SOME MODELS OF RACIAL DISCRIMINATION IN THE LABOR MARKET

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PREFACE

The research for this Memorandum was done as part of a Rand study on the measurement of racial discrimination. The author is Professor of Economics at Harvard University and consultant to The Rand Corporation.

The following text is slightly altered from that of Marshall lectures delivered by the author on invitation of the Faculty of Economics and Political Science of the University of Cambridge, April 14 and 15, 1970. The supporting analysis is presented as Technical Notes following the main text. This Memorandum is being adapted for inclusion in a forthcoming Rand book, The American Economy in Black and White: Essays on Race Discrimination in Economic Life.
The intention of this report is to demonstrate the advantages and disadvantages of neoclassical analysis as a tool for studying racial discrimination in the economic sphere and to suggest possible areas of fruitful research.

The author concentrates on racial discrimination effected through wage differentials. On the labor supply side, neoclassical methods explain only about 60 percent of black/white income differentials. On the demand side, the tastes of employers offer the simplest explanation of wage differences. Wages for black workers will fall short of their marginal product by the marginal rate of substitution between black workers and profits, the rate being computed at the black/white ratio in the labor force. Tastes of employees may also enter the picture; the costs of hiring white labor may be relatively high where whites are expected to work for black supervisors. Neoclassical theory, then, can offer a coherent and not implausible explanation of the impact of racial discrimination, accounting in a gross way for the known facts.

Of course, there are difficulties. Utility explanations of economic behavior tend to lack specificity, and there is some argument over whether profit maximization does not tend to overshadow utility maximization, even in imperfect markets. Under the usual assumptions of economic theory, competitive pressures should work toward the elimination of racial differences in income in the long run. From the employer's point of view, it is hard to understand how discriminatory behavior could persist in the face of such pressures.

The author describes a simple model by which an employer can purchase black labor at a fixed price, but for which labor he must choose some point on an indifference curve between wages and the proportion of whites in the firm. The implications -- no wage differentials, on the one hand, and segregation, on the other -- are respectively contrary to, and harmonious with, observation. In short, we experience a failure of convexity -- extreme alternatives are preferred to compromises.
If the non-convexities are small on the scale of the entire economy, then something like a competitive equilibrium is still possible. We must look at the long-run adjustment processes. If we start from a position where black workers enter an essentially all-white world, the racist feelings of employers and of employees will lead to a difference in wages by race. The forces of competition and the tendency to profit-maximization operate to mitigate these differences. However, the basic fact of a personnel investment on the part of employers prevents these counteracting tendencies from working with full force. In the end, we remain with wage differences coupled with tendencies to segregation.

This central model may be supported by two additional factors, tentatively advanced: (1) Skin color is a cheap source of information and may therefore be used by an employer in discriminating against what he believes to be inferior workers. (2) The qualities of an individual worker may not be known to the employer beforehand.

The foregoing model is elaborated in Technical Note F, the last of six such notes attached to the main text of this report.
SOME MODELS OF RACIAL DISCRIMINATION IN THE LABOR MARKET

The real subject of this Memorandum is economic theory itself, or more precisely, the use and meaning of neoclassical price theory in application to the allocation of resources and the distribution of income in the real world. More specifically, these are some reflections that have grown out of attempts to analyze the differentials in income between blacks and whites in the United States with the tools of economic theory. The phenomenon of income differentials is, after all, an economic phenomenon, however much it may be linked with other social dimensions. There is no reason to impose upon economic theory the burden of a full explanation, but it should provide insight into the links between the social, cultural, and individual facts on one hand and the economic facts on the other, just as the theory of production is supposed to provide a link between the facts of technology and the uses and rewards of factors.

My discussion will therefore be programmatic and methodological rather than a confident analysis. My intention is to present the deficiencies of neoclassical analysis as brought out by the attempt to use it as a tool for the analysis of racial discrimination in the economic sphere and by so doing to suggest the areas in which further research may be more fruitful.

To avoid misunderstanding, let me make clear my general attitude toward the fruitfulness and value of marginal analysis. On one hand, I believe its clarifying value in social thought is great. Especially when dealing with problems central to economics, the difference in approach between trained economists and others, however able, is enormous. The importance of the search for possible alternatives, the value of consistency in different contexts as a guide to judgment, and above all the appreciation that the workings of institutions may be such that the outcomes are very different from the intentions of the agents are among the lessons of economic theory. So long as scarcity is an issue and social organizations for coping with it are complex, these principles and their logical elaboration and empirical implementation will be important. Though this is not the place for an elaborate defense, I reject, on both
logical and historical grounds, the widespread suspicion that neoclassical economics is simply an apology for the status quo.

On the other hand, everyone knows that neoclassical economics is seriously deficient in two directions: (1) its implications, though often exemplified in the real world, are also often falsified there, mass unemployment and the failures in economic development being the most conspicuous examples; (2) the implications of neoclassical economics are frequently very weak, so that it has nothing to say about important economic phenomena. Thus, a highly disaggregated Walrasian model implies a distribution of income; but it would be difficult indeed to say if the observed facts are or are not compatible with the model.

Let us turn to the case at hand. Today, mean earnings of blacks in the United States are about 65 percent of those of whites. This ratio has varied over time; it is certainly cyclical, being higher in prosperity than in recession, and seems to show a very slow upward trend, though one cannot be sure. The tight labor markets of World War II brought a sharp rise of about ten percentage points; the ratio remained near that level until the slackening of employment in the 1950s, after which there was a decline, until about 1963.\(^1\) The present higher levels may be due to the change in political climate, through fair employment laws and through changes in attitudes by economic agents, employers, unions, and individual employees, or again it may simply be due to a high demand for labor. We really don't know.

There are differences in unemployment rates partly because of the concentration of blacks in occupations with high employment rates, but a good part of the difference remains even after correction for the occupational distribution.\(^2\) Nevertheless, the differential unemployment rates are not a major explanation for black-white income differentials. If the unemployment rates were equalized, the earnings


differentials would be reduced by only a few percentage points. The bulk of the difference is accounted for by differences in wage rates, partly because blacks are concentrated in low-income occupations and partly because they receive lower wages even within given occupations, at least as conventionally classified.\(^3\) In what follows, I will therefore speak of racial discrimination in the labor market as being evidenced only through wage differentials.

What would a disciple of Marshall and Walras have to say by way of economic analysis? The most obvious explanation goes back to Cairnes' noncompeting groups; that is, it concerns the supply. For one reason or another, it can be argued, the marginal productivity of black labor is lower than that of whites on the average. There are indeed some supply factors whose existence is indisputable. The educational level of blacks in the labor force is lower, and we know from many studies that earnings are correlated with educational level. (As an aside, I am not persuaded that differences of earnings with educational level are entirely due to increases in productivity, but that is a different story.) The educational gap is being rapidly reduced; indeed, there is only about a 6-month difference in the median numbers of years of education between the races among those leaving school today.\(^4\) But of course this change has not yet had time to have much effect on the comparative average educational levels of the entire labor force. It is also undoubtedly true that the quality of education received by blacks is inferior, though understanding of this fact is not easy to come by. Age distribution is another supply factor; blacks are on the average younger, and, up to a certain point, age is a positive factor in earnings. Less well-known supply factors also have their role. More black families are headed by women. Black families are somewhat larger, and it is apparently a well-established fact that individuals with many siblings earn less.


\(^4\)Ibid.
Various authors have made corrections to the income differential based on these factors. The analysis is indeed reminiscent of that of sources of growth. The studies tend to show that these factors will, taken together, account for half or more of the observed income differential, but there remains at least 40 percent unexplained. No doubt failure to explain is not the same as proof of non-explanation. There may easily be other supply factors overlooked or not easily quantifiable; motivational differences due to cultural variation and especially the heritage of slavery have often been cited by popular writers and by some social scientists, though the evidence is less than compelling. Thus, for instance, it is frequently held that blacks have, because of cultural and historical conditioning, a stronger tendency to discount the future and, because of this, a lower propensity to make investments in themselves. It may indeed be true that they make less personal commitment with a view toward later reward, and I will return to this point later, but I doubt that this behavior is due to a basic difference in attitude toward the future. If it were, it should also be reflected in lower propensities to save; but in fact repeated studies have shown that at any given income level blacks save, if anything, a higher proportion of their income than whites.

Since it appears that supply considerations can explain only part of the black-white income differential, it is advisable to turn to the demand side, which is in any case what I am primarily interested in. There are some obvious positive reasons for expecting the demand for black labor to differ from that for white labor of the same productivity. For one thing, we have other evidence that on the average whites act as if they dislike association with blacks. Residential segregation is an obvious and well-documented example. There is no way of explaining it other than the desire of whites to avoid blacks. The only possible

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5 For example, O. D. Duncan, "Inheritance of Poverty or Inheritance of Race?" in D. P. Moynihan (ed.), On Understanding Poverty, New York and London, Basic Books, Chapter 4, especially Tables 4-3 and 4-4, pp. 98 and 106.

alternative hypothesis would be segregation by economic status; but comparison between blacks and whites of equal income shows conclusively that blacks are far more segregated. Also, at least in the recent past, discrimination in some labor markets, particularly those where unions controlled entry, has been completely overt.

Another positive reason for arguing that there is a racial discrimination in the demand for labor is that the measured income differentials are greater at higher educational levels. For example, among males aged 35 to 44 in the North of the United States in 1959, the ratio of mean nonwhite to mean white income was 79 percent for those with elementary school education, 70 percent for those with high school education, and only 59 percent for those with college education. Indeed, the mean income of nonwhite college graduates is or was as of 1959 no greater than that of white high school dropouts. (Incidentally, my shift in reference from "blacks" to "nonwhites" has no deep significance. The Census figures I have just been quoting give only the white-nonwhite breakdown, but in fact blacks constitute the overwhelming majority of nonwhites in the United States.) Since the successive stages of schooling select those most in tune with the needs of the dominant culture in all aspects, including the economic, it is hard to give any explanation for these figures based on supply considerations. It is most reasonable to explain them on the hypothesis of a racial discrimination in demand that is more intense for higher economic positions, the jobs into which the more educated ordinarily go.

From now on, I will speak of black and white as being interchangeable in production, at least within given skill levels, so as to emphasize the demand determinants of wage differentials. The relevant theoretical literature is surprisingly small in view of the importance of the subject and the great attention it has received by the public.

