USAAVLABS TECHNICAL REPORT 69-49

EFFECT OF LOADING SYSTEM RIGIDITY ON THE INITIAL BUCKLING LOAD OF UNREINFORCED CIRCULAR CYLINDRICAL SHELLS UNDER HYDROSTATIC PRESSURE

By

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November 1971

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U. S. ARMY AIR MOBILITY RESEARCH AND DEVELOPMENT LABORATORY
FORT EUSTIS, VIRGINIA

CONTRACT DA 44-177-AMC-115(T)
STANFORD UNIVERSITY
STANFORD, CALIFORNIA

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(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Stanford University, Department of Aeronautics and Astronautics, Stanford, California	2a. REPORT SECURITY CLASSIFICATION Unclassified
	2b. GROUP

3. REPORT TITLE

EFFECT OF LOADING SYSTEM RIGIDITY ON THE INITIAL BUCKLING LOAD OF UNREINFORCED CIRCULAR CYLINDRICAL SHELLS UNDER HYDROSTATIC PRESSURE

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Final Technical Report

5. AUTHOR(S) (First name, middle initial, last name)

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S. C. Bailey

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
November 1971	34	11

8a. CONTRACT OR GRANT NO. DA 44-177AMC-115(T)	9a. ORIGINATOR'S REPORT NUMBER(S) USAAVLABS Technical Report 69-49
b. PROJECT NO. 1F162204A17002	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) SUDAAR No. 289
d.	

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Eustis Directorate, U.S. Army Air Mobility R&D Laboratory, Fort Eustis, Virginia

13. ABSTRACT

In this report, all the data pertinent to an experimental investigation of the effect of loading system rigidity on the initial buckling load of unreinforced circular cylindrical shells under hydrostatic pressure are presented. According to the commonly accepted Tsien criterion (Reference 2), the buckling pressure should be higher in a rigid system than in a soft system. The research denies this axiom by generating strong evidence that the stiffness characteristic of the loading system does not alter the initial buckling stress, which fully supports the results obtained by Kaplan and Fung (Reference 3) on spherical caps, by Krenzke (Reference 4) on plastic spheres, and by Carlson, Sendelbeck, and Hoff (Reference 5) on nickel-plated spherical shells.

However, in the case discussed in this report, the conclusion is based upon a statistical interpretation of some 100 tests on very nearly identical vehicles; whereas in the contemporary researches, a more limited number of vehicles were used. Moreover, it concurs with the similar findings by Horton, Bailey, Cox, and Smith (Reference 6), that extensional rigidity of the testing machine does not influence the initial buckling load for circular cylindrical shells loaded in axial compression.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

Unclassified
Security Classification

Unclassified
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Initial Buckling Load						
Loading System Rigidity						
Standard Errors						
Level of Significance						
Test Statistic						
Operating Characteristic Curve						

Unclassified
Security Classification

DEPARTMENT OF THE ARMY
U. S. ARMY AIR MOBILITY RESEARCH & DEVELOPMENT LABORATORY
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This research effort was carried out under Contract DA 44-177-AMC-115(T) by Stanford University to evaluate the effect of loading system rigidity on the initial buckling load. The effort was applied to unreinforced circular cylindrical shells under hydrostatic pressure.

The report has been reviewed by this Directorate and is considered to be technically sound. It is published for the exchange of information and the stimulation of future research.

This program was conducted under the technical management of Mr. James P. Waller, Structures Division.

Task 1F162204A17002
Contract DA 44-177-AMC-115(T)
USAAVLABS Technical Report 69-49
November 1971

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SUMMARY

In this report, all the data pertinent to an experimental investigation of the effect of loading system rigidity on the initial buckling load of unreinforced circular cylindrical shells under hydrostatic pressure, already reported by the authors (Reference 1) in condensed form are presented. According to the commonly accepted Tsien criterion (Reference 2), the buckling pressure should be higher in a rigid system than in a soft system. The research presented in this report denies this axiom by generating strong evidence that the stiffness characteristic of the loading system does not alter the initial buckling stress, which fully supports the results obtained by Kaplan and Fung (Reference 3) on spherical caps, by Krenzke (Reference 4) on plastic spheres, and by Carlson, Sendelbeck, and Hoff (Reference 5) on nickel-plated spherical shells.

However, in the case discussed in this report, the conclusion is based upon a statistical interpretation of some 100 tests on very nearly identical vehicles; whereas in the contemporary researches, a more limited number of vehicles were used. Moreover, it concurs with the similar findings by Horton, Bailey, Cox, and Smith (Reference 6), that extensional rigidity of the testing machine does not influence the initial buckling load for circular cylindrical shells loaded in axial compression.

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LIST OF SYMBOLS

R/t	ratio of radius to skin thickness of a shell
α	level of significance
$\bar{P}_{cr}$	mean buckling load
n	sample size
$\nu\%$	coefficient of variation
s	sample standard deviation
s^2	sample variance
n_x, n_y	sample sizes
σ	standard deviation
β	parameter
$\bar{x}$, $\bar{y}$	arithmetic means
$\sigma(x_m)$, $\sigma(y_m)$	standard errors of the m^{th} observation
x	variable buckling load
y	deviation in multiples of standard deviation about the mean cumulative probability point
σ_n	normal standard deviation as a function of sample size

INTRODUCTION

In a recent paper (Reference I), the authors reviewed the general question of the influence of machine stiffness and the validity of the Tsien criterion (Reference 2) in stiffness computation. The present report presents all the test data for one series of experiments discussed in that presentation. The question herein examined is, "Is Tsien's criterion pertinent to the instability of a closed thin-walled vessel under external pressure loading?" Limited experimental data by Kaplan and Fung (Reference 3) and by Krenzke (Reference 4) suggest that Tsien's criterion is not applicable in the cases of shallow spherical caps and plastic spheres. The present study was aimed at an extensive test series using thin cylindrical shells with end caps which could provide the basis of a thorough statistical evaluation.

