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TRACE66

Trajectory Analysis and Orbit Determination Program

Volume I: General Program Objectives Description, and Summary

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13. ABSTRACT

The TRACE66 Trajectory Analysis and Orbit Determination Program is a general-purpose orbital analysis program. It was written specifically for the CDC 6000 series computers to assist The Aerospace Corporation personnel in the analysis and design of satellite orbits and tracking systems. Volume I is a reference for general program objectives; it describes and summarizes the program. A comprehensive description of its areas of application is presented. An overview of the major capabilities of the program is emphasized.

The TRACE66 documentation series is summarized as follows:

Volume I:	General Program Objectives, Description, and Summary
Volume II:	Coordinate & Time-Keeping Systems with Associated Transformations
Volume III:	Trajectory Generation Equations & Methods
Volume IV:	Measurement Data Generation & Observational Measurement Partials
Volume V:	Differential Correction Procedure and Techniques
Volume VI:	Orbital Statistics Via Covariance Analysis
Volume VII:	Usage Guide

(cont.)

14. KEY WORDS
Batch Weighted Least Squares Coordinate and Time-Keeping Systems Covariance Analysis Differential Correction Ephemeris Generation Interplanetary Trajectories Lunar Orbits Measurement Data Generation Multiple Arc Fitting Orbit Determination Recursive Filter Segmented Drag Sequential Batch Weighted Least Squares Simultaneous Vehicle Single Arc Fitting TRACE66 Trajectory Analysis Variational Equations

Distribution Statement (Continued)

Abstract (Continued)

Volume VIII:	Usage Overview - not to be published. This information has been included in Vol. VII
Volume IX:	Detailed Program Structure
Volume X:	Lunar Gravity Analysis
Volume XI:	LGA Data Processor
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FOREWORD

This report is published by The Aerospace Corporation, El Segundo, California, under Air Force Contract Nos. F04701-70-C-0059 and F04701-71-C-0172.

This report, which documents research carried out from July 1970 through July 1971, was submitted for review and approval on 7 October 1971 to Capt. James L. Warwick, SAMSO/IND.

The Aerospace Corporation TRACE66 Trajectory Analysis and Orbit Determination Program has evolved during the past six years from the design and implementation efforts of many individuals. Recent analysis and programming contributions have been made by G. Buechler, W. D. Downs, E. H. Fletcher, P. T. Gray, and A. J. Rusick. In addition, consultations with W. T. Kyner and L. Wong have led to many significant improvements and added capabilities within the program. The TRACE66 documentation series is, like the program's growth and development, a joint effort.

Approved by

A. R. Sims

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Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

James L. Warwick, Captain, USAF
Project Officer

NOTICE

Some of the previously published volumes of the TRACE66 documentation series have been published by The Aerospace Corporation as Technical Operating Reports. Volume III: Trajectory Generation Equations and Methods was published as TOR-0066(9320)-2, Vol III; Volume V: Differential Correction Procedure and Techniques as TOR-0066(9320)-2, Vol V.

Volume VII: Usage Guide was published as TR-0059(9320)-1, Vol VII, and Volume X: Lunar Gravity Analysis as TR-0059(9320)-1, Vol X. Future volumes in this series will be published as Technical Reports.

The information that was to appear in Volume VIII is included in Volume VII; Volume VIII will therefore not be published.

ABSTRACT

The TRACE66 Trajectory Analysis and Orbit Determination Program is a general-purpose orbital analysis program. It was written specifically for the CDC 6000 series computers to assist The Aerospace Corporation personnel in the analysis and design of satellite orbits and tracking systems. Volume I is a reference for general program objectives; it describes and summarizes the program. A comprehensive description of its areas of application is presented. An overview of the major capabilities of the program is emphasized.

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1. INTRODUCTION AND SUMMARY

1.1 INTRODUCTION

This document is the first volume of the TRACE66 Trajectory Analysis and Orbit Determination Program documentation series. It is directed towards the non-user or the potential user interested in obtaining an overview of the capabilities of the TRACE66 program.

Section 2 discusses the characteristics of the trajectory integrator with emphasis on the following:

- Formulation of the equations of motion and variational equations
- Numerical technique to obtain solutions
- Accuracy of technique
- Representative solution times
- Coordinate and time-keeping systems
- Completeness of force model

Sections 3 and 4 discuss the ephemeris and data measurement generation functions for single and simultaneous vehicle modes.

Section 5 presents an overview of the orbit determination function of TRACE66. The following procedures may be used:

- Batch differential correction by weighted least squares
- Sequential least squares
- Recursive Potter square-root filter

Section 6 describes orbital statistics or covariance analysis capabilities of the program, and Section 7 presents an overview of the capabilities of the lunar gravity field analyzer.

1.2 BACKGROUND INFORMATION

TRACE66 is a computer program that simulates orbital motion and tracking operations. In the past, orbit determination was performed at The Aerospace Corporation by the TRACE-D Orbit Determination Program (Ref. 1), which was superseded in 1967 by TRACE66. The current TRACE66 program has capabilities significantly beyond those of TRACE-D.

1.3 APPLICATIONS IN ORBITAL ANALYSIS FOR TRACE66

TRACE66 is The Aerospace Corporation's trajectory analysis and orbit determination program; its applications encompass a wide range of problems in orbital mechanics. In general, TRACE66 is a general-purpose orbital analysis program used to assist corporate personnel in the analysis of tracking operations and orbital motion of artificial satellites about the earth, moon, and other bodies within the solar system. The term "orbit determination" will be defined and discussed later.

A hypothetical situation is used to illustrate some typical applications of the TRACE66 program. Some problems not addressed by TRACE66 are also noted. Suppose you are the manager of a satellite program and face the problems discussed in the following sections.

1.3.1 Ephemeris Generation

Your orbital designers have somehow (without using TRACE66, which does not select or optimize orbits) picked the desired orbit for the first flight. You must direct your recovery ships to their proper stations.

Ground track coordinates are some of the outputs of ephemeris generation, which is the computation of position and velocity components and related quantities as a function of time. Here, the required initial conditions are those of the nominal orbit. In this instance, the ephemeris can be generated without using the sophisticated force models and coordinate systems available in TRACE66.

1.3.2 Measurement Data Generation

You must also alert the tracking network and provide each station with the predicted rise times and associated azimuth and range.

The data generation capability of the program will provide the desired predictions. Here again, nominal initial conditions and simple models will suffice for the accuracy required. As a little plus for yourself, get the rise-set-bar chart off the visibility printer plot. It will show at a glance which stations can see the bird at any given time.

1.3.3 Measurement Error Modeling

For their dress rehearsal, the tracking analysts need some realistic simulated tracking data for exercising the operational software and their own analytic techniques. Another data generation run will do most of the job. Biases and random noise should be added to the generated data, and perhaps the environmental model (i.e., the gravitational model, drag coefficient, and station locations) should be intentionally falsified to provide further realism. The simulated data will be in TRACE66 format (no alternatives are available), but it may be placed either on cards or on magnetic tape for subsequent conversion to another format. No really "wild" points will be generated; if these are needed to test editing features in the operational program, manual methods will be required.

1.3.4 Orbit Determination

The launch is successful, but the tracking analysts are complaining. Their orbit determination program has, according to plan, adjusted the initial condition, drag, and bias parameters so that the computed orbit and observations best match the actual data received from the tracking sites, but too many iterations are required, and many of the final discrepancies (residuals) are larger than the radar engineers had expected.

The process described above, the estimation of unknown parameters from actual observations, is the orbit determination function of TRACE66. The other capabilities of TRACE66 are provided because they satisfy naturally related requirements and, in many instances, use the same basic computations. TRACE66 is well suited to this kind of problem. Because of the great flexibility in its modeling, it can simulate the computations of the operational software. Because of its extensive list of parameters for differential correction, TRACE66 can (in the hands of a skilled analyst) identify previously unsuspected systematic errors.

The first step is to place the tracking data on magnetic tape for efficient processing. TRACE66 accepts tracking data in a variety of formats, so reformatting is probably unnecessary. One iteration of orbit determination provides a check on the operational program. TRACE66 can use the same model and biases and start from the same initial conditions. It will (presumably) generate the same trajectory and derive the same residuals if the prelaunch checks were thorough. The printer plot of residuals may disclose some bad observations that weren't edited by the operational program. The editor in TRACE66 isn't perfect, but bad observations can be rejected on subsequent runs even if they are buried in a magnetic tape.

Suppose the problem is not simply bad data, and the printer plot of residuals shows an obviously systematic pattern. The analyst must nominate a probable cause, although this may be difficult. He must then appropriately modify the model (perhaps also specifying further parameters for differential correction) and try again.

1.3.5 Covariance Analysis

Since reconstruction and prediction are not going well, someone reopens the argument for rapid deployment of the spare sensor. Although you feel that this is the wrong solution to the problem, or perhaps no solution at all, you promise to look into it.

A covariance analysis may provide some objective support for your intuition. A baseline computation will estimate the potential tracking accuracy of the present system. Even when appropriate uncertainties in station locations are factored in, the estimate will be optimistic due to the many unmet assumptions implicit in such calculations. The analysis will, however, show whether significant improvements can be expected if another sensor, with its attendant random errors and uncertainties in station location, time bias, etc., is added to the system.

1.3.6 Orbit Determination-Postflight Mode

The early tracking problem described in Sec. 1.3.4 has been solved by using TRACE66 in its postflight mode of orbit determination; meanwhile, the problem has vanished from the real-time operation.

It seems that the cause was an incompletely spent fuel tank that was supplying some unexpected thrust. Good fits were obtained when an appropriate thrust model was incorporated into the trajectory, and the rate of fuel flow was determined by differential correction. Residual patterns now appear random; more important, acquisition predictions are being met.

