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This paper proposes a method of structural design which evaluates the risk of failure. Whereas conventional design is based upon safety factors which help reduce risk, but do not evaluate it, the probabilistic approach estimates the actual level risk for the structural model. The method of the paper is applicable when major parameters follow the Normal (Gaussian) distribution. Reliability is used as the measure of risk. For the purposes of the paper, reliability is defined as the probability that a structure withstands design loads for a specified time in a specified environment. A reliability of 0.99 indicates a one percent chance that the structure will fail. Basic examples are included to demonstrate the use of the method.



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# PROBABILISTIC STRUCTURE DESIGN

RESEARCH REPORT

Presented in Partial Fulfillment of the Requirements
For the Degree Master of Engineering, Industrial
Engineering Department of Texas A&M University


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1970

# ABSTRACT

## Probabilistic Structure Design

This paper proposes a method of structural design which evaluates the risk of failure. Whereas conventional design is based upon safety factors which help reduce risk, but do not evaluate it, the probabilistic approach estimates the actual level of risk for the structural model. The method of the paper is applicable when major parameters follow the Normal (Gaussian) distribution. Reliability is used as the measure of risk. For the purposes of the paper, reliability is defined as the probability that a structure withstands design loads for a specified time in a specified environment. A reliability of 0.99 indicates a one percent chance that the structure will fail. Basic examples are included to demonstrate the use of the method.

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# CHAPTER I

## INTRODUCTION

Statistical treatment of design has long since demonstrated its importance. Most notably, the United States Government's efforts in defense and space exploration have shown the significance of first reliability, then maintainability. An important area of engineering, structural design, has nearly escaped this trend. Focus upon the probabilistic design of structures may serve to bring the powerful tools of reliability and maintainability to bear upon this challenging problem.

This paper investigates the design of structures by focusing upon evaluation of the risk of failure. Conventional and unconventional design approaches are discussed, and a probabilistic method is proposed. The probabilistic approach requires use of probability distribution functions; hence, these, too, are discussed. A simple beam design example is worked by two methods, beginning with a conventional analysis and progressing through stages of probabilistic analysis. Structural design is a tremendously complex problem, therefore the proposed approach incorporates as much simplicity as possible. The probabilistic approach to structural design is important because it offers a closer evaluation of design parameters and risks. A closer evaluation of these variables should produce a positive economic result: money saved. The problems associated with implementing probabilistic design are admittedly immense. However, the possible benefits are also large. Because of its very high expenditures for structures, the Federal Government could realize significant savings by implementing a probabilistic approach to design.

Probabilistic design of buildings differs from conventional design in one basic way. Statistical distributions, rather than single values, are used to describe important factors such as material strength and structural loadings. A probability distribution function estimate of a quantity inherently contains more information than does a single point estimate. If this additional information can be correctly utilized, it is reasonable to expect improved design. Use of statistical distributions for important parameters requires the use of slightly altered methods. Statistical methods produce an estimate of reliability which is valid for the structure under the assumptions which governed its design. Reliability is defined for this paper as the probability that a structure withstands design loads in a specified environment for a specified length of time. A reliability of 0.99 indicates a one percent chance that the structure will fail to meet design loads. If one hundred structures were so designed, then one failure would be expected. Failure is defined for this paper as complete collapse. Reliability measures the expected performance of the structure.

In contrast to the probabilistic approach, the conventional design method produces no precise estimate of the probability of success. Usual design practice embodies determining near-maximum loads and near-minimum strengths. Safety factors applied to the design load increase the strength of the resultant structure. However, the strength of a particular type of member is not a single-valued quantity, as handbooks might lead us to believe. A certain level of risk always exists in the structure.

An explicit measure of risk enables one to make more intelligent decisions regarding construction. Incremental gains in reliability become more and more costly as reliability approaches unity. Striving for safety alone may produce high costs with only small decreases in risks. Thus, the main benefit of the probabilistic approach is that it provides an explicit measure of risk useful for informed economic decisions.

Some individuals feel that safety factors used in conventional design are arbitrary and, in reality, "factors of ignorance" to cover up for lack of knowledge and possible mistakes. Others hold that safety factors are "factors of experience" resulting from solid engineering judgment and a wealth of experience. Both conceptions are probably true in part and both help illustrate the lack of precise risk evaluation with the safety factor approach.

Freudanthal (11)* succinctly stated the problem of safety: "The difference between safe and unsafe design is the degree of risk considered acceptable, not in the delusion that such a risk can be completely eliminated." Since a reliability of unity can never be achieved, a more realistic reliability goal must be set. The same economic rationale which leads government and industry to utilize the techniques of sampling inspection is relevant here. Just as money can be saved through the use of a sampling plan if a small risk is tolerated, so, too, can money be saved in structural construction if a small risk can be tolerated.