The results obtained were in full agreement with the conclusion of the researchers referenced above. They have been confirmed further by subsequent researches made by Carlson, Sendelbeck and Hoff on spherical shells Reference (5).

EXPERIMENTAL APPROACH

Statistical studies of the behavior of structures are feasible only if a large number of nominally identical test vehicles are available at reasonable cost. Thus, the vehicles chosen were right-circular cylinders with end caps. Such specimens are available from the beverage can industry and are, as previous researches have demonstrated (References 6 and 7), remarkably consistent in character.

Prior experiments had illustrated that a test series of 50 specimens in each of two environments should prove to be adequate for statistical study; as a consequence, this number of vehicles was chosen. The choice was further reinforced by the known fact that cylindrical shells under external pressure loading conditions are more consistent in behavior than similar structures under axial load. This is clear from the work of Strum (Reference 8) and from the extensive study of Cleaver (Reference 9). Thus, the cylindrical shell appeared to be a potentially better choice of test vehicle than a sphere, since it is well recognized that minor imperfections in such vehicles have extreme significance in buckle behavior. A great quantity of data in evidence of this point is available in the current literature.

DETAILS OF THE TEST SPECIMENS

The test vehicles were, as explained in the preceding section, standard beverage cans. Such cans are manufactured in a fully automated process and are remarkably consistent with regard to overall geometry. All specimens were made from the same batch of material. Blanks for the cylindrical bodies were punched from twice cold-rolled steel sheet and then were formed into a cylindrical shape in a completely automatic process.

The overall dimensions were as follows:

Diameter	-	2.63 inches
Length	-	4.75 inches
Thickness	-	0.0058 inch
R/t	-	226

In the interest of highest consistency, both ends were capped in the same automatic machinery.

TEST ENVIRONMENTS

The two environments chosen were obtained by change of fluid within the specimen. In one case, air was used; in the other, hydraulic fluid. In the air tests, the port was large; in the hydraulic fluid tests, small. Thus, in the former case, the volume was permitted to change as rapidly as possible; in the latter, it was severely restrained.

METHOD OF PRESSURE APPLICATION

For the soft system, the method of application of load was as delineated in Figure 1. It is clear also from this sketch that the pressure differential was measured by means of a standard differential manometer. The seal system and arrangement of the specimen in the test apparatus are clearly apparent from Figures 2a through 2b. Typically specimens collapsed in the same manner as in the hard system (Figure 4).

In the hard system, the vessel was filled with hydraulic fluid, extreme care being exercised to ensure that there was no air entrainment. The filled vehicle was then submerged in a bath of hydraulic fluid to mitigate the effect of hydraulic head. This system was then connected to an oil-mercury differential manometer. The full details of the pressure loading system are clear from the sketch shown in Figure 3. The arrangement of the specimen in the test apparatus is portrayed in Figures 4a through 4b. A typical buckled specimen is depicted in Figure 4c.

