Contract No. DAHC04-69-C-0077
ARPA Order No. 1442, Amendment 2
Program Code 9E30

EFFECT OF COLLISIONS ON ELECTROSTATIC PROBE MEASUREMENTS

by

J. Bornstein and S. Lederman

POLYTECHNIC INSTITUTE OF BROOKLYN

DEPARTMENT
of
AEROSPACE ENGINEERING
and
APPLIED MECHANICS

NOVEMBER 1971

R

Approved for public release;
distribution unlimited.

Reproduced by
NATIONAL TECHNICAL
INFORMATION SERVICE
Springfield, Va. 22151

PIBAL REPORT NO. 71-32

25

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*
Polytechnic Institute of Brooklyn
Department of Aerospace Engrg. & Appl. Mech.
Rt. 110, Farmingdale, New York 11735

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3. REPORT TITLE
EFFECT OF COLLISIONS ON ELECTROSTATIC PROBE MEASUREMENTS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Research Report

5. AUTHOR(S) *(First name, middle initial, last name)*
J. Bornstein
S. Lederman

6. REPORT DATE
November 1971

7a. TOTAL NO. OF PAGES
18

7b. NO. OF REFS
12

8a. CONTRACT OR GRANT NO.
DAHC04-69-C-0077

b. PROJECT NO.

c. ARPA Order No. 1442, Amendment 2

d. Program Code No. 9E30

9a. ORIGINATOR'S REPORT NUMBER(S)
PIBAL Report No. 71-32

9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

10. DISTRIBUTION STATEMENT
"Approved for public release; distribution unlimited."

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
U.S. Army Research Office-Durham
Box CM, Duke Station
Durham, North Carolina 27706

13. ABSTRACT

An experimental investigation of the length effects as a functio of collisions in the sheath of a slightly ionized hypersonic flow regime for cylindrical ion collecting probes is undertaken. The results indicate a strong dependence of the length effect on the number of collisions in the sheath and also on the degree of ionization of the media.

Unclassified
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Electrostatic Probes						
Collisions						
Free Molecular						
Degree of Ionization						
Hypersonic Flow						

Unclassified
Security Classification

EFFECT OF COLLISIONS ON ELECTROSTATIC PROBE MEASUREMENTS

by

J. Bornstein and S. Lederman

This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by the U.S. Army Research Office-Durham, under Contract No. DAHC04-69-C-0077.

POLYTECHNIC INSTITUTE OF BROOKLYN
Department
of
Aerospace Engineering and Applied Mechanics
November 1971

PIBAL Report No. 71-32

EFFECT OF COLLISIONS ON ELECTROSTATIC PROBE MEASUREMENTS[†]

by

J. Bornstein[*] and S. Lederman[‡]

Polytechnic Institute of Brooklyn
Preston R. Bassett Research Laboratory
Farmingdale, New York

ABSTRACT

An experimental investigation of the length effect as a function of collisions in the sheath of a slightly ionized hypersonic flow regime for cylindrical ion collecting probes is undertaken. The results indicate a strong dependence of the length effect on the number of collisions in the sheath and also on the degree of ionization of the media.

[†] This research was supported by the Advanced Research Projects Agency, the Department of Defense and was monitored by U.S. Army Research Office-Durham, Box CM, Duke Station, North Carolina 27706, under Contract No. DAHC04-69-C-0077.

[*] NASA Fellow.

[‡] Associate Professor of Aerospace Engineering.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

I. INTRODUCTION

The electrostatic probe has found many applications recently, not only as a measuring device of ionized particles, but also as a diagnostic instrument in flow field investigations. As the scope of applicability of the electrostatic probe widened, it was found that the collisionless theory relating the electron current to the electron number density of an ionized medium as originally developed by Langmuir[1], was not sufficient. Scores of papers have been written since, treating the ionized particle collecting probes theoretically as well as experimentally. As work on probes progressed, their behavior observed, it became clear that in spite of their attractiveness due to simple construction, simple external circuitry and wide dynamic range, the electrostatic probe is far from a simple instrument. It was found that besides operational regimes, degree of ionization, temperature ratio of the ionized species, etc., the geometry of the probes and their relation to other parameters of the plasma in question has a significant influence on the relation between the collected current and the corresponding ionized species number density. Two particular parameters of interest in the case of cylindrical probes is the ratio of the radius of the probe to the Debye shielding distance r_p/λ_D, and the ratio of the length to the radius of the probe ℓ/r_p. This problem has been treated experimentally in Refs. 2, 3, and theoretically in Refs. 4,5, and 6. In the experimental investigations mentioned above[2], the plasma parameters were kept constant and the probe size, that is, r_p and ℓ were varied; thus, the major parameter r_p/λ_D could only be varied by changing r_p. Furthermore, the probes were operated in an essentially collisionless regime. In Ref. 3, the regime of operation of the probes was changed from a collisionless to a transitional and finally to a collision dominated regime. At the same time, the electron number density was increased, and the electron temperature maintained, thus decreasing λ_D. In both of the above investigations, the degree of ionization of the plasma was maintained constant. Although this parameter does not seem to enter explicitly into the relation between the probe collected current and the corresponding ionized particle number density, the number of collisions in the sheath of an ionized particle will be affected. In this work, an attempt is being made to investigate experimentally this effect upon the current collection of negatively biased electrostatic probes. To achieve this, the degree of ionization as well as the electron temperature of the plasma have been varied. This was achieved by varying the ratio of the driver to driven pressure of the hypersonic shock tunnel as well as the composition of the driven gas. Using this method, it was possible to achieve a range of electron densities from 10^7el/cm^3 to 10^{10}el/cm, maintaining the same order of magnitude of the neutral density and thus λ_{n-m}.

II. THE ELECTROSTATIC PROBE

The electrostatic probe consists essentially of a conducting electrode to which a bias voltage is applied. The bias attracts and collects charged particles from the surrounding gas. The rate at which charge is collected is a function of the bias, the thermal energy of the particles, the size of the probe and the electron density of the gas. The probe may be used to collect either electrons or positively charged ions. The lower mobility of the positive ions, however, causes fewer to be collected for a given absolute value of the field strength. Thus, the plasma is disturbed less when ions are collected, making this mode of operation more attractive for fluid dynamics measurements.