Considering structural design on this basis, the acceptable risk of failure can be adjusted for each application to provide the optimum

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*Numbers in parentheses refer to numbered references in the List of References.

economic balance between cost of increased reliability and expected cost of failure. Money spent on unnecessary over-design represents more than just an economic loss. This excess spending might well have been diverted into the construction of more buildings or the provision of more services. In the case of government, the misuse of this money might have significant social consequences.

This paper makes no charge that past methods be abandoned, only that new, supplemental methods be considered. Conventional methods have served well. However, it is desirable that they be examined periodically, not so much to question their validity but rather as a search for a better approach. Thus, the paper presents one alternative to standard structural design.

A structure designed by probabilistic methods is not inherently more or less safe than a conventionally designed structure. The safety of a structure is always determined by its design, construction, and use. The probabilistic approach simply aims to provide the engineer with a method for more precise description of the important factors and, thereby, more control of his design. By better control of his design, the engineer can avoid unbalanced designs, over-designs, and the accompanying waste of materials and money. Perhaps analysis of necessary building safety could lead to less restrictive codes for some applications.

Again, the idea of this paper is not to seek a lowering of safety factors and the accepting of higher risks. Rather, it is the offering of a design aid which may help designers more accurately achieve their goals. In the process of explaining the probabilistic approach, Chapter

III contains a design example. Chapter IV illustrates a more complex example. Chapter II establishes a background for the consideration of the example by discussing related work.

## CHAPTER II

## REVIEW OF RELATED WORK

In reviewing previous efforts in this field, several significant areas are investigated. It is necessary to look beyond just design practice because structural reliability depends heavily upon other factors. Thus, this chapter begins with discussions of material strength and loading such as wind. It will then cover design practice contributions of other authors by starting with the simpler methods and ending with a combined probabilistic-deterministic method. Chapters III and IV contain an example problem which relies upon much of the information discussed in this Chapter.

Johnson (18) utilizes a random-products method to generate a distribution of lumber strengths. This was accomplished by multiplying a randomly selected strength ratio and an assumed distribution of strength for clear wood. Many such calculations produce a distribution of theoretical lumber strengths which agrees well with actual test values. This provides a valuable method for obtaining strength distributions for lumber and shows that this distribution closely follows a Normal distribution. Comparing Johnson's various strength values with a standard Normal distribution indicates an approximate mean and standard deviation of 4480 and 1117 psi, respectively for construction grade lumber. Because these strengths represent actual breaking strengths which account for size deviations, no additional consideration need be made for dimensional variability.

Hasofer (15) has investigated statistical models for live floor loads. His preliminary investigation indicates that several types of loads can be described by a Pareto type formula ($P(w>x)=cx^{-\alpha}$), but does not provide design guidelines. Hasofer notes the lack of knowledge in this area and calls for more investigation.

Thom (26) states that extreme winds in the United States follow a Fisher-Tippitt Type II extreme value (Frechét) distribution ($F(x)=\exp-(\frac{x}{B})^{-\alpha}$). Wind speed charts are given for several percentile levels for the United States. Thom's formulae and the included maximum-value probability paper for the Frechét distribution enable conversion to the time and confidence level desired. This paper fills an important gap in structural reliability: high quality wind data in an exact form. Appendix B contains additional discussion and utilization of Thom's work.

Personal interviews with steel company quality control personnel indicate a basic misunderstanding in strength utilization. The strength of steel test specimens was usually well above minimum strength requirements. This extra strength above the minimum was looked upon as a 'bonus' for the customer. Since only specified strength is guaranteed, designers are unaware or unsure of the 'bonus' strength. Consequently, most designs do not utilize this extra strength. Since the costs related to producing this 'bonus' strength are surely passed on to the customers, the 'bonus' actually is a penalty.

This unfortunate situation is even more undesirable when the simplicity of a solution is considered. Tests currently in use produce approximately ten test specimens per heat. When combined with historical information on distributional form, these ten values provide enough information for reasonable statistical analysis of steel strength (see Appendix A). Thus, a simple change in recording of data obtained from present test could provide the designer with reliable information and enable more complete utilization of steel strengths. While this situation may not be representative of the entire steel industry, it does point out a perplexing problem.

Haugen's paper (16) is based primarily upon the assumed Normal distributions for loads and strength. He gives general procedures for calculating reliability of mechanical elements in aerospace applications. His justification for this approach includes a good coverage of algebra of Normal distributions.

In his recent text (17), Haugen moves through elementary probability theory to reliability oriented design of several individual elements. This text provides a good basic coverage of structural reliability, forming a firm background for analysis of multicomponent structures. As such, it represents a worthwhile contribution to structural reliability.

Cable and Virene (7) assume Normally distributed parameters and explain structural reliability as it relates to the aerospace industry. They provide the important extension of finding confidence intervals on reliability, rather than point estimates alone.