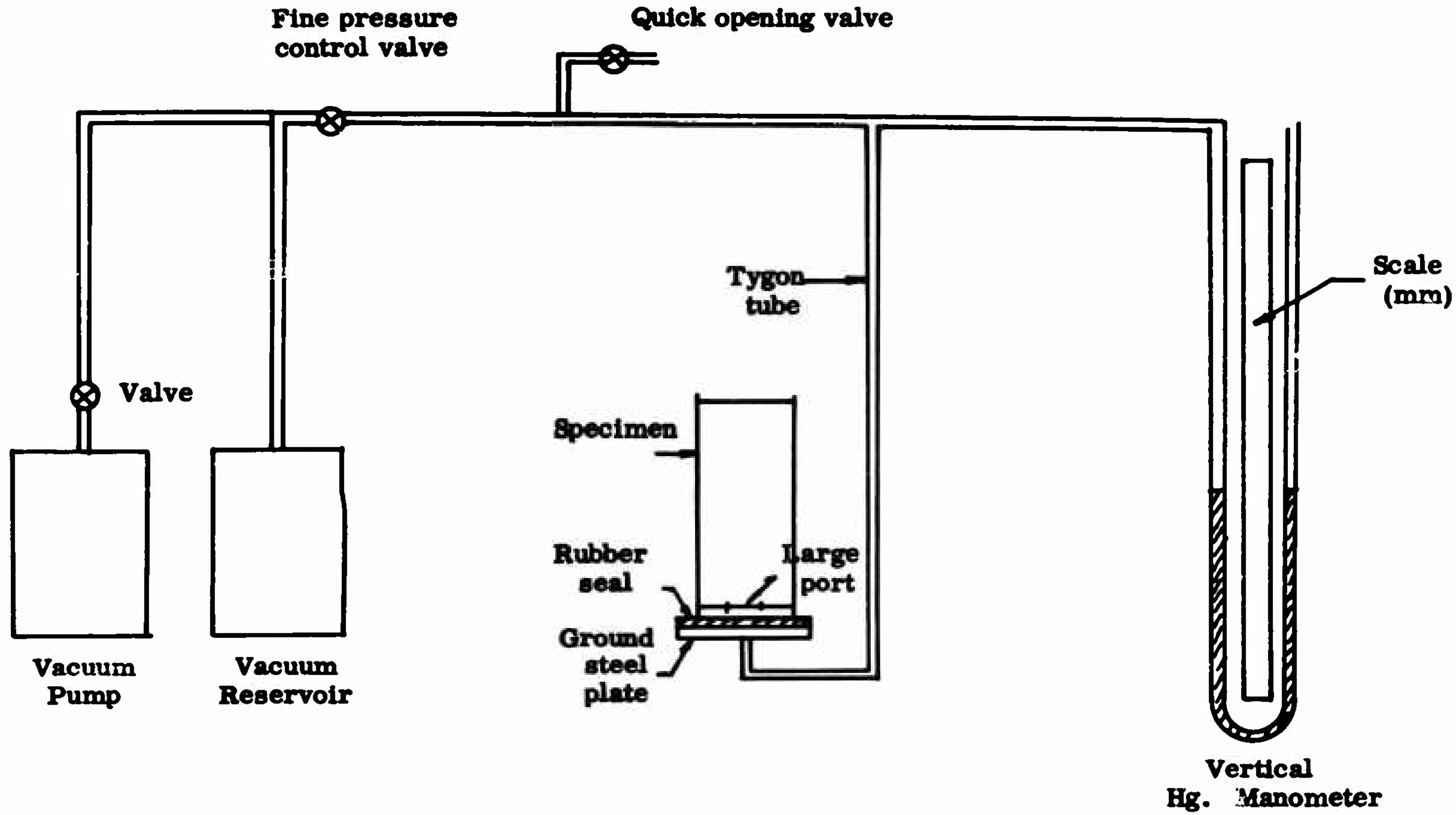
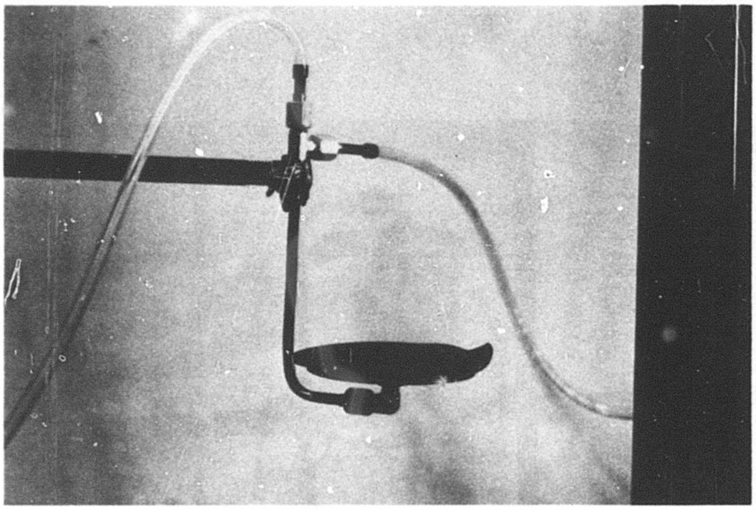
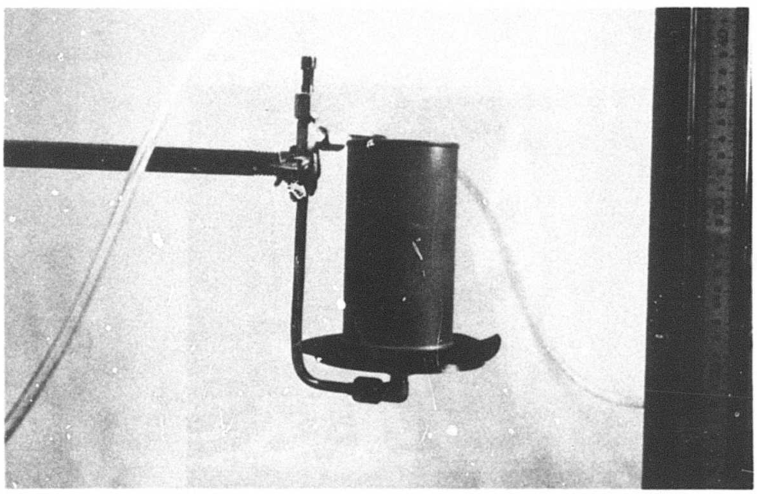


Figure 1. Method of Application and Measurement of External Pressure in a Soft System - Air.

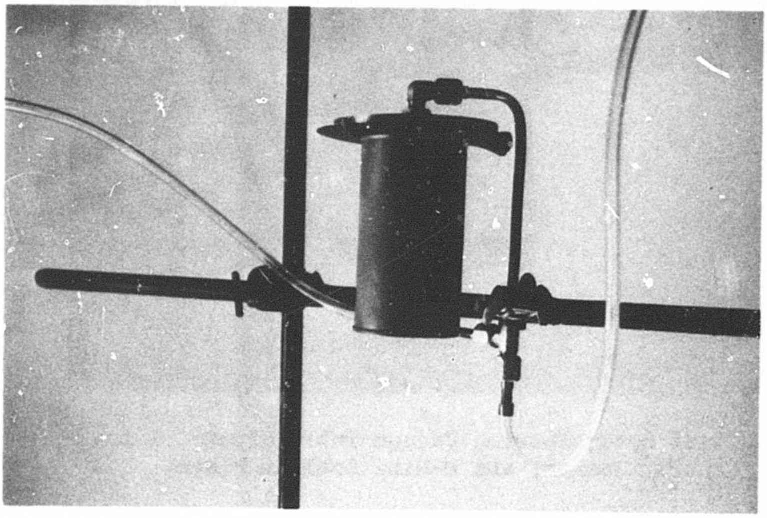


a. Test Setup Showing Vacuum Tube, Ground Plate, Rubber Gasket, and O-Ring Seal in Place.

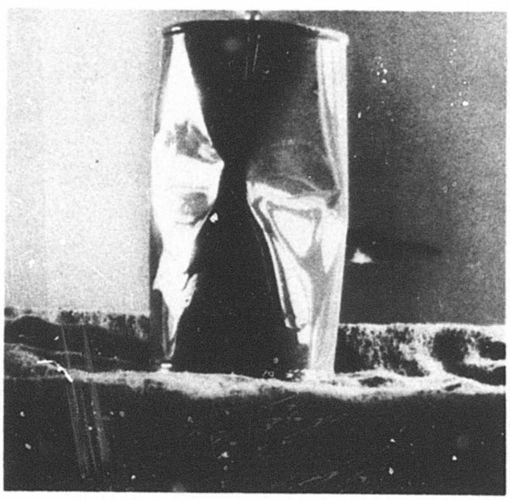


b. Shell Assembled.