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The main study is that of Gary Becker some thirteen years ago; still earlier, Edgeworth had written on some aspects of wage discrimination according to sex. The possible channels by which discriminatory attitudes come to affect wages are well stated by Becker, but what might be termed the general equilibrium aspects are largely ignored; that is, the effects of wage differentials on the stimulation of compensating behavior are slighted, and, as will be seen, these create a crucial dilemma for an appreciation of the value of economic theory.

The most natural starting point for analysis is to look at the proximate determinant of the demand for labor, the employer's decisions. If we assume away productivity differences between black and white employees, the simplest explanation of the existence of wage differences is the taste of the employer. Formally, we might suppose that the employer acts so as to maximize a utility function that depends not only on profits but also on the numbers of white and black employees. (See Technical Note A, p. 29.) Presumably, other variables being held constant, the employer has a negative marginal utility for black labor. It might also be expected that there is a positive marginal utility for white labor, if only in some sense to offset and dilute the black labor. A specific version of this hypothesis would be that the employer's utility depends only on the ratio of black to white workers and is independent of the scale of operations of the firm.

Under these circumstances, the employer will hire white workers up to a point somewhat beyond where their marginal productivity equals their wage, since he is also rewarded through their positive marginal utility. Similarly, he will stop hiring black laborers at a point somewhat before the point that equates their marginal productivity to their wage. Under the assumption that the two kinds of workers are perfect substitutes in production, the marginal productivities of the two kinds of workers are equal. Their common value depends only on the total number of workers of both races hired. It follows then that equilibrium is possible only

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when the wages of white workers are above the marginal product of labor and the wages of black workers below. To be precise, white wages will exceed marginal product by the marginal rate of substitution between white workers and profits, the rate being computed at the white-black ratio in the labor force. A similar statement holds for black wages.

Under this model, it is clear that there is of course a loss to black workers, as compared with the competitive level in the absence of discrimination. On the other hand, white workers are likely gainers relative to the non-discriminatory level. It can be shown that aggregate output is unaffected if all employers discriminate equally; otherwise, there may be some efficiency loss in total output. Whether employers gain or lose in the aggregate is a quantitative question about which a priori theory gives no definite answer in general. However, in the special case where it is assumed that an employer's utility depends only on the ratio of the two kinds of workers, the employer neither gains nor loses, as compared with a non-discriminatory situation.

Once we start applying utility analysis to racially discriminatory behavior, we may extend it to other members of the productive team. In those cases where the entry of workers into jobs is controlled directly by unions, as in the building trades, discriminatory attitudes by fellow workers become decisive. The results are more apt to be total or partial exclusion rather than wage differentials. However, I am more interested in pressures that work through the market.

Consider white workers who supply services complementary to those of another class of workers, for example, white foremen working with a floor force of mixed race. (See Technical Note B, p. 33.) If the foremen dislike working with blacks, they may offer their services at a higher wage to those firms with higher proportions of black workers. That is, given the choice of working for different firms with different proportions of black workers and different wages, they will choose according to some utility function that represents the trade-off between wages and the number of white and black workers. The firm's offers of employment to foremen will then have two dimensions, the wages and the proportion of blacks in the floor force. But this in turn means that
the firm will have a different demand for black laborers than for white, even if they are perfect substitutes in production and even if the employer himself has no discriminatory feelings.

It should be understood that the wage differentiation for foremen according to the proportion of black floor workers may in practice appear in a disguised form. The cost to the employer of increasing the number of blacks may be measured not in statistically observed higher wages for his foremen but in lower morale, lower productivity, or simply lower quality of personnel.

If, parallel to our earlier assumption about the utility functions of employers, we assume that the discriminatory tastes of foremen are determined by the ratio of blacks to whites under them, rather than by the amounts, it can be shown that in equilibrium the black workers lose, the white floor workers gain an equal amount, and neither the foremen nor the employers gain or lose money income.

I have spoken thus far of foremen and floor workers, but obviously the analysis applies to any two complementary forms of labor. A particularly interesting possibility is discrimination by lower level worker against supervisors. That is, the costs of hiring labor may be higher if they have to work for black supervisors. Indeed, it may be expected that the effects of discrimination of this type are greater than the reverse, and this for two reasons. The first is that the resentment against working under a supervisor belonging to a despised group may be more intense than the simple dislike of having them close by. Indeed, sufficiently superior social status can certainly completely compensate for nearness, as in the master-servant relation. The second reason why effects of discrimination by lower echelons against higher may be greater than the reverse is simply that there are so many more of the former. Thus even if the wage compensation needed to work with blacks is the same in two situations, lower working with higher and vice versa, the cost to the employer is much greater in one case than the other.

I find this last observation especially interesting, because it explains why more highly educated blacks are more heavily discriminated
against. They would expect to go into the higher level jobs where the discrimination may be greater.

Parenthetically, let me say that I have omitted still another taste element in the explanation of discrimination, namely, discrimination by customers. If whites dislike associating with blacks in any capacity, they may in particular dislike to deal with them when purchasing goods. There are several interesting questions here, particularly with regard to the exact social nature of the buyer-seller relation in different contexts, but in any case this aspect is irrelevant to the more normal situation in which those who make the goods do not meet buyers face to face.

At a certain level, then, we have a coherent and by no means implausible account of the economic implications of racial discrimination. In the grossest sense, it accounts for the known facts. For example, the fact that discrimination against blacks increases with the level of education implies that the rate of return to the investment in human capital is lower for blacks than for whites, explaining in turn why the proportion of blacks in college is lower than that of whites.

Still, I think we are not too satisfied. To begin with, we can be troubled by the lack of specificity in the hypotheses being advanced. This is, of course, a defect common to all utility explanations of economic behavior. The theory does not give any quantitative clues. A marginal productivity theory of demand for labor, true or false, asserts a highly specific relation between the production function and the demand for labor, each of which is observable under ideal conditions. A utility theory in and of itself asserts much vaguer connections, usually of a qualitative nature and frequently not even that. To take a parallel case, we know that as per capita incomes increase, the proportions in which different commodities are purchased alters. This generalization about behavior is in fact of the greatest importance from the practical point of view. It can only be explained by invoking the nature of tastes, in technical language the non-homotheticity of the indifference surfaces. Have we explained anything? I don't want to get involved in the meaning of explanation as an epistemological
concept, but it is fair to say that the explanation in terms of tastes is not useless. If we add the assumption that the tastes of individuals are similar, at least in a statistical sense, then we may be able to make inferences from the history of demand patterns in one country to that of another. Similarly, in the case of racial discrimination, we may be able to infer from the behavior of employees of one type to those of another on the hypothesis that their tastes are similar.

There is a second sense in which the hypotheses of the theory lack specificity. They invoke a dislike of association with blacks, but as I have already suggested, the dislike may depend upon the nature of the association. Physical proximity is probably significant only because of its implications for status and for feelings of superiority and of fear. The slave owner and his overseer felt no reluctance to work with an all-black labor force. Railroad and airline porters tend to be blacks. Still the matter is not just one of status; detailed studies show wage differences even in narrowly specified low-level occupations, though these differences are much smaller than the average in the economy. No doubt the general concept of association with blacks has to be broken down into several dimensions. But the fact that utility analysis leads to such more detailed questioning is, in my view, an evidence of its fruitfulness.

The excessive generality of utility hypotheses about economic behavior is, then, a drawback, but one that seems intrinsic in the nature of the case. There is a second objection to this and other utility explanations which I will discuss more briefly; namely, that we offer no explanation of racial discrimination but simply refer the problem to an unanalyzed realm. We all remember Molière's intellectual who explained that opium produces sleep because it contains a great deal of the dormitive principle. Yet there is a sense in which all scientific explanation involves the same process of musical chairs; all we ask is that the explanatory principles have some degree of generality and parsimony. But in the context of racial discrimination, one may worry that this advice is too cheap. Explaining an economic phenomenon such as the impact of attitudes, taken as given, on the workings of the economic
system is legitimate enough; but what if those attitudes are themselves the result of economic behavior? Specifically, and in more emotional language, what if racial discrimination and the tastes that underlie it are tools of economic exploitation?

I have mentioned two possible difficulties with accepting utility explanations. There is a third, which I wish to emphasize most strongly of a very different nature and with different implications. The question can be raised whether the economic system has not other forces that counteract the tendency towards wage discrimination. Sherlock Holmes, a man much concerned with the formulation of hypotheses for the explanation of empirical behavior, once asked about the barking of a dog at night. The local police inspector, mystified as usual, noted that the dog had not barked at night. Holmes dryly noted that his silence was precisely the problem.

Have we some dog whose silence should be remarked? Yes; those vast forces of greed and aggressiveness which we are assured and assure our students are the mainsprings of economic activity in a private enterprise economy; not the best but the strongest motives of humanity, as Marshall had said. For some employers, the trade-off between discrimination and profits is less than for others. There need be no assumption of higher morality; if interpersonal comparisons are admitted, it might simply be that some employers are greedier than others. Presumably they will take advantage of the gap between black and white wages by demanding the black labor. (See Technical Note C, p. 36.) In the long run, the less discriminatory will either drive the more discriminatory out of business or, if not, will cause the wage difference to fall. If we suppose that there are some actual or potential employers who do not discriminate at all, then the wage difference should, in the long run, fall to zero. The discriminating employers may possibly continue to operate, but they will employ only white labor.

This kind of argument is not unfamiliar in other fields of application. As soon as utility-maximizing behavior is introduced into the productive side of the economy, the question arises of its relation to profit-maximization and particularly to the role of competition. The
theory of the firm, particularly under imperfect competition, has found a considerable, if fitful, place for tastes. In 1935, Hicks noted that a monopolist might prefer a quiet life to maximum profits. Herbert Simon and his students, especially Oliver Williamson, have suggested that entrepreneurs might seek to maximize a utility function in which other variables entered besides profits: the emoluments of the higher officers and the sheer size of the firm, as well as avoidance of decision making. Marrls has taken up a dynamic version of the size theory; his entrepreneurs have tastes for growth as well as profits.10

But there has also been a countervailing current of opinion that argues in effect that the utility functions of entrepreneurs don't matter. Competition will force firms to maximize profits, since otherwise they won't survive. Even under imperfect competition, profit maximizers will find it profitable to take over firms from utility maximizers. I should note here that from the viewpoint of formal analysis, this case is not as different as might appear from the first; it still presupposes a considerable amount of competition in the capital market.

