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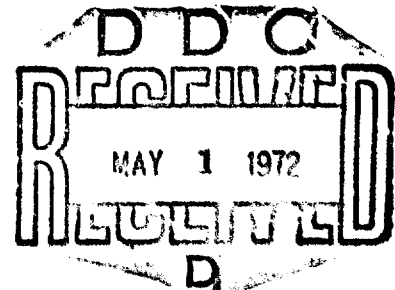
Calculation of the Radar Cross Section of a Perfectly Conducting Sphere

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*Radar Geophysics Branch
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13. ABSTRACT A Fortran computer program has been written to compute the radar cross section of a conducting sphere. The program is useful when a metal sphere is used as a standard target for calibrating a radar for target-cross-section measurement. It has been used to machine-plot with high precision a curve of cross section (normalized to the optical cross section) as a function of the radius/wavelength ratio. The computed data used to plot the curve are also presented in tabular form. The mathematics of the problem are briefly reviewed, and a listing of the computed program is given.			

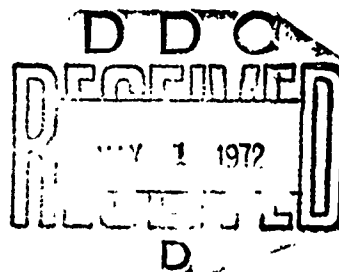
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ABSTRACT

A Fortran computer program has been written to compute the radar cross section of a conducting sphere. The program is useful when a metal sphere is used as a standard target for calibrating a radar for target-cross-section measurement. It has been used to machine-plot with high precision a curve of cross section (normalized to the optical cross section) as a function of the radius/wavelength ratio. The computed data used to plot the curve are also presented in tabular form. The mathematics of the problem are briefly reviewed, and a listing of the computer program is given.

AUTHORIZATION

NRL Problem R02-55
Project RF-151-402-4011

CALCULATION OF THE RADAR CROSS SECTION OF A PERFECTLY CONDUCTING SPHERE

INTRODUCTION

This calculation is of interest because spheres of high-conductivity material (metal) are used as standard targets for calibrating a radar for target cross-section measurement. Published curves based on calculations are unsatisfactory for precise work because they appear to be imprecisely plotted (1) or the scale size is too small for accurate reading of values (2,3). Therefore the problem of writing a computer program to calculate the cross section and to plot it accurately by machine was undertaken.

The computation requires summing a series whose terms involve spherical Bessel and Hankel functions. These terms are complex quantities. Manual calculation would be extremely laborious and computer calculation also poses some difficult problems. After a successful program had been written, it was learned that others had also done it (3). Their programming approach was somewhat different, and their actual program is not presented in their paper. The program developed here is presented in a form that allows it to be used for the calculation of cross sections of specific sphere sizes, by reading in data giving the radius of the sphere and the radar frequency. This procedure would be used if the curve and table do not provide a sufficiently accurate number.

RESULTS

The computer program was used to machine plot a curve, Fig. 1, of the normalized radar cross section $\sigma/\pi a^2$, where σ is the actual cross section and a is the radius of the sphere, as a function of the ratio a/λ , where λ is the wavelength. The normalizing quantity πa^2 is of course the optical cross section of the sphere. The plot was made in a large size (20 x 25 inches) on the NRL Gerber Model 875 Automatic Drafting Machine, with an accuracy of the order of .001 inch. The coordinate grid was plotted by the machine also, so that there are no registration errors, as might occur if the plot were made on standard graph paper.

The computations were made for values of a/λ from .05 to 5.045 in steps of .005. These results are tabulated in Table 1. The plot was also made from these values. Even though the a/λ interval .005 is quite small, a direct digital plot does not result in a perfectly smooth curve. Therefore a "smoothing" subroutine, described in a previous NRL report (4), was used for

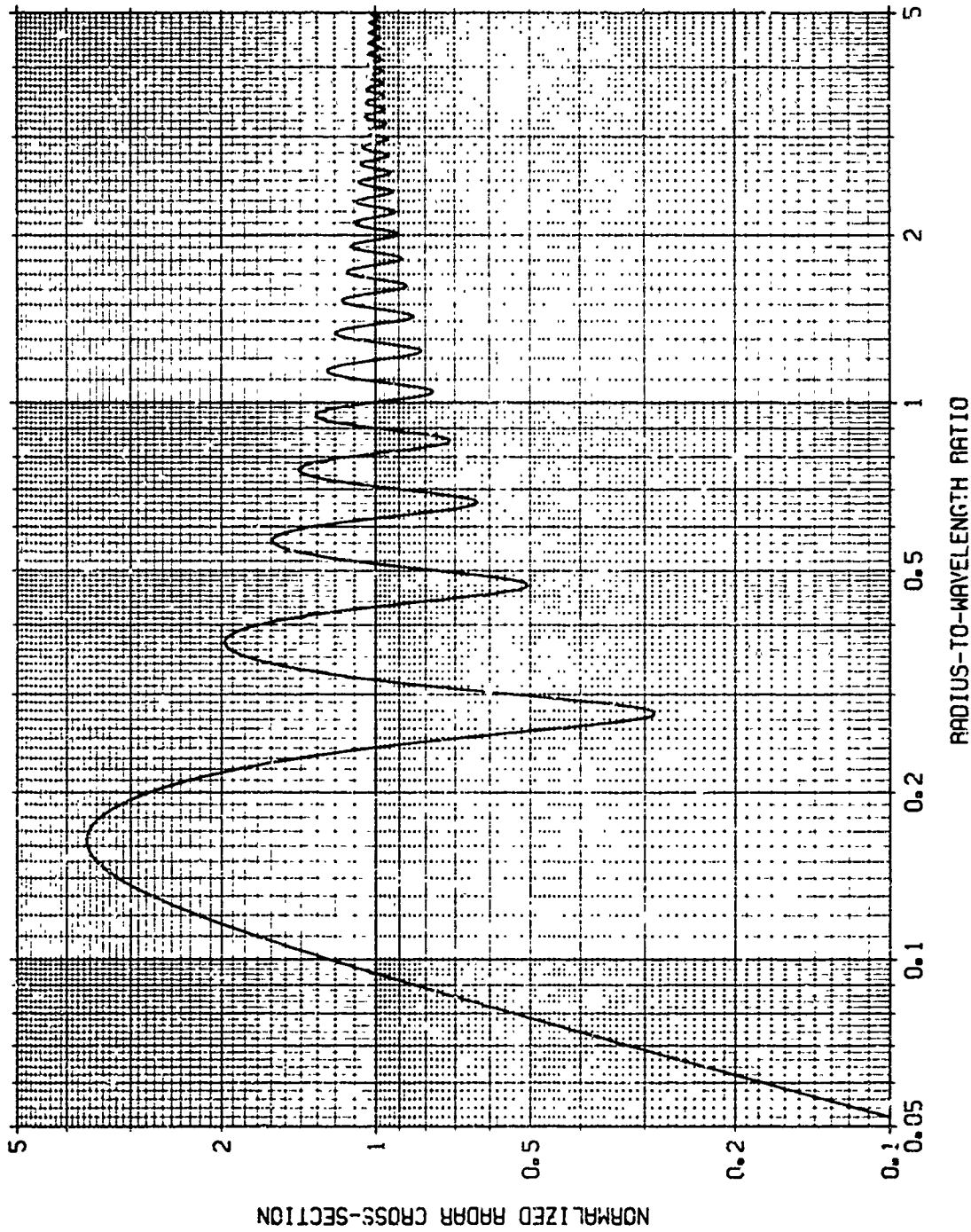


Fig. 1 - Radar cross section of a perfectly conducting sphere, normalized to the optical cross section πa^2 , as a function of the radius-to-wavelength ratio a/λ .