Figure 2. Test Setup for Buckling of a Thin Circular Cylindrical Shell Under External Pressure in a Soft System - Air.



c. Typical Specimen During Test Before Collapse.



d. Typical Specimen After Collapse From External Pressure.

Figure 2. Continued.

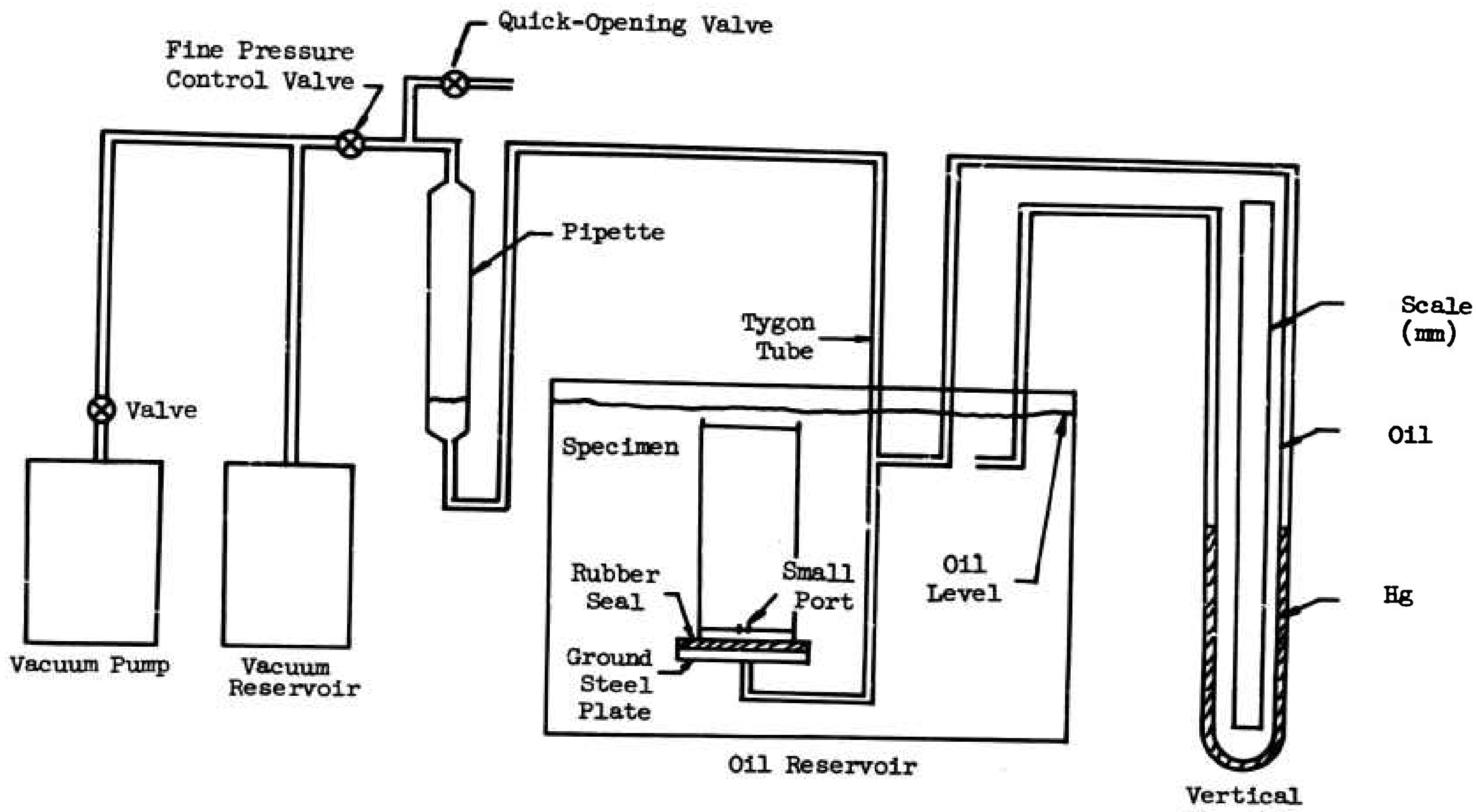
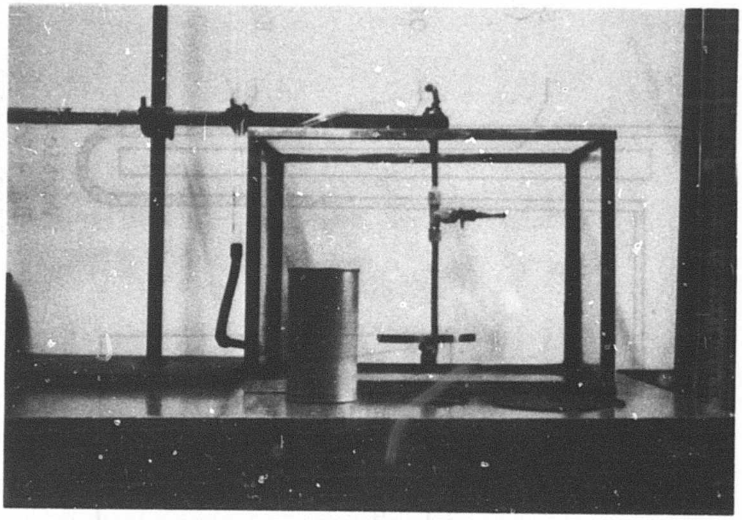
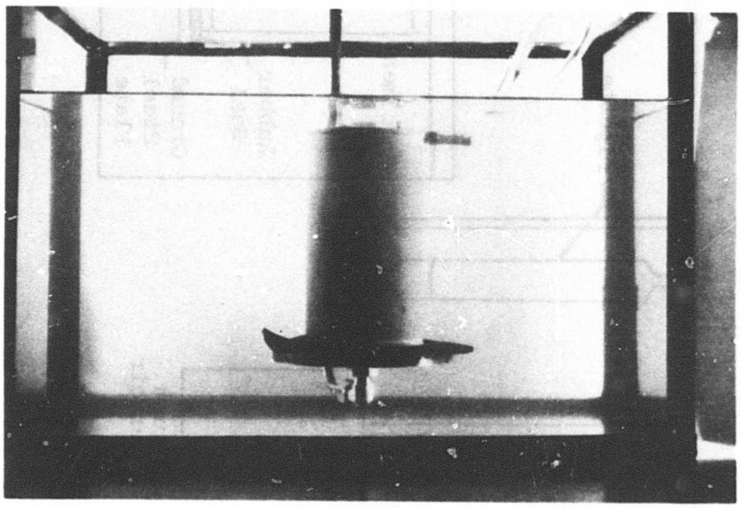


Figure 3. Method of Applications and Measurement of External Pressure in a Hard System - Oil.

