ON "SECOND-ORDER STATE-SPACE FORMULATION OF SYSTEMS"

Krishnaswamy et al[1] propose a second order state space equation for describing a system and claim that this new method offers certain advantages. At first sight, their claims appear to be valid. However, a closer examination reveals that one cannot expect any such advantages with this new formulation of state space equations.

According to Krishnaswamy et al, the first advantage offered by the new formulation of state space equations is that the order of matrices involved is reduced. This is true. But this reduction of the order of matrices involved does not seem to simplify the involved computations, because if this were true the original nth order differential equation which contains only scalar coefficients would be preferred to the state variable equations in order to describe the system.

Secondly, that the characteristic equation and the solution to the system equations can be evaluated easily, is a question of opinion in the light of phase variable canonical form and the Lur'e (Jordan) canonical forms that are available for system description.

Thirdly, that the second order state space formulation provides an insight into the relative stability of the system, appears to be a conjecture and not based on any valid proof. The following two examples contradict the claims made by Krishnaswamy et al.

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[1] P.S.Krishnaswamy, George T.Manohar and V.Seshadri, "Second-Order State-Space Formulation of Systems", IEEE Transactions on Automatic Control, Vol.AC-15, No.1, February 1970, pp.126-128.

Example 1.

Consider a fourth order system described by the following differential equation.

$$\ddddot{c}(t) + a_3 \dddot{c}(t) + a_2 \ddot{c}(t) + a_1 \dot{c}(t) + a_0 c(t) = u(t)$$

where a_1 , a_2 , a_3 and a_0 are constants.

A second order state space representation of this system is given by

$$\ddot{\underline{x}} = \underline{A}_1 \underline{x} + \underline{A}_2 \dot{\underline{x}} + \underline{b}_1 u(t) ,$$

where

$$\underline{A}_1 = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_2 \end{bmatrix} , \quad \underline{A}_2 = \begin{bmatrix} 0 & 0 \\ -a_1 & -a_3 \end{bmatrix} \text{ and } \quad \underline{b}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} .$$

Now, if one lets $a_1 = a_3 = 0$, $\underline{A}_2$ becomes a null matrix. But the characteristic equation of the above system, namely,

$$s^4 + a_2 s^2 + a_0 = 0$$

does not have roots on the imaginary axis of the complex plane, when $a_2 = a_0 = 1$. (The four roots of the characteristic equation are $\pm(1/2) \pm j\,(\sqrt{3}/2)$.) $\underline{A}_2$ being a null matrix is therefore no indication of absence of damping and consequent oscillations, contrary to the conjecture made by Krishnaswamy et al.

Example 2.

Consider the same example treated above, and let $a_0 = 1$, $a_2 = 6$,

and $a_1 = a_3 = 4$.

It is obvious that the system is stable. But

$$\underline{A}_2 = \begin{bmatrix} 0 & 0 \\ -4 & -4 \end{bmatrix}$$

has one of its eigenvalues equal to zero which is not negative. Thus the necessary condition proposed by Krishnaswamy et al is violated.

Krishnaswamy et al say that work in the direction of development of criteria for controllability and observability of the system in terms of $\underline{A}_1$ and $\underline{A}_2$ matrices is progressing. If necessary, a simple method which solves this problem can be proposed as follows.

$$\text{Let } \underline{x} = \underline{z}_1 \; ; \; \dot{\underline{x}} = \dot{\underline{z}}_1 = \underline{z}_2 \quad \text{and} \quad \underline{z} = \begin{bmatrix} \underline{z}_1 \\ \underline{z}_2 \end{bmatrix}$$

Then the so-called second-order state-space equation proposed by Krishnaswamy et al, is contained in the complete description of the system in terms of the conventional state variable equations given by the following.

$$\dot{\underline{z}} = \begin{bmatrix} \dot{\underline{z}}_1 \\ \dot{\underline{z}}_2 \end{bmatrix} = \begin{bmatrix} \underline{0} & \underline{I} \\ \underline{A}_1 & \underline{A}_2 \end{bmatrix} \underline{z} + \begin{bmatrix} \underline{0} \\ \underline{b}_1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} \underline{c}_1^T & \underline{c}_2^T \end{bmatrix} \underline{z}$$

The criteria for controllability and observability in terms of the matrices $\underline{A}_1$, $\underline{A}_2$, $\underline{b}_1$, $\underline{c}_1^T$ and $\underline{c}_2^T$ follow immediately from the above complete description of the system.

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13. ABSTRACT

Krishnaswamy et.al.* propose a second order state space equation for describing a system and claim that this new method offers certain advantages. At first sight, their claims appear to be valid. However, a closer examination reveals that one cannot expect any such advantages with this new formulation of state space equations. Counter examples are given to substantiate this statement.

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