DETECTION SCHEME FOR JUMP OF INPUT WITH KNOWN MEAN AND COVARIANCE MATRIX*

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Abstract

This paper discusses the use of Baye's rule for the detection of the time of application of an impulse input in a system, along with the Kalman Filter algorithm for estimation of the system parameters.

INTRODUCTION

In the tracking of parameters of a system, there are situations when the system undergoes an abrupt change of parameters, but the exact time of this change is not known a priori. This paper assumes that the mean and covariance of the jump of input is known. We use Bayes rule to detect the time of application of this jump.

PROBLEM STATEMENT

Consider a linear system, the dynamics of which is given by $x_i = Fx_{i-1} + \Gamma\omega\delta_{ij}$ (1)
The jump quantity ω is a Gaussian random vector with mean $\bar{\omega}$ and covariance Q, which are known to the observer. The δ_{ij} implies that this jump quantity applies at a certain time instant j which is not known a priori. The observations made on the system are also linear functions of the states and are given by $z_i = H_i x_i + v_i$ (2)
where H_i is a known observation matrix and v_i is a Gaussian measurement noise vector with zero mean and covariance R. It is assumed that a jump, if any, occurs prior to a measurement.

TRACKING

The Kalman Filtering is used to find the estimate $\hat{x}_i = E(x_i|z_i)$ where z_i is the collection of observations to the i^{th} instant, i.e., $z_i = (z_1, z_2, z_3, \ldots, z_i)$. A standard Kalman Filter algorithm[1] is used. The estimation is started with a given initial estimate $\hat{x}_0$. Based on this quantity, two predictions are made for the states at the next stage, one assuming that there has been no jump and the other assuming that there has been a jump before the 1^{st} observation is made. These will be called $\bar{x}_1^0$ and $\bar{x}_1^1$ respectively, as shown in Fig. 1. The subscript denotes the stage referred to, while the superscript denotes the stage before which jump is supposed to have occurred. The superscript zero denotes that no jump has occurred so far. The superscript 1 indicates jump just before stage 1. Fig. 1 shows how the different estimates may be obtained by assuming the jump to have occurred at different points. Proceeding in this manner, at every stage i, we shall have i+1 estimates, one of which is more correct than the others. A number of stages can be considered together, to determine whether there has been a jump. A large number of stages will increase the accuracy of detection. But as a tradeoff between computational load and accuracy, only three stages together will be considered here. Thus, there will be no more than four alternate estimates at any stage.

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PROBABILITY DENSITY FUNCTION OF z_i

The observation z_i is a Gaussian random vector[2]. The mean and covariance are easily shown to be

$$E(z_i)=\bar{z}_i=H\bar{x}_i \quad (3)$$

and

$$\text{cov}(z_i)=H_iM_iH_i^T+R_i \quad (4)$$

where M_i is the pre-measurement covariance of x. We shall write $z_i^j \sim N(H_i\bar{x}_i^j,\ H_iM_i^jH_i^T+R_i)$ $j\leq i$, where the superscript j on z_i, x_i and M_i denotes that the jump is supposed to have occurred just before the j^{th} measurement.

JOINT PROBABILITIES AND DETECTION

With the third measurement, consider the joint probabilities $p(z_1^1, z_2^1, z_3^1)$, $p(z_1^2, z_2^2, z_3^2)$, $p(z_1^3, z_2^3, z_3^3)$ and $p(z_1^0, z_2^0, z_3^0)$, one of which will be highest depending on when the jump occurred or didn't occur at all[3]. Also, it is expected that the probabilities will be graded according to the nearness of the hypothesis to the true hypothesis. This allows one to compare two adjacent probabilities at a time instead of all four. For example, compare $p(z_1^1, z_2^1, z_3^1)$ and $p(z_1^0, z_2^2, z_3^2)$. If the former is larger, then conclude that the jump was indeed before the 1st measurement. If not, then process one more measurement and make similar comparisons for the next set of joint probabilities which does not include $p(z_1^1, z_2^1, z_3^1)$, and so on.