The prevailing opinion seems to be that the question of utility maximization can be raised only under conditions of imperfect competition. Those who defend the importance of tastes for size and growth usually are first concerned to argue that the firm has potential access to monopoly profits, and it is these that might be dissipated in seeking after non-pecuniary goals.

Upon reflection, I believe the relevant distinctions are wrongly drawn. Even under perfect competition, if I have a taste for size and derive pleasure from it, I might perfectly well accept a rate of return below the competitive level in order to indulge my taste. Indeed, all

the statistical evidence I know of suggests that self-employed businessmen in general are accepting less than a competitive rate of return (or alternatively less than their competitive wage) for such pleasures. A perfectly competitive equilibrium is compatible with utility maximization by entrepreneurs; of course, the price they have to pay for their tastes will depend on the tastes of others in the market, but they are not driven out as sharply as might be supposed.

But I want to argue that the hypothesis of competitive elimination might have more force in the case of racial discrimination. More generally, I would suggest, rather tentatively, that this hypothesis might be more likely to hold when the non-pecuniary variables have negative marginal utilities than when they have positive ones. The reason is simple enough; the employer can always avoid the negative utilities and still achieve a competitive rate of return by simply becoming a pure capitalist, a stockholder.

Before going into more details and qualifications, let me again draw an analogy, this time with the spread of innovations. In explaining a failure to introduce an innovation historians frequently invoke a conservative spirit on the part of the entrepreneurs in question; for example, Landes in comparing English and French attitudes toward innovation at the end of the eighteenth century. 11 Theorists find themselves puzzled. No doubt it is possible for French entrepreneurs to have, on the average, a utility function that has a negative weight for innovation. But if even a few entrepreneurs for some eccentric reason lack this distaste, they will introduce the innovation, and the forces of competition will force the others to follow suit, at the peril of elimination. These competitive tendencies operate through the capital market as well as the product market, of course; new capital will flow to the successful innovators.

No doubt this argument has to be modified in the case where monopoly profits are earned. It will pay the firm to remain in business and indulge its distaste for innovation or for hiring blacks. But the

fundamental point is that the competitive pressures, to the extent that
they are decisive, work toward the elimination of racial differences in
income, under the usual assumptions of economic theory.

Thus, after building up a more or less reasonable mechanism that
gives a rationale for linking economic discrimination with other social
attitudes, I now argue that if the logic of the competitive system is
accepted, discrimination should still be undermined in the long run.
This forces us to rethink the meaning of long-run competition, and this
task will be faced tomorrow. I must also call to your attention that
the negative discussion has so far only concerned discrimination by
employers. I must also ask whether discrimination by other employees
is also eroded over time. This raises some other questions of a more
technical nature.

A model in which white employers and employees were motivated by
a dislike of association with blacks as well as more narrowly economic
motives would give a satisfactory qualitative account of observed racial
discrimination in wages but, at least as far as employers are concerned,
it is hard to understand how discriminatory behavior could persist in
the long run in the face of competitive pressures. Several assumptions
have been made, implicitly or explicitly, and perhaps should be restated
here: constant returns to scale in the long run, a sufficiently wide
spectrum of tastes toward discrimination and in particular a sufficient
number of actual or potential non-discriminating employers, and an ade-
quate freedom of entry. The last condition, let me stress, is consistent
with a certain amount of imperfect competition. If there is enough entry
by non-discriminating entrepreneurs to absorb the entire black labor
force and some more, then wages would be equalized, but the surviving
discriminating firms would now be completely segregated. Obviously the
degree of freedom of entry necessary to eliminate racial wage differentials depends upon the proportion of blacks in the labor force. But,
in the United States, the black workers constitute some fifteen percent
of the labor force; if employer discrimination were the sole cause of
wage differences, it is hard to believe that competitive forces are
inadequate to eliminate racial wage differentials.
What then of employee discrimination? Let me take up a case not touched on explicitly before. (See Technical Note D, p. 39.) Because of its extreme nature, it lends itself to simple analysis. I refer to discrimination by white employees who are perfect substitutes for blacks. The discussion itself is due to Becker, but I want to draw attention to its wider implications.

Suppose for a simple model that there is only one kind of utility function expressing a trade-off between wages and the proportion of white workers in the labor force of the firm. Any employer can purchase black labor at a fixed price, but for white labor he must choose some point on an indifference curve between wages and the white proportion. A little reflection makes it obvious that if the wages required by whites for an all-white labor force are lower than black wages, total segregation for whites is optimum for the firm; while in the contrary case, an all-black labor force is cheapest. We are, of course, still assuming equal productivity for the two races. At a general equilibrium where there is full employment of both types of labor, it must be that some firms are segregated in one direction and some in the other. It would never pay a firm to have a mixed labor force, since they would have to raise the wages of their white workers above the level for the all-white option. But also the firms would have to find the two types of segregation equally profitable; otherwise, they would all switch to one or the other. This requires that the wages paid to whites in the all-white firms equal that paid to blacks in the all-black firms. There would be again no wage differentials.

The relation of this result to the possibility of discrimination by complementary types of labor will be discussed shortly. But the model and the kinds of processes of which it is symbolic deserve some attention. Obviously, we are concerned that we have drawn an implication, no wage differentials, that is contrary to observation. But we have drawn another implication, segregation, that is very much a fact. Indeed, some 70 percent of the small firms in Chicago have no black workers at all, although about 14 percent of the Chicago labor force is black. The evidence is that even in large firms blacks tend to be
separated by department and by occupation. Thus, the pure theory turns up with tantalizing results, partly clearly false, partly yielding unusual insights.

The analysis just used, simple as it is, is not typical of economic theory. We tend to infer that conflicting forces will come to balance somewhere in the middle. Here, on the contrary, it is of the essence that firms prefer extreme alternatives to compromises. In technical language, we have a failure of convexity. The situation is similar to, though not identical with, a famous crux of economic theory, the relation between increasing returns and competitive equilibrium. Here too under competitive conditions the firms will either shut down completely or go to some high level of activity, possibly too high to be compatible with resource limitations.

The recognition of non-convexities and their importance in economic life is hardly new; we all recall the central role that Adam Smith gave to division of labor and its relation to the size of the market. Indeed, Smith's ideas of specialization among individuals, firms, and even nations are exactly analogous in formal structure to the occurrence of racial segregation in production. But it has proved very difficult to incorporate non-convexities in systematic general theories. Marx, for example, talks a good deal about concentration of ownership of capital, based on what we would call increasing returns; but his models of simple and expanded reproduction display perfectly orthodox constancy of returns. In the last 20 years, the increasing formalization of economic theory has made more prominent than ever the role of the convexity assumptions that literary economists have always used freely. However, there is now a growing body of literature, starting with Farrell's paper of 1959.


that is seriously attempting to wrestle with the relaxation of convexity hypotheses. At least this much seems to be possible to assert: If the non-convexities are small on the scale of the entire economy, then something like a competitive equilibrium is still possible. But the structure of that equilibrium may be different from what would obtain under convexity. There will be a tendency toward specialization, in the present context toward segregation. Though price levels may not be so much different than they would be in a comparable convex world, the distribution of individuals among occupations and of output among products may display much more concentration on widely separate positions.

Let me return to the problem at hand. The vision of firms rushing from one kind of segregation to another in response to small wage changes is troublesome, and I will come back to that point. Meanwhile, let us ask if the analysis of discriminatory feelings by perfect substitutes has any lessons for discrimination by complementary types of labor. I think the answer is clearly yes, if we suppose that there are black workers available at both higher and lower levels. For then the employer can exploit any racial wage differentials by hiring labor force that is black at all levels. If the proportions of the different skills in the black labor force are different from those desired, the resulting equilibrium will not necessarily equate wages at each level. But there will be a tendency to equate wages on the average. It is possible, for example, that black foremen, presumably scarce, will be paid more than their white counterparts because they are willing to work with a black floor force, which is cheaper to the employer.

We thus see that the structure of tastes that seems adequate to give a short-period explanation does not seem to resist the operations of competitive pressures in the long run. One might search for other and more stable explanatory structures. But I know of none that have been proposed or that seem at all credible. Instead, I propose that we look more closely at the long-run adjustment processes. In particular, as I have already suggested, when dealing with non-convexities, the adjustment processes may have to be very rapid indeed. You must recall that in these circumstances marginal adjustments are punished, not
rewarded. If the firm is to gain by a change, it has to go all the way. Intuitively, we are not surprised that a firm will hesitate to scrap its entire labor force and replace it by another. The problem is to give an acceptable formalization of this intuition.

In several different contexts, there has been a recognition that adjustment, even when convexity is not an issue, is costly in itself. Edith Penrose and Marris have made costs of growth an intrinsic part of the dynamic theory of the firm.\(^{14}\) By this I mean that if a firm grows in size and capital, the cost to the firm is the accumulation of capital plus an additional term that depends on the rate at which the firm grows. The latter can be explained in several different ways. One is that the organization of the firm has to alter with its size and there is a cost to acquiring new channels of communication and control within the firm. Another is that the firm needs to expand its markets; but a customer, once acquired, will remain one cheaply, so that the cost is that of acquiring the customer and therefore is determined by the rate of growth. Note that in both cases, we are really saying that there is an acquisition of some kind of intangible capital goods, either communication channels within the firm or goodwill among customers, and these capital goods are costly.

The same principle, that capital costs of an unconventional kind play an important role in economic behavior and decisions, has been applied to the study of labor turnover, a problem more closely connected with ours. Operations researchers, in trying to draw up plans for the hiring of personnel, have incorporated in their models a fixed cost of hiring an individual. Sometimes it is also held that there is a cost attached to firing as well. These costs are partly in administration, partly in training. Even in the case of workers who have already been generally trained in the kind of work to be done, there is a need for learning the ways of the particular firm. This approach, it has been argued by some, has important general economic implications; it implies

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that firms should not adjust their labor force very rapidly to cyclical shifts in demand, since they may incur both hiring and firing costs if they do, costs that are avoided if the worker is retained during slack periods. This hypothesis provides some explanation for the well-known fact that the average productivity of labor falls in slack periods. Workers are being held in employment even though they contribute little to output to avoid the costs of rehiring them in the expected future boom. I do not myself know whether this explanation is in fact adequate but merely note that it is seriously considered.

