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A Simple Model to Elucidate the Utility of Weather Forecasting in Military Operations: Weather and Warplanes III

R. R. Rapp

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10. ABSTRACT

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PREFACE

This Report represents another step in Rand's continuing effort to explicate the effects of weather and weather information on Air Force operations. The model developed in this Report demonstrates how the amount of time and effort required to complete a weather-sensitive operation can vary with the skill and bias of the weather forecast. The model produces results that are easily understood by forecaster and decisionmaker alike; and, therefore, it could be a very useful device to help increase their understanding of each other's problems and products.

SUMMARY

A simple model is constructed to demonstrate the influence of weather forecasts on operational decisions. The model indicates that the present forecasting capability can reduce the number of futile attempts to destroy targets, with only a slight cost in time to accomplish the objective. Some implications are given for optimizing the method of presenting weather forecasts.

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I. INTRODUCTION

If weather has a strong effect on an Air Force operation, the accuracy of advance information about the weather should have a strong effect on the decision to proceed, or not to proceed, with the operation. To date, little has been done to quantify this relationship between forecast accuracy and the outcomes of operational decisions based on those forecasts. What little has been done, however, has shown that the forecaster can make rather impressive contributions.

In a study of a decision interface between the Air Weather Service and the U.S. Strike Command (Huschke and Rapp, 1970), a fairly elaborate numerical scheme was developed that reproduces forecast probabilities, at different skill levels, of several weather events that affect a complex strike operation. That forecast model is intended for use only when a computer is available for the processing of climatological records, for skill is taken as a variable function of the occurrence probabilities of the weather events.

We felt that a more convenient method, one that could be hand calculated from standard forecast-verification tables and climatological summaries, could serve three very useful purposes: (1) to make quick estimates of forecast value in decisions involving a tradeoff between elapsed time and effort (costs); (2) to demonstrate that the probabilistic nature of forecasts could serve a decisionmaker; and (3) to demonstrate the functional relationship between forecaster and decisionmaker, and help either one to better understand the other's role.

To do this, we have set up a rather simple model of a "go/no-go" decision, which will show not only what benefits ensue from using weather forecasts (say 24 hours in advance) in making decisions, but also the magnitude of the reward from very good forecasts. Comparing our model with forecasts of today's quality, we try to indicate how probabilistic forecasts can be useful by bringing the forecaster and the decision-making commander to a mutual understanding.

DESCRIPTION

In this model the situation has been simplified by leaving unspecified the nature of the target and the surrounding terrain. Other studies have included and will include other facets of the decision process, but our present aim is to highlight the weather effects in a very simple manner. The model presumes that the commander of an air group has been assigned the task of destroying a certain target as soon as, in his judgment, he can do so. We assume attacks on the target are successful if the ceiling is at 5000 feet or higher and visibility is 3 miles or greater. These limits, which define "good weather" in this model, were derived from discussions with pilots about the probability of success of dive-bombing attacks in a permissive air-defense environment.

On the basis of these assumptions, the commander may order a flight once in a given day, and he must decide 24 hours in advance whether he will send a mission to destroy the target. The forecaster provides his best estimate of the expected weather conditions, but the commander, knowing that weather forecasts are not infallible, must evaluate the urgency of destroying the target and weigh the possible attrition of his own forces against the importance of an early success. To determine what incremental benefits the commander can gain, we assume on the one hand that he ignores the weather forecast and on the other that he follows a categorical forecast slavishly. It is doubtful that either of these situations ever applies in practice, but the dichotomy will indicate how the forecast can affect the decision.

We also assume in the model that the weather shows no persistence from one day to the next, to imply not that there is no physical connection between the weather from one day to the next, but that there is no statistical dependence of weather from one day to the next. [Models that allow for the statistical persistence of the weather are in the planning stage.]