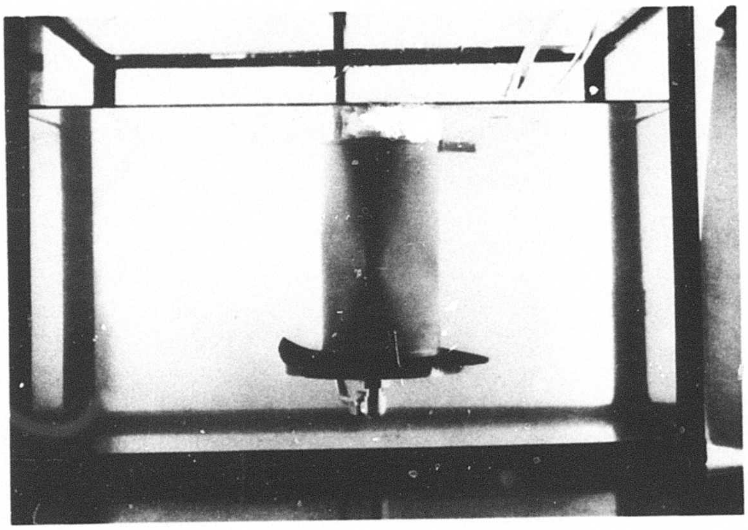


a. Detail of Test Apparatus Showing Pipette, Oil Tank and the Oil - Mercury Differential Manometer.



b. Typical Specimen During Test Before Collapse.

Figure 4. Test Setup for Buckling of a Thin Circular Cylindrical Shell Under External Pressure in a Hard System - Oil.



c. Typical Specimen After Collapse From External Pressure.

Figure 4. Continued.

DETERMINATION OF INSTABILITY PRESSURE

The determination of the critical pressure was readily and simply made by two observers who continually monitored the pressure indicating device. Collapse in either system was readily apparent from the direction change which occurred in the mercury column. However, the buckling pressure obtained from the oil-mercury differential manometer was corrected to eliminate the effect of weight of oil on the observed pressure head.

BEHAVIOR UNDER TESTING

In all cases the failures which took place were of identical character, in the sense that, irrespective of the stiffness of the system, the shell distorted from a circular cross section to a square cross section. However, in the soft system this change in geometric form occurred with great rapidity, whereas in the hard system it was very slow. The final buckle pattern referred to above is apparent from Figure 2d for the soft system and from Figure 4c for the hard system.

DISCUSSION OF EXPERIMENTAL DATA

The values of critical pressure for the soft and hard systems are given in Tables I and II in the appendix. The data have been presented, not in the order of tests, but according to magnitude of loads. This has been done to simplify the analysis. From this presentation, it is simple to devise cumulative probability distributions. These are also given in the tables. They have been graphically portrayed in Figures 5 and 6. It is immediately apparent from these figures that both distributions are essentially normal. As a consequence, the data are analyzable by standard statistical procedures.

The mean values of critical pressure are derived in the computations given in the appendix. It is shown there that the critical levels are 53.58 cm of Hg in the soft system and 53.73 cm of Hg in the hard system. The standard deviations are 2.27 cm of Hg and 2.11 cm of Hg, respectively.

From the close identity of these results, it might be stated that the tests have demonstrated equality of performance under the different test conditions. However, the point has been considered in depth in order to establish a bond of confidence within the confines of normal statistical testing.

The mean buckling pressures have been compared by means of the student's "t" test. This comparison is made in the appendix. In the computation given in detail, the 5% level of significance ($\alpha = 0.05$) was used, and the standard derivations were assumed to be equal. The calculations show that for a sample size of 50 and a 5% risk of accepting a false hypothesis, there is a 95% probability of detecting a difference between the means as small as 1.6 cm, or about 3% of the average buckling pressure. The t test statistic is found to be $|t| = 0.349$. Thus, since $0.349 < t_{0.025;98} = 1.99$, we accept the hypothesis that there are no differences in the mean buckling pressures obtained from the variations in test condition at the 5% level of significance.

CONCLUSION

The results presented in this report provide substantial experimental evidence that the stiffness characteristic of the loading system does not affect the initial buckling stress of unreinforced circular cylindrical shells under hydrostatic pressure. This conclusion supports the experimental findings of Kaplan and Fung; Krenzke; and Carlson, Sendelbeck, and Hoff on the buckling of sphere caps and complete spheres by external pressure in loading systems of different rigidities. Thus, the commonly used Tsien energy criterion for the stability studies of thin-walled shells, which predicts a higher buckling pressure in a more rigid loading system, must be an improper hypothesis to explain the observed buckling phenomenon of shell bodies.

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APPENDIX

STATISTICAL ANALYSIS OF THE EXPERIMENTAL DATA

In this appendix, all test data are presented and analyzed. The individual critical loads for the tests performed in the soft system (air), in accordance with the procedures described in the main text, are listed in order of magnitude in Table I and graphically portrayed in Figure 5.

The buckling loads are arranged in increasing numerical sequence. In order to plot all data values, the cumulative probability of the m th observation is determined from m/(n + 1).

