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Application of the Ring Vortex Method to
Aerodynamic Design of Lifting Rotor Systems

by

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13. ABSTRACT

The operation of the lifting rotor during flight with a horizontal velocity component has been the subject of considerable study. The ring vortex method is a simplified procedure for accurately determing the aerodynamic characteristics of the lifting rotor.

The replacement of the vortex cylinder with a system of discrete vortex rings is equivalent to expansion of the vortex cylinder into rings and longitudinal vortices. Ignoring the longitudinal vortices is equivalent to ignoring the closure of the stream behind the rotor. For low loaded lifting rotors, no significant error is thus created.

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NOTICE

Application of the Ring Vortex Method to Aerodynamic Design of Lifting Rotor Systems

By: V. I. Shaydakov

Introduction:

The operation of the lifting rotor during flight with a horizontal velocity component has been the object of continuous studies by a number of foreign and Soviet scientists, particularly in the past decade. At the present time, great successes have been achieved in the area of determination of the forces and moments acting on the lifting rotor in a slanted stream, on the basis of both flat and three-dimensional models of the vortex shroud.

However, the conclusions of the existing theory cannot be extended to the mode of rapid descent of a helicopter with and without application of power to the lifting rotor shaft. These are the vortex ring and turbulent (autorotation) modes. As we know, the stream theory of Glauert [1] produces results for these modes which do not agree with experimental results. One frequent case of vortex ring and autorotation modes is vertical descent of a helicopter, which has been studied by the author of the present article on the basis of general theories of aeromechanics [5] and by Ye. S. Vozhdayev on the basis of vortex theory.

Steep glide modes of a helicopter correspond to an extremely complex picture of flow around the lifting rotor, which makes experimentation difficult and does not allow the vortex model of the phenomenon to be constructed.

All of this led us to the idea of simplifying the existing vortex models of the lifting rotor as much as possible and finding the simplest method of determination of its aerodynamic characteristics, in order to use this method later to investigate the modes of steep descent of a helicopter in the first approximation.

This article presents a method (which we will refer to as the ring vortex method), providing the same accuracy as the theory of the ideal rotor or the vortex NEZH rotor theory.

1. Content of suggestion method

N. Ye. Zhukovskiy proved a theorem [4] for determination of the resistance of a flat circular plate placed perpendicular to a flow, which can be formulated as follows: the momentum (M) imparted by a closed, flat vortex filament to a limitless liquid mass in the direction perpendicular to the plane in which it is located is equal to the product of the density of the fluid (ρ) times the circulation of the vortex (Γ) and the area which limits it (F) (Figure 1)

$$M = \rho \Gamma F. \qquad (1)$$

It has been determined that this theorem can be used in the solution of certain problems of aerodynamics. If we know how many closed vortices are formed per unit time or the extent to which the area F limited by them increases per unit time we can calculate the change in the momentum and conse :ting on the system.

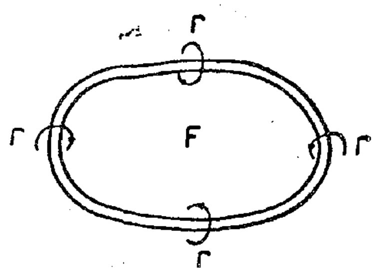

Figure 1.

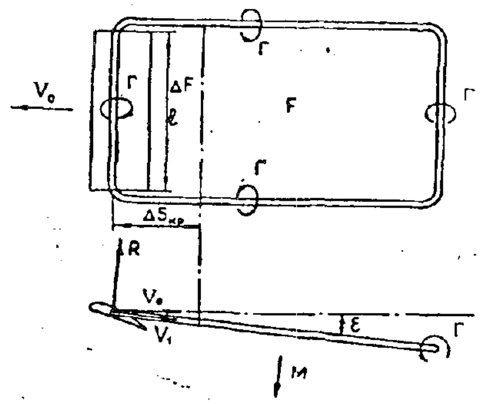

Figure 2.

To illustrate this, let us analyze the case of a wing of finite length with a constant circulation along the span. Figure 2 shows the vortex system of a wing moving at velocity V_0. The attached, acceleration and free vortices make one closed vortex, the plane of which is inclined to the direction of the unperturbed flow by angle ε (ε is the inductive taper angle of the flow). The lift R is perpendicular to the plane of the closed vortex and produces a component in the direction of the velocity of the unperturbed flow (inductive drag). We will base our considerations on an ideal fluid, in which case this vortex system can exist for an infinite time.

