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SOUND ATTENUATION IN MARINE SEDIMENTS

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13. ABSTRACT

In situ measurements of compressional (sound) velocity and attenuation were made in the sea floor off San Diego in water depths between 4 and 1100 m; frequencies were between 3.5 and 100 kHz. Sediment types ranged from coarse sand to clayey silt. These measurements, and others from the literature, allowed analyses of the relationship between attenuation and frequency, relationships between attenuation and other physical properties, and study of appropriate viscoelastic models which can be applied to saturated sediments. Some conclusions are: (1) attenuation in dB/unit length is approximately dependent on the first power of frequency; (2) velocity dispersion is negligible, or absent, in water-saturated sediments; (3) intergrain friction appears to be, by far, the dominant cause of wave-energy damping in marine sediments; viscous losses due to relative movement of pore water and mineral structure are probably negligible; (4) a particular viscoelastic model (and concomitant equations) is recommended; the model appears to apply to both water-saturated rocks and sediments; and (5) a method is derived which allows prediction of compressional-wave attenuation, given sediment mean grain size or porosity.

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Viscoelastic models for sediments						
Causes of attenuation in sediments						
Prediction of attenuation in sediments						

SUMMARY

PROBLEM

Determine and study those characteristics of the sea floor that affect the propagation of acoustic energy in the sea. Support underwater acoustics experiments and theory by furnishing information on the mass physical properties of sea-floor sediments; specifically, study the relationships between frequency and sound attenuation, furnish values of attenuation in specific sediment types, and develop a method to predict attenuation at any frequency, given common sediment properties.

RESULTS

In situ measurements of sound velocity and attenuation were made in the sea floor off San Diego from the research submersible *Deepstar*, and by scuba diving in shallow water, during the period 1966-70. Measurements were made at 3.5, 7, and 14 kHz. Additional measurements are reported from the 1962 *Trieste* program. These measurements, and others from the literature, allowed analysis of the relationship between attenuation and frequency, and study of appropriate viscoelastic models which can be applied to saturated sediments. An understanding of the relationships between attenuation and other physical properties, such as grain size and porosity, allows study of the causes of attenuation, upon which predictions can be based.

The following are important conclusions concerning sound attenuation and related properties of sea-floor sediments.

- A method is derived which allows prediction of attenuation, given sediment mean grain size or porosity.
- Attenuation is related, approximately, to the first power of frequency, that is, in the equation, $\alpha = kf^n$, n is about 1, where α is attenuation of compressional (sound) waves in decibels per meter, k is a constant which applies to a given sediment type, f, the frequency in kilohertz, and n is the exponent of frequency.
- There is no significant dependence of sound velocity on frequency ("velocity dispersion") from a few hertz to the megahertz range.
- A viscoelastic model is recommended in which the Lamé elastic moduli, μ and λ, are replaced by complex moduli $(\mu + i\mu')$ and $(\lambda + i\lambda')$, in which μ, λ, and density govern wave velocity and $i\mu'$ and $i\lambda'$ govern energy damping. In this

model, after eliminating negligible factors, energy damping (as expressed by the quality factor, Q, the specific attenuation factor, $1/Q$, or the logarithmic decrement) is independent of frequency (in the range of most interest in underwater acoustics), and linear attenuation (expressed, for example, in decibels per meter) is proportional to the first power of frequency, and velocity dispersion is negligible or absent.

- Energy losses due to intergrain friction appear to be, by far, the dominant cause of attenuation in water-saturated sediments. Viscous losses due to relative movement of pore water and mineral frame are probably negligible.

RECOMMENDATIONS

Additional *in situ* measurements should be made of attenuation in sea-floor sediments, in various sedimentary environments, and at low frequencies (a few hertz to 5 kHz). Such measurements will allow (at frequencies of most interest): (1) better definition of the relations between attenuation and frequency; (2) improvement of the method, proposed herein, of prediction of attenuation; and (3) determination of environmental differences, if any, of attenuation in various common sediment types.

The gradients of attenuation with depth in thick sediment sections and the filter-effects of various sediment layers should be measured and studied.

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INTRODUCTION

In the sea floor off San Diego, during the period 1966 to 1970, *in situ* measurements were made of the attenuation of compressional (sound) waves in marine sediments. Measurements in deep water were made from aboard a deep-diving submersible. In shallow water, scuba divers were used. In this program, probes were inserted into the sea floor and compressional-wave velocity and attenuation (hereafter called "velocity" and "attenuation") were measured at three frequencies (3.5, 7, and 14 kHz) without removing the probes from the sediment. This report also includes a few measurements at 25 kHz from the bathyscaph *Trieste* program in 1962, and re-evaluated measurements of attenuation at 100 kHz from an earlier scuba diving program (Ref. 1).

In situ measurements from the above programs and other laboratory and *in situ* measurements of attenuation found in the literature allow study of the dependence of attenuation on frequency and dispersion (if any) of compressional-wave velocity.

The relationships between frequency and compressional-wave velocity and attenuation have important implications in forming parameters for permissible theoretical elastic or viscoelastic models for water-saturated, porous sediments.

The objectives of this study are to: (1) report *in situ* measured values of velocity, attenuation, and associated physical properties (*i.e.*, density, porosity, grain size) of marine sediments; (2) assemble and analyze pertinent literature data on attenuation; (3) discuss the relationships between frequency, velocity, attenuation, and other physical properties; (4) discuss the causes of attenuation in saturated sediments; (5) discuss elastic and viscoelastic models which can be applied to marine sediments; and (6) suggest a method to predict attenuation in marine sediments, given frequency and common physical properties, such as porosity and grain size.

EXPERIMENTAL FINDINGS

INTRODUCTION

Three sets of *in situ* measurements (made by the writer) of the attenuation of compressional waves are listed in Tables 1 and 2 and plotted versus frequency in Figs. 1 and 2. In addition, some *in situ* and laboratory measurements from the literature are listed in Tables 3 and 4, and plotted in Fig. 2. For comparison and later discussions, all values are listed (many re-computed) in decibels per meter. The equations showing dependence of attenuation on frequency are listed in the form

$$\alpha = kf^n \tag{1}$$

Table 1. *In Situ* Attenuation and Velocity, and Other Physical Properties of Sediments Off San Diego: A. 1966–1970 Program, B. 1962 Program.*

Sediment Type	No. of Stations†	Water Depth (m)	Mean Grain Diameter		Density (gm/cm³)	Porosity (%)	Velocity (m/sec)	Attenuation (dB/m) at f (kHz)			k‡	n‡
			φ	mm				3.5	7.0	14.0		
A. 1966–1970 Program												
Sand												
Coarse	1a	32	0.81	0.5704	2.060	38.0	1817	...	3.4	6.6	0.53 ...	0.96 ...
Medium	2a	20	1.02	0.4931	2.008	39.2	1798	1.5	3.8	6.8	0.41 ± 0.12	1.09 ± 0.14
Fine	4a	8	2.55	0.1708	1.967	45.6	1686	1.7	3.2	7.2	0.45 ± 0.07	1.04 ± 0.07
Very fine	1a	13	3.30	0.1015	1.933	47.0	1708	1.5	3.5	7.0	0.38 ± 0.05	1.11 ± 0.06
Sand-silt-clay	1a	4	6.27	0.0130	1.512	72.3	1483	...	...	0.7	... ...	... ...
Clayey silt	1b	1030	7.52	0.0055	1.270	83.8	1453	...	...	1.0	... ...	... ...
Clayey silt	1b	1110	7.64	0.0050	1.300	82.5	1457	...	...	0.6	... ...	... ...
Clayey silt	1b	1087	7.42	0.0058	1.374	77.8	1459	...	1.2	2.3	0.19 ...	0.94 ...
Clayey silt	1b	1012	7.39	0.0060	1.349	79.1	1441	...	...	2.4	... ...	... ...
B. 1962 Program									f = 25 kHz			
Fine sand	3a	9	2.08	0.2365	1.973	44.0	1645		9.4		...	...
Sandy silt	1a	20	5.13	0.0286	1.702	60.9	1572		8.0		...	...
Clayey silt	1c	951	7.50	0.0055	1.380	78.9	1449		4.0		...	...

*Averaged values for number of stations indicated.

†Letter after number of stations indicates station occupied by: a-scuba drivers, b-*Deepstar* 4000, or c-bathyscaph *Trieste*.

‡In $\alpha = kf^n$ where attenuation, α, is in dB/m; frequency, f, is in kHz; k is a constant; and n is the exponent of frequency.

Table 2. *In Situ* Attenuation, Velocity, and Other Physical Properties of Sediments Off San Diego; 1956 Program.*

Sediment Type	No. of Stations	Water Depth (m)	Mean Grain Diameter φ	Mean Grain Diameter mm	Density (gm/cm^3)	Porosity (%)	Velocity (m/sec)	Attenuation (dB/m) at 100 kHz
Sand								
Coarse	1	20	0.79	0.5783	2.080	38.3	1752	53.1
Medium	1	19	1.99	0.2517	2.000	40.9	1630	47.3
Fine	17	10	2.46	0.1817	1.926	46.7	1684	52.1
Very fine	4	16	3.16	0.1119	1.938	47.4	1667	55.9
Sandy silt	2	13	4.23	0.0533	1.860	51.2	1619	74.3
Silty sand	2	17	5.07	0.0298	1.680	61.3	1537	60.9
Silt	1	15	5.53	0.0216	1.690	60.9	1465	15.9
Sand-silt-clay	1	17	5.05	0.0302	1.721	58.0	1490	59.7
Clayey silt	1	22	6.10	0.0146	1.600	65.6	1464	18.0

*Averaged values for number of stations indicated. All stations occupied by scuba divers.

where

α is attenuation of compressional waves, db/m

k is a constant

f is frequency, kHz

n is the exponent of frequency

IN SITU MEASUREMENTS

1966-1970 Program.

The equipment and methods used to make *in situ* measurements of velocity and attenuation from a submersible have been described and diagrammed in a previous report (Ref. 2). In general, the equipment consisted of three stainless-steel probes, 7 cm in diameter, fastened to a 2-m-long, rigid beam in such a manner that when the beam is on the sea floor, the probes are inserted a variable, pre-set depth into the sediment. This depth varied from 30 to 60 cm during the several submersible dives. Three barium-titanate transducers were used, one as a sound source and two as receivers. Velocity was determined by measuring travel time over a 1-m path between the receivers. Attenuation was measured in decibels relative to that in the bottom water (assumed to be zero for 1 m). Both velocity and attenuation were measured at 3.5, 7, and 14 kHz without disturbing the probes.* Coring tubes attached to each end of the rigid beam obtained sediment samples for laboratory analyses. Sediment properties were obtained by standard laboratory procedures. Variations and inter-relationships of these properties as well as sediment nomenclature were discussed in Ref. 3.

* The submersible *Deepstar* 4000, from which some of the measurements were made, could not always insert the probes to maximum penetration required to obtain measurements at 3.5 kHz.

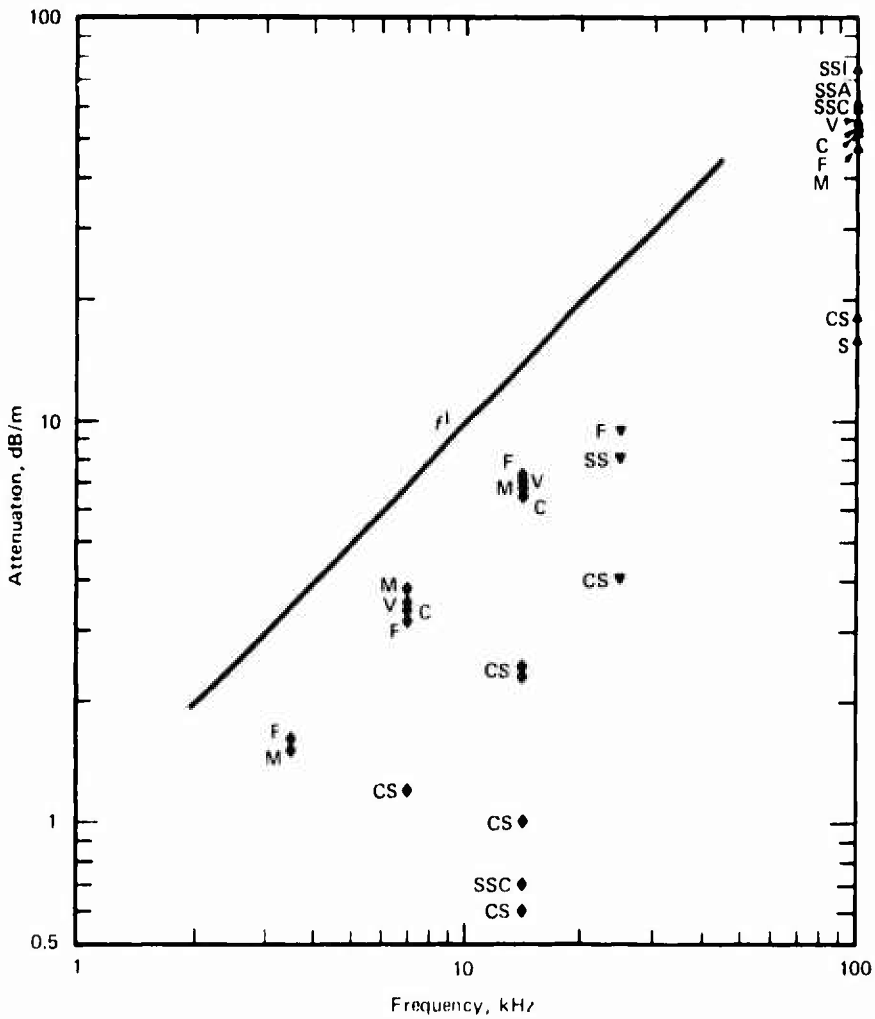

Figure 1. *In situ* measurements of compressional-wave attenuation off San Diego. Symbols: diamonds–1966-1970, Table 1; inverted triangles–1962, Table 1; triangles–1956, Table 2. Letters indicate sediment type: C-coarse sand; M-medium sand; F-fine sand; V-very fine sand; SSI-sandy silt; SSA-silty sand; SSC-sand-silt-clay; S-silt; CS-clayey silt. Line labeled "f^1" indicates slope of any line having a dependence of attenuation on the first power of frequency.