I suggest that a similar consideration explains why the adjustments which would wipe out racial wage differentials do not occur or at least are greatly retarded. (See Technical Note E, p. 41.) We have only to assume that the employer makes an investment, let us call it a personnel investment, every time a worker is hired. He makes this investment with the expectation of making a competitive return on it; if he himself has no racial feelings, the wage rate in full equilibrium will equal the marginal product of labor less the return on the personnel investment. Let us consider the simplest of the above models, that of discrimination by fellow employees who are perfect substitutes. If the firm starts with an all-white labor force, it will not find it profitable to fire that force, in which its personnel capital has already been sunk, and hire an all-black force in which a new investment has to be made simply because black wages are now slightly less than white wages. Of course, if the wage difference is large enough, it does pay to make the shift.

Obviously, in a situation like this, where there are costs to change, history matters a good deal. A full dynamic analysis appears to be very difficult, but some insight can be obtained by study of a very special case. Suppose initially there are no blacks in the labor force at all. Then some enter; at the same time there is an additional entry of whites, and some new equilibrium emerges. Under the kinds of assumptions we have been making, a change, if it occurs at all, must be an extreme change, but there are now three kinds of extremes, or corner maxima. The typical firm may remain segregated white though possibly adding more white workers, it may switch entirely to a segregated black
state, or it may find it best to keep its present white working force while adding black workers. In the last case, of course, it will have to increase the wages of the white workers to compensate for their feelings of dislike; but it may still find it profitable to do so because replacing the existing white workers by blacks means a personnel investment. If we stick closely to the model with all of its artificial conditions, we note that only the all-white firms are absorbing the additional supply of white workers, so there must be some of those in the new equilibrium situation. On the other hand, there must be some firms that are all black or else some integrated firms whose new workers are black in order to absorb the new black workers. It can be concluded in either case, however, that there will always remain a wage difference between black and white workers in this model. Further, there will be some segregated white firms. Whether the remaining firms will be segregated black or integrated will depend on the degree of discriminatory feeling by white workers against mixing with blacks.

I have not worked out the corresponding analysis for the case where there are several types of workers with different degrees of discriminatory feelings against racial mixtures in the complementary types. Nevertheless one easily surmises that similar conditions will prevail.

The generalization that may be hazarded on the basis of the discussion thus far can be stated as follows. If we start from a position where black workers enter an essentially all-white world, the social feelings of racialism by employers and by employees, both of the same and of complementary types, will lead to a difference in wages. The forces of competition and the tendency to profit-maximization operate to mitigate these differences. However, the basic fact of a personnel investment prevents these countering tendencies from working with full force. In the end, we remain with wage differences coupled with tendencies to segregation.

This concludes what may be thought of as the central model. I cannot help but feel that there are still other factors. I have two suggestions to make, both of a very tentative nature. The first suggests that what I have referred to as the discriminatory tastes of the
employer might in fact be better described as a problem in perception. (See Technical Note F, p. 48.) That is, employers discriminate against blacks because they believe them to be inferior workers. Notice that in this view the physical prominence of skin color is highly significant. As an employer, I might have all sorts of views about the relative productivities of different kinds of workers. But to determine what kind of a worker he is may be a costly operation in information gathering; even if I hold my beliefs strongly, it may not, in many circumstances, be worthwhile in my calculations to screen employees according to them. But skin color is a cheap source of information and may therefore be used. In the United States today, I believe it fair to say that school diplomas are being widely used by employers for exactly that reason; it is believed that schooling has something to do with productivity, and asking for a diploma is an inexpensive operation.

The structure of this argument and the range of its applicability need to be considered with some care. It only applies if the employer incurs some personnel investment cost. Presumably after a worker is hired, his performance is or can easily be made to be a matter of known fact. If there were no personnel investment, the employer would hire everyone who applied and simply fire those unqualified. But presumably any testing operation, even a trial period, is some form of personnel investment.

The second assumption that must be made is that the qualities of the individual are not known to the employer beforehand. The most interesting case of that kind is that in which the worker must make some investment in himself but one which the employer can never be sure of. I am thinking here not of the conventional types of education or experience, which are easily observable, but more subtle types the employer cannot observe directly: the habits of action and thought that favor good performance in skilled jobs, steadiness, punctuality, responsiveness, and initiative. A worker who has made the requisite investment will be said to be qualified.

The inefficiency that arises here because employers do not know the qualifications of workers as well as the workers do is the same in
principle as that caused by "adverse selection" in insurance. The insured may represent different degrees of riskiness, and each may have some perception of his own degree, but in many cases the insurance companies have much poorer ability to differentiate. If the insurance companies set rates corresponding to average riskiness, the less risky will eliminate or curtail their purchases of insurance, so that the actual experience of the company will be less favorable than the mean in the population. The rates will have to be raised further, thereby eliminating still more of the favorable risks; either the given type of insurance will eventually be eliminated altogether, or an equilibrium will be reached that is inefficient relative to one in which different premiums are charged to those of different riskiness.

We have two primary elements in this model: The employer's investment of personnel capital will be wasted if the employee turns out not to have made his investment; and the employer cannot know beforehand whether or not the employee is qualified. But the employer does know the race of the individual, and he holds some subjective beliefs about the respective probabilities that white and black workers are qualified. It is of course immediately obvious that if the subjective probability in the mind of an employer that a white worker is qualified is higher than that a black worker is qualified, there will have to be a wage difference if the employer is to hire any blacks at all.

The effects of this model are similar to those based on tastes, but the causes are different. We would still want to know why the subjective probabilities are different. The simplest explanation is prejudice, in the literal sense of that term; that is, a judgment about abilities made in advance of the evidence and not altered by it. Of course, the persistence of prejudice really should not be left unexplained. One possible explanation is to be found in theories of psychological equilibrium, such as Festinger's theory of cognitive dissonance.  

15 If an individual acts in a discriminatory fashion, he would, according to this theory, tend to have beliefs that justify his
actions. Indeed, precisely the fact that discriminatory behavior is in conflict with an important segment of our ethical beliefs will, according to this theory, intensify the willingness to entertain cognitive beliefs that will supply a socially acceptable justification for this conduct.

There is also a more narrowly economic model of this type. Suppose that employers do not misperceive, that they know correctly the proportions of black and white workers who are qualified. Further suppose that the acquisition of human capital in the form of qualifications by workers is costly and that they face an imperfect capital market in any effort to finance this acquisition. Then the actual proportion of whites who are qualified is a function of white wages, and similarly with blacks. I assume here as always that blacks and whites are essentially identical, so these two functional relations are the same.

It is clearly true that there is a non-discriminatory set of wages that will be an equilibrium. But it is also possible that this equilibrium may not be stable. Suppose that somehow to begin with the proportion of qualified whites is slightly higher than that of blacks. Then white wages will be higher. In response to this differential there will be an incentive for whites to increase their qualifications relative to blacks, thereby accentuating the initial discrepancy.

This verbal argument is not conclusive, and the formal discussion is more complex. However, the stability of the non-discriminatory equilibrium depends on quantitative values of the parameters; that is, on the supply functions for qualified labor and on the personnel investments needed by the firms.

Since personnel investments are greater at higher levels, this model of personnel investment and uncertainty about qualifications also helps to explain the increasing discrimination against blacks in higher level jobs. Indeed, the motive for developing the observed model was to explain the observation that much discrimination occurred in the form of a disproportionate representation of blacks in lower wage
occupations, analogous in many ways to the dual economies characteristic of underdeveloped countries. The analogy has been suggested by some who have made detailed studies of local labor markets, Baron and Hymer for Chicago and Doeringer and his students and Piore with reference to Boston. Without going into detailed discussion of the somewhat variant viewpoints, there is a common view that blacks are largely, though not exclusively, confined to marginal jobs, marked by low wages, low promotion possibilities, and instability of employment. The instability, incidentally, is in large part voluntary; it is interpreted as a rational response to limited opportunity, which both increases the value and decreases the cost of search.

In particular, both research groups feel that coexistence of segregation and discrimination is in some sense an equilibrium condition, that no employer or employee will find it individually profitable to depart from the existing situation. Within conventional deterministic models, it is hard to formalize this possibility, as indeed is true in dual economy models for underdeveloped countries; why does not competition from the victims of discrimination reduce wages in the preferred occupations and permit them to enter?

The foregoing model, as elaborated in Technical Note F, is designed to suggest a mechanism in terms of which partial occupational segregation is nevertheless an equilibrium condition. In view of its desperately oversimplified character, it is perhaps best thought of as a metaphor.

Finally, a comment on the question of group interests. It is certainly a common view that in some sense racial discrimination is a device by which the whites in the aggregate gain at the expense of the blacks. Hence, the whole problem is to be interpreted as an exploitative relation. There is a stable relation here; the values inherent in discrimination uphold a structure that is profitable to those holding those values.

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16 Baron and Hymer, "The Negroes in the Chicago Labor Market."
On purely methodological grounds, I do not think such a view can be denied, provided it works, though it is contrary to the tradition of economics. Economic explanations for discrimination or other phenomena tend to run in individualistic terms, and the models presented earlier are no exception. Economists ask what motivates an employer or an individual worker. They tend not to accept as an explanation a statement that employers as a class would gain by discrimination, for they ask what would prevent an individual employer from refusing to discriminate if he prefers and thereby profit. Economists do indeed recognize group interests if they appear in legal form, as in tariffs, licensing, or legally enforced segregation. But the distinction between the legal structure and other social pressures is hardly a sharp dichotomy. If perceived group interests can lead to legislation, they can also lead to other social pressures.

I think there is something in views of this kind, but their mechanism needs careful exploration. We must really ask who benefits, and how are the exploitative agreements carried out? In particular, how are the competitive pressures that would undermine them held in check? The exploitation of blacks can work only if the tendency of individual employers to buy the cheapest labor is somehow suppressed. Recall the great difficulty that producers of rubber and of coffee have had in their efforts to create a mutually beneficial monopoly.

Obviously, from the preceding analysis, the whites certainly gain by discrimination. I must add, though, that it seems very difficult to construct a model in which employers gain in any obvious way; the gains to the whites appear to accrue to white workers primarily. This fact, if it be one, already creates difficulties for the group interest hypothesis; after all, the employers are the most direct possible agents of exploitation, and it would be better for the theory if they were beneficiaries.