The motion of the wing is accompanied by an increase in area F, limited by the closed vortex, causing a change in the momentum. For example, over time Δt the wing travels a distance of

$$\Delta s_{\text{кр}} = \Delta t \cdot V_0,$$

and the free vortices are elongated by Δs_{CB}:

$$\Delta s_{\text{св}} = \frac{\Delta s_{\text{кр}}}{\cos \varepsilon} = \frac{\Delta t V_0}{\cos \varepsilon} = \Delta t V_1.$$

In this same time, a change in the momentum by ΔM occurs:

$$\Delta M = \rho \Gamma \Delta F = \rho \Gamma \Delta s_{\text{св}} \cdot l = \rho \Gamma V_1 l \Delta t.$$

The force R in one second is equal to the change in momentum per second:

$$R = \frac{\Delta M}{\Delta t} = \rho \Gamma V_1 l. \qquad (2)$$

This formula is known as the Zhukovskiy formula for the lift of a wing.

It is particularly convenient to use the method suggested to determine the aerodynamic characteristics of the lifting rotor of a helicopter in axial and inclined streams.

2. Statement of Problem. Thrust Problem

Let us study a NEZH rotor with constant circulation Γ over the radius.

A free vortex leaves the tip of each blade with the same circulation Γ enclosed in the ring as soon as the blade completes one rotation. In one rotation, a rotor with k blades emits k ring vortices, limiting area πR^2, equal to the area of the rotor disk.

Thus, an operating rotor forms a vortex track in its wake, consisting of discrete ring vortices, moving in the direction of the stream striking the rotor. The vortices have identical circulation Γ and are located in planes parallel to the plane of rotation of the lifting rotor. In an ideal fluid the vortices do not break up; therefore, the vortex track has infinite length. In the general case, the ring vortices, moving over the flow surface, change their dimensions due to the presence of constriction of the stream following the rotor.

Figure 3 shows the vortex model which we have described for the case of operation of a rotor in an inclined flow.

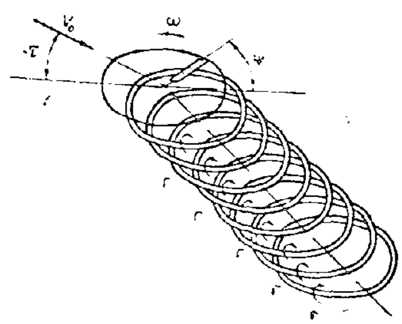

Figure 3.

Let us now go over to calculation of the thrust of the lifting rotor (T).

The time of formation of one ring will be

$$\Delta t = \frac{2\pi}{k\omega}, \tag{3}$$

ω is the angular velocity of rotation of the rotor.

In time Δt, momentum ΔM is created, equal to the force T:

$$T\Delta t = \Delta M = \rho\Gamma\pi R^2.$$

Considering (3), we have:

$$T = \frac{1}{2}\rho k\Gamma\omega R^2. \tag{4}$$

As we can see, (4) is suitable for any mode of operation of the rotor. This same formula, although somewhat longer, can be produced from the theory of an isolated blade element.

We must now find the relationship between circulation Γ and the inductive velocity in the plane of the rotor disk. To do this, let us go over to study of the vortex track behind the blade.

The formulas produced are known and indicate that the method suggested is sound in its basis, simple and contains no grossly incorrect assumptions.

In the next section, we will study the modes of a lifting rotor, the mathematical description of which has always involved significant difficulty.

3. Rotor in Slanted Flow Mode

The first theoretical studies of these modes were performed in 1928 by Glauert, who studied the average inductive velocities v_1 over the disk. He based his calculations on the hypothesis that the lifting rotor in operation acts on a stream of air, the cross section of which has a diameter equal to the diameter of the lifting rotor. This hypothesis is similar to the assumption of Prandtl for a wing [1].

The thrust formula in this case will be

$$T = 2\rho\pi R^2 V_1 v_1, \tag{5}$$

where V_1 is the rate of flow of air through the lifting rotor disk. This formula is broadly used in aerodynamic calculations involving lifting rotors acting in slanted flows [2,6] and for determination of the slope of the vortex solenoid (e.g., see [3]).

The vortex ring method allows us to give a simple proof for Glauert's formula. Figure 4 shows the vortex track resulting from operation of a rotor in the slanted flow mode. Let us separate two characteristic cross sections. Section 1-1 lies in the plane of the rotor disk. Here the air flow rate through the disk is represented as the vector sum

$$\vec{V}_1 = \vec{V}_0 + \vec{v}_1, \tag{6}$$

where v_1 is the mean inductive velocity in cross section 1-1, resulting from the vortex track.