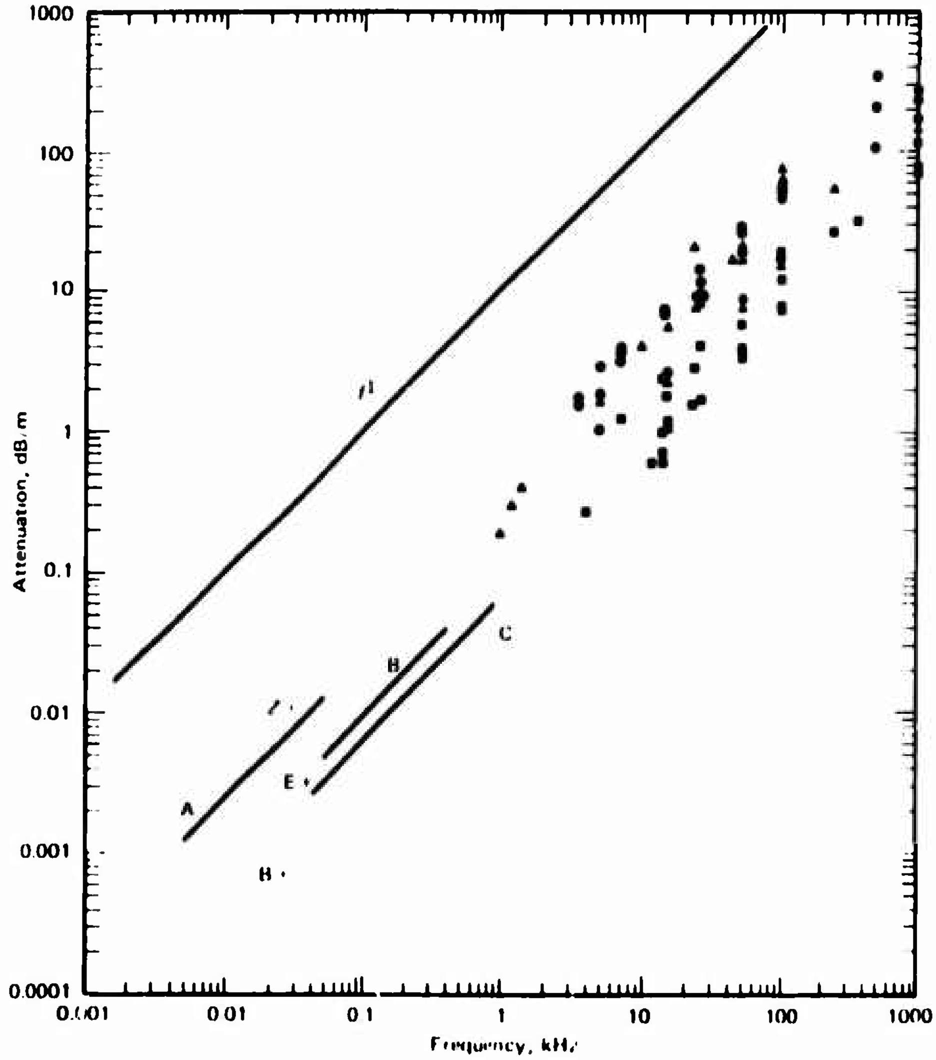

Figure 2. Attenuation versus frequency in natural, saturated sediments and sedimentary strata; data from Tables 1–4b. Symbols: circles-sands (all grades), squares-clayey silt, silty clay; triangles-mixed sizes (e.g., silty sand, sandy silt, sand-silt-clay); sand data at 500 and 1000 kHz from Ref. 21. Low-frequency data: line "A"-Ref. 16 (land, sedimentary strata), "B"-Ref. 13 (Gulf of Mexico coastal clay sand), "C"-Ref. 12 (sea floor, reflection technique). Z-Zhadin (in Ref. 15); E and B-Epinstveva *et al.*, and Berzon (in Ref. 16). Line labeled "f^1" indicates slope of any line having a dependence of attenuation on the first power of frequency.

Table 3. Laboratory Attenuation and Other Physical Properties of Sediments Off San Diego From Ref. 18.*

Sediment Type	No. of Stations	Mean Grain Diameter		Density (gm/cm^3)	Porosity (%)	Velocity 23°C (m/sec)	Attenuation	
		ϕ	mm				dB/m	kHz
Sand								
Medium	13	1.44	0.3686	2.004	39.1	1737	9.4	26.6
Fine	13	2.49	0.1780	1.960	43.6	1693	9.6	26.0
Silty sand	1	2.99	0.1259	1.796	53.2	1551	20.6	23.9
Sandy silt	4	4.57	0.0421	1.737	57.4	1541	11.8	23.6
Silt	8	5.27	0.0259	1.653	61.5	1548	7.8	24.2
Clayey silt	4	6.31	0.0126	1.605	65.5	1508	2.8	23.2
Clayey silt	11	7.51	0.0055	1.336	80.0	1495	1.6	22.8

*Averaged values for number of stations indicated; velocity and attenuation for the second resonant mode (201) only.

The same probes were used in shallow water. Operating from a small boat, divers observed the sea floor, took samples, and helped insert the probes into the sediment.

The results of the above measurements and subsequent sediment laboratory analyses are listed in Table 1, and the attenuation versus frequency data are plotted in Figs. 1 and 2.

1962 Program

In 1962, velocity and attenuation measurements were made at 25 kHz from the bathyscaph *Trieste* and from a small boat in shallow water. The equipment, methods, and results of the velocity measurements were reported in Ref. 4. The probes used for these measurements were an earlier model of those used in the 1966-70 program; field methods were the same.

The few attenuation measurements made during this program (not previously reported) are listed in Table 1 and plotted versus frequency in Figs. 1 and 2.

1956 Program

In 1956, sound velocity and attenuation measurements were reported for a number of *in situ* and laboratory measurements in shallow water off San Diego (Refs. 1 and 5).

In situ velocity measurements were made at 100 kHz. The equipment and methods have been reported (Ref. 1). In general, the equipment consisted of two probes secured through a rigid base so that, when inserted 6 in. (15 cm) into the sea-floor sediment (by divers), velocity and wave-amplitude measurements could be made over a 1-ft (30 cm) path length. At the same stations, small containers (10 cm long and 5 cm in diameter)

Table 4a. Attenuation and Other Properties of Natural, Saturated Sediments (From the Literature)*

Sediment Type	Mean Grain Diameter		Density (gm/cm^3)	Porosity (%)	Velocity 23°C (m/sec)	k†	n†	Notes
	ϕ	mm						
Fine sand	2.5	0.177	(1.99)	41.0	(1742)	0.13	1.26 ± 0.13	1
Very fine sand	3.05	0.121	(1.95)	43.3	(1716)	0.27	1.17 ± 0.13	1
Very fine sand	3.8	0.072	(1.97)	42.3	(1613)	0.56	1.00 ± 0.01	1
Silt	5.32	0.025	(1.90)	46.6	(1622)	0.30	1.05 ± 0.15	1
Medium sand	1.32	0.401	1.99	(40.8)	(1815)	0.164	1	2
Silty sand	...	.....	1.32	(82.2)	(1540)	0.151	1	2
Clayey silt	...	.....	1.20	(89.2)	(1530)	0.118	1	2
Clay-silt	...	.....	1.15	(92.1)	(1525)	0.075	1	2
Silty clay	...	.....	1.13	(93.3)	(1525)	0.072	1	2
"Mud"	(7.5)	(0.006)	1.32	76.0	(1480)	0.066	1	3
(Clayey silt)	(7.6)	(0.005)	...	(80.0)	.	0.049	1	4a
(Clayey silt)	...	.....	...	...	....	0.966	1	4b
Sand-silt-clay	6.35	0.012	1.75	59.3	1527	(0.393)	(0.99)	5
Clay	9.93	0.001	(1.45)	74.8	....	31.9 dB/m at 368 kHz		6
	...	.....	...	...	(1615)	54 dB/m at 250 kHz		7a
(Clayey silt)	(7.5)	.....	...	(75.0)	(1463)	26.2 dB/m at 250 kHz		7b
	...	.....	...	...	(1621)	0.2 dB/m at 1 kHz		8

*Values in parentheses computed or estimated by the writer of this report.

†In, $\alpha = kf^n$, where attenuation, α, is in dB/m; frequency, f, in kHz; k is a constant, and n is the exponent of frequency.

Notes:

1. McCann and McCann (Ref. 8); *in situ*, land and beach; velocity given at 20°C; 5 to 50 kHz.
2. McLeroy and DeLoach (Ref. 20); laboratory; beach sand and St. Andrew Bay, Florida; 15 to 1500 kHz, velocity from measured ratio.
3. Wood and Weston (Ref. 9); *in situ*; tidal mud flat, Emsworth, England, 4 to 50 kHz; velocity from measured ratio.
4. Bennett (Ref. 12); *in situ* (reflection technique); North Atlantic and Mediterranean; 4a 12 kHz; 4b 40 to 900 Hz.
5. Lewis (Ref. 14); *in situ* (best measurements); shallow water off Puerto Rico; 5 to 50 kHz.
6. McCann and McCann (Ref. 8); laboratory, eight core samples, North Atlantic; carbonate < 5%; 368 kHz.

7a. Ulonska (Ref. 10); *in situ*, shallow Baltic Sea; velocity from measured ratio (Station 32).

7b. Ulonska (Ref. 10); *in situ*, shallow North and Baltic Seas; average for all stations with velocity ratios less than 1.00; 250 kHz; velocity from measured ratio.

8. Schirmer (Ref. 11); *in situ*; shallow Baltic Sea (1 station); velocity from measured ratio.

were inserted by hand into the sediment for subsequent laboratory measurements of velocity and attenuation by means of a resonant-chamber method (Refs. 6 and 7).

In Ref. 1, velocities were reported for both the *in situ* probe and laboratory resonant-chamber methods. Values of attenuation (at 23 to 40 kHz) were reported for the resonant-chamber method.

During the *in situ* probe measurements at 100 kHz, the amplitude of the received wave was measured on the oscilloscope face in both bottom water and sediment; three to five separate measurements were made at each station. These amplitudes were reported (Ref. 1, Table II) as "20 log (amplitude water/amplitude sediment)", and not as linear attenuation, because of uncertainties concerning impedance loading on the transducers in

Table 4b Attenuation in Sediments and Sedimentary Strata (From the Literature)*

Sediment Type	k[†]	n[†]	Frequency (Hz)	Notes
Clay-sand	0.093	1	50 to 400	1
Water-saturated clay	0.326	1	(seismic)	2
Sedimentary strata	0.243	1	5 to 50	3
Sedimentary strata	0.119	1	(seismic)	4
Sedimentary strata	0.035	1	(seismic)	5

*(Seismic); assumed

†In, $\alpha = kf^n$, where attenuation, α, is in dB/m; frequency, f, in kHz; k is a constant, and n is the exponent of frequency.

Notes:

1. Tullos and Reid (Ref. 13); Gulf of Mexico coastal sediments; depth interval 2 to 34 m.
2. Zhadin (in Ref. 15); depth interval 0 to 20 m.
3. Zemstov (Ref. 16).
4. Epinatyeva *et al.* (in Ref. 16).
5. Berzon (in Ref. 16).

the two media. Re-evaluation of the method and results indicates these data can be identified as attenuation in the sediments. The basic data have been re-analyzed and extrapolated to 1 m and are listed in the form of decibels per meter in Table 2; attenuation versus frequency is plotted in Figs. 1 and 2.

MEASUREMENTS FROM THE LITERATURE (*IN SITU*)

In situ measurements of sound attenuation in marine sediments are relatively rare compared with measurements of other properties of sediments. Those of the writer are noted above. In this section, brief resumés of other *in situ* measurements in the literature will be presented. Included are *in situ* measurements on land in some water-saturated sediments and sedimentary rocks, which may be pertinent to lower layers of the sea floor or to later discussions. These attenuation measurements are plotted versus frequency in Fig. 2, and listed in Tables 3 and 4a, b. For those studies in which equations are given (*e.g.*, Refs. 8 and 9), points are plotted in Fig. 2 for the lowest and highest frequencies and an intermediate frequency.

Wood and Weston (Ref. 9) made *in situ* measurements of attenuation (4 to 50 kHz) in tidal mud flats in Emsworth Harbor, England. Maximum distance between probes was 180 ft (55 m); probe penetration was about 0.5 m.

Ulonska (Ref. 10) and Schirmer (Ref. 11) measured, *in situ*, compressional velocity and attenuation in shallow-water sediments in the Baltic Sea at sediment depths of about 2 m. Ulonska's measurements were over a 0.5-m path at 250 kHz; Schirmer's over a 12-m path at about 1 kHz.

McCann and McCann (Ref. 8) reported *in situ* measurements in water-saturated land and beach sediments at a mean sediment depth of 2 m. Frequencies were between 5 and 50 kHz.

Bennett (Ref. 12), using reflection techniques from a surface vessel, computed values of attenuation at 12 kHz, and between 40 and 900 Hz, for sediments in the North Atlantic and Mediterranean.

Tullos and Reid (Ref. 13) measured attenuation in Gulf of Mexico coastal sediments (clay-sand) of Pleistocene age in three depth intervals to 1000 ft (305 m); frequencies were between 50 and 400 Hz. Measurements in the depth interval of 7.5 to 110 ft (2 to 34 m) are listed in Table 4b.

Lewis (Ref. 14) measured attenuation, *in situ*, at one station in shallow water off Puerto Rico by inserting probes into the sediment; frequencies were 5 to 50 kHz. The data were scattered, but Lewis believed they represented a dependency of attenuation on frequency to the first power.