In any case, we are not to imagine conspiracies in which 170,000,000 white Americans put their heads together. The process of communication by which the white race agrees on means to further its collective interests must operate unconsciously through its value-forming and
allocating social institutions. The argument would have to be that the discriminatory tastes that I have taken as given up to this point are themselves the mechanism by which discrimination profitable to the whites is carried out. These discriminatory values must themselves be internalized and felt to be genuine by those holding them. It was an obligation of conscience for Huckleberry Finn to turn over the runaway slave, Jim, to the authorities for return to his master, and he resolved to do so for inner peace. Finally, Huck could not return his friend to such misery, but he well knew that his failure was only another proof of his fundamental depravity and that anyone with a stake in society would return a runaway slave rather than suffer the disutility of a failure to carry out his social duty. But the process by which these discriminatory values are formed and transmitted is certainly complex and lengthy in time, and we may easily suppose that the exploitation that results is far from optimum for the exploiters.

Notice that the question at issue is not whether racist utility functions are socially conditioned. We accept that the tastes for material goods are affected by the surrounding culture; and how much more so tastes about status relations. The crucial question, to my mind still an open one, is whether the acceptance and preservation of racial attitudes are in some way related to their profitability to the group. One might hypothesize some sort of Darwinian process for utility functions in which those economically profitable for the group have a greater chance of survival. But all this is at the moment merely speculative, at best a suggestion for research.

There is, however, one further point which should be made here. I do not see how the process of racial discrimination can begin in the economic sphere or out of purely economic motives. It always pays any group with enough power to discriminate against some other. But redheads or blue-eyed individuals do not seem to suffer much. The fact that color is seized on as a basis for discrimination must mean that there is an extra-economic origin, although it is not precluded that its economic profitability reinforces the discrimination once started. 18

18. Hodge and Hodge advanced the hypothesis that, other things being equal, wages in an occupation were lower the greater the number of blacks
I have taken a topic on which many of us feel the greatest moral outrage and have analyzed it most dispassionately. Neither the moral indignation nor the cool analysis is misplaced; their juxtaposition is one of those paradoxes inherent in the nature of human society of which only the naive are ignorant. Our mastery of ourselves as social beings needs all the reinforcement it can get from study of ourselves in all contexts. Indeed, in the absence of analysis from a self-imposed and sometimes painful distance, our moral feelings can lead us to actions whose effects are the opposite of those intended. This is not intended to imply that social action must wait on adequate analysis. Inaction may be, and in this case surely is, as dangerous as any likely alternative. Indeed, social action may be indispensable to increasing our knowledge when the consequences are subjected to adequate study. But a firm commitment to ends must not preclude a tentative, questioning attitude to particular means of achieving them.

and suggested this might make social barriers to entry of blacks a rational procedure and a possible cause of prejudice. (Their interpretation of the empirical evidence is far from conclusive, but that is another question.) In reply, Taeuber, Taeuber, and Cain argued that it would pay the members of any occupation to bar any group of people; the selection of blacks as the target could be explained only on non-economic grounds. (See R. W. and P. Hodge, "Occupational Assimilation as a Competitive Process," American Journal of Sociology 71(1965), pp. 249-285; A. F. and K. E. Taeuber and G. G. Cain, "Occupational Assimilation as a Competitive Process: A Reanalysis," ibid. 72(1966), pp. 273-285. There seems to be considerable confusion in this controversy. An individual has many interests, and for each interest he may find a different set of other individuals who share them. Why certain kinds of groups perceive themselves as having common interests and not others is a question on which economics does not seem likely to throw much light. But given group identification, it is not so unreasonable that the members of the group will work together to promote group interests, even though it would pay any individual to depart from them.
For a starkly dramatic model to bring out some broad tendencies, assume all firms have identical utility functions and identical production functions. There is only one commodity, other than labor, and white and black labor are perfect substitutes in production. The supplies of the two kinds of labor are both perfectly inelastic. Finally, we start with a short-run analysis in which the supply of capital to each firm is given, so that output is a function of the labor employed. Let \( W \) and \( N \) be the amounts of white and black labor hired by a representative firm; then output is given by \( f(W + N) \), where the function \( f \) is strictly concave and increasing. The profits of the firm are given by

\[
\pi = f(W + N) - w_W W - w_N N,
\]

where \( w_W \) and \( w_N \) are the wages of white and black workers, respectively. The aim of the firm is to maximize,

\[
U(\pi, W, N), \text{ where } U_W \geq 0, U_N < 0.
\]

One of the two inequalities must be strict if there is in fact discrimination.

This model is a straightforward generalization of that in Becker.\(^{19}\)

Since all firms are identical and all utility and production functions have the appropriate convexity properties, the choice of \( W \) and \( N \) will be the same for all firms at any given set of wage rates. Since total supplies of \( W \) and \( N \) are given, it follows that at equilibrium each firm will demand \( W \) and \( N \) equal to the respective total supplies divided by the number of firms; assume these values for \( W \) and \( N \) in

\(^{19}\)Becker, The Economics of Discrimination.
what follows. The maximization of (2) with respect to \( W \) and \( N \),

with due account being taken of (1), implies

\[
U_n (f' - W) + U_\pi = 0, \quad U_\pi (f' - N) + U_N = 0,
\]

or

\[
(3) \quad f' = W + d_w = N + d_N,
\]

where \( d_w = -U_n/U_\pi \), \( d_N = -U_N/U_\pi \); these are Becker's discrimination coefficients against white and Negro labor, respectively. From the inequalities in (2), \( d_w < 0 \) (Becker uses the term, nepotism coefficient, for \( -d_w \), \( d_N > 0 \), so that,

\[
(4) \quad W > f' > N,
\]

so that, as is obvious, this model does imply higher wages for whites than for Negroes of identical productivity.

It should be remarked that in this model production is thoroughly efficient. Efficient production requires only that each firm get an equal amount of labor. It may be noticed that this conclusion is not completely robust under relaxation of the assumption of identical production functions. From (3) it is clear that the general condition for efficiency is that \( d_w \) and \( d_N \) be the same for all firms (more strictly, for all firms that in fact employ both kinds of labor).

This condition need only hold at equilibrium; however, a sufficient condition is that \( d_w \) and \( d_N \) be constants, independent of \( \pi, W, \) and \( N \), and, of course, be the same constants for all. This condition is equivalent to stating that the utility function can be linear in \( \pi, W, \) and \( N \), which is the particular form of the model set forth by Becker.

If allocation is efficient, then the presence of discrimination has a purely redistributive effect. Since \( f' \) is the same as it would have been in the absence of discrimination, white workers gain
only if there is positive preference for them, not merely a distaste on the employer's part for Negroes. The effect on employers' profits can be seen by substituting (3) into (1):

\[ \pi = f(W+N) - f'(W+N) (W+N) + d_w W + d_N N. \]

If \( \pi_o \) is the volume of profits in the absence of discrimination,

\[ \pi_o = f - f'(W+N) (W+N), \]

so that,

\[ \pi - \pi_o = d_w W + d_N N. \]

The right-hand side has the following simple interpretation: It is the amount of profits needed to compensate the employer for a unit increase in his labor force that preserves its racial proportions.

One possible hypothesis is that the employers' satisfactions are governed by the proportion of Negro workers, i.e., an increase in labor force scale that preserves racial proportions leaves him indifferent. In that case, (5) tells us that employers do not profit by discrimination, the net effect of which is a transfer from black to white workers. On the other hand, if the primary motivation of the employer is a distaste for black workers, and this is little offset by increased numbers of white workers, then \( d_w \) is small in absolute value, \( d_N \) large, and the effect of discrimination is primarily a pecuniary transfer from black workers to employers. In any case, however, it is elementary that the white community (employers plus white workers) gains in pecuniary terms precisely the gap between marginal product and wage for black workers. This simple but important point has been brought out by Thurow. ²⁰

It is important to emphasize the incidence of racial discrimination, in particular, the possibility that employers may actually gain in

pecuniary terms by their discrimination. This point seems not always
to be grasped; of course, any individual employer would gain by a
reduction in discrimination, but it is at least plausible that employers
collectively gain by discrimination.  

of Political Economy 71(1963), pp. 481-486, for a related argument.
Becker has shown that if there is a third factor of production (e.g., another type of labor such as management) that discriminates against black workers and is complementary to or imperfectly substitutable for them, it will follow that Negro wages will fall below those of perfectly substitutable whites. Similarly, Welch has suggested that the possibility of observed discrimination may arise because white and black workers are not perfect substitutes for each other, but, because of different educational levels, are also complementary. Complementarity creates a motive for the employer to integrate, which offsets the other tendencies to segregation and therefore can lead to wage differentials.

The following model elaborates Becker's and seeks to catch the spirit of Welch's ideas. Assume now that there are two types of labor. White and black workers are perfect substitutes in type 1 labor (which might be thought of as unskilled). However, there is a complementary type of labor, type 2 (perhaps foremen) who prefer to work with whites rather than blacks. We assume that,

\[ w_2 = w_2(L_{1W}/L_1) \]

where \( L_{1W}, L_{1N}, L_1, \) and \( L_2 \) are the respective amounts of type 1 white workers, type 1 black workers, all type 1 workers, and type 2 workers hired by the firm. Profits are given by

\[ \pi = f(L_1, L_2) - w_{1W} L_{1W} - w_{1N} L_{1N} - w_2 L_2, \]

where \( w_{1W} \) and \( w_{1N} \) are the wages of type 1 white and type 1 black

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workers, respectively, and \( f(L_{1w} + L_{1N}, L_2) \) is output. Maximization of (2) with respect to the three types of labor yields,

\[
\begin{align*}
(3a) \quad f_1 &= w_{1w} + (\partial w_2 / \partial L_{1w}) L_2, \\
(3b) \quad f_1 &= w_{1N} + (\partial w_2 / \partial L_{1N}) L_2, \\
(3c) \quad f_2 &= w_2.
\end{align*}
\]

Since \( \partial w_2 / \partial L_{1w} < 0, \partial w_2 / \partial L_{1N} > 0 \), it immediately follows from (3a-b) that \( w_{1w} > w_{1N} \). Since \( w_2 \) is homogeneous of degree 0 in \( L_{1w} \) and \( L_{1N} \),

\[
(\partial w_2 / \partial L_{1w}) L_{1w} + (\partial w_2 / \partial L_{1N}) L_{1N} = 0.
\]

If we multiply (3a) by \( L_{1w} \) and (3b) by \( L_{1N} \), we find,

\[
l_1 (L_{1w} + L_{1N}) = w_{1w} L_{1w} + w_{1N} L_{1N}.
\]

It then follows from (3c) and (2) that the profits of the firm are exactly what they would be with no discrimination, if the firm had the same quantities of the two types of labor in the two situations.