1.3.7 Improved Modeling

Because you anticipate similar problems on subsequent flights, you modify the operational program to include a thrust model and the appropriate flow-rate parameter.

TRACE66 can assist in checking out this new feature by generating test tracking data, by printing a trajectory and its partial derivatives with respect to the new parameter (for comparison purposes), and by demonstrating graphically (via printer plots) the correctness of the partial derivatives.

1.3.8 Multiple Arc Fitting

Although operations are now going smoothly, you notice that the biases determined for one of your stations vary erratically from pass to pass. Because there is no indication of faulty hardware, you suspect that the station location is erroneous. Large quantities of data are available, including some taken by a nearby sensor that has tracked another agency's satellites. Some careful postflight orbit determination is required, using a variety of data types from multiple arcs to determine common parameters (in this case, station location).

This application is probably TRACE66's forte. The program features that contribute to this analysis are: sophisticated force models including radiation pressure, solar-lunar perturbations, and a wide selection of models of the atmosphere and the geopotential; the ability to handle several arcs and data types; precise coordinate and time-keeping systems; and proper use of constraints and a priori estimates.

The proper use of constraints and a priori estimates merits some elaboration. Presumably, both your station and the one nearby are mislocated; their separation is known precisely, but their absolute locations are given only approximately (within certain standard deviations). The constraint on their relative locations can be imposed exactly, and the previous estimate can be weighted-in with the current solution. Appendix F in Vol. VII of this documentation series (Ref. 2) gives some other examples of the use of the constraint capability of TRACE66.

1.3.9 Simultaneous Vehicle Mode of TRACE66

Proponents of a new navigational satellite program claim that it offers more tracking coverage for less money. By operating a constellation of synchronous satellites, they offer to furnish ephemeris data on their birds and a black box to be carried on yours that will return both direct and multipath ranges between their vehicles and yours.

The simultaneous vehicle mode of TRACE66 can be applied in this situation. Not only are several satellites in simultaneous operation, but the observational measurements are functions of the positions of two or more.

Explicit problems similar to those of the previous sections could be posed in the simultaneous vehicle environment, but for this introduction it is sufficient to note that all four of the typical TRACE66 functions (ephemeris generation, data generation, orbit determination, and covariance analysis) can be exercised in this mode and situation.

1.3.10 Other TRACE66 Capabilities

This hypothetical situation has been used to describe briefly a few of the applications of TRACE66; some of its other capabilities are listed below:

- TRACE66 can perform its four basic functions using lunar and interplanetary trajectories.
- In both earth- and moon-centered applications, the gravitational attraction can be computed from spherical-harmonic or point-mass models.
- Orbit determination can be performed using a batch least squares, sequential least squares, or recursive filter algorithm.

1.4 STRUCTURAL CHARACTERISTICS

1.4.1 Functional Characteristics

The functional structure of TRACE66 has been designed and developed around its four major applications, which are orbit determination, ephemeris generation, simulated data measurement generation, and covariance analysis. Figure 1 schematically illustrates the functional design concept of the program; note that the input data processor and the trajectory generator are common to all four major functions. This structure allows for the execution of several functions sequentially against the same data base. To illustrate the potential of this framework, some examples and usage concepts associated with the functional aspects of TRACE66 follow.

In a typical simple case, the program reads all input data, generates the trajectory file, and processes it for the desired output. This may be a printed ephemeris, a covariance analysis, or simulated tracking data.

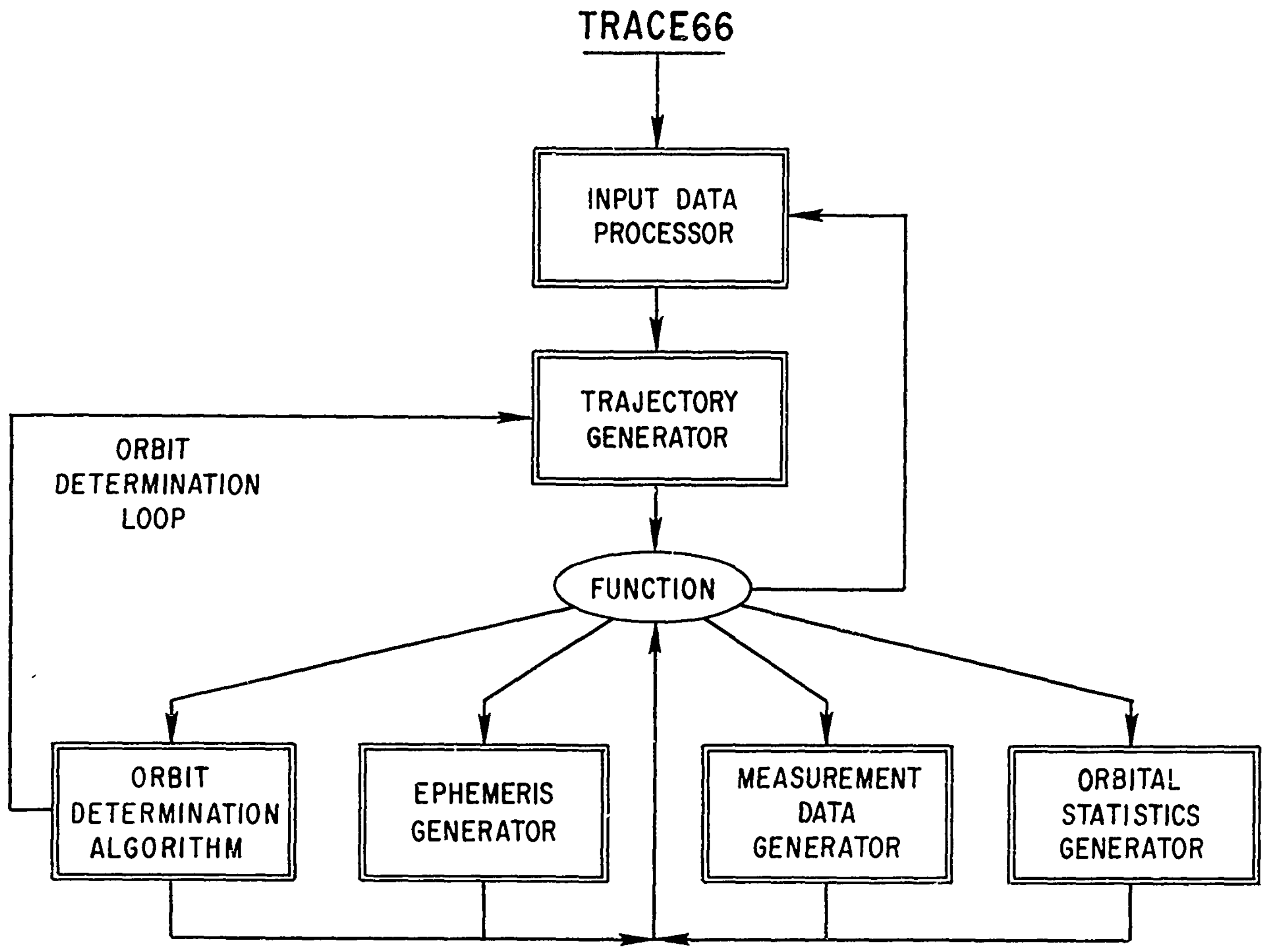


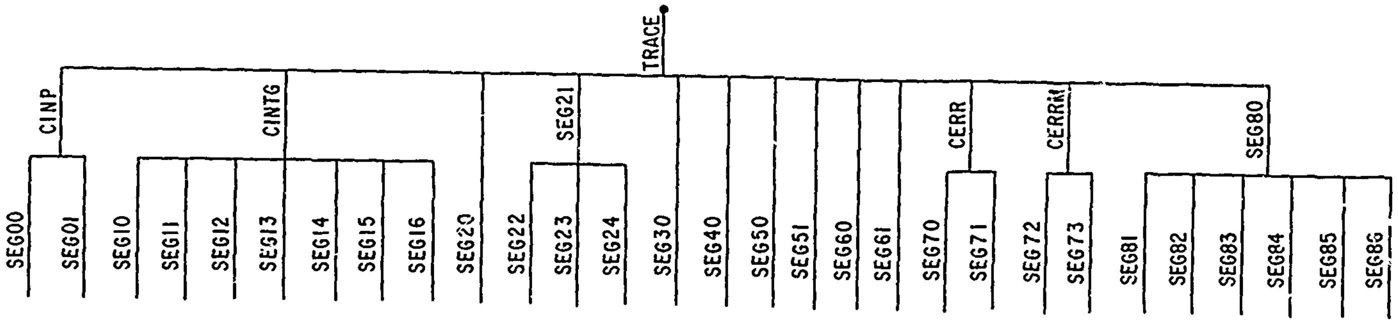
Fig. 1. Schematic of Major Functions

More complicated cases (or a series of cases) can also be executed. The trajectory file may be simply an integration from given initial values, or it may be the result of a trajectory reconstruction from observational measurement data; in fact, both kinds of trajectory generations may appear in a single case. Several processing functions may be executed with each case. Finally, cases may be stacked indefinitely within a single job on the computer.

In its most elementary mode, TRACE66 generates or processes data from a single vehicle. The observational measurement data in this instance pertains only to the space vehicle and its relation to the tracking stations or to the earth itself.

In the multiple arc mode, TRACE66 can generate or process data from several space vehicles, but each data item is associated with only one vehicle. The vehicles are independent and need not be in orbit simultaneously. The data may be associated with only one object, yet be processed separately in different arcs. A proper solution for a sensor or model parameter (for example, a station location or a gravitational anomaly) can be derived from data obtained from several vehicles during different time periods. Normally, the multiple arc reconstruction has common parameters, either naturally or as the result of an imposed constraint; otherwise, the reconstruction could be done separately. Note that the multiple arc mode is a simple extension of the single vehicle mode and requires no special identification or handling.