CLIMATOLOGY

Climatology is represented in the model in the form of the probability p of a day chosen at random having good weather and probability

1 - p of a day chosen at random having bad weather. If the successive days are indeed statistically independent, we can determine the probable number of days until good weather occurs. The probability of good weather on the first day, P(1), is simply the climatological probability of good weather, so that P(1) = p. The probability of good weather first occurring on the second day equals the probability of bad weather on the first day times the probability of good weather on the second day; P(2) = p(1 - p).

NO FORECAST USED

In this model, we first consider the situation that no forecast is used, and that, consequently, planes are dispatched every day. In general, then, the probability of completing the mission in exactly k days is

$$P(k) = p(1 - p)^{k-1} . \tag{1}$$

The expected number of days until one of the missions is successful is given by

$$E[k] = \sum_{k=1}^{\infty} kP(k) ,$$

which becomes

$$E[k] = \frac{1}{p} .^{*} \tag{2}$$

We will be able to compare results of no-forecast and forecast schemes either by comparing the full probability curves, or more simply, by noting the difference in the expected number of days.

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*For the benefit of those who know as little of statistics as the author, some of the derivations are given in Appendixes A, B, and C.

COMPLETE RELIANCE ON FORECAST

Suppose that planes are dispatched only if the forecast is "good weather." This supposition gives rise to three possible outcomes: (1) no mission is dispatched, (2) a mission is dispatched and fails, and (3) a mission is dispatched and succeeds. The skill of the forecaster in predicting "good" days can be best represented in a two-by-two contingency table (Table 1).

Table 1

SCHEMATIC REPRESENTATION OF FORECASTS VS. OBSERVATIONS

(Number of Days)

Observed	Forecast: Good	Forecast: Bad	Total Observed
Good	a	b	a + b
Bad	c	d	c + d
Total Forecast	a + c	b + d	N = a + b + c + d

From Table 1 the climatological probability of a good day is given by $p = (a + b)/N$. It is also possible to define the probabilities of the three possible outcomes noted above. Thus the probability on any day of not dispatching a mission is $p_1 = (b + d)/N$; the probability on any day of dispatching a mission that fails is $p_2 = c/N$; and the probability on any day of dispatching a mission that succeeds is $p_3 = a/N$. Thus $p_1 + p_2 + p_3 = 1$, and with the assumption of independence, the distribution of the three events is given by the multinomial distribution

$$P(n_1, n_2, n_3) = \frac{k!}{(n_1!)(n_2!)(n_3!)} \left(p_1\right)^{n_1} \left(p_2\right)^{n_2} \left(p_3\right)^{n_3}, \qquad (3)$$

where n_1 = number of times no mission is dispatched,

n_2 = number of times a mission is dispatched and fails,

n_3 = number of times a mission is dispatched and succeeds, and

$$n_1 + n_2 + n_3 = k \ .$$

In this part of the problem we are interested not only in the number of days until a task is completed, but also in the number of missions dispatched during this time. If Eq. (3) is solved for $n_3 = 1$, the result is the probability that there will be at least one success in the sequence of k days. This would be the sum of the probabilities of success on days 1,...,k. Since a single success is equally likely on any day, this can be expressed as k times the probability of success on exactly one day. To get the probability that success will occur only on the last day, it is necessary to divide the coefficient of Eq. (3) by k.* If we denote this probability as $P(n_1, n_2)$, since n_3 is no longer a variable, the equation becomes

$$P(n_1, n_2) = \frac{(k-1)!}{n_1!\, n_2!}\left(p_1\right)^{n_1}\left(p_2\right)^{n_2}\left(p_3\right). \tag{4}$$

Noting that $n_1 + n_2 = k - 1$ and that $n_2 + 1 = n$, this can be written as

$$P(n, k) = \frac{(k-1)!}{(n-1)!\,(k-n)!}\left(p_3\right)\left(p_2\right)^{n-1}\left(p_1\right)^{k-n} , \tag{5}$$

where $P(n, k)$ is the probability of sending exactly n missions in exactly k days.