The data pertaining to the tests made in the hard system (oil) are likewise given in Table II. This information is graphically portrayed in the probability plot of Figure 6. It is immediately apparent from Figures 5 and 6 that the distributions are essentially normal; therefore, the data are analyzable by standard statistical methods.

When these procedures are followed with the data relevant to the soft system, the mean buckling load $\bar{P}_{cr}$ is derived as follows:

$$\bar{x} = \bar{P}_{cr} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{2678.9}{50} = 53,578\text{-cm Hg} \qquad (1)$$

The corresponding sample variance (s^2) and the appropriate standard deviation (s) are derived from the equation

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} - \bar{x}^2 \qquad (2)$$

as follows: $s^2 = 2875.751 - 2870.602 = 5.149$
and $s = 2.268$-cm Hg
while the coefficient of variation ν, being the ratio of the standard deviation to the mean, is given by

$$\nu\% = \frac{s}{\bar{x}} \times 100\% = \frac{2.268}{53.578} \times 100\% = 4.23\% \qquad (3)$$

TABLE I. CRITICAL PRESSURE FOR 50 NOMINALLY IDENTICAL CYLINDERS TESTED UNDER EXTERNAL PRESSURE - AIR PRESSURE SYSTEM. (Values arranged in order of magnitude)

Load No. (m_i)	P_{cr} Critical Pressure cm Hg (x_i)	Cumulative Probability % m/(n + 1)	Load No. (m_i)	P_{cr} Critical Pressure cm Hg (x_i)	Cumulative Probability % m/(n + 1)
1	48.6	1.96	26	53.5	51.0
2	49.1	3.92	27	53.9	52.9
3	49.4	5.88	28	54.0	54.9
4	50.3	7.85	29	54.1	56.9
5	50.6	9.80	30	54.2	58.9
6	50.7	11.8	31	54.2	60.8
7	50.9	13.7	32	54.3	61.6
8	50.9	15.7	33	54.4	64.6
9	51.1	17.6	34	54.5	66.6
10	51.5	19.6	35	54.6	68.5
11	51.5	21.6	36	55.1	70.5
12	51.7	23.5	37	55.1	72.5
13	52.1	25.5	38	55.1	74.5
14	52.2	27.4	39	55.2	76.5
15	52.5	29.4	40	55.2	78.5
16	52.6	31.4	41	55.5	80.5
17	52.7	33.3	42	55.8	82.4
18	52.8	35.1	43	55.8	84.4
19	52.9	37.2	44	56.2	86.2
20	53.1	39.2	45	56.6	88.2
21	53.2	41.1	46	56.6	90.1
22	53.3	43.1	47	57.5	92.1
23	53.3	45.1	48	57.7	94.0
24	53.4	47.1	49	57.8	96.0
25	53.5	49.1	50	58.1	98.0

$$\sum P_{cr} = 2{,}678.9$$

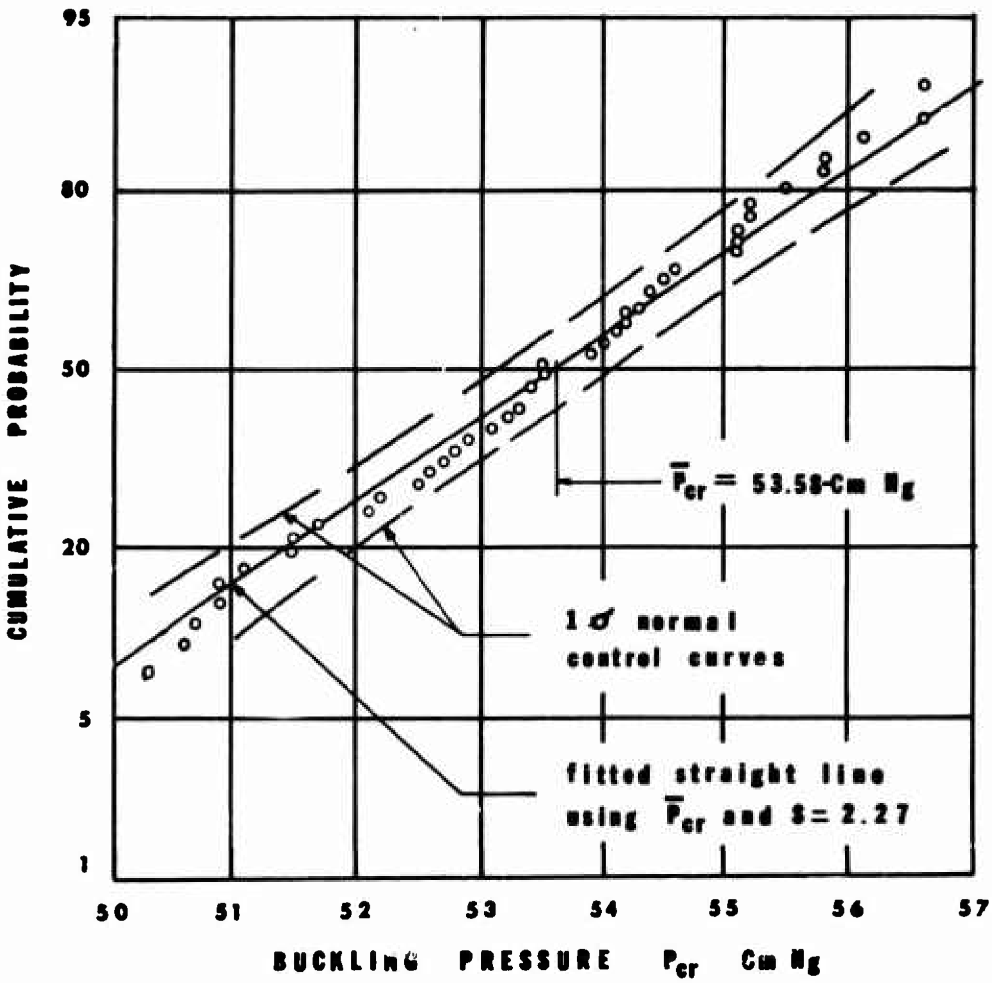

Figure 5. Distribution of Buckling Pressures for Thin Circular Cylindrical Shells When Tested in a Soft System - Air.