For simplicity, one may write

$$J = eN_e v \alpha_p / 4 \qquad (1)$$

which relates the current density collected by a probe to the electron density and the thermal drift current, by a normalization parameter, α_p. Therefore, α_p represents the ratio of the current collected by the probe to the current which would be collected due to the thermal motion alone.

Previous studies of the behavior of ion collecting probes have concentrated primarily on one of two sets of conditions:

(a) where the fluid could be considered to be a continuum with respect to the radius of the probe,

(b) where the gas may be considered to be free molecular with respect to this same characteristic size.

This paper will treat the case of cylindrical probes in the transitional regime where there is assumed to be only a small finite number of collisions within the probe's sheath.

Through experiment, it has been found that for ion collection by probes of finite length for cases in which the flow velocity is of the same order of magnitude or larger than the thermal velocity, an additional length effect is present. The phenomenon was first treated experimentally by Lederman, Bloom and Widhopf[2,3] and more recently a theoretical analysis by Bettinger and Chen[6] who were attempting to explain the dependence of the collected current density upon the probe's angle of attack. Their explanation was that the current increase was due to the collection of ions entering the probe's sheath through the end, normal to the flow direction. As in the above, to calculate this current, a collisionless type of analysis was employed. For the case having no angle of attack, the resulting equations were, from Refs. 6 and 7,

$$j/j_\infty = 1 + \frac{\pi}{2\sqrt{2}}\left(\frac{r_p}{\lambda_D}\right)\left[1 + \left(\frac{\lambda_D}{r_p}\right)^2\left(\frac{r_B}{\lambda_D}\right)^2 \cdot \beta\right]\frac{1}{\varphi^{\frac{1}{2}}\tau_\ell} \qquad (2)$$

where $\beta = \begin{Bmatrix} 1 & b_a > 1 \\ b_a(2-b_a) & b_a < 1 \end{Bmatrix}$

and $b_a = \left(\varphi_p \frac{T_e}{T_i}\right)^{\frac{1}{2}}\left(\frac{r_p}{\lambda_D}\right)\left(\frac{\lambda_D}{r_B}\right)$

The value of the sheath thickness, r_B, is obtained from an equation previously derived by Bettinger and Walker[8]. A plot of the r_B as a function of the probe radius with the number density as a parameter is shown in Fig. 1. A plot of j/j_∞ as a function of the ionized particle number density with the ratio of length-to-diameter as a parameter for 0.002" and 0.01" diameter probes is shown in Figs. 2 and 3, respectively. As can be seen in these figures, j/j_∞ is inversely dependent upon the length of the probe and its sheath thickness which is in turn dependent upon both the electron density of the gas and the probe's radius. The effect is then more pronounced for cases where the probe radius is small or the electron density is relatively low. It was found in the present tests that for probes having diameters on the order of 0.08 inches, the effect under the present conditions was negligible.

A second analysis in which the steady three-dimensional problem was transformed into an analogous, unsteady, one-dimensional problem, was performed by Sonin[7]. Again, it was felt that the "length effect" was being caused by the collection of ions which passed into the sheath through the face normal to the flow. This method, however, took into consideration the axial velocity dependence which had been neglected by Bettinger and Chen. The initial condition for the unsteady problem, that no current is collected, placed a lower limit of about τ_1=0.5 on the length of the probe for which the theory was applicable. The τ_1

represents a non-dimensional parameter characteristic of the plasma under investigation. The results also break down for values of τ_1 greater than 3. Because this value of τ_1 also represents the lower limit of applicability of the Bettinger and Chen model, it appears from calculations that these two analyses compliment one another.

A later paper by Shih and Levi[9] has utilized the coordinate transformation in order to transform a finite length cylindrical probe into a spherical probe in the transformed plane. Then applying a small perturbation analysis to a model similar to that of Allen, Boyd and Reynolds[10], they were able to determine the collected current for finite length cylindrical probes under conditions where each collected ion undergoes a small number of collisions in the probe's sheath. Results obtained from the analysis were said to be in close agreement with the results obtained through experiment by Lederman, Bloom, and Widhopf[2].

III. EXPERIMENTAL PROCEDURE

It is the primary purpose of the present experiment to relate the current collected by a cylindrical probe of finite length to the current collected by an infinitely long probe. One of the governing parameters in this context is the length-to-diameter ratio of the probes. To accomplish this, a rake of essentially the same construction as the one used in Ref. 2 was utilized. The probes of cylindrical type, varying in diameter from 0.002 to 0.08 inches of varying length, were constructed. The ratio of ℓ/d varied from 750 to 20.

Tests were carried out with the probes mounted in different axial positions in the secondary nozzle of the PIB hypersonic shock tunnel[11]. With these changes, variations in the electron density of only an order of magnitude were practical. To achieve larger variations for the electron density, it was necessary to use different initial driver-driven gas combinations. Those used were:

(1) Helium at 1800 psia as the driver gas and air at a pressure of 70 mm(Hg) as the driven gas.

(2) Driver of Helium at 1800 psia and as the driven gas a mixture of air (8 mm) and argon (30 mm).

Changes in these conditions yielded a variation in the ion number density, while the neutral density remained of the same order of magnitude. For this reason, at a given station in the nozzle of the shock tunnel, the value of λ_{n-n} and therefore, λ_{i-n} did not vary greatly between the two test conditions. The large variation in the ion density, however, caused a variation in the sheath thickness and thus provided a means for creating a variation of the number of collisions occurring within the probe's sheath.

In addition, a few tests were run with a driver of 1800 psia of helium and 38 mm(Hg) air as the driven gas for the purposes of comparison with previous experiments carried out at this facility. Test conditions resulting from each of the above are tabulated in Table I.

Measurements of the electron temperature was obtained from the current voltage characteristic of the probe assuming a Maxwellian electron energy distribution. Ion temperatures were estimated from calculations carried out using the Cornell Aeronautical Lab computer program[12] for chemically reacting nozzle flows. To determine the length effect, a series of tests were carried out in which the collected current densities of probes having the same radii but different lengths were compared. By normalizing all the current densities by that of the longest probe, a curve of j/j_{ref} as a function of ℓ/d could be drawn.

A sample type of probe response obtained during a typical test is pictured in Fig. 4. It can be broken up into two portions, a spike-like transient response and a smoother steady-state response. Lederman, Bloom and Widhopf have shown that either of the two responses may be used to obtain relative electron density data. In the current work, only the steady state portion of the response has been used.