EVALUATION OF JOINT PROBABILITIES

Since the observations are statistically independent[3], the joint probabilities are the products of the individual probabilities. We use the logarithm of the probabilities, for convenience in computation. Thus, one needs compute, with each new estimate, the following quantity: $\text{Ctn}[\prod_{k=1}^{i} P(z_k^j)] =$

$$\sum_{k=1}^{i} \{(z_k^j-H_k\bar{x}_k^j)^T(H_kM_k^jH_k+R_k)^{-1}(z_k^j-H_k\bar{x}_k^j)+\ln|H_kM_k^jH_k^T+R_k|\}]$$

where C is a negative constant for different $j\leq i$. The $\bar{x}_k^j$ is obtained by Kalman Filtering according to Fig. 1. For a certain j, the negative of the logarithm of the joint probabilities, i.e., the right side of the above equation is the smallest. That j denotes the time instant of jump. Note, that any $i\geq 3$, the only j's considered are j=i, i-1, i-2 and i-3, since the likelihood of a jump at an earlier stage has already been discarded in the previous step. Thus at any stage, there are no more than 4 estimates to be made and four sums to be updated. This restricts the size of memory required.

EXAMPLE

A simple example was chosen with $F=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, $\Gamma=\begin{bmatrix}0\\ 1\end{bmatrix}$, H=J, $R=\begin{bmatrix}.08 & 0\\ 0 & .025\end{bmatrix}$ and $x_0\begin{bmatrix}0\\ .5\end{bmatrix}$ Kalman Filtering was carried over 30 stages. The jump was introduced at one of the 30 stages. The probability of success, as obtained from computor runs, is given in Table 1.

CONCLUSIONS

This paper gives a scheme for detecting a sudden change in a system. A previous paper by Schagiri Prabhu[3] does not involve any filtering, the present scheme does. A major difference between the two is, thus, that while the former assumes known distributions of the measurements, this scheme recalculates the distribution at every stage. The scheme, as tried, was quite successful, though naturally sensitive to initial errors of estimation.

REFERENCES

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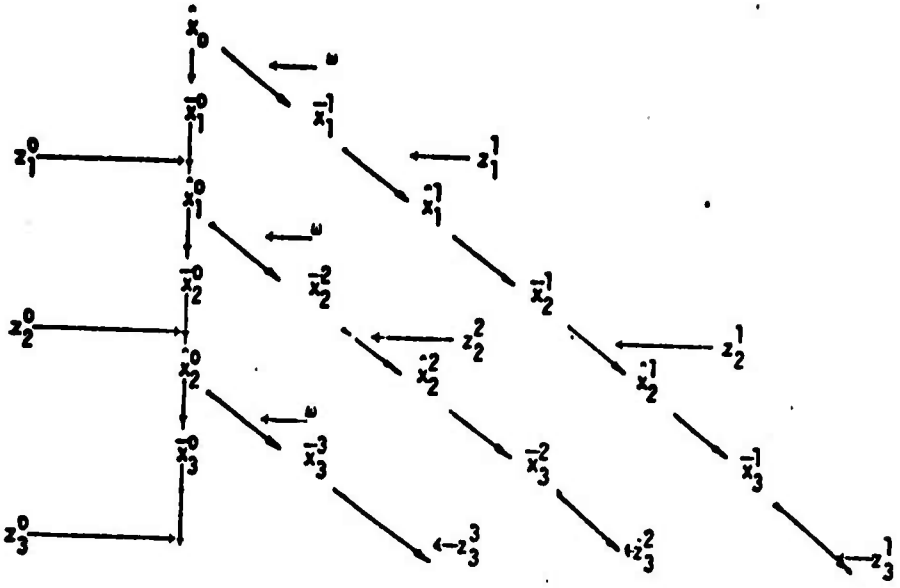

FIGURE 1

DIAGRAM SHOWING THE ALTERNATIVE ESTIMATES

Sr.No.	Value of $\bar{\omega}$	Value of Q	Correct Detection Ratio=No. of trial
1	0.01	0.25	29/40
2	0.05	0.25	32/40
3	0.10	0.25	32/40
4	0.25	0.5	32/38
5	0.25	0.0	40/40
6	0.50	0.5	71/78
7	1.0	0.5	21/22

TABLE 1

RESULTS OBTAINED FROM COMPUTER RUNS