Table 1

Normalized Values of Radar Cross Section (SIGMA) of a Perfectly Conducting Sphere
for Specified Radius/Wavelength (RATIO) Values

RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA
0.0500	0.0860	0.3000	0.5412	0.5500	1.5211	0.8000	1.1222
0.0550	0.1254	0.3050	0.6544	0.5550	1.5567	0.8050	1.0659
0.0600	0.1767	0.3100	0.7206	0.5600	1.5793	0.8100	1.0096
0.0650	0.2418	0.3150	0.9158	0.5650	1.5884	0.8150	0.9550
0.0700	0.3229	0.3200	1.0557	0.5700	1.5840	0.8200	0.9034
0.0750	0.4218	0.3250	1.1963	0.5750	1.5663	0.8250	0.8551
0.0800	0.5404	0.3300	1.3337	0.5800	1.5360	0.8300	0.8143
0.0850	0.6800	0.3350	1.4641	0.5850	1.4941	0.8350	0.7790
0.0900	0.8417	0.3400	1.5845	0.5900	1.4417	0.8400	0.7510
0.0950	1.0258	0.3450	1.6919	0.5950	1.3805	0.8450	0.7310
0.1000	1.2315	0.3500	1.7839	0.6000	1.3122	0.8500	0.7194
0.1050	1.4572	0.3550	1.8587	0.6050	1.2388	0.8550	0.7163
0.1100	1.6999	0.3600	1.9150	0.6100	1.1622	0.8600	0.7218
0.1150	1.9549	0.3650	1.9517	0.6150	1.0845	0.8650	0.7355
0.1200	2.2164	0.3700	1.9684	0.6200	1.0080	0.8700	0.7569
0.1250	2.4772	0.3750	1.9653	0.6250	0.9344	0.8750	0.7855
0.1300	2.7294	0.3800	1.9427	0.6300	0.8658	0.8800	0.8203
0.1350	2.9646	0.3850	1.9015	0.6350	0.8037	0.8850	0.8603
0.1400	3.1747	0.3900	1.8431	0.6400	0.7498	0.8900	0.9043
0.1450	3.3524	0.3950	1.7691	0.6450	0.7053	0.8950	0.9512
0.1500	3.4921	0.4000	1.6815	0.6500	0.6711	0.9000	0.9996
0.1550	3.5898	0.4050	1.5827	0.6550	0.6440	0.9050	1.0482
0.1600	3.6435	0.4100	1.4753	0.6600	0.6362	0.9100	1.0957
0.1650	3.6533	0.4150	1.3620	0.6650	0.6359	0.9150	1.1407
0.1700	3.6208	0.4200	1.2458	0.6700	0.6468	0.9200	1.1822
0.1750	3.5490	0.4250	1.1296	0.6750	0.6685	0.9250	1.2191
0.1800	3.4418	0.4300	1.0165	0.6800	0.7001	0.9300	1.2501
0.1850	3.3038	0.4350	0.9092	0.6850	0.7407	0.9350	1.2753
0.1900	3.1397	0.4400	0.8106	0.6900	0.7889	0.9400	1.2932
0.1950	2.9542	0.4450	0.7228	0.6950	0.8435	0.9450	1.3039
0.2000	2.7522	0.4500	0.6481	0.7000	0.9028	0.9500	1.3069
0.2050	2.5380	0.4550	0.5881	0.7050	0.9653	0.9550	1.3025
0.2100	2.3160	0.4600	0.5439	0.7100	1.0290	0.9600	1.2908
0.2150	2.0900	0.4650	0.5164	0.7150	1.0925	0.9650	1.2721
0.2200	1.8638	0.4700	0.5057	0.7200	1.1539	0.9700	1.2477
0.2250	1.6412	0.4750	0.5118	0.7250	1.2116	0.9750	1.2166
0.2300	1.4255	0.4800	0.5339	0.7300	1.2641	0.9800	1.1813
0.2350	1.2202	0.4850	0.5710	0.7350	1.3101	0.9850	1.1423
0.2400	1.0284	0.4900	0.6217	0.7400	1.3483	0.9900	1.1007
0.2450	0.8533	0.4950	0.6841	0.7450	1.3780	0.9950	1.0575
0.2500	0.6976	0.5000	0.7564	0.7500	1.3983	1.0000	1.0140
0.2550	0.5641	0.5050	0.8363	0.7550	1.4089	1.0050	0.9712
0.2600	0.4548	0.5100	0.9216	0.7600	1.4095	1.0100	0.9304
0.2650	0.3718	0.5150	1.0098	0.7650	1.4004	1.0150	0.8925
0.2700	0.3161	0.5200	1.0986	0.7700	1.3819	1.0200	0.8586
0.2750	0.2884	0.5250	1.1856	0.7750	1.3545	1.0250	0.8294
0.2800	0.2886	0.5300	1.2686	0.7800	1.3193	1.0300	0.8057
0.2850	0.3159	0.5350	1.3454	0.7850	1.2771	1.0350	0.7881
0.2900	0.3688	0.5400	1.4142	0.7900	1.2293	1.0400	0.7770
0.2950	0.4449	0.5450	1.4732	0.7950	1.1772	1.0450	0.7725

Table 1 (Continued)

Normalized Values of Radar Cross Section (SIGMA) of a Perfectly Conducting Sphere
for Specified Radius/Wavelength (RATIO) Values

RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA
1.0500	0.7158	1.3000	1.0951	1.5500	1.1124	1.8000	0.8977
1.0550	0.7276	1.3050	1.1209	1.5550	1.0934	1.8050	0.8911
1.0600	0.7387	1.3100	1.1436	1.5600	1.0723	1.8100	0.8876
1.0650	0.7495	1.3150	1.1626	1.5650	1.0496	1.8150	0.8873
1.0700	0.7605	1.3200	1.1774	1.5700	1.0260	1.8200	0.8901
1.0750	0.7758	1.3250	1.1876	1.5750	1.0022	1.8250	0.8959
1.0800	0.9096	1.3300	1.1931	1.5800	0.9787	1.8300	0.9047
1.0850	0.9460	1.3350	1.1936	1.5850	0.9562	1.8350	0.9160
1.0900	0.9840	1.3400	1.1893	1.5900	0.9353	1.8400	0.9297
1.0950	1.0224	1.3450	1.1803	1.5950	0.9165	1.8450	0.9452
1.1000	1.0603	1.3500	1.1669	1.6000	0.9003	1.8500	0.9622
1.1050	1.0966	1.3550	1.1495	1.6050	0.8871	1.8550	0.9802
1.1100	1.1305	1.3600	1.1286	1.6100	0.8772	1.8600	0.9988
1.1150	1.1620	1.3650	1.1047	1.6150	0.8709	1.8650	1.0173
1.1200	1.1872	1.3700	1.0786	1.6200	0.8682	1.8700	1.0354
1.1250	1.2088	1.3750	1.0510	1.6250	0.8693	1.8750	1.0525
1.1300	1.2250	1.3800	1.0225	1.6300	0.8741	1.8800	1.0683
1.1350	1.2355	1.3850	0.9940	1.6350	0.8823	1.8850	1.0822
1.1400	1.2460	1.3900	0.9663	1.6400	0.8938	1.8900	1.0940
1.1450	1.2536	1.3950	0.9400	1.6450	0.9082	1.8950	1.1032
1.1500	1.2613	1.4000	0.9158	1.6500	0.9252	1.9000	1.1099
1.1550	1.2683	1.4050	0.8944	1.6550	0.9442	1.9050	1.1136
1.1600	1.2701	1.4100	0.8763	1.6600	0.9646	1.9100	1.1145
1.1650	1.1773	1.4150	0.8620	1.6650	0.9851	1.9150	1.1124
1.1700	1.1503	1.4200	0.8517	1.6700	1.0079	1.9200	1.1075
1.1750	1.1201	1.4250	0.8458	1.6750	1.0295	1.9250	1.0998
1.1800	1.0875	1.4300	0.8443	1.6800	1.0503	1.9300	1.0897
1.1850	1.0533	1.4350	0.8473	1.6850	1.0697	1.9350	1.0774
1.1900	1.0184	1.4400	0.8545	1.6900	1.0873	1.9400	1.0633
1.1950	0.9839	1.4450	0.8659	1.6950	1.1027	1.9450	1.0477
1.2000	0.9506	1.4500	0.8809	1.7000	1.1153	1.9500	1.0311
1.2050	0.9193	1.4550	0.8993	1.7050	1.1249	1.9550	1.0140
1.2100	0.8910	1.4600	0.9204	1.7100	1.1313	1.9600	0.9966
1.2150	0.8663	1.4650	0.9437	1.7150	1.1343	1.9650	0.9797
1.2200	0.8458	1.4700	0.9685	1.7200	1.1338	1.9700	0.9635
1.2250	0.8307	1.4750	0.9942	1.7250	1.1300	1.9750	0.9485
1.2300	0.8194	1.4800	1.0200	1.7300	1.1229	1.9800	0.9352
1.2350	0.8141	1.4850	1.0454	1.7350	1.1128	1.9850	0.9237
1.2400	0.8141	1.4900	1.0695	1.7400	1.0999	1.9900	0.9145
1.2450	0.8194	1.4950	1.0918	1.7450	1.0846	1.9950	0.9078
1.2500	0.8301	1.5000	1.1117	1.7500	1.0674	2.0000	0.9037
1.2550	0.8455	1.5050	1.1296	1.7550	1.0486	2.0050	0.9022
1.2600	0.8652	1.5100	1.1423	1.7600	1.0289	2.0100	0.9035
1.2650	0.8887	1.5150	1.1522	1.7650	1.0087	2.0150	0.9075
1.2700	0.9152	1.5200	1.1582	1.7700	0.9887	2.0200	0.9140
1.2750	0.9441	1.5250	1.1602	1.7750	0.9692	2.0250	0.9229
1.2800	0.9745	1.5300	1.1581	1.7800	0.9509	2.0300	0.9338
1.2850	1.0056	1.5350	1.1520	1.7850	0.9342	2.0350	0.9466
1.2900	1.0367	1.5400	1.1422	1.7900	0.9195	2.0400	0.9607
1.2950	1.0667	1.5450	1.1289	1.7950	0.9072	2.0450	0.9760

Table 1 (Continued)

Normalized Values of Radar Cross Section (SIGMA) of a Perfectly Conducting Sphere
for Specified Radius/Wavelength (RATIO) Values

RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA
2.0500	0.9918	2.3000	1.0838	2.5500	0.9681	2.8000	0.9609
2.0550	1.0079	2.3050	1.0798	2.5550	0.9588	2.8050	0.9687
2.0600	1.0237	2.3100	1.0739	2.5600	0.9507	2.8100	0.9772
2.0650	1.0389	2.3150	1.0660	2.5650	0.9440	2.8150	0.9864
2.0700	1.0530	2.3200	1.0566	2.5700	0.9389	2.8200	0.9959
2.0750	1.0657	2.3250	1.0458	2.5750	0.9355	2.8250	1.0055
2.0800	1.0767	2.3300	1.0339	2.5800	0.9339	2.8300	1.0149
2.0850	1.0857	2.3350	1.0213	2.5850	0.9341	2.8350	1.0240
2.0900	1.0924	2.3400	1.0082	2.5900	0.9361	2.8400	1.0324
2.0950	1.0968	2.3450	0.9951	2.5950	0.9398	2.8450	1.0399
2.1000	1.0986	2.3500	0.9824	2.6000	0.9452	2.8500	1.0464
2.1050	1.0979	2.3550	0.9702	2.6050	0.9521	2.8550	1.0516
2.1100	1.0947	2.3600	0.9590	2.6100	0.9602	2.8600	1.0555
2.1150	1.0890	2.3650	0.9490	2.6150	0.9694	2.8650	1.0579
2.1200	1.0812	2.3700	0.9405	2.6200	0.9794	2.8700	1.0589
2.1250	1.0714	2.3750	0.9337	2.6250	0.9900	2.8750	1.0583
2.1300	1.0598	2.3800	0.9286	2.6300	1.0008	2.8800	1.0562
2.1350	1.0468	2.3850	0.9259	2.6350	1.0115	2.8850	1.0527
2.1400	1.0328	2.3900	0.9251	2.6400	1.0220	2.8900	1.0478
2.1450	1.0181	2.3950	0.9263	2.6450	1.0318	2.8950	1.0417
2.1500	1.0031	2.4000	0.9295	2.6500	1.0408	2.9000	1.0347
2.1550	0.9882	2.4050	0.9347	2.6550	1.0487	2.9050	1.0267
2.1600	0.9738	2.4100	0.9416	2.6600	1.0553	2.9100	1.0182
2.1650	0.9604	2.4150	0.9501	2.6650	1.0605	2.9150	1.0093
2.1700	0.9481	2.4200	0.9599	2.6700	1.0641	2.9200	1.0002
2.1750	0.9374	2.4250	0.9706	2.6750	1.0660	2.9250	0.9912
2.1800	0.9286	2.4300	0.9825	2.6800	1.0662	2.9300	0.9825
2.1850	0.9218	2.4350	0.9946	2.6850	1.0647	2.9350	0.9744
2.1900	0.9172	2.4400	1.0068	2.6900	1.0615	2.9400	0.9670
2.1950	0.9150	2.4450	1.0189	2.6950	1.0568	2.9450	0.9606
2.2000	0.9151	2.4500	1.0304	2.7000	1.0506	2.9500	0.9554
2.2050	0.9175	2.4550	1.0411	2.7050	1.0432	2.9550	0.9514
2.2100	0.9222	2.4600	1.0508	2.7100	1.0348	2.9600	0.9487
2.2150	0.9290	2.4650	1.0590	2.7150	1.0255	2.9650	0.9474
2.2200	0.9378	2.4700	1.0658	2.7200	1.0156	2.9700	0.9476
2.2250	0.9482	2.4750	1.0708	2.7250	1.0055	2.9750	0.9492
2.2300	0.9600	2.4800	1.0739	2.7300	0.9953	2.9800	0.9522
2.2350	0.9729	2.4850	1.0751	2.7350	0.9853	2.9850	0.9565
2.2400	0.9865	2.4900	1.0744	2.7400	0.9759	2.9900	0.9619
2.2450	1.0004	2.4950	1.0718	2.7450	0.9672	2.9950	0.9684
2.2500	1.0143	2.5000	1.0674	2.7500	0.9594	3.0000	0.9757
2.2550	1.0278	2.5050	1.0612	2.7550	0.9529	3.0050	0.9836
2.2600	1.0406	2.5100	1.0535	2.7600	0.9476	3.0100	0.9920
2.2650	1.0523	2.5150	1.0446	2.7650	0.9439	3.0150	1.0005
2.2700	1.0626	2.5200	1.0345	2.7700	0.9417	3.0200	1.0090
2.2750	1.0712	2.5250	1.0237	2.7750	0.9411	3.0250	1.0173
2.2800	1.0780	2.5300	1.0124	2.7800	0.9421	3.0300	1.0251
2.2850	1.0827	2.5350	1.0008	2.7850	0.9447	3.0350	1.0323
2.2900	1.0853	2.5400	0.9894	2.7900	0.9488	3.0400	1.0386
2.2950	1.0856	2.5450	0.9784	2.7950	0.9542	3.0450	1.0438