Bonnicksen (5) develops a theoretical approach to structural design based upon assumed Normally distributed loads and strengths. In addition, he solves a simplified example to illustrate these concepts. Bonnicksen also discusses several concepts such as weak-linked and warning sensor considerations. The sensing of impending failure provides warning, thus lowering the reliability required of the structure. Weak-linked design allows for partial failure to lessen the load without complete destruction in a manner similar to a fuse. With his approach, buildings might be constructed as a combination of reparable modules. These concepts, plus the trade-off of return on investment against risk, are among the important contributions made by Bonnicksen's paper.

Yao (29) formulates his safety analysis based upon counting all possible failure modes. Investigation of mode and probability of each possible failure points out the importance of considering the combinations of events which produce failure. Unfortunately, for all but simple structures, counting failure modes becomes unmanageably complex. Yao's solution is also limited by his assumption of deterministic load, a condition rarely encountered in practice.

Moses and Kinser (20), working on the basis that strengths of members in elastic structures are independent, draw several general conclusions: "If the load has small standard deviation compared to (that of) the strength, then the failure modes are close to being independent. This is because for a fixed load, the probability of failure depends only upon the strength, and if the strengths are statistically independent, then the failure modes are also independent. If the strengths have a small standard deviation compared to (that of) the loads, then the failure modes are close to being statistically dependent."

Moses and Kinser discard both the Monte Carlo and numerical integration techniques in favor of an ordering approach. The ordering approach requires enumeration of all possible modes of failure. Approximations convert this set of all possible failure modes to a set of independent failure modes. This set is then ordered by decreasing probability of occurrence. The most likely mode of failure is then examined as the critical parameter. Moses and Kinser provide a procedure for examining failure modes in depth. However, this method, like Yao's counting approach, requires extensive effort.

Ang and Amin (1) utilize a combined probabilistic-deterministic approach. They point out the important distinction between parameters which are stochastic and parameters about which knowledge is incomplete. Probabilistic parameters such as strength distributions can be handled statistically to find the associated risks. Risks resulting from various assumptions and from approximations in modeling and structural theory are not so easily calculated. To solve this problem, Ang and Amin introduce a factor of uncertainty to account for the undeterminable errors and an acceptable risk associated with statistical parameters. Significantly, this approach relies upon both conventional and probabilistic concepts.

Review of these contributions to structural reliability indicates needs: weight distributions of construction materials, loading distributions, a simple workable design method, and statistical consideration of wind loadings. The next chapter utilizes known strength distributions (18) and wind distributions (26) in offering a possible design approach. In Chapter III, a conventional design is shown first for comparison. After that example, a basic probabilistic method is shown. Finally, the proposed probabilistic method is offered. Chapter IV illustrates a more complete design example.

## CHAPTER III

## PROBABILISTIC DESIGN METHOD

Synthesis of a structural design method will begin by consideration of a conventional design example. For this example a simply supported, uniformly loaded beam will be analyzed. The beam is a 2x4 of construction grade Douglas fir, ten feet in length. The problem is to find the maximum load which this beam will carry without failing by bending.

Conventional design utilizes either a safety factor applied to load and a margin of safety applied to strength or it utilizes a factor of safety applied to load and an allowable stress. This example uses the safety factor and allowable stress approach. From Roark (22), the following formulae are obtained:

$$s = \frac{m}{I/c}$$

$$m_{max} = \frac{(W)\ (L)}{8}$$

where

s is the extreme fiber bending stress, psi,

m is the bending moment, in-lbs.,

I/c is the section modulus, in,

L is the length of beam, inches,

W is the total load, pounds.

Using Scofield's (24) values for allowable stress, s, and for I/c for a construction grade 2x4, the solution is found to be 356 pounds. Applying a safety factor of 1.7 gives the final solution of 209 pounds.

The risk of failure of the beam while supporting the load of 209 pounds is not apparent. The risk would not have been any more apparent if a different safety factor had been used. In general, as the safety factor increases, the structure becomes stronger; the safety factor is a general indication of safety or risk. The question of precisely how safe is considered by the following probabilistic approach.

The probabilistic approach will utilize Johnson's (18) lumber strength results. A strength reduction factor of 1/1.7 for long-time loading produces a Normal strength distribution with mean of 2630 psi and standard deviation of 657 psi. No study of weight distribution of structures has been found. However, the common range of densities for Douglas fir is 28-36 lbs./ft.$^3$. The most widely accepted value is 32 lbs./ft.$^3$. The distribution of weight of one cubic foot of Douglas fir is assumed to be Normally distributed with mean of 32 and standard deviation of 1.33. The direct relationship between density and strength, which is Normally distributed, justifies the assumption of Normality. The standard deviation is obtained by assuming extremes of common values to be plus or minus three standard deviations from the mean. This assumption may be somewhat tenuous, but it will be used until more exact weight distributions are available.

In the first probabilistic example, loading upon the beam is assumed to result only from the weight of the structure above it. The structure is built entirely of Douglas fir. Hence the total dead load, W, upon the beam will be some multiple, M, of the basic weight of one cubic foot of Douglas fir. The resulting distribution of dead load, W, is therefore Normal with mean, $\overline{W}$, of (M)(32) and standard deviation, $\sigma_W$, of (M)(1.33).