There are several studies from Russian literature listed in Table 4b. Zhadin (in Ref. 15) reported attenuation in "water-saturated clay" in the depth range of 0–20 m. No frequency is given, but is presumed to be in the seismic range. Zemtsov (Ref. 16) reports his own studies of attenuation in the frequency range of 5 to 50 Hz in "sedimentary strata" and notes attenuation versus frequency equations from Epinatyeva and Berzon in similar materials; values are shown at 30 Hz in Fig. 2.

MEASUREMENTS FROM THE LITERATURE (LABORATORY)

It is possible, but difficult, to make valid laboratory measurements of attenuation in natural or artificial water-saturated sediments. Even when valid measurements are made in some experimental materials, the results cannot always be extrapolated to natural marine sediments. Some of these difficulties are noted below.

One of the chief difficulties in measuring attenuation in "water-saturated" laboratory samples is air entrapped in the pore spaces of artificial sediments, or air or gas from organic materials in natural sediment samples transported to the laboratory. When air or gas is present in "water-saturated" sediments, the measured attenuation is apt to be too high and velocity too low (depending on frequency and bubble size). The experiences of Wood and Weston (Ref. 9) with sands and muds are instructive in this matter.

Some artificial sediments are not truly analogous to natural sediments. For example, kaolinite in distilled water containing a deflocculent acts as a Newtonian fluid without rigidity, whereas in a flocculated structure, rigidity is present (Ref. 17). As discussed in a later section, the relation of attenuation with frequency differs in the two cases. A favored laboratory material is round-grained quartz sand (e.g., St. Peter's sand).

This material has less rigidity and attenuation than the more common, angular-grained natural sands (discussed below).

Some laboratory measurements in natural marine sediments of particular significance to the present study are noted and briefly discussed in this section.

Shumway (Ref. 18) used a resonant-chamber technique to measure attenuation and velocity at various frequencies between 20 and 40 kHz. The basic theory and equipment were reported in Refs. 6 and 7. These measurements have been widely referenced, and values extrapolated to other frequencies.

Unfortunately, Shumway's measurements, alone, cannot be used to determine frequency and attenuation relationships, although in a number of samples, measurements were made at two or three frequencies. A detailed examination of the published data and the unpublished records of measurements (in the writer's files) reveals numerous experimental errors. These errors are evident for the following reasons: (1) in individual samples where attenuation was measured at two or three frequencies, there are 16 cases (in Ref. 18) in which attenuation *decreased* with increasing frequencies, a result inconsonant with all theory and other experiments in similar materials, and (2) even with these 16 cases eliminated, computations of the frequency dependence of attenuation (in the form: $\alpha = kf^n$) revealed that the exponent n varied in similar sediment types (having similar physical properties) as follows: sand (0.9 to 3.1), sandy silt and silty sand (0.8 to 2.9), and clayey silt (0.6 to 3.4). Neither theory nor other experiments in similar materials support these variations.

The causes of the above-noted experimental errors are unknown. Shumway's samples were excellent. Most were taken *in situ* by divers and never exposed to air prior to measurement. These errors may be, in part, related to the method. Shumway noted (Ref. 6, p. 318) that the resonant-chamber method assumes samples with no, or negligible, rigidity. This may be true in some shallow-water silt-clays, but not in sands and silty sands. (See, for example, data in Ref. 19.) In addition, not all resonant modes were of equal reliability. Toulis (Ref. 7) suggested that the second resonant mode (201) was most reliable.

However, Shumway's attenuation data (Ref. 18) can probably be used, selectively, by averaging values for general sediment types; hopefully, experimental errors will cancel out. For comparison with the *in situ* measurements off San Diego (presented herein) and with other values from the literature, the writer averaged attenuation values (and other properties) at the second resonant mode for various sediment types off San Diego; these are listed in Table 3 and plotted in Fig. 2.

McLeroy and DeLoach (Ref. 20) measured (in the laboratory) sound speed and attenuation, from 15 to 1500 kHz, in natural sediments from sites in St. Andrew Bay, Panama City, Florida, and in sand from a Gulf of Mexico beach. They noted no indication of entrapped gases. These measurements are listed in Table 4a and plotted in Fig. 2.

The unusually low densities and computed (by the writer) high porosities of the Bay sediments are not typical of most open-ocean sediments.

The excellent study of attenuation in marine sediments by McCann and McCann (Ref. 8) has been noted in the section on *in situ* measurements. These writers also measured attenuation at 368 kHz in sediment cores from the North Atlantic. Most of these sediments contained appreciable amounts of calcium carbonate. There is some indication that attenuation in this type of sediment may be higher than in non-calcareous clay-silts with similar grain sizes and porosities. Because calcareous sediments are not included in this report, an average value of attenuation for eight clay samples with less than 5 percent carbonate is listed in Table 4a and plotted in Fig. 2.

DISCUSSION AND CONCLUSIONS

INTRODUCTION

The study of elastic and viscoelastic models which can be applied to dry and saturated porous rock and sediments has concerned a large number of scientists and engineers. Many of these studies have considered the extent to which these media can be described by the equations of Hookean elasticity or by those of the Kelvin-Voigt, Maxwell, or other viscoelastic models. It is somewhat surprising, therefore, to discover such a wide diversity of models, equations, and opinions on such an important subject. In the case of water-saturated natural sediments, the reason for this diversity appears to be that, until recently, experimental evidence capable of restricting model parameters has been scarce. In the following discussion (unless otherwise noted) it is assumed that the medium is a porous, uncemented mineral structure, fully saturated with water. The stress is that of a compressional or shear wave of low amplitude, and strains are of the order of 10^{-6}, or less. Wavelengths are much greater than grain size, otherwise Rayleigh scattering can occur; and attenuation is related to the fourth power of frequency (see, for example, Ref. 21).

The equations of Hookean elasticity do not account for energy damping. Consequently, an adequate model must be anelastic if energy damping is considered. In the selection of an appropriate anelastic model, a critical factor is the extent of relative movement of pore water and mineral particles. If the pore water moves significantly relative to mineral structure, then viscous damping and velocity dispersion must be considered. If the pore water does not move significantly with respect to the solids, there is negligible, or no, velocity dispersion, and energy damping is not dependent on viscosity of pore water and permeability of the mineral structure. The dependence of energy damping on frequency is different in the two cases. Consequently, two critical parameters for anelastic models are velocity dispersion, if any, and the dependence of energy damping on frequency. In the next section, the experimental evidence concerning these two factors will be reviewed. In following sections, a particular anelastic model is recommended, and the relationships between attenuation and other physical properties and the causes of attenuation will be discussed. In the last section, a method for predicting attenuation (given grain size and sediment porosity) will be discussed.

SOME PARAMETERS OF ELASTIC AND VISCOELASTIC MODELS FOR SATURATED SEDIMENTS

Introduction

Recent experimental studies in wave velocities and attenuation and in dynamic rigidity have placed important restrictions on probable (and possible) elastic and viscoelastic models for water-saturated sediments. These parameters merit a more extended review because of their importance in geophysical studies. Additionally, the concepts and statements of this paper require documentation. These restrictions apply to porous sediments saturated with water, and without a gas phase, when wavelengths are much greater than grain sizes. The frequency range is from a few hertz to several hundred kilohertz, or into the megahertz range. These restrictions are concerned with the question of the dependence, if any, of velocity on frequency ("velocity dispersion"), and the dependence of wave-energy damping, or attenuation, on frequency. In the following sections, these restrictions will be documented and discussed.

Velocity Dispersion

A number of investigators have measured compressional- and shear-wave velocities in rocks (laboratory and *in situ*). Most have concluded that there is no (or negligible) measurable velocity dispersion in the range from seismic frequencies into the megahertz range. Examples include work and reviews by Wylie *et al.* (Ref. 22), Birch (Ref. 23), Peselnick and Outerbridge (Ref. 24), White (Ref. 25), and Press (Ref. 26).

Most viscoelastic models requiring a dependence of velocity on frequency include movement of pore water relative to the mineral frame, which, in turn, causes water viscosity and sediment permeability to be important factors. Sands have relatively high permeability, large grain size, and interconnecting pores; consequently, these models indicate maximum velocity dispersion in sands. It is therefore instructive to examine the experimental evidence regarding velocity dispersion in sands across wide frequency ranges.

Because permeability is a factor in some models, it is pertinent that Wylie *et al.* (Ref. 22) found that velocity through water-saturated glass beads of various sizes was unaffected as permeability varied by a factor of 4.6×10^4.

No velocity dispersion was measured in clean, round-grained sands in the laboratory by Hunter *et al.* (Ref. 27) over the range 7 to 73 kHz; Hardin and Richart (Ref. 28), 0.2 to 2.5 kHz; Nolle *et al.* (Ref. 29), 200 to 1000 kHz; and Schön (Ref. 30), 20 to 64 kHz.

In round-grained, pure quartz sands in distilled water at porosities of 36 percent, Shumway (Ref. 18, p. 463) and Nolle *et al.* (Ref. 29) measured compressional velocities of 1744 m/sec (26 kHz) and 1740 m/sec (400 to 1000 kHz), respectively.

In soil mechanics investigations *in situ* in sands, low-frequency vibrations were used in studies by Barkan (Ref. 31) and Jones (Ref. 32) to measure shear-wave velocities (Jones' measurements also included clay-silt). No velocity dispersion was measured in the frequency range from 10 to 400 Hz.

Ideally, to test for velocity dispersion, measurements should be made on the same sample in the laboratory, or *in situ* in the same sediment, by merely changing the frequency. In natural sands this has been done in the laboratory and *in situ*. In the laboratory, McLeroy and DeLoach (Ref. 20) measured a ratio (velocity in sediment/velocity in water) of 1.189 in medium sand over a frequency range of 15 to 1500 kHz. *In situ* McCann and McCann (Ref. 8) reported no velocity dispersion in fine or very fine sands between 5 and 50 kHz. During the 1970 measurements off San Diego reported herein, special tests were made in fine sand at four stations to determine the presence or absence of velocity dispersion between 3.5 and 14 kHz. The results indicated no measurable velocity dispersion between these frequencies.

In natural saturated sands it is difficult to compare velocity measurements in different samples or from different stations because of the effects on velocity of grain size and shape, porosity, and other factors. (For a discussion of these, see Ref. 3.) However, the following *in situ* measurements (corrected to 23°C) indicate no significant velocity dispersion in natural fine sand over a frequency range from 14 to 100 kHz:

Source	No. of Stations	Velocity (m/sec)	Porosity (%)	Frequency (kHz)
Present study	6	1712	46.7	14
Ref. 1	17	1704	46.7	100
Ref. 8	..	1742	41.0	5-50

In very fine sand, McCann and McCann measured a velocity of 1716 (porosity, 43.3 percent) at frequencies between 5 and 50 kHz.

In high-porosity silt-clays in the laboratory and in the field, compressional velocity is usually less in the sediment than it is in the water, and no velocity dispersion is indicated over very wide frequency ranges. The evidence is summarized in Table 5. Velocity data are presented as the ratio: velocity in sediment/velocity in water. As discussed in Ref. 3, this ratio remains the same in both the laboratory and the surficial sediments of the sea floor.

Some recent experiments in artificial clays in the laboratory have implications for studies of possible shear-wave velocity dispersion. Cohen (Ref. 17) measured complex rigidity ($\mu + i\mu'$) in flocculated kaolinite in distilled water. Both μ and $i\mu'$ were independent of frequency in the range 8.6 to 43.2 kHz. Hardin and Black (Ref. 40), using a vibration technique to measure dynamic rigidity in kaolinite in distilled water, demonstrated that dynamic rigidity was independent of wave amplitude, in their samples, at amplitudes less than 10^{-4}, and independent of frequency between 200 and 300 Hz. No dispersion in dynamic rigidity, μ, indicates no dispersion in shear-wave velocity.

Table 5. Summary of Ratios of Compressional-Wave Velocities (V_p Sediment/V_p Water) in High-Porosity Silt-Clays at Various Frequencies.

Material	Velocity Ratio	Frequency	Notes
Kaolinite in distilled water	0.97 to 0.99	1 MHz	1
Kaolinite in distilled water	0.97 to 0.99	9 to 43 kHz	2
Deep-sea clay slurry	0.97	28 kHz	3
Sand-silt-clay	0.98	14 kHz	4
Clayey silt, San Diego Trough	0.98	23 to 40 kHz	3
Clayey silt, San Diego Trough	0.98	25 kHz	5
Clayey silt, San Diego Trough	0.98	14 kHz	4
Clay-silt, St. Andrew Bay	0.997	15 to 1500 kHz	6
Silt, clayey silt, continental shelf	0.992	2 MHz	7
Silt-clays, continental shelf	0.994	2 MHz	8
Silty clay, continental shelf	0.994	200 kHz	9
Deep-sea silt-clay	0.977	30 to 200 Hz	10
Deep-sea silt-clay	0.980	30 to 200 Hz	11
Deep-sea silt-clay	0.985	200 kHz	9
Deep-sea clay	0.975	200 kHz	9
Deep-sea silt-clay (1 micron)	0.977	400 kHz	12
Deep-sea silt-clay (2 microns)	0.987	400 kHz	12
Deep-sea silt-clay	0.986	400 kHz	13
"Low-velocity layer," continental shelf	0.980	250 kHz	14

Notes:
1. Urick (Ref. 33); ratio dependent on concentration of solids.
2. Cohen (Ref. 17); ratio dependent on concentration of solids.
3. Shumway (Ref. 18); resonant chamber; laboratory.
4. Hamilton, this report; *in situ* measurements.
5. Hamilton (Ref. 3); *in situ* measurements.
6. McLeroy and DeLoach (Ref. 20); laboratory samples from St. Andrew Bay, Fla.
7. Bieda (Ref. 34); tops of 5 cores off Southern California.
8. Lasswell (Ref. 35); 9 cores; tidal mud flat, Southern California.
9. Hamilton (Ref. 3); laboratory; samples from North Pacific.
10. Fry and Raitt (Ref. 36); reflection technique; deep Pacific.
11. Houtz and Ewing (Ref. 37); reflection technique; deep Atlantic.
12. Horn *et al.* (Ref. 38); laboratory; core samples from North Pacific.
13. Schreiber (Ref. 39); tops of 10 cores off Hawaii.
14. Ulonska (Ref. 10); *in situ* measurements; shallow North and Baltic Seas.