One could look at both the forecast case and the no-forecast case with the assumption that the probability of success is given by $p_3 = a/N$; then the probability of success in k days is

$$P_f(k) = p_3(1 - p_3)^{k-1} , \tag{6}$$

---

*It should be noted that this formulation would apply for the single success to occur on precisely one day in the sequence, but as far as the mathematics is concerned, it need not be on the last day.

where $P_f(k)$ is the probability that k days will be required for a successful mission when the forecast is heeded. It can be shown by summing Eq. (4) from $n_2 = 0$ to $n_2 = k - 1$ that the same result is obtained. The expected number of days according to Eq. (6) is now $1/p_3$. To determine the expected number of missions dispatched, it is necessary to compute the conditional probability of n_2 missions, given that k days are required. If $P(n_2|k)$ is the probability that n_2 unsuccessful missions will be dispatched, given that k days are required to complete the job, the joint probability distribution can be written

$$P(n_2, k) = P(n_2|k)P_f(k) ,$$

and by Bayes's rule,

$$P(n_2|k) = P(n_2, k)/P_f(k) . \tag{7}$$

Multiplying Eq. (7) by n_2 and summing from $n_2 = 0$ to $n_2 = k - 1$ yields the expected value of n_2 for any fixed k. This formulation (see Appendix C) gives an expected value of

$$E[n_2|k] = \frac{(k - 1)p_2}{(1 - p_3)} . \tag{8}$$

Since the total number of missions flown is n_2 (the number that failed) plus the one that succeeded, the expected number of missions flown for k days until success will be $E(n_2|k) + 1$:

$$E[n|k] = \frac{(k - 1)p_2}{(1 - p_3)} + 1 . \tag{9}$$

II. CALCULATIONS WITH VIETNAM DATA

From the climatology of Pleiku, assuming that a day is good if ceilings are 5000 feet or higher and visibility 3 miles or greater, it was found that $p = 0.36$ at noon during the summer monsoon. The estimated contingency table (Table 2) below was made from data on the skill of forecasters in an area of Southeast Asia with a similar climatology.

Table 2

ESTIMATED FORECAST SKILL

(Number of Days)

Observed	Forecast Good	Forecast Bad	Total Observed
Good	34	2	36
Bad	7	57	64
Total Observed	41	59	100

From Table 2, $p_1 = 0.59$, $p_2 = 0.07$, and $p_3 = 0.34$.

Substituting the climatological probability into Eq. (1) and the value of p_3 into Eq. (6), the probability that the task would be completed in exactly k days may be computed, following the two alternative decision strategies ("go every day" or "follow forecast"). Table 3 shows these probabilities, together with the cumulative probabilities for enough days to ensure that both strategies yield a cumulative probability of over 95 percent. The largest difference in P(k) (on the first day) is so small that we may conclude, for this climatology and this kind of forecasting skill, that ignoring the forecasts saves an insignificant amount of calendar time.

The next question is, "How many missions would be dispatched in the two alternative strategies?" Of course, if one attempt were made each day, the number of missions would equal the number of days until success. And if the forecasts were followed, we could find the probabilities of sending n missions with success in k days. Table 4 gives these probabilities.

For comparison with a corresponding number in Table 3, the probabilities for $n \leq 2$ and $k \leq 7$ can be summed to give a probability of 93 percent that the mission would be successful in 7 days or less with up to 2 missions; with the go-every-day strategy, 93 percent probability of success requires only six days, but at a cost of up to six missions.