However, when observational data concern the position or velocity of two or more space vehicles orbiting simultaneously, the program is run in its simultaneous vehicle mode. The program handles simultaneous vehicle analyses differently from the way it handles single vehicle or multiple arc problems. Input processing and numerical integration are common to both



TRACE	MAIN CONTROL PROGRAM
CINP	DATA BASE GENERATION CONTROL
SEG00	INPUT NON-VEHICLE DATA
SEG01	INPUT VEHICLE DATA
CINTG	INTEGRATION CONTROL
SEG10	INTEGRATION SETUP
SEG11	ECI INTEGRATOR
SEG12	MCI INTEGRATOR
SEG13	BCI INTEGRATOR
SEG14	ECI (S-VARIABLE) INTEGRATOR
SEG15	SPECIAL MCI (DISK) INTEGRATOR
SEG16	ECI (LGA) INTEGRATOR
SEG20	MEASUREMENT PARTIALS AND RESIDUALS
SEG21	SIMULTANEOUS VEHICLE MEASUREMENT PARTIALS AND RESIDUALS CONTROL
SEG22	SIMULTANEOUS VEHICLE MEASUREMENT PARTIALS & RESIDUALS SETUP
SEG23 SEG24	COMPUTES MEASUREMENT PARTIALS & RESIDUALS
SEG30	SOLUTION OF THE NORMAL MATRIX
SEG40	DATA BASE RESET
SEG50	SINGLE VEHICLE EPHEMERIS (TRAJECTORY) OUTPUT
SEG51	SIMULTANEOUS VEHICLE EPHEMERIS OUTPUT
SEG60	SINGLE VEHICLE SIMULATED DATA GENERATOR
SEG61	SIMULTANEOUS VEHICLE SIMULATED DATA GENERATOR
CERR	COVARIANCE ANALYSIS CONTROL
SEG70	COVARIANCE ANALYSIS MATRIX INVERSION
SEG71	COVARIANCE ANALYSIS OUTPUT
CERRM	SIMULTANEOUS VEHICLE COVARIANCE ANALYSIS CONTROL
SEG72	SIMULTANEOUS VEHICLE COVARIANCE ANALYSIS SETUP
SEG73	SIMULTANEOUS VEHICLE COVARIANCE ANALYSIS MATRIX INVERSION AND OUTPUT
SEG80	SEQUENTIAL LEAST SQUARES & RECURSIVE FILTER DRIVER
SEG81	SEQUENTIAL LEAST SQUARES & RECURSIVE FILTER SETUP
SEG82	SEQUENTIAL LEAST SQUARES PARTIALS AND RESIDUALS
SEG83	SEQUENTIAL LEAST SQUARES SOLUTION
SEG84	SEQUENTIAL LEAST SQUARES AND RECURSIVE FILTER UPDATE, DATA BASE RESET AND CLEAN UP
SEG85	RECURSIVE FILTER
SEG86	SEQUENTIAL LEAST SQUARES COVARIANCE ANALYSIS

Fig. 2. TRACE66 Overlay Structure

simultaneous vehicle and single vehicle (multiple arc) modes. However, the four functional processors indicated in Fig. 1 are separate and independent, depending on the mode of program operation.

1.4.2 Physical Structure

TRACE66 is written entirely in CDC 6000 series FORTRAN and runs under the SCOPE operating system. In its standard configuration, the program requires approximately 34,000 sixty-bit words. Various numerical limitations (e.g., 350 spherical harmonic terms or 100 ground stations) stated throughout this document refer to this standard configuration. The majority of these restrictions may be relaxed, with the penalty of additional core requirements. On the other hand, the program may be run with full sophistication, but reduced capabilities, in as few as approximately 25,000 sixty-bit words.

TRACE66 is modular in design and employs main, primary, and secondary overlay levels to perform various functions (see Fig. 2). For example, the input data processor and the integrator use all three levels; the ephemeris generator resides only in a primary overlay level. The total number of modules (overlays) in the program is 36; it is not expected that any one application would use all the overlays. The average number of modules employed for particular functions is tabulated as follows:

FUNCTION	NUMBER OF MODULES
Orbit Determination	10
Covariance Analysis	10
Data Generation	8
Ephemeris Generation	8

TRACE66 is a file-oriented system and uses mass storage devices (disk and/or magnetic tape) extensively for communication among modules. The program has significant restart capabilities through the use of saved files.

In addition, varied output files are available as options; they may then be used as input to other programs, such as plotting or statistical analysis programs.

Much of the flexibility of TRACE66 is due to its modular structure and its file orientation. For example, significant modifications may be performed within a particular overlay with little or no effect on the remainder of the program. Furthermore, one may add new functions simply as a separate overlay and thereby utilize the applicable capabilities (e.g., the input processor or the numerical integrator) of the remainder of the program. Because of this, TRACE66 is commonly used as a test bed for new algorithms and techniques and as a simulator of other orbit determination and trajectory analysis software.

1.5 DOCUMENTATION SERIES

The TRACE66 documentation series is summarized as follows:

Volume I: General Program Objectives, Description, and Summary is directed towards the non-user or potential user interested in obtaining an overview of TRACE66 capabilities.

Volume II: Coordinate & Time-Keeping Systems with Associated Transformations is a technical reference for the coordinate and time-keeping systems and related transformations used within TRACE66.

Volume III: Trajectory Generation Equations & Methods (Ref. 3) serves as a technical reference for the trajectory generation function of TRACE66.

Volume IV: Measurement Data Generation & Observational Measurement Partials is a technical reference for the generation functions of the measurement data and associated observational measurement partial derivatives of TRACE66.

Volume V: Differential Correction Procedure and Techniques (Ref. 4) serves as a technical reference for the batch differential correction procedure and associated techniques used within TRACE66.

Volume VI: Orbital Statistics Via Covariance Analysis is a technical reference for the orbital statistics generation or covariance analysis function of TRACE66.

Volume VII: Usage Guide (Ref. 2) serves as a reference defining all input data required to perform any of the TRACE66 functions. Each input item is defined, and all basic data deck structures necessary to execute TRACE66 are described. Note, however, that constant changes and improvements are being made to the program; this volume may not include a description of every input quantity available in the current version of TRACE66.

Volume VIII: Usage Overview will not be published; the information that was to appear in this volume has been included in Volume VII.

Volume IX: Detailed Program Structure describes the program structure to the subroutine level.

Volume X: Lunar Gravity Analysis (Ref. 5) serves as a technical reference for the Lunar Gravity Field Analyzer of TRACE66.

Volume XI: LGA Data Processor serves as a technical reference for the LGA data processing function of TRACE66.

Volume XII: Sequential Least Squares & Recursive Filter Procedures and Techniques is a technical reference for the sequential least squares and recursive square-root Potter filter procedures and associated techniques used within TRACE66 to perform orbit determination.

2. TRAJECTORY GENERATION OVERVIEW

The trajectory generation function of TRACE66 is common to all program applications. This is the process of computing the position and velocity of the space vehicle as a function of time, given the position and velocity at some initial time and a dynamic model that accounts for the accelerations acting on the vehicle. A reference frame is defined within which the components of the various vector quantities are expressed and the accelerations are numerically integrated, subject to the given initial conditions. This section presents an overview of the formulation and numerical integration method and of the reference coordinate systems and force models available in TRACE66.

2.1 FORMULATION AND NUMERICAL METHOD

TRACE66 numerically integrates the total vehicle acceleration vector in a Cartesian coordinate system (Cowell formulation of the equations of motion). The numerical technique used to perform the integration is a predictor-corrector tenth-order, Gauss-Jackson differencing scheme. The integrator is operated in either a fixed-step or variable-step mode with automatic local truncation error controls. A fourth-order Runge-Kutta method is used for integrator starting and halving procedures. When the fixed-step mode is used, the set of equations being integrated can be divided into two groups. The second group of equations may be integrated at a step size that is a multiple (by some positive power of two) of the step size used for the first group. This is particularly advantageous for an orbit determination application, in which variational equations can be integrated at a step size larger than that of the equations of motion without loss of accuracy.

Time (t) is the normal independent variable in a Cowell formulation. For most problems, precise and efficient geodetic, lunar, and interplanetary trajectories are obtained using the variable-step mode. However, in the case of highly eccentric orbits, it has been demonstrated that regularization of the