Hampton (Ref. 41) measured compressional velocity and attenuation in artificial sediments and reported a marked velocity dispersion (4 to 6 percent) in silt-clays between frequencies of 3 to 200 kHz. In his low-frequency measurements (3 to 30 kHz) in flocculated kaolinite in distilled water, he reported (Ref. 41, Fig. 11) anomalously low velocity ratios (0.93 to 0.94) and unusually high attenuation values (Ref. 41, p. 886). The experimental evidence of other investigators (*e.g.*, Refs. 17 and 33) in the same, or similar, materials and at the same frequencies, in both the laboratory and field (see Table 5), does not support Hampton's reported low velocity ratios or velocity dispersion. There is a possibility that his clay-water, artificial sediments were not air or gas free.

In summary, the following can be concluded in regard to velocity dispersion in saturated sediments. In sands, a number of studies (cited above) have reported no dispersion over restricted frequency ranges. These studies and comparison of the values of velocity in similar sands at different frequencies, as above, indicate that velocity dispersion, if present, is negligibly small from a few kilohertz to the megahertz range. The evidence (as in Table 5) indicates that velocity dispersion in higher porosity silt-clays, if present, is negligible over a frequency range from less than 1 kHz to 2 MHz. However, it must be stated (as a reviewer pointed out) that most of the quoted tests of velocity dispersion were made over only an order of magnitude of frequencies or less, which is not necessarily enough to show dispersion. In other words, it cannot be stated on the basis of present experimental evidence that velocity dispersion is non-existent, especially over very wide frequency ranges (from a few hertz to several megahertz).

Energy Damping

The relationship between frequency and wave-energy loss, or damping, is a critical parameter in selection of an appropriate anelastic model for any medium. Recent reviews have summarized a large number of laboratory and field studies of wave-energy losses in rocks in which the specific attenuation factor, $1/Q$, and the logarithmic decrement have been shown to be approximately independent of frequency over a range of at least 10^8 Hz (Refs. 25 and 42-45). Attwell and Ramana (Ref. 45) included some sediment data.

Evidence that the specific attenuation factor, $1/Q$, is independent of frequency implies that attenuation, α, in decibels/unit length, increased linearly with frequency, f (*e.g.*, Ref. 25, p. 98). Recent summaries of work in this field, in the case of compressional waves (Refs. 25, 43, and 45), indicate that, for most rocks, there is a small variation around linearity in the range of frequencies of most interest in underwater acoustics and marine geophysics; that is, in the relationship $\alpha = kf^n$, the exponent n is approximately 1. These studies included dry rocks (usual in the laboratory) and *in situ* measurements in rock strata which, below groundwater levels, are saturated.

Some recent studies of attenuation in saturated sediments which are pertinent to the present report have been noted in a previous section and their results presented in tables and figures. Of special interest in these studies, and in the measurements of this report, is the dependence of attenuation on frequency. In the following paragraphs the current experimental evidence on this subject will be reviewed.

In examining the dependence of attenuation on frequency, it is important to recognize that, in natural marine sediments, each set of measurements and resulting equation (*e.g.*, $\alpha = kf^n$) is apt to be unique. The reasons for this phenomenon will be discussed below; but in general, the important variables involve the varying sediment structures, porosity, grain size and shape, interparticle contacts and surface areas in sands and coarse silts, and physicochemical forces (cohesion) in the fine silts and clays. These variable factors result in much scatter of measured values of attenuation in similar sediments (Tables 1–4), and consequent scatter in computed values of the constant k. Also, because of the difficulties of making accurate measurements of attenuation in natural sediments (*in situ* or laboratory), it is to be expected that computed values of the exponent of frequency, n, will vary. As a result, generalized statements about the exact dependence of attenuation on frequency must be qualified and, as far as justifiable, a statistical approach followed in studying data (which was the approach of Attwell and Ramana, Ref. 45).

The important factor in predicting attenuation and extrapolating to various frequencies is the exponent of frequency, *n* (in the equation above). Also, this exponent is a critical parameter for selecting appropriate anelastic models which can be applied to saturated sediments. In Tables 1 and 4a are values of *n* for measurements made at two or more frequencies in a single sample or sediment type by an individual experimenter. In Table 4b, values are given for some low-frequency measurements in sediments or sedimentary rock sections (some of the reports did not specify the degree of lithification). Some of the measurements in Table 4b may be pertinent to lithified or semi-lithified, deeper layers in the sea floor. As can be seen in Tables 1 and 4, measured values of *n* in these natural sediments, from widely scattered geographic areas, vary closely around 1 for a wide variety of deepwater and shallow-water sediments and over a wide frequency range.

Computations were made (Table 6, Part A) which interrelated *in situ* measurements in similar sediments from various experiments conducted by the writer in the sea floor off San Diego (Tables 1, 2). This allowed extension of the frequency range; frequencies usually included 3.5, 7, 14, 25, and 100 kHz. Off Mission Beach, attenuation data for two stations came from Ref. 1, Table II. The *in situ* data for "Same area off Mission Beach" and "San Diego Trough" involved only two or three stations in each area. In these locations the values of *n* (rounded to two decimals in the tables) are 1.007, 1.024, 0.947, and 0.969. The data for "General sediment type" for fine sand involved 24 stations; the value of *n* is 1.007.

In Table 6, Part B, the writer's *in situ* measurements (Tables 1, 2, and 6, Part A) were combined with averaged laboratory values for general sediment types off San Diego (Table 3) from Ref. 18. The value of *n* for 37 stations in fine sand is 0.992; for medium sand at 16 stations, *n* is 0.982.

Because of the larger number of stations, the data for "General sediment type" in Table 6 are considered more reliable: 1.007, 0.992, 0.982. As in previous tables, the values of *n* vary closely around 1, with the more reliable data at, or slightly below, 1.

In reconciling acoustic theory with experimental acoustic energy losses of sound incident on the sea floor, it is usually necessary to assume values of attenuation at frequencies of interest for various sediment types and layers. Such reconciliation has been successful in several studies (frequencies from 0.1 to 4 kHz) in which a first-power dependency of attenuation on frequency was assumed (*e.g.*, Refs. 46–49). Cole concluded that the first-power dependency can be extended over the range 100 to 900 Hz

Compressional velocity and attenuation data from Refs. 10 and 11 are possibly significant in determining the relationship between attenuation and frequency. Schirmer, at a water depth of 20 m in the Baltic Sea, measured attenuations of 0.2 to 0.4 dB/m at 1.0 to 1.4 kHz; an average velocity ratio (in sediment/in water) of 1.086 (1.05 to 1.14)

Table 6. Attenuation and Other Physical Properties of Sediments Off San Diego; Combined From Various Programs.

Sediment Type	Mean Grain Diameter		Density (gm/cm^3)	Porosity (%)	Velocity* (m/sec)	k†	n†	Frequencies‡ (No. of Stations)	Notes
	ϕ	mm							
A. *In situ* (Tables 1 and 2)									
Sand									
Medium	1.51	0.351	2.00	40.1	1714	0.47 ± 0.06	1.01 ± 0.05	a,b,c(2); g(1)	1
Fine	2.40	0.190	1.94	46.4	1680	0.46 ± 0.08	1.01 ± 0.06	a,b,c(4); d(3); g(17)	2
Very fine	3.23	0.107	1.94	47.3	1725	0.45 ± 0.04	1.02 ± 0.03	a,b,c(1); g(4)	1
Clayey silt	7.44	0.006	1.37	78.6	1450	0.19 ± 0.01	0.95 ± 0.03	b,c(2); d(1)	3
Clayey silt	7.41	0.006	1.36	78.5	1450	0.18 ± 0.03	0.97 ± 0.05	b(1); c(2)	3
B. *In situ* (Part A, above) combined with laboratory (Ref. 18), Table 3									
Sand									
Medium	1.45	0.366	2.00	39.2	1740	0.48 ± 0.10	0.98 ± 0.07	a,b,c(2); f(13); g(1)	2
Fine	2.42	0.187	1.93	46.5	1703	0.46 ± 0.09	0.99 ± 0.07	a,b,c(4); d(3); e(13); g(17)	2

* Part A: Velocities *in situ*; Part B: Velocities at 23 C.
† In $\alpha = kf^n$; where, attenuation, α, dB/m; frequency, f, kHz; k is a constant; n, the exponent of frequency.
‡ Frequencies (kHz): a-3.5; b-7; c-14; d-25; e-26; f-26.6; g-100.

Notes: 1. Same area off Mission Beach; 2. General sediment type; 3. San Diego Trough.

was measured in a core from the site. Ulonska (Ref. 10), at a water depth of 22 m in the Baltic Sea, measured (Station 32) a velocity ratio of 1.082 and attenuations of 50 to 58 dB/m at 250 kHz. Both sets of measurements were at about 2 m depth in the sea floor. The velocity ratios indicate the sediment types were about the same. If a first-power dependency of attenuation on frequency is assumed, extrapolation of Ulonska's average data (54 dB/m at 250 kHz) to 1 kHz yields an attenuation of 0.22 dB/m (Schirmer measured 0.2 dB/m at 1 kHz).

In a recent study of attenuation in quartz sand in distilled water (which has been widely referenced), Hampton (Ref. 41) reported attenuation dependent on the square root of frequency. The measurements do not support this conclusion. The data (Ref. 41, Fig. 9) are more in accord with a first-power dependency (as noted, also, by Mizikos in Ref. 50).

The values of n which were measured, computed, and assumed above are close to that statistically computed by Attwell and Ramana (Ref. 45) for frequencies between 1 and 10^8 Hz, *i.e.*, 0.911. Strick (Ref. 51) made a case that the dependence of attenuation on frequency should be close to, but less than, 1 to satisfy causality. The writer believes the best of his experimental data are in fine sand off San Diego (n = 1.007 ± 0.060). When these data are combined with Shumway's (Ref. 18) averaged data for fine sand, n = 0.992 ± 0.065.

Data listed in Tables 1 through 4 and in 6 are plotted in Fig. 2 (frequency versus attenuation). It can be seen that most of the data are consistent with an approximate first-power dependency of attenuation on frequency over a wide frequency range. The upper and lower bounds of the data plot probably define the area in which most natural marine sediments will lie. With regard to sediment type, the silt-clays lie in a narrow band at the lower side of the data plot, and very fine sands, silty sands, and sandy silts at the top. Extrapolation of the silt-clay data to frequencies below 1 kHz, using a first-power dependency, results in attenuation values in accord with the data of Bennett (Ref. 12; 40 to 900 Hz) and those of Tullos and Reid (Ref. 13; 50 to 400 Hz).

Two recent studies of complex rigidity and energy damping in artificial clays have important implications in determining parameters for elastic and viscoelastic models in saturated sediments. Cohen (Ref. 17) measured both μ and $i\mu'$ in complex rigidity, $(\mu + i\mu')$, in artificial, laboratory sediments composed of kaolinite and bentonite in distilled water, with and without a deflocculating agent. Cohen demonstrated that both μ and $i\mu'$ were independent of frequency in the range 8.6 to 43.2 kHz in flocculated clay; but when a deflocculating agent was added, the flocculated structure of the clay sediment dispersed, the material lost all rigidity, μ, and behaved as a Newtonian fluid in which there was viscous damping of wave energy that was linearly dependent on frequency. The addition of 35.5 ppt of NaCl caused reflocculation, and complex rigidity was the same as before. Krizek and Franklin (Ref. 52), in studies of shear-wave energy damping in flocculated kaolinite in distilled water, demonstrated that $1/Q$ was independent of frequency in the range 0.1 to 30 Hz, and that the stress-strain hysteresis loop for a given cycle was that of a linear viscoelastic medium.

The two studies noted above (Refs. 17 and 52) have several important implications for saturated clay sediments, at least in the frequency range covered (0.1 Hz to 43 kHz), and probably at much higher frequencies: (1) saturated, flocculated clay sediments respond to shear-wave energy as linearly viscoelastic media; (2) the independence of energy damping ($1/Q_s = \mu'/\mu$) from frequency implies that linear attenuation of shear waves should be proportional to the first power of frequency; and (3) suspensions (without flocculated structures) do not respond to wave energy as do flocculated clay structures, and almost all natural, high-porosity silt-clays have this general type of structure.

Recent measurements of attenuation in water-saturated, natural sediments over the frequency range 3.5 to 1500 kHz can be summarized as follows. Tables 1, 4, and 6 list 25 values of the exponent of frequency, n, between 0.94 and 1.26; however, all but two of the values fall between 0.94 and 1.11. The experimental evidence indicates that the dependence of attenuation on frequency is close to f^1, and does not support any theory calling for a dependence of attenuation on $f^{1/2}$ or f^2. However, as in the discussion of velocity dispersion, the case should not be overstated. As a reviewer pointed out, there is no single data set covering more than two orders of magnitude in frequency. While these data are enough to show that the dependence of attenuation on frequency is more nearly f^1 than $f^{1/2}$ or f^2, it is not enough to verify an exact dependence.

REVIEW OF ELASTIC AND VISCOELASTIC MODELS

In the field of soil mechanics, large static or dynamic stresses have to be considered; and over the full range of stresses, sediments may be elastic, viscoelastic, or plastic. Yong and Warkentin (Ref. 53, p. 80–94) present a good discussion of the various models and elements within the models which describe this behavior.

In the fields of soil mechanics and foundation engineering, the Hookean model and equations are commonly used for derivations of dynamic elastic constants and studies of vibrating loads (*e.g.*, Refs. 28, 31, 32, and 54–57). However, the dynamic moduli from most velocity data are for very small strains, on the order of about 10^{-6}, and corrections to moduli should be made for greater strains (Ref. 58 presents a correction curve).