IV. RESULTS AND DISCUSSION

As has been previously mentioned, a quantitative determination of the length effect was made by comparing the collected current density of probes having the same diameter but different lengths. Probes having diameters of 0.002", 0.005", 0.01", 0.02" and 0.08" were used in these experiments. The results for the case in which the electron density is 7.5×10^7el/cm^3 appear in Figs. 4,5, and 6. Also displayed are the appropriate results obtained from the theoretical analyses of Bettinger and Chen, and Sonin for the collisionless case.

In the case of the smaller radii probes, the experimentally obtained points appear to be consistently lower than the value which is predicted by the Bettinger and Chen analysis. This does not, however, appear to be the case for the thicker probes, where the results are consistent with the collisionless type calculation. Also, the behavior which was predicted by Sonin for very short probes having τ_1 less than 3 is absent.

Similar data for the case in which $N_e=3 \times 10^9$el/cm^3 is presented in Fig. 7. Measured values of j/j_{ref} appear uniformly higher than those predicted by a collisionless analysis.

An examination of these results reveal several interesting points concerning the behavior of cylindrical electrostatic probes, relative to the theoretical predictions of Bettinger and Chen as well as Sonin. In those figures, 3 parameters have been varied. In Figs. 4 and 7, the probe diameter was kept constant and the electron density was changed.

In Figs. 4 and 5, the electron density remained constant and the diameter of the probes was different. It appears that the collected relative current density was consistently higher with the higher absolute electron density than the theoretical predictions of Bettinger and Chen, and consistently lower than those predictions for the lower electron number density. The experimental data seem to agree with the theoretical predictions only in the case of a larger diameter probe. An examination of Figs. 2 and 3 provides a partial answer to this apparent agreement. It is clear that the length effects become less significant as the diameter of the probes increases. At an ℓ/d of 100 at 10^8 el/cm^3, for example, the current density increases by about a factor of 20 for the -.002" diameter probe and only by a factor of less than 2 for a 0.01" diameter probe. In view of the scatter of the data, the apparent agreement between experiment and theory in Fig. 5 is reasonable. The same two figures, 2 and 3, indicate also that this length effect decreases as the measured electron density increases. Yet, the relative current density as measured in these experiments does not appear to be affected by the number density as evident from Figs. 4 and 7, where the diameter of the probe is kept constant. On the other hand, the relative current density decreases with the increase of the diameter of the probe in agreement with the theoretical predictions, Fig. 8. As has been mentioned previously, the number density was varied in order to explore the possible effects of collisions in the sheath. In Fig. 1, a plot of the sheath thickness as a function of the probe diameter with the collected ion density number is plotted. Since in both cases considered λ_{n-n} and λ_{i-n} were of the order of 1 mm and the probe diameters were, at most, .5 mm, the conventional criteria for free molecular operation were met. However, at the lower ion number densities, sheath thickness of about 4 mm was obtained. The probability of an ion colliding with a neutral particle on the way of being collected is much higher than in the higher ion density case, where the sheath thickness is of the same order as λ_{i-n}.

This parameter of sheath thickness to neutral mean free path, although not explicitly appearing in any probe formation, thus appears to have an effect on the relation between the collected current density and the ion density in the plasma, by virtue of altered collisions in the sheath.

V. CONCLUSIONS

From the above experiments, it is clear that collisional processes have a great influence on the behavior of ion collecting probes. They affect not only the behavior of the non-dimensional current parameter, α_p, at low values of r_p/λ_D, where, according to the collisionless theory, the orbit motion limited regime exist (see Ref. 2) but they also influence the length effect behavior of probes. Thus, for a proper utilization of the ion current collecting probe, a prior knowledge of the operation regime is imperative.

VI. REFERENCES

1. Langmuir, I. and Mott-Smith, H.M., Theory of Collections in Gaseous Discharges, The Physical Review, 1926.
2. Lederman, S., Bloom, M.H. and Widhopf, G.F., Experiments On Cylindrical Electrostatic Probes in a Slightly Ionized Hypersonic Flow, AIAA J., 6, 11, pp. 2133-2139, November 1968.
3. Lederman, S., Bloom, M.H. and Avidor, J., The Electrostatic Probe: Some Applications to Hypersonic Flow Diagnostics, Israel J. Tech., 8, No. 1-2, pp. 73-85, 1970.

4. Laframboise, J.G., Theory of Spherical and Cylindrical Langmuir Probes in a Collisionless Maxwellian Plasma at Rest. University of Toronto, UTIAS Report No. 100, June 1966.

5. Sonin, A.A., The Behavior of Free Molecular Cylindrical Langmuir Probes in Supersonic Flows and Their Application to the Study of the Blunt Body Stagnation Layer, AIAA J., 4, 9, pp. 1588-1596, September 1966.

6. Bettinger, R.T. and Chen, A.A., An End Effect Associated with Cylindrical Langmuir Probes Moving at Satellite Velocities, J. Geophys. Res., 73, 1, January 1968.

7. Sonin, A.A. and Hester, S.D., An Ion Temperature End Effect in Cylindrical Langmuir Probe Response at Ionospheric Satellite Conditions, M.I.T. Fluid Mechanics Laboratory Publication No. 69-9, September 1969.

8. Bettinger and Walker, Relationships for Plasma Sheaths About Langmuir Probes, Phys. Fluids, 8, p. 748, 1965.

9. Shih, C.H. and Levi, E., The Effect of Collisions on Cold Ion Collection by Means of Langmuir Probes, Presented at the AIAA Third Fluid and Plasma Dynamics Conference, Los Angeles, California, June 28-July 1, 1970, Paper No. 70-757.

10. Allen, Boyd and Reynolds, The Collection of Positive Ions by a Probe Immersed in a Plasma, Proceedings of Physics Society B, 70, p. 297, 1957.

11. Bloom, M.H. and Lederman, S., Measurements of the Ionized Flow in a Shock Tunnel by Means of Resonant Cavities and Electrostatic Probes, Polytechnic Institute of Brooklyn, PIBAL Report No. 1019, May 1967.

12. Lordi, J.A., Mates, R.E. and Moselle, J.R., Computer Program for the Numerical Solutions of Non-Equilibrium Expansion of Reacting Gas Mixtures, Cornell Aero. Lab., Report AD-1689-A6, October 1965.