TABLE II. CRITICAL PRESSURE FOR 50 NOMINALLY IDENTICAL CYLINDERS TESTED UNDER EXTERNAL PRESSURE - OIL PRESSURE SYSTEM. (Values arranged in order of magnitude)

Load No. (m_i)	P_{cr} Critical Pressure cm Hg (x_i)	Cumulative Probability % m/(n + 1)	Load No. (m_i)	P_{cr} Critical Pressure cm Hg (x_i)	Cumulative Probability % m/(n + 1)
1	49.1	1.96	26	54.1	51.0
2	49.3	3.92	27	54.1	52.9
3	49.3	5.88	28	54.1	54.9
4	51.1	7.85	29	54.1	56.9
5	51.3	9.80	30	54.4	58.9
6	51.5	11.80	31	54.6	60.8
7	51.6	13.7	32	54.6	61.6
8	51.7	15.7	33	54.7	64.6
9	51.8	17.6	34	54.7	66.6
10	52.1	19.6	35	55.0	68.5
11	52.1	21.6	36	55.1	70.5
12	52.1	23.5	37	55.2	72.5
13	52.1	25.5	38	55.4	74.5
14	52.3	27.4	39	55.4	76.5
15	52.3	29.4	40	55.5	78.5
16	52.3	31.4	41	55.6	80.5
17	52.4	33.3	42	55.9	82.4
18	52.5	35.1	43	55.9	84.4
19	52.6	37.2	44	55.9	86.2
20	52.9	39.2	45	56.2	88.2
21	53.1	41.1	46	56.4	90.1
22	53.1	43.1	47	56.8	92.1
23	53.4	45.1	48	57.1	94.0
24	53.8	47.1	49	58.1	96.0
25	53.9	49.1	50	58.1	98.0

$$\sum P_{cr} = 2,686.7$$

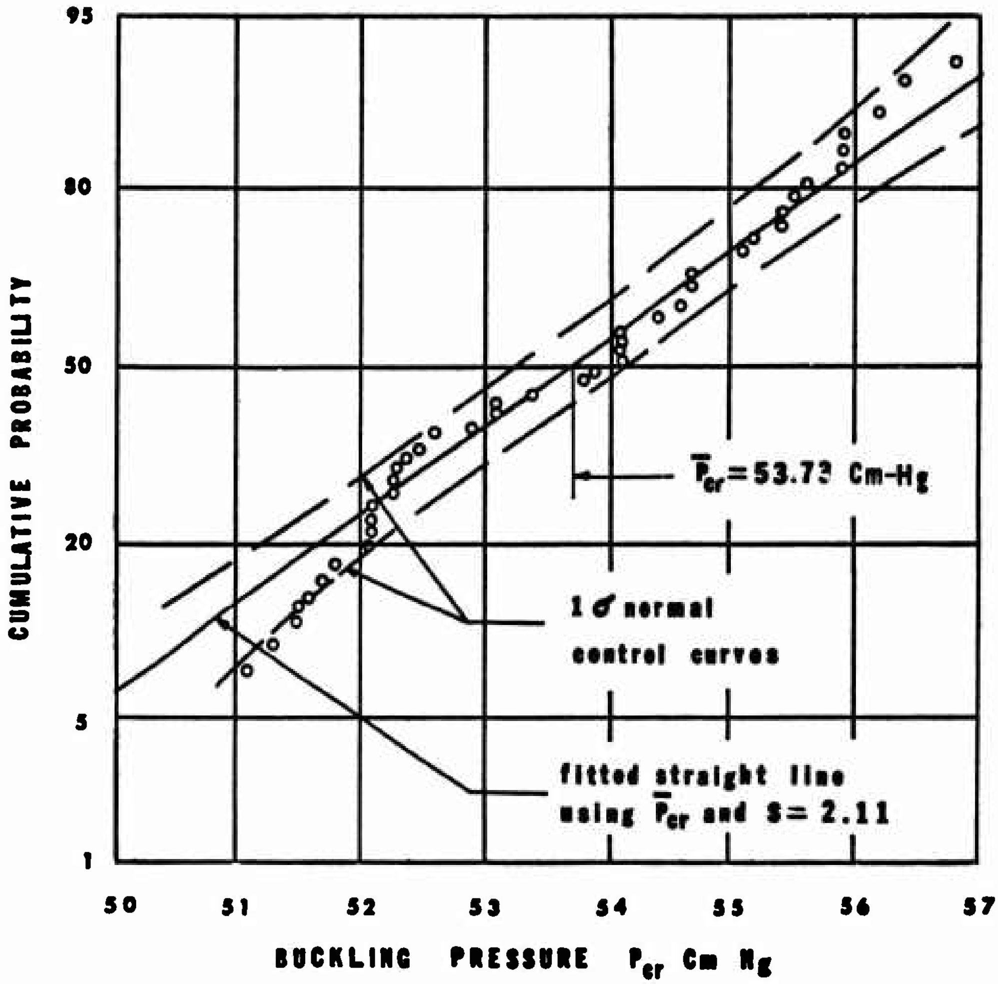

Figure 6. Distribution of Buckling Pressures for Thin Circular Cylindrical shells When Tested in a Hard System - Oil.

The theoretical curve for the data displayed in the probability plot of Figure 5 is

$$x = \beta + (1/\alpha)y \qquad (4)$$

where x is the variable buckling load and y is the deviation in multiples of standard deviation about the mean cumulative probability point. These values correspond to cumulative probabilities as follows in Table III.