Table 1 (Continued)

Normalized Values of Radar Cross Section (SIGMA) of a Perfectly Conducting Sphere
for Specified Radius/Wavelength (RATIO) Values

RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA
3.0500	1.0479	3.3000	1.0075	3.5500	0.9620	3.8000	1.0159
3.0550	1.0508	3.3050	1.0002	3.5550	0.9636	3.8050	1.0207
3.0600	1.0523	3.3100	0.9930	3.5600	0.9662	3.8100	1.0249
3.0650	1.0524	3.3150	0.9860	3.5650	0.9697	3.8150	1.0285
3.0700	1.0513	3.3200	0.9794	3.5700	0.9740	3.8200	1.0314
3.0750	1.0487	3.3250	0.9735	3.5750	0.9789	3.8250	1.0334
3.0800	1.0450	3.3300	0.9683	3.5800	0.9845	3.8300	1.0344
3.0850	1.0401	3.3350	0.9640	3.5850	0.9904	3.8350	1.0348
3.0900	1.0342	3.3400	0.9607	3.5900	0.9966	3.8400	1.0341
3.0950	1.0275	3.3450	0.9585	3.5950	1.0029	3.8450	1.0326
3.1000	1.0201	3.3500	0.9575	3.6000	1.0090	3.8500	1.0302
3.1050	1.0123	3.3550	0.9576	3.6050	1.0150	3.8550	1.0271
3.1100	1.0042	3.3600	0.9588	3.6100	1.0205	3.8600	1.0233
3.1150	0.9961	3.3650	0.9612	3.6150	1.0255	3.8650	1.0189
3.1200	0.9881	3.3700	0.9646	3.6200	1.0298	3.8700	1.0140
3.1250	0.9806	3.3750	0.9689	3.6250	1.0333	3.8750	1.0088
3.1300	0.9737	3.3800	0.9741	3.6300	1.0359	3.8800	1.0034
3.1350	0.9675	3.3850	0.9800	3.6350	1.0376	3.8850	0.9980
3.1400	0.9622	3.3900	0.9864	3.6400	1.0384	3.8900	0.9927
3.1450	0.9580	3.3950	0.9931	3.6450	1.0381	3.8950	0.9876
3.1500	0.9550	3.4000	1.0000	3.6500	1.0368	3.9000	0.9829
3.1550	0.9532	3.4050	1.0069	3.6550	1.0346	3.9050	0.9787
3.1600	0.9527	3.4100	1.0136	3.6600	1.0315	3.9100	0.9750
3.1650	0.9535	3.4150	1.0200	3.6650	1.0276	3.9150	0.9721
3.1700	0.9556	3.4200	1.0258	3.6700	1.0230	3.9200	0.9670
3.1750	0.9588	3.4250	1.0309	3.6750	1.0179	3.9250	0.9626
3.1800	0.9632	3.4300	1.0352	3.6800	1.0123	3.9300	0.9581
3.1850	0.9685	3.4350	1.0385	3.6850	1.0065	3.9350	0.9538
3.1900	0.9747	3.4400	1.0409	3.6900	1.0005	3.9400	0.9498
3.1950	0.9815	3.4450	1.0422	3.6950	0.9946	3.9450	0.9478
3.2000	0.9888	3.4500	1.0424	3.7000	0.9889	3.9500	0.9474
3.2050	0.9964	3.4550	1.0415	3.7050	0.9835	3.9550	0.9481
3.2100	1.0041	3.4600	1.0395	3.7100	0.9786	3.9600	0.9482
3.2150	1.0117	3.4650	1.0365	3.7150	0.9743	3.9650	0.9486
3.2200	1.0189	3.4700	1.0326	3.7200	0.9707	3.9700	0.9491
3.2250	1.0257	3.4750	1.0279	3.7250	0.9680	3.9750	0.9496
3.2300	1.0317	3.4800	1.0225	3.7300	0.9661	3.9800	1.0018
3.2350	1.0370	3.4850	1.0165	3.7350	0.9651	3.9850	1.0069
3.2400	1.0412	3.4900	1.0102	3.7400	0.9651	3.9900	1.0118
3.2450	1.0443	3.4950	1.0036	3.7450	0.9661	3.9950	1.0164
3.2500	1.0463	3.5000	0.9971	3.7500	0.9679	4.0000	1.0206
3.2550	1.0471	3.5050	0.9906	3.7550	0.9706	4.0050	1.0242
3.2600	1.0467	3.5100	0.9845	3.7600	0.9741	4.0100	1.0272
3.2650	1.0450	3.5150	0.9788	3.7650	0.9783	4.0150	1.0295
3.2700	1.0422	3.5200	0.9737	3.7700	0.9831	4.0200	1.0310
3.2750	1.0384	3.5250	0.9694	3.7750	0.9883	4.0250	1.0316
3.2800	1.0335	3.5300	0.9660	3.7800	0.9938	4.0300	1.0315
3.2850	1.0279	3.5350	0.9634	3.7850	0.9995	4.0350	1.0306
3.2900	1.0215	3.5400	0.9619	3.7900	1.0051	4.0400	1.0288
3.2950	1.0147	3.5450	0.9615	3.7950	1.0106	4.0450	1.0263

Table 1 (Continued)

Normalized Values of Radar Cross Section (SIGMA) of a Perfectly Conducting Sphere
for Specified Radius/Wavelength (RATIO) Values

RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA	RATIO	SIGMA
4.0500	1.0232	4.3000	0.9770	4.5500	0.9945	4.4000	1.0223
4.0550	1.0195	4.3050	0.9751	4.5550	0.9984	4.4050	1.0219
4.0600	1.0153	4.3100	0.9739	4.5600	1.0024	4.4100	1.0208
4.0650	1.0107	4.3150	0.9734	4.5650	1.0063	4.4150	1.0192
4.0700	1.0059	4.3200	0.9736	4.5700	1.0100	4.4200	1.0172
4.0750	1.0010	4.3250	0.9745	4.5750	1.0135	4.4250	1.0147
4.0800	0.9960	4.3300	0.9761	4.5800	1.0166	4.4300	1.0118
4.0850	0.9913	4.3350	0.9774	4.5850	1.0192	4.4350	1.0084
4.0900	0.9867	4.3400	0.9812	4.5900	1.0214	4.4400	1.0053
4.0950	0.9826	4.3450	0.9845	4.5950	1.0230	4.4450	1.0018
4.1000	0.9790	4.3500	0.9882	4.6000	1.0239	4.4500	0.9983
4.1050	0.9760	4.3550	0.9923	4.6050	1.0243	4.4550	0.9949
4.1100	0.9736	4.3600	0.9965	4.6100	1.0240	4.4600	0.9916
4.1150	0.9720	4.3650	1.0008	4.6150	1.0232	4.4650	0.9886
4.1200	0.9711	4.3700	1.0051	4.6200	1.0217	4.4700	0.9859
4.1250	0.9710	4.3750	1.0093	4.6250	1.0196	4.4750	0.9836
4.1300	0.9717	4.3800	1.0132	4.6300	1.0171	4.4800	0.9817
4.1350	0.9731	4.3850	1.0167	4.6350	1.0141	4.4850	0.9804
4.1400	0.9753	4.3900	1.0198	4.6400	1.0108	4.4900	0.9796
4.1450	0.9781	4.3950	1.0224	4.6450	1.0072	4.4950	0.9793
4.1500	0.9815	4.4000	1.0244	4.6500	1.0035	4.5000	0.9796
4.1550	0.9854	4.4050	1.0257	4.6550	0.9997	4.9050	0.9804
4.1600	0.9897	4.4100	1.0264	4.6600	0.9959	4.9100	0.9818
4.1650	0.9942	4.4150	1.0264	4.6650	0.9923	4.9150	0.9836
4.1700	0.9989	4.4200	1.0257	4.6700	0.9889	4.9200	0.9859
4.1750	1.0036	4.4250	1.0244	4.6750	0.9858	4.9250	0.9886
4.1800	1.0082	4.4300	1.0224	4.6800	0.9831	4.9300	0.9915
4.1850	1.0126	4.4350	1.0198	4.6850	0.9809	4.9350	0.9947
4.1900	1.0167	4.4400	1.0168	4.6900	0.9792	4.9400	0.9980
4.1950	1.0203	4.4450	1.0133	4.6950	0.9781	4.9450	1.0014
4.2000	1.0233	4.4500	1.0095	4.7000	0.9776	4.9500	1.0047
4.2050	1.0258	4.4550	1.0055	4.7050	0.9776	4.9550	1.0079
4.2100	1.0276	4.4600	1.0014	4.7100	0.9783	4.9600	1.0109
4.2150	1.0286	4.4650	0.9973	4.7150	0.9796	4.9650	1.0136
4.2200	1.0289	4.4700	0.9932	4.7200	0.9814	4.9700	1.0159
4.2250	1.0285	4.4750	0.9894	4.7250	0.9837	4.9750	1.0178
4.2300	1.0273	4.4800	0.9859	4.7300	0.9864	4.9800	1.0192
4.2350	1.0254	4.4850	0.9828	4.7350	0.9895	4.9850	1.0202
4.2400	1.0229	4.4900	0.9802	4.7400	0.9928	4.9900	1.0206
4.2450	1.0198	4.4950	0.9781	4.7450	0.9964	4.9950	1.0205
4.2500	1.0162	4.5000	0.9766	4.7500	1.0000	5.0000	1.0198
4.2550	1.0122	4.5050	0.9758	4.7550	1.0037	5.0050	1.0187
4.2600	1.0079	4.5100	0.9756	4.7600	1.0072	5.0100	1.0170
4.2650	1.0034	4.5150	0.9761	4.7650	1.0105	5.0150	1.0149
4.2700	0.9989	4.5200	0.9772	4.7700	1.0136	5.0200	1.0125
4.2750	0.9945	4.5250	0.9784	4.7750	1.0163	5.0250	1.0097
4.2800	0.9902	4.5300	0.9812	4.7800	1.0185	5.0300	1.0067
4.2850	0.9862	4.5350	0.9840	4.7850	1.0203	5.0350	1.0036
4.2900	0.9826	4.5400	0.9872	4.7900	1.0216	5.0400	1.0004
4.2950	0.9795	4.5450	0.9907	4.7950	1.0222	5.0450	0.9971

the plotting; this subroutine interpolates additional points where they are needed, using a cubic interpolating polynomial.

The program was then modified so that it calculates the radar cross section, in both square meters and square feet, of a specific sphere at a specific frequency. Printouts of sample calculations using this program are shown as Figs. 2, 3, and 4. A listing of the program is given later in this report.

MATHEMATICAL FORMULAS

Kerr¹ gives the following equation (p. 451) for the radar cross section σ of a sphere of radius a :

$$\frac{\sigma}{\pi a^2} = \frac{1}{\rho^2} \left| \sum_{n=1}^{\infty} (-1)^n (2n+1) (a_n^s - b_n^s) \right|^2 \quad (1)$$

where $\rho = 2\pi a/\lambda$, λ is the wavelength, and the a_n^s and b_n^s are terms of a "multipole expansion" -- that is, these terms are proportional to the amplitudes of magnetic and electric multipoles induced in the sphere by the incident wave. When the sphere is perfectly conducting (Kerr, p. 452):

$$a_n^s = - \frac{j_n(\rho)}{h_n^{(2)}(\rho)} \quad (2)$$

$$b_n^s = \frac{-[\rho j_n(\rho)]'}{[\rho h_n^{(2)}(\rho)]'} \quad (3)$$

where the primes denote differentiation with respect to the argument. The functions j_n and $h_n^{(2)}$ are, respectively, the spherical Bessel function of the first kind, and the spherical Hankel function of the second kind.

A subroutine to evaluate j_n , j_n' , and also the spherical Bessel function of the second kind and its derivative, y_n and y_n' , was obtained from the NRL CDC-3800 program library.* No subroutine for evaluating the

*Library Catalog Identification C3-UCSD-BFFGH. Subroutine BFFGH was written by Frank Hagin of Texas Instruments, Dallas Texas.

CALCULATION OF RADAR CROSS SECTION OF SPHERE
REFERENCE - KERR, PROPAGATION OF SHORT RADIO WAVES
RAD LAB SERIES VOL, 13, P, 451, EQ, 29,

INPUT DATA AND DERIVED QUANTITIES --

RADIUS, METERS	1,524
RADIUS, FEET	5,000
RADAR FREQUENCY, MEGAHERTZ	138,60
WAVELENGTH, METERS	2,16
RADIUS/WAVELENGTH RATIO	0,7046
CIRCUMFERENCE/WAVELENGTH RATIO	4,4270

OUTPUT DATA --

NUMBER OF SERIES TERMS ADDED	12
RADAR CROSS SECTION, NORMALIZED TO OPTICAL	0,9599
RADAR CROSS SECTION, SQUARE METERS	7,0038
RADAR CROSS SECTION, SQUARE FEET	75,3884

Fig. 2 - Computer printout from Program SPHERE for radius
5 feet and frequency 138.6 MHz.

CALCULATION OF RADAR CROSS SECTION OF SPHERE
 REFERENCE - KERR, PROPAGATION OF SHORT RADIO WAVES
 RAD LAB SERIES VOL, 13, P, 451, EQ, 29,

 INPUT DATA AND DERIVED QUANTITIES --

RADIUS, CENTIMETERS	50,00
RADIUS, FEET	1,640
RADAR FREQUENCY, MEGAHERTZ	1000,00
WAVELENGTH, CENTIMETERS	29,98
RADIUS/WAVELENGTH RATIO	1,6678
CIRCUMFERENCE/WAVELENGTH RATIO	10,4792

OUTPUT DATA --

NUMBER OF SERIES TERMS ADDED	20
RADAR CROSS SECTION, NORMALIZED TO OPTICAL	0,9984
RADAR CROSS SECTION, SQUARE METERS	0,7841
SQUARE METERS, EXPONENT FORMAT	7,8411-001
RADAR CROSS SECTION, SQUARE FEET	8,4401

Fig. 3 - Computer printout from Program SPHERE for radius
 0.5 meter and frequency 1000 MHz.