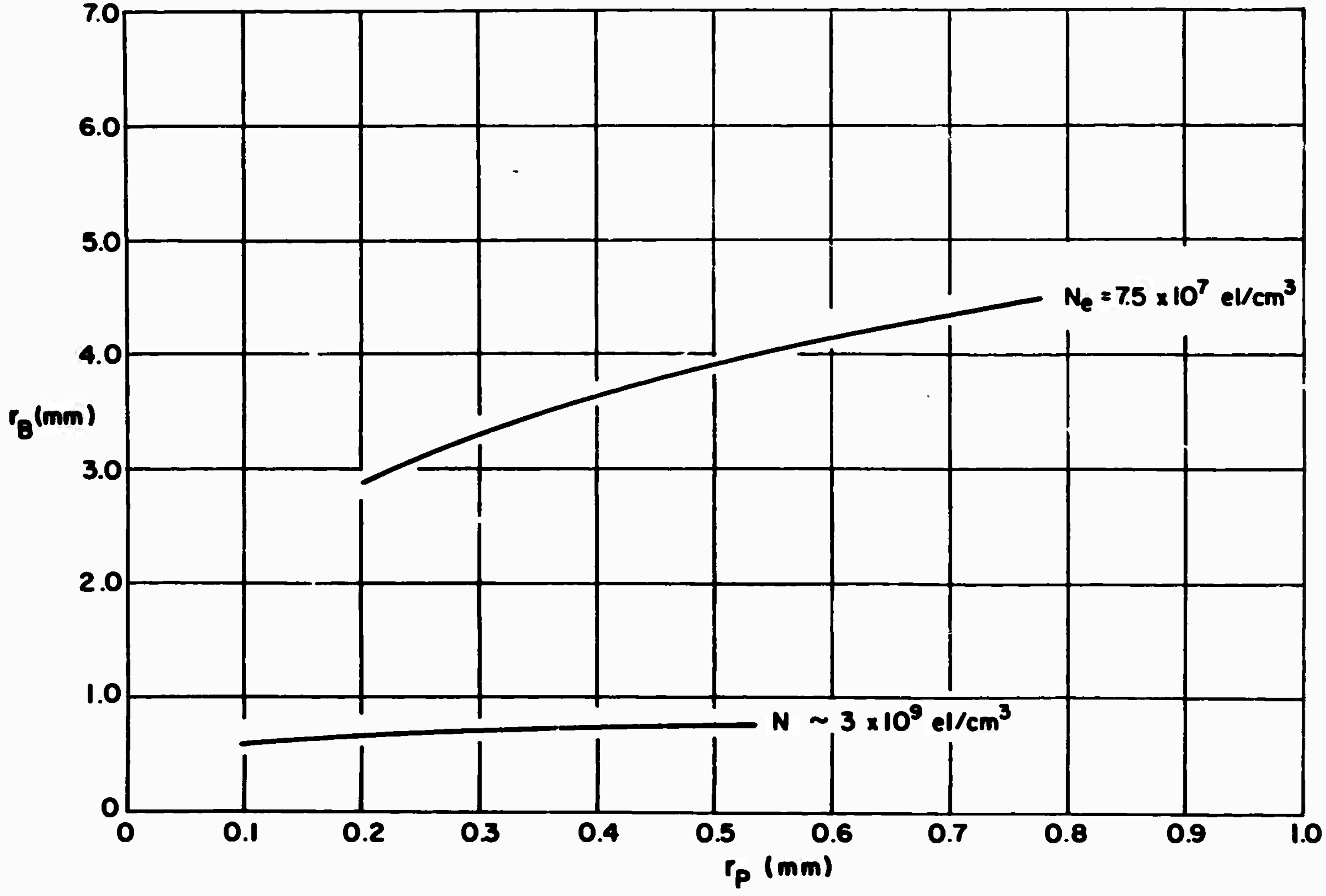

FIG. 1 SHEATH THICKNESS AS A FUNCTION OF PROBE RADIUS

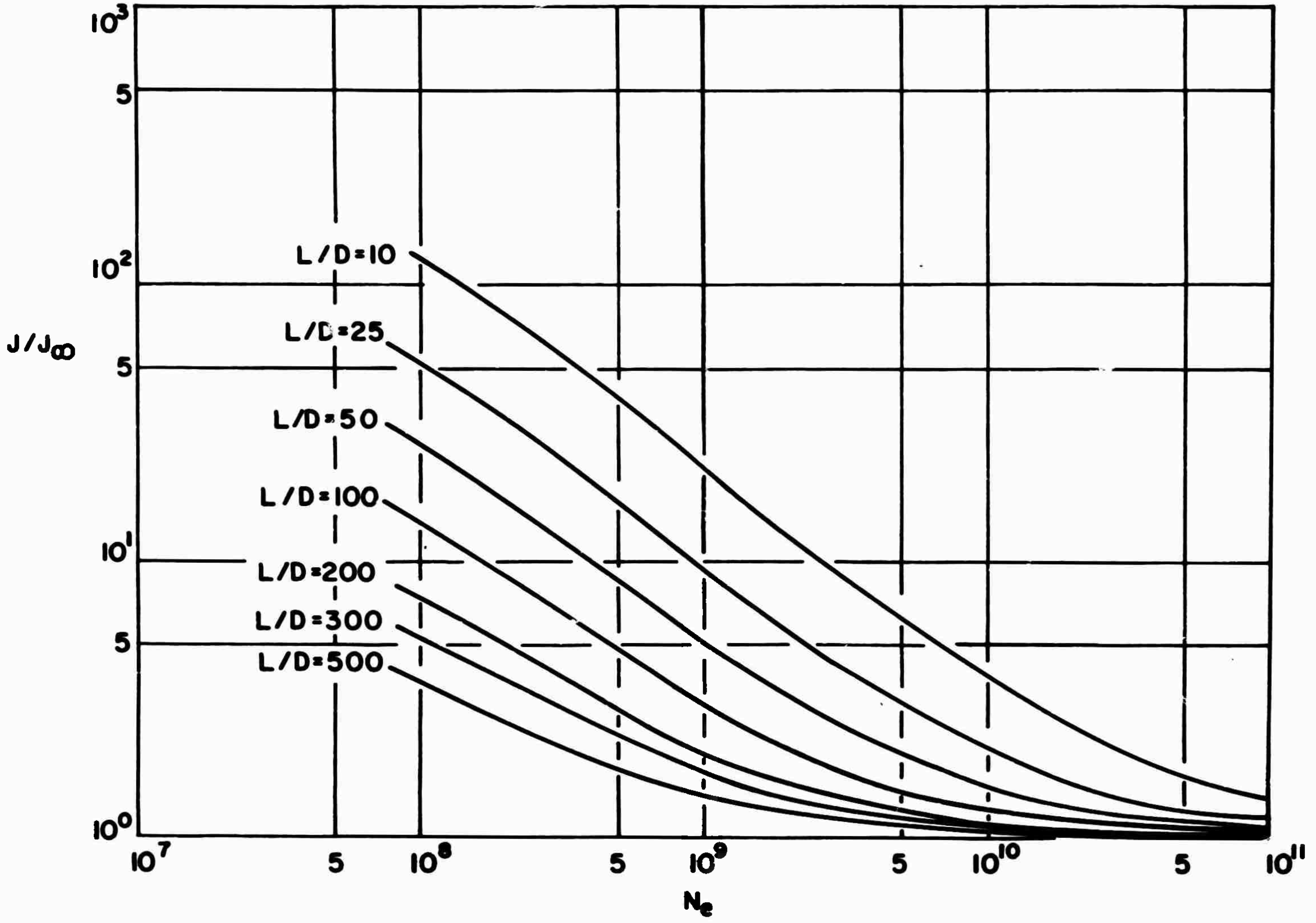

FIG. 2 J/J_∞ FOR 0.002" DIAMETER