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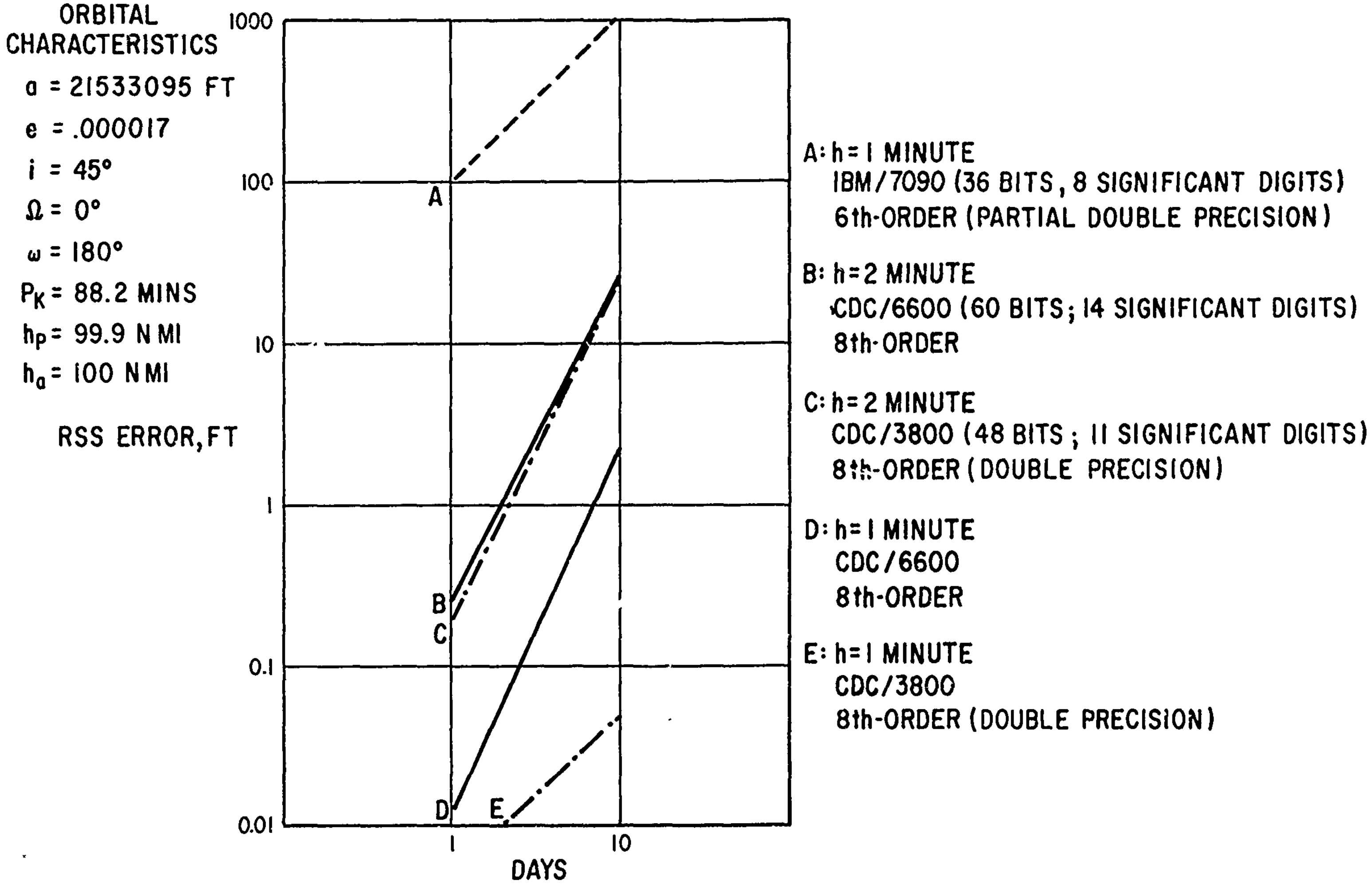


Fig. 3. Accuracy of Numerical Integration: Analytic Solution, Keplerian Model

independent variable leads to more rapid and accurate solutions (see Ref. 6). TRACE66 can perform its numerical integration with a regularized independent variable s defined such that

$$\frac{dt}{ds} \propto r^n$$

where r is the magnitude of the vehicle position vector at time t, and $1 \le n \le 2$.

The numerical accuracy of the TRACE66 integrator is documented in Ref. 3 and is indicated here by a representative example of RSS (root-sum-square) error obtained by differencing the numerical solution with a known analytical solution. The results are shown (see Fig. 3) as a time history of RSS error on a log-log scale.

Table 1 presents representative solution times for the numerical integration. Note that the eccentric orbits, cases 1 and 2, have comparable results for the regularized independent variable s.

2.2 COORDINATE AND TIME-KEEPING SYSTEMS

2.2.1 Reference Coordinate Systems

An inertial coordinate frame is usually chosen as the reference system for a problem in orbital mechanics so that the equations of motion are in a simple form. Regardless of the system chosen, it is necessary to transform to other coordinate systems; for example, earth-fixed coordinates are needed to evaluate the geopotential and to locate trackers. Therefore, in selecting a reference coordinate system and associated transformations, a compromise is made between extreme accuracy and ease of computation. A choice completely free of known errors and approximations is computationally time-consuming and is unnecessary for many applications. TRACE66 allows the user to select a reference coordinate frame and transformation equations according to the precision required for the particular application. For example, for near-earth

Table 1. Representative Numerical Integration Solution Time Examples Via CDC/6600

		CASE I	CASE 2	CASE 3	CASE 4	SOLUTION TIME PER INTEGRATION STEP
EQUATIONS OF MOTION (ONLY)	t	14 sec	10 sec	2 sec	4.5 sec	10 m sec
	s	16 sec	2.0 sec			
EQUATIONS OF MOTION AND SIX VARIATIONAL EQUATIONS	t	21 sec	15 sec	12 sec	12 sec	15 m sec
	s	20 sec	2.5 sec			

CASE I : ECI LOW-ALTITUDE
90 MINUTE PERIOD
NEAR CIRCULAR
8TH ORDER GRAVITY MODEL
DRAG
LUNAR & SOLAR PERTURBATIONS
I DAY (16 REVS)

CASE 2: ECI 12-HOUR ORBIT
HIGH ECCENTRICITY
8TH ORDER GRAVITY MODEL
DRAG
LUNAR & SOLAR PERTURBATIONS
I DAY (2 REVS)

CASE 3 : MCI LUNAR ORBITER
TRIAXIAL MOON
EARTH & SUN PERTURBATIONS
2 HOURS (I REV)

CASE 4: BCI INTERPLANETARY FLIGHT
EARTH-TO-MARS
SUN, EARTH, MOON, VENUS, MARS, JUPITER, SATURN PERTURBATIONS
156 DAYS

satellites with primarily electronic observations recorded, it is usually not necessary to employ rigorous transformations to the celestial system. (The celestial system is used for the tabulation of lunar, solar, and planetary ephemerides; star catalogues; and as a reference for optical measurements.) However, for precise computations over extended time periods and for the analysis of optical measurements, rigorous transformations must be used.

All TRACE66 coordinate systems originate at the center of gravity of the central body. The central body can be any body for which there is available a tabulated ephemeris relative to the earth. The fundamental plane and the principal axis may be defined as parallel to any of the following:

- True earth equator of date (instant) and mean equinox of midnight date of epoch
- Mean earth equator and mean equinox of midnight date of epoch
- Mean earth equator and mean equinox of 1950.0 (celestial system)
- Mean earth equator and mean equinox at a specified base date.

The first of these systems is simplest in terms of the transformation matrices employed. Precession of the equator and equinox from midnight date of epoch to date and nutation of the earth equator are ignored in all transformation equations. Furthermore, though the system is noninertial (fixed to the true earth equator), additional terms to account for its motion are not included in the equations of motion of a satellite.

The three remaining systems are inertial, and the transformations include the effects of precession and nutation. The actual transformation matrices are recomputed at a user-supplied time interval. Therefore, it is still possible to trade computation time for accuracy by varying the length of the recomputation interval. For example, it can be done as frequently as once per integration step or as infrequently as once during the numerical integration process. In addition to considering precession and nutation effects, TRACE66 can, optionally, account for the difference between the instantaneous pole (or spin axis) of the central body and a point attached to the crust by computing a pole

TYPICAL COORDINATE TRANSFORMATIONS

- **ROTATION** FROM AN INERTIAL SYSTEM TO A TIME DEPENDENT ROTATING FRAME

$\underline{r}_{BF} = [F][N][P]\,\underline{r}_{BCI}$

WHERE:

$\underline{r}_{BCI}$ = INERTIAL VEHICLE POSITION VECTOR RELATIVE TO A MEAN EQUATOR & EQUINOX OF SOME REFERENCE TIME (EPOCH)

$[P]$ = GENERAL PRECESSION MATRIX TO MEAN EQUATOR & EQUINOX OF DATE

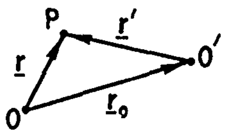
$[N]$ = NUTATION MATRIX TO TRUE EQUATOR & EQUINOX OF DATE

$[F]$ = BODY-FIXED MATRIX TO LOCAL MERIDIAN

$\underline{r}_{BF}$ = BODY-FIXED VEHICLE POSITION VECTOR RELATIVE TO A TRUE EQUATOR AND REFERENCE MERIDIAN OF DATE

- **TRANSLATION** FROM ONE INERTIAL FRAME TO ANOTHER INERTIAL FRAME

$\underline{r}' = \underline{r} - \underline{r}_0$



WHERE:

$\underline{r}$ = POSITION VECTOR OF SOME POINT P RELATIVE TO THE ORIGIN 0

$\underline{r}_0$ = POSITION VECTOR OF THE OTHER ORIGIN 0' RELATIVE TO 0

$\underline{r}'$ = POSITION VECTOR OF POINT P RELATIVE TO ORIGIN 0'

Fig. 4. TRACE66 Coordinate Systems

wander matrix. This transformation matrix is updated at the same time-frequency as the precession and nutation matrices. Figure 4 indicates the typical coordinate transformations available in TRACE66.

2.2.2 Time-Keeping Systems

The foundations of dynamics demand that "uniform" time exist and that it correspond identically to the time variable in dynamical equations. Any physical measure of time is established by definition; it usually represents an attempt to approximate uniform time more closely than did its predecessor, i.e., to reduce discrepancies between observations and dynamic theories. Though each of the definitions is nonuniform to some degree, many have areas of continuing application. Four time-keeping systems may be used simultaneously in TRACE66 to solve a problem in orbital dynamics. These systems are:

- IT: Uniform integration time is used as the independent variable in the equations of motion. A typical example is IT = A1 (atomic time).
- ET: Ephemeris time is needed to relate to the planetary ephemerides.
- UT1: Universal time is based on the rotation rate of the earth and is required to compute the sidereal angle.
- OT: Observation time is used as the time tag identifying the instant at which a measurement is recorded at a tracking station. A typical example is OT = UTC (broadcast time).