Table 3

PROBABILITY OF A SUCCESSFUL MISSION IN k DAYS BY TWO ALTERNATIVE DECISION STRATEGIES

	Go Every Day		Follow Forecast	
k	P(k)	Cum	P(k)	Cum
1	0.3600	0.3600	0.3400	0.3400
2	0.2304	0.5904	0.2244	0.5644
3	0.1474	0.7378	0.1481	0.7125
4	0.0943	0.8321	0.0977	0.8102
5	0.0604	0.8925	0.0645	0.8747
6	0.0386	0.9311	0.0426	0.9173
7	0.0247	0.9558	0.0281	0.9454
8	0.0158	0.9716	0.0185	0.9639

Table 4

JOINT PROBABILITY OF SUCCESS USING FORECAST FOR EXACTLY k DAYS WITH EXACTLY n MISSIONS

	k								
n	1	2	3	4	5	6	7	8	9
1	0.3400	0.2006	0.1184	0.0698	0.0412	0.0234	0.0138	0.0081	0.0048
2		0.0238	0.0280	0.0248	0.0195	0.0144	0.0101	0.0070	0.0047
3			0.0017	0.0030	0.0035	0.0035	0.0031	0.0025	0.0020
4				0.0001	0.0002	0.0003	0.0004	0.0004	0.0004

III. SOME VARIATIONS OF THE PROBLEM

To compare several different situations, we look at the expected number of days and the expected number of missions for a few cases. Using the no-forecast scheme, the expected number of days equals the expected number of missions, which can be calculated from Eq. (2). For the decision based on forecasts (as in Table 2) the expected number of days can be computed from Eq. (2), but with p_3 replacing p. The expected number of missions can then be calculated by using the expected value of k in Eq. (7).

Three situations can be calculated immediately. If the forecasts are ignored, the expected number of missions equals the expected number of days — for Pleiku climatology, 2.78. If the forecasts are perfect, the first good day will be selected for a single mission and the expected number of days until the mission is dispatched will be the same as if no forecast were used. For Vietnam forecasting skill, the expected number of days until mission succeeds is 2.94, and the expected number of missions dispatched, 1.20. We assume that the field commander's objective is to optimize the time and the number of missions. Within the constraints of our model, the minimum number of missions will be one. If the data on forecast accuracy used with the Pleiku climatology are correct, the forecasters could assist in making great strides toward the objective.

That the forecasts will always be so successful is not axiomatic. Suppose that the forecaster makes twice as many mistakes as are indicated in Table 2, with the same climatology and the same ratio between pessimistic and optimistic statements. The relations between observations and forecasts would be as shown in Table 5, where $p_1 = 0.54$, $p_2 = 0.14$, and $p_3 = 0.32$. Since the climatology remains the same, the expected days to completion following the forecasts increase to 3.12 and the missions expected increase to 1.44. Thus even much worse forecasts than the Vietnam data indicate can save effort with a seemingly small cost in task completion time.

Table 5

FIRST VARIATION OF FORECAST SKILL

(Number of Days)

Observed	Forecast Good	Forecast Bad	Total Observed
Good	32	4	36
Bad	14	50	64
Total Forecast	46	54	100

Now suppose the forecaster who scored lower tried to balance his forecasts by calling more of the marginal cases "bad." His results might be as shown in Table 6, where $p_1 = 0.63$, $p_2 = 0.08$, and $p_3 = 0.29$. The expected days to completion increase to 3.45, but the expected missions decrease to 1.28. There are many more possible variations, but the ones presented sufficiently demonstrate a few pertinent points.

Table 6

SECOND VARIATION OF FORECAST SKILL

(Number of Days)

Observed	Forecast Good	Forecast Bad	Total Observed
Good	29	7	36
Bad	8	56	64
Total Forecast	37	63	100

Table 7 summarizes results from the five cases just discussed, all based on the weather statistics of the summer monsoon at Pleiku. Comparison of the first two rows reveals the rather substantial cost (in number of missions flown) imposed by complete ignorance of the weather. In such a predicament, a mission will be dispatched every day. With perfect forecasts -- perfect foreknowledge of the weather --

all of this loss could be recouped (point B versus point A in Fig. 1). Using forecasts of the excellence of those now available in Vietnam (line 3 of Table 7), much of this cost in missions flown is recouped, the price being a small increase of elapsed time required to complete the task (point C).