If, for given \( w_{1w} \), \( w_{2N} \), and schedule \( w_2 (L_{1W} / L_1) \), each firm had a unique optimum, then, under the assumptions made, all firms would have the same amounts of \( L_{1w}, L_{1N}, \) and \( L_2 \), which would be the same as in the absence of discrimination. Hence, allocation would be efficient; profits would be the same in the two situations; from (3c), the wages of type 2 labor would be the same, and the net effect would be a transfer of income from Negro to white type 1 workers, even though any discriminatory feelings the latter might have are irrelevant to the final equilibrium.
However, it is possible that the shape of the function $w_2(L_{1W}/L_1)$ is such that there can be multiple optima for the firm. The equal allocation might not even satisfy the second-order conditions for an optimum. It is therefore possible that at equilibrium there will be a number of different niches for firms. Each niche is characterized by a different value of $L_{1W}/L_1$ and therefore a different value of $w_2$. This possibility requires further investigation.

Some insight can also be obtained into the determinants of the magnitude of racial wage differences in type 1 labor. If (3b) is subtracted from (3a), we deduce,

$$w_{1W} - w_{1N} = \left(\frac{\partial w_2}{\partial L_{1N}}\right) - \left(\frac{\partial w_2}{\partial L_{1W}}\right)L_2.$$  

On the other hand, by setting $L_1 = L_{1W} + L_{1N}$ in (1) and then finding the partial derivatives with respect to $L_{1W}$ and $L_{1N}$, it is easy to see that,

$$\frac{\partial w_2}{\partial L_{1N}} - \frac{\partial w_2}{\partial L_{1W}} = -\frac{w_2}{L_1},$$  

where prime denotes differentiation with respect to $L_{1W}/L_1$. Thus,

$$w_{1W} - w_{1N} = -\frac{w_2}{L_1}L_2/L_1.$$  

Since $f_1$, the marginal productivity of type 1 labor, is the wage in the absence of racial discrimination, the relative racial wage differential is,

$$\frac{w_{1W} - w_{1N}}{f_1} = -\frac{w_2}{w_2} \frac{w_2L_2}{f_1L_1} = -\frac{w_2}{s_2} \frac{s_2}{s_1},$$

where $s_1$ and $s_2$ are total payments to type 1 labor and to type 2 labor, respectively. Thus, the more important type 2 labor is as an input, the greater the discrimination in the payment to type 1 labor.
The analysis of Technical Note A is strictly short run, but so long as the assumptions of the model are literally adhered to, the extension to long-run equilibrium offers no difficulty. Assume that firms have access to capital on perfect markets. Some long-run equilibrium rate of interest prevails at which firms can borrow freely. Then for given \( W \) and \( N \), assume that the firms borrow optimally. Hence, we need only reinterpret the production function \( f(W + N) \) as representing output after optimal acquisition of capital; the rate of interest then enters the production function as a parameter, but this point does not affect any conclusions previously arrived at. If the production function displays constant returns to capital and labor, then the derived function, \( f \), now displays constant returns to labor. Then \( f' \) is a constant, independent of \( W \) and \( N \), though in general dependent on the rate of interest. All previous conclusions hold.

However, if the condition of the model that states that utility functions are identical is relaxed, the model may have some implications that are not acceptable factually. Specifically, except under improbable conditions, we would expect that the less discriminatory firms should drive out the more discriminatory, so that discrimination should have been eliminated or reduced over time. Let us spell this argument out a bit. Assume that all firms have identical production functions and operate under constant returns to scale in the long run. Then, in (3) of Note A, \( f' \) is a given constant. However, it is no longer necessary that each firm employ both types of labor; corner maxima are possible, so that (3) must be replaced by:

\[
\begin{align*}
(1a) \quad f' &< w_w + d_w \text{ with equality if } W = 0, \\
(1b) \quad f' &< w_N + d_N \text{ with equality if } N > 0.
\end{align*}
\]
Relations (1a) and (1b) hold for each firm; since utility functions vary over firms, \( W, N \), and the discrimination coefficients \( d_W \) and \( d_N \) vary from firm to firm. Since equilibrium implies full employment of both types of labor, the equality must hold in (1a) for at least one firm and similarly for (1b). Hence, all whites are employed in those firms for which \( d_W \) is the algebraic minimum, and similarly all blacks in those firms for which \( d_N \) is a minimum.

First, suppose there are some firms that do not discriminate against blacks, i.e., \( d_N = 0 \). Then \( d_N = 0 \) for all firms for which \( N > 0 \). The only way, then, that there can be any black-white wage differential is for \( d_W < 0 \), i.e., nepotism. But it is reasonable to postulate that any preference a firm might have for the hiring of whites per se arises as an offset to the presence of disliked blacks. That is, for a firm that has no black employees, \( d_W = 0 \). On the other hand, for a firm that does not discriminate against blacks, there will also be no reason to pay anything extra for white employees. That is, we assume,

\[
(2) \quad \text{if either } N = 0 \text{ or } d_N = 0, \text{ then } d_W = 0.
\]

Since it has been shown that either \( d_N = 0 \) or \( N = 0 \) for all firms, \( d_W = 0 \) for all firms. Therefore \( w_W = f' = w_N \), and there is no observed discrimination (there may, however, be some segregated white firms).

(3) If (2) holds and there are some firms that do not discriminate against blacks, then there is no market discrimination against blacks in the long run.

This conclusion suggests some limits of the employer discrimination model. It predicts the absence of the phenomenon it was designed to explain.

It may be worthwhile to generalize the analysis of the long-run case a bit before drawing even tentative conclusions. Suppose then
we drop the assumption that there is any firm that fails to discriminate. At equilibrium the minimum value of $d_N$ among all firms is now positive. Let $N$ be the set of firms that hire some blacks; then $d_N$ is at its minimum value for all firms in $N$. Let $SW$ be the set of firms that hire no blacks (segregated white). From (2), $d_w = 0$ for such firms. If $d_w < 0$ for any firm, then, it must be for a firm in $N$. Then it would follow that no white workers are in $SW$ firms, i.e., we would have the remarkable conclusion that there are no segregated white firms, though now market discrimination would exist.

If we insist that both market discrimination and the existence of segregated white firms are empirical facts that must be explained by any model, we are forced then to agree that $d_N > 0$ and $d_w = 0$ for all firms. This implies that a firm that discriminates against blacks nevertheless derives no satisfaction from "diluting" the black labor force with white employees. The explanation of segregation in this model is, however, a little weak; the allocation of the white labor force between segregated and integrated firms is in neutral equilibrium; it would be consistent with all equilibrium conditions for all whites to work for integrated firms.
It is straightforward and intellectually appealing hypothesis that discrimination against blacks arises from the dislike of white employees for working along side of them. This hypothesis may be considered either as an alternative or as a supplement to that of employer discrimination. But, as Becker has shown, it is difficult to set forth a model in which employee discrimination can induce market discrimination through ordinary economic channels, though it is easy to explain segregation. This argument will now be reviewed; Technical Note E will show that it needs modification if we recognize that there are costs associated with white workers by blacks in response to wage differentials.

To begin with, assume, as in Becker, that white workers have an indifference map between wages, $w_w$, and the proportion, $W/L$, of white workers in the firm. For any fixed level of satisfaction, $w_w$ is a decreasing function of $W/L$. The cost to any given firm of hiring $W$ white and $N$ black workers is, then,

$$C(W, N) = w_w(W/L) W + w_N N,$$

where $L = W + N$.

which is homogeneous of degree one. For fixed $L$, the cost of an all-white labor force is $C(L, 0) = w_w(1)L$ and that of an all-black labor force is $C(0, L) = w_N L$. Since $w_w(1) < w_w(W/L)$ for $W < L$, it is obvious that an integrated labor force is more expensive than the cheaper of the two possible segregated labor forces.

Suppose as before that whites and blacks are perfect substitutes in production, but now assume that firms do not have discriminatory tastes. Then clearly any firm will maximize profits by complete segregation. Those firms that have only white employees will pay a wage rate, $w_w(1)$, while those with only black employees will pay $w_N$. Then it must be that $w_w(1) = w_N$, for otherwise it would pay a

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24 Becker, The Economics of Discrimination, Chapter 4; Welch, "Labor Market Discrimination."
firm segregated in one way to switch to the opposite. Hence, as far as the argument has gone, employee discrimination produces segregation but not discrimination in observed wage rates.

Welch has proposed a somewhat different mechanism, which, however, has the same implications. Suppose that when white and black workers are in the same plant, there is sufficient dissatisfaction and loss of morale that production is adversely affected. Together with the assumption of equal ability of white and Negro workers, it is implied that, for given $L$, output is the same when $W = 0$ as when $N = 0$ but is less if both $W$ and $N$ are positive. Then clearly if $w_W(1) \neq w_N$, each firm will segregate completely in the cheaper type of labor, a condition not compatible with equilibrium. Then $w_W(1) = w_N$; each firm will find it profitable to segregate, though it will be indifferent in which type of labor to specialize. Hence again equilibrium implies segregation but equal wages.
The argument of Technical Note D hinges strongly on a complete flexibility of the firm with regard to its labor force; it must be prepared to fire its entire labor force and replace it by one of the opposite color if this act will cheapen its costs. Suppose we assume instead that there is a capital cost associated with the addition of a worker to the labor force. (The capital costs may be hiring costs, training, or more subtle kinds of organizational adjustment.) Then replacement of white by black workers involves a sacrifice of this capital and may therefore be avoided.

A full analysis of this possibility in a dynamic context where both production functions and the supplies of the two kinds of labor are changing is rather complex. To indicate the possible implications for the analysis of racial discrimination, I consider here only a very simple situation in which initially there are no blacks in the labor force. Then some enter, and, at the same time, there is an additional entry of whites, and a new equilibrium emerges. As before, firms are assumed to have identical production functions; it is also assumed that no new firms enter in response to the increased labor force.

In accordance with the previous remarks, we now assume that a return, \( r \), must be earned on each additional worker hired. Thus if a firm now hires \( N \) black workers, it will have to incur a flow cost of \( rN \). There is no corresponding gain by releasing workers.