In the fields of physics and geophysics, studies of the elasticity of minerals and rocks have demonstrated that the elastic equations of the Hookean system adequately define the velocities of compressional and shear waves. These equations are conveniently interrelated in a table by Birch (Ref. 23, p. 2206). This field has been summarized by Birch (Ref. 59) and by Anderson and Lieberman (Ref. 60). Papers of special interest are by Christensen (Refs. 61, 62), Brace (Refs. 63, 64), and Simmons and Brace (Ref. 65).

The question of water movement relative to mineral frame is a critical key to whether or not the equations of elasticity can be used in studies of wave velocities in rock and sediments. If the pore water does not move significantly with respect to the solids, then the effective density of the medium is the sum of the mass of the water and the solids in a unit volume, there is negligible or no velocity dispersion, energy damping is negligibly dependent—or independent—of frequency, and the equations of Hookean elasticity can be used to study wave velocities and elastic constants within the frequency range in which these parameters are effective, unless attenuation is involved in the study. This is the "closed system" of Gassmann (Ref. 66). The "closed system," as a special case in studies of the elasticity or viscoelasticity of saturated porous media, has been noted in many experimental and theoretical studies (Refs. 19, 22, 23, 25, 31, 32, 42, 59, 65-81).

Although the elastic equations of the Hookean model adequately account for wave velocities in most earth materials, they do not provide for wave-energy losses in these media. To account for energy losses, various anelastic (viscoelastic and "near-elastic") models and equations have been proposed. Viscoelastic models frequently favored are the Kelvin-Voigt, Maxwell, or some other combination of Hookean elastic springs and Newtonian dashpots (see Ref. 53 for a concise resumé), or some variation of Biot's models (Ref. 68) in which a basic assumption involves movement of pore water of the Poiseuille type (at lower frequencies).

In his various papers, Biot (*e.g.*, Refs. 68, 69) discussed the full range of systems in which water within pore spaces does or does not move with the solids upon imposition of a small stress, such as that produced by a sound wave. In some of these acoustic models, this movement or flow of water through the sediment mineral structure was considered to be of the Poiseuille type. In the last several decades, it has been determined that the simple flow equations of the Poiseuille type (derived from flow of water through tubes) do not hold for real, *in situ* sediments. These equations have to be considerably altered, even for clean sands, and are not applicable to relatively impermeable clays (*e.g.*, Ref. 53). In other words, models based on Poiseuille-type flow of pore water are probably not applicable to natural sediments.

One model which has been especially studied in connection with rocks and sediments is the Kelvin-Voigt model, in which, as originally defined, compressional-wave velocity varies with frequency; and attenuation, at frequencies of most interest in underwater acoustics and geophysics, increases with the square of frequency. White (Ref. 25, p. 110-112) has a thorough discussion of theory and experimental evidence on this subject and concludes (p. 112) that neither velocity nor attenuation shows this frequency dependence and that the Voigt solid cannot be considered an adequate model of earth materials. The evidence of this report and earlier ones (Refs. 2 and 19) are in accord with this conclusion.

A VISCOELASTIC MODEL FOR WATER-SATURATED SEDIMENTS

In the absence of sufficient experimental evidence, it has been possible to construct rather elegant theoretical approaches, altered—if necessary—by constants to fit available data. To derive such theoretical models, one must start with assumptions. In the case of water-saturated sediments, some of the less tenable of these assumptions have been that (1) all water-saturated sediments are analogous to suspensions of mineral particles in fluids; (2) all the mineral particles are spheres; (3) Poiseuille flow operates in natural sediments; (4) pore water necessarily moves relative to the mineral frame or sediment structure; and (5) sediments lack rigidity, in which case the shear modulus is zero, and Poisson's ratio is 0.5.

All of the above assumptions are invalid in part or in whole. Two recent papers (Refs. 3 and 19) discussed several aspects of sediment structure and elasticity. Some conclusions are pertinent: almost all saturated sediments have mineral particles which are not spheres; near suspensions are unusual; and almost all sediments have sufficient rigidity to allow transmission of shear waves. As noted previously, Yong and Warkentin (Ref. 53) indicate Poiseuille flow (through small tubes) does not hold for natural sediments.

Given macroscopic isotropy, small, sinusoidal stresses, wavelengths much greater than grain size, and frequencies from a few hertz to at least several hundred kilohertz (and probably into the megahertz range for most natural sediments), some parameters in addition to those in the preceding paragraph are as noted in previous sections: attenuation in decibels per linear measure is approximately dependent on the first power of frequency; and velocity dispersion, if present, is small. Some relative movement of pore water and mineral frame cannot be excluded on the basis of present evidence, although the above parameters indicate that, if present, it should be small.

The model proposed below is intended as a tentative, working model. It should be emphasized that other models are not excluded if they are within the stated parameters. The whole subject merits much more experimental and theoretical study.

A model and concomitant equations within the parameters noted above is a case of linear viscoelasticity. The basic equations of linear viscoelasticity have been summarized in an excellent treatise by Ferry (Ref. 82). For the model recommended in this paper, the basic equations (Ref. 83) have been discussed in different form, including neglect of negligible factors, in Refs. 2, 25, 52, 82 (pp. 93, 4), 84 and 85.

In the above model, the Lamé elastic moduli μ and λ are replaced by complex moduli, $(\mu + i\mu')$ and $(\lambda + i\lambda')$, in which μ, λ, and density govern wave velocity, and the imaginary moduli, $i\mu'$ and $i\lambda'$, govern energy damping. The following (Ref. 82, pp. 11-13) illustrate the stress-strain relations in this model. For a sinusoidal wave, if the viscoelastic behavior is linear, the strain will be out of phase with stress. The stress can be vectorially decomposed into two components: one in phase with strain and one 90 deg out of phase. For a shear wave, the complex stress/strain ratio is $\mu^* = \mu + i\mu'$. The phase angle, ϕ, which expresses energy damping is, in this case: $\tan \phi = \mu'/\mu$.

The basic derivations of the above model are in Refs. 25 and 82 and will not be repeated here. Without assumptions as to negligible factors, the equations of the model in the form used by Bucker (in Ref. 2, p. 4046) or by Ferry (Ref. 82, p. 94, 419), reduce to the following for both compressional and shear waves (with some changes in notation).

$$\frac{1}{Q} = \frac{aV}{\pi f - \dfrac{a^2V^2}{4\pi f}} \tag{2}$$

where

$1/Q$ is the specific attenuation factor (or specific dissipation function)

a is the attenuation coefficient

V is wave velocity

f is frequency (circular frequency, $\omega = 2\pi f$)

Subscripts (p or s) can be inserted into Eq. (2) when referring to compressional or shear waves.

When energy damping is small (i.e., $\lambda' << \lambda$ and $\mu' << \mu$; Ref. 25, p. 95, Ref. 82, p. 123; $r << 1$, where $r = aV/2\pi f$), the term in the denominator of Eq. (2), $a^2V^2/4\pi f$, is negligible and can be dropped. This leaves the more familiar expression (e.g., Refs. 25, 42, 44, and 45).

$$\frac{1}{Q} = \frac{aV}{\pi f} \tag{3}$$

$$\frac{1}{Q} = \frac{2aV}{\omega} = \frac{\Delta}{\pi} = \tan\phi \tag{4}$$

Additionally

$$\frac{1}{Q_p} = \tan\phi_p = \frac{\lambda' + 2\mu'}{\lambda + 2\mu} \tag{5}$$

$$\frac{1}{Q_s} = \tan\phi_s = \frac{\mu'}{\mu} \tag{6}$$

$$\frac{\Delta E}{E} = \frac{2\pi}{Q} \tag{7}$$

$$\alpha = 8.686a \tag{8}$$

Where (in addition to those symbols already defined)

Δ is the logarithmic decrement (log of the ratio of two successive amplitudes in an exponentially decaying sinusoidal wave)

$\tan \phi$ is the loss angle

$\Delta E/E$ is fraction of strain energy lost per stress cycle

α is attenuation in decibels per linear measure (e.g., dB/cm)

Equations involving compressional- and shear-wave velocities in Ref. 2, or in Ref. 76 are (in Ferry's notation):

$$(\lambda + 2\mu) = \rho V_p^2 (1 - r^2)/(1 + r^2)^2 \tag{9}$$

$$\mu = \rho V_s^2 (1 - r^2)/(1 + r^2)^2 \tag{10}$$

where

$r = \alpha V/2\pi f$

λ = Lamé's constant

μ = rigidity

ρ = density

In Eqs. (9) and (10), the term $(1 - r^2)/(1 + r^2)^2$ indicates the degree of velocity dispersion for linear viscoelastic media. When damping is small (defined above), this term is negligible and can be dropped, as implied in Ref. 82, p. 94. This leaves the more familiar Hookean equations:

$$(\lambda + 2\mu) = \rho V_p^2 \tag{11}$$

$$\mu = \rho V_s^2 \tag{12}$$

This means that if the factor $(1 - r^2)/(1 + r^2)^2$ in Eqs. (9) and (10) is considered negligible and dropped, wave velocity, $1/Q$, and the log decrement are independent of frequency, and linear attenuation is proportional to the first power of frequency.

Computations performed with the data of this report and from the literature indicate that most water-saturated rocks and sediments qualify under the above definitions as media with "small damping." Therefore, Eqs. (3), (4), (11), and (12) should apply to both water-saturated sediments and rocks. However, those investigators who wish to include velocity dispersion and $1/Q$ or a log decrement dependent on frequency can consider Eqs. (2), (9), and (10).

$1/Q$, Q, and Δ

In the sediments discussed in this report, the quality factor, or specific dissipation function, Q, $1/Q$, and the logarithmic decrement, Δ, are approximately independent of frequency. This follows from the dependence of attenuation (approximately) on the first power of frequency. Table 7 lists these properties for the *in situ* measurements in Tables 1 and 2. A conclusion from the data of this report is that the approximate frequency independence of $1/Q$, Q, and Δ can be extended from rock (*e.g.*, Ref. 43) through most natural, water-saturated sediments.

Table 7. The Quality Factor, Q_p, the Specific Attenuation Factor, $1/Q_p$, and the Logarithmic Decrement, Δ_p, From *In Situ* Measurements of Compressional-Wave Velocity, V_p, and Attenuation, α_p, in Sediments Off San Diego.*

Sediment Type	Q_p	$1/Q_p$	Δ_p	Notes
Sand				
Coarse	32	0.031	0.099	1
	29	0.034	0.107	2
Medium	31	0.032	0.101	1
	35	0.028	0.089	2
Fine	31	0.032	0.100	1
	31	0.032	0.101	2
	44	0.023	0.071	3
Very fine	32	0.031	0.093	1
	29	0.034	0.107	2
Sandy silt	23	0.044	0.138	2
	54	0.018	0.058	3
Sand-silt-clay	31	0.033	0.102	2
Clayey silt	104	0.010	0.030	2
	111	0.009	0.028	1
	114	0.009	0.028	1
	118	0.009	0.027	3
	263	0.004	0.012	1
	437	0.002	0.007	1
Sand-silt-clay	368	0.003	0.009	1

* Computed from equations

$$1/Q_p = \alpha_p V_p/\pi f = \Delta_p/\pi;\ \text{attenuation},\ a_p = 8.686\ \alpha_p$$

Notes: 1. Table 1 (Part A); frequency: 14 kHz.
2. Table 2; frequency: 100 kHz.
3. Table 1 (Part B); frequency: 25 kHz.

RELATIONSHIPS BETWEEN ATTENUATION AND OTHER PROPERTIES

Introduction

The relationships between attenuation and other physical properties in saturated sediments are of considerable importance in determining the causes of attenuation and in selecting appropriate anelastic models. In this section, various relationships will be briefly noted and illustrated prior to discussions of the causes of attenuation.

As discussed, attenuation in decibels per linear measure is approximately dependent on the first power of frequency: that is, in the equation $\alpha = kf^n$, n is close to 1. If n is taken as 1, the only variable in the equations for various sediments is the constant k. This constant is particularly useful in relating attenuation to other sediment properties, such as grain size and porosity. The relations between k and other physical properties give an insight into the causes of attenuation, which allows prediction of attenuation (as discussed in the last section) after deriving a value of k from its relationships with mean grain size and porosity.

With an assumption that linear attenuation is dependent on the first power of frequency, values of k can be easily computed by dividing attenuation by frequency. This was done for all data in Tables 1 to 4 except as follows: (1) the values of k from Table 1 (Part A) were determined from the 14-kHz measurements (considered most reliable); and (2) for three measurements in Table 4a which did not show a first-power dependency, k was determined from an attenuation computed at a frequency near the mid-range of the measurements. These values of k were then plotted versus mean grain size and porosity in Figs. 3, 4, and 5.

Relationships between k (or attenuation) and mean grain size, porosity, and dynamic rigidity are illustrated and discussed in the following sections. The data are for natural sediments; both *in situ* and laboratory measurements are included. Averaged values from Tables 1 through 3 were used as primary data to establish regression equations (in the figure captions), but the individual measurements which were averaged are shown to better illustrate the trends and scatter of the data. These regression equations are included for use in predicting attenuation when grain size and porosity are known. These equations are strictly empirical and are recommended only within the limiting values indicated. The values of k so obtained are approximations, but it is predicted that most future measurements of attenuation will result in k values which fall within the indicated "envelopes."