PROBE FROM BETTINGER AND CHEN

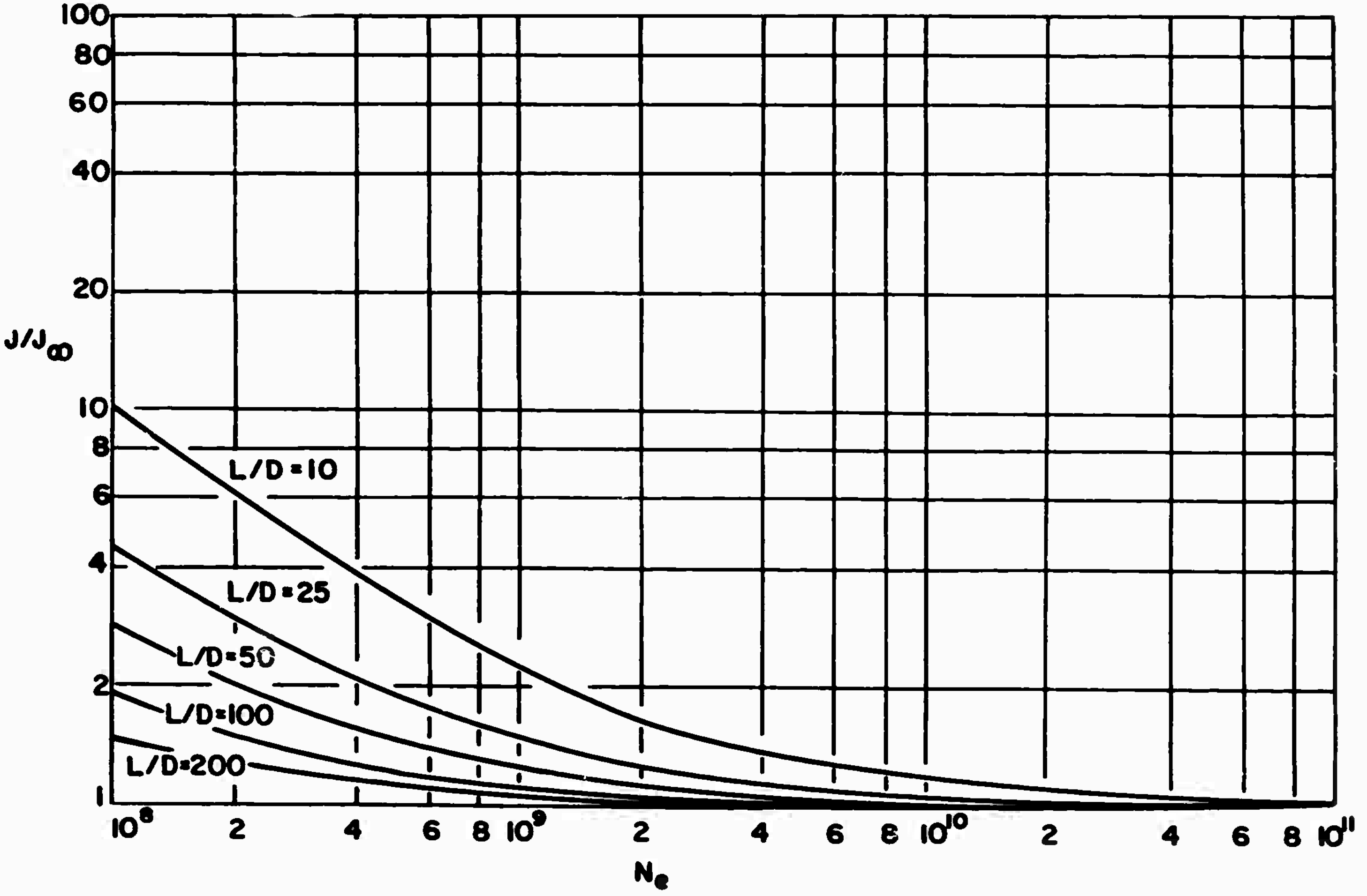

FIG. 3 J/J$_\infty$ FOR 0.01" DIAMETER PROBE FROM BETTINGER AND CHEN

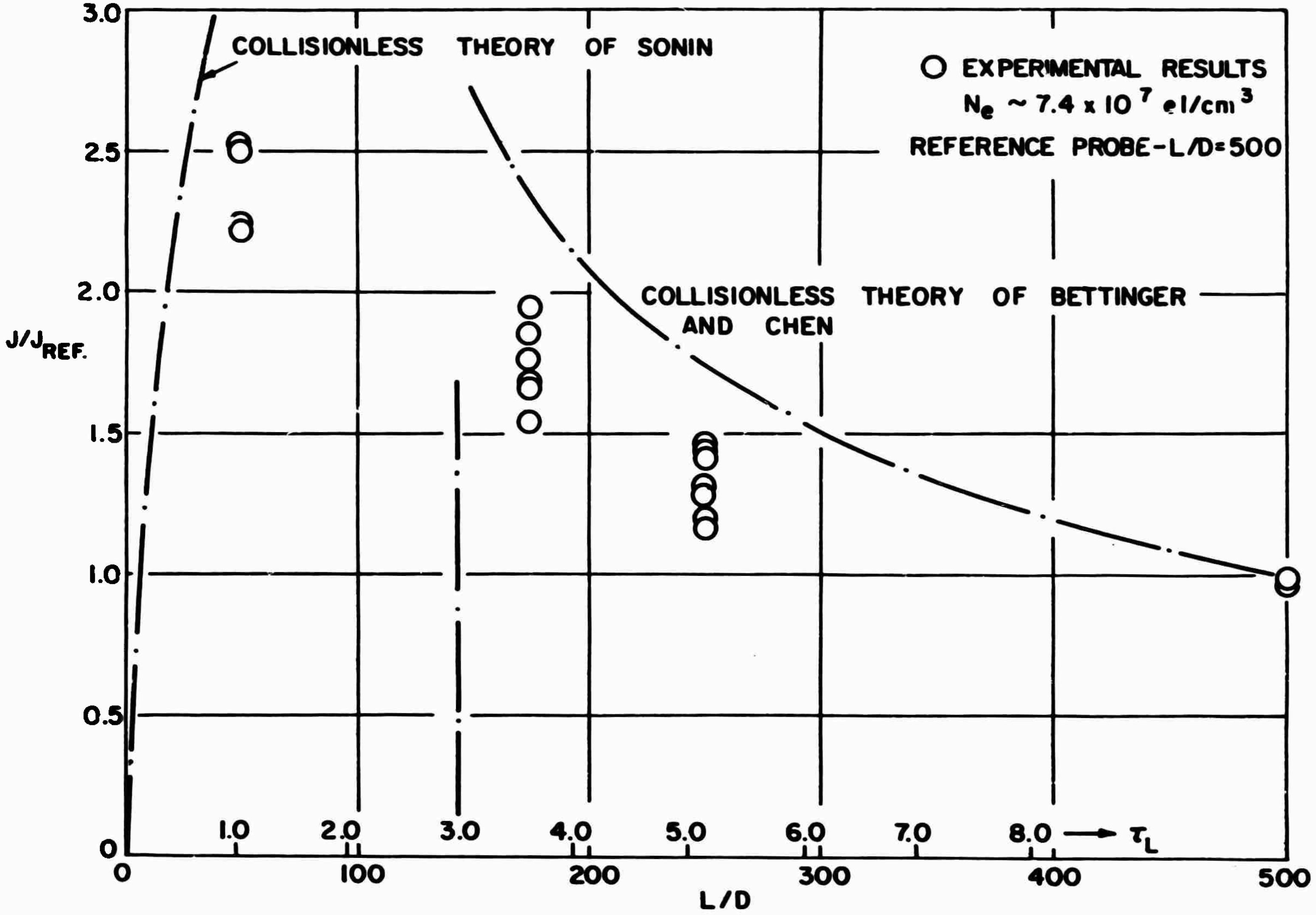