Finally, with regard to the function, \( C(W, N) \), which gives wages costs as a function of \( W \) and \( N \), we make a stronger assumption than hitherto; we assume that

\[
C(W, N) \text{ is a concave function of } W \text{ for fixed } L = W + N. \tag{1}
\]

Assumption (1) has the following interpretation: if we add any linear function of \( W \) and \( N \), say \( aW + bN \), to \( C(W, N) \), then as \( W \) and \( N \) change, \( L \) remaining constant, the total, \( C(W, N) + aW + bN \),
is either monotone increasing or monotone decreasing or rises to a maximum and then decreases. This is stronger than the previously observed property that minimum cost is always found at one of the two segregated extremes.

Before the introduction of black labor into the market, each firm has an equal number, $L_0$, of white workers. Consider a firm that, after the change, has decided to have a labor force of $W$ white and $N$ Negro workers. If $W \geq L_0$, then the firm is adding $N + (W - L_0)$ and thereby incurring a training cost of $rN + (W - L_0)$. If, however, $W < L_0$, then the firm is adding $N$ workers for a training cost of $rN$ (there is no rebate for the $L_0 - W$ white workers released). Therefore the total costs are

$$
C(W, N) + rL - rL_0 \quad \text{if} \quad W \geq L_0,
$$

$$
C(W, N) + rN \quad \text{if} \quad W < L_0,
$$

where $L = W + N$. Now for any fixed $L$ the firm will certainly seek to minimize its costs. If in fact $W \geq L_0$, then (2) tells us that costs will be minimized at one of the extreme values for $W$, i.e., either $W = L_0$ (and therefore $N = L - L_0$) or $W = L$ (and $N = 0$). If $W < L_0$, then costs are minimized for either $W = L_0$ or $W = 0$ ($N = L$). Thus any firm will be in one of the three situations:

(SN) $W = 0, \quad N = L$;

(1) $W = L_0, \quad N = L - L_0$;

(SW) $W = L, \quad N = 0$.

Let $v(W/L)$ be the difference between white and black wages if the proportion of whites in the labor force is $W/L$, i.e., $v(W/L) = w_w(W/L) - w_N$. Then the costs for each of the above situations can be written, from (1) of Technical Note D and (2) of this Note,
The profits for a given total labor force in each situation are then given by

\[
(4) \quad \pi_{SN}(L) = f(L) - (w_N + r) L - f'(L) \frac{v(L_o/L)}{L} - r L_o - \frac{w_l}{L_o} L + rL_o.
\]

If a firm is in situation SN, it will choose \( L \) so as to maximize \( \pi_{SN}(L) \); call this value \( L_{SN} \). Similarly, let \( L_I \) and \( L_{SW} \) be the values of \( L \) that maximize \( \pi_I(L) \) and \( \pi_{SW}(L) \), respectively. Note that

\[
\begin{align*}
\pi'_{SN}(L) &= f'(L) - (w_N + r) , \\
\pi'_{I}(L) &= f'(L) - (w_N + r) - L_o \frac{v(L_o/L)}{L} , \\
\pi'_{SW}(L) &= f'(L) - [w_l + r] .
\end{align*}
\]

The magnitudes \( L_{SN} \), \( L_I \), and \( L_{SW} \) are obtained by setting these three derivatives respectively equal to zero. Hence, by subtraction,

\[
\begin{align*}
&f'(L_I) - f'(L_{SN}) = L_o \frac{v(L_o/L)}{L} , \\
\Rightarrow & L = L_I \\
&f'(L_{SW}) - f'(L_{SN}) = v(1).
\end{align*}
\]

Since \( v(L_o/L) \) differs from \( w_l(L_o/L) \) only by \( w_N \), a constant from the viewpoint of the firm, and \( w_l(L_o/L) \) is a decreasing function of \( L_o/L \) and therefore an increasing function of \( L \), it follows that,
\[3v(L_0/L) > 0,\]

and therefore \(f'(L_I) - f'(L_{SN}) > 0.\) Since \(f'\) is decreasing this means that

\[L_I < L_{SN}.\]

Similarly,

\[L_{SW} < L_{SN} \text{ if } v(1) > 0.\]

Under the assumptions made here, only a firm in the SW situation is hiring more whites than before while only those in the SN or I situations are hiring blacks. But the general equilibrium of the labor market requires that more whites be hired than before and that blacks be hired. Hence, some firms must be in situation SW while others are in situation SN or I. This requires that the firms in the SW situation be as profitable as the more profitable of those in the SN or I situations. There are then two possibilities: (SW & SN); SW and SN firms are equally profitable while I firms are no more profitable than SN firms; (SW & I) SW and I firms are equally profitable while SN firms are not more profitable than I firms.

(SW & SN): It must certainly be true that if \(L > L_{SN}\), a firm following policy I cannot have lower costs of operation than an SN firm. From (3), a comparison of the costs of SN and I firms shows that this condition can be written,

\[v(L_0/L_{SN}) \geq r.\]

Since \(\tau_{SN}(L_{SW}) = \tau_{SN}(L_{SN})\), we have, from (4),

\[f(L_{SN}) - (\nu_N + r) L_{SN} = \tau_{SN}(L_{SW}).\]
By definition,
\[ f(L_{SN}) - [w_w(1) + r]L_{SN} = \pi_{SW}(L_{SW}) - rL_o. \]

Therefore subtraction of the last equation from the previous yields
\[ (8) \quad v(L) L_{SN} = \pi_{SW}(L_{SW}) - \pi_{SW}(L_{SN}) + rL_o. \]

Since \( L_{SW} \) was the value of \( L \) that maximized \( \pi_{SW}(L) \), we know that \( \pi_{SW}(L_{SW}) \geq \pi_{SW}(L_{SN}) \). Hence, \( v(L) L_{SN} \geq rL_o > 0 \), so that \( v(L) > 0 \), that is, \( w_w(L) > w_w(L_{SN}) \).

We thus conclude it is indeed possible to have total segregation and wage discrimination simultaneously.

Some idea of the conditions under which the general equilibrium has the configuration (SW & SN) can be derived. If we use the definition of \( v(W/L) \), we can write \( v(L_o/L_{SN}) = w_w(L_o/L_{SN}) - w_w(L_{SN}) \), \( v(L) = w_w(L) - w_w(L_{SN}) \). Then (7) can be interpreted as an inequality in \( w_w(L_{SN}) \), (8) as an equation. In combination, we have,
\[ w_w(L_o/L_{SN}) - r \geq w_w(L_{SN}) - r(L_o/L_{SN}) - \pi_{SW}(L_{SW}) - \pi_{SW}(L_{SN})/L_{SN}. \]

Add \( r - w_w(L) \) to the extreme terms, and divide by \( 1 - (L_o/L_{SN}) \).
\[ \frac{w_w(L_o/L_{SN}) - w_w(L_{SN})}{1 - (L_o/L_{SN})} \geq r - \frac{\pi_{SW}(L_{SW}) - \pi_{SW}(L_{SN})}{L_{SN} - L_o}. \]

The left-hand side is a measure of employee discrimination; it is the rate of change of wages demanded by white workers with respect to the proportion of white workers in the labor force. The case (SW & SN) will then arise when employee discrimination exceeds an adjusted version of the capital cost per new worker.

(SW & I): Now it must be true that if \( L = L_I \), a firm following policy SN cannot have lower costs than an I firm. Again, from (3),
we have the condition,

(9) \( v(L_o/L_I) < r \).

Since \( \pi_I(L_I) = \pi_{SW}(L_{SW}) \), we have, from (4),

\[
(f(L_I) - (W + r) L_I - [v(L_o/L_I) - r] L_o = \pi_{SW}(L_{SW})).
\]

By definition,

\[
f(L_I) = [v(1) + r] L_I + rL_o = \pi_{SW}(L_I) .
\]

Subtract this last equation from the previous one, and then add \( v(L_o/L_I) L_o \) to both sides.

(10) \( v(1) L_I = v(L_o/L_I) L_o + [\pi_{SW}(L_{SW}) - \pi_{SW}(L_I)] \).

As before, the expression in brackets is non-negative since \( L_{SW} \) maximizes \( w_{SW} \).

(11) \( v(1) L_I \geq v(L_o/L_I) L_o \).

Since all blacks are being absorbed into the \( I \) firms, these must be doing some net hiring, so that \( L_I > L_o \). Since \( w_w(W/L) \) is a decreasing function, \( w_w(1) < w_w(L_o/L_I) \), and therefore \( v(1) < v(L_o/L_I) \).

From (11),

\[
v(L_o/L_I) L_I > v(L_o/L_I) L_o .
\]

With \( L_I > L_o \), it must be that \( v(L_o/L_I) > 0 \), and, from (11), \( v(1) > 0 \).

Thus, in this case, some firms are integrated and some are segregated white. Wages of white workers are higher in integrated firms than in segregated white firms, and the latter are, in turn, higher than black wages.
As in the previous case, we can get some idea of the conditions under which the (SW & I) case will occur. Equation (9) can be used to derive a lower bound on \( w_N \), while we can solve for \( w_N \) in (10).

\[
\frac{w_W(1) L_I - w_W(L_o/L_I) L_o - [\pi_{SW}(L_{SW}) - \pi_{SW}(L_I)]}{L_I - L_o} = w_N
\]

\[
\geq w_W(L_o/L_I) - r.
\]

Add

\[
r = \frac{w_W(1) L_I - w_W(L_o/L_I) L_o}{L_I - L_o}
\]

to the first and third expressions

\[
r = \frac{[\pi_{SW}(L_{SW}) - \pi_{SW}(L_I)]}{L_I - L_o} \geq \frac{w_W(L_o/L_I) \& w_W(1)}{1 - (L_o/L_I)}
\]

Thus the case where blacks are hired in integrated firms is that for which the rate of employee discrimination does not exceed an adjusted version of the capital cost per new worker.

The crudity of the foregoing model needs no emphasis; in particular, the existence of normal turnover in the labor force means that the opportunity cost of introducing black labor may be less than suggested here. But the model puts in evidence the strong possibility that, because of costs of addition to the labor force, discriminatory attitudes of white employees can result in wage differentials as well as some degree of segregation.
Suppose there are two types of labor, 1 and 2. Type 1 labor is unskilled. Type 2 labor, however, is created only if both the employer and the worker invest some human capital. A worker who has made his investment is said to be qualified. An employer cannot know whether or not a worker is qualified, but he holds subjective beliefs about the respective probabilities, to be denoted by $p_W$ and $p_N$, that white and black workers, respectively, are qualified.

