NR 046-x

Final Report for Work on Contract Nonr-08400

In this project, we have calculated the nine direction cosines for the conversion to rectangular galactic components of motion for all the stars in the General Catalogue. For a given star of right ascension α and declination δ, the direction of increasing α, that of increasing δ, and that of the radius vector, are at right angles. The two rectangular proper motion components are customarily observed in right ascension and declination.

The observations of proper motions result in rectangular components in right ascension, μ_α, and in declination, μ_δ. These are at right angles to the line of sight and are given in seconds of arc per annum. The third component of the motion of a star in space is the radial velocity, V, measured in km./sec. along the radius vector. The axes of the rectangular galactic coordinate system are fixed in space. The first two are parallel to the galactic plane toward longitudes 0° and 90° respectively, and the third is toward the galactic north pole.

The nine direction cosines pertain to the angles between the axes μ_α, μ_δ and the radius vector, which are functions of α, δ, and the galactic axes which are not. We call x_1, x_2, x_3 the cosines with the first, y_1, y_2, y_3 with the second, and z_1, z_2, z_3 with the third galactic axis. The pole of the galactic plane has been taken to be at $12^h\ 40^m$ +28° (1900), as is common practice.

In order to find the space velocity of a star, it is necessary to know its parallax with a much greater percentage of accuracy than we do at present (for all except comparatively few stars). If p is the parallax, the galactic components of the space velocity in km./sec. are:

$$x = \frac{4.74}{p}\mu_\alpha x_1 + \frac{4.74}{p}\mu_\delta x_2 + V x_3$$

$$y = \frac{4.74}{p}\mu_\alpha y_1 + \frac{4.74}{p}\mu_\delta y_2 + V y_3$$

$$z = \frac{4.74}{p}\mu_\alpha z_1 + \frac{4.74}{p}\mu_\delta z_2 + V z_3$$

These equations are frequently used to compute a mean parallax for a number of stars, substituting for the left-hand members the known components of the velocity of the sun (with opposite signs). They are also used in correcting spectroscopic parallaxes. In problems of this kind, one needs the quantities $\mu_\alpha x_1 + \mu_\delta x_2$, $\mu_\alpha y_1 + \mu_\delta y_2$ and $\mu_\alpha z_1 + \mu_\delta z_2$, and these have been computed under the present project. Finally, the radial velocities, for about 7,000 stars for which these are known, have been multiplied by x_3, y_3, and z_3.

The basic data for computing the nine direction cosines and the contributions to the galactic rectangular velocity components resulting from the proper motions of the 33, 342 stars in the General Catalogue was on hand at Columbia University in the form of two catalogs of punched cards. These are called Card Catalog A and Card Catalog B. A mimeographed description of these cards is available.

To compute the nine direction cosines, the following information from Catalog B was reproduced onto a set of blank cards which will be referred to as GMR Card Set I:

> GC number, sin b (b is the galactic latitude), sin $(\alpha + 80°)$, 0.4695 sin δ, 0.883 cos δ, sec $(\alpha + 80°)$, ℓ (ℓ is the galactic longitude; not used in computations)

The formulae used to compute the cosines to make convenient use of the way the data were punched on Catalog B are:

$$x_1 = -\sin(\alpha + 80°)$$
$$x_2 = (-0.4695 \sin\delta)(P)$$
$$\text{where } P = 1/\sin 28° \sec(\alpha + 80°) = 2.1300545/\sec(\alpha + 80°)$$
$$x_3 = 0.5317\,(0.883 \cos\delta)\,P$$

$$y_1 = 0.2204\,P$$
$$y_2 = -0.4695 \sin\delta \sin(\alpha + 80°) + 0.883 \cos\delta$$
$$y_3 = 0.5317\,(0.883 \cos\delta) \sin(\alpha + 80°) + 1.8807\,(0.4695 \sin\delta)$$

$$z_1 = -0.4145\,P$$
$$z_2 = 1.8807\,(0.4695 \sin\delta) \sin(\alpha + 80°) + 0.5317\,(0.883 \cos\delta)$$
$$z_3 = \sin b$$

The computation was divided into three parts, each part computed during one run of the cards through the calculating machine as follows:

Run I

(1) y_2 was computed and punched — y_2

(2) the quotient 2.1300545/sec $(\alpha + 80°)$ was developed and punched — intermediate result

(3) 0.5317 (0.883 cos δ) was computed and punched — intermediate result

(4) sin $(\alpha + 80°)$ was rounded to 3 decimal places and punched with opposite sign — x_1

This computation was begun on the 602-A multiplier but took so much time per card that it was broken up into two parts. Computations (1) and (4) were done on the 602 multiplier and (2) and (3) on the 602-A, the two machines running simultaneously. The entire Run I was then checked on the 604 multiplier.

Run II: The quotient (which we have called P) developed in (2) of Run I, was multiplied on the 602-A by:

(5)	0.5317 (0.883 cos δ)	x_3
(6)	0.2204	y_1
(7)	-0.4145	z_1
(8)	-0.4695 sin δ	x_2

This computation was checked on the 604 multiplier.

Run III

(9)	1.8807 (0.4695 sin δ) was computed and stored in the machine for use in the next two steps	
(10)	1.8807 (0.4695 sin δ) sin (α + 80°) + 0.5317 (0.883 cos δ) was computed and punched	z_2
(11)	0.5317 (0.883 cos δ) sin (α + 80°) + 1.8807 (0.4695 sin δ) was computed and punched	y_3
(12)	sin δ was rounded to three decimals and punched. On cards where it was given in complement form it was converted to true figures.	z_3

This computation was then checked on the 604 multiplier.

The following information from Catalog A was reproduced onto a set of blank cards which will be referred to as OMR Card Set II:

GC number, magnitude, spectrum, α (1900), δ (1900), μ_α, μ_δ

Then, the nine direction cosines and ℓ were transferred onto this set from OMR Card Set I.

The contributions to the galactic rectangular velocity components resulting from proper motions were computed in one run on the 602-A multiplier.

Run IV

(13) $\mu_\alpha x_1 + \mu_\delta x_2$
(14) $\mu_\alpha y_1 + \mu_\delta y_2$
(15) $\mu_\alpha z_1 + \mu_\delta z_2$

This was checked on the 604 multiplier.

A list of all the information on ONR Card Set II was made on the 405 tabulating machine, two lines for each star. The first line contains all the identification data of the star, the second line the computed quantities. The size of this list is such that it cannot be included in this report.*

ONR Card Set III is a set of about 7,000 cards which are the General Catalogue stars for which we have radial velocities. The contributions to the galactic rectangular velocity components resulting from radial velocity were computed on the 602-A (Run V) by multiplying the radial velocity by:

(16) x_3
(17) y_3
(18) z_3.

The machines used in the computations were made available through the generosity of the Watson Scientific Computing Laboratory.

* We have one carbon copy of the list which is available for the ONR, if desired. The cards, or part thereof, are available for reproduction at cost to qualified persons.

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