Since the mean load, $\overline{W}$, is equal to M32, determining M will determine $\overline{W}$. As in conventional design, $s = \frac{W\ L}{(8)(I/c)}$. Substitution of the above values yields the mean, $\overline{S}$, of the stress distribution to be $\frac{\overline{W}\ (120)}{(8)(3.56)}$ which can be simplified to $\overline{S} = 4.22\overline{W}$.

In this probabilistic example A and S are considered to be random variables of strength and stress, respectively. A is Normally distributed with mean $\overline{A}$ and standard deviation $\sigma_A$. S is Normally distributed with mean $\overline{S}$ and standard deviation $\sigma_S$. Success is achieved when A is greater than S, where S = 4.22 W, that is, when (A-4.22W>0). The quantity (A-4.22W) is Normally distributed with mean of ($\overline{A}$-4.22$\overline{W}$) and standard deviation of $(\sigma_A{}^2 + \sigma_S{}^2)^{1/2}$ which is $(\sigma_A{}^2 + (4.22)^2\sigma_W{}^2)^{1/2}$.

Reliability is equal to the probability of success which is the probability (A-4.22W>0). The following relation is the basis for the design:

$$\text{Reliability} = \Phi\left[\frac{\overline{A} - \overline{S}}{\text{standard deviation of } (A-4.22W)}\right]$$

where $\Phi$ denotes the standardized Normal distribution. Using the assumed values for this example, the design basis becomes:

$$\text{Reliability} = \Phi\left[\frac{2630 - 4.22(32M)}{\sqrt{657^2 + (4.22M\ 1.33)^2}}\right]$$

At this point the desired reliability is chosen, for example 0.99. Because 0.99 is equal to $\Phi[2.33]$, bracketed quantities are equated as follows:

$$2.33 = \frac{2630 - (4.22)(32M)}{\sqrt{657^2 + (4.22\ M\ 1.33)^2}}\ .$$

Solution by standard methods yields M equal to 8.20. From this, allowable load mean, $\overline{W}$, is found to be 262 with standard deviation, $\sigma_W$, equal to 10.9.

It is important to note that this solution is in the form of a distribution of loads rather than a single load estimate. A solution of this type accounts for the fact that many different actual loads are possible. If the actual loads occur with probabilities consistent with a Normal distribution of mean 262 and standard deviation 10.9, then the reliability of that particular structure is 0.99. If actual loads follow a different Normal distribution, the reliability of the structure will not be 0.99, but may be obtained by this approach. Several loads usually combine to form the total load. This adding of loads often produces a total load which is approximately Normally distributed. In this case the methods of this paper provide approximate solutions.

The additional problem of wind loading will now be considered. For short-time loading such as wind, the allowable strength of wood can be increased by approximately one-third (23). Thus, the relevant distribution of strengths for a combined dead-load and wind load analysis is Normal, with mean 3510 psi and standard deviation 877 psi. Because wind speed follows a Frechét distribution, it cannot be combined readily with the previous Normal (strength minus dead load) distribution to form another Normal distribution. To circumvent this difficulty, a particular wind load, WL, will temporarily be treated as a constant and combined into the Normal distribution. The design basis is, thus, changed from Reliability equal to the Probability $(A-4.22W>0)$, to Reliability is equal to Probability $(A-4.22W-4.22WL>0)$. Each particular wind load has a certain probability of being exceeded. Let this probability be denoted as $\alpha$. Table 3.1 indicates wind loadings for certain levels of risk (see Appendix B for derivation and assumptions).

TABLE 3.1

WIND LOAD VS. RISK

| $\alpha$ | F(X) | $X(V_{30})$ | $V_{15}$ | Nominal Load | Expected Return Period |
|---|---|---|---|---|---|
| .20 | .9855 | 77 MPH | 65.5 MPH | 11 PSF | 65 YR |
| .15 | .989 | 80 | 68 | 12 | 99 |
| .10 | .995 | 86 | 73.2 | 13 | 200 |
| .05 | .9965 | 88 | 74.4 | 14 | 286 |
| .03 | .998 | 92 | 78.2 | 16 | 500 |
| .02 | .9985 | 94 | 80 | 17 | 667 |
| .015 | .999 | 102 | 87 | 19 | 1,000 |
| .01 | .9993 | *** | *** | *** | 1,430 |

*** Values not obtainable from probability paper for F(S)>.999.

In Table 3.1 $\alpha$ is the risk, F(X) is the cumulative frequency distribution, X is the wind speed at a 30 foot elevation, $V_{15}$ is the wind speed at an elevation of 15 feet which produces the nominal load.