As in Technical Note E, let $r$ be the return per worker that the employer must earn on his human capital investment. Assume further that the employer is risk-neutral. Then, in the notation previously used, the equilibrium condition for hiring of both white and black workers is

$$r = (f_2 - w_{2W}) p_W = (f_2 - w_{2N}) p_N,$$

where $w_{2W}$ and $w_{2N}$ are white and black wages for type 2 labor, respectively. It follows that

$$w_{2W} = q w_{2N} + (1-q) f_2,$$

where $q = p_N/p_W$. Then if $p_N < p_W$, we find discrimination in wages in type 2 labor.

If for reasons of social pressure or administrative convenience, it is not easy to maintain an adequate differential between $w_{2W}$ and $w_{2N}$, blacks will be excluded from occupation 2.

It can be argued within the model that, if one considers the factors governing the supply of human capital by the workers, the realized equilibrium is very likely to result in market discrimination. Suppose now that employers do not misperceive, that $p_W$ and $p_N$ are indeed the actual proportions of "qualified" whites and blacks in the relevant population. Since the employer cannot directly observe the
possessing qualifications by workers, the relevant population in each case is the totality of workers. Finally, suppose that the acquisition of human capital (qualification) by workers is costly and that they face imperfect capital markets in any effort to finance this acquisition. Then \( p_W \) will be an increasing function of \( w_{2W} - w_1 \) and \( p_N \) of \( w_{2N} - w_1 \), since these determine the returns to the investment in qualifications. If we assume no difference in the basic structure of motivation between whites and blacks, the two functional relations will be the same.

Let \( L_1 \) and \( L_2 \) be the numbers of workers of types 1 and 2, respectively; in the absence of employer misperception,

\[
L_1 = (1 - p_{W}) W + (1 - p_{N}) N, \quad L_2 = p_{W} W + p_{N} N
\]

where \( W \) and \( N \) are the total supplies of white and Negro labor, respectively. Then \( f \) and therefore \( f_2 \) are functions of \( p_W \) and \( p_N \), respectively. With \( w_{2W} \) and \( w_{2N} \) functions of \( p_W \) and \( p_N \), (1) constitutes a pair of equations in \( p_W \) and \( p_N \) for given \( w_1 \).

Under all these hypotheses, the equations are symmetric in the two variables, and therefore they have a solution for which \( p_W = p_N \). It might appear then that in long-run equilibrium, the absence of misperceptions would imply the absence of discrimination.

But the non-discriminatory equilibrium may well not be stable. Intuitively, a possible sequence of events might be described as follows. If \( p_W \) is, for some reason, slightly greater than \( p_N \), then, from (2), \( w_{2W} \) will slightly exceed \( w_{2N} \). In response to this differential there may be some incentive for \( p_W \) to rise relative to \( p_N \), thereby reinforcing the original discrepancy. At the same time, the rise in \( p_W \) will have a negative effect on \( w_{2N} \) since it means increased competition from whites and thereby also serves to discourage an increase in Negro attempts at qualification.

The verbal account of instability is, of course, by no means conclusive or even very convincing. To develop a formal model, let
us suppose that the labor markets and the determination of wage levels are short-run phenomena, which come into equilibrium quickly relative to changes in the supplies of the two kinds of labor between the two races. Assume then that \( p_W \) increases or decreases as the desired supply of type 2 labor among whites is above or below the actual. That is, we postulate an increasing function \( \varphi(w_{2W} - w_1) \), the desired supply of type 2 labor, and a dynamic adjustment relation,

\[
(3) \quad \dot{p}_W = k [\varphi (w_{2W} - w_1) - p_W];
\]
similarly,

\[
(4) \quad \dot{p}_N = k [\varphi (w_{2N} - w_1) - p_N].
\]

Since it is assumed that white and Negro workers have the same motivation, the adjustment coefficients \( k \) and the supply functions \( \varphi \) are assumed the same for both races.

Write the marginal productivity relations (1) together with that for type 1 labor as,

\[
w_1 = f_1, \quad w_{2W} = f_2 - (r/p_W), \quad w_{2N} = f_2 - (r/p_N),
\]

so that,

\[
(5) \quad w_{2W} - w_1 = (f_2 - f_1) - (r/p_W), \quad w_{2N} - w_1 = (f_2 - f_1) - (r/p_N).
\]

For a fixed total labor force, \( L = W + N \), let

\[
F(L_2) = f(L - L_2, L_2),
\]

the output obtained by allocating \( L_2 \) laborers to type 2 labor and the rest to type 1 labor. Then \( F \) is a strictly concave function if \( f \) is, so that
\[ F'' < 0, \]
\[ F' = f_2 - f_1, \]

and,
\[ \frac{\partial F'}{\partial p_W} = F''(\frac{\partial L_2}{\partial p_W}) = F''W, \quad \frac{\partial F'}{\partial p_N} = F''N. \]

From (5) and these remarks,
\[ (6) \quad \frac{\partial (w_2W - w_1I)}{\partial p_W} = (\frac{\partial F'}{\partial p_W}) + (\frac{\tau}{p_W^2}) = F''W + (\frac{\tau}{p_W^2}), \]
\[ \frac{\partial (w_2W - w_1I)}{\partial p_N} = F''N. \]

Interchanging \( W \) and \( N \) in these expressions yields
\[ (7) \quad \frac{\partial (w_2N - w_1I)}{\partial p_W} = F''W, \quad \frac{\partial (w_2N - w_1I)}{\partial p_N} = F''N + (\frac{\tau}{p_N^2}). \]

We use (6) and (7) in the analysis of the stability of the system (3) and (4). Since the adjustment coefficient, \( k \), is the same for both differential equations, it plays no role in the stability analysis and can be set equal to 1 without loss of generality. Form the matrix whose elements are \( \frac{\partial p_i}{\partial p_j} \), where \( i, j \) range over \( W, N \). The condition for stability is that the characteristic roots of this matrix are both negative (or they are complex-conjugate with negative real parts), when the matrix is evaluated at equilibrium. We are here considering the non-discriminatory equilibrium. If we define
\[ a_W = \phi'(F''W), \quad a_N = \phi'(F''N), \quad b = \phi'(\frac{\tau}{p^2}) - 1, \]
\[ c = \phi'[F''L + (\frac{\tau}{p^2})] - 1 = a_W + a_N + b, \]

where all functions are evaluated at the non-discriminatory equilibrium values and \( \tau \) is the common value of \( \tau_W \) and \( \tau_N \) there, then the matrix is
\[
\begin{pmatrix}
aw + b & a_N \\
aw & a_N + b
\end{pmatrix}
\]

The sum of the characteristic roots is the trace (sum of diagonal elements) of this matrix, which is \((aw + b) + (aN + b) = (aw + aN + b)\) + \(b = c + b\); the product of the characteristic roots is the determinant, which is \((aw + b)(aN + b) - awaN = b(aw + aN) + b^2 = b(aw + aN + b) = bc\). Since the sum of the roots is \(b + c\) and their product is \(bc\), the roots must be the real numbers \(b\) and \(c\). Stability requires that both be negative. But since

\[
c = b + \phi'F''L,
\]

and \(\phi' > 0, F'' < 0, L > 0\), we must have \(c < b\); hence, the condition \(b < 0\) is necessary and sufficient for stability, i.e., the non-discriminatory equilibrium is stable if and only if,

\[
(8) \quad \phi'r < p^2.
\]

Whether or not this condition is apt to be met in practice obviously depends on the three magnitudes involved, which are \(\phi'\), the supply responsiveness, \(r\), the cost of the employer's investment, and \(p\). Thus, if type 2 labor is relatively rarely used, so that \(p\) is small, we would expect \(\phi'\) to be correspondingly small, but \(p^2\) to be much smaller yet, so that (8) might well be violated. Again, a large \(r\), i.e., a large investment by the employer in his potential employees, makes for instability.

A rewriting of (8) may help understand the condition. Let \(E\) be the elasticity of supply of type 2 labor; under our assumptions, this must be taken as an elasticity with respect to the difference of wages between type 2 and type 1 labor,
\begin{align*}
E = (w_2 - w_1) \varphi'/\varphi.
\end{align*}

Since \( p = \varphi(w_2 - w_1) \), we can divide through in (8) by \( p^2 \) and
obtain, \( E[(r/p)/(w_2 - w_1)] < 1 \) which is necessary and sufficient for
stability. Note here that \( r/p \) is the amount the firm must recover
on its human investment per type 2 laborer finally employed, that is,
the equilibrium gap between marginal productivity and wages for type
2 labor. Thus, the second factor in (9) is the ratio between the
extent to which type 2 wages fall short of marginal productivity and
the extent to which they exceed type 1 wages.

As a clarifying remark, it might be noted that the two character-
istic roots correspond to two kinds of movements. The root \( b \), which
is dominant, corresponds to the motions of the difference \( p_W - p_N \)
between the proportions of qualified workers in the two races. In
what follows, let asterisks refer to equilibrium values; the symbol
\( \approx \) means "equivalent up to linear approximations,"

\[
\frac{d(p_W - p_N)}{dt} = k\left[\varphi'(w_2 - w_1) - \varphi'(w_2 - w_{1*}) - (p_W - p_{N*})\right]
\approx k\left[\varphi'(w_2 - w_{2N}) - (p_W - p_N)\right].
\]

But from (5),

\[
w_2 - w_{2N} = (r/p_N) - (r/p_W) \approx (r/p_*^2)(p_W - p_*) - (r/p_*^2)(p_N - p_*)
\]

so that

\[
\frac{d(p_W - p_N)}{dt} \approx k\left[\varphi'(r/p_*^2)(p_W - p_N) - (p_W - p_N)\right] = kb(p_W - p_N),
\]

and within linear approximations, the convergence of the discriminatory
elements of the system to 0 depends on \( b \). The characteristic root
c governs rather the movement of the total proportion of qualified workers. For if we now define

\[ p = \frac{(W_p + N_p)}{L} \]

it can easily be calculated that, to a first-order approximation,

\[ \dot{p} = k c(p - p^*) \]

for which the stability condition, \( c > 0 \), is less stringent. This is also the stability condition for a corresponding model in which there is in fact only one kind of worker.