It is assumed in TRACE66 that IT and A1 are related to ET by constants and that UT1 and OT are both related to A1 by quadratic polynomials, with time in seconds from a fixed epoch as the independent variable. Complete flexibility is afforded the user by varying input. For example, measurements may be time-tagged in any time system by using the polynomial relating OT to A1. For a short-duration run, it can be assumed that UT1 is uniform; UT1 can then be used as the independent variable for the equations of motion if the relationships between IT and ET, ET and A1, and A1 and UT are set to the

TYPICAL TIME-KEEPING SYSTEM RELATIONSHIPS

$ET = IT + \Delta T$ (COMPATIBILITY WITH PLANETARY EPHEMERIDES)

$UT1 = A1 + \alpha_0 + \alpha_1 T + \alpha_2 T^2$ (COMPUTATION OF SIDEREAL ANGLE)

$OT = A1 + \beta_0 + \beta_1 T$ (OBSERVATION DATA TIME TAGS)

$ET = A1 + 32.15$

WHERE:

IT = A1 = UNIFORM TIME USED FOR INTEGRATION (ATOMIC TIME)

ET = EPHEMERIS TIME

UT1 = UNIVERSAL TIME

OT = UTC = BROADCAST TIME

T = TIME REFERENCED TO SOME EPOCH

$\alpha_0, \alpha_1, \alpha_2, \beta_0, \beta_1$ = TIMING POLYNOMIAL COEFFICIENTS

Fig. 5. TRACE66 Time-Keeping Systems

appropriate constants. Figure 5 shows a typical set of relationships among time-keeping systems. Reference 7 contains a more complete description of this subject.

2.3 FORCE MODEL

The force model representation within TRACE66 can range from a simple central gravitational term to a highly sophisticated model. For example, the motion of the vehicle can be considered to be influenced by the following perturbative effects:

- Gravitational potential of the central body expressed as spherical harmonics, using up to 350 terms of C_{nm}, S_{nm}
- Up to 50 point masses (μ_i, ρ_i, ϕ_i, λ_i) imbedded in the central body
- Gravitational attraction from other bodies in the solar system
- Atmospheric density
  - Ballistic coefficient(s):
    - * Constant
    - * Time-segmented
    - * Represented as a polynomial in time
    - * Represented as a tabular function of altitude, time, Mach number, or angle-of-attack
  - Density models:
    - * ARDC 1959
    - * U.S. Standard 1962
    - * Lockheed-Jacchia
    - * Exponential
    - * LMSC 1967
    - * Jacchia 1964 (log ρ, Walker analytic, or Walker modified by Bruce)
    - * AFCRL (Champion 1968)
    - * NWL
- Seven finite thrust models with achieved velocity increment cutoff option

$$\underline{\ddot{r}} = \sum_{i=0}^{6} \underline{\ddot{r}}_i$$

WHERE:

$\underline{\ddot{r}}$ = TOTAL INERTIAL ACCELERATION OF VEHICLE

$\underline{\ddot{r}}_0$ = ACCELERATION DUE TO POINT MASSES IMBEDDED IN CENTRAL BODY

$\underline{\ddot{r}}_1$ = GRAVITATIONAL ACCELERATION DUE TO CENTRAL BODY

$\underline{\ddot{r}}_2$ = GRAVITATIONAL ACCELERATION DUE TO OTHER BODIES IN THE SOLAR SYSTEM

$\underline{\ddot{r}}_3$ = ACCELERATION DUE TO ATMOSPHERIC EFFECTS

$\underline{\ddot{r}}_4$ = ACCELERATION DUE TO THRUST

$\underline{\ddot{r}}_5$ = ACCELERATION DUE TO SOLAR RADIATION PRESSURE

$\underline{\ddot{r}}_6$ = ACCELERATION DUE TO LOCAL GRAVITATIONAL ANOMALIES

Fig. 6. Force Model Representation of the Equations of Motion

- Solar radiation pressure constant with a shadow-dependent scale factor
- Orbit adjusts at specified times by instantaneous change in velocity

The total acceleration acting on the vehicle is given symbolically as

$$\ddot{\underline{r}} = \sum_{i=o}^{6} \ddot{\underline{r}}_i$$

This representation includes central gravitation and all the perturbative effects defined in Fig. 6. Each of the acceleration components $\ddot{\underline{r}}_i$ is evaluated in its appropriate reference frame, rotated if necessary, and accumulated in a body-centered rectangular coordinate system.

2.4 TRAJECTORY PARTIAL DERIVATIVES VIA VARIATIONAL EQUATIONS

The solution of an orbit determination or covariance analysis problem requires the ability to predict the effects of perturbations of various parameters on the orbit. TRACE66 estimates these effects by numerically integrating the partial derivatives of the total acceleration with respect to the parameter of interest. These partial derivatives are called variational equations, and the resulting integrals (solutions to the variational equations) are the trajectory partial derivatives. Functionally, the total vehicle acceleration vector can be expressed as

$$\ddot{\underline{r}} = \ddot{\underline{r}}(\underline{r}, \dot{\underline{r}}, p)$$

where p is an explicit or implicit equation-of-motion parameter. Figure 7 symbolically illustrates (employing the above functional notation) the formation of a variational equation and its solutions.

$$\boxed{\partial(\underline{r},\underline{\dot{r}})/\partial p}$$

$$\frac{\partial \underline{r}}{\partial p} \longleftarrow \int \partial\underline{\dot{r}}/\partial p \longleftarrow \int \partial\underline{\ddot{r}}/\partial p$$

OBTAINED FROM VARIATIONAL EQUATIONS OF THE FORM:

$$\frac{\partial \underline{\ddot{r}}}{\partial p} = \left[\partial\underline{\ddot{r}}/\partial\underline{r}\right]\left(\frac{\partial \underline{r}}{\partial p}\right) + \left[\partial\underline{\ddot{r}}/\partial\underline{\dot{r}}\right]\left(\frac{\partial \underline{\dot{r}}}{\partial p}\right) + \frac{\partial \underline{\ddot{r}}(\underline{r},\underline{\dot{r}},p)}{\partial p}$$

$$= \left[\frac{\partial \underline{\ddot{r}}_0}{\partial \underline{r}} + \frac{\partial \underline{\ddot{r}}_1}{\partial \underline{r}} + \frac{\partial \underline{\ddot{r}}_2}{\partial \underline{r}} + \frac{\partial \underline{\ddot{r}}_3}{\partial \underline{r}}\right]\frac{\partial \underline{r}}{\partial p} + \left[\frac{\partial \underline{\ddot{r}}_3}{\partial \underline{\dot{r}}}\right]\frac{\partial \underline{\dot{r}}}{\partial p} + \frac{\partial \underline{\ddot{r}}}{\partial p}$$

WHERE:

p IS AN EQUATION OF MOTION PARAMETER

Fig. 7. Generation of Trajectory Partials

3. EPHEMERIS GENERATOR OVERVIEW

The ephemeris generation function of TRACE66 produces vehicle trajectory information at prespecified or requested event times. The integrator generates the vehicle state S ($\underline{r}$, $\underline{\dot{r}}$, $\underline{\ddot{r}}$) at integration times; to obtain the vehicle state S($\underline{r}$, $\underline{\dot{r}}$; t) at a particular time, Hermitian interpolation formulas are used (see Ref. 8). This print time could result from prespecified print times such as:

- Epoch event
- Print time vector
- Stop event

or from the occurrence of trajectory events such as:

● Instantaneous orbit adjusts	specified
● Ballistic coefficient changes	
● Start and stop of thrust periods	
● Observational measurement times	
● Crash altitude event	detected
● Particular latitudes, longitudes, or altitudes	
● Ascending and descending nodes	
● Apsis points	
● Eclipsing (entry/exit) points	
● Points of closest approach	
● Local moon or midnight	

3.1 SINGLE VEHICLE EPHEMERIS GENERATION

The types of trajectory information available in TRACE66 for a single vehicle are described in this section.

The vehicle state vector is characterized as:

• Bodycentric	$x, y, z, \dot{x}, \dot{y}, \dot{z}$
• Body-fixed	$\hat{x}, \hat{y}, \hat{z}, \dot{\hat{x}}, \dot{\hat{y}}, \dot{\hat{z}}$
• Spherical	$\alpha, \delta, \beta, Az, R, V$
• Orbit-plane	$R, T, C, \dot{R}, \dot{T}, \dot{C}$
• Classical	$a, e, i, \Omega, \omega, \tau$

Related quantities are represented as:

• Latitude, longitude, and altitude	Φ, λ, h
• Subvehicle latitude and geomagnetic latitude and longitude	$\Phi^*, \Phi_g, \lambda_g$
• Rev number and apocenter and pericenter altitudes	Rev, h_a, h_p
• Empirical nodal regression, period, and period decay	$\dot{\Omega}', P'_{\Omega}, \dot{P}'_{\Omega}$
• Secular rates of node and pericenter	$\dot{\Omega}, \dot{\omega}$
• Eccentric, mean, and true anomalies	E, M, ν
• Keplerian, anomalistic, and nodal periods	P_K, P_A, P_N
• Right ascension and declination of planetary bodies (e.g., the sun or moon)	α', δ'
• Relative position and velocity vectors to planetary bodies and/or to another vehicle (e.g., trajectory differences)	$\underline{\Delta}, \underline{\dot{\Delta}}$

3.2 SIMULTANEOUS VEHICLE EPHEMERIS GENERATION

Frequently, it is necessary to analyze trajectory information for a vehicle system that consists of more than one vehicle. A convenient way of presenting this information is to time-collate the vehicle ephemeris output at prespecified print times that result from the following:

- Epoch event
- Print time vector
- Crash altitude event
- Stop event

This time-collated output is intermixed with individual vehicle ephemeris output (see Sec. 3.1) at specified trajectory events (thrust start/stop, orbit adjusts, ballistic coefficient change, etc.) and at detected trajectory events (nodes, apsis points, etc.).