Let the allowable risk for the design wind loading of the example be 0.15. The total risk incurred in this approach will be discussed after the load calculations. For an allowable wind loading risk of 0.15, Table 1 indicates a loading of 12 pounds per square foot (PSF). Assume the beam supports a surface with a wind shape coefficient of 0.7. Assume further that the beam supports a surface sixteen inches wide along its entire length. The beam is ten feet long, thus the total wind load, WL, is equal to (12)(0.7)(16/12)(10) or 112 pounds. The basic design formula now takes the form:

$$R^* = \Phi\left[\frac{3510 - 4.22(32M) - 4.22WL}{(877^2 + (4.22(1.33M))^2)^{1/2}}\right]$$

where $R^*$ is a partial measure of reliability but is not equal to the total reliability. Substituting 112 for WL and solving for M as before yields a value of 7.62 for M. From this, the new allowable dead load is Normally distributed with mean of 244 and standard deviation of 10.1. Note that this is not the same as the allowable distribution for dead load alone. The smaller of the two calculated dead loads is always the proper one to use.

Given that the partial reliability measure, $R^*$, is 0.99 and $\alpha$ is 0.15 for the example, what is the true reliability, R? True reliability is not equal to $R^*$ because WL was combined into the design formula as a constant which it is not. There exists probability, $\alpha$, that wind loading will exceed WL, thus reducing the true reliability below $R^*$. The wind loading may be less than WL with probability $(1-\alpha)$. If the actual wind load is less than WL, the true reliability is greater than $R^*$. Thus the lower limit for R is $(1-\alpha)(R^*)$. The range of possible values for R is, therefore, $(1-\alpha)(R^*) < R < R^*$. In most design situations, $R^*$ will be close to unity and $\alpha$ will be small. Ang and Amin (1) analyze risk in a similar situation: their analysis would conservatively indicate the risk here to be approximately $R^* - (\alpha)(1-R^*)$. Under these conditions, the reliability for this example is 0.9885 with regard to bending.

Thus a possible method of solution has been illustrated for a simple beam. However, only one mode of failure, bending, has been investigated. Other modes of failure would be treated in an analogous manner; combine load and strength distributions to form a reliability estimate for each mode of failure. Solution for other modes of failure will not be given here because strength distributions for other types of failure are not available.

Treatment of several modes of failure for each structural element is a problem which has been almost completely neglected in most probabilistic approaches. If further analysis of the beam example were to give reliability for horizontal shear of 0.9999 and reliability for deflection of 0.95, what would be the overall reliability of the beam? The failure modes are not independent since the same load distribution acts in all cases. The beam would not be more reliable than the worst case indicates (0.95). The actual difference between the true reliability and the reliability of the worst case depends upon the nature of the resistance of the particular beam to various forms of failure.

Material strengths might be related directly so that a specimen which is strong in one mode is also strong in all other modes. In this case, the weakest specimen in one mode would be weakest in all modes; this specimen would be the first to fail. Furthermore, this specimen would fail in its weakest mode, the worst case mode. If the nature of material strength did conform to the preceding assumptions, all but the worst case analysis could be neglected in risk calculations. The reliability of the element would then be the same as that of its least reliable mode.

Material strength is grossly determined by material composition and specifically determined by variations in size, presence of defects, and similar factors. A defect would usually reduce strength in all modes. This gives some justification for the worst case approach.

However, if the defect or other strength reducing factor does not reduce all strength modes in a proportionate manner, the worst case theory becomes less accurate. This lack of a clear course of action indicates the need for fundamental research and testing to determine various material strength distributions and their interrelationships. Until better knowledge becomes available, this report recommends the worst case type analysis.

The design example was presented in a manner intended to foster its understanding. This resulted in an order of steps nearly opposite that of a true design problem. The actual order would likely be

1. Find load and strength distributions. May want strength distributions for several grades of material and structural shapes.
2. Set reliability goal utilizing economic study of risk and considering expected structure life.
3. Perform the actual design. This involves reliability calculations for various configurations and may require allocating reliability to subsystems.

Admittedly, the design of a simple beam is far less complex than the design of most structures. However, the design example did illustrate the basic method. Many structures can be idealized to fit a simple series chain model if the links of such a chain model are properly defined functional units. Other authors such as Bonnicksen (5) furnish theoretical assistance in expanding the analysis to more complex structures. A more complex design example is given in Chapter IV to more fully illustrate the procedure. Much more research is needed to establish desirable reliability goals.