Lines 4 and 5 show that even with less skilled forecasting, the number of expected missions can be approximately halved, although time to completion is perceptibly longer (points D and E).

Table 7

RESULTS OF FIVE VARIATIONS OF FORECASTING SKILL

Variations in Forecasting Skill	Expected Number of Days (k)	Expected Number of Missions (n)	Percent of Forecasts Correct
No forecasts	2.78	2.78	36*
Perfect forecasts	2.78	1.00	100
Vietnam forecasts	2.94	1.20	91
First variation	3.12	1.44	82
Second variation	3.45	1.28	85

*Assume a "good-weather" forecast every day.

The last column of Table 7 is included as a rough measure of the skill of the forecaster. It is worth noting that the "fudging" done to produce the second variation raises this score and reduces the number of missions, although at the same time it increases the calendar time required to complete the task.

INTERACTION WITH THE DECISIONMAKER

The decisionmaker's first concern is to balance the "cost" of dispatching a mission to destroy a target against the "cost" of permitting

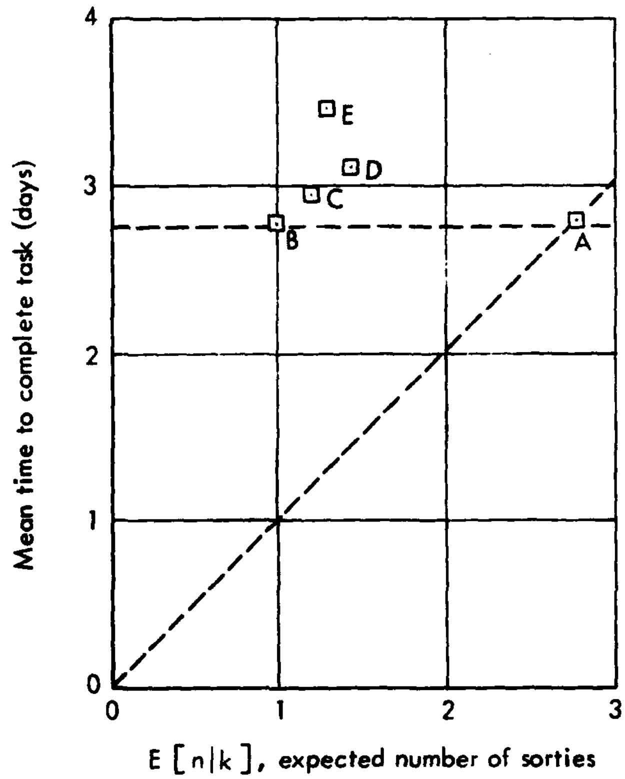

Fig. 1 -- Illustration of trade-offs between calendar time to complete task and number of sorties flown, for various assumptions about weather forecasting skill and weather/decision options; computed for the summer monsoon in Vietnam.

A. Try daily regardless of forecasts.
B. Perfect forecast; try when forecast is favorable.
C. Vietnam skill; try when forecast is favorable.
D. First variation; try when forecast is favorable.
E. Second variation; try when forecast is favorable.

The horizontal dashed line is a minimum limit imposed by the Vietnam climatological statistics used here.

the target to exist. The word cost has been placed in quotation marks because there is more to be considered than the pure economic value. Under the stress of battle conditions, such things as the availability of reserves or the plight of a given unit under fire may be far more important than the dollar value of the equipment and POL required for the mission. The "costs" referred to here are strictly the value judgments of the commander. Although such value judgments will vary considerably from day to day in a combat situation, let us assume that a value D can be specified which is the commander's judgment of the daily cost, in units of the mission cost, to permit a target to exist. The total expected cost, T, to destroy the target would then be given by the expected number of days that the target could remain, k, multiplied by the daily value, D, plus the cost of the number of missions sent in attempts to destroy it, n.