Optional information available in TRACE66 during the use of this function consists of the following:

- Trajectory differences between reference vehicles (maximum of five) and the remaining vehicles of the system.
- Vehicle-to-vehicle visibility information
  - Visibility status array
  - Range and range-rate measurements
- Station-to-vehicle visibility information
  - Visibility status array
  - Range, elevation, and azimuth measurements
- Vehicle-to-vehicle encounter detection
  - Time and distance of the point of closest approach between two vehicles

The basic restrictions and limitations associated with the generation of simultaneous vehicle ephemeris information are:

- Maximum of 20 vehicles.
- Maximum of 5 reference vehicles allowed within a system of n vehicles when orbit differences are computed.
- Epoch-stop time interval for the first vehicle must span (or have coincident end points) with all other vehicle epoch-stop time intervals.
- All trajectories must be generated by an earth-centered-inertial (ECI) integrator.

4. DATA MEASUREMENT GENERATOR OVERVIEW

The data measurement generation function of TRACE66 produces simulated observational measurements and visibility information. Hermitian interpolation formulas are used to obtain the vehicle state $S\ (\underline{r},\ \dot{\underline{r}};\ t)$ at a particular measurement time.

4.1 SINGLE VEHICLE DATA MEASUREMENT GENERATION

The single vehicle measurement data generator is characterized by the following:

- 38 single vehicle measurement types (see Table 2)
- Maximum of 100 stations
- Vehicle-to-station rise, maximum elevation, and set detection
- Vehicle-planetary body (or earth-fixed station) occultation detection
- Station visibility constraints on range, elevation, and azimuth
- Simulated measurement noise and bias
- Simulated refraction and light-time corrections

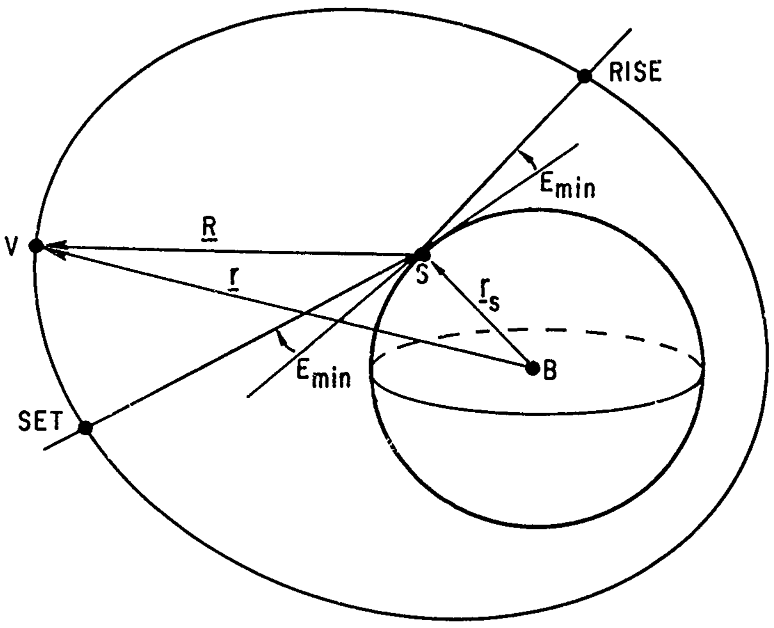
Figure 8 shows a visibility example of station-to-vehicle rise/set occurrence based on minimal station elevation constraint and vector geometry related to the body, stations, and vehicle.

4.2 SIMULTANEOUS VEHICLE DATA MEASUREMENT GENERATION

The simultaneous vehicle measurement data generator is characterized by the following:

- 19 simultaneous vehicle measurement types (see Table 3)
- Maximum of 100 stations
- Station visibility constraints (e.g., minimum and maximum range, azimuth, and elevation)
- Simulated measurement noise, white or colored
- Simulated station-dependent and vehicle-dependent measurement biases

- Simulated station-clock error models with both deterministic and statistical terms
- Simulated tropospheric and ionospheric refraction corrections for some measurement types
- Simulated light-time corrections for some measurement types
- Station-to-vehicle and vehicle-to-vehicle visibility status as a function of time is available in matrix and printer plot formats
- Mutual visibility status involving sets of two prespecified station-satellite combinations.



V: VEHICLE

B: BODY CENTER

S: STATION

$\underline{R}$: RANGE VECTOR

$\underline{r}$: VEHICLE POSITION VECTOR

$\underline{r}_s$: STATION POSITION VECTOR

E_{min}: MINIMUM ELEVATION ANGLE

Fig. 8. Data Measurement Generator Visibility Geometry

Table 2. Single Vehicle Measurement Types

Symbol	Definition	Symbol	Definition
$R, \dot{R}, \ddot{R}$	Slant Range, Rate, Acceleration	ϕ ε	Aspect Angles
$Az, \dot{Az}, \ddot{Az}$	Azimuth Angle, Rate, Acceleration	SA	Signal Attenuation
$E\ell, \dot{E\ell}, \ddot{E\ell}$	Elevation Angle, Rate, Acceleration	$\hat{x}$ $\hat{y}$ $\hat{z}$	Earth-Fixed Rectangular Coordinates
$\dot{P}$ $\dot{Q}$ P Q	Interferometer Data	α_T δ_T	Topocentric Right Ascension and Declination
Φ^*	Geodetic Vehicle Latitude	α δ	Geocentric Right Ascension and Declination
λ	Vehicle Longitude	HA	Topocentric Hour Angle
SR	Surface Range from Station	u	Vehicle-Centered Argument of Latitude
H	Vehicle Height	v	Vehicle Cross-Plane Angle
D	Doppler Rate		
f_2	Two-Way Doppler	A_x A_y	Antenna x and y Angles
f_T	Tranet Doppler		
$\dot{R}_{SGLS}$	SGLS Range Rate	Δ	Geoceiver Range Differences
LA	Look-Angle		
K	Angle between Station-Line-of-Sight and Geocentric Vehicle Radius Vector		

Table 3. Simultaneous Vehicle Measurement Types

Symbol	Definition
R, $\dot{R}$	Station-to-Vehicle Slant Range and Range Rate
$\dot{R}_{SGLS}$	SGLS Range Rate (Range Differences)
Az	Station-to-Vehicle Azimuth Angle
$E\ell$	Station-to-Vehicle Elevation Angle
Ax Ay	Station-to-Vehicle Antenna x and y Angles
V2, $\dot{V}2$	Vehicle-to-Vehicle Range and Range Rate
S2, $\dot{S}2$	Station-to-Vehicle-to-Vehicle Range and Range Rate Sums
V3, $\dot{V}3$	Vehicle-to-Vehicle-to-Vehicle Range and Range Rate Sums
S3, $\dot{S}3$	Station-to-Vehicle-to-Vehicle-to-Vehicle Range and Range Rate Sums
TOA	Time of Arrival
TDOA	Time Difference of Arrival
3WR	Three-Way Range
MP	Multipath

5. ORBIT DETERMINATION OVERVIEW

The basic orbit determination problem is to estimate values for a set of observational and model parameters that, in some way, best represent a given set of measurements. The choice of parameters is dictated by the purpose and objectives of the user and, to some extent, by the nature of the available measurements. The two criteria used in TRACE66 for determining parameter values that represent observables are:

- Minimization of a prespecified cost function
- Production of a minimum-variance (optimal) estimate

The first criterion is empirical, whereas the minimum-variance approach is formulated in statistical terms. Additional empirical devices are available in the program to assist the user in achieving the desired minimum (see Sec. 5.1). In most instances, the empirical approach is used in solving an orbit determination problem. However, if certain assumptions are made (particularly with regard to the statistical significance of contributors to the cost function), it can be shown that the minimization of a cost function produces a minimum-variance estimate.

The second criterion used for estimation allows statistical considerations to be imposed on the solution with either a recursive or nonrecursive algorithm.

In TRACE66, three different computational algorithms are available to process a given set of measurements (Volumes V and XII of this documentation series are the technical manuals for these algorithms). These three orbit determination algorithms are listed as follows:

- Batch differential correction by weighted least squares
- Sequential differential correction by weighted least squares
- Potter square-root filter

The first two algorithms are nonrecursive; that is, they update parameter values at one time point based on a set of data that may be collected over a relatively long time-arc. One obtains an estimate at any other time point by integrating the trajectory, using the updated parameter values as initial conditions. The third algorithm is an example of recursive estimation, or filtering, in which a set of measurements is processed successively in its natural time order, and an estimate is produced at each measurement time.

All three estimation algorithms are described in the following sections, with emphasis on the following:

- Basic equations (cost functions) and methods
- Convergence characteristics
- Parameter constraints
- Statistical considerations.

The limitations in the application of these algorithms are listed below:

- Single- or multi-vehicle fit with no limit on the number of vehicles for single-vehicle measurement types. There is a limit of 20 vehicles when simultaneous vehicle types are considered.
- Total of 100 parameters
- Maximum of 60 equation-of-motion parameters for any vehicle

The orbit determination parameters available with these algorithms are summarized in Table 4.