Attenuation and Grain Size

When wavelengths approach grain size, Rayleigh scattering takes place, and attenuation is proportional to the fourth power of frequency. This effect has been discussed by Knopoff and Porter (Ref. 86) in the case of granite. Rayleigh scattering appears to be a

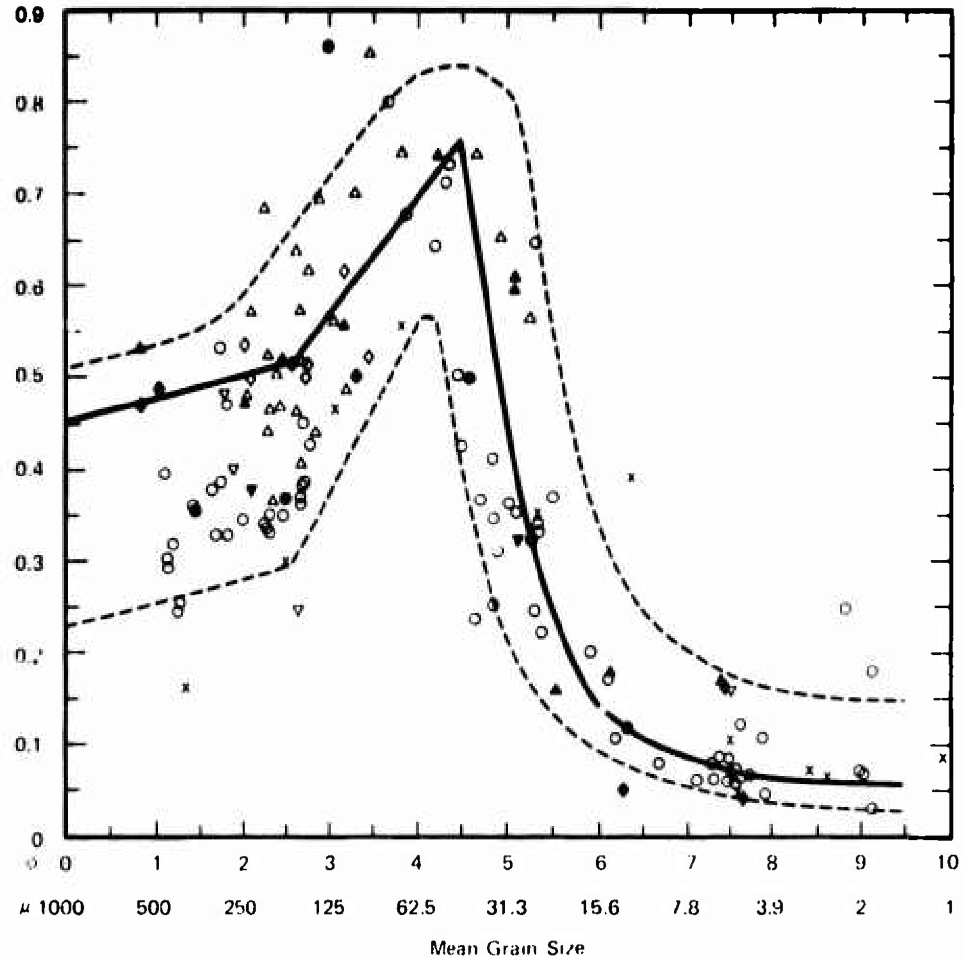

Figure 3. Mean grain size, M_z, in ϕ units and microns, μ, versus k (in $\alpha = kf^1$) in natural, saturated sediments (Tables 1–4a); see text. Solid symbols are averaged values for data off San Diego: diamonds–1966–1970; triangles–1956; inverted triangles–1962; circles–Shumway (Ref. 18), Table 3; open symbols–data in averages; X–literature value from Table 4a. Area between dashed lines: predicted area within which most data should fall. Regression equations for solid lines (for data off San Diego and selected literature values–recommended for use in similar sediments): Coarse, medium, and fine sand, in part (0 to 2.6 ϕ): $k = 0.4556 + 0.0245(M_z)$. Fine sand (in part), very fine sand, and mixed sizes (2.6 to 4.5 ϕ): $k = 0.1978 + 0.1245(M_z)$. Mixed sizes (4.5 to 6.0 ϕ): $k = 8.0399 - 2.5228(M_z) + 0.20098(M_z)^2$. Silt-clays (6.0 to 9.5 ϕ): $k = 0.9431 - 0.2041(M_z) + 0.0117(M_z)^2$.

factor in Busby and Richardson's (Ref. 21) measurements of attenuation in the megahertz range in sands. For most sediments, Rayleigh scattering is not a factor to at least several hundred kilohertz, and probably into the megahertz range.

Figure 3 illustrates the relationships between the constant k (in $\alpha = kf^1$) and mean grain size for most of the sediments listed in Tables 1 to 4. Mean grain size is plotted in logarithmic phi units ($\phi = -\log_2$ of grain diameter in millimeters; see Ref. 87 for discussion). In studying Fig. 3, one can easily translate k into linear attenuation in decibels per meter, because at any given frequency the only variable is k.

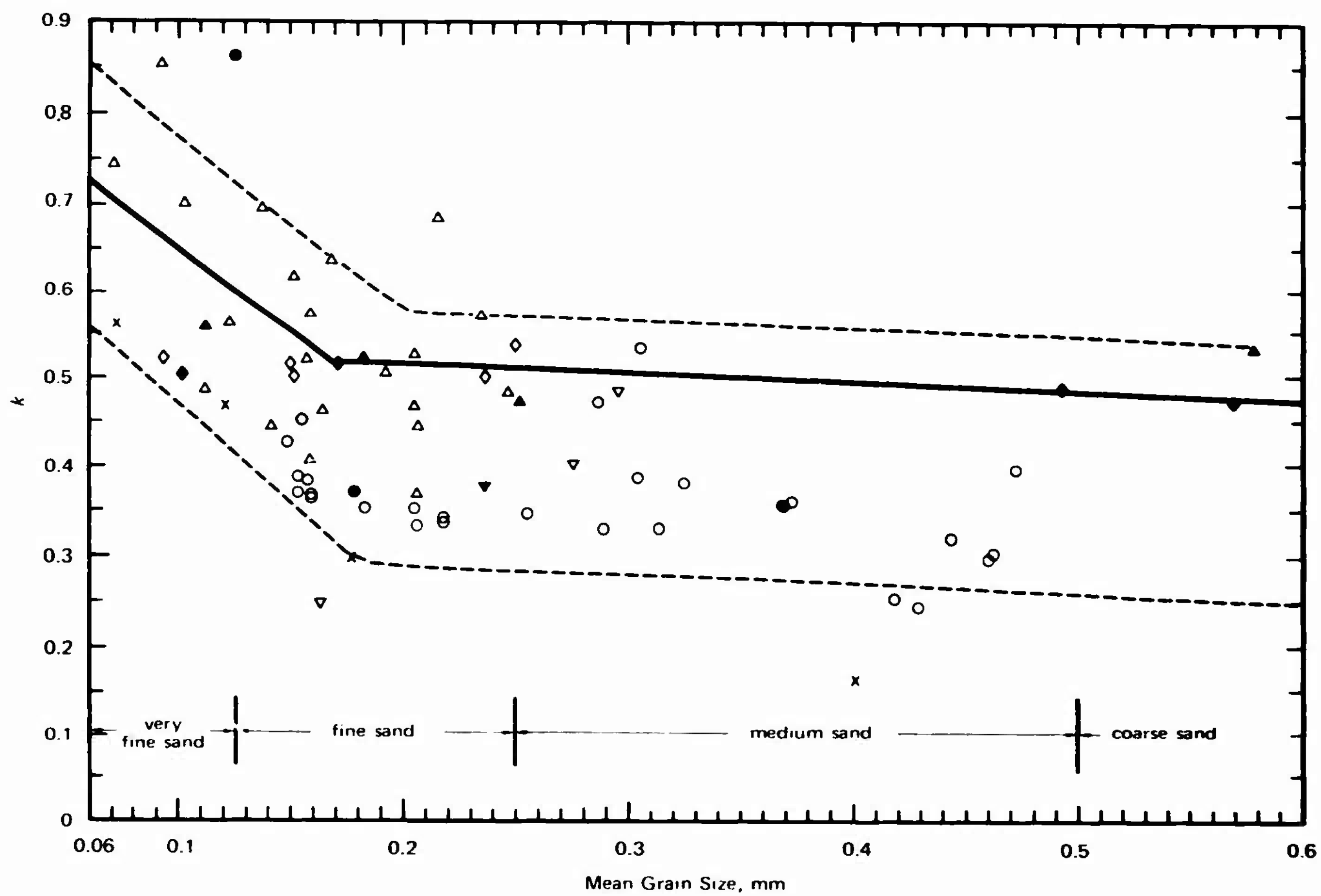

Figure 4. Mean grain size, mm, versus k (in $a = kt^1$). Data, symbols, and remarks: same as in caption for Fig. 3. Plot of mean grain size in mm emphasizes relationships in sands (see text). Regression equations for solid lines: Coarse, medium, and fine sand, in part (0.6 to 0.167 mm): $k = 0.5374 - 0.1113\ (M_z)$. Fine sand (in part), and very fine sand (0.167 to 0.063 mm): $k = 0.8439 - 1.9431\ (M_z)$.

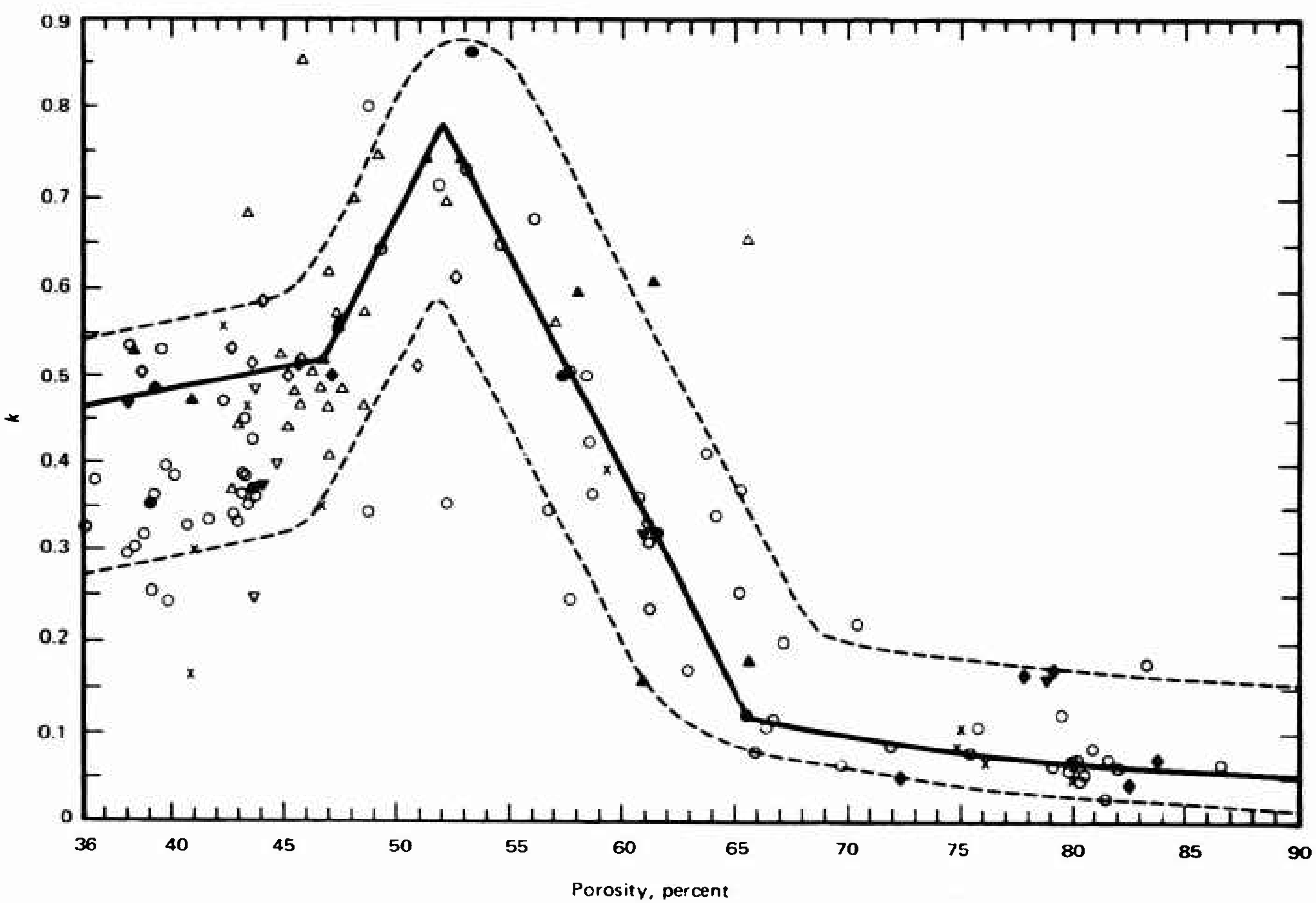

Figure 5. Porosity, n, percent, versus k (in $a = kf^1$). Data, symbols, and remarks same as in caption for Fig. 3. Regression equations for solid lines: Coarse, medium, and fine sand (36 to 46.7 percent): $k = 0.2747 + 0.00527\,(n)$. Very fine sand and lower porosity mixed sizes (46.7 to 52 percent): $k = 0.04903(n) - 1.7688$. Mixed sizes (52 to 65 percent): $k = 3.3232 - 0.0489(n)$. Silt-clays (65 to 90 percent): $k = 0.7602 - 0.01487(n) + 0.000078(n)^2$.

Figure 3 illustrates the distinct difference between mean grain size, M_z, and the constant k (or attenuation) in sands (M_z = 0 to 4 ϕ, or 1 to 0.0625 mm) and in the silt-clays ($\phi > 4$). In sands (Figs. 3 and 4), k increases gradually with ϕ (or decreasing grain size) from coarse into fine sands (to about 2.6 ϕ), and then k increases rapidly into the finer sand sizes. Other investigators have measured increased attenuation in sands with decreasing grain size (*e.g.*, Refs. 8, 18, and 41, Fig. 9). The maximum values of k are in very fine sand and in mixtures of sand, coarse silt, and clay (*e.g.*, silty sand, sandy silt) in the grain-size range of 3.5 to 4.5 ϕ (0.09 to 0.04 mm). Equivalent lower values of k are in coarser grained sand, in silt, and in finer sized sandy silt. Attenuation decreases with decreasing grain size (increasing ϕ) from about 4.5 ϕ to about 6 ϕ, and then gradually declines with decreasing grain size into fine silts and clays.

A semi-log plot (using phi units: Fig. 3) tends to clarify relationships between grain size and other properties in the finer sizes (silt-clays), but obscures relationships in sands. Therefore, mean grain size in millimeters is plotted versus k in Fig. 4. Figure 4 illustrates better than Fig. 3 the gradual increase of k from coarse and medium sand sizes into fine sands, and the marked increase in k from about 0.17 mm (2.6 ϕ) into the finer sand sizes.

