30	
Electric current supply	Through liquids				
Footnotes					1

[Continuation of table from previous page]

МзВ	МВ-1	4) МГ	5) АГ	МВ2М	МВ-2	MW-63	[illegible]	[illegible]
1956	1957	1955	1959		1963	1958	1961	1962
3) ФРГ	2) СССР	2) СССР	2) СССР	2) СССР	2) СССР	3) ФРГ	3) ФРГ	3) ФРГ
25	45	80	60	35	35	40	30	15
30*	14	18	12	12,5	14	11*		7*
560	165	65	93	55	175	77	60*	75
	Cylinder					Cylinder		
	On a pivot					On a torsion element		
								0,25 - 0,035 - 150
	24	4	4	11,0	24			3,7
Through liquids								
2								

Key: 1) MUG-3
2) USSR
3) Federal Republic of Germany
4) MG
5) AG

1 The optical telescope is common to observation of the ЧЗ and terrain features.

2 A pneumatic gyroscope motor is used for explosion proving.

Pendulum gyrotheodolites with torsion suspension in air

Technical characteristics	KT-1	LMK-604	GI-B1	MT-1
Date of manufacture	1959	1930	1963	1363
Country	[2] ФРГ	[3] США-[3] ФРГ	[4] ВНР	[5] СССР
Accuracy (angular sec)	20	10	20	10
Period of oscillation (min) (at a latitude 60°)	6	9	11,5	11
Weight data for complete set (kg)	[illegible]	60 *	65	57
Dimensions of torsion element (mm)	0,62 * 0,03 * 150	0,62 * 0,03 * 150	0,37 0,05 150	
Kinetic moment (g-cm sec)·10^3	3,7	2	4,3	10, 23
Type of system for tracking twisting of twisting of the torsion element	—	[7] Э/механическая	—	[8] Фотоэлектрическая
Weight of oscillating system (kg)	1,5	1,5	1,25	
Electric power supply	100 в = V 400 гц = hz 80 мА = milliamp.		30 в = V 416 гц = hz	
Number of revolutions (rpm)·10^3	24	24	23	
Electric current feed			[10] Ленточные спиральные	
Footnotes				

1 In determining the equilibrium position by the "passing" method,(while the alidade is fixed), the period of oscillations is 2.5-3.5 min; there is a mechanism for ascertaining the amplitude of oscillations of the ЧЭ by the liquid short time damping method.

2 The measurement is accomplished with two rotor positions and, therefore, does not require introduction of the instrumental correction.

3 Manufactured by Fennel Co. for military purposes (Kleingerät) [43].

4 12 min--total time of measurement.

[Continuation of table from previous page]

МБК-1	КТ-1А	МБК-2	ГБ-Б2	КТ-2	1) [illegible] «ГИМРАДА»
1953	1964	1955	1962	197[illegible] *	
6) ГДР	2) ФРГ	6) ГДР	4) ВНР	2) ФРГ	3) США
20	20	20	10		[illegible]
8	6	6,5	10,5		12
70	70	62	62		[illegible]
0,5 * 0,05 * 110	0,62 0,08 150	0,5 0,05 110	0,[illegible] 0,05 150		
1,7	3,7	2	4,3		
—	—	—	8) Фотоэлектрическая	—	9) [illegible]
0,8	1,5	1,50	1,25		
36 сек 400 гц		35 сек 100 гц	30 416		
23	21	21	21		
11) Ртутные	12) Ленточные	11) Ртутные	10) Ленточные спиральные		
1	2	1		3	4

Key: 1) developed by GIMRADA
2) Federal Republic of Germany
3) USA
4) Hungarian People's Republic
5) USSR
6) German Democratic Republic
7) electromechanical
8) photoelectric
9) electrical damping until oscillations are quieted down
10) spiral ribbon
11) mercury 12) ribbon

Gyroscopic attachments (to a theodolyte) with torsion suspension of the ЧЭ

Technical characteristics	TK-2	TK-3	OAK-1	GI-C1	GI-D1	TK-4
Date of manufacture	1961	1963	1963	1965	1965	1965
Country	1) ФРГ	1) ФРГ	2) Швейц.	3) ВНР	3) ВНР	1) ФРГ
Accuracy (angular sec)	60	30	25	30	60	30
Period of oscillation at latitude $\phi \approx 60°$ (min)	6	8	8	7	6	8
Weight of gyroscope attachment (kg)	2		1,8			
Dimensions of torsion element (mm)			0,4 0,02	0,4 0,05	0,4 0,05	
Kinetic moment (g-cm-sec)$\cdot 10^3$		2	2			1,2
Oscillation damping	—	4) Воздушное				
Number of revolutions of the rotor (rpm)$\cdot 10^3$		21	24	22		
Electric power supply		115 в (v) 399,99 гц (hz)	115 в (v) 400 гц (hz)	34 в (v) 410 гц (hz)	34 в (v) 300 гц (hz)	
Weight of oscillating system (kg)		0,55	0,55			
Electric power supply				10) Ленточные спиральные		
Footnotes		1	1			1

1 There is a mechanical braking system for establishing small amplitudes of oscillation of the ЧЭ.

2 A double image prism is used in the optical telescope.

3 The time for complete measurement is shown instead of the period of oscillation; ЧЭ oscillation is quickly damped and the gyroscope spin axis is brought to the rest position.