FIG. 4 J/J$_{REF.}$ AS A FUNCTION OF L/D FOR 0.002" DIAMETER PROBE

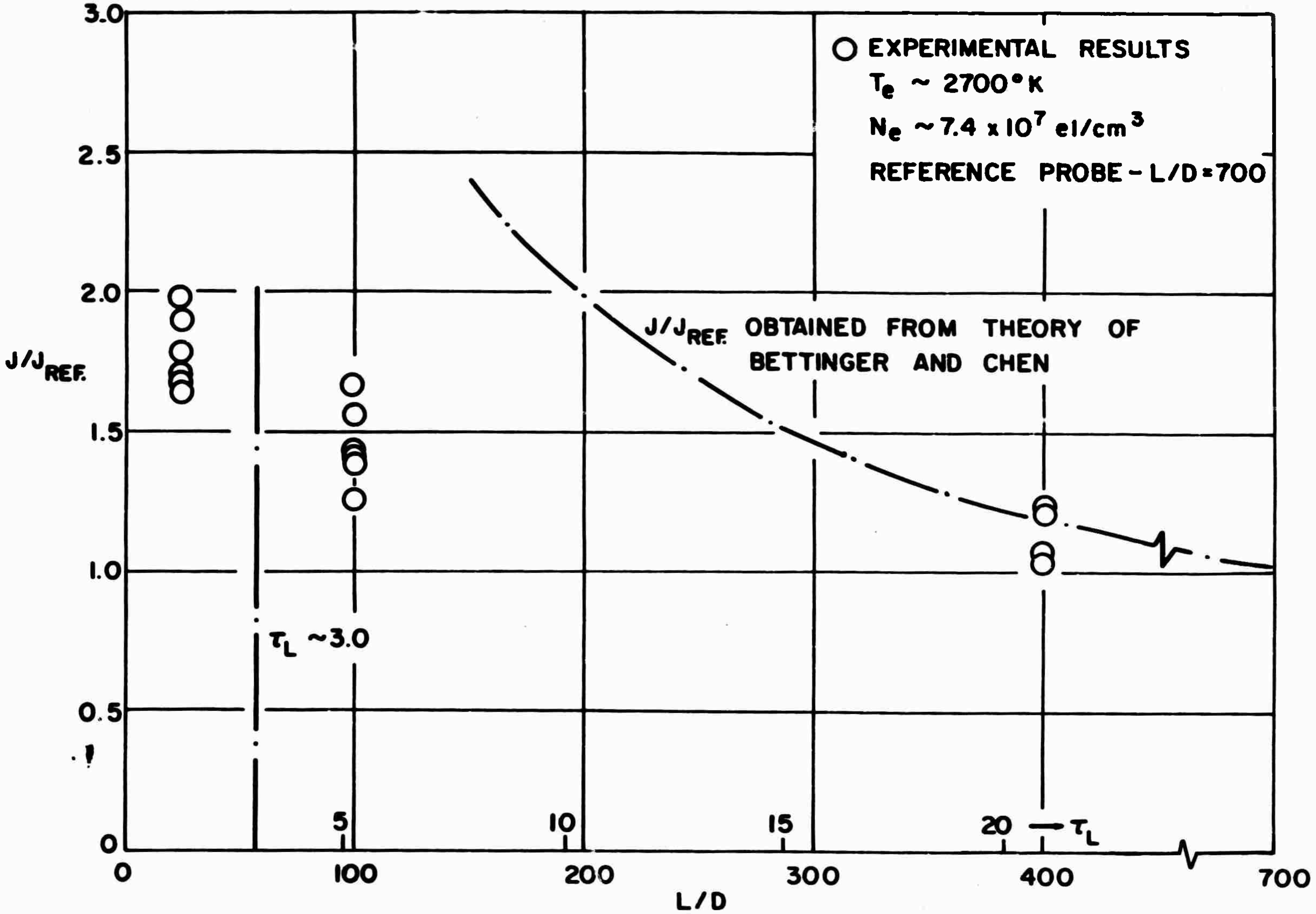

FIG. 5 $J/J_{REF.}$ AS A FUNCTION OF L/D FOR 0.005" DIAMETER PROBE