$$T = kD + n \ .$$

Using the expected values of k and n from the five lines in Table 7, five linear equations result.

$$T_1 = 2.78D + 2.78$$

$$T_2 = 2.78D + 1.00$$

$$T_3 = 2.94D + 1.20$$

$$T_4 = 3.14D + 1.44$$

$$T_5 = 3.45D + 1.28$$

Comparing all other cases to T_1, it can be seen that T_2 is parallel to T_1 and with a smaller intercept is always a superior strategy. With a steeper slope and larger intercept, T_3 intersects T_1 at a value of $D = 9.9$. This suggests that the decisionmaker should choose to heed forecasts of this quality unless the target was judged to be more valuable than 10 times the value of the mission.

Applying the same type of reasoning to the other two variations, it is found that such forecasting variations as the first one should be heeded unless the value of the target is 4 times the cost of the mission and the second variation is useful if the value of the target is 2 times the cost of the mission.

IV. [illegible]

Based on several sweeping generalizations, the model presented was intended merely to be a quickly understood device that will illustrate the usefulness of preparing such models and clarify the kind of insights that can be gained from them.

IMPLICATIONS FOR THE COMMANDER

In the case illustrated, not the least important implication is the following. A commander whose weather advisor has the skill represented by point C or point D in Fig. 1 need not be content with the weather forecast as it is. He is entitled to -- and this study should encourage him to -- discuss his specific needs for his specific tasks with his forecaster. If it seems called for, he can then ask for a biased forecast that might, for example, tend to conserve personnel and materiel (point E) or he might prefer one that would tend to shorten the time to completion of the task, sometime between C and the line AB. At the same time, this model makes it evident that the forecaster who has been given an understanding of the factors entering into a commander's decision is better prepared to meet the specific forecasting needs for a given task.

IMPLICATIONS FOR THE WEATHER FORECASTER

The forecaster predicting for military operations should constantly be aware of the importance of weather input to any decision, as well as of the numerous other factors that bear on the decision. The urgency to destroy the target may cause a commander to make a decision on a very low probability of success, but a shortage of supplies may restrain him until the probability of success is very high. Unless the forecaster is told specifically of the commander's desire for a biased categorical forecast, it might be more practical for him to make a probability forecast (Nelson and Winter, 1960). Some commanders may not like a strictly probabilistic forecast, but are likely to appreciate some measure of the forecaster's confidence.

These ideas are engendered by my experience as a military forecaster. When the demands of the military situation dictated that a mission be attempted at all costs, my weather predictions were received with a curt "Thank you." If the weather could be exploited, my predictions were subjected to sharp scrutiny, and every attempt was made to determine my confidence in them. A commander who must take an action that may be influenced by the weather needs not only good predictions but also some measure of confidence in them. If the forecaster does not provide these, the commander must make his own judgments, which often results in his downgrading the importance of weather prediction and hence of the service that the forecaster has to offer.

Appendix A

EXPECTED NUMBER OF DAYS TO A SUCCESSFUL MISSION

The expected value of a discrete variable is defined as the sum of the product of the variable and its distribution function.

$$E[k] = \sum_{k=1}^{\infty} k\ P(k)$$

Let $P(k) = p(1 - p)^{k-1}$ according to Eq. (1). Now consider the sum of the geometric series for $q < 1$

$$S = \sum_{n=0}^{\infty} q^n = (1 - q)^{-1}$$

and its derivative

$$\frac{dS}{dq} = \sum_{n=0}^{\infty} q^{n-1} = (1 - q)^{-2}$$

Let $n = k;\ q = 1 - p$

Then

$$\sum_{k=1}^{\infty} k(1 - p)^{k-1} = \frac{1}{p^2}$$

Multiplying by p,

$$\sum_{k=1}^{\infty} kp(1 - p)^{k-1} = \frac{1}{p} = \sum_{k=1}^{\infty} kP(k) = E[k]$$