Attenuation and Porosity

Porosity varies with k (Fig. 5) in the same way that mean grain size varies with k (Figs. 3 and 4). Equivalent values of k are apt to be found in the coarser sizes of sand and in higher porosity silts and sandy silts. The highest values of k are in very fine sands, silty sands and sandy silts in the porosity range of 50 to 54 percent. In silt-clays, k decreases with increasing porosity. Shumway (Ref. 18, Fig. 6) indicates a similar variation between attenuation and porosity at frequencies between 20 and 40 kHz. Relationships between grain size and porosity, as they affect attenuation, will be discussed in a section below.

Attenuation and Dynamic Rigidity

Figures 6 and 7 (reproduced from Ref. 19) illustrate the probable variations of computed values of dynamic rigidity, μ, with grain size and porosity. Comparison of these figures with Figs. 3, 4, and 5 indicates that k and values of rigidity respond in the same way to variations in grain size and porosity.

PROPOSED PREDICTION METHOD

CAUSES OF ATTENUATION

Introduction

In studies of the attenuation of compressional and shear waves in saturated porous media, there are usually two (or a combination of two) common viewpoints: (1) a visco-elastic model is used in which pore water moves relative to mineral grains, and pore-water

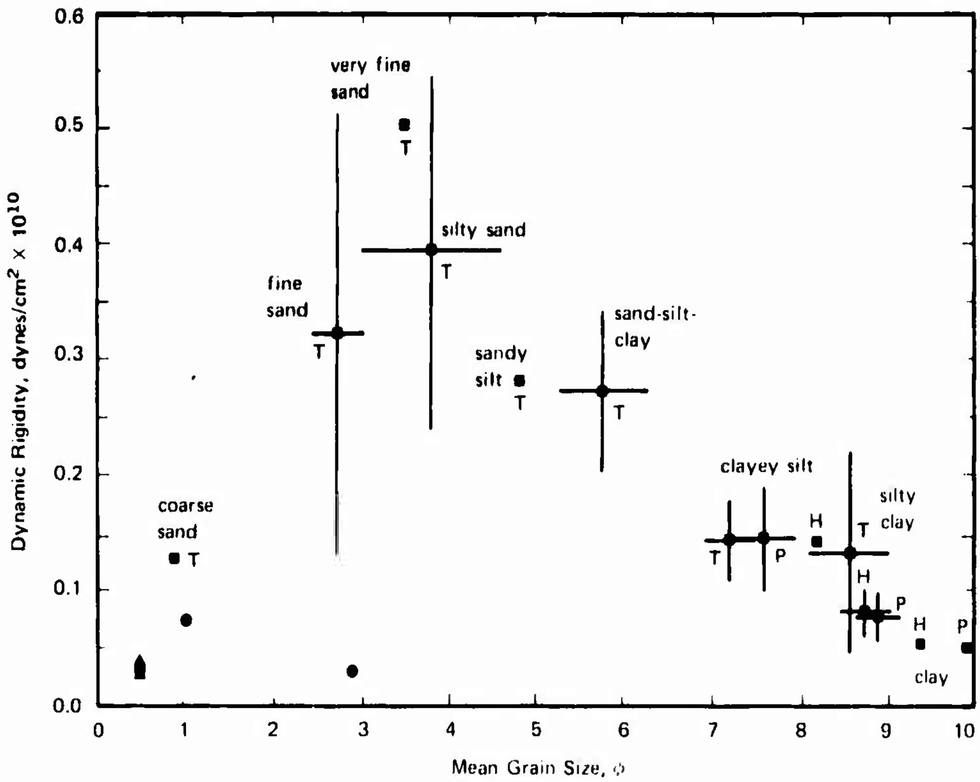

Figure 6. Mean grain size versus computed values of the shear modulus (dynamic rigidity); from Hamilton (Ref. 19). Squares are mean values; crosses are 3 times the standard error of the mean. T-samples from the continental terrace (shelf and slope); P-abyssal plain; H-abyssal hill. Lower left: St. Peter's sand (circles) and Ottawa sand (triangles). See Ref. 19 for discussion.

viscosity and media permeability are dominant factors in viscous sound absorption; or (2) a linear viscoelastic, or "nearly elastic," model is favored in which attenuation is mostly, or entirely, due to internal friction (*i.e.*, energy is lost in intercrystalline or intergrain movements).

The case has been made that in crustal rocks, both dry and saturated, internal friction is by far the most probable dominant process in wave-energy damping at seismic frequencies to at least several hundred kilohertz, and that viscous losses are probably negligible (*e.g.*, Refs. 25 and 88). The writer believes, as discussed below, that internal friction is by far the dominant dissipative process in water-saturated sediments.

It is apparent from the present study that sands with grain-to-grain contacts and no cohesion (physicochemical net attraction) should be studied separately from silt-clays with cohesion and mineral particles probably separated by layers of adsorbed water. Consequently, the following discussion will deal separately with these two structural types. The structures of these and other sediment types were reviewed in Refs. 3 and 19.

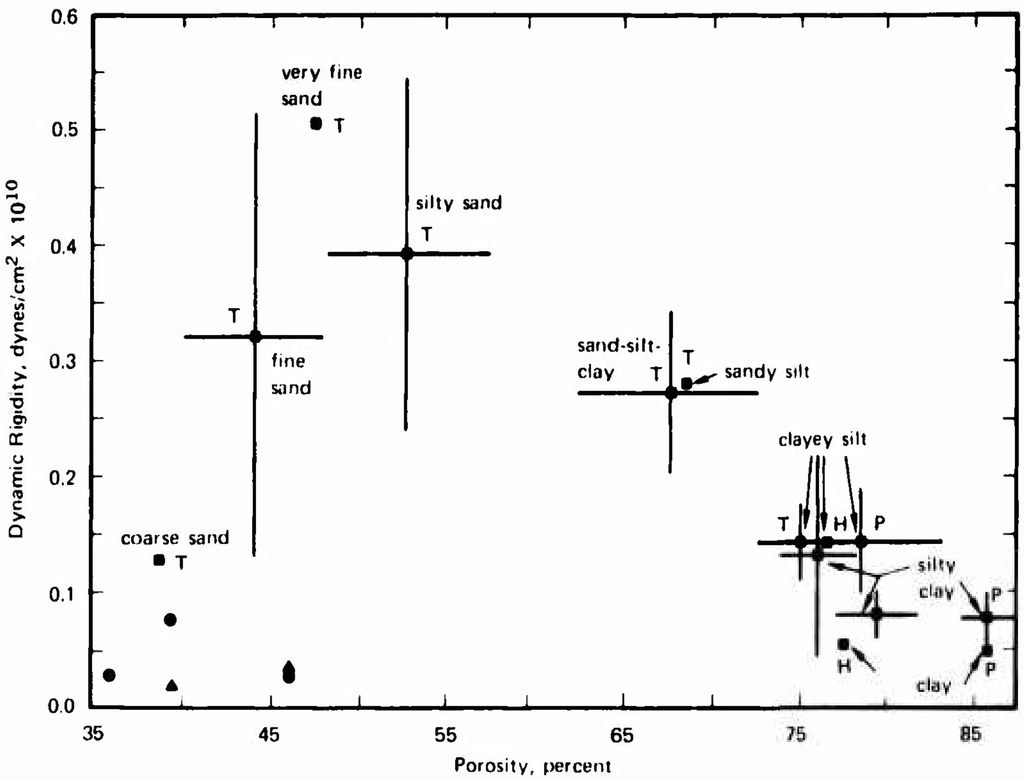

Figure 7. Sediment porosity versus dynamic rigidity; symbols and remarks as in Fig. 6.

Attenuation in Sand

Study of attenuation in sands is of particular interest in determining the causes of attenuation because these sediments are the most permeable, and viscous losses, if present, should be higher in sand than in other sediment types. On the other hand, if frictional energy losses are dominant, the intergrain reactions to stress which cause attenuation should be related to the same complex factors which affect dynamic rigidity and static shear strength. These factors will be briefly reviewed. Reference 19, a more extensive review, contains numerous references to items outlined below.

Sands have grain-to-grain contacts between mineral particles, and resistance to shear stress is related to sliding and rolling friction between grains, to the number of intergrain contacts, and interlocking between grains. See Ref. 53 for a good resumé.

The number of interparticle contacts in sands depends on grain size and density of packing (*i.e.*, loose, dense, or—most frequently—an intermediate packing). At any given particle size, porosity is a measure of packing. Interlocking of grains increases with density of packing and angularity of grains.

Laboratory experiments with clean quartz sand grains (St. Peter's or Ottawa sand) have indicated the following (Ref. 28): (1) at the same grain size, dynamic rigidity increases with decreasing porosity in sands with round grains because of more interparticle contacts with denser packing; (2) at the same grain size and porosity, sands with angular grains have higher dynamic rigidities and shear-wave velocities than sands with round grains because of interlocking between grains; and (3) at the same grain size, increased angularity of grains (causing greater rigidity) can be more important than increases in porosity (causing lesser rigidity).

The grains of natural marine sands are much more angular than those in St. Peter's or Ottawa sand at any given grain size. In natural sands, angularity increases and sphericity decreases with decreasing grain size (Ref. 89). Interlocking of grains should be greater when coarse silt particles are present.

The discussion above and the conclusions presented in Ref. 19 imply and predict for natural marine sediments:

(1) Coarse sand has fewer intergrain contacts and the grains are rounder than in the finer sizes; thus, dynamic rigidity should be a minimum in coarse sand because of lesser intergrain friction.

(2) Dynamic rigidity should be less in sands composed of the highly rounded grains of St. Peter's or Ottawa sand than in natural sands of the same density, porosity, and grain size; thus tests with St. Peter's or Ottawa sand cannot always be directly related to natural sands.

(3) Hardin and Richart (Ref. 28) observed that an increase in porosity will cause a decrease in rigidity. This is only true in sands of the same grain size and angularity.

(4) In natural sands, porosity increases with finer sizes which have more numerous intergrain contacts and more angular grains, which may cause rigidity to increase with increasing porosity.

(5) At some relatively fine sand-silt size, intergrain friction and interlocking will reach a maximum and rigidity will be at a high point for natural, uncemented sediments.

(6) When sand grains are no longer in contact because of increased amounts of fine silt and clay, intergrain friction and interlocking are no longer relatively effective and rigidity is mostly due to cohesion between finer particles.

(7) Cohesion between fine particles varies with the numerous factors discussed below; but, in general, cohesion and rigidity decrease with increasing porosity and decreasing grain size in higher porosity silt-clays.

Figures 6 and 7 illustrate the phenomena outlined above. Of the sands, coarse sand has the least rigidity, and natural sands have higher rigidities than round-grained St. Peter's sand. Rigidity increases sharply with decreasing grain size and increasing porosity to maximum values in very fine sands, silty sands, and coarse silts. These maximum values of dynamic rigidity occur between mean grain sizes of 3.5 to 4.5 ϕ (0.09 to 0.04 mm) and porosities between 55 and 60 percent. In a density versus dynamic rigidity plot (not shown), maximum rigidities occur between densities of 1.7 and 1.8 g/cm^3. As grains become finer, rigidity decreases with increasing porosity and decreasing density (Fig. 7) and decreasing grain size (Fig. 6). Scatter diagrams of percent sand size by weight versus rigidity (not shown) indicate maximum rigidity normally occurs when a sediment is composed of about 60 to 65 percent sand by weight.

Dynamic rigidity is a measure of resistance (friction and interlocking between grains) to shearing forces, which tend to move grains. If attenuation is due to energy lost by friction between grains, then rigidity and attenuation should vary because of the same factors. However, as with rigidity, there is complex interaction between grain sizes, shapes, density of packing, or porosity.

Mean grain size is an important factor in both rigidity and attenuation. In coarse sands, the grains are larger and more rounded, interparticle contacts fewer, and surface areas smaller, than in the finer sands. Consequently, rigidity is relatively low in coarser grained sands; and when grains are moved, the fewer interparticle contacts produce only a small attenuation that can be attributed to friction (Figs. 3 and 4). As grain size decreases, the grains become more numerous in a unit volume, they are more angular, and consequently, have larger surface areas and more interparticle contacts. Therefore, rigidity increases. When grains are relatively moved, however, the more numerous interparticle contacts and greater surface areas result in greater energy losses. This has been shown in the laboratory (*e.g.*, Ref. 41, Fig. 9) and *in situ* (this report and Ref. 8).

The change in rate of increase of k from coarser sands at about 2.6 ϕ, or 0.17 mm, into the finer sand sizes (Figs. 3, 4) requires further discussion. The change is apparently related to the relationships between grain size, porosity, and surface area, or the number of interparticle contacts.

A semi-log plot of mean grain size (in phi units) versus porosity (Ref. 3, Fig. 2) indicates an almost linear relationship. This type of plot obscures the real relationship. Figure 8, an arithmetic plot of grain size (in millimeters) versus porosity, illustrates the very gradual increase of porosity with decreasing grain size in the coarse and medium sand sizes, and the marked increase in porosity which occurs as grain sizes decrease from fine sands into the silt-clay sizes (porosities greater than about 60 percent). There is much scatter in the relationships between grain size and porosity, because at any given grain size, porosity varies with packing (*i.e.*, dense, loose).