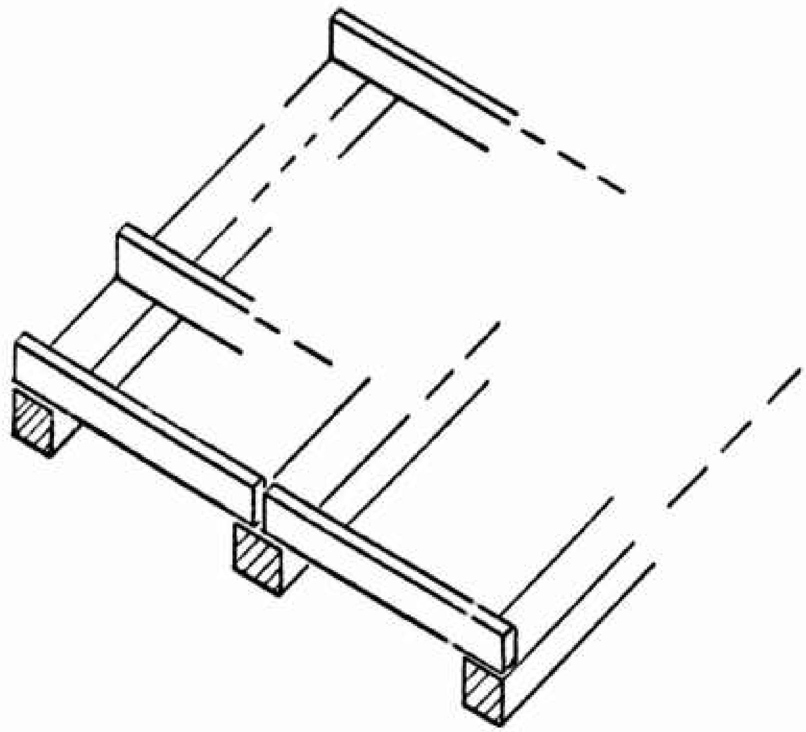
CHAPTER IV

DESIGN EXAMPLE

This Chapter will demonstrate the design of a subfloor system using the probabilistic method. A sketch of the proposed subfloor follows:

FIGURE 4.1

SUBFLOOR



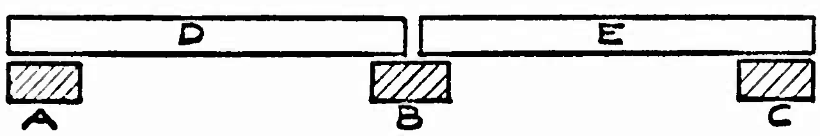
Assumptions to be considered are: the subfloor is to be 20 feet by 16 feet; along the 20 foot dimension are to be a number of equally spaced, identical joists with one joist being at each end; the joists are to be 8 feet long; there will be three identical girders with 8 foot spacing; the girders will be supported by piers at 5 foot intervals; the uniform load transmitted to the joists from the floor above is Normally distributed, with mean 22,400 pounds and a standard deviation of 1,600 pounds; the basic distributions of strength are Normal, with mean of 2,630 psi and standard deviation of 657 psi for long-time loading (18). The desired reliability goal for this subfloor is to be 0.95.

The solution desired is the optimum spacing of the joists and their size plus the size of the girders. It is assumed that the cost of the materials is directly proportional to the nominal size, such that a 2 x 4 has a cost of 1 unit and a 2 x 6 has a cost of 1.5 units.

The first step in the design is finding the spacing, cost, and reliability for each size joist. The optimum will be selected on this basis. The two identical systems of joists and each of the three girders are considered as functional units. The following sketch labels each functional unit with a letter:

FIGURE 4.2

SUBFLOOR - SIDE VIEW




If any one of the functional units fails, the system fails; hence, this forms a chain type structure which has its reliability determined by the Product Rule, $R = R_a R_b R_c R_d R_e$ (26). Units D and E are identical, as are A and C; thus $R = R_a{}^2 R_b R_d{}^2$.

The desired R goal for the subfloor is 0.95. This can be achieved in accordance with the Product Rule by several different apportionments of unit reliability. As one possible starting point, each functional unit is assigned an equal reliability goal of 0.99 ($0.9895^5 = 0.95$).

Based on the reliability goal of 0.99, consider the design of one of the joist systems. Because a joist system is considered as one functional unit, total load and total strength for that unit is the basis for decision. The total load for one joist system is Normal, with mean 11,200 pounds and a standard deviation of 800 pounds. The total strength of the joist system is Normal, with mean (NJ)(2630) and a standard deviation of (NJ)(657), where NJ is the number of joists minus one. The number of joists minus one is used to account for the fact that in uniform loading the two end joists each support only one-half as much as any other joist.

Using the design basis of Reliability = Probability

$(A - \frac{12W}{I/c} > 0)$, the following results are obtained:

A. 2x4's - 26.8 required - 30 used - 8 inch spacing - cost = 30

B. 4x4's - 16.4 required - 20 used - 12 inch spacing - cost = 40

C. 2x6's - 14.4 required - 16 used - 16 inch spacing - cost = 24

D. 2x8's - 9.36 required - 11 used - 24 inch spacing - cost = 22

There is a fixed labor cost associated with installing each joist; therefore, the fewer joists the lower the installation cost. Convenient spacing for construction was the basis for rounding to obtain the above results. In lieu of these considerations, eleven 2x8's with spacing 24 inches were used.