CALCULATION OF RADAR CROSS SECTION OF SPHERE
REFERENCE - KERR, PROPAGATION OF SHORT RADIO WAVES
RAD LAB SERIES VOL, 13, P. 451, EQ, 29,

INPUT DATA AND DERIVED QUANTITIES --

RADIUS, CENTIMETERS	6,00
RADIUS, INCHES	2,36
RADAR FREQUENCY, MEGAHERTZ	1000,00
WAVELENGTH, CENTIMETERS	29,98
RADIUS/WAVELENGTH RATIO	0,2001
CIRCUMFERENCE/WAVELENGTH RATIO	1,2575

OUTPUT DATA --

NUMBER OF SERIES TERMS ADDED	6
RADAR CROSS SECTION, NORMALIZED TO OPTICAL	2,7464
RADAR CROSS SECTION, SQUARE METERS	0,0311
SQUARE METERS, EXPONENT FORMAT	3,1061-002
RADAR CROSS SECTION, SQUARE FEET	0,3343
SQUARE FEET, EXPONENT FORMAT	3,3434-001

Fig. 4 - Computer printout from Program SPHERE for radius
0.06 meter and frequency 1000 MHz.

spherical Hankel function was available. However, the problem was solved by the use of an expression relating the spherical Hankel function to the spherical Bessel functions. The necessary relation is given by Messiah (5):

$$h_k^{(2)} = j_k - iy_k$$

$$\therefore h_k^{(2)'} = j_k' - iy_k'$$

Since Subroutine BFFGH gives j_k , j_k' , y_k , and y_k' , it is a simple matter to obtain $h_k^{(2)}$ and $h_k^{(2)'}$. (To interpret the relations given by Messiah, it is necessary to note that his function n_k is the negative of y_k , and his functions $h_k^{(+)}$ and $h_k^{(-)}$ are equal to $ih_k^{(1)}$ and $-ih_k^{(2)}$, respectively.)

For very small values of ρ , the approximation*

$$\frac{\sigma}{\pi a^2} = 9\rho^4 = 1.4027 (a/\lambda)^4 \times 10^4 \quad (4)$$

can be used; this is the well known Rayleigh scattering law (for back scattering).

For very large values of ρ (or a/λ), the asymptotic result is

$$\frac{\sigma}{\pi a^2} \approx 1$$

As will be discussed in some detail, this approximation is valid to 4 significant figures for $a/\lambda > 44$ (and possibly for somewhat smaller values).

FORTRAN PROGRAM

The program written to compute the cross section is simply a Fortran algorithm to evaluate Eq. (1). The summation of multipole terms (which are complex numbers) is carried to the point to which the fractional change in σ

*Reference 1, page 452.

due to the last term added is less than 10^{-6} . No exact analysis of the accuracy obtained by this procedure was made; however, a check on accuracy is afforded by the knowledge that as a/λ becomes large, the envelope of the oscillations of σ asymptotically approaches πa^2 . The numerical results obtained for large a/λ agree with this prediction. To test this, the program was run with the value of a/λ doubled for each successive calculation. The results are shown in Table 2. The initial value of a/λ was .0850 and this was doubled until the value 21.76 was reached, at which time the run terminated. An additional run was then made for the values $a/\lambda = 44$ and $a/\lambda = 88$. The second column of the table gives the number of series terms summed before the above-stated criterion was met (fractional change in $\sigma < 10^{-6}$).

Table 2
Results for Large Values of a/λ

a/λ	No. of Terms Summed	$\sigma/\pi a^2$
.0850	5	0.6800
.1700	6	3.6208
.3400	7	1.5845
.6800	11	0.7001
1.3600	17	1.1286
2.7200	28	1.0156
5.4400	47	0.9929
10.8800	83	0.9970
21.7600	158	0.9997
44.0000	303	1.0000
88.0000	586	1.0000

It is seen that for $a/\lambda \geq 44$, the values of $\sigma/\pi a^2$ differ from 1 by less than 5×10^{-5} . This suggests that the calculation is accurate to at least 4 significant figures for values of a/λ up to at least 88. To make sure that the values 1.0000 were not obtained "by accident" at $a/\lambda = 44$, the program was run with values $a/\lambda = 44.04$ and 44.045 . These numbers were chosen to insure that if a/λ just happened to result in $\sigma/\pi a^2 = 1.0000$ when the value is still actually oscillating appreciably, the oscillation would be revealed. The results, printed out to additional decimal places, were $\sigma/\pi a^2 = 0.999982$ and 0.999981 , respectively. It thus seems reasonable to conclude that the calculation does have at least 4-significant-figure accuracy.

Since for large values of a/λ the number of terms summed almost doubles with each doubling of a/λ , a point would eventually be reached at which the accuracy would be reduced by computer round-off error. However, in the CDC-3800 this event occurs at a value of a/λ well above the point at which the approximation $\sigma/\pi a^2 \approx 1$ can be assumed to 4 significant figures. It is therefore recommended that for values of $a/\lambda > 44$, the optical approximation $\sigma/\pi a^2 = 1.000$ be used instead of making an actual computer calculation.

The time required for the calculation with the CDC-3800 is approximately 58 milliseconds per series term, not including compilation time and input-output operations. Therefore calculation of single values of σ for the usual range of values of a/λ requires only a few seconds at most.

A listing of the Fortran program that has output of the type shown in Figs. 2-4 follows. The input to this program is a data card or cards (one for each combination of sphere size and frequency to be calculated). Any number of data cards can be stacked at the appropriate place in the deck, and followed by an end-of-file card. The program will then read each card, do the appropriate computations, and print out the results. The data card format is:

<u>Card Columns</u>	<u>Quantity</u>	<u>Format Specification</u>
1-10	Radius of sphere	F10.
11-20	Radar frequency	F10.
30	Radius units designator	I1

The radius of the sphere can be given in either meters or feet. If it is given in meters, Column 30 is left blank. If it is given in feet, a 1 is punched in Column 30. The radar frequency is given in megahertz. Both the radius and the frequency can be punched as floating-point numbers anywhere within the specified fields; a decimal point must be included unless the numbers are integers and are right-adjusted in the field.

As mentioned in the introduction, the computer program that follows was written before the paper of Adler and Johnson (3) was called to the author's attention. They describe a method of computing the spherical Bessel and Hankel functions based on a recursion relation. Use of this method instead of Subroutine BFFGH would have been somewhat more direct and possibly would reduce computing time; however, since the program as it stands works well and runs without excessive use of computer time, it was decided not to change it.