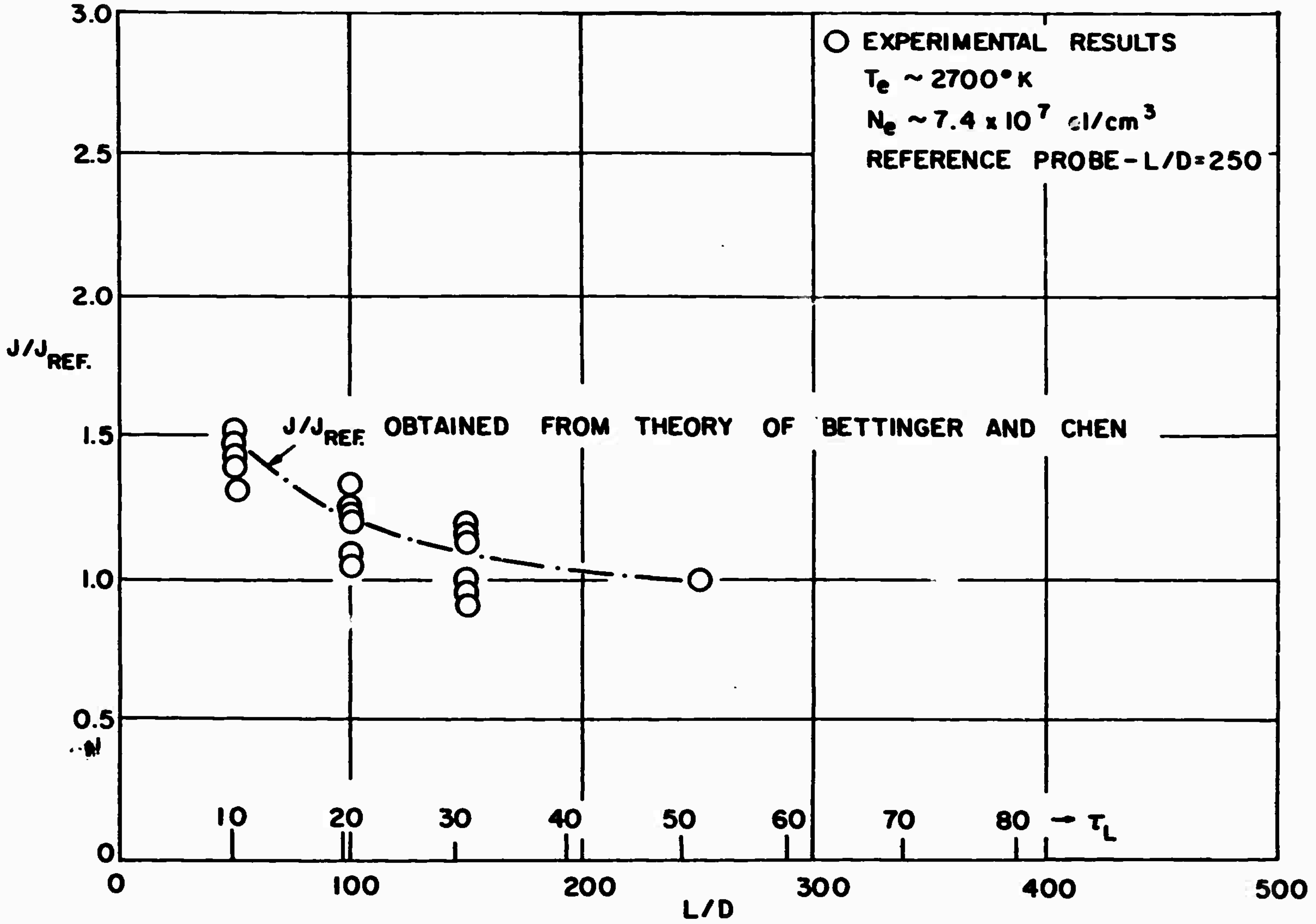

FIG. 6 $J/J_{REF.}$ AS A FUNCTION OF L/D FOR 0.02" DIAMETER PROBE

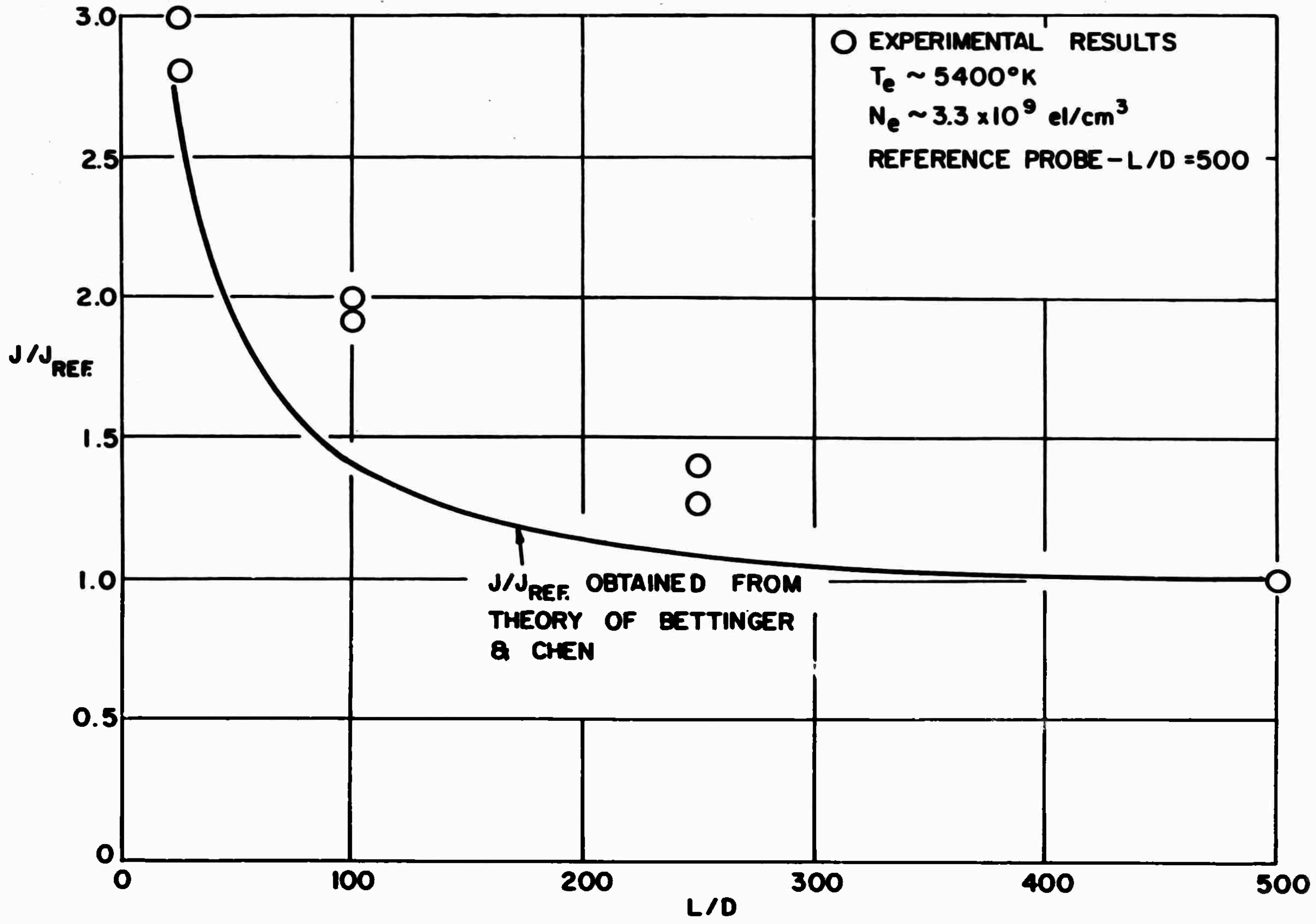