4 Gyroscope assembly is located under the theodolite (gyroscope attachment).

5 Cylindrical ЧЭ in a supporting liquid, centered on a pivot.

[Continuation of table from previous page]

GI-C2	GI-D2	TK-5	ARK-4	Girolit	Girolit II	NSK	TKW	МВТ2 MVT2	[illegible] MVSh3
1966	1966	1966	1966	1960	[illegible]		1957	[illegible]	
3) ВНР	3) ВНР	1) ФРГ	2) Швейц.	1) ФРГ	1) ФРГ	4) Бельгия	1) ФРГ	5) СССР	
20	20	10	20	180	160	60		45	[illegible]
7	6		8	15	15	19		7	11
								3,5	4,5
0,4 0,02	0,4 0,02		0,4 0,02						—
4,3			2					1,1	[illegible],0
				7) Жидкостные	8) Магнитные	9) Следящей системой			
			22						
31 в (V), 410 гц (hz)	31 в (V), 300 гц (hz)		115 в (V), [illegible] гц (hz)						
			0,55						
10) Ленточные спиральные									
2	2			3	3	3		4	5

Key:
1) Fed. Rep. of Germany
2) Switzerland
3) Hungarian People's Republic
4) Belgium
5) USSR
6) air
7) liquid
8) magnetic
9) with tracking system
10) spiral ribbon

Two-degrees-of-freedom gyrotheodolites and other instruments, for determining the direction of a meridian

Technical characteristics	1) „Эйбл.“ (Able)	2) „Ориентир“	GYTAR	3) Малый гироскоп	ПИМ (PIM)
Date of manufacture	1958	1960	1962 *	1963 *	1959
Country	5) США	6) США	7) США	8) США	9) Англия
Accuracy (angular sec)	120	60	20	10	15
Time for one measurement (min)	30	30	20	10	20
Weight data (kg)	50	70	55	9.5	
Support Characteristics	14) Стальной керн на сапфировом подшипн				15) Оси из корунда в сапфировом подшип.
Kinetic moment (g-cm sec)·10^3					2
Damping					20) Жидкостное
Footnotes					

1 References to literature in the book are given in square brackets; numerical values for the instruments are calculated.

[Continuation of table from previous page]

4) ЛИТМО	[11]	[23]	[35]	КАТ (CAT)	5) Вибро-гиро	6) КД	7) ТКД
1960 *	1965	1965				1965	[illegible]
10) СССР	10) СССР	10) СССР	11) Италия	8) США	8) США	12) ФРГ	12) ФРГ
120	30	15	15	20		13) Гироскоп типа КТ-1	ТК-3
30	15	8	8	7			
						21) Прибор для измерения магнитного склонения	
16) Воздушное	17) Торсионное вектор $\overline{H}$ вертикально	18) Торсионное	17) Торсионное вектор $\overline{H}$ вертикально			19) Точность 30"	
110	4	4	4				
	20) Жидкостное						
	1	1	1				

Key: 1) "Eybl" 2) "Orientor" 3) small gyroscope 4) LITMO 5) Vibrogiro 6) KD 7) TKD 8) USA 9) England 10) USSR 11) Italy 12) Fed. Rep. of Germany 13) Gyroscope model 14) steel core on sapphire bearing 15) corundum axis on sapphire bearing 16) air 17) vertical torsion vector $\bar{H}$ 18) torsion suspension 19) accuracy, 30" 20) liquid 21) device for measuring magnetic declination

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[1] [Term unknown.]

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13. ABSTRACT

An attempt to systematize the uncoordinated information in gyrotheodolites and methods of working with them is offered. In setting forth the principle of operation and the functional connections of the various gyrotheodolite system the physical processes taking place in the instrument are concentrated on. Together with the above the first chapter presents information on the theory of gyroscopes. Also discussed are gyrotheodolites in the research stage. Part of the book is devoted to measurements with gyrotheodolites, methods of appraising the quality of the work of the instrument and its errors. Considered are questions connected with automation of measurement, distant transmission of the results of measurements, and the possibility of constructing these instruments with aperiodic transitional processes and without introduction of a constant correction.

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KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Gyroscope						
Gyrotheodolite						
Gyroscope System						
Gyroscope Component						
Gyroscope Motion						
Gyroscope Stability						
Subject Code: 17						
Country Code: UR						

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