63/06/72

PROGRAM SPHERE

C
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COMPUTES RADAR CROSS SECTION OF A PERFECTLY CONDUCTING SPHERE,
MATHEMATICS GIVEN BY KERR, PROPAGATION OF SHORT RADIO WAVES
RAD LAB SERIES VOL. 13 (MCGRAW-HILL, 1950), P. 451, EQ. 29,
PROGRAM WRITTEN BY L. V. BLAKE, NRL CODE 5370, THIS VERSION 3/1/72

COMPUTES AS MANY CASES (SPHERE SIZES, FREQUENCIES) AS THERE ARE
DATA CARDS, RADIUS OF SPHERE, COLS, 1-10, FREQUENCY MEGAHERTZ,
COLS. 11-20, IF RADIUS IS GIVEN IN FEET, PUNCH A 1 IN COL. 30,
IF RADIUS IS GIVEN IN METERS, NO PUNCH (OR ZERO PUNCH) IN COL. 30.

```
TYPE COMPLEX HN2,DHN2,ANS,BNS,TERM,SUM
DATA (PI=3.1415926536), (M=0)
500 READ J0, RADIUS, FRQNCY, IUNITS
IF (EOF,60) 88,89
88 STOP
89 IF (IUNITS,EQ,1) J1,32
J1 RADFT=RADIUS
RADIUS=RADIUS*.3048
GO TO 33
J2 RADFT=RADIUS/.3048
J3 WVLGTH=299.7925/FRQNCY
RATIO= RADIUS/WVLGTH
RH0 = 2.*PI*RATIO
N = 0
QSQ = 1.
IKO = 1
Y = 1.
N = 0
SUMLAST = 0.
ISIGN = 1
SUM = CMPLX(0.,0.)
1 N = N+1
ISIGN = ISIGN*(-1)
CALL BFFGH(RH0,N,BJ,DBJ,DDRJ,BY,DBY,DDRBY,QSQ,IKO,Y)
HN2 = CMPLX(BJ,-BY)
DHN2 = CMPLX(DDRJ,-DDRBY)
ANS = -BJ/HN2
BNS = -(RH0*DBJ+BJ)/(RH0*DHN2+HN2)
TERM = ISIGN*(2*N+1)*(ANS-BNS)
SUM = SUM+TERM
ABSUM2 = (ABS(SUM))**2
DIFFR=ABS ((ABSUM2-SUMLAST)/ABSUM2)
IF (DIFFR .LT. 1.E-6) GO TO 9
SUMLAST = ABSUM2
GO TO 1
9 SIGNML = ABSUM2/(RH0*RH0)
SIGMA=SIGNML*PI*RADIUS*RADIUS
SIGFT=SIGMA*10.76391
IF (M,EQ,0) 80,81
80 M=1
GO TO 82
81 PRINT 998
82 PRINT 992
PRINT 99
```