5.1 BATCH DIFFERENTIAL CORRECTION BY WEIGHTED LEAST SQUARES

The criterion used in this algorithm minimizes the differences (residuals) between the actual measurements and the corresponding values computed from the mathematical model in a generalized least squares sense By using a weighting matrix, one may assign the proper relative importance to measurements of various types and qualities. Measurements are usually related to

Table 4. Orbit Determination Parameters

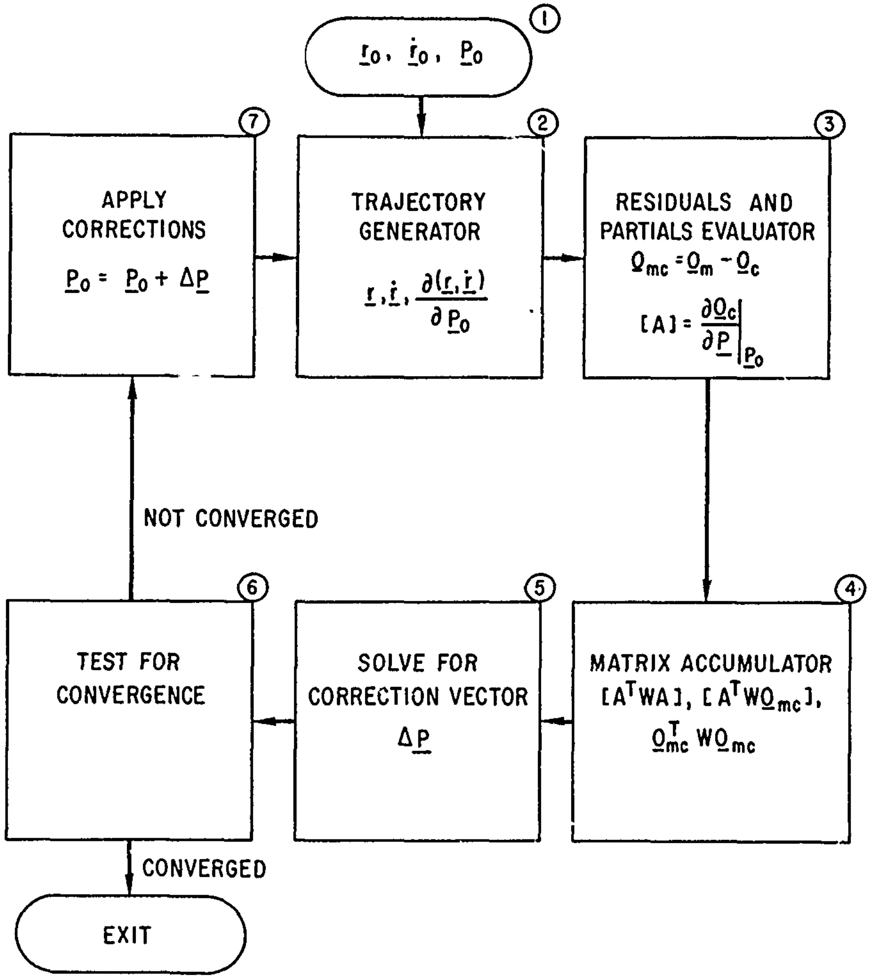
Model-Dependent Parameters	Vehicle-Dependent Parameters	Sensor Parameters
μ	$\underline{r}_o, \dot{\underline{r}}_o, t_o$	Station Locations
$C_{nm}; S_{nm}$	$C_D A/W$ (20 Segments)	Time Bias
ω_a	Orbit Adjusts (10)	Measurement Biases
Atmospheric Constants	Accelerometer Scale Factor and Bias (Segmented 20)	Measurement Scale Factors
Point Masses (μ, ρ, ϕ, λ)	Solar Radiation Pressure Coefficient	
	Thrust (15 Thrust Intervals)	

equation-of-motion and observational parameters by complicated and highly nonlinear functions. To make the minimization computationally tractable, one usually linearizes the problem and uses an iterative technique.

An initial value for each parameter is required to start the procedure. Each iteration solves for a set of corrections to the currently estimated values. The computational steps involved in batch weighted least squares are illustrated in Fig. 9. Note that in this context, the word "batch" implies the processing of all available measurements on each iteration of the differential correction procedure.

The basic equation of weighted least squares is written:

$$[A^T WA]\Delta P = A^T WO_{mc}$$



COMPUTATIONAL STEP	DESCRIPTION
1	SET INITIAL VALUES FOR THE STATE $\underline{r}_0, \dot{\underline{r}}_0$ AND PARAMETER SET $\underline{P}_0$
2	GENERATE THE TRAJECTORY $\underline{r}, \dot{\underline{r}}$ AND ASSOCIATED PARTIALS $\partial(\underline{r}, \dot{\underline{r}})/\partial \underline{P}_0$
3	EVALUATE OBSERVATION RESIDUAL SET $\underline{O}_{mc}$ AND $[A] = \partial \underline{O}_c / \partial \underline{P}_0$
4	PERFORM RESIDUAL EDITING AND ACCUMULATE THE $[A^T W A]$ MATRIX
5	SOLVE FOR THE PARAMETER CORRECTION VECTOR $\Delta\underline{P}$
6	TEST FOR CONVERGENCE
7	APPLY CORRECTIONS; THAT IS, RESET $\underline{r}_0, \dot{\underline{r}}_0$ AND $\underline{P}_0 = \underline{P}_0 + \Delta\underline{P}$ AND CONTINUE WITH STEP 2 UNTIL CONVERGENCE IS OBTAINED

Fig. 9. Schematic of the Differential Correction Procedure

where

A = the matrix of partial derivatives of measurement with respect to the parameters

W = the weighting matrix, which is assumed to be diagonal or block-diagonal with block dimensions ≤ 60

O_{mc} = the vector of measurement residuals

ΔP = the vector of parameter corrections to be determined

This matrix equation is referred to as the normal equation, and the matrix $[A^T WA]$ is called the normal matrix.

When weighted least squares is applied to an orbit determination problem, it is not uncommon for the iterative process to exhibit poor convergence, or even divergent characteristics. The following conditions could lead to such circumstances: inadequacies in the observational model and/or a poor initial approximation of the parameter values. In such situations, the analyst may aid in the eventual convergence of the differential correction process by imposing a side condition on the solution that bounds the magnitude of the correction vector on any iteration. The bounds, which are specified individually for each parameter, are adjusted dynamically as the successive iterations converge or diverge.

A related aid to convergence is the addition of a constant conditioning matrix to the least squares normal matrix. A good choice for the conditioning matrix is an a priori parameter variance/covariance matrix. Note that both of these convergence aids are strictly empirical and have no statistical ramifications. Since the cost function being minimized is unchanged, the final parameter values are unrestricted and will coincide with the ordinary weighted least squares solution.

If one assumes that the mathematical model is exact, that the observational errors are random with mean zero and variance/covariance matrix W^{-1}, and that the observations are linear functions of the parameters, it can be shown that the weighted least squares solution is a minimum-variance solution (see Ref. 9). Furthermore, the inverse normal matrix $(A^TWA)^{-1}$ equals the parameter error variance/covariance matrix. This matrix is computed; the resulting TRACE66 output serves as an estimate of the reliability of the orbit determination results.

When the initial parameter values are not merely guesses, it may be desirable to limit the parameter corrections according to the confidence level associated with the initial estimate. The reason for limiting these corrections may stem entirely from empirical considerations, based on confidence in past values and the continuity of the process. Or the reason might stem from statistical considerations; in this case the objective is to statistically update parameter estimates according to physical measurements and their associated statistics. With either interpretation, a slight modification to the basic equation of weighted least squares is required.

When one views orbit determination by the batch weighted least squares method as an empirical process, one may use either of two approaches to derive the applicable equations. The first approach is to augment the measurement vector and its weighting matrix with the initial estimate and an appropriate set of weights. The second approach is to redefine the cost function as the weighted sum of squares of the measurement residuals plus the weighted sum of squares of the total parameter corrections. Either way, one obtains the same equations.

On the other hand, if one regards the orbit determination process as statistical, the problem is that of optimal combination of statistical estimates (i.e., an a priori estimate and its covariance matrix are to be combined with the estimate and covariance matrix produced by the previously described minimum-variance algorithm). If optimal combination is defined as a linear, unbiased, minimum-variance combination, the resulting equations correspond to those of the empirical algorithm, with the inverse a priori covariance matrix chosen as the weighting matrix for the total parameter correction term in the cost function. A minimum-variance estimate without a priori statistics is called an absolutely unbiased estimate; when a priori statistics are included, it is referred to as an unbiased estimate.

It may be that constraints among the parameters are part of the physical problem. For example, suppose that precise knowledge of the relative locations of two nearby observing stations is available. If such locations were among the parameters in a differential correction, it would be important to constrain the parameter corrections to preserve the relative locations. In TRACE66, this can be accomplished by introducing linear constraints of the form $P = BP' + C$. In terms of corrections, this reduces to $\Delta P = B\Delta P'$. The weighted least squares problem may be solved in terms of $\Delta P'$, and the above matrix equation used to compute ΔP.

5.2 SEQUENTIAL DIFFERENTIAL CORRECTION BY WEIGHTED LEAST SQUARES

Systematic repeated application of batch weighted least squares is generally referred to as sequential weighted least squares. This method can be used with or without including a priori statistics. However, the usual manner of processing sequential least squares includes a priori statistics, and the result is interpreted as a minimum-variance estimate.

In the computational procedure, the data is separated into a series of batches, or stages. After each stage is processed iteratively to convergence, the epoch time, vehicle position and velocity vectors, any remaining parameters, and the parameter error variance/covariance matrix produced by the estimation process are updated (propagated forward) in time. For example, the previous epoch quantities are updated to the end of the particular data batch. These quantities then comprise the initial conditions and a priori statistics for the next data stage. In this fashion, an essentially unlimited amount of data over an unlimited time period can be processed, maintaining dependence through the propagated covariance matrix.

The a priori covariance matrix is used as the weighting matrix for the parameter correction term in the cost function. The use of the inverse normal matrix as a covariance matrix is based on several assumptions which are, in general, not rigorously satisfied. Therefore, an optimistic estimate of the parameter error variance/covariance matrix is produced, and systematic errors may be introduced into the algorithm. To reduce the effects of such errors, TRACE66 is capable of using additive and multiplicative deweighting matrices to alter the a priori covariance statistics before processing any data in a particular stage. This is normally used as a limited-memory function, so that older data measurements have less influence on the current solution.