Section 2-2 lies far behind the rotor. Here the stream has undergone maximum constriction, and the vortex track has been converted to an inclined cylindrical vortex solenoid. The velocity within the solenoid

$$\vec{V}_2 = \vec{V}_0 + \vec{v}_2, \tag{7}$$

where v_2 is the mean inductive velocity resulting from the vortex solenoid.

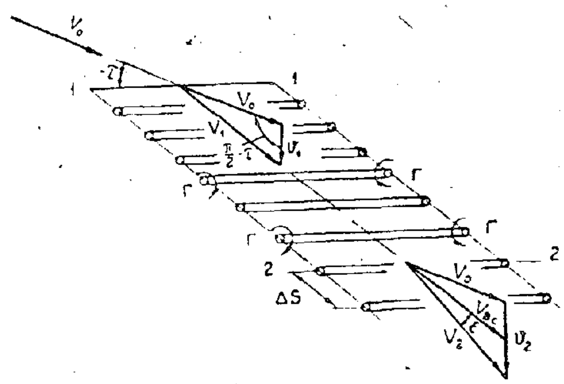

Figure 4.

It can be considered with a high degree of accuracy that the vortex rings follow each other closely, forming a continuous vortex layer. Then the inductive velocity within the vortex track increases continually and monotonically with increasing distance from the rotor, reaching its maximum at cross section 2-2.

Outside the vortex solenoid in cross section 2-2, the mean inductive velocity is equal to zero, and the total velocity is equal to the velocity of the unperturbed stream V_0. The mean running circulation of the vortex layer around the circumference in the direction of the axis of the inclined vortex cylinder

$$\gamma_{\text{в. c}} = |\vec{V}_2 - \vec{V}_0| = |\vec{v}_2|, \tag{8}$$

while the velocity of motion of the vortex layer (i.e., the velocity of motion of the ring vortices) is the vector half sum between the velocities V_2 and V_0:

$$\vec{V}_{\text{в. c}} = \frac{\vec{V}_2 + \vec{V}_0}{2} = \vec{V}_0 + \frac{\vec{v}_2}{2}. \tag{9}$$

It follows from the Froud-Finsterwalder theorem [7] that for all operating modes the rotor

$$v_2 = 2v_1. \tag{10}$$

This theorem is called the theorem of velocities in the stream of a rotor and is proven on the assumption that the vortex column behind the rotor is not constricted. (In our case, the vortex rings do not change their dimensions as they move away from the rotor.) In the practice of aerodynamic calculation, constriction of the stream beyond the rotor is generally ignored. This produces no great divergence with experimental results. Therefore, we can consider that

$$\gamma_{\text{в. c}} = 2v_1 \tag{11}$$

and

$$\vec{V}_{\text{в. c}} = \vec{V}_0 + \vec{v}_1 = \vec{V}_1. \tag{12}$$

The running circulation of the vortex layer in Section 2-2 can be produced by another method as well.

During the time of formation of one ring (Δt) the vortex ring in Section 2-2 moves over a path of

$$\Delta s = \Delta t \cdot V_{\text{в. c}}. \tag{13}$$

Substituting formulas (3) and (12) here, we produce

$$\Delta s = \frac{2\pi V_1}{k\omega}. \tag{14}$$

The running circulation of the vortex layer in the direction of the axis of the vortex track will be:

$$\gamma_{\text{в. c}} = \frac{\Gamma}{\Delta s} = \frac{k\Gamma\omega}{2\pi V_1}. \tag{15}$$

Setting equations (11) and (15) equal to each other, we find:

$$k\Gamma\omega = 4\pi V_1 v_1. \quad (16)$$

Substituting (16) into formula (4) for the thrust, we find:

$$T = 2\rho\pi R^2 V_1 v_1 = 2mv_1. \quad (17)$$

Here m is the mass flow rate of air through the disk of the lifting rotor per second. As we can see, the formula produced corresponds fully with (5), which was to be proven. We will relate all velocities to the inductive velocity of hovering (v_{1B}):

$$v_{1B} = \sqrt{\frac{T}{2\rho\pi R^2}},\ \tilde{v}_1 = \frac{v_1}{v_{1B}},\ \tilde{V}_1 = \frac{V_1}{v_{1B}},\ \tilde{V}_0 = \frac{V_0}{v_{1B}} \text{ etc.}$$

Then formula (17) becomes:

$$1 = \tilde{V}_1 \cdot \tilde{v}_1. \quad (18)$$

It follows from Figure 4 that[1]

$$\tilde{V}_1 = \sqrt{\tilde{V}_0^2 + \tilde{v}_1^2 - 2\tilde{V}_0 \tilde{v}_1 \sin\tau}. \quad (19)$$

Let us substitute expression (19) into (18):

$$\tilde{v}_1^4 - 2\tilde{V}_0 \sin\tau \tilde{v}_1^3 + \tilde{V}_0^2 \tilde{v}_1^2 - 1 = 0. \quad (20)$$

The equation produced gives us the relationship between inductive velocity, the velocity of the oncoming stream V_0 and angle of attack τ. In [6] we find the graph of

$$v_1 = f(\tilde{V}_0, \tau).$$

We can attempt to estimate the influence of the nonlinearity of the axis of the vortex cylinder on the inductive characteristics of the rotor.