03/06/72

```
PRINT 990
PRINT 991
PRINT 999
PRINT 100
IF (RADIUS ,GE. 1.) 40,41
40 PRINT 101, RADIUS
43 PRINT 102, RADFT
GO TO 42
41 RADCM=RADIUS*100,
PRINT 1011, RADCM
IF (RADFT ,GE. 1.) 43,44
44 RADIN=RADFT*12.
PRINT 1012, RADIN
42 PRINT 103,FRONCY
IF (WVLGTH,GE.1.) 50,51
50 PRINT 104, WVLGTH
GO TO 52
51 WVLGHH=WVLGTH*100,
PRINT 105,WVLGHH
52 PRINT 106, RATIO
PRINT 107, RH0
PRINT 108
PRINT 1080, N
PRINT 109, SIGNML
PRINT 110, SIGMA
IF (SIGMA ,LT. 1. .OR. SIGMA ,GT, 99999,9999) PRINT 112, SIGMA
PRINT 111, SIGFT
IF (SIGFT ,LT. 1. .OR. SIGFT ,GT, 99999,9999) PRINT 113, SIGFT
GO TO 500
30 FORMAT(2F10,2,9X,11)
998 FORMAT(1H1)
992 FORMAT(////////)
99 FORMAT(15X,*CALCULATION OF RADAR CROSS SECTION OF SPHERE* )
990 FORMAT(15X,*REFERENCE - KERR, *PROPAGATION OF SHORT RADIO WAVES*)
991 FORMAT(15X,*RAD LAB SERIES VOL, 13, P, 451, EQ, 29, *//)
999 FORMAT(15X,*-----*//)
100 FORMAT(15X,*INPUT DATA AND DERIVED QUANTITIES --*//)
101 FORMAT(20X,*RADIUS, METERS .....*,F10.3 )
1011 FORMAT(20X,*RADIUS, CENTIMETERS .....*,F10.2 )
1012 FORMAT(20X,*RADIUS, INCHES .....*,F10.2 )
102 FORMAT(20X,*RADIUS, FEET .....*,F10.3 )
103 FORMAT(20X,*RADAR FREQUENCY, MEGAHERTZ .....*,F10.2 )
104 FORMAT(20X,*WAVELENGTH, METERS .....*,F10.2 )
105 FORMAT(20X,*WAVELENGTH, CENTIMETERS .....*,F10.2 )
106 FORMAT(20X,*RADIUS/WAVELENGTH RATIO .....*,F10.4 )
107 FORMAT(20X,*CIRCUMFERENCE/WAVELENGTH RATIO .....*F10.4//)
108 FORMAT(15X,*OUTPUT DATA --*//)
1080 FORMAT(20X,*NUMBER OF SERIES TERMS ADDED .....*,5X,15 )
109 FORMAT(20X,*RADAR CROSS SECTION, NORMALIZED TO OPTICAL *,F10.4 )
110 FORMAT(20X,*RADAR CROSS SECTION, SQUARE METERS .....*,F10.4 )
112 FORMAT(23X,*SQUARE METERS, EXPONENT FORMAT .....*,E10.4 )
111 FORMAT(20X,*RADAR CROSS SECTION, SQUARE FEET .....*,F10.4 )
113 FORMAT(23X,*SQUARE FEET, EXPONENT FORMAT .....*,E10.4 )
END
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C SUBROUTINE BFFGH (Z, L, BA, BJP, BJPP, BB, BVP, BYPP, QSQ, IKQ, Y)
C3 UCSD BFFGH
DIMENSION RJ(5), BJL(5), BY(5), BYL(5), R(3)
YSAVE = Y
IF (Y = 1,) 102, 102, 101
101 Y = 1,
102 FL = L,
L1=L+1
ZB = Z
ZE = 0,
FL1 = FL + 1,
ZSQ = Z * Z
FLR = SQRTF (FL * FL1)
SINE = SIN(Z)
COSE = COS(Z)
IF (IKQ = 1) 104, 104, 108
100 IF (Z = FLR) 800, 800, 400
104 IF (Z=1,) 300, 200, 200
108 IF (QSQ) 110, 110, 104
110 IF (Z = 1,) 500, 400, 400
200 BJ(1) = SINE / ZB
SIGN = 1,
ER = -Y
BY(1) = ER * COSE
210 BY(1) = BY(1) / ZB
ER = BJ(1) / ZB
IF (SIGN) 211, 212, 212
211 ER = -ER
212 BJ(2) = (BY(1)/Y + ER) / Y
BY(2) = (BY(1)/Z - Y*BJ(1))*Y
FLX = 2,
221 TEMP = (2, *FLX = 1,)/Z
BJ(3) = (TEMP * BJ(2) - BJ(1)/Y)/Y
IF (SIGN) 223, 224, 224
223 BJ(3) = -BJ(3)
224 ER = TEMP * BY(2)
BY(3) = Y * BY(1)
IF (SIGN) 225, 226, 226
225 BY(3) = -BY(3)
226 BY(3) = ((ER - BY(3)))* Y
IF (FLX - FL1) 222, 228, 228
228 IF (SIGN) 430, 430, 100
222 BJ(1) = BJ(2)
BY(1) = BY(2)
BJ(2) = BJ(3)
BY(2) = BY(3)
FLX = FLX + 1,
GO TO 221
390 LX = L - 1
SEXT = 1,
391 DO 320 I = 1, 3
301 SUM? = 1,
SUM1 = 1,
TERM1 = 1,
TERM2 = 1,
FM = 1,
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TL = 2 * LX
302 TM = 2. * FM
TERM1 = -TERM1 * ZSQ / (TM * (TL * TM + 1.)) * SEXIT
TERM2 = TERM2 * ZSQ / (TM * (TL * TM + 1.)) * SEXIT
OLD1 = SUM1
OLD2 = SUM2
SUM1 = SUM1 + TERM1
SUM2 = SUM2 + TERM2
DIFF = ABSF((OLD1 - SUM1) / SUM1) * ABSF((OLD2 - SUM2) / SUM2)
IF (DIFF > .000001) 306, 306, 304
304 FM = FM + 1,
GO TO 302
306 IX = 2 * LX - 1
ASM = -1,
IF (LX) 310, 310, 307
307 DO 308 J = 1, IX, 2
AJ = J
308 ASM = ASM * AJ
310 ZYL = (Z / Y) ** LX
ZYL1 = ZYL * (Z / Y)
IF (LX) 312, 312, 314
312 ASM1 = 1,
GO TO 316
314 ALX = LX
ASM1 = -ASM * (2. * ALX + 1.)
316 BJ(1) = ZYL * SUM1 / ASM1
BY(1) = ASM * SUM2 / ZYL1
320 LX = LX + 1
IF (SEXIT) 430, 430, 900
400 SIGN = -1,
ER = EXPF (Z9)
ERL = 1. / ER
SNH = .5 * (ER + ERL)
CSH = .5 * (ER - ERL)
BJ(1) = SNH / ZB
RY(1) = Y * CSH
GO TO 210
500 LX = LX - 1
SEXIT = -1,
GO TO 391
430 KNT = 2
BY(1) = Y/ZB
BY(2) = Y*Y*(1.+ZB) / (ZB*ZB)
583 BY(3) = (2.*FL0AT*(KNT) -1.)*Y*BY(2) / ZB + Y*Y*BY(1)
IF (KNT-L-1) 582, 581, 581
582 BY(1) = BY(2)
RY(2) = RY(3)
KNT = KNT + 1
GO TO 583
581 EXZ = EXPF(+ZB)
DO 584 N = 1, 3
584 RY(N) = BY(N) * EXZ
IF (ZB-1.) 600, 580, 580
580 IF (Z=4.*FLR) 700, 600, 600
600 ER = 2. * FL + 1,
RJP = (FL * BJ(1) / Y + FL1 * BJ(3) * Y) / ER

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BYP = (FL * BY(1) * Y + FL1 * BY(3) / Y) / ER
LOC=1
IF (L, EQ, 0) GO TO 1100
BJPP = 2. * BJP / Z - (1. + FL * FL1 / ZSQ) * BJ(2)
BYPP = 2. * BYP / 7 - (1. + FL * FL1 / ZSQ) * BY(2)
GO TO 950
800 A = 0.1
    IZE = 0
    B = 0.35
    L1 = L + 1
    FN1 = L
    IDX = -1
    FN2 = Z - 0.5 + SQRTF(30, 0 * B * Z)
    ZE = 0.
    U1 = 2. * Z / (2. * FN1 + 1.)
    U2 = 2. * Z / (2. * FN2 + 1.)
    M1 = FN1 + 30. * (A + B * U1 * (2. - U1 * U1) / (2. * (1. - U1 * U1)))
    M2 = FN2 + 30. * (A + B * U2 * (2. - U2 * U2) / (2. * (1. - U2 * U2)))
    AL = L
    IF (FN2 - AL) 801, 801, 802
801 AM = M1 + 1
    M = M1
    GO TO 803
802 IF (M1 - M2) 801, 801, 805
805 AM = M2 + 1
    M = M2
803 R(3) = Z / (2. * AM + 3.)
    N = M
804 AN = N
    R(2) = Z / (2. * AN + 3. - Z * R(3))
    N = N - 1
    IF (L - N) 806, 806, 807
806 R(3) = R(2)
    GO TO 808
807 BJ(3) = R(2)
    BJ(2) = 1.
    AL = 2 * L + 1
    BJ(1) = AL / Z - R(2)
809 LA = L
    LA1 = L - 1
    ALPHA = Z * Z * (BJ(2) * BY(1) * Y * LA - BJ(1) * BY(2) * Y * LA1)
    NA = L - 1
    DO 812 N=1, 3
    BJ(N) = (1. / (Y * NA * ALPHA)) * BJ(N)
812 NA = NA + 1
    GO TO 900
700 A = 0.1
    B = .35
    FN = .5 + SQRTF(30. * B * ZB)
    IF (FN = FL) 701, 702, 702
701 FN = FL
702 U = 2. * ZB / (2. * FN + 1.)
    M = FN + 30. * (A + B * U) + 1.
    R(3) = ZB / 2. * FLOATF(M) + 1.
    M = M - 1
705 AN = M
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R(2) = ZB/(2. * AN + 1. + ZB * R(3))
M = M - 1
IF (M - L - 1) 703,704,704
704 R(3) = R(2)
GO TO 705
703 RJ(3) = R(2)
BJ(2) = 1.
BJ(1) = (2. * FL + 1.) / ZB * R(2)
ALPHA = ZB * ZB * (BJ(2) * BY(1) / Y * L + BJ(1) * BY(2) / Y * L1)
NA = FL * 1.
DO 712 N=1,3
RJ(N) = (1. / (Y * NA * ALPHA)) * RJ(N)
712 NA = NA + 1
GO TO 600
900 ER = 1. / (2. * FL + 1.)
BJP = ER * (FL * BJ(1) / Y + FL1 * BJ(3) * Y)
BYP = ER * (FL * BY(1) * Y + FL1 * BY(3) / Y)
LOC = 2
IF (L, EQ, 0) GO TO 1100
BJPP = (FL * FL1 / ZSQ - 1.) * BJP(2) * 2. * BJP / Z
BYPP = (FL * FL1 / ZSQ - 1.) * BY(2) * 2. * BYP / Z
950 BA = BJ(2)
BB = BY(2)
1000 Y = YSAVE
RETURN
1100 IF (IKQ, LE, 1) 1120, 1110
1110 IF (OSQ, LT, 0) 1150, 1120
1120 BA = SIN(Z) / Z
BB = -COS(Z) / Z
IF (LOC - 1) 1130, 1130, 1140
1130 BJPP = 2. * BJP / Z - (1. + FL * FL1 / ZSQ) * BA
BYPP = 2. * BYP / Z - (1. + FL * FL1 / ZSQ) * BB
RETURN
1140 BJPP = (FL * FL1 / ZSQ - 1.) * BA - 2. * BJP / Z
BYPP = (FL * FL1 / ZSQ - 1.) * BB - 2. * BYP / Z
RETURN
1150 BA = SINH(Z) / Z
PI = 3.1415926535
BB = EXP(-Z) * PI / (2. * Z)
IF (LOC - 1) 1130, 1130, 1140
END
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FUNCTION SINH(Z)
SINH = (EXP(Z) - EXP(-Z)) / 2.
END
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