FIG. 7 $J/J_{REF.}$ AS A FUNCTION OF L/D FOR 0.002" DIAMETER PROBE

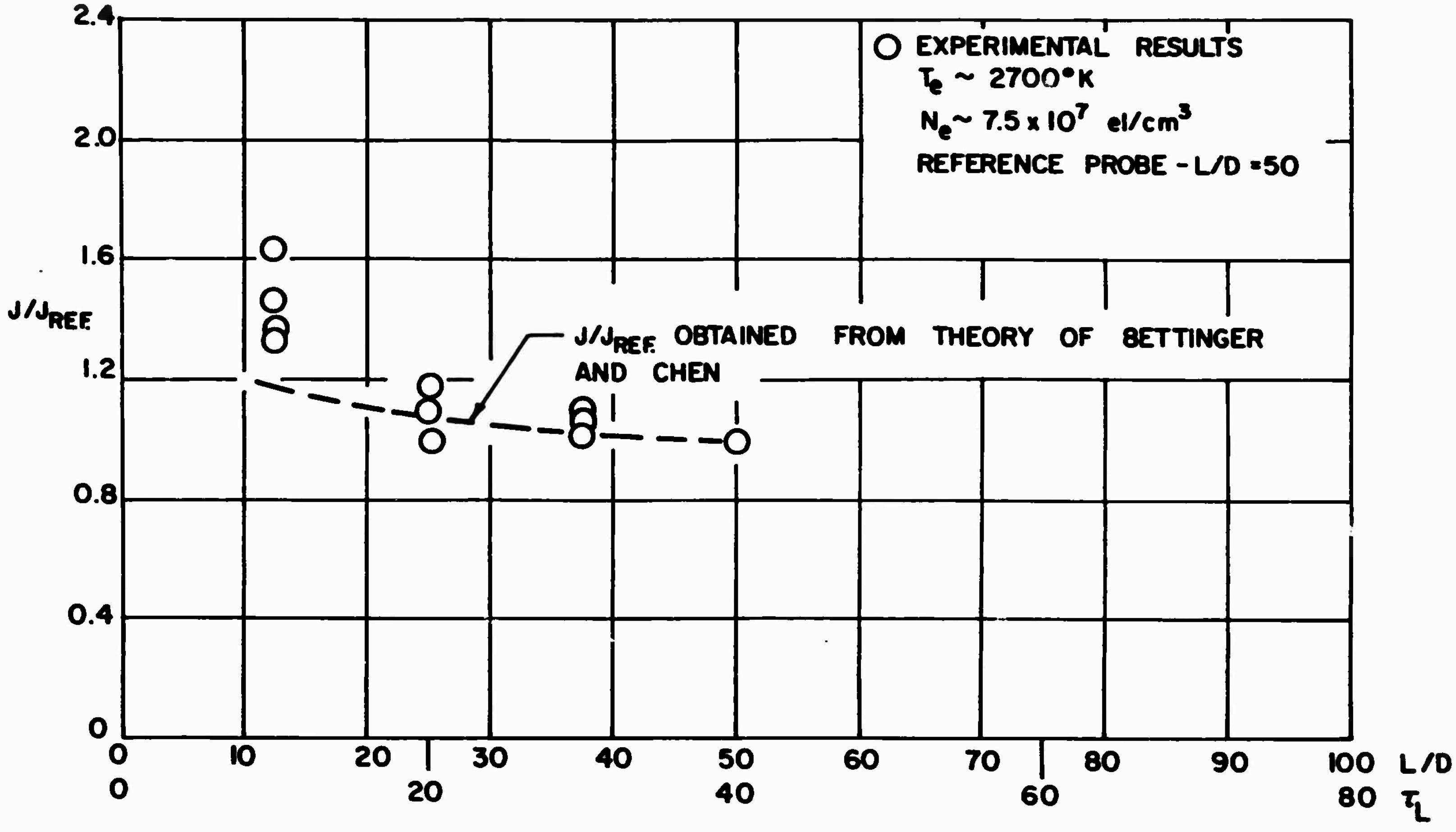

FIG. 8 $J/J_{REF.}$ AS A FUNCTION OF L/D FOR 0.08" DIAMETER PROBE