Appendix B

DERIVATION OF PROBABLE NUMBER OF DAYS FOLLOWING FORECAST

$$P(n_2, k) = \frac{(k - 1)!}{n_2!(k - 1 - n_2)!} p_3 \, p_2^{n_2} \, p_1^{k-1-n_2} .$$

To find the sum of $P(n_2, k)$ from $n_2 = 0$ to $n_2 = k - 1$,

$$\sum_{n_2=0}^{k-1} P(n_2, k) = \sum_{n_2=0}^{k-1} \frac{(k - 1)!}{n_2!(k - 1 - n_2)!} p_3 \, p_2^{n_2} \, p_1^{k-1-n_2} ,$$

note that p_3 is independent of n_2 and can be taken out of the summation sign. Let $k - 1 = m$, then the right-hand side of the above equation becomes

$$= p_3 \sum_{n_2=0}^{m} \frac{m!}{n_2!(m - n_2)!} p_2^{n_2} \, p_1^{m-n_2} .$$

Also note that the summation defines the binomial expansion so that

$$\sum_{n_2=0}^{m} \frac{m!}{n_2!(m - n_2)!} p_2^{n_2} \, p_1^{m-n_2} = (p_1 + p_2)^m .$$

But, by definition, $p_1 + p_2 + p_3 = 1$ so that $p_1 + p_2 = 1 - p_3$

Therefore,

$$\sum_{n_2=0}^{k-1} P(n_2, k) = p_3(1 - p_3)^m = p_3(1 - p_3)^{k-1} .$$

Appendix C

DERIVATION OF EXPECTED NUMBER OF DAYS FOLLOWING FORECAST

From Eqs. (4), (5), and (6),

$$P(n_2|k) = \frac{(k-1)!}{n_2!(k-1-n_2)!} \frac{p_3 \, p_2^{n_2} \, p_1^{k-1-n_2}}{p_3(1-p_3)^{k-1}} .$$

Then,

$$E[n_2|k] = \sum_{n_2=0}^{k-1} n_2 P(n_2|k) .$$

Since when $n_2 = 0$, no contribution is made to the sum, we sum for $n_2 = 1$:

$$E[n_2|k] = \frac{1}{(1-p_3)^{k-1}} \sum_{n_2=1}^{k-1} \frac{n_2(k-1)!}{n_2!(k-1-n_2)!} p_2^{n_2} \, p_1^{k-1-n_2} .$$

Let $m = k - 1$; and note that $\frac{n_2}{n_2!} = \frac{1}{(n_2-1)!}$.

Then,

$$E[n_2|k] = \frac{1}{(1-p_3)^{k-1}} \sum_{n_2=1}^{m} \frac{m!}{(n_2-1)!(m-n_2)!} p_2^{n_2} \, p_1^{m-n_2} .$$

Let $n_2 = j + 1$, $j = n_2 - 1$;

then,

$$E[n_2|k] = \frac{1}{(1 - p_3)^{k-1}} \sum_{j=0}^{m-1} \frac{m(m - 1)!}{j!(m - j - 1)!} p_2^j \, p_2 \, p_1^{m-j-1} .$$

Let $\ell = m - 1$; then

$$E[n_2|k] = \frac{mp_2}{(1 - p_3)^{k-1}} \sum_{j=0}^{\ell} \frac{\ell!}{j!(\ell - j)!} p_2^j \, p_1^{\ell-j} .$$

The summation is again the binomial expansion

$$E[n_2|k] = \frac{mp_2}{(1 - p_3)^{k-1}} (p_1 + p_2)^{\ell} .$$

But, $p_1 + p_2 = 1 - p_3$, $\ell = m - 1 = k - 2$, $m = k - 1$,

therefore,

$$E[n_2|k] = \frac{(k - 1)p_2}{(1 - p_3)^{k-1}} (1 - p_3)^{k-2}$$

$$= \frac{(k - 1)p_2}{(1 - p_3)} .$$

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