Recalculating the reliability of a joist system of 11 joists requires utilizing the equivalent strength of 10 joists as before. In addition, the previously neglected load from the weight of the joists themselves must be included. The appropriate Normal strength distribution has mean 26,300, and a standard deviation of 6,570. The appropriate Normal load distribution has mean of 11,438 pounds, and a standard deviation of 810 pounds. This reflects a Normal weight distribution for the eleven joists of mean 238 pounds and a standard deviation of 10 pounds. The design formula has the form

$$R = \Phi\left[\frac{26,300 - (.787)(11,438}{\sqrt{(6570)^2 + [(.787)(810)]^2}}\right]$$

The reliability of one joist system is found to be .9957. Both joist systems have this same reliability because they are identical.

The design of the joist system completed, attention is turned to the design of the girder system. The Product Rule $(0.95 = (.9957)^2 R_{girdersystem})$ indicates that the reliability of the girder system must be at least 0.9581. The three girders must be the same size; however, the center girder supports one-half the total load. The other two girders each support one-fourth of the total load. Thus, the reliability of the center girder will be lower than that of the other girders. A possible method of reliability allocation sets $R_b$ equal to $R_a^2$, under the constraint that $R_b R_a^2$ is equal to or greater than 0.9581. This sets the reliability goal for $R_b$ at a value of 0.979. This reliability value is considerably less than the original estimate of 0.99 for each functional unit. The reason for this change is the conversion to an integer number of joists in the solution of the joist system. The trading of functional unit reliabilities in this manner is often required in meeting the overall reliability goal.

The design formula reflecting this design goal is

$$R = \Phi\left[\frac{.2630 - \dfrac{(18.35)(2288)}{I/c}}{\sqrt{657^2 + \left[\dfrac{(18.35)(162)}{I/c}\right]^2}}\right]$$

solving for I/c yields a desired value of 33.6. This corresponds very closely to the section modulus of a 4x8 (33.9). Substituting this value for I/c and the relevant loading for the exterior girders into the design formula yields $R_a = 0.9943$. Checking the results with the Product Rule indicates an achieved reliability, R, equal to $(.9943)^2(.979)(.9957)^2$ or 0.9596.

This Chapter combined the reliability of subgroups to achieve the desired reliability goal. The method of this Chapter was basically the same as that of Chapter III, however it was applied to a more complex example. Chapter III and Chapter IV have illustrated the proposed probabilistic approach and have laid the groundwork for the discussion of Chapter V which follows.

# CHAPTER V

## CONCLUSION AND RECOMMENDATIONS

This paper has investigated the factors affecting structural design and risk. Wood strength values were found to be distributed according to the Normal probability density function. From the strength distribution and its relationship to density, the weight of wood was estimated following the Normal probability density function. The difference in the strength and weight was obtained as a linear combination of independent Normally distributed variables. This combination, therefore, itself formed a Normal distribution. By choosing the desired probability of structural success, reliability, the design of the structure was accomplished.

Further investigation found wind speeds to be distributed according to a Fisher-Tippitt Type II (Frechét) distribution. The difficulty of combining this with the Normal distribution was circumvented by treating design wind speed as a constant. An approximate method was used to evaluate the risks encountered in this approach.

The scope of this paper is specific in that its results are valid only for the situation covered: Normally distributed strengths and dead load and wind loads from a Frechét distribution. The methods of the paper should be used only when the assumptions of the paper are applicable. Use in different situations is valid only insofar as necessary changes in method account for the differences.

The scope of this paper is general, however, inasmuch as the situation discussed herein is general. Strengths of wood are Normally distributed and wood is an important and widely used construction material. The Frechét distribution is generally applicable for extreme winds in all of the United States. Many of the assumptions and procedures of this report are applicable to varied situations.

This report is significant in that it offers a straightforward method for utilizing statistical factors in structural design. Utilization of statistical factors is important as a means for more precise design and for obtaining a more exact measure of risks. More precision in design and risk evaluation offers the possibility of improved design. A straightforward method for accomplishing this goal is important because methods heretofore available were exceedingly complex. In avoiding this complexity, a certain sacrifice in accuracy was possibly admitted. However, it is believed that this sacrifice has been kept small so that a worthwhile, workable method is offered.

Specific contributions of this report include:

1. Introduction of a specific engineering problem and a proposed method for its solution.

2. Collection and discussion of many current related works, thus providing a substantial bibliography.

3. Utilization of the latest work in this field to provide a method based upon information currently available.

4. Illustration of a simple design example to illustrate the basic design method and risk evaluation procedure.

5. Provision of a more extensive design model to further illustrate principles of probabilistic design and to incorporate basic economic considerations of risks.

In the preparation of this paper, several related problems were uncovered as were several problem areas basic to this report. The following list enumerates those areas in need of work, in the hope that appropriate future effort will be stimulated:

1. Fundamental research and testing to determine the strengths of engineering materials in the form of statistical distributions. This should include the study of the relationship between various modes of failure for each particular material.

2. Studies of desired reliabilities of buildings of different types based upon risks and costs of failure.

3. Assurance of quality control in all phases of a structure's life: production of construction material, actual construction, and maintenance.