Actually, it must be considered that in cross section 2-2, the vortex rings continue their motion in the direction of velocity V_2, not V_1, as occurs in cross section 1-1. In this case, the rate of displacement of the vortices in cross section 2-2 will be (Figure 4):

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[1] In this and all subsequent formulas, angle τ should be substituted with the proper sign, in our case with the minus sign. (The angles and velocities are taken according to B. N. Yur'ev [6].)

$$V_{в.с} = V_1 \cos \varepsilon, \tag{21}$$

where ε is the angle between axes of the vortex solenoid in cross sections 1-1 and 2-2 or the angle between velocities V_1 and V_2.

Then formula (18) becomes

$$1 = \tilde{V}_1 \tilde{v}_1 \cos \varepsilon, \tag{22}$$

while equation (20) is rewritten as:

$$\tilde{v}_1^4 - 2\tilde{V}_0 \sin \tau \tilde{v}_1^3 + \tilde{V}_0^2 \tilde{v}_1^2 - \frac{1}{\cos \varepsilon} = 0. \tag{23}$$

As we can see, consideration of nonlinearity slightly increases the inductive velocities. Let us determine how significant this increase is.

It follows from the triangle of velocities in cross section 2-2 (Figure 4) that

$$\cos \varepsilon = \frac{V_1^2 + V_2^2 - v_1^2}{2V_1V_2}$$

or after simple conversions

$$\cos \varepsilon = \frac{\tilde{V}_0^2 + 2\tilde{v}_1^2 - 3\tilde{V}_0\tilde{v}_1 \sin \tau}{\sqrt{(\tilde{V}_0^2 + \tilde{v}_1^2 - 2\tilde{V}_0\tilde{v}_1 \sin \tau)(\tilde{V}_0^2 + 4\tilde{v}_1^2 - 4\tilde{V}_0\tilde{v}_1 \cdot \sin \tau)}}. \tag{24}$$

In order to determine $\tilde{v}_1 = f(\tilde{V}_0, \,)$, we must solve equations (23) and (24) jointly. The solution is quite cumbersome. We can use a much simpler approach. As will be demonstrated, consideration of nonlinearity of the axis of the vortex track changes the inductive characteristics of the lifting rotor very little; therefore, $\cos \varepsilon$ will be calculated using formula (24), substituting the values of $\tilde{v}_1 = f(\tilde{V}_0, \tau)$ preliminarily found from equation (20), which ignores this nonlinearity, into formula (24).

The greatest influence of nonlinearity of the stream axis beyond the rotor is on the mode of a flight with zero angle of attack ($-\tau = 0$).

We will estimate this influence using coefficient ξ, which shows the number of times by which the inductive velocity calculated according to equation (20) must be increased in order to consider this nonlinearity.

In the table we present the results of calculation for the case $-\tau = 0$:

$\tilde{V}_0$	0	0,25	0,5	0,75	1,0	1,25	1,5	1,75	2,0	2,5
$\cos \varepsilon$	1,0	0,980	0,974	0,954	0,942	0,940	0,950	0,965	0,977	0,989
ξ	1,0	1,003	1,007	1,014	1,021	1,024	1,022	1,017	1,012	1,006

As we can see from the table, the maximum influence of nonlinearity of the axis of the vortex cylinder on inductive velocity does not exceed 2.4%. Therefore, the nonlinearity of the stream axis behind the rotor can be ignored in vortex theories, which is usually done.

Thus, the method we have presented can be successfully used in aerodynamic design of lifting systems, particularly the lifting rotors of helicopters. The replacement of the vortex cylinder with a system of discrete vortex rings is equivalent to expansion of the vortex cylinder into rings and longitudinal vortices, which is always performed in vortex rotor theories. Ignoring the longitudinal vortices is equivalent to ignoring the closure of the stream behind the rotor. For low-loaded lifting rotors, as we know, no great error is thus produced.

This method can be used for rotors with variable circulation over their radius. In this case, integral formulas are produced.

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