5.3 POTTER SQUARE-ROOT FILTER

The weighted least squares algorithms are particularly valuable when the number of measurements being processed in one update interval is large in comparison to the number of parameters. When the opposite is true, it may be computationally advantageous to use a recursive estimation algorithm for an orbit determination (Ref. 10). For this purpose, TRACE66 has a square-root form of a minimum-variance filter similar to that originally introduced by Potter (Ref. 11). The vehicle trajectory and associated partial derivatives may be generated analytically (only vehicle position and velocity may be selected as parameters), or by numerical integration (full parameter capability). In addition, the program may be instructed to periodically switch between the Potter square-root filter and the sequential least squares algorithm during the processing of a given data set.

6. COVARIANCE ANALYSIS OVERVIEW

The performance of an estimation algorithm is best evaluated in the operational environment for which it is intended. Since this is frequently impractical, linearized covariance analyses have been constructed. This essentially entails a determination of the effects that specified error sources have on the precision of the estimation procedure. These error sources may be, for example, inaccuracies in station locations, random errors in measurements, or errors in differential equation parameters. Covariance analysis of the batch and sequential least squares estimators is available in TRACE66.

Note that the covariance analysis procedure does not require an actual orbit determination, nor does it require actual measurements. It does require the specification or simulation of the tracking system, i.e., tracker locations and characteristics, visibility constraints, measurement types and rates, a list of the parameters to be studied, and a reference orbit. A linearized covariance analysis may then be obtained by matrix manipulations. The basic output consists of covariance matrices as a function of time, which provide uncertainties in the orbit determination (P) parameters and in the satellite position and velocity based on the following:

- A priori information on the P parameters
- Uncertainties in the measurements
- Uncertainties in a set of nonestimated (Q) parameters

6.1 BATCH WEIGHTED LEAST SQUARES COVARIANCE ANALYSIS

The batch weighted least squares covariance analysis algorithm is summarized in Fig. 10. Note that because weighted least squares is a nonrecursive procedure, the P parameter covariance matrices, $C(P)_p$ and $C(P)_{p+q}$, provide uncertainties in the P parameters at a fixed time only. The matrices depend

- FORM OF NORMAL MATRIX: $A^TWA = \left[\begin{array}{c|c} A_p^T W A_p & A_p^T W A_q \\ \hline A_q^T W A_p & A_q^T W A_q \end{array}\right]$ WHERE:

  A_p = PARTIALS OF MEASUREMENTS WRT P

  A_q = PARTIALS OF MEASUREMENTS WRT Q

- VARIANCE/COVARIANCE MATRICES OF INTEREST:

$$C(P)_p = \left[A_p^T W A_p\right]^{-1}$$

ORBIT DETERMINATION PARAMETER COVARIANCE MATRIX BASED UPON P-PARAMETER A PRIORI INFORMATION $C(P)_0$ AND MEASUREMENT COVARIANCE MATRIX Σ

$$C(P)_{p+q} = C(P)_p + \left(\frac{\partial P}{\partial Q}\right) C(Q) \left(\frac{\partial P}{\partial Q}\right)^T$$

P-PARAMETER COVARIANCE MATRIX WHICH INCLUDES EFFECTS OF UNCERTAINTIES IN Q-PARAMETERS

WHERE: $\left(\frac{\partial P}{\partial Q}\right) = -C(P)_p \left[A_p^T W A_q\right]$

NOTE THAT:

1. VARIANCE/COVARIANCE MATRICES APPLY TO PARAMETER VALUES AT EPOCH TIME ONLY
2. P-PARAMETER A PRIORI INFORMATION $C(P)_0$ IS INCLUDED IN A^TWA BY ADDING $C(P)_0^{-1}$ TO $\left[A_p^T W A_p\right]$
3. MEASUREMENT UNCERTAINTIES DEFINED BY Σ ARE INCLUDED IN A^TWA BY SETTING $W=\Sigma^{-1}$
4. Q-PARAMETER COVARIANCE MATRIX $C(Q)$ IS PRESPECIFIED AT EPOCH

- VEHICLE STATE COVARIANCE MATRICES AS A FUNCTION OF TIME ARE OBTAINED BY TRANSFORMING THE PARAMETER COVARIANCE MATRIX AS FOLLOWS

$$C(X) = \left(\frac{\partial X}{\partial P}\right) C(P)_p \left(\frac{\partial X}{\partial P}\right)^T + \left(\frac{\partial X}{\partial Q} + \frac{\partial X}{\partial P}\frac{\partial P}{\partial Q}\right) C(Q) \left(\frac{\partial X}{\partial Q} + \frac{\partial X}{\partial P}\frac{\partial P}{\partial Q}\right)^T$$

WHERE:

$\left(\frac{\partial X}{\partial P}\right)$ & $\left(\frac{\partial X}{\partial Q}\right)$ ARE TRAJECTORY PARTIAL DERIVATIVES (SOLUTIONS OF THE VARIATIONAL EQUATIONS)

NOTE THAT THE FIRST TERM OF C (X) GIVES THE EFFECT OF MEASUREMENT UNCERTAINTIES AND THE SECOND TERM GIVES THE EFFECT OF Q-PARAMETER UNCERTAINTIES

TO TRANSFORM C (X) TO ANY OTHER COORDINATE SYSTEM R APPLY THE MATRIX EQUATION

$$C(R) = \left(\frac{\partial R}{\partial X}\right) C(X) \left(\frac{\partial R}{\partial X}\right)^T$$

Fig. 10. Orbital Statistics Via Covariance Analysis

on the amount of tracking data included in the computation and will therefore vary with time even though they relate to a parameter value at a fixed time. On the other hand, the vehicle state covariance matrices C(X) and C(R) give uncertainties in the satellite position at the current time.

The batch least squares covariance analyzer can be operated in three modes: real-time, postflight, and update. In the real-time mode, all covariance matrices are based on tracking data up to and including the time of the output. Therefore, the matrices are representative of the uncertainties that would result from processing data in a real-time fashion. In the postflight mode, covariance matrices are based on all tracking data available. Therefore, there is just one $C(P)_p$ and $C(P)_{p+q}$. The satellite state vector matrices represent the uncertainties that would result from an attempt to reconstruct a trajectory in a postflight analysis fashion. Finally, the update mode consists of updating satellite position uncertainties in time based on an a priori covariance matrix, not on tracking data. Therefore, the satellite state vector covariance matrices represent the uncertainties in satellite positions that would result from a simple trajectory generation.

6.2 SEQUENTIAL WEIGHTED LEAST SQUARES COVARIANCE ANALYSIS

A sequential least squares covariance algorithm takes on a different form because of the periodic updates intrinsic to the algorithm. Two matrices, corresponding to $C(P)_p$ and $C(P)_{p+q}$ must be maintained and propagated according to the updating process. The notations "C" for the former, and "P" for the latter are used; Q=C(Q), the Q parameter variance/covariance matrix. The lower-case letters "p" and "q" are used to denote the parameter vectors. The algorithm is then summarized in Fig. 11. Note the multiplicative and additive deweighting matrices F and M that are applied to C. A possible use of this algorithm, not applicable to batch weighted least squares, would be an adaptive determination of the best F and M for a particular satellite and tracker geometry.

INITIALIZE

C = p PARAMETER CONVARIANCE MATRIX USED BY ALGORITHM

P = ACTUAL p PARAMETER CONVARIANCE MATRIX

$S = 0$

$i = 0$

UPDATE AND DEWEIGHT:

$$C_0^{-1} = F^T\left[\phi C\phi^T + M\right]^{-1}F$$

$$P_0 = \phi P\phi^T + \psi Q\psi^T - \phi S\psi^T - \psi S^T\phi$$

$$S_0 = \phi S - \psi Q$$

where $\phi = \dfrac{\partial P_{t_{i+1}}}{\partial P_{t_i}}$ and $\psi = \dfrac{\partial P_{t_{i+1}}}{\partial q}$

ADD NEW DATA:

$$C^{-1} = C_0^{-1} + A_p^T W A_p$$

$$P = C\Big[C_0^{-1}P_0C_0^{-1} + A_p^TWA_p + A_p^TWA_qQA_q^TWA_p + C_0^{-1}S_0A_q^TWA_p + A_p^TWA_qS_0^TC_0^{-1}\Big]C$$

$$S = C\left[C_0^{-1}S_0 + A_p^TWA_qQ\right]$$

INCREMENT:

$i = i + 1$

REPEAT PROCESS FROM UPDATE AND DEWEIGHT STEP

Fig. 11. Schematic of Sequential Least Squares Covariance Analysis Algorithm

7. LUNAR GRAVITY FIELD ANALYZER OVERVIEW

In addition to the TRACE66 trajectory analysis and orbit determination capabilities previously discussed (see Sec. 5), a particular portion of the program has been designed and developed specifically to assist in the dynamical determination of lunar gravitational constants and in the reconstruction of lunar satellite orbits (Ref. 5). The ultimate purpose of this software is to obtain a lunar gravity model from the JPL/NASA Deep-Space Network two-way doppler tracking data. The force model can be expressed in terms of a spherical harmonic representation of the central body, a surface distribution of up to 750 disks or point masses, and gravitational potential due to the earth and sun.

Within TRACE66 proper, the key features of this option are as follows:

- Mass parameter selection algorithm by arc
- Printer plot of doppler residuals versus the vehicle's selenographic ground trace
- Residual and data edit summaries
- Complete orbit determination if the numer of parameters is less than 100
- Accumulation of a large (256 parameters/arc) normal matrix that can be output onto an auxiliary file with specific modifications

Note that if there are more than 100 parameters in the parameter set of interest, the merging of normal matrices and the solution of the normal equations is performed outside of TRACE66, using auxiliary software and techniques written explicitly for a large, sparse linear system. This software has been employed along with the TRACE66 normal matrix accumulation to solve a system of the approximate order 2000.

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