The classical method of least squares is applied to estimate the parameters β and $1/\alpha$. This leads to the values which follow

$$\beta \approx \bar{x} = 53.578 \text{ cm. Hg.} \qquad (5)$$

$$1/\alpha \approx \frac{s}{\sigma_n} = \frac{2.268}{0.932} = 2.433 \qquad (6)$$

where σ_n is the normal standard deviation as a function of sample size. With $n = 50$; $\sigma_n = 0.932$ (Reference 10, Table 1. 2. 9, p. 39). Thus the empirical line is

$$x = 53.58 + 2.43y \qquad (7)$$

This is plotted in Figure 5.

The degree of fit of the straight line and the data is determined graphically using control curves in the following manner. The standard errors $\sigma(x_m)$ of the m^{th} observations are added to and subtracted from the m^{th} values x_m as determined from the fitted straight line. The points $x_m \pm \sigma(x_m)$ are joined to form these curves. The standard errors $\sigma(x_m)$ are derived from the usual statistical formula

$$\sigma(x_m) \quad \frac{\sigma(y_m)\sqrt{n}}{\alpha\sqrt{n}} = 0.344 \quad \sigma(y_m)\sqrt{n} \qquad (8)$$

where the values $\sigma(y_m)\sqrt{n}$ are pure numbers independent of the parameters (Reference 10, Table 2. 16, p. 52). Probability values ranging from 0.15 to 0.50 are obtained from symmetry. The control curves are plotted in Figure 5. The data for the hard system (Oil) is treated in an identical manner and the various parameters of importance are derived as follows:

$$\text{Mean Buckling Load} = \bar{P}_{cr} = \frac{2686.7}{50} = 53.734 \text{ cm. Hg.}$$

$$\text{Variance} = s^2 = 2891.791 - 2887.343 = 4.448$$

$$\text{Standard Deviation} = s = 2.110 \text{ cm. Hg.}$$

$$\text{Coefficient of variation } v\% = \frac{2.110}{53.734} \times 100\% = 3.93\%$$

TABLE III. CUMULATIVE PROBABILITIES FOR ONE STANDARD DEVIATION ABOUT THE MEAN POINT.

Deviations in σ y	Cumulative Probability % $F(x)$
+1	84.13
0	50.00
-1	15.87

TABLE IV. CONTROL CURVE DATA FOR THE SOFT-SYSTEMS - AIR.

Probability	$\sigma(y_m)\sqrt{n}$	$\frac{1}{\alpha\sqrt{n}}$	$\sigma(x_m)$
0.5	1.253	0.344	0.431
0.6	1.268	0.344	0.436
0.7	1.318	0.344	0.453
0.8	1.429	0.344	0.492
0.85	1.532	0.344	0.427

TABLE V. CONTROL CURVE DATA FOR THE HARD-SYSTEM - OIL.

Probability	$\sigma(y_m)\sqrt{n}$	$\frac{1}{\alpha\sqrt{n}}$	$\sigma(x_m)$
0.5	1.253	0.320	0.401
0.6	1.268	0.320	0.406
0.7	1.318	0.320	0.422
0.8	1.429	0.320	0.457
0.85	1.532	0.320	0.490

The coefficients $\beta + 1/\alpha$ are given by

$$\beta \approx \bar{x} = 53.734 \text{ cm. Hg.} \tag{9}$$

$$1/\alpha \approx s/\sigma_n = \frac{2.110}{0.932} = 2.264 \text{ cm. Hg.} \tag{10}$$

The line defined by these coefficients is plotted in Figure 6. The appropriate control curves are derived as before. The necessary values of $\sigma(x_m)$ being listed below.

$$\sigma(x_m) = \frac{\sigma(y_m)\sqrt{n}}{\alpha/n} = 0.320 \quad \sigma(y_m)\sqrt{n} \tag{11}$$

These control curves are shown in Figure 6.

It is readily apparent from the probability plots of Figures 5 and 6 and their appropriate control curves that both sets of observations can be assumed samples from normal distributions. The standard deviations of these distributions are extremely close and therefore the means are compared using the student 't' test. The hypothesis of equality is examined.

The criterion for acceptance of this premise is

$$|t| \leq t_{\alpha/2;\ n_x + n_y - 2} \tag{12}$$

where α is the level of significance
and $n_x + n_y$ are the sample sizes.

Here

$$n_x = n_y = 50$$

$$n_x = n_y - 2 = 98 \tag{13}$$

The t test statistic is calculated from (Reference 11, Table 7.2, p. 171).

$$t = (\bar{x} - \bar{y}) \sqrt{\frac{n_x n_y (n_x + n_y - 2)}{(n_x + n_y)\,(n_x s_x^2 + n_y s_y^2)}} \tag{14}$$

where x = sample data from soft system

y = sample data from hard system

$\bar{x} = 53.578$ $\quad \bar{y} = 53.734$

$s_x^2 = 5.1486$ $\quad s_y^2 = 4.448$

$n_x = 50;\ n_y = 50$

$$t = (0.156) \frac{48}{9.5966} = 0.349$$

Choosing a 5 percent level of significance a table of percentage points of the t distribution for 98 degrees of freedom gives

$$t_{0.025;\ 98} = 1.984$$

Since

$$|0.349| < 1.984$$

the hypothesis of equality of the means is accepted at the 5 percent level.

An indication of the sensitivity of the analysis is obtained from an examination of the operating characteristic curve at this level. The curve corresponding to a sample size of 50 indicates a 95 percent probability of detecting a difference $d = 0.35$. (Reference 12, Figure 6.1C, p. 129). For this test

$$d = \frac{|\mu_x - \mu_y|}{2\sigma} \tag{15}$$

Using the average sample standard deviation from the two tests as an estimate for σ,

$$|\mu_x - \mu_y| = 0.70(2.189) \approx 1.6 \text{ cm. Hg.} \tag{16}$$

Thus, a difference between the means as small as 1.6 cm. Hg. or approximately 3 percent of the average buckling load could be detected at the 95 percent level.

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