If attenuation is related to energy lost in intergrain friction, a critical factor involves the number of intergrain contacts, or surface areas involved, per unit volume. If

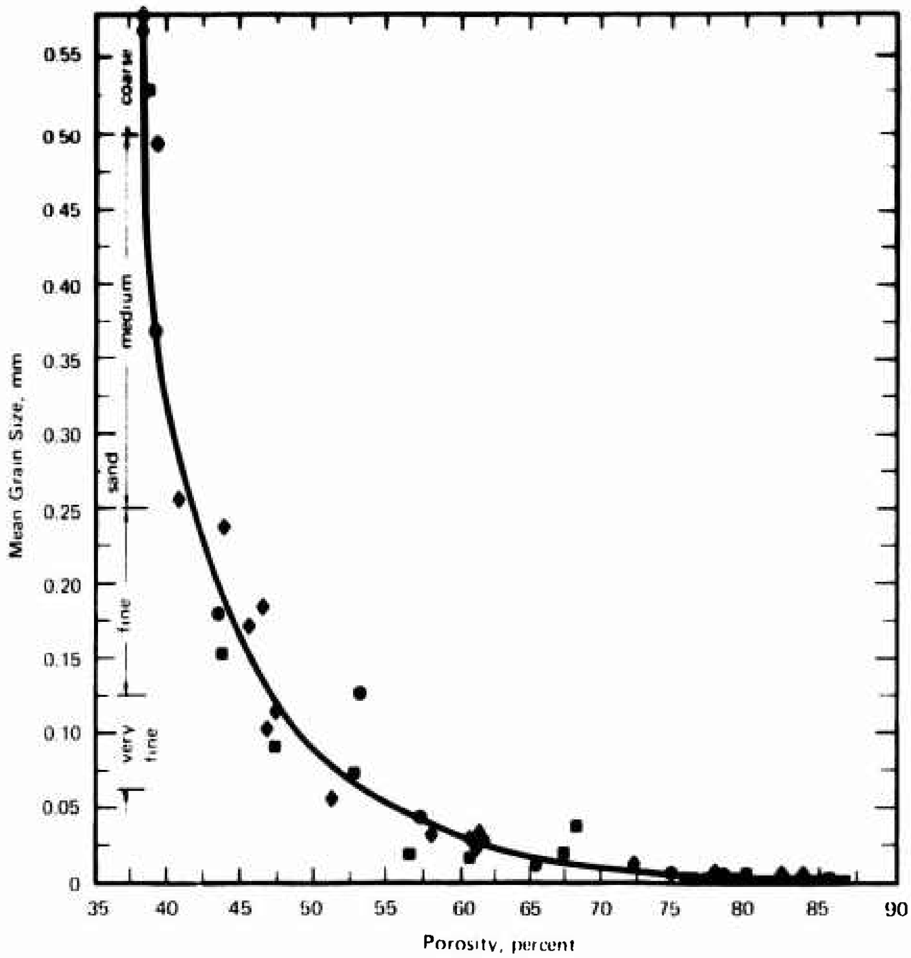

Figure 8. Sediment porosity versus mean grain size in mm. Symbols: diamonds-from Tables 1 and 2; circles-Table 3, from Ref. 18; squares-from Ref. 3.

grain sizes decreased without a change in porosity, the more numerous contacts should lead to an increase in attenuation. If porosity increased without a change in grain size, the fewer intergrain contacts should lead to decreased attenuation. In coarse and medium sands there is a relatively small increase in porosity with decrease in grain size (Fig. 8). As grain sizes decrease from 0.500 to 0.180 mm (1 to 2.5 ϕ), there is an increase in porosity of about 5 to 6 percent, and, according to Shumway and Igelman (Ref. 90), grain surface areas increase by about 240 percent. From 0.180 to 0.063 mm (2.5 to 4 ϕ), porosity increases about 9 to 10 percent, but grain surface area increases by about 320 percent. In summary, as grain sizes decrease from 0.500 to 0.063 mm (1 to 4 ϕ) there is an increase of about 15 percent porosity, but grain surface areas increase on the order of 600 to 700 percent. If attenuation is due to energy lost between mineral particles in contact, then the increase in surface area (more mineral particles in contact in a unit volume) should cause increased attenuation (k higher) which is not offset by greater porosity. In addition, the rate of increase in attenuation should be less from 0.500 to

0.180 mm (coarse-medium sand into fine sand) than the rate from about 0.180 mm into finer sizes (the increased rate actually occurs at about 0.177 mm: Figs. 3, 4).

Figure 3 (porosity versus k) illustrates these same relationships: k increases gradually as porosity increases from 38 to about 47 percent, and then increases sharply from this porosity range to porosities of about 52 percent. These increases in attenuation with increasing porosity in sands are related through grain size and the more numerous particles and greater grain surface areas, as discussed above.

Attenuation in Fine Silts and Clays

In fine silts and clays, shear strength, dynamic rigidity, and attenuation are apparently related to cohesion between fine particles. Cohesion is the resistance to shear stress which can be mobilized between adjacent fine particles which stick, or cohere, to each other. Cohesion is considered to be an inherent property of fine-grained, clayey sediments which is independent of stress; it is caused by physicochemical forces of an interparticle, intermolecular, and intergranular nature. This subject is reviewed in Ref. 19, which contains numerous references to the following items. Clay particles may not be in contact; they are apparently surrounded by layers of adsorbed water through which they interact with other particles. The amount of pore water, the distance between particles, and the number of interparticle contacts are important influences on cohesion. At points of near contact between clay particles, there is often bonding akin to cementation, especially in the presence of iron oxides, calcium, silica, and other minerals in solution in interstitial waters. Where sediments have been exposed to overburden pressures, there is apt to be pressure-point solution and redeposition. The structure of the mass of clay particles is important; for example, it has been demonstrated that the flocculated, or "cardhouse" structure (Ref. 3, Fig. 1d) is the strongest. These structures are largely determined by interparticle forces and the number of interparticle contacts. Differing clay minerals affect cohesion because of particle size and differing interparticle forces; for example, Na-montmorillonite has stronger cohesive bonds than kaolinite. Shear stress in clayey sediments occurs between particles and not through them; near-contact points will deform elastically or plastically (depending on stress) by an amount to sustain the effective stress. Homogeneous clays are practically impervious; for example, the coefficient of permeability (centimeters per second) in clean sand is of the order of 10^{-2}, whereas this coefficient in homogeneous clay is of the order of 10^{-8} (Ref. 91).

The highest attenuation in marine sediments is in very fine sand and coarse silt and in mixtures of sand and silt. As grain sizes become smaller, the sand and coarser silt grains become fewer and separated, and relative movement between grains is controlled by complex interparticle forces (cohesion). Cohesion, dynamic rigidity, and attenuation become less as grain sizes decrease and porosities increase to the level of high-porosity silt-clays. These effects are apparently due to the same causes: the strength of interparticle, net attractive forces and the number of interparticle contacts. These forces become weaker and contacts fewer with higher porosities, given the same mineral and structural type.

In the curves relating k to porosity, the sharp inflection which occurs at about 65 percent (Fig. 5) and at a mean grain size of about 6 ϕ (0.016 mm; Figs. 3, 4) indicates the probable range of porosity and mean grain size where the separation of larger grains (sand and silt) occurs. McCann and McCann (Ref. 8) also selected 6 ϕ as the grain size at which interparticle forces become dominant.

It is important in studies of attenuation in clay-water systems to understand the different mineral structures. Clay, in the presence of an electrolyte (e.g., seawater), forms flocculated structures which have a finite rigidity and transmit shear waves (see Ref. 19). When this flocculated structure is dispersed and the physicochemical bonds are broken, the mixture is an actual or near suspension. Attenuation in dispersed or flocculated clay-water systems is distinctly different. Cohen (Ref. 17) demonstrated that both μ and $i\mu'$ were essentially independent of frequency in flocculated kaolinite in distilled water; but when a deflocculating agent was added, the mixture lost rigidity, μ, and behaved as a Newtonian fluid. This means that $1/Q$ for shear stresses (μ'/μ) was independent of frequency, and attenuation dependent on the first power of frequency in the flocculated structure; but in the dispersed mixture, attenuation was dependent on f^2. When Cohen added NaCl, the mixture reflocculated and complex rigidity was the same as before. De Graft-Johnson (Ref. 92) experimentally showed that dispersed clay structures have higher log decrements than flocculated structures at the same water content. McCann (Ref. 93) explained this effect theoretically. Except as an approach to understanding theory, experimental data on attenuation in dispersed clay-water systems are not applicable to almost all marine sediments.

The data listed in the tables and plotted in the figures do not include attenuation in calcareous sediments. There is a small amount of evidence which indicates that high-porosity calcareous "oozes" may have higher attenuation values than sediments formed by clay minerals of the same general porosity and grain size. This might be due to the nature of interparticle bonds and the presence of foraminiferal tests in the calcareous oozes.

Effects of Wetting

When a dry sand is wetted, shear velocity and rigidity are both reduced (Ref. 19, Figs. 8 and 9, and Ref. 28), and the log decrement increases (Ref. 56). Pilbeam and Vaisnys (Ref. 94) also noted that lubrication of grains in a granular aggregate generally increased relative energy losses. The role of rigidity can be seen by substituting $(\mu/\rho)^{1/2}$ for V_s in Eq. (4).

$$a_s = \Delta_s (\rho)^{1/2} f/\mu^{1/2} \qquad (13)$$

Dynamic rigidity is less in wet sand than in dry apparently because lubrication allows grains to move under less shear stress (Ref. 28). Rigidity is also less in saturated porous rock than in dry rock (Ref. 95), p. 578). Hardin and Richart noted that a mere 1.4 percent increase in water content can reduce rigidity by as much as 15 percent. Experimentally, the log decrement of shear waves in sand and rock increases upon wetting

(Refs. 56 and 95), but shear-wave velocity decreases because of the decrease in rigidity. Density would increase with wetting. Thus, all factors in Eq. (13) result in increase in a_s (after wetting) at any given frequency. A variation of Eq. (4), $\Delta_s = a_s V_s / f$, indicates that the increase in the log decrement upon wetting is due entirely to the increase in a_s, because V_s decreases. The apparent reason for the increase in attenuation of shear waves upon wetting or lubrication is that, given the same grain size and porosity (or the same number of interparticle contacts), the reduced rigidity allows greater slippage of grains under any given shear stress, resulting in greater energy losses because of greater intergrain friction.

Effects of Pressure

Although attenuation decreases with increasing effective pressure (*i.e.*, overburden pressure), as discussed below, it is believed that the laboratory and *in situ* measurements of this report are comparable because of the low or negligible pressures involved. This was also a conclusion of McCann and McCann (Ref. 8) after comparing their *in situ* measurements, at depths of 2 m in sediments, with the laboratory measurements of Shumway (Ref. 18). The *in situ* measurements using probes (listed in Tables 1-4a) were at sediment depths from about 0.3 to 2 m; thus, effective pressures were very small. Laboratory samples had negligible effective pressures. Consequently, the data and conclusions of this report are concerned with surficial sediments under very low, or negligible effective pressures. Some of the measurements at seismic frequencies in Table 4b involved greater effective (overburden) pressures, but these were not involved in calculations of equations relating attenuation and frequency.

Increase of effective pressure increases rigidity and shear- and compressional-wave velocity in rocks (Refs. 59 and 96) and sand (Refs. 19 and 28); but in both rocks and sand, attenuation and log decrement decrease (for rocks, see Refs. 97, 98, 99; for sand, Refs. 27 and 56). As various authors have noted, these effects are apparently due to pressure effects on grain elastic moduli and grains in harder contact, a situation that offers greater resistance to shear stress (shear modulus, or rigidity, greater), but allows less intergrain movement, which constraint reduces energy loss through intergrain friction (attenuation less).

PREDICTION OF COMPRESSIONAL-WAVE ATTENUATION

Estimation or prediction of compressional-wave attenuation in saturated sediments is of considerable importance in geophysics and underwater acoustics. The relationships between attenuation (as expressed by the constant k) and grain size and porosity previously discussed and illustrated (Figs. 3, 4, and 5) afford a very simple method for prediction of approximate values of attenuation at most frequencies of interest in geophysics and underwater acoustics in all major sediment types (with the possible exception of calcareous sediments).

In this report, attenuation is related to frequency in the form

$$\alpha = kf^n \tag{1}$$

where

α is attenuation, dB/m
f is frequency, kHz

The case has been made that the exponent of frequency, n, is close to 1. If n is taken as 1, the only variable in the equation is the constant k. This constant varies with mean grain size (Figs. 3 and 4) or porosity (Fig. 5).

Mean grain size of a sediment can be easily determined in both wet and dry sediment; porosity can be determined in saturated materials. In the absence of measurements, both properties can be predicted, given general environment and predicted sediment type. References 3 and 100 discussed and listed averages of these properties for various sediment types in the major environments and discussed methods of prediction. Technically, porosity should be a better index of attenuation because it is a better measure of the number of interparticle contacts (at the same grain size, the degree of packing determines porosity and the number of interparticle contacts).

The method for estimating or predicting approximate values of attenuation (given mean grain size or porosity) follows; a numerical example is included:

(1) Determine or predict mean grain size or porosity of the sediment.

Example: mean grain size, $M_z = 7.5\ \phi$.

(2) Refer to the mean grain size or porosity versus k diagram (Figs. 3, 4, 5), and determine the value of k. Regression equations for these data are listed in the figure captions.

Example: Fig. 3 (mean grain size versus k diagram) yields: $k = 0.07$.

(3) Insert k into Eq. (1), reproduced above.

Example: insertion of the value of k yields: $\alpha = 0.07f^1$

(4) Compute attenuation (in dB/m) at the desired frequency (kHz)

The experimental evidence indicates that the exponent of frequency probably varies in saturated sediments between 0.9 and 1.1. Theoretically, or statistically, the value might be slightly less than 1 (Refs. 45 and 51). Use of a value of k, assuming n is 1 (rather than 0.9 or 1.1), results in small differences in computed attenuation at lower frequencies, those of most interest in geophysics and underwater acoustics. For example, at 3 kHz, if 0.07 is taken as k, and n is varied between 0.9, 1.0, and 1.1, the computed attenuations are 0.19, 0.21, and 0.23 dB/m; at f = 50 Hz, the values of attenuation are 0.005, 0.004, and 0.003 dB/m. In a fine sand, if k is 0.5, the values of attenuation at 3 kHz (and n = 0.9, 1.0, and 1.1) are 1.34, 1.50, and 1.67 dB/m; at 50 Hz, attenuations are 0.034, 0.025, and 0.019 dB/m.

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