4. Extend present reliability methods to include confidence intervals upon reliability estimates.

5. Incorporation of maintainability considerations into structure design: planned preventive maintenance schedules, increased ease of repair of most likely failure units, design of specific units to fail under high stress so as to relieve the loading on the remaining structure (fuse concept), and modularized construction.

APPENDIXES

## APPENDIX A

## ANALYSIS OF A STEEL PLANT TEST PROCEDURE

The particular steel plant examined for this report primarily produced pipe for construction of petroleum pipelines. Although pipelines were not a main concern of this paper, the results obtained from this plant should provide an indication of the basic testing problem. The quality control and testing procedures in use were based upon American Petroleum Institute (API) Standard 5L. Products tested under this standard validly meet the minimum strength requirements with desired confidence.

Basically, the test procedure consisted of dividing each heat or production quantity of approximately two thousand specimens into about five lots of four hundred specimens or less. Each of the lots were tested independently. Two specimens from the lot were tested; if both met the specified strength requirement, the lot was considered acceptable.

However, the strength of the test specimens was usually far above the minimum. A typical case was a steel with specified yield strength of 55 KSI. The test specimens had strengths in the range of 60 to 66 KSI. Because engineers design upon the basis of specified strength, any strength above this is wasted for design purpose.

To overcome this waste, one method of wringing more information from the same amount of testing would be as follows:

1. Record all test results for historical indication of the type of distributional form followed by steel strengths.

2. Obtain an estimate, S, of the standard deviation of strengths from test results.

3. For a particular heat of steel, combine all (usually ten) test specimen results.

This would permit specifying the distribution of strengths with a certain confidence.

If the resulting distribution were Normal, the statistical analysis of confidence must utilize the t-distribution because the standard deviation is unknown (see ref. 6). The confidence interval of the true mean can be derived from the expression

$$\text{Probability}(-t_{*/2,\ n-1} \le t \le t_{*/2,\ n-1}) = 1-*.$$

This yields the similar expression:

$$\text{Probability}(\bar{x} - (t*/2,\ n-1)(S)\sqrt{n} \le \text{mean} \le \bar{x} + (t_{*/2,\ n-1})(S)/\sqrt{n}) = 1-*.$$

From this, the range of the true mean would be

$$(2)(t_{*/2,\ n-1})(S)/\sqrt{n}.$$

Using the example of test specimens in the range 60 to 66 KSI, assume that this indicates an estimate of standard deviation of 1 KSI. Assume further that the confidence in the result should be 0.95. Under these conditions the range of the true mean of the distribution would be 1.66 KSI.

Hence, a method of providing increased strength information from the same amount of testing has been shown. Under the assumptions of this method a rather close estimate of the mean of the distribution can be provided. This Appendix indicates the general approach to the problem of testing and is by no means complete. Reference 9 which was unavailable at this writing should provide additional information on this point.

APPENDIX B

DERIVATION OF TABLE 1 (WIND LOAD vs. RISK)

This derivation is based upon the work of Thom (26) who determined that wind speed in the United States follows a Fisher-Tippitt Type 11 (Frechêt) distribution. The Frechet distribution is given by $F(x) = \exp - (\frac{x}{B})^{-\alpha}$ where x is the extreme wind speed, B and $\alpha$ are the scale and shape factors. F(x) is the probability that an extreme value is less than x. Thom does not provide values for the parameters, but instead provides quantile maps for the United States and maximum-value probability paper for the Frechêt distribution. Thom gives the probability of no extreme wind having speed greater than x during the first m years to be $\alpha = 1 - F(x)^{\underline{m}}$. From this the relation $F(x) = (1-\alpha)^{1/\underline{m}}$ is obtained.

The design service life of the examples in this paper is assumed to be fifteen years. Hence, $F(x) = (1-\alpha)^{1/15}$ for this paper. If the risk of wind speed exceeding x during the first fifteen years is to be $\alpha$ = 0.15, then F(x) is equal to 0.989.

For the location of interest, Oklahoma City in this case, pairs of points (x,F) from the quantile maps are plotted upon the probability paper. Then, knowing the desired value for F(x) is 0.989, a value of x is read from the probability paper. The wind speed so obtained for the example is 80 MPH.

Wind speeds, however, are given by the quantile maps for a height of thirty feet. The velocity is corrected to an assumed height of fifteen feet by the standard formula: $V_z = V_{30}(\frac{z}{30})^{1/n}$

where z is the height at which the speed is desired, $V_z$ is the wind speed at height z, and n is a constant. For example, $V_{15} = V_{30}(\frac{z}{30})^{1/5}$ yielded the solution $V_{15}$ equal to 68 MPH.

A standard chart from Richey (21) based upon the formula pressure = $0.00256\ W^2$ was used to convert the wind speed to a nominal air pressure of 12 PSF.

Return period, RP, for a wind of speed x is obtained from the relation $RP = \frac{1}{1 - F(x)}$. The expected return period for a corrected wind of 68 MPH is 99 years.

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